MOHAMED JAFFAR, MOHAMED ZUBAIR. Modeling and Analysis of Saturated Inductances and Torque Ripple Components in Interior Permanent Magnet Motors. (Under the direction of Dr. Iqbal Husain).

Interior Permanent Magnet motors (IPMs) have far-reaching applications in industry and electric traction. Significant research has been invested on this technology over the past 40 years, addressing issues in IPM analysis and design. Analytical and hybrid analytical-FEA methods are key to assess torque behavior and overload operation during design stage, before large-scale manufacturing. The $q$-axis inductance of an optimally designed IPM exhibits a saturating behavior as the current increases. This dissertation proposes two analytical models to study the behavior by using path permeance function, Ampere’s law and flux continuity law.

Pulsating torque in IPMs is an undesirable effect causing mechanical vibration and premature aging of drivetrain. Based on electromagnetic sources, the torque is separable into cogging torque, reluctance torque and mutual torque. This research presents a method to decompose these components with the aid of two synthesized machines derived from a reference IPM using Finite Element Analysis (FEA). The benefit is two-fold: (i) independent control of magnet flux linkage and saliency ratio to achieve targeted cost reduction, torque density improvement and wide speed range; (ii) and insights for torque ripple minimization.

The torque and inductance analysis methods are applied to IPMs with different distributed winding topologies, rotor geometries and over wide power range. Comprehensive experimental testing is conducted on two fabricated prototypes – IPM & a synchronous reluctance motor (SyRM) to validate the theory and to refine the FEA models. Conventional and proposed experimental practices are implemented that provide engineering lessons for future motor testing.
Modeling and Analysis of Saturated Inductances and Torque Ripple Components in Interior Permanent Magnet Motors

by
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A dissertation submitted to the Graduate Faculty of North Carolina State University in partial fulfillment of the requirements for the degree of Doctor of Philosophy

Electrical Engineering

Raleigh, North Carolina
2019

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DEDICATION

To my father, Mr. M. Mohamed Jaffar and my mother, Mrs. Shamima, for their unconditional love, support and sacrifice at every step of my life
Mohamed Zubair M Jaffar completed his Bachelor and Masters in Electrical Engineering from the Indian Institute of Technology – Madras (IITM) in 2013. He received his Ph.D. degree in Electrical Engineering from North Carolina State University, Raleigh, NC in 2019. He was a IEEE student member since 2012 and IEEE graduate student member since 2014. Currently, he is an Electric Powertrain Project Engineer at FEV North America, Inc. in Auburn Hills, MI.

During his doctoral study in the ECE department, NCSU, from 2014 to 2019, he worked on analysis, design, controls and experimental evaluation of Interior Permanent Magnet motors (IPM), Synchronous Reluctance motors (SyRM) and pole-phase modulated even-phase induction motors (IM). He focused on physics based modeling and Finite Element Analysis methods to study problems in traction IPMs including iron losses at high switching frequency, overload operation and torque ripple mitigation. He also invested effort and engineering in building a dynamometer testbed at the FREEDM center, and conducting experiments on IPMs and IMs.

He completed a 7 month internship at ABB US Corporate Research Center, Raleigh, NC during 2016, where he worked on simulation and testing of axial flux motors. His research interests include multi-physics design of motors and system level design of drive technologies for emerging markets including electric vehicle traction, more-electric aircraft and integration of renewables such as wind and wave.
ACKNOWLEDGMENTS

I express my sincere gratitude to Dr. Iqbal Husain for accepting me as a PhD student and guiding me through the years of this journey. This PhD has been a key learning step in my life. He has provided mentorship, support, motivation and regular technical feedback to keep my interest in the research. He has helped me understand the large-picture view of how my research fits into industrial applications, which is very valuable for me.

I would like to acknowledge the funding sources for my research assistantship over the semesters at FREEDM center. I very much appreciate the pedagogy of professors, both at NC State University and IIT Madras, through which I view Electrical Engineering as not only a profession, but also an enjoyment. During my junior years in PhD, I was fortunate to interact and learn about motor analysis and design from fellow students – Ashfanoor, Zhao, Adeeb, Chandan and Rajib. In recent years, I enjoyed discussing machines research, building hardware and experimental testing with Taohid, Ritvik, Sarif and Sodiq. I was lucky to have worked on interesting projects with exchange scholars - Simone (Italy) and Natsuki (Japan). I wish to thank all of these people.

I appreciate the help provided by Mr. Rodney for the stator rewinding, Mr. Hendershot for the rotor manufacturing, Chris for machining work and Hulgize for assisting in lab setup. I thank the students of FREEDM center and support staff – in particular, Karen and Rebecca for making my stay here enjoyable. I thank the friends I made in Raleigh, with whom I had good experiences in the US away from home. It has been a long journey, I may have forgotten to acknowledge some people and I hope they excuse me.

I am indebted to my father, Mr. M. Mohamed Jaffar (B.A. Economics) and my mother, Mrs. Shamima (B.Sc. Physics) for nurturing me from birth to this point in PhD. I can never
repay the affection and sacrifices they have made for my sake. It was my father’s dream that I pursue my Doctorate in USA after my Bachelors & Masters at IIT Madras. Words are not enough to describe the motivation, guidance, inspiration and meticulous life coaching that my father imparted me at every stage of my life, which has enabled me to stand strongly on my feet. My mother’s tasty cooking and lively stories have given much relaxation when I was stressed with work. I thank my brother, Khalid for making my life colorful with his camaraderie.

I would also like to express gratitude to my extended family members – grandparents, uncles, aunts and cousins, and all my friends in school and college, who have supported me at different steps of my life.

Finally, I thank the Almighty.
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Chapter 1. Introduction

1.1 Interior permanent magnet motor and drive

1.2 Principles of operation
   1.2.1 Machine \( d-q \) model
   1.2.2 Operating regions of IPM drive

1.3 Applications and desirable features
   1.3.1 Saturated operation
   1.3.2 Low torque ripple applications

1.4 Research motivation & objective

1.5 Outline of dissertation
The field of electric motors has progressed by leaps and bounds since the invention of the first electric motor in the late nineteenth century. Starting from squirrel cage AC induction motor, the technology has grown to include a rich variety of machines – wound field synchronous motors, rotor excited induction motors, DC commutator motors, surface permanent magnet (SPM) motors, synchronous reluctance motors (SyRM), switched reluctance motors (SRM) and interior permanent magnet motors (IPM) [1]. The drive system for these motors has also seen tremendous advancement with advent of cost-effective and high-quality power semiconductor devices, power electronics and signal processing systems.

Across all segments, 40% of electricity generated in the U.S. is consumed by electric motors [2]. Majority of these motors are connected to grid-tied transformers and use gearboxes to vary the speed of the mechanical load. This is a highly inefficient method resulting in huge electrical and mechanical losses. Since the development of low-cost power electronics in 1970s and 1980s, the variable speed drive (VSD) has become an attractive alternative for industrial motion needs. Concerns of global warming and depleting oil resources have driven many countries to invest heavily in hybrid electric vehicles (HEVs), electric vehicles (EVs) and even electric aircrafts [3]. Thus, despite electric motors being an old technology, economic and environmental considerations are providing renewed impetus to the research in electric motors and drives.

1.1 Interior permanent magnet motor and drive

Interior Permanent Magnet motor (IPM) consists of permanent magnets embedded in each pole of the rotor, which acts as the excitation source for torque production [4]. Figure 1.1 shows some popular rotor geometries. The stator consists of either concentrated windings or
distributed windings; only the distributed configuration is considered here. The stator and rotor core material is magnetic steel, which has high permeability. The key features of IPM are high torque density, high power factor, wide constant power speed range, low volume and mechanical ruggedness [5]. This gives the IPM an edge over induction motors and synchronous reluctance motors. Increasingly, IPMs have become the motor of choice for traction in EVs & HEVs. Other traditional applications are spindle motors, servo motors for precision position control and robotics. Some of the drawbacks of the IPM are demagnetization risk to the magnets and high, volatile cost of the rare earth magnets – Neodymium Iron Boron (NdFeB).

**Figure 1.1** Interior Permanent Magnet motor designs
The magnetic field source in IPMs interacts with armature reaction to produce a magnet (or alignment) torque; which is similar to surface permanent magnet machines (SPMs). The embedded nature of the magnet in the IPM lends a saliency to the rotor, in that the rotor presents a variable reluctance along the direct axis (d-axis) and the quadrature axis (q-axis). This gives rise to a reluctance torque that is similar to torque production in a synchronous reluctance machine (SyRM). Thus, the IPM can be conceived as a hybrid between an SPM and a SyRM, in regard to electromagnetic performance (Figure 1.2). This is the reason behind higher torque in smaller volume and field weakening capability [6].

In all applications, the IPM motor is integrated into a variable speed drive system (inverter-fed operation). Direct connection to AC mains supply (line-fed operation) is not a viable

Figure 1.2 The IPM – a hybrid between SPM and SyRM
option, as single frequency voltage source cannot startup the IPM, unlike induction motors. A standard three-phase inverter provides pulse-width modulated (PWM) voltages to the IPM. The DC bus of the inverter is supplied by a AC/DC rectifier connected to the AC mains. The instantaneous phase currents feedback and position feedback from an encoder or resolver are provided to the control circuitry, which usually consists of digital signal processor (DSP). Gate driver circuitry interfaces the switching signals from the DSP with the inverter switches – MOSFETs or IGBTs.

Internal to the DSP, the torque control is enabled by a field oriented control (FOC) technique that independently decides the d- and q-axes currents [7]. The phase voltage

![Figure 1.3 A general IPM motor drive](image)
commands are modulated with sine PWM or space vector modulation (SVM) scheme to switch at a high frequency (5 kHz – 15 kHz). In order to have a speed controlled drive, a speed loop is built as outer loop to the torque/current loop. For position controlled drive (e.g. servo applications), a position loop is built as outer loop to the speed loop. In applications for extended speed range, the field-weakening control is built as additional component in the FOC [8].

\textbf{Principles of operation}

There are two key concepts in the principle of operation of the IPM drive. The first is the behavioral modeling of the motor itself under steady state and dynamic conditions. The synchronous frame d-q model relates the terminal voltages, phase currents and generated torque. Second, when the motor is part of a drive, there are limits to the torque generated at a given speed, which are derived from allowable operating region curves.

\textit{1.2.1 Machine d-q model}

The IPM model is conventionally modeled in the rotor fixed synchronous reference frame [9]. In this frame, the direct axis (d-axis) is aligned with the centerline of the magnet in a pole. The quadrature axis (q-axis) lies 90 electrical degrees apart; passing through the centerline of the interpolar region. The Park’s forward and inverse transforms are used to switch between three phase (abc) sinusoidal quantities and two phase (dq) DC quantities. Neglecting iron saturation, air gap harmonics and iron losses, the machine dynamic model is comprised of the flux linkage equation, voltage equation and torque equation:

\begin{align}
\lambda_d &= \lambda_m + L_di_d \\
\lambda_q &= L_qi_q
\end{align}  

(1.1a)
(1.1b)
Where $\lambda_d$ & $\lambda_q$ are the $d$-$q$ axes flux linkages, $\lambda_m$ is the magnet flux linkage, $i_d$ & $i_q$ are the $dq$ currents, $L_d$ & $L_q$ are the $d$-$q$ axes inductances.

\[
v_q = R_s i_q + \frac{d}{dt} \lambda_q + \omega_r \lambda_d \quad (1.2a)
\]
\[
v_d = R_s i_d + \frac{d}{dt} \lambda_d - \omega_r \lambda_q \quad (1.2b)
\]

where $v_d$ & $v_q$ are the $dq$ voltages and $\omega_r$ is the synchronous speed.

\[
T_{em} = \frac{3p}{2} \left[ \lambda_m i_q + (L_d - L_q) i_d i_q \right] \quad (1.3)
\]

Where $T_{em}$ is generated electromechanical torque and $p$ is the pole-pair number.

The vector diagram in Figure 1.4 shows the relations in steady state. The $(i_d, i_q)$ currents can also be denoted by the magnitude of current vector and current advance angle with respect to the $q$-axis $(I, \gamma)$.
1.2.2 Operating regions of IPM drive

The IPM motor is driven by a PWM Inverter that has a finite DC bus voltage and current rating. The power output from the drive is about 70% – 90% of the inverter kVA rating, as the motor has a power factor less than one and efficiency less than 100%. It is desirable to extract maximum torque over the entire speed range of the drive, without exceeding the inverter and motor limits or inducing current regulator saturation. Flux weakening operation above the rated speed is possible in IPM drives, so that voltage limit is satisfied [10]. The trace of maximum torque as function of speed is referred to as the torque-speed envelope; it is a valuable tool to evaluate a drive for a particular application, like traction, rolling mills or tool spindles.

The circle diagram (Figure 1.5(a)) is a powerful graphical technique to arrive at the operating limits of the IPM drive [11]. It includes the current limit curves, voltage limit curves and constant torque curves of the drive in the \( i_d - i_q \) plane. It is allowable to operate in regions within the limit curves; current controller can receive commands \((i_d^*, i_q^*)\) lying within these regions. This diagram is drawn by assuming constant parameters, steady state operation and negligible iron losses and stator resistance.

The current limitation is a circle centered at the origin with radius decided by the rated current, \( I_r \).

\[
i_d^2 + i_q^2 = I_r^2
\]  \(1.4\)

The voltage constraint, \( V_r \) gives a family of ellipses, whose sizes shrink with operating speed, \( \omega_m \) and center at the infinite speed operating point, \( \lambda_m L_d \).

\[
(\lambda_m + L_di_d)^2 + (L_di_q)^2 = \left(\frac{V_r}{\omega_m}\right)^2
\]  \(1.5\)

\[
(\lambda_m + L_di_d)^2 + (L_di_q)^2 = \left(\frac{V_r}{\omega_m}\right)^2
\]  \(1.5\)
A command torque can be realized by several combinations of \((i_d, i_q)\) that trace out a hyperbola. There exists a family of hyperbolae for different torque outputs:

\[
\frac{3p}{2} [\lambda_m i_q + (L_d - L_q)i_d i_q] = \text{const}. \tag{1.6}
\]

The torque-speed envelope in the \(T_e - \omega\) plane of an infinite maximum speed drive has three operating modes (Figure 1.5(b)), corresponding to following curves in the circle diagram:

**Mode 1: Current limited**

From zero speed to rated speed \((\omega_r)\), the voltage limit ellipse encloses the current limit circle completely. So maximum or rated torque \((T_r)\) is obtained at the point where constant
torque hyperbola is tangent to the rated current circle. This is referred to as maximum torque per ampere (MTPA) operation, and it minimizes the copper losses.

**Mode 2: Current and Voltage limited**

At a speed $\omega$ greater than rated speed, the intersection of the voltage ellipse with the rated current circle gives the operating point with maximum torque possible. This mode is the first part of the flux-weakening region (Figure 1.5(b)).

**Mode 3: Voltage limited**

Beyond a certain speed $\omega'$, the voltage ellipse falls within the rated current circle. In this case, the tangent of torque hyperbola to the ellipse gives the maximum torque. This mode is the second part of the flux-weakening region. Practically, the neglected iron losses, mechanical losses and PWM effects cause the torque output to fall zero at a high speed.

If the characteristic current, $\lambda_m/L_d$ is greater than the rated current, the drive has a finite speed (green curve in Figure 1.5(b)). Thus, the parameters of the IPM motor - $\lambda_m, L_d, L_q$ are closely related with the inverter ratings to attain optimum performance for a given cost.

### 1.3 Applications and desirable features

The IPM has a leading edge over induction motor, SRM and Surface PM motor in applications such as EV & HEV traction motor, spindle motors and servo motor. Despite the benefits of high torque density, wide constant power speed range and high power factor, there are certain issues in the IPM drive that need careful engineering to satisfy application requirements. Two of the issues that have been given emphasis in this dissertation are saturated operation and low-torque ripple design.
1.3.1 Saturated operation

The core of the IPM motor consists of magnetic steel that exhibits a non-linear relation between magnetic flux density (B) and magnetic field intensity (H). From the B-H curve in Figure 1.6, the steel has a high permeability ($\mu_{Fe}$) below the saturation flux density, $B_{sat}$; above it, $\mu_{Fe}$ falls down rapidly. This material effect translates to diminished torque output at high currents. Aggressive motor designs have this effect even at 50% of the rated current. Speaking in electromagnetic terms, some sections of the stator teeth, rotor pole face and stator yoke are saturated, depending on the operating current phasor and rotor position.

Ideally, constant magnet flux linkage and d-q axes inductances are assumed while characterizing steady-state and transient behaviors of a motor design. Considering all saturation and cross-saturation effects theoretically, $\lambda_m, L_d$ & $L_q$ are all functions of the exciting currents, $(i_d, i_q)$ [12]. The magnetic model of IPM can be simplified considering that

![B-H curve of M 19 steel](image)

**Figure 1.6** B-H curve of M 19 steel
the d-axis magnetic circuit consists of a larger effective airgap, hence lower flux density than $q$-axis magnetic circuit. Thus, $\lambda_m$ and $L_d$ are less prone to saturation, and hence constants. Only the $q$-axis inductance is given as a function of $q$-axis current.

The other important effect of saturated operation is on controller development in the IPM drive. Low-complexity controllers have constant $\lambda_m$, $L_d$ & $L_q$ values, that work fine under low loading conditions. When saturation kicks in, the MTPA point in $(i_d, i_q)$ plane and the torque-speed envelope are miscalculated by ignoring $L_q(i_q)$ [13]. Also, the commanded terminal voltages exceed the inverter limits and current regulators saturate, leading to unstable transient operation. Some researchers [14] provide methods to consider saturation effect of IPM in the vector control algorithm, which results in improved and optimal drive performance. The $L_q(i_q)$ data is built-in while arriving at optimal $i_d$ & $i_q$ commands at a particular $(T, \omega)$ operating point. The controller can be tuned either using saturation data from FEA model of the IPM or from actual experiment.

1.3.2 Low torque ripple applications

The IPM motor belongs to the class of sinusoidal PM AC machines. Ideally, it has a sinusoidal winding distribution and a sinusoidal flux density of the rotor magnet, which results in a flat output torque. This is not true in a practical motor where winding is placed in discrete slots and rotor magnets are placed in one, two or utmost three layers. Thus, the torque output has oscillations riding on a mean value, which is defined as pulsating torque. Pulsating torque is the sum of torque ripple and cogging torque. Torque ripple is attributed to the stator electrical loading, so occurs only in loaded condition [15]. Cogging torque, on the other hand, is due to interaction of rotor’s field with stator slotting structure, and occurs under both no-load and loaded conditions[16].
Apart from the electromagnetic motor design, torque ripple could also be the result of the controller that is part of the IPM drive. Rotor position feedback errors, imperfect machine model, DSP and inverter time lags result in differences between actual and ideal exciting current waveforms leading to controller-induced torque ripple [15].

Some applications such as robotic servo systems, satellite tracker, spindles and electric power steering require high performance motor drives that have minimum speed oscillations. They require a torque ripple less than 5%. EV traction motors also specify an upper limit of 10% for torque ripple. High torque ripple results in mechanical vibrations, noise and resonances. This causes inferior user experience and faster aging of mechanical components like gearboxes, couplings and housing. Thus, efforts are made to have a low torque ripple IPM drive using motor design or controller design.

In the motor design stage, traditional approaches use one or a combination of stator skewing, rotor step skewing and short-pitched windings. The target is to reduce interaction between harmonics in stator MMF and rotor electromagnetic response. Specifically, cogging torque in IPM can be mitigated by proper pole-slot combination, slot opening tuning and skewing [16]. In synchronous reluctance motors (SyRM) and PM assisted SyRM, rules are specified for rotor barrier numbers and position in relation to stator slot number [17]. Another innovative technique is to have different rotor barrier positions in adjacent poles (Machaon design), which causes cancellation of two torque harmonics simultaneously [18]. In IPMs, a winding with odd number of slots per pole-pair gives lower torque ripple with a symmetric rotor design [19]. Modifications on the rotor include asymmetric placement of magnet layers [20]. The tradeoff with these designs in general, is a lowering of mean torque and increase in manufacturing cost.
Controller based techniques include programmed current control and selective harmonic injection methods [15, 21]. These employ feed-forward control based on pre-knowledge of motor behavior and as such, are sensitive to the motor parameters and aging. Interestingly, the commonly used speed controlled drive minimizes speed oscillations caused due to torque harmonics, due to noise rejection capability of the speed loop. A drawback is that this works only under motor speeds lower than a threshold, set by the control loop bandwidth. In all, it is always advisable to minimize torque ripple in the IPM design stage, and use controller methods as a supplement only under special scenarios.

1.4 Research motivation & objective

In the design stage, analytical modeling and FEA are the two tools available for modeling saturated inductances under overload conditions. FEA gives accurate results for a given geometry, but to initialize a candidate geometry requires high computing resources and many iterations. On the other hand, analytical models aid in arriving at an approximate estimate of the performance characteristics of a wide pool of designs. This is subsequently used in optimization studies before implementing a full-fledged FEA study. The parallel benefit is that the derivation of the models is based on physics, hence provides opportunities to analyze interesting phenomenon.

The separation of the average torque components provides design insights to enable optimization of the rated torque, flux weakening performance, overloading capability and manufacturing cost. A study of the segregated pulsating torque components holds the promise of providing ideas for torque minimization. Under loaded conditions, the interaction of the
fields produced by the interior magnet and the windings with the complex non-linear reluctance network in the core, makes it challenging to separate the components.

There are three main objectives of this dissertation:

1. To analytically solve for saturated $q$-axis inductance and leakage inductance in IPM motors
2. To separate the components of electromagnetic torque based on their source of generation and with innovative use of FEA
3. To design, prototype and comprehensively test a SyRM and an IPM to verify modeling and simulation, and evaluate good experimenting techniques

1.5 Outline of dissertation

Chapter 2 reviews the literature in magnetic equivalent circuit (MEC) based modeling of flux linkage and inductances in PM based machines. An alternative technique is proposed based on the path permeance approach for saturated $q$-axis inductance modeling. An advanced model is developed that accounts for non-linear superposition of the magnetizing and slot leakage fluxes, when stator tooth is saturated. The models are applied to sample 48-slot/8-pole traction motors with I-type and U-V type rotors, and verified with FEA.

Chapter 3 elaborates on the physics behind torque generation in electric motors and the FEA-intensive Frozen Permeability method of torque segregation in PM motors. A method utilizing fictitious derived rotors from the reference IPM is used to separate cogging torque, mutual torque and reluctance torque. The Torque Separation with Synthesized Rotors (TSSR) method is applied to 18-slot/4-pole and 24-slot/4-pole stators with I-type and V-type rotors. The results are validated with FEA under different loading conditions. Sensitivity analysis of
the torque harmonics to certain geometry parameters is explored, to suggest some ripple minimization strategies.

Chapter 4 presents the design and analysis of a SyRM and IPM, prototyping and experimental dynamometer platform. A modified winding function to get the MMF of multi-layer windings is proposed and used for the prototype winding design. A combination of basic hand-rules and commercial optimization software is used to obtain the candidate geometry. The processes involved in the prototyping are stator rewinding, rotor manufacturing, magnet insertion and assembling all components into the motor housing. Mechanical setup, electrical hardware and DSP logic are implemented to obtain valuable experimental data with high fidelity.

Chapter 5 describes the experimental results collected from the 18-slot/4-pole I-type SyRM and IPM. Inductance tests methods are implemented on the SyRM. SyRM is operated over a wide operating speed to extract the operating envelope of the drive. Torque characteristics are collected on both prototypes with low speed testing, and experimental torque separation at certain operating points is presented. Heat run tests are also conducted on the IPM. The results provide the real-world parallel to the modeling and FEA research on torque and inductances.

Chapter 6 summarizes the contributions in this dissertation and provides suggestions for future research work in this field on IPM modeling, analysis and design.
Chapter 2. Modeling of IPM Inductances under Saturation

2.1 Research Background

2.2 Motor $d$-axis parameters

2.3 Standalone saturated $q$-axis inductance model (Model 1)

2.3.1 Solution of the $q$-axis magnetic circuit

2.3.2 Implementation algorithm

2.4 Parallel magnetizing and leakage inductances model (Model 2)

2.4.1 Solution for slot leakage inductance

2.4.2 Auxiliary part of algorithm

2.5 Validation of analytical models with FEA

2.5.1 FEA Post-processing for inductances

2.5.2 48-slot/8-pole IPMs with I-type & U-V-type rotors (Baseline designs)

2.5.3 18-slot/4-pole IPM with I-type rotor (Prototype design)

2.6 Sensitivity analysis of inductances with slot geometry

2.6.1 Variation of magnetizing $q$-axis inductance

2.6.2 Variation of leakage inductance

2.7 Conclusion
The magnetizing $q$-axis inductance of an optimally designed interior permanent magnet machine (IPM) exhibits a saturating behavior as the current excitation increases. This chapter proposes an analytical method to study this behavior by expressing armature reaction as the product of stator MMF and flux path permeance function. An improved model accounting for superposition between slot leakage and magnetizing fluxes is developed that provides leakage inductance also. The two analytical models are applied to I-type and U-V-type traction IPMs and verified against FEA. Sensitivity analysis of inductances with slot dimensions is conducted and its utility in motor design is illustrated.

2.1 Research Background

Interior Permanent Magnet synchronous machines have wide applications in industry and electric traction. Depending on the application and inverter compatibility, the basic requirements on the IPM design are rated torque, maximum phase current, armature voltage at corner speed, efficiency and torque ripple. In Section 1.2, the $d$-$q$ synchronous reference frame theory was introduced, whereby the average steady state torque ($T_{em}$) and armature voltage ($v_{arm}$) are obtained from magnet flux linkage ($\lambda_m$), fundamental $d$-$q$ magnetizing inductances ($L_{md}, L_{mq}$), leakage inductance ($L_{lkd}$) and the exciting $d$-$q$ currents ($i_d, i_q$).

The basic machine parameters directly influencing the $d$-$q$ axes inductances are winding configuration, air-gap diameter, stack length, pole number, and magnet dimensions [4]. Well-established models based on magnetic equivalent circuit (MEC) are available for estimating $L_{md}$ and $L_{mq}$ [22]. Also, the $d$-axis reluctance path is dominated by the cavity geometry, and these two parameters have a weak dependence on the exciting currents. The
The $q$-axis reluctance path is primarily occupied by steel, and hence it exhibits a saturating behavior - $L_{mq}(i_q)$. The unsaturated $L_{mq}$ is equal to the round-rotor air-gap inductance [23].

Analytical modeling [22 - 24] and FEA [25] are the two tools available for saturated $q$-axis inductance modeling. Analytical approaches typically involve a reluctance network having saturating and non-saturating elements with MMF sources on the stator [22-24, 26]. The accuracy depends on the number of these elements. This research work proposes an alternative technique based on the path permeance approach, which was introduced in [27] for the study of stator slotting. Concepts of stator and rotor path permeance functions are introduced which help in comparing the levels of saturation in the stator and rotor. Due to unpredictability of the flux paths in certain regions, a minimum number of reluctances are used. Hence, the estimates are within an error margin when compared with FEA.

A second model is developed that accounts for a subsidiary phenomenon, wherein the stator tooth saturates with both the magnetizing and slot leakage fluxes. There exists an interdependence between $L_{mq}(i_q)$ and $L_{lkq}(i_q)$ solutions. This effect has not received sufficient attention in previous work on saturated $q$-axis inductance modeling [23,24,28]. A simple iterative algorithm has been implemented using MATLAB scripts and functions to solve for the inductances.

### 2.2 Motor d-axis parameters

A baseline IPM, intended for traction applications, is chosen for demonstration of the concept (Table 2.1). The baseline stator geometry is designed with Toyota Prius 2004 as the reference [29]. The winding configuration is a conventional distributed, single layer with 48 slots and 8 poles. The targeted power rating is 30 kW at 1500 rpm and rated current is 150 A-
peak. For the slot dimension optimization study, the slot area is kept constant to ensure a current density of 18 $\frac{A}{mm^2}$. Two baseline rotors with different magnet configurations (Figure 2.1) are considered a single layer I-shaped rotor and a double layer U-V shaped rotor. The materials used are M-19 steel and NdFeB magnet with remanence of 1.237 T and recoil permeability of 1.032.

Among the three parameters - $\lambda_m$, $L_{md}$, $L_{mq}$; the unsaturated $q$-axis inductance is less sensitive to cavity shape and magnet placement as the $q$-axis excited flux paths flow tangential to the cavity’s length. Thus, a simple strategy is to arrive at the baseline rotor geometry considering only $\lambda_m$ and $L_{md}$. It follows from the linear torque expression in $d$-$q$ frame that

**Table 2.1 Stator Geometry Variables – Fixed**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air gap radius ($R_g$)</td>
<td>80.625 mm</td>
</tr>
<tr>
<td>Stack length ($l_{stk}$)</td>
<td>50 mm</td>
</tr>
<tr>
<td>Air gap thickness ($g$)</td>
<td>0.75 mm</td>
</tr>
<tr>
<td>Stator yoke height ($h_{ys}$)</td>
<td>15 mm</td>
</tr>
<tr>
<td>Slot Area ($A_{slot}$)</td>
<td>149 mm$^2$</td>
</tr>
<tr>
<td>Turns/phase ($N_{ph}$)</td>
<td>88</td>
</tr>
</tbody>
</table>
\( \lambda_m \) be maximized and \( L_{md} \) be minimized to gain on magnet torque and reluctance torque, respectively. This optimization exercise was done with magnet volume as a constraint through parametric sweeps in FEA. The resultant baseline rotor geometry is listed in Table 2.2. It also provides the \( d \)-axis parameters - \( \lambda_m \) and \( L_{md} \), and the leakage inductance - \( L_{lk}\). The notations in the geometry are given in Figure 2.1.

### Table 2.2 Rotor geometry variables & \( d \)-axis parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>I-type IPM</th>
<th>U-V-type IPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_m ) &amp; ( (h_{m1}, h_{m2}) )</td>
<td>6 mm</td>
<td>(4, 4) mm</td>
</tr>
<tr>
<td>( w_m ) &amp; ( (w_{m1}, w_{m21}, w_{m22}) )</td>
<td>16.4 mm</td>
<td>(12, 16.3, 18.1) mm</td>
</tr>
<tr>
<td>( \theta_m ) &amp; ( (\theta_{m1}, \theta_{m2}) )</td>
<td>28.2°</td>
<td>(21.3°, 33.1°)</td>
</tr>
<tr>
<td>( \lambda_m )</td>
<td>120.7 mWb</td>
<td>212.8 mWb</td>
</tr>
<tr>
<td>( L_{md} )</td>
<td>0.97 mH</td>
<td>0.75 mH</td>
</tr>
<tr>
<td>( L_{lk} )</td>
<td>0.5 mH</td>
<td>0.46 mH</td>
</tr>
</tbody>
</table>

2.3 *Standalone saturated q-axis inductance model (Model 1)*

In a machine with no excitation sources in the rotor, the air gap flux density due to stator currents can be expressed as a product of stator MMF, \( F_{st}(\theta) \) and flux path permeance function, \( P_{q}(\theta) \) (unit: \( T/A \) – turns) [30]. The MMF function for a generic winding configuration and given excitation is obtainable by the winding function theory. An alternate and simple method is to use the fact that the spatial function - \( F_{st}(\theta) \) is piecewise constant with a jump of total slot current occurring at each slot location on the stator periphery. The saturated conditions in the rotor steel, stator teeth and back iron are captured in the path permeance function. It can be imagined as a spatial function representing an equivalent variable thickness air gap, sandwiched between a non-salient rotor and a slot-less stator, both of which are unsaturated. With knowledge of the paths taken by the \( q \)-axis flux lines, the
network of bulk reluctances is defined as shown in Figure 2.2. Their values have a non-linear dependence on the averaged operating flux density in the elements cross-section [23].

Certain assumptions are made to simplify the magnetostatic conditions and make it amenable to apply the models: (a) The permanent magnets excitation does not affect the response of the reluctance network to the stator current excitation. This is approximately valid, as the peak flux paths due to d-axis and q-axis excitation sources are apart by $90^\circ$ elec. (b) The flux bridges in the rotor periphery, provided for mechanical integrity, are saturated with the magnet’s flux and hence are equivalent to air. (c) The variation of d-q flux linkage with relative position between stator and rotor, or in other words, the time dependence, is negligible for the distributed winding. The small variation is due to stator slotting. (d) Cross-saturation effect is influence of d-axis current on $L_{mq}$ and vice-versa [26]. This is neglected on the same grounds as assumption (a).
2.3.1 Solution of the q-axis magnetic circuit

The stationary phase-A axis is aligned with the rotor q-axis and three phases are excited with sinusoidal currents such that \( i_q = I_0 \) and \( i_d = 0 \) are asserted in the synchronous reference frame. This sets up the armature reaction, \( B_g^q(\theta) \) in the airgap, which is calculated iteratively using the path permeance formula. The fundamental component of this waveform, \( B_{mq1} \) is obtained by Fourier series. The flux per pole, the flux linkage and finally the inductance are calculated as shown in Eqn. (2.1). It is sufficient to evaluate the spatial waveforms over half pole pitch, in the region between \( d \)-axis and \( q \)-axis (defined as \( \theta \)-domain), using their quarter wave symmetry.

\[
B_g^q(\theta) = \mathcal{F}_{st}^q(\theta) \times P^q(\theta) \tag{2.1a}
\]
\[
\phi_{mq1} = \frac{2B_{mq1}R_{st}}{p} \tag{2.1b}
\]
\[
\lambda_{mq1} = k_w N_{ph} \phi_{mq1} \tag{2.1c}
\]
\[
L_{mq} = \frac{\lambda_{mq1}}{i_q} \tag{2.1d}
\]

The path permeance function is segregated into three components - \( P_{rot}^q \), \( P_{st}^q \) and \( P_g^q \) corresponding to portions of the reluctance network in the rotor, stator and air gap, respectively.

**Figure 2.3** Stator MMF and path permeance components – I-type IPM at rated q-axis current
Each is a piecewise constant function, with the edges in $\theta$-domain decided by the boundary between adjacent reluctances. Thus the resultant $P^q$ and $B_g^q$ are also piecewise constant.

$$P^q(\theta) = P^q_{rot}(\theta) || P^q_g(\theta) || P^q_{st}(\theta) ; \quad P^q_g(\theta) = \frac{\mu_0}{1.7 g_e}$$

In the $\theta$-domain, $P^q_{st}$ is a 4-level function to account for the teeth regions; $P^q_g$ is a constant function due to uniform air gap thickness; $P^q_{rot}$ has 3 levels for single layer IPM, 5 levels for double layer IPM, to account for intervening steel and cavity tips. The levels change with saturation in the series reluctances over each $\theta$-span. For example, a differential flux packet about $q$-axis will encounter $\mathcal{R}_{ts3}$ and $\mathcal{R}_{ys3}$, in the stator. Hence, the level of $P^q_{st}$ in that region is given by

$$\frac{h^t_{ts3}}{\mu_0 R_g \left( \frac{g_{sp}}{2} \right)_{stk}} = \left( \mathcal{R}_{ts3} + \frac{\mathcal{R}_{ys3}}{2} \right) ; \quad P^q_{st}[ts3] = \frac{\mu_0}{h^t_{ts3}}$$

### 2.3.2 Implementation algorithm

A simple iterative algorithm is developed to solve these equations. Two variants of $B_g(\theta)$ are used the calculated waveform, which comes from the path permeance formula; and the estimated waveform, which is updated at each iteration. The logic begins by initializing the estimate with the path-permeance solution of the unsaturated machine. At each iteration, the averaged operating flux density and hence the non-linear reluctances are obtained from the estimate using flux continuity law. These reluctances are substituted to get $P^q$ and thus, the calculated $B_g$. Comparing the two variants, the estimates level in the $i^{th}$ $\theta-$span, $B_g[i]$ is incremented or decremented by $\Delta B = 0.01 \ T$. This serves as the feedback mechanism in the algorithm. The logic is said to converge when the calculated and estimated variants of $B_g$ match within an error limit. The flowchart is presented as the left branch of Figure 2.4.
2.4 Parallel magnetizing and leakage inductances model (Model 2)

The total synchronous $d$-$q$ inductance is the sum of the magnetizing and leakage inductances. The leakage is classified into four types based on the flux path tooth tip, slot side, harmonic and end-winding leakages [4]. These fluxes flow in auxiliary loops that do not link the rotor. Leakage flux does not contribute to torque production, but occurs in the terminal voltage expression. Of the four components, slot leakage is the dominant one in small air gap, slotted IPM machines. It is excited by the MMF of slot current and characterized by flux paths crossing slot walls. The conventional method (referred to as Model 0) (Figure 2.5(a)) presumes
that steel in the leakage path is unsaturated and that the complete slot MMF drops in the slot air gap [23]. This gives a constant value for $L_{lk,g}$. The current research attempts to include tooth saturation in the slot leakage calculation and study its interdependence with the $q$-axis armature reaction. This results in a saturable leakage inductance - $L_{lk,g}(i_q)$.

2.4.1 Solution for slot leakage inductance

Consider the tooth, $ts1$ in Figure 2.5(b). It is shared by three flux packets - magnetizing flux passing through the air gap, slot leakage flux from slot 1, and slot leakage flux from slot 2. If the steel in $ts1$ has crossed the saturation limit, these three fluxes superpose in a non-linear manner. Thus, the magnetizing component, $\phi_{mag-ts1}$ affects the leakage component, $\phi_{lkg-ts1}$ and vice-versa. Note that the slot flux lines are distributed over the entire slot height and aggregate at the slot’s root. The mathematical derivation follows. The annotation - $lkg$ is omitted from the subscripts in the following equations, since all of them refer to leakage quantities.
Consider slot 2 that is sandwiched between *ts1* and *ts2*. At height *x* from slot bottom, a differential flux *dΦ* flows due to the enclosed current - *N(x)I_sl2*. From flux continuity law, the following relation exists between flux densities in *slot 2* (*B_{sl2}*) in *ts1* (*B_{ts1}*) and in *ts2* (*B_{ts2}*)..

\[ d\Phi = B_{ts1} \cdot dy = B_{ts2} \cdot dy = B_{sl2} \cdot dx \]  
(2.4a)

\[ \frac{dx}{dy} \approx \frac{h_{sl}}{w_{ts}} = r \]  
(2.4b)

According to Ampere’s law, the enclosed current equals the sum of potential drops along the path of *dΦ*. The drops in *ts1* and *ts2* are included with their respective relative permeabilities, *μ_{ts1}* and *μ_{ts2}*. 

\[ N(x) \cdot I_{sl2} = B_{sl2} \cdot w(x) + B_{ts1} \cdot \frac{x}{\mu_0 \mu_{ts1}} + B_{ts2} \cdot \frac{x}{\mu_0 \mu_{ts2}} \]  
(2.5a)

\[ N(x) \cdot I_{sl2} = \frac{B_{sl2}(x)}{\mu_0} \left[ w(x) + \frac{x}{\mu_{ts1}} \cdot r + \frac{x}{\mu_{ts2}} \cdot r \right] \]  
(2.5b)

Now, consider tooth *ts1*. At height *x* from the tooth bottom, leakage flux *ϕ_{ts1}(x)* flows with contributions from *slot 1* and *slot 2*. It is obtained by integrating the differential fluxes from the tooth tip, *h_{sl}*, to the height *x*. Note the direction of flux superposition.

\[ \phi_{ts1}(x) = l_{stk} \int_x^{h_{ts}} B_{sl1}(x) \cdot dx - l_{stk} \int_x^{h_{ts}} B_{sl2}(x) \cdot dx \]  
(2.6)

### 2.4.2 Auxiliary part of algorithm

The above equations applied to all slots and teeth in one pole-pitch along with the equations in Section 2.3 are used to solve for the magnetostatic condition. Iterative solution method is required due to the non-linear steel. A parallel branch is introduced in the previous flowchart (Figure 2.4). The common section between the two branches includes two steps - superposition and finding the relative permeability in the stator teeth. When the solver converges, the *d-q* leakage flux is obtained from the leakage flux linked by turns in each slot, *λ_{lk-g-sl1}* [25].
\[ \lambda_{tkg-sl1} = l_{stk} \int_0^{h_{ts}} N(x) \cdot B_{s1}(x) \cdot dx \]  

(2.7)

The above method can be equivalently viewed as an MEC problem with a discretized reluctance in the teeth and, a series of variable MMF source and slot reluctance at each height \( x \) (Figure 2.5(c)). In comparison, the conventional method neglects reluctances in the tooth and has one slot reluctance at the slot tip.

### 2.5 Validation of analytical models with FEA

Finite element analysis divides the geometry domain into a large number of mesh regions and solves for the magnetic field in each node to a good degree of accuracy. Hence, it is considered an acceptable benchmark, before an experimental validation [25].

#### 2.5.1 FEA Post-processing for inductances

The baseline machine topologies (48-slot/8-pole stator) for the I-type and U-V-type IPMs are implemented in Altair’s FLUX2D software. Magnetostatic simulations are conducted with \( q \)-axis excitation and rotor \( d \)-axis aligned with the stator \( q \)-axis. An \( i_q \) value of 50 \( A - pk \) is considered as the unsaturated case, \( i_q \) value of 150 \( A - pk \) or the rated current is considered as the saturated case. The upper limit for the model comparison is an overload condition with \( i_q = 200 \ A - pk \). The magnet region is occupied by isotropic material of relative permeability, 1.032; it is a pure reluctance rotor. This is in line with assumption (a).

After solving, the FEA internally computes the \( q \)-axis flux linkage, \( \lambda_q \) through the path integral of magnet vector potential over coil loop [27]. Fundamental of the radial air gap flux density in the middle of the air gap, \( B_g(\theta) \) is obtained. Then, it is integrated to get the magnetizing flux/pole, \( \phi_{mq} \) and hence, \( L_{mq} \):

\[ \phi_{mq} = R_g l_{stk} \int_0^{\pi/p} B_{g1}(\theta) \cdot d\theta \]  

(2.8)
The difference between $\lambda_q$ and $\lambda_{mq}$ gives the flux linkage due to slot leakage:

$$\lambda_{tkg} = \lambda_q - \lambda_{mq} \quad (2.9)$$

Note that the end winding leakage is another significant leakage component, but it is not part of a 2D FEA solution.

The inductances from the FEA and analytical models are plotted against $q$-axis current for validating and evaluating the nature of their variation. In the plots, the standalone approach is referred to as *Model 1* and parallel approach is referred to as *Model 2*. It is noted that in the double layer IPM reluctance network, numerical correction factors were added to some reluctances in the rotor to match the model under unsaturated case. This was necessary since finding accurate values for the cross-sections and flux path lengths is difficult.

### 2.5.2 48-slot/8-pole IPMs with I-type & U-V-type rotors (Baseline designs)

An inspection of Figure 2.6 shows that the FEA curve is closer to the Model 2 curve, rather than the Model 1 curve, which is as expected. This points to the importance of incorporating slot leakage in the solution stage of $L_{mq}$. The model curves are an overestimation at all values of $i_q$. The above two observations are not true for $i_q$ range below $75 A - pk$; the reason may be a limitation of the algorithm or inaccurate modeling of non-saturating reluctances. The prediction of the numerical inductance values is within acceptable limits, as shown in Table 2.3. The variation between the saturated and unsaturated values is significant, hence there is value for such dedicated modeling. A shortcoming is that $L_{tkg} (i_q)$ has a huge error, when compared with FEA. This calls for a better understanding of the phenomenon. One possibility is to consider the tooth permeability as a function of the height $x$, $\mu_{ts} (x)$.

The saturating behavior of the $q$-axis magnetic circuit results in the saliency ratio dropping from 2.7 to 1.5 at rated current. Hence, relative contribution of magnet and reluctance
torques is expected to have wide variation with current level. While evaluating torque and armature voltage over the allowable region in the \((i_d, i_q)\) plane, a higher error is expected in comparison to evaluations on the \((i_d = 0, i_q)\) line. This is owing to negligence of cross-

![Figure 2.6 Variation of inductances with current](image)

**Figure 2.6** Variation of inductances with current

<table>
<thead>
<tr>
<th>Table 2.3 Figures of Merit – Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quantity</strong></td>
</tr>
<tr>
<td>Error in (L_{mq}) at rated current</td>
</tr>
<tr>
<td>Error in (L_{kg}) at rated current</td>
</tr>
<tr>
<td>Variation in (L_{mq}) with saturation</td>
</tr>
<tr>
<td>Variation in (L_{kg}) with saturation</td>
</tr>
</tbody>
</table>
saturation effect and the magnets field [26]. It was observed that the spatial harmonics in the air gap flux density waveform do not match well with FEA. Thus, this model is limited in its application to machines with a sinusoidal winding. It is expected to work well for distributed windings, but not concentrated windings, as they generate prominent harmonics.

2.5.3 18-slot/4-pole IPM with I-type rotor (Prototype design)

In this dissertation, a 2 HP IPM motor was prototyped and tested comprehensively for inductance as well as general performance. The results are covered in Chapter 5. The geometry of the prototype is depicted in Figure 2.7; it is an 18-slot/4-pole I-type design. Model 1 is
applied to this motor and compared against FEA. The rated current is 4 Arms and test current $i_q$ has been varied from 0.2 pu to 1.5 pu (Figure 2.8). The analytical prediction matches well with FEA except for a divergence at overload condition. Interestingly, the $L_{mq}(i_q)$ behavior of the prototype is flat below 0.8 pu current and falls linearly beyond that. This is different from the baseline motor results of Figure 2.6.

Also, the difference between $L_q$ and $L_{mq}$ from FEA appears constant. This means that the leakage inductance is not saturating. The slot width is larger in comparison to the slot height, hence slot leakage is low. As a result, any improvement in results using Model 2 would be negligible for this motor.

2.6 Sensitivity analysis of inductances with slot geometry

There is scope for optimization both on the rotor and stator geometries. The first step is to investigate the sensitivity of the inductances - $L_{mq}$ and $L_{ukg}$ to the geometry. The stator tooth width, $w_{ts}$ and slot height, $h_{sl}$ are selected for this analysis (Figure 2.7). Inspection of the segregated path permeance functions helps in identifying specific geometric regions that tend to saturate more. From Figure 2.3, it is observed that the rotor permeance function, $P_{rot}^q$ is very high in most regions. Hence, the stator permeance function, $P_{st}^q$ acts as the limiting factor.

The fundamental reason is that the ratio of cumulative sums of $\theta$ — spans occupied by an air region to that occupied by steel region is higher on the stator side. A narrower steel region has a higher chance of saturation for a given amount of flux. The stated ratio for the rotor depends on the number of magnet layers and cavity heights; the ratio for the stator
depends on number of slots per pole and slot width. For the studied single and double layer IPMs with 6 slots per pole, \( B_{st}^q \) could be assumed to be the limiting factor. The inverse may hold true for PM assisted synchronous reluctance machines with higher number of rotor layers than stator slots per pole.

Keeping air gap radius, number of turns and winding configuration fixed, the primary factor to influence \( I_{st}^q \) and \( L_{mq} \) is the tooth width. The slot area is held constant, to maintain the limit on slot current density. The yoke height is also held constant. These constraints are given by

\[
\begin{align*}
    w_{sp} &= \text{constant} \quad ; \quad h_{ys} = \text{constant} \quad (2.10a) \\
    (w_{sp} - w_{ts}).h_{sl} &= A_{\text{slot}} \quad (2.10b) \\
    w_{sp} \left( 1 - \frac{w_{ts}}{w_{sp}} \right).h_{sl} &= A_{\text{slot}} \quad (2.10c)
\end{align*}
\]

Thus, pairs of the sweeping variables \( (\frac{w_{ts}}{w_{sp}}, h_{sl}) \) are supplied to the functionalized form of the models. FEA values at few data points are also added for crosscheck.

**2.6.1 Variation of magnetizing q-axis inductance**

Under a low current excitation, the teeth are unsaturated; there is no dependence of \( L_{mq} \) on the tooth width. This is confirmed by the flat shape of unsaturated curves over major part
of the range (Figure 2.10). Under a high current excitation, a higher tooth width is expected to give less saturated teeth. This beneficial behavior is shown by the *Model 1* saturated curves which increase monotonically. However, in the practical scenario, the slot leakage is also sharing the tooth. The geometric region of $w_{ts}/w_{sp}$ higher than 55% is characterized by a narrow and long slot, which results in considerable leakage. Thus, the expected benefit of a higher $L_{mq}$ is partially counterbalanced. This is conveyed by the *Model 2* curves which show a moderate increase. Also, there are ranges where the models fail to capture the magnetic condition properly.

![Figure 2.10](image1.png)  
(a) Baseline I-type IPM

![Figure 2.10](image2.png)  
(b) Baseline U-V-type IPM

**Figure 2.10** $q$-axis inductance variation with tooth width
2.6.2 Variation of leakage inductance

It is interesting to observe the leakage inductance variation for the U-V-type IPM (Figure 2.11). The Model 0 (conventional method) curve lies between the saturated and unsaturated curves. All three show a quadratic variation with $w_{ts}/w_{sp}$. If the $L_{lk}$ value becomes comparable to that of $L_{mq}$ at a particular geometry, it signifies a poor conversion between the applied terminal voltage to the delivered torque. From an optimization point of view, three factors - $L_{mq}, L_{lk}$ and stator lamination volume can be tracked independently. The $d$-axis inductance and the magnet flux linkage are constant since, the rotor geometry is held constant. Hence, a higher $L_{mq}$ would result in higher reluctance torque. The stator lamination volume increases linearly with the increase of the $w_{ts}/w_{sp}$ ratio, because the yoke height is held constant. For a given inverter, the deliverable terminal voltage can be assumed to be constant. These qualitative observations and constraints are summed up to suggest that an optimum value of tooth width exists with material cost and torque output as the trade-offs.

![Figure 2.11 Leakage Inductance and steel volume variation with tooth width: U-V-type IPM](image-url)

**Figure 2.11** Leakage Inductance and steel volume variation with tooth width: U-V-type IPM
2.7 Conclusion

This chapter proposes the use of flux path permeance function to study the saturating characteristics of the \( q \)-axis inductance and the slot leakage inductance. It provides a detailed analysis of the superposition of magnetizing and leakage fluxes in the stator tooth, by treating it as a basic physics problem. A simple iterative algorithm is developed to solve the magnetostatics problem. The approach is verified with FEA to give results within acceptable error margins. To gain design insight and identify performance tradeoffs, a sensitivity analysis of inductances with slot dimensions in I-type and U-V-type IPM is conducted. Although there are certain limitations, the proposed path permeance based model is expected to be useful in rapid optimization of IPM lamination geometries. This approach is applicable to machines with a distributed winding and a salient rotor construction, such as IPMs, Synchronous Reluctance Machines and PM assisted Synchronous Reluctance Machines.
Chapter 3. Separation of IPM torque components with synthesized rotors

3.1 Research Background

3.1.1 Components of torque

3.1.2 Frozen permeability method

3.2 Physics of torque generation

3.2.1 Stator winding MMF

3.2.2 Air-gap flux density and electromagnetic torque

3.3 Torque separation with synthesized rotors (TSSR)

3.3.1 Reference motor and motor with cavity rotor

3.3.2 Motor with current sheet rotor

3.4 Verification of TSSR with FEA on sample motors

3.4.1 18-slot/4-pole V-type and 24-slot/4-pole I-type at constant current vector

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3.5 IPM Design with TSSR

3.6 Dependence of pulsating torque components on geometry

3.6.1 Variation of Slot opening angle (18-slot/4-pole IPM)

3.6.2 Variation of Pole arc (18-slot/4-pole IPM)

3.6.3 Variation of Pole offset in asymmetric rotor designs (24-slot/4-pole IPM)

3.7 Conclusion
Understanding the torque behavior of Interior Permanent Magnet (IPM) machines is critical for the motor design and the end application. The net torque in IPMs is separable into cogging torque, reluctance torque and mutual torque. This chapter presents the Torque Separation by Synthesized Rotors (TSSR) technique with the aid of Finite Element Analysis (FEA). It basically proposes two fictitious models derived from the reference IPM - the ‘Cavity Rotor’ machine and the ‘Current sheet’ machine. The physics involved in the generation of electromagnetic torque including stator magnetomotive force (MMF) and the air gap flux density are explained to appreciate this method better.

By comparing superposed torque profile with reference motor’s profile, the method is validated on an 18-slot/4-pole motor and a 24-slot/4-pole motor with I-type rotor, operating under varying current commands. In the design stage, the two synthesized machines enable independent control of the magnet flux linkage and saliency ratio to achieve targeted performance. The second utility is in torque ripple minimization, for which trends in the dominant harmonic of each torque component are obtained as functions of slot opening angle, pole arc and pole offset angle.

3.1 Research Background

The net torque in practical motors consists of the useful mean torque and the pulsating torque. Pulsating torque is an undesirable effect causing mechanical vibration and premature aging of drivetrain. Design and control methods to mitigate pulsating torque are discussed in Section 1.3.2. The production of torque in electric motors is due to the interaction between the stator magnetomotive force (MMF) and the air gap field. Conversely, it can be viewed as an interaction between rotor MMF and the air gap field.
3.1.1 Components of torque

Based on the electromagnetic origins, cogging torque ($T_{cog}$), mutual torque ($T_{mutl}$) and reluctance torque ($T_{rel}$) are the three components of the net torque in IPMs [15]:

(a) **Cogging torque** : $T_{cog}$ is generated as an interaction between the permanent magnet rotor’s magnetic field with the stator permeance variation due to the slotted structure [16]. It exists under both no-load and loaded conditions.

(b) **Reluctance torque** : The interaction of the stator MMF with the rotor saliency variation due to the barriers results in $T_{rel}$. It exists in IPMs as well as synchronous reluctance motors (SyRMs) [17].

(c) **Mutual torque** : $T_{mutl}$ is a result of the interaction between the stator MMF with the PM rotor’s magnetic field.

Cogging torque is purely oscillating, whereas $T_{mutl}$ and $T_{rel}$ consist of the average torque and the torque ripple.

3.1.2 Frozen permeability method

In existing literature [31-34], the frozen permeability (FP) method is used to separate the torque components. This is an FEA based procedure that separates fields due to the armature excitation and magnet, without neglecting the cross-influences between them. Under loaded condition, the steel core’s flux density is high in saturated regions, so the assumption of steel relative permeability being high ($\sim 10^5$) is not true. Three FEA simulations are involved. The loaded simulation (FEA step 1) of the reference IPM gives a flux ($B_{net}, H_{net}$) and relative permeability spatial map ($\mu_{net}$) at each time step. This map is assigned to the whole of lamination. It also gives the net torque, $T_{net}$. Then the PM is de-activated and the
armature current excited (FEA step 2); this gives the reluctance torque, $T_{rel}$. The armature current is kept zero, and the PM activated (FEA step 3); this gives the cogging torque, $T_{cog}$. Mutual torque is obtained as:

$$T_{mutl} = T_{net} - T_{rel} - T_{cog} \quad (3.1)$$

**Figure 3.1** Frozen permeability relations – Magnetic flux density – Field intensity (B-H) steel curve

**Figure 3.2** Comparison of torque reconstruction with FP and TSSR in a sample IPM
The B-H relation in the FP method is depicted in Figure 3.1. With non-linear superposition of fields enabled by FP method, $B_{\text{curr-FP}}$ and $B_{\text{PM-FP}}$ are the core flux densities, during FEA steps 2 and 3, respectively (green lines). This is lower than $B_{\text{curr}}$ and $B_{\text{PM}}$ which are under assumption of linear superposition (blue lines).

Though FP is accurate under heavy loaded conditions by accounting for saturation, it is time consuming and cumbersome to execute on a series of geometries in the initial design stage. In comparison, the proposed Torque Separation by Synthesized Rotors (TSSR) method has two sets of motor geometries that can be subjected to parameter sweeps in FEA. The TSSR provides simplicity and intuition on the average reluctance to mutual torque contribution and relative torque ripples, rendering it a quick and valuable analysis tool in the design filtering stage.

The means to validate a new approach is to reconstruct the net torque waveform and compare it against the results from the reference motor. The results for a sample IPM at two current values is depicted in Figure 3.2 using FP and using TSSR. The average torque seems to be accurately predicted by both methods. TSSR provides a marginal overestimation of the pulsating torque in comparison to FP, which is higher at 150 $A$ than 100 $A$. Thus, TSSR is a good enough competitor to FP.

3.2 Physics of torque generation

The study of torque ripple generation requires a good knowledge of the magnetic field quantities in the machine air gap and the steel-air interfaces. In this section, definition and equations are provided for the physical quantities at the airgap interface with rotor and stator:

(i) Linear current density [Unit: $Amp/m$]
These physical quantities, in general, are dependent on both the angular position in the
air gap, \( \theta \) and time, \( t \). Hence they are denoted as functions of \((\theta, \omega t)\), where \( \omega \) is the electrical
frequency.

### 3.2.1 Stator winding MMF

The concept of MMF comes from Ampere’s law, by line-integrating field intensity, \( H \)
from a reference point in the motor’s geometry. It is also known as the magnetic scalar
potential. The stator winding of an electric machine supplied with balanced three phase
sinusoidal currents, produces an MMF in the airgap which is a summation of forward traveling
and reverse traveling harmonic waves [35].

Consider a machine with \( p \) pole-pairs, \( q \) slots/pole/phase and \( z_Q \) conductors in each
slot. The winding factor, \( k_{wv} \) describes the harmonic characteristics of the winding dependent
on the distribution of coil sides and on short-pitching. The winding function, \( n(\theta) \) describes the shape of the MMF in the airgap due to windings of one phase:

\[
n(\theta) = \sum_{\nu=1}^{\infty} n_{\nu} \cos(\nu\theta) = \frac{2}{\pi} qzq \sum_{\nu=1}^{\infty} \frac{k_{w\nu}}{\nu} \cos(\nu\theta) 
\]  
\[ 3.2 \]

(a) Space vector representation (\( \nu = 1, -2, 4 \))

(b) Space-time representation (\( \nu = -2, 4, -5, 7 \))

**Figure 3.4** Position of harmonic MMF waves for a sample winding
The set of windings for phases $A, B & C$ are spatially laid $\frac{2\pi}{3}$ $rads$ apart from each other. The current advance angle is $\gamma$ and current magnitude is $\hat{I}$. The net MMF is a summation of MMFs due to each phase (with $\phi = \omega t + \gamma$):

$$F(\theta) = n(\theta)\hat{I}\cos(\phi) + n\left(\theta - \frac{2\pi}{3}\right)\hat{I}\cos\left(\phi - \frac{2\pi}{3}\right) + n\left(\theta - \frac{4\pi}{3}\right)\hat{I}\cos\left(\phi - \frac{4\pi}{3}\right) \tag{3.3}$$

Processing with trigonometric identities and simplifying, we get two sets of waves - forward traveling and reverse traveling. Spatial variable, $\theta = 0$ position is depicted in Figure 3.3(a). The alignment between $d-q$ frame and stator $a-b-c$ at time, $t = 0$ is depicted in Figure 3.3(b). The total stator MMF is given by:

$$F(\theta, t) = \frac{3}{2} \hat{I} \left[ \sum_{\nu=3k-2,k\in\mathbb{N}} n_{\nu} \cos(\nu \theta - \omega t - \gamma) + \sum_{\nu=3k-1,k\in\mathbb{N}} n_{\nu} \cos(\nu \theta + \omega t + \gamma) \right] \tag{3.4}$$

This expression shows that $3n$ harmonics are absent in 3 phase windings. Harmonic waves of order, $\nu = \{1,4,7,\ldots\}$ are forward traveling; harmonic waves of order, $\nu = \{2,5,8,\ldots\}$ are reverse traveling. Figure 3.4(a) shows space vector notation of the waves of order, $\nu = 1,2,4$. Figure 3.4(b) shows two forward traveling waves ($\nu = 4,7$) and two reverse traveling waves ($\nu = 2,5$) at different time instants ($\alpha = \omega t$) for a double layer, short pitched winding. The shape of the net MMF waveform keeps evolving over one electrical cycle, as the component waves travel at different speeds.

### 3.2.2 Air-gap flux density and electromagnetic torque

The torque in an electric machine is generated as an interaction between the radial airgap flux density, $B_g$ and linear current density on the airgap-stator periphery, $A_{st}$. The rotor of an IPM rotor has an intrinsic MMF, $F_{rot}$.
The $B_g$ is related to the difference in MMFs of the rotor and stator [18] ($g =$ air gap thickness):

$$B_g = \frac{\mu_0}{g} (F_{\text{rot}} - F_{\text{st}}) \quad (3.5)$$

Maxwell’s stress tensor theory [34] states that an electromagnetic stress ($\sigma$) exists on an element carrying current density ($A$) due to a perpendicular flux density ($B_g$), given by

$$\sigma = B_g \cdot A \quad (3.6)$$

Consider a differential area element on the stator, $R_g d\theta dz$ (Figure 3.5). It experiences a tangential shear force, $dF$, which causes a torque, $dT_{em}$ about the axis of rotation. The net torque acting on the stator is obtained by integrating $dT_{em}$ over a cylindrical surface at stator inner diameter. For a machine with stack length $l_{\text{stk}}$, air gap radius $R_g$ and pole-pair number $p$, the net torque generated is given by,

$$T_{em} = R_g^2 l_{\text{stk}} p \int_0^{2\pi/p} B_g \cdot A_{\text{st}} \, d\theta = R_g^2 l_{\text{stk}} p \int_0^{2\pi/p} B_g \cdot A_{\text{rot}} \, d\theta \quad (3.7)$$

The important conclusion is that the current density function can either belong to the stator ($A_{\text{st}}$) or rotor ($A_{\text{rot}}$).
3.3 Torque separation with synthesized rotors (TSSR)

The separation of mean torque components and pulsating torque components is conducted by FEA solutions at a specific operating current vector on the reference IPM (also, Complete or Machine I) and two derived motors. The motor with cavity rotor (Machine II) and motor with current sheets are proposed to isolate reluctance torque and mutual torque generation mechanisms. An 18-slot/4-pole distributed winding, I-type IPM is used to demonstrate the TSSR concept.

3.3.1 Reference motor and motor with cavity rotor

Loaded current mode simulation of the reference IPM (Figure 3.6(a)) under certain current magnitude, $I_{mag}$ and current angle, $\gamma$ gives the sum of net mean torque ($T_{mean}$) and

![Motors with reference & synthesized rotors](image)

**Figure 3.6** Motors with reference & synthesized rotors
pulsating torque ($T_{puls}$). Cogging torque ($T_{cog}$) is defined as the interaction between the PM rotor’s field with the variable airgap permeance caused by stator slotting [16]. Its profile is obtained by no-load simulation of the complete IPM, which is zero currents supplied to the stator windings and the shaft rotated at constant speed externally.

For a salient type machine, the variation in rotor reluctance from $d$-axis to $q$-axis causes the rotor to have an induced MMF as reaction to the stator MMF [18]. The induced MMF reacts with the winding current profile to produce reluctance torque ($T_{rel}$), with the mean and ripple. To eliminate the PM’s field and account for the reluctance only, the machine with cavity rotor (Machine II) is derived by removing the pole magnets and retaining the air cavities in the rotor lamination (Figure 3.6(b)). It is essentially a synchronous reluctance machine, but with a sub-optimal design. In the TSSR technique, reluctance torque, $T_{rel}(t)$ is obtained through FEA loaded simulation of Machine II at operating current phasor – ($I_{mag}, \gamma$) and under constant speed.

3.3.2 Motor with current sheet rotor

Mutual torque or alignment torque is a result of interaction between the PM rotor’s field and the stator winding currents. The machine with current sheets (Machine III) is proposed (Figure 3.6(c)), wherein the effect of reluctance variation is eliminated by emulating the PM’s field via current sheets ($A_{mag}$) placed at the flux bridge locations. The sum of mean and ripple mutual torque ($T_{mutt}$) in the TSSR method, is obtained from FEA simulation of Machine III at operating current phasor - ($I_{mag}, \gamma$) and under constant speed. It is crucial to note that under no-load condition, both Machine I and Machine III produce the same air gap field (Figure 3.7(a)). The difference occurs with non-zero stator currents, when the cavity
structure in Machine I starts producing additional reactionary MMF; this is absent in Machine III.

The linear current density, $A_{mag}$ in the current sheets is dependent on the motor geometry and magnet properties. The magnetic equivalent circuit under no-load condition for unit stack length is depicted in Figure 3.7(b); it is a half circuit, using the symmetry condition between adjacent poles. The variables used to derive $A_{mag}$ are listed in Table 3.1. The pole
The magnet’s MMF source, magnet’s equivalent flux source and constant leakage flux in the bridge are defined as,

$$F_m = H_c h_m; \phi_m = \frac{F_m}{R_m}; \phi_{\text{kg}} = B_{\text{sat}} w_{\text{brg}}$$

(3.9)

The face air-gap reluctance, magnet body’s reluctance and the cavity’s non-magnetic section reluctance are respectively given by,

$$R_g = \frac{g'}{\mu_0 R_g(\theta_m/2)}; R_m = \frac{h_m}{\mu_0 \mu_r (w_m/2)}; R_{nm} = \frac{h_m}{\mu_0 (w_{nm}/2)}$$

(3.8)
The total flux passing through the pole face, $\phi_g$, is obtained from the MEC, which in turn is used to get the fictitious current sheet’s strength using Ampere’s law,

\[
\phi_g = \frac{1}{\mathcal{R}_g} \frac{1}{\mathcal{R}_g + 1/\mathcal{R}_m + 1/\mathcal{R}_{nm}} (\phi_m - \phi_{lk}) \quad (3.10a)
\]

\[
A_{mag} = \frac{g}{\mu_0 R_g^2 \theta_{brg} (\theta_m/2)} \cdot \phi_g \quad (3.10b)
\]

(a) With magnet width and PM remanence

(b) With pole arc and magnet V-angle

Figure 3.9 Linear Current Density variation
The variation of linear current density in the current sheet rotor (Machine III) with parameters of the reference motor is studied, as obtained from the above equations.

In Section 3.5, $A_{mag}$ is used as a design variable, so geometries yielding a requisite $A_{mag}$ can be obtained from these plots. The analysis is also applicable to V-type IPMs for generality; a magnet V-angle of $\theta_v = 180^0$ gives the I-type IPM. The varied dimensions are depicted in Figure 3.8(a) and (b). Figure 3.9(a) depicts the curves as the magnet remanence $B_r$ and width of magnet in the cavity are changed, for constant pole arc, $\theta_m$ of $56^0$ and $\theta_v$ of $130^0$. There is almost linear progression of current sheet strength with both variables. The increase is directly due to the higher magnet strength and magnet volume. In contrast, Figure 3.9(b) shows that higher pole arc results in a slight decrease in $A_{mag}$ for a constant $B_r$ of 1.22 $T$ and $w_m/w_{tyr} = 1$. Though magnet volume increases with $\theta_m$, the flux emanating from the pole face is distributed over a wider cross section. A lower value of $\theta_v$ means more magnet volume can be accommodated within same pole arc, and hence an increase in current sheet strength.

3.4 Verification of TSSR with FEA on sample motors

The accuracy of proposed torque component separation method (TSSR) is verified in this section on three sample motors and under varying operating points for the prototype motor. The time domain torque profiles from Machines II and III are added to get the superposed torque ($T_{mean} + T_{puls}$), which is the sum of the three components

$$ T_{mean} + T_{puls} = T_{cog} + T_{mutt} + T_{rel} $$

(3.11)

Theoretically, this should be equal to the reference motor’s (Machine I) torque profile ($T'_{mean} + T'_{puls}$). This is called the superposition condition, conventionally used to verify frozen permeability method [32].
3.4.1 18-slot/4-pole V-type and 24-slot/4-pole I-type at constant current vector

An 18-slot/4-pole IPM with I-type rotor was designed and prototyped to verify the presented technique. The decomposition was conducted at the rated current of 4 Arms and current angle $\gamma$ of $45^\circ$ in Figure 3.10(a). Mutual torque has bigger contribution to the net average torque. Cogging torque and mutual torque also contribute more to the ripple than the

![Figure 3.10 Superposition of torque components](image)
reluctance torque. All three components have a significant 18\textsuperscript{th} harmonic as this winding has an odd number of slots (\(= 9\)) per pole pair. The superposed and reference torque profiles appear close, with a small phase offset. The mean torques \(T_{\text{mean}}\) and \(T'_{\text{mean}}\) are within 1.4 \%. The superposed pulsating torque is 21.9 \% peak to peak, which is close to reference pulsating torque of 19.2 \%.

To demonstrate the generality of the TSSR approach, the torque separation method is also tested on an 18-slot/4-pole V-type IPM and a 24-slot/4-pole I-type IPM (Figure 3.11), at the same operating point of \((4 \text{ Arms, } 45^\circ)\). The comparisons are summarized in Table 3.2. Machines \(I, II\) and \(III\) have a very good correlation between them, as seen from the bottom two rows.

**Table 3.2 Verification of TSSR on Sample Motors**

<table>
<thead>
<tr>
<th>Torque component</th>
<th>18-slot/4-pole V-type</th>
<th>24-slot/4-pole I-type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean torque (Nm)</td>
<td>Ripple (pk-pk) (Nm)</td>
</tr>
<tr>
<td>(T_{\text{cog}})</td>
<td>0</td>
<td>1.15</td>
</tr>
<tr>
<td>(T_{\text{mutt}})</td>
<td>6.41</td>
<td>0.95</td>
</tr>
<tr>
<td>(T_{\text{rel}})</td>
<td>1.49</td>
<td>0.26</td>
</tr>
<tr>
<td>(T_{\text{mean}} + T_{\text{puls}})</td>
<td>7.91</td>
<td>1.39</td>
</tr>
<tr>
<td>(T'<em>{\text{mean}} + T'</em>{\text{puls}})</td>
<td>7.78</td>
<td>1.73</td>
</tr>
</tbody>
</table>

**Figure 3.11** Geometry of 24-slot/4-pole I-type IPM
The number of turns in the 24-slot/4-pole winding was tuned so that its fundamental back EMF matches with the prototype motor. This is to have similar saturated conditions between the two. In Figure 3.10(b), the 24-slot/4-pole IPM is having very high ripple, as the rotor was not optimized in relation to the winding layout. Also, the cogging torque and mutual torque have two dominant harmonics, as inferred from their non-sinusoidal shape.

3.4.2 Prototype IPM at varying current vector

For the prototype IPM, the method is further investigated at different operating conditions. In Figure 3.12(a), the current magnitude is varied from 20% to 160% of the rated current, while maintaining current advance angle of 45°. The mean torque increase is approximately linear, as mutual torque dominates over reluctance torque. With regards to the average, the superposed torque also tracks the reference accurately. The pulsating torque normalized to the mean torque is higher at lower currents, as cogging torque is a constant, irrespective of operating current. The superposed and reference pulsating torques are within 10% in the whole range.

Maintaining rated current, the current angle is varied in the positive-\(i_q\) negative-\(i_d\) quadrants in Figure 3.12(b). The mean reference torque is higher than mean superposed torque, with maximum difference of 0.9 Nm. The maximum torque per ampere (MTPA) region is flat and within 5° for the two curves. The pulsating torque is within 12% till the \(\gamma\) value of 68°; the mismatch is high above this angle. From the two plots in Figure 3.12, it can be seen that the decomposition method overestimates pulsating torque at higher current magnitudes and lower \(\gamma\) values. This is due to negligible saturation in the tooth tips and rotor cavity tips in the synthetic machines, which results in sharper edges in air gap flux density waveform \(B_g\).
Whereas, the reference machine exhibits saturation and smoother edges in $B_y$, and hence lower torque ripple.

(a) For varying current magnitudes (at $\gamma = 45^0$)

(b) For varying current angles (at $I_{mag} = 4$ Arms)

Figure 3.12 Verification of TSSR on prototype IPM
3.5 IPM Design with TSSR

This section focuses on the mean parameters, the saliency ratio \( (L_q/L_d) \) and magnet flux linkage \( (\lambda_m) \), neglecting the harmonics. The cavity type motor (Figure 3.6(b)) and current sheet motor (Figure 3.6(c)) that are integral to TSSR, are shown to be useful in the FEA based design procedure of a V-type IPM. The key performance figures such as rated torque, constant

![Saliency ratio as function of \((\theta_m, \theta_v)\)](image1)

![Magnet flux linkage as function of \((\theta_m, A_{mag})\)](image2)

**Figure 3.13 Design Planes with TSSR**
power speed range, uncontrolled generator operation and material cost are directly linked to $L_q/L_d$ and $\lambda_m$ [11,36]. The constants in the design are air gap thickness, stator dimensions, cavity height and PM remanence. Only the magnet V-angle ($\theta_v$), pole arc ($\theta_m$) and magnet to cavity width ratio ($w_m/w_{lrr}$), are varied for the tradeoff study (refer to Figure 3.8). It is noted that such an exercise with frozen permeability method would be more accurate in handling saturation, but requires significant computational time as several steps are run at each design point [31-34].

Figure 3.13(a) and (b) constitute the design planes from which the designer can choose based on the application requirements. The $q$-axis inductance is constant with geometry and is equal to the round rotor inductance. With operating point $i_q = 0, i_d = I_0$, $d$-axis inductance is collected from FEA of Machine II. Thus plots of $L_q/L_d$ as function of ($\theta_m, \theta_v$) are obtained in Figure 3.13(a). With increase in pole arc, $L_q/L_d$ increases till $\theta_m = 55^\circ$, but levels after that. This is due to higher reluctance of a wide cavity leading to lower $d$-axis inductance. There is a small variation with magnet V-angle as well. No-load FEA of Machine III gives $\lambda_m$ as function of line current density, $A_{mag}$ and $\theta_m$. Correspondence between a design point ($\theta_m, \theta_v, A_{mag}$) and dimensions of Machine I is arrived at using equations in Section 3.3.2.

Two designs are selected to compare their performances at MTPA. The maximum saliency ratio in the design plane is assigned to Motor A (yellow circle) and the lowest value

<table>
<thead>
<tr>
<th>Table 3.3 Comparison of two IPM designs</th>
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<tbody>
<tr>
<td><strong>Metrics</strong></td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Motor A</td>
</tr>
<tr>
<td>Motor B</td>
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</tbody>
</table>
is assigned to *Motor B* (green square). In relation to *Motor A*, *Motor B*’s cavity spans small angle at rotor periphery and has a sharper V angle. With the constraint of having equal rated torque and $\theta_m$ determined from Figure 3.13(a), the $A_{mag}$ values are tuned and determined on Figure 3.13(b). Table 3.3 compares the two designs extensively. *Motor B* has higher $\lambda_m$ than *Motor A*. Thus, higher saliency ratio motor has a higher component of reluctance torque contribution in the overall torque. The magnet volume is marginally different. For two or three magnet layer IPMs, higher $L_q/L_d$ ratio is achievable, and better tradeoff with magnet volume and cost is possible. In [37], a hierarchical design procedure is proposed, wherein, the saliency ratio is optimized in first stage and the PM usage is optimized in the second stage. Such procedures can also utilize the synthesized rotors.

### 3.6 Dependence of pulsating torque components on geometry

The second benefit of the TSSR method is to observe independent variation of the cogging, mutual and reluctance torque ripples with a particular geometric parameter. This is expected because the electromagnetic source of each component is fundamentally different. Three phase machines with distributed windings exhibit $6n$ harmonics. Depicted in Figure 3.14(a), the Fourier spectrum of the prototype machine’s (18-slot/4-pole) torque over one electrical cycle has the harmonics in decreasing order of magnitudes - 18, 6, 36, 12. From these, the $18^{th}$ and $36^{th}$ are attributed to slotting harmonics in the air gap flux density. The interaction of winding harmonics and rotor MMF harmonics results in the $6^{th}$ and $12^{th}$ of torque [15]. From the phase spectrum, the three $18^{th}$ harmonics are within $90^0$ of each other. In Sections 3.6.1 and 3.6.2, the effect of stator and rotor geometry on the most significant $18^{th}$ harmonic is investigated in detail.
A similar exercise is repeated on the 24-slot/4-pole IPM with respect to pole offset angle in asymmetric rotors. The Fourier spectrum depicted in Figure 3.14(b) is for the symmetric (conventional) rotor IPM; it shows harmonics in decreasing order – 12, 24, 6, 36. The significant 12th harmonic is the subject of the sensitivity analysis in Section 3.6.3.

Figure 3.14 Fourier spectrum of torque components
3.6.1 Variation of Slot opening angle (18-slot/4-pole IPM)

The ratio of slot opening angle to slot pitch ($\alpha_{so}$) is taken as the varying parameter (Figure 3.15(a)), which is known to be a major factor in pulsating torque [15,16]. The mean torque of the reference motor falls almost linearly from 9 Nm to 5 Nm as $\alpha_{so}$ varies from 10% to 70%. This is because the Carter adjusted airgap increases with wider slot opening. The ratio of harmonics of $T_{cog}$, $T_{mutt}$ and $T_{rel}$ to the mean base torque at each geometry point is plotted in Figure 3.15(b), to have a fair comparison. Cogging torque is major contributor over most region. It is clearly seen that normalized cogging torque harmonic, $T_{cog}^{18}$ peaks around 45% of
\( \alpha_{so} \), whereas the mutual and reluctance torque harmonics (\( T_{mutl}^{18}, T_{rel}^{18} \)) encounter minima at 50\% of \( \alpha_{so} \). The phase plots also show dissimilar trends. Of particular interest are the crossover points of magnitude curves of \( T_{mutl}^{18} \) and \( T_{rel}^{18} \) (encircled). At \( \alpha_{so} = 60\% \), the phase values of \( T_{mutl}^{18} \) and \( T_{rel}^{18} \) are apart by 122\(^0\), which results in partial cancellation. At \( \alpha_{so} = 20\% \), the difference is 90\(^0\) and, hence, there is no cancellation but an out-of-phase addition. Indeed, the minima in overall pulsating torque was observed at \( \alpha_{so} \) of 58\%, and the maxima at 40\%.

### 3.6.2 Variation of Pole arc (18-slot/4-pole IPM)

The pole arc (\( \theta_m \)) of the IPM is varied (Figure 3.8(a)), keeping the other geometric parameters constant. The mean reference torque increases from 5.2 Nm to 8.4 Nm from \( \theta_m \) value of 30\(^0\) to 70\(^0\). Interestingly, the normalized torque component harmonics - \( T_{cog}^{18}, T_{mutl}^{18} \) and \( T_{rel}^{18} \) exhibit a periodic behavior (Figure 3.16). This was shown analytically for single flux barrier synchronous reluctance motors in [38]. All of the curves have four cycles in the studied \( \theta_m \) range, but the maxima and minima of each curve do not coincide, pointing to the different

![Figure 3.16 Variation of 18th torque harmonics with pole arc](image)

61
MMF interactions in the no-load rotor, cavity rotor and current sheet rotor. The phase spectrum is also dissimilar. Possible high ripple and low ripple design points are individuated. At $\theta_m = 55^0$, $T_{18}^{cog}$ and $T_{18}^{rel}$ are near the maxima, $T_{18}^{mut}$ is near its minima and their phases are within $90^0$. At $\theta_m = 60^0$, $T_{18}^{cog}$ and $T_{18}^{rel}$ are near their minima, $T_{18}^{mut}$ is near its maxima, and the phases span $180^0$. This suggests and also it is observed that overall pulsating torque has a maximum of 32% at $\theta_m = 54^0$, and minimum of 9.8% at $\theta_m = 59^0$.

3.6.3 Variation of Pole offset in asymmetric rotor designs (24-slot/4-pole IPM)

A technique in low torque ripple designs of IPM is to offset the magnet cavity from the pole centerline. This reduces the interaction between the stator MMF and rotor MMF harmonics [39]. This beneficial effect cannot be observed in the 18-slot/4-pole configuration, hence this study is conducted on the 24-slot/4-pole winding. Figure 3.17 shows that $T_{12}^{cog}, T_{12}^{rel}$ & $T_{12}^{mut}$ decrease monotonically as pole offset angle, $\theta_{off}$ is varied from $0^0$ to $3.5^0$. In the same range, the mean torque falls from 9.32 Nm to 8.9 Nm, which is a small tradeoff in

![Figure 3.17 Variation of 12th torque harmonics with pole offset angle](image-url)
comparison to the improvement in torque ripple. The phases of $T_{cag}^{12}$, $T_{rel}^{12}$ & $T_{mutt}^{12}$ are within $90^\circ$ of each other, hence there is no cancellation between them. There are no extremum points in this sensitivity analysis in comparison to slot opening coefficient and pole arc analyses.

3.7 Conclusion

In this chapter, a novel approach with synthesized FEA geometries is proposed to segregate torque components in IPMs and its utility in motor design is elaborated. Whereas the torque separation by FP is FEA-empirical based, the TSSR technique gives physics based relationships with the synthesized torques. In addition, it consumes much less time in comparison with the FP method. The validation is conducted on 18-slot/4-pole IPMs and a 24-slot/4-pole IPM over a wide operating current range. This method can be extended to concentrated winding machines and rotors with multiple magnet layers.

This has dual benefits in the design stage: Facilitating tradeoff studies with magnet flux linkage and saliency ratio; and exploring opportunities to cancel a significant harmonic of cogging torque, reluctance torque ripple or mutual torque ripple. The ripple sensitivity analyses is studied for variation of slot opening coefficient, pole arc and pole offset angle. The motor designer can take aid of analyses in Sections 3.5 and 3.6 to co-ordinate IPM designs satisfying mean torque and ripple torque requirements. The FEA of the three machines can be run in few hours for hundreds of designs and post-processed with simple routines to generate such performance curves.
Chapter 4. SyRM & IPM design, analysis, prototyping and testbed

4.1 Modified winding function for multi-layer windings

4.2 Optimization of SyRM & IPM prototype designs
   4.2.1 Design requirements and choices
   4.2.2 Stator winding design with modified winding function
   4.2.3 Rotor design with Global Response Search Method

4.3 Construction of prototypes
   4.3.1 Stator rewinding
   4.3.2 Rotor manufacturing – SyRM & IPM

4.4 Dynamometer testbed

4.5 Field oriented control with DSP

4.6 Conclusion
This chapter covers the analysis and design of Synchronous Reluctance Motor (SyRM) and Interior PM motor (IPM) prototypes and the complete test setup built to validate the theory. The purpose of the hardware is to conduct experimental testing for inductance, power factor, efficiency and torque-speed operating envelope. Furthermore, it is used for experimental torque component separation, as a parallel to the FEA method in Chapter 3 [40].

A modified winding function with an adjusted pitch factor is derived that is applicable to multi-layer windings. It is illustrated for a sample slot-pole combination, and then applied for the prototype winding design. The rotor lamination geometry for the Synchronous reluctance motor (SyRM) and IPM is obtained by coupling electromagnetic FEA with an optimization software that runs Global Response Search Method (GRSM). The construction of the prototypes involving the stator winding, rotor stack, magnets, frame and bearings is described.

The driving motor (DM) and its drive were assembled and tuned to extract reliable constant speed results from the prototypes. The motor under test (MUT) (terminology for the prototype being tested) is operated with field-oriented control (FOC) with an off-the-shelf inverter kit and DSP card.

4.1 Modified winding function for multi-layer windings

A new multi-layer (ML) distributed winding configuration was presented in [41], after considering pros and cons of the fully-pitched distributed, concentrated and fractional pitch overlapped windings. The ML winding has advantages of highly sinusoidal stator MMF, in comparison to distributed winding, and shorter end-turn lengths in comparison to fractional pitch ones. The ML winding consists of variable slot pitch coils starting from the concentrated
A 3-phase ML winding has 3 layers, and each slot houses coil sides from all of the phases. Figure 4.1 shows the ML layout for one phase in a 36-slot/4-pole stator. Each phase winding occupies all stator slots in varying number of turns, \( N_{\text{turn}} \). Because of the unconventional structure, the winding function theory [35] with standard distribution and pitch factors is not applicable here. Hence a modified winding function is derived in this section for the analysis of ML windings.

In the ML winding of Figure 4.1, a particular coil \( -x \) in a phase winding group is designated by \( C_x \) with its number of turns being defined as \( N_x \). The harmonic characteristics
of the stator MMF $\hat{F}_{st}^v$ is defined by its winding factor $k_{wv}$ (here, $v$ is harmonic number) which is the product of its distribution factor $k_{dv}$ and pitch factor $k_{pv}$ and expressed as

$$k_{wv} = k_{dv} \times k_{pv} \quad (4.1a)$$

$$\hat{F}_{st}^v = \frac{3}{\pi v} \cdot \frac{N_{ph}}{p} \cdot k_{wv} \cdot \hat{i}_s \quad (4.1b)$$

Here, $N_{ph}$ is the total number turns/phase, $p$ is the number of pole pairs, and $\hat{i}_s$ is the peak value of phase current. The peak of harmonic MMF due to coil $C_x$ is referred to as $\hat{F}_{C_x}^v$ and these MMFs are superimposed in the airgap. For the ML winding, MMFs from all coil sets are aligned on the $\theta$-axis, making $k_{dv} = 1$ for each harmonic and its pitch factor, $k_{pv}$ needs to be accounted for the winding factor calculation. This coil distribution of Figure 4.1 results in 9 coil sets, $C_1$ to $C_9$, centered on its peaks. Winding pitch to pole ratio, $W/\tau_p$ is $(2x - 1)/9$ for $C_x$ ($x = 1, 2, \ldots, 9$). Hence MMF due to each coil set is given by

$$\hat{F}_{C_x}^v = \frac{2}{\pi v} \sin \left( v \cdot \frac{2x-1}{9} \cdot \frac{\pi}{2} \right) \cdot N_x \cdot \hat{i}_s \quad (4.2)$$

The harmonic stator MMF of each phase, $\hat{F}_{ph}^v$ is obtained by adding the MMF due to each coil set, $\hat{F}_{C_x}^v$,

$$\hat{F}_{ph}^v = \sum_x \hat{F}_{C_x}^v = \frac{2}{\pi v} \cdot \hat{i}_s \sum_x \sin \left( v \cdot \frac{2x-1}{9} \cdot \frac{\pi}{2} \right) \cdot N_x \quad (4.3a)$$

$$\hat{F}_{ph}^v = \frac{2}{\pi v} \cdot \frac{N_{ph}}{p} \cdot \hat{i}_s \sum_x \sin \left( v \cdot \frac{2x-1}{9} \cdot \frac{\pi}{2} \right) \cdot \frac{N_x - p}{N_{ph}} \quad (4.3b)$$

$$\hat{F}_{ph}^v = \frac{2}{\pi v} \cdot \frac{N_{ph}}{p} \cdot k_{pv} \cdot \hat{i}_s \quad (4.3c)$$

$$N_{ph} = p \cdot \sum_x N_x \quad (4.3d)$$

The modified pitch function, $k_{pv}$ is obtained as the weighted sum of the MMFs of individual coil sets by their corresponding number of turns, $N_x$. Its formulation is as follows,

$$k_{pv} = \frac{p}{N_{ph}} \sum_x N_x \cdot \sin \left( v \cdot \frac{2x-1}{\tau_p} \cdot \frac{\pi}{2} \right) \quad (4.4)$$
Single phase winding excited with a sinusoidal current produces a set of harmonic standing MMF waves with peaks $\{\tilde{F}_{ph}^v\}$, while three-phase windings excited with balanced three-phase currents produces a set of harmonic travelling MMF waves, $\{\tilde{F}_{st}^v\}$. The superposition of the three phases also introduces a scaling factor of $3/2$.

The peak of harmonic waves generated by three phases $\{\tilde{F}_{st}^v\}$ is represented as follows,

$$\tilde{F}_{st}^v = \frac{3}{\pi v} \frac{N_{ph}}{p} k_{pv} \hat{I}_s$$  \hspace{1cm} (4.5)

![Figure 4.2 Stator MMF due to one-phase in the 36-slot/4-pole multi-layer winding](image)
Stator MMF is calculated analytically for the 36-slot/4-pole multilayer winding distribution and its harmonics are calculated with the above equations. In addition, 2D finite element analysis (FEA) model of the ML winding is developed with slotless stator and solid round rotor to consider only the effect of stator winding excluding slot harmonics. Stator MMF harmonic characteristics for both analytical and FEA based models are presented in Figure 4.2. Both the MMF waveform of one phase in space and its Fourier spectrum are depicted; the analytical and FEA show good correlation. The derived net stator MMF is near sinusoidal as the triplen harmonics \( \nu = 3n, n \) is an integer in Figure 4.2(b) will also be absent under balanced three phase excitation.

4.2 Optimization of SyRM & IPM prototype designs

For illustration of torque component separation, two prototypes were built with the same lamination geometry – (i) SyRM prototype – without magnet; (ii) IPM prototype – with NdFeB magnets. The idea is that the SyRM will give reluctance torque only (Cavity Rotor concept in Section 3.3); the IPM will give the sum of cogging torque, reluctance torque and mutual torque. The starting point for the prototype was an off-the-shelf servo motor rated at 5 kW. Several design choices were considered and a targeted rigorous optimization was carried out in a commercial software. Both the SyRM & IPM are required to satisfy certain objectives.

4.2.1 Designs requirements and choices

An off-the-shelf servo motor was dismantled and the stator, rotor, endplates, frame and shaft were separated (Figure 4.3). The idea was to re-use all of these parts except the rotor, to save engineering effort and cost. Hence the prototype geometry was constrained to a great extent. The significant parameters such as air gap radius, stack length and slot-pole
combination were fixed as shown in Table 4.1. The winding configuration is 18-slot/4-pole double layer type. The lamination material is M19 steel with 29 gauge. The magnet is NdFeB – 38UH.

Starting with drive and motor requirements, hand calculations are used to fix the rated quantities: Power ($P_r$), corner speed ($\omega_r$), voltage ($V_r$), rated current ($I_r$) and rated torque ($T_r$) [4]. A reasonable initial requirement for the IPM design at the corner point:

$$P_r = 1.5 \, kW \, \text{ at } \omega_r = 1800 \, \text{rpm} \rightarrow T_r = 8 \, Nm$$

(4.6)

The drive inverter is assumed to have a DC bus voltage of 400 V. With space vector PWM, the available line-to-neutral voltage ($V_r$) is 127.7 Vrms. Assuming an efficiency of 90% and power factor ($\cos \theta$) of 0.85, the rated current is:

$$3.V_r I_r \cdot \cos \theta \cdot \eta = P_r \rightarrow I_r = 4 \, Arms$$

(4.7)

The coil number of turns ($N_{coil}$) is tuned next. The slot current density ($J_{slot}$) is limited to 6.8 Arms/mm$^2$. Double layer windings that are machine wound typically have low fill factor ($FF$) around 0.4.

$$J_{slot} = \frac{2N_{coil} I_r}{A_{slot \cdot FF}} \rightarrow N_{coil} = 32$$

(4.8)
Moving onto the rotor side of the SyRM/IPM, three options were considered for the cavity shapes: I-type, U-type and V-type (Figure 4.4). An additional target is set – to maximize the torque of the SyRM prototype. This is difficult as it has only one layer. Several FEA simulations showed that there is not much difference between torques generated by the three

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>No. of poles (2p)</td>
<td>4</td>
</tr>
<tr>
<td>No. of slots (Qs)</td>
<td>18</td>
</tr>
<tr>
<td>Rotor outer radius (Rro)</td>
<td>26.65 mm</td>
</tr>
<tr>
<td>Stator Inner radius (Rsi)</td>
<td>27.35 mm</td>
</tr>
<tr>
<td>Stator outer radius (Rso)</td>
<td>54 mm</td>
</tr>
<tr>
<td>Air gap thickness (g)</td>
<td>0.7 mm</td>
</tr>
<tr>
<td>Lamination stack length (lstk)</td>
<td>110 mm</td>
</tr>
<tr>
<td>Fill factor (FF)</td>
<td>0.4</td>
</tr>
<tr>
<td>Slot area (Aslot)</td>
<td>95.6 mm^2</td>
</tr>
</tbody>
</table>

Table 4.1 Geometry Constraints – Prototype SyRM & IPM

Figure 4.4 Considered sample rotors
configurations without magnet. Also, shaft radius was large limiting the space for U- and V-type rotors. Hence, I-type was finalized for further tuning.

### 4.2.2 Stator winding design with modified winding function

In the available 18-slot/4-pole stator, the slot/pole/phase is 1.5, which is a fraction. Hence single layer distributed winding is not possible. Three different double layer distributed winding configurations were considered, with different coil spans. The fundamental pitch factor, $k_{p1}$ is calculated with aid of equation (4.4). Higher the value of $k_{p1}$, higher is the mean torque for the same number of turns. The fundamental pitch factor is 0.831, 0.945 and 0.945 for windings with coil spans of 3, 4 and 5, respectively. It is also desirable to have a lower copper loss, that is a function of the end-winding overhang. A coil span of 5 will result in higher stator resistance than coil span of 4. Hence, we finalize a double layer winding configuration of coil span – 4, after taking the two factors into account. The resultant cross-

![Figure 4.5 18-slot/4-pole double layer winding (prototype stator)](image-url)
sectional view of the layout is depicted in Figure 4.5. Six from the 18 slots are shared between coil sides of different phases.

The stator MMF under a sample excitation of $i_q = 4\, Arms$ & $i_d = 0$ is depicted in Figure 4.6 over two pole-pitches. This is the result of all three phases, hence triplen harmonics are absent in the spectrum plot. The MMF is rich in harmonics, suggesting that the motor will

![Stator MMF](image)

(a) Space domain

![Fourier spectrum](image)

(b) Fourier spectrum

**Figure 4.6** Net stator MMF of the prototype winding, $i_q = 4\, Arms, i_d = 0$
exhibit high torque ripple. It is noted that high ripple is a peculiar requirement for our experimental testing purposes. The rotor MMF with its odd symmetry exhibits \((2n \pm 1)\) harmonics. So there is no interaction of the prominent stator MMF harmonics – 8th and 10th. Instead, the 17th and 19th harmonics of stator and rotor interact to generate high torque ripple of 18th order [19]. This was observed from FEA data in Chapter 3.

4.2.3 Rotor design with Global Response Search Method

A commercial software, Altair Hyperstudy coupled with electromagnetic FEA, FLUX2D is employed to optimize the I-type rotor design [42]. We intend to maximize the mean torque and torque ripple of the SyRM prototype. This is because higher ripple would aid in its experimental measurement, overcoming noise issues. The procedure consists of two main steps: Design of experiments (DOE) and optimization with Global Response Search Method (GRSM) [43].

In DOE stage, the sensitivity of mean torque \((T_{mean})\) and ripple \((T_{rpt})\) to the geometry variables and exciting current vector – \((I_r = 4\, Arms, \gamma)\) are assessed. It generates Pareto plots from which the user can choose a subset of variables for the optimization study. Figure 4.7 shows evaluation of 50 FEA models, from which the software learns. Wide swings in \(T_{mean}\) and \(T_{rpt}\) are observable.

![Figure 4.7 Evaluations of SyRM in DOE stage - Hyperstudy](image)

(a) Mean torque  
(b) Ripple torque

**Figure 4.7** Evaluations of SyRM in DOE stage - Hyperstudy
Mono-objective optimization with constraints is executed in Hyperstudy. The pole arc ($\theta_m$), span of bridge tip ($\theta_{brg}$) and magnet height ($h_m$) are selected for the optimization at the exciting current vector of ($I_r = 4\, Arms, \gamma = 45^0$). The constraints on geometry are:

\[
30^0 < \theta_m < 75^0 \ ; \ 2^0 < \theta_{brg} < 10^0; \\
\theta_m + 2.\theta_{brg} < 78^0; \ 2 < h_{mag} < 5.5
\]

(4.9)

The torque ripple has a minimum constraint and the objective is to maximize mean torque:

\[
T_{rpl} > 20 \% \ ; \quad \text{Maximize} \ (T_{mean})
\]

(4.10)

Figure 4.8 shows the evolution of the geometry over 25 iterations of the FEA model. Figure 4.9 shows the corresponding evolution of the torque characteristics. The settled value for the SyRM predicted by Hyperstudy is:

\[
T_{mean} = 2.1\, Nm \ ; \ T_{rpl} = 20.4 \%
\]

(4.11)
The final geometry is listed in Table 4.2. Inserting NdFeB magnets into the SyRM results in the IPM prototype. The torque values for the IPM are:

\[ T_{\text{mean}} = 8.28 \, Nm \; ; \; T_{\text{rpt}} = 21.7 \% \]  \hspace{1cm} (4.12)

4.3 Construction of prototypes

The stator stack, endplates, bearings and shaft of a standard frame servo motor were reused in the construction of the prototypes. Only the stator winding and rotor lamination stack were replaced following the metrics in Section 4.2. Usually the stator stack is press-fitted onto the housing.

4.3.1 Stator rewinding

The existing PM rotor stack was pulled out and separated from the stator. The stator was torched at a high temperature to loosen the existing old coils; then they were cut open and

![Figure 4.9](image-url)
disposed off. The new coil winding layout is depicted in Figure 4.10. In the cross-sectional view, it can be seen that each slot carries two coil sides, that is the definition of double layer winding. For each phase, six coils with 32 turns of 20.5 AWG Inverter grade wire, are wound in a winding tool. Each coil has a coil throw of 4 slot pitches; maintaining equal coil throws reduces complexity and tooling effort.

The stator slot is lined with thin insulation paper to prevent short circuit faults [44]. Two coil sides in each slot are also separated with the same insulation paper. The six coils are laid out in the slots, in a lap winding fashion as shown in Figure 4.10; this is a smart concept to avoid overlapping between end-windings and to have a symmetric layout. Figure 4.11 shows

Figure 4.10  Lap winding layout for the prototype stator
the two ends of the stator after the layout is completed. Three of the six terminals of phases $A, B$ and $C$, are soldered together to have a star configuration; the other three terminals are brought out of the motor frame. The end-windings are neatly tied at the end and then the whole setup is cured by dipping in hot resin (Figure 4.12).

4.3.2 Rotor manufacturing – SyRM & IPM

The bearing is pressed out from one end of the shaft. The existing rotor laminations are pulled out from that end. The rotor geometry from Section 4.2.3 is used to obtain laser cut laminations of M19 29 gauge Silicon Steel. The new laminations are stacked onto a support
structure, bonded together and then heated to get a rigid stack [45]. The shaft is press fitted onto the rotor stack replacing the support structure. The rotor assembly is inserted into the stator maintaining air-gap thickness and avoiding eccentricity. This gives the SyRM prototype.

Tests were conducted with the SyRM; magnets inserted and then tests for the IPM are conducted. Two rectangular NdFeB magnets were used per pole of the 110 mm stack. The rotor is unskewed; so two magnets are sufficient. After inserting 8 magnets into the 4 cavities, the rotor is warmed up to $100^\circ C$ and Loctite is poured into the cavity cracks; this ensures the magnets don’t displace with high speed rotation. Figure 4.13 shows the finished rotor. Next, it is inserted carefully into the stator with an assembly tool, as the PM rotor tends to stick to a side of the stator.

4.4 Dynamo meter testbed

The conventional method of testing motors is to load them with a VFD fed machine capable of both motoring and generating [46]. A 5 HP dynamometer testbed with a maximum speed of 5000 rpm and maximum torque of 10 Nm was built in-house for experimenting with the prototypes. Sufficient engineering care was taken to ensure match between the various components for adequate accuracy of test results.
Figure 4.14 Elements of the dynamometer testbed

(a) MUT, torque transducer & DM – Mechanically coupled

(b) Drive & resistor bank for DM

(c) TI Inverter kit & DC supply

(d) Oscilloscope, power analyzer & comput
The driving motor (DM) is from ABB – BSM90C-375AF, which is a 5 HP servo motor with accurate speed and position control (Figure 4.14(a)). The DM is controlled in speed mode or torque mode by a Baldor/ABB commercial drive – Motiflex e100 [47]. It is supplied by 480 V 3-phase supply. It has an uncontrolled front-end diode rectifier, a DC bus and bidirectional inverter; thus it is a 2 quadrant 10 HP drive. A 100 Ω, 8 A resistor bank is connected to internal DC bus of the drive, to allow sinking maximum generated power of 5 HP (Figure 4.14(b)).

The motor under test (MUT) is driven by a high voltage digital motor control kit by Texas Instruments – TMDSHVMTRPFCKIT, that has an inverter supplying 1 kW motor load. The switching signals and controls comes from TI fixed point DSP card – F28035 that is mated onto the main board [48]. The flash on the card is programmed from a computer through a USB/JTAG interface. The kit has a digital input interface for the encoder signals and 4 digital-to-analog (DAC) outputs to probe internal signals. The power is output from the board via three phase terminals. The input power used for these experiments is provided by connecting the DC bus to a programmable TDK Lambda DC supply (500 V, 20 A) (Figure 4.14(c)).

From Figure 4.14(a), the setup has two rubber couplings connecting the MUT, the torque transducer and the DM. The baseplates for mounting the MUT and shaft adapters were fabricated at the NCSU Precision Machine Shop. An incremental quadrature encoder from Timken (THS 25) provides 2560 lines per mechanical revolution of the shaft; which is essential feedback for the vector control. The time-domain torque profile is collected by a high precision torque transducer by Interface (T25-20-F3A), with a sampling rate of 2.5 kHz and resolution of 0.01 Nm.
The line-to-line armature voltage is measured with isolated differential voltage probes; phase currents are probed with clamp type current transducers. They are observed in the Tektronix oscilloscope on the testbench (Figure 4.14(d)) for debugging and recording purposes. A Yokogawa WT3000 power analyzer is connected in series between the inverter kit and the motor, to measure input power and power factor. A laptop computer is used to control the TI DSP card, the ABB DM drive, and collect torque data.

4.5 Field oriented control with DSP card

TI’s fixed point DSP – F28035 provides the programmability and computing power necessary for implementing the field oriented control (FOC) of a synchronous machine – both SyRM & IPM [49]. Figure 4.15 describes the complete system – the blocks within the big dotted rectangle are internal to the DSP logic, and referred to as macros. The other blocks are the peripheral signal-level hardware and power-level inverter and motor. The relevant peripherals on the TI board consist of the gate driver IC, current sensor, Analog-to-Digital (ADC) converter and Quadrature Encoder Processor (QEP).

The software process consists of 4 build levels, whereby the functioning of each macro in Figure 4.14 is verified and debugged if necessary. The C code is written and modified with the Code Composer Studio interface by TI. The current controller is in the rotor aligned $d$-$q$ frame. It consists of two Proportional-Integral (PI) elements with an output saturation limit. The $k_p$ and $k_i$ values have to be tuned for optimum response. The reference values – $(i_d^*, i_q^*)$ are provided by the user and feedback obtained from on-board current sensors. The Park’s transform and inverse transform blocks receive calibrated rotor position from the encoder through the QEP macro. The space vector PWM macro generates duty ratios for phases – $A, B, C$
based on the reference $\alpha - \beta$ voltages. This is used by the gate driver circuit to generate switching pulses for the 6 power MOSFETs in the inverter.

### 4.6 Conclusion

This chapter discussed in detail the IPM & SyRM analysis, prototyping efforts, dynamometer test-bed assembly and the digital vector control methodology. Two 18-slot/4-pole prototypes with I-type rotor – a SyRM and an IPM, are designed to test for saturated inductance and pulsating torque decomposition. A modified winding function is derived for unconventional multi-layer windings and validated with FEA. It is used to design the prototype double layer winding by maximizing fundamental pitch factor and reducing copper loss due to end-winding. A commercial optimization software piloting electromagnetic FEA is used to arrive at rotor lamination geometry maximizing mean torque and torque ripple.
The double layer lap winding with a simple coil layout scheme is wound on an available stator stack. The rotor lamination is manufactured, with and without rare-earth magnets for the two stages of testing. A dyno testbench with test motor drive, load motor drive and data acquisition for position, torque, and electrical quantities is built in-house from scratch. Sensored vector control of the SyRM and IPM is implemented on hardware with a DSP. A step-by-step description is provided of the engineering decisions and invested efforts to get a fully functional test platform. One can appreciate the benefits and limitations of this setup by an analysis of the test results in Chapter 5.
Chapter 5. Experimental Validation

5.1 Experiments on SyRM prototype
   5.1.1 \textit{d-} & \textit{q-} axes inductance tests
   5.1.2 Mean torque & torque ripple maps
   5.1.3 Torque – speed operating envelope
   5.1.4 Mechanical loss of the dyno setup

5.2 Experiments on IPM prototype
   5.2.1 No-load test
   5.2.2 Mean torque & torque ripple maps
   5.2.3 Thermal test

5.3 Experimental separation of torque components
   5.3.1 Results for mean torque components
   5.3.2 Hybrid method: Alignment of torque ripple data from SyRM & IPM
   5.3.3 Results for pulsating torque components

5.4 Conclusion
This chapter covers the comprehensive testing of the SyRM and IPM prototypes on the assembled dynamometer testbed. The purpose is twofold: to verify the inductance modeling [30] presented in Chapter 2; and to experimentally separate the mutual torque, reluctance torque & cogging torque [40] as presented in Chapter 3. The methods used to obtain the torque, power factor, efficiency and torque ripple are described. The practices followed to minimize the measurement errors are also detailed here.

The $d$- and $q$-axes inductances for SyRM are extracted and compared with FEA model. Within the constraints of voltage rating and current rating of the motor drive, the torque speed envelope is obtained for SyRM. The mechanical losses of dyno systems was obtained by running the SyRM MUT with no loading.

On the testing of IPM prototype, no-load test was conducted to get the Back EMF constant and cogging torque. The mean torque and torque ripple maps of the IPM was obtained in the $(i_d, i_q)$ plane. Then, thermal test was conducted to measure temperature rise in the frame and end-winding regions.

Finally, a hybrid torque decomposition method utilizing experimental and FEA data is implemented. Discussion of the results and comparisons with FEA models provides interesting insights into the design and analysis of SyRMs and IPMs.

5.1 Experiments on SyRM prototype

The SyRM prototype has a 18-slot/4-pole double layer distributed winding with a single cavity 4-pole rotor. It is referred to as Motor Under Test (MUT) in this section. It serves the purpose of extracting the reluctance torque component – with mean and harmonics, and
obtaining inductances with an unsaturated core. Complete details of design and manufacturing of the MUT are provided in Sections 4.1 and 4.2.

5.1.1 d- & q-axes inductance tests

Experimental testing of d- and q-axes inductances ($L_d$ & $L_q$) is important for machine modeling, drive controls and validating existing FEA models. Several methods are available in literature for IPM motors such as AC standstill (locked rotor) method [50], open circuit test and short circuit test, which fall under the IEEE Standard 1812 [51]. A small modification to the expressions makes them amenable to SyRM inductance testing. In the setup at hand, a vector controlled drive system is available, wherein defined ($i_d, i_q$) currents can be supplied to the motor. So, we can take advantage of it following [50].

The SyRM motor model in synchronous d-q frame is reproduced here (from Sec 1.2.1) under steady state conditions:

$$v_d = i_d R_s - \omega_e \lambda_q = i_d R_s - \omega_e L_q i_q \tag{5.1a}$$
$$v_q = i_q R_s + \omega_e \lambda_d = i_q R_s + \omega_e L_d i_d \tag{5.1b}$$

For d-axis inductance testing, we assert the current vector as follows:

$$i_d = I_0, i_q = 0 \tag{5.2a}$$

(a) $L_d$ testing

(b) $L_d$ testing

(c) $L_q$ testing

**Figure 5.1 d-q plane vector diagram**
This results in isolating the $L_d$ as:

$$L_d = \frac{v_q}{\omega_e i_d} \quad (5.2b)$$

Similarly, q-axis inductance is obtained as follows:

$$i_q = I_0 \quad , \quad i_d = 0 \quad (5.3a)$$

$$L_q = \frac{v_d}{\omega_e i_q} \quad (5.3b)$$

Figure 5.1 clearly depicts the current vector, flux linkage vector and voltage vector in the d-q plane, for the two scenarios.

**Figure 5.2** Scope images in q-axis inductance calculation steps
The vector controlled MUT is run with DC bus voltage of 160 V and speed of 600 rpm. The implementation of this theory is enabled by collecting data from two isolated voltage probes to measure line-to-line voltages - $v_{ab}, v_{bc}$, and the $dq$-aligned rotor position ($\theta_{rot}$) from DSP board. This data is post-processed with MATLAB routines to get the fundamental line-to-neutral voltages - $v_{an}, v_{bn} & v_{cn}$:

\[
v_{ab} + v_{bc} + v_{ca} = 0 \quad (5.4a)
\]

\[
v_{an} = \frac{2}{3} v_{ab} + \frac{1}{3} v_{bc} \quad (5.4b)
\]

\[
v_{bn} = \frac{2}{3} v_{bc} + \frac{1}{3} v_{ca} \quad (5.4c)
\]

\[
v_{cn} = \frac{2}{3} v_{ca} + \frac{1}{3} v_{ab} \quad (5.4d)
\]

Figure 5.2(a) shows the raw line-to-line voltages with the PWM harmonics and the filtered fundamental. Figure 5.2(b) shows the time aligned waveforms of line-to-neutral voltages and rotor position. Now, the task is to obtain $V_d$ and $V_q$ which are constants with time if $v_{an}, v_{bn}, v_{cn}$ are purely sinusoidal.

For the case of d-axis inductance testing, the $q$-axis is aligned with $a$-axis when $\theta_{rot} = 0$. Hence,

\[V_q = v_{an} \quad \text{when } \theta_{rot} = 0 \quad (5.5)\]
For the case of q-axis inductance testing, the \( d \)-axis is aligned with the \( as \)-axis when \( \theta_{rot} = \frac{\pi}{2} \). Hence,

\[
V_d = v_{an} \quad \text{when} \quad \theta_{rot} = \frac{\pi}{2}
\]  

(5.6)

The analytical modeling of saturated q-axis inductance was covered in Chapter 2. It was found to be in good correlation with the FEA model. Here, the fidelity of the FEA model against the experimental SyRM prototype is verified. So, the above procedure is repeated for

![Figure 5.4 Final experimental results – SyRM – compared with FEA](image)

(a) \( d \)-axis inductance

(b) \( q \)-axis inductance

**Figure 5.4** Final experimental results – SyRM – compared with FEA
a range of current magnitudes, as we want to observe the saturating behavior of $d$- and $q$-axis inductances. Figure 5.3 shows the scope waveforms of $v_{ab}, v_{bc}, \theta_{rot}$ and phase-$A$ current under the 25% overload condition.

The final results are plotted in Figure 5.4. The $L_d$ from FEA follows the experimental value, closely. It falls from 65 mH to 32 mH in an almost exponential manner. This is due to saturation in the rotor bridges. In comparison, the experimental $L_q$ is flat around 85 mH till $i_q = 0.75 \ pu$, and then decreases linearly to 63 mH at $i_q = 1.25 \ pu$. This is due to saturation in the stator teeth and parts of rotor core. This was modeled in detail in Chapter 2. The FEA compares well with experimental $L_q$ with a maximum error of 6.25% under unsaturated case.

5.1.2 Mean torque & torque ripple maps

The torque capability and ripple profile of the SyRM MUT is evaluated on the dyno testbed. A high precision torque transducer from Interface – T25-20-F3A measures the shaft torque transmitted between the MUT and DM. A contour map of mean torque - $T_{em}$ as function of the $d$- and $q$-axes currents - $(i_d, i_q)$ gives a clear picture of the optimum operating regions of the drive system [46]. These include maximum torque per ampere (MTPA), maximum ampere (MA) and maximum torque per volt (MTPV) trajectories.

The mean torque test was conducted with the DM maintaining a low speed of 100 rpm and MUT under current control. The DC bus voltage was held at 80 V. The $(i_d, i_q)$ combination was swept over a $10 \times 10$ matrix as follows:

\[
\begin{align*}
    i_d &= (-0.5 : -0.5 : -5) \ A \\
    i_q &= (0.5 : 0.5 : 5) \ A
\end{align*}
\]
Figure 5.5 shows the torque map results. The iso-torque lines are hyperbolic in nature, which is expected from the SyRM equation:

$$\frac{3p}{2} (L_q - L_d)i_d i_q = \text{constant} \quad (5.8)$$

Torque values ranging from 0.3 Nm to 2.7 Nm have been observed. To evaluate fidelity of the FEA model, few torque points on the $i_d - i_q$ plane are obtained from the simulation models. The red markers in Figure 5.5 denote values from FEA; they show good match with experiment.

The torque ripple measured from the SyRM prototype is equal to the reluctance torque ripple component ($T_{rel}$) of the corresponding IPM. Thus, its measurement in this step helps to evaluate the torque separation mechanism proposed in Chapter 3. The torque profile as function of time was captured under varying operating currents – ($i_d, i_q$) over an 8 X 8 matrix:

$$i_d = (-0.6 : -0.6 : -4.8) \, A \quad (5.9a)$$
$$i_q = (0.6 : 0.6 : 4.8) \, A \quad (5.9b)$$
Experimental measurement of ripple is challenging, as the torque transducer provides sum of torques from the MUT, the DM and the mechanically coupled system [52]. The bandwidth and accuracy of the transducer also matters, in relation to the shaft speed and maximum torque harmonic to be measured. Hence, industrial test rigs are equipped with a gear box to filter DM harmonics and to maintain a low speed of 5 – 10 rpm [53]. The setup in this

Figure 5.6 Contour map of torque ripple (Nm) in $(i_d, i_q)$ plane - (SyRM at 50 rpm)
work consists of only the conventional dyno testbed. Hence, several trial runs and innovations in raw data capture and processing were needed. For instance, the DM drive was able to maintain a stable shaft speed of 50 rpm, only. The recorded torque exhibited a large oscillation at the mechanical frequency. The MUT’s torque was filtered from the net recorded torque, by summing the $12n$ mechanical frequencies only. These are essentially $6n$ electrical frequencies, as MUT has two pole-pairs.

Figure 5.6(a) depicts the peak-to-peak torque ripple map in the $(i_d, i_q)$ - plane following the processing of raw data. Minimum value of 0.05 Nm and maximum value of 0.3 Nm are observed. Incidentally, the low ripple regions overlap with the MTPA trajectory ($\gamma \sim 40^0 - 50^0$). This is good, as usually those are the low speed operating points of the drive. Higher ripple is encountered in the MTPV trajectory. But operating in deep flux weakening regions ($\gamma > 70^0$) gives a very low mean torque, hence is avoided. Comparison of experiment with sample points from FEA (red markers) shows acceptable errors. The maximum error point is attributed to measurement limitations and inaccurate end-winding parameters in the FEA model. One possible reason is that the mechanical system’s transfer function subdues higher torque harmonics at 50 rpm.

From a Fourier analysis of $T_{rel}(\theta)$ over one electrical cycle, it was observed that the significant harmonics are $18^{th}$, $6^{th}$ and $12^{th}$ in that order. So, the $18^{th}$ component ($T_{rel}^{18}$) is singled out and mapped onto the $(i_d, i_q)$ plane in Figure 5.6(b). Pockets of high ripple are observed at high negative $i_d$ values. This approach of mapping a specific harmonic is useful to the motor designer, in that a targeted minimization can be attempted.
5.1.3 Torque – speed operating envelope

A synchronous reluctance motor (SyRM) drive has limits of operation in the torque-speed ($T - \omega$) plane [54, 55]. A standard constraint is the rated voltage and current rating of the drive. These are, in turn determined by maximum available DC bus voltage, inverter rating and motor rating. The theory is similar to that of an IPM drive, full details of which are provided in Section 1.2.2.

In this testing stage, the limits of the MUT are extracted through experiments. The DC bus voltage is maintained at its maximum, that is $300\, V$ (= $1\, pu$), and armature current is limited to $4\, Arms$ (= $1\, pu$). The outputs of the PI $d$-$q$ current regulators give voltage references - $v_d^*$ and $v_q^*$, which is supplied to the Space Vector PWM. The MUT-DM pair’s speed is stepped in increments from $300\, rpm$ to $4100\, rpm$. The maximum torque available without over-modulation of the SVPWM stage is recorded. The criteria are as follows:

\[
\sqrt{(i_d^*)^2 + (i_q^*)^2} \leq 1\, pu \tag{5.10a}
\]
\[
-0.7\, pu \leq v_d^* \leq 0.7\, pu \tag{5.10b}
\]
\[
-0.7\, pu \leq v_q^* \leq 0.7\, pu \tag{5.10c}
\]

The experimental plots are presented in Figure 5.7 with the exciting current, torque output, power output, efficiency & power factor for two speed ranges: $300\, rpm - 2500\, rpm$ (Range I) and $3100\, rpm - 4100\, rpm$ (Range II).

Range I consists of the Maximum Torque-per-Ampere (MTPA) operation till corner speed of $1500\, rpm$ followed by field weakening in the Maximum Ampere (MA) operation till $2500\, rpm$ [55]. The current angle is increased from $\gamma_{MTPA} = 50^0$ till $\gamma = 71^0$, to satisfy criteria in Equation (5.10). The electromagnetic torque is flat in MTPA and falls down in MA operation; correspondingly, the output power increases linearly then tends to flatten. A
comparison with FEA shows 2.8 % error in low speed region and 27.6 % error at 2500 rpm. This points to refinements needed in the FEA model. The efficiency increases with speed and reaches 70%; this is because the copper losses stay constant with speed whereas power keeps increasing. The iron losses are negligible for this SyRM, the mechanical losses increase, but are less dominant than the copper losses. The power factor falls off from 0.68 to 0.53 in MTPA and then flattens out in MA operation.

<table>
<thead>
<tr>
<th>Exciting current vector</th>
<th>Range I</th>
<th>Range II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque and power output</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Efficiency and power factor</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.7 Experimental plots of SyRM performance over operating envelope
Range II consists of the Maximum-Torque-per-Voltage (MTPV) operation from 3100 \( rpm \) – 4100 \( rpm \) [55]. Here, the current is below the rated value of 4 Arms, and decreases further with speed. The \( \gamma \) value varies weakly about 68\(^\circ\). Both the power and torque reduce with speed. Output power reaches minimum value of 275 \( W \). The FEA shows good

\[3000 \ A_r\] Rated current, rated armature voltage
\[4100 \ A_r\] below Rated current, rated armature voltage

\[3000 \, rpm\] 3000 \( rpm \), Rated current, rated armature voltage
\[4100 \, rpm\] 4100 \( rpm \), below Rated current, rated armature voltage

**Figure 5.8** Scope images : Line-line voltage, phase currents and \( d-q \) frame angle
prediction of the variation with maximum error of 19% at 4100 rpm. The power factor varies from 0.46 to 0.38. The efficiency increases dramatically, there is possibility of error in input power readings from the power analyzer at high frequencies. The motor drive is capable of higher speed, the MUT’s frame is rated for 5000 rpm and DM is rated for 7000 rpm. Yet, the mechanically coupled system exhibited large vibrations and noise at 4100 rpm, so it was not practically safe to continue further.

The phase currents - \( i_a, i_b \), line-to-line voltage - \( v_{ab} \) and the rotor position feedback - \( \theta_{dq} \) from TI DSP are captured on the oscilloscope. Figure 5.8(a) shows the waveforms at junction of MA & MTPV operation, that is 3000 rpm. Current frequency is 100 Hz and amplitude is 4 Arms. At the highest speed point – 4100 rpm, the current frequency is 136 Hz and amplitude is 2.7 Arms (Figure 5.8(b)).

**5.1.4 Mechanical loss of the dyno setup**

The dyno system consists of a chain of mechanical components in addition to the DM and MUT, resulting in sizeable friction and windage losses. These are assumed to be the same when the MUT is SyRM or IPM, as both have similar geometries. Generally, the mechanical losses, \( P_{mech} \) are a function of the shaft speed, \( \omega_{shaft} \).

The SyRM MUT is driven by current control from inverter kit. The DM is off, so that there is no load on the shaft. Power input, \( P_{in} \) to the MUT is measured with the power analyzer; phase current, \( I_{rms} \) is measured with current sensor. The torque transducer gives the shaft speed. The iron losses are negligible for the SyRM motor as it has low flux density. Only the copper losses is dissipated in the resistance of the coils, \( R_{ph} \). Thus, mechanical loss is obtained as:

\[
P_{mech} = P_{in} - 3I_{rms}^2 R_{ph} \tag{5.11}
\]
The data is collected in the range 200 rpm – 2500 rpm. Polynomial curve fitting is applied to the raw data (Figure 5.9). The quadratic term is negligible. The final expression is:

\[ P_{\text{mech}}(\omega) = 0.0106 \times \omega + 6.46 \text{ (in W)} \]  \hspace{1cm} (5.12)

5.2 Experiments on IPM prototype

In the second stage of testing, NdFeB magnets were inserted into the SyRM lamination to get the IPM motor. The IPM MUT is subjected to no-load test, loaded tests to get mean torque and torque ripple maps, and a temperature rise test to observed thermal design of the frame.  5.2.1 No-load test: Back EMF & cogging torque.

5.2.1 No-load test

The no-load test of PM motors helps in validating and retuning the FEA model. Also, the fidelity of the electric and torque measurements can be observed. To characterize cogging torque and back EMF, the IPM MUT is open circuited [43]. It is rotated at different speeds by the DM in the range 50 rpm – 1000 rpm. The line-to-line voltage is measured by isolated
voltage probes. This is generated due to the magnetized rotor, and does not include inductive elements. Figure 5.10 shows few cycles of the voltages, $V_{ab}$ & $V_{bc}$ at 700 rpm. Both are similar in magnitude and frequency is proportional to shaft speed. They have a pure sinusoidal shape devoid of any low order harmonics. This means the magnets are healthy and balanced in strength across the 4 poles.

![Figure 5.10 No-load test of IPM: Back EMF waveform on scope at 700 rpm](image)

![Figure 5.11 No-load test of IPM: Back EMF magnitude as function of shaft speed](image)
The back EMF constant, \( k_{BEMF} \), relates the open circuit voltage to the motor speed, \( \omega_{rpm} \): 

\[
V_{l}(rms) = k_{BEMF} \cdot \omega_{rpm} \quad (5.13)
\]

The raw data shows an almost linear variation with speed, as expected. From Figure 5.11, the value of \( k_{BEMF} \) is \( 0.118 \, \text{Vrms/rpm} \). The no-load magnet flux linkage is derived from this data to be \( 460 \, \text{mWb} \). This compares well with the initial FEA model that gives \( 526 \, \text{mWb} \).

5.2.2 Mean torque & torque ripple maps

Similar to Section 5.1.2, the torque characterization test was conducted with the DM maintaining a low speed of 50 rpm and MUT under current control. Care was taken to ensure the rotor magnets do not demagnetize at high negative \( d \)-axis currents. The \( (i_d, i_q) \) combination was swept over a \( 8 \times 8 \) matrix as follows:

\[
i_d = (-0.6 : -0.6 : -4.8) \, A \quad (5.14a)
\]
\[
i_q = (0.6 : 0.6 : 4.8) \, A \quad (5.14b)
\]

Figure 5.12 Contour map of mean torque (Nm) in \( (i_d, i_q) \) plane - (IPM at 50 rpm)
Figure 5.12 shows the torque map results. It is interesting to note that the iso-torque lines of the IPM are hyperbolic in nature, but there is a noticeable difference from the iso-torque lines of the SyRM (Figure 5.5). It is because the vertical asymptote for the IPM is on the positive $i_d$ axis at $\lambda_m/(L_q - L_d)$ as shown by the equation of an iso-torque line,

$$i_q[\lambda_m + (L_q - L_d)i_d] = \text{constant}$$  \hspace{1cm} (5.15)
As a result, the MTPA line for the IPM is closer to the $i_q$ axis compared to the SyRM, which is closer to the $i_q = -i_d$ line. Torque values ranging from 1 $Nm$ to 8.5 $Nm$ have been observed. Some aberrations in the contour map with 3 $Nm$ value are due to measurement errors.

Figure 5.13(a) depicts the peak-to-peak torque ripple map of IPM in the $(i_d, i_q)$ - plane following the processing of raw data. The data here includes all three components of torque ripple - $T_{cog}$, $T_{rel}$ and $T_{mutl}$. Minimum value of 0.2 $Nm$ and maximum value of 0.7 $Nm$ are observed. This high ripple is expected, as we intentionally designed a high ripple motor in Section 4.2. The low torque ripple section resembles a triangular region above the $T_{rpt} = 0.3$ $Nm$ iso-torque line. Similar to the SyRM contour map, highest ripples are observed in deep flux weakening regions. The 18$^{th}$ component ($T_{rpt}^{18}$) is mapped in the $(i_d, i_q)$ plane in Figure 5.13(b). Pockets of high ripple around 0.12 $Nm$ and low ripple around 0.08 $Nm$ are visible. In fact, more accurate torque ripple maps could be obtained with an advanced dyno setup and sensitive transducer.

5.2.3 Thermal test

The thermal behavior of rotor lamination, stator lamination, coils and frame is important test while characterizing high-performance motors [46]. A simple heat run test is conducted on the IPM MUT at half the rated current (2 $A$ rms) and maintained at 100 rpm speed by the DM. The frame is totally enclosed non-ventilated (TENV) type. J-type thermocouples are glued with thermal paste on the frame and end-winding portion. Keysight data acquisition unit 34972A is used to record data from the thermocouples.
Figure 5.14 shows that the thermal steady state is attained in approximately 130 mins. The steady state temperature of the frame is 39°C and that of the end winding is 48°C. The frame is cooler than the end-winding because the winding region is the heat source, whereas the frame is closest to the ambient. Of course, these values will differ as the armature current changes and as continuous duty operation is replaced with intermittent operation. The heat generation is primarily due to copper losses; iron losses are small for the prototype design. The

Figure 5.15 Hotspot measurement on IPM frame with thermal camera
hotspot location is inspected with an FLIR thermal camera. From the thermal image in Figure 5.15, the temperature distribution is fairly uniform on the finned frame. The maximum temperature point at steady state is 42.6°C.

5.3 Experimental separation of torque components

Low speed loaded test of the IPM MUT is conducted following same practices as in Section 5.1.2. The torque waveform captured by the torque transducer is selectively filtered to capture \(6n\) harmonics only. The separation of cogging torque, mutual torque and reluctance torque is done by a hybrid approach using experimental data for the magnitudes and FEA data for the relative phases.

5.3.1 Results for mean torque components

The average of the time domain torque waveform - \(T_{IPM}(t)\) over one electrical cycle is recorded under an exciting current vector - \((|I|, \gamma)\). Under the same conditions, torque data from SyRM prototype, \(T_{SYRM}(t)\) was also collected. This is purely reluctance torque, \(T_{rel}(t)\). The cogging torque has zero mean component. Hence net mean torque of the IPM consists of average reluctance torque, \(|T_{rel}|\) and average mutual torque, \(|T_{mut}|\).

So the mean mutual torque is obtained as follows,

\[
|T_{mut}| = |T_{IPM}| - |T_{SYRM}|
\]  
(5.16)

Figure 5.16 shows the decomposed mean components for fixed current magnitude of 4 Arms and varying current angle, \(\gamma\). The mutual or magnet torque is highest when current vector is along \(i_q\) axis and falls monotonically with \(\gamma\). On other hand, the reluctance torque is low at \(\gamma = 10^0 \& 80^0\) and has maxima at \(\gamma = 50^0\). This is as expected from the IPM torque equation.
The experiments on SyRM and IPM were conducted during different periods. There was no reliable way of aligning the torque waveform with the \(d-q\) currents. With this limitation in mind, a hybrid approach was used. Only the 18\(^{th}\) harmonic is considered. The magnitudes of this harmonic are obtained from the experiment - \(|T_{cog}^{18}|, |T_{SyRM}^{18}|, |T_{IPM}^{18}|\). The phase values are obtained from the FEA models of corresponding motors - \(\phi_{cog}^{18}, \phi_{SyRM}^{18}, \phi_{IPM}^{18}\). The torque waveform as function of rotor position (\(\omega t\)) is constructed from the magnitude and phase data.

\[
T_x^{18}(t) = |T_x^{18}| \cdot \cos(18\omega t + \phi_x^{18}) \quad ; \quad x = cog, SyRM, IPM
\]  

Next, the mutual torque is isolated as follows:

\[
T_{mut}^{18}(t) = T_{IPM}^{18}(t) - T_{cog}^{18}(t) - T_{rel}^{18}(t)
\]  

The result for a sample current vector – (\(|I| = 4\) Arms, \(\gamma = 30^0\)) is depicted in Figure 5.17. At this operating point, the phases of \(T_{cog}^{18}\) and \(T_{rel}^{18}\) are close to each other, but apart by 160\(^{0}\) with the phase of \(T_{mut}^{18}\).
5.3.3 Results for pulsating torque components

The hybrid torque segregation is conducted at the rated current of 4 Arms and varying current angle, $\gamma$. The cogging torque is collected at no-load, so it is a constant curve in Figure 5.18. The reluctance torque ripple, $T_{rel}^{18}$ and mutual ripple, $T_{mut}^{18}$ vary with $\gamma$ in different fashion. Both of them increase after $\gamma = 70^0$, which is the onset of deep flux weakening. The overall ripple is also higher in that region as seen in the contour maps of IPM and SyRM. With

![Figure 5.17 18th harmonic component torques – time domain at $I_{mag} = 4$ Arms & $\gamma = 30^0$](image1.png)

![Figure 5.18 18th harmonic component torques – for $I_{mag} = 4$ Arms & varying $\gamma$](image2.png)
$i_d$ current opposing the magnet’s field, the rotor pole face has random patterns of saturation. This results in a distorted air gap flux density waveform; the large harmonics in $B_g(\theta)$ cause higher torque ripple.

The accuracy of this experimental data is possibly low, keeping in view of the mechanical design constraints of the dyno testbed. For instance, the setup has two sets of flexible rubber couplings that introduce damping [52]. The bandwidth and resolution of the torque transducer play an important role. It is rated for $20 \, \text{Nm}$, and we are measuring ripples in range of $0.01 - 0.1 \, \text{Nm}$. The data is being collected for $36^{th}$ harmonic in 1 mechanical revolution at 50 rpm. The drive of the DM was operating at its limit of low speed regulation. Nevertheless, the trends of $T_{\text{mutt}}^{18}$ and $T_{\text{rel}}^{18}$ in Figure 5.18 give valuable information on effect of current vector on drive performance.

### 5.4 Conclusion

This chapter presented the comprehensive experimental results from the SyRM and IPM prototypes, as a step to verify the analytical modeling and FEA simulations of the previous chapters. The methods followed to collect inductance data, torque ripple data, efficiency and power factor were clearly discussed. The low speed region with MTPA and field weakening with MA and MTPV upto $4100 \, \text{rpm}$ was experimentally characterized for the SyRM prototype. These results compare to a good degree with the FEA models; mismatches point to necessity of retuning. No-load testing of IPM was conducted to collect Back EMF constant and cogging torque. The IPM torque characteristics and thermal behavior were collected to better understand the design. A novel torque component separation method utilizing experimental data and FEA data was proposed and implemented. The limitations in torque
ripple measurement in the built platform were considered, suggesting better experimental apparatus in the future. Overall, the time-intensive process of prototyping and experimental testing provides the key engineering dimension, to add onto the modeling research of this dissertation.
Chapter 6. Summary of Contribution and Future Work

This dissertation broadly dealt with the analytical modeling, hybrid analytical-FEA methods, fabrication and hardware testing of IPM motor with emphasis on the saturated inductance and pulsating torque. The contributions from this research are as follows:

1. The path-permeance function is proposed to study the saturating behavior of $q$-axis inductance in aggressively designed IPMs and under overload operation. The path permeance is a function of both the lamination geometry and the saturated conditions in steel regions. In slotted IPMs, the stator tooth conducts both the magnetizing flux and slot leakage flux, which superpose with each other to determine the saturation condition. This subsidiary phenomenon is investigated in detail using Ampere’s law and flux continuity law. Two models are developed and applied to 30 kW traction IPMs with 48-slot/8-pole stator, and both single layer and double layer rotor topologies. The results are validated with FEA and a single variable optimization of slot dimensions is illustrated.

2. The Torque Separation with Synthesized Rotors (TSSR) method is developed to decompose the cogging torque, mutual torque and reluctance torque components of an IPM. It is obtained through FEA simulation of two synthesized machines derived from a reference IPM. By comparing superposed torque profile with reference motor’s profile, the method is validated on an 18-slot/4-pole and a 24-slot/4-pole motor with I-type and V-type rotor magnets, operating under varying current commands. In the design stage, the two synthesized machines enable independent control of the magnet flux linkage and saliency ratio to achieve targeted performance. The second utility is in torque ripple minimization, for which trends
in the dominant harmonic of each torque component are obtained as functions of pole arc to pole pitch ratio and slot opening to slot pitch ratio.

3. Two prototypes – \( \frac{1}{2} HP \) SyRM and 2 HP IPM are designed with 18-slot/4-pole stator and I-type rotor. A modification to the conventional winding function is derived with an altered pitch factor so that it is applicable to novel multi-layer windings. It is applied to the design of the prototypes’ double layer winding configuration. The rotor design is optimized using a commercial software that applies Global Response Search Method to a series of FEA models.

4. A step-by-step procedure is implemented and described to build a fully-functional dynamometer testbed with a driving motor (DM), torque transducer, encoder, test motor (MUT), mechanical couplings, power analyzer, oscilloscope, DSP, inverter kit and current/voltage probes. The prototyping of an electric motor including rotor stack, stator winding and assembly is presented, which is useful knowledge for a new motor design engineer. Best practices in engineering were used at each stage.

5. Experiments are conducted to investigate inductances, efficiency, power factor and torque-speed envelope of the SyRM prototype. Mean torque and torque ripple maps in the \( (i_d, i_q) \) plane are extracted from low speed testing of both SyRM and IPM. Furthermore, the IPM is subjected to no-load tests to get cogging torque and magnet flux linkage; and thermal test to observe temperature of frame and end-winding. Time-tested and acceptable methods in motor testing are implemented in collection of raw data, and post-processing is done with MATLAB routines. A novel and interesting approach is executed for separation of reluctance torque, mutual torque and cogging torque profiles, using experimental harmonic torque magnitude and
FEA based harmonic torque phase. Limitations in accuracy of the collected results is discussed, in light of the practical constraints faced by the author.

Suggestions for future work related to this research topic are:

1. Explore applicability of the path-permeance and advanced inductance models to concentrated winding machines and multi-barrier PM assisted motor; evaluate shortcomings and suggest improvements.

2. Research torque ripple minimization methods in multiple-layer rotors with innovations in stator and rotor geometry by taking aid of the TSSR method.

3. Incorporate the inductance model and mean torque separation as part of a rigorous iterative design procedure for IPMs, in the pre-FEA design stage.

4. Improve the apparatus and testing method to separate torque components by obtaining both magnitude and phase data from experiment only, and with better accuracy.

5. Investigate the effect of saturated inductance and high torque ripple on the speed and torque control of the IPM drive from a system level point of view.
REFERENCES


