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STOCHASTIC SITE RESPONSE ANALYSIS THROUGH UNCERTAIN ELASTOPLASTIC SOIL

Fangbo Wang[1], Hexiang Wang[2], Han Yang[2], Yuan Feng[3], Boris Jeremić[4,5]

[1] Assistant Professor, Tianjin University, Tianjin, China
[2] PhD Candidate, University of California, Davis, CA, USA
[3] Research Scientist, TuSimple, San Diego, CA, USA
[4] Professor, University of California, Davis, CA, USA
[5] Faculty Scientist, Lawrence Berkeley National Laboratory, Berkeley, CA, USA

ABSTRACT

Presented is a stochastic site response analysis with uncertain seismic motion and uncertain elastoplastic material. The uncertain soil parameters and seismic motion are modelled as random fields and random process, and both represented by Hermite polynomial chaos. The site response, also represented by Hermite polynomial chaos, is solved for following a developed intrusive stochastic finite element formulation based on stochastic Galerkin method. Risk implications from the analysis are also discussed. Presented methodology is implemented in the Real ESSI Simulator system.

INTRODUCTION

A site response analysis, i.e., determines the response of soil deposit from the input bedrock motion, is able to predict ground surface motions, to evaluate soil dynamic stress and strains, and to assess the stability of earth-retaining structures. Material parameters and input motions of conventional site response analysis [3] are mostly deterministic. However, due to limited data, spatial non-uniformity of soil has been long recognized by civil engineering community and soil parameters are considered to be uncertain [4]. In addition, uncertainties of seismic motion arises from the seismic source, wave propagation path, etc. Also, it is computationally challenging to consider the uncertainties in the structural system.

Among the available numerical techniques, Monte Carlo method [5] is the most commonly used approach. It repeatedly calls the deterministic solver with generated samples of uncertain parameters. Statistics of structural response maybe post-processed after collecting results from all sample runs. However, Monte Carlo method is notorious for slow convergence and requires a large number of samples to reach acceptable accuracy but computationally intractable. In this paper, Hermite polynomial chaos is employed to represent the uncertainties of material parameters and seismic motion, and a time-domain stochastic dynamic finite element formulation is employed to propagate the input uncertainties through the soil deposit. We will first present the stochastic dynamic finite element formulation, and the probabilistic constitutive model of soil. A stochastic site response analysis with uncertain soil deposit and bedrock motion will be conducted using the developed formulation.

STOCHASTIC DYNAMIC FINITE ELEMENT FORMULATION

In the developed time domain stochastic Galerkin formulation, uncertain material parameters and forcing are simulated as non-Gaussian random fields and stationary/non-stationary random process, which can be quantified by Hermite polynomial chaos (PC) expansion. In addition, the response processes, displacement, acceleration, are also represented with Hermite PC. Then, stochastic Galerkin projection is applied to minimize the error on estimating response PC coefficients. The statistics and distributions of displacement, acceleration maybe post-processed for design and risk analysis purposes.

Hermite PC representation of random process

The weak form of deterministic, dynamic finite elements [2] can be written as:

$$\int_{D_e} N_m(x)\rho(x)N_n(x)d\Omega\ \ddot{u}_n(t) + \int_{D_e} \nabla N_m(x)D(x)\nabla N_n(x)d\Omega\ u_n(t) - f_m(t) = 0 \tag{1}$$

where N_m is the finite element shape function, Ω and $f_m(t)$ incorporates the various elemental contributions to the global force vector.

Next, we assume the (tangent) stiffness, $D(x)$, and the forcing function, $f_m(t)$, to be a heterogeneous random field and a non-stationary random process, respectively and represent them in terms of multidimensional, Hermite PC expansions with known coefficients. Details of Hermite polynomial chaos quantification of random field/process can be found in [6, 7].

$$D(x,\theta) = \sum_{i=1}^{P_1} a_i(x)\Psi_i(\{\xi_r(\theta)\}) \tag{2}$$

$$f_m(t,\theta) = \sum_{j=1}^{P_2} f_{mj}(t)\Psi_j(\{\xi_r(\theta)\}) \tag{3}$$

where $P_1 = (M_1 + p_1)!/(M_1!p_1!)$ and $\{\Psi_i\}$ are multidimensional, orthogonal and uncorrelated, Hermite polynomials of zero-mean, unit variance Gaussian random variables, $\{\xi_r\}$, while M_1 and p_1 are the corresponding dimension and order in the PC representation. Note that θ is introduced to denote uncertainty. Similarly, $P_2 = (M_2 + p_2)!/(M_2!p_2!)$. As a result, the nodal displacement, $u_n(t)$, and nodal acceleration, $\ddot{u}_n(t)$, will also become random processes. They will also be represented using multidimensional, Hermite PC expansions but with unknown coefficients which will be computed using a stochastic Galerkin approach.

Stochastic Galerkin approach to compute PC coefficients of displacement, acceleration response processes

The PC representation of response process should include all the input uncertainties, therefore, the PC dimension should be $M_1 + M_2$ and order Q is the maximum of p_1 and p_2. Accordingly, let's represent the nodal displacement, $u_n(t)$, in terms of a multidimensional Hermite PC expansion of dimension $M_1 + M_2$ and order Q as:

$$u_n(t, \theta) = \sum_{k=1}^{P_3} d_{nk}(t)\Psi_k(\{\xi_l(\theta)\}) \tag{4}$$

where $P_3 = (M_1 + M_2 + Q)!/((M_1 + M_2)!Q!)$. Twice differentiating Eq. 4, we obtain a multidimensional Hermite PC representation of nodal acceleration, $\ddot{u}_n(t)$, as:

$$\ddot{u}_n(t, \theta) = \sum_{k=1}^{P_3} \ddot{d}_{nk}(t)\Psi_k(\{\xi_l(\theta)\}) \tag{5}$$

Substitute Eqs. 2, 3, 4, and 5 into Eq. 1, and denote the shape function gradients $\nabla N_n(x)$ as $B(x)$, we obtain

$$\begin{aligned} &\sum_{k=1}^{P_3} \int_{D_e} N_m(x)\rho(x)N_n(x)d\Omega\ \Psi_k \ddot{d}_{nk}(t) + \\ &\quad \sum_{k=1}^{P_3} \sum_{i=1}^{P_1} \int_{D_e} B_m(x)a_i(x)B_n(x)d\Omega\ \Psi_i\Psi_k d_{nk}(t) - \sum_{j=1}^{P_2} f_{mj}(t)\Psi_j = 0 \end{aligned} \tag{6}$$

Multiplying both sides of Eq. 6 by Ψ_l and taking ensemble average, namely stochastic Galerkin projection [1], we obtain the following system of ordinary differential equations:

$$\begin{aligned} &\sum_{k=1}^{P_3} \langle \Psi_k \Psi_l \rangle \int_{D_e} N_m(x)\rho(x)N_n(x)d\Omega\ \ \ddot{d}_{nk}(t)\ \ + \\ &\quad \sum_{k=1}^{P_3} \sum_{i=1}^{P_1} \langle \Psi_i \Psi_k \Psi_l \rangle \int_{D_e} B_m(x)a_i(x)B_n(x)d\Omega\ \ d_{nk}(t) = \sum_{j=1}^{P_2} \langle \Psi_j \Psi_l \rangle f_{mj}(t) \end{aligned} \tag{7}$$

with $l = 1, 2, ..., P_3$ and $m = 1, 2, ..., N$ where N is the number of finite element nodes. Note that Eq. 7 is identical to the deterministic finite element system of equations when P_1, P_2, P_3 are equal to 1. Transform Eq. 7 into matrix-vector form:

$$\boldsymbol{M}\ddot{\boldsymbol{d}} + \boldsymbol{K}\boldsymbol{d} = \boldsymbol{F} \tag{8}$$

where M and K may be termed as the generalized stochastic mass and stiffness matrices, while F, d, and $\ddot{d}$ may be termed as the generalized stochastic force, displacement, and acceleration vectors, respectively. Note that the ensemble averages of the double and triple products of the PC basis functions appearing within M, K, and F may be pre-computed symbolically. Rayleigh damping might be added into Eq. 8, and we can get:

$$\boldsymbol{M}\ddot{\boldsymbol{d}} + \boldsymbol{C}\dot{\boldsymbol{d}} + \boldsymbol{K}\boldsymbol{d} = \boldsymbol{F} \tag{9}$$

where $\boldsymbol{C} = \alpha\boldsymbol{M} + \beta\boldsymbol{K}$, with α and β being the Rayleigh damping parameters. Eq. 8 or Eq. 9 may be solved using any of the available time integration schemes of the deterministic dynamic finite element method.

Note that the size of the stochastic finite element system of equations can be substantially larger than the corresponding deterministic finite element system of equations, depending upon the number of PC terms used to represent the displacement random process.

After solving Eq. 8 or Eq. 9, PC coefficients of displacement and acceleration random processes can be substituted into Eqs. 4 and 5 to synthesize the random processes, and realizations of the processes can be simply generated through random sampling. In addition, the evolutionary mean and standard deviation of response processes can be computed using special properties of the PC basis functions. For example, the evolutionary mean and standard deviation of displacement time history at node n may be estimated as:

$$\mu_{u_n}(t) = \langle u_n(t,\theta)\rangle = d_{n1}(t) \tag{10}$$

and,

$$\sigma_{u_n}(t) = \sqrt{\sum_{k=2}^{P_3} \langle \Psi_k^2 \rangle (d_{nk}(t))^2} \tag{11}$$

CONSTITUTIVE SIMULATION

In classical plasticity, sign of loading index is the criteria to apply elastic loading or plastic loading, i.e., the 'if' condition. By assuming uncertain material parameters, the distribution of loading index would have probability of positive loading index and probability of negative loading index at one time step, and the distribution of stress and updated tangent modulus would be multi-modal distributions. However, multi-modal distributions requires Hermite PC with very high order which is impractical for computations.

In order to overcome the difficulties of 'if' condition, the assumption of zero elastic region of material is utilized to allow material yielding in all time steps. For a one-dimensional von-Mises material with Armstrong-Frederick kinematic hardening, the evolution of stress can be fully represented by the back stress of the Armstrong-Frederick hardening equation, and the incremental update of stress, $\Delta\sigma$, can be written as:

$$\Delta\sigma = H_a \Delta\epsilon - C_r \sigma |\Delta\epsilon| \tag{12}$$

where H_a and C_r are the two parameters in Armstrong-Frederick kinematic hardening rule with the material strength to be H_a/C_r. Without any loss of generality, for the case of $\Delta\epsilon > 0$,

$$\Delta\sigma = H_a \Delta\epsilon - C_r \sigma \Delta\epsilon \tag{13}$$

and the tangent stiffness can be computed as:

$$E = \frac{\Delta\sigma}{\Delta\epsilon} = H_a - C_r \sigma \tag{14}$$

The stress-strain hysteretical behavior of the material is shown in Figure 1 with parameters $H_a = 60MPa$, $C_r = 100$, and hence material strength is 100 kPa.

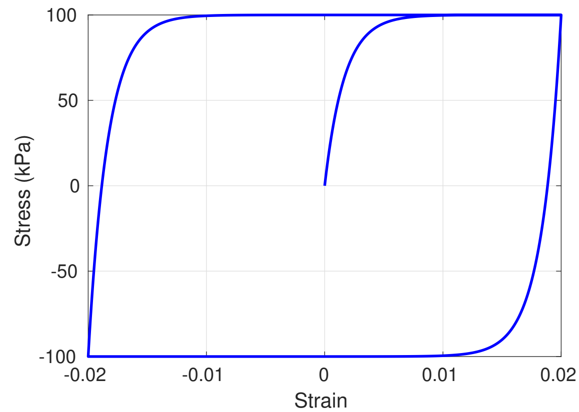


Figure 1: Hysteretical stress-strain behavior of the 1-D material model with $H_a = 60MPa$, $C_r = 100$.

Extension to stochastic 1-D material model

The material parameters, H_a, C_r, maybe assumed uncertain with limited measurements. In addition, the incremental strain, $\Delta\epsilon$, from global level may also be uncertain. Let us represent H_a, C_r, $\Delta\epsilon$ with Hermite PC:

$$H_a = \sum_{i=1}^{P} H_{a_i} \Psi_i(\{\xi_r\}) \tag{15}$$

$$C_r = \sum_{i=1}^{P} C_{r_i} \Psi_i(\{\xi_r\}) \tag{16}$$

$$\Delta\epsilon = \sum_{i=1}^{P} \Delta\epsilon_i \Psi_i(\{\xi_r\}) \tag{17}$$

$$\sigma = \sum_{i=1}^{P} \sigma_i \Psi_i(\{\xi_r\}) \tag{18}$$

$$\Delta\sigma = \sum_{i=1}^{P} \Delta\sigma_i \Psi_i(\{\xi_r\}) \tag{19}$$

$$E = \sum_{i=1}^{P} E_i \Psi_i(\{\xi_r\}) \tag{20}$$

where P is the total number PC terms in the stochastic system, and $H_{a_i}, C_{r_i}, \Delta\epsilon_i, \sigma_i, \Delta\sigma_i$ are corresponding PC coefficients. Then, substitute Eqs. 15 to 20 into Eq. 13, 14, and apply stochastic Galerkin projection on both sides of equations, we can obtain PC coefficients of E and $\Delta\sigma$:

$$\Delta\sigma_i = \frac{H_{a_j}\Delta\epsilon_k\langle\Psi_i\Psi_j\Psi_k\rangle - C_{r_l}\sigma_m\Delta\epsilon_n\langle\Psi_i\Psi_l\Psi_m\Psi_n\rangle}{\langle\Psi_i^2\rangle} \tag{21}$$

$$E_i = H_{a_i} - \frac{C_{r_j}\sigma_k\langle\Psi_i\Psi_j\Psi_k\rangle}{\langle\Psi_i^2\rangle} \tag{22}$$

Note that index notation is used in the above equations with index ranging from 1 to P. In addition, the negative sign should be switched to positive if the mean of incremental strain is negative.

To illustrate the stress-strain behaviors of probabilistic constitutive model, material initial stiffness, H_a, is assumed log-normal distribution with mean and coefficient of variation (COV) to be 60 MPa, 40%, respectively, while strength parameter, H_a/C_r, is assumed log-normal distribution with mean and COV to be 100 kPa, 20%, respectively. Note that PC dimension 2 should be used for stress output since H_a and H_a/C_r are two independent random variables. In addition, PC order 6 is used for the stress output. It is observed that the stress-strain behavior from intrusive simulation is in good agreement with Monte Carlo analysis using 10,000 samples. In addition, standard deviation of the shear strength in Figure 2 is 20%, which is the same as input uncertainty of H_a/C_r.

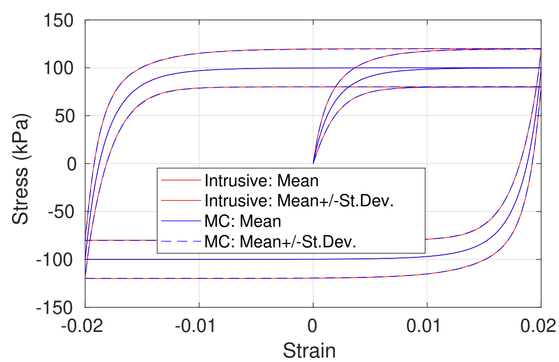


Figure 2: Hysteretical stress-strain behavior of 1-D material with uncertain H_a and H_a/C_r.

RESULTS AND ANALYSIS

By keeping the 1-D site response analysis in mind, this section presents stochastic finite element simulations with uncertain nonlinear material parameters and bedrock motion. The soil deposit, as shown in Figure 3, is 10 m deep and discretized into 10 shear beam elements. The material model, as formulated in previous section, is the 1-D von-Mises model with Armstrong-Frederick kinematic hardening and zero-elastic region.

The initial stiffness, H_a, of the 1-D shear beam model is assumed to be a random field with log-normal distribution (mean 60 MPa, COV 20%), exponential correlation structure with correlation length 10 m. We use Hermite polynomial chaos expansion with dimension 4 order 2 to represent the random field of H_a. The material strength parameter, H_a/C_r, is also uncertain but fully correlated with the random field of H_a. Therefore, the marginal distribution of H_a/C_r is also log-normal but with mean 100 kPa and COV 20%. The correlation structure of H_a/C_r is identical to that of H_a. Since we use dimension 4 for the random field and dimension 150 for the seismic motion, the PC dimension for the response process, displacement,

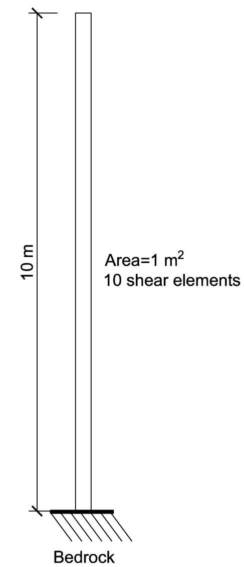


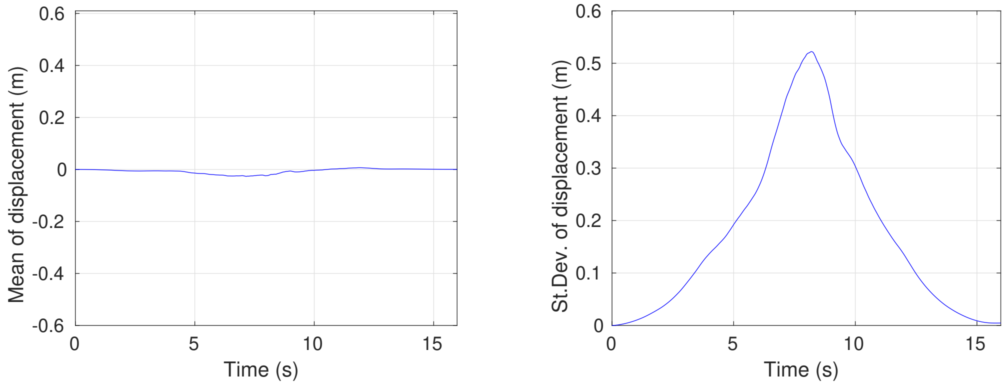
Figure 3: 1-D shear beam model.

acceleration, should be 154.

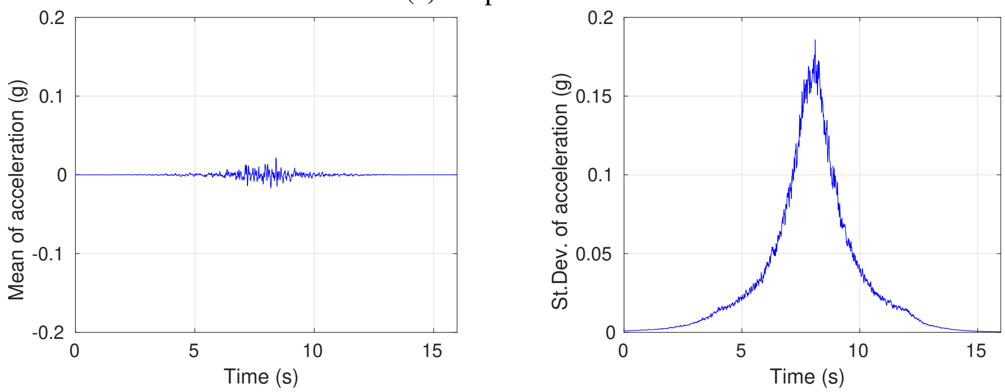
Uncertain bedrock motion, modelled as random process, are developed from stochastic Fourier amplitude spectra and Fourier phase spectra. The inter-frequency correlation structure of Fourier amplitude spectrum is captured, and the non-stationarity of ground motion is quantified by statistical phase derivative model. Hermite polynomial chaos expansion is employed to represent the random process with the correlation structure discretized by Karhunen-Loève expansion. For the earthquake scenario of Magnitude 7, epicenter distance 20km, source stress drop 5MPa and site attenuate Kappa 0.02s, the statistics of the seismic motion, marginal mean, standard deviation of displacement and acceleration is shown in Figure 4. Since the Kolmogorov-Smirnov test shows that its marginal distribution is Gaussian, PC order 1 is able to completely quantify its marginal information. However, a very high PC dimension, i.e., a large number of independent Gaussian variables, should be employed to accurately capture its non-stationary correlation structure in Figure 5. Here we use PC dimension 150 order 1 in order to capture the seismic motion random process.

Figure 6 shows the simulated marginal mean and standard deviation of displacement, acceleration at the ground surface. Note that a case with uncertain elastic material is also performed for comparison. Magnitude of mean response is very small compared with those of standard deviation, and both the mean and standard deviation of response is similar to the mean and standard deviation of input bedrock motion as shown in Figure 4. It indicates the uncertainty of response mostly results from the uncertainty of seismic motion. Due to material nonlinearity, permanent deformation is nontrivial with both the mean and standard deviation of displacement are non-zero at the end of seismic loading. In addition, standard deviation of acceleration decreases significantly due to probabilistic plastification of soil.

Figure 7 shows the stress evolution of soil at ground surface. The mean of stress is trivial compared to standard deviation of stress. In addition, for uncertain elasto-plastic soil, standard deviation of stress drops significantly due to soil plastification in comparison with results of elastic soil.

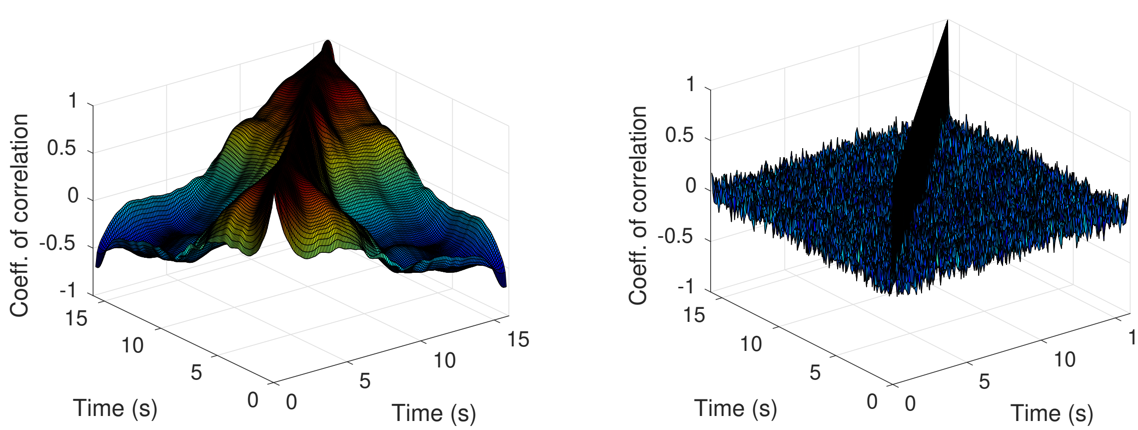


(a) Displacement



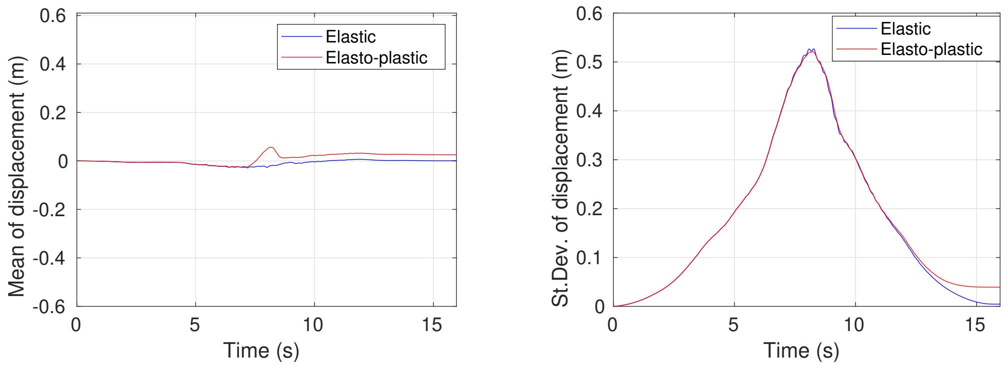
(b) Acceleration

Figure 4: Marginal statistics of the seismic motion random process.

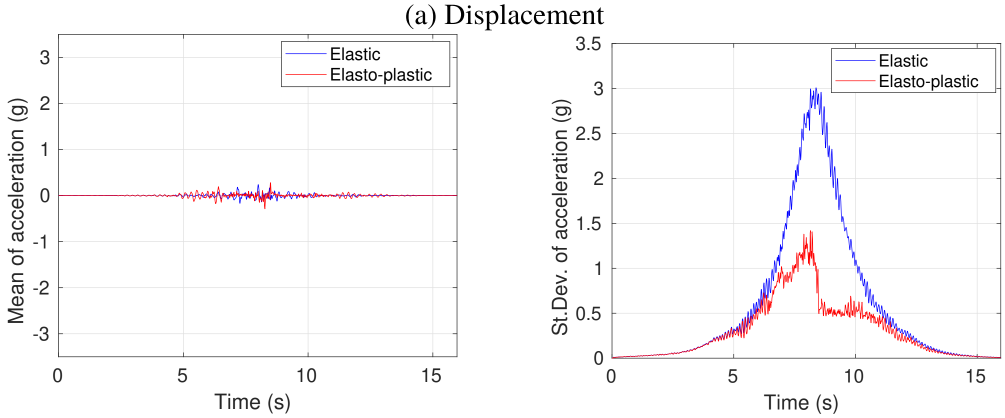


(a) Displacement (b) Acceleration

Figure 5: Correlation structure of the seismic motion random process.



(a) Displacement



(b) Acceleration

Figure 6: Simulated surface response statistics of the soil deposit with uncertain elastic/elasto-plastic material and seismic motion.

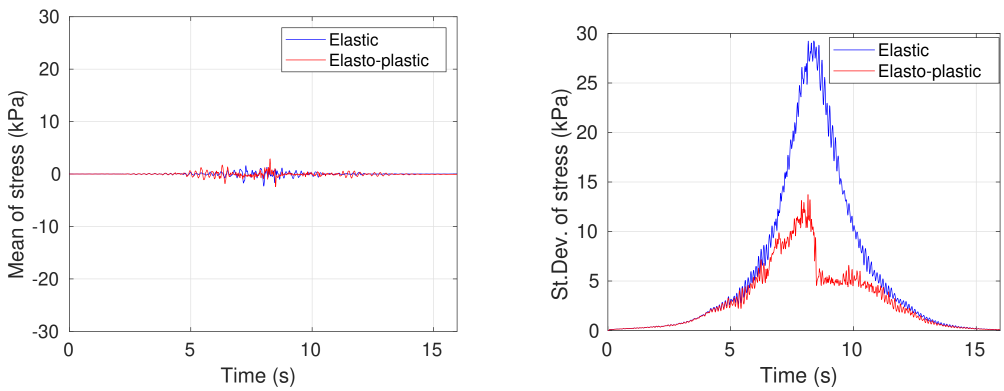


Figure 7: Stress evolution of the surface of soil deposit with uncertain elastic/elasto-plastic material and seismic motion.

CONCLUSIONS

This paper presents a stochastic site response analysis with uncertain elasto-plastic soil and seismic motion. The material parameters of soil deposit are assumed as non-Gaussian random fields while the uncertain seismic motion is considered as a non-stationary bedrock motion. Hermite polynomial chaos is employed to represent the soil random fields and seismic motion random process. The probabilistic constitutive model of soil is von-Mises model with kinematic hardening and zero-elastic region. A stochastic finite element analysis based on stochastic Galerkin method is performed to evaluate the ground surface response.

Uncertainties of simulated ground surface response mostly comes from uncertainties of input seismic motion, and the mean response is trivial compared to standard deviation of response. Compared with the case with elastic soil, permanent deformation is evident and standard deviation of acceleration drops significantly due to probabilistic plastification of soil.

ACKNOWLEDGEMENT

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