Method of Seismic Response Reduction for Crossover Piping
Using Inertial Mass Damper
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ABSTRACT

Piping systems crossing between isolated and non-isolated structures are designed to have relatively long length not to restrict the displacement of the isolated structure during earthquakes. Therefore, the natural frequencies of these piping systems are not high enough to avoid the resonances due to earthquake motions. In such cases, supports are required not to restrain the displacement of isolation, and to suppress the resonances of the piping systems. Because of this requirement, rigid supports cannot be applied to crossover piping systems. In this paper, the method of seismic response reduction for crossover piping using inertial mass damper is proposed. Inertial mass damper generates the force which is proportional to the relative acceleration acting on the damper. In this paper, it is shown that the participation factors of the target vibration modes are minimized by optimizing the parameter of the inertial mass damper. Additionally, by earthquake response analysis, it is confirmed that the resonances of piping systems are suppressed effectively.

INTRODUCTION

An application of seismic isolation system is considered for improvement of a seismic resistance, in nuclear power plants. An isolation system composed of an isolated reactor building and a non-isolated turbine building is assumed as a rational constitution. In this constitution, the vibrations of the crossover piping systems caused by earthquakes are problem. The crossover piping systems are installed between isolated and non-isolated structure as shown in Figure 1. In the isolation system of a nuclear power plant, for example, main stream piping systems and water supply piping systems correspond to crossover piping systems. Crossover piping systems are designed to have relatively long length not to restrict the displacement of the isolated structure. Therefore, the natural frequencies of these piping are not high enough to avoid the resonances due to earthquake motions. But if rigid supports are applied to crossover piping, large loads are generated to the piping systems against the displacement of isolation, so rigid supports cannot be applied. Therefore, the support method different from rigid support is required.

In this paper, the method of seismic response reduction for crossover piping using inertial mass damper is proposed. Inertial mass damper generates the force which is proportional to the relative acceleration acting on the damper. As a type of an inertial mass damper, that using rotor was developed in the field of architecture, for example, Hanzawa and Isoda (2009). The schematic of the example of the inertial mass damper using rotor is shown in Figure 2. In this type, the axis of damper has the ball screw, and the flywheel as a rotor is connected to the axis by the ball nut. By this composition, the translation of the axis is exchanged for the rotation of the rotor. By the moment of the inertia of the rotor, the force of the inertial mass damper has the characteristic proportional to the acceleration of axis.

In the vibration system composed of the isolated structure and the piping systems, the first vibration mode is the motion of the isolated structure, and the higher vibration modes are the resonances of the piping systems. The resonances of the piping systems are targeted in the proposed vibration reduction method. In this method, the participation factors of the vibration modes of the target modes are
minimized by optimizing the inertial mass of the inertial mass dampers. In this paper, it is shown the
effects of this method, by vibration analysis using the two-degrees-of-freedom model and the crossover
piping system model.

ANALYSIS OF 2DOF VIBRATION MODEL

This section discusses eigenvalue analysis using the two-degrees-of-freedom (2DOF) model for simplify.
The analytical model is shown in Figure 3. In this model, the inertial mass damper is set between the
piping system and the non-isolated structure. The isolated structure and the piping system are represented
as a 1DOF vibration system. It is assumed that the non-isolated structure is rigid enough, so the motion of
the non-isolated structure is set equal to the ground motion. In this 2DOF vibration model, 1st vibration
mode is motion of the isolated structure. On the other hand, 2nd mode is the resonance of the piping
system. The 2nd vibration mode is the target of the vibration reduction. In the model, mI is the mass of
the isolated structure, mp is the equivalent mass of the piping, kp is the equivalent stiffness of the piping
system, kI is the stiffness of the isolated structure. The inertial mass damper generates the force which is
proportional to the relative acceleration acting on the damper. In this model, the force of the inertial mass
damper is expressed as follows:

\[ F = -m_s (\ddot{z}_p - \ddot{z}_0). \]  

(1)

Because the force is proportional to acceleration, the coefficient of the force, m_s, has a dimension of a
mass. This is called inertial mass. In Figure 3, m_s represents the inertial mass of inertial mass damper. The
equation of motion of this model is expressed as

\[ ([M] + [M_s]) \{\ddot{z}\} + [K] \{z\} = -[M] \{e\} \ddot{x}_0 \]  

(2)

where the matrices and vectors are expressed as follows:

\[ [M] = \begin{bmatrix} m_p & 0 \\ 0 & m_p \end{bmatrix}, \quad [K] = \begin{bmatrix} k_I + k_p & -k_p \\ -k_p & 2k_p \end{bmatrix}, \quad [M_s] = \begin{bmatrix} 0 & 0 \\ 0 & m_s \end{bmatrix}, \quad \{z\} = \begin{bmatrix} z_I \\ z_p \end{bmatrix}, \quad \{e\} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \]  

(3)

Here, z_I and z_p are the relative displacements of the piping system and the isolated structure, respectively.
By normalization, the Equation (2) is transformed as
where the matrices, vectors and other parameters expressed as follows:

\[
[M] = \begin{bmatrix} \mu_I & 0 \\ 0 & 1 \end{bmatrix}, \quad [K] = \begin{bmatrix} \mu_I \nu_I^2 + 1 & -1 \\ -1 & 2 \end{bmatrix}, \quad [M_s] = \begin{bmatrix} 0 & 0 \\ 0 & \mu_s \end{bmatrix}
\]  

(5)

\[
\mu_I = \frac{m_I}{m_p}, \quad \mu_s = \frac{m_s}{m_p}, \quad \nu_I = \frac{\omega_I}{\omega_p}, \quad \omega_I = \sqrt{\frac{k_I}{m_I}}, \quad \omega_p = \sqrt{\frac{k_p}{m_p}}, \quad \tau = \frac{d}{d\tau}, \quad \tau = \omega_p f.
\]  

(6)

Next, it is shown that the participation factor of the target mode (2nd mode) can be 0 by optimization of the inertial mass. The participation factor of the 2nd mode is calculated by Equation (8):

\[
\beta_2 = \frac{\{Z_2\}^T [M] \{e\}}{\{Z_2\}^T [M] + [M_s] \{Z_2\}}.
\]  

(7)

Here, \(\{Z_2\}\) is the mode vector of the 2nd mode, which is normalized so that the displacement of the piping system \(z_p\) is 1 as follows:

\[
\{Z_2\} = \begin{bmatrix} Z_{f2} \\ Z_{p2} \end{bmatrix} = \begin{bmatrix} \alpha \\ 1 \end{bmatrix}.
\]  

(8)

The participation factor of the target mode (2nd mode) calculated by Equation (8) is shown in Figure 4. The horizontal axis is the mass ratio \(\mu_s\), which is the ratio of the inertial mass \(m_s\) and the piping system equivalent mass \(m_p\). The mass ratio \(\mu_I\) is set to 611 0 \(\times\), since the mass of the isolated structure is much larger than the mass of the piping system. Assuming a case where the natural frequency of the piping system is ten times larger than that of the isolated structure, the ratio of the natural frequencies \(\nu_I\) is set to 0.1. In Figure 4, it is shown that the value of the participation factor of the 2nd mode becomes 0 at \(\mu_s\) is about 1. Therefore, it is possible to minimize the resonance of the piping system by tuning the inertial mass of the inertial mass damper. This suppression effect is obtained in the only case that the one side of the piping system is connected to the isolated structure like a crossover piping system, so is not obtained in the case that general piping systems are targeted.

In the following, the optimum value of the inertial mass which sets the participation factor of the 2nd mode to 0 is derived analytically. From a relationship between eigenvalue and mode vector, this equation is obtained.

\[-\nu_{22}^2 \{M\} + [K] \{Z_2\} = \{0\}\]  

(9)

Here, \(\nu_{22}\) is the dimensionless natural frequency of 2nd mode expressed in the following equation.
Here, \( \omega_{n2} \) is the natural frequency of the 2nd mode with the inertial mass damper. Substituting Equation (5), (8) into Equation (9), following equation is obtained.

\[
\begin{align*}
-1 + \alpha + \alpha \mu_i v_i^2 - \alpha \mu_i v_n^2 &= 0 \\
2 - \alpha - v_n^2 - \mu_i v_n^2 &= 0
\end{align*}
\]

Also, the condition that the participation factor of the 2nd mode is 0, expressed as follows

\[
\begin{bmatrix} Z_2 \end{bmatrix}^T \begin{bmatrix} M \end{bmatrix} \mathbf{e} = 0. \quad (12)
\]

Substituting Equation (5), (8) into above equation, following equation is obtained.

\[
1 + \alpha \mu_i = 0 \quad (13)
\]

Solving the simultaneous equations of Equation (11) and (13), the solutions of the parameters are obtained as follows:

\[
\nu_{n2} = \left( 1 + v_i^2 + \frac{1}{\mu_i} \right)^{1/2}, \quad \alpha = -\frac{1}{\mu_i}, \quad \mu_i = \frac{\mu_i \left( 1 - v_i^2 \right)}{1 + \mu_i \left( 1 + v_i^2 \right)}. \quad (14)
\]

\( \mu_i \) expressed as above equation is the optimum value. In general, the natural frequency of the isolated structure is much lower than that of the piping system, and the mass of the isolated structure is much larger than that of the piping system. Considering these characteristics, the limit as \( v_i \to 0 \) and \( \mu_i \to \infty \) is calculated as follows:

\[
\lim_{v_i \to 0, \mu_i \to \infty} \mu_i = \lim_{v_i \to 0} \left( \lim_{\mu_i \to \infty} \mu_i \right) = 1. \quad (15)
\]

From above equation, the optimum value of \( \mu_i \) is 1, so optimum value of \( m_s \) is equal to the equivalent mass of the piping system. This result is in agreement with the results shown in Figure 4.

**VIBRATION MODEL OF CROSSOVER PIPING SYSTEM**

In the following sections, the analyses using the crossover piping system model are shown. The outline of the piping system used for analysis is shown in Figure 5. This piping system crosses between the non-isolated structure and the isolated structure, and has the two elbow parts so as not to restraint the displacement of the isolation. The piping is modelled by the beam element of FEM, and isolated structure is modelled as 2DOF spring-mass system which has the horizontal two degrees of freedom. In this analysis, the horizontal isolation system is supposed and the vertical excitation is not considered. The
specifications of the piping model are shown in Table 1. The natural frequencies of the isolated structure are set to 0.3 Hz. Firstly, the natural frequencies and the participation factors under the condition without the inertial mass dampers are shown in Table 2. Also, vibration modes under the same condition are shown in Figure 6. The 1st and 2nd modes are motions of the isolated structure in Y and X direction, respectively. The other modes over 3rd are motion of the resonances at the natural frequencies of the piping system itself. Here, from the 3rd to 8th modes with the natural frequencies not high enough to avoid the resonances due to earthquake motions are targeted for vibration reduction.

OPTIMISATION OF INERTIAL MASS

Next, it is shown that the participation factors of target vibration modes are minimized by optimizing the parameter of the inertial mass damper. In this vibration reduction method, the number of the necessary inertial mass dampers is the same as that of the target vibration modes. In this case, the 6 inertial mass dampers are installed to the piping system, because the number of the target vibration mode is 6 as mentioned above. As shown in Table 2, the 4th, 6th and 8th modes have the nonzero participation factor in the X direction and the 3th, 5th and 7th modes have in the Y direction. Therefore the 3 inertial mass dampers

![Figure 5. Analytical model of crossover piping system.](image)

<table>
<thead>
<tr>
<th>Vibration mode No.</th>
<th>Natural frequency [Hz]</th>
<th>Participation factor [-] X direction</th>
<th>Participation factor [-] Y direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.31</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
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</tr>
<tr>
<td>3</td>
<td>4.6</td>
<td>0.00</td>
<td>0.66</td>
</tr>
<tr>
<td>4</td>
<td>4.8</td>
<td>0.19</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>7.0</td>
<td>0.00</td>
<td>0.50</td>
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<tr>
<td>6</td>
<td>8.0</td>
<td>0.47</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>13.8</td>
<td>0.00</td>
<td>0.27</td>
</tr>
<tr>
<td>8</td>
<td>23.7</td>
<td>0.33</td>
<td>0.00</td>
</tr>
<tr>
<td>9</td>
<td>29.3</td>
<td>0.00</td>
<td>0.14</td>
</tr>
</tbody>
</table>
are installed in the X and Y directions, respectively. The optimal conditions about the inertial mass of the dampers are calculated by the numerical method shown below. First, the vector composed of the inertial masses of the inertial mass dampers is defined as \( \mathbf{\mu} \) as following equation.

\[
\mathbf{\mu}_s = \begin{bmatrix} \mu_s^1, \mu_s^2, \mu_s^3, \ldots, \mu_s^n \end{bmatrix}^T
\]

(16)

Here, \( n \) is the number of the inertial mass dampers. By giving the value of \( \mathbf{\mu} \), the matrix \( \mathbf{M_s} \) composed of the inertial masses is decided, and the participation factors obtained by the eigenvalue analysis. Next, the vector composed of the participation factors of the target vibration modes is defined as \( \mathbf{\beta} \) as following equation.

\[
\mathbf{\beta} = \begin{bmatrix} \beta_1, \beta_2, \beta_3, \ldots, \beta_n \end{bmatrix}^T
\]

(17)

Because \( \mathbf{\beta} \) is obtained by giving \( \mathbf{\mu} \), \( \mathbf{\beta} \) can be regarded as a function of \( \mathbf{\mu} \). Therefore the relationship between \( \mathbf{\beta} \) and \( \mathbf{\mu} \) can be expressed by the function \( f \).

\[
\mathbf{\beta} = f(\mathbf{\mu})
\]

(18)

When \( \mathbf{\mu} \) changes by slight value \( \delta \mathbf{\mu} \), \( \mathbf{\beta} \) changes by \( \delta \mathbf{\beta} \) and the following equation is obtained.

\[
\mathbf{\beta} + \delta \mathbf{\beta} = f(\mathbf{\mu}) + \mathbf{J}(\mathbf{\mu}) \delta \mathbf{\mu}
\]

(19)

Here, \( \mathbf{J} \) is jacobian obtained by differentiating \( f \) by \( \mathbf{\mu} \). Newton-Raphson method is used in this optimization. The correction value of \( \mathbf{\mu} \) in the \( k \)th step of the iterative calculation is expressed as following equation.

\[
\delta \mathbf{\mu}_s^{(k)} = -\mathbf{J}(\mathbf{\mu}_s^{(k)})^{-1} f(\mathbf{\mu}_s^{(k)})
\]

(20)

The optimal value of \( \mathbf{\mu} \) can be obtained by iterative calculating using the above equation until \( \delta \mathbf{\mu}_s^{(k)} \) becomes sufficiently small. In the calculation of the jacobian, \( df/d\mathbf{\mu} \) is calculated as numerical differentiation, because \( df/d\mathbf{\mu} \) cannot be obtained analytically. In deciding the positions of dampers, the pattern that the necessary inertial masses are minimized is selected. The selected positions of the dampers are shown in Figure 7, and optimized inertial masses are shown in Table 3. Next, the calculated natural
frequencies and participation factors are shown in Table 4 when the inertial masses are set to the optimum values shown in Table 3. For comparison, the results in the case without the inertial mass dampers are also shown in the same Table. It is confirmed that the participation factors of the target vibration modes (hatched in Table 4) are set to 0 by applying the dampers. The maximum value of required inertial masses is $23.3 \times 10^3$ kg larger than the total mass of the piping system (about $19 \times 10^3$ kg). But it is thought that the necessary inertial mass can be secured by selecting inertial mass dampers using rotor. Because this type of dampers can generate the inertial mass several hundred times larger than the mass of the rotor.

**FREQUENCY RESPONSE ANALYSIS**

This section shows the result of frequency response analysis on the condition that the inertial mass dampers are optimized. The node point near the center of the straight part of the piping system is selected as an evaluation point (point A shown in Figure 7). The calculated frequency response curves in the both cases with and without the inertial mass dampers are shown in Figure 8. It is shown that the resonance peaks of all the target modes are minimized by applying the inertial mass dampers.

![Figure 7. Positions of inertial mass dampers.](image)

<table>
<thead>
<tr>
<th>Table 3: Values of optimized inertial masses</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inertial mass damper No.</strong></td>
</tr>
<tr>
<td>-------------------------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
</tbody>
</table>

![Table 4: Natural frequency and participation factor under the condition with the inertial mass dampers.](image)
SEISMIC RESPONSE ANALYSIS

Finally, the seismic response analysis by the piping system model is shown. The maximum acceleration of the input earthquake wave is 9.0 m/s², and the directions of excitation are two directions of X and Y. A modal analysis is used in this calculation. The node points A and B are selected as the evaluation points. In the case without the inertial mass dampers, the response accelerations at evaluation points A and B become maximum in X and Y directions, respectively. The calculated time histories of response accelerations are shown in Figure 9 and Figure 10. It is confirmed that the response accelerations are suppressed effectively by applying the inertial mass dampers. In this response wave, the accelerations by the resonances of the piping system are minimized, and the acceleration by the motion of the isolated structure is only remained.

Figure 9. Calculated seismic response accelerations in X direction at evaluation point A.
CONCLUSION

The method of seismic response reduction using the inertial mass dampers is proposed, for crossover piping systems installed between isolated and non-isolated structures. The results obtained by the analyses are shown below:

- The effect of the proposed method was examined using a two-degrees-of-freedom model composed of the isolated structure and the crossover piping systems expressed as one degree-of-freedom system respectively. It was shown that the participation factor of the target vibration mode can be set to zero, by optimizing the inertial mass of the inertial mass damper.

- Vibration analysis was carried out using the three-dimensional crossover piping model. It was shown that the participation factors of the all target modes can be set to zero, by optimizing the inertial masses of the multiple inertial mass dampers. Also frequency response analysis was carried out, and it was shown that the resonance peaks of the piping are suppressed by inertial mass dampers. Additionally, the seismic response analysis was carried out, and it was confirmed that the response acceleration of the piping system are suppressed efficiently.

REFERENCES