DUCTILE TEARING OR PLASTIC COLLAPSE?

Philippe Gilles¹, Edith Marques-Viera², Olivier Aneilet³, Patrick Le Delliou⁴, Thierry Le Grasse⁵

¹Consultant in structural integrity, GEP-INT, Paris, 75017, France (gilles.filip2@orange.fr)
²Engineer, Framatome, Paris-La-Défense, France
³Expert in structural integrity, Framatome, Paris-La-Défense, France
⁴Research & Development Engineer, EDF, Moret-Sur-Loing, France
⁵Engineer, CEA, Saclay, France

ABSTRACT

Ductile fracture is characterized by large amount of deformation. Two mechanisms may lead to ductile failure: local tearing may appear in front of a stress concentration region like a crack front or large geometry changes in a weak part of the structure can lead to global collapse of the structure. For large cracks such as those considered in Leak Before Break studies, tearing analyses and net-section collapse criteria have been used. The present paper examines the link between these two mechanisms for a through-wall cracked pipe under bending and concludes that the transition from ductile tearing to collapse depends not only on material properties but also on geometrical parameters. The reference stress approach is a promising approach for predicting the transition, provided the transferability issue of J-R curve has been solved.

INTRODUCTION

Structures containing large cracks and made in ductile materials may experience two types of failure mechanisms: ductile tearing or plastic collapse. Under displacement controlled loading ductile tearing is a stable crack growth mechanism. Plastic collapse leads to a much faster damage evolution.

Ductile tearing is the result of void growth and coalescence ahead of the crack front under the high strain concentration. This mechanism is slowed down by a high material hardening and under a high constraint. The global deformation of the structure is limited.

Plastic collapse is induced by plastic strain accumulation along slip lines. Slip lines depends on the geometry of the cracked structures and of the type of loading. Therefore plastic collapse produces large deformations of the structure.

The aim of this paper is to identify the critical crack size for which ductile tearing becomes plastic collapse. Several experiments conducted on through-wall cracked or deeply cracked structures made in very tough materials have shown such changes of behaviour. The paper analyses these experiments and simulates the apparition of the collapse mechanism using J estimation schemes based on the reference stress approach.

EXPERIMENTAL RESULTS

In the frame of the French research program on Leak Before Break, a series of tests has been performed to determine the conditions of crack initiation and crack propagation in pipes of 316L austenitic stainless
steel and subjected to a bending load (Moulin, 1996). A large circumferential through-wall crack is machined in the central cross section of the pipe. Figure 1 describes the pipe geometry and shows the variation of the bending moment as a function of the crack angle. Tests are conducted using a 4-point test rig. In the present case it seems that a collapse mechanism develops shortly after the maximum load. Other configurations should be analysed to define the effect of material properties (strain hardening and tearing resistance) and constraint on the ductile fracture behaviour of structures containing a large crack. The total crack angle named \( \theta \) in the experiments is named \( 2\beta \) in the present paper.

Tests on Fixed Grip SE(T) specimens in Nickel base alloy 600 were performed by CEA showing a similar transition between ductile tearing to collapse. O. Ancelet performed a thorough interpretation of the test which is of value for the analysis of the other configurations.
In both series of experiments, the load variation with crack growth follows a line similar to the collapse curve. The crack extension is accompanied by a large deformation of thickness of the cracked part and the cracked section.

PRESENTATION OF THE APPROACH

The driving idea of the paper consists in modelling the through-wall cracked pipe tearing behaviour using estimation schemes.

Tearing model based on the crack driving force $J$

In the $J$ based approach, ductile tearing is governed by the two following equations:

- Energy balance equation
  \[ J = J_{1c} \]  \(1\)
  Where $J$ is the crack driving force and $J_{1c}$ the material toughness.

- Crack stability equation
  \[ \frac{dJ(X>M)}{dX} = \frac{dJ_R(X>X_0)}{dX} \]  \(2\)
  Where $a = r_m \cdot \pi \cdot X$ and $X_0$ characterizes the initial crack size. $J_R$ is the crack resistance curve.

Collapse criterion

For Elastic-Plastic Materials, plastic collapse is a failure mode characterized by limit load formulae. The classical limit load theory is based on the initial geometry of the structure. The limit load formula in plane stress condition, takes the form:

\[ Q_{LY} = \sigma_y \cdot Z_{P,ns} \cdot \mu_c \text{(Cracked geometry)} \]  \(3\)
  Where $\sigma_y$ is the limit stress of the material, $Z_{P,nc}$ is the plastic modulus of the uncracked geometry and the crack factor $\mu_c$ is a function of the crack size relative to the dimensions of the uncracked section. A common approach consists in using a net section yielding model. For a through-wall cracked pipe under bending moment $M$, the net section formula is assuming a thin shell condition:

\[ M_{LY,NSC} = 4\sigma_y \cdot r_m^2 \cdot t \cdot \mu_f \]  \(4\)
  The reduction section factor $\mu_f$ is a function of the relative crack size $X = \frac{\beta}{\pi}$.

\[ \mu_f = \cos \left( \frac{\pi X}{2} \right) - \frac{1}{2} \sin(\pi \cdot X) \]  \(5\)

For strain-hardening materials, the collapse criterion is based on an extension of the previous formula.

\[ M = M_f = 4\sigma_f \cdot r_m^2 \cdot t \cdot \mu_f \]  \(6\)
  Where $\sigma_f$ is assumed to be a material characteristic. For stainless steel $\sigma_f$ is usually taken as the average of the yield and ultimate stresses ($\sigma_y$ and $\sigma_u$). For ferritic steels, $\sigma_f = \sigma_u$.

From the previous experimental results we consider that the transition from ductile tearing to plastic collapse occurs when the slope of the moment-crack size curve during tearing equals the slope of the collapse moment crack size curve. A tangency point is obtained by scaling the flow stress.

SIMULATION OF THE TEARING OF A THROUGH-WALL CRACKED PIPE UNDER BENDING

The reference stress approach is applied to model the behaviour of a circumferentially through-wall cracked pipe tested by CEA. The CEA test number 26 is selected. Starting from the GE-EPRI estimation...
scheme expressions developed by Kumar at al. (1981, 1984, see Appendix), R. Ainsworth (1984) applied the reference stress concept to propose a more general estimation scheme. This approach may be applied to all fracture parameters J, rotations and displacements.

**J derivation**

For elastic-plastic materials, the crack driving force J may be separated in three terms (Kumar, 1981):

\[ J = J_e + J_z + J_p \]  

Where \( J_e \) and \( J_p \) are respectively the elastic and fully plastic contributions. \( J_z \) is a corrective term accounting for small scale yielding effects. This term is neglected in the present study.

The elastic contribution is given by the formula:

\[ J_e = \frac{M^2}{E} \left( \frac{r_m}{t} \right)^2 \pi \cdot \beta \cdot F^2 \left( \frac{\beta}{\pi} \frac{r_m}{t} \right) = \frac{M^2}{E} \cdot X \cdot \frac{\beta^2}{r_m^4} \]  

- \( r_m \) and \( t \) are respectively the pipe mean radius and the pipe thickness.
- \( I \) is the moment of inertia of the uncracked section.
- \( a = r_m \cdot \beta \) is the crack length.
- \( X = \beta / \pi \)
- \( F \) is the shape factor of the SIF. We have selected the Klecker formula (1986) for bending:

\[ F_B \left( \frac{\beta}{\pi} \frac{r_m}{t} \right) = 1 + A_b \cdot \left( \frac{\beta}{\pi} \right)^{1.5} + B_b \cdot \left( \frac{\beta}{\pi} \right)^{2.5} + C_b \cdot \left( \frac{\beta}{\pi} \right)^{3.5} \]  

Where \( A_b, B_b, C_b \) are polynomial functions of the ratio \( r_m / t \)

The reference stress approach relates the J to a ratio of plastic to elastic reference strains

\[ \frac{J_p}{J_e} = \frac{\sigma_{R_{p}}(\beta r_m / t)}{\sigma_{R_{0}}} \]  

The reference stress for J is

\[ \sigma_{R_{0}} \left( \frac{\beta}{\pi} \frac{r_m}{t} \right) = \sigma_0 \cdot \frac{M}{M_{R_{0}}(\beta r_m / t)} = \frac{M}{\sigma_{p_{a}} \cdot \mu(X)} \]  

The reference load \( M_{R_{0}} \) depends on the geometry, but also on the strain hardening represented here by the exponent \( n \) of a power law fitted on the stress-strain curve. In the literature, this material dependence is usually neglected. We selected an expression obtained by Kim et al. (2001):

\[ \mu_c = \left( 0.82 + 0.75 \cdot X + 0.42 \cdot X^2 \right) \cdot \mu_t (X) \]  

The reference strain \( \varepsilon_{R_{0}}(\sigma_{R_{0}}) \) is related to the reference stress by the material stress-strain law. The true stress-strain law has to be preferred to the actual law.

Neglecting the plastic zone correction, J is given by:

\[ J = J_e \cdot \frac{\sigma_{R_{1}}}{\sigma_{R_{0}}} \]  

**Similarly, for the rotations:**

\[ \frac{\varphi_{c_{p}}}{\varphi_{c_{e}}} = \frac{E \cdot \varepsilon_{R_{p}}(\sigma_{R_{0}})}{\sigma_{R_{0}}(\beta \frac{r_m}{t} + n)} \]  

with \( \varphi_{c_{e}} = \frac{M}{E \pi r_m t} \cdot \varphi_{c_{p}} \)  

The compliance functions for through-wall-cracked pipe under bending derived by Klecker et al. (1986)

\[ I_B \left( \frac{\beta}{\pi} \frac{r_m}{t} \right) = 2 \cdot \beta^2 \cdot \left[ 1 + 8 \cdot \left( \frac{\beta}{\pi} \right)^{1.5} \cdot I_{b_1} \left( \frac{\beta}{\pi} \frac{r_m}{t} \right) + \left( \frac{\beta}{\pi} \right)^3 \cdot \left( I_{b_2} \left( \frac{\beta}{\pi} \frac{r_m}{t} \right) + I_{b_3} \left( \frac{\beta}{\pi} \frac{r_m}{t} \right) \right) \right] \]
Where the three coefficients $I_{b1}$ are functions of the pipe curvature parameter $r_m / t$.

The reference stress for the rotation due to the presence differs from the one related to $J$. The formula has been fitted on Finite Element computations.

$$
\mu_{cp} = a_0 + a_1 \cdot X + a_2 \cdot X^2 + a_3 \cdot X^3 + a_4 \cdot X^4
$$

(16)

With $a_0 = 0.71743 \quad a_3 = -3.1777 \quad a_4 = 1.7640 \quad a_1 = -2a_0 + a_3 + 2a_4 \quad a_2 = a_0 - 2a_3 - 3a_4$

This formula is not valid for short cracks ($X < 1/32$)

Neglecting the plastic zone correction, the élasto-plastic rotation due to the crack is given by:

$$
\varphi_e = \varphi_{ec} \cdot \frac{E \cdot \varepsilon_{Rt}(\sigma_{R\varphi 0})}{\sigma_{R\varphi 0}(r_m/r_m)}
$$

(17)

For the rotation ones needs to take into account the contributions of the uncracked parts:

$$
\varphi_{e,nc} = \frac{M \cdot 2L}{E \cdot t} \quad \text{and} \quad \varphi_{nc} = \varphi_{e,nc} \cdot \frac{E \cdot \varepsilon_{Rt}(0,r_m)}{\sigma_{R\varphi 0}(0,r_m)}
$$

(18)

$$
\sigma_{R\varphi 0}(0, \frac{r_m}{t}, n) = Z_{p,nc} \cdot \mu_{cp}(0) \quad \text{with} \quad \mu_{cp}(0) \cong 0.98
$$

(19)

**Simulation of the CEA26 test**

The values of the material and geometrical characteristics are given in the following tables.

<table>
<thead>
<tr>
<th>Young’s modulus</th>
<th>Poisson’s ratio</th>
<th>Yield stress</th>
<th>Ultimate stress</th>
<th>0.224</th>
<th>0.207</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (MPa)</td>
<td>ν</td>
<td>$\sigma_y$ (MPa)</td>
<td>$\sigma_u$ (MPa)</td>
<td>P-4</td>
<td>P-5</td>
</tr>
<tr>
<td>142900</td>
<td>0.3</td>
<td>233.18</td>
<td>554</td>
<td>2.424</td>
<td>2.378</td>
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**Table 2: rationale stress-strain curve**

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<tr>
<th>0,001</th>
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<th>0,0012</th>
<th>0,0013</th>
<th>0,0015</th>
<th>0,0016</th>
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<th>0,0097</th>
<th>0,0157</th>
<th>0,0263</th>
<th>0,0356</th>
<th>0,0563</th>
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<td>0,09615</td>
<td>0,11227</td>
<td>0,12816</td>
<td>0,14378</td>
<td>0,15918</td>
<td>0,17433</td>
<td>0,18927</td>
<td>0,20399</td>
<td>0,21851</td>
<td>0,23279</td>
<td>0,24687</td>
<td></td>
<td></td>
</tr>
<tr>
<td>404</td>
<td>429,3</td>
<td>453</td>
<td>477,2</td>
<td>498,5</td>
<td>521,4</td>
<td>541,2</td>
<td>561,4</td>
<td>580</td>
<td>598,09</td>
<td>615,22</td>
<td>631,54</td>
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<td>0,30133</td>
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<td>0,40133</td>
<td>0,45133</td>
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<td>0,65133</td>
<td>0,70133</td>
<td>0,85133</td>
<td>1,00133</td>
<td></td>
<td></td>
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<tr>
<td>647,14</td>
<td>662,09</td>
<td>690,25</td>
<td>739,15</td>
<td>784,33</td>
<td>826,49</td>
<td>866,13</td>
<td>939,3</td>
<td>973,36</td>
<td>1005,99</td>
<td>1096,8</td>
<td>1179,11</td>
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</table>

<table>
<thead>
<tr>
<th>Mean radius</th>
<th>Thickness</th>
<th>Location of rotation measurement</th>
<th>Half-crack angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_m$ (mm)</td>
<td>$T$ (mm)</td>
<td>$L$ (mm)</td>
<td>$\beta$ (Degree)</td>
</tr>
<tr>
<td>48,365</td>
<td>8,27</td>
<td>85</td>
<td>37,5</td>
</tr>
</tbody>
</table>

The value of the moment measured at initiation is 17.7 kN.m.
J-R curve

A crack resistance curve has been measured on CT12 specimen by CEA using a J modified formulation.

\[
J = J_c + C \cdot (a - a_0)^m
\]

\[
C = 675.82 \text{ MPa/mm}^m
\]

\[
a \text{ and } a_0 \text{ in mm}
\]

Figure 3: CT12 J-M-R curve

The simulation of crack initiation and growth under rotation control \( \varphi_t \) requires solving a set of two non-linear equations of unknowns the bending moment \( M \) and the relative crack size \( X \):

\[
\varphi_{e,nc} + \varphi_{p,nc} + \varphi_{c,c} + \varphi_{p,c} = \varphi_t
\]

\[
J = J_R(X - X_0)
\]

Transferability issue

There is a transferability issue of JR curves from specimen to structure. Ductile fracture is driven by strain concentration represented by J, but also by stress triaxiality the higher value of which constraint the yielding around the crack tip. Depending on the material, J-R curves for through-wall cracks are similar or higher to the curves determined on specimens (Pavankumar 2002, Bethmont, 1994, Brust, 1995).

We used the pipe bending moment value at initiation to determine a representative \( J_c \) and we defined two types of J-R curve for the cracked pipe analysis: the CT power-law curve shifted by changing \( J_c \) from 474 to 1900 kJ/m\(^2\) and a bilinear J-R curve with the same \( J_c \) and a slope of 200 MPa.

Crack growth simulation

The simulation is conducted under control of total rotation. The two unknowns, moment and increment of crack size are determined using the solver of Excel. The results are shown in Figures 4 and 5.
The power-law J-R curve gives excellent results up to a rotation of 20 degrees. For larger rotation the scheme does not converge well. The slope of the linear J-R curve seems to be slightly too high. A bilinear curve should improve the predictions.

The collapse curves (dotted lines) are not exactly tangent to the tearing curves (solid lines) but close to. The slopes of the computed tearing curves are higher than the experimental tearing curve because the final slope of the resistance curve is too high. Anyway the value of the collapse stress $\sigma_f$ associated to the tangency point is very sensitive to the tearing modulus value.
The collapse moment curves tangent to the moment-crack size correspond respectively to $\sigma = 647$ MPa and $\sigma_f = 1066$ MPa to be compared with $\sigma = 0.85 \cdot (\sigma_y + \sigma_u)/2 = 335$ MPa given by Moulin (1996). However, the coefficient of 0.85 is a lower bound and the value $\sigma_f$ corresponding to the tangency of the tearing and the collapse curve is 478 MPa.

We derive below the relationship between $\sigma_f$ and the tearing characteristics. At the tangency point defined by the ratio $Y = \sigma_f/\sigma_y$ and the crack size $X_f$:

$$M = M_f = \sigma_f \cdot Z_{pl,nc} \cdot \mu_r$$

After some algebra, we get:

$$J_e \left[M_{LY,NSCH\mu_r(X_f)} \right] = \frac{E}{\mu_r} \cdot \frac{\mu_{cl}(X_f)}{\mu_{cl}(X_f)} \cdot J_R(X_f - X_0)$$

For deep defects $X \approx 0.5$

$$\mu_{cl}(X_f) \approx \mu_r(X_f)$$

This means that the flow stress $\sigma_f$ depends on the crack geometry and the tearing resistance and not only on tensile characteristics.

SIMPLIFIED ANALYSIS OF THE RELATIONSHIP BETWEEN TEARING AND COLLAPSE CHARACTERISTICS

The derivation of formulae giving the coordinates of the tangency point between the moment-crack size curve and the collapse moment-crack size curve cannot be achieved for general material characteristics. However, considering a stress-strain power law $\varepsilon_{RJ}(Y) = \alpha \cdot \sigma_y \cdot \left(\frac{\sigma}{\sigma_y}\right)^n$ and making some approximations, it is possible to get explicit expressions of the coordinates of the tangency point. These coordinates are obtained by solving the two basic equations (1) and (2).

Energy balance

$$J_p(X_f, M_f) = J_R(X_f - X_0)$$

$$\left[4 \cdot Y \cdot \mu_r(X_f) \cdot F\right]^2 \cdot r_m \cdot X_f \cdot \alpha \cdot \left(\frac{\mu_{cl}(X_f)}{\mu_{cl}(X_f)}\right)^n = \frac{E}{\sigma_y} \cdot J_R(X_f - X_0)$$

When neglecting $J_e$ before $J_p$, we get a lower bound for $Y$

$$Y_{sup} = \left[\frac{\mu_{cl}(X_f)}{\mu_{cl}(X_f)}\right]^{n+1} \cdot \left[\frac{1}{\sigma_y} \cdot \frac{J_R(X_f - X_0)}{16 \cdot r_m \cdot F^2 \cdot X_f}\right]^{n+1}$$

For a Perfectly Plastic material

$$n \to \infty \quad \mu_{cl}(X) \to \mu_r(X) \quad \text{and} \quad Y = 1 \quad \text{which was expected.}$$
Stability equation

\[
\frac{\partial I(XM_0)}{\partial X} = \frac{dI_R(X_f-X_0)}{dX}
\]

(30)

\[
J_p \cdot \left( \frac{2}{M} \frac{\partial M}{\partial X} + \frac{1}{X} + \frac{2}{F} \frac{\partial F}{\partial X} \right) + (n-1) \cdot J_p \cdot \left( \frac{1}{M} \frac{\partial M}{\partial X} - \frac{1}{Mu F_1} \frac{\partial \mu F_1}{\partial X} \right) = \frac{dI_R}{dX}
\]

(31)

Using (27), we obtain:

\[
J_R \cdot \left[ \frac{1}{M} + \frac{2}{F} \frac{\partial F}{\partial X} + \frac{(n+1)}{M} \frac{\partial M}{\partial X} - \frac{(n-1)}{\mu c_f} \frac{\partial \mu c_f}{\partial X} \right] = \frac{dI_R}{dX}
\]

(32)

The tricky point is to estimate the logarithmic derivative of the moment \( \frac{1}{M} \frac{\partial M}{\partial X} \).

For low tearing modulus, we consider

\[
\frac{1}{M} \frac{\partial M}{\partial X} = \frac{1}{\sigma f} \frac{\partial \sigma f}{\partial X} + \frac{1}{Mu R} \frac{\partial Mu R}{\partial X} = \frac{1}{\mu c_f} \frac{\partial \mu c_f}{\partial X}
\]

(34)

since \( \sigma f \) is independent of \( X \).

We get the following relationship:

\[
\frac{\partial J_p}{\partial X} = J_R \cdot \left[ \frac{1}{M} + \frac{2}{F} \frac{\partial F}{\partial X} + (n+1) \frac{1}{\mu c_f} \frac{\partial \mu c_f}{\partial X} - (n-1) \frac{1}{\mu c_f} \frac{\partial \mu c_f}{\partial X} \right] = \frac{dI_R}{dX}
\]

(35)

For large tearing modulus, the derivative is obtained from the \( \phi_{pc} = \text{Constant} \) condition.

\[
\frac{1}{M} \frac{\partial M}{\partial X} \cong -\frac{1}{n} \left[ \frac{1}{M} \frac{\partial J_R}{\partial X} \right] - (n-1) \frac{1}{\mu c_f} \frac{\partial \mu c_f}{\partial X}
\]

(36)

Form these equations we get the variations of \( Y \) and \( X_f \) as a function of the tearing modulus. Three strain hardening exponents are considered 8.5 and 3.7 which fits the stress-strain curve at strains smaller or larger than 0.0336. The value 6.1 corresponds to the best fit of \( J_p \) in the rage of \( X = 0.2 \) to 0.5.

The more representative combination corresponds to \( n = 6.1 \). The prediction of \( X_f \) is less accurate, but the variations are decreasing when \( DJ/da \) exceeds 150 MPa. We have to stress that the expressions developed in the present paragraph are very approximate and namely do not take into account the effect of the length of the uncracked part at the end of which rotation is applied.
CONCLUSION

The collapse criterion is usually described by a limit load formula where a “flow stress” is substituted for the limit stress of the Elastic Perfectly Plastic material. The present paper shows that this flow stress is not a material constant, but depends also on the geometry of the flawed structure. Furthermore, the material dependence combines stress-strain and tearing characteristics.

The lower tearing moduli, the lower the flow stress and the earlier is the transition from ductile tearing to plastic collapse.

The reference stress approach is a promising approach for predicting the transition, provided the transferability issue of J-R curve has been solved.

It is worth to analyse CEA experiments on deeply cracked specimen to identify the key geometric parameters governing the transition from tearing to collapse.

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REFERENCES


