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Natural gas is an energy source, which has increased in importance in recent years and has become more and more popular in international trade with its increasing usage areas in terms of being cleaner and safer than other fossil fuels. On the other hand matching algorithms and its special subtype matching with contracts have been used for economic theory and beyond this have important practical applications. In this context, this study aims to transform the international natural gas trade with an alternative method with the matching with contracts mechanism, specifically cumulative offer process. Although choice functions of the model have not satisfy the most desired outcome of stability of the model, I show that cumulative offer process selects stable matching in any problem. Since violation of substitutes condition is never observed during the mechanism, for that model, the process is stable in the case.
A Market Design Approach to Natural Gas Trade

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To my amazing husband,

to my precious kids,

to my biggest supporter my brother and

to my parents who always support me materially and morally.
BIOGRAPHY

Fatma Bihter Kalkan was born in 1981 in Ankara, Turkey. Her Bachelor of Science Degree in economics is from Ankara University. After graduating, in 2004 she started to work as a banking specialist at Ziraat Bank which is the biggest bank of Turkey by assets. Then, in 2009 she changed her job and succeeded in a highly selection pool for a position with Energy Market Regulatory Authority (EMRA) as an energy expert. She has been working on Natural Gas Market Department of EMRA for nine years. In this regard, she is quite well equipped on both domestic and international natural gas markets. She graduated from the NC State University Poole College of Management in 2020 with a high degree of success in all courses and earned a Master of Science Degree in Economics.
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1. Introduction

Natural gas is considered a lower-carbon fuel and one of the main energy sources. Demand and consumption rates of natural gas increase promptly in the time. Also, the desire of countries for generating clean and secure energy highlights the natural gas to meet the energy needs and achieve an affordable energy source [1]. Therefore, the natural gas’s importance increasing in the world, especially in Europe. The development of national economies and the capacities of competition are considerably related to the energy sectors [2]. Natural gas export market dominated by only 20 countries with the ratio of %92. On the import side, the situation is similar that the first 20 country’s import share is almost %85.

There are various steps on producing natural gas, such as search, extraction, production, transportation, storage, distribution and sale until the natural gas reaches to the end user. On the other side, trade of the natural gas is involved in all these steps to ensure the safe production. So, we can define that the trade of natural gas is the coordination of all steps within the production and marketing stages from wells to end users.

Natural gas trade has not had a global market structure yet. For this reason, there are three main regional markets, which the natural gas is priced. The first one is dominated by the gas markets in North America. This market is very competitive that the price is determined by the gas trade and distribution center called “Hub”. This model is a typical spot market model which can be defined as a gas to gas competition. The second one is the Long-Term Contract model which is in use in the European market including Turkey and in which the price of gas is indexed to petroleum products. It provides to producers and consumers the protection from the price fluctuations like 'take or pay' model. The third model is the Japanese Crude Cocktail model,
where gas prices are directly linked to oil price averages in long-term LNG contracts and reflect the freight / freight costs [3].

Therefore, the economic operations of the American, Asian and European gas markets have developed geographically as disjointed throughout the years. The pricing mechanism in long-term contracts has become widespread in the European gas trade with the name of Groningen as of the 1950s and has reached today with the slightest change in contract structures. However, this model, settled in Europe, has been questioned by the restructuring of the European gas market as a result of the developments in 2008 after 50 years of sovereignty. Today, gas trading in European markets is no longer done through long-term import-export contracts, but rather through short-term spot market transactions.

As the purpose of this thesis is to make a theoretical model that includes the structure of European and Asian markets, it is considered useful to look at the European gas markets development process.

Competition for natural gas is started in the early 2000’s in the European market. After then all the gas market, involving the organization, coordination and the actors are changed to adapt with new competitive situation. Europe’s gas markets are one of the steady markets in the world that it covers many populated areas and supplies gas more than 90 percent of households. From 1990 to 2010, the natural gas consumption of the EU27 countries grew from 333 to 520 billion cubic meters per year. Since the power generation sector is the main customer, it accounted for more than 60 percent of incremental demand growth by years.

It is hard to say that all the end user can buy the gas at the same price. We can see lot of different prices in different EU countries. So, EU tries to establish a European energy market which keeps the prices same level for all the countries in the union [4].
Like EU, many union or region in the world aims to form a common energy market which provides a stable market in terms of both supply and demand and help them take a position on the global energy market [2].

European Union sets some general principles to ensure a common market in the union which are market pricing, competitive market structure and remove the obstacle to trade in energy resources. By this way, it aims these principles to align with the national regulations to maintain a competitive business infrastructure.

There has been so many EU Gas Directives in order to form a common European gas market. European Union Commission has announced a new Energy Union Policy in February 2015 which aim to transform the European energy market. This policy aims to provide customers secure, sustainable energy with affordable prices as well as keeping its climate policy. This policy gives attention to the following topics: (1) energy security; (2) an integrated European energy market; (3) energy efficiency; (4) decarbonizing the economy and (5) innovation and research. It can be said that the main shift in the EU Energy Policy is moving away from a fossil fuel economy to a more diversified model which has a new business model.

EU mainly focuses on the energy security issue, so it determines the objectives to meet the energy needs of region. There is huge need to diversify the energy resources as well as the supplier and transport routes.

In this regard EU aims to diversify energy as follows: first alternative is Southern Energy Corridor which enable to transfer the central Asia gasses to Europe; secondly constitute a liquid gas market in various parts of Europe; thirdly development of the transportation facilities supported with financial instruments; fourthly in case of emergency to keep the usual activities prepare a LNG (liquified natural gas) strategy and construct gas storage facilities; fifthly as a part
of strategy of moving away from fossil fuel economy paying attention to increase the share of renewable energy for both to reduce the import dependence and to meet the environmental expectations.

On the other side, while the physical natural gas trade increased all around the Europe, all the actor in the market such as producers, distributors and even end users try to manage their risks by using derivative financial instruments like future contracts to avoid the price risk. By this way, actors can fix the price of the natural gas today for delivery for a future day. Recent years there is noticeably increase in future contracts especially on UK National Balancing Point (NBP) and Dutch Title Transfer Facility (TTF) hubs. These contract provides producers, end users, institutions, energy corporations to predictable, safe and transparent price opportunity [5].

However, in this study, it is tried to be modeled by analyzing the physical gas trade and not the virtual gas trade. It can be said that the competition which is started almost 20 years ago in the European natural gas market, still continues to reshape and reorganize today. Since the world changes very promptly in terms of such as environmental, political, geopolitical or technological, the policy making is required to be more updated in line with the developments in the market [6].

As a result of the economic and political transformations of the global markets, the supply side of the natural side is also affected as well as the demand side in the European gas market. Naturally, these changes shaped the European gas market context and are reflected to the gas directives as well.

Europe has its own indigenous gas reserves in Netherlands, Norway, UK, Germany, France and Italy for the supply side of gas. However, apart from Norway, potential of gas supply is decreasing every year in Europe region. While the potential is decreasing, some other factors
can arise such as, in the Netherland’s Groningen field. Due to the gas production sourced
earthquakes government had to limit its gas production in that field.

Therefore, European countries import natural gas from Norway, Russia, Algeria and
Libya and it is transferred to Europe via pipelines. Although these imports are set by the long-
term contracts, these countries are open to renegotiate the terms and conditions almost any time
to keep their competitive position against the alternative suppliers. The natural gas is not an
indefinite source, sure these import facilities have limited capacity for the future. Since new gas
resources, especially in Russia and Norway, are in difficult areas, it requires a lot of investment
and is expensive to operate. So, finding new natural gas resources for the current streams are
very harder than operating current gas opportunities.

As an alternative gas source, new potential suppliers for Europe may be Iraq, Iran,
Azerbaijan and Turkmenistan. In recent years, East Mediterranean area which includes Israel,
Egypt and Cyprus, has a growing importance in terms of finding new gas resources. With adding
South East countries and the Mediterranean countries to the game, it helps the diversification of
players in terms of supply and demand side. However, although the number of players is
increasing, the situation is not getting smooth because of the instabilities and conflicts in the
region. Also, there are some transit issues arising from the many counties on the path. The
political complexities due to the position of Russia also make it difficult to construct in new
pipelines through the south.

In the light of this information, it can be said that the LNG could be a good alternative for
the huge natural gas and energy demand of Europe. LNG market is growing promptly in terms of
both the current resources and the new reserves after the exploration of US shale gas. Also
relatively short transfer distance, low political risk and predictable, transparent price structure
make the US shale gas very attractive for the European market. The current infrastructure for re-gasification of the LNG is quite enough to meet the expectation of LNG trade between EU and US.

European customer mainly focusses on the price, volume, flexibility, and availability of the LNG export form North America. Also, this export has importance in terms of the possible consequences on the European markets. However, the export shale gas in form of LNG from North America needs a more detailed review for its sustainability and the ability to meet the demand.

The demand for the natural gas in Europe also needs to be evaluated. Some of the European countries uses the natural as a traditional source of energy for long years. In these countries, the growth for the demand is slow. However, for some countries like Spain, the natural gas is relatively new source of energy. In Spain, the demand and the market share are increasing promptly for the ease of use and the environmental expectation. In this respect, although the outlook for the natural gas was encouraging at first, for the long term it is not clear to make strategic decisions.

One of the reasons, probably the most important, is the economic slowdown in Europe after the 2008 economic crisis which caused a decrease in consumption of the natural gas. It brings some uncertainties for the demand of the gas in the future. For the environmental consequences, natural gas lost its reputation especially after the Paris agreements in December 2015. Some of the European countries, especially Germany and Spain turned its face to the renewable energy such as wind and solar.
In the light of all these supply and demand informations and supply and demand side constraints, a matching strategy with contracts will be designed to have the features compatible with the European gas market structure.

The original matching with contracts models is a two-sided matching model, and as such, each country whether importer or exporter has preferences over other countries. “The theory of two-sided matching markets attracts attention for its theoretical appeal and its applicability to the design of real-world institutions” [7]. Therefore, it is thought that examining this market, which has many constraints, players, and alternative matching opportunities, under this theory will contribute to both the market and the theory itself.

The paper is organized as follows. After this section where I describe the features of the natural gas market, in section 2, I mention about the related literature. In Section 3, I formally introduce the matching game with general framework along with the definitions of the desirable properties of choice functions. In Section 4, I give results including stability condition, condition guaranteeing existence of a stable matching, COP and stability of COP. In Section 5, I conclude.

In this thesis, I will focus on trade of natural gas in European and Asian Markets. The purpose of this thesis is to contribute to the market design aspects of real life matching markets by establishing a theoretical matching model in the light of all these real market information and data. Such an approach to the natural gas trade has not been done before. So it can be considered as a contribution to the literature in terms of showing gas trade as many-to-many model for the first time. Besides, reaching a stable matching can be considered as a contribution although the conditions do not provide stability.
2. Related Literature

The matching market first published by David Gale and Lloyd Shapley by defining it with a simple mechanism has developed in fifty years and has found a wide working area. Gale and Shapley were especially interested in the stability of the matching they built on man and woman choices for each other which is named as marriage market. They made a great contribution by finding that there was a stable matching, which is referred to the literature as deferred acceptance algorithm. Many versions of this algorithm are used today to match hospitals with residents and students with public schools in New York City and Boston [8]

Lloyd Shapley and Herbert Scarf then published a somewhat related paper which was the simplest exchange economy one could think of. It was a matching algorithm with indivisible goods and they presented a Top Trading Cycles algorithm.

Matching algorithms have been used for economic theory and though have important practical applications. Alvin E. Roth and Elliott Peranson [9] explain how a certain two-sided matching procedure, which is similar to the college admissions algorithm introduced by David Gale and Lloyd Shapley [10], has been adapted to match 20,000 doctors per year to medical residency programs [8]

Atila Abdulkadiroğlu and Tayfun Sönmez [11] used the same algorithm for school choice programs, adopted to New York City schools [12] and another is being evaluated by the Boston schools [13]. They presented a Top Trading Cycles algorithm. Almost thirty years later, the basic ideas of Gale's Top Trading Cycles algorithm adopted and used for organized kidney exchange in the world and saved and is still saving thousands of lives.
There are several surveys on two-sided matching markets especially until 1990. The best known of these are by Roth and Sotomayor [14], Roth [15]. Both these surveys are essentially of two-sided matching markets.

Hatfield and Milgrom [16] formalized a general matching with contracts framework in which a contract is fully identified by a doctor, a hospital and possibly a wage. In our model we also focus on a matching with contracts with money (price). The Kelso-Crawford labor market matching, package auctions and the college admission problem are embodied in this unified framework as its special cases [17].

Hatfield and Milgrom subsequently introduced a different mechanism named Cumulative Offer Process (COP). This mechanism is consistent with Gale-Shapley’s doctor offering algorithm under some conditions. In COP hospitals are able to choose among all the offers they have received previously including the current offers however the necessary condition is substitutes condition. Using this algorithm, Hatfield and Kojima [7] introduced new conditions of different types of substitutes, which are bilateral and unilateral substitutes. They also showed that bilateral condition is a sufficient condition for a stable allocation. Additionally, they also prove that along with the law of aggregate demand condition, the unilateral substitutes guarantees the group strategy proofness of the doctor-optimal stable mechanism. In a different study by Aygun and Sonmez [18], it is shown that without Irrelevance of Rejected Contracts (IRC) condition, the majority of their theorems are not hold.

A different study by Sonmez [19], is another example that shows that matching with contracts model have great potential to prescribe solutions to real-life resource allocation problems.
In Contract design and stability in many-to-many matching, Hatfield and Kominers [20] develop a model of many-to-many matching with contracts that subsumes as special cases many-to-many matching markets and buyer–seller markets with heterogeneous and indivisible goods. In a recent study, Dur, Morrill and Phan show that a stable outcome exists without substitutes condition [21].

3. Model

The matching-with-contracts notation treats an allocation as a set of contracts and characterizes the stable allocations in terms of the solution of a certain system of two equations. An allocation is a set of contracts such that capacity constraints are satisfied.

In the natural gas market, there are two sides – importer countries and exporter countries. Each importer is matched to several exporters (indicating that it is not unmatched) in order to meet their natural gas needs and vice-versa (each exporter is matched to several importers in order to sell all its natural gas capacity). Thus, more than one importer is assigned more than one exporter through contracts. So this is an example of a “many-to-many” matching market.

In this model there is no unmatched pair. Here, unmatched does not mean that the countries necessarily reach an agreement. The definition of this concept is that even if importers or exporters cannot make a deal, even if they do not reach a match in line with the price level they set, there is no blocking pair either. The stability of the model will be examined in detail in the following section.

In the model both importers and exporters have their maximum and minimum acceptable price levels and their own ranking orders on countries. Country’s rankings are formed by their decisions and choices on pipeline presence, historic and political experiences etc.
Each importer and exporter has their own capacities. Importers have either 1-unit or 2-units capacities. There are also two kinds of importers who have 2-units capacities, one of which is allowed to buy all its gas needs from a single exporter, the other, however, is not allowed to do this. Just the opposite it has to meet the goal of diversity by purchasing gas from two different exporters. The importer countries allocate their demands at least 1 unit to exporter countries. Decimal numbers are not allowed for this model.

A simple example of natural gas market matching model could expect to obtain a stable outcome where only importers are allowed to make proposals. The choice functions of countries are affected by natural gas supply and demand capacities, the price of natural gas and the rankings.

**Ranking over partners:**

To express the rankings between countries explicitly, the rankings of each country will be represented by a list of ranking orders, $P_m$ and $P_e$.

We also assume that remaining unmatched is the last choice for each importer/exporter.

$P_m$ on the set $M \cup \{m\}$.

$$P_m = e_1, e_2, \ldots, e_n$$

$P_e$, on the set $E \cup \{e\}$

$$P_e = m_1, m_2, \ldots, m_n.$$

**3.1 Formal Model:**

The elements of our model are as follows:
There are two finite and disjoint sets: M and E: M=\{m_1,m_2,...,m_n\} is the set of natural gas importer countries, and E=\{e_1,e_2,...,e_n\} is the set of natural gas exporter countries. Each importer and exporter country has restrictions described above. The choice functions of countries are affected by natural gas supply and demand capacities, the price of natural gas and the preferences which is a function of different characteristics and priorities of countries such as the presence of pipelines, positive or negative international relations between countries. Ranking orders are meant to represent how importer and exporter countries would choose among different alternatives with all these restrictions and when they confronted with a situation that all the other things are equal.

So, this natural gas matching problem starts with:

1. A finite set of natural gas importers M= \{m_1, m_2, m_p\}
2. A finite set of natural gas exporters E= \{e_1, e_2, e_n\}
3. A vector of quantity of natural gas that is sold/purchased by exporters/importers $q_e=\{q_e^1, q_e^2, ..., q_e^n\}$ and $q_m=\{q_m^1, q_m^2, ..., q_m^p\}$
4. Price $\Pi=\{\pi^1, \pi^2, ..., \pi^c\}$

### 3.2 Set of Contracts and Choice Functions:

The sets of importers and exporters are denoted by M and E, respectively, and the set of contracts is denoted by X. For this model, a contract specifies an importer, an exporter, amount of gas sold, and a price, $X \in M \times E \times Q \times P$.

We formalize set of contracts as follows:

Given a set of contracts $X' \subset X$ offered in the market,
\(X'_m=\) set of contracts including importer \(m\), it also implies the contracts of importer \(m\) in \(X'\).

\(x \in X'\) and \(x = (m, e, q, \pi)\)

\(m(x)\) represents the importer country of the contract \(x\).

\(e(x)\) represents the exporter country of the contract \(x\).

\(q(x)\) gives the capacity of the contract \(x\).

\(\pi(x)\) represents the price of the contract \(x\).

**Importers:**

Importers take into account two criteria when creating a set of choices. First criterion is the capacity they need. Some of them need 1 bcm (billion cubic meter) gas, others need 2 bcm. Some of the importers in need of 2 bcm gas are allowed to buy all of its gas need from a single country and others have to buy gas from at least two countries in accordance with their legislation. The second criterion for importers is the price. They want to buy their capacity needs as cheap as possible therefore offer to buy gas first from the lowest priced exporter. Among the same prices, importer countries’ ranking orders on country choices will be decisive. Each country has its own ranking order as mentioned above.

When we say an importer country prefer with a choice between two, it would choose \(a\) and not \(b\), and if faced with a choice from a set of alternatives that include \(b\), then it would not choose \(b\) if \(a\) were also available. There is not a situation being indifferent between two, the country has to choose its most preferred country, when all the other things are equal. To sum up, preferences are to be made by taking into account capacity ratios established within the scope of
security of supply in the legislation of countries, pipeline presence preferences, historical and political issues and etc.

To express these ranking orders between countries explicitly, the preferences of each country will be represented by an ordered list of rankings, \( P_m \) on the set \( M \cup \{m\} \). That is, an importer country’s preferences might be of the form;

\[
P_m = e_1, e_2, \ldots, e_p
\]

which indicates that when price level is equal, the importer country’s first choice is to import natural gas from the exporter country \( e_1 \), its second choice is to be \( e_2 \) and so on. Another country \( m' \) has its own preferences among exporter countries. So, its ranking order list will be denoted as follows:

\[
P_{m'} = e_3, e_1, \ldots, e_n
\]

indicates that country \( m' \) prefers to import natural gas from the exporter \( e_3 \) first, then \( e_1 \) and so on.

**Exporters:**

For exporters, there is not a restriction on importers and choices. The capacity of the exporter is the only restriction for exporters. The only characteristic is the prioritization of the importer who offers the highest price. Exporters should of course accept bids made in accordance with the supply capacity. Exporters distribute the capacity starting from the top one in the order of ranking among the bids made at the same price level.

Just like importer countries, exporters have country rankings over a choice between two, it would choose a and not b, and if faced with a choice from a set of alternatives that include b, then it would not choose b if a were also available. Again, an exporter country does not face with
a situation being indifferent between two, we mean that it would choose one of them. Hence, ranking orders are to be made by taking into account capacity ratios established within the scope of security of demand.

Similarly, each exporter country has an ordered list of ranking order $P_e$, on the set $E \cup \{e\}$. So, one of the exporter countries’ ranking list can be indicated as;

$$P_e = m_1, m_2, \ldots, m_k.$$ 

Another exporting country’s ranking order list can be indicated as follows:

$$P_{e'} = m_4, m_2, \ldots, m_n.$$ 

To sum up, in this system, both importers have ranking orders over exporters and exporters have ranking orders over importers. That is, each importer/exporter has well-defined ranking orders over exporters/importers alone independent of the price. We also assume that remaining unmatched is the last choice for each importer/exporter.

**Choice functions for importers;**

Each importer country has a choice function which selects the chosen set of contracts among a given subset of contracts.

$$X_m \subset X; \text{ such that } \sum_{x \in C_m(X')} q(x) \leq Q_m$$

Given $X'$, we denote the set of contracts chosen by $m$ as;

$$C_m(X') \subset X'_m$$

$$|C_m(X')| \leq Q_m$$
Each importer $m$ can sign more than one contract. Set of contracts chosen by $m$ is a subset of all contracts that include $m$. Each importer country has a choice function which selects the chosen set of contracts among a given subset of contracts. Total of every single quantity of chosen set of contracts for an importer must be less than or equal to total capacity of that importer.

- **For importers:** (with 1-unit capacities, 2-units capacities with one exporter-unrestricted)
  
  • Importer $m$ considers all contracts in $X'$ with price $\pi_1$, which is the lowest price. It fills its capacity among lowest price according to $P_m$.
  
  • Contracts are added to chosen set one by one
  
  • Checking all contracts in this price level, if it still has capacity remaining, it will now consider at the next price level, $\pi_2$, according to $P_m$.
  
  • $C_m(X')$ is determined through this procedure until the capacity is reached.

Among lowest price it fills its capacity according to $P_m$. Each importer $m$ can sign more than one contract. Its preferences over possible contracts, including the null contract $\emptyset$, are described by the total order $\succ_m$. The null contract represents unmatched and not blocking pair of contracts, and contracts are acceptable or unacceptable according to whether they are more preferred than $\emptyset$. When we write preferences as $P_m$: $x \succ_m y \succ_m z$, we mean that $P_m$ names the ranking order of $m$ and that the listed contracts (in this case, $x$, $y$, and $z$) are the only acceptable ones.

One important thing is that, for an importer with 2 bcm capacity, if the importer has 2 and 1 unit contracts to be chosen, then importer will choose 2-unit contracts.
- For importers: (with 2 unit capacity-2 exporters-restricted)

  • Importer $m$ considers all 1-unit contracts in $X'$ with the lowest price. It fills its capacity with contracts including two different exporters at the lowest price.

  • If there exist no 2 different exporter contracts at the lowest price, considers contracts with different countries at a higher price according to $P_m$.

  • Contracts are added to chosen set one by one.

  • Importer checks different exporter contracts in all steps until the capacity is reached.

  • $C_m(X')$ is determined through this procedure until the capacity is reached.

**Choice functions for exporters;**

$X'_e$ = set of contracts including exporter $e$

$X_e' \subseteq X'$;

$C_e(X') \subseteq X'_e$

Given $X'$, we denote the set of contracts chosen by $e$ as;

$C_e(X') \subseteq X'_e$ such that $\sum q(x) \leq Q_e$

$x \in C_e(X')$

Each exporter $e$ can sign more than one contract. Set of contracts chosen by $e$ is a subset of all contracts that include $e$. Each exporter country has a choice function which selects the chosen set of contracts among a given subset of contracts.

  • For a given set of contracts exporter $e$ first considers the one with the highest price.
• Such contracts are added to the chosen set one by one as long as the capacity is not exceeded according to the ranking over the importer countries.

• After checking all contracts in this price level, if it still has capacity remaining, it will now consider at the next price level according to \( P_e \).

• Chosen set of contracts for exporter \( e \) is determined through this procedure until the capacity is reached.

Among the highest price, it fills its capacity according to \( P_e \). Price is the most important and almost single criterion for exporters in the process of accepting offers. The supply capacity is the other criterion. The features of the choice functions will be examined in detail in the next section. It is also valid for exporters that if the exporter is offered 2-unit and 1-unit contracts to be chosen, then exporter will choose 2-unit contracts.

4. Result

In this two-sided market, stability is the most desired goal. I formally explain what stability means first. After the definition of stability, I introduce the condition on the choice functions which plays key role in the existence of stable matching. And check whether they are stable or not.

Then I define COP for our model. I look at whether the solutions are stable or not in COP.

4.1 Stability

Definition: A set of contracts \( X' \subseteq X \) is a stable allocation (or a stable set of contracts) if

1. \( C_m(X') = C_e(X') = X' \), and
2. There exists no exporter $e \in E$ and set of contracts $X'' \neq C_e(X')$ such that $X'' = C_e(X' \cup X'') \subseteq \bigcup_{m \in M} C_m(X' \cup X'')$. 

An allocation is a set of contracts such that capacity constraints are satisfied. When the first condition fails, the allocation $X'$ fails individual rationality and there is a blocking importer or an exporter. When the second condition fails, there is a blocking coalition that consists of an exporter $e$ and a subset of importers $\{x_m\}_{x \in X''}$. In this case we say that $X''$ blocks $X'$ [23].

4.2 Substitutes Condition:

Hatfield and Milgrom [16] present a unified framework of matching with contracts, which includes the two-sided matching models and package auction models as special cases. They introduce the substitutes condition, which is a natural extension of the substitutability condition in the matching literature to matching with contracts and show that there exists a stable allocation of contracts if contracts are substitutes [7]. 

In the original matching with contracts model, and in all of the settings just described, **cumulative offer mechanism**—ascending auction-like mechanisms in which agents on one side of the market successively propose contracts to the other side—have been found to be stable and strategy-proof [16].

Contracts are substitutes if a contract that is chosen from a larger set $X''$ is chosen from any of its subsets $X' \subset X''$ as well. Equivalently, any contract that is rejected from a smaller set $X'$ is also rejected from any larger set $X''$ that contains $X'$. If elements of $X$ are substitutes, then the set of stable allocations is nonempty [22]. In other words, in matching with contracts settings, contracts are substitutes if the set of rejected contracts does not shrink whenever the firms’ choice set expands [17]. This abstract substitute’s condition coincides exactly with the demand theory condition for standard models with prices.
**Definition:** Contracts are **substitutes** for exporter e if there do not exist a set of contracts $Y \subset X$ and a pair of contracts $x, z \in X \setminus Y$ such that

$$z \notin C(Y \cup \{z\}) \text{ and } z \in C(Y \cup \{x, z\}) \quad [18]$$

The substitutes condition captures the intuitive idea that a contract that is rejected from a set of contracts shall remain to be rejected when there is increased competition [18].

In another definition, elements of X are substitutes for exporter e if for all subsets $X' \subset X'' \subset X$, we have $R_e(X') \subset R_e(X'')$.

In our model, substitutability must be examined for importers and exporters separately. For importers it will be analyzed for two kinds of importer separately too.

**Proposition 1:** Choice function of importers with quota 1 and restricted importers satisfies substitutes condition.

**Proof:** Let $C_m(X')$ be the chosen set of contracts when $m$ considers $X'$. Suppose $x \notin C_m(X')$ but $x \in X'$. Then if $C_m(X')$ is substitutable, $x \notin C_m(X' \cup x)$. In order to understand this, it is necessary to examine two cases why $x \notin C_m(X')$.

1. Let $\pi_m^i$ be the threshold price (maximum price level to be accepted by the importer) for importer $m$. Let $\pi(x)$ is above threshold price for $m$, $\pi(x) > \pi_m^i$. If this is the case, adding any other contracts will not change the fact that $\pi(x)$ is above threshold price. Then $x \notin C_m(X' \cup x)$.

2. $\pi(m)$ is below threshold price for $m$, $\pi(x) < \pi_m^i$, but $\pi(x)$ is either higher than the maximum price of contracts in $C_m(X')$ or $\pi(x)$ is equal to that price but all chosen contracts are from more preferred countries.
In these two cases, adding new contracts will not change the situation that this contracts with $\pi(x)$ price level is rejected.

**Proposition 2:** Choice function of unrestricted importers does not satisfy substitutes condition.

**Proof of Proposition 2:** Let importer $m$’s ranking over exporters be $\{e_1, e_2\}$.

Let $X' = \{(m, e_2, q=2, \pi); (m, e_2, q=1, \pi)\}$

$X'' = \{(m, e_1, q=1, \pi)\} \cup X'x$

$C_m(X') = \{(m, e_2, q=2, \pi)\}$

$C_m(X'') = \{(m, e_1, q=1, \pi), (m, e_2, q=1, \pi)\}$

In the first set of contracts, $X'$, when an importer faces two different contracts with the same exporter and same price, it chooses one with 2-units capacity by assumption. So, the contract with 1 unit is rejected. However, if we expand the set of contracts by adding a better contract, $X''$, which implies a contract with a more preferred exporter, the previously rejected contract will be chosen by the importer to fill the capacity.

Any contract that is rejected from a smaller set $X'$ must be also rejected from any larger set $X''$ that contains $X'$ however in this case this condition does not hold. Thus, these two choice functions are not substitutes.

**Proposition 3:** Choice function of exporters does not satisfy substitutes condition.

**Proof of Proposition 3:** Let exporter $e$’s ranking over importers be $\{m_1, m_2\}$.

Let $X' = \{(m_2, e, q=2, \pi); (m_2, e, q=1, \pi)\}$

$X'' = \{(m_1, e, q=1, \pi)\} \cup X'$
\[ C_e(X') = \{(m_2, e, q=2, \pi)\} \]

\[ C_e(X'') = \{(m_2, e, q=1, \pi), (m_1, e, q=1, \pi)\} \]

Any contract that is rejected from a smaller set \( X' \) must be also rejected from any larger set \( X'' \) that contains \( X' \), however, in this case this condition does not hold. So again, for this case, choice functions of exporter \( e \) are not substitutes.

Hatfield and Kominers [20] study matching with contracts in many-to-many matching markets. They show that if any agent’s preferences are not substitutable, then the existence of a stable outcome cannot be guaranteed. Since the choice function of unrestricted importers and exporters do not satisfy substitutes condition, existence of a stable outcome cannot be guaranteed. In contrast, we show that cumulative offer process always selects stable outcome in our setting.

### 4.3 Cumulative Offer Processes

The Cumulative Offer Process was introduced in Hatfield and Milgrom [16]. The COP is a generalization of the Deferred Acceptance mechanism of Gale and Shapley’s to the matching with contracts framework [16]. The COP allows the offer-receiving party (in our definition exporters) to choose from cumulatively expanding set of contracts [16]. In other words, our model is an importer proposer model. In this regard, we denote the cumulative offer process as follows:

**Step 1:** Let \( X_m^1 \) be the all contracts related to importer \( m \). Importer \( m \) offers contracts in \( C_m(X_m^1) \) to the corresponding exporter countries.

Let \( X_e^1 \) be the set of contracts received by exporter \( e \), \( C_e(X_e^1) \). The exporter \( e \) accepts the contracts in \( C_e(X_e^1) \) and rejects \( X_e^1 \setminus C_e(X_e^1) \). In every step exporter accepts contracts in the
chosen set, among these, it accepts the best contract which imply that of with the highest price or among the same price level contract with the higher ranking country.

**Step 2:** Let \( X_m^2 \) be the all contracts related to importer \( m \) which have not been rejected yet. Importer \( m \) offers contracts in \( C_m(X_m^2) \) to the corresponding exporter countries.

Let \( X_e^2 \) be the set of contracts received by exporter \( e \), \( C_e(X_e^2) \). The exporter \( e \) accepts the contracts in \( C_e(X_e^2) \) and rejects \( X_e^2 \setminus C_e(X_e^2) \).

**In general;**

**Step \( k \):** Let \( X_m^k \) be the all contracts related to importer \( m \) which have not been rejected yet. Importer \( m \) offers contracts in \( C_m(X_m^k) \) to the corresponding exporter countries.

Let \( X_e^k \) be the set of contracts received by exporter \( e \), \( C_e(X_e^k) \). The exporter \( e \) accepts the contracts in \( C_e(X_e^k) \) and rejects \( X_e^k \setminus C_e(X_e^k) \).

The algorithm terminates when either every importer is matched to at least an exporter or every unmatched importer has had all acceptable contracts rejected. Since each contract is offered at most once, the algorithm terminates in some finite Step \( K \). The outcome is the accepted contracts in the last step.

The outcome;

\[
\bigcup_{e \in E} C_e(X_e^K).
\]

**Theorem 1:** Cumulative offer process selects stable matching in any problem

**Proof Idea:** We provide the formal proof in Appendix A.

- An importer never considers two contracts with the same exporter and same price at the same time.
• An exporter never considers two contracts of with same importer and same price at the same time.

Hence, violation of substitutes condition is never observed during the mechanism. We can see this easily with an example of matching game scenario as follows.

4.4 Matching Game Scenario

The auction algorithm begins with each importer country proposing natural gas import to its most preferred set of exporter countries at the one possible price. When some exporters turn it down, the importer makes with new contracts to exporters to fill its remaining demand gap. We specify that a contract is fully identified by the importer, the exporter, and the price (preferences over countries and other constraints are also valid for the consideration). We also specify that a contract is identified by the bidder (importer), the preferences that the bidder will acquire, and the price to be paid for that bid. Several additional variations can be encompassed by this model, however in this section we set a specific supply-demand quantity-price combination.

Each importer’s preferences and their price proposals for our specific game are as follows;

In this case only one of the importers has to buy natural gas from more than one exporter. We can also run the scenario where all of these importer countries will be under the restriction of gas supply from more than one country. In all these cases, the cumulative offer process operates.

Germany has a demand capacity of 2 bcm with a maximum price offer of $9 per bcm. In order to meet supply security goal, it has to buy natural gas from more than one exporter to meet its natural gas demand (restricted). When making decision, it will take into account the pipeline presence whereas LNG shipping alternative. All countries should trade natural gas up to 1 bcm
and its multiples. Decimal numbers are not allowed. While choosing from the same price levels, Germany's ranking order over exporter countries is as follows:

\[ P_{\text{Germany}} = \{ \text{Russia, Iran, Nigeria} \} \]

Italy has a demand capacity of 2 bcm with a maximum price offer of $8 per bcm. Italy will try to make agreements to meet its natural gas demand need. Italy can buy its whole needed capacity from one single exporter (unrestricted). When making decision, it will take into account the pipeline presence whereas LNG shipping alternative. Due to the geographical situation, Italy has comparative advantages in term of access to the LNG market in comparison with Germany. When confronted with the same prices, Italy’s ranking order over exporters is as follows:

\[ P_{\text{Italy}} = \{ \text{Russia, Iran, Nigeria} \} \]

Turkey has a demand capacity of 2 Bcm with a maximum price offer of $7 per Bcm. Turkey can also buy its whole needed capacity from one single exporter (unrestricted). When making decision, it will take into account the pipeline presence whereas LNG shipping alternative. Due to the geographical situation, Turkey also has comparative advantages in term of access to the LNG market in comparison with Germany.

\[ P_{\text{Turkey}} = \{ \text{Iran, Russia, Nigeria} \} \]

Thereafter, we can look at the exporters' assumptions.

Russia has a supply capacity of 2 bcm with a minimum price offer of $7 per bcm. In order to maximize its profit/income, Russia aims to purchase all its gas supply. Its purpose is to export all its supply resources with a maximum price offer. Russia can trade natural gas up to 1 bcm and its multiples. Decimal numbers are not allowed. When confronted with the same prices, Russia’s ranking order over importers is as follows:
\[ P_{\text{Russia}} = \{\text{Germany, Italy, Turkey}\} \]

Nigeria has a supply capacity of 2 bcm of LNG with a minimum price offer of $7 per bcm for Germany, $6 for Turkey and $6 for Italy. Since Nigeria is far from the main natural gas consumer countries, the transport cost of the gas is slightly over the market prices. Nevertheless, other countries have to buy gas from Nigeria due to price advantage and insufficient sources. Due to the transfer costs, Nigeria has a preference of selling gas to the nearest one between the two countries (Nigeria has set a rate lower minimum price level for Turkey and Italy than Germany because of this preference). While confronted with the same prices from all three importers, Nigeria’s ranking order is as follows:

\[ P_{\text{Nigeria}} = \{\text{Italy, Germany, Turkey}\} \]

Iran has supply capacity of 2 bcm with a minimum price offer of $6 per bcm. Just like other exporters, Iran also aims to sell all its natural gas supply to maximize its profit/income. If it meets a demand exceeding its capacity, Iran uses the ranking order to distribute the demands according to its preference function over importers which is as follows:

\[ P_{\text{Iran}} = \{\text{Germany, Italy, Turkey}\} \]

**In the first round;**

1- Germany makes an offer to Russia of 1 bcm for $7 and to Iran of 1 bcm for $6 in the line with the goal of diversity. Since Germany can trade gas in quantities of 1 bcm and multiples of one and it has to buy gas from at least two countries, Germany prefers Iran among the two countries, Nigeria and Iran, that cost 6 dollars. On the other hand, Italy and Turkey do not have such a restriction so;

2- Italy makes an offer to Iran of 2 bcm for $6.
3- Turkey makes an offer to to Iran of 2 bcm for $6.

4- For the first round as Iran is facing a demand of $6 for a total of 5 bcm gas, it accepts contracts in accordance with the ranking order over three countries. Germany has 1 bcm, Italy and Turkey cannot find response to their contracts.

**In the second round**, countries reconsider their offers and take new positions.

As Germany meets its goal from Iran and Russia it does not change these offers for the second round. However, Italy and Turkey make new offer for their needs.

1- Turkey renews its offer to meet its natural gas needs toward Nigeria because of its price advantage. At the price level of $6, Turkey makes an offer to Nigeria for its 1 bcm gas need and to Iran for the remaining 1 bcm.

2- Italy also renews its offer to meet its natural gas needs toward Nigeria because of its price advantage. At the price level of $6, Italy makes an offer to Nigeria for its 1 bcm gas need and to Iran for the remaining 1 bcm.

In this round, as Iran is facing a demand for a total of 3 bcm gas which exceeds its 2 bcm capacity, it accepts contracts in accordance with its ranking order over three countries (Germany, Turkey and Italy) because both country offer at the same price level of $6. Turkey’s 1 bcm offer is rejected by Iran. At the end of this round, Turkey has a lack of natural gas demand of 1 bcm.

The price mismatch still prevents to make the deals between countries.

**In the third round**, countries reconsider their offers and take new positions.

As Germany meets its goal from Iran and Russia and Italy meets its 2 bcm need from Iran and Nigeria, they do not change these offers for the second round. However Turkey makes new offer for its need.
1- Turkey renews its offer to meet its natural gas needs toward Nigeria because of its price advantage. At the price level of $6, Turkey makes an offer to Nigeria for its 2 bcm gas need.

In this round, as Nigeria is facing a demand for a total of 3 bcm gas which exceeds its 2 bcm capacity, it distributes gas in accordance with its preference function between two countries (Turkey and Italy) because both country offers at the same price level of $6. Italy has all its 1 bcm demand from Nigeria but Turkey’s contract with 2 bcm capacity is rejected. At the end of this round, Turkey has a lack of natural gas demand of 2 bcm.

The price mismatch still prevents to make the deals between countries.

In the fourth round, while other countries do not alter the bids accepted temporarily by exporting countries, Turkey makes a new offer to Iran by raising price to $7 for its capacity of 1 bcm and make an offer of a contract with 1 bcm capacity to Nigeria (this contract was accepted by Nigeria in previous rounds but has not been rejected yet). Now Iran faces an exceeding gas demand from both 3 countries. As Turkey offers the highest price, Iran accepts this offer of 1 bcm for $7. Then between two countries with the same price level, Iran prefers Germany than Italy.

1. Italy is rejected by Iran for its 1 bcm offer with the price of $6. So, Italy still faces this amount of gas deficit.

In the fifth round, Germany do not change its offers to Iran and Russia. Italy has to offer for its 1 bcm needs of natural gas.

1. Italy now offers its whole capacity need of 2 bcm to Nigeria for $6.

2. Turkey repeats its contract offers to Iran and Nigeria.
3. This time Nigeria has an exceeding demand of 3 bcm from Italy (2 bcm) and Turkey (1 bcm). According to Nigeria’s preference function, at the same price level, it prefers Italy to Turkey.

At the end of this round only Turkey cannot meet its 1 bcm need.

In the sixth round, Turkey raise prices again for its remaining capacity of 1 bcm of gas. Turkey will make a $7 offer to Iran for this 1 bcm gas due to country preference. So, a new contract for 2 bcm for $7 is proposed to Iran.

Now, Iran is again facing a demand of a total of 3 bcm gas, 2 of which is from Turkey at $7 and 1 of which is from Germany at $6. It accepts contract in accordance with the price level, so Germany’s first offer is now rejected by Iran. Turkey temporarily accepted by Iran for its 2 bcm offer of $7.

In the seventh round, it’s time for Germany to raise the price. At the same price level, the order of preference for Germany is Russia, Iran and Nigeria. Accordingly, it will make its offer at the level of $7 for 1 bcm to Iran as he is obliged to buy gas from two different countries.

Iran facing with a 3 bcm demand, now accepts Germany’s 1 bcm offer and rejects Turkey’s 2 bcm offer. In this case Turkey couldn’t meet its gas need yet. So, in the next round it will make a new offer.

In the eighth round, Turkey offers Russia for its 1 bcm for $7 and Iran for its 1 bcm for $7 according to its country preference order. Since the contract including Iran with 1 bcm for $7 has not been rejected by Iran in the previous rounds, Turkey can propose it for sure.

At the end of this round, the market reaches the general optimum deal point. In this case, no country wants to change its agreement. All countries are satisfied with their agreements.
There is also no justified envy in any of the exporting and importing countries, which implies stability up to a point.

Final sales status:

<table>
<thead>
<tr>
<th>Importer</th>
<th>Purchased Gas Amount (Bcm)</th>
<th>Exporter</th>
<th>Final Price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>1</td>
<td>Russia</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>Iran</td>
<td>7</td>
</tr>
<tr>
<td>Italy</td>
<td>2</td>
<td>Nigeria</td>
<td>6</td>
</tr>
<tr>
<td>Turkey</td>
<td>1</td>
<td>Russia</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>Iran</td>
<td>7</td>
</tr>
</tbody>
</table>

Since violation of substitutes condition is never observed during the mechanism, importer proposing COP matching model is stable in our case.

5. Conclusion

In the past ten years, the attention has shifted to mechanism/market design aspects of real-life matching markets, and not only have these applications been influencing the direction of the theory but also has matching theory been making an important impact on policy [11]. Based on this discourse, this model, which can be thought to contribute as an example to the models prepared and that can be designed within the framework of the market mechanism, has produced satisfactory results for COP. As Hotfield and Kominers [20] mentioned that contract design can determine which—and even more importantly, if—stable relationships can be found, it is believed that this model will both offer a closer to real life model and provide the necessary features by adding more features and constraints.

Choice functions are most important part of the model and are examined whether they satisfy the required condition of substitutability for stability. For importers which have 1-unit
capacity and unrestricted type and exporters, substitutability does not hold. However, cumulative offer process which represents the importer proposing model provides stable solutions for our scenario. Since violation of substitutes condition is never observed during the mechanism, importer proposing COP matching model is stable in our case. It also applies to different samples.

Future work with unlimited capacities or additional features can be examined for further many-to-many matching models for natural gas trade.
REFERENCES


APPENDIX
**Proof of COP is Stable**

**Case 1:** $m$ has a capacity of 1 or needs to diversify its exporters.

Then, for contract $x$; $q(x)=1$.

First notice that, under mechanism, $m$ proposes contracts with capacity 1 only. Moreover, since $m$ prefers $x$ to its current match, it has proposed during mechanism. Suppose $m$ was rejected in step $k$. Then, in step $k$, all contracts $e$ is holding are better than $x$, i.e.

- either with better prices or
- same price but higher ranking countries.

Moreover, in the future steps, $e$ will fill its quota with better contracts than $x$.

**Case 2:** $m$ has a capacity of 2 and does not need to diversify its exporters.

If $m$ is not matched with another contract $x'$ such that $e(x')=e$, then we refer to Case 1. If $m$ is matched with another contract $x'$ such that $e(x)=e$, then $q(x')=1$. Moreover, as explained in case 1, $x$ has been proposed during mechanism and $e$ is holding contracts which are better than $x$.

Finally, we would like to point out one important feature of our mechanism. An unrestricted importer $m$ proposes to exporter country $e$ a contract with price $\pi$ and quota 1, if either $e$ has rejected contract with price $\pi$ and quota 2 or $m$ is also proposing another contract $x'$ which is better than any contract including exporter $e$ and price $\pi$. Similarly, $m$ proposes to $e$ with price $\pi$ and quota 2, if all its better contracts without price $\pi$ and country $e$ have been rejected.

Since these observations are true for all importers, there is no complimentary between countries, the outcome of COP is stable.