ABSTRACT

DU, YUHUA. Distributed Control of Networked Microgrids with Dynamic Boundaries. (Under the direction of Srdjan M. Lukic).

The world has witnessed a dramatic growth of microgrid (MG) in the last decade. The development of enabling technology and the increasing market demand are leading MG to a more important role in the future electric power systems. As the MG systems are getting more complex and dedicated, it is crucial to come out with MG control systems that are generic, reliable and ensure proper MG operation. To standardize such need, IEEE Std 2030.7 defines the minimum functionalities MG control systems need to provide so that the MG system under control can manage itself when connected to the grid, operate autonomously when islanded and transit between the two operation modes. On the other hand, advanced MG structures are developed as the number of on-line MGs increases. Coordination among adjacent MGs should be enabled instead of managing them individually. Utilization of MGs with dynamic boundaries, also know as dynamic MGs, have been showing great potentials in system optimal operation and distributed system restoration. In this work, a set of distributed control controllers are proposed to provide the standardized core functionalities to meet the minimum operation requirements for dynamic MGs. This work focuses on achieving system regulation by utilizing dispatchable inverter-based distributed generations (DGs).

A distributed control strategy for unbalanced dynamic MGs operation under both steady state mode and transition mode is first proposed. A novel system structure is proposed to manage the dynamic MG operation in a distributed manner. The proposed controllers focus on providing secondary control for dynamic MGs operation. They are able to achieve seamless transition during system topology variations and dynamic grouping for connected DGs.
for proper power sharing. A pinning-based distributed control strategy is then proposed that is able to perform a fast and seamless reconnection of an islanded MG to the main grid by enabling islanded MG to operate synchronously with the main grid. The proposed controllers enable explicit regulation on the islanded MG frequency, phase and voltage magnitude while ensuring proper power sharing among DGs. At last, a novel distributed average observer algorithm is proposed to achieve accurate average tracking in the presence of time varying communication delays among agents.

It is noteworthy that the effectiveness of each control algorithm proposed in this work have been validated in a controller hardware-in-the-loop (CHIL) MG testbed. The developed CHIL testbed utilizes a novel software platform, called Resilient Information Architecture Platform for Smart Grid (RIAPS) platform to have the proposed control algorithms implemented and executed using hardware controllers.
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Distributed Control of Networked Microgrids with Dynamic Boundaries

by

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DEDICATION

To my father and mother

Suigeng Du

Xiaoxia Li

And my uncle, Yaofang Ma who left us too early to read this
Yuhua Du was born in Xi’an and received his bachelor degree from Xi’an Jiaotong University, China in 2013 in electrical engineering. He purchased his M.S and Ph.D. degree with the Future Renewable Electric Energy Delivery and Management (FREEDM) Systems Center, Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC, USA.

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1.1 Introducing MG

1.1.1 Concept of MG

The development of distributed generations (DGs) are bring both benefits and challenges to modern power system operation. On the one hand, utilization of DGs are able to increase system efficiency and resiliency: renewable DG penetrations (e.g. PV farm) are able to ease the loading requirement from utility and dispatchable DG units (e.g. energy storage system) could act as UPS unit to keep critical loads from black-out [Gue09]; on the other hand, proper DG operation requires dedicated control efforts as the ones designed for conventional generation units (e.g. diesel generators) are usually not applicable as DG units usually have small system inertia [Gue04].
To integrate DG units with the external electric system, power electronic based interfaces (e.g. inverter) are favored due to its extensive controllability and fast dynamic responses [Gue13]. To efficiently manage inverter-based DG units, there are in general two types of inverter-interfaced DGs: grid following DG and grid forming DG. Grid following DG can be classified as current-source inverter by injecting grid-synchronized current to the external system. Such type of DG is usually used to interface resources that are intermittent and undispachable (e.g. PV and wind). Grid following DGs, especially when interfacing intermittent resources, usually have two operation modes: 1) Maximum power point tracking (MPPT) mode to maximize power extraction under all conditions and 2) Curtailed mode to set bound to the system injection power. Grid forming DG can be classified as voltage-source inverter that stabilizes the external system frequency and voltage. Such type of DG is used to interface resources that are dispatchable, like energy storage system or generator. Grid forming DGs can be operated under voltage control mode (VCM) as a slack bus or under current control mode (CCM) as a controllable power point (PQ bus) [HL11].

Besides various approaches on operating different types of DG units under device level, a lot of works have been done on managing DG units under system level. As DGs are making up an increasing portion of power generation, such transformation motivates the development of MGs (MGs). A MG is defined as a group of interconnected loads and DGs within clearly defined electrical boundaries that acts as a single controllable entity with respect to the grid [TS12]. MGs can be generally classified into AC MG, DC MG and hybrid AC/DC MG. By far, most research interests have been laid on AC MG due to its practicality in existent AC power system. Nevertheless, DC MG and hybrid AC/DC MG are witnessing a growing attention, especially with the development of shipboard power system [Jin16]. A general structure of a AC MG is presented schematically in Fig. 1.1.

Compared to conventional power systems, MGs requires complex control to
Figure 1.1: General structure of an AC MG system with inverter-based DGs

Maintain stable and optimized operation. Operation and control challenges of the state-of-the-art MGs are distinguished from the ones of conventional power grid system from following perspectives:

- MGs should be able to operated under both islanded mode and grid-connected mode, which provides more flexible solutions for system operators: Under grid-connected mode, power flow between MG and the main grid could be controlled to help improving system efficiency from economic perspective; Under islanded mode, the MG could isolates from the main grid and operated as an independent entity to keep the load within the MG energized.

- Due to lacking generators with large inertia, islanded MGs are usually energized by inverter-based DGs and have small system inertia. Islanded MGs are more vulnerable towards system power flow variations, which could be caused by intermittent resources or step load change. Also, convention generation units are usually assumed
to be equipped with enough fuel supplement, while grid forming DGs usually have limited power/energy capacity (e.g. battery energy storage system). Such limitation makes the DG power/energy management becomes an important research topic on MG operation [Han17].

- Unlike conventional power systems where there exists only one slack bus in the system, an islanded MG could be energized by several grid-forming DGs, which makes the conventional power flow calculation using Newton-Raphson method unfeasible in islanded MG analysis [Saa99]. Such limitation makes power system optimal reconfiguration problem becomes very challenging when trying to take MG into consideration without making certain simplifications (e.g. only one pre-defined DG is allowed to be presented in one MG [Che18a]).

- Adaptive protection strategies designed for islanded MGs remain challenging [Jai17]. Conventional power system protection schemes assume a radial system topology where the directions of power flow are fixed and the magnitudes of fault currents are dependent on the fault locations. However, once the MG is islanded and energized by multiple DGs, protection schemes designed for grid-connected MG would fail as the direction of system power flow could change and the fault becomes hard to localize.

- Communication and information flow of conventional power systems are usually done using SCADA system, which represents a centralized structure with a center computer at the highest control level. However, communication networks for a MG are more flexible and could vary from highly centralized ones with MG centralized controller (MMC) to completely distributed ones that require only sparse communication links. Additionally, due to the small system inertia, timely and robust communication networks are more crucial towards MG operation and regulation.
At last, it is noteworthy that there are often confusions on the definitions of MG and Smart Grid (SG). In general, SG is a vision of future power system structure while MG can be viewed as a building block of SG.

### 1.1.2 Brief History on MG Development

Although the concept of MG was not introduced until recent decades, power systems sharing the similar construct of a MG have already been installed in practice at the initial stage of power system development, before the structure of centrally controlled and operated utility grids were not yet formed. Back to 1882, Thomas Edison built his first power plant, the Manhattan Pearl Street Station, and his company installed 50 DC MGs in four years. Similar to most modern MGs, Edison's electric system was small with localized (or distributed) generation and limited and clearly defined electric boundaries (several blocks). Interestingly, Edison's system also have steam-heating pipes transferring heat from its steam generation to his company, which is called combined heat and power (CHP) today [LS08].

Shortly thereafter, the electric services industry evolved to a state-regulated monopoly market. With the utility grid subsequently utilizing large centralized power plants which benefits from the economic perspective and the increasing concerns of transmission reliability [Par15]. Isolated power systems are connected into a monopolistic utility and these 'MGs' fades away. The conventional electric grid, or marcogrid, has been planned and structured for centralized generation of electricity predominantly from fossil fuels using MW level synchronous generators. However, with the growth of electric demand and the development of power electronics devices, traditional power systems are facing the following challenges:

- Restrictions on expansion of centralized power generation and delivery system makes
it hard to meet the growing electric demand at acceptable cost [Mar01]. Additionally, the environment issues caused by utilization of fossil fuels are gaining more and more attentions, which is holding back the expansion of convention power generations.

- Conventional distribution systems usually have radial topology, where any contingency at upstream will result in significant power quality issues to downstream customers. As the electric systems are getting more complex, maintaining system resiliency becomes challenging, especially during extreme cases like nature disasters.

- Asynchronous and distributed generation sources (DERs) are taking a significant portion of the total electric generation. There are a significant amount of DERs at kW level dispersed throughout distribution systems, which brings new challenges to system operation stability and essentially changes the structure of future electric market.

Driven by the above mentioned challenges, the concept of MG is developed by the researchers. In 2001, some of the essential concepts of MG that are still kept today were first pointed out in [Ram01]:

- Interface between MG and the main grid should isolates the two sides electrically, and yet connects them economically.

- MG should be capable of operating as an autonomous power system and providing service of proper power quality that meet the need of users.

- Communication of information may be used to enhance system performance, but DERs must be capable of stabilizing system using local controller.

In the same year, Consortium for Electric Reliability Technology Solutions (CERTS) first implemented the concept of MG [PL01] in integrating DERs within an industrial site. The
The site has an overall demand of 1 MW and is connected to the main grid with a series of transformers. The whole system is divided into three distinct groups with transformers, loads (both sensitive and non-sensitive) and DERs. Whenever the connection with the main grid fails, the DERs are able to pick up load automatically using P-f droop control and keep the system operated in islanded mode with zero power flow in certain branches.

The development of MG technique witnessed a dramatic growth in 2004~2011. In 2003, there was a widespread power outage throughout parts of the Northeastern and Midwestern United States, which affected an estimated 45 million people in eight U.S. states. The blackout brought not only enormous economic losses but highlighted the ease with which the power grid could be taken down. This event has significantly stimulated researchers on developing MGs techniques in order to improve power system resiliency. In 2004, the CERTS MG (CM) Concept was first proposed as an alternative approach for integrating small scale distributed energy resources (DER of <500 kW) into electricity distribution systems[MB04]. Unlike traditional approaches that requires instantaneously disconnection of onsite micro-generators in the event of system outage, CM envisages systems designed to operate semi-independently, usually operating connected to the macrogrid but separating (islanding) from it when cost effective or necessary. The key features of CM are: (1) single common coupling with respect to the main grid; (2) generator controlled by on-board power electronics that are of fast dynamic response speed; (3) slow supervisory control for system energy management; (4) CHP applications which permits useful application of the wast heat; (5) heterogeneous power quality and reliability to ensure the energization of critical load. In the same year, the concept of peer-to-peer and plug-and-play functionality for each component of the MG that are still adopted today were first clearly defined in [LP04]: The peer-to-peer concept insures that there are no components that is critical for operation of the MG; The plug-and-play implies that a unit should be placed at any point on
the electric system without re-engineering the controls. Meanwhile, initial ideas on master-slave inverter control structure \cite{Lia03} and DG management using multi-agent-system (MAS) \cite{Mac04} were also proposed by that time.

Several MG projects were delivered in the following years in Europe and US. In the European Union (EU), the *MGs: Large Scale Integration of Micro-Generation to Low Voltage Grids* activity was funded at €4.5 million. The project was successfully completed and some of the highlights include:

- Development and enhancement of micro-source controllers to support frequency and voltage based on droops. Application of software agents for secondary control.

- Development of hierarchical MG control architecture and the corresponding communication requirements.

- Development of appropriate protection and grounding policies that will assure safety of operation and capability of fault detection, isolation and islanded operation.

A follow-up project titled *More MGs: Advanced Architectures and Control Concepts for More MGs* was funded at €8.5 million. This project provides a comprehensive study on the economic benefits of MG investments from the following perspectives: 1) MG benefits identification; 2) market and regulatory settings for MGs; 3) sensitivity analysis of market price and pricing policy variations. This project also pointed out several potential services MGs could provide with respect to utility: 1) peak shaving; 2) voltage regulation; 3) loss reduction; 4) reliability improvement. In US, the most well-known MG project by that time was pursued by CERTS. The concept of CM, as previously introduced, was carried out under a full-scale testing. The test site, which was installed in Columbus Ohio and operated by American Electric Power, is by far the most comprehensive and productive MG testbed and
is still delivering significant research outcomes [Las11]. There is no master controller or source in CM and each source is connected in a peer-to-peer fashion with localized control scheme. CM testbed uses a central communication system to dispatch DG set points to improve overall system operation (not used for dynamic operation of the MG). DC storage is used to provide short bursts of power required by the MG independent of the rate of the prime mover, and a charger ensures the energy is slowly replenished into the batteries after the burst.

Another remarkable achievement proposed by that time was the concept of hierarchical control. As previously introduced, researchers generally agree that MG should be stabilized by DGs using only local controller with no need for communication, yet proper coordination among DGs could potentially improve the system steady-state performance and economic efficiency. However, there was no common standard to classify different MG control approaches. In 2011, the concept of hierarchical control was first introduced in [Gue11b]. Hierarchical control consists of three levels of control:

- The primary control is based on the droop control to make the system stable and more damped. Primary control requires fast dynamic response and thus runs on local controllers with no communication.

- The secondary control ensures the system operation states are regulated as desired. It also includes mechanism that ensures the MG to seamlessly connect to or disconnect from the main grid.

- The tertiary control manages the power flow between the MG and the external electrical distribution system. Control under this level is mainly driven by improving system economic efficiency and executed by system operators.

The concept of hierarchical control standardized the control approaches of MG, which
makes the analysis of MG control become more efficient and provide significant guidance in MG future development.

By that time, basic principles/assumptions of MG operation and control that are still closely kept nowadays had been proposed. Some of them includes: 1) MG should be able to operated under both grid-connected mode and islanded mode, and perform seamless transition between the two modes; 2) there is usually only one PCC between the MG and the main grid and the electric boundary of a MG is clearly defined; 3) all the DGs are managed under hierarchical control and the MG need to be stabilized using decentralized control; 4) plug-and-play and peer-to-peer are very important features and should be kept by all DGs within a MG; 5) there needs to be at least one dispatchable DG (battery or CHP) operated in an islanded MG as slack bus. The original definition of MG, along with the aforementioned prerequisites, construct a universal MG architecture that is still serving as preliminary knowledge/guidance nowadays in the field on both academic and industry.

Since then, development of MG has been growing at a stable pace. The industry have been enthusiastically investing in the MG market. By the end of 2017, the existing MG capacity have been identified as 3.2 GW and is expected to grow to 6.5 GW by 2022, at a 14.4 percent compound annual growth rate [Met17]. Reported by Global Market Insights, the US MG market is expected to surpass $17.51 Billion By 2025. New types of MG have been proposed in practice for different purposes. The concept of community MG was introduced in [Bur17]: A community MG is a local energy system that’s designed to support vital community services during a utility outage. While not necessarily technically different from MGs, they are fundamentally different from MGs from a regulatory and business model as utility regulation comes much more significantly into play in this type of MG. Another type of power system that follows similar operation rules of MG is the so-called remote power system (rgrids). Referring to the definition of MG, rgrids is true MG since
they are not able to operate under grid-connected mode. However rgrids shares the similar technology and can sometimes be loosely described as MGs (one good example is the shipboard power system [Jin16]).

Meanwhile, researchers in academic have been putting great efforts in improving MG technology from different aspects. Some of the main motivations includes:

- **Innovations on power electronics technology**  As previously reviewed, power electronic devices have played an important role in interfacing DERs with the main grid. With the development of power electronics technology, researchers are able to equip the MG with more advanced functionality. One good example is the development of The Future Renewable Electric Energy Delivery and Management System (FREEDM System) [Hua10]. Unlike conventional AC MG where all the DERs need to be interfaced with the LV (e.g. 480 V) AC bus, FREEDM system enables interfacing LV (e.g. 400 V) DC bus with HV (e.g. 12 kV) AC bus, which enables integration of DERs who are mostly DC in nature in LV DC system and a smart and efficient connection with the external HV AC system. The key features of the FREEDM system are enabled by the revolutionary Solid State Transformer (SST) interface.

- **Development on interdisciplinary research**  As the power system is getting more complex, there exist many problems with respect to MG that can not be solved properly by the knowledge from traditional electric engineering. However, with the help from other research areas (e.g. control theory, economy, etc.), more flexible solutions are enabled to make MG operation more efficient and robust. For example, conventional hierarchical control utilizes either decentralized control where no communication is presented or centralized control where there exists a MCC to process all the information. Both methods are not efficient enough to achieve coordination among DGs
as the size of MG increases. However, with the development of multi-agent system (MAS) from control area, distributed control has been widely adopted nowadays by researchers [Sha14c; SP15b], especially on improving MG secondary control.

- **New challenges on power system operation** Just as the development of MG was first stimulated by the Northeast blackout of 2003, so far there are several events that have created new branches of MG research. For example, with the development of electrical vehicle (EV) and especially the booming success of Tesla in the EV market, there have been a significant amount of focus on EV integration and management with respect to MG. Another example is the utilization of MG in distribution system restoration. US has witnessed several severe power outages due to extreme natural disasters in the past years, which calls for MGs’ being actively participating in distribution system restoration, instead of acting as an isolated entity [Che16].

## 1.2 Introducing Dynamic MG

### 1.2.1 Advanced MG Structures

Conventional vision of MG generally focus on a single group of loads and DGs that forms a single electric power system with clearly defined electric boundaries and single PCC. In case there are multiple MGs exists in the distribution system, they are treated as independent entities individually and would not be physically interconnected to each other, except insofar as they all are embedded in the distribution system of the local utility.

However, as more and more MGs being created and integrated into the grid, a viable advanced MG model that interconnects the MG with the utility and additional MGs is proposed in [Bow14]: Advanced hardware, intelligent inverters, smart controllers, and
compatible communications will be the enabling technology mix used to maximize a MG system's economic and operational benefits. Advanced communication interfaces and smart controls will increase the value of the energy provided by these advanced MG systems. The reliability, resilience, and interoperable electrical service for conventional and advanced MG customers will be vastly improved over results of today's installed MGs.

The desire to link multiple MGs gives rise to a new concept of MGs, called Nested MGs [Sam16]. Nested MGs, also called Interconnected MGs, MG Cluster or Aggregated MGs in different works, refer to the interconnection of multiple adjacent MGs into one network. Compared to conventional MG that operates independently and only interact with the external system through single PCC, Nested MG could have multiple PCC and PCC locations. A Nested MG structure envisioned by DOE is presented in Fig.1.2. Cooperated operations among each Nested MG are enabled under both grid-connected and islanded mode, which could potentially improve system resiliency, operation efficiency and economic benefit. The cooperation among MGs enables the utilization of the most efficient generation methods and have the system run optimally. System security could be improved by having areas with deficit generation supported by other Nested MGs that are connected. However, Nested MGs management is also bringing several challenges:

- By connecting multiple MGs together, system operation and coordination become more complicated as the number of assets increase.

- Managing multiple PCC results in additional operation complexity (e.g. protection adjustment) which has not been exclusively studied.

- Interactions among Nested MGs becomes critical and requires extra control efforts as the Nested MG network becomes complex.
Besides the constantly increasing interests from the academic, there have been several real world projects that put the concept of Nested MGs into practice. For example, The Bronzeville MG is initiated by ComED to install a MG in a location that would allow for the interconnection with a nearby MG, which has already been implemented by Illinois Institute of Technology. The two NMs are connected to the grid through different substations, however, they have a tie line allowing them to be connected electrically. Another example is the Olney Town Center MG project funded by DOE. The MG is arranged into several zones based on the distribution of DERs and critical loads and a MG controller is designed such that the system is able to meet the following power quality requirements: SAIDI of less than 2 minutes, reduction in emissions of 20% and an improvement in efficiency of over 20% [Lon18].
1.2.2 Concept of Dynamic MG

In addition to the notion of Nested MG, there is also another type of advanced MG structure that has been proposed, which is called dynamic MG. A dynamic MG can be defined as “a MG with flexible boundaries that expand or shrink to keep the balance between generation and load at all times[NS16].” The boundaries of these dynamic MGs would be the substation breakers and distribution automation switches and isolators. Compared to conventional MG that has static electric boundaries, the structure of dynamic MG could be actively varied as per request to the system operator. For example, the topology of a dynamic MG could be varied due to a power balancing request for DER integration [NS16] or participation of distributed system restoration during nature disaster [Kim16].

The differences between conventional static MGs and emerging dynamic MGs have been briefly presented in Fig.1.3. A typical multi-MG structure is shown in Fig.1.3a. The electric boundaries of each static MG are pre-defined. Interconnections between each static MG is achieved through static breaker at static PCC. When the breaker is open, the whole MG operates as an independent entity with respect to external system. Structure of a dynamic MGs system is shown schematically in Fig.1.3b. As discussed, dynamic MGs have varying electric boundaries. Interactions between each dynamic MG is achieved using dynamic breakers (e.g. SSW) [NS16]. As the topology of each dynamic MG varies, their interconnection points become dynamic.

As above-mentioned, dynamic MGs can be effectively used in distribution system restoration. After severe damage like natural disasters, the distribution grids are disconnected from upstream grids. DGs with grid-forming capability should be utilized to energize the system and actively managed to maintain system resiliency. Multiple MGs could be formed to pick up critical loads in different areas. Furthermore, the MGs should be able to
actively change their electric boundaries to restore as much load as possible or mitigate power demand for DGs in heavily loaded areas. Additionally, since distribution systems are usually unbalanced, dynamic MGs should also be able to handle system unbalance mitigation under varying topology [Du19c; Du18c].

1.2.3 Dynamic MG Control Framework

Generally speaking, topology variation of dynamic MGs starts with the reconfiguration request sent from system operator to the target SSWs that need to change status. Conventional operation strategy changes the on-off status of the target SSWs directly once the
reconfiguration request is received without proper regulation, unbounded transients will be introduced during system transition which could cause serious stability issue to the system. In this paper, the proposed control strategy is designed to achieve transient-free topology variation: the identified reconfiguration request will be passed to every controllable DGs within dynamic MGs. These DGs will be controlled coordinately to minimize the through power at the target SSWs or minimize the voltage phasor differences across the target SSWs. Once the reconfigure request is received, SSW is able to function independently using local measurements and change status once the criteria for seamless system transition are met. Therefore, the system is operated under new topology without any undesired transients.

Particularly, the configuration of an autonomous distribution grid is presented schematically in Fig. 1.4. The concept of Minimum MG (Min-MG) is introduced in this framework and employed in the proposed control strategy. A Min-MG is defined with pre-grouped DGs and SSWs that are able to support local critical loads by forming a self-sustained and islanded MG. Min-MG is treated as the basic building block that constructs dynamic MGs, and each MG in the dynamic framework is comprised with one or multiple neighboring
Min-MGs. It is assumed that the distribution system has been pre-divided into several Min-MGs. Compared to conventional MG, the electric boundary of a Min-MG is defined based on the positions of SSWs. Following rules should be met when locating the SSWs: 1) each Min-MG only interconnects with (either isolates from or connects to) external system through SSWs; 2) two adjacent Min-MGs should share only one SSW (this can be easily achieved in a radial distributed system). SSW defines the connectivity between neighboring Min-MGs and is in response to the reconfiguration request sent from system operator. In this study, two neighboring Min-MGs is denoted as connected Min-MGs if the SSW they share are closed.

The information flow in the proposed control strategy are enabled by three independent communication channels, namely DG to DG channels (D2D), SSW to DG channels (S2D) and SSW to SSW channels (S2S). As defined, DGs and SSWs within the same Min-MG are pre-grouped and equipped with proper grouping information. It is assumed that the SSW shared by two neighboring Min-MGs could identify and communicate with the DGs in each Min-MG respectively. D2D channel is designed for communication among DGs within the same Min-MG and could be modeled as a connected undirected graph. S2D channel is designed for the communications between DGs and SSWs within the same Min-MG and could be modeled as a \( K_{D,Y,D_X} \) complete bipartite graph. To be more specific, each SSW is designed to communicate with all the DGs in the neighboring Min-MGs through S2D channel. S2D channel is static once the Min-MGs are set and on-line DG dynamic management and regrouping is achieved by the proposed controller. S2S channel is designed for communication between SSWs and could be modeled as a connected undirected graph. The developed communication network is presented in Fig. 1.5.
Figure 1.5: Communication network for the proposed control framework
As the technologies and operational concepts to integrate and manage MGs interconnected to the grid are getting diversified, it is critical to come out with MG control strategies that are:

- **Generic**  The MG control systems need to be scalable and their applicability should not be limited by system size, topology and physical property.

- **Fundamental**  The MG control systems could be designed with different objectives but should all be met with basic MG operation requirements.

- **Reliable**  The MG control systems should be designed in plug-and-play and peer-to-peer fashion with no master controller.

In this work, the focus is on providing a set of MG solutions to MG secondary control
satisfying the above mentioned requirements. The application of the proposed controller is not limited to conventional MG structure but can also be extended to MGs with advanced structure. To be more specific: 1) The designed control algorithm are focused on DGs with grid-forming capability as they are the essential asset that ensures MG operation reliability; 2) The proposed control strategies are designed to provide the standardized functionalities that ensures MG operation; 3) The proposed control strategies are applicable to advanced MG architectures with multiple PCC and varying topology; 4) Distributed control approaches are utilized in the proposed controller design to avoid system single point of failure.

2.1 IEEE Standard for the Specification of Microgrid Control System

In 2018, IEEE Std 2030.7-2017 IEEE Standard for the Specification of Microgrid Controllers is published to 'define the functional requirements of the MG controller in a manner that can be universally adopted' [829]. The main purpose of this standard is to provide the minimum functional requirements for MG controller that ensure a technically sound operation of the MG at the POI. The standard is functionality driven and provides modular approach, i.e. the defined minimum functions are applicable to MG operations regardless of system topology, configuration or jurisdiction. The requirements and functions should be applicable to a large range of MGs and MG controllers, which are generic and commonly applicable.

In the standard, a MG control system is defined as 'A system that includes the control functions that define the MG as a system that can manage itself, operate autonomously, and connect to and disconnect from the main distribution grid for the exchange of power
and the supply of ancillary services. An electrical system consists of DER and aggregated controllable loads is considered to be a **MG** in the standard if it has the following characteristics:

- Clearly defined electrical boundaries.
- A control system to manage and dispatch resources as a single controllable entity.
- Installed generation capacity that exceeds the critical load.

The above-mentioned MG definition will be adopted in this paper. Compared to the MG definition in [TS12], this definition has highlighted the importance of MG control system and the requirements on installed generation capacity within an MG. There are in total six MG operation modes defined in the standard, namely: 1) **steady state connected** (SS1), 2) **steady state islanded** (SS2), 3) **unplanned islanding** (T1), 4) **planned islanding** (T2), 5) **black start** (T3) and 6) **reconnect** (T4).

The interconnection requirements of a MG control system are satisfied using functions at the high level, **core level** and low level of its functional framework, as presented in Fig. 2.1. The functions designated as **core functions** (marked in red in Fig. 2.1) in this standard are of the highest importance for modular design of MG control systems and are under study in this work. Two core functions are defined in this standard:

- The **dispatch** function, which dispatches individual devices in given operating modes and with specified set points.
- The **transition** function, which supervises the transitions between connected and disconnected states, and ensures the dispatch is appropriate for the given state.

The dispatch function determines the dispatching of MG assets (inverter-based DGs in this work) in each modes and provides them with proper power set points, as shown in Fig.
2.2. As defined in the standard, the dispatch core function should provide the following functionalities for each operation mode:

- Balancing generation and load under normal islanded operating conditions;
- Re-dispatching controllable resources in response to internal events related to the load and generation profiles;
- Responding to external orders, for example interconnection agreement requirements, and external events by re-dispatching resources.

The functionalities covered in this work with respect to dispatch function are marked in red in Fig. 2.2. In this work, I focus on providing adaptive dispatch rules designed for inverter-based DGs towards MG dispatch function under steady state islanded (SS2), planned islanding (T2) and reconnect (T3). Unplanned islanding (T1) and steady state connected (SS1) operation modes are enabled in this work using conventional control approaches.
The concept of emergency dispatch order (EDO) is introduced in the standard (see Fig. 2.2): EDO is a continuously updated order that enables immediate non-critical load shedding to match available generation upon an unplanned islanding. This function belongs to load response and is not discussed in this work.

The transition function provides the logic to switch the dispatch function between one of the relevant dispatch modes, which includes the four transition modes (T1 to T4) and two steady state modes (SS1 and SS2). The transition logic for MG operation modes switch is shown in Fig. 2.3 and the operation modes and sequences covered in this work are marked in red.

As defined in the standard, the MG control system shall be able to carry out the operations for three transitions: unplanned islanding (T1), planned islanding (T2) and reconnect (T4) whose operations are characterized by the following steps. The process and steps for planned islanding (T2) includes: a) Receive islanding command b) Balance load and
generation (adjust P and Q to both be 0 at the POI) c) Set local controllers and protection devices appropriately d) Create island e) Transition to steady state islanded dispatch mode.

The process and steps for unplanned islanding (T1) include: a) Detect islanded conditions b) Create island c) Set local controllers and protection devices appropriately d) Execute required pre-planned actions such as load shedding (and/or implement a black start if required) e) Transition to steady state islanded dispatch mode. The process and steps for reconnect (T4) include: a) Resynchronize, set/match voltage, phase angle, and frequency within prescribed limits specified by applicable grid codes or requirements b) Set local controllers and protection devices appropriately c) Reconnect d) Transition to steady state connected dispatch mode and restore non-critical loads as appropriate. Black start (T3) steps are unique to each MG and are not specified in the standard.
In conclusion, the core functionalities (minimum requirements) of MG control systems are classified as dispatch functions and transition functions. Dispatch functions include the operation functionalities one MG control system need to provide under each MG operation mode, while transition functions provides the transition logic and process between each operation mode. In this work, the DG controllers I proposed covers the core functions for **T1**, **T2**, **T3**, **SS1** and **SS2** operation modes. **Black start** and load demand operation are not discussed. As previously reviewed, MG control follows a three-level hierarchical control. Conventionally, primary control requires fast dynamic response (millisecond) and can only be achieved using decentralized approach, while secondary control and tertiary control operate at relatively slow dynamics (second to days) and are usually achieved using centralized control, as shown in Fig. 2.4. As previously stated, tertiary control is not discussed in this work. Secondary control is in charge of regulating MG operation states as desired, which presents a very close match with respect to the standardized core functions defined in [829] and will be the main focus of this work.

### 2.2 Microgrid Hierarchical Control

As previously reviewed, hierarchical control plays a vital role in MG operation and control. It acts as the backbone of MG operation and has been guiding the direction of MG researches for decades. Most of the works done with respect to MG control fall into one of hierarchical control levels. In this section, hierarchical control implementation in inverter-based DGs are briefly reviewed (i.e. DGs that are motor-interfaced, e.g. CHP, are not covered). The traditional structure of an inverter-based DG is presented in Fig. 2.4.
Inner Control Loops

Inverter-based DG can be treated as controllable voltage source whose operation points (voltage magnitude, frequency and phase) are directly determined by its inner control loops. It needs to be pointed out that: 1) Inner current/voltage control loop as part of inverter control is used to ensure the DG output voltage and current follow the given reference using feedback and feed-forward control. It is considered as part of the inner control loops in this paper; 2) Although inner control loops is sometimes considered as part of primary control, its control objective is essentially different from primary control: inner control loops provides certain control patterns over the DG terminal voltage so that the DG power outputs could track the power set points determined by primary control (or higher level of hierarchical control).

There are two types of inner control loops for voltage source inverter (VSI), namely current-control-mode (CCM) and voltage-control-mode (VCM) [Gue13] and their control
Figure 2.5: Inner control loops of (a) VSI-CCM (b) VSI-VCM

Loops are presented in Fig. 2.5, respectively.

VSI-CCM is designed to have the DG generate desired output power follow given reference by injecting grid-synchronized current to the external system. It consists of an inner current loop and a phase-locked loop (PLL) to constantly track the external phasor. The inner current loop is designed to run under the reference frame measured by PLL, so that the magnitude and phase of DG output current can be controlled under a static dq0 frame [Bla93]. The reference current is generated using the following relationships in dq0 frame:

\[
P = \frac{3}{2}(E_d i_d + E_q i_q + 2E_0 i_0), \quad Q = \frac{3}{2}(E_q i_d - E_d i_q)
\]

(2.1)

where \(P\) and \(Q\) present the DG active and reactive power outputs, respectively; \(E_{d,q,0}\) present...
the DG terminal voltage under dq0 axis, respectively and \(i_{d,q,0}\) present the DG output current under dq0 axis, respectively. Under a balanced three phase three line electric system, \(E_0\) and \(i_0\) do not exist. Control variate method is usually used to simplify the power output control by setting \(E_q = 0\). Thus the relationships shown in 2.1 are reduced to:

\[
i_d = \frac{3}{2E_d}P, \quad i_q = -\frac{3}{2E_d}Q
\]

where \(P\) and \(Q\) are the reference power outputs and \(E_d\) is the terminal voltage measured by PLL. It can be observed that the terminal voltage phasor of VSI-CCM is controlled passively while its current output is under active regulation. This makes VSI-CCM operates like a controllable current source (VSI-CCM is also called current-source inverter in some works), and provides no active regulation/stabilization over the system voltage and frequency.

VSI-VCM is designed to have the DG operate as a slack bus to regulate/stabilize the system voltage and frequency by regulating its terminal voltage phasor, so that the whole system is able to reach power balance automatically and enter steady state. It consists of an inner voltage and current loop, both run under dq0 axis. Unlike VSI-CCM, there is no PLL presented in the inner loop of VSI-VCM and the DG terminal voltage phasor is continuously adjusted till the system reaches steady state. The DG power flow can be described using following relationships:

\[
P = \left(\frac{E_{PCC}}{|Z|} \cos \varphi - \frac{E_{PCC}^2}{|Z|}\right) \cos \theta + \frac{E_{PCC}}{|Z|} \sin \varphi \sin \theta
\]

\[
Q = \left(\frac{E_{PCC}}{|Z|} \cos \varphi - \frac{E_{PCC}^2}{|Z|}\right) \sin \theta + \frac{E_{PCC}}{|Z|} \sin \varphi \cos \theta
\]

where \(E\) and \(E_{PCC}\) present voltage magnitude of voltage phasor at DG terminal and PCC; \(Z = R + jX = |Z|\angle \theta\) presents the line impedance and \(\varphi\) presents the phase angle difference.
between the two voltage phasors. In practical systems, the phase angle difference is typically small and thus \( \sin \varphi = \varphi \) and \( \cos \varphi = 1 \). For a line impedance that is mainly inductive, the line resistance \((R)\) could be neglected. The power flow relationship shown in (2.3) could be simplified as:

\[
P = \frac{E_{PCC}}{X} \varphi = \frac{E_{PCC}}{X} (\varphi_0 + \int (\omega - \omega_G) d t), \quad Q = \frac{E_{PCC}(E - E_{PCC})}{X}
\]  

(2.4)

where \( \omega \) and \( \omega_G \) present the DG operation frequency and grid frequency, respectively. In steady states, the DG power flow should be stationary as the power balance between generation and consumption is met within the system. VSI-VCM and the grid then share the same frequency, as \( \omega = \omega_G \). It can be concluded from (2.4) that by actively adjusting DG terminal voltage phasor \((\omega \) and \(E)\), VSI-VCM is able to adjust its power outputs to seek for the system power balance point and in turn stabilize system frequency and voltage. This makes VSI-VCM operates like an ideal controllable voltage source.

**Primary Control**

Primary control is used to stabilized system frequency and voltage relaying on decentralized approach. Droop control (or droop relationship) has been adopted in most works as primary control. The initial idea of this control level is to mimic the behavior of a synchronous generator, which reduces its operation frequency when its active power output increases. There are different types of droop control designed for different operation scenarios (e.g. inductive system, resistive system or hybrid system) and different control purposes (e.g. enhanced dynamic response, improved power sharing, etc.). In this paper, the conventional P-f and Q-V droop control for an inductive system (P/Q droop in short [ZW09]) is discussed.

P/Q droop control for an inductive system is proposed based on the relationship shown
in (2.4). It can be concluded that in an inductive microgrid, DG active power flow is proportional to phase angle difference ($P \propto \varphi$), while reactive power flow is proportional to voltage magnitude difference ($Q \propto (E - E_{PCC})$). The general mathematical expression of P/Q droop control is:

\[ -m(P - P_{ref}) = \omega - \omega^* \]

\[ -n(Q - Q_{ref}) = E - E^* \]

where $m$ and $n$ present $P - f$ droop gain and $Q - V$ droop gain, which are usually designed to be inversely proportional to DG's rated capacity ($m \propto \frac{1}{P_e}$ and $n \propto \frac{1}{Q_e}$); $P_{ref}$ and $Q_{ref}$ present the DG active and reactive power reference under rated system operation states; $\omega^*$ and $E^*$ present rated operation frequency and voltage. Droop control is in the form of Proportional control and takes system operation states as feedback, which makes it applicable for DG operation control under both grid-connected and islanded modes.

As mentioned in previous section, the power outputs of VSI-CCM ($P$ and $Q$) are under active regulation while its terminal voltage phasor ($\omega$ and $E$) are controlled passively. Thus from (2.5), the power outputs of VSI-CCM determined by droop control ($P_{droop}$ and $Q_{droop}$) can be expressed as:

\[ P_{droop} = -\frac{1}{m}(\omega - \omega^*) + P_{ref} \]  

\[ Q_{droop} = -\frac{1}{n}(E - E^*) + Q_{ref} \]

When the VSI-CCM is grid connected, system frequency is at rated, as $\omega = \omega^*$ and $P = P_{ref}$, while the DG terminal voltage can not be guaranteed to be at rated, as $E \neq E^*$ and $Q \neq Q_{ref}$. To have VSI-CCM output accurate amount of active/reactive power as desired, droop control is always bypassed for grid-connected VSI-CCM. Under islanded mode, although VSI-CCM is not capable of providing voltage and frequency support for system
stabilization, it is still able to adjust its power outputs to help the system reach power balance by increasing power outputs when system frequency and voltage drops and reducing its power outputs otherwise. Control loops of VSI-CCM droop control implementation are presented in Fig. 2.6.

Unlike VSI-CCM, VSI-VCM is able to actively regulate its terminal voltage phasor ($\omega$ and $E$) so that its power outputs ($P$ and $Q$) are automatically adjusted till the system reach power balance. Thus from (2.5), the terminal voltage phasor of VSI-VCM determined by droop can be expressed as:

$$\omega_{droop} = -m(P - P_{ref}) + \omega^* \quad (2.7a)$$

$$E_{droop} = -n(Q - Q_{ref}) + E^* \quad (2.7b)$$

When the VSI-VCM is grid connected, the main grid should be the prime source that ensures system power balance and VSI-VCM should be operated as controllable power source. When the system is under rated frequency, as $\omega^* = \omega$ and $P = P_{ref}$, VSI-VCM is able to output referenced active power. Reactive power control of grid-connected VSI-VCM is also tricky, as in steady state $E \neq E^*$ and $Q \neq Q_{ref}$. Additionally, unlike VSI-CCM, the power output of VSI-VCM can not be directly controlled by bypassing droop control. Thus to ensure accurate reactive power output, grid-connected VSI-VCM usually uses a PI controller instead of droop control (which essentially is a P controller) to regulate
its terminal voltage [HL11]. When the VSI-VCM is islanded, it is able to actively adjust its operation frequency and terminal voltage till its power outputs are stationary, which indicates that the system reaches power balance. It should be noted that under islanded mode, the reactive power output of VSI-VCM should be automatically adjusted to balance the reactive power mismatch instead of outputting at a constant value. Thus under islanded mode the reactive power control of VSI-VCM should be switched back to droop control. Control loops of VSI-VCM droop control implementation are presented in Fig. 2.7.

It is noteworthy that the droop relationships presented in (2.7) are only validated when the approximations made in (2.4) are satisfied, i.e. $X \gg R$ so that the active power $P$ is predominantly dependent only on the phase angle $\varphi$ while the reactive power $Q$ mostly depends on output voltage $E$ [Gue11b]. Otherwise the regulations on $P$ and $Q$ will be coupled, as shown in (2.3) and cause undesired system oscillations. To enable the P/Q droop control on VSI-VCM in system that is not inductive enough, virtual impedance is introduced to mimic the behavior of an impedance that is 'connected' between VSI-VCM and the external grid (between $L_2$ and $Z_E$ as shown in Fig. 2.4). The block diagram of virtual
impedance loop is presented in Fig. 2.8. Virtual impedance loop is designed to emulate extra virtual inductive response for better DG dynamic and steady state response and could be described by the following relationship:

$$E_{\text{cap,ref}} = E_{\text{droop}} - I_{L2} \times Z_v$$  \hspace{1cm} (2.8)$$

where $E_{\text{droop}}$ presents the reference voltage magnitude generated by droop control, as shown in (2.7); $E_{\text{cap,ref}}$ presents the final inverter voltage magnitude reference; $I_{L2}$ is the measured line current and $Z_v = R_v + j \omega L_v$ presents the designed virtual impedance. Practically, virtual impedance impact is usually implemented under dq frame, as shown in Fig.2.8. With the introduction of virtual impedance, the equivalent impedance under which the VSI-VCM operates on could be modified to be inductive enough. Detailed analysis of virtual impedance design and corresponding system response have been well presented in [HL11].
Secondary control

When a MG is grid-connected, the system frequency and voltage are controlled by the main grid. When the MG is islanded, VSI-CCMs are able to act as reserve loads but do not provide any voltage support. System frequency and voltage should be stabilized by VSI-VCMs using droop control. However, droop control would bring steady state errors on system frequency and voltage and secondary control is needed to eliminate the deviations and have the system operate as desired.

The P-f and Q-V droop relationships on a single VSI-VCM are shown in Fig. 2.9, respectively. The original droop curves are plotted in black in Fig. 2.9. It can be observed that as the $i$th droop controlled VSI-VCM adjusts its power outputs to meet the system power balance, as $P_i \neq 0$ and $Q_i \neq 0$, its steady state operation frequency and voltage will be deviated from the rated values, as $\omega_{SS} \neq \omega^*$ and $E_{SS} \neq E^*$. In steady state, system frequency is global and thus $\omega_G = \omega_{SS} \neq \omega^*$, indicating a steady state deviation on system operation frequency. Unlike frequency, system voltage could be naturally different and varies from bus to bus, especially in the case where multiple VSI-VCMs are not connected to the same bus. However, it can still be concluded that the system voltage is deviated from its rated value as there is a steady state voltage deviation on each VSI-VCMs who act as slack bus. To eliminate the deviations introduced by droop control and have the system operate under desired frequency and voltage, secondary control is introduced. Regardless of how different secondary controllers are designed for different control objectives, secondary control is essentially done by vertically shifting the conventional droop curve so that the islanded MG is able to operate under new steady state operation states (system frequency and voltage) without compromising system power balance. As shown in Fig. 2.9, the droop curves shifted by secondary control are plotted in green. It can be observed that when operate under the
shifted droop curves, the VSI-VCMs are able to operate under rated frequency and voltage while keeping the system power balance. Such approach could also be expressed using the following relationships:

\[ \omega_{SS} = -m(P - P_{ref}) + \omega^* + \Delta \Omega \]  

\[ E_{SS} = -n(Q - Q_{ref}) + E^* + \Delta E \]  

where \( \Delta \Omega \) and \( \Delta E \) present the vertical shifts, or secondary control variables on P-f droop and Q-V droop, respectively. In practice, system power consumption might vary with system operation states and thus the secondary control variables are usually calculated using feedback control. For example, a system frequency/voltage restoration controller using PI control is presented in [Sha14c]:

\[ \Delta \Omega = k_{pf}(\omega^* - \omega) + k_{if} \int (\omega^* - \omega) \, dt \]  

\[ \Delta E = k_{pv}(E^* - V) + k_{IV} \int (E^* - V) \, dt \]  

Another issue with droop control is the undesirable power sharing among DGs. For
Figure 2.10: Multiple VSI-VCM power sharing under (a) P-f droop and (b) Q-V droop

simplification $P_{ref}$ and $Q_{ref}$ are set to be zero in subsequent discussion. Fig. 2.10 presents operation of two DGs (DG1 and DG2) with different power capacities ($S_1 < S_2$) under droop control. It can be observed from Fig. 2.10(a) that under the original P-f droop curves (plotted in solid lines), proportional active power sharing between the two DGs is kept in steady state, as $\frac{P_1}{P_2} = \frac{P_{1*}}{P_{2*}} = \frac{(\omega^* - \omega_{SS})m_1^{-1}}{m_1^{-1}}$. Shifted P-f droop curves by secondary control are plotted in dashed lines. It can also be observed that as long as both P-f curves are equally shifted ($\Delta \Omega_1 = \Delta \Omega_2$), proportional active power sharing can still be kept, as $\frac{P_1}{P_2} = \frac{(\Delta \Omega_1 + \omega^* - \omega_{SS})m_1^{-1}}{m_1^{-1}} = \frac{P_{1*}}{P_{2*}}$, while still ensuring system frequency regulation.

However, this is not the case with reactive power sharing. Due to the fact that different DGs might have different equivalent impedance with respect to PCC, their steady state terminal voltages could be different. As shown in Fig. 2.10(b), under original Q-V droop curves, due to the voltage difference ($E_{SS1} \neq E_{SS2}$), proportional reactive power sharing are not achieved, as $\frac{Q_1}{Q_2} = \frac{(E^* - E_{SS1})n_1^{-1}}{(E^* - E_{SS2})n_2^{-1}} \neq \frac{Q_{1*}}{Q_{2*}}$. What’s more, DG with higher capacity is usually connected to the external system with higher impedance, which would lead to greater voltage drop and make the reactive power sharing even worse. As shown in Fig. 2.10(b), as $E_{SS1} < E_{SS2}$, it can be observed that $Q_1 < Q_2$, which is undesirable as DG1 has greater
capacity and should be sharing more power. Shifted Q-V droop curves by secondary control are plotted in dashed lines. It could be observed if both Q-V curves are shifted equally to have the terminal voltages of both DGs restored as rated, the reactive power sharing would be even worse, as $Q'_1 < Q_1$ and $Q'_2 > Q_2$. It could be concluded that conventional secondary control on voltage restoration could make reactive power sharing even worse and extra control efforts are needed. State of the art technique on power sharing among DGs in an islanded MG will be discussed in later sections.

As defined in [Gue11b], another topic that should be covered by secondary control is the MG seamless connection and disconnection from the main grid. There are in general two types of MG islanding, namely planned islanding and unplanned islanding: 1) A planned islanding start with a grid-connected MG receives islanding request. Then the power generation and consumption within the MG should be balanced to eliminate the power flow at point of interconnection (POI). Local controllers and protection devices should be pre-set to islanded operation mode and then the main breaker shall open and create island; 2) An unplanned islanding starts with a MG that has already entered islanded condition due to the main breaker opened by unplanned event (e.g. disturbance on the main grid). The islanded conditions shall be detected and the local controllers and protection devices should be set to islanded operation mode. Minimum requirements of seamless MG transition from grid-connected to islanding includes [829]: 1) Fast load shedding (alternative is hardening of the loads via backup power supply); 2) Closed transition (Enabling utility and backup power supply overlap); 3) Synchronization operation for grid reconnection. To reconnect an islanded MG back to the main grid, the voltage phasors on both sides of POI need to be resynchronized so that the mismatch on voltage magnitude, phase angle and frequency are kept within prescribed limits. Then the local controllers and protection devices shall be pre-set to grid-connected and the main breaker on POI shall close and reconnect the
MG with the main grid. State of the art technique on MG seamless transition between grid-connected and islanded mode will be discussed in later sections.

**Tertiary Control**

As defined in [Gue11b], tertiary control manages the power flow between the MG and the external distribution system, where the MG is considered as an controllable power source and its power flow can be controlled by changing the DG power set points. However, with the increasing penetration of renewables, power flow within a MG becomes more stochastic, which makes tertiary control more challenging. Tertiary control is usually described by an optimal power flow (OPF) problem to find an optimal operation solution for DGs subject to system power quality requirements and operation limits. Some of the control objectives that have been frequently considered are listed: 1) minimize the cost of energy purchase from the main grid [Oua15]; 2) minimize the power losses within the MG [Mor18]; 3) maximizing the generation renewables [Zha13]; 4) regulate DGs within the MG to be operated around optimal operation point [Oua15]. Also, AC MG OPF problem is usually non-convex due to the non-linear relationship between bus voltage and system power flow, as shown in (2.3). Such non-convexity make the computation of OPF problem become challenging and makes the solutions available are either approximation or heuristic [Mor16]. At last, how to properly model and handle the stochastic characteristics of DERs are also challenging [SS16].

The focus of this work is on the control of inverter-based DG within an MG and solving OPF problem is out of scope. Regardless of how the DG power outputs are optimized, the control commands received by each inverter-based DG are essentially the desired power set points by system operators and the power tracking performance of each DG is mainly determined by its primary and secondary control. No further discussion on MG tertiary
control will be presented in this work.

### 2.3 Overview on Microgrid Control Architecture

Conventional electric power systems are usually managed under centralized control, where one main controller is responsible for managing the execution of each controllable unit using exclusive communication links. Centralized control has been an effective and useful control schema that has dominated MG control for years. This is mainly caused by the generally provide significant controllability due to the limited system complexity. However, with the increasing penetration of DERs that are inverter-interfaced, MG systems are getting more stochastic and delicate, which could significantly limit the applications of centralized secondary control due to the following reasons:

- Centralized control approach relies on MCC to process the control commands to all the controllable units, which requires significant computation capability as the size of MG increases.

- Centralized control runs on dedicated communication networks to link the MMC with all the controllable units, which requires high bandwidth network and is hard to be spanned.

- Single MCC operates as a master controller and could potentially introduce single point of failure to the system.

- The performance of centralized control is vulnerable to communication delay and the asynchronous operation of each controllable units.

Therefore, non-centralized control approaches are favored by most researchers on MG control. Non-centralized MG control can be either decentralized or distributed [YMS14]. A
decentralized control system is a system in which the execution of each component does not depend on the knowledge of the states of all the other components and there is no information exchange between any components, while in a distributed control system, states of all the components are required but the computation is spread over the system by enabling communication among neighbouring components. In conclusion:

- Compared to centralized control, both decentralized control and distributed control utilize localized controllers to accomplish global goals, which avoids system single point of failure and ease the computational burden on individual controller;

- Unlike decentralized control where only local information are utilized, execution of distributed control are computed based on global information, which provides more controllability over the system but requires extra efforts for information collection;

- Compared to centralized control where global information is directly collected and processed by central controller, the information sharing in distributed control is usually implicit, which ease the requirements on communication system.

The state of the art decentralized secondary control usually requires pre-knowledge of system physical structures, either by measurements [Gu17a] or prediction [Mor17], which are undesirable as they are highly model-dependent. With the development of MAS, distributed control becomes promising in MG secondary control applications. Each controllable units in the MG (inverter-based DG in this work) can be treated as an agent that operates as independent entities with proper intelligence and communication capabilities. Operations made by each agent are based on the implemented distributed control strategies and information exchange from other neighbouring agents. Although agents can communicate, a large part of control is based on the autonomy of the agent and is performed locally.
In conclusion, distributed control is favored in this work due to the following reasons:

- There is no master controller presented in distributed control, which significantly improves system resiliency and robustness against communication delay.
- Distributed control usually runs on sparse communication network, which reduces the cost of communication system and enables system spanning.
- Information exchange between neighbouring units are enabled, which makes coordination among controllable units scalable and not model dependent.

### 2.4 Microgrid Control Systems Implementation and Validation

MGs are ideally suited for distributed control solutions. However, implementation and validation of the developed distributed control algorithms are quite challenging. In this section we propose a Controller Hardware-in-the-Loop (CHIL) testbed for MG distributed control applications that satisfy the requirements of *IEEE Std. 2030.7* for MG control systems. We describe two main features of the proposed platform: 1) a software platform that enables the implementation of control algorithms that have been developed analytically and 2) a real-time MG platform that replicates practical MG operation environment by using real-time communication network and grid solutions. Implementation and validation of a distributed MG synchronization operation control strategy are used to demonstrate the performance of the proposed CHIL testbed.
2.4.1 Validation Techniques for Microgrid Control Algorithms

Despite significant efforts to develop distributed MG control algorithms, few were tested in the deployment environment, due to the inherent cost and risk associated with testing algorithms in actual MGs [PL01]. Hardware testbeds are typically small-scale prototypes, with limited components and simple system topology [Lou18a]. Additionally, fault scenarios are difficult to test in the field without risk of equipment loss, for example, distribution system reconfiguration under abnormal operating conditions [Kim16]. As a result, software validations (e.g. MATLAB/Simulink) have been widely adopted. Compared to a hardware implementation, such an approach shows significant advantages due to its easy and economic access to most researchers. Moreover, most simulation software provides models that can be easily adopted and modified for any use case, which greatly reduces the development time. However, such approach has the following disadvantages: 1) the simulated system is usually too ideal to provide convincing results for safe system deployment; 2) most models are simplified to a level where they do not accurately represent all the features of the component, making edge conditions difficult to test; 3) practical communication networks rely on multiple protocols and time-varying latency, while data exchanges in a purely simulation environment are usually ideal; and 4) time synchronization is guaranteed in a simulation due to the pre-determined simulation time step.

To overcome the above mentioned drawbacks, real-time hardware-in-the-loop (HIL) simulations provide a higher level of fidelity [Bur17]. Commercial simulators are able to model the MG system in real-time and integrate actual equipment and communication links into the simulation. Controllers of critical components, whose operating modes may be proprietary, can be integrated into the HIL simulation, while the non-critical components are modeled in the simulator. Still, the control algorithms that need to be validated,
are typically modeled in the real-time simulator, reducing the fidelity of the resulting simulation approach. Since the main goal of the research effort is to validate the MG controller operation, simulating the control algorithm in the real-time simulator does not allow for evaluating possible design issues including: 1) distributed controller designed with complex structure could require too much computing power and becomes economically unfeasible; 2) algorithm scalability may be difficult to evaluate if the number of participating agents in a distributed control algorithm is large and variable; and 3) information exchange among distributed hardware controllers requires complementary communication network which presents its own challenges and limitations.

To further overcome the above-mentioned limitations and fully utilize the capability of real-time simulators, it is favored to execute the control algorithms using practice hardware controllers that interact with the simulated system models in real-time as a special case of HIL (also called CHIL). However, implementing the analytically developed control algorithms at the hardware level present additional hurdles beyond the traditional control system challenges. We describe some of these challenges below [Du18b]:

- Because of the nature of digital control, the hardware controllers need to have sufficient computational capability, so that the selected sampling time step can be made small enough to minimize the impact caused by transforming from continuous mode to discrete mode.

- The developed distributed control algorithm utilizes multi-agent control approach where the number of participating agents can be large and variable. Therefore, the controller and the hardware implementation must be scalable and should support plug-and-play capability.

- Each agent has to have the capability to interact with physical devices such as PMU.
Developing reusable device interaction code that is modular presents additional challenges that are orthogonal to the development of the core control algorithm.

- Though distributed control algorithms do not require delicate communication network compared to centralized ones, reliable and accurate information exchange are still crucial towards system stability because of the small system inertia. Information exchange among distributed hardware controllers requires complement communication system which is also challenging

In this section, a CHIL testbed for MG distributed control applications is proposed. The proposed platform provides a MG testbed to validate the performance of any MG controller that follows the requirements defined in IEEE Std 2030.7. In addition to the stated advantages of a conventional HIL simulation, the proposed CHIL testbed enables the implementation and testing of distributed control strategies using real hardware controllers. We introduce a software platform, called Resilient Information Architecture Platform for Smart Grid (RIAPS) to implement distributed control algorithms in hardware, using a scalable and modular approach [Du17b; Tu18].

### 2.4.2 RIAPS Platform Architecture Overview

The RIAPS platform [Dub18] allows for efficient implementation of distributed control algorithms using a reusable development framework, which can be deployed on real hardware operating on the grid’s edge. The RIAPS platform was first introduced in [Eis17] as an open-source software platform; it provides a run-time and design-time software environment for building applications that execute on computing devices of the Smart Grid\(^1\). Applications

\(^1\)Find more about RIAPS in [https://riaps.isis.vanderbilt.edu/](https://riaps.isis.vanderbilt.edu/) and get access to the open source files in [https://riaps.github.io/](https://riaps.github.io/)
include, but are not limited to, monitoring and control, data collection and processing, energy management, and safety applications. The key concept is to provide a “middleware” and various support service functions that enable each “actor” (i.e. an application process) to communicate with others so that the developers can focus on distributed application logic instead of messaging and networking. Compared to existing solutions, the RIAPS platform distinguishes itself with the following features:

- Dispersed fog-computing architecture with multi-tenant hosts;
- High-precision time synchronization and time-sensitive messaging;
- Coordination services and synchronized control actions across the network;
- Built-in resilience to faults anywhere in the system.

The RIAPS platform has a three-layer architecture, as shown in Fig. 2.11: Component is the reusable building block for applications and is used to provide specific physical func-
tionality, like computation or measurement; **Components** are composed to form **Actors** that realize an abstract function, like a control algorithm or state estimation. In RIAPS the distributed algorithms are implemented as **Applications** and are composed of **Actors**. Each **actor** encapsulates run-time layers of RIAPS that provide:

- Component framework that defines a concurrent model of computation for building distributed applications;
- Resource management framework for controlling the use of computational resources;
- Fault management framework for detecting and mitigating faults in all layers of the system;
- Security framework to protect the confidentiality, integrity, and availability of a system under cyber attacks;
- Fault tolerant time synchronization service;
- Coordination framework for coordinated computations and actions across the network.

Note that the **Application**’s business logic can be separated from the low-level details the framework.

A developed **Application** is distributed to each computing node through RIAPS’ deployment mechanism, as shown in Fig. 2.11. The distributed algorithms could be coded in Python or C++ by the developer using a single development machine and downloaded to all the distributed nodes running RIAPS. The RIAPS platform provides programming APIs to help development of device wrappers and provides a ZeroMQ-based messaging layer for information exchange between various RIAPS nodes. The RIAPS discovery service
controls the information flow. Communication patterns available to applications include group-based publish-subscribe as well as point-to-point client-server mechanisms.

### 2.4.3 Microgrid Controller Hardware-in-the-Loop Testbed Setup

To replicate the environment in which the MG distributed controllers operate, a real-time CHIL MG testbed is proposed in Fig. 2.12. The testbed consists of three major parts, namely real-time simulators, hardware-in-the-loop and controller-in-the-loop:

- The real-time simulator models the MG response in real-time. Switching components (e.g. power electronic converters) are modeled in FPGA-based simulator (Opal 5607) that runs with the simulation time step of a few hundred nanoseconds, while the non-switching components (e.g. power transformer) are modeled in CPU-based simulator (Opal 5031), and execute with a time step of a few microseconds. The simulated MG system operates in both grid-connected (SS1) and islanded modes (SS2) and transition between the two modes.

- The hardware-in-the-loop part provides integration of hardware devices with the
simulated MG system. They can be either industry-standard devices that provide realistic responses to disturbances, or customized devices that await validation. Currently in our lab, MCUs from Texas Instruments (F28377S) are interfaced to the DG inverter switching models and function as the primary controllers. A SEL 451 Relay is integrated with the MG distribution network to emulate the behavior of a practical relay.

- The controller-in-the-loop enables the comprehensive validation of distributed algorithms. BeagleBone Black boards (BBBs) are selected as the hardware to implement local intelligence in the form of a control algorithm. Each BBB represents a distributed controller that can be attached to any controllable hardware device or the simulated MG device directly. In our lab, each MCU and relay is assigned a BBB. The execution and coordination of BBBs are realized by the RIAPS platform.

The proposed CHIL testbed supports multiple communication protocols. The simulated voltage are read by the relay from the real-time simulator as analog signals. The relay reacts to the analog measurements and communicate with its assigned BBB using IEEE C37.118.2 communication protocol. Each DG inverter modeled in the FPGA is directly controlled by the gate signals generated by its assigned MCU using PWM digital signals. Meanwhile, the MCUs are able to take the local measurements from the real-time simulator (e.g. inverter output current and voltage) as analog signals. Communication between the MCU and its assigned BBB uses Modbus communication protocol. The distributed control algorithm runs in the BBB and the communication among BBBs is enabled through the RIAPS platform using the messaging layer. The messaging architecture can be configured in the development environment, typically a Linux machine.

Although the delays introduced to the data exchange in each communication channel
Table 2.1: MG Testbed Communication Delay

<table>
<thead>
<tr>
<th>Communication Form</th>
<th>Time Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analog/Digital signals</td>
<td>$\Delta T_1 \approx 0$</td>
</tr>
<tr>
<td>C37.118.2 protocol</td>
<td>$\Delta T_2 = 10 \sim 100$ ms</td>
</tr>
<tr>
<td>The RIAPS platform</td>
<td>$\Delta T_3 = 1 \sim 25$ ms</td>
</tr>
<tr>
<td>Modbus protocol</td>
<td>$\Delta T_4 = 4.5 \sim 5$ ms</td>
</tr>
</tbody>
</table>

Figure 2.13: Distribution of communication delays in different forms

depend on the real-time network traffic, expected ranges of time delay are summarized in Table 2.1: 1) The analog/digital signals are transferred using cables that are less than one meter long and thus the communication delay is ignored; 2) IEEE C37.118.2 protocol delay is estimated referring to IEEE Standard for Synchrophasor Data Transfer for Power Systems [611]; 3) Communication delay introduced in the RIAPS platform is measured experimentally. The amount of time it takes for the data package transfers from one BBB to another BBB has been measured using 10K data packages. The distribution of measured communication delay in RIAPS is shown in Fig. 2.13a; 4) Communication delay introduced by Modbus protocol is measured experimentally. The amount of time between the phasor regulation DG BBB sending one data package to the MCU and get the receiving confirmation back from the MCU has been measured for 10K data packages. This round-trip time is
then halved to get the average Modbus protocol delay in Table 2.1. The distribution of measured communication delay in Modbus is shown in Fig. 2.13b. More information about the testbed can be found in [Tu18a; Du18a; Tu18b; Du18; Du17a; Xu19]
Dynamic MGs are able to support critical loads without energization from utility and allow system topology variation upon request. Utilization of dynamic microgrids can provide more flexible solutions towards distribution system restoration from natural disasters. Moreover, dynamic MGs in the context of distribution system reconfiguration are usually unbalanced. Unlike balanced system where only positive-sequence components exist, proper operation of unbalanced dynamic MGs presents additional challenges.

This section proposes a distributed secondary control strategy for dynamic MGs operation by providing coordinated regulation on both system positive- and negative-sequence operation. System frequency and voltage are under constant regulation, along with voltage unbalance (VU) management for multiple critical load buses (CLB). The proposed control strategy enables seamless system transition during dynamic MGs reconfiguration, and
guarantees proportional positive- and negative-sequence power sharing among connected
DGs with respect to system topology variation. Detailed controller designs are provided
and stability analysis are derived. The proposed control strategy is fully implemented in
hardware controllers, and its effectiveness is validated using IEEE 34-bus system on the
developed CHIL MG testbed.

3.1 Motivations in Dynamic Microgrid Utilization

United States has witnessed several severe power outages due to extreme natural disasters
in the past years. In 2017, hurricane Irma hit Puerto Rico and Florida and left about 1 million
customers without power. Hurricane Harvey, formed at almost the same time, caused about
0.3 million power outages across Texas and Louisiana [DOE17]. Conventional outages at
distribution level would only affect portions of the feeders, and the restoration can be
done using adjacent feeders to re-energize the isolated branches through tie lines [Shi92].
However, conventional network reconfiguration techniques rely heavily on the energization
from utility, which might not be available after natural disaster. The distribution system
could be completely isolated from substation [PM17] or even the substation itself could be
at fault [Kwa12]. In either case conventional network reconfiguration will not be able to sup-
port the critical load downstream from the fault. There is a growing interest in self-healing
distribution systems resilient to natural disasters [Wan16b]. Network reconfiguration has
been adopted widely for distribution system operation after conventional power system
contingency (e.g., power outages at electrical components) [Que15]. Distribution system
should be actively and promptly reconfigured to tackle the impacts of power outages and
pick up critical loads [Shi92].

The feasibility of utilizing MGs as resiliency resource has been discussed in [Sch17]. As
per request by *IEEE Std. 2030.7*, MG control systems shall dispatch MG assets to perform BS in MG without grid connection or operating self-generation. Instead of staying in black until the transmission system becomes available, the outage area could be sectionalized and re-energized using BS DGs by forming networked autonomous MGs [Che14]. Many works have been proposed for coordinated operation and management in the context of multi-MGs structure to improve system resiliency. The authors in [Wan16] propose a scheduling strategy that utilizes resources in normal MGs to support the on-emergency portion of the system during fault period. In [Far16], a hierarchical outage management framework is proposed to schedule the resources within each MG and coordinates the possible power transfer among MGs. Detailed sequence of actions for multi-MG service restoration after a complete power system black out is suggested in [Res11].

With the development of distribution automation, utilization of dynamic MGs can provide flexible solutions to grid service restoration [Che17], the electric boundaries of dynamic MGs shall vary as per system operator’s request to pick up as much loads as possible and mitigate the power demand for DGs in heavily loaded areas. However, restoration of a complete blackout system in the context of dynamic MGs operation have not been properly discussed. The authors in [Ma18] propose a controller design that helps integration of dynamic MGs into the existing distribution system. However, the proposed restoration scheme still involves energization from neighbouring substations. A scheduling method that optimizes the boundaries of dynamic MGs in the presence of demand response programs under system contingencies is proposed in [Moh18]. However, the proposed work is only applicable for islanded system with restored electric network. In [Kim16], a framework to determine dynamic MG formation is proposed for fault isolation and load restoration. However, the proposed framework is not designed for restoration of a complete blackout system.
3.2 Dynamic Microgrids Operation Challenges

3.2.1 Steady State V.S. Transient Response

Using MGs for network reconfiguration has been mainly formulated as an optimization problem with constraints concerning system steady state operation stability [Wan16c; Li14; Che17], i.e., the steady state operation status of system after reconfiguration should not violate any operating limits. However, little work has been done on providing proper regulation on system operation status during the transition of reconfiguration. Unlike conventional distribution systems that have large inertia due to large online synchronous generators, MGs are mainly supported by inverter-based DGs and have very small inertia. Due to the current continuity of the inductors that form the distribution line, undesired transients will be introduced to the MG during reconfiguration in case of non-zero power flow at the moment breaker opens and asynchronous voltage phasors at the moment breaker closes [Tan15]. If the transients are out of bounds, inverter-based DGs will be forced to trip and the MG will be de-energized [481].

In [Xu18], system dynamic operation constraints were taken into consideration when solving network reconfiguration problem with MG utilization. The potential reconfiguration options are evaluated using both power flow calculation and dynamic simulation, and would not be considered as feasible if the estimated operation status during and after reconfiguration violate either steady state or dynamic constraints. Such approach could effectively circumvent the reconfiguration options that might lead to operation violations, but it would further limit feasible reconfiguration options and significantly increase the computation time due to the need for dynamic simulations. In [Kim16], instead of pre-screening before reconfiguration, system frequency/voltage are constantly regulated...
utilizing synchronous generators. Such approach could bound the transients during recon-
figuration and takes less operation time. However, the proposed control strategy presents
only passive regulation effort on system operation status, i.e., the regulation would take
effect only after the deviations have occurred and been identified. Transients still exist at
the instant when the SSW changes status and system dynamic response mainly depends
on controller design. Such approach could put bounds to system transients but does not
guarantee seamless reconfiguration. The system operation deviations and transients could
be essentially eliminated if the system is properly regulated. Taking MG reconnection as
example, the system could perform a seamless transition if the voltage phasors on both
sides of the breaker are perfectly synchronized before breaker closes.

3.2.2 Static Topology V.S. Dynamic Topology

Hierarchical control has been widely adopted for MG operation, as reviewed in Section 2.2.
Regulation for dynamic MG reconfiguration falls under secondary control. The concept of
networked MGs or MG clusters that comprise several interconnected individual MGs is
introduced in [Hat14]. Secondary control of a multi-MG system has been developed and
is realized by either each MG operating as controllable entities [Wan16a; Sha14b; Lu18]
or all the MGs collectively as a whole [JL17],[Bui18]. However, secondary control strategy
designed for dynamic MGs operation has not been exclusively studied and the conventional
approaches are not always applicable for dynamic MGs.

Dynamic MGs have shifting electric boundaries. This is in contrast to conventional
multi-MG structure, shown in Fig. 1.3a [Wan16a]. The electric boundaries of each static MG
are pre-defined. Interconnections among static MGs are achieved through static breakers
at PCC. When the breaker is open, the whole MG isolates from external system and operates
autonomously. Structure of a dynamic MGs system is shown in Fig. 1.3b [Min10]. As the topology of each dynamic MG varies, its points of interconnection (POI) change [481]. Interactions among dynamic MGs is enabled by SSWs [NS16]. Secondary control designed for static MGs are not applicable for dynamic MGs for two main reasons.

1. Conventional secondary control is designed for MGs with static boundaries and PCC, while dynamic MGs operate with varying topology and dynamic POIs. In addition to conventional secondary control objectives (e.g., frequency/voltage regulation, DG power sharing, etc.), secondary control for dynamic MGs needs to coordinate the interconnection of MGs at varying POIs to ensure seamless dynamic MGs topology variation.

2. Conventional secondary control for static MGs assigns a particular DG to a given microgrid with pre-determined boundaries. This approach fails when the boundaries of microgrids shift when the system is reconfigured. As shown in Fig. 1.3b, DG grouping needs to be managed dynamically according to system topology variation, in real time.

3.2.3 Balanced System V.S. Unbalanced System

Distribution systems are usually unbalanced, due to the common use of single- and two-phase laterals. Voltage unbalance (VU) in three-phase systems is usually measured by voltage unbalanced factor (VUF), which is defined as the ratio between the negative sequence component of applied voltage and the nominal voltage. As specified in [833], three-phase DG may trip if its operating VUF is greater than 3%. Additionally, there also exist some particular buses, e.g. critical load buses (CLB), where the connected loads are sensitive to unbalanced voltage and require strict VU regulation. Most works done so far with respect to
MG utilization in distribution system restoration focus on unbalance compensation in individual MGs along with the corresponding steady state stability analysis [Ach18]. Compared to the distributed secondary control algorithms designed for balanced system where only positive-sequence components are presented [Guo15b], the ones for unbalanced system require extra control efforts. In [Men16], a hierarchical control strategy is proposed for grid-forming DG operation in unbalanced system. The proposed strategy achieves accurate VU regulation at single CLB and negative-sequence current sharing among DGs. In [Guo15a], a finite-time average consensus algorithm is proposed to achieve VU regulation and share the regulation efforts dynamically among DGs in distributed fashion.

Despite the existing secondary control algorithms designed for unbalanced system operation, the ones for unbalanced dynamic MGs has not been extensively discussed. Conventional secondary control approaches are mainly designed for single MG with static topology, which are not applicable for dynamic MGs operation. Conventional secondary control on system VU mitigation usually considers single CLB in a given MG, while multiple CLB could be connected in the context of dynamic MGs, and the VU mitigation of which has not been explicitly considered. For an islanded system, the negative-sequence voltage (and thus VUF) on a particular CLB is regulated by all the dispatchable DGs collectively, and VUF regulations on multiple CLB requires coordinated control efforts. Also, for an unbalanced inductive system under normal condition, its positive- and negative-sequence voltage magnitudes could be treated as decoupled [Men16], and thus simultaneous regulations on both states are feasible. However, voltage phases on positive- and negative-sequence are coupled as their first order derivatives are both determined by system frequency, which is global. Simultaneous regulations of voltage phase on both sequences could be conflicting and destabilize the system. The above mentioned challenges are critical and should be properly addressed by the operative secondary control algorithms.
3.3 Distributed Secondary Control for Dynamic Microgrid

As discussed, proper regulation for dynamic MGs provides more feasible reconfiguration solutions with less operation time and enables seamless system transitions. MG secondary control has been studied for a long time and recent research interests have evolved from centralized control schemes to decentralized or distributed ones. Fully decentralized secondary control scheme are highly model-dependent and thus have limited applications [Gu17b]. Distributed control strategy (e.g., multi-agent system) has been widely employed to avoid system single point of failure [Sha14c].

Referring to the dynamic MGs control framework proposed in Section 1.2.3, a group of connected Min-MGs that comprise a dynamic MG is denoted as $\Omega$. For each Min-MG, i.e. Min-MG$_X$ for $X \in \Omega$, its neighboring Min-MGs are denoted as $\Omega_X$ and they are connected by SSWs, i.e. SSW$_{XY}$ for $Y \in \Omega_X$. It is assumed that there are $D_X$ on-line DGs and $D_{S,X}$ closed SSWs in Min-MG$_X$. The D2D channel of Min-MG$_X$ is presented by graph, $G_{D,X}$. $L_X$ presents the Laplacian matrix of $G_{D,X}$ and $A_X = \{a_{ij}\}$ is the corresponding adjacency matrix.

The proposed controller design is characterized into regulations on positive- and negative-sequence, respectively. In this work, an inductive three-phase three-wire islanded system is under study, and no zero-sequence components is considered. Hierarchical control structure is kept as a basic framework of the proposed control strategy, while the proposed control focuses on resilient secondary operation of unbalanced dynamic MGs. Three controller operation modes are proposed, namely Type I, Type II and Type III mode:

1. Type I mode is designed for unbalanced dynamic MGs operation when no reconfiguration is requested. Proportional power sharing on both positive- and negative-sequence among connected DGs is achieved.
2. Type II mode is designed for unbalanced dynamic MGs reconfiguration when a closed SSW is requested to open. The current (and thus power) that flows through the target SSW are minimized before the SSW opens.

3. Type III mode is designed for unbalanced dynamic MGs reconfiguration when an open SSW is requested to close. Voltage phasors on both sides of the target SSW are synchronized before the SSW closes.

Operation mode of the proposed controller is indicated by designed state variables, \( \eta_{XY}, \lambda_{XY} \): 1) \( \eta_{XY} = 0, \lambda_{XY} = 0 \) indicates the controller is in Type I mode and no status change is requested for SSW\(_{XY}\); 2) \( \eta_{XY} = 1, \lambda_{XY} = 0 \) indicates the controller is in Type II mode and SSW\(_{XY}\) is requested to open; 3) \( \eta_{XY} = 0, \lambda_{XY} = 1 \) indicates the controller is in Type III mode and SSW\(_{XY}\) is requested to close. It is noteworthy that only one operation type can be activated at a time.

### 3.3.1 Positive-sequence Controller Design

Controllers for the \( i \)th DG in min-MG\(_X\) on frequency and positive-sequence voltage regulations are provided below:

\[
\begin{bmatrix}
\omega_i \\
E_i
\end{bmatrix} = \begin{bmatrix}
\omega^* \\
E^*
\end{bmatrix} - \begin{bmatrix}
m_i P_i \\
n_i Q_i
\end{bmatrix} + \begin{bmatrix}
\Delta \Omega_i \\
\Delta E_i
\end{bmatrix} \tag{3.1a}
\]

\[
\begin{bmatrix}
\Delta \dot{\Omega}_i \\
\Delta \dot{E}_i
\end{bmatrix} = -\begin{bmatrix}
\Delta \Omega_O \\
\Delta E_O
\end{bmatrix} - (1 - \eta_{XY}) \begin{bmatrix}
\Delta \Omega_I \\
\Delta E_I
\end{bmatrix} - \eta_{XY} \begin{bmatrix}
\Delta \Omega_{II} \\
\Delta E_{II}
\end{bmatrix} - \lambda_{XY} \begin{bmatrix}
\Delta \Omega_{III} \\
\Delta E_{III}
\end{bmatrix} \tag{3.1b}
\]
where power loop of droop controlled DG is presented in (3.1a) with \( m_i \) and \( n_i \) as the \( P-f \) and \( Q-V \) droop gains, respectively, and \( \Delta \Omega_i \) and \( \Delta E_i \) as the designed secondary control variables; \([ \omega_i, E_i ]\) and \([ \omega^*, E^* ]\) present DG operated and rated frequency/positive-sequence voltage; \( k \) denotes designed control gains; \( \Delta P'_{ij} = P'_{ij} - P_{ij} \) and \( \Delta Q'_{ij} = Q'_{ij} - Q_{ij} \) present per unit positive-sequence active/reactive power outputs mismatch between the \( i \)th and \( j \)th DG, respectively; \( \Delta \omega_i = \omega_i - \omega^* \) and \( \Delta V_i = \psi_{\Omega} - E^* \) present system frequency and voltage regulation error where \( \psi_{\Omega} \) presents average positive-sequence voltage among connected DGs. The proposed secondary control variables are defined in (3.1b) to (3.1f):

1. \( \Delta \Omega_O \) and \( \Delta E_O \) in (3.1c) represent regulations on system frequency, voltage and power sharing among DGs within the same min-MG. The average DG voltage, \( \psi_{\Omega} \) can be observed using the distributed average observer from [Du19b];

2. \( \Delta \Omega_I \) and \( \Delta E_I \) in (3.1d) represent Type I mode regulations on DG power sharing between adjacent min-MGs, where \( \Delta P'_{iY} = P'_{i} - \text{avg}(P'_{Y}) \) and \( \Delta Q'_{iY} = Q'_{i} - \text{avg}(Q'_{Y}) \),
$a v g(P'_Y)$ and $a v g(Q'_Y)$ present per unit average DG power in neighboring min-MGs;

3. $\Delta \Omega_{I_{II}}$ and $\Delta E_{I_{II}}$ in (3.1e) represent Type II mode regulations on system transit as SSW opens, where $\Delta P'_{XY}$ and $\Delta Q'_{XY}$ present positive-sequence active/reactive power flows through SSW$_{XY}$ in per unit.

4. $\Delta \Omega_{I_{III}}$ and $\Delta E_{I_{III}}$ in (3.1f) represent Type III mode regulations for system transit as SSW closes, where $\theta_{XY}$ and $|V_X|$ present positive-sequence voltage phase mismatch and magnitude at SSW$_{XY}$ on min-MG$_X$ side.

A distributed observer is proposed to observe the average voltage of DGs that are connected. The proposed observer consists of two independent distributed controllers to observe the voltage summation of DGs that are connected and the total number of DGs that are connected, respectively. Detailed controller designs are provided as follows:

$$
E_{\Omega,XY} = E_{XY} + \hat{E}_{\Omega,XY}
$$

(3.2a)

$$
\dot{\hat{E}}_{\Omega,XY} = -k_U \sum_{Y,Z \in \Omega_X} \alpha_{XY} \alpha_{XZ} (E_{\Omega,XY} - E_{\Omega,XZ})
$$

(3.2b)

$$
D_{\Omega,XY} = D_{XY} + \hat{D}_{\Omega,XY}
$$

(3.2c)

$$
\dot{\hat{D}}_{\Omega,XY} = -k_U \sum_{Y,Z \in \Omega_X} \alpha_{XY} \alpha_{XZ} (D_{\Omega,XY} - D_{\Omega,XZ})
$$

(3.2d)

where (3.2a) and (3.2b) represent the voltage summation observer and (3.2c) and (3.2d) represent DG total number observer. $E_{\Omega,XY}$, $\hat{E}_{\Omega,XY}$, $D_{\Omega,XY}$ and $\hat{D}_{\Omega,XY}$ represent state variables of the distributed observer and $k_U$ is a positive control gain; $E_{XY}$ equals to $\frac{1}{D_{S,X}} \sum_{i=1}^{D_X} E_{i,X}$ and $D_{XY}$ equals to $\frac{1}{D_{S,X}} D_X + \frac{1}{D_{S,Y}} D_Y$; term $\alpha_{XY} \alpha_{XZ}$ is used to identify the connection among Min-MGs: it is only when both SSW$_{XY}$ and SSW$_{XZ}$ are closed ($\alpha_{XY} = \alpha_{XZ}$).
\( \alpha_{xz} = 1 \) will the DG voltages in Min-MG\( _x \), Min-MG\( _y \) and Min-MG\( _z \) be included in average DG voltage observation, otherwise they will not be calculated together as they are not connected. Denote \( \psi_{\Omega,xy} = \frac{E_{\Omega,xy}}{D_{\Omega,xy}} \). Both the proposed voltage summation observer in (3.2a) and (3.2b) and the proposed DG total number observer in (3.2c) and (3.2d) are in the standard formation of dynamic consensus algorithm, it can be concluded that:

\[
E_{\Omega,xy}(\infty) = \frac{1}{D_{s,\Omega}} \sum_{x,y \in \Omega} E_{xy}(\infty)
\]

\[
D_{\Omega,xy}(\infty) = \frac{1}{D_{s,\Omega}} \sum_{x,y \in \Omega} D_{xy}(\infty)
\]

where \( D_{s,\Omega} \) represents the total number of connected SSWs in dynamic MG \( \Omega \). Considering the fact that the weighted voltage summation and total number of DGs in Min-MG\( _x \) (\( x \in \Omega \)) are counted by \( D_{s,X} \) SSWs, it can be concluded that term \( \frac{1}{D_{s,X}} \sum_{i=1}^{D_{x}} E_{i,x} \) and \( \frac{1}{D_{s,X}} \sum_{i=1}^{D_{y}} D_{xy} \) are counted \( D_{s,X} \) times in \( \sum_{x,y \in \Omega} E_{xy}(\infty) \) and \( \sum_{x,y \in \Omega} D_{xy}(\infty) \), respectively. \( E_{\Omega,xy}(\infty) \) and \( D_{\Omega,xy}(\infty) \) can then be expressed as:

\[
E_{\Omega,xy}(\infty) = \frac{1}{D_{s,X}} \left( \sum_{i=1}^{D_{x}} E_{i,x}(\infty) + \sum_{i=1}^{D_{y}} E_{i,y}(\infty) + \cdots \right)
\]

\[
D_{\Omega,xy}(\infty) = \frac{1}{D_{s,X}} \left( D_{x} + D_{y} + \cdots \right)
\]

And the average DG voltage of dynamic MG \( \Omega \) can be observed by:

\[
\psi_{\Omega} = \frac{E_{\Omega,xy}(\infty)}{D_{\Omega,xy}(\infty)} = \frac{\sum_{x \in \Omega} \left( \sum_{i=1}^{D_{x}} E_{i,x}(\infty) \right)}{\sum_{x \in \Omega} D_{x}}
\]

Term \( D_{s,X} \) has been cancelled out and not required in the proposed observer. The numer-
ator represents the voltage summation of DGs that are connected and the denominator represents the total number of DGs that are connected. It is noteworthy that the proposed distributed observer requires only voltage measurements that are locally accessible by each SSW using S2D channel and can automatically identify the connection of Min-MGs so that only voltage from DGs that are connected are calculated.

As the controller converges, all the terms on the right side of (3.1b) become zero. System frequency and average DG voltage are regulated as rated by (3.1c). Proportional positive-sequence power sharing among connected DGs are ensured by (3.1c) and (3.1d). At SSW\(_{XY}\), the positive-sequence power flow are minimized by (3.1e) before SSW opens and positive-sequence voltage phasors are synchronized by (3.1f) before SSW closes. At last, control diagram of the proposed positive-sequence controller designs are presented in Fig. 3.1.

### 3.3.2 Negative-sequence Controller Design

Controllers for the \(i\)th DG in min-MG\(_X\) on negative-sequence current and voltage regulations are provided below:

\[
I_i^N = E_i^N(-n_i^N Q_i^N + \Delta G_i^N) + \Delta I_i^N 
\]  \hspace{1cm} (3.3a)

\[
\Delta \dot{G}_i^N = \Delta G_o^N + (1 - \eta_{XY})\Delta G_i^N + \Delta G_{III}^N 
\]  \hspace{1cm} (3.3b)

\[
\Delta G_o^N = -(1 - \lambda_{XY})k_V^N \Delta U F - k_Q^N \sum_{j=1}^{D_X} a_{ij} \Delta Q_{ij}^{N'} 
\]  \hspace{1cm} (3.3c)

\[
\Delta G_i^N = -k_Q^N \sum_{y \in B_X} a_{xy} \Delta Q_{iy}^{N'} 
\]  \hspace{1cm} (3.3d)

\[
\Delta \dot{I}_{II}^N = -\eta_{XY} k_{i,XY}^N \Delta I_{XY}^N 
\]  \hspace{1cm} (3.3e)

\[
\Delta G_{III}^N = -\lambda_{XY} k_V^{N'} \Delta U F_X^{'N} 
\]  \hspace{1cm} (3.3f)
where \( Q^N - G \) negative-sequence virtual conductance control is presented in (3.3a) with \( \Delta G_i^N \) and \( \Delta I_i^N \) as the designed secondary control variables; the definition of negative-sequence reactive power from [Che09] is adopted in this paper, i.e., \( Q_i^N = 3 E_i I_i^N \) to better describe the compensation efforts of individual DG; \( \Delta Q_{ij}^N = Q_i^N - Q_j^N \) presents per unit negative-sequence reactive power outputs mismatch between the \( i \)th and \( j \)th DG; \( I_i^N \) and \( E_i^N \) present DG output negative-sequence current and voltage; \( \overline{VUF} \) presents the average VUF of connected CLB and can be observed using the previously proposed distributed observer, \( VUF^* \) is the designed regulation threshold; \( \Delta VUF \) presents the regulation error: \( \Delta VUF = \overline{VUF} - VUF^* \) if \( \overline{VUF} > VUF^* \); otherwise \( \Delta VUF = 0 \). The proposed secondary
control variables are defined in (3.3b) to (3.3f):

1. $\Delta G_N^O$ in (3.3c) represents regulations on coordinated VU mitigation among connected CLB and power sharing among connected DGs within the same min-MG. When average VUF is below the designed threshold, the system VU is under accepted range. In that case, the VUF regulation is disabled and no reversing regulation effort towards system VU is introduced.

2. $\Delta G_N^I$ in (3.3d) represents Type I mode regulations on DG power sharing between neighboring min-MGs, where $\Delta Q_{iY}^{N'} = Q_{iY}^{N'} - a v g(Q_{iY}^{N'})$, $a v g(Q_{iY}^{N'})$ presents per unit average DG power in neighboring min-MGs.

3. $\Delta I_{II}^N$ in (3.3e) represents Type II mode regulations on system transit as SSW opens, where $\Delta I_{XY}^{N'}$ presents the negative-sequence current that flows through SSW$_{XY}$.

4. $\Delta G_{III}^N$ in (3.3f) represents Type III mode regulations on system transit as SSW closes, where $\Delta VUF_X' = VUF_X - VUF'$, $VUF_X$ presents the VUF measured at SSW$_{XY}$ on min-MG$_X$ side and $VUF'$ presents the reference VUF for SSW reclose. As previously mentioned, phase mismatch of both positive- and negative-sequence voltage are coupled by system frequency deviation, and simultaneous control on both sequences could be conflicting. In this work, the negative-sequence voltage phase is not directly regulated, instead the voltage magnitude (i.e. VUF) on both sides of the SSW are reduced to minimize the transients.

As the controller converges, all the terms on the right side of (3.3b), (3.3e) and (3.3f) become zero. System VU is regulated under desired range by (3.3c). Proportional negative-sequence reactive power sharing among connected DGs are ensured by (3.3c) and (3.3d). At SSW$_{XY}$, the negative-sequence current (power) flow is minimized by (3.3e) before SSW
Figure 3.2: Control diagram of the proposed negative-sequence controller

opens and the negative-sequence voltage magnitudes on both sides of SSW are are minimized by (3.3f) before SSW recloses. It is noteworthy that when implemented under balanced system where the system negative-sequence operation states are naturally close to zero, the proposed negative-sequence controller is automatically disabled to avoid introducing unnecessary negative-sequence components to the system, which makes it applicable for both balanced and unbalanced system. At last, control diagram of the proposed negative-sequence controller is presented in Fig. 3.2.

### 3.4 Small Signal Stability Analysis

Sufficient conditions for system exponential stability are derived. Following lemmas are used for subsequent analysis:
Lemma 1: [Du19b] $M = \text{diag}(M_i)$ is positive definite scalar matrix and $L$ is a Laplacian Matrix of a connected undirected graph. $L + r[1]_nM$ is positive definite when $r$ is a positive constant.

Lemma 2 [Hor90]: If both $M$ and $N$ are positive definite, then $M + N$ is positive definite.

Lemma 3 [Hor90]: If $M$ is positive definite and $N$ is a positive definite scalar matrix, then $MN = NM$ is positive definite.

3.4.1 System Exponential Stability with Positive-sequence Controller

The system is assumed to be stable with primary control. Compared to the dynamics of power loop, the dynamics of inverter inner current/voltage loop is fast enough and can be ignored [MES08]. Since system frequency can be regarded as a global signal that converges fast enough across the entire system, the delay in adjusting DG output frequency can be ignored. The delay in adjusting DG output voltage magnitude is modeled as a first-order low-pass filter for simplification [SP15b]. The system operation status are modeled as:

$$\Delta \dot{\delta}_i = \omega_i - \omega^* = -m_i \frac{E_i E_p}{X_i} \sin(\Delta \delta_i) + \Delta \omega_i$$  \hspace{1cm} (3.4a)

$$\omega_p^{-1} \dot{E}_i = E^* - E_i - n_i \frac{(E_i \cos(\Delta \delta_i) - E_p)E_p}{X_i} + \Delta E_i$$  \hspace{1cm} (3.4b)

where $E_p$ presents the voltage magnitude at PCC; $X_i$ and $\Delta \delta_i (= \delta_i - \delta_p)$ present the equivalent reactance and voltage phase mismatch between the $i$th DG and PCC and $\omega_p$ presents cut-off frequency of the equivalent low-pass filter. To derive system small signal stability, it is assumed that $\sin(\Delta \delta_i) \approx \Delta \delta_i$ and $E_p \approx E^*$. Recall the fact that droop control for inductive electric system is validated when the active power $P_i$ is predominantly dependent only on the phase angle $\theta_i$ while the reactive power $Q_i$ mostly depends on output voltage $E_i$. 

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[Gue11b], it is assumed that in (3.4a), \( E_i \approx E' \) is constant and in (3.4b), \( \cos(\Delta \delta_i) \approx \cos(\delta') \) is constant. Therefore, (5.5) can be simplified as:

\[
\Delta \hat{\delta}_i = -m_i M_i \Delta \delta_i + \Delta \omega_i
\]  
(3.5a)

\[
\omega_t^{-1} \dot{E}_i = -E_i - n_i N_i E_i - \left( \frac{N_i}{\cos \delta'} - 1 \right) E^* + \Delta E_i
\]  
(3.5b)

where for the \( i \) th DG, \( m_i \) and \( n_i \) are the designed \( P-f \) droop gain and \( Q-V \) droop gain, respectively and are designed to be proportional to its rated capacity in this work; \( M_i = \frac{E' E^*}{X_i} \) and \( N_i = \frac{E^*}{X_i} \cos \delta' \) are treated as constants.

Two adjacent min-MGs are considered, min-MG\( X \) and min-MG\( Y \) which are connected by SSW\( _{XY} \). Define the following diagonal matrices regarding min-MG\( X \): \( M_X = diag(M_{i,X}) \), \( N_X = diag(N_{i,X}) \), \( m_X = diag(m_{i,X}) \) and \( n_X = diag(n_{i,X}) \). Define the following diagonal matrices for presentation purpose: \( M_{XY} = diag(M_X, M_Y) \), \( N_{XY} = diag(N_X, N_Y) \), \( m_{XY} = diag(m_X, m_Y) \) and \( n_{XY} = diag(n_X, n_Y) \). Let \([1]_{m,n}\) presents the \( m \) by \( n \) all-ones matrix, otherwise \([1]_n\) presents the \( n \) by \( n \) all-ones matrix.

In Type I and Type II mode, SSW\( _{XY} \) is closed and the operation status of min-MG\( X \) and min-MG\( Y \) are coupled by the power balance constraint presented below:

\[
S_X + S_Y = S_{L,Total}
\]  
(3.6)

where \( S_X \) and \( S_Y \) presents the total power generation within min-MG\( X \) and min-MG\( Y \); \( S_{L,Total} \) presents the total apparent power demand within the connected min-MGs and is treated as constant for analysis.

System small signal stability with proposed frequency/active power flow regulation

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controller is first derived. The linearized system equations are presented in matrix form as:

\[ \dot{X} = WX + U \]  \hspace{1cm} (3.7)

where \( X = [\Delta \delta_{XY}, \Delta \omega_{XY}] \) presents system states, \( W \) presents corresponding system matrix and \( U \) presents variables that are independent from \( X \).

In Type I mode, the corresponding system matrix is derived:

\[
W_N = \begin{bmatrix}
-\frac{m_{XY}M_{XY}}{k_f} & I \\
\frac{k_f m_{XY} M_{XY} - H}{M_{XY}} & I \\
-W_1 & I \\
-W_2 & -k_f I
\end{bmatrix},
\]

where \( H = diag(H_X, H_Y) \) with \( H_X = k_p (L_X + I)M_X + k_p \frac{[1]D_X M_X}{D_Y} \) and \( H_Y = k_p (L_Y + I)M_Y + k_p \frac{[1]D_Y M_Y}{D_X} \). The system is exponentially stable if and only if the eigenvalues of \( W \) have negative real parts. The characteristic polynomial of \( W_N \) can be simplified by Schur complement:

\[ \det(sI - W_N) = \det(sI + k_f I) \det(s^2 I + sW'_1 + W'_2) = 0, \]

where \( W'_1 = W_1 + k_f I \) and \( W'_2 = W_2 + k_f W_1 = H \). It is observed that \( \det(sI + k_f I) = 0 \) satisfied \( Re(s) < 0 \) for all its roots. For \( \det(s^2 I + sW'_1 + W'_2) = 0 \), referring to Routh-Hurwitz stability criterion, it is proved in [SP15b] that roots of such characteristic polynomial satisfies \( Re(s) < 0 \) if and only if the following conditions are satisfied:

\[ \lambda_{min}(W'_1 + W'^T_1) > 0 \] \hspace{1cm} (3.8a)
\[ \lambda_{min}(W'_2 + W'^T_2) > 0 \] \hspace{1cm} (3.8b)

Condition (3.8a) is satisfied as \( W'_1 = m_{XY} M_{XY} + k_f I \) is positive definite. For \( W'_2 = H \), it is observed that when \( M_i \approx M_j \), \( M_X \) could be treated as a positive definite scalar and \( k_p(L_X + \ldots \)
\[
\frac{[1]_{D_X}}{D_Y}M_X \text{ is positive definite referring to Lemma 1. } H_X = k_p (I_X + \frac{[1]_{D_X}}{D_Y})M_X + k_p M_X \text{ is also positive definite referring to Lemma 2. The same conclusion can be made for } H_Y \text{ and thus } W'_2 \text{ is positive definite. Condition (3.8b) is satisfied under the approximation that } M_i \approx M_j \text{ for the system, which presents the sufficient condition for system exponential stability in Type I mode.}
\]

In Type II mode, the system matrix is found to be:

\[
W_S = \begin{bmatrix}
-m_{XY}M_{XY} & I \\
k_f m_{XY}M_{XY} - H & -k_f I
\end{bmatrix},
\]

where \( H = \text{diag}(H_X, H_Y) \), \( H_X = k_p L_X M_X + 2k_p X_Y k_p \frac{[1]_{D_X}}{D_Y} M_X \), \( H_Y = k_p L_Y M_Y + 2k_p X_Y k_p \frac{[1]_{D_Y}}{D_Y} M_Y \). Referring to the similar derivation done for Type I mode, it is concluded that eigenvalues of \( W_S \) have negative real parts if \( H \) is proved to be positive definite. It is observed that when \( M_i \approx M_j \), \( H_X \) is positive definite referring to Lemma 1. The same conclusion can be made to \( H_Y \) and thus \( H \) is positive definite. \( M_i \approx M_j \) presents the sufficient condition for system exponential stability in Type II mode.

System small signal stability with proposed voltage/reactive power flow regulation controller is then derived. For simplification, it is assumed that \( \psi_{\Omega}(t) = \text{avg}[E_{\Omega}(t)] \). The linearized system equations are presented in matrix form as:

\[
\dot{X} = \omega_v W_X + \omega_v U \quad (3.9)
\]

where \( X = [E_{XY}, \Delta E_{XY}] \) presents system states.

In Type I mode, the corresponding system matrix is derived:

\[
W_N = \begin{bmatrix}
-I - n_{XY}N_{XY} & 1 \\
-k_e T - J & 0
\end{bmatrix} = \begin{bmatrix}
-W_1 & 1 \\
-W_2 & 0
\end{bmatrix},
\]
where \( T = \begin{bmatrix} [1]_{Dx} & [1]_{Dx,Dy} \\ D_X & [1]_{Dx,Dy} \\ D_Y & [1]_{Dx,Dy} \\ D_X & [1]_{Dx,Dy} \end{bmatrix} \) and \( J = diag(J_X, J_Y) \) with \( J_X = k_Q (L_X + I) N_Y + k_Q [1]_{Dx} \) and \( J_Y = k_Q (L_Y + I) N_Y + k_Q [1]_{Dy} \). The characteristic polynomial of \( W_N \) can be simplified by Schur complement and is derived as:

\[
det(sI - W_N) = \det(sI + W_1) \det(s^2 I + sW_1 + W_2) = 0.
\]

It is observed that \( W_1 = I + n_{XY} N_{XY} \) is positive definite. The roots of \( \det(sI - W_N) = 0 \) satisfies \( Re(s) < 0 \) if and only if the following conditions are satisfied:

\[
\lambda_{min}(W_1 + W_1^T) > 0 \quad (3.10a)
\]
\[
\lambda_{min}(W_2 + W_2^T) > 0 \quad (3.10b)
\]

Condition (3.10a) is satisfied as \( W_1 \) is positive definite. To derive the eigenvalue of \( W_2 \), set \( k_E = 0 \) and \( W_2 \) is reduced to \( W_2 = J \). \( J \) is in the similar form as \( H \) in frequency/active power case and it could be concluded that \( W_2 \) and thus \( W_2 + W_2^T \) is positive definite when \( N_i \approx N_j \) and \( k_E = 0 \). Consider the fact that eigenvalues are continuous functions of matrix parameters, condition (3.10b) can be satisfied under the approximation that \( N_i \approx N_j \) for the system and \( k_E > 0 \) is sufficiently small, which present the sufficient conditions for system exponential stability in Type I mode.

In Type II mode, the system matrix is found to be:

\[
W_S = \begin{bmatrix} -I - n_{XY} N_{XY} & I \\ k_E T - J & 0 \end{bmatrix},
\]

where \( J = diag(J_X, J_Y) \), \( J_X = k_Q L_X N_Y + 2\mu_Q k_Q \sum Q_{XY} N_Y \), \( J_Y = k_Q L_Y N_Y + 2\mu_Q k_Q \sum Q_{XY} N_Y \). Referring to the similar derivation done for Type I mode, it is concluded that when \( k_E > 0 \) is
sufficiently small, eigenvalues of $W_s$ have strictly negative real parts if $J + J^T$ is proved to be positive definite. It could be observed that when $N_i \approx N_j$, $J_X = k_Q L_X N_Y + 2 \mu Q k_Q \sum Q_{XY}^1 N_Y$ is positive definite referring to Lemma 1. The same conclusion can be made to $J_Y$. $J$ and thus $J + J^T$ is positive definite. $N_i \approx N_j$ and $k_E > 0$ is sufficiently small present the sufficient conditions for system exponential stability in Type II mode.

In Type III mode, $SSW_{XY}$ is open and operation status of the two neighboring min-MGs are independent, thus the system stability analysis can be applied to each min-MG respectively. Taking min-MG $X$ as an example, according to Millman's theorem [Mil40], the voltage phasor measured by $SSW_{XY}$ could be expressed in a complex form:

$$|V_X|(1 + j \theta_X) = \sum_{i=1}^{D_X} c_i E_i (1 + j \delta_i)$$  \hspace{1cm} (3.11)

where $c_i = X_p / X_i$ and $(X_p)^{-1} = \sum_{i=1}^{D_X} (X_i)^{-1}$ presents system equivalent parallel reactance. Both real and imaginary parts on both sides of (5.10) should be equal respectively and the voltage magnitude and phase at $SSW_{XY}$ can be expressed as $|V_X| = [1]_{1,D_X} C E_X$ with $C = diag(c_i)$ and $\theta_X = [1]_{1,D_X} D \delta_X$ with $D = diag(E_i / |V_X|_i c_i) \approx diag(E' / |V_X|_i c_i)$.

For system with proposed frequency/active power flow regulation controller, the linearized system equations could be expressed under the same form as (5.8) with $X = [\Delta \delta_X, \Delta \omega_X]$ and $W_M = \begin{bmatrix} -m_X M_X & I \\ k_f m_X M_X - H & -k_f I \end{bmatrix}$ with $H = k_p L_X M_X + \mu \theta [1]_{D_X} D$. Referring to the similar derivation done for Type I mode, it could be concluded that eigenvalues of $W_M$ have strictly negative real parts if $H + H^T$ is proved to be positive definite. It could be observed that when $M_i \approx M_j$, $H$ can be written as: $H = k_p (L_X + \mu \theta [1]_{D_X} D M_X^{-1}) M_X$. Recall the definition of $M_X$ and $D$, it is found that $D M_X^{-1} \approx diag(X_p / |V_X| E^*)$ can be treated as a scalar matrix and term $L_X + \mu \theta k_p^{-1} [1]_{D_X} D M_X^{-1}$ is positive definite referring to Lemma 1. Referring to
Lemma 3, $\mathbf{H}$ and thus $\mathbf{H} + \mathbf{H}^T$ is positive definite. $M_i \approx M_j$ presents the sufficient condition for system exponential stability in Type III mode.

For system with proposed voltage/reactive power flow regulation controller, the linearized system equations could still be expressed under the same form as (5.11) with $\mathbf{W}_M = \begin{bmatrix} -1 - n_X N_Y & 1 \\ -k_0 L_X N_Y - \mu_V [I] D_x C & 0 \end{bmatrix} = \begin{bmatrix} -W_1 & 1 \\ -W_2 & 0 \end{bmatrix}$ and $\mathbf{X} = [\mathbf{E}_X, \Delta \mathbf{E}_X]$. Referring to the similar derivations done for Type I mode, it could be concluded that eigenvalues of $\mathbf{W}_M$ have strictly negative real parts if $\lambda_{\text{min}}(W_2 + W_2^T) > 0$ is satisfied. It can be proved that when $N_i \approx N_j$, $W_2 + W_2^T$ is positive definite by the same reasoning as previously presented. $N_i \approx N_j$ presents the sufficient condition for system exponential stability in Type III mode.

In conclusion, sufficient conditions under which system is exponentially stable in each operation modes are found to be: $M_i \approx M_j$, $N_i \approx N_j$ and $k_E > 0$ is sufficiently small.

3.4.2 System Exponential Stability with Negative-sequence Controller

Sufficient conditions under which the proposed controllers are exponentially stable are discussed in this section. As aforementioned, regulations over system positive- and negative-sequence voltage are decoupled, while system frequency is regulated with respect to only system positive-sequence operation states. For unbalance system analysis, the unbalance in power cables is usually ignored and unbalanced loads are considered [Men16]. Y-connected unbalanced loads are considered in this work, as delta-connected loads can be transferred to the Y-connected ones. An electric system with an unbalanced load is shown in Fig. 3.3, where the unbalanced load, $Y_u$ is between phase a and b. $Y_a$, $Y_b$ and $Y_c$ present load on each phase, and they can be either balanced ($Y_a = Y_b = Y_c$) or unbalanced ($Y_a \neq Y_b \neq Y_c$). Let $\mathbf{T}$ presents the transform matrix from three-phase components to symmetric components, the symmetrical components of such system could be calculated by:
Figure 3.3: Y-connected unbalanced load equivalent circuit

\[ I_{012} = Y_{012} \cdot E_{012} = T^{-1} \cdot Y_{abc} \cdot T \cdot E_{012} \]  
(3.12)

where \( Y_{abc} \) is given by:

\[
\begin{bmatrix}
Y_a + Y_u & -Y_u & 0 \\
-Y_u & Y_b + Y_u & 0 \\
0 & 0 & Y_c
\end{bmatrix}
\]

and the system negative-sequence current is obtained as:

\[ I^N = \frac{1}{3}[(Y_a + \alpha Y_b + \alpha^2 Y_c) - 3\alpha^2 Y_u]E + \frac{1}{3}[(Y_a + Y_b + Y_c) + Y_u]E^N = Y^P \cdot E + Y^N \cdot E^N \]  
(3.13)

where \( Y^P \) and \( Y^N \) are equivalent admittance and \( \alpha = e^{2\pi i/3} \).

The delay in adjusting DG output negative-sequence current magnitude is modeled as a first-order low-pass filter for simplification [SP15b]. For the \( i \)th DG, its negative-sequence operation states could be described as:

\[ \omega_i^{-1} I_i^N = -I_i^N + E_i^N (-n_i^N Q_i^N + \Delta G_i^N) + \Delta I_i^N \]  
(3.14)

where \( \omega_i \) presents cut-off frequency of the equivalent filter.

Two adjacent min-MGs, min-MG\(_X\) and min-MG\(_Y\) are considered with one equivalent CLB. The positive-sequence voltage in this case is treated to be constant as \( E_i \approx E' \),
and $E_i^N = E_i V U F \approx E' V U F' = E''$ is constant in (3.14). Recall $Q_i^N = 3E_i I_i^N$, and the system negative-sequence operation states could be modeled as:

$$\omega_i^{-1} I_{XY}^N = -(I + N^N) I_{XY}^N + E'' \Delta G^N + \Delta I^N$$

(3.15)

where $I_{XY}^N = [I_{X,1}^N, \ldots, I_{X,Dx}, I_{Y,1}^N, \ldots, I_{Y,Dy}]$ is the states vector, $N^N = diag(N_i^N)$ where $N_i^N = 3E' E'' n_i^N$ is constant. Balance between system power generation and consumption should always be met, which is expressed as:

$$\sum Q_{i,X}^N + \sum Q_{i,Y}^N = Q_L^N$$

(3.16)

where $Q_L^N$ presents the system total negative-sequence reactive power demand and is treated as constant for analysis. Following relationship can be derived by substituting $E_i \approx E'$ and $Q_i^N = 3E_i I_i^N$ into (3.16):

$$\sum I_{X}^N + \sum I_{Y}^N = \frac{Q_L^N}{3E'}$$

(3.17)

Referring to Millman’s Theorem [Mil40], the negative-sequence voltage on CLB can be expressed as:

$$E_L^N = \sum c_i^L E_i^N = C^L E$$

(3.18)

where $c_i^L = X_{p,i,t}^{-1}$, $X_{i,t}$ is the reactance between the $i$th DG and CLB, $(X_{p,t})^{-1} = \sum(X_{i,t})^{-1}$ and $C^L = diag(c_i^L)$. System voltage at POI can be expressed similarly:

$$E_{PCC}^N = \sum c_i^P E_i^N = C^P E^N$$

(3.19)

where $C^P = diag(c_i^P)$ and $c_i^P = X_{p,i}^{-1}$, $X_{i,p}$ is the reactance between the $i$th DG and POI. Combining (3.3), (3.17) and (5.18), the dynamics of $\Delta G^N$ and $\Delta I^N$ are derived (states
independent to $I_{XY}^N$ are ignored):

$$\Delta \dot{G}^N = G^N_O + (1 - \eta_{XY})G^N_I + G^N_{III}$$  \hspace{1cm} (3.20a)$$

$$G^N_O = (\lambda_{XY} - 1)\frac{k^N_{V}}{E'} \tilde{C}^L_{XY} E^N_{XY} - 3k^N_{Q} E' L_{XY} I^N_{XY} \hspace{1cm} (3.20b)$$

$$G^N_I = -3a_{XY} k^N_{Q} E'(1 + D_{XY}) I^N_{XY} \hspace{1cm} (3.20c)$$

$$G^N_{III} = -\lambda_{XY} k^N_{V} \tilde{C}^p_{XY} E^N_{XY} \hspace{1cm} (3.20d)$$

$$\Delta I^N = -2\eta_{XY} [1]_{D_X, D_Y, k^N_{I, XY}} I^N_{XY} \hspace{1cm} (3.20e)$$

where $\tilde{C}^L_{XY} = [1]_{D_{XY}}, C^L_{XY}, L_{XY} = diag(L_X, L_Y), D_{XY} = diag[1]_{D_X, [1]_{D_Y}}$ and $\tilde{C}^p_{XY} = diag(C^p_X, C^p_Y)$. At last, the linear relationship between $I_{XY}^N$ and $E_{XY}^N$ is derived from (3.13):

$$E_{XY}^N = \frac{I_{XY}^N - Y^M E'}{Y^{NN}} = Z_{NN}^N I_{XY}^N - \frac{Y^M}{Y_{NN}^N} E'$$ \hspace{1cm} (3.21)$$

Combining (3.15), (3.20) and (3.21), the linearized system equations could be expressed in matrix form as:

$$\dot{X}^N = W^N X^N$$ \hspace{1cm} (3.22)$$

where $X^N = [I_{XY}^N, \Delta G^N, \Delta I^N]$ is states vector and $W^N$ presents system matrix. It can be
concluded that the negative-sequence controller is exponentially stable when all the eigenvalues of $W^N$ under each mode have strictly negative real parts.

Under Type I operation mode, the system matrix could be found as:

$$W^N_I = \begin{bmatrix} -\omega (I + N^N) & \omega I E'' I \\ \frac{k_v}{E'} Z^{NN} \hat{C}^{L}_{XY} - G & 0 \end{bmatrix}$$

where $G_I = \frac{3k_N}{Q} E'(L_{X+Y} + I + D_{X+Y})$. Denote $W^N_I = \begin{bmatrix} -H^I_1 & 1 \\ -H^I_2 & 0 \end{bmatrix}$ and the characteristic polynomial of $W^N_I$ can be simplified by Schur Complement: $\det(sI - W^N_I) = \det(sI + H^I_1) \det(s^2 I + sH^I_1 + H^I_2) = 0$. It is observed that $\det(sI + H^I_1) = 0$ satisfy $Re(s) < 0$ for all its roots as $H^I_1$ is positive definite. For $\det(s^2 I + sH^I_1 + H^I_2) = 0$, the roots of such characteristic polynomial satisfies $Re(s) < 0$ if and only if the following conditions are satisfied:

$$\lambda_{min}(H^I_1 + (H^I_1)^T) > 0$$  \hspace{1cm} (3.23a)

$$\lambda_{min}(H^I_2 + (H^I_2)^T) > 0$$  \hspace{1cm} (3.23b)

Condition (3.23a) is satisfied as $H^I_1$ is positive definite. For $W_2$, set $k_v^N = 0$ first and $H^I_2$ reduced to $H^I_2 = H = diag(H_X, H_Y)$, where $H_X = 3k_N E' (L_X + I + \frac{[1]D_X}{D_X})$ and $H_Y = 3k_N E' (L_Y + I + \frac{[1]D_Y}{D_Y})$.

Referring to Lemma 1 and Lemma 2, it can be proved that $H_X$ is positive definite. Then $H_Y$ and thus $H$ can be proved to be positive definite. $W_2 + W_2^T$ is positive definite when $k_v^N = 0$. Consider the fact that eigenvalues are continuous functions of matrix parameters, condition (3.23b) can be satisfied under when $k_v^N > 0$ is sufficiently small, which present the sufficient conditions for system exponential stability in Type I mode.

Under Type II operation mode, set $k_v^N = 0$ and denote $\Delta H = E'' \Delta G + \Delta I$, the system matrix could be found as:

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\[
W_{II}^N = \begin{bmatrix}
-\omega_j(I+N^N) & \omega_jI \\
-3k_q^N E'E''L_{XY} - G_{II} & 0
\end{bmatrix}
\]

where \( G_{II} = 2k_{I,XY}^N[1]_{D_x,D_y} \). The characteristic polynomial of \( W_{II}^N \) could be calculated as:

\[
det(sI - W_{II}^N) = det[sI + \omega_j(I+N^N)]det(s^2I + sH_{II}^I + H_{II}^I) = 0,
\]

where \( H_{II}^I = \omega_j(I+N^N) \) and \( H_{II}^I = 2\omega_j k_{I,XY}^N[1]_{D_x,D_y} + 3\omega_j k_q^N E'E''L_{XY} \). For \( det[sI + \omega_j(I+N)] = 0 \), it can be easily observed that all its roots satisfy \( Re(s) < 0 \) as term \( \omega_j(I+N) \) is positive definite. For \( det(s^2I + sH_{II}^I + H_{II}^I) = 0 \), it has been proved that all its roots satisfy \( Re(s) < 0 \) if both \( H_{II}^I \) and \( H_{II}^I \) are positive definite. It can be easily observed that \( H_{II}^I \) is positive definite. It can also be proved that \( H_{II}^I \) is positive definite referring to Lemma 1. Following the previous derivations, it is concluded that the system in Type II mode is exponentially stable when \( k_v > 0 \) is sufficiently small.

Under Type III operation mode, denote \( \Delta H = E''\Delta G + \Delta I \) and the system matrix could be found as:

\[
W_{III}^N = \begin{bmatrix}
-\omega_j(I+N^N) & \omega_jI \\
-3k_q^N E'E''L_{XY} - G_{III} & 0
\end{bmatrix}
\]

where \( G_{III} = Z_{NN}^N k_{v,XY}^N \tilde{C}_{XY}^p \). The characteristic polynomial of \( W_{III}^N \) could be calculated as:

\[
det(sI - W_{III}^N) = det[sI + \omega_j(I+N)]det(s^2I + sH_{III}^I + H_{III}^I) = 0,
\]

where \( H_{III}^I = \omega_j(I+N^N) \) and \( H_{III}^I = \omega_j k_{v,XY}^N Z_{NN}^N \tilde{C}_{XY}^p + 3\omega_j k_q^N E'E''L_{XY} \). It has been proved that all the roots of \( det[sI + \omega_j(I+N)] = 0 \) satisfy \( Re(s) < 0 \). For \( det(s^2I + sH_{III}^I + H_{III}^I) = 0 \), it can be easily observed that \( H_{III}^I \) is positive definite and \( H_{III}^I \) is positive definite referring to Lemma 1, and thus all its roots satisfy \( Re(s) < 0 \). Following the previous derivations, it is concluded that the system is exponentially stable under Type III. It is noteworthy that VUF regulation on CLB is disabled in this operation mode and thus the selection of \( k_v^N \) does not effect the system stability.
Table 3.1: Controller Gain Designs

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Symbol</th>
<th>Value</th>
<th>Symbol</th>
<th>Value</th>
<th>Symbol</th>
<th>Value</th>
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<td>$k_p$</td>
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<td>0.75</td>
<td>$k_{θ,XY}$</td>
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<td>$k_Q$</td>
<td>7.5</td>
<td>$k_{Q,XY}$</td>
<td>15</td>
<td>$k_{V,XY}$</td>
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<td>12.5</td>
<td>$k_Q^N$</td>
<td>7.5</td>
<td>$k_{I,XY}^N$</td>
<td>0.75</td>
<td>$k_{V,XY}^N$</td>
<td>12.5</td>
</tr>
</tbody>
</table>

Figure 3.4: Modified IEEE 34-bus test system

3.5 Hardware Experiments and Case Study

The proposed controller is validated using a 34-bus system with following modifications over the standard IEEE 34-bus test feeder: 1) All the single-phase line-to-neutral loads are modified into three phase balanced loads with equal power rating to eliminate the system zero-sequence components; 2) Grid-forming DGs with the same power rating ($S^* = 300$ kVA) are introduced at bus 814, 890, 844, 838 and 836. SSWs are introduced between bus 816 and 824, bus 832 and 858 and bus 834 and 860. The test system is divided into four min-MGs by three SSWs ($SSW_{12}$, $SSW_{23}$ and $SSW_{34}$) and five DGs (DG1 to 5), as shown in Fig. 3.4. Spot loads at bus 830, 848 and 840 are balanced and defined as CLB. Selected control gains are presented in Table 3.1.
In this case, the test system is islanded due to severe events and only DG primary control on both positive- and negative-sequence are enabled at initial stage. As shown in Fig. 3.5, the system initially operates under Topology #1 with $SSW_{12}$ and $SSW_{23}$ being closed and $SSW_{34}$ open. System restoration is activated in the context of dynamic MGs and the proposed controller under Type I mode is initiated at $t=t_1=5$ s. The system is then requested to transit from Topology #1 to #2 at $t=t_2=27$ s and then to #3 at $t=t_4=92$ s.

The system frequency and positive-sequence voltage measured at bus 830 are presented in Fig. 3.6, along with the VUFs at each CLB. Before Type I mode is initiated ($t<t_1$), the system operates under primary control with steady state operation errors ($\Delta f_{SS}=1.36$ Hz and $\Delta V_{SS}=0.06$ p.u.), and the VUFs at each CLB are above threshold. After Type I controller is energized ($t\geq t_1$), the system frequency and voltage are regulated as rated and the VUFs at each CLB are minimized as desired. Without the proposed control strategy, the system operates constantly with Type I controller, and the reconfiguration is done by directly closing
SSW\textsubscript{34} at \( t = t_3 = 60 \text{ s} \), and opening SSW\textsubscript{23} at \( t = t_5 = 121 \text{ s} \). As shown in Fig. 3.6, it could be observed that without the proposed control strategy, significant transients will be generated each time the system transits (\( |\Delta f_{\text{max}}| = 1.6 \text{ Hz} \) and \( |\Delta V_{\text{max}}| = 0.2 \text{ p.u.} \)). To achieve seamless system reconfiguration, the proposed control strategy is implemented. The reconfiguration is done by first initiating Type III mode at \( t = t_2 \). It is noteworthy that at this stage (\( t_1 < t < t_2 \)), the control objective of system voltage regulation has switched from average DG voltage to the voltage at SSW\textsubscript{23}, which would result in bus voltage deviations accordingly to the rest of system, as shown by black curve in Fig. 3.6. After SSW\textsubscript{34} closes at \( t = t_3 \), the controller switches back to Type I mode before the next reconfiguration request is received. At \( t = t_4 \), the proposed controller switches to Type II mode. After SSW\textsubscript{23} opens at \( t = t_5 \), the controller
switches back to Type I mode again and manages the system under new topology. It is observed from Fig. 3.6 that with the proposed control strategy, no significant transients are introduced during system reconfiguration and VUF at each CLB has been kept below threshold.

To further validate the effectiveness of the proposed controller under Type II and III mode, real-time phase A voltage on both sides of SSW_{34} at \( t=t_2 \) and \( t=t_3 \) are presented in Fig. 3.7a and Fig. 3.7b, respectively. It could be observed that the voltage phasors are asynchronous before Type III mode is enabled: the phase mismatch, \( |\theta_{34}| \) is about 30° and the voltage peak value mismatch, \( |\Delta V_{34}| \) is about 0.04 p.u. With Type III control, the mismatches
Figure 3.9: Recorded DG power outputs under varying system topology

are minimized and the voltage phasors are synchronized before SSW34 closes. Additionally, real-time three-phase current flow through SSW23 at $t=t_4$ and $t=t_5$ are presented in Fig. 3.8a and Fig. 3.8b, respectively. Before Type II controller is enabled, a considerable amount of current flow through the SSW could be observed, as $\Delta I_{23} \approx 4$ A in RMS value. With Type II control, the current flow is minimized, as $\Delta I_{23} \approx 0.28$ A in RMS value before SSW23 opens.

At last, the measured DG power outputs are presented in Fig. 3.9. It is observed that proportional power sharing among connected DGs are achieved during system reconfiguration: The system initially operates under Topology #1 ($t<t_2$), and DG1 to 3 are grouped together while DG4 to 5 belong to another group. After SSW34 closes and the system transits to Topology #2 ($t_3 \leq t < t_5$), DG1 to 5 are connected and share the demands together. It is noteworthy when Type II control is activated ($t_4 \leq t < t_5$), though the system still operates
under Topology #2, power sharing between DGs that belong to min-MG$_2$ and min-MG$_3$ is disabled, while the ones for the rest connected min-MGs are still enabled (min-MG$_1$ and min-MG$_2$, and min-MG$_3$ and min-MG$_4$). After SSW$_{23}$ opens and the system transits to Topology #3 ($t \geq t_4$), DG1 and 2 are connected while DG3 to 5 are connected, respectively.

### 3.6 Conclusion

In this section, a distributed secondary control strategy for grid-forming inverter interfaced DGs in unbalanced dynamic MGs is proposed. System frequency, voltage and VU at CLB are under constant regulation. Under static system topology, the proposed control strategy is able to automatically group and manage connected DGs to proportionally share total power demands. When system topology variation is requested, the proposed control strategy is able to satisfy the criteria of seamless system transitions before the reconfiguration is physically fulfilled and the VUF is minimized to the desired level. Additionally, the proposed control strategy utilizes fully distributed control approach that requires only sparse communication network and avoids system single point of failure. The proposed controller designs are implemented under hardware level and validated on a 34-bus system using a HIL MG testbed. It is proved that the proposed control strategy is able to improve system resiliency by providing coordinated regulations on both system positive- and negative-sequence operation states, and achieve seamless system transition.
To seamlessly transition a MG from islanded to grid-connected mode, it is necessary to synchronize the magnitude, frequency, and phase of the MG voltage to the voltage of the main grid. In this section, we propose a distributed control strategy to achieve synchronized operation of an islanded MG supported by multiple VSI-VCMs. The proposed method utilizes a pinning-based consensus algorithm to ensure explicit coordination between magnitude, frequency and phase angle regulation, while ensuring proportional power sharing. System frequency is regulated by all the DGs in proportion to their capacity, while a selected DG eliminates the phase and magnitude regulation errors. Controller design criteria is based on small-signal stability analysis. The proposed control strategy is implemented in hardware controllers and its effectiveness is demonstrated using a real-time hardware-in-the-loop MG testbed.
4.1 Review of Microgrid Synchronization Approaches

As defined in [829], a MG must be able to operate in islanded and grid connected modes, and to seamlessly transition between the two operating modes. Before an islanded MG reconnects to the main grid, voltage phasors on both sides of the PCC (or POI) must be synchronized to each other. Any voltage phasor mismatch introduces transient currents into MG after reconnection, where the severity of the transient is a function of both voltage phasor mismatch and MG topology at the instant when the relay at the PCC closes [Tan15]. Considering the grid voltage phasor to be uncontrollable, the synchronization is done by regulating the MG voltage phasor to match the grid voltage phasor. Voltage phasor is composed of three elements: magnitude, frequency, and phase shift with respect to a reference (i.e. main grid). For an inductive MG, system voltage magnitude is not coupled with the other states and can be directly regulated. Frequency regulation and phase regulation are naturally coupled: phase shift can only be controlled by mismatched system frequency, while eliminating the frequency mismatch will keep the phase mismatch constant.

Despite many problem formulations and implementations (centralized vs. decentralized, regulation using a single DG operation vs. multiple DGs, etc.), the ultimate control objective of MG synchronization is to match the voltage phasors on both sides of PCC. State of the art technique for MG synchronization, which is adopted in many industrial implementations, uses a single DG to regulate the MG voltage phasor while other DGs always operate under grid following mode [Sgi]. The DG responsible for regulating the MG voltage phasor operates in grid forming mode and ensures that its voltage phasor matches that of the main grid. This approach reduces the communication requirement and computational burden of the synchronization controller. However, the DG operating in grid forming mode must have significant spare capacity capable of compensating for any load
variation, without the help of the grid following DGs. Therefore, the capacity of the grid forming DG needs to be large and its dynamic response very fast. This solution becomes economically unfeasible for large MGs, or systems where there is no single large-capacity DG.

An alternative way of achieving MG synchronization is by allowing multiple DGs to coordinate and contribute corporately. The existing approaches differ in how they formulate the problem of frequency regulation. In [AT12], frequency regulation is first initiated, and a small frequency mismatch is preserved for phase regulation. Since the frequency deviation is never eliminated, the proposed strategy is best suited for systems where a small frequency mismatch will not result in undesired transients after grid connection. In [Jin12], when phase mismatch falls in a certain range, the control switches back to frequency regulation. This approach assumes that the frequency regulation is instantaneous, so that the phase mismatch regulation is maintained, which is hard to guarantee. In [Lee13], both frequency and phase mismatch at PCC are measured respectively and eliminated using two independent controllers. Such approach could achieve both frequency and phase regulation simultaneously but does not solve the natural conflict between the two regulation loops that could result in system instability. Additionally, it has been assumed in most works that the main grid operates constantly as rated and thus the islanded MG is regulated to 60 Hz by default [Pap17]. However, such assumption might not necessarily be the case and the main grid frequency should be actively followed in case of any variation.

Another approach, presented in [Tan15; Sun17; Shi17], is to focus on eliminating the phase mismatch, since eliminated phase mismatch will also result in a matched frequency. Phase mismatch is captured from cross product of the main grid and MG voltage phasors and is used as the feedback control parameter in the MG synchronization algorithm. By regulating only phase mismatch while ignoring the dynamic response of the frequency
mismatch, this approach is able to circumvent the conflict caused by frequency and phase coupling. However, such approach has two main drawbacks. First, without any restriction on system frequency variation, the islanded MG could be destabilized. The unbounded frequency variation could potentially jeopardize components that are frequency sensitive like synchronous motors. Second, since the cross product calculates the \( \sin() \) function, the discontinuity caused by a mismatch around 90 degrees may cause the convergence speed to be undesirably slow.

Beyond phase and frequency regulation, voltage magnitude regulation is necessary for a successful transition to grid connected mode. This problem is closely linked to the MG voltage restoration in the islanded mode [TA16; Lee13; AT12], where the main grid voltage phasor is measured and sent to all participating DGs. However, this approach is only effective for systems where the entire MG is connected to a single bus, or where the impedance between the MG buses are negligible. Otherwise, a control approach that attempts to bring the voltage at each bus to that of the grid may cause undesired reactive power flow. In [Nas16], a distributed control approach is proposed to adjust the average voltage of the MG to the grid voltage, allowing for natural voltage variations between the MG nodes, while ensuring all DGs maintain proportional reactive power sharing. This approach improves system robustness, but may not eliminate the voltage magnitude mismatch at the node closest to the PCC.

Additional control objectives for the islanded MGs include: automatic compensation on system unbalance and harmonic distortion [Tan15], accurate power sharing among all the DGs [Sun17], improved controller dynamic response [Tah17], robustness against communication delay [Abh17], etc. Based on the existing control approaches, desired features of MG synchronization controller are listed below, and marked bold in Fig.4.1:
• The islanded MG should always be operated synchronously with the main grid. This allows the microgrid to connect to the main grid as soon as the connection command is issued.

• Frequency and phase synchronization should be coordinated to improve reliability. Regulating only the phase can potentially destabilize the system due to the unbounded system frequency variation.

• Voltage magnitude regulation must eliminate the voltage magnitude mismatch at PCC while ensuring proportional reactive power sharing between DGs to avoid circulating reactive power flow.

• All controllable DGs should contribute proportionally to support the total system

![Figure 4.1: Review of MG synchronization approaches](image-url)
consumption and therefore to eliminate the phasor mismatch.

- Compared to centralized control, distributed control avoids single point of failure in the MG.

### 4.2 Proposed Microgrid Synchronization Control Strategy

As previously reviewed, dispatchable resources within an islanded MG need to actively and accurately regulate their operation states to achieve MG synchronization. In this work, we consider the DGs under discussion are all VSI-VCMs equipped with droop controller for inductive system as primary control\[^{[HL11]}\]. Grid following DGs are accounted for in the model as being a part of the stochastic load profile.

Due to the nature of MG synchronization problem, the voltage phasor mismatch on both sides of PCC in frequency, phase and magnitude are usually measured by the main relay and not locally accessible to all the DGs. Pinning-based consensus algorithm is utilized in this paper to avoid additional communication channels. The measured voltage phasor mismatch is only shared to the pinned DG(s) while all the DGs operate coordinately through a sparse communication network. A DG is called phasor regulation DG (PR-DG) if it is pinned, otherwise it is called supporting DG.

#### 4.2.1 Frequency/phase regulation controller

The proposed frequency/phase regulation controller is defined as follows:

$$\omega_i = \omega^* - m_i P_i + \Omega_i$$  \hspace{1cm} (4.1a)
\(- \frac{d \Omega_i}{d t} = k_f (\omega_i - \bar{\omega}_i) + k_p \sum_{j=1}^{N} a_{ij} (\Omega_i - \Omega_j) + r_i \Delta \delta_C \) \hspace{1cm} (4.1b)

where for the \( i - t \)th DG \((i = 1, \cdots, N)\), \( \omega_i \) and \( \omega^* \) present its measured and rated operation frequency; \( m_i \) presents the designed droop control gain; \( P_i \) presents the measured active power output; \( \Delta \delta_C \) presents the measured phase mismatch between the islanded MG and the main grid at PCC; \( k_f, k_p \) and \( r_i \) present the designed control gains: \( r_i = r > 0 \) if the \( i - t \)th DG is selected to be the phasor regulation DG, otherwise \( r_i = 0 \); \( \Omega_i \) presents the frequency/phase control variable; \( \bar{\omega}_i \) presents the corrected frequency regulation term that is used to track the main grid frequency and is defined as:

\[
\bar{\omega}_i = \begin{cases} 
\omega_G & r_i > 0 \\
\omega^* - k_\omega \sum_{j=1}^{N} a_{ij} (\bar{\omega}_i - \bar{\omega}_j) d t & \text{otherwise}
\end{cases}
\] \hspace{1cm} (4.2)

where \( \omega_G \) presents the measured main grid frequency from the main relay and \( k_\omega \) presents the designed controller gain. (4.2) indicates that the measured main grid frequency will be utilized directly by the phasor regulation DG(s) for correction and eventually shared to all the supporting DGs using a standard pinning-based consensus algorithm [Liu13], where the supporting DGs that have direct communication links with the phasor regulation DG(s) are pinned.

In steady state, the frequency correction terms in each DG are uniform and converge to the measured main grid frequency \((\bar{\omega}_i = \bar{\omega}_j = \omega_G, \text{ for } i, j = 1, \cdots, N)\). The derivation in (4.1b) becomes zero and the following statements are true: 1) \( \omega_i = \bar{\omega}_i = \omega_G \), meaning that the frequency mismatch between the main grid and islanded MG is eliminated; 2) \( \Omega_i = \Omega_j \), meaning that the droop curves in each DG have been shifted equally, indicating proportional active power sharing among DGs [SP15b]; 3) \( \Delta \delta_C = 0 \), meaning that the phase
mismatch between the main grid and islanded MG is eliminated.

### 4.2.2 Voltage regulation controller

As reviewed, voltage regulation is decoupled from frequency/phase regulation and thus can be done separately. The proposed voltage regulation controller is defined as follows:

\[
E_i = E^* - n_i Q_i + e_i
\]  \hspace{1cm} (4.3a)

\[
- \frac{d e_i}{dt} = k_Q \sum_{j=1}^{N} a_{ij} (\frac{Q_i}{Q_i^*} - \frac{Q_j}{Q_j^*}) + \beta_i \Delta E_C
\]  \hspace{1cm} (4.3b)

where \(E^*\) represents the rated voltage; \(e_i\) is the voltage regulation variable; \(\kappa_i\) is the designed regulation gain; \(Q_i^*\) represents the rated reactive power of the \(i\)-th DG; \(\Delta E_C\) represents the voltage magnitude mismatch between the main grid and the islanded MG; and \(\beta_i\) is the designed magnitude regulation gain. If the \(i\)-th DG is selected to be the PR-DG \(\beta_i > 0\), otherwise \(\beta_i = 0\). The adjacency matrix of system communication topology \(B = \{b_{ij}\}\) is assumed to be the same as the one for the frequency/phase regulation, in order to avoid additional communication channels. Therefore, \(b_{ij} = a_{ij}\) and \(\beta_i = \beta > 0\) if \(r_i = r > 0\). The control loop of the entire synchronization algorithm, and specifically, the implementation of the PR-DG controller is shown in Fig. 4.2.

The presented voltage regulation controller is constructed based on the distributed average consensus control proposed in [SP15b] and modified into the form of pinning-based consensus for MG synchronization problem. In steady state, the following statements are true: 1) \(\frac{Q_i}{Q_i^*} = \frac{Q_j}{Q_j^*}\), meaning that proportional DG reactive power output is achieved; 2) \(\Delta E_C = 0\), meaning that the voltage magnitude mismatch between the main grid and islanded MG is eliminated.
Figure 4.2: Control loop for the PR-DG
4.3 System Stability Analysis

To derive the stability conditions for the proposed controller operating in an islanded MG and to develop the design criteria for the frequency/phase regulation approach, the system small-signal model is developed and presented in this section.

4.3.1 Linearized System Model

The inverter dynamic response is determined by its outer power loop, which can be modeled as a first order low pass filter (LPF). This approach ignores the dynamics of inverter inner current/voltage loop, since the inner control loops are fast enough to be ignored. Referring to (4.1a), the inverter under proposed controller design can be modeled as:

\[ \dot{\delta}_i = \omega_i - \omega^* = -m_i P_i + \Omega_i \]  \hspace{1cm} (4.4)

And the dynamics introduced by the first order LPF can be described as:

\[ \dot{P}_i = -\omega_F P_i + \omega_F \frac{E_C E_i}{X_i} \sin(\delta_i - \delta_C) \]  \hspace{1cm} (4.5)

where \( \omega_F \) represents the LPF cut-off frequency. Recall that droop control is valid only when the active power \( P_i \) is predominantly dependent only on the phase angle \( \theta_i \) [Gue04] and the exponential stability of system frequency/active power flow is decoupled from system voltage variation. Additionally, voltage variation within an electric system is required to be bounded within \( \pm 5\% \) [Ass96]. Thus it can be assumed that \( E_i \approx E_j \) when analyzing the exponential stability of system frequency/active power flow. The system small-signal model is developed by taking the approximation that \( \sin(\delta_i - \delta_C) \approx \delta_i - \delta_C \) and \( E_i \) and \( E_C \) being
treated as constants:

\[ \dot{\delta}_i = -m_i P_i + \Omega_i \]  \hspace{1cm} (4.6a)

\[ \dot{P}_i = -\omega_F P_i + \omega_F M_i (\delta_i - \delta_C) \]  \hspace{1cm} (4.6b)

where \( M_i = \frac{E_i E_C}{X_i} \) is constant.

To ensure system stability, balance between power generation and consumption should always be met within an islanded MG. System power balance constraint is presented as:

\[
\sum_{i=1}^{N} \frac{E_i E_i}{X_i} \sin(\delta_i - \delta_C) + \sum_{l=1}^{L} \frac{E_C E_l}{X_l} \sin(\delta_l - \delta_C) = 0 \]  \hspace{1cm} (4.7)

where \( \delta_i \) and \( E_i \) represents phase angle and voltage magnitude at the \( l \)-th load; \( X_l \) represents the equivalent reactance between the \( l \)-th load and PCC \( (l = 1 \text{ to } L) \). The first term on the left side of equation represents system total power generation and the second term represents system total power consumption. The small-signal representation of (4.7) is:

\[
\sum_{i=1}^{m} \frac{1}{X_i} (\delta_i - \delta_C) + \sum_{l=1}^{L} \frac{1}{X_l} (\delta_l - \delta_C) = 0 \]  \hspace{1cm} (4.8)

For simplicity, constant power loads are considered in the small-signal analysis and the second term in (4.8) is treated as constant. Equation (4.8) can be rewritten as:

\[ \delta_C = \sum_{i=1}^{m} d_i \delta_i + \Delta_L \]  \hspace{1cm} (4.9)

where \( d_i = \frac{X_p}{X_i}, X_p \) is the equivalent parallel reactance calculated as \( X_p = (\sum_{i=1}^{N} \frac{1}{X_i})^{-1}; \Delta_L = \sum_{l=1}^{L} \frac{X_p}{X_l} (\delta_l - \delta_C) \) is a constant determined by system load consumption. The small-signal model of the proposed frequency/phase regulation approach is derived by substituting
(4.10)

From (4.5), (4.9) and (4.10), the small-signal model of multi-inverter system with the proposed controller is summarized as:

\[
\begin{align*}
\dot{\delta}_i &= -m_i P_i + \Omega_i \\
\dot{k_i} \Omega_i &= m_i P_i - \Omega_i - \sum_{j=1}^{N} a_{ij}(\Omega_i - \Omega_j) - r_i \delta_C + \Delta_L \\
\dot{P}_i &= -\omega_F P_i + \omega_F M_i(\delta_i - \sum_{i=1}^{N} d_i \delta_i - \Delta_L)
\end{align*}
\] (4.11)

4.3.2 Small-signal Stability Analysis

Sufficient conditions under which the system is exponentially stable are derived. Without losing generality, the regulation gain in each inverter is set to be identical, \(k_1 = \ldots = k_N = k\) and \(r_i = r\) if the \(i\)th DG is selected as PR-DG. The Laplacian matrix of the communication network is denoted as \(L\). Let \(I_N\) (respectively, \(0_N\)) represents a \(N\)-by-\(N\) identity matrix (respectively, zero matrix) and \([1]_N\) represents a \(N\)-by-\(N\) all ones matrix. Define following matrices for presentation purpose: \(m = diag(m_i)\), \(M = diag(M_i)\), \(d = [1]_N diag(d_i)\) and \(R = diag(r_i)\). Following lemma is introduced for subsequent analysis.

**Lemma 4:** If \(L\) is the Laplacian matrix of an undirected and connected graph with \(N\) vertices, then \(I_N + L\) is positive definite [RB08]. The system small-signal model in (4.11) could be written in matrix form:

\[
\dot{x} = Wx + u
\]

(4.12)

where \(x = [\delta, \Omega, P]\) represents system states under analysis, \(u\) contains states that are inde-
dependent from $X$, and $W = \begin{bmatrix} 0_N & I_N & -m \\ -k^{-1}Rd & -k^{-1}(I_N + L) & k^{-1}m \\ \omega_F M(I_N - d) & 0_N & -\omega_F I_N \end{bmatrix}$ is the system matrix. We define the system matrix as:

$$W = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad (4.13)$$

where $A = -k^{-1} \begin{bmatrix} 0_N & -kI_N \\ Rd & I_N + L \end{bmatrix}$, $B = k^{-1} \begin{bmatrix} -k m \\ m \end{bmatrix}$, $C = \begin{bmatrix} \omega_F M(I_N - d) & 0_N \end{bmatrix}$, and $D = -\omega_F I_N$.

The system is exponentially stable if and only if all the eigenvalues of $W$ have strictly negative real parts. The characteristic polynomial of $W$ can be simplified by Schur complement:

$$\det(sI_N - D)\det[(sI_{2N} - A) - B(sI_N - D)^{-1}C] = 0 \quad (4.14)$$

It can be easily observed that $\det(sI_N - D) = 0$ satisfies $Re(s) < 0$ for all its root and (4.14) can be reduced to:

$$\det \begin{bmatrix} W_1 & W_2 \\ W_3 & W_4 \end{bmatrix} = 0 \quad (4.15)$$

where

$W_1 = sI_N + \omega_F m(sI_N + \omega_F I_N)^{-1}M(I_N - d)$,

$W_2 = -I_N$,

$W_3 = k^{-1}Rd - k^{-1}\omega_F m(sI_N + \omega_F I_N)^{-1}M(I_N - d)$,

$W_4 = sI_N + k^{-1}I_N + k^{-1}L$.

Referring to Lemma 4, $\det(W_4) = \det(sI_N + k^{-1}I_N + k^{-1}L) = 0$ satisfies $Re(s) < 0$ for all its roots. Applying Schur complement again, (4.15) can be further simplified to:
\[
\text{det}(a_3 s^3 + a_2 s^2 + a_1 s + a_0) = 0 \tag{4.16}
\]

where
\[
a_3 = I_N, \\
a_2 = (k^{-1} + \omega_F)I_N + k^{-1}L, \\
a_1 = k^{-1} \omega_F (I_N + L) + \omega_F mM(I_N - d) + k^{-1}Rd, \\
a_0 = k^{-1} \omega_F (mML + Rd).
\]

For a matrix characteristic polynomial that can be written in following form: \(\text{det}(a_3 s^3 + a_2 s^2 + a_1 s + a_0) = 0\), the matrix \(a_3 s^3 + a_2 s^2 + a_1 s + a_0\) must be singular, and therefore, the polynomial has a solution if and only if \(x^T (a_3 s^3 + a_2 s^2 + a_1 s + a_0) x = 0\) for some real vector \(x\) of unit length. According to Routh-Hurwitz criterion, if it is true that:

\[
\lambda_{\text{min}}(a_3 + a_3^T) > 0 \\
\lambda_{\text{min}}(a_2 + a_2^T) > 0 \\
\lambda_{\text{min}}(a_1 + a_1^T) > 0 \\
\lambda_{\text{min}}(a_0 + a_0^T) > 0 \\
\lambda_{\text{min}}(a_2a_1 + a_1^T a_2^T - a_3a_0 - a_0^T a_3^T) > 0
\]

where \(\lambda_{\text{min}}(\cdot)\) is the smallest eigenvalue of the matrix argument, then all the roots of \(a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0\) satisfy \(\text{Re}(s) < 0\). Equation (4.17) represents the conditions under which the small-signal system is exponentially stable. Sufficient conditions under which the system is exponentially stable is presented:

**Theorem 1**[Du19a] For a given system, there exists a constant \(r_{\text{max}} > 0\) such that when
4.4 Hardware Experiments and Case Study

The proposed distributed control strategy is validated using the developed CHIL MG testbed as shown in Fig. 2.12. The MG topology under study is presented in Fig. 4.3. Detailed system physical settings can be found in [Du19a]. Four DGs (DG\(_1\), DG\(_2\), DG\(_3\) and DG\(_4\)) and three critical loads (\(S_L = S_{L1} = S_{L3} = 0.5S_{L2}\)) are presented and DG\(_2\) is selected as the phasor regulation DG. Communication links among relay and DGs are presented in red dashed lines. Detailed testbed parameters are provided in Table 4.1 and the control gains are designed based on the stability analysis derived in previous section.

The dynamic performance of the proposed MG synchronization controller with main grid frequency variation is simulated and the recorded system operation states measured at PCC are presented in Fig. 4.4 and 4.5. As shown in Fig. 4.4, the MG initially operates...
Table 4.1: MG Testbed Parameters

<table>
<thead>
<tr>
<th>DG (kW/kVAR)</th>
<th>Critical Load (kW/kVAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P^* = 100$</td>
<td>$Q^* = 100$</td>
</tr>
<tr>
<td>$P_L = 50$</td>
<td>$Q_L = 24$</td>
</tr>
</tbody>
</table>

Control Gain Design

<table>
<thead>
<tr>
<th>$m^{-1} = 100000$</th>
<th>$n^{-1} = 10000$</th>
<th>$k_f = 0.033$</th>
<th>$k_p = 0.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = 0.01$</td>
<td>$k_\omega = 0.1$</td>
<td>$k_Q = 0.001$</td>
<td>$\beta = 0.03125$</td>
</tr>
</tbody>
</table>

under grid connected mode and the main relay opens at $t = t_1$. The islanded MG is first stabilized by droop control which results in steady state deviations on system frequency and voltage. The proposed MG synchronization controller is initiated at $t = t_2$, it can be observed that as the system converges, the islanded MG and the main grid operate under the same frequency ($\omega_G = 60$ Hz) and voltage mismatch measured at PCC on phase and voltage magnitude are eliminated. A step change on the main grid frequency ($|\Delta \omega_G| = 0.05$ Hz) is introduced at $t = t_3$. It can be observed that as the system converges, the proposed controller is able to have the islanded MG operates synchronously with the main grid even if the main grid is not operated under rated frequency. Fig. 4.5 shows the zoomed in system response during main grid frequency deviation. It can be observed that as the main grid frequency changes, the islanded MG frequency is able to accurately follow the one of the main grid. New phase mismatch is introduced due to the instant frequency mismatch between the main grid and islanded MG but such mismatch is eliminated eventually. It is noteworthy that no deviations on voltage mismatch is observed in Fig. 4.5, which validates the assumptions made in system stability analysis that system voltage/reactive power variation and frequency/active power variation are decoupled. Additionally, active and reactive power sharing among all the DGs are always maintained in steady state, as shown in Fig. 4.6.
4.5 Conclusion

A distributed MG synchronization operation algorithm is proposed. The proposed algorithm is able to keep the voltage phasor at the MG side of PCC synchronized with the one on the main grid side. System frequency, phase and voltage magnitude mismatch are regulated explicitly at the same time, and the system total load are proportionally shared by all the DGs. Exponential stability of the proposed controller is derived and its dynamic performance is validated using a CHIL MG testbed. The proposed MG synchronization controller is proved to be effect by the experimental results.
Figure 4.5: Recorded system operation states experimental results (zoom-in)

Figure 4.6: Recorded DG power outputs experimental results
Beyond Convergence - Accurate Consensus-based Distributed Averaging with Variable Time Delay

It can be observed that distributed secondary control has been widely used in hierarchical control structures, where multiple DGs need to coordinate to regulate system operation states. In these systems, the adopted consensus algorithms, despite the specific forms they are developed, generally provide two major functions: 1) a distributed comparator that eliminates the mismatched state of interest on each agent (e.g. power outputs of each DG); and 2) a distributed observer that observes the average of a group of dynamic states (e.g. voltages measured by a group of DGs). To be useful, consensus algorithms must be computationally efficient, stable and accurate. In practice, numerous practical implementation challenges significantly affect the consensus equilibrium. In this section, a novel distributed average observer is proposed to achieve accurate average tracking in the
presence of time varying communication delays among agents. The proposed distributed average observer is implemented on hardware controllers and its effectiveness is validated in a controller hardware-in-the-loop testbed.

5.1 Introduction

The average value of a group of system states is often used as control variable in secondary control. In this context, the average value consists of weighted local information. When accurate regulation on each state is not feasible, regulation over the average provides a compromise solution. For example, in a system with significant impedances (i.e. voltage differences between buses), accurate regulation of both local bus voltages and DG reactive power sharing is not feasible [SP15a]. To guarantee accurate DG reactive power sharing while maintaining the bus voltages of all nodes in an acceptable range, the average voltage of all DGs are regulated to the nominal value [Zho18; Lou17]. Regulating the average value has other applications: in [Men16], the negative-sequence output current of each DG tracks the average, thus achieving equal current sharing; in [Lu14], a group of DGs in a DC MG share current proportionally by regulating their distributed average output voltage to a reference value.

Distributed secondary control in the context of multi-agent systems (MAS) provides a number of benefits [McA07]; it delivers the same controllability compared to centralized control while enabling scaling and improving reliability and resiliency [Che18b]. Typical average calculation in a MAS requires the system state to be globally accessible, which relies on a fully-connected communication network among all agents [Sha14a]; however this approach becomes impractical as the number of agents increases. Consensus-based algorithms are able to compute the average value in a distributed manner and require only a
sparse communication network [OS07]. For a MAS, when multiple agents agree on the value of a particular variable, they are said to have reached consensus. The common value is called consensus equilibrium and the appropriate algorithm for negotiation is called consensus algorithm [RB08]. Consensus-based average algorithms reach consensus on the average value of a set of states measured locally by multiple agents [Men16; Lou18b]. Computational convergence of the consensus-based average observers has been extensively analyzed in face of changing network topology [Lai16] and communication delays [Dou17]. In general, the convergence of a dynamic consensus algorithm is guaranteed if the communication delays are bounded [SM03a].

Beyond computational convergence, numerical accuracy is also critical, i.e., the consensus equilibrium needs to be the exact average value. The estimation error is difficult to detect by any individual agent, meaning that the accuracy of the algorithm must be high and the observation error bounded in the algorithm design. In practice, communication delays cannot be avoided in distributed control systems. Real-time data transmission rates are determined by many factors (e.g. communication protocol, real-time network traffic, etc.), and in many implementations communication delays are time varying [KB12]. The effects of communication delays on the average consensus equilibrium have been considered in the literature: [SM03a] presents a general form of average observer in a MAS with communication delays; [BFT08] shows that the tracking accuracy in such a system cannot be guaranteed. Authors in [Lai16] analyze the tracking accuracy of dynamic consensus algorithms, and show that when all states are delayed uniformly, accurate average tracking can be achieved. However, this condition is difficult and expensive to implement with the state-of-the-art communication networks. Authors in [Ata12] derive a general expression that quantifies the deviation of the calculated consensus equilibrium from the true average.

Another challenge to determining an accurate consensus equilibrium is ensuring time
synchronization among all agents. Many researchers assume that the distributed algorithms can be executed synchronously by each controller [Sha14a; Zho18; Lou18b]. However, synchronizing distributed controllers in practice is non-trivial and time asynchronous operation influences the observer accuracy. There exist methods that achieve time synchronization analytically [Qin16]; however these approaches require a communication channel that guarantees a fixed communication delay. In most practical applications, GPS timing technology is utilized to provide a time synchronization signal to all the agents [ZM12]. Other practical challenges include the need for sufficient computational capability of individual agents and a toolkit for scalable distributed controller implementation, among others.

In this work, a distributed average observer that achieves accurate average tracking is proposed. Compared to the existing work in the literature, the main contributions and notable features of this work are listed below:

1. We provide a comprehensive literature review of the existing consensus-based distributed averaging algorithms and identify the main challenges that impair the performance of distributed average observers: time varying communication delay and time synchronization among agents. Importantly, a detailed performance analysis of ratio consensus algorithms with time varying delays is presented, which has not been extensively discussed in the literature.

2. We propose a distributed average observer that guarantees accurate average tracking in the presence of time varying communication delays, which can not be done using the state-of-the-art consensus-based averaging algorithms. The proposed algorithm is an improvement on the ratio consensus algorithm proposed in [KK17], which loses accuracy in face of time varying communication delays with variable probability.
distribution – a realistic condition in real communication networks.

3. We implement the state-of-the-art and the proposed algorithms in controller hardware using a novel software platform, called Resilient Information Architecture Platform for the Smart Grid (RIAPS). Going beyond the state-of-the-art where ideal simulation-based controller implementation is presented [Lou17; Zho18; Men16; Lu14], the proposed distributed average observer is implemented on single board computers (BeagleBone Black) to evaluate implementation challenges. Time synchronization among hardware controllers is achieved using the RIAPS platform. We compare the performance of the state-of-the-art and the proposed algorithm in a controller hardware-in-the-loop testbed.

5.2 Problem Statement

In this work, the communication delay, \( \tau \) is time varying and bounded as \( \tau \in [0, \tau_u] \), where \( \tau_u \) represents the upper bound that ensures system convergence. The probability distribution function that describes the delay distribution over the interval \( [0, \tau_u] \) is denoted as \( f(s) \) and the mean of delay is calculated as \( \bar{\tau} = \int_0^{\tau_u} s f(s) \, ds \). The communication network of the multi-agent system under study is assumed to be a graph, \( G = (V, \mathcal{E}, A) \) where \( V = \{v_1, v_2, \ldots, v_N\} \) denotes the set of vertices (agents), \( \mathcal{E} \subseteq V \times V \) denotes valid connectivity (communication) between the vertices and \( A = \{a_{ij}\} \) is the adjacency matrix defined as \( a_{ij} = 1 \) if and only if the edge \( \{v_i, v_j\} \in \mathcal{E} \), otherwise \( a_{ij} = 0 \). A graph is defined as undirected if for \( \{v_i, v_j\} \in \mathcal{E} \), \( a_{ij} = a_{ji} \), otherwise it is defined as directed. The Laplacian matrix of the communication network, \( L \) is defined as \( L = D - A \) where \( D = \{d_{ii}\} \) is the degree matrix and \( d_{ii} = \sum_{j=1}^{n} a_{ij} \). \( G \) is defined as a connected graph if there is a path between every pair of vertices, i.e. for \( v_i \in V \), there exists at least one \( v_j \in V \) (\( i \neq j \)) so that \( \{v_i, v_j\} \in \mathcal{E} \). The
communication networks under discussion are assumed to be undirected and connected in the subsequent analysis. Denote $[1]_{m,n}$ as a m-by-n all ones matrix; and $\text{ceil}(x) = \lfloor x \rfloor$ as the ceiling function that maps $x$ to the least integer greater than or equal to $x$.

### 5.2.1 Average Consensus Algorithm

Average consensus algorithm was first introduced in [SM03a]. The average consensus algorithm in the absence of communication delay can be expressed as [Ata12]:

$$
\dot{x}_i(t) = -\kappa' \sum_{i,j \in \mathcal{E}} a_{ij} [x_i(t) - x_j(t)]
$$

(5.1)

where $x_i(t)$ represents the real-time state of interest of the $i$th agent at time instant $t$ and $\dot{x}_i(t)$ represents its first-order derivation; $x_i(0) = x^0_i$ represents the initial value of $x_i(t)$ at $t = 0$; the normalized coupling coefficient $\kappa' = \frac{\kappa}{d_{ii}}$ represents the interaction strength among agents and $\kappa \geq 0$ is a designed control gain. In [Ata12] authors show that all states in (5.1) reach the average consensus asymptotically, i.e., let $x^*_i = \lim_{t \to \infty} x_i(t)$, then $c_x = x^*_i = \frac{1}{N} \sum_{i \in \mathcal{E}} x_i(0)$, and $c_x$ represents the consensus equilibrium of $x_i(t)$.

If a fixed communication delay is present, the average consensus algorithm can be expressed as:

$$
\dot{x}_i(t) = -\kappa' \sum_{i,j \in \mathcal{E}} a_{ij} [x_i(t - \tau) - x_j(t - \tau)]
$$

(5.2)

where $x_i(t - \tau)$ represents the state of interest of the $i$th agent delayed by period $\tau$. This algorithm converges and reaches the accurate average consensus at each agent if $\tau_u = \frac{\pi}{2\kappa \lambda'_1}$, where $\lambda'_1$ represents the largest eigenvalue of the normalized Laplacian matrix $L' = D^{-1}L$ [SM03b]. Although the tracking accuracy of (5.2) is proved to be robust against communication delay, such protocol is hard to implement in practice because:
1. Both local states, $x_i(t)$ and external states, $x_j(t)$ must be delayed by the exact same period, which is hard to implement when the delays are time varying;

2. Local states $x_j(t - \tau)$ need to be accessible at the time instant $t$, requiring data storage and buffering.

A more practical form of average consensus, presented in [Nic01; Ata12] formulates the algorithm as:

$$\dot{x}_i(t) = -\kappa' \sum_{i, j \in \mathcal{E}} a_{ij} \left[ x_i(t) - x_j(t - \tau) \right]$$  \hfill (5.3)

The upper bound of $\tau$ ensures system convergence [Nic01]. The consensus equilibrium of (5.3) is calculated as [Ata12]:

$$c_x = \frac{1}{1 + \kappa \tau} u \left( x(0) + \kappa \int_0^\tau \int_{-\tau}^0 f(s) x(\xi) d\xi d s \right)$$  \hfill (5.4)

where $x(0) = [x_0^0, x_0^1, \ldots, x_0^N]$ represents vector of initial conditions and $x(t) = [x_1(t), x_2(t), \ldots, x_N(t)]$ represents vector of the real-time state of interest; $u = \frac{1}{N} [1, 1, \ldots, 1]$ represents the left eigenvector of $L$ that corresponds to the zero eigenvalue. From (5.4), non-zero communication delay will result in deviated consensus value, and the deviations depends on the distribution of $\tau$ and the dynamics of $x(t)$.

### 5.2.2 Dynamic Consensus Algorithm

The average consensus algorithm calculates the average of initial values, $ux(0)$ and is only applicable for static inputs. To track the average of a group of dynamic states, dynamic consensus algorithm with the presence of communication delay is expressed as:

$$\dot{x}_i(t) = \ddot{x}_i(t) - \kappa' \sum_{i, j \in \mathcal{E}} a_{ij} \left[ x_i(t) - x_j(t - \tau) \right]$$  \hfill (5.5)
where \(z_i(t)\) represents the real-time dynamic states of the \(i\)th agent being tracked at time instant \(t\) and \(\dot{z}_i(t)\) represents its first-order derivation; \(z_i(0)\) represents the initial value of \(z_i(t)\) at \(t = 0\) and \(x_i(0) = z_i(0)\); \(\mathbf{z}(t) = [z_1(t), z_2(t), \ldots, z_N(t)]\) represents vector of the real-time dynamic states being tracked. In [Spa05] authors show that under ideal conditions (i.e. \(\tau = 0\)), (5.5) must converge to a steady-state and each agent tracks the dynamic consensus with zero steady-state error, i.e., \(x_i(t) = \frac{1}{N} \sum_{i \in \mathcal{F}} z_i(t)\) for \(t \to \infty\).

To understand the deviations brought by non-zero communication delay on the consensus equilibrium of (5.5), it is assumed that the system is initially under equilibrium and the dynamic inputs, \(\mathbf{z}(t)\) vary at \(t = t^0\), i.e., for each agent, \(x_i(t) = x_j(t) = c'_x\) and \(z_i(t) = z'_i\) for \(t \in [-\tau_u + t^0, t^0)\), where \(c'_x\) and \(z'_i\) are constant. The variations of \(z_i(t)\) at \(t = t^0\), \(\Delta z_i(t)\) are modeled as a set of step functions \(\mathbf{z}(s) = \frac{\Delta \mathbf{z}}{s}\), where \(\Delta \mathbf{z} = [\Delta z_1, \Delta z_2, \ldots, \Delta z_N]\) represents vector of step variations of each dynamic states and the consensus equilibrium of (5.5) is calculated as:

\[
c_x = \frac{1}{1 + \kappa \tau} \mathbf{u} \left( c'_x + \Delta \mathbf{z} + \kappa \int_{-\tau+ t^0}^{t^0} c'_x d\xi \right) = c'_x + \frac{1}{1 + \kappa \tau} \mathbf{u} \Delta \mathbf{z}
\]  

(5.6)

Therefore, the consensus value in dynamic consensus depends not only on the system momentary performance but also the previous ones, which makes accurate average tracking more challenging.

5.2.3 Ratio Consensus Algorithm

To overcome the delay effects on consensus equilibrium, ratio consensus algorithm was proposed in [KK17]. Ratio consensus algorithm that tracks the average of dynamic states with the presence of communication delay is expressed as:
\[
\dot{x}_i(t) = \dot{z}_i(t) - \kappa' \sum_{i,j \in \mathcal{E}} a_{ij} [x_i(t) - x_j(t - \tau)]
\]
\[
\dot{y}_i(t) = -\kappa' \sum_{i,j \in \mathcal{E}} a_{ij} [y_i(t) - y_j(t - \tau)]
\]
\[
p_i(t) = \frac{x_i(t)}{y_i(t)}
\]

where \( y_i(t) \) is state variable and \( p_i(t) \) represents local average observation. The initial conditions are set to be \( x_i(t) = x_j(t) = 0 \) and \( y_i(t) = y_j(t) = 0 \) for \( t \in [-\tau_u, 0) \), \( x_i(0) = z_i(0) \) and \( y_i(0) = 1 \).

The ratio consensus protocol in (5.7) consists of two independent functions, and it follows that the algorithm will asymptotically converge if both sub-protocols in (5.7a) and (5.7b) converge. To calculate the consensus equilibrium in (5.7), we assume that the system is initially in equilibrium: \( x_i(t) = x_j(t) = c'_x, \ y_i(t) = y_j(t) = c'_y \) and \( z_i(t) = z'_i \) are all constant for \( t \in [t^0 - \tau^u, t^0) \). The variations of \( z(t) \) at \( t = t^0 \) are modeled using a step function. The consensus equilibrium of (5.7) is calculated as:

\[
c_x = \frac{1}{1 + \kappa \tau} \left( c'_x + u \Delta z + \kappa \int_{-\tau+t^0}^{t^0} c'_x d\xi \right) = c'_x + \frac{1}{1 + \kappa \tau} u \Delta z
\]
\[
c_y = \frac{1}{1 + \kappa \tau} \left( c'_y + \kappa \int_{-\tau+t^0}^{t^0} c'_y d\xi \right) = c'_y
\]
\[
c_p = \frac{c_x}{c_y} = \frac{c'_x}{c'_y} + \frac{(c'_y)^{-1}}{1 + \kappa \tau} u \Delta z
\]

From (5.8c), the protocol in (5.7) is able to accurately track the variation of the average, \( u \Delta z \) when \( c'_y = \frac{1}{1 + \kappa \tau} \) are always true. Referring to (5.4) and the initial conditions of (5.8b), this condition is achievable only when \( \tau \) remains unchanged since \( t = 0 \) and the
consensus equilibrium of (5.8b) equals to \( c_y = c'_y = \frac{1}{1 + \kappa \tau} \). Recall that \( \overline{\tau} = \int_0^{\tau_u} s f(s) ds \), any variation in the probability distribution of communication delay, \( f(s) \) after (5.7b) reaches equilibrium will result in mismatched \( \overline{\tau} \) and lead to a tracking error. Therefore, the algorithm in (5.7) can accurately track the average of the dynamic states at the initial stage (i.e. before (5.7b) converges), but it cannot accurately track the average when faced with time-varying communication delays.

### 5.2.4 Distributed Controller Time Synchronized Operation

To implement the distributed averaging algorithms on multiple nodes, the algorithm is implemented on state-of-the-art digital controllers, which exchange relevant information with each other and iterate the algorithms at a fixed time interval (i.e. iteration step time). The iteration and data exchange process between two nodes is presented schematically in Fig. 5.1. The communication delay, \( \tau \) is defined as the time it takes the \( i \)th controller to receive updated data sets from the \( j \)th controller.

Besides agent-to-agent communication delay, one major issue that has not been adequately addressed in past work is the effect of controller clock drift. The process of asynchronous operation among controllers with communication delays are presented in Fig. 5.1a: Each controller is able to iterate accurately by the same period, \( \Delta T \). However, the execution time points of each controller are asynchronous, due to lacking time synchronization. Take the \( j \)th controller as reference, the clock drift on the \( i \)th controller is denoted as \( \tau_i \). Note that the clock drift, \( \tau_i \) can be both positive and negative, while the communication delay, \( \tau \) is always positive. In practice, both the clock drift and the communication delays will vary with time [KI15]. Considering both effects, the discrete form of algorithm in (5.5)
becomes:

\[ u_i(n) = -\kappa' \sum_{i,j \in \mathcal{E}} a_{ij} [x_i(n-k_i) - x_j(n-k_j)] \]  

\[ (5.9a) \]

\[ x_i(n+1) = z_i(n-k_i) + \Delta T \sum_{r=1}^{n} u_i(r) \]  

\[ (5.9b) \]

where rectangle method is adopted for computing definite integrals [FV12]; \( n \) represents the \( n^{th} \) iteration time step; \( u_i(n) \) represents the cumulative difference between two controllers.
and $u_i(0) = 0$; $\Delta T$ is a scalar iteration time step which is often omitted in the algorithm formulation; $k = \lceil \frac{\tau}{\Delta T} \rceil$ and $k_i = \lceil \frac{\tau_i}{\Delta T} \rceil$ represent the delayed iteration time steps by $\tau$ and $\tau_i$, respectively.

In this work, RIAPS platform is adopted to synchronize controller clocks, and eliminate controller clock drift [Tu18b]. Figure 5.1b represents time synchronized operation (i.e. $\tau_i = 0$) among controllers with communication delays. It can be observed that if the bounded communication delays are smaller than the iteration time step (i.e. $\tau < \Delta T$), the resulting discretized, time-synchronized consensus algorithm with bounded delay becomes:

\begin{equation}
    u_i(n) = -\kappa' \sum_{i,j \in \mathcal{E}} a_{ij} [x_i(n-1) - x_j(n-1)] \tag{5.10a}
\end{equation}

\begin{equation}
    x_i(n+1) = z_i(n) + \Delta T \sum_{r=1}^{n} u_i(r) \tag{5.10b}
\end{equation}

where at execution step $n$, a constant delay, $\Delta T$ is introduced to $x_i(n)$, and $x_i(n-1)$ is accessible from a data buffer. The above control protocol guarantees accurate average tracking, since it represents a discrete formulation of (5.2) with $\tau = \Delta T$. As noted earlier, the conditions for implementing (5.10) are: 1) distributed controllers are time synchronized, i.e., $\tau_i = 0$ and 2) the iterative time step is larger than the worst-case communication time delay, i.e., $\Delta T > \tau_u$.

The above approach has practical limitations:

1. To minimize the effect brought by operating a continuous algorithm in its discretized form, the value of $\Delta T$ should be selected as small as possible [FV12], which is in contradiction with the operation requirements of (5.10).

2. For most communication networks, it is not feasible to design $\Delta T$ to be large enough to cover all the possible communication delays.
3. To ensure accurate average tracking, past internal states, \( x_i(n-1) \) and \( z_i(n) \) are used to compute \( x_i(n+1) \) in (5.10). This could result in poor tracking performance if \( \Delta T \) is selected to be large.

A more practical scenario, shown in Fig. 5.1b, allows the communication delay to vary and to be longer than the iteration time step (i.e. \( \tau(n-1) \neq \tau(n) \) and \( \tau(n) > \Delta T \)). As shown in Fig. 5.1b, at execution time step \( n \), \( \tau(n-1) < \Delta T \) and \( x_j(n-1) \) is accessible to the \( i \)th controller. Protocol in (5.10) can be implemented. However, at execution time step \( (n+1) \), \( \tau(n) > \Delta T \) and \( x_j(n) \) becomes unavailable. The latest available data, \( x_j(n-1) \) in this case, is used for computation and (5.10) can no longer be implemented.

### 5.3 Proposed Distributed Average Observer

In this paper, a distributed average observer inspired by ratio consensus algorithm is proposed to track the average of a set of dynamic states with the presence of time varying delay. The proposed observer consists two control protocols that operate independently:

\[
\dot{x}_i(t) = \dot{z}_i(t) - \kappa' \sum_{i,j \in \epsilon} a_{ij} [x_i(t) - x_j(t - \tau)] \tag{5.11a}
\]

\[
\dot{y}_i(t) = \dot{g}_i(t) - \kappa' \sum_{i,j \in \epsilon} a_{ij} [y_i(t) - y_j(t - \tau)] \tag{5.11b}
\]

where for the \( i \)th distributed controller, \( x_i(t) \) and \( y_i(t) \) are state variables; \( z_i(t) \) represents the dynamic states whose average is tracked and \( g_i(t) \) represents a pre-defined square-wave function that is uniform on each agent:

\[
g_i(t) = 0.5 \text{sgn} (\sin2\pi \frac{t}{T_g}) + 1 \tag{5.12}
\]
where \( sgn() \) represents sign function and \( T_g \) is the designed period of square wave. It can be observed that \( g_i(t) \) varies every \( 0.5T_g \) and the variation magnitude is \( |\Delta g| = 1 \). The initial conditions in (5.11) are \( x_i(t) = x_j(t) = 0 \) and \( y_i(t) = y_j(t) = 0 \) for \( t \in [-\tau_u, 0) \), \( x_i(0) = z_i(0) \) and \( y_i(0) = g_i(0) = 1 \).

Equation (5.11) converges asymptotically if both (5.11a) and (5.11b) are stable. Both (5.11a) and (5.11b) are in standard form of dynamic consensus algorithm with presence of communication delay, whose convergence conditions have been properly discussed in [Lai16] and are adopted in this work: If the time varying delays, \( \tau \) satisfy \( \tau < d, \dot{\tau} < d_1 < 1 \) and \( d < \frac{(1-d_1)\pi}{2\kappa \max_{i=2,\ldots,N}(|\lambda_i(L')} \) with certain positive constants \( d \) and \( d_1 \), then the dynamic consensus algorithms in both (5.11a) and (5.11b) can asymptotically converge. Term \( \max_{i=2,\ldots,N}(|\lambda_i(L')} \) represents the second largest eigenvalue of \( L' \).

Based on previous discussion, it can be observed that the state \( y_i(t) \) in (5.11b) represents averaging of a group of dynamic states \( g(t) = [g_1(t), g_2(t), \ldots, g_N(t)] \), where each dynamic states are identical, as \( g_i(t) = g_j(t) \) and performs step variation every \( 0.5T_g \). The equilibrium consensus of (5.11b) can be calculated referring to (5.6):

\[
c_y = c_y' + \frac{1}{1 + \kappa \tau} |\Delta g|
\]

(5.13)

And the following relationship can be derived:

\[
|c_y - c_y'| = \frac{1}{1 + \kappa \tau} |\Delta g|
\]

(5.14)

where term \( |c_y - c_y'| \) represents observed square wave magnitude and can be observed every time (5.11b) reaches consensus. Both \( |c_y - c_y'| \) and \( |\Delta g| \) are locally accessible for each distributed controller. The mean of delay, \( \bar{\tau} \) can be estimated by each controller locally and
regularly as:
\[ \bar{\tau} = \kappa^{-1}\left(\frac{1}{|c_y - c'_y|} - |\Delta g|\right) = \kappa^{-1}\left(\frac{1}{|c_y - c'_y|} - 1\right) \] (5.15)

Similarly, the variation of \( z(t) \) at \( t = t^0 \) is modeled as \( z(s) = \frac{\Delta z}{s} \) and the equilibrium consensus of (5.11a) is calculated as:

\[ c_x = c'_x + \frac{1}{1 + \kappa \bar{\tau}} u \Delta z \] (5.16)

where \( c_x \) and \( c'_x \) are locally accessible and \( u \Delta z \) can be calculated by substituting (5.15) into (5.16):

\[ u \Delta z = \frac{c_x - c'_x}{|c_y - c'_y|} \] (5.17)

The variation of the average, \( u \Delta z \) is accurately observed using (5.17) regardless of the communication delay variation. Additionally, it follows that when \( t^0 = 0 \), the initial average under track, \( u z(0) \) can also be accurately observed using (5.17). Thus by implementing (5.11) and (5.17), the average of a group of dynamic states, \( u z = u(z(0) + \sum \Delta z) \) can be accurately observed in face of a varying communication time delay, as long as the delay is bounded and requires no direct measurement of the delay. The discrete form of the proposed distributed average observer is presented below:

\[ x_i(n + 1) = z_i(n) - \kappa \sum_{r=1}^{n} \sum_{i,j \in \mathcal{E}} a_{ij} [x_i(r) - x_j(r-k)] \] (5.18a)

\[ y_i(n + 1) = g_i(n) - \kappa \sum_{r=1}^{n} \sum_{i,j \in \mathcal{E}} a_{ij} [y_i(r) - y_j(r-k)] \] (5.18b)
where $z_i(n)$ is the input dynamic state under track; $p_i(n)$ is the output observed average; $k = \lceil \frac{\tau}{\Delta T} \rceil$ represents the time varying delay; $x_i(0) = z_i(0)$ and $y_i(0) = g_i(0)$; $\Delta c_y(n) = |c_y - c_{y'}|$ is updated every time $g_i(n)$ varies and $y_i(n)$ converges to new equilibrium. Compared to the discrete protocols discussed in (5.9) and (5.10), term $\Delta T$ is scaled in $\kappa'$ and not explicitly presented in (5.18).

It is noteworthy that term $|c_y - c_{y'}|$ is not available until at least one variation in $g_i(n)$. To ensure proper initial operation of the proposed observer before $\Delta c_y(n)$ is available, it is required that $g_i(0) = 1$ and $\Delta c_y(n) = y_i(n)$ for $n = 0, \ldots, n_0$, i.e., the proposed controller should be initiated at the moment $g_i(t) = 1$ and operated under conventional ratio consensus algorithm before $|c_y - c_{y'}|$ is available at execution step $n_0$. Additionally, to ensure proper update on $\Delta c_y(n)$, a convergence check function is developed to improve the performance of (5.18b): The developed function shall detect the convergence of $y_i(n)$ locally at every execution step and update $c_y = y_i(n + 1)$ when $|y_i(n + 1) - y_i(n)| < \delta$, where $\delta$ is a designed threshold. Once $c_y$ is updated, $c_{y'}$ is accessible from a data buffer and $|c_y - c_{y'}|$ can be updated locally. The flow chart of the convergence check function is presented in Fig. 5.2.

At last, the ‘plug-and-play’ operation of the proposed observer is presented. It is assumed...
that there are in total $N$ controllers in the system and they have reached consensus as
$x_i(t) = x_j(t) = c'_x$ and $y_i(t) = y_j(t) = c'_y$. To introduce a new controller into the system
at $t = t_{pnp}$ and properly observe the average of the dynamic states from the total $N+1$
controllers, the initial conditions of the $N+1$th controller should be set as $x_{N+1}(t) = c'_x$
and $y_{N+1}(t) = c'_y$ for $t \in [-\tau + t_{pnp}, t_{pnp})$, $x_{N+1}(0) = z_{N+1}(0)$ and
$y_{N+1}(0) = g_{N+1}(0)$. The values of $c'_x$ and $c'_y$ can be observed by the $N+1$th DG through communication with the operating controllers.

Besides the aforementioned analytic controller design criterion, following requirements
also need to be properly addressed when implementing the proposed observer:

1. The external states in (5.18), $x_j(n)$ and $y_j(n)$ need to share the same communication
delay, $k$ even though the delay can vary from one data transmission to the next. This
is achieved by transferring the data sets in the same packet using the RIAPS platform so that they can be received with uniform delay.

2. The function $g_i(n)$ generated by each distributed controller needs to be identical, requiring all controllers to be synchronized (also applied during 'plug-and-play' operation). This is achieved using the RIAPS platform time synchronization function. More information regarding RIAPS platform will be introduced in the following sections.

Control diagram of the proposed average observer for distributed digital controller implementation is presented in Fig. 5.3. Procedure regarding parameter design of the proposed average observer is presented. Referring to (5.11) and (5.12), values of the following parameters need to be designed:

- $\kappa$ Referring to the aforementioned stability condition, set $d_1 \approx 0$ and the upper bound of $\kappa$ can be approximated by $\kappa < \frac{\pi}{2\tau_M \max_{i=2,\ldots,N} (|\lambda_i(L)|')}$ where $\tau_M$ represents the expected upper bound of communication delay. On the other hand, it is proved in [XB03] that $\kappa = \frac{2}{\lambda'_1 + \lambda_{N-1}}$ gives the fastest convergence speed, where $\lambda'_{N-1}$ represents the least non-zero eigenvalue of $L'$. Thus, the value of $\kappa$ should be designed as $\kappa = \min\left(\frac{\pi}{2\tau_M \max_{i=2,\ldots,N} (|\lambda_i(L)|'), \frac{2}{\lambda'_1 + \lambda_{N-1}}}\right)$ with sufficient margin.

- $T_g$ The sub-protocol in (5.18b) needs to converge on each controller respectively in less than $0.5T_g$ period, which requires a sufficiently large $T_g$. The convergence rate/time of (5.18b) is geometric and determined by several factors in a highly non-linear relationship that does not have an analytic solution [Zha17]. For a given communication network, the convergence rate/time of consensus decreases due to time delays [Zha17]. The selection of $T_g$ is done experimentally: the worst case scenario is simulated in advance to estimate the longest convergence time, $T_M$ and the value of
\( T_g \) should be selected as \( T_g > 2T_M \) with sufficient margin.

At last, it is noteworthy that the iterative time step \( \Delta T \) is mainly limited by the operation power of the adopted hardware controller (BBB board in this work), and should be selected as small as possible without jeopardizing controller operation reliability.

### 5.4 Hardware Experiments and Case Study

As previously reviewed, average DG voltage is frequently used as a control objective in secondary control for MG voltage regulation, and thus the accuracy of the average observation is critical to system operation stability. For validation purpose, the proposed observer is implemented in hardware controllers using RIAPS platform and utilized to track the average operation voltage of five DGs in the IEEE 34 bus feeder, implemented in a HIL testbed. The entire feeder is operated as an islanded MG, and feeder shunt capacitors are switched on and off from the circuit in order to cause voltage variations at each DG. For the sake of validating the voltage averaging algorithm, secondary control (that would restore the voltage along the feeder) is disabled and the DGs are operated under droop control mode to stabilize the system frequency and voltage [Gue11a]. The feeder voltage variations, which are governed by droop relationship, are allowed to vary significantly beyond the IEEE standards for validation purpose.

#### 5.4.1 RIAPS Platform and Time Synchronization Functionality

RIAPS platform is an open source platform that distributes the intelligence and control capability to local endpoints. It provides an environment to implement the analytically developed distributed control algorithms on hardware controllers.
The RIAPS time synchronization service aims at creating a universal clock that is common to all the RIAPS nodes (agents managed under RIAPS platform). One RIAPS node’s system (or GPS-synchronized) clock is selected as the universal clock to which clocks of the other nodes are synchronized. All the RIAPS nodes are connected to a local-area network (LAN). Two open-source tools, PHC2SYS and PTP4L, are used by the RIAPS platform to align the slaves’ clocks to the master. Both tools are part of Linux PTP project which implements the Precision Time Protocol (PTP) according to IEEE standard 1588 in Linux. In case the selected reference clock fails, RIAPS platform is able to automatically detect such failure and refer to another available clock. Such increased operation reliability is enabled by the fault tolerance capability of RIAPS platform. With accurate synchronization between each step, the clock difference between RIAPS nodes is controlled within 1 μs. More details about the RIAPS time synchronization service can be found in [Vol17]. At last, it is noteworthy that the proposed distributed average observer is not designed to provide analytic solution towards time synchronization. Time synchronized operation...
among controllers is a prerequisite and the adopted time synchronization technique does not introduce master-slave control approach to the proposed control strategy.

### 5.4.2 Case Study

In this section, the effectiveness of the proposed distributed observer is validated, and the communication delay effects on the accuracy of average observers based on both dynamic consensus and ratio consensus are presented. IEEE 34 bus test feeder is adopted as the islanded three-phase AC MG and detailed system parameters can be found in [Dis]. Five identical grid-forming DGs are connected to the test feeder (see Fig. 5.4). The DGs operate as voltage control mode voltage source inverters (VCM-VSIs) [HL11]. A schematic control diagram of VCM-VSI operation in droop control mode is presented in Fig. 5.5 and detailed DG operation parameters can be found in Table 5.1. In this work, $P-f$ and $Q-V$ droop is adopted in the droop control. Virtual impedance loop is implemented on each DG for the
Table 5.1: DG operation parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Droop control</td>
<td></td>
</tr>
<tr>
<td>$P$-f droop gain ($m$)</td>
<td>$6.2832 \times 10^{-5}$</td>
</tr>
<tr>
<td>$Q$-V droop gain ($n$)</td>
<td>$10 \times 10^{-5}$</td>
</tr>
<tr>
<td>Rated RMS voltage ($E^*$)</td>
<td>480 V</td>
</tr>
<tr>
<td>Rated frequency ($\omega^*$)</td>
<td>377 rad/s</td>
</tr>
<tr>
<td>Inner loop</td>
<td></td>
</tr>
<tr>
<td>Voltage loop PI gains ($k_{P,V}$, $k_{I,V}$)</td>
<td>0.14, 120</td>
</tr>
<tr>
<td>Current loop PI gains ($k_{P,I}$, $k_{I,I}$)</td>
<td>2.5, 400</td>
</tr>
<tr>
<td>LCL filter</td>
<td></td>
</tr>
<tr>
<td>Inverter side inductor ($L_1$)</td>
<td>750 uH</td>
</tr>
<tr>
<td>Capacitor ($C$)</td>
<td>100 uF</td>
</tr>
<tr>
<td>Grid side inductor ($L_2$)</td>
<td>500 uH</td>
</tr>
<tr>
<td>Virtual impedance</td>
<td></td>
</tr>
<tr>
<td>Virtual resistor/inductor ($R_V$, $L_V$)</td>
<td>0 Ohm, 1 mH</td>
</tr>
<tr>
<td>Power circuit</td>
<td></td>
</tr>
<tr>
<td>Switching frequency ($f_{SW}$)</td>
<td>5000 Hz</td>
</tr>
<tr>
<td>DC link voltage ($V_{DC}$)</td>
<td>1200 V</td>
</tr>
<tr>
<td>Rated power output ($S^*$)</td>
<td>300 kVA</td>
</tr>
</tbody>
</table>

sake of droop-based VCM-VSI operation stability [HL11].

In this work, the application uses C++ implementation of the Modbus protocol in order to improve the accuracy and the speed of the communication between the BBB boards and the MCUs, while the distributed averaging algorithm is written in Python for better code readability. Since the inherent communication delays are relatively small, due to the fact that they are running on the same local network with a single router, additional delays were introduced in the RIAPS platform itself to emulate more realistic communication networks.

Each algorithm is executed on the HIL MG testbed simultaneously to ensure the same communication delays during operation. The algorithms are used to observe the average DG voltage and do not introduce any regulation towards DG operation. The DG operation states are determined by droop control. It is noteworthy that since DG operation states (e.g. DG power outputs) are not determined by the implemented averaging algorithms, their variations do not provide any validation on the performance of the proposed observer, which disagrees with the purposed of this case study. Operation states that are irrelevant to
DG voltage averaging are beyond discussion and not included in this paper.

The system operates in islanded mode and is supported by grid-forming DGs operating in droop control mode. The average observers are enabled at $t_1 = 2$ s. To generate system voltage variation, shunt capacitors at Bus 848 and Bus 844 are switched off at $t_2 = 11.1$ s and $t_5 = 31.1$ s and on at $t_3 = 21.1$ s and $t_6 = 41.1$ s. The communication network topology among DGs is presented in Fig. 5.4 and the corresponding normalized Laplacian matrix is:

$$L' = \begin{bmatrix}
1 & -0.25 & -0.25 & -0.25 & -0.25 \\
-0.5 & 1 & -0.5 & 0 & 0 \\
-0.5 & -0.5 & 1 & 0 & 0 \\
-0.5 & 0 & 0 & 1 & -0.5 \\
-0.5 & 0 & 0 & -0.5 & 1
\end{bmatrix}.$$  

Referring to the developed design procedure, the optimal value of coupling coefficient, $\kappa$ is calculated as $\kappa = 1$. In order to observe more significant deviations brought by delays for presentation purpose, the value of $\kappa$ in this case study is intentionally selected as $\kappa = 4$, which is larger than the optimal one. The upper bound of acceptable communication delay is estimated as $\tau_u = 785$ ms. The algorithm execution time step in each BBB is selected as $\Delta T = 50$ ms. The designed period of the predefined square-wave inputs, $g_i(t)$ is selected as $T_g = 6$ s. For validation purpose, the communication delays are intentionally extended. Data exchanges among each BBB board are delayed by 50 ms. Further, to demonstrate the effects brought by communication delay distribution variation, data exchanges on DG1 are further delayed by 100 ms at $t_4 = 26$ s to emulate a network congestion. Recall the measurements shown in Fig. 2.13, the range of time varying communication delay could be estimated approximately as $\tau \in (50, 80)$ ms when $t < t_4$ and $\tau \in (117, 147)$ ms when $t \geq t_4$. 

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The voltages measured by each DG, $E_i$, is recorded in p.u. and presented in Fig. 5.6; they represent the dynamic states whose average is being tracked, i.e., in this case $z_i = E_i$ where $E_i$ represents operation voltage of the $i^{th}$ DG. It can also be observed in Fig. 5.6 that the voltage variation at $t_2$, $t_3$, $t_5$ and $t_6$ represent step changes of $z_i$, and thus the subsequent experimental results and corresponding discussion are all derived under step changes of $z_i$. State variation of the proposed distributed observer in (5.18) is presented in Fig. 5.7 and Fig. 5.8. In Fig. 5.7, the state $x_i$ on each DG is able to capture the system voltage variations and converge in less than 1 s after network congestion at $t = t_4$. In Fig. 5.8, the state $y_i$ on each DG is able to respond to the communication delay variation every time it reaches equilibrium: $|c_y - c'_y| = 0.83$ before the network congestion while $|c_y - c'_y| = 0.68$ after the communication delays are extended. Additionally, state $y_i$ on each DG are able to converge in less than 1 s after network congestion. Figure 5.9 presents the average voltage calculated at DG1’s BBB board by different types of distributed average observers.

1. The average voltage observed by dynamic consensus in (5.5) is shown in Fig. 5.9a. It can be observed that the estimated average contains significant steady state errors and the estimation errors depend on network communication delay. As shown in
Fig. 5.9a, the average DG voltage observed by average consensus algorithm after the initialization at $t_1$ is $V_{\text{avg}, 1} = 0.69$ p.u., while the accurate average is $V_1 = 0.82$ p.u. Referring to (5.6), in this case $c_x' = 0$, $c_x = 0.68$ and $u'z = 0.82$. The deviation factor is calculated as $\frac{1}{1 + \kappa \overline{\tau}} = \frac{c_x}{u'z} = 0.83$. The mean of delay is calculated as $\overline{\tau} = 51.5$ ms, which falls in the estimated range of communication delay at this stage.

2. The average voltage observed by ratio consensus in (5.7) is shown in Fig. 5.9b. After the observer is initialized, the state in (5.7b), plotted in blue, converges to $c_y = c_y' = 0.83$, which agrees with the deviation factor introduced to average consensus. Referring to (5.8), ratio consensus can accurately track the average before the network congestion.
However, after the delay distribution is extended at $t_4$, the observer can no longer accurately track the average. Figure 5.9b shows the actual average voltage variation at $t_5$ is $|\Delta V| = 0.14$ p.u., while the observed variation is $|\Delta V_{est}| = 0.115$ p.u. which represents a 21% observation error. Referring to (5.8), in this case, $c'_y = 0.83$, $c_p = 0.115$ and $u'z = 0.14$. The deviation factor is calculated as $\frac{1}{1 + \kappa^2} = \frac{c_p c'_y}{u'z} = 0.68$ when $t \geq t_4$. The mean of delay is calculated as 118 ms, which falls in the estimated range of communication delay at this stage.

3. Tracking performance by the proposed observer in (5.18) is presented in Fig. 5.9c. The proposed observer is able to track the average DG voltage with high accuracy...
Figure 5.9: Average voltage observation done on DG1 BBB board by (a) dynamic consensus (b) ratio consensus and (c) proposed observer in presence of time varying delay. As shown in Fig. 5.8, the observed square wave magnitude is calculated as $|c_y - c'_y| = 0.83$ when $t < t_4$ and $|c_y - c'_y| = 0.68$ when $t \geq t_4$, both of which agree with the deviation factors observed in previous cases.

Finally, we validate the time-synchronization performance of the HIL testbed enabled by RIAPS platform. The function $g_i(t)$ generated by each BBB board is presented in Fig. 5.10. The plot shows close synchronization of the distributed nodes, with a maximum operation time mismatch $\Delta T_{sync}$ of about 0.2 ms, as shown in Fig. 5.10b. As discussed in Section
5.4.1, the RIAPS platform synchronization feature will synchronize system clocks of the BBB boards within microseconds. However, the voltage measurements that need to be averaged are measured by the TI MCU and need to be communicated to the BBB via the Modbus link. While the Modbus protocol has some inherent time delay variations, the voltage measurement time drift is still bounded within 0.2 ms in this case. Compared to the 50 ms controller iterative time step, the 0.2 ms time synchronization error can be ignored. Additionally, such error is also negligible for power system secondary control in practice (see IEEE Std. 2030.7 [829]). Therefore, it can be concluded that synchronous sampling of the measurement data and thus the synchronous operation among distributed hardware controllers is achieved in the proposed HIL testbed.
5.5 Conclusion

In this work, a distributed control algorithm is proposed to track the average of a group of dynamic states. Compared to conventional observers that are based on dynamic consensus or ratio consensus, the proposed average observer is able to achieve accurate average tracking in presence of time varying communication delays. The proposed observer relies on time synchronous operation among the distributed controllers and requires no direct measurement of the real-time communication delay. The effectiveness of the proposed algorithm is validated by tracking the average voltage measurements of five DGs in a HIL testbed using industrial hardware controllers. The hardware implementation and time synchronous execution of the proposed algorithm is enabled by RIAPS platform. In conclusion, the proposed observer is able to accurately track the average DG voltage with varying delay caused by network congestion.
In this work, VSI-VCM DGs have been utilized for system operation regulation due to their increasing system integration and desirable controllability. The proposed controllers are all designed in distributed manner to be scalable and avoid system single point of failure. The proposed MG control system is able to ensure a dynamic MG being properly operated following the requirements defined in IEEE Std 2030.7. Dispatch rules for each MG operation mode are defined (excepted for black start (T3) as it has been identified as system-dependent): 1) In steady state connected (SS1), VSI-VCMs operate as controllable power source by injecting desired active/reactive power into the system; 2) In steady state island (SS2), VSI-VCMs operate as slack bus. System frequency and average voltage are well-regulated and proportional power sharing are achieved among connected DGs; 3) In case of unplanned islanding (T1), the VSI-VCMs are able to switch from grid-connected
mode to islanded mode to keep energizing the loads and provide voltage support; 4) In case of **planned islanding (T2)**, the VSI-VCMs are able to actively eliminate the power flow through the breaker that is about to open to ensure a seamless system transition; 5) In case of **reconnect (T4)**, the VSI-VCMs are able to have the islanded MG operated synchronously with the main grid before the main breaker closes. The proposed MG control system can be applicable to not only static MGs individually but also dynamic MGs with interactions.

In Chapter 1, the concept of dynamic MGs, as an advanced MG structure, is introduced and a novel control framework for dynamic MGs operation is proposed. The proposed system structure manages dynamic MG operation in a distributed manner utilizing SSWs.

In Chapter 2, standardized requirements for MG control systems are reviewed. A novel software platform, called RIAPS platform is introduced as a toolkit for implementing distributed control algorithm under hardware level. A CHIL MG testbed is proposed to better emulate the operation environment of a MG system and provides reliable validation of the analytically developed control algorithms.

In Chapter 3, a set of distributed controllers are proposed providing secondary control for both balanced and unbalanced dynamic MGs operation and serve **steady state island (SS2)**. The proposed controllers are able to provide constant system frequency and average voltage regulation, along with active system VU compensation, and ensure seamless system transition as system topology varies. After the reconfiguration is done and the system operates under new topology, the proposed controller is able to identify DGs that are connected together and automatically perform DG re-grouping if needed. At last, it is noteworthy that the proposed controller that enables seamless system transition as SSW opens can also be adopted to serve in **planned islanding (T2)**.

In Chapter 4, a distributed controller utilizing pinning-based consensus algorithm is proposed to achieve islanded synchronous operation with the main grid and serves
The proposed controller is able to explicitly regulate system frequency, phase and voltage magnitude at PCC while ensuring proportional power sharing among DGs. Unlike most existing works that assume a constant and rated main grid frequency, the proposed controller is able to actively capture and respond to any frequency variation at the main grid.

In Chapter 5, the discussion focus on the accuracy of consensus-based distributed average observer. The deviations brought by time-varying communication delays to the equilibrium values of three types of consensus algorithms are analyzed, and a distributed average observed that guarantees accurate average observation is proposed.

The work presented in this paper mainly focus on developing secondary control for dynamic MGs operation. The proposed control algorithms enables reliable system operation regarding network reconfiguration, while still rely on the system operator to decide the topology under which the system should be operated. In the future, more works could be done under tertiary level regarding the proposed dynamic MG control framework: e.g., how the min-MGs could utilize information from both local measurement and peer-to-peer communication to decide the optimal system topology collectively in a distributed fashion.
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