Transactions, SMiRT-25
Charlotte, NC, USA, August 4-9, 2019
Division VII

Improvement in Ground Motion Model Selection Method for Probabilistic Seismic Hazard Analysis

Masato Nakajima[1]

[1] Senior Research Associate, Central Research Institute of Electric Power Industry, Abiko-shi, Chiba-ken, Japan (masato@criepi.denken.or.jp)

ABSTRACT

This paper proposes an improved method for selecting ground motion models for probabilistic seismic hazard analysis. First, a brief overview of previous selection methods (e.g., Cotton et al.'s method, Sherbaum et al.'s method, South Western United States (SWUS) Senior Seismic Hazard Analysis Committee (SSHAC) Project method) is presented. In this study, we focus on the SWUS's method using Sammon's map representation as we believe that this method is the most rigorous from the viewpoint of mathematics. Then, their approach is analysed on the basis of a numerical simulation. Finally, their method using Sammon's map representation is improved so that it can consider cases where the magnitude for candidate ground motion prediction equations (GMPEs) needs to be unified. There exists strong nonlinearity among the parameters of these equations.

It is concluded that the Sammon' map representation method provides a sophisticated approach in selecting ground motion model; further it is possible to improve it with respect to target ranges and computations.

INTRODUCTION

The evaluation of epistemic uncertainties is a key issue in probabilistic seismic hazard assessment (PSHA). The Senior Seismic Hazard Analysis Committee (SSHAC) first published NUREG-2117 guideline methodologies in 2012; these have been widely adopted in the US and other countries (e.g., in the Swiss PEGASOS Project). The SSHAC guideline provides a logical and rational procedure to assess epistemic uncertainties that arise from ground motion characterisation (GMC) and seismic source characterisation (SSC). However, although a few methods and techniques have been developed, certain issues still remain to be addressed when selecting ground motion models.

With respect to GMC, it is necessary to build logic trees on the basis of appropriately selected ground motion models that can represent ground motion accurately for a target site/area. In general, the backbone approach (e.g., Atkinson (2014)) and the selection criteria approach (e.g., Cotton et al. (2006)) have been widely used to select ground motion models; further, more quantitative selection methods using information regarding consistency of data (e.g., Sherbaum et al. (2009)) have also been discussed.

The SWUS SSHAC Project (GeoPentech (2015)) developed a novel and innovative selection method for PSHA using a non-linear mapping technique on the basis of Sherbaum's work. The method developed can rigorously meet following necessary and sufficient conditions for building logic trees:

- Logic tree branches must be mutually exclusive in the same hierarchy
- Logic tree branches must be collectively exhaustive in the same hierarchy

However, there is still room to develop the Sammon's map representation method in terms of the following points: (i) considerations of parameter uncertainties, and (ii) expansion of the maximum dimensions.
In this study, the characteristics of the mapping-based method developed through the SWUS SSHAC Project are analysed based on case studies of numerical calculations. The method is then improved to accurately select ground motion models in a case where strong non-linearity that arises from the parameter uncertainties exists among candidate ground motion prediction equations (GMPEs). Specifically, we overcome the gap addressed above by embedding an advanced searching algorithm into the Sammon's map representation. The improved method can be applied to ground motion model selection when conducting PSHA for highly-seismicity areas.

GROUND MOTION SELECTION METHOD USING SAMMON'S MAP TECHNIQUE

First, we examine ground motion selection methods using Sammon's map representation of the alternative magnitude and the site to source distance scaling of the GMPEs.

Evaluation of the Ground Motion Median Model using Sammon's Map Representation

The SWUS SSHAC Project developed a method that makes it possible to select ground motion models more quantitatively and subjectively. The method employs a nonlinear mapping technique developed by Sammon (Sammon (1969)).

The distance between GMPE i and GMPE j in probabilistic space is calculated using Equation (1):

$$E = \frac{1}{\sum_{i<j} \overline{\Delta_{GM}}_{ij}} \cdot \sum_{i<j} \frac{\left(\overline{\Delta_{GM}}_{ij} - \overline{\Delta_{map}}_{ij}\right)^2}{\overline{\Delta_{GM}}_{ij}} \tag{1}$$

where $\overline{\Delta_{GM_{ij}}}$ denotes the distance between the two GMPEs in original high-dimensional space, and, $\overline{\Delta_{map_{ij}}}$ denotes the distance in a low-dimensional space that maintains the same relationship in a high-dimensional space. We employ the three distances, which are defined as follows:

$$\overline{\Delta_{GM_{ij}}}(L_1) = \frac{1}{N}\sum_{k}^{N} \left|\mathrm{GMPE}_{ik} - \mathrm{GMPE}_{jk}\right| \tag{2}$$

$$\overline{\Delta_{GM_{ij}}}(L_2) = \sqrt{\frac{1}{N}\sum_{k}^{N} \left(\mathrm{GMPE}_{ik} - \mathrm{GMPE}_{jk}\right)^2} \tag{3}$$

$$\overline{\Delta_{GM_{ij}}}(L_\infty) = max_k \left|\mathrm{GMPE}_{ik} - \mathrm{GMPE}_{jk}\right| \tag{4}$$

To calculate the distances among candidate GMPEs in probability space, the following steps are applied:

Step 1: Calculation of ground motion level by GMPEs

The ground motion level is calculated by the selected GMPE for sets of magnitude M and site-to source distance R obtained by discretisation. The faulting type and shear wave velocity V_S are set considering the conditions of the target site and seismic sources.

Step 2: Setting of reference model

We refer to the average of all ground motion models "mix" hereafter. Three indexes are calculated as a reference point from the following equations:
(a)Index regarding the increment/decrement of the predicted value

$$mix + \ln(\alpha) \cdots \alpha = 0.67, 0.8, 1.25, 1.5 \tag{5}$$

The values obtained above are denoted as $S_{--}, S_{-}, S_{+}, S_{++}$.

(b)Index regarding the sensitivity of the earthquake magnitude

$$mix + \beta(M_w - 6) \cdots \beta = 0.67, 0.8, 1.25, 15 \quad (6)$$

The values obtained above are denoted as $M_{--}, M_{-}, M_{+}, M_{++}$.

(c)Index regarding the sensitivity of the site to source distance

$$mix + \gamma\left(R_{JB} - 30\right) \cdots \gamma = -0.01, -0.005, 0.005, 0.01 \quad (7)$$

The values obtained above are denoted as $R_{--}, R_{-}, R_{+}, R_{++}$.

Step 3：Setting of axes in contracted space

The map is centred on the average model at the point (0,0). Then, the map is rotated so that the model S_{++} is to the right, and the line from S to S_{++} is approximately horizontal. Finally, the map is mirrored such that M_{++} is located in the upper half of the model.

Step 4: Computation of candidate GMPE distance in contracted space

Using the equations listed above, the distances among candidate GMPES in a contracted space are computed. A set of vectors $X_i (i = 1, \ldots, N)$ is assumed to exist in an L-th dimensional space. This space is referred to as the "Original high-dimensional space" hereafter. The data structure analysis method by Sammon (Sammon (1969)) makes it possible to obtain a set of vectors $Y_i (i = 1, \ldots, N)$ in d (less than L)-th dimensions in a contracted space that has an equivalent positional relationship in the original space. In general, d is set to as 2 or 3 to easily visualise and grasp what the map indicates.

The vector X_i $(i = 1, \cdots, N)$ is assumed to exist in the L-th dimensional space, which is referred to as "original space". The objective of the calculation is to search vector Y_i $(i = 1, \cdots, N)$ in a d-th dimensional space that is a two- or three-dimensional space, which is referred to as the "contracted space". We define the distance of each vector as follows:

$$\text{Original space: } d_{ij}^* \equiv \mathrm{dist}\left[X_i, X_j\right] \quad (8)$$
$$\text{Contracted (Reduced) space: } d_{ij} \equiv \mathrm{dist}\left[Y_i, Y_j\right]$$

As an initial d-th dimensional space configuration, we randomly select a Y vector as follows:

$$Y_1 = \begin{bmatrix} y_{11} \\ \vdots \\ y_{1d} \end{bmatrix} \; Y_2 = \begin{bmatrix} y_{21} \\ \vdots \\ y_{2d} \end{bmatrix} \; \ldots \; Y_N = \begin{bmatrix} y_{N1} \\ \vdots \\ y_{Nd} \end{bmatrix}$$

The Y vector assembly is searched that minimises the error E by changing the candidate y_{pq}; $(p = 1, \cdots, N)(q = 1, \cdots, d)$

$$E = \frac{1}{\sum_{i<j} d_{ij}^*} \cdot \sum_{i<j}^{N} \frac{\left(d_{ij}^* - d_{ij}^*\right)^2}{d_{ij}^*} \quad (9)$$

For example, in a case where we select three GMPEs as a ground motions models in the two-dimensional space, we set the Y vector assembly as:

$$Y_1 = \begin{bmatrix} y_{11} \\ y_{12} \end{bmatrix} \; Y_2 = \begin{bmatrix} y_{21} \\ y_{22} \end{bmatrix} \; Y_N = \begin{bmatrix} y_{31} \\ y_{32} \end{bmatrix} \quad (10)$$

In a case where there are four GMPEs candidates, the Y vectors are set as follows:

$$Y_1 = \begin{bmatrix} y_{11} \\ y_{12} \end{bmatrix} \; Y_2 = \begin{bmatrix} y_{21} \\ y_{22} \end{bmatrix} \; Y_3 = \begin{bmatrix} y_{31} \\ y_{32} \end{bmatrix} \; Y_4 = \begin{bmatrix} y_{41} \\ y_{42} \end{bmatrix}$$

Step 5: Verification of search algorithm for optimal solution

It is necessary to verify a computation algorithm when the scaling factor assigns different values to a candidate GMPE. Specifically, the relationship between $\overline{\Delta_{GM_{ij}}}(L_\infty)$ and $\overline{\Delta_{map_{ij}}}(L_\infty)$ is invariant and does not depend on the GMPE.

NUMERICAL SIMULATIONS

Problem Settings of Sammon's Map Representation for Ground Motion Models

In this paper, Sammon's map representation of GMPEs is discussed based on numerical simulations in a case where uncertainties in magnitude are inherent. In other words, ground motion model selections is conducted when there are strong nonlinear relationships among candidate GMPEs.

We consider an inland crustal earthquake with the parameters set as shown in Table 1. The following five GMPEs are employed as ground motion models.

- Abrahamson and Silva's GMPE (2008) : denoted hereafter as "eq.1"
- Noda et al.'s GMPE (1999) : denoted hereafter as "eq.2"
- Kataoka and Sato's GMPE (2006) : denoted hereafter as "eq.3"
- Matsumoto et al.'s GMPE (2004) : denoted hereafter as "eq.4"
- Si et al.'s GMPE (2013) : denoted as hereafter"eq.5"

Table 1: Parameter settings of seismic sources

Inland crustal earthquake	Dip angle= 90°, point source ● Mw=5.0,5.25,5.5,5.75,6.0,…,7.0,7.25,7.5 (11 patterns) ● R=1,5,10,15,20,25,30,35,…,65,70 km (10 patterns)
Interplate earthquake	Focal depth=30 km ● Mw=6.0,5.25,5.5,5.75,6.0,…,7.75,8.0 (11 patterns) ● R=1,5,10,15,20,25,30,35,…,65,70 km (10 patterns)
Intraplate earthquake	Focal depth= 30km ● Mw=6.0,5.25,5.5,5.75,6.0,…,7.75,8.0 (11 patterns) ● R=2,10,20,30,40,60,80,100,120,140 km (10 patterns)

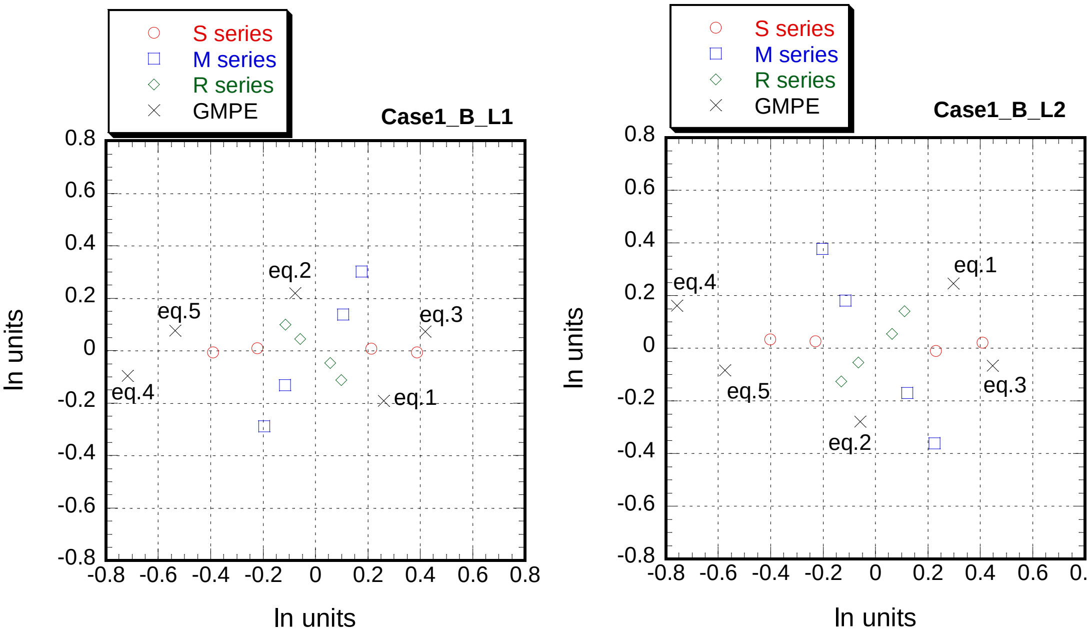
Fundamental Characteristics of Sammon's map Representation

On the basis of the problem settings described above, Sammon's map of ground motion model is computed (Figure 1). Figure 1 shows the Sammon's map when the error E is a minimum, and Figure 1(i)(ii)(iii) illustrate maps corresponding to a distance index L_1, L_2 and L_∞ norm respectively.

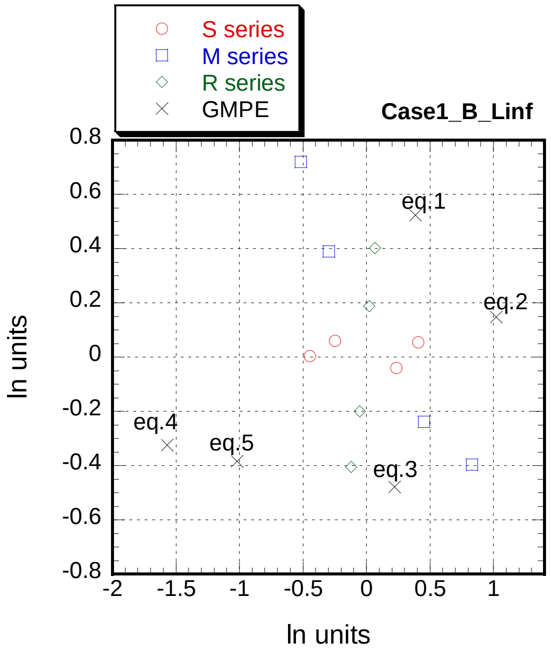
An optimising search needs to be conducted for non-linear problems. To this end, the conjugate gradient method (Press et al. (1996)) has been adopted to search the optimising vector: the error E is minimised. The numerical outputs can vary depending upon the initial value of the random number in case of a non-linear problem. Therefore, 10000 trials are performed to observe the differences in simulation results, particularly in cases where the error E is either minimised or maximised. Figure 2 shows the Sammon's map when the error E is at a minimum

We can represent Figure 1 diagrammatically such that the positional relationship among GMPEs using L_1norm and L_2 norm are almost the same. Further, a diagram using L_∞ norm differs from diagrams using other the norms. It suffices to say that the differences are caused by the distance index defined by Equations (4), (5), and (6). The distance index L_1norm and L_2 norm indicate an average norm in a space however, the distance index L_∞ norm prescribes a maximum distance between GMPEs in a space.

A diagram for which an error is maximal in 10000 cases is shown in Figure 2. A comparison between Figure 1 and Figure 2 corresponding to the same distance index shows that a positional relationship between the M-axis prescribed by $M_{--}, M_{-}, M_{+}, M_{++}$ and the S-axis prescribed by $S_{--}, S_{-}, S_{+}, S_{++}$ differs, and the positions of some GMPEs vary drastically.

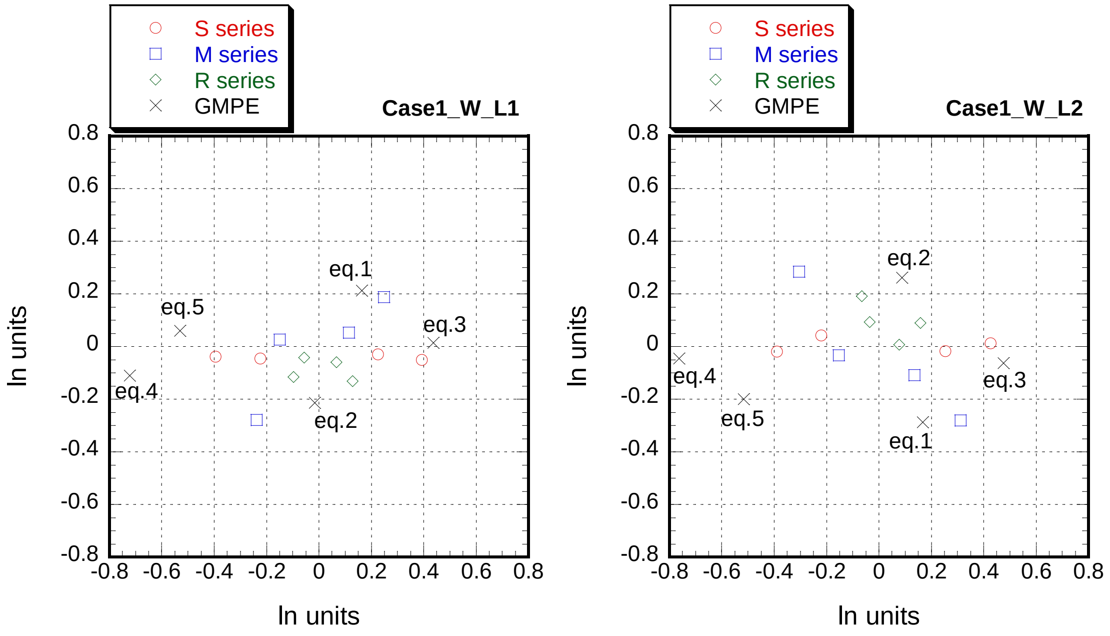


(i)Distance index: L_1 norm (ii) Distance index: L_2 norm

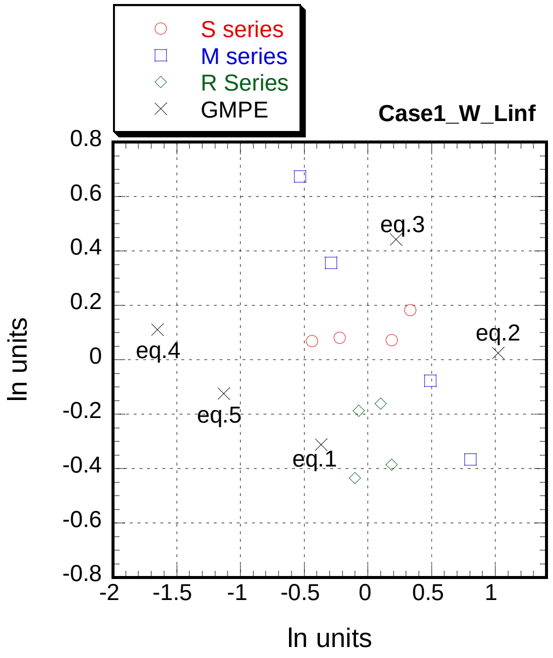


(iii) Distance index: L_∞ norm

Figure 1. GMM Sammon's Map (error E is minimum)



(i)Distance index: L_1 norm (ii)Distance index: L_2 norm



(iii)Distance index: L_∞ norm

Figure 2. GMM Sammon's Map (E is maximum)

Improvements in Ground Motion Model Selection Method Using Sammon's Map

As described in previous sections, the method for selecting ground motion models using Sammon's map representation technique is built using a rigorous mathematical formulation, and it is entirely fair to say that it can provide more quantitative and subjective output compared to existing methods.

However, there is a room to improve the method when it is applied to the selection of ground motion models. To be specific, it is necessary to overcome the following technical issues:

- The parameter uncertainties need to be considered, for example, the uncertainty that arises in magnitude conversion
- The maximum dimensions of Sammon’s map technique should be expanded

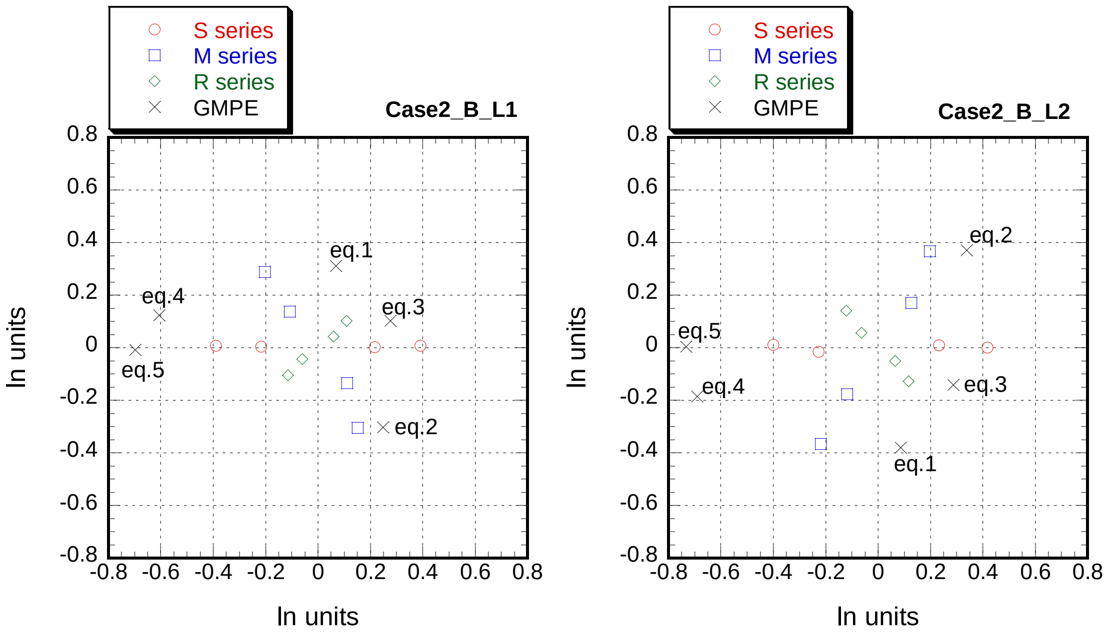
A Sensitivity Analysis of Uncertainties Caused by Magnitude Conversion

A GMPE is mathematically formulated using several parameters (e.g., earthquake magnitude, site-to-source distance, focal depth, and faulting type); these parameters play the role of explanatory variables.
In generating GMM Sammon’s map, it is necessary to convert the magnitude to unify its type in a case where some GMPE candidates are formulated using different magnitude units. Here, we confine a target parameter to the magnitude, and conduct a sensitivity analysis to investigate the effect of uncertainties that arise from magnitude conversion on the result, namely the GMM Sammon’s map.
Takemura (1990) established two relationship equations with respect to inland shallow earthquakes: one is an equation between seismic moment M_0 and Japan Meteorological Agency (JMA) magnitude M_J, and the other is an equation between M_0 and moment magnitude M_w. On the basis of these two equations, the relationship equation between M_0 and M_J is derived as follows:

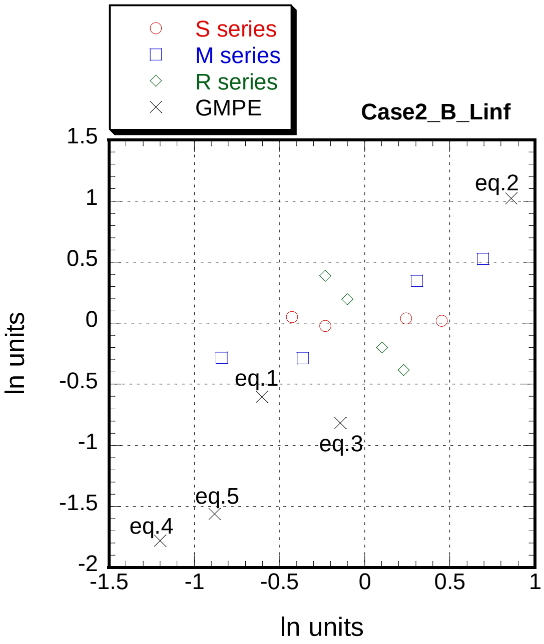
$$M_W = 0.78M_J + 1.08 \qquad (11)$$

We unify the magnitude to the moment magnitude M_w by using Equation (11) in a case where a candidate GMPE uses the JMA magnitude M_J as an explanatory variable. It is assumed that the above equation has an uncertainty in converting magnitude, and the uncertainty is modelled by a normal (Gaussian) distribution with coefficient of variation (COV) =0.3.

Figure 3 shows the calculation results, namely Sammon’s map of ground motion model in a case where the error E is at a minimum. Three distance indexes defined by Equations (2), (3), and (4) are employed in this map (Figure 3(a)(b)(c)). A comparison of their maps shows two obvious characteristics: (i) geometrical relationship between GMPEs and the axis differs corresponding to the selected distance index, (ii) and the axis scale when the L_∞ norm is adopted is larger than when the L_1 norm and L_2 norms are adopted.

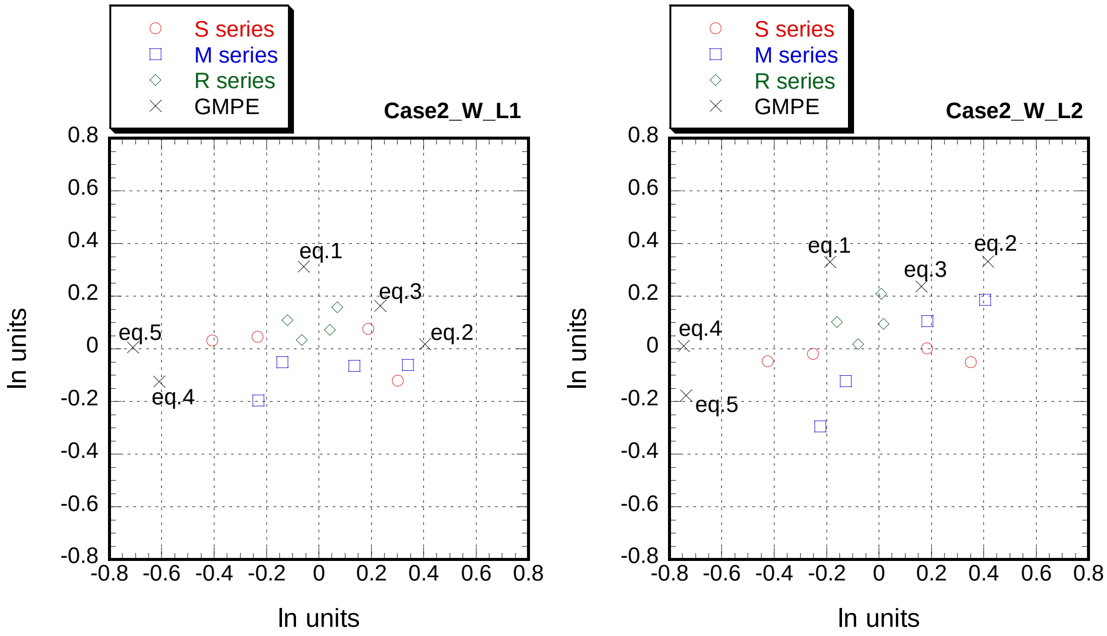


(i)Distance index: L_1 norm (ii)Distance index: L_2 norm

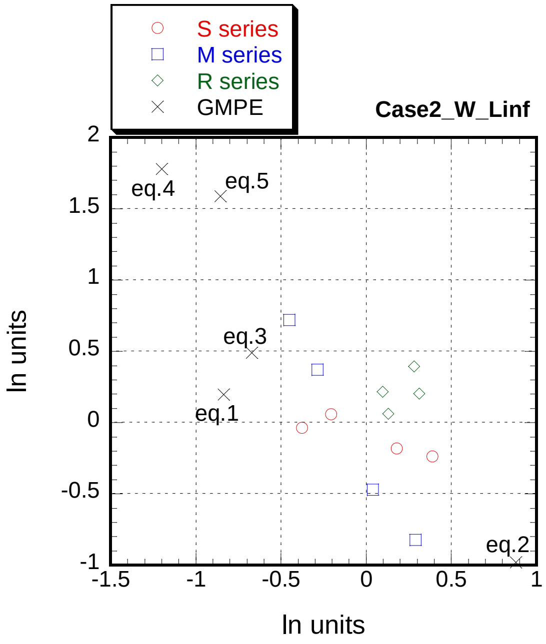


(iii)Distance index: L_∞ norm

Figure 3. GMM Sammon's map (when error E is minimum)



(i)Distance index: L_1 norm (ii)Distance index: L_2 norm



(iii)Distance index: L_∞ norm

Figure 4. GMM Sammon's map (where the error E is a maximum)

DISCUSSION

Through a sensitivity analysis based on numerical simulations, we found that the uncertainty owing to magnitude conversion causes changes in GMPEs relations in the probabilistic model space.
We have to consider the uncertainty-induced variations in the Sammon's map.

- The values obtained as $M_{--}, M_{-}, M_{+}, M_{++}$ vary in a case where we consider uncertainties arising from magnitude conversion, and this variation greatly affects the ground motion level calculated from the GMPEs and "*mix*". The variation in sensitivity means a variation in the scaling factor. These variations lead to changes in the positional relationship among GMPEs and changes in the relationship between GMPEs and the axes.
- The uncertainty of magnitude conversion is modelled using a Gaussian distribution with COV = 0.3 in this analysis. This modelling should be conducted considering not one but several scaling law relationships. Specifically, a probability distribution model to represent the uncertainty should be discussed.
- The uncertainties that arises from the distance index conversion should be examined similarly because some GMPEs adopt different distance indexes (e.g., the GMPE given in Noda et al. (2001)). The results from additional numerical simulations to examine the effect of the distance index conversion will be shared in the presentation.

Although these results are derived from a limited number of calculations, they are considerably important to stress. Therefore, in order to validate this method, we need to examine the sensitivity for different periods of time, and with other parameters.
Furthermore, it is necessary to examine the effect of the variation in Sammon's map representation on the weight value, which is obtained using the residual and likelihood. Some of the additional examinations will be presented in the conference.

CONCLUSION

Quantitative selection method for ground motion models for PSHA was discussed on the basis of SWUS SSHAC project works. First, we provided a brief overview of existing selection methods (e.g., the method of Cotton et al., which is the most rigorous from a mathematical viewpoint.

We examined the possibility of improving the method by focusing on the uncertainty in conversion of different types of magnitude adopted in candidate GMPEs.

Numerical simulation results indicated that the uncertainty concerning the magnitude conversion considerably affects the relations among GMPEs in the probabilistic space. Because space is limited, we have concentrated on the GMPE and its one parameter. We could not examine the cases using fault rupture models as ground motion models.

ACKNOWLEDGEMENTS

I gratefully acknowledge the helpful and valuable discussions with Dr Norman A. Abrahamson on the Sammon's map representation for ground motion models.

REFERENCES

Abrahamson, N.A. and Silva, W.J. (2008). "Summary of the Abrahamson & Silva NGA ground motion relations", *Earthquake Spectra* **24**, pp.67-97

Atokinson, G.R., Bommer, J.J. and Abrahamson, N. (2014). "Alternative approaches to modelling epistemic uncertainty in ground motions in probabilistic seismic-hazard analysis", *Seismological Research Letter*, 85(6), 1141-1144, doi: 10.1785/0220140120.

Cotton, F., Sherbaum, F., Bommer, J.J. and Bungun, H. (2006). "Criteria for selecting and adjusting ground-motion models for specific target regions: Application to Central Europe and rock sites", *Journal of Seismology*, **10**: pp.137-156.

GeoPentech (2015). "Southwestern United States Ground Motion Characterization SSHAC Level 3", Technical Report, Revision 1

Kataoka, S., Satoh, T., Matsumoto, S. and Kusakabe, T. (2006). "Attenuation Relationships" of Ground Motion Intensity Using Short Period Level as a Variable, *Journal of Japan Society of Civil Engineers* A **62** (4), pp.740-757 (in Japanese)

Nishimura, I., Noda, S., Takahashi, K., Takemura, M., Ohno, S., Tohdo, M. and Watanabe, T. (2001). "Response Spectra for Design Purpose of Stiff Structures on Rock Sites", *16th Conference of Structural Mechanics in Reactor Technology* (SMiRT 16), Washington DC, Paper#1133.

Pacific Gas and Electric Company (2018). "Seismic Probabilistic Risk Assessment in Response to 50.54 (f) Letter with Regard to NTTF 2.1: Seismic Summary Report", Enclosure 1, PG&E Letter DCL-19-027.

Press, W.H., Teukolsky, S.A., Vetterling, W.T. and Flannery, B.P. (1996). "Numerical Recipes in Fortran 90 –The Art of Parallel Scientific Computing-, *Cambridge University Press*"

Sammon, J.W. (1969). "A Nonlinear Mapping for Data Structure Analysis", *IEEE Transactions on Computers*, Vol. **C-18** (5), pp.401-409

Sherbaum, F., Delavaus, E. and Riggelesen, C. (2009). "Model Selection in Seismic Hazard Analysis: An Information-Theoretic Perspective", *Bulletin of the Seismological Society of America*, Vol. 99, No. 66, pp.3234-3246, 2009.

Takemura, M (1990). "Magnitude – Seismic Moment Relations for the Shallow Earthquakes in and around Japan", *Zishin* (*Journal of Seismological Society of Japan*), 2nd ser. Vol. 43, pp.257-265. (in Japanese)

U.S.NRC (2012). "Practical Implementation Guidelines for SSHAC Level 3 and 4 Hazard Studies", NUREG-2117, Rev.1