# REDUCTION OF A MECHANICAL MODEL DEDICATED TO THE STUDY OF ASSEMBLY BOW

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## ABSTRACT

In the pressure vessel of a PWR, the core is filled of up to 241 fuel assemblies. Under operation, they are subjected to irradiation growth, creeping, tightening relaxation and rod slipping. Combined with the hydraulic forces induced by the coolant flow, these phenomena lead to assembly lateral deformation. Between two operating cycles, when changing the assembly position in the core, this deformation may sometime create difficulties with regard to handling operations. In order to prevent this situation, it would be useful to simulate the thermomechanical behaviour of the core, taking into account the previously mentioned phenomena. For that purpose, we present a 3D modelling approach in which each fuel assembly is represented by a reduced model that uses shape functions for kinematics as well as internal variables. The constitutive equations are then expressed in the reduced basis by defining and coupling the modal thermodynamic forces. We finally analyse the accuracy and performance of that new method through a simple case.
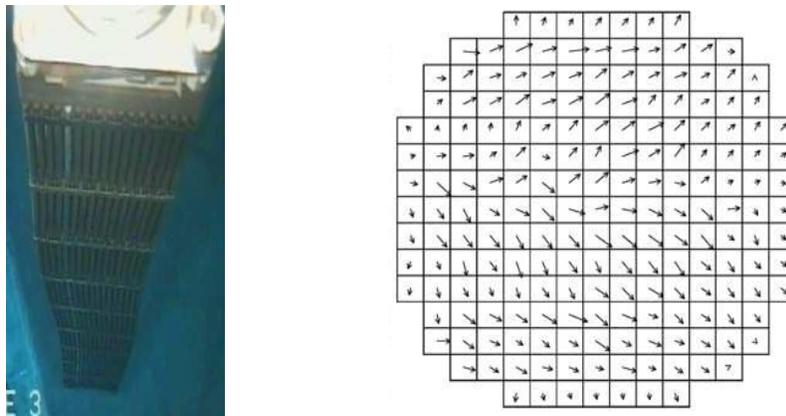
## MODELLLING DIFFICULTY



Figure 1. Picture of a deformed fuel assembly and horizontal displacements at mid core height.

Given the total number of rods, grids, springs and other components that should be represented in a complete PWR pressure vessel, and also considering the extended areas subsect to creeping, contact and friction between rods and grids as well as between neighbouring grids (cf. Figure1), it is unlikely to perform a detailed calculation of a complete core over a few cycles length. The solution usually applied to lighten such a calculation is to create simplified models for fuel assemblies, with less rods and liaisons, as do Wanninger A. et al. (2018). With such simplified models, one can simulate FSI of a complete core, which

is already a significant achievement. Nevertheless, in order to limit the number of hypotheses necessary to build these simplified models, it might be interesting to use recent techniques of "Model Order Reduction". The approach hereafter presented is strongly inspired by the Nonlinear Transformation Field Analysis (NTFA), developed by Michel and Suquet (2003), which is an a posteriori homogenisation technique for nonlinear materials.

**MACROSCOPIC AND MICROSCOPIC BASES FOR THE REDUCED ORDER MODEL**

The NTFA, like other homogenisation techniques, is based on the analysis of a Representative Volume Element (RVE), the deformation of which is driven by its mean strain tensor $\bar{\bar{E}}$. Here we consider the whole fuel assembly as a RVE of the core in order to reach a significant model reduction. Given the shape of a fuel assembly and its ability to curve, we choose to replace the mean strain tensor $\bar{\bar{E}}$ by a series of mean displacement modes.$\widehat{U_j^{ext}}$, that describe the position of centre of the grids. So, for 10 grids and 3 degrees of freedom per grid, a maximum of 30 modes $\widehat{U_j^{ext}}$ is used. The macroscopic displacement of the fuel assembly is decomposed in this basis, as shown in equation 1 below.

$$\widehat{U^{ext}}(t) := \sum_{j=1,J} \alpha_j^{tot}(t)\, \widehat{U_j^{ext}}$$

( 1 )

The microscopic fields of internal variables, such as the viscoplastic strain tensor, are also decomposed, in equation 2, on an orthonormal basis of tensor modes $\widehat{\bar{\bar{\varepsilon}}_i^{vp}}$. This basis is built by the Proper Orthogonal Decomposition described by Karhunen (1946).

$$\widehat{\varepsilon^{vp}}(t) := \sum_{i=1,I} \alpha_i^{vp}(t)\, \widehat{\bar{\bar{\varepsilon}}_i^{vp}}$$

( 2 )

**LOCAL PROBLEM**

We limit the present description to an elastic viscoplastic problem, which obeys the local laws 3. $\varphi^*(\bar{\bar{\sigma}})$ is the dual dissipation potential for creeping and $a$ is a material parameter depending on temperature and fast neutron flux (>1MeV).

$$
\begin{cases}
\bar{\bar{\sigma}}(x) = \dfrac{\partial w}{\partial \bar{\bar{\varepsilon}}} = \bar{\bar{\bar{\bar{C}}}} : \left(\bar{\bar{\varepsilon}}^{tot}(x) - \bar{\bar{\varepsilon}}^{vp}(x)\right) = \sum_{j=1,J} \alpha_j^{tot}\, \bar{\bar{\sigma}}_j^{tot}(x) + \sum_{i=1,I} \alpha_i^{vp}\, \bar{\bar{\sigma}}_i^{vp}(x) \\[4mm]
\dot{\varepsilon}^{vp}(x) = \dfrac{\partial \varphi^*\big(\bar{\bar{\sigma}}(x)\big)}{\partial \sigma} \\[4mm]
\varphi(\sigma) = a\, \sigma_{eq}^m
\end{cases}
$$

( 3 )

**LOCAL STRESSES**

The system condition is fully determined by the displacement field $\widehat{U^{ext}}$ (1) and by the state variables $\widehat{\bar{\bar{\varepsilon}}^{vp}}$ (2), hence by the reduced variables vector $(\overline{\alpha^{tot}}, \overline{\alpha^{vp}})$. One can then deduce the stress field $\hat{\sigma}$ all over the RVE by superposition of stress modes $\widehat{\bar{\bar{\sigma}}_i^{vp}}$ and $\widehat{\bar{\bar{\sigma}}_j^{tot}}$, respectively characterised as *elastic* solutions of

elementary problems with internal unit loading $\widehat{\bar{\bar{\varepsilon}}_i^{vp}}$ or external unit loading $\widehat{U_j^{ext}}$. This superposition used by the NTFA method is made possible by the linearity of elasticity operator $\bar{\bar{\bar{C}}}$ in relation 3.

$$\widehat{\bar{\bar{\sigma}}}(t) := \sum_{i=1,I} \alpha_i^{vp}(t)\, \widehat{\bar{\bar{\sigma}}_i^{vp}} + \sum_{j=1,J} \alpha_j^{tot}(t)\, \widehat{\bar{\bar{\sigma}}_j^{tot}} \qquad (4)$$

In this manner, at any time t, knowing the reduced variables $(\overline{\alpha^{tot}}, \overline{\alpha^{vp}})$ allows to know the local stress field $\widehat{\bar{\bar{\sigma}}}$ everywhere in the fuel assembly.

**REDUCED FORCES**

The free energy in the complete basis (detailed model) is equal to the free energy in the reduced basis. The internal forces $\overline{f^{int}}$, associated to the macroscopic modes $\widehat{U_j^{ext}}$, are easily deduced. Doing the same with the dissipated power, the reduced thermodynamic forces $\overline{\beta^{vp}}$, associated to the viscoplastic modes $\widehat{\bar{\bar{\varepsilon}}_i^{vp}}$ are also identified in equation 5. The relations are linear and stored once in the matrix $\bar{\bar{D}}$ of dimension $(I + J) \times (I + J)$.

$$\begin{cases} (\overline{f^{int}}, \overline{\beta^{vp}}) := \bar{\bar{D}}.(\overline{\alpha^{tot}}, \overline{\alpha^{vp}}) \\ D_{kl} = \iiint \widehat{\bar{\bar{\sigma}}_l} : \widehat{\bar{\bar{\varepsilon}}_k}\ dV \quad k, l = 1:(I+J) \end{cases} \qquad (5)$$

**REDUCED EVOLUTION LAW**

The evolution law from equation 3, transposed into the reduced basis, connects forces $\overline{\beta^{vp}}$ to the flow velocities $\overline{\alpha^{vp}}$ and remains a power law. It is thus given the form of equation 6, for each single modal direction of viscoplastic flow, in order to respect the local problem 3.

$$\alpha_i^{\dot{v}p} = a_i\, m\, (\beta_i^{vp})^{m-1} \qquad (6)$$

The modal characteristic $a_i$ should be precised. Since, in this case, $a(x)$ depends on the thermal field $\hat{T}$ and the neutron flux field $\hat{\emptyset}$, $a_i$ can be spatially heterogeneous. Equalling the dissipated power in both bases leads to the equation 7 for $a_i$.

$$a_i(\hat{T}, \hat{\emptyset}) = \frac{\iiint a(T(x), \emptyset(x)) \left[\bar{\bar{\sigma}}_i^{vp}(x)\right]_{eq}^m dV}{\left[\iiint \bar{\bar{\sigma}}_i^{vp}(x) : \bar{\bar{\varepsilon}}_i^{vp}(x)\ dV\right]^m} \qquad (7)$$

Michel and Suquet (2004) showed that is necessary to couple the thermodynamic forces in order to obtain a better approximation of the viscoplastic flow. The easiest way is to take the Euclidian norm of vector $\overline{\beta^{vp}}$.

$$\alpha_i^{\dot{v}p} = a_i(\hat{T}, \hat{\emptyset})\, m\, \left\|\overline{\beta^{vp}}\right\|^{m-1} \frac{\beta_i^{vp}}{\|\beta^{vp}\|} \qquad (8)$$

Equation 8, although a necessary approximation, constitutes the second level of reduction of our reduced order model.

## EQUILIBRIUM

Since the behaviour of the structural RVE is now solved in a reduced base, we can go back to the internal forces with equation 15 below, then to the average movements of the grids, used in homogenized modelling of the core. In the end, the internal forces of the reduced fuel assembly model must balance the external forces in the chosen displacement base of equation 1:

$$\widehat{F^{ext}} = \sum_{j=1,J} f_j^{int} \, \widehat{U_j^{ext}}$$

( 9 )

## FIRST VALIDATION OF THE REDUCTION METHODOLOGY ON A CREEP TEST

The test case chosen takes the form of a 3-storey structure, of dimensions 0.4m x 0.4m x 3m, the base of which is clamped. The mesh consists of quadratic bricks (14748 modes and 57672 integration points). The elastic characteristics are those of steel and a Norton creep law is imposed $\dot{\varepsilon}_{eq}^{vp} = 10^{-21} \, \sigma_{eq}^{1.4}$ in order to obtain significant creep. A series of 12 preliminary calculations is carried out over $10^8$ seconds, using 10 equal time steps, with loads of constant forces distributed on the floors of each floor. A POD analysis then extracts the viscoplastic modes and helps create the reduced model. Table 1 provides an overview of the accuracy and time savings achieved using the reduced model presented.

Table 1 – Reduced order model versus detailed model on a simple creep test

| Calculation type | CPU time (ms) | Relative displacement error | Deformed mesh (x120) |
|---|---|---|---|
| Reference FE calculation with Cast3M | 78962 | Reference. |  |
| Hybrid Reduced Order Model | 56098 | 0.35% | |
| Reduced Order Model | 45 | 0.45% | |

The hybrid reduced model uses an integration of creep into the complete model. It therefore indicates the best precision that can be obtained with the microscopic basis built by POD. This shows again if needed the importance of choosing relevant snapshots prior to perform the POD analysis. In this case, the ROM displacement error of 0.45% is acceptable, considering the gain in time, which amounts to almost 2000. With a bigger mesh, one can hope even better results for this type of problem.

## CONCLUSION

We have proposed a method inspired by the NTFA (Nonlinear Transformation Field Analysis), which allows the model reduction of a complete elastic viscoplastic structure, rather than that of a classical representative volume element. It is based on two decompositions in spatial modes: on the one hand, that of the mean macroscopic displacements and, on the other hand, that of the microscopic viscoplastic deformations. Once identified, this reduced structure model will be the elemental brick of a reduced core model of PWR, which consists of up to 241 ROM. Finally, the writing of a behavior law in the reduced basis brings the second level of reduction and guarantees a significant decrease in the duration of the calculation.

## PERSPECTIVES

An important perspective concerns the model reduction of internal slippage, which may occur between the fuel assembly components (the fuel rods are simply pinched by springs at the passage of the grids, which represents several thousand zones of contact with friction). To deal with this problem, we plan to decompose the sliding on a sliding mode basis, following the method described in this work to treat viscoplastic deformations of the fuel assembly.

Secondarily, an improvement might be performed on the creeping assessment. The reduced evolution law 8 is a first order approximation, which already gives quite good results. It is nevertheless desirable to estimate it more precisely in order to gain precision. For example, using a second-order development of dissipation potential like Ponte Castaneda (1996) or Michel and Suquet (2016). The idea is then to use the mean and variance of each stress mode to perform a Taylor expansion of the dissipation potential to the second order, leading to a better approximation of the reduced evolution law.

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