

ABSTRACT

BELCHER, MICHAEL JAMES. Examining Middle Graders' Experiences with a STEM Entrepreneurial-Based Curriculum and its Impact on Mathematics Learning: A Design Study. (Under the direction of Dr. Jere Confrey and Dr. Erin Krupa).

To increase interest and engagement in STEM, researchers have begun building curricula that situate STEM learning within entrepreneurial pitch competitions (see e.g. Newton et al., 2017). However, little is known about how these types of entrepreneurial-based STEM challenges support students' engagement or STEM learning, especially with respect to mathematics. This study sought to explore and understand the processes by which an entrepreneurial-based STEM challenge can support students' engagement and mathematics learning. A design research methodology (Cobb, Confrey, et al., 2003; Confrey, 2006) was used to explore students' experiences with the Design & Pitch (D&P) Challenges in STEM, a curricular framework that situates STEM learning within entrepreneurial pitch competitions, and specifically with the *Building Algorithms* challenge (Confrey, Krupa, & Belcher, 2019). Twenty-one students, grades 6 through 8, participated across two iterations of the design study: the first iteration (n=6) was conducted at the team's research office, and the second (n=15) was conducted in a suburban charter school. Qualitative data (video of students' daily work, team interviews, post-challenge focus groups, task-based interviews, and daily work samples) were collected and analyzed using a combination of *a priori* and open coding. The results of the study showed that the D&P challenge framework and the *Building Algorithms* challenge supported engagement by empowering students to choose authentic and relevant contexts and through the appeal and pressure of pitching to external judges. Additionally, through entrepreneurial processes (e.g. iterating and prototyping) and features unique to the challenge (e.g. the use of spreadsheets), students were supported to engage in a functional approach to algebraic expressions, as they

grappled with how to define and operationalize variables for use in a spreadsheet algorithm. This study demonstrated how entrepreneurship and entrepreneurial-based STEM challenges can support engagement and STEM learning, especially mathematics.

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Examining Middle Graders' Experiences with a STEM Entrepreneurial-Based Curriculum and
its Impact on Mathematics Learning: A Design Study

by
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DEDICATION

To Liz and Charlotte, for everything.

BIOGRAPHY

Michael James Belcher was born in Arlington, Virginia on July 18, 1983 to Jack and Nancy Belcher. The third of four children, Michael graduated from Wake Forest University in Winston-Salem, NC, in 2005, earning a Bachelor of Arts degree in mathematics, with minors in secondary education and physics. After graduating from Wake Forest, Michael taught high school mathematics for three years in Chapel Hill-Carrboro City Schools. During his time teaching in Chapel Hill, Michael met the two loves of his life: his wife Liz and his dog Lucy. In 2008, he enrolled at Teachers College, Columbia University. After earning his Master's degree in 2009 from Teachers College, Michael returned to North Carolina and accepted a position in Durham Public Schools teaching high school mathematics in a project-based learning school. During those three years, Michael and Liz married and added their imaginative, brilliant, and loquacious daughter Charlotte to their team. Michael and Liz were, and continue to be, bewildered by the source of Charlotte's glowing and extroverted personality.

In 2012, Michael began working as a curriculum developer at Amplify Learning in Durham. He worked at Amplify for three years on a team led by Dr. Jere Confrey, developing a digital middle grades mathematics curriculum. In 2016, Michael was accepted to the Learning and Teaching in STEM doctoral program at North Carolina State University.

Michael currently lives in Durham with his wife Liz, their daughter Charlotte, and their two pit bulls, Iggy and Smudge. Sadly, Lucy, who stuck with Michael for 13 years through many highs and lows, was unable to make it through to the end of the doctoral program and passed away in 2018. She is missed.

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CHAPTER 1: INTRODUCTION

Innovations in science, technology, engineering, and mathematics (STEM) are drastically altering the career landscape, with many jobs being eliminated or becoming obsolete due to automation. As a result, the skills and knowledge required for today's jobs are likely to be vastly different from those necessary for the jobs current K-12 students will be applying for when they enter the workforce (OECD, 2018). Recognizing this shifting landscape, STEM education researchers and educators have long argued that changes to STEM education are necessary to prepare students to fill the jobs of tomorrow, even though it is not clear what those jobs will be or what specific skills they will require.

Instead of training students to *fill* the jobs of tomorrow, some argue that STEM educators should be preparing students to *create* the jobs of tomorrow. This sentiment is highlighted in a 2018 publication by the Organization for Economic Cooperation and Development (OECD).

To prepare for 2030, people should be able to think creatively, develop new products and services, new jobs, new processes and methods, new ways of thinking and living, new enterprises, new sectors, new business models and new social models. Increasingly, innovation springs not from individuals thinking and working alone, but through cooperation and collaboration with others to draw on existing knowledge to create new knowledge (p. 6).

One approach that places job creation at its center and emphasizes similar characteristics and ways of thinking is entrepreneurship (Filion, 1994; venturelab.org). Entrepreneurship emphasizes using innovative and business-oriented thinking to invent solutions to messy, ill-defined problems that create value for users (or customers).

But entrepreneurship alone will not be enough to prepare students to invent solutions to these challenging and pressing problems. Students will also need disciplinary knowledge, such as science, technology, engineering, and mathematics (OECD, 2018). Thus, new curricular approaches are needed that authentically support students to learn to think and act like entrepreneurs, while also developing a deep understanding of disciplinary content, especially in STEM.

Many students perceive STEM, especially mathematics, as disconnected from their lived experiences or future aspirations and, as a result, are disengaged and disinterested in the mathematics classroom (Boaler, 2002; Bouillion & Gomez, 2001; Gutstein, 2003; Hill, Corbett, & St. Rose, 2010; Samuelson & Litzler, 2016). Project-based learning (PBL) and design-based learning (DBL) are two curricular frameworks that, when used in STEM, 1) highlight the relevance of the STEM content taught in school; 2) engage students in creative and authentic problem solving activities representative of those carried out by STEM professionals; and 3) provide a framework that can support the learning of specific STEM content (Apedoe & Schunn, 2013; Doppelt, Mehalik, Schunn, Silk, & Krysinski, 2008; Krajcik & Blumenfeld, 2006). More recently, attention is being paid to entrepreneurial-based learning (EBL), which teaches students entrepreneurial skills and processes (Lackeus, 2015). Given the compelling and motivating nature of entrepreneurship and the focus of PBL and DBL on engaging and supporting students to learn specific content, it is likely that a framework that combines features of EBL, PBL, and DBL can create an educational environment that is engaging for students and promotes interest and learning in STEM.

While some curricular approaches involve situating the teaching and learning of STEM within entrepreneurial pitch competitions (see e.g. Newton et al., 2018), few of these approaches

describe alignment to specific mathematics content standards or an explicit curricular framework that would support their integration within a STEM curriculum. Furthermore, little is known about the mechanisms by which a curricular approach that is built around entrepreneurship and entrepreneurial pitch competitions can support the learning of specific mathematics concepts.

Statement of the Problem

Students are disengaged in the mathematics classroom (Boaler, 2002) and this lack of engagement can be detrimental to their learning (Fredricks, Blumenfeld, & Paris, 2004). To increase students' engagement in STEM, researchers have begun situating STEM activities within entrepreneurial contexts (Moore, Newton, & Baskett, 2017). However, few of these approaches describe alignment to specific mathematics content standards or include an explicit curricular framework that would support their integration within a STEM or mathematics curriculum. Thus, new approaches to curricula are needed that leverage entrepreneurship to generate excitement and engagement in the mathematics classroom and that promote and support the learning of specific and standards-aligned mathematics concepts.

Purpose Statement

To address the above stated problem, a team from North Carolina State University, led by Dr. Jere Confrey and Dr. Erin Krupa, partnered with JASON Learning to develop the Design & Pitch (D&P) Challenges in STEM curricular framework (Confrey, Krupa, & Belcher, 2019). This novel curricular framework aims to increase students' interest and engagement in mathematics, while also supporting mathematics learning. It includes nine challenges which situate STEM learning within entrepreneurial pitch competitions. This study draws from Dr. Confrey's and Dr. Krupa's larger D&P Challenges in STEM project (NSF Grant No. 1759167) and is a design study exploring students' experiences with one D&P challenge. Specifically, the

study reported on in this paper explored whether and how the *Building Algorithms* challenge, a STEM entrepreneurial challenge, supports engagement in the mathematics classroom and the learning of targeted mathematics content. In this challenge, students were tasked with building a spreadsheet algorithm that rated or ranked something that they found important and that could be the foundation of a business.

Given the novelty of the D&P Challenges in STEM as a curricular framework, there is a need for a “greater understanding of the learning ecology” it creates (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003, p. 9). This study examined how the D&P Challenges in STEM framework and the specific components of the *Building Algorithms* challenge interact to promote engagement and support the learning of algebraic expressions. Thus, the goal of this study is to inform the design of future D&P challenges and their eventual implementation by teachers. The D&P challenge selected for this study (the *Building Algorithms* challenge) was built to leverage a functional approach (Blanton, Brizuela, Gardiner, Sawrey, & Newman-Owens, 2015) to the teaching and learning of algebraic expressions, within the context of building, testing, and refining a spreadsheet algorithm.

The following research questions were explored in this study:

1. How and to what extent does the *Building Algorithms* challenge promote and support students’ engagement in a mathematics classroom setting?
2. How and to what extent does the *Building Algorithms* challenge promote and support the learning of specific mathematics concepts?
 - a. How and to what extent do students engage with algebraic expressions as they research, develop, describe, and justify their entrepreneurial solutions to the *Building Algorithms* challenge?

- b. How and to what extent does the structure of the *Building Algorithms* challenge support mathematics learning?

Structure of the Remaining Chapters

In the chapters that follow, I start by presenting a review of the literature relating to the design of the intervention and to the constructs that were explored in the study. First, I define and present the research relating to student engagement and the rationale for including it as a target outcome of this study. Next, I discuss the research relating to the three instructional frameworks that formed the foundation of the Design & Pitch Challenges in STEM, making them likely to support engagement and STEM learning: entrepreneurial-based learning (EBL), design-based learning (DBL), and project-based learning (PBL). Finally, I discuss the research that informed the design of the *Building Algorithms* challenge, focusing specifically on how specific components of the challenge could support a functional approach to learning to think and reason algebraically. In Chapter 3, I discuss the methods that were used to explore the two research questions and the design of the present study, before presenting and discussing the results of the study in chapters 4 and 5 respectively. I conclude this paper by discussing the implications and limitations of the study and presenting a practical guide intended to support teachers to implement the Design & Pitch Challenges in STEM with their students.

CHAPTER 2: LITERATURE REVIEW

The Design & Pitch Challenges in STEM curricular framework incorporates features of project-based learning (PBL), design-based learning (DBL), and entrepreneurial-based learning (EBL) to support middle grades students' engagement in a mathematics classroom setting and their learning of specific mathematics content. In the sections that follow, I start by briefly defining engagement and discussing the research justifying its inclusion as a target outcome of D&P Challenges in STEM. Then, I discuss the theory and research supporting the incorporation of specific features of EBL, PBL, and DBL in the D&P Challenges in STEM. Because the primary focus of this study was on how entrepreneurship could support engagement and mathematics learning, I chose to begin the discussion of these three curricular frameworks with an overview of EBL, before discussing PBL and DBL. After discussing the research on these broad approaches to curriculum, I review the literature on mathematics learning that is specific to the challenge that is the focus of this study. Specifically, I discuss how spreadsheets and algorithms can support a functional approach to learning to reason algebraically.

Engagement

Researchers have long recognized the foundational role of engagement in students' learning. Engagement refers to the behaviors, emotions, and cognitive processes that indicate the extent of students' investment in their learning. According to Marks (2000), engagement represents "...the attention, interest, investment, and effort students expend in the work of learning...[It] implies both affective and behavioral participation in the learning experience" (p. 154-155). This definition alludes to a distinction that is often made in research on student engagement between the behavioral, affective (or emotional), and cognitive forms of engagement. Behavioral engagement generally refers to students' conduct in a school setting and

in relation to learning activities. It can relate to following directions, paying attention, “and contributing to a class discussion” (Fredericks et al., 2004, p. 62). Emotional engagement refers to students’ “affective reactions in the classroom, including interest, boredom, happiness, sadness, and anxiety” (Fredericks et al., 2004, p. 63). Finally, cognitive engagement has been defined as “the deliberate, task-specific thinking that a student undertakes while participating in a classroom activity” (Helme & Clarke, 2001, p. 136). More recently, social engagement was added as a fourth type of engagement. Social engagement relates to students’ interaction with their classmates around an academic task (Rimm-Kaufman, Baroody, Larsen, Curby, & Arby, 2014).

Although these four types of engagement differ in their indicators (e.g. the presence of behavioral engagement is often identified using student behaviors, such as participation and persistence, while emotional engagement is identified using students’ affect), they are deeply interconnected, with each type reinforcing the other. “In reality, [behavior, emotion, and cognition] are dynamically interrelated within the individual; they are not isolated processes” (Fredericks et al., 2004, p. 61). For example, students who are willing to “exert the effort necessary to comprehend complex ideas” (cognitive engagement; Fredericks et al., 2004, p. 60) are also likely to demonstrate persistence (behavioral engagement), report a positive affect (emotional engagement), and engage in meaningful social interactions (social engagement) relative to their learning of those ideas. Given the interconnectedness of these four types of engagement, some researchers have opted to use the more general construct, “engagement,” in place of the finer grained behavioral, emotional, cognitive, or social engagement. Regardless of the term researchers use or the granularity of their definitions of engagement, most researchers agree that it represents the degree to which 1) students are invested in their school or school

work and 2) are willing to work beyond the minimum requirements for learning, performance, or participation. When applied to curriculum, students' investment and willingness to work beyond the minimum requirements are defined in relation to curricular tasks or activities (Lamborn, Newmann, & Wehlage, 1992).

Students who are engaged in school are more likely to experience greater academic success and are less likely to drop out of school (Fredricks et al., 2004; Sciarra & Seirup, 2008). Actively engaged students “have the chance to develop interest in and positive attitudes toward STEM topics, as well as improved perception of their ability to participate in STEM” (NSTC, 2013). In addition to supporting academic performance (Fredericks et al., 2004; Reyes, Brackett, Rivers, White, & Salovey, 2012; Sciarra & Sierrup, 2008), engagement creates a classroom environment in which students are willing to invest in a lesson or task and, thus, are better able to fully experience the benefits of that lesson or task (Lamborn et al., 1992). “Ultimately, although engagement might begin with liking or participating, it can result in commitment or investment and thus may be a key to diminishing student apathy and enhancing learning (Helme & Clarke, 2001, p. 82). When students are engaged, they are more likely to persist, self-regulate, and spend more time on task while learning challenging content (Helme & Clarke, 2001; Reschly & Christenson, 2012). Students are also more invested in their learning and are “...willing to exert the effort necessary to comprehend complex ideas and master difficult skills” (Fredericks et al., 2004).

Engagement can be improved under the right conditions, such as an autonomous, supportive, collaborative, and cognitively demanding learning environment (Lamborn et al., 1992). In a mathematics classroom, the design of authentic curricular tasks could be particularly effective for increasing students' engagement (Helme & Clarke, 2001; Marks, 2000), as students

often perceive the subject as disconnected from their cultures, lived experiences, and future aspirations (Boaler, 2002; Gutstein, 2003). “Authentic academic work involves students intellectually in a process of disciplined inquiry to solve meaningful problems, problems with relevance in the world beyond the classroom and of interest to them personally” (Marks, 2000, p. 158). Authentic tasks can help students connect the content they are learning in school to situations that they find important and worth pursuing. These authentic tasks help to enhance students’ perceptions of the utility of the content they are learning (Reschley & Christenson, 2012) and establish a clear and tangible purpose for that content (Blumenfeld, Kempler, & Krajcik, 2006).

Summary

Students are generally disengaged in school, which negatively affects their achievement and learning, especially with respect to STEM. Researchers have identified strategies for improving students’ engagement at the classroom level, including creating environments in which students collaboratively grapple with authentic and cognitively complex tasks that promote autonomy (Lamborn et al., 1992). In the following sections, I discuss three instructional frameworks that could support such a classroom environment and increase students’ engagement in a mathematics classroom.

Entrepreneurial-Based Learning

In the past few decades, the U.S. and global economies have undergone considerable shifts, affecting both the types of jobs that are available and in demand, and the skills and knowledge people need to fill those jobs (Aspen Institute, 2016; NSTC, 2013; OECD, 2018). Recently, STEM education researchers have begun to recognize the importance of preparing students to create their own career opportunities through entrepreneurship (OECD, 2018). In

addition to providing a flexible career connection, entrepreneurship and entrepreneurial competitions have become increasingly popular. For example, entrepreneurs like Oprah Winfrey and Elon Musk have become objects of public fascination and shows like SharkTank in which contestants compete in entrepreneurial pitch competitions are experiencing sustained success. If leveraged appropriately, entrepreneurship has the potential to capture students' interest, while also equipping them with essential skills for a rapidly changing economic landscape.

Attempting to capitalize on the rising popularity of entrepreneurship and recognizing the importance of teaching students to think innovatively to solve open-ended problems, researchers have begun exploring how to leverage entrepreneurship in STEM education (see e.g. Moore et al., 2017). Leveraging entrepreneurial contexts to teach content from other disciplines is a type of entrepreneurial-based learning (EBL; Lackeus, 2015; Yuste, Diez, Cotano, Fernandez, & de Diego Martinez, 2014). In this section, I describe EBL and discuss its potential for increasing student engagement in the mathematics classroom. I then present a framework for entrepreneurship and discuss its potential for also supporting math learning.

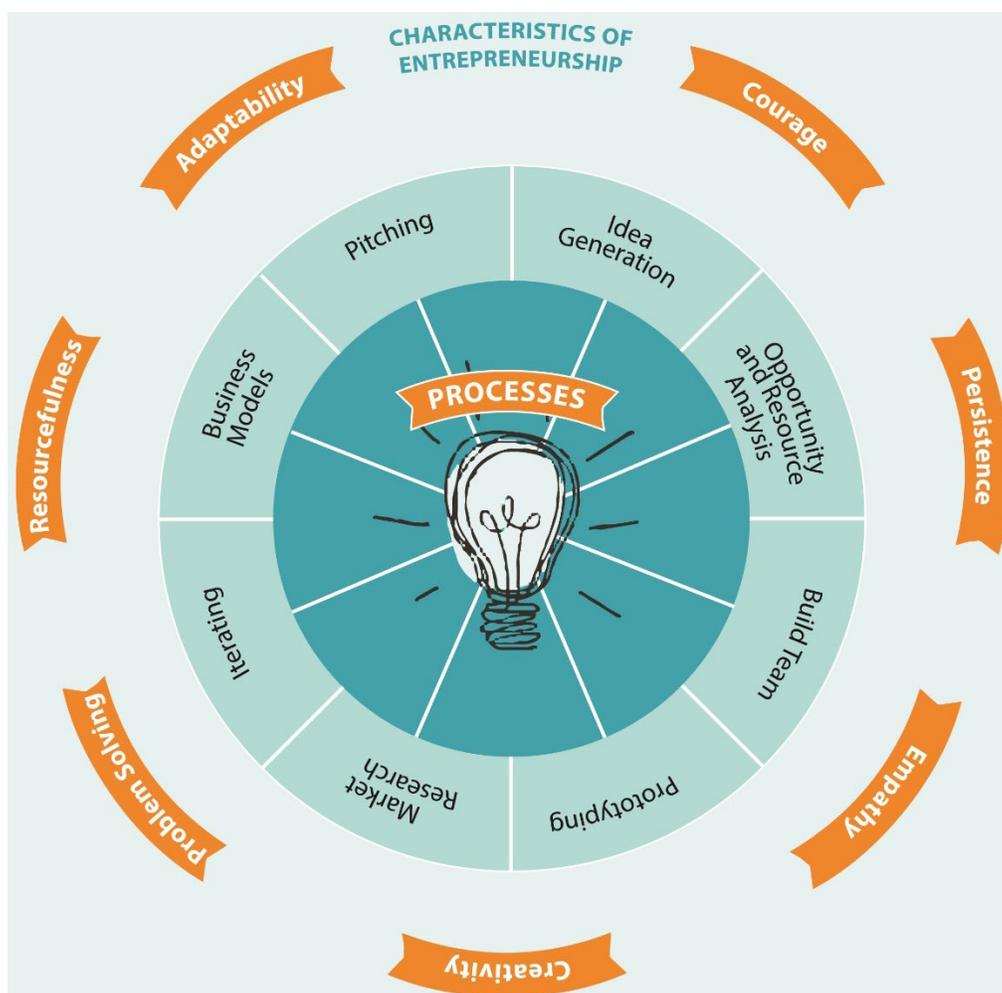
Researchers often classify approaches to EBL into three categories: “teaching *about* entrepreneurship...teaching *for* entrepreneurship...[and] teaching *through* entrepreneurship” (Lackeus, 2015, p. 10). Approaches to EBL that teach *about* entrepreneurship are typically confined to use in higher education and focus on giving students “...a general understanding of the phenomenon” (Lackeus, 2015, p. 10). These approaches explore the process of entrepreneurship from a largely research perspective. EBL programs that teach *for* entrepreneurship are “...occupationally oriented approach[es] aiming at giving budding entrepreneurs the requisite knowledge and skills” to be entrepreneurs (Lackeus, 2015, p. 10). In these approaches, entrepreneurial knowledge is of greatest concern and the development of

discipline-specific content knowledge is secondary. Finally, approaches that teach *through* entrepreneurship focus on “...connecting entrepreneurial characteristics, processes and experiences to [a] core subject” (Lackeus, 2015, p. 10). According to Moberg (2014), “the main idea of education *through* entrepreneurship...is to focus on the pupil’s interests and motivation as the basis for the learning process” (p. 515). In this approach, curricula situate the learning of non-entrepreneurial and disciplinary content knowledge within inherently motivational entrepreneurial contexts.

However, entrepreneurship is not merely a contextual driver when teaching *through* entrepreneurship. Even though the entrepreneurial context motivates the learning of the disciplinary content, the goal is also to support the learning of entrepreneurial skills and knowledge, such as how to develop business plans, identify business opportunities, leverage available resources, and understand and create value for customers (Filion, 1994; Lackeus, 2015, venturelab.org). At its foundation, the goal of EBL and teaching *through* entrepreneurship is to help students think and act like entrepreneurs to solve authentic and relevant problems. Entrepreneurs possess “the ability to tackle ambiguous problems and to think innovatively” (Bilen, Kisenwether, Rzasa, & Wise, 2005, p. 240). They identify problems lacking an adequate solution and innovate to develop new or better ways to solve those problems. Entrepreneurs are opportunity seekers (venturelab.org), viewing these problems as opportunities to produce innovations that offer value to specific customers or users (Borasi & Finnigan, 2010; Lackeus, 2015). They consistently express persistence and grit as they embrace and learn from failure as a step towards building new and better solutions (venturelab.org). As such, the problems and opportunities that interest entrepreneurs are often ill-defined and lacking of a clear solution path (Bilen et al., 2005).

Entrepreneurs must also possess the humility necessary to accurately assess their own skills and knowledge, recognize when they lack specific skills and knowledge relative to a problem, and actively seek out opportunities to learn those skills and that knowledge (T. Confrey-Maloney, personal communication, December 6, 2018). Entrepreneurs are action oriented. “They imagine visions of what they want to do. They then build a relational system that allows them to realize their visions” (Filion, 1994, p. 69). Finally, they recognize and act on potential business opportunities. This requires not only analyzing situations to identify opportunities, but also being able to investigate and define areas in which new designs can offer value to customers (Borasi & Finnigan, 2010). Entrepreneurs must have empathy for their customers and must constantly work to better understand customer needs through research and iteration (venturelab.org).

In summary, thinking and acting like an entrepreneur means seeing oneself as capable of enacting change through the design of economically viable solutions to messy and persistent problems. It comprises a range of characteristics and a variety of processes. Figure 1 represents a framework that synthesizes these characteristics (blue, outer circle) and processes (green, inner circle), and acts as a foundation for the Design & Pitch Challenges in STEM (Confrey et al., 2019).

Figure 1*D&P Entrepreneurial Framework*

Note. This figure shows the entrepreneurial framework for the Design & Pitch Challenges in STEM (Confrey et al., 2019)¹.

Given the recent appeal of entrepreneurship and EBL's emphasis on authentic activity (Passaro, Quinto, & Thomas, 2017), it is reasonable to expect that teaching STEM, specifically mathematics, through entrepreneurship could support improved engagement in a mathematics classroom setting. Researchers and private companies have begun designing EBL programs and

¹ This framework is based on the work of venturelab.org and was developed through a partnership between NCSU and JASON Learning.

entrepreneurial competitions for use in K-12 settings. Many of these programs, such as the Lemelson InvenTeams competition (<http://lemelson.mit.edu/inventeams>) and the Blue Ocean Competition (blueoceancompetition.org), are stand-alone competitions open to a wide range of age groups. Although the designers of several of these programs claim they can be used in STEM classrooms, the materials do not describe alignment to specific STEM content standards or learning goals. For these programs, entrepreneurship is the primary focus and STEM learning is secondary. Other approaches, such as Google's Science Fair or the SparkLab InventIt Challenge, leverage the competition aspect of entrepreneurship, but lack any other explicit entrepreneurial component.

One approach that closely resembles the D&P Challenges in STEM framework is the InVenture Challenges developed by Moore and colleagues (2017) at Georgia Tech University. The InVenture Challenges engage students in entrepreneurial pitch competitions and are designed to be used in K-12 STEM classrooms. Although they include support materials, such as lessons on the entrepreneurial process, and describe alignment to STEM practices, they are not aligned to specific STEM content standards. This is likely because students are free to pursue the development of any idea that is of interest, which makes it difficult for teachers to anticipate the STEM content with which students will engage. To date, no research has been published on whether participating in the InVenture Challenges promotes the learning of specific STEM content.

Lastly, because EBL does not require a specific instructional model for teaching through entrepreneurship, little is known about how to best implement EBL within a STEM or, more specifically, a mathematics classroom. To develop entrepreneurial skills and an entrepreneurial approach to problem-solving, students need opportunities to 1) engage in authentic

entrepreneurial tasks (Passaro et al., 2017), such as “...defining situations, imagining scenarios and deciding what is to be done while minimizing risks” (Filion, 1994, p. 70); 2) collaborate, argue, and debate ideas and processes with peers (Passaro et al., 2017); 3) reflect on their knowledge and skills relative to a specific entrepreneurial opportunity; and 4) consider ways of providing value to customers (Lackeus, 2015). Given these conditions, it is not surprising that researchers have suggested situating entrepreneurship learning in project-based or design-based settings. However, research is needed to better understand how an instructional framework that combines features of PBL and DBL can support students to think and act like entrepreneurs.

Summary

Given the novelty of the approach and the paucity of research on mathematics learning in relation to EBL, little is known about whether or how entrepreneurial approaches can support mathematics learning. While it seems likely that entrepreneurship and entrepreneurial pitch competitions could generate student engagement in a mathematics classroom, the evidence is lacking. Moreover, even in cases where student engagement is attributed to the entrepreneurial pitch competition (e.g. Moore et al., 2017), little is known about how entrepreneurship engages students, or what about the pitch competition, specifically, contributed to that engagement.

Project-Based Learning

One approach that aims to support students’ engagement in school is project-based learning (PBL): an instructional framework that motivates the learning of new content by situating instruction within active projects and relevant and authentic contexts. PBL may be ideally suited to generating engagement and interest in STEM by providing a structured way to allow students to recognize its utility in solving real world problems, including problems that could be related to their career aspirations. One way to increase student engagement in school

learning is to design activities that establish purpose for that learning (Ainley, Bills, & Wilson, 2005; Blumenfeld et al., 2006). In terms of PBL, this purpose takes the form of addressing the authentic and relevant driving question, the creation of the tangible artifact, and the final presentation.

PBL projects begin with a driving question that piques students' interest and focuses their attention on the relevant aspects of a task (Krajcik & Blumenfeld, 2006; Schneider, Krajcik, Marx, & Soloway, 2002; Thomas, 2000). Driving questions are "ill-defined" in that strategies for addressing questions are not immediately obvious and multiple pathways and points of entry exist for addressing them. Driving questions are not "solved," but rather are addressed through the creation of a final product, or artifact. Rather than the application of previously learned content, adequately addressing the driving question in a STEM project requires learning and applying new content (Barron et al., 1998; Thomas, 2000).

In the project titled, "Estuarine Ecosystems," learning about concepts relating to surface area (mathematics), water sampling in estuaries (science), the behavior of barnacles (science), and the engineering design process (engineering) was motivated by a need to design a system for water sampling in estuarine ecosystems (McCulloch & Ernst, 2012). The task of designing a solution to a relevant problem served to increase student engagement and to motivate learning new STEM content. Thus, the project supported both engagement and STEM content learning.

The project launch and corresponding driving question is more than just a superficial hook designed to momentarily occupy student interest (Blumenfeld et al., 2006). Rather, they support meaningful content learning and sustained student engagement. During the launch of the project, students participate in formulating the problem to be solved or task to be completed. They collectively identify the information they have and the information they need relative to the

task (Blumenfeld et al., 2006). In this way, students help to refine and focus the task into something that is manageable and that can be investigated (Krajcik & Blumenfeld, 2006; Rivet & Krajcik, 2004). By including students in the problem formulating process, PBL promotes meaningful cognitive engagement by supporting them to take ownership of their learning (Blumenfeld et al., 2006). Additionally, the driving question creates a need for students to pursue learning of new content (Barron et al., 1998; Kanter, 2009; Krajcik, McNeil, & Reiser, 2008). In this way, PBL is consistent with the entrepreneurial framework, promoting resourcefulness and persistence. Students must be able to self-assess, identify knowledge they are lacking, seek out opportunities to build that knowledge (resourcefulness), and work diligently to learn what is needed to complete the challenge (persistence).

Project work typically extends over multiple class sessions, as students work collaboratively in “communities of learners...includ[ing] peers, teachers, and members of the community” (Schneider et al., 2002, p. 3) to address the driving question through the creation of a tangible artifact and a final presentation. The creation of the final product provides a concrete outcome for students’ schoolwork (Blumenfeld et al., 2006), which can create an immediate purpose (Ainley et al., 2005) for learning that drives student engagement. Within a mathematics classroom, the creation of the final artifact provides feedback for students’ mathematical thinking as students test their ideas against tangible context criteria. Similarly, the final presentations offer students the opportunity to share their ideas and evaluate and critique the ideas of their peers (Krajcik & Blumenfeld, 2006; Schneider et al., 2002; Thomas, 2000). The presentation component is essential to PBL by providing a way to support the learning of STEM content, and by providing a structured opportunity for students to reflect on their process and the connections between the project task and the underlying STEM content (Krajcik et al., 2008).

While much research on project-based instruction has been conducted in relation to science or STEM, considerably less has been conducted in relation to mathematics. Even those studies exploring projects that claim to be interdisciplinary, give a superficial treatment to mathematics learning. For example, in a 2001 study, Bouillion and Gomez (2001) present the case of an interdisciplinary STEM project that addressed multiple STEM curricular goals, “including learning about pollution and ecosystems, developing a conservancy plan, collecting and analyzing data, [and] sharing findings” (Bouillion & Gomez, 2001, p. 884). The researchers reported that students demonstrated a heightened interest and greater efficacy in learning about specific concepts in science, specifically citing the role played by the project in “...making connections between school-based science and their community-based experiences and concerns” (p. 890). However, the authors did not explore whether or how the project supported mathematics learning. Thus, although the project increased students’ interest relative to the learning of STEM concepts through addressing the authentic and relevant goal of cleaning up a local river, little is reported about what mathematics students learned or how the project supported them to learn it.

Other studies of PBL, despite reporting quantitative data that demonstrate improved academic performance in science and math, do not report on whether those gains can be attributed to the design of the project or how the observed learning occurred. For example, in their study of a project-based science (PBS) curriculum, Schneider and colleagues (2002) compared scores on the National Assessment for Educational Progress (NAEP) for students who received PBS instruction with students who received an alternative form of instruction. The authors found that the students who engaged in PBS demonstrated higher performance than the control group (Schneider et al., 2002). However, they do not report details on students’

experiences participating in specific projects or make claims about how the curriculum supported higher NAEP performance for PBS students.

In a similar study of a PBS curriculum, Kanter and Konstantopoulos (2009) found positive effects of participation. Comparing gains in pre- and post-test scores on a test of comparable science content, the authors found that minority students who participated in PBS exhibited greater growth from pre-test to post-test than minority students who did not participate in PBS (Kanter & Konstantopoulos, 2009). Like the study by Schneider and colleagues (2002), this study did not explore, in depth, the specific projects or possible mechanisms for the observed assessment performance.

Other studies on PBS have attempted to more closely relate learning gains to project activity through pre and post-intervention achievement tests aligned to the targeted science content. In one example, Rivet and Krajcik (2004) explored the effects of a 6th grade PBS unit on students' understanding of "...balanced and unbalanced forces and the functioning of simple machines" (p. 683). These researchers found significant improvements on achievement test scores across all four years of the study.

Although many of these science-based or STEM-based studies of PBL claim alignment to math, it is often relegated to generic content, such as data analysis (see e.g. Bouillion & Gomez, 2001), or the standards for mathematical practice (CCSSO, 2010). In the few studies that do integrate meaningful mathematics, such as McCulloch and Ernst's (2014) study on the project "Estuarine Ecosystems," the researchers do not discuss the process through which the mathematics learning occurs. In this study, the authors report that students were engaged and that they learned math concepts, such as surface area, but they do not report on the extent of the

learning, the mechanism by which the project supported that learning, or how it developed over the course of the project.

Similarly, several studies report quantitative data on students' mathematics learning in relation to project-based instruction, but do not provide sufficient detail about the projects to allow for an analysis of how the PBL approach is or is not effective for certain students or groups of students. In a 2015 study, Han, Capraro, and Capraro investigated the effects of STEM PBL on students' mathematics and science achievement, as measured by results on a high stakes standardized assessment. The researchers found that, when compared to other ethnicities, "Hispanic students had a higher growth rate on mathematics tests for 3 years during the implementation of STEM PBL activities" (p. 1109). They argued that "STEM PBL provided more opportunities for Hispanic students to communicate with peers and teachers than would traditional lecture" (p. 1109). However, the researchers also reported that, when controlling for ethnicity, a student's mathematics performance was negatively correlated with their socioeconomic status (SES). Students labeled as "low SES" were found to have experienced diminished performance over the course of the study. These findings underscore that while measuring student learning using high stakes assessments can be informative, it provides an incomplete picture of how STEM PBL affects student learning. This is especially problematic given the negative findings for students labeled as low SES, whose scores dropped over the course of the study. Without knowing anything about the specific projects that were used in the study, the processes by which students addressed the driving questions, or the supports teachers provided to connect the project activities to the target STEM content, it is difficult to draw meaningful conclusions about the effectiveness of PBL for supporting mathematics learning. Moreover, although the researchers conjectured that the collaborative nature of PBL supported

students' mathematics learning, they did not include an analysis of how the features of PBL supported the conceptual development of specific mathematics content.

In another study, Boedeker and colleagues (2015) explored the effects of STEM PBL instruction on students' course taking patterns and performance on high stakes mathematics and science assessments. Although they found that female students experienced statistically significant gains in performance on both the math and science assessments, and experienced higher enrollment in non-required advanced STEM courses such as Precalculus, Physics, and Advanced Placement Calculus, they provided no discussion of the specific projects or the processes that influenced mathematics learning.

Both the Han and colleagues (2015) study and the Boedeker and colleagues (2015) study paint an incomplete picture of students' mathematics learning in STEM PBL and neither focuses specifically on STEM projects designed to promote mathematics learning. One study that does attempt to understand how PBL supports mathematics learning is a 2000 study by Stevens. In this study, Stevens (2000) reports on a project-based math (PBM) unit in which students were tasked with designing a housing structure for a research team in Antarctica. Instead of reporting quantitative data relating to students' mathematics learning, Stevens (2000) reports observational data describing mathematics problems that arose during students' work, whether they chose to engage with those problems, and how they made those decisions. For Stevens (2000), understanding how and why students selected mathematics problems to solve during a project-based unit was more important than quantifying mathematics learning gains. Additionally, Stevens suggests students' mathematics learning in a PBM setting should be evaluated according to "1) their success in finding, formulating, and solving emergent math problems, and 2) their success in making and using project-relevant...generalizations" (p. 137).

There are other curricular approaches in mathematics that are designed to support engagement and learning through the use of specific PBL features, such as the use of authentic, meaningful, and relevant contexts and promoting prolonged and sustained engagement with those contexts. For example, the company Mathalicious (www.mathalicious.com) developed a set of tasks that are aligned to mathematics content standards and are designed to support engagement by situating instruction in authentic and relevant contexts. In a study on the effect of Mathalicious tasks on students' mathematics achievement, Jackson and Makarin (2018) found that students who received instruction using the Mathalicious tasks outperformed a control group on the mathematics portion of the Virginia Standards of Learning state assessment. They also found that the Mathalicious students reported greater interest in mathematics and greater agreement with the view that it is applicable in real-world situations, both of which the researchers conclude could positively affect students' engagement. This study suggests that incorporating features of PBL, such as using real-world contexts and allowing for prolonged engagement with those contexts, could be an effective strategy for supporting learning and engagement in a mathematics classroom setting.

In another study, Boaler (1998) explored students' experiences and mathematics learning within a "reform-oriented" curriculum that, while not entirely project-based, emphasized the use of open-ended and context-based tasks similar to those used in PBL settings. Boaler (1998) found that students instructed using the reform-oriented curriculum out-performed students instructed using a traditional curriculum on open-ended tasks and performed at an equivalent level on closed tasks. Boaler (1998) attributed these results, in part, to the reform-oriented curriculum's use of open-ended tasks. Boaler also found that students instructed using the

reform-oriented curriculum were more likely to recognize a connection between the math they were learning in school and its use in an applied, out-of-school setting.

Finally, Terry (2011) reports on students' experiences with a mathematics task built around a real-world context that was both authentic and relevant to his students. In Terry's (2011) study, African American high school students used data analysis and ratio reasoning to craft counterstories (a tangible artifact) to a commonly and misrepresented statistic about higher education and the rates of incarceration among African American males. While the author does not explicitly classify this work as PBL, the shared components between this activity and PBL make it worth considering in the development of the D&P Challenges in STEM. For example, the use of an authentic and relevant issue created the need for specific curricular mathematics concepts and drove the learning of that content. Terry (2011) reports high levels of student engagement and interest among students as they grappled with a rich, complex, and meaningful mathematics task.

The studies by Terry (2011), Boaler (1998), and Jackson and Makarin (2018) are three examples that demonstrate how mathematics curricula that incorporate features of PBL can have a positive effect on students' engagement and mathematics learning in a mathematics classroom setting. What is lacking from these studies, however, is an in-depth analysis of how these PBL features, such as the use of real-world contexts and open-ended tasks, support the conceptual development of specific mathematics topics. Research is needed to better understand how student thinking develops as they engage with challenges that incorporate features of PBL and how that development is supported by those challenges.

Summary

Project-based learning is an instructional approach designed to increase students' engagement by using authentic contexts and practices to motivate the learning of new content. Projects provide students with a clear, tangible, and immediate purpose for their learning through the creation of artifacts and the delivery of presentations. Furthermore, by encouraging collaborative group work, supporting self-assessment and resourcefulness, and offering opportunities to share and reflect on learning, PBL has the potential to support meaningful mathematics learning in addition to entrepreneurial processes and characteristics. Thus, these features of PBL have been incorporated into the D&P Challenges in STEM framework, with the goal of increasing students' engagement and interest in STEM, while also supporting the learning of specific mathematics content. However, much of the research on project-based learning has been conducted in science or STEM with projects that underemphasize the role of mathematics. Finally, no research has been conducted on students' mathematics learning resulting from participation in a STEM PBL unit situated within an entrepreneurial context.

Design-Based Learning

Like PBL, design-based learning (DBL) promotes student engagement by providing an immediate purpose (Ainley et al., 2005) for student learning as they engage in authentic practices characteristic of STEM careers. In DBL the student takes on the role of an engineer, using an authentic practice, design thinking (Dym, Agogino, Eris, Frey, & Leifer, 2005), to design or create a physical artifact (or a two-dimensional sketch of a physical artifact), that solves an "ill-defined," authentic, and relevant problem (Fortus, Dershimer, Krajcik, Marx, & Mamlok-Naaman, 2004; Wendell & Rogers, 2013). The criteria for the culminating artifact are based on the needs of a user or group of users (Dym et al., 2005). Thus, like PBL, DBL supports student

engagement and interest in STEM through establishing an immediate purpose for learning STEM content (Kolodner, 2002; Mehalik, Doppelt, & Schunn, 2008; Wendell & Rogers, 2013).

The development of a design solution, in response to a design challenge, requires students to “use and extend their current content knowledge (often science or math)” (Apedoe & Schunn, 2013, p. 774). As students work to generate, test, and revise their design ideas, they identify gaps in their understanding (relative to the content, context, and original designs) that need filling. “[T]he need to make one’s design ideas work provides opportunities and reasons for students to identify incomplete and poor conceptions of science content and to debug those conceptions” (Kolodner, 2002, p. 2). Thus, the design process serves to not only motivate the learning of new content, but also to focus the learning on the specific needs for the design, clearly demonstrating the connection between curricular content and authentic situations (Mehalik et al., 2008).

In a study by Penner and colleagues (1998), students were tasked with designing a working model of an elbow. After creating initial designs that visually resembled human arms but that behaved differently (e.g. bending backwards), the whole class conducted structured experiments to better understand the behavior of the human elbow and the mechanics of simple machines. The design challenge and the process of prototype, test, experiment, and refine created the need to learn about specific science content: simple machines. Likewise, in a study by Kolodner (2002), students were tasked with building a parachute out of coffee filters. Students engaged in a continual process of planning, building, and testing coffee filter parachutes, the results of which informed and motivated guided inquiry activities exploring force, drag, and the mechanics of parachutes (Kolodner, 2002).

A typical DBL unit starts with the launching of the design challenge that will structure students' work for the duration of the unit. During the launch, students (either individually, in groups, or as a class) engage in a process of “problem scoping” or “problem formulating” in which they begin “clarifying and restating the goal of the problem, identifying constraints to be met in problem solution, exploring feasibility issues, drawing on related context to add meaning, experimenting with materials, and establishing collaborative group work” (English & King, 2015, p. 4). This problem scoping stage focuses students' attention on the specific criteria that will determine success of a design. It helps orient the class, supports students to attend to the relationship between the problem and any proposed solutions (Penner, Lehrer, & Schauble, 1998), and can initiate student action, engagement, and interest through their familiarity with the context (Fortus et al., 2004). Additionally, by leveraging familiarity, DBL contexts support multiple points of entry to engage with the challenge.

After launching the design challenge and establishing initial conditions and criteria against which designs will be evaluated, students engage in an iterative process of gathering and analyzing information (Fortus et al., 2004), generating possible solutions (Apedoe & Schunn, 2013), building, testing, presenting, and refining prototypes (Doppelt et al., 2008; Kolodner, 2002; Penner et al., 1998; Razzouk & Schute, 2012), and reflecting on their process (Fortus et al., 2004; Kolodner, 2002). Throughout the challenge, students continually make their thinking public and expose it to critique (Penner et al., 1998), critique supports students to reflect on the connection between the design task and the underlying STEM content targeted by the challenge. At the end of this process, students produce a final design artifact, which provides a tangible purpose for their work (Ainley et al., 2005).

How students decide on and make sense of design criteria is another important component of the DBL process. As is characteristic of an ill-defined and open-ended problem, design challenges lack an objective determination of what constitutes a “good” solution (Fortus et al., 2004). The process of exposing prototypes and design decisions to critique is central to defining an effective solution for a specific user in a target environment. Consistent with entrepreneurship, although “...judgment of designs is based on how well and broadly they work, designs are conceived in conjunction with clients, and communication of ideas is critical to achieving a solution that fulfills the client’s needs” (Kolodner, 2002, p. 4). In terms of entrepreneurship, the client is the target customer and students must demonstrate empathy as they define criteria for success (venturelab.org). Furthermore, when prototypes and decision-making are made public, students can collectively decide whether the agreed upon criteria have been met, whether those criteria remain valid, and whether the decision-making that led to valuing one criterion over another is justified (Apedoe & Schunn, 2013). In this way, DBL engages students in developing a shared and “functional” (Burkhardt, 2007) understanding of the STEM concepts targeted by the challenge.

Studies on the use of DBL in STEM classrooms have shown some evidence of its effectiveness for supporting the learning of specific content and increasing students’ engagement in STEM. In one study (Wendell and Rogers, 2013), researchers compared DBL instruction with what they called the district’s “status quo” curriculum, using pre and post-intervention assessments that used a combination of multiple choice and open-ended items to assess students’ understanding of the science content targeted in the DBL unit. The researchers supplemented these data with surveys measuring students’ attitudes about science. They found that students experienced statistically significant learning gains in science content within the four domains

targeted by the challenge: animals, material properties, simple machines, and sound (Wendell & Rogers, 2013). Although the researchers did not observe differences in students' attitudes towards science, both groups of students reported similarly positive attitudes. The researchers attribute this finding to the presence of similar design-based activities in the "status quo" (Wendell & Rogers, 2013, p. 516) curriculum that may have impacted students' attitudes toward science. Thus, given students' engagement in the DBL unit and their observed learning gains relative to the targeted science content, this study suggests that DBL can support meaningful content learning while also maintaining high levels of interest in science.

Fortus and colleagues (2004), in their study of a design-based science (DBS) unit, incorporated knowledge tests to measure gains in students' understanding of science. These knowledge tests included multiple choice and open-ended items that were aligned to the unit content. Analysis revealed that both groups of students labeled as "low achievers" and those labeled as "high achievers" based on pretest scores experienced statistically significant gains from pretest to posttest. To gain a more complete picture of students' learning gains during the unit, Fortus and colleagues (2004) supplemented the knowledge test data with analyses of students' artifacts using researcher-developed rubrics. The rubrics were used to determine "...which concepts were considered in creating and testing the artifacts, and whether the conclusion resulting from the use of these science concepts were correct" (Fortus et al., 2004, p. 1090). The researchers found evidence that students were using the targeted science content in their solutions, suggesting DBL could be used to provide an engaging learning environment that results in the meaningful use and learning of targeted science content.

Doppelt and colleagues (2008) used a similar approach in their study of a DBS unit centered on the design of alarm systems. The researchers evaluated the effectiveness of the DBS

unit using pre and post knowledge tests aligned to the science content of the design challenge; teacher and peer assessments of students' oral presentations; and analyses of students' design challenge portfolios. Almost all subgroups in the study experienced statistically significant gains in achievement from pretest to posttest, except the group of students labeled as "low achievers." However, from their analysis of students' portfolios and oral presentations, the researchers found that, contrary to the results of the knowledge tests, students in the low achiever class performed better than the high achiever class on the oral presentations. Additionally, Doppelt and colleagues (2008) found that teachers reported greater learning and engagement in their lower level classes than in their higher-level classes. Combined with the observed learning gains, this study provides further evidence that DBL, in a science setting, can be engaging for all students while supporting the learning of specific science content.

Summary

Design-based learning has been shown to support student engagement in science (Doppelt et al., 2008; Fortus et al., 2004) and the learning of specific and targeted content. DBL uses the creation of a tangible artifact to add purpose to students' science learning, which can increase students' engagement in science, and motivate the learning of new science content (Kolodner, 2002). Additionally, its emphasis on collaboration and building, testing, communicating, and refining prototypes can support the learning of STEM content (Kolodner, 2002; Penner et al., 1998). The D&P Challenges in STEM framework leverages these features with the intent of creating an educational environment that promotes students' engagement in mathematics, while supporting the learning of specific mathematics content. However, research on DBL has largely been confined to science, technology, or engineering classrooms. Little attention has been given to the use of DBL in mathematics classrooms and within entrepreneurial

settings. Research is needed on whether select components of DBL, when combined with components of PBL and EBL can support student engagement, interest, and learning in a mathematics classroom.

Research on Learning to Reason Algebraically

This study focused on students' experiences and mathematics learning while participating in the *Building Algorithms* challenge. Specifically, through the task of building a spreadsheet algorithm, the challenge was designed to support students to learn about algebraic expressions through functional thinking. "Functional thinking entails (a) generalizing relationships between covarying quantities; (b) representing and justifying these relationships in multiple ways using natural language, variable notation, tables, and graphs; and (c) reasoning fluently with these generalized representations in order to understand and predict functional behavior" (Blanton et al., 2015, p. 512). The task of building a functioning algorithm engages students in these three criteria.

According to Knuth (1974), "an algorithm is a precisely-defined sequence of rules telling how to produce specified output information from given input information in a finite number of steps" (Knuth, 1974, p. 323). For students to build a functioning algorithm, they must consider and construct a generalized relationship (process) between covarying quantities (input information and output information). Thus, the *Building Algorithms* challenge, through the task of designing an algorithm-based business could be a powerful approach to supporting students' functional thinking and their learning of algebraic expressions.

Two types of algorithms that are the foundation of many businesses and that could be accessible, appealing, and familiar to students are ranking and rating algorithms. A rating algorithm is a process that uses input information to assign a numerical and evaluative score

(rating) to an item, based on some criteria defined by the rater. Two common types of input information used in a rating algorithm are evaluative measures (e.g. user-generated or expert ratings) and descriptive measures (e.g. quantitative data used to summarize a characteristic of an item). Using evaluative measures as input information often involves asking people or users to evaluate an item on a quantitative scale based on established criteria and relating to a target construct. For example, many websites provide user-generated product ratings by asking people to evaluate products on a scale from one to five based on their opinions of the quality of the product. In these cases, the criteria that define “quality” is left to the user. The algorithm is the process by which all user ratings of a product are combined to generate a single rating for the product.

Using descriptive measures as input information involves identifying data that are indicative of the target construct. For example, during the COVID-19 pandemic, the company Unacast (<https://www.unacast.com/covid19/social-distancing-scoreboard>) developed a rating algorithm that would assign grades to counties based on the extent to which residents were practicing social distancing. In the first iteration of their algorithm, Unacast used cellphone data to calculate the change in average distance traveled (a descriptive measure) to assign letter grades to counties. Counties were assigned a letter grade of “A” if the change in average total distance traveled by residents decreased by greater than 70%; “B” if the decrease was between 55% and 70%; “C” if the decrease was between 40% and 55%; “D” if the decrease was between 25% and 40%; and “F” if the decrease was less than 25%. In later iterations they introduced two new descriptive measures: “percent change in non-essential visitation” and “decrease in human encounters.” For Unacast’s algorithm, these three descriptive measures were used as the input information and the final social distancing grades represented the output information. The

algorithm was the process by which the three descriptive measures were combined to generate a final social distancing grade for each county.

An important characteristic of rating algorithms is the use of equal interval scales. That is, the difference in quality (with respect to some construct) between an item assigned a rating of 5 and one assigned a rating of 4 is the same as the difference in quality between an item assigned a rating of 4 and one assigned a rating of 3 (Warner, 2008). This equal interval scale allows users to determine 1) the overall quality of an item with respect to some construct (e.g. product quality or how well a location is social distancing) and 2) how much better one item is than another (Warner, 2008).

Alternatively, a ranking algorithm is a process that uses input information to assign numerical values that indicate an item's position in a list ordered according to an underlying construct and based on criteria defined by users or the algorithm builder. For example, weekly college basketball rankings are generated using a ranking algorithm. College basketball coaches assign numerical rankings to teams based on criteria they define and according to the underlying construct, "which team is better?" The ranking algorithm is the process by which all coaches' lists are combined to create an aggregate and ordered list of the top 25 teams. In many cases, these aggregated and ordered lists rely on a system in which points are assigned to teams according to the rankings they received. For example, in the women's college basketball coaches' poll, teams are assigned 25 points for every coach who ranked them 1st, 24 points for every coach who ranked them 2nd, and so on. Teams are then ranked according to the total number of points they received (<https://www.usatoday.com/sports/ncaaw/polls/coaches-poll/>).

Rankings are useful for directly comparing items within a set (e.g. college basketball teams). However, because rankings rely on ordinal measurement, they are less useful for

evaluating those items based on some external criteria. In ordinal measurement, “numbers represent ranks, but the differences between scores do not necessarily correspond to equal intervals with respect to any underlying characteristics” (Warner, 2008, p. 35). For example, college basketball rankings are useful for deciding if one team is better than another, but cannot determine how much better one team is than another or even how good the best team is. In other words, the gap between the 1st place team and the 2nd place team might be considerably smaller than the gap between the 2nd place team and the 3rd place team. Additionally, the 1st place team one year might be much worse than the 1st place team in another year. Thus, although ranking algorithms are useful for direct comparison, they are limited in their capacity to inform an overall evaluation of an item being ranked or of the gap between ranked items. Nevertheless, ranking algorithms provide a rich and authentic context in which students’ can leverage their functional thinking and engage in rich mathematical reasoning.

The task of building, testing, and refining a rating or ranking algorithm has the potential to support a functional approach to the teaching and learning of algebraic expressions. A functional approach, which has been shown to be accessible early elementary-aged students (Blanton et al., 2015; Carraher, Schliemann, Brizuela, & Earnest, 2006), views algebra as “...generalized arithmetic of numbers and quantities” (Carraher et al., 2006, p. 88). The functional perspective holds that moving from arithmetic to algebra requires a move from considering operations on particular numbers to operations on sets of numbers (Carraher et al., 2006). In terms of algorithms, this means recognizing that an algorithm not only describes a sequence of arithmetic operations to calculate a rating or ranking for one item in a set, but that it describes the same sequence of operations for all other items in the set. Furthermore, the algorithm uses variables that represent quantified characteristics for any item. Thus,

understanding algorithms requires an understanding of variable as representing a range of possible values and not just one specific value, which, while essential for algebraic reasoning, can be challenging for many middle-grades students (Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005).

Rojano (1996) highlights the existence of a "didactic cut along the evolutionary line from the child's arithmetic to algebraic thought produced by the difficulty in operating on the unknown quantities" (p. 139). Rojano (1996) argues that spreadsheets can be a powerful tool for helping students overcome this didactic cut by connecting the specific in an arithmetic expression to the unknown in an algebraic (Rojano, 1996; Filloy, Puig, & Rojano, 2008). In a spreadsheet, writing a formula (algorithm) requires students to write a symbolic expression in which values are represented using the column letter and row number of the cell in which the value is located. This naming convention connects the variables in a formula to a specific value in the arithmetic expression. In this way, the spreadsheet offers an intermediate step between the specific and unknown (Rojano, 1996). Applying those formulas across multiple cells or rows supports to generalize those specific values to the quantities those values represent (Rojano, 1996; Filloy et al., 2008). In other words, the spreadsheet helps students develop the understanding that a variable in an algebraic expression can represent a range of possible values (Ainley, 2005; Carraher et al., 2005;) and that those values carry contextual meanings that should be considered (Filloy et al., 2008).

Lastly, by providing immediate and meaningful feedback relative to a tangible purpose (successfully programming the spreadsheet), the spreadsheet encourages students to iterate on their algorithms, which can support learning. Students can automatically test an algorithm and assess whether it returns the expected results. This affordance of spreadsheets allows students to

manipulate their algorithms and explore how changing conditions in the expression (e.g. weights/coefficients, operations, number of variables) changes the output, supporting them to interpret and understand the structure of an algebraic expression (Filloy, Puig, & Rojano, 2008).

Summary

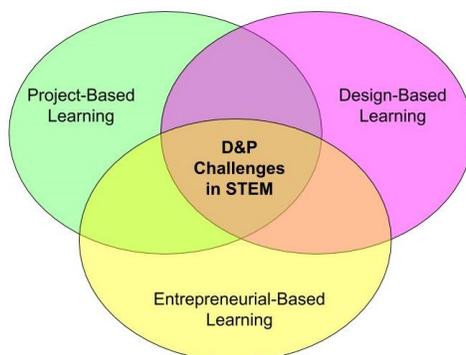
Authentic, collaborative, and cognitively complex tasks have the potential to increase students' engagement (Lamborn et al., 1992). When those tasks represent rich and research-based approaches to learning specific mathematics concepts, they can also support mathematics learning. One such task involves using spreadsheets to build, test, and refine a rating or ranking algorithm. Algorithms represent a timely, authentic, and potentially relevant context. When built within a spreadsheet environment, algorithms could support a functional approach to algebraic expressions that creates a need for students to shift from considering the known (arithmetic) to considering the unknown (algebraic; Rojano, 1996). In addition to requiring students to use and understand symbolic representations, the spreadsheet and its affordances support students to engage in behaviors that are foundational for mathematics learning, such as persistence, iteration, and generalization. Thus, a challenge in which students build, test, and refine algorithms as part of an entrepreneurial pitch competition, could support increased engagement and deep mathematics learning.

Chapter Summary

EBL, PBL, and DBL are three curricular frameworks that have the potential to increase students' engagement, while also supporting them to learn specific and targeted mathematics concepts. The framework resides in the intersection of entrepreneurial-based learning (EBL), project-based learning (PBL), and design-based learning (DBL; see Figure 2).

Figure 2

EBL, PBL, DBL, and the D&P Challenges in STEM



Note. This figure illustrates the relationship between the D&P Challenges in STEM, EBL, PBL, and DBL.

That is, each challenge fits within EBL, PBL, and DBL, and, by leveraging features of these three curricular frameworks, is designed to support engagement and learning in a mathematics classroom setting. However, the extent to which the D&P Challenge framework can support engagement and mathematics learning likely depends on the challenge or task with which students engage. A challenge designed around building, testing, and refining rating or ranking algorithms, such as the *Building Algorithms* challenge, could be especially effective for supporting engagement and rich mathematics learning relating to algebraic expressions and functions. In the next chapter, I present the methodology used to explore students' experiences with the D&P Challenge framework and the *Building Algorithms* challenge.

CHAPTER 3: METHODOLOGY

The purpose of this study was to explore whether and how participating in the *Building Algorithms* challenge, an example of the D&P Challenges in STEM curricular framework, influenced middle grades students' engagement in a mathematics classroom setting, while also supporting and creating opportunities for mathematics learning. This study employed a design-based research (DBR) methodology (Cobb, Confrey, et al., 2003). DBR was selected for exploring the research questions of interest for several reasons. First, "a design study...is particularly appropriate when there is little knowledge about the development and typical forms of student thinking, perhaps because the structure of the subject matter is being...taught in a new way" (Lehrer & Schauble, 2004, p. 641). Prior to this study, little was known about students' experiences learning mathematics within a design-based entrepreneurial challenge and pitch competition. Even less was known about whether or how entrepreneurship would affect the development of students' algebraic and functional reasoning. The study proceeds by proposing a set of conjectures, and then testing and revising the conjectures through the design study (Confrey & Lachance, 2000).

Second, the "highly interventionist" (Cobb, Confrey, et al., 2004, p. 10) nature of design studies allows researchers to develop and test conjectures in real-time to explore the underlying theory that explains students' development. Specifically, design studies allow for "...both 'engineering' particular forms of learning and systematically studying those forms of learning within the context defined by the means of supporting them" (Cobb, Confrey, et al., 2004, p. 9). Given the novelty of the D&P challenge framework, especially as it relates to mathematics learning, it was necessary to experiment with in-class interventions to establish the conditions for successful implementation.

Finally, the purpose of the research study was to develop a deeper understanding of how an entrepreneurial-based curriculum (i.e. the D&P Challenges in STEM) could support learning and engagement in a mathematics setting, under “good, but not unique conditions” (Lehrer & Schauble, 2004, p. 640). The study sought to establish a theory that could explain how the D&P Challenges in STEM work (or why they do not work) before progressing to exploring how they compare to other similar curricular approaches. Given this goal of “...develop[ing] theories, not merely to empirically tun[ing] ‘what works’ developing and testing theories” (Cobb, Confrey, et al., 2003, p. 9), DBR was identified as the optimal methodology for the study.

For these reasons, this study employed a DBR methodology to better understand how a STEM design challenge, situated within an entrepreneurial pitch competition and implemented using a project-based approach, can support students’ engagement and mathematics learning (see [Appendix A](#) for the study diagram). Qualitative data were collected and analyzed to explore how the *Building Algorithms* challenge and the D&P Challenges in STEM framework promoted engagement, while also creating opportunities for rich mathematics learning. These qualitative data included 1) video records of students working in teams on the *Building Algorithms* challenge and participating in whole-group discussions, daily team interviews, post-competition task-based interviews, and post-competition focus groups; 2) daily student work samples; and 3) a detailed study log that was maintained throughout the study to keep track of the development, testing, reflecting on, and revising of the design study conjectures.

In the sections that follow, I explain how the study design and methods were used to explore the two target research questions and their corresponding design study conjectures, which were:

1. How and to what extent does the *Building Algorithms* challenge promote and support students' engagement in a mathematics classroom setting?

Conjecture 1: the entrepreneurial framing of the challenge will empower students to perceive their solutions as actionable and will help them maintain their engagement during the challenge.

Conjecture 2: the challenge materials and support resources will help students perceive the challenge as authentic, which will increase their engagement.

Conjecture 3: practicing for and participating in the pitch competition will increase students' engagement and enthusiasm during the challenge.

2. How and to what extent does the *Building Algorithms* challenge promote and support the learning of specific mathematics concepts?

- a. How and to what extent do students engage with algebraic expressions as they research, develop, describe, and justify their entrepreneurial solutions to the *Building Algorithms* challenge?

- b. How and to what extent does the structure of the *Building Algorithms* challenge support mathematics learning?

Conjecture 4: the task of building a working algorithm that automatically produces rankings or ratings, combined with the affordances of spreadsheets, will support students to demonstrate a functional understanding of algebraic expressions.

Theoretical Framework

The study design, data collection, data analysis, and interpretation were all informed by a theoretical framework built upon one global theory of learning, constructivism, and three specific curricular frameworks: entrepreneurial-based learning (EBL), project-based learning (PBL), and

design-based learning (DBL). In the following sections, I provide a brief overview of each framework and discuss how it informed the design of the study.

Constructivism

The research paradigm for this study is constructivism. From a learning perspective, constructivism holds that students hold internally consistent models that are viable for particular situations and settings (Confrey, 2006; Confrey & Kazak, 2006; von Glasersfeld, 1982). Learning involves establishing for students a perceived problematic that they can work to resolve through developing, testing, and refining models of a given situation (Confrey, 1991). Key to this is that students need to 1) have a felt need to act to resolve the problematic, which means the problematic must be meaningful to them and be something against which they can evaluate the viability of their models; and 2) they need to have opportunities to apply and build from their prior knowledge and experiences (Confrey, 1991).

This constructivist perspective on learning as occurring through building and testing the viability or fit (von Glasersfeld, 1982) of conceptual models provided the framework for defining and analyzing students' mathematics learning during the study. The constructivist theory of learning tracks closely with the learning perspective of mathematical modeling (Confrey & Kazak, 2006), which describes how learning occurs through a process of building, testing, and revising mathematical representations for the purpose of understanding the underlying structure of a situation (Lehrer & Schauble, 2007). Mathematical modeling frames learning as an iterative process through which indeterminate situations are made more determinate (Confrey & Maloney, 2007; Dewey, 1938/1981) using mathematics. It requires one to continually build mathematical models, test their fit, coherence, and consistency, and make revisions to improve their viability. As students progress through this iterative cycle of building, testing, and refining their

mathematical models of a given target situation, they develop a deeper understanding of both the mathematical representation and the structure of the target situation (Confrey & Maloney, 2007). In this study, evidence of students' mathematics learning was collected and analyzed through the lens of constructivism and mathematical modeling.

Finally, as a research paradigm, constructivism stresses understanding not just whether students learn, but also the processes through which that learning happens, which requires one to elicit and investigate students' perceptions of their experiences (Confrey & Kazak, 2006; Schutz, Chambliss, & DeCuir, 2003; von Glasersfeld, 1982). The student voice is essential to the process of conducting research within the constructivist paradigm. Student voice should be recruited, listened to, and interpreted through the researcher's perspective to develop insights into the novel and often unanticipated paths students take in building an understanding of the target content (Confrey, 1991; Confrey & Lachance, 2000).

In this way, constructivism informed 1) the design of the D&P challenge framework; 2) the design of the *Building Algorithms* challenge; 3) the selection of the DBR methodology; 4) the instructional interventions that took place during the study; and 5) the analysis of student learning that occurred during the study.

Entrepreneurial-Based Learning

Entrepreneurial-based Learning (EBL) provides the overarching curricular framework for this study. The D&P Challenges in STEM are based on the theory that engaging in entrepreneurial processes and embracing entrepreneurial characteristics are essential experiences for students. Students who can think innovatively and creatively, identify and act on opportunities, consider and respond to the needs of customers, and grapple with messy and ill-defined problems will be better prepared for a rapidly changing career landscape (Filion, 1994;

Lackeus, 2015; OECD, 2018). The design of the intervention materials was based on the D&P Challenges in STEM entrepreneurial framework (see Figure 1) and is intended to increase students' engagement in mathematics and interest in STEM-related careers. As such, the collection and analysis of much of the qualitative data will consider features of the entrepreneurial framework.

Project-Based Learning

PBL is the second curricular framework that guided the planning and design of the study. PBL emphasizes engaging students in school by motivating the learning of new content through authentic contexts, the creation of tangible artifacts, and the delivery of presentations (Krajcik & Blumenfeld, 2006; Thomas, 2000). PBL provides a structure for implementing the curricular intervention and offers opportunities for students to learn new content as they identify gaps in their understanding and work to fill those gaps (Blumenfeld et al., 2006; Krajcik & Blumenfeld, 2006). PBL supports learning by creating a need for specific content. How that need is met by teachers, students, or curriculum designers, will determine the extent to which the new content is learned. As will be described below, the specific D&P Challenge that will be explored in this study is designed to support students to identify gaps in their understanding of algorithms and algebraic expressions, and to fill those gaps through accessing the supporting resources. The collection and analysis of data was guided by this framework, paying particular attention to the ways in which 1) students recognized a need for the targeted mathematics content; 2) identified and filled gaps relative to that content; and 3) whether and how those efforts result in improved understanding of the content.

Design-Based Learning

DBL is the final curricular framework that guided this study. In DBL, students engage in design thinking as they work to develop tangible solutions to authentic design challenges. DBL is intended to increase student engagement and interest in STEM through establishing an immediate purpose (Kolodner, 2002; Mehalik et al., 2008; Wendell & Rogers, 2013) for their work through the creation of a tangible artifact to address a design challenge. In DBL, the iterative process of prototyping, testing, communicating, and refining solutions to the challenge creates an environment that supports students to identify and work to fill gaps in their understanding of concepts relevant to the design challenge. How those gaps are filled is likely to determine the extent of the content learning during the challenge. The iterative and design-based framework of DBL informed both the design of the challenge used in this study and the collection and analysis of observation and focus group data.

Design of the Challenge

This study will explore students' experiences with a single challenge within the Design & Pitch Challenges in STEM framework: The *Building Algorithms* challenge. In this section, I first present an overview of the *Building Algorithms* challenge. Then, I describe the specific features of the *Building Algorithms* challenge within the more general Design & Pitch Challenges in STEM framework. For each component, I explain how it is designed to 1) promote entrepreneurial skills and processes, as described in the entrepreneurial framework; 2) support students' engagement during the competitions; and 3) create opportunities for mathematics learning. Finally, I conclude the section with a brief description of the study plan, organized by day.

The Building Algorithms Challenge

This study explored students' participation in the *Building Algorithms* challenge, which tasks students with building an algorithm that ranks or rates something they care about and around which they can build a business. A central goal of the challenge is to use the authentic, timely, and entrepreneurial context of algorithm building to motivate student learning relative to algebraic expressions and functions. Specifically, the challenge was designed to create opportunities for students to engage with algebraic expressions as they build, test, and refine functioning spreadsheet algorithms. This challenge is aligned to the 6th grade Common Core State Standards in Mathematics (CCSS-M) represented in Table 1.

Table 1

Standards Alignment for the Building Algorithms Challenge.

Standard	Description
6.EE.A.2	Write, read, and evaluate expressions in which letters stand for numbers.
6.EE.A.2A	Write expressions that record operations with numbers and letters standing for numbers.
6.EE.A.2B	Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
6.EE.A.2.C	Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
6.EE.B.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
6.EE.C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

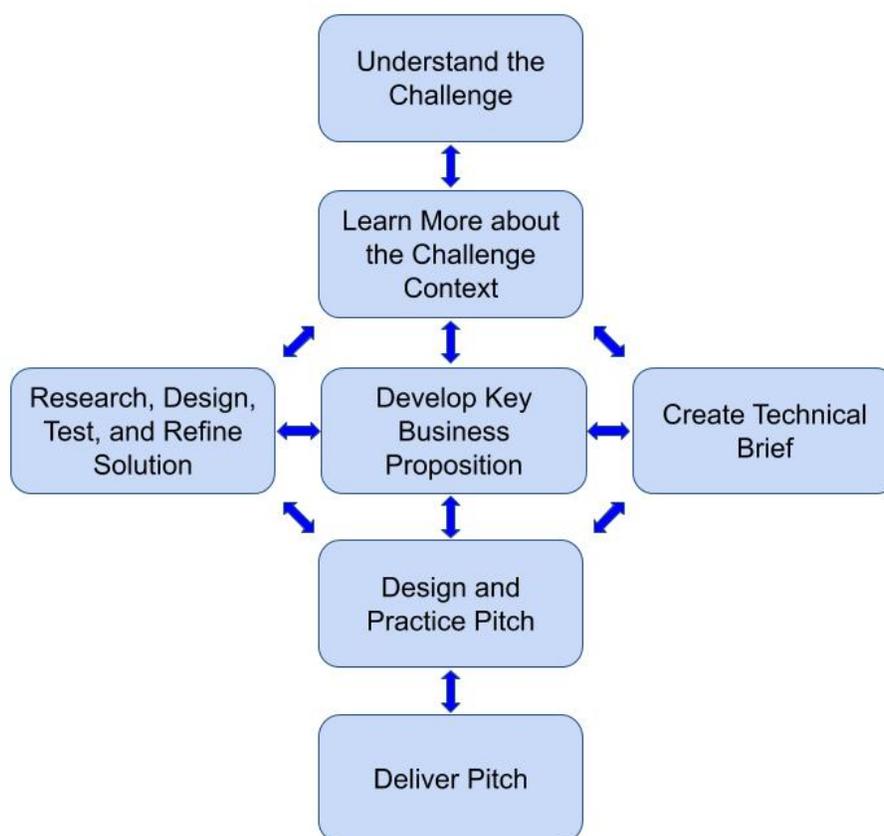
Note. Strikethrough text is not addressed by this challenge.

Design & Pitch Challenge Process and Components

The D&P challenge framework promotes an iterative process (see figure 3; Confrey et al., 2019) through which students develop entrepreneurial solutions and pitches to address a specific design challenge.

Figure 3

D&P Challenge Process



Note. This figure shows the student process for completing a D&P challenge.

In this section, I describe the D&P challenge process, using specific components of the *Building Algorithms* challenge to illustrate the various steps in the process. It is important to note that, although figure 2 suggests the components of the D&P challenge process are sequential, students are not expected to progress through them sequentially. That is, the D&P challenge

process encourages and expects students to continually cycle between steps as they develop and improve upon their solutions.

Competition Launch. Each challenge begins with a launch of the pitch competition, which includes helping students understand what it means to think and act like an entrepreneur and to participate in a pitch competition. The overarching pitch competition engages students in authentic entrepreneurial processes (e.g. idea generation, opportunity and resource analysis, building business models, iterating, problem solving, building diverse teams, and pitching), while supporting the development of entrepreneurial characteristics, such as resourcefulness, adaptability, and courage.

In the competition launch, students are introduced to entrepreneurship, the entrepreneurial framework, and the rules and specifications for the pitch competition. The challenge launch also includes watching a role model video in which a professional entrepreneur discusses her/his experiences in entrepreneurship². The competition launch serves to 1) orient students to the general goals of the activity (i.e. developing a solution to a challenging problem and pitching it to potential investors); and 2) get students excited about entrepreneurship. Lastly, and most important for engagement and mathematics learning, the competition launch establishes a tangible and immediate purpose for students' work, while emphasizing the integration of the three primary aspects of a complete solution: a physical solution or artifact (Fortus et al., 2004; Wendell & Rogers, 2013), evidence of its economic viability (Lackeus, 2015), and a final presentation to investors (Krajcik & Blumenfeld, 2006; Thomas, 2000).

Understand the Challenge and Learn More about the Challenge Context. After launching the pitch competition, the specific design challenge is introduced. This component

² At the time of the study, the entrepreneur role model videos were under development and were not used in the study.

draws from both DBL and PBL and relates to the specific design challenge that students are tasked with completing during the broader pitch competition. The challenge launch includes showing students a challenge champion video and discussing the challenge statement document. In the challenge champion video, a person connected to the challenge context introduces the challenge and describes the general criteria, which are described in greater detail in the challenge statement. In the *Building Algorithms* challenge, the challenge of building an algorithm-based business is introduced by Cathy Yee (https://www.jason.org/design-and-pitch?sub_resource=30642), the Chief Executive Officer (CEO) and founder of Includive, a company that rates movies according to their treatment of diversity (www.includive.com).

Students then read the challenge statement (see [Appendix B](#)), which adds detail to the challenge champion video and includes 1) a summary of the challenge; 2) a detailed description of the context in which the design challenge is situated; 3) a statement of the specific design challenge; and 4) a description of general criteria against which final designs will be evaluated.

In the *Building Algorithms* challenge, the general solution criteria specify that algorithms must 1) allow users put in data and automatically rate or rank the thing they care about; 2) include weighted categories; 3) be transparent; and 4) include a way to make money. Like the competition launch, the challenge launch is designed to engage students in entrepreneurial processes, highlight entrepreneurial characteristics, and support mathematics learning. First, the authentic context (in this case algorithms) connects the work students will be doing to a specific and real context. In addition to providing a purpose for their work, the design challenge and authentic context engage students in entrepreneurial processes (e.g. idea generation and opportunity and resource analysis) and provide opportunities for them to practice entrepreneurial characteristics (e.g. creativity and empathy).

Second, the challenge context may “trigger” certain student ideas or actions (Fortus et al., 2004), giving them an entry into the challenge. In the *Building Algorithms* challenge, the challenge launch references algorithms for comparing basketball players and restaurants as a way of triggering students to consider characteristics that should be considered in those rankings and begin engaging in their own idea generation. In this way, the challenge launch engages students in an entrepreneurial and design process: students operate in a familiar context, are supported to narrow their focus to a specific problem that needs innovation, and are provided general criteria relevant to the intended users.

Challenge Work. Once the challenge has been launched, teams work collaboratively to develop entrepreneurial solutions to the challenge. Group work is a key process of entrepreneurship. Depending on the setting in which the challenges are implemented, students may not have the opportunity to build diverse teams. The D&P challenge framework leaves to teachers the decision of how to best form teams. However, it is likely that teams will include students possessing diverse interests and skills. For teams to be successful, students will need to learn to effectively and efficiently leverage that diversity: a skill that is essential to entrepreneurship (T. Graves-Manns, personal communication, September 19, 2019) and one that promotes engagement by creating multiple points of entry to for the competition.

The D&P challenge process includes three parallel steps for students to complete as they build their solution. These three components are 1) research, design, test, and refine solutions; 2) build the technical brief; and 3) develop the key business proposition. As students complete these steps, the teacher (or researcher) plays a critical role in facilitating the process. In the D&P challenge framework, the teacher (or researcher) needs to introduce the steps and support materials as well as ask probing questions to support teams as they build their solutions.

Additionally, halfway through the challenge work time, the teacher, researcher, or a community member should check-in with teams to conduct an “expert” check-in. During the “expert” check-in, teams are questioned about their entrepreneurial solutions and are provided feedback for how to improve those solutions. These check-ins focus on the real-world application and feasibility of teams’ solutions and provide an opportunity to drive further iteration on teams’ solutions.

Research, Design, Test, and Refine Solutions. During challenge work, students leverage their diverse skills and interests to research, design, test, and refine their entrepreneurial solutions to the challenge. In the *Building Algorithms* challenge, this process includes working to identify the context for their algorithms, collecting data relative to that context, and building, testing, and refining their algorithms in the spreadsheet tool. To support students in this process, the *Building Algorithms* challenge includes three additional resources designed to support students’ engagement and mathematics learning by scaffolding their work and strengthening the connection between the challenge and the targeted STEM content.

First, a challenge background video featuring the challenge champion, gives students background information relating to the context and suggests things to consider in designing their solution. In the *Building Algorithms* challenge, the challenge background video (https://www.jason.org/design-and-pitch?sub_resource=30645) features Cathy Yee giving additional detail about the meaning of the word “algorithm” and discussing how her company uses algorithms to rate movies. Second, the challenge context document (see [Appendix C](#)) includes a set of links to real-world and entrepreneurial uses of rating and ranking algorithms. These links are design to jumpstart students’ brainstorming process, while also demonstrating the authenticity of the challenge by connecting students’ work to real and, in some cases, familiar businesses. For example, the document includes a link to an article on how the video games use

algorithms to rate characters (<https://fivethirtyeight.com/features/madden/#>) and an article on how the Eurovision Song Contest uses an algorithm to rank contestants (https://en.wikipedia.org/wiki/Voting_at_the_Eurovision_Song_Contest).

Finally, an interactive spreadsheet resource (see [Appendix D](#)) is included that 1) explains the difference between rating and ranking algorithms, 2) presents tips for writing spreadsheet formulae, and 3) gives examples for students to practice programming a spreadsheet to carry out a variety of rating and ranking algorithms. The spreadsheet resource was designed to engage students in the relevant mathematics (algebraic expressions via spreadsheets), while also providing alternative models on which they can base their own algorithms. Additionally, during a pilot study, it was revealed that few students had prior experience with spreadsheets and most lacked the skills necessary to build a spreadsheet algorithm. The spreadsheet resource was designed to support students' engagement during the challenge by equipping them with the minimum skills necessary to engage with the tool and make progress towards a functioning spreadsheet algorithm.

Develop the Key Business Proposition. Thinking and acting like an entrepreneur involves reframing problems as opportunities to create value for customers (Lackeus, 2015). In the Design & Pitch Challenges and *the Algorithms Challenge*, this focus on value creation is operationalized through completing the key business proposition (see [Appendix E](#)). In both DBL and EBL, students play a central role in defining the criteria for a successful solution and making necessary adjustments to those criteria based on value judgments (Dym et al., 2005). Where EBL differs from DBL, is in its emphasis on not just the needs of the customer, but on identifying and communicating the value proposition of a design (Lackeus, 2015). The key business proposition

provides a structure for students to identify their target customers, define their needs, and map how the features of their solution meets those needs.

The Key Business Proposition supports learning and promotes entrepreneurial characteristics and processes in several ways. First, it encourages empathy, as students consider the needs of customers, and problem solving, as they consider how to adapt their ideas to create value for customers. Second, it supports learning by leveraging the evaluation and critique component of DBL. In DBL, it is important for students to consider alternative designs, to make value judgments relative to the criteria for success, and to select the optimal or preferred design for the final product (Dym et al., 2005). Finally, through the elevator pitch framework, the Key Business Proposition prompts students to consider how to concisely describe their business and algorithm, while positioning it in the market relative to competitors. By prompting students to consider their competitors' designs, the key business proposition engages them in the evaluation and critique of those designs, which supports the development of a deeper understanding of both the solution and the underlying mathematics content. In the *Building Algorithms* challenge, this process involves students comparing their algorithms and corresponding rankings with those of a competitor. This comparison focuses students' attention on the structure of their algorithm and, likely, results in modifications.

Build the Technical Brief. In both PBL and DBL, reflecting on the process of developing the final artifact is essential for connecting the design or project tasks with the underlying STEM content and skills (Kolodner, 2002; Krajcik et al., 2008; Penner et al., 1998). In PBL, this reflection may occur through scaffolded lessons implemented during the unit, through the culminating presentation (Kanter, 2009; Krajcik et al., 2008), or through project journals in which students document their process and the problems they choose to solve (or not solve;

Stevens, 2000). In DBL, reflection might involve whole class presentations (Fortus et al., 2004; Penner et al., 1998), gallery walks, design journals, or guided inquiry lessons during the design process (Kolodner, 2002). One way the D&P Challenges support students' reflection is through the creation of the Technical Brief. In the technical brief, students describe the specifics of their solutions, including mathematical and scientific justifications, and discuss the process through which they developed their solutions. The goal of the technical brief is to encourage students to demonstrate how they know a solution will work or that it meets the specific criteria, ideally using the targeted STEM content.

In the *Building Algorithms Challenge*, the Technical Brief (see [Appendix F](#)) makes explicit the requirement for students to represent their algorithms using algebraic expressions. Students are also prompted to describe how they arrived at their final algorithm design, including any variables or weights they considered but did not include, and to demonstrate how their algorithms work using sample data. These requirements connect students' algorithm work to the underlying mathematical content relating to algebraic expressions. By representing their algorithms using algebraic expressions, the connection between algorithms and algebraic expressions is made explicit. By requiring students to describe their process including variables and weights they considered, they need to interpret the algebraic expression and understand its structure. Finally, the requirement that students demonstrate how their algorithms work using sample data creates an opportunity for students to write and evaluate their algebraic expressions, which can support their transition from numerical and arithmetic representations to algebraic and functional representations (Blanton et al., 2015; Rojano, 1996).

Design, Practice, and Deliver the Pitch. Each D&P challenge culminates with a five-minute pitch to a panel of judges. To help students learn to build a pitch deck and plan their

entrepreneurial pitch, the D&P Challenge framework provides a set of pitch resources. These resources include the How to Build a Pitch document (see [Appendix G](#)), which outlines the key components of an effective pitch, a set of sample pitches from real companies, and the Pitch Judging Sheet (see [Appendix H](#)), which lists the criteria judges will use to evaluate pitches.

In the days leading up to the final pitch competition, students are also given an opportunity to complete a semi-formal practice pitch to an external practice judge. Depending on the setting and the resources available, the panel of judges and the external practice judge could be members of the community, local entrepreneurs, or school stakeholders. The only requirement for judges is that they not be familiar with students' entrepreneurial solutions prior to the practice or final pitch. The decision to use external judges for both the practice pitch and the final pitch competition was based on the conjecture that external judges would provide a measure of external accountability that would result in increased student engagement.

In the D&P challenge framework, the practice pitch occurs one to two days prior to the final pitch competition and is designed to create an additional opportunity for students to expose their solutions and pitches to critique and to identify ways to improve their solutions, pitch decks, and pitches. After completing the practice pitch, teams are given the remaining time to make necessary revisions and to practice for the final pitch competition.

The final pitch competition situates and reframes the presentation components of both PBL and DBL within an authentic, exciting, and high-pressure entrepreneurial experience: the start-up competition (Moore et al., 2017; Passaro et al., 2017; startupweekend.org). In addition to engaging students in an entrepreneurial process (pitching) and encouraging entrepreneurial characteristics (courage and creativity), the pitch competitions have an added benefit for learning: developing and delivering a 5-minute pitch requires students to have a clear and deep

understanding of their solution and the rationale for why their solution works. In the *Building Algorithms* challenge, this means understanding what the algorithm does, how it does it, and how it meets the needs of a customer or group of customers. It means being able to communicate what the algorithm values, what it devalues, and how it incorporates those values into its calculations.

Sample and Participants

Context

To adequately address the target research questions, two iterations of the design study were conducted in two different contexts. The first iteration of the study was conducted in the summer of 2019 in our research office on the campus of North Carolina State University. Students were given iPads to access the challenge materials and to research and build their solutions. After completing the first iteration of the design study it was determined that a second iteration was needed to explore two new research conjectures³. The second iteration of the design study was conducted in a mathematics elective course at a suburban public charter school also in North Carolina. The charter school is a K-8 school that advertises project-based instruction. Although students' access to technology was not uniform across classes, the school was technology-rich with students in the participating mathematics elective class having daily access to Ideapads and Chromebooks⁴ and the teacher having daily access to a SmartBoard.

³ Because the later conjectures emerged from the first iteration of the design study and thus are part of an emerging understanding of the results of the study, a decision was made to delay introducing them until after the results of the first iteration have been presented.

⁴ Students were not guaranteed access to the same laptop each day. Students had to save their work daily to a flash drive provided by the research team. The flash drives were stored between classes in a locked cabinet in the classroom.

Participants

For the first iteration of the design study, local middle grades students were recruited to participate using a recruitment flyer (see [Appendix I](#)) that was shared on social media and posted in public libraries and on message boards on local home school websites. This recruitment process resulted in six students, entering grades 6 through 8, volunteering to participate the week-long study (see Table 2). All students and parents completed IRB approved consent forms (see [Appendix J](#)) and agreed to attend all five 3-hour testing sessions. Table 2 lists the students who participated in the first iteration, organized by team. All names listed in the table are pseudonyms.

Table 2

Students Participating in the First Iteration of the Design Study.

Team	Student Pseudonyms	Grade
Team A	Denise	7 th
	Donald	8 th
Team B	Beatrice	6 th
	Maria	8 th
Team C	Kim	6 th
	Shaun	7 th

Note. Grades represent the grades students would be entering in the fall.

Participants for the second iteration of the study were identified using opportunity sampling. A parent of a participant in an earlier pilot study of the D&P challenges recommended the challenges to the principal at her child's school, who then invited our research team to lead a three-hour workshop with the school's elementary and middle grades math and science teachers. Following the workshop, a teacher volunteered to test the *Building Algorithms* challenge with fifteen students in her mathematics elective course. Table 3 lists the students who participated in the second iteration of the study, organized by team. All student names listed in the table are

pseudonyms. The team names are the names students selected for their algorithm-based businesses.

Table 3

Students Participating in the Second Iteration of the Design Study.

Team	Student Names	Grade
RestaurantX	Paul	6 th
	Phil	6 th
	Patrick	7 th
Pizza Hunters	Brad	6 th
	Brie	7 th
	Beth	7 th
Discover.GE	Ari	6 th
	Alex	6 th
	Alma	8 th
The Amigos	Cynthia	7 th
	Charles	8 th
	Carla	8 th
Gamer's Paradise	Ethan	6 th
	Edgar	6 th
	Earl	8 th

Data Collection

During the first iteration of the study, all three teams of students were video-recorded as they worked to complete the challenge and participated in whole-group discussions, daily team interviews, post-competition focus groups, and post-competition task-based interviews. Daily work samples were also collected from each team to document the progression of their solutions and to analyze the extent to which they engaged with algebraic expressions and functions in building those solutions. Except for the daily team interviews and the post-competition task-based interviews, the same data were collected during the second iteration of the study. Additionally, given the greater number of teams in the second iteration, only one team, Discover.GE, was video-recorded throughout the week. The other four teams were video-recorded during whole-group discussions and when I interacted with them directly.

Procedures

The *Building Algorithms Study Plan* (see [Appendix K](#)) was developed in the lead-up to the first iteration of the study. The design of the study plan was based on 1) a theorized five-day implementation model for the D&P challenges in STEM and 2) goals for testing the four research conjectures that informed the targeted research questions. For each day, the plan included 1) a description of daily student benchmarks, such as selecting variables for their algorithms (day 1) and beginning work on the Key Business Proposition (day 2) and 2) sets of questions, aligned to the research conjectures, to use during whole-group discussions and daily team interviews. In this way, the *Building Algorithms Study Plan* (see Table 4 for an abbreviated version of the *Building Algorithms Study Plan*) acted as a daily guide for conducting the study in a way that was consistent with a design study methodology. The goal of a design study is to establish “what can occur under good, but not highly unusual, instructional circumstances” (Lehrer & Schauble, 2004, p. 640) and to understand how student thinking develops by eliciting, listening to, and learning from student voice (Cobb, Confrey, et al., 2004; Confrey, 1991; Lehrer & Schauble, 2004). The *Building Algorithms Study Plan* provided a structure within which I could experiment with teacher interventions to support student thinking and explore unanticipated lines of student thinking relative to entrepreneurship, algorithms, and algebraic expressions.

Table 4*Abbreviated Version of the Building Algorithms Study Plan.*

Day	Challenge Activity	Research Objectives
1	Launch the competition and challenge. Students brainstorm contexts and variables for their algorithms.	Explore 1) students' conceptions of entrepreneurship, pitch competitions, and algorithms; and 2) students' perceptions of the support videos. Team Interviews: Teams A and B
2	Introduce the Key Business Proposition and the Business Models documents. Encourage students to use the spreadsheet resource.	Explore 1) how the spreadsheet resource and algorithm task support algebraic and functional reasoning; and 2) how the entrepreneurial framing of the challenge supports engagement and perceptions of actionability. Team Interviews: Teams A and C
3	Introduce the technical brief and the pitch resources. Conduct "expert" check-ins with teams. Students continue building out their algorithms and businesses.	Explore 1) students' conceptions of the role of pitching in entrepreneurship; 2) how students explain and justify their algorithms; and 3) how entrepreneurship supports iteration. Team Interviews: Teams B and C
4	Introduce the pitch judging sheet and discuss the final pitch competition. Students continue building out their solutions and complete a practice pitch with an external practice judge.	Explore 1) how pitching and the entrepreneurial framing of the challenge support engagement and enthusiasm; 2) how students explain and justify their algorithms and algebraic expressions.
5	Students prepare for and participate in the final pitch competition.	Explore 1) how pitching and the entrepreneurial framing of the challenge support engagement and enthusiasm; 2) how students explain and justify their algorithms and algebraic expressions; and 3) how students used the support materials to learn about pitching.
5 - Post Study	Teams participate in the post-competition focus group and students (individually) participate in the post-competition task-based interview.	Explore 1) students' perceptions of the challenge and their use of mathematics in building their solutions and 2) students' understanding of algorithms and algebraic expressions.

Deviations from the study plan, including researcher interventions, were made based on attempts to explore student thinking and/or experiment with strategies for supporting students'

engagement and mathematics learning. At the end of each day, I debriefed with members of the research team and documented my justification for instructional interventions or deviations from the original study plan. During the first iteration of the study, the only significant deviation occurred between days 3 and 4, when I decided to postpone the discussion of pitching. This decision was made on day 3 to avoid disrupting what appeared to be a productive workflow for all teams. At the conclusion of the study, teams of students participated in a focus group and individual students completed a Task-Based Interview (see [Appendix A](#) for a diagram of the study).

The second iteration of the study followed the same sequence of events that was described in the *Building Algorithms Study Plan*. However, because of the shorter sessions (daily sessions lasted 45 minutes in the second iteration) and unanticipated interruptions (e.g. schoolwide testing and field trips), several events were spread across multiple days, resulting in a ten-day duration to the study. In addition to the daily researcher interventions, there were also two significant deviations from the *Building Algorithms Study Plan*. These deviations were the result of newly identified research conjectures that emerged during the first iteration of the study and will be discussed in the results section after the results of the first iteration have been presented.

Instrumentation

To explore the research objectives described in the *Building Algorithms Study Plan*, three instruments were used. First, during each of the first three days of the first iteration of the study, daily semi-structured interviews were conducted with teams using the Building Algorithms Daily Team Interview protocols (see [Appendix L](#)). These semi-structured interview protocols were only used in the first iteration of the study. Second, the Design & Pitch Challenges in STEM

Focus Group Protocol (see [Appendix M](#)) was used after the completion of the final pitch competition to elicit reflections from teams of students regarding their experiences participating in the challenge and their perceptions of both entrepreneurship and their use of mathematics in building their solutions. Given differences in the time and resources available between the two iterations of the study, the focus group protocol was used with all teams in the first iteration and two teams in the second iteration. The two teams chosen to participate in the focus group during the second iteration were chosen based on two different factors. One team (Discover.GE) were selected after the classroom teacher identified the team members as likely to actively participate in the focus group. The second team (the Amigos) were chosen because of the uniqueness of their solution context. Finally, the Building Algorithms Task-Based Interview (see [Appendix N](#)) was administered in the first iteration of the study following the final pitch competition to explore students' thinking relative to algorithms, algebraic expressions, and functions.

Building Algorithms Daily Interview Protocols. During each of the first three days of the first iteration of the study, daily interviews were conducted to explore students' experiences with the challenge materials and the progression of their algorithms. To limit interruptions to teams' daily work, two teams were interviewed each day for 15 minutes each. Across the first three days of the study, each team was interviewed twice. These interviews were semi-structured and followed Daily Interview Protocols (see [Appendix L](#)) that were developed the previous day. Making the interviews semi-structured allowed for the addition or removal of questions based on interactions with teams as they worked on the challenge. Because the second iteration was intended to test two specific conjectures relating to the introduction of the spreadsheet resource and the use of the technical brief on day 10 of the study, daily team interviews were only used during the first iteration of the study.

The *Building Algorithms Task-Based Interview*. Following the completion of the pitch competition, the *Building Algorithms Task-Based Interview* (see [Appendix N](#)) was administered to each student individually. This task-based interview was designed to explore how students' experiences building spreadsheet algorithms would inform their interpretation of algorithms when represented as a generalized algebraic expression and as a formula in a spreadsheet. To that end, the *Building Algorithms Task-Based* interview included two parts. In the first part of the task-based interview, students were asked to work with a rating algorithm represented only as a single algebraic expression. Students needed to 1) explain how the algorithm calculated ratings, 2) determine, from the expression, which variable was considered most important in the algorithm; 3) evaluate the algorithm for specific values of each variable; and 4) consider effects of changing the structure of the expression on the overall scores produced by the algorithm.

In the second part of the task, students were presented with a restaurant rating algorithm presented in a spreadsheet table and represented using a spreadsheet formula. Students were asked to 1) describe how the spreadsheet algorithm calculated overall scores for restaurants, 2) calculate a score for one of the given restaurants, and 3) represent the three spreadsheet formulae as a single algebraic expression. Taken together, the two parts of the *Building Algorithms Task-Based Interview* were designed to explore whether students could interpret and evaluate algebraic expressions when presented in the context of algorithms and aligned to the CCSS-M standards targeted by the challenge (see Table 1). For example, in asking students to explain how the algorithm calculated ratings, the *Building Algorithms Task-Based Interview* was assessing whether students could interpret the meaning of an algebraic expression (CCSS-M standard 6.EE.A.2) and recognize the relationship between input and output variables in that expression (CCSS-M standard 6.EE.C.9). In asking students to use the given expressions to determine

ratings for specific values of the variables, students were being assessed on whether they could evaluate algebraic expressions (CCSS-M standard 6.EE.2.C).

The Design & Pitch Challenges in STEM Focus Group Protocol. A semi-structured focus group interview was conducted at the end of both iterations of the study using the Design & Pitch Challenges in STEM Focus Group Protocol (see [Appendix M](#)). In the first iteration, all three teams participated in the post-competition focus group. In the second iteration, because of the greater number of teams and students, two teams were selected to participate in the focus group. Based on the original design of the study, the team that were followed throughout the study (Discover.GE) on the classroom teacher's recommendation, also participated in the post-competition focus group. The other team (the Amigos) were selected because their algorithm context (Hispanic foods) was unique and I wanted to learn more about their experiences with the competition.

The questions on the Design & Pitch Challenges in STEM Focus Group Protocol explored whether students enjoyed participating in the competition, whether and how the structure of the D&P Challenges influenced students' enjoyment and engagement, and their perceptions of entrepreneurship, STEM, and mathematics. The interview was conducted with teams of students to allow them to build off each other's answers and remind each other of specific experiences during the challenge. The focus group protocol included a list of open-ended questions that prompted students to reflect on specific aspects of the D&P Challenge, such as the design challenge, group work, the Key Business Proposition, the Technical Brief, the role of external judges, and the pitch competition. Lastly, the protocol included questions that prompted students to reflect on their processes developing algorithms, relating specifically to STEM learning as described in the theoretical framework. For example, students were asked to reflect

on times during the challenge when they identified gaps in their understanding and how they worked to fill those gaps. The interview was designed to be semi-structured to allow for a consistent administration and to guarantee that every team of students were asked the same questions, while also opening the space to explore unanticipated responses and interesting paths of students' thinking (DeCuir-Gunby & Schutz, 2017).

Data Analysis

Following the completion of the study, the qualitative data were cleaned and converted into a form that would support in-depth analysis. All video records were transcribed verbatim and imported into the qualitative data analysis tool Atlas.ti Cloud (<https://atlasti.com/cloud/>), to prepare for coding. Before coding began, I read each transcript and drafted memos (Creswell & Poth, 2018) summarizing the content of the transcripts with respect to students' engagement and mathematics learning. I then analyzed the transcripts using a combination of open and *a priori* coding. The set of *a priori* codes (see figure 5) were developed using 1) Helme & Clarke's (2001) work studying student actions indicative of cognitive engagement in a mathematics classroom; 2) the entrepreneurial characteristics and processes as described in the entrepreneurial framework; 3) the components of the D&P challenge framework and *Building Algorithms* challenge; 4) hypothesized student actions that would indicate students' engagement with algebraic expressions and functions; and 5) hypothesized student actions consistent with a constructivist and mathematical modeling perspective of learning (Confrey & Maloney, 2007; see Table 5).

Table 5*Codes from a Priori and Open Coding.*

Category	A Priori Codes	Emergent Codes
Engagement	<p>Students in Parallel*: Concentration; Gestures; Seeking Information and Feedback; Self-Monitoring; Verbalizing Thinking</p> <p>Students in Small Groups*: Completing Utterances; Giving Directions; Justifying Arguments; Questioning</p> <p>Students in Small Groups with Teacher*: Answering Questions from Teacher; Asking Teacher Questions; Explaining and Justifying</p> <p>Whole-Class Discussions*: Asking and Answering Questions; Completing Teacher Utterances; Contributing Ideas; Making Evaluative Comments</p> <p>Student Claims*: Attempted to Learn; Engagement; Learned Something; Student Recall</p>	Energy and Enthusiasm
Entrepreneurial Framework	<p>Entrepreneurial Characteristics: Adaptability; Courage; Creativity; Empathy; Persistence; Problem Solving; Resourcefulness</p> <p>Entrepreneurial Processes: Building Diverse Teams; Business Models; Idea Generation; Iterating; Market Research; Opportunity and Resource Analysis; Pitching; Prototyping</p>	Conceptions of Entrepreneurs, Referring to Real Businesses
D&P Challenge Framework	<p>Challenge Structure: Challenge Launch; Competition; “Expert” Check-in; Practice Pitch; Pitching</p> <p>Challenge Materials: Business Model Types Document; Challenge Videos; Context Document; Key Business Proposition; How to Build a Pitch Document; Pitch Judging Sheet; Spreadsheet Resource; Technical Brief</p>	Authentic and Actionable, Pressure, Productive Failure (Kapur, 2012)

Table 5 (continued.)

Mathematics Learning	Coefficients and weights; Describing inputs and outputs; Describing variables; Exposing Ideas to Critique; Making Assumptions and Simplifying; Multiple meanings of variables; Numerical to Algebraic; Spreadsheet Affordances; Testing Algorithms; Verbal to algebraic	Converting data; Making Assumptions and Simplifying; Predicting and Restricting Outputs
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**Note:* Engagement codes came from Helme & Clarke’s (2001) study on cognitive engagement.

These *a priori* codes were used to identify instances and potential sources of math learning and engagement (or the absence of engagement) relative to the *Building Algorithms* challenge. Open coding was used to document unanticipated sources of students’ engagement and mathematics learning. The set of open and *a priori* codes were then categorized and themes were identified that could explain how the D&P Challenge framework and the *Building Algorithms* challenge supported and promoted students’ engagement and mathematics learning.

After coding and analyzing the video transcripts, students’ daily work samples were analyzed to develop deeper insight into whether and how students’ utterances and behaviors manifested in tangible work products. Together, the video transcripts and daily work samples were used to address the two research questions and their corresponding design study conjectures.

Research Question 1

Research question 1, “how and to what extent does the *Building Algorithms* challenge promote and support students’ engagement in a mathematics classroom setting?” was answered by testing three corresponding research conjectures. First, it was conjectured that the entrepreneurial framing of the challenge would empower students to perceive their solutions as

actionable and would help them maintain their engagement during the challenge. This conjecture was tested by analyzing students' engagement and enthusiasm as they considered the entrepreneurial feasibility of their algorithms. I also explored the extent to which students referenced real businesses and the needs of consumers in building, testing, and refining their algorithms. Team worktime, whole-group discussions, daily team interviews, and post-competition focus groups were analyzed to understand how the entrepreneurial framing of the competition affected students' engagement.

Second, it was conjectured that the challenge materials and support resources would help students perceive the challenge as authentic, which would increase their engagement. This conjecture was tested by exploring the sources of students' engagement (both observed and self-reported) as they watched the challenge videos and accessed the support materials (e.g. the Business Model Types document, the Context Document, etc.) as they completed the challenge. The daily team interviews and the post-competition focus group were also analyzed to explore whether the support resources and materials affected students' perceptions of the authenticity of the challenge and whether those perceptions supported their engagement.

Finally, it was conjectured that practicing for and participating in the pitch competition would increase students' engagement and enthusiasm during the challenge. This conjecture was tested by exploring the overlap (or lack thereof) of the a priori engagement codes and codes relating to the process of students building, practicing, and refining their pitches and pitch decks. Additionally, whole-group discussions, daily team interviews, and post-competition focus groups were analyzed to explore students' understanding and perceptions of pitching and its role in supporting their engagement during the competition.

Research Question 2

Research question 2 stated, “how and to what extent does the *Building Algorithms* challenge promote and support the learning of specific mathematics concepts?” It included two corresponding sub-questions: 1) how and to what extent do students engage with algebraic expressions as they research, develop, describe, and justify their entrepreneurial solutions to the *Building Algorithms* challenge? and 2) how and to what extent does the structure of the *Building Algorithms* challenge support mathematics learning?

Research Question 2a. Research question 2a was answered by testing the conjecture that the task of building a working algorithm that automatically produces rankings or ratings, combined with the affordances of spreadsheets, would support students’ mathematics learning relative to algebraic expressions and functions. This conjecture was tested by analyzing video transcripts and student work samples to understand how the challenge task and the spreadsheet tool created opportunities for students to grapple with symbolic representations. A combination of *a priori* and open coding was used to identify 1) students’ utterances and considerations consistent with a functional approach to algebraic expressions and 2) students’ behaviors that were consistent with a constructivist and mathematical modeling perspective of learning. The daily team worktime video data were supplemented with daily work samples, final pitch decks, and final spreadsheet algorithms to gain a more complete picture of students’ engagement with algebraic expressions and functions as they built, tested, and refined their algorithms and business.

Research Question 2b. The second sub-question for research question 2 explored how and to what extent the structure of the *Building Algorithms* challenge, which included opportunities for students to explain and defend their ideas and the expectation that the challenge

would expose gaps in understanding that students would then work to fill, supported mathematics learning. This sub-question did not have a corresponding research conjecture. However, following the completion of the study, a retrospective conjecture was developed, which posited that the entrepreneurial framing of the competition and the challenge resources would create a need for students to revise and refine their algorithms, which would create opportunities for math learning. This conjecture was tested by coding the video transcripts to determine the extent to which students 1) recognized and acted on a need to learn content relating to algebraic expressions and functions; and 2) revisited and revised their algorithms based on the needs of consumers, feedback from “experts,” or feedback on their pitches.” Additionally, daily work samples, including the Technical Brief, were collected and analyzed to document the progression of students’ mathematical thinking and use of algebraic expressions and functions over the course of the competition.

Credibility and Trustworthiness of the Results

Reliability/Credibility. The reliability/credibility of the results (Creswell & Poth, 2018) was established in several ways across the different qualitative instruments. First, protocols for the daily team interviews and the post-competition focus group were developed to establish consistency in the collection of interview data (Creswell & Poth, 2018). Although the interviews were semi-structured to allow me to follow student thinking and adapt questioning to pursue conjectures in real-time, the protocols supported consistency in administration by dictating specific questions that would be asked to each team (DeCuir-Gunby & Schutz, 2017). Second, pilot studies were conducted with the *Building Algorithms* challenge. These pilot studies provided opportunities to practice implementing the challenge, conducting team interviews, and

administering the focus group protocol, which supported consistent implementation across both iterations of the design study.

Validity/Trustworthiness. Several approaches were used to establish the validity/trustworthiness (Creswell & Poth, 2018) of the qualitative findings. First, multiple sources of data were triangulated to gain for a more complete picture of students' experiences with the *Building Algorithms* challenge. These data sources included 1) video records of all team and whole-group interactions, 2) daily interviews with student teams, 2) daily debrief sessions with the research team, 3) daily student work samples, 4) post-competition focus groups, and 5) post-competition task-based interviews with individual students.

Secondly, "rich, thick description[s]" were used in reporting the results to allow "readers to make decisions regarding transferability" (Creswell & Poth, 2018, p. 263). Students' actions and utterances were described in detail and via a "rational reconstruction" (Confrey & Lachance, 2000, p. 259) of the study process and events. This rational reconstruction allows readers to evaluate "...the quality of the process...in relation to the coherence of the story recounting the dialectical relationship between the events in the classroom and the conjecture[s]" (Confrey & Lachance, 2000, p. 259). Additionally, students' voices were included and were "extensive and authentic...to convince a reader of the depth of students' commitment to and ownership of the ideas" (Confrey & Lachance, 2006, p. 259). Finally, daily debriefing sessions with the research team provided "a 'devil's advocate'...who kept the researcher honest [and] ask[ed] hard questions about methods, meanings, and interpretations" (Creswell & Poth, 2018, p. 263). During these daily debriefing sessions, which are characteristic of the design study methodology (Cobb, Confrey, et al., 2003; Confrey, 2006), the research teams helped to reflect on evolving research conjectures and identify new avenues for testing those conjectures.

CHAPTER 4: RESULTS

The results from the first iteration of the design study uncovered insights into how the Design & Pitch Challenges in STEM, specifically the *Building Algorithms* challenge, can promote students' engagement and mathematics learning. In general, the combination of the D&P Challenge framework and the specific components of the *Building Algorithms* challenge supported students' sustained engagement during the competition, while also creating opportunities for students to engage with symbolic expressions and learn important ideas about variables, algebraic expressions, and functions.

In this section, I start by presenting a historical reconstruction (Lehrer & Schauble, 2004) of events and students' experiences of those events during the week-long first iteration of the design study. Consistent with the design research methodology and the constructivist conceptual framework, I place students' experiences (voices) at the center of this historical reconstruction and pair the descriptions of those experiences with my retrospective analysis (perspective; Confrey, 1991). At the conclusion of the first iteration of the study, two new conjectures were formulated and tested in a second iteration of the study conducted with students at a suburban charter middle school in North Carolina. I conclude the chapter by presenting results from this second iteration of the study, relating specifically to the two new research conjectures.

Historical Reconstruction

This section begins with a "historical reconstruction" (Lehrer & Schauble, 2004, p. 646) of the first iteration of the design study. Organized chronologically by day, this historical reconstruction describes and analyzes the events of the week. The events described in this section were selected because they shed light on how the D&P challenge framework and the *Building Algorithms* challenge affected students' engagement (positively and negatively) during

the competition and how students' mathematical and entrepreneurial thinking developed over the course of the study. To construct a coherent narrative of students' experiences, I drew upon several data sources, including daily notes, video transcripts of team and whole-group interactions, daily debriefs, and daily students work samples (see Table 6).

Table 6

Use of Data Sources in Reporting Results

Day	Data Sources
1	Daily Notes
	Video of team and whole group interactions
	Daily Work Samples
	Interviews with teams A and C (Iteration 1 only)
	Debrief Notes
2	Daily Notes
	Video of team and whole group interactions
	Daily Work Samples
	Interviews with teams A and B (Iteration 1 only)
	Debrief Notes
3	Daily Notes
	Video of team and whole group interactions
	Daily Work Samples
	Interviews with teams B and C (Iteration 1 only)
	Debrief Notes
4	Daily Notes
	Video of team and whole group interactions
	Daily Work Samples
	Debrief Notes
5	Daily Notes
	Video of team and whole group interactions
	Daily Work Samples
	Debrief Notes
	Task-Based Interviews with each student (Iteration 1 only)
	Focus Groups

I also included two additional data sources, collected after the conclusion of the competition in Iteration 1, to gain a clearer picture of students' engagement and mathematics learning during the competition. The results of the post-competition task-based interviews, which were conducted with each student in the first iteration of the design study, are presented after the

historical reconstruction of that first iteration. Results from the post-competition focus group, which was conducted with each team during the first iteration of the design study and two teams during the second iteration of the design study, were drawn upon in the discussion chapter.

These data were analyzed using a combination of *a priori* and open coding. The *a priori* codes came from several sources. First, *a priori* codes for engagement were identified based on the framework developed by Helme and Clarke (2001), which describes behaviors indicative of cognitive engagement in a mathematics setting. Open coding was used to supplement these *a priori* codes, by identifying indicators of engagement that were not anticipated prior to the study, such as when students referred to real businesses in building their solutions or independently referred to the authenticity or actionability of their solutions. A second set of *a priori* codes were used to identify student behaviors, utterances, and work products indicative of a constructivist and mathematical modeling perspective of mathematics learning. These *a priori* codes were identified based on 1) components of functional and algebraic reasoning, such as describing inputs and outputs and describing; and 2) components of a mathematical modeling cycle, such as making assumptions and simplifying, exposing ideas to critique, and testing algorithms. Open coding was used to allow for the emergence of unanticipated functional and algebraic reasoning.

Finally, a third set of *a priori* codes, based on the entrepreneurial framework (see Figure 1), were included to allow for the engagement and mathematics learning codes to be cross-referenced with the processes and characteristics of entrepreneurship. After completing the coding process and cross-referencing the codes with the entrepreneurial framework, themes were developed to explain how the *Building Algorithms* challenge and the D&P Challenge framework supported and promoted students' engagement and mathematics learning. These themes will be reported at the start of the Discussion chapter.

Monday (Day 1)

Understanding Entrepreneurship. I started the study by exploring students' familiarity with pitching and entrepreneurship. During this discussion, it was revealed that students had little familiarity with either pitching or entrepreneurship. Shaun, a member of Team C and a rising 7th grader, described an entrepreneur as "someone that likes to make stuff different...they use their ideas, but they also use other's ideas to make them better." Other students built on Shaun's characterization of entrepreneurs as combining ideas to make something new. Building on Shaun's characterization of entrepreneurs as combining ideas to make something new, Denise, a member of Team A and a rising 7th grader, introduced the potential for entrepreneurs to have a social impact. After watching a video of a young entrepreneur discussing her lemonade business, Denise stated, "I think what makes [her] an entrepreneur is that she used her, she used a cookbook recipe for lemonade, but then she also used something that she likes, to add something else to the lemonade, which made it better. And she used it for a good cause." Kim, a member of Team C and a rising 6th grader, added actionability to the group definition of entrepreneurship, stating, "...entrepreneurs actually use [their idea] and put it into action."

Although students latched onto this characterization of entrepreneurs as problem solvers who take action and consider social impact, they either did not consider business as part of entrepreneurship or did not connect the business aspect of entrepreneurship with "taking action." That is, students did not immediately recognize that the business component of entrepreneurship makes it possible for entrepreneurs to build, market, and sell their solutions. For example, when asked why entrepreneurs start businesses and make money, Donald, a member of Team A and a rising 8th grader, responded, "maybe it's just to, well, she's using it to donate, to donate to like people who are also helping to save them." Kim added, "I don't know, maybe just make money

that she can use and spend.” For students, the purpose of making money was to help a good cause (e.g. through donations) and not to support business activities, such as paying for employees or infrastructure: a perception that, for the rest of the competition, contributed to how they assessed the actionability of their solutions.

Understanding Algorithms. Following the discussion of entrepreneurship and a brief overview of the goals of the competition, I facilitated a whole-group discussion about algorithms to probe students’ familiarity with algorithms and to leverage students’ out-of-school knowledge to establish the problematic (Confrey, 1991) of the challenge. Two students stated that they had heard the term “algorithm” in their math classes, but only Maria, a member of Team B and a rising 8th grader, could elaborate on how they used algorithms in her math class, stating, “...well, we would have like standard algorithms, like for geometric sequences, I think...it kind of gives you a basis for what you ought to do.” Maria’s description suggested an awareness that an algorithm represented a process, which she called a “basis for what you ought to do.”

Of the six students in the first iteration of the study, Maria had the most developed understanding of algorithms and, other than Denise, was the only one who had even heard the term prior to the study. However, when framed within the context of familiar companies that use algorithms, such as Apple Music and Netflix, students were eager to share their knowledge, suggesting they had some interest in and familiarity with algorithms, despite not being able to explain the meaning of the word. For example, when asked whether they were familiar with companies like Apple Music that give personalized content recommendations, and whether they were ever surprised by the accuracy of those recommendations, students said, “no.” Kim explained her lack of surprise, stating,

Because they take the things that you're already watching or you're already listening to... and take that into account (the type of thing, the band or the actors) and then they use all of that to generate another thing that they think you might like.

Kim's description, based on her familiarity with Netflix and Apple Music, implicitly referenced the functional relationship between the inputs (characteristics of what a user previously listened to) and outputs (what users should listen to next) of an algorithm and the need for that relationship to be consistent and coherent (Confrey & Maloney, 2006, 2007). In other words, users should be able to understand the relationship between what they have listened to in the past (inputs) and what Apple Music recommends they listen to next, and that relationship should be consistent across all users. Kim's ability to use her contextual knowledge of a real company to explain how algorithms work was important for the development of her team's algorithm. From this first description of algorithms, Kim could describe what her team's algorithm should do and that description guided the decisions they made throughout the week as they built their algorithm.

Introducing concrete and familiar examples of algorithm-based businesses resulted in heightened student engagement and provided an opportunity for students to share their informal understanding of algorithms. This use of concrete examples was also used to introduce and create a need for the three components of algorithms as described by Knuth (1974): input information, output information, and "a precisely-defined sequence of rules telling how to produce specified output information from given input information" (Knuth, 1974, p. 323). In planning the implementation of the *Building Algorithms* challenge, I mistakenly believed that for students to identify these components for a rating or ranking algorithm, they first needed to perceive a need for systematic comparison. That is, I expected students would see rating and

ranking as a process for comparison, which would then create a need for a set of input information that could be generated for the complete the set items.

Thus, in an effort to draw students' attention to a need for systematic comparison, students were asked to rank the top 5 places they would want to visit and to describe their reasons for ranking one location ahead of another. During the discussion, all students contributed ideas and were eager to share their lists, both with their partners and with the rest of the group. However, the activity was not successful in creating a need for systematic comparison. Instead of comparing their locations using a shared set of factors, students identified different factors for each location. An example of this can be seen in Shaun's explanation of why he selected Peru and "the mountains" as his top two locations.

Okay, so first one I said was mountains because it's pretty high up and not too hot all the time and you can see everything. Number two, I said Peru, because the food there is really great and when I visit in summer it's winter over there, so it doesn't get really hot.

Shaun referred to one common factor (temperature) in comparing two of his locations. However, he did not use the other factors he mentioned (e.g. "you can see everything" for the mountains and "the food...is really great" for Peru) for both locations. That is, Shaun used altitude to evaluate the mountains and food to evaluate Peru, suggesting that the activity did not create a need for a systematic comparison based on a shared set of input information. In fact, by the end of the competition, it appeared that students' algorithms did not emerge from a need to systematically compare objects. As will be discussed later, students' algorithms instead emerged from a need to inform users' decision making.

Launching the Challenge. To launch the *Building Algorithms* challenge, students were shown the challenge statement video, in which Cathy Yee, the founder and Chief Executive

Officer (CEO) of a company that uses algorithms to rate movies, describes her company and the challenge. Students then read and discussed a written version of the challenge statement. In the whole-group discussion students appeared to understand the requirements of the challenge. They all recognized that they needed to build an algorithm that rates or ranks something they care about and that can be the start of a business.

According to Denise, their challenge was to "...build an algorithm that uses other people's opinions to rank something...that we care about." Shaun built on Denise's description, stating "...we've got to make an algorithm to rate what people think of something and we've got to make money off of it, and have a way so people know how it works." After not recognizing the role of business in his initial description of entrepreneurship, Shaun highlighted it in the context of the algorithm challenge. Additionally, in explaining that people need to know how their algorithm works, Shaun added the criterion that their algorithm be transparent, in that users should be able to understand how their algorithms work.

After watching the challenge statement video and discussing the challenge statement document, students had a baseline understanding of the expectations for the challenge. In addition to establishing a goal for students' work, the challenge statement video also supported students' engagement by enhancing the authenticity of the challenge. According to Kim,

...the first time, like, I read about the challenge, it was kind of just like okay you have to, okay you have to make an algorithm and apply it to something. But when, but when we watched the video it kind of showed me how...[Cathy Yee] used the algorithm...with her company and how her company still does, like, things that we use every day...[I]t kind of connected everything for me. Like how...I can use an algorithm to make something and it's like not going to just be like, Oh, just apply this to this.

It was important for Kim to know that the work she was doing during the competition was like the work being done by real companies, and this knowledge was gained via the challenge video.

Brainstorming Solutions. Teams started their work on the challenge by brainstorming ideas for what their algorithm would rate/rank and the variables on which those ratings/rankings would be based. To allow the authenticity of the situation and context to drive the design of teams' algorithms and create a need for the support resources, students were encouraged to brainstorm contexts and variables before delving too deeply into the details of their algorithm.

Team A. Beginning on day 1, Team A struggled to collaborate effectively. Donald was reluctant to propose ideas for his team's algorithm, leaving the responsibility for generating ideas to Denise. Donald attributed his reluctance to a wish to remain private, stating, "Yeah, I'm just going to go with the flow with other people. I usually just go with...other people's ideas...Everything I do is...basically private." As a result, Donald did not engage during the brainstorming process. Left to brainstorm on her own, Denise did not know how to begin and struggled to come up with ideas for her team's algorithm. Donald's reluctance to participate in the brainstorming process and Denise's difficulty identifying a context highlights several important insights relating to this type of open-ended entrepreneurial challenge. First, in an open-ended challenge, especially one in which students must pick their own context, students are likely to have difficulty knowing how to begin. Students may need support to jump start their creative energy. Second, brainstorming innovative ideas and exposing those ideas to critique inherently involves making oneself vulnerable. Brainstorming is a risky proposition and one that takes courage. Thus, Donald's desire for privacy is not surprising and Denise showed considerable courage in exposing her out-of-school interests to critique.

After seeing that Donald and Denise were making little progress brainstorming contexts, I asked Denise to talk about her interests, which led to her to identify a context for her team's algorithm. Based on her interest in horses and her concern for racehorse safety, Denise suggested building an algorithm around horse racing tracks, with the goal of improving racehorse health and safety. Denise explains their decision and rationale, stating, "I was thinking, like, how they take care of the horses...That's a really important thing. So, I think like rating them on how well they treat their horses. I want to see how healthy the horses, on the health of the horses." Denise's interest in horses and her knowledge of horse racing tracks drove her identification of a context in need of an algorithm and, thus, an entrepreneurial opportunity.

Additionally, both Denise and Donald leveraged their knowledge of horses to identify variables on which they would judge a racetrack's treatment of horses. According to Denise, when evaluating the health and treatment of horses, one should consider factors like, "if there's any injuries or if their tendons or anything are like swollen or anything like that and their backs too, 'cause sometimes they could have like strain on their backs." At this stage in the competition, neither Donald nor Denise knew how to build an algorithm and their selection of variables was based on identifying the factors that they perceived to be authentic to the task of evaluating horses' health. Although they had not yet considered how to operationalize their variables, the process of defining those variables was central to Team A's process of building their algorithm and, thus, algebraic expression.

Team A spent the rest of this first day working to define their variables and their algorithm, but their understanding of algorithms impeded their progress. At this point in their work, Team A conceived of an algorithm as a process by which they used their variables (e.g.

horse health) to assign a rating to a racetrack. When asked to explain her team's algorithm, Denise stated,

Well, first we do, like, a lot of research, like, past history and we host a few horse examinations and then we look into, like, treatments, health, and um, we also use some of the horse's owners, like their opinions about the track to help us. And then we think about all of this and then we create a rating that we think is most suitable for it.

Denise's description includes important components of algorithms, such as inputs (e.g. "past history" and opinions of horse owners) and outputs ("a rating that we think is most suitable). However, at this point in the competition, Team A had not yet defined a process by which they would turn those inputs into outputs. Moreover, because Team A's description of their algorithm included all relevant parts that were identified in the challenge statement video, they did not perceive a need to make their process consistent or operationalize their variables.

Believing they had sufficiently described their process, neither Denise nor Donald knew how to proceed, and they began seeking out helpful resources, which included the spreadsheet resource: an interactive spreadsheet with guided examples to help students 1) learn about ranking and rating algorithms, 2) learn the language and syntax necessary for programming a spreadsheet, and 3) explore example algorithms. When Team A accessed the spreadsheet resource, they used it as a textbook: reading and taking notes, but not attempting to complete the algorithm examples. Although Denise claimed the resource was helpful, stating, "it helped a little bit," she did not, after finishing her notes, suggest changes to her team's algorithm or begin building an algorithm in the spreadsheet. When accessed independently, the spreadsheet resource initially did not support Team A's cognitive engagement and did not help them make progress on their algorithm.

Team B. Team B began their search for a context by considering their shared interests, with Maria stating, “so I think the first thing we should do is figure out what we want to be ranking in general. So, what are some things that you like to do?” Maria then suggested, “we could have people rank⁵ YouTube channels.” After selecting their context, Beatrice and Maria then began brainstorming variables to use in their algorithm, focusing more on whether a variable was indicative of the quality of a YouTube channel than on whether the variable would work for their algorithm. In this way, the context, YouTube channels, and the meaning of the variables drove the mathematical development of their algorithms, even when those variables introduced complexity. For example, Beatrice and Maria originally considered the following variables for their algorithm: type of content, the language spoken in the video, the number of times a YouTube channel’s videos were viewed, and the number of users who have subscribed to a YouTube channel. These variables included both quantitative (number of views, number of subscribers) and categorical (content of the video, language) variables, which, while authentic to the context of rating YouTube channels and reasonable to include in a single algorithm, adds complexity to the process of building an algorithm.

Team B also considered whether some variables should be more important than others in determining the quality of a YouTube channel. They identified the variable “content” as being the most important variable and, thus, the one that should be, in Maria’s words, “weighted most heavily.” By type of content, Beatrice and Maria were referring to how they would classify YouTube videos according to their content, such as “reacting to and gaming” videos (Beatrice) and “cake decorating” videos (Maria). While it is not clear the extent to which Beatrice and

⁵ At this early stage of the competition, team B had not yet discussed the difference between ratings and rankings, and their use of the word “rank” did not appear to indicate a consideration of whether ratings or rankings were better suited to their context.

Maria, on day 1, understood how to weigh variables differently in an algorithm, they both agreed that not all variables should be of equal importance and that the type of content should be the most important variable. In this way, the context and students' out-of-school expertise supported them to begin considering the structure of their algorithm.

After further discussion, Beatrice and Maria decided to include the following variables in their algorithm: 1) content, 2) uploads (i.e. how frequently a YouTube channel uploads videos), 3) length (i.e. the length of an average video on a YouTube channel), and 4) views (i.e. how many a YouTube channel's videos have been viewed). As Team B considered how to operationalize their variables, they recognized the challenge of including both categorical and quantitative variables in their algorithm. They also recognized the dangers of including quantitative variables measured on vastly different scales. For example, Beatrice and Maria determined that including both views (a quantitative variable that could have a magnitude in the millions) and uploads per month (a quantitative variable whose magnitude was unlikely to exceed 30) could impact the results of their algorithm. Their first attempt at operationalizing their four variables involved quantifying the lone categorical variable (content), by applying an evaluative scale, and using a binning system (see figure 4) to lessen the impact of the much larger (or smaller) quantitative variables.

Figure 4

Recreation of Team B's Binning Approach

Uploads – a month Content – 0-10 enjoyment Length – 0-10 mins, 10-20 mins, 30-40 mins, 40-50 mins, 50-60 mins, 60+ mins Views/Likes – 0-100k, 100k-500k, 500k-1 million, 1 million - 5

Note. This figure shows how Team B's binning approach for operationalizing their variables.

In their system, content, a categorical variable, would be rated on a scale of zero to ten based on users' "enjoyment" of the content. Length would be assigned a numerical value from one to six based on whether the average video length was 0-10 minutes, 10-20 minutes, 20-30 minutes, 30-40 minutes, 40-50 minutes, or 50-60 minutes. The number of views would be assigned a numerical value from one to five based on whether the channel had been viewed 0 to 100,000 times, 100,000 to 500,000 times, 500,000 to 1,000,000 times, or 1,000,000 to 5,000,000 times. The "uploads" variable would be assigned a numerical value equal to the average number of times the YouTuber uploaded a video per month. This method of converting seemingly inconsistent variables for use in their algorithm demonstrates how the authenticity of the context and the need for the variables to be similarly authentic drove students' mathematical thinking and creativity.

As Team B moved towards building their spreadsheet algorithm, they realized that they did not know how to program a spreadsheet and independently accessed the spreadsheet resource. Team B also did not engage with the interactive algorithm examples in the spreadsheet resource and, although it initially did not help them gain the necessary knowledge or skills to program a spreadsheet, it did introduce additional considerations for their algorithm. Specifically, after reading the spreadsheet resource, Maria and Beatrice discussed whether to build a ranking algorithm or a rating algorithm and, as they considered the tradeoffs of each type of algorithm, they began to clarify the purpose of their algorithm. For example, Beatrice claimed, "...I think if we rate [YouTube channels], [users would] be most likely to get something that they would enjoy than something that is ranked." Maria then referenced two YouTube channels, adding,

Because if I say that I think that Jacksepticeye is better than Markiplier, I mean that just tells me that I'd rather watch Jack, it doesn't tell me what I like about his videos and what I would want to see in other people's videos and so I'll just keep like watching more Jacksepticeye instead of, 'here, look at this. We've found that this video has something similar that you might enjoy as well.'

In arguing for a rating algorithm, Maria and Beatrice referenced an important difference between ranking and rating algorithms. A ranking algorithm allows for a direct comparison of objects by establishing ordinal relationships. For example, a ranking algorithm would allow users to see that Channel A is better than Channel B because Channel A has a rank of 1 and Channel B has a rank of 2. However, it would not allow users to determine how much better Channel A is than Channel B because the magnitude of the difference between two ranks is not meaningful. Furthermore, users would not be able to determine if they would like Channel A, they would only be able to determine whether they would like Channel A more than Channel B.

Rating algorithms, while not allowing for direct comparison, produce numerical values that can inform users about 1) how much better one channel is than another and 2) when paired with a maximum rating, how "good" those channels are. In stating, "...[a ranking algorithm] doesn't tell me what I like about [Jack Septiceye's] videos and what I would want to see in other people's videos..." Maria is referencing a limitation of ranking algorithms that is a strength of rating algorithms. A ranking algorithm would allow users to know if Jack Septiceye's videos are better than Markiplier's, in terms of their content, length, uploads, and views. It would not allow users to know if they would like Jack Septiceye's videos, based on those same four variables. Thus, Maria and Beatrice were considering the tradeoffs between rating and ranking algorithms and recognized that, while a ranking algorithm is useful for comparing and ordering YouTube

channels, it would be less useful in helping users decide whether they would like a YouTube channel and, thus, whether they should watch it. By considering the tradeoffs between ranking and rating algorithms, Beatrice and Maria began to gain clarity about the purpose of their algorithm.

Team C. Team C began brainstorming by creating a table of potential topics for their algorithm context. This table included three columns. In the first two columns Shaun and Kim listed topics that they found interesting. In the third column, they listed topics that they thought the “general public” would find interesting (see figure 5). Using this process, Team C selected music as the context for their algorithm. Team C planned to have users respond to a survey and, based on those responses, their algorithm would return a list of songs that would be of interest to the users.

Figure 5

Team C's Brainstorming Chart

		Idea:
restraunts restraunts attractions housing/hotels music/ instruments	music art shopping	actors/movies social media music artists Use music people/the consumer are listening to, to recommend new music/artists

Note. This figure shows the chart Team B used to brainstorm ideas for their algorithm context.

Whereas both teams A and B leveraged their personal interests to identify a context and thus engaged in the entrepreneurial process of resource analysis, Team C leveraged both their personal interests and the interests of potential users and customers. They integrated opportunity

analysis (i.e. what will potential customers care about?) and resource analysis (i.e. what is our expertise and what do we care about?) in selecting the context for their algorithm.

Additionally, Team C's familiarity with algorithm-based businesses, such as Netflix and Apple Music, influenced their brainstorming process. For example, as they were brainstorming ideas, Kim suggested an algorithm that resembled the business algorithms (e.g. Netflix and Spotify) she referenced during the whole-group discussion, stating "so, maybe, like, do a survey of what people are listening to, ...the music that they like and are already listening to...and using that information...to recommend new music, new artists." Whereas Team B leveraged their understanding of algorithms and YouTube while brainstorming and operationalizing their variables, Team C instead approached the challenge from the business side, leveraging their familiarity with similar algorithm-based businesses. Furthermore, considering these algorithm-based businesses helped Team C define a purpose for their algorithm: helping users decide what songs to listen to.

Having selected their context and identified a purpose for their algorithm, Team C then began constructing a survey that would be given to users and that would be used to generate a list of recommended songs. The survey included the following four questions: 1) what genres of music do you like? 2) what artists do you like? 3) what instruments do you like? and 4) what artists do you dislike? Team C's focus in selecting variables was whether those variables were authentic to their stated purpose of helping users decide what music to listen to instead of whether those variables would work in their algorithm. For example, Team C included a question asking users to select music genres that they liked.

Kim: So, like, also, should we...be doing something for, like, a year...kind of like [what] decades, like, do you like? I know that sounds weird, but, like, I know some people like old stuff or some people...new age stuff.

Shaun: That's like the genre, like 80s or 90s.

Kim: Okay. Any other genres you want to add?

For Kim and Shaun, identifying the possible categorical values for the variable “genre” was more important, initially, than how they would or whether they could use it in their algorithm.

As Team C transitioned to building their algorithm in the spreadsheet resource, Kim stated that she was “not good at technology” and quickly became frustrated over not knowing how to make the algorithm do what she wanted it to do. When asked what it was that she wanted their algorithm to do, Kim explained that she wanted it “...to be able to...take these, these categories that we've written down and take that into account and then have it, have it supply another thing that we've like we've put in information about that kind of goes with that stuff. But I don't know how to do that.” For Kim and Shaun, their algorithm represented a sequential process for which the survey responses (or variables) were inputs and the list of recommended songs were outputs. Kim’s frustration was over not being able to program the spreadsheet to carry out the rule for mapping their inputs onto their outputs. Based on this confusion and frustration over not knowing how to make the algorithm do what they wanted it to do, Kim and Shaun accessed the spreadsheet resource. For Team C, as well as for Team A and Team B, the challenge created a need for additional learning, which they sought via the spreadsheet resource. Unlike Team A and Team B, Kim attempted to work through the examples in the spreadsheet resource, but in her head and not by programming the spreadsheet. Shaun, who had prior

experience with spreadsheets, did not work on the spreadsheet resource and, instead, began experimenting with Google Sheets functions in a blank workbook.

This split in their use of the spreadsheet resource represents a division of Team C's work responsibilities that was beginning to take shape by the end of day 1. Shaun, having experience with spreadsheets and programming, assumed the role of technology expert and took charge of building his team's spreadsheet algorithm. Kim, who stated she was, "not good at technology," assumed the role of the music expert and took charge of building her team's survey and music library.

By the end of day 1, all teams had selected a context for their algorithm and had begun identifying variables to use in their algorithm. Although the three teams selected different contexts (horse racing tracks, YouTube channels, and music) and were building what appeared to be qualitatively different algorithms, they all included or were considering components that are central to the definition of algorithm. These components included, 1) a shared set of well-defined input information, 2) a set of well-defined output information, and 3) a process for mapping the input information onto the output information (Knuth, 1974).

Tuesday (Day 2)

Understanding Business Models and the Role of Investors. Day 2 began with a whole-group discussion designed to 1) probe students' understanding of investors and how they evaluate businesses, and 2) introduce and create a need for the Key Business Proposition. During the discussion, only Maria could explain the role of investors, stating, "...[an investor is] somebody that can, like, give you money for you to work on a project and in return gets a certain percentage of the profits." Once the group had a shared and basic understanding of investors and their role in business, students were eager to participate in the discussion, suggesting things that

would help them decide whether to invest in a company. For example, Shaun highlighted the importance of having a customer base, arguing that investors will want to know "...if [the business is] going to become popular and people [will] spend money on it." The identification of a target customer base is an essential characteristic of entrepreneurship. In an entrepreneurial setting, students need to consider how to solve problems in ways that meet the needs of customers. The Key Business Proposition was designed to support them to do this by providing a structure for explicitly mapping solution features onto customer needs.

Shaun then added the importance of positioning an entrepreneurial solution in relation to its competition, arguing that investors would want to know how one's product compares to existing solutions. As Shaun explained, "investors might want to see if...other people are doing what they're doing. 'Cause if a lot of other people are doing the exact same thing, then it's not going to fit in with all the other ones." He recognized that, for a business to succeed, it needed to improve on or be unique from existing solutions.

Finally, Denise and Maria suggested components of pitching as integral to business success, a process also scaffolded by the Key Business Proposition in the elevator pitch framework. For example, Denise suggested that investors would want to gauge entrepreneurs' confidence in their business ideas, stating, "...I also probably see like how serious they are about it...like if they really think that their idea is going to, like, make a really, like, a really good business, that brings in a lot of money." Maria built on Denise's suggestion, stating, "one of the factors that I would take into account would be how well they present their idea and how they pitch it, whether they're like focused and they're well thought out and like if they know what they're doing." Denise and Maria viewed the ability to clearly, concisely, and confidently pitch one's idea as central to entrepreneurship.

Although they recognized the importance of identifying target customers, students did not independently consider the role played by the type of business model a company uses to sell their product or service. To probe students' understanding of business models, I asked students to describe how familiar businesses make money. Students were eager to participate in this part of the discussion, describing familiar businesses and their business models, some of which corresponded to the business models included in the business model types document. For example, Donald explained how Subway makes money by selling a physical product (sandwiches) to customers. Both Kim and Maria demonstrated an understanding of how web-based services make money through revenue from advertisers and from users. Kim described how Apple Music uses a subscription model that requires users to pay a monthly fee to access their library of music, and Maria explained YouTube's freemium model, through which users can access YouTube's services for free and YouTube's revenue comes from advertisers, or pay a monthly fee for premium services, such as ad-free viewing.

Building Solutions and Defining the Business. After the whole-group discussion, teams returned to their workstations⁶ and continued building out their solutions.

Team A. The business model discussion offered an opportunity for Donald to become more engaged. Immediately following the whole-group discussion, Donald stated, "I have an idea for our business: subscription." On day 1, Denise and Donald had struggled to find a way for their algorithm, which rated horse racing tracks, to make a profit, specifically because they could not identify their customers. They knew who they wanted to help (horses and horse owners), but they were having difficulty deciding who would pay for their service. On day 2,

⁶ Halfway through day 1, I was concerned teams' physical proximity was hindering collaboration. After consulting with members of the research team, I assigned teams to separate workstations (cubicles) in the research office.

Denise and Donald invented a creative solution to this problem by identifying the racetrack owners as their customers and the horse owners as their users. As Denise explained,

...we help [racetracks] get better ratings, more business and like we're, since we look at like, horse health and treatment and like that general idea of we kind of help them, um, we help other people kind of trust their racetrack to take care of their horses. Cause I know I personally would not let like someone anywhere near my horse.

Denise and Donald recognized that, for their business to work and have a positive impact on the health of horses, they needed to find a way for the racetracks, as Denise described it, "to get something out of it, too." Team A identified the problem they wanted to solve (poor treatment of horses at horse racing tracks) and, in considering business models, found a creative way to turn the problem into an entrepreneurial opportunity without exploiting the people their algorithm was designed to help. Denise justified her team's decision to rate horse racing tracks, stating,

Because um, racetracks, like, I know that they need their ratings to be good. Like, um, there was one racetrack, I think it's Santa Anita, something like that. Uh, they started getting pretty bad ratings cause a lot of horses...started dying by them. They killed more than 10 horses. I know that. So now everyone wants it to be shut down but that racetrack's pretty lucky cause for some reason they can't just be shut down. I don't remember what the reason was. But, like rating is a big impact on their business, too, and the money they make.

Based on her knowledge of horse racing tracks and her realization that racetracks will care about receiving good ratings, Denise discovered an entrepreneurial opportunity for her team's algorithm. Their algorithm would help horse owners decide whether their horses would be safe at a track, by providing ratings on the treatment of horses at those tracks. However, instead of

charging the horse owners for this service, Team A decided their customers would be the racetracks that were being rated. Thus, Team A demonstrated creativity in finding a way to make their solution actionable, without exploiting the suffering of either horses or horse owners. Instead, the racetracks would bear the cost of the service.

Having decided on a business model type and identified their customers, Team A spent most of day 2 describing their algorithm: a five-step process by which racetracks would be assigned ratings based on three variables (horse examinations, horse treatment, and horse owners' opinions; see figure 6).

Figure 6

Team A's Initial Algorithm

-
1. We research about the track (research)
 2. We ~~will~~ host a horse examination.
 3. We look into how they treat their horses
 4. We use the horse owners opinion of the track to help us
 5. We factor this all in and create a rating (we do all this while offering a rate)

Note. This figure shows Team A's description of their algorithm on day 2.

Although Team A's description of their algorithm now included inputs, outputs, and a general process for using the inputs to generate outputs, their process did not appear to be systematic and their inputs had not yet been operationalized. For example, it was not clear how they would "...factor [horse examinations, horse treatment, and horse owners' opinions] in and create a rating" for racetracks (step 5 in their process). Team A's conception of algorithms did not require them to make step five systematic and, as a result, they were content with their algorithm as described and felt they had completed their goal of building an algorithm. Thus, instead of working on building their algorithm into the spreadsheet or using the spreadsheet resource, Team A spent much of day 2 writing and rewriting their five-step process. In doing so,

Team A refined the inputs and outputs of their algorithms and, by the end of day 2, described their outputs as the ratings assigned to racetracks, and had started considering how those ratings should look, specifically referencing Google's five-star rating system.

Compared to their description on day 1, Team A's day 2 description of their algorithm reflected an improved understanding of what input information they would use in their algorithm and a more precise description of how the process by which their input information would be used to generate their output information. This improvement in their description demonstrates how the multiple opportunities for students to practice explaining their algorithm supported their developing understanding of algorithms.

Team B. Team B began day 2 with Maria excitedly reporting that she had been thinking about their business for rating YouTube channels the night before and had an idea for how to change their algorithm.

...So, the original way we had thought about ranking was by like numerical statistics instead of based on customers' opinion. So, I feel like we should keep the same categories, 'cause I feel like that's, like, I feel like those are good categories, but I feel like we should do it based on how the customers feel about it. Like, whether they think they upload not enough or whether they upload, like, way too much.

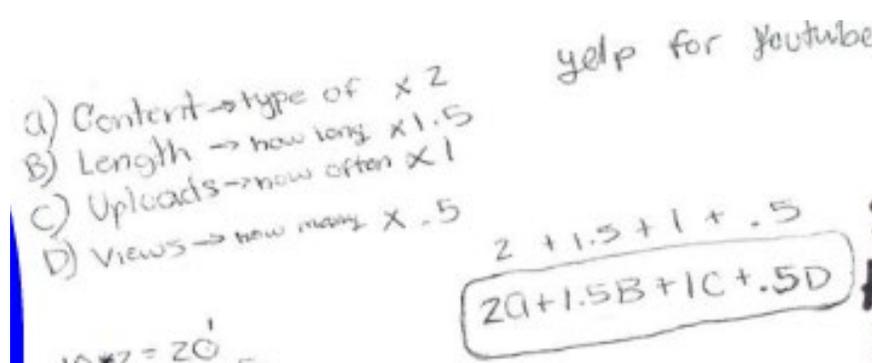
Based on her perception of the needs of users, Maria proposed changing her team's variables from representing descriptive measures (e.g. number of views or number of uploads) about the YouTube channel to representing "consumer ratings," or evaluative measures, of the YouTube channel. Maria justified her suggestion by claiming, "...it would be better to have a product that's less aimed at the YouTube channels and more aimed at what the consumers want to see." Their knowledge of consumers' needs, gained from their experience as YouTube viewers,

combined with the purpose they identified for their algorithm, led Team B to change how they had operationalized their variables. In this way, the perceived needs of the users influenced the mathematical development of their algorithm.

Next, having previously decided that some variables (e.g. content) should be more important than others (e.g. views), Team B discussed the specific weights that would be applied to each variable, with Maria communicating her thinking using an algebraic expression (see figure 7) and explaining that content should be “twice as important as uploads.”

Figure 7

Team B's Weighted Variables



Note. This figure shows the weights Team B assigned to their variables and the algebraic expression they used to represent their algorithm.

Maria and Beatrice then began working to program their spreadsheet to carry out their algorithm. Neither Maria nor Beatrice had programmed a spreadsheet before the start of the competition and neither student knew how to program the spreadsheet to carry out their algorithm. As Maria explained, they wanted their spreadsheet to:

...take these four different categories and have them, like, have different weights. And we figured out that we want [content] to have twice as much and [length] would have, like, one and a half times as much, and [views] to have like, I want to have one half but we don't know how to put that in the spreadsheet.

Identifying this gap in their knowledge and recognizing a need to fill that gap to be able to complete the challenge, Beatrice and Maria consulted the spreadsheet resource and began experimenting, with Maria taking the lead and Beatrice looking on. Unaware of the syntax or language necessary for programming a spreadsheet, Maria was unable to program the spreadsheet to carry out her team's algorithm, explaining,

...we want to do it by like A, B, C, D, so we just do like cell A. What, would we do 2 times cell A or would it work like that? How do we put that into the spreadsheet? Is there a way we can, like, take it so that when you enter the numbers it like automatically does it? Or, do we have to multiply ourselves?

Team B wanted the spreadsheet to automatically apply their algorithm to multiple YouTube channels. As Maria explained, "I want to [have the spreadsheet calculate ratings] for like a whole column." Maria understood how to use an algebraic expression to represent their algorithm but, because she did not know the language or syntax needed to program a spreadsheet, she was not able to build their algorithm into the spreadsheet. For Team B, the shift from an algebraic expression to spreadsheet formula presented a barrier. In an algebraic expression, a single symbol can be used to represent any possible value from within a domain for a given variable. For example, in Team B's algebraic expression, the symbol "A" represented any possible user rating from 1 to 10, and the same algebraic expression (i.e. $2A+1.5B+1C+0.5D$) could be used to represent the relationship between the variable ratings and overall scores for any YouTube channel. In programming a spreadsheet, a Maria needed to use a different symbol to represent the content rating for each YouTube channel. For example, in figure 8, the variable representing "content" in Team B's spreadsheet formula changes from row

to row. Maria's algebraic expression could be thought of as a generalization of the successive spreadsheet formulae, where the symbol "A" is used in place of B2, B3, B4, and B5.

Figure 8

Spreadsheet Supporting Shift to Algebraic Reasoning

	A	B	C	D	E	F
1		Type of Content	Length	Uploads	Quality	Total Score
2	SsniperWolf	10	9	9	10	=2*B2+1.5*C2+...
3	James Charles	4	7	6	9	=2*B3+1.5*C3+...
4	Kyle Exum	10	9	6	10	=2*B4+1.5*C4+...
5	Daniel Howell	10	9	4	10	=2*B5+1.5*C5+...

Total Score = 2A + 1.5B + 1C + 0.5D

Note. This figure shows how spreadsheets can support students to shift from numerical expressions to algebraic.

Although this feature of spreadsheet language and syntax can be helpful in introducing algebraic expressions by linking students' arithmetic reasoning to algebraic reasoning (Blanton et al., 2015), it presented a barrier for Maria. Maria's prior experience with algebraic expressions made this intermediate step unnecessary.

After observing Team B struggling and failing to program the spreadsheet to carry out their algorithm and sensing a drop in engagement due to their rising frustration with the spreadsheet, I decided they needed additional support. I recognized that their inability to make progress could be attributed to them not knowing the language or syntax needed to program a spreadsheet, and I concluded that they needed more explicit support to use and understand the spreadsheet tips section of the spreadsheet resource. Thus, I supported Team B to learn the necessary programming language and syntax, including how to name variables and use the

autofill command. When Team B finally succeeded at programming the spreadsheet, Maria reacted with visible excitement and relief, stating, “I know how to program a spreadsheet!” This excitement carried over to the rest of day 2, with Maria excitedly showing off her team’s working spreadsheet algorithm.

Team B’s eagerness to share their algorithm, created opportunities for them to expose their ideas to critique and, in doing so, to reflect on the meaning and structure of their algorithm. For example, after developing an initial prototype for their algorithm, Maria excitedly explained how their algorithm works to a member of the research team, stating,

So, let's say you gave [a YouTube channel] a nine for the type of content. For the length of [the channel's] videos, they thought the videos were a bit too short, so they give it a six. They're still like good videos, you know, hold on a second, that doesn't seem right. Oh yeah, that is right. All right. Then they keep doing that. That's the third category. They think they upload really nicely. They have their schedule worked out perfectly, down to the second...they give it a 10. Okay. And then they get a good amount of views, not like hundred million billion, like every single video, like, enough people watch it. Okay. And they give it like, you know, good. It's pretty good. Give it an eight and bam, the overall score is 37.

In her explanation, Maria 1) described the meaning of the input variables in relation to the context, including what different values tell you about the YouTube channel; 2) expressed a functional and sequential interpretation of algorithms; and 3) evaluated the coherence of the algorithm (i.e. do the results make sense?) in real-time by examining the automatically generated ratings. In this way, the spreadsheet supported Team B to attend to the structure and meaning of their algorithm and to practice explaining their algorithm.

Team B also spent time on day 2 working on their Key Business Proposition, focusing specifically on the elevator pitch framework and using it to justify their algorithms and businesses. When asked whether the Key Business Proposition influenced the design of their algorithm, Maria explained that it did not, stating, “we had already decided...that we should make [our algorithm and business] more like consumer-focused for the consumers rating it.” However, while engaging with the elevator pitch framework, Team B refined how they described their business and algorithm, making both more precise and distinct from potential competition. As Beatrice explained, “...if you're an entrepreneur, you have to make sure that you're sharing what your company or your product does...like you should be able to explain it and, like, kind of quickly and...get to the point of what it is.” In other words, being able to clearly and concisely describe one’s business and algorithm is essential and, for Team B, the Key Business Proposition served this purpose.

For Team B, becoming more precise in how they defined their business and algorithm involved researching their competition: companies that also rated YouTube channels. As they discovered companies that did similar work, like Social Blade, they found ways to distinguish their company. Thus, the relationship between the Key Business Proposition and Team B’s algorithm-based business was bidirectional. That is, their algorithm and business informed how they completed the Key Business Proposition and completing the Key Business Proposition informed revisions to their algorithm and business.

Team C. Team C spent most of day 2 building and testing their music filtering algorithm, with Kim building their survey and their library of songs, and Shaun programming their spreadsheet. From his prior experience with spreadsheets, Shaun had a basic understanding of spreadsheet programming language and syntax. However, he did not know how to program a

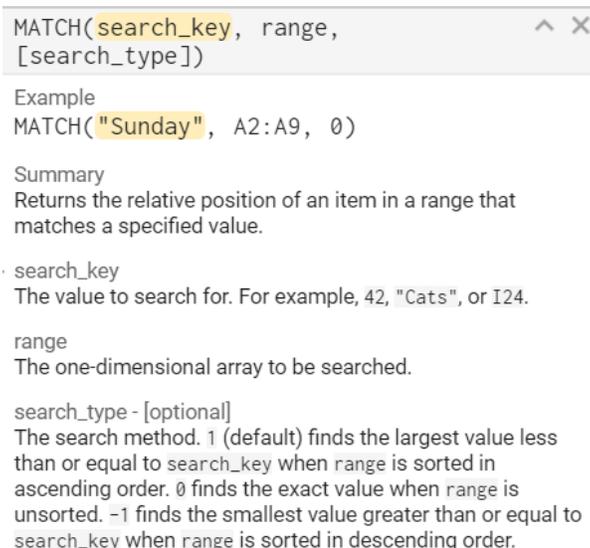
spreadsheet carry out his team's algorithm that would filter a list of songs based on users' music preferences, such as genre (e.g. pop, jazz, rock), instruments, and artists they like or dislike.

Although Team C accessed the spreadsheet resource, it did not include examples of filtering algorithms. Recognizing that he needed additional knowledge and that the available resources would not help him gain that knowledge, Shaun searched the list of functions available in Google Sheets, testing the ones that looked promising until he decided the "MATCH" function would meet his team's needs.

The "MATCH" function searches a specified range for a value (number, word, or phrase) and returns the row number of the last cell in the range that contains that value. For example, if a user selected "pop" from the list of possible music genres, Team C's algorithm would search the genre column for the complete list of songs and return the row number of the last song in the list that was classified as "pop." Driven by his investment in building a functioning spreadsheet algorithm, Shaun independently identified a necessary skill he was lacking and took steps to learn that skill. Furthermore, through evaluating the available spreadsheet functions, Shaun engaged in symbolic reasoning. Take for example the description of the MATCH command provided by Google Sheets (see figure 9).

Figure 9

Description of the MATCH Function



Note. This figure shows the Google Sheets description of the MATCH function.

In evaluating whether this description met his team’s needs, Shaun needed to 1) recognize that a cell location could be used to represent a range of possible survey responses and act as the “search_key,” 2) interpret a verbal description of the function rule to determine if it would produce the desired output for their algorithm (i.e. a list of songs), and 3) interpret the meaning of “range” and recognize that the phrase “A2:A9” could represent his team’s list of songs that would be filtered. Whereas Team B operationalized their categorical variables by quantifying them, Team C did so by leveraging the available spreadsheet functions and, as a result, engaged with and interpreted symbolic representations.

Continuing the division of labor they had established on day 1, as Shaun took the lead on programming the spreadsheet, Kim took the lead on completing the KBP and building the survey and music library. Completing the elevator pitch framework supported Team B to more precisely define and justify the features of their business, while also introducing new considerations for

their algorithm. Their familiarity with companies like Spotify and Apple Music informed their elevator pitch, which stated,

Our company helps anyone who need [sic] new music find the right thing for their specific taste in music. Unlike Apple Music, which only plays songs you already listened to and doesn't suggest new songs, our product gives you new music. Find My Music is an app where you fill out a simple four question survey and from what you put in, you get a recommendation for new music. Once you listen to the new music, you can rate it. You can change the answers in your survey to get new recommendations. We make money by selling ad space on our app.

Although Team C's claim about Apple Music not suggesting new songs was inaccurate (Apple Music does in fact recommend new music to listen to), it demonstrates how they used the Key Business Proposition and the elevator pitch framework to more precisely define the problem they were trying to solve and position themselves relative to other music services. Additionally, the process of building and refining their elevator pitch, informed Team C's understanding of their algorithm. For example, the statement that "...you can change the answers in your survey to get new recommendations," informed their conceptualization of their algorithm and introduced new mathematical considerations. Specifically, it prompted Team C to informally consider whether their algorithm should represent a one-to-one, one-to-many, or a many-to-one mapping of survey responses onto sets of songs.

Kim's understanding of the context created opportunities for further exploration and STEM learning. When asked whether their algorithm would map a user's survey response to a single song or to a list of songs, both students confidently stated that their algorithm should return a list of songs. Later, after discovering that the MATCH function would only map a

survey response onto a single song, both Kim and Shaun began exploring Google Sheets to identify a different function for their algorithm. This was the first instance in which Kim engaged with the technical aspects of building her team's algorithm and it was driven by her investment in her team's business and a concern for the needs of users. Additionally, as both students practiced describing and explaining their algorithm, those descriptions became clearer, reflecting a deeper understanding of their algorithm.

Wednesday (Day 3)

Creating a Need for the Technical Brief. To motivate the need for describing the specifications of their product (part 7 of the Technical Brief), at the start of day 3, students were asked to discuss their experiences shopping online for a product, focusing specifically on what information they needed to help them decide whether to buy it. As evidenced by students' comments, shopping online was not a familiar experience. As Donald explained, "I don't buy anything online...It's usually my parents [who] do the planning." Even for those students who had experience shopping online, the product specifications were irrelevant. Denise reported basing her decisions on whether she wanted the product or liked the brand: two concerns that did not justify the need for creating a detailed description of how a product works. However, when viewed from the perspective of an investor, students were better able to explain why part 7 of the Technical Brief was necessary. As Shaun explained, "it's important to describe the key details [of the algorithm] so investors know what they're working with, if they should support it." Kim agreed, arguing,

...[investors] are going to want to know all about the algorithm because if they don't...know about, like, what you're actually doing and about what goes into it, then...that might change their decision because, because they're going to want to know

what's happening, like other than just what the consumer's going to see. I think they're going to want to know like everything that goes into it.

The goal of convincing investors to fund a business served to motivate the need for part 7 of the technical brief, which most directly connects students' work to the targeted STEM content. Once the discussion shifted from online shopping to considering the perspectives of investors, students were able to identify a purpose for the first six parts of the Technical Brief, as well. As Maria explained, the first six parts are important because, "if something goes wrong, you can figure out where you should go to kind of fix it. And, so, documenting your process, like, [helps you] keep track of that." Despite identifying a purpose for the technical brief, once students started working, it became something they had to complete and not something that was integral to their entrepreneurial process.

Completing the Technical Brief and Building Solutions. Following the whole-group discussion, teams returned to their stations and began working on completing the Technical Brief and programming the spreadsheet to carry out their algorithms.

Team A. After two days of work, Team A had not made progress on programming the spreadsheet to carry out their horse racing track algorithm and had not yet operationalized their variables. Based on Team B's success with the spreadsheet resource after receiving support and Team A's lack of success when accessing the resource independently, I used the start of Team A's work time to guide them through the spreadsheet resource. Unlike Team B, who had specific questions relating to the spreadsheet resource and appeared to understand what it meant to build an algorithm, Team A were still having difficulty knowing what to do for the challenge. Thus, the guided introduction of the spreadsheet resource for Team A was more thorough, starting with a summary of the contents and purpose of the spreadsheet resource. Denise and Donald were

then directed to read and summarize the first example in the rating algorithms worksheet (see figure 10).

Figure 10

Example Rating Algorithm 1

Example: Suppose a company is hiring middle school students for a summer internship. They've narrowed their list of candidates to three students: Jay, Ebony, and Vania. The company gave each student a score out of 10 (with 10 being the best) in four categories and will use an algorithm to decide who to hire.

Algorithm 1: One algorithm the company can use to compare candidates is to find the sum of the three ratings for each student. In this algorithm, each category has the same impact on the total score. Use this algorithm to complete the table. Try writing a formula that tells the spreadsheet how to carry out the algorithm. Based on this algorithm, which student will the company hire?

Note. This figure shows example 1 from the rating algorithms worksheet in the spreadsheet resource.

According to Denise, the first algorithm was "...saying to add all three of [the candidates'] scores and then, like, make a final score out of it." By asking Denise and Donald to summarize the first example algorithm, I focused their attention on the algorithm as a systematic process for combining the values of the different variables: a characteristic that was missing from their earlier conceptions of algorithms.

Next, Denise and Donald were prompted to use the table in the spreadsheet resource to carry out the first example algorithm (see figure 11), which they did by mentally calculating the sum of the three scores for Jay (21) and typing it in cell J30. Denise and Donald were using the spreadsheet as a static table and not leveraging the affordances of the tool, which allowed them to avoid engaging with a symbolic representation of the algorithm.

Figure 11*Rating Algorithms Table*

	E	F	G	H	I	J
28						
29		Student	Interview Performance	Enthusiasm for the Position	Eagerness to Learn	Score
30		Jay	10	6	5	
31		Ebony	4	9	9	
32		Vania	9	6	8	
33						

Note. This figure shows the table from example 1 in the rating algorithms worksheet of the spreadsheet resource.

To encourage Team A to engage with more of the affordances of the spreadsheet, I showed Denise and Donald how to program the spreadsheet to do the calculations for them by teaching them the language (how do you write a variable in a spreadsheet formula?) and the syntax (where do you place the equals sign?) needed to program the spreadsheet. During this discussion, both students engaged with the spreadsheet tool, writing and testing formulas in the table. Their heightened engagement, compared to days 1 and 2, can likely be attributed to several factors. First, by allowing them to struggle for the first two days of the challenge, Denise and Donald were likely more receptive to instruction and benefited more from it (Kapur, 2012). They had spent considerable time grappling with the relationships between the variables in their algorithm and how to represent them in a spreadsheet, and they were now able to contextualize what they were learning during the tutorial. Second, their investment in their context and their desire to be ready for the competition further enhanced their receptivity to instruction. Finally, the hands-on nature and the affordances of the spreadsheet tool encouraged Denise and Donald to learn through experimentation and established a purpose for their activity (Ainley et al., 2005).

After learning that the spreadsheet could be programmed to carry out calculations and after learning that they could program a spreadsheet by tapping the location of the numbers they

wanted to operate on, Denise and Donald each entered the formula, “=G30 + H30 + I30” by tapping on the cells G30, H30, and I30. After tapping “Return” on their keyboards, Denise’s spreadsheet returned a value of 21, while Donald’s spreadsheet returned an error message stating “circular dependency detected.” Examining his spreadsheet formula revealed that Donald had entered the formula “=G30 + H30 + J30,” which led to a discussion about the meanings of the three variables.

Researcher: Do you know why that's G 30?

Denise: No,

Researcher: Donald, do you know why the 10 is G 30?

Donald: [shakes head no]

Researcher: Can you see a G anywhere in the spreadsheet?

Denise: Oh, it's in the column...Maybe it's like the 30th row.

Asked if they could explain why “G30” appears in the formula, Denise discovered that it represented the number that was in row 30 of column G and was then able to determine that Donald had mistakenly included J30 in the formula, which was the cell they were typing in and, hence, caused the circular dependency. The combination of immediate feedback and the spreadsheet programming language that defined variables using their location in the table supported Denise and Donald to connect the variables in their expression to the numbers they represented in the table.

After writing a successful formula using the spreadsheet language and syntax, Denise’s description of the algorithm changed from adding specific values to adding the values contained in specific cells. According to Denise, “[the algorithm] add[s] what’s ever in that box and what’s in that box and what’s in that box,” pointing to the cells representing interview performance,

enthusiasm for the position, and eagerness to learn. To further support Team A’s understanding of algorithms and to help them recognize the usefulness of algorithms, Denise and Donald were then shown how to use the autofill command to automatically calculate the overall ratings for the other two job candidates. Additionally, they were shown how they could check whether their formula worked by changing values in the table and checking if the overall ratings changed automatically.

Once they had successfully programmed the spreadsheet to carry out the first example algorithm, Team A began working on the second example: an algorithm that included weighted variables (see figure 12).

Figure 12

Weighted Rating Algorithm Example

Algorithm 2: The company decides “Eagerness to Learn” is more important than “Interview Performance.” Their new algorithm uses weights to make “Eagerness to Learn” have a greater impact on a candidate’s score. The new algorithm multiplies a candidate’s “Interview Performance” rating by 1, their “Enthusiasm for the Position” rating by 1.5, and their “Eagerness to Learn” rating by 2.

Note. This figure shows an example of a weighted rating algorithm from the spreadsheet resource.

In this algorithm, students needed to multiply the ratings for interview performance, enthusiasm for the position, and eagerness to learn by 1, 1.5, and 2 respectively, before calculating the sum of the three terms. Here, Team A’s focus on the values within specific cells interfered with their ability to automatically apply their algorithm across the three rows of the table. Denise, building on her success with the first algorithm asked whether she should change the values in each cell, stating “okay, so, I do change some of these right?...The interview performance just multiply by one so it stays the same. Should I multiply [the enthusiasm for the position] by 1.5?” Denise was trying to decide whether she needed to change the values in the

table, which would allow her to use the same formula she used for the first algorithm. Although this process would work in calculating the overall rating for each candidate, it does not allow one to input ratings for each variable and have the overall ratings automatically update. Furthermore, this process obscures the structure of the algorithm by embedding the coefficient within the variable and making it more difficult to distinguish between variable value and the coefficient value when looking at the spreadsheet formula.

I reminded Team A about the autofill command, specifically the ability to change the value of a variable and have the overall ratings automatically update, explaining,

You're trying to make it so that if you just change any of these numbers, the score just changes automatically, and you don't have to do any extra work with it. So, how could you make the spreadsheet do the multiplying for you?

Denise and Donald responded by looking at the spreadsheet tips to learn the syntax that would allow them to program the spreadsheet to multiply the value of a cell by a number. After reading the spreadsheet tips and after some experimentation, Team A successfully programmed the spreadsheet to carry out the second algorithm, finding a score of 29 for Jay. When they applied their formula to the other two candidates, Denise and Donald found that the scores were the same. After examining their formula, Denise identified the problem, explaining, "I see what I did...I should have used like the, the G like 41 and stuff like that." Despite knowing the correct language and syntax, they had reverted to writing a numerical expression for the spreadsheet formula instead of using the cell names.

When asked to explain why they should use the cell name instead of the numerical values the cells represent, Denise stated, "It gives it more of like an exact number and it tells it to do the same thing for the other ones too...it gives it more of like, like an exact little, like, a plan kind of,

of what I want it to do.” Denise then elaborated, “I want it to multiply this column with one this column with 1.5 and this column with two.” Denise’s description of what she wanted the algorithm to do (the “plan”), demonstrates her evolving conception of algorithms and algebraic expressions, shifting from operating on the specific values in a cell to operating on the “column,” or, what those cells represent. This shift was supported by the immediate feedback provided by the spreadsheet and the appeal of the autofill. Additionally, because they wanted to be able to change the ratings for each candidate and see the scores update automatically, Denise and Donald needed to write the formula in a way that separated the multipliers for each variable from their numerical ratings. In this way, the spreadsheet affordances created a situation in which Team A had to attend to the structure of their algebraic expression. Their scaffolded exploration of the spreadsheet resource supported Team A to understand algorithms, engage with algebraic expressions and functions, and make progress on the challenge.

After learning how to program a spreadsheet and with a clearer understanding of algorithms and the challenge, Team A continued working through the examples in the spreadsheet resource with the goal of identifying the best model for their racetrack rating algorithm. By the end of day 3, Team A had revised their algorithm to include a description of a systematic process for assigning ratings to horse racetracks and had found a way to operationalize their three variables (see figure 13).

Figure 13*Team A's Day 3 Algorithm*

Algorithm EXAMPLE (Not Final)

#	Track Name	Health	Treatment	ESTIMATE examination results	Score
3	Santa Anita	2.5	4	4.5	
	Belmont Park				
	Churchill Downs				

EXAMPLE #2 ESTIMATE examination results SCORE

Note. This figure shows Team A's revised algorithm as they described it in their technical brief and as they represented it in a table.

Specifically, they operationalized their variables by assigning ratings out of a maximum score of 5 in each variable for each racetrack. Then, as they explained in their technical brief, “if [their] numbers were 3, 6⁷, and 5, [they] would add those numbers up and...then [they] would divide...by 3.”

After working through the spreadsheet resource, Team A participated in an “expert” check-in, during which they described their business and algorithm, and received feedback. I used “expert” check-ins to give teams feedback on their ideas from an entrepreneurial perspective and to test whether this kind of feedback could be used by teachers to encourage students to reflect and iterate on their solutions. This check-in created an opportunity for Team A to demonstrate their improved understanding of algorithms and to discuss the tradeoffs they considered in selecting the optimal algorithm for their context and business. When asked how they would decide on the “right” algorithm from the examples in the spreadsheet resource, Denise explained that they wanted an algorithm that was,

⁷ Team A used the number “6” as an example for how their algorithm would combine the values of their three variables, despite it being greater than the maximum possible score (5). It is not clear why they made this mistake as, once they decided on their algorithm, they consistently referred to all scores being out of a maximum score of 5.

Kind of something like [the third algorithm in the spreadsheet resource], like how it gave us like smaller numbers. There's like number four too. Like those, the last two, something small and a little bit more easy to manage. Kind of like how they would, you know, like I said like a few days ago, like that Google thing, like the stars. Yeah, something like that. Keep it like a 5 max kind of.

For Team A, it was important that their algorithm produce numerical ratings that were the right magnitude in that the ratings would be “easy to manage.” By evaluating potential algorithms in this way, Denise and Donald were using alternative models to inform their algorithm and were considering the relationship between the inputs, outputs, and function rule for those models. Additionally, they evaluated the fit of each algorithm against the needs of their users, an example of how the context and challenge leveraged the entrepreneurial situation to establish a standard against which they could evaluate the “correctness” of their solutions.

In addition to considering the magnitude of the ratings produced by an algorithm, Team A also considered whether the ratings met their expectations for how familiar tracks should be rated. Team A’s decision to build an algorithm that rates horse racing tracks was based on their familiarity with the Santa Anita racetrack: a track that had recently received publicity for its high number of racehorse deaths. When asked what ratings they expected for their three selected tracks, Denise stated,

Um...well, I'll say I need a little bit more research on this because I don't know much about [Churchill Downs and Belmont Park], but lately [Santa Anita] hasn't been doing so well. It has a good rating as being a race track, but we're not rating it on being a racetrack, we're rating it on how it treats their horses and how healthy those are. But there have

been so many deaths there lately, so I just have to like, we have to do a little more research and look into it more.

Denise was then asked if she had a prediction for the rating Santa Anita would receive, to which she responded, “Um, I don't think they're very compliant...Probably like 2.5 or something.” When asked what would happen if Santa Anita received a 5 (out of 5), Denise stated, “[I would] definitely reconsider how I was doing it because, not being judging here, but just they haven't been doing very well. So yeah, I would want to look at it again. Just go over it and figuring out what I did wrong.” Thus, the characteristics of the racetracks determined the reasonableness of their algorithm.

Given their improved understanding of algorithms, Denise and Donald were able to consider the meaning of an algorithm and the tradeoffs of selecting one algorithm over another. Furthermore, the context and spreadsheet created a situation in which testing and revising an algebraic expression was natural and appealing to Team A's process.

Team B. By the end of day 2, Team B had a working spreadsheet algorithm for rating YouTube channels. They used day 3 to work on their technical brief and build their pitch deck. Beatrice and Maria did not communicate with each other as they worked on the Technical Brief and neither their algorithm nor their business changed as a result of working on the Technical Brief. For Team B, the Technical Brief served more as an opportunity to reflect on their process than as a support resource that could help them build their algorithm or business.

Later in the day, I conducted a check-in with Team B, with the goal of understanding Team B's rationale for how they selected and assigned weights to their variables. Previously, Team B had expressed that content was more important than the other variables, but it was not clear whether the selection of the weights reflected a consideration of how much more important

content should be than the other variables. Maria's initial explanation for the weights in her team's algorithm suggests that they had only considered whether one weight was greater than another and not how much greater. As Maria explained,

We feel like these were the best weights that we can figure out based on our experiences with YouTube and what's important to us when we look at a YouTube channel...To me, the type of...content is infinitely more important than the amount of views a video gets because it could be amazingly planned, well thought out. You can tell the production quality's so high and it's just so amazing that it's, and it's just an amazing video, but it only gets about 20 views. Yeah. So I feel like the quality of the video and the content that it has would be a lot more important than the amount of views.

In this first interaction during the check-in, Maria explains that they selected weights so that "more important" variables were assigned greater weights than less important variables.

However, in a later conversation with a different member of the research team, Maria discussed their selection of the weights differently, citing instead a need to make content more important than the other variables, but not so much more important that it overwhelms the overall rating. According to Maria, "we figured out what kind of different weights we would want to use on different categories so that there would be ones that more heavily influenced the final score, but not so much that...this one category that decided everything." In selecting the weights for their algorithm, Team B considered how the structure of their algorithm (specifically, the relative magnitude of the coefficients) impacted the relationship between the inputs and outputs of their algorithm, as well as the meaning of those outputs. In other words, they recognized that changing the value of the coefficients would impact the overall ratings generated by their algorithm and, if one coefficient were too much greater than the others, then the meaning of those ratings would

change. Additionally, through repeated opportunities to describe their algorithm, Beatrice and Maria improved the clarity of their descriptions and of their understanding of their algorithms.

During the check-in, Beatrice and Maria were also questioned about their selection of variables. Specifically, it was unclear how a YouTube channel's upload schedule would influence its quality. Thus, I asked Team B to explain why they included uploads as a variable. Maria defended their inclusion of uploads, stating,

Well, some YouTubers only upload two or three, four times a month. But they have really amazing videos that you can tell they put so much work and effort into. And some YouTubers upload every single day and the quality of the videos, it's just kind of not that good...And so if they think they upload, like they upload a perfect amount so that they always get good quality videos and you don't have to wait too long in between them, um, that would be a higher score than if you think they don't have good videos but they upload every single day... so, if you think that their upload schedule is fitting with the type of content that they make.

Maria's justification for including uploads as a variable is important for two reasons. First, it demonstrates her deep understanding of what the variable means and represents. Maria was able to explain that "uploads" represented a user's evaluation of how frequently a YouTube channel uploads videos and that a higher score did not represent a higher frequency of uploads but rather a more suitable frequency of uploads. Second, Maria's deep understanding of her team's context empowered her to explain, justify, and defend her team's selection of variables. Beatrice, it should be noted, did not contribute to the discussion during the "expert" check-in. As was the case for much of the study, Maria was the more vocal of the two and was more eager to take the lead. After the check-in, Team B used the remainder of day 3 to work on their pitch deck.

Team C. Following the whole-group discussion of the technical brief, Team C continued their work from the previous day, testing and refining their music filtering algorithm. They did not start working on the technical brief until after the “expert” check-in. When they got to their workstation, Kim reiterated to Shaun that it was important for their algorithm to return a list of songs instead of a single song, stating,

...I think we should do it where, like, if it's possible, where...they select the ones that they want and then, like, it searches all of them up maybe.... It gives us more than just one thing? There might be like something other than just the than like what we've been doing that might be doing that might do it better, but I don't know.

At this point in the competition, Kim and Shaun were still using the “MATCH” function to search the list of songs, which would not return a list of songs. Neither Kim nor Shaun were aware of a Google Sheets function that would meet their needs, but they were confident one existed. Recognizing that the helpful resources provided in the challenge did not include anything that would help them gain the knowledge they needed, Kim searched for solutions online and Shaun searched through the list of functions included in Google Sheets. After several minutes of searching, Shaun identified the “LOOKUP” function as a possible solution and began experimenting with it in his team’s spreadsheet. Kim explained how their algorithm used the LOOKUP function.

So, basically, you put in like the, the row and column of...the word that you want to match...and it basically...finds all the...ones that matched the word...When it searches it in the range, when it finds one there and matches it with the second range which is right next to it and it gives the one that's the same row number.

Team C’s algorithm used the “LOOKUP” command to search a list of song characteristics (e.g. genre and instruments) within a column and within a given range, both specified by the programmer. When it finds a cell containing the specified characteristic, the spreadsheet returns the name of the song, which corresponds to that cell. For example (see figure 14), if a user responds to their survey stating they like “pop” as a genre, then the LOOKUP function searches column C (“Genre”) for the last cell that contains the word “pop” (cell C36). When it finds that cell, the function returns the contents of the cell located in the same row (36) and in the column labeled “Name.” In this example, the LOOKUP function would return the song “Firework,” by Katy Perry.

Figure 14

Team C’s Song Library

	A	B	C	D
30				
31				
32		Name	Genre	Artist
33		What A Wonderful	Jazz	Louis Armstrong
34		Fly Me To the Moon	Jazz	Frank Sinatra
35		Me	Pop	Taylor Swift
36		Firework	Pop	Katy Perry
37		All the Small Things	Punk	Blink-182
38		Sugar We're Going	Punk	Fall Out Boy
39		Bohemian Rhapsody	Rock	Queen
40		Another Brick In	Rock	Pink

Note. This figure shows the music library Team C’s algorithm filtered to generate a list of recommended songs.

According to the description in Google Sheets, the LOOKUP function requires that the contents of the column being searched be listed in order (numerical or alphabetical, depending on the contents of the cell) and it returns only the last matching cell in the list. Thus, the LOOKUP function does not, when used in isolation, return a list of songs, it only returns one song. To address this issue with their algorithm, Team C devised a strategy that segmented the

search range and used multiple LOOKUP functions across cells to return a list of multiple songs. They planned to use multiple LOOKUP functions, each in a different cell, to generate their list of multiple songs. For example, to find pop songs, Team B would write two LOOKUP functions in two different cells: one that searches cells C33 to C35 and one that searches cells C36 to C40 (see figure 13). Although there are flaws⁸ with this strategy, Team C demonstrated an understanding of their inputs (the survey responses), outputs (the lists of recommended songs), and the rule by which those outputs are generated. Additionally, the immediate feedback provided by the spreadsheet, combined with the programming language and syntax, supported Shaun and Kim to engage with the tool and continuously reflect on the meaning and structure of their algorithm.

Team C's process involved Shaun researching Google Sheets functions and then testing them to see if they produced the desired results. This process created an iterative cycle in which Team C would write a formula, using the cell names as variables, test it against their expectations for what the algorithm should do, and then revise their formula (or expression) and test it again. Through this process, Team C continually evaluated the structure of their expression and checked the meaning of their variables. For example, to test the LOOKUP function, Shaun needed to 1) identify the cell containing the search term; 2) define the range of cells in which the spreadsheet would look for the search term; 3) define the range of cells in which the spreadsheet would look for the cell corresponding to the matching cell; and 4) describe, using spreadsheet language, what to report back. This following exchange between Shaun and Kim is an example

⁸ Team C's strategy of segmenting the search range would require them to continually revise how they segmented the search range whenever songs are added. Additionally, they had yet to develop a strategy for how they would apply their algorithm to the other three columns, representing instruments, musician likes, and musician dislikes.

of the extent to which they engaged in symbolic reasoning as they tested and discussed their algorithm.

Shaun: So, here it matches this cell and that cell...and then E9, that's Pop. It matches that to

Kim: So, like we can change this to something else?

Shaun: Yeah, it can change it to a different response.

Kim: Oh, wait, I want to play with that

Shaun: D to D, which is this entire row, it searches it all up for the first pop it finds.

Kim: All right, I'm gonna put it on B88

Shaun: B88?

Kim: yes, I am putting it on B88. What did it do? So, I check it?

Shaun: Perry, Katy and I gotta change it to E to E because it's a different section.

Kim: Okay. So I have to change it to E to E.

Kim and Shaun were able to seamlessly integrate the symbolic representation into their verbal descriptions of the algorithm and their context, using cell and column names interchangeably with songs, artists, and genres. Furthermore, each cycle of experimentation and revision drove Team C to consider the meaning of their variables and the structure of their expression. In this example, Team C tested their algorithm using a value of “Pop” for the variable “genre.” When they saw that it did not work as expected, Shaun evaluated their expression and discovered that they needed to change the search range for the LOOKUP function from column D to column E. To be able to make this revision, Team C needed to 1) understand the meaning of the variables in their spreadsheet formula, 2) interpret the structure of their spreadsheet formula, and 3) understand how the variables in the expression are mapped onto

their desired outputs via their function rule. As an example, the final spreadsheet algorithm Team C submitted contained the following spreadsheet formula (figure 15):

Figure 15

Team C's Final Spreadsheet Algorithm

```
=IF(LOOKUP(I105, C101:C103,B101:B103)=J104,0, IF(LOOKUP(I105, C101:C103,B101:B103)
=J105,0,LOOKUP(I105,C101:C103,B101:B103)))
```

Note. This figure shows the final version of Team C's spreadsheet algorithm.

For the sake of brevity, consider the final conditional statement in their expression (see figure 16).

Figure 16

Last Conditional Statement from Team C's Spreadsheet Algorithm

```
LOOKUP(I105, C101:C103, B101:B103)
```

Note. This figure shows the final conditional statement from Team C's spreadsheet algorithm.

In building, testing, and revising this component of their algorithm, Team C needed to understand 1) the process followed by the LOOKUP function rule, 2) recognize "I105" as representing the search term, 3) recognize "C101:C103" as representing the range in which the function rule searches for the search term, and 4) recognize "B101:B103" as representing the range of cells from which the LOOKUP function will select a cell corresponding to the search term. Although Team C did not submit a completed spreadsheet algorithm that matched their description and although their algorithm is not typically considered part of a middle grades treatment of algebraic expressions, it nevertheless represents sophisticated symbolic and functional reasoning.

Halfway through day 3, Team C participated in their "expert" check-in, during which they were asked to explain what would happen in their algorithm if a user responded that they

liked every genre and every instrument. The goal of this question was to prompt Team C to consider the relationship between the inputs (survey responses) and outputs (lists of songs) of their algorithm. Kim responded that users would “get a lot of music to listen to” and went on to explain,

that's one thing we're actually working on. Like, it'd be...recommending, like, a lot of things...I think there should be a limit, a limit. Like, maybe...cap it at like 20 because I feel like it could be overwhelming. Like you don't want to have like 700 songs thrown at you.

Based on Kim’s claim that users might not want to see “700 songs thrown at [them],” they considered revising their algorithm by 1) restricting the number of options a user can select for the genre and instrument questions, which would then reduce the number of songs that could be identified using the LOOKUP function, and 2) applying a process by which they could restrict the range (the number of recommended songs) after they have been identified. In a follow-up question, Shaun and Kim were asked to explain the process through which they would restrict the range, to which Shaun responded, “...once they do the survey and you get the results, like, it can show which ones are popular between like random people that search up on YouTube...it’ll just find which ones appear on, like, a Google search, but those ones are more popular.”

Through the first three days of the competition, Team C considered, at various times, adding a rating or ranking component to their algorithm (i.e. something that would evaluate users’ agreement with the statement, “I like this song”), but they never progressed beyond discussing it. However, examining the features of Team C’s algorithm suggests that, even without the rating or ranking component, it satisfied the three conditions for an algorithm and supported them to engage with the STEM content targeted by the challenge. Specifically, Team

C's filtering algorithm included inputs (survey responses), outputs (list of recommended songs), and a consistent process (LOOKUP function) for mapping inputs onto outputs. Moreover, the purpose of Team C's algorithm was to provide users with information that would help them make decisions relative to their context (music). Thus, despite the apparent inconsistency between Team C's filtering algorithm and the rating algorithms created by teams A and B, it nevertheless fit within the established criteria for an effective solution, absent the use of weighted categories. Allowing Team C the space to innovate with respect to their algorithm supported their sustained engagement, while also revealing new insights into what it means to build an algorithm.

Thursday (Day 4)

Understanding Pitching. Day 4 started with a whole-group discussion of pitching, which began with an online video of the first half of an entrepreneurial pitch delivered by Cathy Yee, the CEO of Includie, to actual investors, and included an overview of the rules of the competition (i.e. pitches cannot exceed five minutes and will be delivered to external judges). After watching the video, students discussed the key features of Cathy Yee's pitch and made suggestions for what should happen during the second, unseen part of her pitch. From this discussion, students generated a list of the features that should be included in the beginning, middle, and end of an effective entrepreneurial pitch. At the start of a pitch, students argued, entrepreneurs should 1) briefly describe their product, 2) introduce themselves, 3) give more detail about their product, 4) describe the problem their product solves, and 5) describe their target users. In the middle of the pitch, entrepreneurs should 1) describe how they plan to use investors' money, 2) give more detail about how their product works, and 3) describe how their

company will make money. At the end of the pitch, students suggested presenting a brief summary in case the investors missed anything during the beginning or middle.

Although students were not able to discuss pitching in the abstract and seemed to have a limited understanding of the important components of a pitch, the discussion demonstrated that they did have familiarity with how to make a convincing argument. Additionally, this comfort with convincing and the concrete example in the form of the Cathy Yee pitch, helped to establish the norms and expectations for their final pitch competition. These norms and expectations, together with the pitch resources, guided students' work as they built their pitch decks and scripted and practiced their pitch with practice judges.

Practicing the Pitch. All three teams used day 4 to prepare for their pitch practice with an external practice judge. This involved building their pitch decks and scripting what they would say during the pitch. The pitch practice was the first opportunity for students to share their business and algorithm ideas with someone other than a member of the research team and to do so in a semi-formal setting. All six students expressed nervousness about having to pitch their ideas to an external judge, who they worried would unfairly judge their work. Beatrice verbalized her nervousness, stating,

I'm kind of excited about tomorrow, but I'm kind of nervous. Public speaking is not really a strength of mine. I don't like to talk in front of other people unless it's like a family member or something...I mean like I know what I want to say, but then my mind just goes blank when everyone's staring at me and I keep staring at my paper.

In response to this nervousness, all three teams worked diligently to prepare their pitches and pitch decks in preparation for the practice pitch.

Team A. Denise and Donald spent the first half of the day working on their technical brief and building their pitch deck for their horse racing track algorithm. Continuing the trend of the three previous days, Denise shouldered much of the workload, while Donald was largely disengaged and off-task. As Denise worked on her team’s technical brief, she discovered a potential problem with their algorithm, stating, “...since we’re not, like, a real business yet, I can’t make, like, a rating for the examination results ‘cause we don’t really have examination results.” Because their algorithm relied on the results of horse examinations and because they had not yet done those examinations, Denise was worried that they would not have the data necessary for completing a prototype. As a result, Team A decided to replace the “examinations” variable with a variable they called, “physical appearance,” which allowed them to complete a functioning prototype (see figure 17).

Figure 17

Recreation of Prototype 1 of Team A’s Algorithm

Track Name	Health	Treatment	Physical Appearance	Score
Santa Anita	2.5	4	4.5	3.5
Belmont Park	3	4.5	5	4
Churchill Downs	1.5	3	3	2.5

Note. This figure shows a first prototype of Team A’s algorithm, represented using a table.

In their prototype, Team A assigned ratings for each variable based on their online research and determined an overall rating for each horse racing track by calculating the mean of the three variable ratings.

Once they had a functioning prototype of their algorithm, Denise and Donald began building their pitch deck in preparation for the practice pitch. As they worked, I checked in with

them to gauge their progress and probe their understanding of their prototype. Specifically, I wanted to check whether the overall scores produced by their algorithm met their expectations for the three sample racetracks. Examining their prototype revealed that Santa Anita had received a rating of 3.5 out of 5, while Churchill Downs had received a rating of 2.5 out of 5. I asked Denise and Donald whether they were surprised by that Santa Anita was not the worst of the three tracks. Donald responded, “so, now we can say Churchill downs is horrible. I looked it up and there were 16 fatalities in 2018.” Donald went on to explain that “the health depends on the number of horse deaths. The greater number in horse deaths, the lower rating it has on health. But, the less amount of deaths, the higher the rating.” In other words, instead of blindly accepting the results of their prototype, Denise and Donald revised their prediction and justified the result using their understanding of the relationship between the inputs and the outputs of their algorithm. Specifically, they recognized that a greater number of horse deaths would result in a lower rating for “health,” which would then result in a lower overall rating for the track.

Following their discussion with the researcher, Team A returned to building their pitch deck in anticipation of their practice pitch. Although Team A consulted the How to Build a Pitch resource to help them structure their pitch, it was not until after delivering their practice pitch that they were able to identify key components that were missing. For example, following their practice pitch, the judge pointed out to Team A that they did not introduce themselves, describe their target customers, or state the problem they were trying to solve with their business: all essential components of an effective pitch.

During their practice pitch, Team A also did not describe their algorithm. In a pitch competition, entrepreneurs must give enough detail about their product to help investors feel confident it can succeed, while also maintaining the excitement and enthusiasm of the pitch. By

omitting a description of their algorithm, Team A did not achieve this important balance. Without understanding how the algorithm works, investors would lack the information necessary to make an informed decision about whether the business can succeed. Furthermore, by not describing their algorithm during their practice pitch, Team A missed an opportunity to reflect on the structure and meaning of their algorithm and engage with the targeted mathematics and STEM content.

Following the practice pitch, the practice judge asked Denise and Donald whether each variable in their algorithm was equally important to the overall score. As she explained, "...treatment and Health are more important than, like, physical appearance." Based on her knowledge of horses, Denise argued that a horse's treatment and health should be more important to a track's overall rating than a horse's physical appearance. However, after looking at the tabular representation of their prototype, Denise recognized that this was not reflected in their algorithm, which prompted them to revise their algorithm and pitch deck.

Although the practice judge's feedback was brief, by situating that feedback in the goals of the pitch competition, she created a need for Team A to revise and clarify the structure of their algorithm. Team A then changed their algorithm from a simple mean to a weighted mean, in which they attempted to make the health variable twice as important as the treatment and physical appearance variables. They represented this new algorithm in a table (see figure 18) and using the expression, $(h \times 2) + T + P$, where h represents the health of the horses, T represents the treatment of the horses, and P represents the physical appearance of the horses.

Figure 18*Recreation of Prototype 2 of Team A's Algorithm*

Track name	Health	Treatment	Physical Appearance	Score
Santa Anita	2.5	4	4.5	4.5
Belmont Park	3	4.5	5	5
Churchill Downs	1.5	3	3	3

Note. This figure shows a second prototype of Team A's algorithm, represented using tables.

Although Team A made an error in using a weighted mean (they divided the sum by 3 instead of by 4) and in their algebraic expression (they did not represent dividing by 3) their effort to develop a more sophisticated algorithm emerged from their investment in the context and the feedback they received during the pitch practice.

Following the practice pitch, both Denise and Donald reported viewing the experience positively, despite their initial nervousness. Specifically, they reported feeling more comfortable and better prepared for the actual pitch competition. As Denise explained, “[the practice pitch] gave great feedback and the feedback really helped and made me a little more prepared. And I’m not as nervous.” Building on Denise’s comment, Donald stated, “...it gave us a chance to make some improvements and have some tips on performance.” Both students valued the opportunity to practice their pitch and appreciated the feedback they received. Furthermore, the roughness of their practice pitch demonstrates the importance of giving students opportunities to practice delivering a pitch. Although nervous about pitching, Denise and Donald appeared to understand both their algorithm and their business in the lead-up to the practice run. However, when attempting to deliver a concise and convincing pitch, they struggled to effectively communicate key features of their algorithm and business. It was not until they practiced their pitch in a semi-formal setting that they understood the challenges of pitching or could identify how to improve their pitch. Additionally, the process of repeatedly practicing their pitches, resulted in an

improved pitch, which reflected Team A's deeper understanding of their algorithms and businesses. Recognizing a need to improve their pitch, Team A used the rest of day 4 to write a script and finish building their pitch deck.

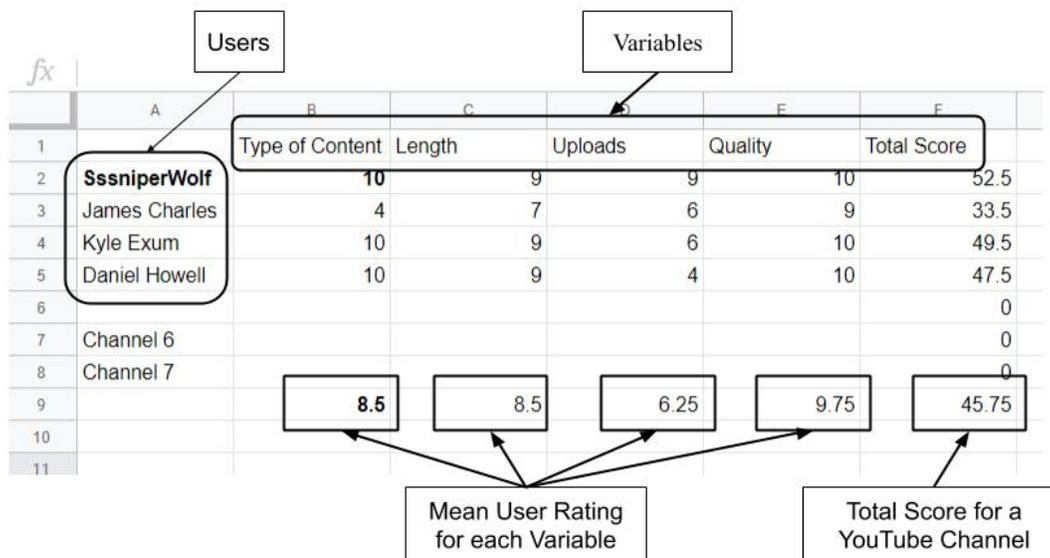
Team B. After the whole-group discussion, Team B started working on their YouTube algorithm pitch deck and preparing for their practice pitch. Based on her review of the Pitch Judging Sheet on day 3, Maria made suggestion for how they could start their pitch, stating, "...one of the things [the Pitch Judging Sheet] says is, like, describe the target audience... So, I thought that we could include, like, before we get into what YouRate is, we could talk about some of, like, the YouTube statistics." Maria's use of the *Pitch Judging Sheet* helped her realize that they needed to describe their target customers during the pitch. Team B then began researching YouTube statistics to use at the start of their pitch.

As they worked to build their pitch and pitch deck, Team B also made revisions to their algorithm, swapping out the variable "views" for a new variable, "quality," which represented users' ratings of the technical quality of the videos uploaded to a YouTube channel. As Beatrice explained, this decision was based on the needs of users, stating, "we decided instead of having views as something that people could rate, 'cause that'd be kind of hard, um, we changed it to the quality." Even though they had been content with their algorithm at the end of day 3, Beatrice and Maria continued considering their users and tinkering with their algorithm. Views, they reasoned, would be harder for users to rate than the quality of the video. Preparing for the pitch practice created a need for Team B to review the Pitch Judging Sheet, which focused their attention on their target audience and led them to improve their algorithm by finding a variable that provided more useful information for evaluating the quality of a YouTube channel.

They also reported that the new “quality” variable would now have a weight of one. When asked how one could see the weights of the different variables in their algebraic expression, Maria pointed to the coefficients, stating, “based on the values they have...um...what are they called? I should know them. The constants, I think. Yeah. I don't know...You're looking for the highest number.” Maria could point to where she could see the different variable weights in their algebraic expression, but she was not able to remember how to name them. Recognizing that Team B knew how to interpret the structure of their expression, but lacked the formal language, I introduced the term “coefficient” to help them verbalize their understanding.

During this discussion, Beatrice and Maria were also asked how their algorithm would combine multiple users' ratings for each variable. At this point in the competition, they had defined a process for combining the ratings for each variable to produce an overall rating for a YouTube channel. They had also claimed that the variable ratings would come from users. However, they had not yet explained how they would combine multiple users' ratings to produce the variable ratings. Team B explained that they would do this by calculating the mean of user ratings for each variable. Figure 19 shows how Beatrice and Maria adapted their spreadsheet to demonstrate how their algorithm calculates a score for one YouTube channel based on multiple users' ratings.

Figure 19

Team B's Annotated Spreadsheet

Note. This figure shows an annotated version of Team B's spreadsheet algorithm.

Whereas Team B originally used column A to represent multiple YouTube channels, they now used column A to represent multiple users, with columns B, C, D, and E representing ratings assigned by those users to a single YouTube channel. Team B used row 9 to calculate a mean for each of the four variables and, using their algorithm, they combined those ratings to determine an overall score, which appears in cell F9. In making this change, Team B demonstrated a flexible use and interpretation of the variables in their spreadsheet. Using the same workbook, Beatrice and Maria were able to conceive of their variables as representing both specific values (any number from 1 to 10) and as expressions (means of user ratings, from 1 to 10, for each variable).

Soon after this conversation, Team B started their practice pitch. Their external judge, James, was a software engineer, who focused his feedback on the structure and quality of the pitch, as well as the feasibility of Team B's spreadsheet algorithm and business. James's questions and feedback following Team B's practice pitch pushed Beatrice and Maria to explain

and justify the decisions they made in building their algorithm, such as the variables they included, the values those variables could represent, and the ratings their algorithm produced. In one exchange, James asked Beatrice and Maria, “how do you guys feel about, um, your algorithm being too subjective? ...On the length category, um, what would be the basis on which you would rate a video in which you would rank it higher than other videos?” By asking about the subjectivity of the variables, James’s question probed Team B’s understanding of the meaning of the variables in their algebraic expression and did so from an entrepreneurial perspective. Maria responded to James’s question, explaining,

Well, let's say a YouTuber creates a two-minute video... The video, you know, it's amazing quality. It's very funny. It gets to the point, it's kind of nice. It's like a good video then that's okay. That's a good length for the video if it fits correctly. But if the video is like 45 minutes long and you don't really get to the point of the video ‘til the last five minutes and it's just kind of a waste of time, then that would be a lower rating.

Maria’s response demonstrated her understanding of the meaning of the “length” variable and its relationship to the overall rating assigned to a YouTube channel. James’s question, which was about whether the “length” variable was too subjective (an entrepreneurial concern), prompted Maria to reflect on the meaning and structure of their algorithm (a mathematical concern). Thus, the mathematical practice of reflecting on and explaining the meaning of an algebraic expression was elicited through entrepreneurial considerations.

Later, James addressed the feasibility of Team B’s algorithm-based businesses, focusing specifically on the “quality” variable, which Maria defined as, “...technical quality...like, whether the sound sounds good, the camera quality, just that kind of technical thing.” Similar to

how he approached the “length” variable, James leveraged his knowledge of the context and his expertise as an engineer, while also bringing up important entrepreneurial considerations, asking,

So how would you get that information? Would you have like, would you hire, like, a bunch of people that just watch videos as their full time job and then they're like, ‘Oh, this is good quality sound,’ you know, ‘this is a good quality video,’ or would you, like, try to invest in some kind of, like, sound recognition system that differentiates between, like, low quality sounds and high quality samples?

Again, James’s question, posed from an entrepreneurial perspective, created an opportunity for Team B to define their variables and reflect on the structure and meaning of their algorithm. In response, Maria stated,

Well our entire, um, our entire website, it's going to be a website, is based on consumer opinions. So, if the consumers think that the camera quality is perfect, it's got like, I don't know, like really high resolution, the sound isn't like murky or like static, yeah, and the consumer like sees that and they're like, ‘Oh wow, that's a really good quality video,’ then that would be a higher rating.

Pressed on whether the variable “quality” was feasible, Maria defended both its relevance to their algorithm’s purpose and the method by which it would be assigned values. Team B’s investment in and expert knowledge of their algorithm supported them to defend their choices, even in response to a STEM professional’s expert feedback. The entrepreneurial context created a situation in which they felt equivalently expert, which empowered them to explain and justify the meaning of their variables.

Team B's willingness to engage with their expert judge and defend their ideas was not limited to the mathematical components of their algorithm. James also questioned Team B on the technical aspects of their business, asking,

...How would you guys protect yourself against...internet trolls that, you know, they just want to...break your system. You know, they get, like, a really good video and then they're like, 'Oh I'm going to give this a bad rating...' to, like mess up your algorithm or your system.

In this situation, it would have been reasonable for Team B to cede expertise to James and accept his feedback. However, Team B, invested in their algorithm and business, again felt empowered to disagree with the expert and to justify their reasoning. As Maria explained,

I mean, I feel like that could be a problem with any business that's that, like, allows people to rate, like, just allows...anybody that has an account...to rate things because if they just go online and they rate everything, like it's zeros, no stars, everything like that, that could happen on anything that could happen on like a lot of websites. But I think it's more of a problem of identifying who it is and making sure that they don't do that than it is stopping them before they [do it].

Maria was able to use her out-of-school expertise about a real and current issue, internet trolling, to creatively respond to James's question. More importantly, Maria's response demonstrated an awareness of factors that can influence a mean. Understanding and responding to James's question required an awareness that a mean can be manipulated by increasing the number of data points at the low (or high) end of a range, something that Maria understood and was able to respond to. Additionally, Maria's response demonstrated that she recognized that user ratings, while an essential component of their algorithm, can also undermine their

algorithm's purpose: informing users' decisions about whether to watch a video from a YouTube channel.

Finally, James questioned Team B's decision to make 55 be the maximum score a YouTube channel could receive, stating,

...I thought it was kind of weird that the max score was like 55 or something. Yeah. I thought that maybe you might want to weigh it differently so that the max score becomes, like, a hundred something that's more commonly used and more easily understood by your users.

To which Maria responded, "I think we can switch that." Beatrice and Maria did not defend this aspect of their algorithm. Instead, they agreed that changing the maximum score a YouTube channel could receive to 100 would make more sense to users. Although Team B never did change their algorithm so that the maximum score would be 100, this interaction is further evidence of how the entrepreneurial framing of the challenge can drive mathematics learning, by referencing the needs of users to promote iterative design. Furthermore, it is evident from Maria's description and Team B's practice pitch that they became more confident and clearer in describing their algorithm and business, with repeated practice. Following their pitch practice, Team B spent the remaining time making small changes to their pitch decks, revising their pitch script, and cleaning up their spreadsheet algorithm.

Team C. On day 4, Kim and Shaun did not make any significant changes to either their music filtering algorithm or their business. Instead, they used the bulk of day 4 to tinker with their algorithm and prepare for the practice pitch, with Shaun working on the spreadsheet and Kim working on building out their pitch deck.

I used the lead-up to the pitch practice to better understand Team C's algorithm. Specifically, I wanted to know whether they described their algorithm as a sequential process and, if so, what that sequence was. Shaun explained that their algorithm, "...looks up [a genre response] and if [it is] equal to [likes] or equal to [dislikes]. If it equals to this, which is the dislikes...then it'll turn into zero...So, it won't give you that, 'cause it's in your dislikes." According to Shaun, the algorithm first filtered the list of songs by a user's responses to the genre and instrument questions and then filtered the resulting list according to the responses to the likes and dislikes questions.

I then asked a follow-up question to explore how the algorithm used the likes to filter the list of songs. Particularly, I wanted to know if the likes outweighed the other three variables. In other words, if a user lists an artist that they like, would the algorithm only recommend songs by that artist? I asked Team C what would happen if a user responded to the "likes" question with only one artist (Katy Perry). Shaun immediately replied "yes," while Kim replied, "I mean, we won't make all the songs be Katy Perry. There might be a few thrown in there, but it won't make like every single response Katy Perry, maybe." The disconnect between their two responses demonstrates how Team C's understanding of the needs of users informed the iterative development of their algorithm and how Shaun and Kim contributed to that development differently. Shaun's response was based on his knowledge of the mechanics of the spreadsheet algorithm, which filtered songs so that users would only see their "likes" in the list of suggested songs. Kim's response was based on her knowledge of music and her familiarity with similar music services. Through Shaun and Kim's collaboration, their algorithm and business co-developed.

After this exchange, Team C returned to tinkering with their algorithm and building out their pitch deck. As they prepared for their practice pitch, Team C consulted the pitch resources, which helped them structure their pitch and identify important components that were missing. For example, while Shaun and Kim were referencing the Pitch Judging Sheet, Kim specifically highlighted the requirement that they show how their solution works, stating, “right, so we have to show how [our algorithm] works. So maybe we can make, like, a video, like you know how there are things where you can record what's going on on your screen.” Although describing one’s solution during a pitch might seem obvious, for Team C, who had little familiarity with pitching or entrepreneurship at the start of the study, it was not. The Pitch Judging Sheet was essential for clarifying the expectations for the components of an effective pitch.

Halfway through day 4, Team C delivered their practice pitch. Despite their ease and confidence informally discussing their algorithm and business with the researcher, in this more semi-formal setting with an unfamiliar judge, Kim and Shaun were unprepared and disorganized. For Team C, practicing their pitch in a more formal setting was a different experience than explaining their idea to the researcher and, despite consulting the Pitch Judging Sheet, this process helped them uncover essential components that they had neglected to include in their pitch. For example, immediately after they finished their practice pitch, Team C realized they had forgotten to include a conclusion, with Shaun stating, “I forgot the conclusion part...Like, I forgot what that was.” Team C’s difficulty highlights the importance of the pitch practice: even when students feel confident informally talking about their solutions, delivering a clear and concise description in a high-pressure setting is challenging and requires practice.

Following their practice pitch, Team C received feedback from their practice judge, who recommended that Team C be confident in their delivery and that they try not read from their

notes during the pitch. The opportunity to practice their pitch and to receive feedback from a practice external judge were both instrumental in helping Team C prepare for the pitch competition. The practice pitch helped Team C recognize that they needed to prepare for the pitch competition, and it helped them identify critical aspects, such as an introduction and a conclusion, that were missing from their pitch. As Kim explained after the practice pitch, “we need to...work on the script. We need to work all with memorizing it. That's what we need to work on now because we have everything else down. But we just need to work on that...”

Additionally, Team C’s nervousness in the lead-up to the practice pitch prompted them to access the *Pitch Judging Sheet*, which helped them to begin planning their actual pitch. Finally, the use of an external practice judge helped Team C recognize the importance of being thorough in describing their algorithm and business. According to Kim,

[the practice external judges are] going in with no background information or little background information. So, they're kind of, so it's kind of like if they're not understanding something, it's going to be better. It's going to be better for them to explain, like, what someone might not get at because it's like...[they] haven't heard this stuff before, so they might not be understanding what we're saying.

Kim’s comment demonstrates why the pitch practice was important from an engagement perspective. The pitch practice helped establish students’ expectations for the actual pitch competition by making apparent the extent to which they needed to explain their ideas in order to make them clear to an external judge. This expectation drove Team C’s engagement for the remainder of day 4 as they worked to practice and revise their pitches. These expectations were also important from a mathematics learning perspective. Repeatedly explaining their algorithms

and practicing their pitches helped students to more clearly and thoroughly describe their solutions, which reflected a deeper understanding of their algorithms.

Friday (Day 5)

Preparing for the Final Pitch. All three teams used day 5 to prepare for and participate in the pitch competition. Unlike the previous days, day 5 did not begin with a whole-group discussion. As soon as they arrived, teams began practicing and refining their pitches.

Team A. Donald and Denise did not make any changes to their horse racing tracks algorithm as they prepared for the pitch competition. The two students used the time to practice delivering their pitch and to make final modifications to their pitch deck. At the start of the day, Denise, who was visibly excited and nervous throughout the lead-up to the pitch competition, suggested reviewing the pitch resources again to make sure they had everything they needed. Specifically, she was worried that they were not discussing the problem their business was designed to solve, stating, “I think we should use the examples a little more... ‘cause we don’t have like a problem in there and how we solved it.” For Team A, the pitch resources supported their refined understanding of pitching and promoted engagement by establishing expectations for what constituted a complete and effective pitch.

Team A’s efforts to practice and improve on their pitch appeared to be driven by their nervousness about the final pitch competition and their desire to win. For example, after stating, “I’m so nervous. Do you think we have a good chance of winning?” Denise more forcefully attempted to engage Donald in preparing for the pitch competition, something she had been reluctant to do during the previous four days. In one exchange, Denise repeatedly asked Donald what he was working on and if he had read the script.

Denise: Okay. What do you think about this for the outline of our pitch? It's like cut off like right there. It's like I stopped right there and you could start for right there. So, I split it up evenly. So, where it's kind of fair.

Donald: Okay.

Denise: Are you working on something? Are you working on this?

Donald: I'm actually like, if we work on this at the same time then we're always going to get the resolve conflict screen.

Denise: So, did you read this? I'm just going to add another slide.

In her efforts to be prepared for the pitch competition, Denise directed Donald to stop working on an animation he claimed was for their pitch deck and read the script she drafted. She then led two practice runs of their pitch, with each leading to specific changes to both their pitch deck and pitch. For example, after their first practice run, Denise realized that the table showing their algorithm did not reflect their new weighted mean, stating, “okay, there’s just one more thing that I have to do...Remember how we had to change our algorithm to make it weighted more?...We totally forgot the changing the slides. I'm gonna have to do that.” Thus, practicing their pitch provided another opportunity for Denise to connect her description of her team’s algorithm to the tabular representation of that algorithm in the pitch deck. This also presented an opportunity to engage Donald, who was tasked with changing the visual representations of the ratings to match their updated algorithm (see figure 20).

Figure 20

Tabular Representation of Team A's Algorithm

TRACK NAME	HEALTH	TREATMENT	PHYSICAL APPEARANCE	RATING
SANTA ANITA	●●●●●●●●	●●●●●●●●	●●●●●●●●	●●●●●●●●
BELMONT PARK	●●●●●●●●	●●●●●●●●	●●●●●●●●	●●●●●●●●
CHURCHILL DOWNS	●●●●●●●●	●●●●●●●●	●●●●●●●●	●●●●●●●●

Note. This figure shows Team A's final algorithm applied to three horse racetracks.

During their second practice run, Team A recognized that their pitch deck was missing their problem and solution slides, which they were able to add before the pitch competition. The pressure of the pitch competition motivated Team A to continuously practice and revise their pitch up until the last possible moment before the competition began. Although Donald demonstrated limited engagement throughout the competition, the urgency of the final pitch presented opportunities for him to interact with their horse racing track algorithm and pitch deck. Additionally, because he was responsible for describing part of his team's algorithm during the actual pitch and because Denise encouraged him to repeatedly practice that part, Donald had multiple opportunities to reflect on and improve his understanding of their algorithm, demonstrating how the competition provided students with several chances to engage (or re-engage) during the process.

Team B. Beatrice and Maria came into day 5 with a completed pitch deck for their YouTube algorithm and ideas for their pitch. Having considered their pitch the previous night, Maria came in with an idea for how to structure their pitch, stating

...I kind of came with like a little script in my head last night because I over think everything. So, what I was thinking is trying to start off by asking, 'how many of you guys have heard of YouTube?' Then, when they, like, raise their hands we can be like, 'great, great. So our product for you today is YouRate, I'm Maria,' and you'll say, "I'm Beatrice,' or something. You just introduce yourself and then we'll say, 'and today we're here to talk to you about YouRate,' and we can, like, get the statistics about YouTube and then we'll talk about YouRate and kind of go through the different slides, you know? But I was wondering how you think we should, like, do you think we should divide the slides up? Like um, like I could do this one, then you could do that one, and I could do this one.

Maria's suggestion reflected her refined understanding of pitching as a process distinct from typical presentations. Maria recognized that they needed to engage the audience (and judges) first to sell them on the importance of the situation, before launching into the specifics of their solution and business.

From there, Beatrice and Maria proceeded to practice their pitch three times before the actual competition, with both students becoming more comfortable and better able to explain the features of their algorithm and business each time they practiced.

Team C. Kim and Shaun spent the first 35 minutes of day 5 practicing and refining their pitch for their music filtering algorithm. At the start of the session, Kim suggested an idea for how to structure their pitch, stating,

Alright, so for the start, [we say,] 'who here likes to listen to music? Have you ever thought, well have you ever found yourself looking for new music but not knowing because actually what to hear, what to listen to. Hi, we're Sh, we're Shaun and Kim and we're the inventors of Find My music and we're here to solve just that.'...I quickly

identify as a problem and, and then we go into our solution, which is one thing that they're going to be judging us.

Kim's suggestion reflects a refined understanding of what it means to deliver an entrepreneurial pitch. On day 4, Team C forgot to include a description of the problem their business was designed to solve. Now, on day 5, Shaun and Kim were considering not only how to describe the problem, but also how to help the judges feel the relevance and importance of that problem.

Shaun and Kim then began practicing and refining their pitch, using the Pitch Judging Sheet to make sure they were meeting all requirements for the pitch competition, stating, "...we should, like, we should make sure that we have all threes." Their investment in winning the pitch competition created a need for the Pitch Judging Sheet, which supported their engagement by establishing expectations for their work.

As they practiced, Kim and Shaun focused primarily on improving their delivery, with Kim leveraging her singing experience to help Shaun improve his enthusiasm. For example, during one practice run, Kim explains to Shaun, "Be dramatic! But, seriously, like, you don't want them thinking that's just something that, like, you don't really care. Like, it's just like something you had to do." Thus, Kim recognized the performative aspect of pitching, which had not been emphasized in the pitch resources, but which is essential to engaging an audience. In the remaining time, Kim and Shaun repeatedly practiced and refined their pitch.

Delivering the Final Pitch.

Team A. Denise started her team's pitch by briefly defining the problem their business was designed to solve, explaining to the judges why she cared about that problem, describing her team's business, and comparing that business to a competitor, stating

I have always been interested in horses, even racehorses, but unfortunately, not all horses are treated very well. So, we made a website where horse owners can help save racehorses. It's called Racey, a website that rates racetracks on how well they treat horses. While Google usually rates racetracks on how well the track is, our Racey rates tracks on how they treat their horses.

Denise's introduction contained several key components of an effective pitch including 1) a description of the problem her team's business was designed to solve; 2) an explanation for why it was important for them to solve the problem; 3) a description of how their business improved on a competitor's (Google) model; and 4) the identification of their target users (horse owners). However, this part of the pitch could have been made stronger had Denise shared more of what she had discussed previously with the research team. Specifically, Denise could have discussed her concern about the number of horses dying at the Santa Anita racetrack or her teams' rationale for identifying racetracks as their target customers. Given Team A's concern over running out of time during the pitch, it is possible they did not think they had enough time to include this information.

Following her introduction, Denise and Donald proceeded to describe their algorithm and briefly discuss how they would use their projected revenue, before concluding the pitch.

Denise: We searched up information about three major tracks: Churchill downs, Belmont park, and Santa Anita. We counted up the number of deaths from this year and from last year. This influences our health category.

Donald: While researching, we also looked into how they treated their horses. Finally, we looked at the horse's physical appearance from recent races. We gave them ratings for are all three categories...health, treatment, and physical

appearance. Then we multiply the health category by two, add it with the other two categories and divided by three. This is our basic algorithm. We charge as this money to care for the horses we have saved, so that they have good basic living conditions. We believe that our website can help millions of horses out there or that might not have the best living conditions. We can really help up horses out there. Thank you for listening.

Team A's description of their algorithm demonstrates the progress they made over the course of the week. During the first two days of the study, Team A's description of their algorithm did not include a systematic process. By the end of the study, their description was more precise and included a systematic process through which their ratings of horses' health, treatment, and physical appearance (inputs) would be mapped onto racetrack ratings (outputs). Furthermore, Donald's repeated practice describing his team's algorithm created opportunities for him to engage with and reflect on his team's algorithm.

Team B. Beatrice and Maria began their pitch for their YouTube algorithm by engaging the audience and quantifying their potential user base, before introducing their business.

Maria: How many of you guys have heard of YouTube? Good. Good. That's all of you. That's great. So today we're here to talk to you about our product YouRate. That is our lovely logo designed by Beatrice. YouTube has 1 billion, 300 million users. That's 16% of the total world population. It's one of the world's largest online platforms right up there with Instagram, Twitter and Snapchat and allows users to upload videos of any kind for the rest of the world to see

Beatrice: YouRate is a consumer focus website. That allows you to rate YouTube channels based on four categories to give it an overall rating. It helps people find content that they may or that they are or could be interested in and avoid content that may make them feel uncomfortable.

Maria: Because of the amount of people who use YouTube, you are. It has a potential to have millions of users all around the globe.

Beatrice: Unlike our competitor, Social Blade, we use consumer opinion to set up new numerical statistics to rate YouTubers. This gives us more consumer-focused product which will bring in more business and revenue.

Instead of helping the audience relate to the problem they wanted to solve (i.e. helping people find content), Beatrice and Maria focused on helping the audience recognize the entrepreneurial opportunity by describing the size of YouTube's user base. Next, Beatrice and Maria briefly described their business and distinguished it from a potential competitor, Social Blade. In doing so, Beatrice and Maria transitioned into describing their algorithm. Specifically, by explaining that YouRate uses consumer opinions instead of "numerical statistics," Beatrice was beginning to describe how they had operationalized their variables for use in their algorithm. Beatrice and Maria went on to describe, in greater detail, how their algorithm assigned scores to YouTube channels.

Maria: On the YouRate website, users can view ratings of you up given to YouTubers and can rate each YouTuber on a scale of one to 10 in four categories is to get an overall score, um, based on the different weights of the category.

Beatrice: The categories, which are the four categories. Content, users can rate how they feel about YouTubers' content. This category is weighted twice. Length,

users can rate how they feel about how long the average length of YouTubers' videos are, too short, too long, average things like that. This category is weighted 1.5 times Uploads, users can rate how they feel about the YouTubers' upload schedule – too often, not enough, just right, things like that – this category is weighted once. Quality, users can rate the quality of a YouTuber's videos, sound, visuals. This category is weighted once.

Maria: So here we have an example of our spreadsheet. We couldn't show you the real thing and you know, but here you can tell it's kind of cut-off but you can kind of see the four different categories that the YouTubers are rated on. So we have four different examples. YouTubers up here and we've given them ratings in four of the categories based on our opinions with the YouTubers. And this is what these are, what their total scores add up to based on the ratings. And what we did, forget the different names up here right now. Let's say this is all one YouTube and four different people have given these YouTubers ratings. So, these are the average ratings in each of the categories. And this would be the average rating of the YouTube that would show up on the website. Um, you would also be able to see the different ratings of the categories so that you can tell what other people like about the YouTubers and they don't like.

For much of the pitch, Beatrice and Maria were reading from a script they had prepared earlier. However, when they started discussing how their algorithm combines multiple user ratings, Maria appeared to go off-script, though she was able to confidently and clearly explain how their algorithm 1) combines the ratings for each variable to create a total score for each YouTube

channel, and 2) combines multiple user ratings to generate the ratings for each variable. The comfort they demonstrated while explaining both their business and their algorithm reflected a deep understanding, gained through sustained engagement with and repeated practice explaining and justifying their algorithm and business.

Team C. In the time between their first practice pitch on day 4, which was disorganized and missing components, and their final pitch on day 5, Team C's pitch of their music filtering algorithm saw significant improvement. Kim began her team's pitch by engaging the audience and defining the problem their algorithm was designed to solve, stating,

How many of you like to listen to music? Have you ever found yourself wanting new music to listen to but not knowing what to listen to? Well, we're here to solve just that.

Hi, I'm Kim and this is Shaun and we're the inventors of Find My Music.

After this introduction, Shaun and Kim equally and enthusiastically participated in describing their business and giving an overview of their algorithm.

Shaun: Find My Music is an app where you take a simple survey and get a list of new songs for you to listen to.

Kim: Our survey consists of four questions. What genres of music do you like to listen to, do you like to listen to, what artists do you listen to, What instruments do you like to hear when you're listening to music, and do you have any dislikes? So how does it work?

Shaun: Find My Music is for people of all ages, since everybody loves music and sometimes does not know what to listen to.

Kim: with your help, we believe that we could have over 50 million users. So, we make money by selling ad space we, which means, which means the consumer

doesn't have to pay for it, so it doesn't scare them away from the app. We donate 40%...of our, of our income to Hungry for Music.

Shaun: Hungry for Music is a, is a company that donates musical instruments to children in less fortunate places. Find My Music works with an algorithm that allows it to take answers from the survey and give you a list of new music for you to listen to by simply matching the genres, instruments, and slash or the artists. It gets rid of any song dislikes.

Kim: In conclusion, Find My Music is an app that uses an algorithm to give you new recommendations for music and donates to charity. Thank you.

Additionally, although Team C described their algorithm at two different points in their pitch, those descriptions did not include a discussion of the process by which users' survey responses would be mapped onto lists of songs. This omission could be the result of Team C not completing a functioning prototype by the time of the pitch competition. However, given their ability before the pitch to describe their algorithm in depth, it is likely that Shaun and Kim had difficulty: 1) anticipating how much information they would be able to present in 5 minutes and 2) knowing what information was most important during an entrepreneurial pitch. Additionally, Shaun and Kim were the only team to discuss how they planned to use their profits, with Kim explaining, "we donate 40%...of our income to Hungry for Music. Although this use of profits demonstrates team C's goal having a positive social impact with their business, it suggests they were still not considering the role of revenue in sustaining their business.

Post-Competition Task-Based Interviews.

On day 5, following the completion of the pitch competition, I conducted a seven-question task-based interview (see [Appendix N](#)), one with each student. The task-based interview

contained two parts. In part 1, students were given an algorithm for rating soccer players represented by the equation, $Overall\ Score = 1.5x + 4y + 2z$, where x represented a player's dribbling score, y represented a player's speed score and z represented a player's agility score. Students were then asked to respond to a set of six questions, which involved 1) interpreting the meaning and structure of the expression and 2) evaluating the expression for given values of the three variables. In part 2, students were given an algorithm for rating restaurants represented using three examples in a spreadsheet table and using the spreadsheet formulas, " $= 3 * B\# + 0.5 * C\# + 5 * D\#$," where $B\#$ represented a restaurant's cleanliness score, $C\#$ represented a restaurant's speed score, and $D\#$ represented a restaurant's menu score. Students then responded to a set of three questions, which again involved evaluating and interpreting the expression.

Table 7 shows the results of the task-based interviews.

Table 7

Results on the Task-Based Interview

Student	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7a	Item 7b	Item 7c	Correct (%)
Denise	+	+	+	+	+	+	+	+	+	100
Donald	+	-	+	-	NA	-	+	+	-	50
Beatrice	+	-	+	-	-	-	+	+	-	44
Maria	+	+	+	+	+	+	+	+	+	100
Kim	+	+	+	+	+	+	+	+	-	89
Shaun	+	+	+	+	+	+	+	+	+	100

“+” = correct response, “-” = incorrect response

In this section, I summarize the findings from those task-based interviews to shed light on students' understanding of algebraic expressions and spreadsheet algorithms.

Task-Based Interview Part 1. Denise, Maria, and Shaun were all able to correctly answer the first four questions without support and Kim was able to do so with minimal support. After reading the task statement, Kim, a rising 6th grader, initially expressed confusion, stating,

“I haven’t learned this.” Kim explained that, while she understood “what the variables stand for,” she did not understand the expression. After I explained that the expression was an algorithm that a company was using to rate soccer players, Kim was able to correctly interpret the meaning of the algorithm and explain how the company uses it to determine ratings for players, stating,

Just like looking at the statistics, like, what their speed is, their dribbling, and their agility, and then combining them and I’m assuming this, like, is how weighted they are... So, the speed score, they take that and multiply it by 1.5, and then multiply dribbling score by four, and agility score by two, and then add it up.

Relating the algebraic expression to the algorithm context helped Kim overcome her discomfort about answering a question relating to something she “[hadn’t] learned yet,” and make sense of the expression.

Denise, Maria, and Shaun all described the algorithm similarly, with Denise explaining, “they have like three different skills that they’re looking for: speed, dribbling, and agility. And I think they...multiply speed with, like 1.5 and then dribbling with four, agility with two, and then they add it all together [to get] the overall score.”

Questions 2 and 3 asked students to interpret the structure of the algebraic expression. Specifically, students needed to explain what the coefficient “1.5” represented and identify which variable the company thinks is most important. Denise, Maria, Shaun, and Kim recognized that the 1.5 in the algebraic expression represented the weight applied to players’ speed scores. Although Denise identified the “1.5” as the weight for a player’s speed score, she was unable to verbalize what she meant by “weight,” stating, “It’s like the weight...Like, um, I honestly don’t know how to explain it.” However, when asked which skill the company thought was most important for a soccer player to have, Denise responded, “dribbling,” explaining,

“...because the number that they’re multiplying it with is a lot higher than the other two.” While she was not able to define weight, she was able to interpret the structure of the expression and identify the variable that was “most important” by finding the variable with the greatest coefficient. Similarly, Maria identified dribbling as the most important score, explaining, “it has the highest, like weight, coefficient,” which she explained was “4.” Finally, on question 4, Denise, Maria, Shaun, and Kim all correctly calculated the overall score for a soccer player given a speed score, dribbling score, and agility score.

Beatrice and Donald had more difficulty with questions 1 through 4. Both students gave seemingly accurate descriptions for how the company uses the algorithm to calculate an overall score for soccer players. Beatrice described the algorithm by stating, “...they take the score from their combined ratings of their speed, dribbling, and agility to find the overall score.” Donald described it as “add[ing] up...all...three ratings to get an overall score on the player.” Both students recognized that the algorithm was combining the scores for each variable to produce an overall score for a player.

As evidenced by their responses to questions 2 through 4, however, Beatrice and Donald appeared to interpret the coefficients of the expression as representing the scores and the variables in the expression as representing labels for those scores (Clement, Lochhead, & Monk, 1981). In response to question 2, Beatrice explained that the 1.5 in the expression represents the player’s speed score because “...it says x equals the speed score.” Donald similarly interpreted the “1.5” in the expression as representing the speed score instead of the weight applied to the speed score in the algorithm, stating, “it represents...the score you got from the speed test.” Then, in question 4, both students substituted the given scores (i.e. 6 for the speed score, 10 for the dribbling score, and 5 for the agility score) for the coefficients instead of the variables. Thus,

Beatrice answered question 4 by adding together all three coefficients, concluding that the player's overall score would be 21 ($6+10+5$). Similarly, Donald concluded that the player's overall score would be " $6x + 10y + 5z$." Even though it resembled the algorithms they built for the challenge, Beatrice and Donald had difficulty interpreting the new algorithm, which was presented without a corresponding spreadsheet and related to a context that they had not previously considered.

Beatrice and Donald's interpretation of the coefficients as representing the skill scores did not affect their ability to answer question 3 correctly. Both Beatrice and Donald identified the greatest coefficient as the skill the company viewed as most important. Donald explained his reasoning, stating, "...I think the dribbling score is most necessary, is the most important, since it has the highest rating." In retrospect, this question may have been misleading and should be revised for future testing, as, "most important" is not clearly defined and, as a result, could have a range of correct answers depending on how it is interpreted by students.

Questions 5 and 6 further probed students' abilities to interpret the structure of algebraic expressions. Students were given a new algorithm that contained the same set of coefficients and variables, but rearranged, so that each variable had a different coefficient than in the original expression. In question 5, students were asked to explain how this new algorithm, $Overall\ Score = 4x + 2y + 1.5z$ would change the soccer player's (from question 4) score. In question 6, students were asked to change the algebraic expression so that a player's agility score would be four times more important than her dribbling score.

For question 5, only Beatrice argued that the player's score would not change, stating, "No, I don't think it would change because you're still adding the numbers together, even if they're in a different order." Given Beatrice's previous statements regarding the coefficients

representing the skill scores, her response to question 5 makes sense. To Beatrice, the algorithm was a sum of the three coefficients, so changing the order of the coefficients would not change the sum. Although Donald shared a similar interpretation of the algorithm, this question was unintentionally omitted in his interview.

The other four students all argued that the player's score would likely change. Kim argued that the score would likely be lower, referencing the magnitude of the skill scores and new coefficients, stating,

...because you would be multiplying by different things. So, like, it would, instead of it being 10 by four, it would be six by four, which is much less and then and then five, and then five by one half is...also less than five by two and, and it's just going to change up like the actual answers...for these, which means it's going to either make it more or less, which for this one, I would say less. Like it would be like they would have a worse overall score.

Denise also argued that the score would change, reasoning that the skill scores were being multiplied by different numbers, so the overall score was likely to also be different. Maria tested the new algorithm with the values from question 4 to see how the score would change. After stating that the score went “down from 59 to 51.5,” Maria then began thinking through the cases in which the score would increase or decrease.

I mean, I think it would depend on like the maximum the variable could be. Because I mean, if it was 50, then obviously that would be much higher. But if it's just like around 10 that I feel like that could be like, that's different.

Finally, Shaun reasoned that, “the overall score would be smaller because the dribbling score for the other one was really high and it had a really high multiplier also, but here, the one with the

big multiplier isn't as great of a number.” All four students were reasoning about the structure of the expression and the relationship between the inputs (skill scores), the outputs (overall scores), and the algebraic expression to determine whether the overall score would change.

Task-Based Interview Part 2. Part 2 of the task-based interview asked students three questions (7a, 7b, and 7c in Table 7) relating to a new algorithm, which was represented in a spreadsheet table and using a spreadsheet formula. All six students were able to explain how the algorithm calculated an overall score for each restaurant and were able to use the algorithm to calculate the rating for one of the three restaurants. Compared with the algorithm in part 1, the spreadsheet representation supported Beatrice to more easily interpret the algorithm.

Beatrice: So, you take the number from where it's like B2 you multiply that by three, and then for C2, you multiply that by 0.5, and then for D, D2, you multiply that by 5.

Researcher 1: And when you said the letters, what did those represent? What are you multiplying? Like, you said you take B2 and multiply by three. What is B2?

Beatrice: Well, it represents 4

Researcher 1: is it just any 4, is it just always 4?

Beatrice: no, it's the, well the number represents the cleanness of each restaurant.

In part 1, Beatrice interpreted the variables as labels for the coefficients. Now, when represented using a spreadsheet table and formula, Beatrice was better able to explain how the algorithm worked and to interpret the structure of the algebraic expression. The convention for naming variables in a spreadsheet, combined with the need to explicitly define any operations, supported Beatrice to interpret those variables as representing values instead of labels

Beatrice then demonstrated how the algorithm works by calculating the overall score for Panera using a sequence of numerical expressions (see Figure 21).

Figure 21

Beatrice's Solution for Panera

$$4 \times 3 + 5 \times 0.5 + 9 \times 5 = 82$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$12 + 25 + 45 = 82$$

Note. This figure shows the numerical expressions Beatrice used to calculate the overall score for Panera.

Although Beatrice made a minor calculation error as she typed the expression into her calculator, writing the product of 5 and 0.5 as 25, she demonstrated an ability to evaluate the symbolic spreadsheet formula. The spreadsheet formula and its corresponding tabular representation connected Beatrice's arithmetic reasoning to more generalized symbolic reasoning. However, on the last part of question 7, Beatrice was not able to write the spreadsheet formula as a single algebraic expression, suggesting that her ability to interpret an algebraic expression depended on seeing an explicit connection between symbolic and numerical expressions. Further research would be needed to establish whether interacting with multiple spreadsheet formulas would support Beatrice to, over time, recognize a need for a more efficient and generalized algebraic expression in which one symbol is used to represent all possible values for a variable.

Kim was also not able to represent the algorithm as a single algebraic expression, but was able to interpret the structure of the expression, stating,

No, this is the weights, and then the star means multiple multiplication. And then this is what, and then this is the square. So like she would, so they, so they would be

multiplying 4, in this case, B2 by three and that and They would be adding C, C2 multiplying that by 0.5, and then D2 multiplying that by five and, and then adding it up to get their overall score.

When Kim said, “the square,” she was pointing at the cell in the spreadsheet table and was explaining that the variables represented the locations of the values that would be used in the calculation. Kim then explained that the locations represented the column headings and showed, through three numerical expressions and by circling the coefficients, that the weights stayed the same for each expression and the variables could represent a range of values (see Figure 22).

Figure 22

Kim's Identification of Weights in Numerical Expressions

7. $2 \times 4 + 0.5 \times 8 + 5 \times 7$
 $3 \times 4 + 0.5 \times 5 + 5 \times 9$
 $3 \times 6 + 0.5 \times 9 + 5 \times 5$
 etc
 ○ = weights

Note. This figure shows the numerical expressions Kim used to answer question 7 on the task-based interview.

Kim's series of numerical expressions demonstrate how spreadsheet algorithms could act as a stepping-stone to writing more efficient and generalized algebraic expressions. By highlighting the invariances between the expressions (i.e. circling the weights) and recognizing the quantities that vary, Kim is on the verge of representing the expression using a single algebraic expression in which the varying quantities are represented symbolically.

Donald also had difficulty representing the spreadsheet algorithm using a single algebraic expression, despite his success describing how the algorithm worked. Donald described the restaurant rating algorithm by stating, "...they multiply three...by the clean, cleanliness of the restaurant...added with point 5 multiplied by the speed of the restaurant and then add it again added again with five multiplied by the menu score, to get an overall score." The spreadsheet formula and its corresponding tabular representation supported Donald to describe how the algorithm operated on the variables to calculate an overall score for the restaurant. Donald was describing the algorithm as operating on the variable quantity (e.g. speed of the restaurant) instead of a specific value. When asked if the spreadsheet is applying the same algorithm for all three restaurants, Donald responded in a way that confirmed his understanding of the variables as representing a range of possible values.

Donald: Well, for the cleanliness of Panera stays the same as for McDonald's. But the speed changes, it has a lower score than McDonald's. The menu has the menu for Panera is greater than McDonald's.

Researcher 1: And what stays the same for the algorithm from row to row from restaurant to restaurant?

Donald: So, the placing is all the same. It just, it's just that the scores are different from each restaurant.

In referencing "the placing," Donald demonstrated how the spreadsheet supported his interpretation of the algorithm by more closely connecting the variables to the numbers they represented. That is, the first algebraic expression (in questions 1 through 6) required Donald to understand that letters represented unseen quantities. In the spreadsheet algorithm, Donald could see the numbers that cell names represented. Although Donald, Kim, and Beatrice were all

unable to write an algebraic expression for the spreadsheet algorithm, they were nevertheless able to write numerical expressions, which suggests the spreadsheet could be a stepping-stone in the process of learning to use and interpret more efficient and generalized algebraic expressions.

Shaun, Maria, and Denise were all able to 1) explain how the spreadsheet algorithm calculated overall scores for restaurants, 2) calculate an overall score for Panera, and 3) write a single algebraic expression to represent the algorithm. Shaun described the algorithm as, “it multiplies the cleanliness for all of them by three and adds it with the speed, which is multiplied by a half, and then adds it to the menu, which is multiplied by five.” Denise described it similarly and added a reference to the weights applied to each variable, stating, “the cleanliness column, they’re weighting that by three and for speed, they’re doing it by point five and then for menu they’re doing it five.” Finally, Maria described the algorithm as “multipl[ying] the cleanliness of the restaurant times three, speed times five, and the menu times five, er, not the speed by point five, not five, I think, then the menu by five and then it gives it an overall score by adding those three together.” In their descriptions, all three students used language consistent with a functional understanding of algebraic expressions. They described the inputs (the cleanliness, speed, and menu of a restaurant), outputs (the overall score for the restaurant), and the systematic process for mapping the inputs onto the outputs. Additionally, all three were able to represent the algorithm as a single algebraic expression.

Summary

The first iteration of the design study shed light on how the *Building Algorithms* challenge and the Design & Pitch Challenges in STEM framework can support students’ engagement, while creating opportunities for mathematics learning. Specifically, the entrepreneurial framing of the challenge and the support resources empowered students to

engage in an iterative design process in which they built, tested, defended, and refined prototype algorithms. Additionally, through their engagement with the spreadsheet tool and the spreadsheet resource, students built, interpreted, and demonstrated flexibility with symbolic expressions.

Second Iteration of the Design Study

After reflecting on the results of the first iteration of the design study, two new research conjectures emerged. The first conjecture related to the use and introduction of the spreadsheet resource. Given the success of the spreadsheet resource when introduced with support, it was conjectured that providing students with explicit instruction on how to interpret and use the spreadsheet resource would help them overcome the spreadsheet language and syntax barrier and more quickly and easily engage with the challenge of building a spreadsheet algorithm. This conjecture was needed to better understand how to effectively implement the *Building Algorithms* challenge and how the challenge resources could support students' engagement. The second conjecture related to the use of the Technical Brief and posited that, when introduced during the competition and returned to after the competition with teacher/researcher support, it would help students to connect their work building spreadsheet algorithms to more generalized algebraic expressions. This conjecture was needed to better understand how to use the technical brief to more strongly and more explicitly connect students' challenge work to the STEM and mathematics content targeted by the challenge. This conjecture was especially important given the novelty and creativity of students' algorithms and the common criticism of PBL that students are unable to connect project work to the underlying STEM content goals (Barron et al., 1991).

To test these two new design study conjectures, I conducted⁹ a second iteration of the design study in the fall of 2019 with 15 students at a suburban charter middle school in North

⁹ The original plan was to have the classroom teacher facilitate the challenge with her students. However, after the first two days of the challenge, it became clear that the training she had been provided relative to the D&P challenge

Carolina. This second iteration of the study was conducted across ten¹⁰ 45-minute class periods. In the following sections, I start by briefly describing the contexts and variables selected by each team during the first two days of the study. Then, I report results relating to the testing of the two new research conjectures.

Brainstorming Contexts and Variables. Before the start of the study, the classroom teacher divided students into five teams of three. Because the class was an elective and not a core requirement, it included students in grades six through eight and teams were grouped heterogeneously by grade. Although this heterogeneous grouping likely meant students differed in their prior knowledge of algebraic expressions, I did not observe grade-level differences affecting students' ability to engage with the challenge.

After launching the competition and the challenge on day 1, teams used the rest of day 1 and all of day 2 to brainstorm contexts and variables for their algorithms. Across the five teams, students pursued algorithms relating to variety of contexts (see Table 8) and included variables representing both quantitative and categorical data.

Table 8

Teams from the Second Iteration of the Design Study

Team Name	Grades of Students	Context
Gamer's Paradise	6 th Grade (2), 8 th Grade (1)	Console Games
Pizza Hunters	6 th Grade (1), 7 th Grade (2)	Pizza Restaurants
Discover.GE	6 th Grade (2), 8 th Grade (1)	Video Games
RestaurantX	6 th Grade (2), 7 th Grade (1)	Restaurants
The Amigos	7 th Grade (1), 8 th Grade (2)	Hispanic Foods

Note. Team names represent the names students gave to their businesses.

framework and the *Building Algorithms* challenge was insufficient to support faithful implementation. Thus, on day 3, after consulting with the classroom teacher and with her approval, I took over responsibility for facilitating the challenge.

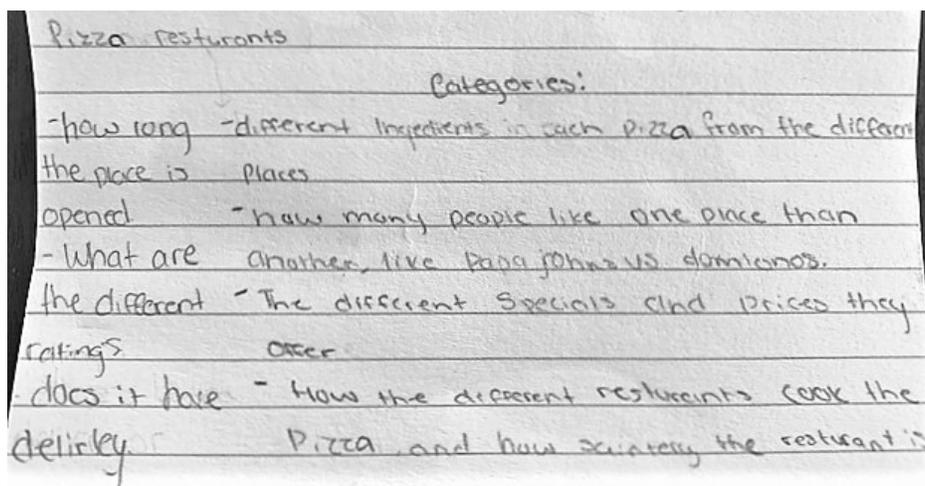
¹⁰ The challenge was planned to extend across five 45-minute class periods. Unanticipated interruptions, such as school-wide testing, fire drills, half-days, and class field trips, resulted in the challenge lasting nine class periods, each 30-45 minutes, with a tenth day added for students to complete their technical briefs.

The Gamer's Paradise team were interested in building an algorithm that would help "gamers" and parents of gamers decide whether to buy a "console video game." They identified four variables (three quantitative and one categorical) for use in their algorithm: accessibility (on what devices can users play the game?), rating, units sold, and a variable they called "extra stuff." For the Gamer's Paradise team, these variables all represented data they would consider when deciding whether to play a specific console video game. The Discover.GE team also selected video games for their context and identified the variables rating, age limit, and graphics as relevant to informing their decisions about whether to play a game.

The Pizza Hunters team wanted to build an algorithm that would help users decide whether to eat pizza at different restaurants and developed a list of potential "categories" that they would consider when deciding where to eat pizza (see Figure 23).

Figure 23

Pizza Hunters' Categories for Pizza Restaurants



Note. This figure shows the list of "categories" considered by the Pizza Hunters team for use in their algorithm.

This list of categories included questions like, "does it have delivery," "how sanitary [is] the restaurant," and "how [do] the different restaurants cook the pizza."

The RestaurantX team began the competition by considering building an algorithm that would rate animals. On day 2, based on their consideration of the entrepreneurial potential of an animal rating algorithm, the RestaurantX team decided to change their context to restaurants and identified the following three variables for use in their algorithm: cleanliness, quality of food, and service time¹¹.

Finally, the Amigos team leveraged their unique (to the class) cultural experiences in identifying variables for their algorithm, which would assign ratings to “Hispanic foods.” Their list of variables included the spiciness, appearance, smell, and serving size of foods: four characteristics that they viewed as relevant to their context and the purpose of their algorithm.

By the end of day 2, all five teams had identified contexts and an initial list of variables for their algorithms. Despite accessing the spreadsheet resource, as directed by the classroom teacher, none of the five teams had begun considering how to build their algorithm or operationalize their variables for use in their algorithm. Similar to teams A (horse racing tracks) and B (YouTube) in the first iteration of the design study, students only briefly reviewed the spreadsheet resource and, when they did, did not complete the interactive examples.

Introducing the Spreadsheet Resource. During the first iteration of the design study, the spreadsheet resource was a powerful tool that not only supported students to learn the spreadsheet programming language and syntax (a significant barrier for two of the three teams), but also helped clarify what it meant to build an algorithm. However, when accessed independently, two of the three teams used the resource statically (i.e. they did not engage with

¹¹ The original study design included the use of a pretest and posttest to quantify students’ math learning. After failing to recruit a large enough sample of students, the quantitative component was dropped from the study and the results of the pretest and posttest were not analyzed. However, the similarity between the RestaurantX team’s algorithm and an algorithm given on the pretest (question 7 on the task-based interviews) suggests the pretest may have influenced RestaurantX’s selection of a context and input variables.

the algorithm examples) and did not benefit from the resource as intended. One possible explanation for this result is that the instructions and examples in the spreadsheet resource were unclear and needed to be revised. Another possible explanation is that students needed explicit support to use, understand, and fully benefit from the spreadsheet resource. To avoid prematurely revising the materials and because the teacher/researcher support was effective during the first iteration of the design study, I decided to test the latter explanation during the second iteration of the design study.

Specifically, it was conjectured that providing students with explicit instruction on how to interpret and use the spreadsheet resource would help them overcome the spreadsheet language and syntax barrier and more quickly and easily engage with the challenge of building a spreadsheet algorithm. This conjecture was needed to better understand how teachers can implement the *Building Algorithms* challenge and effectively leverage the spreadsheet resource to support students' engagement. From a more global perspective, this conjecture could also help inform how teachers can effectively scaffold student work on the D&P Challenges in STEM.

On day 3 of the study, I led a brief guided tutorial on how to use the spreadsheet resource, focusing specifically on helping students to understand the purpose of the interactive algorithm examples and the necessary spreadsheet language and syntax. During this tutorial, I, 1) showed students how to access the spreadsheet resource; 2) briefly explained that the purpose of the algorithm examples was to provide alternative models on which students could base their algorithms; 3) explained the purpose of the spreadsheet tips; and 4) gave students the task of figuring out how to program the spreadsheet to carry out the first rating algorithm example. Once teams understood how to write a spreadsheet formula for the first example, I then demonstrated how to use the "autofill" command to apply the formula across multiple rows.

In the guided tutorial, I only discussed the first example in the rating algorithms worksheet. I made this decision for two reasons. First, given the time constraints of a 45-minute class period, I wanted to keep the guided tutorial short enough to allow students time to apply what they learned to their algorithms before the end of class. Second, the eagerness of Team A (horse racing tracks algorithm) to experiment with the other rating algorithm examples during the first iteration of the design study, suggested that one example could be enough to encourage teams to independently engage in further exploration.

By the end of the guided tutorial, all groups were able to use the appropriate spreadsheet language and syntax to successfully program their spreadsheets to compute a sum of multiple cells and to apply the sum across multiple rows. Additionally, after showing confusion on day 2, the guided tutorial supported teams to begin making progress on their algorithms and had a considerable impact on how they operationalized their variables and built their function rules. The decision to only discuss the spreadsheet tips and the first rating algorithm example during the guided tutorial resulted in a lack of variation across the five teams, in terms of how they defined their function rules and operationalized their variables.

The first rating algorithm example involved evaluating job candidates using expert ratings in three variables: interview performance, eagerness to learn, and enthusiasm for the position. The algorithm calculated an overall score for each job candidate using a sum in which the expert ratings in each of the three variables were equally weighted. Teams appeared to rely heavily on this example in building their algorithms. For example, the Amigos team used ratings on a scale of 1 to 10 to operationalize each of their four variables for rating Hispanic foods. Their algorithm then assigned an overall score to each food by calculating the sum of the four variable ratings (see Figure 24).

Figure 24

The Amigos' Spreadsheet Algorithm.

	C	D	E	F	G	H
4		SPICINESS	SMELL	SERVING SIZE	APPEARANCE	SCORE
5	FJITAS	4	8	8	6	26
6	CARNE ASADA	0	9	8	6	#REF!
7	PUPASAS	1	7	6	7	21
8	LULO	0	6	4	8	18

=D8+E8+F8+G8

Note. This figure shows how the Amigos team operationalized their variables for use in their algorithm for rating Hispanic foods.

In row 9 in figure 23, the four variable ratings for “LULO” (a Latin American fruit) were collaboratively agreed upon by the three team members. They programmed the spreadsheet using the formula “=D8+E8+F8+G8” to calculate the sum of the four variable ratings to assign “LULO” a score of 18.

Similarly, the RestaurantX team operationalized their three variables (service time, food, and cleanliness) by assigning ratings to each variable. Their algorithm determined overall scores for each restaurant by calculating the sum of the three variable ratings (see figure 25). For example, to calculate the total score for Oliver Garden (row 3), one member of the team assigned the ratings of 4.1, 5, and 4.7 to the variables service time, food, and cleanliness, respectively. Then, they programmed their spreadsheet using the formula “=D3+C3+B3” to calculate the sum of the three ratings and assign Olive Garden a total score of 13.8.

Figure 25

Restaurant X's Spreadsheet Algorithm.

	A	B	C	D	E
1	Restrants	service time	food	cleanliness	total score
2	Lugano	4.5	5	4.9	14.4
3	Olive Garden	4.1	5	4.7	13.8
4	Panera	4.2	4.8	4.3	13.3

=D3+C3+B3

Expert Ratings

Note. This figure shows the RestaurantX team's spreadsheet algorithm.

The Gamer's Paradise team used a combination of evaluative measures and descriptive measures to operationalize variables for use in their console games algorithm (see figure 26). Specifically, they operationalized the "rating" variable by assigning each console game a rating out of 5. They used descriptive measures to operationalize the "units sold" variable and a newly added "other versions" variable. The "units sold" variable represented the number of copies sold, in millions, and the "other versions" variable represented the number of versions of the game. Their algorithm then calculated the sum of these three variables to produce a score for each game.

Figure 26

Gamer's Paradise Spreadsheet Algorithm.

	A	B	C	D	E	F	G	H	I	J
1	games		platforms you could play on			Rating	units sold (in millions)	other versions		score
2										
3	apex legends		ps4	xbox one	computer	5	10			15
4										
5	a plaque tale innocence		ps4	xbox one	computer	4	11			15
6	mortal kombat 11	ps4	xbox one	computer	switch	4	11	1		16
7										

Expert Rating

Descriptive Measures

=F6+G6+H6

Note. This figure shows how the console games team operationalized their variables for use in their algorithm, using a combination of evaluative and descriptive measures.

For example, to calculate the total score for “mortal kombat 11” (row 6), they used the formula “=F6+G6+H6” to find the sum of the values for “rating,” “units sold,” and “other versions.”

The Discover.GE team also operationalized their variables using expert measures of how much they liked the game (on a scale from 1 to 5) and the quality of the graphics (on a scale from 1 to 10). Additionally, they were one of only two teams to use a function rule other than an equally weighted sum. The Discover.GE team wanted the “graphics” variable to have a greater impact on a game’s overall score than the “rating” variable. However, rather than using coefficients to reflect this difference in weight, the Discover.GE team rated the “graphics” variable out of a maximum value of 10 instead of 5. In this way, they made “graphics” twice as important to the overall score as “rating.” According to the classroom teacher, this method of weighting variables was consistent with the school’s approach to grading, in which assignments are weighted differently by changing the maximum possible score.

The Discover.GE team also used a piecewise rule for assigning overall scores to video games. Instead of simply adding the ratings of the three variables together, their algorithm sorted ratings based on whether they were greater than or less than a predetermined value (3.5 for the “rating” variable and 5 for the “graphics” variable). They then combined all scores less than 3.5 (for the “rating variable) or less than or equal to 5 (for the “graphics” variable) to determine an overall “negative” score for the game. They combined all scores greater than 3.5 (for the “ratings” variable) or greater than 5 (for the “graphics” variable) to determine an overall “positive” score (see Figure 27). For example, in Figure 27, Discover.GE showed how their algorithm would calculate a positive and negative score for a video game, though they mislabeled their spreadsheet. Discover.GE’s algorithm combined the circled numbers (8, 5, and 10) to calculate a positive score (labeled “negative” in the figure) of 23, and the boxed numbers

(3, 2, and 5) to calculate a negative score (labeled “positive in the figure) of 10. Although the Discover.GE team did not build their algorithm into a spreadsheet, the spreadsheet resource clearly influenced their final algorithm, both in terms of how they operationalized their variables and their use of a sum for their function rule.

Figure 27

Discover.GE’s Final Algorithm, Annotated.

People:	Rating:	Age limit:	Graphics:
Ryan	3	10	8
Beth	2	8	5
Katie	5	13	10

Positive scores:	Negative scores:
10	23

○ Positive scores*

□ Negative scores

* The team switched the labels for positive and negative scores. This was determined based on their statements describing their algorithm.

Note. This figure shows the Discover.GE team’s final algorithm in tabular form with annotations to explain how it worked.

Finally, the Pizza Hunters team also operationalized their variables using a combination of evaluative measures and descriptive measures, but their function rule differed from the other four teams. Initially, the Pizza Hunters team used a sum to calculate the overall rating for a pizza restaurant (see Figure 28) based on the descriptive measure for “How clean is the restaurant” and the evaluative measures for the variables “How good it tastes,” “Different toppings,” and “How good the crust is.”

Figure 28

Pizza Hunters' First Algorithm.

	A	B	C	D	E	F
1	Pizza Restaurant	How clean is the restaurant	How good it taste	Different topping	How good the crust is	Overall score
2	Jonny's Pizza	96.5	8	8	9.5	122
3	Ruckus	95.5	7	10	8	120.5
4	Mellow Mushroom	96.5	9	10	9.7	125.2

Descriptive Measure
(out of 100)

Expert Ratings
(out of 10)

=B4+C4+D4+E4

Note. This figure shows the first iteration of Pizza Hunters' algorithm.

For their final algorithm, the Pizza Hunters team instead used an equally weighted mean to calculate the overall score for each restaurant, but did not program it into their spreadsheet.

The algorithms for all five teams were influenced by the guided introduction of the spreadsheet resource, helping students operationalize their variables and define their function rules. In this way, the guided introduction supported students' engagement during the challenge by helping them to develop the skills and knowledge needed to make progress towards their solution. However, the scaffolded introduction of the spreadsheet resource may have also limited students' creativity and limited their engagement with more complex algorithms. Three of the five teams used algorithms that involved calculating the sum of equally weighted variables. These teams modeled their algorithms on the first rating algorithm example in the spreadsheet resource and were not observed experimenting with the other more complex algorithm examples, suggesting that discussing an additional algorithm during the guided tutorial could have been beneficial.

Using the Technical Brief to Connect Algorithms to Algebraic Expressions. The Design & Pitch Challenges in STEM were designed to engage students in authentic entrepreneurial activity as way of increasing students' interest and engagement in STEM, especially mathematics. Central to this goal was allowing students the freedom to innovate in solving real and messy problems, while also establishing constraints that would allow teachers some level of confidence in the STEM and mathematics content with which students would engage while building their solutions. To that end, the Technical Brief was created as a tool that students would perceive as authentic to their entrepreneurial processes and that would connect their challenge work to the targeted and standards-aligned STEM content.

During the first iteration of the design study, students had difficulty engaging with the Technical Brief. Their enthusiasm and their eagerness to collaborate were both visibly depressed as they worked on the technical brief. As a result, teams did not spend much time considering how to represent their spreadsheet formulas using a more generalized algebraic expression. One possible explanation for their lack of engagement is that there was a disconnect for students between the *Building Algorithms* challenge, which they perceived as different from their typical school math experience, and the technical brief, which was designed to elicit standards-aligned (i.e. school) mathematics. However, given the importance of having students reflect on the connections between their challenge work and the underlying STEM content (Krajcik et al., 2009), the technical brief is essential if the D&P challenges are to be used in a school setting.

Thus, additional testing was needed to understand how to best position, frame, and support the use of the Technical Brief. Specifically, it was conjectured that the Technical brief, when introduced during the competition and returned to after the competition with teacher/researcher support, would help students to connect their work building spreadsheet

algorithms to more generalized and standards-aligned algebraic expressions. This conjecture was needed to determine whether the technical brief would be better positioned as a tool for reflection and whether positioning it in this way would support students to connect their challenge work to the targeted STEM content.

On day 6 of the competition, I briefly introduced the technical brief to the class. With fewer than half of the students in attendance¹², teams made little progress on their technical briefs and did not return to the technical briefs before the final pitch competition. To test the new conjecture, an extra day (day 10) was added to allow teams time to complete the technical brief with teacher/researcher support.

During the competition, teams only used symbolic expressions while programming their spreadsheets and none of the five teams independently represented their spreadsheet formula using a single algebraic expression. It is possible that, like in the first iteration of the design study, there was a disconnect between the expectations of the technical brief (e.g. using a single algebraic expression to represent spreadsheet formulas) and the representations students used in building their algorithms. I used day 10 to work with students specifically on completing part 7 of the Technical Brief which tasks students with representing their spreadsheet algorithms using algebraic expressions.

When teams began working on part 7 the Technical Brief, several reported that they did not know what an algebraic expression was and, thus, did not know how to use one to represent their algorithm. I supported each of these teams by leveraging the components of part 7 and teams' understanding of their algorithms. First, I asked teams to describe, in words, how their algorithm worked. Then, I asked them to show or describe an example calculation from their

¹² Over half the class were either attending a field trip or completing school-wide testing.

spreadsheet and to explain what each number in the expression represented in their algorithm. Finally, I briefly explained how symbols could be used to represent those values, in place of the terms or phrases used in their verbal descriptions.

This approach was effective for four of the five teams. For example, the RestaurantX team explained how their algorithm worked using their rating of Panera as an example, writing the equation, $4.2 + 4.8 + 4.3 = 13.3$. When asked what the 4.2 in their expression represented, one student exclaimed, “Ohh, the service time.” They were then able to write the algebraic expression shown in Figure 29 to represent their algorithm, demonstrating how, with teacher/researcher support, they were able to build from representing their algorithm numerically to representing it algebraically.

Figure 29

RestauranX's Algebraic Expression.

X=Service time Y=Quality Food and Z=Cleanliness
So, our algebraic expression is $X+Y+Z$

Note. This figure shows a recreation of how the RestauranX team represented their algorithm using an algebraic expression.

After receiving similar support, the Amigos team were also able to verbally describe their algorithm, give a numerical expression as an example, and then represent both using a single algebraic expression. They described their algorithm, by stating that it “adds [the ratings] together to get the total score,” then explained using numbers from their spreadsheet, before translating it into the algebraic expression shown in figure 30.

Figure 30

The Amigos' Algebraic Expression.

x -spiciness y -smell m -serving size w -appearance $x+y+m+w = \text{score}$

Note. This figure shows the Amigos' algebraic representation of their algorithm.

While the Pizza Hunters team did not need additional support to represent their algorithm using an algebraic expression, their written response to part 7 of the Technical Brief suggests their thinking followed a progression similar to the support given to the RestaurantX and the Amigos team. They included both a numerical expression, showing how they calculated an overall score for the pizza at the restaurant "Roma's Italian," and a corresponding algebraic expression (see Figure 31).

Figure 31

Pizza Hunters' Algebraic Expression.

$95.5 + 7.5 + 9 + 9 \div 4 = 30.25$ $C + t + d + H \div 4 = O$ C how clean the restaurant is t = how good it taste [sic] d = different toppings H = how good the crust is
--

Note. This figure shows the algebraic expression Pizza Hunters used to represent their algorithm.

Although both their numerical expression and algebraic expression contained a syntactical error (omitting parentheses to show that they were dividing the sum by 4 instead of only dividing H by 4), the two expressions demonstrate how the Pizza Hunters team built from their numerical expression to their algebraic, by replacing the numerical values with symbols representing the quantities those values represented.

Unlike the Amigos, Pizza Hunters and RestaurantX teams, neither the Discover.GE team nor the Gamer’s Paradise team were able to represent their complete algorithm using an algebraic expression, despite receiving the same support. The Discover.GE team’s success representing part of their algorithm algebraically suggests the complexity of their complete algorithm acted as a barrier. Their algorithm classified user ratings of two variables (“rating” and “graphics”) as positive or negative based on whether they were greater or less than a cut point (3.5 for “rating” and 5 for “graphics”). To show how their algorithm worked, the Discover.GE team focused only on how they would determine the total value for the “rating” variable if four different users rated the same game (see Figure 32).

Figure 32

Discover.GE’s Numerical Expression.

$$(3 + 2) = \text{Negative} \quad (5 + 4) = \text{Positive}$$

Note. This figure shows the numerical expression used by the video games team to represent their algorithm on the Technical Brief.

In their example, a video game received two ratings that would be classified as “positive” (4 and 5) and two ratings that would be classified as “negative” (2 and 3). To motivate a need for the algebraic expression, I explained, “the point of the algebraic expression is that you can hand it to somebody and they can reproduce your system and they can come up with the same scores. The problem here is, I don’t know what 3 and 2 are [and] I don’t know what 5 and 4 are.” By referring to the need to reproduce their algorithm, I was trying to help the Discover.GE team understand the importance of representing the values generally. The team responded stating, “um, that’s just like that’s just his rating, ‘cause we have a rating out of 5 stars.” Following this discussion, the video games team then wrote the algebraic expression shown in figure 33, in

which P1, P2, P3, and P4 represented the value of the rating variable for person (or user) 1, 2, 3, and 4 respectively.

Figure 33

Discover.GE's Algebraic Expression.

$$\begin{array}{l} (P1 + P2) = N \\ (P4 + P3) = P \end{array}$$

Note. This figure shows the numerical expression used by the Minecraft team to represent their algorithm on the Technical Brief.

Although this algebraic expression communicates how their algorithm calculates the value of the “rating” variable for four different users, it does not show how that value would be incorporated into the overall score or how the values would be calculated for their other variable (graphics). The Discover.GE team appeared to be grappling with the issue of how to handle multiple users, which was the same issue Team B (YouTube) grappled with in the first iteration of the design study. Team B used the same spreadsheet to show how their algorithm would calculate an overall score for different YouTube channels and how it would calculate an overall score for one YouTube channel with multiple user ratings for each variable. In the same way, the Discover.GE team were showing how they would combine multiple ratings for a single variable and a single video game, but did not show how their algorithm would determine the overall score for the video game. Nevertheless, the Discover.GE team’s experience with the technical brief created an opportunity for the teacher to work with them to connect their algorithm to more formal algebraic expressions.

The Gamer’s Paradise team were also unable to fully connect their verbal and numerical representations of their algorithm to a single algebraic expression (see figure 34).

Figure 34

Gamer's Paradise's Expressions.

- a. Our algorithm had a spread sheet that added the topic we were rating for example
 $4\text{rating sold } 11\text{million } 1\text{ versions} = 16\text{ score}$
- b. We added units sold, the rating of the game and other versions of the game which got us to our total
 $4\text{ sold } 13\text{m } 5\text{v} = 4 + 13 + 5 =$

Note. This figure shows the algebraic expressions used by Gamer's Paradise to represent their algorithm.

The two expressions they produced suggest that the Gamer's Paradise team recognized the goal of using variables to more concisely describe their algorithm. However, like Beatrice and Donald in the first iteration of the study, they appeared to interpret variables as labels instead of as representing the ratings assigned to those variables (Clement et al., 1981). For example, consider the transition from expression 1 to expression 2 in figure 34. Instead of replacing the numerical values with symbols, they replaced the description of those values with symbols. That is, they 1) removed "rating" from their expression, which described what the "4" in the expression represented; 2) replaced "million," which described the number of units sold, with "m;" and 3) replaced "versions," which described what the "1" in the expression represented) with "v." Had I more explicitly connected the meaning of variable to their spreadsheet formula, it is possible the Gamer's Paradise team would have been able to better understand what it meant to represent a quantity using a variable.

Except for when they engaged with the spreadsheet, none of the teams in the second iteration of the study independently used algebraic expressions during the competition to represent their algorithms. As conjectured, the technical brief provided all teams with an opportunity, after the competition, to reflect on their challenge work and connect their

spreadsheet formulas to the types of generalized algebraic expressions (Rojano, 1996) that are more typical in middle grades instruction. Because this reflection occurred after the competition had been completed, it did not influence teams' algorithms as originally intended. However, its effectiveness connecting teams' challenge work to the targeted STEM content suggests this could be a potentially effective model for utilizing the technical brief.

Overall, the second iteration of the design study revealed important insights about the Spreadsheet Resource and the Technical Brief. As conjectured, the early and guided introduction of the spreadsheet resource supported students to quickly overcome the language and syntax barrier that initially prevented teams A and B in the first iteration of the study from engaging with the spreadsheet tool. However, this early and guided introduction of the spreadsheet resource may have restricted students' creativity and willingness to take risks in building their algorithms, which may have also limited the complexity of the expressions that they encountered during the study. The second iteration of the design study also demonstrated that the Technical Brief, when introduced as a tool for reflection with teacher/researcher support, can be an effective tool for supporting students to connect their challenge-specific mathematical activity to the targeted standards-aligned mathematics.

Results Chapter Summary

While participating in the *Building Algorithms* challenge, students explored a variety of contexts for their algorithms and demonstrated consistently high levels of engagement as they built, tested, and refined their algorithms, businesses, and pitches. The open and a priori coding process revealed several themes that shed light on how the Design & Pitch Challenges in STEM curricular framework and the *Building Algorithms* challenge supported students' engagement and

mathematics learning. In the following chapter, I present these themes and use them to answer the two research questions that were the focus of the study.

CHAPTER 5: DISCUSSION

Across both iterations of the design study, the D&P challenge framework and the *Building Algorithms* challenge supported students' engagement, with respect to both the task of designing an algorithm-based business and building a functioning spreadsheet algorithm. Additionally, the dual tasks of designing a business and building a prototype algorithm interacted to create opportunities for rich mathematics learning, relating specifically to algebraic expressions and functions. The coding of video transcripts and analysis of student work samples revealed several themes (see Table 9) which shed light on how the *Building Algorithms* challenge and the D&P challenge leveraged the entrepreneurial processes to support cognitive engagement and mathematics learning.

Table 9

Coding Themes.

Theme	Description	Corresponding Codes
Engagement through Entrepreneurial Processes	The entrepreneurial processes helped students to invest in designing their algorithm-based businesses and promoted persistence and engagement.	Authentic and Actionable; Defining Variables; Enthusiasm; Entrepreneurial Processes and Characteristics codes; Referring to Real Businesses.
Structure and Resources Support Authenticity	The structure of the challenge and the support resources supported engagement by highlighting and enhancing the authenticity of students' work.	Business Model Types Document; Challenge Videos; Context Document; Expert Check-in; How to Build a Pitch; Identifying Gaps in Knowledge; Pitch Judging Sheet; Spreadsheet Resource; Technical Brief.
Engagement through Pitching	The pitch process generated emotions and entrepreneurial characteristics that supported sustained engagement.	Competition; Convincing; Courage; External Judges; Final Pitch; Nervousness; Pitching; Practice Pitch.

Table 9 (continued).

Math Learning through Pitching	Pitching created opportunities for students to reflect on the structure and meaning of their algorithms.	Considering and Justifying Coefficients; Considering Inputs and Outputs; Defining Variables; Describing algorithms; Expert Check-in, Explaining, Defending, Refining, and Justifying; Pitching.
Math Learning through Spreadsheets and the Challenge Structure	The challenge structure and the affordances of spreadsheets created a need for algebraic expressions and supported students to persist in learning about them.	Considering and Justifying Coefficients; Exposing Ideas to Critique; Identifying Gaps in Knowledge; Multiple Meanings of Variables; Numerical to algebraic; Predicting and Restricting outputs; Spreadsheet Affordances; Spreadsheet Resource; Testing and Refining Algorithms; Verbal to Algebraic.
Algorithm Challenge and Entrepreneurial Processes Support Modeling	The combination of the algorithms challenge and the set of entrepreneurial processes supported students to progress through an iterative modeling cycle, which supported mathematics learning.	Considering and Justifying Coefficients; Considering Inputs and Outputs; Converting Data; Defining a Function Rule; Describing Algorithms; Entrepreneurial processes codes; Exposing Ideas to Critique; Predicting and Restricting Outputs; Testing and Refining Algorithms,

In the following sections, I explain how the themes in Table 9 address the two research questions and their corresponding design study conjectures.

Research Question 1

Research question 1 stated, “How and to what extent does participating in the *Building Algorithms* challenge support and promote students’ engagement in a mathematics classroom setting?” The exploration of this question was guided by the following design study conjectures:

- 1) The entrepreneurial framing of the *Building Algorithms* challenge (i.e. situating the challenge in the context of planning an algorithm-based business) will help students to perceive their solutions as actionable and authentic, which will help them maintain their engagement during the challenge.

- 2) The challenge materials and support resources will enhance students' perceptions of the authenticity of the challenge, which will increase their engagement.
 - a. Providing students with explicit instruction on how to interpret and use the spreadsheet resource will help them overcome the spreadsheet language and syntax barrier and more quickly and easily engage with the challenge of building a spreadsheet algorithm.
- 3) Practicing for and participating in the pitch competition will increase students' engagement and enthusiasm during the challenge.

Overall, the D&P challenge framework and the *Building Algorithms* challenge promoted and supported students' engagement by 1) leveraging entrepreneurial processes and students' out-of-school knowledge, expertise, and experiences to empower students and generate investment; 2) establishing expectations for students' work and supporting persistence through the challenge support resources and materials; and 3) providing accountability and building enthusiasm through the pitch process. In this section, I address research question 1 by discussing the results of the four design study conjectures.

Conjecture 1

The first conjecture tested during the first iteration of the design study stated that the entrepreneurial framing of the *Building Algorithms* challenge (i.e. situating the challenge in the context of planning an algorithm-based business) would help students to perceive their solutions as actionable and authentic, which would help them maintain their engagement during the challenge. The results of testing this conjecture can be explained by the theme titled "engagement through entrepreneurial processes" (see Table 9) This theme describes the ways in

which the entrepreneurial processes helped students to invest in designing their algorithm-based businesses and promoted persistence and engagement.

At the start of the study, students demonstrated a limited understanding of entrepreneurship and pitch competitions and expressed little excitement during the initial launch of the competition. This suggests that the prospect of participating in an entrepreneurial pitch competition was not immediately engaging to students and did not initially compel them to consider the actionability of their solutions. However, once students had opportunities to participate in authentic entrepreneurial processes, such as opportunity and resource analysis and selecting business model types, and were able to connect their work to work being done by real businesses, students became much more enthusiastic about the competition and were willing to share their out-of-school interests and expertise.

Two of the most impactful entrepreneurial processes that supported students' engagement by highlighting the authenticity and actionability of their solutions were 1) idea generating and 2) opportunity and resource analysis. The impact of these two processes were most evident in students' selection of contexts for their algorithm-based businesses. In allowing students the freedom to develop innovative solutions to problems they cared about, the *Building Algorithms* challenge unleashed their creativity and invited them to leverage their out-of-school knowledge and expertise in a mathematics setting. As a result, teams selected a variety of contexts for their algorithm-based businesses, all of which were personally relevant, meaningful, and authentically entrepreneurial. For example, although Team B's selection of YouTube channels as their context was initially based only on their enthusiasm for YouTube, their willingness to persist with the context was also based on the number of potential customers for a YouTube-focused business (i.e. its entrepreneurial potential). Similarly, the Amigos' selection of Hispanic foods as their

context was based on their excitement about sharing something that was authentic and personally relevant with the class. Their willingness to persist with the context emerged from their recognition of its entrepreneurial potential: they could identify customers who would want help deciding whether to eat different Hispanic foods.

For several of the teams, the process of selecting a context (idea generating) involved a negotiation between finding topics for which they were uniquely suited to address (resource analysis) and finding topics that carried entrepreneurial potential (opportunity analysis). Students' consideration of a context's entrepreneurial opportunity and whether they possessed the resources needed to address that opportunity helped them to refine and focus the purpose of their businesses. This was especially true for Team A (horse racing tracks) who considered, simultaneously, the resources they had available and the possibility of an entrepreneurial opportunity. Team A selected their context based on 1) Denise's passion for horses and 2) her identification of a specific problem (horses dying at racetracks) that needed solving. They identified a context based on the resources they had available to them (i.e. their knowledge of horses) and the presence of an entrepreneurial opportunity. However, as they began building their algorithm and designing their business, they had difficulty defining their target customer. Once they arrived at charging the horse racing tracks for their service, they reframed how they were describing their business. Thus, for Team A, the entrepreneurial process of opportunity and resource analysis highlighted the importance of making their solution actionable and helped them sharpen the focus of their business idea.

Students' creativity in designing businesses around their contexts supported their investment in building working prototypes of their algorithms and empowered them to explain, defend, and justify their thinking relative to their algorithms and businesses. These two processes

created opportunities for students to leverage their out-of-school knowledge and experiences in the planning of their algorithm-based businesses. For example, the Pizza Hunters team's experiences with pizza restaurants empowered them to justify their selection of variables (e.g. variety of toppings and quality of crust) and their recognition of its actionability and authenticity empowered them to persist in building a prototype of their algorithm.

Another way that entrepreneurship supported students' engagement was through the value it places on diversity of thought and skills. The presence of diverse skill sets in each team and students' abilities to identify and leverage those skill sets created multiple points of entry to engage with the challenge. In several cases, those diverse skills allowed students who had less experience with algebraic expressions and spreadsheets, to contribute to their teams' algorithms, pitch decks, and pitches. Beatrice's artistic skills gave her a way to engage with the challenge by designing her team's logo and her knowledge of different types of YouTube channels (ones Maria was not familiar with) allowed her to contribute to her team's selection and operationalizing of variables. Kim leveraged 1) her skills as a performer to contribute meaningfully to her team's pitch and 2) her knowledge of music to take the lead on building her team's survey and music library. Shaun leveraged his skills with programming and spreadsheets to take the lead on building his team's music filtering algorithm. Denise leveraged her knowledge and expertise relative to horses to take the lead on identifying tracks and defining variables for her team's horse racing track algorithm.

In each of these cases, students' diverse skill sets gave them a way into the challenge, even if that way in was light on STEM content. For both Beatrice and Kim, their identification of their diverse skill sets seemingly allowed them to avoid engaging with the STEM content targeted by the challenge. However, by finding a way to contribute during the competition, both

students were invested in their algorithm-based businesses and this investment allowed them to benefit from the algorithms challenge. This was especially true for Beatrice, who as a rising 6th grader was not familiar with algorithms or algebraic expressions. Beatrice's responses on the task-based interview suggest that, despite her seemingly peripheral engagement during the challenge, the algorithms challenge supported her understanding of algebraic expressions. Beatrice was able to interpret the structure and meaning of an algorithm represented using spreadsheet language and syntax. Thus, despite not engaging directly with the programming of her team's spreadsheet, Beatrice's diverse skills gave her a way to engage with the challenge which allowed her to benefit from participating in the competition.

Conjecture 2

For the second conjecture, I explored how the challenge support materials and resources supported students' engagement. Specifically, I explored whether the *Building Algorithms* challenge materials and support resources would help students perceive the challenge as authentic and whether their perception of its authenticity would increase their engagement. The rationale for this conjecture was that by connecting students' challenge work to real entrepreneurial activity, students would perceive the competition as worthwhile beyond satisfying a school requirement.

The results of testing this conjecture can be explained by the theme titled, "the structure and resources support authenticity." This theme describes the ways in which the structure of the challenge and the support resources supported engagement by highlighting and enhancing the authenticity of students' work. The set of challenge materials and resources can be organized into three categories based on their role in establishing authenticity. The first category includes resources that were designed to demonstrate the authenticity of the challenge task: the challenge

champion videos and the challenge context document. The second category includes resources designed to engage in authentic entrepreneurial processes. This category includes the Key Business Proposition, the Business Model Types document, and the pitch resources. The final category includes resources designed to highlight the authenticity of the challenge by supporting students to build working prototypes of their solutions. This category is intended to highlight the authenticity of the challenge by helping students see that their solution can work under authentic circumstances.

Resources Designed to Highlight Authenticity of the Competition. The challenge champion videos and the Context Document were designed to highlight the authenticity of the competition by showing the connection between the task of building a spreadsheet algorithm and the work done by real companies. The two challenge champion videos that were used during the competition (challenge statement video and challenge background video), both featured Cathy Yee: the CEO of a business that uses rating algorithms. As Kim explained, the challenge champion videos helped students understand how the work they were doing in the competition was the same type of work being done by familiar companies. According to Kim,

[the videos] kind of showed how [Cathy Yee] used the algorithm...with her company and how her company still does, like, things that we use every day...She said in the second video it was kind of like [Yelp] and then I was like, 'Oh...I use that...and it kind of connected everything for me. Like...how I can use an algorithm to make something and it's, like, not going to just be like, 'oh, just apply this to this.'

The challenge videos helped Kim, and the rest of the group, understand how the task of building an algorithm was useful and relevant to real companies, which they cited as contributing to their enjoyment of the activity. This enthusiasm was evident throughout the competition and

led several students to spend time outside of the testing sessions discussing and thinking about how to improve their solutions.

The Context Document was also designed to highlight for students how their challenge work was a timely and authentic entrepreneurial context. Containing internet links to real-world uses of algorithms, the Context Document was designed to jumpstart student thinking and to demonstrate the authenticity of the challenge by referencing real and familiar businesses that also use algorithms. Because the challenge was launched with a whole-group discussion of familiar algorithm-based businesses (e.g. Netflix and Spotify) and because most teams were able to quickly identify contexts for their algorithms, students did not perceive a need for the context document and did not access it during the competition. Nevertheless, students' heightened enthusiasm and eagerness to share their ideas during the discussion demonstrates how helping students consider the authenticity of the challenge can support their engagement.

Additionally, by allowing students to draw on their out-of-school knowledge, the *Building Algorithms* challenge supported students to authentically connect algorithm building to their out-of-school experiences and interests. Both Shaun and Kim referenced this connection as being important to their willingness to persist in completing school activities. As Kim explains, "...if I don't have...a connection to it, I just...kind of get bored or, like, 'Oh, this doesn't really matter.' But if I have a connection to it, I feel like it makes it a lot more interesting." This authentic connection between the task of building an algorithm-based business and students' out-of-school interests was integral to supporting their willingness to engage and persist in building their algorithms.

Resources Designed to Engage Students in Authentic Processes. The D&P challenge framework includes three sets of resources designed to support students to engage in authentic

entrepreneurial processes. These sets of resources include business model resources (the Business Model Types document and the Key Business Proposition), the Technical Brief, and the Pitch Resources.

The business model resources were designed to support students to consider the business side of their solutions and thus highlight the authenticity of the pitch competition and challenge. These resources included the Business Model Types document, which contained description of different types of business models used by companies, and the Key Business Proposition, which included supports for defining one's customer value proposition (A. Parker, personal communication, December 7, 2018). Although students were not observed enthusiastically engaging with either resource, the authentic business considerations that these resources elicited tapped into the excitement and curiosity students felt relating to their favorite businesses.

During the whole-group discussion used to introduce the Business Model Types document, students were eager to share their understanding of how businesses like YouTube, Apple Music, Hot Topic, and Subway make money. Students leveraged their out-of-school knowledge to build on each other's ideas about how businesses make money. Similarly, the Key Business Proposition supported students to precisely define their customers and their product, and to position the product in relation to competitors. Team A (horse racing tracks algorithm), after struggling to define their business model type and customers, were supported by the Key Business Proposition to demonstrate creativity, problem solving, and adaptability in identifying horse racing tracks as their customers, which then informed revisions to how they defined the variables in their algorithm. In this way, the business model resources allowed students to participate in authentic entrepreneurial processes, which supported and created opportunities for rich cognitive engagement.

Similarly, the Technical Brief was intended to support students to document their design process and the specifications of their product, two authentic processes of entrepreneurship. However, students' enthusiasm and collaboration were both visibly dampened as they worked on completing the Technical Brief, as they did not perceive it as necessary for their design process. Instead, students in both iterations of the design study viewed it as an external requirement that would help them during the pitch competition. As Donald explained in the post-challenge focus group, the technical brief was, "...just an afternote. Like after the product's done." This sentiment was echoed by Kim who explained its role in her team's process, stating, "...by the time we got the tech brief, honestly like we were already so far down though. It was like we couldn't, like, change it."

One possible explanation for this perception is that students, in both iterations of the design study, did not have enough time to fully engage with the Technical Brief as intended. Another possible explanation is that there may have been a disconnect between the components of the Technical Brief and the representations students used in building their algorithms, which could have detracted from its authenticity. However, given its role in explicitly connecting students' challenge work to the targeted and standards-aligned STEM content, the Technical Brief is an essential component of the D&P challenge framework. During the second iteration of the design study, the Technical Brief when used as a post-competition tool for reflection and with teacher/researcher support, created an opportunity for students to engage with the targeted and standards-aligned STEM content. This suggests that the Technical Brief could be more effective if utilized as a tool for reflection after students complete the pitch competition.

Finally, the pitch resources, which included the Pitch Judging Sheet, the How to Build a Pitch document, and a set of sample pitch decks, supported students in learning the components

of an effective entrepreneurial pitch, a process none of the students were familiar with at the start of the competition. Learning these components then created opportunities for students to iterate and reflect on their algorithms, businesses, and pitch decks, and to leverage their diverse skill sets and creativity in a mathematics setting. For example, Team B, after reviewing the Pitch Judging Sheet, realized they had forgotten to include their target audience, which was a core feature that drove the design of their algorithm and the selection of their variables. This realization inspired Maria to creatively and seamlessly integrate a description of their target customers into the start of their pitch. Team C used the pitch resources to self-monitor, which led them to discover they had failed to include a description of their problem or their solution in their pitch. Although describing one's solution during a pitch might seem obvious, for students with little familiarity of pitching or entrepreneurship, it was not. The pitch resources supported both teams' engagement by establishing expectations for their work, relative to their pitches.

Although the extent to which students embraced and engaged with the challenge resources varied, by the end of the challenge, all teams used and benefited from the resources. It should be noted that this challenge represented students' first experience with both entrepreneurial pitch competitions and the Design & Pitch Challenges in STEM. Thus, it is reasonable to assume that, with more experience, students will be able to more quickly and productively engage with the complete set of resources.

Resources that Support Students to Build Prototypes. The spreadsheet resource was designed to provide students with the scaffolding necessary to gain the skills essential for building a functioning spreadsheet algorithm. By supporting students to build a solution that works under real-world conditions, the spreadsheet resource was designed to support their perceptions of the authenticity of the challenge. Without having a way to build something real

(i.e. more than an algebraic expression on paper), the *Building Algorithms* challenge would risk “humor[ing students’] interests” (Dewey, 1897/1981, p. 452) which could dampen their enthusiasm and lessen their engagement. During both iterations of the design study, the spreadsheet resource was effective for helping students learn the skills they needed to build functioning prototypes of their algorithms.

At a minimum, the spreadsheet resource, when introduced with teacher/researcher support, supported students’ engagement by clarifying the expectations for the task. In some cases, the spreadsheet resource did more, creating opportunities for students to demonstrate their creativity and resourcefulness as they experimented with different ways to use the spreadsheet tool to build an authentic prototype algorithm. This was especially true for Team A during the first iteration of the study, who used the spreadsheet resource to explore several different rating algorithm examples to find the best model to use for their horse racing track algorithm, based on which one produced the “right”-sized outputs. In this way, the spreadsheet resource created an opportunity for Team A to grapple with symbolic expressions and consider tradeoffs between different expressions.

The spreadsheet resource empowered Team B to experiment with the variables to show how their YouTube rating algorithm would handle multiple users rating a single YouTube channel. Team B’s use of the spreadsheet resource also resulted in a sense of pride and accomplishment when they were finally able to successfully program the spreadsheet, which then supported their continued experimentation with the tool. Though Team C did not use the examples in the spreadsheet resource, they nevertheless demonstrated a similar willingness to experiment with the tool, searching and testing multiple Google Sheets functions for use in their music filtering algorithm.

In the first iteration of the design study, the spreadsheet resource supported students' engagement by providing the tool and the necessary support to use the tool to build an authentic prototype algorithm. Based on Team A's and Team B's need for teacher/researcher support, a new conjecture, relating to the spreadsheet resource, was tested during the second iteration of the design study.

Conjecture 2a: The Spreadsheet Resource Conjecture. The spreadsheet resource was introduced on day 3 of the second iteration of the design study using a brief whole-class tutorial. It was conjectured that providing students with explicit instruction on how to interpret and use the spreadsheet resource would help them overcome the spreadsheet language and syntax barrier and more quickly and easily engage with the challenge of building a spreadsheet algorithm. Although this conjecture was correct and students in the second iteration were able to more quickly engage with the spreadsheet tool, it may have also undercut students' creativity and constricted the creative experimentation that was observed during the first iteration. As a result, the complexity and variety of teams' algorithms were limited. One possible explanation for this finding is that the guided tutorial was introduced too early in the process, which did not allow students enough opportunities to struggle (and fail) with programming their spreadsheets (Kapur, 2012). It is possible that with a greater emphasis on supporting productive struggle, students in the second iteration would have benefited more from the tutorial and would have been more willing to experiment with other algorithm types.

Conjecture 3

This conjecture stated that practicing for and participating in the pitch competition would increase students' engagement and enthusiasm during the challenge. Presentations are a defining feature of PBL, providing accountability and a tangible target for project work. Presentations

also allow students an opportunity to reflect on their work and share their creative final products with their peers and, in some cases, community (Bouillion & Gomez, 2001; Krajcik & Blumenfeld, 2006; Thomas, 2000). The D&P challenge framework attempts to leverage and enhance these features of PBL presentations to generate excitement and enthusiasm by situating them in the context of a pitch competition with external judges. In this design study, it was conjectured that practicing for and participating in the pitch competition would increase students' engagement and enthusiasm. The results of testing this conjecture can be explained by the theme titled, "engagement through pitching." In this conjecture, the pitch process included all instances in which students 1) practiced exposing their ideas to critique, 2) scripted, practiced, revised, and delivered their final pitches, and 3) built, designed, and refined their pitch decks.

The pitch process affected students' engagement and enthusiasm through inspiring a combination of nervousness about exposing their algorithms to critique by unfamiliar (external) judges, excitement about sharing what they accomplished with their algorithms, eagerness to win the competition, and enthusiasm for the opportunity to participate in a new and atypical (for them) type of presentation.

Nervousness was the most overt and most referenced source of engagement relative to the pitch process. Particularly in the lead-up to the pitch practice and the final pitch competition, students reported and exhibited nervousness relating to public speaking and exposing their ideas to critique by their peers and the external judges. Kim explained her nervousness in a post-challenge focus group, stating, "[the external judges are] two whole new people...I probably haven't met before. So, it's just like, 'great, a bunch of, like, strangers are going to be, like, looking at what I've done for the past week and like judging it.'" Students were so invested in their work that this seemingly low stakes environment (e.g. no grades, small audience, etc.) took

on much higher stakes because they were exposing their ideas to critique. This anxiety over pitching to external and unfamiliar judges led all three teams to continuously rehearse and refine their pitches in the lead-up to the both the practice pitch the final pitch.

In addition to nervousness, students' confidence and pride in their solutions also drove their engagement over the course of the week. One of the more compelling examples of students' pride in their solutions can be seen in the sense of empowerment felt by students to explain, defend, justify, and, when they determined it was necessary, revise their algorithms and business plans. For example, when Team B, during their practice pitch, defended the inclusion of "uploads" as a variable in their algorithm, they did so from the dual perspective of user and entrepreneur. They were so confident "uploads" was a relevant variable for evaluating the quality of a YouTube channel that they were willing to justify its inclusion in their algorithm, even when challenged by their practice judge, who was a STEM professional. Similarly, when Team A were questioned about the results produced by their prototype horse racing track algorithm, instead of immediately thinking their algorithm was wrong or needed to be changed, they justified the results by referencing the relationship between the value of their input variables (e.g. horse health) and the overall racetrack rating. In both cases, students' knowledge about their context and their investment in their algorithm-based business empowered them to expose their ideas to critique and defend those ideas by referencing the meaning and structure of their algorithms and algebraic expressions.

As students' confidence and pride in their solutions grew, so too did their enthusiasm about sharing their innovative solutions with their peers and the external judges. Students found the creative and persuasive aspects of pitching especially compelling and cited both aspects as characteristics that distinguished pitching from their typical school presentations. According to

Kim, "...at school, you're not trying to sell them something, you're not trying to get them...to buy into your product. You're just trying to, like, show the information you learned." Shaun concurred, stating "[in other school presentations] you're not trying to convince them of anything." For Kim and Shaun, the opportunity to convince the judges was appealing and offered a new angle on presentations that encouraged their creativity. Maria also referenced the creative aspect of pitching in explaining why she liked pitching, stating,

This feels kind of different to me because usually in school when I do presentations it's something, my entire class was doing a presentation on, but we just do it like on different things. But this is something that only us two really like know all the facts about or at about or at least most of the facts about. And um, so it's kind of us who are like telling people new things and kind of leading a discussion on it instead of just saying facts that everybody else already knows because they've been working on it too.

For Maria, being able to pitch their unique idea was important and, based on observations, contributed to her engagement as she and Beatrice prepared their pitch deck and final pitch.

Students' sustained engagement around preparing for the final pitch competition involved practicing describing their algorithms and businesses clearly, concisely, and completely to each other, researchers, and practice judges. This repeated practice, in turn, supported students to develop a deeper understanding of their algorithms, businesses, and algebraic expressions. Demonstrating an understanding of algorithms, businesses, and algebraic expressions requires more than building a functioning spreadsheet algorithm. Students must be able to effectively and convincingly communicate their understanding to others (von Foerster, 1973/2003). Over the course of the *Building Algorithms* challenge, students were provided multiple opportunities (e.g. teacher/researcher informal check-ins, "expert" check-ins, practice pitches, and final pitches) to

express their understanding of their algorithms, algebraic expressions, and businesses. As a result, their explanations and pitches improved with each successive attempt. As students practiced explaining, defending, and justifying their algorithms and businesses throughout the week, their verbalizations became clearer, more convincing, and more thorough. Students needed opportunities to practice communicating their ideas and to receive feedback on how to do so more effectively. Thus, this engagement around an entrepreneurial process (pitching) created opportunities for students to engage deeply with the targeted mathematics.

Summary of Research Question 1

The *Building Algorithms* challenge and the D&P challenge framework were designed to increase students' engagement in a mathematics classroom by situating mathematics learning in an authentic entrepreneurial experience. Overall, the entrepreneurial framing of the competition, the support resources, and the pitch competition empowered students to persist in pursuing authentic and actionable entrepreneurial opportunities that were personally meaningful and relevant. By allowing and supporting students to be truly innovative in their selection of their context, the design of their algorithm, and the defining of their businesses, the *Building Algorithms* challenge promoted a willingness to persist in learning new skills and in developing their algorithms and businesses. In summary, the *Building Algorithms* challenge demonstrated how the D&P Challenges in STEM framework and entrepreneurship can drive students' engagement in a mathematics classroom by leveraging and enhancing key components of project-based learning, design-based learning, and entrepreneurial-based learning.

Research Question 2

The *Building Algorithms* challenge was written to support students to engage with algebraic expressions and functions as they built spreadsheet algorithms. Research question 2

asked, how and to what extent does the *Building Algorithms* challenge promote and support the learning of specific mathematics concepts? To address research question 2, the following two sub-questions were explored: a) How and to what extent do students engage with algebraic expressions as they research, develop, describe, and justify their entrepreneurial solutions to the *Building Algorithms* challenge? and b) How and to what extent does the structure of the *Building Algorithms Challenge* support mathematics learning?

Overall, as students built their algorithms, engaged with the challenge resources, and practiced for their pitches, multiple opportunities arose for them to engage with and develop a deeper understanding of algebraic expressions and functions. Furthermore, the ways in which students conceptualized the task of building a rating or ranking algorithm varied across teams and shed light on how the process of building an algorithm in an entrepreneurial setting can support mathematics learning.

Research Question 2a

The first sub-question for research question 2 was addressed by testing the conjecture that the task of building a working spreadsheet algorithm that automatically produces rankings or ratings combined with the affordances of spreadsheets would support students to demonstrate functional reasoning in relation to algebraic expressions. In this section, I describe the conceptual development of algorithms that emerged over the course of the study. By listening to and presenting student voice and applying my perspective (Confrey, 1991), presented through the lens of mathematical modeling, I highlight how the *Building Algorithms* challenge and the entrepreneurial framework promoted students' learning of algebraic expressions and functions.

Conjecture 4 and the Conceptual Development of Algorithms. The results of testing this conjecture can be explained by the theme titled, “algorithms task and entrepreneurial

processes support modeling.” This theme describes how the combination of the algorithms task and the set of entrepreneurial processes supported students to progress through an iterative modeling cycle, which supported mathematics learning.

While completing the task of building a spreadsheet algorithm and supported by the entrepreneurial framing of the competition, students engaged in an iterative modeling process of building, testing, and refining algorithms within a spreadsheet environment. This iterative process supported students to develop a deeper understanding of their context and created opportunities for students to engage with algebraic expressions and functions. Across the eight teams in the study (three in the first iteration of the design study and five in the second), students’ processes building their algorithms followed a common progression. In this section, I describe the common steps in students’ progressions, situate those steps relative to mathematical modeling, and discuss the opportunities for mathematics learning that arose as a result of the progression.

Identifying the Context. Building on the work of Dewey (1938/1981), Confrey and Maloney (2006, 2007) describe the intent of mathematical modeling is to take an indeterminate situation and make it more determinate through the development of a mathematical model (Confrey & Maloney, 2006, 2007). In the *Building Algorithms* challenge, students were tasked with identifying their own indeterminate situation (or context) that they would then make more determinate. In this way, students were encouraged to ask legitimate questions (von Foerster, 1973/2003) for which the answers were not known or obvious. Students were supported in this process by the entrepreneurial framing of the challenge, which required that their algorithm and context represent a legitimate entrepreneurial opportunity, and the challenge champion video, which provided students with one example model of an algorithm. Thus, for all teams in both

iterations of the design study, the first step in the development of their algorithms was identifying a context, which would serve as the target of their modeling activity. Once they had identified a general context (e.g. horses, YouTube, or Hispanic foods), teams then considered how to focus those contexts for use in their algorithms. For example, Team A started by identifying animals as their context, before narrowing their focus to racehorses and then, more specifically, the health and treatment of racehorses. In addition to providing an entry point to the modeling process, these contexts also allowed students to select concrete scenarios that would later support their interpretation of their symbolic expressions.

Defining a Purpose for the Algorithm. Having selected a context, students in both iterations of the design study then worked to define a purpose for their algorithms that met the need of specific users or customers. Defining the purpose of their algorithm was foundational for 1) selecting the type of algorithm they would build (e.g. ranking or rating), 2) identifying the variables they would include in their algorithm, and 3) defining the type and characteristics of the outputs their algorithms would generate. To support students to define purposes for their algorithms, they were provided several example algorithms in the support materials, such as a rough overview¹³ of Cathy Yee's algorithm for rating movies, the spreadsheet resource algorithms for rating and ranking job candidates, and the set of sample algorithms presented in the Context Document.

Teams perceived their algorithms as tools for informing decision making relative to their selected contexts. To identify the information necessary for making informed decisions, teams assumed the dual role of entrepreneur and user. That is, because students selected context about which they were knowledgeable (e.g. Team B selected YouTube because they were users of

¹³ Because Inlucvie was in the process of being launched at the time of the study, Cathy Yee was not allowed to give a detailed description of her company's algorithm in the challenge champion videos.

YouTube), they were able to leverage their experiences as users to inform their entrepreneurial decisions. For example, Team B, as users, wanted a way to decide whether they would like a video from a YouTube channel before watching it. This purpose then informed their selection of variables and whether to build a rating or ranking algorithm.

Team B wanted their algorithm to help users decide whether to watch a video from a specific YouTube channel and not to help users compare multiple YouTube channels, and this purpose informed their decision to build a rating algorithm. Using a rating algorithm Beatrice and Maria could give channels a numerical score that would help users decide whether they would like videos from that channel (i.e. a higher score for their content variable indicates greater agreement with the statement “I like the Content of this YouTube channel”). Conversely, a ranking algorithm would allow Beatrice and Maria to assign YouTube channels numerical ranks that would only help users determine whether they would like one channel better than another (i.e. a higher numerical rank for the content variable for a YouTube channel indicates that the content of the videos produced by that channel is better than the content produced by a channel with a lower rank). Thus, when Beatrice and Maria considered the tradeoffs between building a rating algorithm and building a ranking algorithm, they selected a rating algorithm because they argued it was better suited to helping users decide whether they would like a video and what about that video (i.e. the ratings for their four variables) they liked.

Similarly, Team C defined the purpose of their algorithm as helping users decide whether to listen to a given song and, accordingly, they selected variables that they consider when looking for new music. In the second iteration of the design study, the Amigos team defined the purpose of their algorithm as helping restaurants inform customers about different Hispanic

foods and included variables that they perceived as useful for that purpose, such as the foods' spiciness, smell, taste, and appearance.

In terms of mathematical modeling, the process of defining a purpose for an algorithm established a foundation that would support the development of a real model, which could then be represented mathematically (Blum & Borromeo Ferri, 2009; Blum & Leiss, 2005).

Additionally, the algorithm's purpose, which developed in conjunction with the development of students' value propositions (i.e. how does the algorithm help consumers), guided students' work as they built their algorithms and provided a lens through which they could interpret the structure and meaning of their expressions and functions.

Defining Inputs and Outputs and Considering Tradeoffs. After defining the purpose for their algorithms, students then 1) defined variables that would be useful to that purpose (inputs) and 2) considered how to represent results from their algorithms to users (outputs). Again, entrepreneurial considerations informed students' mathematical decisions. For example, Team A (horse racing tracks) initially identified three factors (inputs) that would help a horse owner decide whether to take their horses to a racetrack, based on the track's treatment of horses. They also leveraged their familiarity with how companies like Google assign ratings in deciding that the best way to communicate their evaluation of a track (outputs) to horse owners would be a score out of 5. Team A wanted their algorithm to produce outputs that were the right magnitude and met their expectations for what a rating should look like. Even though they had yet to consider 1) how to operationalize those three factors (inputs) or define a systematic process for generating a score from those factors (outputs), Team A were able to identify variables to include in their algorithm and the outputs they wanted their algorithm to generate. This early decision about what the outputs of their algorithm should look like eventually led to them deciding to base

their algorithm on a model from the spreadsheet resource that produced outputs that were less than 5.

Similarly, Team B's selection of input variables was informed by the purpose they identified for their algorithm. Initially, Team B (YouTube) selected both quantitative and categorical variables before switching to a combination of quantitative user ratings and descriptive measures. After considering the purpose of their algorithm, they decided to change their input variables to only include quantitative user ratings, replacing number of views and number of subscribers with user ratings of video length and upload frequency. Thus, the purpose of students' algorithms and their knowledge of the situation allowed them to create a real model (Blum & Borromeo Ferri, 2009; Blum & Leiss, 2005) of the situation that fulfilled their algorithm's purpose and that would then inform their mathematical model.

In the second iteration of the study, the introduction of the spreadsheet resource may have limited students' ability to deeply consider tradeoffs in defining their inputs and outputs. During the first two days of the study, students identified their contexts and input variables. However, no team was observed considering output variables during the first two days. After introducing the spreadsheet resource, the consideration of ideal outputs still did not appear to factor into teams' algorithm designs. It was not until the final day of the competition that students were observed considering ways of changing their outputs to better fit their algorithms' purpose. In the lead-up to the final pitch competition, the Pizza Hunters team changed their algorithm from a sum to a mean to 1) lessen the impact of the much larger Health Score variable on the overall scores and 2) reduce the magnitude of their outputs. Teams' inattention to their output information may have been the result of all teams using the first spreadsheet example, in which job candidates are assigned ratings by calculating the sum of three variables, as their model algorithm. Had I

discussed more of the examples in the spreadsheet resource during the guided tutorial or, had I allowed students more time to work on their algorithms before the guided tutorial, it is possible more teams would have been observed considering the outputs of their algorithms.

Operationalizing Variables for use in an Algorithm. Having considered their input and output variables, teams began working to create their mathematical models. This process of mathematizing their real models (Blum & Borromeo Ferri, 2009; Blum & Leiss, 2005; Confrey & Maloney, 2006, 2007) proved to be a significant hurdle for students, especially as it related to operationalizing their variables (i.e. measuring or converting to a useable form the variables identified as relevant to the identified purpose). For example, Team A spent the first two days of the challenge unable to operationalize their variables relating to horse racing tracks. As a result, initial iterations of Team A's algorithm included descriptions of the input variables, but lacked a consistent method for turning those inputs into outputs. Through their work with the spreadsheet resource, Team A were able to operationalize their variables for use in their algorithm using evaluative measures. That is, they decided to measure each variable on a scale from 1 to 5, which supported Team A to more precisely and intentionally define the variables in their expressions. This shift is especially important given the tendency for students to interpret variables in algebraic expressions as labels (Clement et al., 1981). The task of building a functioning spreadsheet algorithm supported Team A to clearly define the variables in their expression in terms of the quantities those variables represented.

The task of operationalizing variables also supported teams to attend to the structure and meaning of their algorithms and spreadsheet formulas. For example, as Team B worked to operationalize their variables related to YouTube channels, they had to continually reflect on the structure and meaning of their algorithm and algebraic expression. Based on Maria's prior

experience with algebraic expressions, she started the competition wanting to quantify the variables she and Beatrice had identified as being relevant to deciding whether to watch a YouTube channel. Thus, in the first iteration of their algorithm, Team B included both quantitative variables, including descriptive measures and evaluative measures, and categorical variables and immediately found ways to quantify the categorical variables. For example, they operationalized the variable “content type” by assigning a score from 1-10 based on whether users liked the type of content. They operationalized the quantitative variables by creating a binning system. On day 2, Team B revised their variables, referencing the purpose of their algorithm and intended meaning of their expression. Recognizing that descriptive measures, such as number of views or uploads, were not useful for evaluating the quality of a YouTube channel, Team B changed how they operationalized their variables so that each variable represented a user’s evaluation of that variable. Finally, Team C (music filtering), given Shaun’s familiarity with spreadsheets and programming, operationalized their variables using cell names and the LOOKUP and MATCH commands, which allowed them to use the categorical variables (e.g. genre, instruments, likes/dislikes) that they identified at the start of the competition.

This process of operationalizing variables for use in a spreadsheet algorithm created opportunities for students to clearly and precisely define their variables, while attending to the meaning of their algorithms. Furthermore, this process was driven by students’ real models (Blum & Borromeo Ferri, 2009; Blum & Leiss, 2005), which emerged from their entrepreneurial considerations. In the second iteration of the design study, the guided introduction of the spreadsheet resource may have prevented students from experiencing the same benefits. By introducing an algorithm example too early in the process, students latched onto operationalizing their variables using evaluative measures. Two teams (Pizza Hunters and Gamer’s Paradise)

included variables operationalized as descriptive measures, but, because those values were quantitative and could be used in a sum, neither team considered alternative strategies for operationalizing their variables.

Defining the Function Rule. An essential step for students in building their algorithms was defining the rule by which the inputs would be mapped onto the outputs of the algorithm. For all teams in the study, students were unable to define a function rule until they had operationalized their variables. For example, once Team A (horse racing tracks) had operationalized their variables using evaluative measures, they were able to begin exploring possible function rules for mapping those evaluative measures onto overall track ratings. In considering different possible function rules for use in their algorithm, Team A had to 1) consider the relationship between the inputs and outputs of their algorithm (i.e. how does changing the score for health change the overall score for the track?); 2) consider tradeoffs between different function rules (which function rule produces outputs that are the “right” magnitude?); and 3) interpret the meaning of their algorithms’ outputs.

For Team A and Team B, operationalizing their variables was more difficult than defining their function rules. Conversely, for Team C (music filtering), defining their function rule was considerably more challenging than operationalizing their variables. Having operationalized their variables using the LOOKUP and MATCH commands, Team C spent the majority of their time writing the function rule (spreadsheet formula) that would allow them to map survey responses onto multiple recommended songs. In doing so, Team C continually coordinated between the meaning of their variables in their expression, the structure of their expression, and the outputs of their algorithm. For example, Team C needed to understand how

the placement of a variable (or cell name) in the LOOKUP command related to the list of songs their algorithm was filtering and the eventual list of recommended songs.

In the second iteration of the design study, the guided introduction of the spreadsheet resource may have, again, restricted students' creativity and critical thinking in this stage of the process. Three of the five teams in the second iteration used a simple equally weighted sum for their function rule: the same function rule used in the first example in the spreadsheet resource and the only example discussed in the guided tutorial. Of the two teams that used a different function rule, one (Pizza Hunters) used an equally weighted sum for the bulk of the challenge, before changing to a mean on the day of the final pitch. The other team (Discover.GE) used two equally weighted sums to generate their positive and negative scores.

The process of defining their function rule and programming the spreadsheet to carry it out, created opportunities for students to engage with the structure and meaning of algebraic expressions. Even students in the second iteration, despite the lack of complexity in their algorithms, engaged with the structure of their algebraic expressions as they built their function rules. Once students had operationalized the variables for use in their algorithms and had defined their function rules, they had developed a mathematical model for their situation and were ready to evaluate its effectiveness.

Testing and Refining the Algorithm. After defining the function rule, teams worked to build prototypes of their algorithms in the spreadsheet tool. The appeal of building a working prototype of their algorithm motivated all teams to persist in learning the language and syntax needed to program a spreadsheet. Thus, testing and refining the algorithm created opportunities for students in both iterations of the study to engage with algebraic expressions and functions. Additionally, by testing their algorithms with real cases from their contexts, students were able to

independently evaluate the effectiveness of their solutions and identify necessary revisions to their algorithms. From a modeling perspective, the testing process created an opportunity for students to test the “consistency and coherence” of their algorithms (Confrey & Maloney, 2006, 2007). For example, when Team A prototyped their horse racing track algorithm, they were surprised by the score given to Santa Anita, which prompted them to reconsider their algorithm and reflect on both the meaning of their variables and the relationship between those variables and the overall ratings. Instead of immediately revising their algorithm in response to the surprisingly high score for Santa Anita, Team A justified the result by citing the values of the health variables for Santa Anita and Churchill Downs. Thus, they were able to justify an output of their algorithm based on their understanding of how the values of their input variables impact the values of their output variables.

Similarly, Team C were continually testing the consistency and coherence of their music filtering algorithm and revising it in real-time based on their perception of its success. Shaun would frequently write a spreadsheet formula using the LOOKUP or MATCH command and observe whether it successfully returned the song or songs from their song library that they expected. When it did not, Shaun would reexamine the formula and make changes, including changing the position of the search term in the formula, the name of the cell containing the search term, and the names of the cells that defined the search range. Thus, as Team C prototyped their algorithm, they continually reflected on and refined the structure of their algebraic expression.

Research Question 2a Summary. The task of building a spreadsheet algorithm supported students to engage in an iterative mathematical modeling process and that process was enhanced by the authentic entrepreneurial processes. Although the guided introduction of the

spreadsheet resource during the second iteration of the design study may have limited students' engagement with the modeling process, it nevertheless created multiple opportunities for all students to engage deeply with algebraic expressions and functions. Specifically, as students progressed through identifying their context, defining the purpose of their algorithm, and considering potential inputs and outputs for their algorithm, they developed a real model (Blum & Borromeo Ferri, 2009; Blum & Leiss, 2005) that established a contextual foundation for interpreting the meaning and structure of their algorithms represented as algebraic expressions. As they defined their function rule and prototyped their algorithm, they evaluated the coherence and consistency (Confrey & Maloney, 2006, 2007) of their algorithm relative to their context and purpose, reflected on the structure of their algorithm, and revised their algorithms as needed. In this way, the *Building Algorithms* challenge and the entrepreneurial components of the D&P challenge framework supported students to meaningfully engage with algebraic expressions and functions.

Research Question 2b

The second sub-question that guided the exploration of research question 2 explored how and to what extent the structure of the *Building Algorithms* challenge supported students to learn how to build and interpret symbolic algebraic expressions. Prior to the study, it was conjectured that the task of building a working algorithm that automatically produces rankings or ratings, combined with the affordances of spreadsheets, would support students to demonstrate a functional understanding of algebraic expressions. However, after reflecting on the results of the study, a retrospective conjecture was developed relating to how the structure and the entrepreneurial framing of the challenge supported students' mathematics learning. Specifically, it was conjectured that the structure of the challenge, which leveraged features of PBL and DBL,

its entrepreneurial framing, and the affordances of spreadsheets supported mathematics learning by 1) establishing a tangible purpose for students' work that created a need for symbolic algebraic expressions and functions; 2) supporting students to identify and fill knowledge gaps relating to algorithms, algebraic expressions, and functions; 3) empowering students to explain, defend, and justify the structure of their algebraic expressions and functions; and 4) establishing a foundation for the learning of more abstract and generalized algebraic expressions. In this section, I describe how the theme titled, "math learning through the spreadsheet tool and challenge structure," created a need for algebraic expressions and supported students' persistence in learning to use them.

Creating a Need for Generalized Algebraic Expressions. In PBL and DBL, the learning of STEM content is motivated by authentic driving questions and contexts (PBL) or design challenges (DBL; Kolodner, 2002; Krajcik & Blumenfeld, 2006; Thomas, 2000). The *Building Algorithms* challenge incorporates these features of PBL and DBL to create a need for the learning of algebraic expressions and functions, but does so within an entrepreneurial setting. Specifically, the task of building a working spreadsheet algorithm relating to a context with entrepreneurial potential created a need for students to engage with symbolic algebraic expressions. Of the twenty-one students across the two iterations of the study, only one reported prior experience working with spreadsheet language and syntax. The other twenty students did not know how to use a spreadsheet as anything other than a table for organizing data and sought opportunities to learn how to use symbolic algebraic expressions to program the spreadsheet to carry out their algorithms. Thus, the *Building Algorithms* challenge, through the central task of building a functioning spreadsheet algorithm, created a need for students to engage with expressions through a "spreadsheet-algebraic" (Rojano, 1996, p. 141) approach.

However, in terms of developing a curricular or standards-aligned understanding of algebraic expressions, learning to program a spreadsheet using the symbolic programming language and syntax is only an intermediate step that can connect students arithmetic reasoning to more abstracted and generalized algebraic expressions (Fillooy et al., 2008). The shift from arithmetic to algebraic involves bridging the gap between operating on specific and known values to “operating on [the unknown] as if it were known” (Rojano, 1996, p. 140). When problem solving in a spreadsheet environment, students often employ an arithmetic approach in which they start with specific values for variables before representing those variables in a formula using cell locations as names (e.g. the name “B4” to represent the value in column B and row 4). The last step to the more generalized and abstracted algebraic expression is to move from reasoning about specific cells to reasoning about the quantities those cells represent (Rojano, 1996).

During the first iteration of the design study, this shift was most apparent when Denise, while using the spreadsheet resource, shifted first from describing an algorithm using specific values to describing it using cell names and then to describing it using the quantities those cells represented (i.e. interview performance, eagerness to learn, and enthusiasm for the position). The spreadsheet resource and its affordances supported Denise to shift from using arithmetic reasoning to eventually arriving at the more generalized and abstracted algebraic expressions typical of middle grades mathematics curricula. Denise was the only student from the first iteration of the design study to be observed making this shift *during* the competition. However, based on the results of the task-based interview, the other five students were either already able to use these more generalized algebraic expressions (e.g. Maria and Shaun) or were on the verge of making the shift (e.g. Kim, Beatrice, and Donald). For example, Kim, Beatrice, and Donald

were all able to 1) use their arithmetic reasoning to explain how the spreadsheet algorithm calculated overall scores for restaurants, 2) recognize that each spreadsheet formula, despite including different cell names, represented the same algorithm, and 3) describe the quantities the cell names represented. It is worth noting that Kim, Beatrice, and Donald all benefited, in terms of their mathematics learning, from engaging with challenge, despite seemingly leveraging their diverse skills to avoid the more STEM-heavy requirements of the challenge. This suggests that engagement can be important for mathematics learning, even when that engagement seems to an observer to be peripheral.

During the second iteration of the study, the teacher/researcher-supported use of the technical brief supported students to engage with the more generalized algebraic expressions. Following the completion of the competition, students needed support to shift from describing their algorithms using arithmetic expressions to describing them using generalized algebraic expressions. For example, before receiving support, the Amigos team were able to describe their algorithm for rating Hispanic foods using specific numerical values and could explain the symbolic expressions they used in their spreadsheet. With support from the teacher/researcher, they were able to shift to describing and representing their algorithm as a single generalized algebraic expression. By leveraging students' abilities to reason arithmetically about their algorithms and their success programming their spreadsheets to carry out those algorithms, the Technical Brief and teacher/researcher support helped them to describe their algorithms using the quantities the numerical values and cell names represented.

Supporting Students to Identify and Fill Knowledge Gaps. In both PBL and DBL, opportunities for STEM learning emerge as students identify and seek to fill gaps in their knowledge in order to address the driving question or design challenge (Kolodner, 2002; Krajcik

& Blumenfeld, 2006; Penner et al., 1998). The structure of the *Building Algorithms* challenge leveraged this feature of PBL and DBL to motivate students' mathematics learning. Additionally, the entrepreneurial framing of the challenge (EBL) increased students' engagement, investment, and persistence in building their solutions, while also providing a basis on which students could evaluate the coherence and consistency of their mathematical models (Confrey & Maloney, 2006, 2007).

After identifying a context, defining a purpose for their algorithms, and selecting their input variables, students recognized they needed to learn the specific STEM concepts (i.e. what is an algorithm) and skills (how to program a spreadsheet) that were targeted by the challenge. Teams then worked to learn those concepts and skills by either accessing the challenge resources and support materials, such as the challenge champion videos and the spreadsheet resource, or independently researching available Google Sheets functions. Thus, the structure of the challenge motivated students to engage with the targeted STEM content and students' deep understanding of their selected context supported them to persist in learning how to program their spreadsheet algorithms to work with the variables they identified as authentic to those contexts.

Once students understood how to use and interpret the spreadsheet resource, it was essential for supporting them to engage with and develop a more nuanced understanding of algebraic expressions and functions. Specifically, the affordances of the spreadsheet, such as the ability to receive immediate feedback or apply formulas across multiple cells, created opportunities for mathematics learning. For example, the autofill command created a need for students to use the spreadsheet programming language and syntax. This was especially important for Team A (horse racing tracks), who, in their first attempt to program the spreadsheet to carry

out an algorithm, avoiding the use of variables entirely, instead writing only numerical expressions. Through introducing the autofill command, which allows users to apply formulas across multiple cells, Team A were motivated to begin using the spreadsheet's programming language. Once they had successfully programmed the spreadsheet to carry out the algorithm and had worked through multiple examples, the programming language and syntax further supported Team A to expand their interpretation of variables. By seeing the variables names change between rows, with the column letter staying the same and the row number changing, Team A shifted from describing variables as representing specific values in cells to representing the range of possible values as described by the column heading.

Team B's excitement about the autofill command, as well as the immediate feedback provided by the spreadsheet, motivated them to experiment with the spreadsheet to demonstrate how their YouTube channel algorithm would handle multiple user ratings for each variable. While experimenting, Team B demonstrated a flexible understanding of variables as being able to represent a range of possible values (e.g. ratings from 1-10) or an expression (e.g. the mean of four user ratings, each from 1-10). Without the spreadsheet or the spreadsheet resource, this flexibility might not have emerged.

Although Team C did not spend much time engaging with the examples in the spreadsheet resource, the spreadsheet tool and its affordances nevertheless had a considerable impact on their opportunities to learn the targeted STEM content while building their music filtering algorithm. Specifically, encouraged by the immediate feedback offered by the spreadsheet tool, Team C displayed a willingness to take risks and experiment with unfamiliar and complex Google Sheets functions. Their experimentation created multiple opportunities for

Team C to reflect on and revise the structure of their expression as they continually tinkered with their spreadsheet formula and the Google Sheets commands.

The spreadsheet resource was essential for Team A and Team B during the first iteration of the design study. During the second iteration of the design study, although it provided students with the skills and knowledge they needed to engage with and make progress on their algorithms, it may have also undermined students' creativity and willingness to take risks while engaging with the spreadsheet algebraic expressions and functions. Nevertheless, the affordances of the spreadsheet motivated teams to consider how to write and interpret algebraic expressions. Specifically, the appeal of the "autofill" command created a need for students use the spreadsheet language and syntax, and the immediate feedback of the spreadsheet tool provided teams with a way to evaluate their success in real-time.

Empowering Students to Explain, Defend, Revise, and Justify. Another feature of the D&P Challenge framework that supported students' mathematics learning was the expectation that students frequently explain, defend, revise, and justify the choices they made in their design process, which created opportunities to reflect on and revise the structure of their algebraic expressions. The process of exposing one's ideas to critique is central to DBL and drives STEM learning by creating opportunities for students to identify ways to improve their designs (Kolodner, 2002; Penner et al., 1998). Through the structure and entrepreneurial framing of the D&P challenge framework, the *Building Algorithms* challenge empowered students to not only expose their ideas to critique, but also to leverage their understanding of their contexts, businesses, and algorithms to defend and justify those ideas.

This result is explained by the theme "mathematics learning through pitching," which describes how the pitch process created opportunities for students to reflect on the structure and

meaning of their algorithms. Through the “expert” check-ins and pitches, the *Building Algorithms* challenge provided students with multiple opportunities to explain, defend, and justify their businesses and algorithms. To do this, students needed to understand and communicate the structure of their algebraic expressions. In explaining their algorithms, all teams leveraged their deep understanding of their selected context to assign meaning to the structure of their expressions. For example, during an “expert” check-in, Team C were asked to explain how their music filtering algorithm used the LOOKUP function to recommend songs to users. In responding to this question, Team C needed to reflect on the structure and meaning of their expression to be able to explain 1) how a user’s survey responses are represented in their symbolic expression; 2) what the terms in the symbolic expression represent (e.g. the search term or cell names defining the search range); and 3) how the terms in the expression are related to each other and to the output of the function.

When Team A were asked, following their practice pitch, whether the variables in their horse racing track algorithm were all equally important to the overall score for a track, they responded by examining the structure of their expression. Based on their examination, Team A recognized that all variables were, in fact, weighted equally. Similarly, when Team B were asked to explain their choice of coefficients for their YouTube channel algorithm, Maria reflected on the structure of their algorithm and justified their choice based on the real-world relationship between the variables, arguing that they wanted the content variable to be the most heavily weighted, but not to the point of overwhelming the other variables. The frequent opportunities for students to expose their ideas to critique through the “expert” check-ins, practice pitches, and final pitches, supported students to improve their descriptions and understanding of their solutions.

Discussion Chapter Summary

Combining features of project-based learning (PBL), design-based learning (DBL), and entrepreneurial-based learning (EBL), the structure of the *Building Algorithms* challenge supported students' mathematics learning relative to algebraic expressions and functions in several ways. First students were encouraged to identify personally relevant contexts about which they were knowledgeable and that represented entrepreneurial opportunities. Combined with the task of building a spreadsheet algorithm, these contexts established a tangible purpose for students' work and created a need for learning about algebraic expressions and functions. Second, the challenge of building a spreadsheet algorithm supported students to uncover specific gaps in their knowledge relating to algebraic expressions and functions, which directed them to the spreadsheet resource and enhanced their receptiveness to instruction on how to use the spreadsheet resource. Finally, the challenge structure offered multiple opportunities for students to practice describing their algorithms, which supported students to gain clarity with respect to their algorithms and to reflect on the structure and meaning of their algebraic expressions and functions. The results of the study provide evidence for how the D&P challenge framework can support mathematics learning and promote engagement by using entrepreneurship to enhance select features of PBL and DBL.

CHAPTER 6: CONCLUSION

Summary of Study Findings

The Design & Pitch Challenges in STEM (Confrey et al., 2019) is a novel curricular framework that situates STEM and mathematics learning within entrepreneurial pitch competitions. Currently, nine challenges with corresponding support resources, have been developed for grades 6-8. As part of a larger study on the development and use of the D&P challenges (Confrey, Krupa, & Belcher, 2019), this study sought to explore and understand middle grades students' experiences with one of those nine challenges, the *Building Algorithms* challenge. Specifically, using a design study methodology (Cobb, Confrey, et al., 2003; Confrey, 2006), this study explored the following two research questions and corresponding sub-questions:

1. How and to what extent does participating in the *Building Algorithms* challenge support and promote students' engagement in a mathematics classroom setting?
2. How and to what extent does the *Building Algorithms* challenge promote and support the learning of specific mathematics concepts?
 - a. How and to what extent do students engage with algebraic expressions as they research, develop, describe, and justify their entrepreneurial solutions to the *Building Algorithms* challenge?
 - b. How and to what extent does the structure of the *Building Algorithms Challenge* support mathematics learning?

Across two iterations of the design study, with two different groups of students, the results demonstrated how the *Building Algorithms* challenge, through its entrepreneurial framing, support resources, and challenge materials supported sustained engagement, while creating

opportunities for rich mathematics learning. Specifically, by situating a rich mathematics task (building a spreadsheet algorithm) within authentic entrepreneurial processes focused on making solutions actionable (e.g. business models, opportunity and resource analysis, prototyping, and pitching), the challenge empowered students to assume ownership of their work and to take risks and persist in learning new content and skills. Furthermore, the *Building Algorithms* challenge demonstrated how pairing a powerful learning tool (the spreadsheet) with a challenge perceived as authentic by students could create a purpose for students' mathematics learning and a meaning for algebraic expressions and their symbols.

The findings reported in this study have implications for curriculum design and teaching. In the following sections, I discuss several of these implications, including a practical guide for teachers planning to implement the *Building Algorithms* challenge with their students. I conclude by discussing the limitations of the study and areas for future research.

Implications for Curriculum Designers and Teachers

The results of this study have several implications for designers of middle grades curricula. First, this study suggests that curricula seeking to motivate and engage students in learning rich STEM content, especially mathematics, could benefit by situating that learning in contexts that students select and that they perceive to be authentic and actionable. The *Building Algorithms* challenge, through tasking students with building a functioning spreadsheet algorithm for a context they selected and designing and pitching an algorithm-based business, empowered students to take ownership of their solutions and to take risks and persist in learning new content. Additionally, by providing support materials and resources that connected the challenge work to real businesses and authentic entrepreneurial practices, students' perceptions of the authenticity of the challenge were reinforced, which further motivated them to persist.

Curriculum designers and teachers should consider ways of incorporating STEM challenges that students perceive to be authentic and actionable, while also providing supports that enhance that authenticity.

Secondly, when leveraging authentic career practices in STEM curricula, students will need support to understand and engage in those practices. In the *Building Algorithms* challenge, students were not familiar with entrepreneurship, pitching, or defining a value proposition for a business. However, with support, these concepts drove engagement by helping students perceive their ideas as actionable and authentic, and created opportunities for STEM learning, especially with respect to mathematics. Curriculum designers and teachers, when attempting to demonstrate and leverage career connections, could benefit from providing supports that help students understand what people in those careers do and how those practice relate to the work they are doing in school.

Thirdly, the study demonstrated how the D&P Challenge framework, which combines characteristics of project-based learning, design-based learning, and entrepreneurial-based learning, can open the space for students to explore innovative, authentic, and, in some cases, unexpected solution paths, while also engaging with rich STEM content. In the *Building Algorithms* challenge, students were given the space to define the context, purpose, and input and output information (Knuth, 1974) for their algorithms. This resulted in students selecting a wide range of contexts and pursuing a variety of algorithms, all of which created an environment in which they engaged deeply with the process of building, defining, and reflecting on the meaning and structure of algebraic expressions. This study demonstrates that it is possible to develop authentic and engaging activities that students value and that support them to grapple with rich mathematical concepts. Although additional scaffolding would be needed to help students arrive

at a standards-aligned understanding of algebraic expressions, they nevertheless developed a deep and nuanced understanding of functional relationships and symbolic expressions as they engaged with the spreadsheet tool. The results of this study suggest that curricula could benefit by incorporating more challenges that allow students to pursue contexts and solution paths that are personally relevant, authentic, and actionable, while also incorporating tools and criteria that establish bounds on those solutions.

Finally, the study adds to the body of research demonstrating the powerful effects of using spreadsheets to support the development of algebraic and functional reasoning (Ainley et al., 2005; Filloy et al., 2008; Rojano, 1996). In the *Building Algorithms* challenge, the affordances of the spreadsheet tool and the benefits of the spreadsheet resource provided additional evidence for how spreadsheets can be used to introduce algebraic expressions and functions, bridging the gap between students' arithmetical and algebraic reasoning (Filloy et al., 2008; Rojano, 1996). When paired with the authentic and meaningful contexts students opted to pursue for their algorithms, the spreadsheet tool was especially impactful in supporting students to attend to the structure and meaning behind the symbols in their algebraic expressions (Rojano, 1996). Curricula could benefit from more heavily leveraging this powerful tool in the teaching of algebraic expressions and functions.

Practical Guide for Implementing the Design & Pitch Challenges in STEM

Consistent with a design study methodology, this study was designed and conducted with the hope that it could inform practice (Cobb, Confrey, et al., 2003). In this section, I present a practical guide for teachers hoping to implement D&P challenge with their students, using the *Building Algorithms* challenge as an example.

Timing. There are several possible models for implementing the D&P Challenges in STEM with students. Table 10 shows one model that is possible in a typical classroom setting with 45-minute class periods.

Table 10

Implementation Schedule for the D&P Challenges in STEM

Day	Activities/Benchmarks
0	Teacher launches the competition, describes the challenge process, and discusses entrepreneurship with students using the entrepreneurial mentor videos.
1	Teacher establishes the context and launches the challenge. Students begin researching and brainstorming solutions.
2	Teacher introduces the Key Business Proposition with students. Students begin building prototype solutions and defining their value proposition (the KBP).
3	Teacher introduces the Technical Brief with students. Students continue working on their solutions and Key Business Proposition, and start working on the Technical Brief. During group work, teacher or external community member conducts “expert” check-ins with teams.
4	Teacher introduces the Pitch Resources and discusses pitching with students. Students begin building their pitch decks and continue working on their prototypes, Key Business Propositions, and Technical Briefs.
5	Teacher discusses the rules for the pitch competition. Students practice their pitches with a practice judge and revise their pitches based on feedback from the judge.
6	Teacher facilitates the final pitch competition with external judges.

The amount of time students will need to complete the challenge will likely depend on students’ familiarity with the steps of the D&P challenge. For example, if students have previously completed a D&P challenge or are familiar with entrepreneurship, Day 0 and Day 1 could be combined into a single day and the teacher introductions of the materials (e.g. Key Business Proposition) would likely be shortened or removed from the schedule.

Tips for Launching the Competition and the Challenge. While students may have some familiarity with entrepreneurship and pitch competitions, it is likely limited to watching television shows like Shark Tank. It will be important for teachers to generate enthusiasm for the competition by introducing and discussing what entrepreneurship is, why it is important, and

what it means to participate in a pitch competition. Teachers should allow students opportunities to share what they know about these topics and to discuss familiar businesses and entrepreneurs. Students may not know what the term “entrepreneur” means, but they will likely be able to talk about the businesses they engage with frequently. Teachers could consider using an entrepreneurial mentor video¹⁴ to frame the discussion. Day 0 should also be used to outline the expectations for the week using the Student Instructions.

After launching the competition and discussing the D&P Challenge process on Day 0, teachers should use Day 1 to launch the challenge and allow students to begin researching and brainstorming for their business and solution. When launching the challenge, teachers should take time to discuss the context with students before showing the challenge champion videos or reading the challenge statement. Allowing students opportunities to discuss how they have experienced the context outside of school can help make the experience more authentic by connecting the competition to students’ out-of-school lives and experiences (Boaler, 2002; Bouillion & Gomez, 2001; Stevens, 2000). For example, asking students to think about how companies like Amazon or Google rate products can help connect the algorithm context to their experiences.

Tips for Facilitating the Challenge: Opening the Space for Innovation. Teachers play an integral role in facilitating challenges in the D&P challenge framework. They introduce resources and can support student thinking by asking probing questions that create opportunities for students to practice describing their businesses and to reflect and iterate on their products. At the start of the competition, teachers should allow students space to brainstorm creative and

¹⁴ At the time of the study, the entrepreneurial mentor videos had not yet been completed and so are not reported on in this paper. The mentor videos include STEM entrepreneurs discussing their experiences as entrepreneurs and why entrepreneurship matters.

innovative solutions. This space is essential for students to perceive the challenge as authentic and to be willing to invest in the process and solution. When students are given the space to innovate, they are likely to rise to the occasion and invent creative solutions that meet the challenge criteria. Although their initial ideas may lack clarity and definition, as students engage with the resources and consider their users, those solutions are likely to become more detailed and more clearly defined. Furthermore, allowing students space to innovate can help them recognize a need for the targeted STEM content, which will then motivate them to seek out opportunities to learn that content.

By the end of day 1, students should have a rough idea for their solutions and have begun considering some of the specifics of those solutions. What those specifics entail depends on the challenge. For example, in the *Building Algorithms* challenge, students should have a context for their algorithm and have begun identifying variables that would inform that context. If students are having trouble identifying a context, encourage them to use the Context Document to jump start their thinking.

Tips for Facilitating the Challenge: Challenge Resources. Over the next three days, students should work to build out their solutions, including defining the value proposition for their business (the Key Business Proposition and Business Model Types document), defining the specifications of the product or service (the Technical Brief), and preparing for their pitch. On days 2, 3, and 4, the teacher's role is to introduce the resources (i.e. the Key Business Proposition and Business Model Types document, Technical Brief, and pitch resources) through a whole-class discussion. If students have participated in a D&P challenge previously and are familiar with how to use these resources, the discussion can be brief and be more about establishing a

benchmark for students' work than explaining how to use the resources. Otherwise, teachers should use the discussion to help students understand the rationale for the resources.

One strategy that has been effective for introducing the challenge resources is to have students assume the role of an investor and consider what they would look for when deciding if a business will be successful. In other words, as investors, what would they want to know before investing their money in a business? This discussion can be useful to highlight the importance of defining the business model (Business Model Types Document and Key Business Proposition), demonstrating why and how a product will make money (Key Business Proposition), identifying customers (Key Business Proposition), describing the specifications of the product or service (Technical Brief), and clearly, concisely, confidently, and convincingly describing one's product (Pitch Resources).

Depending on the implementation model and students' familiarity with the resources, the amount of time and discussion the resources require could vary. During students' first experience with the D&P challenges and in a traditional school setting, it would be beneficial for teachers to only introduce one resource per day to avoid overwhelming students. As students become more experienced with the challenges and when using models that allow for longer blocks of time, less discussion may be required to introduce the resources.

Tips for Facilitating the Challenge: Checking in with Students. As students work on their prototypes and pitches, teachers should periodically check-in with students to probe their thinking and monitor their progress. As students begin seeking out opportunities to learn the targeted STEM content, teachers play an important role in supporting students to learn that content. This support can take a variety of forms and it is up to the teacher to decide how to best do it. However, teachers should endeavor to remain learner-centered, while allowing time and

space for students' models to develop. In the case of the *Building Algorithms* challenge, teachers will likely need to support students to use and understand the spreadsheet resource and to introduce important terminology (e.g. variable, coefficient, term, function, function rule) to help students apply a name to their mathematical activity. Teachers should be careful to avoid intervening or introducing formal algebraic expressions too early in the process, as this might undermine students' creativity and autonomy in building their solutions.

On Day 3 of the competition, the teacher should conduct an "expert" check-in with teams. This can be conducted by the teacher or external community members, if possible. The "expert" check-in provides teachers with an opportunity to monitor students' progress, while also using the entrepreneurial framing to highlight the authenticity of the challenge and encourage students to iterate on their designs. During the "expert" check-ins, students should describe and receive feedback on their businesses and solutions. This feedback should come from the perspective of a user who is knowledgeable about the challenge criteria. That is, the "experts" should give feedback on whether the business and solution are feasible (i.e. does it seem like it will work?) and, if the solution does not address the criteria, additional information aligned to the criteria that would help improve the businesses and solution.

Tips for Facilitating the Challenge: Preparing for the Pitch. Although students can work on their pitches from the beginning of the competition, Day 4 is the latest teams should begin building their pitch decks. On day 5 of the competition, teams should complete a semi-formal practice pitch with an external practice judge. The practice judge should be an adult who 1) has not yet seen teams' solutions and 2) will not be a judge for the final pitch competition. The practice pitch is important for two reasons. First, setting a date and time for a semi-formal pitch practice motivates teams to begin developing their pitches and pitch decks well in advance of the

actual pitch competition and gives them opportunities to identify and make revisions. Although some last-minute preparation is to be expected, building in a formal practice time will limit the amount and scope of this preparation. Second, pitching is a challenging process as it requires students to clearly, concisely, and convincingly describe the relevant pieces of their business ideas. Even with a list of expectations and required components, students are unlikely to understand what they are missing or how difficult it is until they complete a practice pitch. The practice pitch offers a lower-stakes opportunity for students to learn how to construct an argument.

Tips for Facilitating the Challenge: The Pitch Competition. During the final pitch competition, it is important that teachers are strict with the rules of pitching: pitches cannot exceed five minutes and the judges and audience are not allowed to ask questions afterwards. These rules are important because they establish the expectation that pitches need to be clear, concise, and contain all relevant information – there are no opportunities to clarify statements once the pitch is over. The rules of the pitch raise the stakes of the pitch competition and students are likely to rise to the occasion.

It will also be important that teachers recruit external judges for the final pitch competition. These external judges should be adults from students' communities (e.g. parents, business leaders, administrators, etc.). The use of external judges can positively impact students' engagement and STEM learning. After a week of working hard on a creative and innovative entrepreneurial solution, students are likely to be excited about sharing their ideas with new people. Second, because the judges were not present when students were building their solutions, they will have not be familiar with the problem students think they are solving, why that problem is important, or how the solution solves it. Thus, knowing that new people will be judging their

work, students will likely focus more on including all necessary and relevant information relating to their solution.

Limitations and Future Research

This study had several limitations. First, this study explored students' experiences with one example of the Design & Pitch Challenges in STEM framework. The goal of the study was to generalize students' experiences with the *Building Algorithms* challenge to the underlying theory guiding the Design & Pitch Challenges in STEM (Cobb, Confrey, et al., 2003). Although the *Building Algorithms* challenge was a representative example of the D&P Challenge framework, it is nevertheless possible that students' experiences would have been different with a different D&P challenge. Based on the results of this study, which revealed how general components of the framework, such as its entrepreneurial framing and the challenge materials, supported both engagement and mathematics learning, it is reasonable to generalize these findings to the D&P Challenges in STEM framework. However, further research is needed with different D&P challenges to better understand how the overarching framework supports middle grades students' engagement and learning in a mathematics classroom setting.

Second, the participants in both iterations of the design study spanned multiple grade levels, which is not typical of a middle grades mathematics classroom. The two iterations included students in (or entering) 6th, 7th, and 8th grade. Given the range of grade levels and the number of students in each iteration of the design study, all groups included at least two students at different grade levels. This heterogeneous grouping may have influenced students' engagement and mathematics learning, though whether this influence was positive or negative is unclear. More research is needed to understand 1) whether and how this type of entrepreneurial challenge, which offers multiple levels at which students can access the central mathematical

task, can support or can benefit from groups of mixed grade students, and 2) whether and how the *Building Algorithms* challenge can support students' engagement and mathematics learning within a same-grade classroom.

Closing Remarks

When today's K-12 students enter the workforce, they will likely face unprecedented global challenges, many of which will require them to invent creative and actionable STEM solutions (OECD, 2018; www.globalgoals.org). In other words, students will need to be able to think and act like entrepreneurs. To prepare students to address these challenges, STEM education, especially mathematics, must create opportunities for students to ask legitimate questions without obvious answers (von Foerster, 1973/2003), and grapple with messy, ill-defined (Burkhardt, 2007), and authentic problems. Students will need opportunities to consider whether the solutions they develop in a classroom setting can be economically viable beyond the classroom and meet the needs of the people and communities those solutions are designed to help. Achieving this goal will require STEM educators, researchers, and curriculum developers to develop ways of supporting these skills without sacrificing conceptual rigor in the learning of discipline-specific content knowledge. This study demonstrates how a STEM entrepreneurial challenge can support middle grades students to develop authentic and actionable solutions to real-world problems, while also engaging with rich STEM content, especially mathematics.

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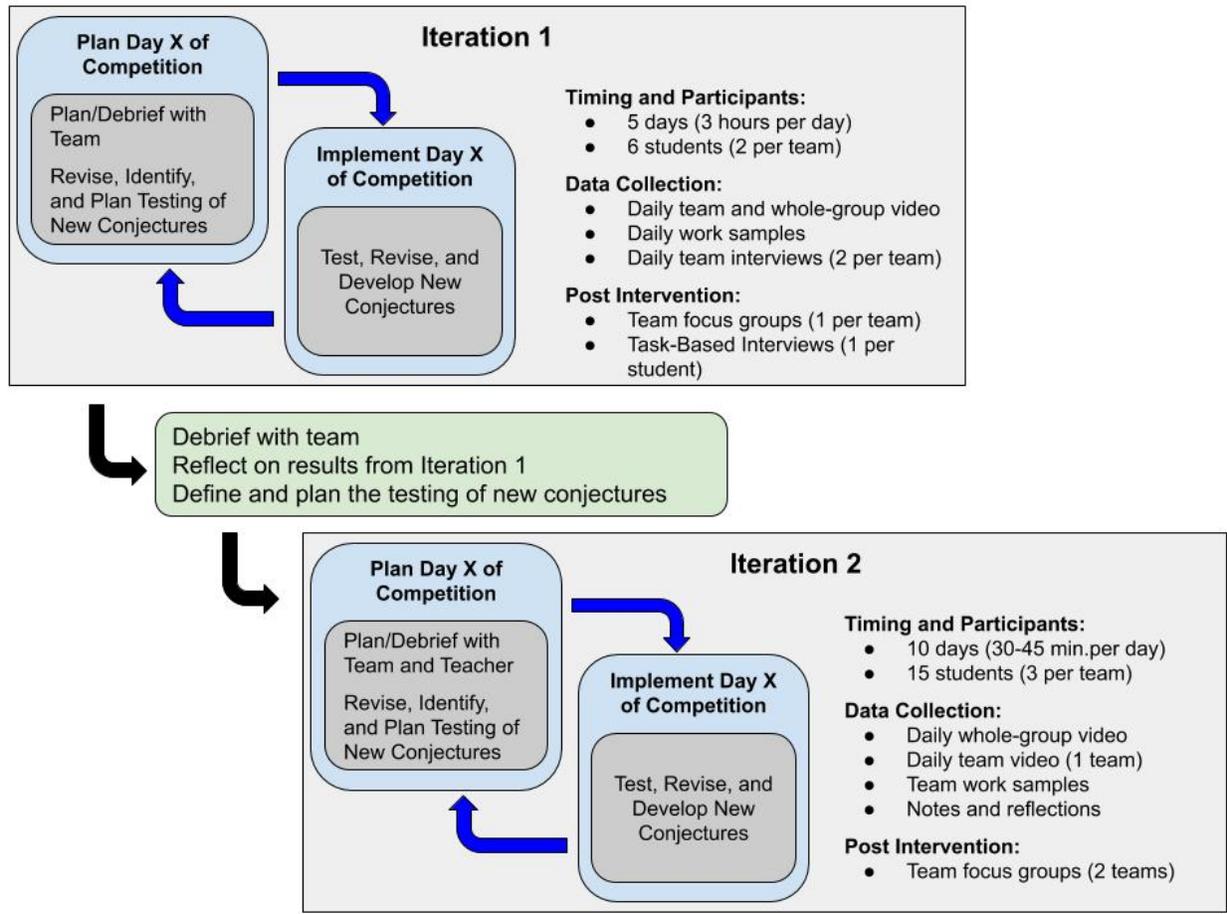
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Technologies (ICTs).

APPENDICES

Appendix A

Study Design Diagram



Click [here](#) to return to previous location in the paper.

Appendix B

The *Building Algorithms* Challenge Statement

An online version of the challenge statement can be found [here](#).

Scenario: Is LeBron James a better basketball player than Michael Jordan? Is Chinese food better than Tex Mex? In today's internet world, data on people's opinions are highly prized. One way to figure out those opinions is to ask people to complete surveys where they rate or rank their favorites and figure out how to use those results to create an ordered list of people's top choices. The way they figure out the final ordered list of favorites is to create formulas that put together those answers in an automated process. That process is called an algorithm.

How one carries out the algorithm on the data is objective because it is a well-defined set of combination of computations. But the way the algorithm was built to weigh some characteristics as more important than others is not objective -- the values of the designer of the algorithm come into play. In the case of basketball players, do you value their scoring more or less than their rebounding? In restaurants, do you like rice with your meal or prefer tortillas?

Challenge:

Many successful businesses are built around algorithms. Your challenge is to build an algorithm that uses people's opinions to rate or rank something you care about **and** that can be the start of a successful business. Your solution must:

1. Allow users to put in data and automatically rate or rank the thing you care about (You are **not** allowed to rate or rank students or physical appearance)
2. Include weighted categories*
3. Be transparent (Users should know how your algorithm works and be able to test it)
4. Include a way to make money

***Weighted Categories:** Algorithms often assign "weights" to categories depending on how important they are to the builder of the algorithm. For example, suppose you were building an algorithm to decide where to live when you graduate from college. You might care more about the average temperature of the city than the size of the city. You can use weights so that average temperature has a larger impact on the ranking than population. Explore the interactive spreadsheet in the *challenge context* document to see how an algorithm that compares dog breeds uses weighted categories.

Click [here](#) to return to previous location in the paper

Appendix C

The *Building Algorithms* Challenge Context Document

An online version of the document can be found [here](#).

Building Algorithms Challenge Context Document

Explore these resources to learn more about building algorithms and designing your business.

Ratings, Rankings, and Weighted Categories:

Use this resource to learn about ranking and rating algorithms, weighted categories, and spreadsheets.

https://docs.google.com/spreadsheets/d/1sVaEmb_90wZk1Sojus4qJ4op7oitfEloebnQVd0rl6Y/e/dit#gid=1309158863

Finding Categories

Algorithms can be used to rate or rank almost anything. All you need are data that can be represented numerically. Explore these examples to get you started. Remember, you should pick something that 1) is important to you, and 2) can be the foundation of a business.

Ratings based on user input:

- Video games combine user ratings to rate characters: (<https://fivethirtyeight.com/features/madden/#>)
- Includie combines user ratings to rate movies: (<https://includie.com/blog>)

Ratings that are not based on user input:

- Great Schools rates K-12 schools based on things like class size and test scores: <https://www.greatschools.org/>
- Baseball and softball uses statistics like ERA and WHIP to rate pitchers: https://en.wikipedia.org/wiki/Walks_plus_hits_per_inning_pitched

Rankings based on user input:

- The National Basketball Association combines media rankings to pick the most valuable player: (https://en.wikipedia.org/wiki/NBA_Most_Valuable_Player_Award)
- A singing competition in Europe (the Eurovision Song Contest) combines rankings to select a winner: https://en.wikipedia.org/wiki/Voting_at_the_Eurovision_Song_Contest

Rankings that are not based on user input:

- The Mohs scale of mineral hardness ranks minerals according to their hardness: https://en.wikipedia.org/wiki/Mohs_scale_of_mineral_hardness
- The English Premier League uses a point system to rank the 20 teams in the league each season: https://en.wikipedia.org/wiki/Premier_League#Competition

Click [here](#) to return to previous location in the paper.

Appendix D

The *Building Algorithms* Challenge Spreadsheet Resource

An online version of the most up to date spreadsheet resource can be found [here](#).

Introduction									
<p>This workbook is designed to help you build your algorithms. Your algorithm must rate or rank things within a category. But what's the difference between ratings and rankings?</p> <p>Rating algorithms assign scores to things using user input or data (like a player's points total). When you know the maximum score, a rating can tell you how good something is. For example, if Includie gives Black Panther a score of 5 out of 5, then you know it is good for diversity. But, if both Black Panther and Crazy Rich Asians receive a score of 5, you can't tell which movie is better for diversity.</p> <p>Ranking algorithms combine rankings to create an ordered list that tells you how one thing compares to another. Ranking algorithms can be great for comparing things, like ice cream flavors, but are not so great for telling you how good something is. For example, a ranking algorithm that compares ice cream flavors might ask users to rank several flavors. The algorithm then combines these rankings to create a final ranking or list. This list can tell you that cotton candy ice cream is better than lobster ice cream, but may not be able to tell you whether cotton candy is a good flavor of ice cream.</p> <p>Use the next two tabs in the worksheet to:</p> <ol style="list-style-type: none">1. Learn how to build algorithms in spreadsheets,2. Explore sample algorithms, and3. Understand how to use weighted categories in your algorithms.									

A	B	C	D	E	F	G	H	I	J																				
Weighted Categories in Rating Algorithms																													
Algorithm builders often use weights to make some variables have a greater impact than others on a final rating or ranking. This worksheet will walk you through how to use weighted categories in your algorithm.																													
<p>Spreadsheet Tip: Cells are named using the letter of the column (vertical) and the number of the row (horizontal). The blue cell to the left is named B11 because it is in column B and row 11.</p> <p>To write a formula in a cell, always start with = =Cell1 + Cell2 - add the numbers in Cell1 and Cell2 =Cell1 - Cell 2 - subtract the number in Cell2 from the number in Cell1 =5*Cell1 - multiply the number in Cell1 by 5 (you can also find the product of cells - Cell1*Cell2) =Cell1/2 - divide the number in Cell 1 by 2 (you can also find the quotient of cells - Cell1/Cell2) =Sum(Cell1:Cell3) - add the numbers in cells 1, 2, and 3 =Sum(Cell1, Cell2, Cell4) - add the numbers in cells 1, 2, and 4 =Count(Cell1:Cell4) - count the number of cells from Cell1 to Cell4</p> <p>You can apply a formula to multiple columns or rows by dragging the square in the bottom right corner of a cell. If you're using an iPad, highlight the cells and select "Autofill". This way you only have to write a formula once.</p>																													
<p>Example: Suppose a company is hiring middle school students for a summer internship. They've narrowed their list of candidates to three students: Jay, Ebony, and Vania. The company gave each student a score out of 10 (with 10 being the best) in four categories and will use an algorithm to decide who to hire.</p>																													
<p>1. One algorithm the company can use to compare candidates is to find the sum of the three ratings for each student. In this algorithm, each category has the same impact on the total score. Use this algorithm to complete the table. Try writing a formula that tells the spreadsheet how to carry out the algorithm.</p> <p>Based on this algorithm, which student will the company hire?</p>					<table border="1"> <thead> <tr> <th>Student</th> <th>Interview Performance</th> <th>Enthusiasm for the Position</th> <th>Eagerness to Learn</th> <th>Score</th> </tr> </thead> <tbody> <tr> <td>Jay</td> <td>10</td> <td>6</td> <td>5</td> <td></td> </tr> <tr> <td>Ebony</td> <td>4</td> <td>9</td> <td>9</td> <td></td> </tr> <tr> <td>Vania</td> <td>9</td> <td>5</td> <td>8</td> <td></td> </tr> </tbody> </table>					Student	Interview Performance	Enthusiasm for the Position	Eagerness to Learn	Score	Jay	10	6	5		Ebony	4	9	9		Vania	9	5	8	
Student	Interview Performance	Enthusiasm for the Position	Eagerness to Learn	Score																									
Jay	10	6	5																										
Ebony	4	9	9																										
Vania	9	5	8																										
<p>2. The company decides "Eagerness to Learn" is more important than "Interview Performance." Their new algorithm uses weights to make "Eagerness to Learn" have a greater impact on a candidate's score. The new algorithm multiplies a candidate's "Interview Performance" rating by 1, their "Enthusiasm for the Position" rating by 1.5, and their "Eagerness to Learn" rating by 2.</p> <p>Try writing a formula that tells the spreadsheet how to carry out the new algorithm. Remember, the asterisk (*) tells the spreadsheet to multiply. Using this algorithm, which student will the company hire?</p>					<table border="1"> <thead> <tr> <th>Student</th> <th>Interview Performance</th> <th>Enthusiasm for the Position</th> <th>Eagerness to Learn</th> <th>Score</th> </tr> </thead> <tbody> <tr> <td>Jay</td> <td>10</td> <td>6</td> <td>5</td> <td></td> </tr> <tr> <td>Ebony</td> <td>4</td> <td>9</td> <td>9</td> <td></td> </tr> <tr> <td>Vania</td> <td>9</td> <td>6</td> <td>8</td> <td></td> </tr> </tbody> </table>					Student	Interview Performance	Enthusiasm for the Position	Eagerness to Learn	Score	Jay	10	6	5		Ebony	4	9	9		Vania	9	6	8	
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Ebony	4	9	9																										
Vania	9	6	8																										

3. The first two algorithms allow the company to compare intern candidates. However, the total scores may be difficult to interpret: each category rating is out of 10 and the scores are not. A mean score could be easier to interpret.

When calculating a mean, each rating is counted once. This means that each rating has an equal **weight** in the algorithm.

For example,

$$\text{Score for Jay} = (9+6+5)/3$$

Try writing a formula that tells the spreadsheet how to find the mean of the three ratings for each student. Remember, the slash (/) tells the spreadsheet to divide. Also, spreadsheets follow the order of operations, so you will need to use parentheses.

Using this algorithm, who should the company hire?

Student	Interview Performance	Enthusiasm for the Position	Eagerness to Learn	Score
Jay	10	6	5	
Ebony	4	9	9	
Vania	9	6	8	

4. The company can also use weights when using means in their algorithm. This is called a **weighted mean**. In a **weighted mean**, scores that will have a greater impact are counted more in the calculation.

For example, this algorithm counts the rating for "Interview Performance" **once**, the rating for "Enthusiasm for the Position" **1.5** times, and the rating for "Eagerness to Learn" **2** times. The total is then divided by **4.5** ($1 + 1.5 + 2$), or the total number of times the ratings are counted.

$$\text{Score for Jay} = (1(10)+1.5(6)+2(5))/4.5$$

Try writing a formula that tells the spreadsheet how to calculate a weighted mean for each student. Based on this algorithm, who will the company hire?

Student	Interview Performance	Enthusiasm for the Position	Eagerness to Learn	Score
Jay	10	6	5	
Ebony	4	9	9	
Vania	9	6	8	

	A	B	C	D	E	F	G	H	I	J
1	Weighted Categories in Ranking Algorithms									
2										
3										
4	Algorithm builders often use weights to make some variables have a greater impact than others on a final rating or ranking. This worksheet will walk you through how to use weighted categories in your algorithm.									
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45										

Spreadsheet Tips:
Cells are named using the letter of the column (vertical) and the number of the row (horizontal). The blue cell to the left is named B11 because it is in column B and row 11.

To write a formula in a cell, always start with =
 =Cell1 + Cell2 - add the numbers in Cell1 and Cell2
 =Cell1 - Cell 2 - subtract the number in Cell2 from the number in Cell1
 =5*Cell1 - multiply the number in Cell1 by 5 (you can also find the product of cells - Cell1*Cell2)
 =Cell1/2 - divide the number in Cell 1 by 2 (you can also find the quotient of cells - Cell1/Cell2)
 =Sum(Cell1:Cell3) - add the numbers in cells 1, 2, and 3
 =Sum(Cell1, Cell2, Cell4) - add the numbers in cells 1, 2, and 4
 =Count(Cell1:Cell4) - count the number of cells from Cell1 to Cell4

You can apply a formula to multiple columns or rows by dragging the square in the bottom right corner of a cell. This way you only have to write a formula once.

Example:
Suppose a company is hiring middle school students for a summer internship. They've narrowed their list of candidates to three students: Jay, Ebony, and Vania. The company owner, a manager, and an employee each ranked the three students, where 1 represents their first choice and 3 represents their last choice. The company will use an algorithm to combine these rankings and pick a candidate to hire.

Algorithms that rely on user rankings need a way to combine those rankings and get a final ranking.

1. One way to combine rankings is to find the sum of the three rankings for each candidate. In this algorithm, lower final scores mean better rankings.

Try writing an algorithm that tells the spreadsheet how to calculate a combined rank for each of the three candidates. Based on this algorithm, who will the company hire?

Student	Owner	Manager	Employee	Combined Rank
Jay	1	1	3	
Ebony	2	3	2	
Vania	3	2	1	

2. The company decides that the owner's rankings should have a greater impact than either the manager's or the employee's rankings. The new algorithm uses **coefficients** to weight the rankings differently. In this new algorithm, the owner's rankings are multiplied by 2.5, the manager's are multiplied by 1, and the employee's are multiplied by 0.5.

Try writing an algorithm that tells the spreadsheet how to calculate combined rankings. Based on this algorithm, who will the company hire?

Student	Owner	Manager	Employee	Combined Rank
Jay	1	1	3	
Ebony	2	3	2	
Vania	3	2	1	

3. The two algorithms above assign a score to each candidate based on user rankings. The company will still need to order the candidates based on these scores. Another way to combine user rankings that gives a score in the same 1-3 point range is to find the mean ranking for each candidate. In an algorithm that uses a **mean**, each user's rankings are counted once. This means their rankings are weighted equally.

Jay's Combined Rank: $(1+1+3)/3$

Try writing an algorithm to rank the three candidates. Based on this ranking algorithm, who will the company hire?

Student	Owner	Manager	Employee	Combined Rank
Jay	1	1	3	
Ebony	2	3	2	
Vania	3	2	1	

4. Like the algorithm in #2, **weights** can be added to an algorithm that uses the **mean**. This is called a **weighted mean**. When calculating a **weighted mean**, some people's rankings are counted more than others. For example, weights can be used to make the owner's rankings have a greater impact. Using the same weights as in algorithm #2, the owner's rankings are counted **2.5** times in the combined rankings, the manager's rankings are counted **once**, and the employee's rankings are counted **0.5** times. The total rankings are then divided by 4 ($2.5 + 1 + 0.5$), or the total number of times the rankings are counted.

Jay's Rank: $(2.5(1)+1(1)+0.5(3))/4$

Try writing an algorithm that tells the spreadsheet how to calculate the combined rankings using this new algorithm. Based on this algorithm, which student will the company hire?

Student	Owner	Manager	Employee	Combined Rank
Jay	1	1	3	
Ebony	2	3	2	
Vania	3	2	1	

Click [here](#) to return to previous location in paper.

Appendix E

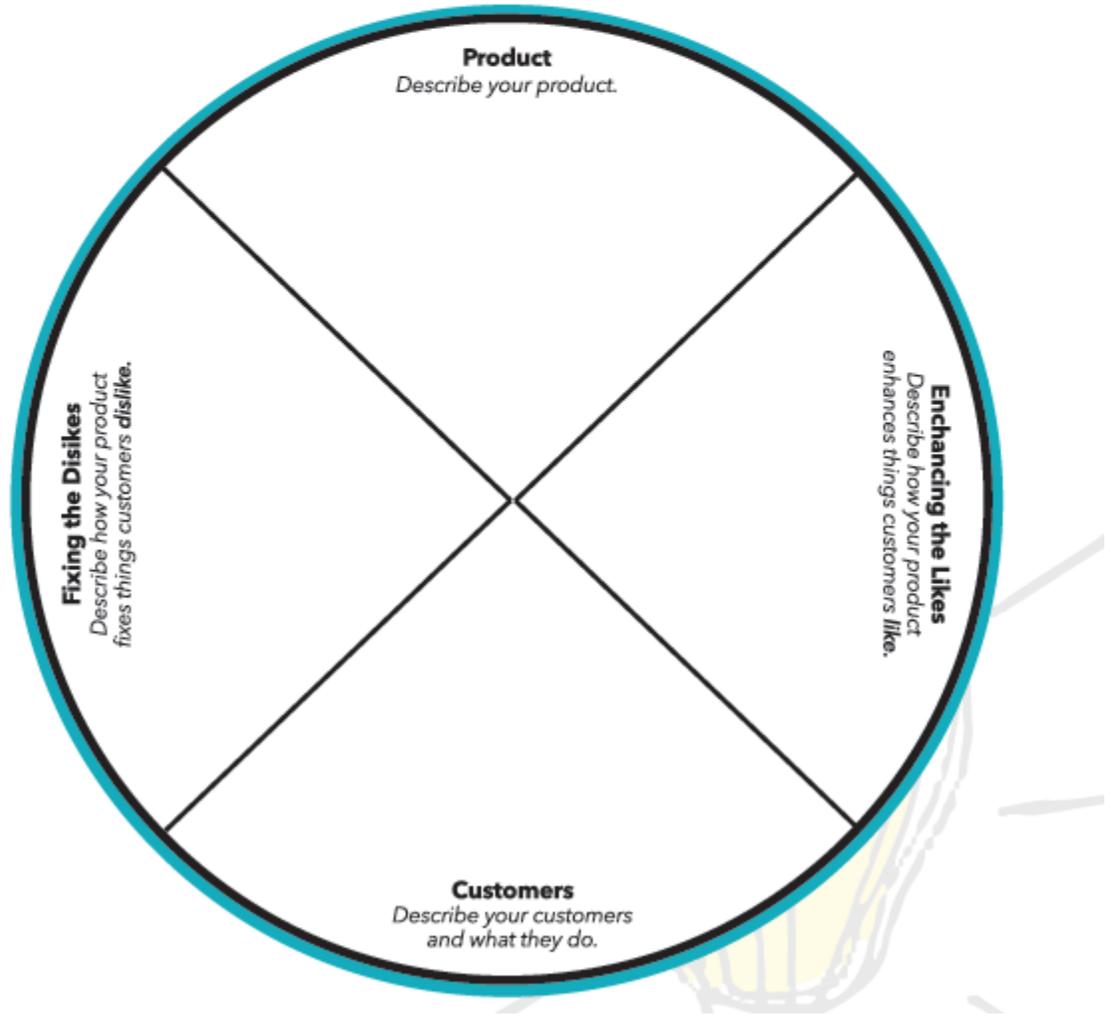
Key Business Proposition

An online version of the Key Business Proposition can be found [here](#).



Entrepreneurs look for ways to create value for their customers. Creating value means making a customer's situation or job better. This can be done by enhancing something that they like about their situation (**likes**) or fixing something they don't like about their situation (**dislikes**).

This chart can help you get clear about who your customers are and how your product creates value for them.



DESIGN & PITCH CHALLENGE

KEY BUSINESS PROPOSITION

Briefly describe the business model type you will use for your design and why you chose that model type.

Practicing your Elevator Pitch

Entrepreneurs must be able to quickly and convincingly explain their product to investors. This is often called an **elevator pitch**, because they should be able to fully explain their product during an elevator ride.

Sentence starters are a great way to practice describing your product. Each blank represents part of your Key Business Proposition. Fill in the blanks to describe what your product is, who it is for, and how it creates value.

Then, everyone in your group should practice saying this statement several times. This will help you be sure that you all agree on how to describe your product.

Hi, our names are _____, _____, and _____.
(group member's name) (group member's name) (group member's name)

Our company, _____, invented _____.
(company name) (product name)

It helps _____, who _____.
(customers) (customer's situation or job)

Unlike _____, which _____, and _____
(competitor) (competitor weakness 1) (competitor weakness 2)

our product _____ the _____, by _____.
(improves, makes easier, lessens, etc.) (a customer dislike)

_____ It also _____
(how the product lessens a customer dislike)

_____ the _____, by _____
(increases, enhances, etc.) (a customer like)

_____.
(how the product enhances a customer like)

Click [here](#) to return to previous location in paper.

Appendix F

The *Building Algorithms* Challenge Technical Brief

An online version of the Technical Brief can be viewed [here](#).

Now that you have analyzed a problem, proposed and refined a solution, and developed a pitch for your idea, you will create a technical brief describing your product and the process you used to find a solution.

Part 1. Briefly describe your solution and how it solves the problem or challenge you identified.

Part 2. Describe the mathematics, science, and engineering you researched to design your product. Include links to websites or other resources you used.

Part 3. Describe the decisions you made to choose your design and the challenges you had to overcome.

Part 4: How did developing your Key Business Proposition and related Business Model Types affect your process?

Part 5. Your final solution probably looked different from your original idea. Describe the process for how you developed your idea from start to finish.

Part 6. How well do you think your solution will work under real-world conditions?

Just a little	Somewhat	Fairly Well	Almost Completely

Explain your reasoning.

Part 7. Fully describe your Algorithms solution based on the questions below.

1. What does your algorithm do?
 - a. Include a description that will allow others to test your algorithm.
 - b. Represent your algorithm as an algebraic expression.
2. How does your algorithm work?
 - a. Describe the variables you used in your algorithm and how you decided on the weights for each variable.
 - b. Explain how your algorithm determines a rating or ranking.
 - c. Show how you would use your algorithm to calculate a rate or rank.
 - d. Include a table showing the ratings or rankings for at least 10 things in the category you care about.
3. How will your company make money?
 - a. Describe the business plan you will use for this company.
 - b. Show, using calculations, that your company will make money.
4. What are the limitations of your algorithm?
 - a. Describe what your algorithm does well.
 - b. Describe what your algorithm doesn't do well.

- c. Describe any extra information would you want to find a way to use in your algorithm.
- d. Explain why you didn't include this information in your algorithm.

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Appendix G

The *Building Algorithms* Challenge – How to Build a Pitch

An online version of the How to Build a Pitch document can be viewed [here](#)

DESIGN&PITCH CHALLENGE

HOW TO BUILD YOUR PITCH

BUILDING YOUR PITCH

1. Before you start, think about the story you want to tell about your product. How would you convince someone that you have a good idea that can make money?
2. It's important to consider your Key Business Proposition as you are planning your pitch. Be flexible. As you start thinking about how to pitch your product, you may want to make edits to your Key Business Proposition. And, as you make changes to your Key Business Proposition, you may also want to make changes to your pitch.

THE PROBLEM

What problem are you solving and why does it matter?

Start by giving investors some background on the problem you wanted to solve. Explain why it matters to you and your customers, and why it should matter to investors. Your goal is to make investors feel the importance of finding a solution for this problem.

Tip: The less writing, the better. If you need help remembering what to say, you can write it out in the speaker notes section in your presentation.

THE COMPETITION

Convince investors that current solutions are not good enough. What other products have been designed for your customers?

Describe competitors' solutions and make it clear to investors what these solutions are missing, from the customer's perspective. This should connect to the likes and dislikes you described in the Key Business Proposition.

Tip: make sure that anything you describe as missing from competitors' solutions is addressed by your solution. For example, if you say "most competitors' products do not result in world peace," then it should be clear how your solution will achieve world peace.



DESIGN&PITCH CHALLENGE

HOW TO BUILD YOUR PITCH

YOUR PRODUCT

Describe your product and how it enhances customers' likes and fixes customers' dislikes.

These should be brief statements of the ways in which your product enhances the customers' likes or fixes the customers' dislikes.

Tip: Use pictures to illustrate your product.

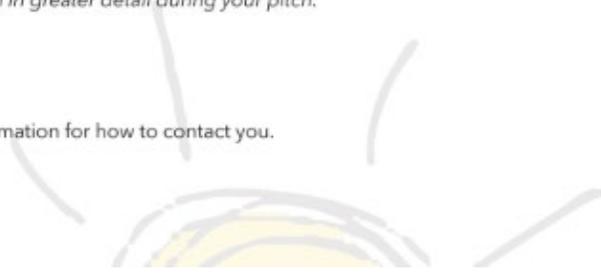
HOW YOUR PRODUCT WORKS

Explain how each part of your product works (these should be the parts you described on 'YOUR PRODUCT').

Tip: You don't have to include everything in every bullet point. Use bullet points to summarize your main points and explain them in greater detail during your pitch.

THANK YOU

Always say thank you and give investors information for how to contact you.



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Appendix H

The *Building Algorithms* Challenge – Pitch Judging Sheet

An online version of the How to Build a Pitch document can be viewed [here](#).



This sheet will be used to judge pitches and pick a winner. Each row will be assigned a score from 0 to 3 using the following key:

3 points: The pitch thoroughly and accurately addresses **all** of the listed qualities.

2 points: The pitch has most of the qualities but is missing one or two key parts.

1 point: The pitch has a few of the listed qualities but is missing major parts.

0 points: The pitch does not contain any of the listed qualities.

EVALUATING THE SOLUTION

Criteria	3 points	2 points	1 point	0 points
The team clearly defines the "problem" and explains how their solution adds value for customers.				
The team describes the target customers, estimates how many potential customers there are, and explains what they need in a solution.				
The team discusses the research they conducted and how it led to their solution.				
The team describes the features of their solution and explains how and why they work.				
The team includes a sketch or prototype of their solution to show its features.				
The team demonstrates that their solution will work under real-world conditions.				
The solution shows creativity and imagination.				
The team identifies limitations of their solution.				

DESIGN&PITCH CHALLENGE

CHALLENGE JUDGING SHEET

EVALUATING THE PREPARATION AND PRESENTATION

Criteria	3 points	2 points	1 point	0 points
The pitch was designed for the appropriate audience (investors).				
The presentation was professional and transitions through the pitch were smooth.				
All team members demonstrated an understanding of their design and participate in the pitch.				
The team displayed a passion for their solution.				
The presentation materials are error-free, with correct grammar usage, punctuation, capitalization, and spelling.				
Images and graphic design are used well and communicate key messages. Text is used well and placed strategically to enhance the presentation.				

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Appendix I

Design & Pitch Challenges in STEM Recruitment Flyer

The Challenges

Students use their knowledge of STEM to invent solutions to some of the world's most pressing issues. What makes these challenges different from other STEM challenges is that we want students to think like entrepreneurs, including developing business plans, because without money, even the best solution is unlikely to succeed!

Who Can Participate?

We're looking for **middle graders** to help us test a new type of activity that we're calling the Design & Pitch Challenges in STEM. Students will use math in solving these challenges but there is no prerequisite knowledge required to participate in the challenge. All who are excited about using STEM to solve real and messy problems are welcome!

Design & Pitch Challenges in STEM for Middle Grades NC State University

When: July 16, 17, and 18
9:30am – 12:30 pm

Where: SUDDS Research Office,
407 Gorman St., Raleigh, NC

**Participation is FREE and light snacks will
be provided**

Sign up no later than Wednesday, July 10

Contact Michael Belcher at mjbelche@ncsu.edu for
more information!



NC STATE
College of Education

Why Participate?

These challenges represent a new way to learn math. Students will learn to work with a team and use their STEM skills to invent entrepreneurial solutions to real-world problems. Along the way, they will strengthen their math skills and see STEM in a new light.

Students will be partners with NC State in the design process and will help us improve the challenges and get them ready for use in real middle school classrooms.

To sign up:



Or visit:
<https://go.ncsu.edu/stemchallengesignup>

Click [here](#) to return to previous location in the paper.

Appendix J

Design & Pitch Challenges in STEM Consent Form

North Carolina State University INFORMED CONSENT FORM for RESEARCH

Title of Study/Repository: Innovation Challenges for Middle School Mathematics in a Digital Learning System: Student Participation and Impact on Achievement, Affect, and STEM Career Interest (IRB #12603)

Principal Investigator: Dr. Jere Confrey, jere_confrey@ncsu.edu, XXX-XXX-XXXX

What are some general things you should know about research studies?

You are being asked to take part in a research study. Your participation in this study is voluntary. You have the right to be a part of this study, to choose not to participate and to stop participating at any time without penalty. The purpose of this research study is to better understand how students work with a new type of science, technology, engineering, and mathematics (STEM) activity.

You are not guaranteed any personal benefits from being in this study. Research studies also may pose risks to those who participate. You may want to participate in this research because you will get to invent solutions to real-world problems using STEM. You may not want to participate in this research because the problems will be challenging and you will be asked to present your ideas to your peers and one or more researchers.

In this consent form you will find specific details about the research in which you are being asked to participate. If you do not understand something in this form it is your right to ask the researcher for clarification or more information. A copy of this consent form will be provided to you. If at any time you have questions about your participation, do not hesitate to contact the researcher(s) named above or the NC State IRB office (contact information is noted below).

What is the purpose of this study?

The purpose of the study is to understand how students use science, technology, engineering, and mathematics to complete a new type of problems. We also hope to understand how participating in these challenges affects how students view and understand mathematics. The overall aim of our study is to develop a new approach to teaching and learning math.

Am I eligible to be a participant in this study?

There will be approximately 2 – 15 participants in this study.

In order to be a participant in this study you must be in middle school or be of middle school age (10 to 14 years old).

You cannot participate in this study if you are not in middle school or are not of middle school age.

What will happen if you take part in the study?

If you agree to participate in this study, you will be asked to work in a team to invent new ways of solving real problems. These problems do not have obvious answers. You will need to work with your team to brainstorm ideas, research your ideas and other ideas, test your ideas, and figure out a way to convince investors that your idea is worth funding. At the end of the work time, you will be asked to present your idea to your peers and the researcher. You will then be asked to take part in a focus group with the other students participating in the study. During the focus group, you will need to answer questions about your participation in the activity and your career interests. You will also be asked to complete a short survey on your career interests before and after you complete the challenge.

The total amount of time that you will be participating in this study is 15 hours, spread across 5 days. Testing will take place at the NCSU SUDDS office located at 407 Gorman St., 2nd Floor, Raleigh, NC 27607.

Photos and video

If you want to participate in this research, you must agree to being video and audio recorded. If you do not agree to being video recorded or audio recorded you cannot participate in this research.

We will videotape you and your group as you work on the challenge, present solutions, and participate in the focus group. These video and audio recordings are important for our research: they help us study how you work on the problems and the ideas you come up with to solve the problem. Because in our research we often study these kinds of video recordings again and again, we plan to keep the recordings for use in other research studies. Also, if you provide permission, we may use parts of the recordings in professional presentations and publications (e.g. professional journals or book chapters). We will keep any video and audio recordings on secure servers or secured hard drives, managed by the research team.

I consent to be video recorded and/or audio recorded

I do not consent to be video recorded or audio recorded

I also consent for the research team to use short excerpts of video recordings in professional research presentations and teacher professional development materials (materials to help teachers understand the activities). Recordings will be kept indefinitely and may be used in publications, as long as I am not identified by name in such presentations.

Risks and benefits

There are minimal risks associated with participation in this research. The particular activities you may work on will be similar to those that you do in school. You will be asked to present your solutions to your peers and researchers, which may be uncomfortable. We try to make presenting easier by helping you build your presentation and guiding you through the process. Also, this is a friendly and safe environment. We are excited to see what you come up with and will only offer constructive feedback. The purpose of the presentation is to help us understand how to support students before and during the presentations.

There are no direct benefits to your participation in the research. The indirect benefits include the ability to participate in developing new ways of teaching and learning math. Also, because you are using science, technology, engineering, and mathematics in new ways and are using drawings and models to explain your thinking, you are likely to recognize that you have good ideas and to become more confident in math.

Right to withdraw your participation

You can stop participating in this study at any time. In order to stop your participation, please tell the researcher and you can stop. If you choose to withdraw your consent and stop participating, you will be allowed to stop working on the activity or participating in the focus group and wait for your parents to pick you up. In the event that you want to stop participating, we will not use any of your data.

Confidentiality

The information in the study records will be kept confidential to the extent allowed by law. The research team will retain all digital data and non-digital work products the subjects generate (drawing, paper-and-pencil solutions/Attempts, and other artifacts). All data (including video interviews) will be stored electronically on file storage leased by NC State University. Access is restricted by user name and

password. With your consent (signature below) particular clips from the video recordings may be shared with people beyond our project team (for instance, professional meetings and teacher professional development classes or materials for teachers.). Project reports will include individual student responses to challenges, strategies with and reactions to challenges. Children will not be identified by name in such presentations or materials, or in any written or oral reports of the research.

Compensation

You will not receive any compensation for participating in this study.

What if you have questions about this study?

If you have questions at any time about the study itself or the procedures implemented in this study, you may contact the researcher, Dr. Jere Confrey, 407 Gorman St. 2nd Floor, Raleigh, NC 27607, jere_confrey@ncsu.edu, (XXX) XXX-XXXX

What if you have questions about your rights as a research participant?

If you feel you have not been treated according to the descriptions in this form, or your rights as a participant in research have been violated during the course of this project, you may contact the NC State IRB (institutional Review Board) Office via email at irb-director@ncsu.edu or via phone at 1.919.515.8754. You can also find out more information about research, why you would or would not want to be a research participant, questions to ask as a research participant, and more information about your rights by going to this website: <http://go.ncsu.edu/research-participant>

Consent To Participate

"I have read and understand the above information. I have received a copy of this form. I agree to participate in this study with the understanding that I may choose not to participate or to stop participating at any time without penalty or loss of benefits to which I am otherwise entitled."

Participant's printed name _____

Participant's signature _____

Date _____

Parent's or Guardian's printed name _____

Parent's or Guardian's signature _____

Date _____

Investigator's signature _____

Date _____

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Appendix K

The *Building Algorithms* Challenge Study Plan

Daily Plan

Day 1 (Monday, July 22)

Activity Goals:

- Give students an overview of the activity
- Engage in team-building activity to highlight the importance of brainstorming, collaboration, and iteration
- Discuss entrepreneurship
- Introduce the algorithms challenge
- Begin identifying categories for rankings/ratings, brainstorming characteristics to use in their algorithms, and begin building their spreadsheet algorithms.

Study Goals:

- Explore students' familiarity and experiences with algorithms (Conjecture 4)
- Explore students' perception of entrepreneurship (Conjecture 1)
- Explore students' perception of the video resources and whether they perceive those resources as adding to the authenticity of the challenge task (Conjecture 2)

Detailed Schedule:

1:00 – 1:10 students arrive, make name tags, turn in consent forms, get snacks, go over logistics (e.g. bathroom, water, snacks).

1:10 - 1:20 students complete the STEM CIS on paper

1:20 – 1:25 Go over the goals of the study and their ability to opt out. Give a brief summary of the plan for the week:

1:25 – 1:45 tower building activity and discussion

2:05 – 2:20 Discuss entrepreneurship, using the lemonade video as an example.

[Think – Pair – Share] What is an entrepreneur? (**Conjecture 1**)

Show the lemonade stand video and talk about what it means to be an entrepreneur.

What is an entrepreneur?

Is an entrepreneur the same as a problem solver? Why or why not?

Why do you think entrepreneurs want to make money?

Introduce the pitch competition

[Conjecture 3 - how does the pitch competition affect engagement and excitement?] What is a pitch competition? What do you think makes a good pitch?

2:20 - 2:40 Introduce the challenge: Discuss the context, watch challenge statement video, and read the challenge statement.

Have you ever heard of an algorithm? If so, where? What is it? Give an example of a simple rating algorithm and ask if they know how ratings are generated (e.g. dunk contest, unless I can come up with something better before testing)

[Think – Pair – Share] Imagine you and your family won a trip to anywhere in the world. What would be your top 5 choices for where you would want to go? Make an ordered list of your top 5 locations (in other words, rank your top 5 choices). Share with your neighbor. Discuss: How did you rank your 5 locations? How did you decide one place was “better” than another?

(Conjecture 4) *How are students describing how they ranked their vacation choices? What evidence is there of students applying a systematic process to compare locations?*

Ask “Is there anything else you have done in math that is like this?”

One way to compare choices is to use an algorithm. Your challenge will be to build an algorithm that helps people rate or rank something they care about. I’m going to show you a video of an entrepreneur discussing how she built a company that uses algorithms to assign ratings to movies.

Show video the Cathy Yee video to launch the challenge.

Discuss video: Turn to your neighbor. What does Includive do? Why did Cathy Yee think it was important to start her company?

(Conjecture 1 - do they see algorithms and the challenge as authentic and does that increase their engagement and enthusiasm?) Includive is a start-up, which means it’s just getting started - do you think her business can succeed? Why or why not?

Read the challenge statement to yourself (silently first): Show them how to access the challenge statement using their iPads.

Discuss challenge statement: What is your challenge? What are the criteria for a good solution?

Show students how to access the background resources, including the background video. Tell them they can access these materials at any time and that they are designed to help them build their algorithms.

2:40 - 2:45 Quickly go over student instructions.

2:45 - 4:00 Start working with your team. Remember, on Friday, you will be pitching your business idea and algorithm to two investors. By the end of this first day, you should aim to have 1) selected your category (e.g. movies) and 2) begun building your algorithm in a spreadsheet. **Show students how to access their spreadsheet resource** (in their Sheets app or in the Background folder on the drive). Remember, you need to have a spreadsheet algorithm and it should include weighted categories. The spreadsheet resource will help you learn more about these concepts.

3:30 - 3:45 and 3:45 - 4:00 student interviews (2 teams, 1 team in each time slot).

What is the challenge you're working on? (*Conjecture 1*)

What is an algorithm? Had you heard of algorithms before today? (*Conjecture 1 and 4*)

How did Cathy Yee's video impact how you viewed the challenge? (*Conjecture 2*)

What is an entrepreneur? Is it important? If so, why? If not, why not? (*Conjecture 1*)

Day 2 (Tuesday, July 23)

Activity Goals:

- Give students time to continue working with their teams to build their algorithms
- Work with the spreadsheet resource to learn more about spreadsheets, algorithms, and how to build an algorithm in a spreadsheet
- Introduce the key business proposition and business model types document
- By the end of the day, teams should have a prototype algorithm and should have begun defining their key business propositions for their algorithms.

Study Goals:

- Explore Conjecture 1 - how the spreadsheet and algorithm task supports functional thinking in algebra
 - Describe your algorithm - how does it work? What does it do? What variables does it use to calculate a rating/ranking? - is there evidence of students using functional language as they describe their algorithms?
- Explore Conjecture 4 - how the entrepreneurial framing helps students perceive their algorithms as actionable and promotes engagement.

1:00 - 1:10 students arrive, get settled, and get their assigned iPads.

1:10 - 1:45 refresher of the previous day's activities. Remember, your goal is to build a rating/ranking algorithm that can be the start of a successful business. On Friday, you will need to pitch your business idea to two investors.

Introduce the key business proposition and the business model types document.

How do you know if an idea is a good business idea?

If you were an investor, what would you need to know before you decide to invest in a company?

The Key Business Proposition will help you get clear on who your customers are, how your product helps those customers, and how your business will make money. *[Show and discuss]*

How do websites and apps make money?

The Business model types document describes a few possible business models.

There are many others. *[Show and discuss]*

Where have you seen some of these business models before?

How do you think a company like Netflix makes money? What about something like Spotify?

Observations: *to what extent do students participate in the discussion? Are they excited to share their familiarity with website/app business models?*

By the end of today, you should have a prototype algorithm and should have begun defining your key business proposition for your algorithm.

Observations as they work:

Are students engaging with the business aspect of their algorithm solutions? How does their engagement change relative to the algorithm after they begin considering their customers and their business models?

1:30 - 2:25 Check in with groups about their spreadsheets

[Conjecture 4] How does your algorithm calculate a rating/ranking?

How does the spreadsheet algorithm work? How did you program it to give ratings/rankings for [insert thing they're rating/ranking]?

How does your algorithm use weights in calculating a rating/ranking? Where can I see the weights in your spreadsheet formula? Where can I see the characteristics (use student language) in your spreadsheet formula? What is the relationship between the weights and the overall rating/ranking?

How would you explain your algorithm to a friend so that they could test it in their spreadsheet?

2:25 - 2:40 Bring students together to view the Includie website.

Show them the website. Ask them to suggest movies to explore. Ask them how they think the ratings are generated.

Show them one example that has a diversity and general rating.

What type of algorithm are they using (rating or ranking)?

How do you think they calculate a rating or ranking?

How do you think Includie makes money? Who are Includie's target customers?

[to target Conjecture 1 and 2 and the authenticity of the activity - are they connecting the work they're doing with Includie's website and using their understanding to analyze/interpret Includie's algorithm? To target conjecture 1 - is the entrepreneurial component something that students are interested in discussing relative to an unknown business model applied to an algorithm business.]

2:40 - 3:45 students return to their algorithms and key business propositions

3:15 - 3:30 and 3:30 - 3:45 Interviews with teams

- How did the key business proposition influence your algorithm? What do you think the importance is of a key business proposition?
- Who are your customers? Why is it important for you to have customers?
- How did thinking about the needs of your customers influence your algorithm?
- What did you think of the Includie website? Did seeing it influence how you thought about your challenge? If so, how? If not, why not?

[Testing whether entrepreneurship components add to the authenticity of the task and help students consider whether their solutions are actionable]

Day 3 (Wednesday, July 24)**Activity Goals:**

- Give students time to continue working with their teams to build their algorithms and their key business proposition
- Introduce the technical brief and the pitch resources
- By the end of the day, teams should have a working algorithm, a draft Key Business Proposition, and a draft technical brief. Teams should have also begun planning their pitches and building their pitch decks.

Study Goals:

- Explore Conjecture 3 - the role of the pitch
- Explore Conjecture 1 and 4 - understanding how the entrepreneurial focus on the needs of customers informs students' algorithms

1:00 - 1:10 students arrive, get settled, and get their assigned iPads.

1:10 - 1:20 introduce and discuss the technical brief in relation to product specifications

Have you done online shopping? Have you ever noticed the product specifications section? What does it tell you?

Why is that section important? Why might it be important for an algorithm company? Think about Includie, why do you think people would want to know the specifics of how their algorithm rates the diversity of movies?

1:20 - 1:30 remind students of the final pitch on Friday and discuss the features of a successful/effective pitch.

Have you ever seen or done an entrepreneurial pitch? What was it like? Was it convincing? Why or why not?

What do you think a convincing pitch looks like?

By the end of the day, you should have a working algorithm, a draft Key Business Proposition, and a draft technical brief. You should have also begun planning your pitches and building your pitch decks.

1:30 - 2:30 students work on building out their algorithms, completing their key business propositions, building their pitch decks, and completing their technical briefs.

2:30 - 2:50 check-ins with resident experts

Observations: how do students respond to the product-specific feedback they receive from the resident experts?

2:50 - 4:00 continue working on pitches, technical briefs, and key business propositions

3:10 - 3:30 and **3:30 - 3:50** interviews with students

What is a pitch? Compare it to other presentations you have done.

Have you done a pitch before? If so, what do you like about pitching?

What don't you like?

On Friday, you will be pitching your ideas to two expert judges. What worries you about your pitch on Friday? What excites you about your pitch?

What do you still need to do to be ready for Friday?

How will you prepare for Friday?

What do you think of the competition side of the pitch? Do you like competing?

Day 4 (Thursday, July 25)

Activity Goals:

- Finish key business proposition, technical brief, algorithm, and pitch
- Introduce and discuss the pitch judging sheet.

Study Goals:

- Explore Conjecture 3 - the role of the pitch
 - Describe your algorithm - how does it work? What does it do? What variables does it use to calculate a rating/ranking? - is there evidence of students using functional language as they describe their algorithms?
- Explore Conjecture 1 and 2 - understanding how the entrepreneurial focus on the needs of customers informs students' algorithms and how entrepreneurial components (KBP, pitch, business model types, technical specifications) supports students' engagement and enthusiasm for the challenge.

1:00 - 1:10 students arrive and get settled

1:10 - 1:20 Discuss pitches [*exploring conception and role of pitches*]

What is a pitch?

How is a pitch different from other presentations?

What makes a pitch convincing?

What do you think investors will want to hear in your pitch? Why?

1:20 - 1:25 goals for the day:

- Finish your algorithm, key business proposition, technical brief, and pitch decks

- Make sure students know how to access KeyNote for their pitch decks and the supporting pitch resources.

1:25 - 3:00 put finishing touches on algorithms, key business proposition, technical brief, and pitch decks.

Check in with groups as they work on their pitch decks

*(Conjectures 1 and 2) ask questions about the economic viability of their algorithms, their key business propositions, and the persuasiveness of the pitches. **Conjecture - by giving feedback relative to the entrepreneurial components of the solutions, students will feel more engaged in the process. That is, the focus is on making a solution that works for investors instead of focusing only on the underlying mathematics.***

In observations, do we see heightened engagement/increased willingness to engage with the researcher relative to their designs?

3:00 - 3:20 pitch practice

Same conjecture relative to feedback on entrepreneurship and pitching as a means of increasing students' engagement and enthusiasm for the process. The underlying mechanism is that students view their ideas as authentic and actionable - does this help them see the math as central to this particular entrepreneurial innovation?

3:20 - 4:00 revisions to pitch decks and pitch plan

Day 5 (Friday, July 26)

1:00 - 1:40 Final preparation for the pitch competition

Students work on preparing for the pitch competition, making final revisions to their pitch decks, and practice delivering their pitches.

1:40 - 2:00 pitch competition and announcing winners

2:30 - 3:00 Focus groups

2:30 - 3:45 task-based interviews with teams of students (20 minutes per team). This will be to explore conjecture 1.

Conjectures

Conjecture 1: the entrepreneurial framing of the challenge will empower students to perceive their solutions as actionable and will help them maintain their engagement during the challenge.

Questions to Explore:

1. How do students conceptualize entrepreneurship and its role in developing their algorithms?
 - a. What do students think entrepreneurship is?
 - b. How do students perceive the role of entrepreneurship in their process of developing a solution to the challenge?

- c. *To Test: Interview questions on students' understanding of entrepreneurship and their reflections on its role in the challenge?*
2. To what extent do the needs of target customers inform the design of students' algorithms?
 - a. What evidence is there of students considering the key business proposition in the design of their algorithms and pitches?
 - b. What evidence is there of students discussing how to turn their algorithms into a business?
 - c. What evidence is there of students refining their algorithms based on the economic viability of their algorithms?
 - d. How do students describe the role of the key business proposition in their solutions?
 - e. *To Test: Observations and probing questions as students work to see how they are using the KBP and how they are discussing the business aspects of their algorithms. Interview questions around a hypothetical algorithm – would you pursue this business idea? Why or why not?*
3. How does considering the monetization of their algorithms using real business model types as presented in the business model types document, help students to view their algorithms as actionable?
 - a. *To Test: Observations and interview questions relating to how the business considerations are informing the design of the algorithms. Interview questions on students' perceptions of whether they perceived their solutions as actionable and whether that matters to them.*

Interview question giving another hypothetical algorithm, this time with a business model type – is this a business idea you would pursue? Why or why not?

Conjecture 2: the challenge materials and support resources will help students perceive the challenge as authentic, which will increase their engagement.

Questions to Explore:

1. How does connecting the challenge to the work of a real CEO and an actual company (challenge champion videos and Inlucvie website) affect students' perceptions of the authenticity of the challenge?
 - a. What evidence is there of students discussing examples of algorithms in businesses?
 - b. What evidence is there of students' heightened engagement during the videos?
 - c. What evidence is there of students' heightened engagement during the sharing of the Inlucvie website? How do students discuss their experience viewing the challenge champion videos and Inlucvie website, and is there evidence that they view the challenge as more authentic?
 - d. *To Test: Use of challenge champion video, resources, and sharing of Inlucvie website. Interview with 2 students on day 1. Observations on day 1.*

2. How does the authenticity of the challenge affect students' engagement and enthusiasm during the challenge?
 - a. *To Test: focus group questions on day 5.*

Conjecture 3: Practicing for and participating in the pitch competition will increase students' engagement and enthusiasm during the challenge.

Questions to Explore:

1. How does participating in a pitch affect students' engagement and enthusiasm during the challenge?
 - a. How do students conceptualize pitching?
 - b. What evidence is there that students view pitching and the pitch competition as something different than a typical school presentation?
 - c. How does the use of expert judges for the pitch competition affect students' engagement and enthusiasm during the challenge?
 - d. What evidence is there of increased engagement and enthusiasm in response to the pitch competition and pitch practice?
 - e. How does the competition aspect influence students' engagement and enthusiasm during the challenge?
 - f. *To Test: Observations and focus group. Observations during day 4 or 5. Focus group on day 5.*
2. What evidence is there that the focus of the feedback given to students following their pitch practice affected their engagement and enthusiasm during the challenge?
 - a. How do students describe the pitch practice?
 - b. What evidence is there that students valued their pitch practice feedback?
 - c. *To Test: Observations and focus group. Observations during day 4 or 5. Focus group on day 5.*

Conjecture 4: the task of building a working algorithm that automatically produces rankings or ratings, combined with the affordances of spreadsheets, will support students to demonstrate a functional understanding of algebraic expressions.

Questions to explore:

1. How do students describe their algorithms, and how do their descriptions relate to functional thinking in algebra?
 - a. What do students think an algorithm is?
 - b. What evidence is there of students describing the characteristics they used in their algorithms (e.g. cleanliness, speed, etc. in restaurants) in terms consistent with inputs, their final ratings or rankings in terms consistent with outputs, and the algorithm in ways consistent with the notion of a function rule as generalizing the process of mapping inputs onto outputs? *[not the exact terminology, but in ways that reflect a notion of an input and an output]*

- c. What evidence is there of students describing their algorithms in terms consistent with an understanding of algebraic expressions as generalizing a relationship between two sets of quantities? *[Look for whether students are describing their algorithms as representing a relationship between quantities in the table]*
 - d. ***To Test: Listening to students and probing their thinking during launch and work time, and post-session interviews with students***
 - i. ***Questions: explain the meaning of algorithms, describe their team's algorithms, explain how their algorithms work, and describe the relationship between the characteristics and the overall rating/ranking.***
 - ii. ***Interviews should happen at end of day 1, to see how students conceptualize algorithms, and after day 4, to explore how students describe their algorithms.***
2. How do the affordances of spreadsheets and the requirement that students' algorithms automatically produce a ranking or rating from given data support a functional understanding of variable?
 - a. What evidence is there of students describing the characteristics they used as inputs in their algorithms in terms of the columns in their spreadsheets?
 - b. What evidence is there of students describing the weights they used in their algorithms in terms consistent with parameters or coefficients in a function rule?
 - c. What evidence is there of students describing their algorithms using symbolic representations?
 - d. ***To Test: Listening to students and probing their thinking during launch and work time, and post-session interviews.***
 - i. ***Questions: How does your algorithm calculate a rating/ranking? How does the spreadsheet algorithm work? How did you program it to give ratings/rankings for [insert thing they're rating/ranking]? How does your algorithm use weights in calculating a rating/ranking? Where can I see the weights in your spreadsheet formula? Where can I see the characteristics (use student language) in your spreadsheet formula? What is the relationship between the weights and the overall rating/ranking? How would you explain your algorithm to a friend so that they could test it in their spreadsheet? Is there another way to write the same algorithm or is this the only way?***
 - ii. ***Interviews should happen at end of day 1 and end of day 4.***
 3. How does the algorithm context support students' interpretations of the structure of algebraic expressions?
 - a. What evidence is there of students referring to the context or their selected category in describing the pieces of their spreadsheet formula?
 - b. What evidence is there of students leveraging their understanding of their algorithm to interpret a new algebraic expression?

- c. *To Test: Listening to students and probing their thinking during launch and work time, and post-session task-based interview*
- i. *Probing questions: Why did you select [insert category] to rate/rank? How did you decide on the variables to use in your algorithm? How does your spreadsheet calculate a rating/ranking?*
 - ii. *Task-based interview on day 5: give students an algorithm represented algebraically and ask them to 1) explain how it works, 2) identify the weights and variables, and 3) state which characteristic has the biggest impact on the overall rating/ranking. Ask students what would happen to the overall rating/ranking if the weights were moved around in the expression. Ask students to change the expression so that one characteristic (or variable) is 3 times as important as another.*

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Appendix L

Daily Team Interview Protocols

The *Building Algorithms* Challenge Daily Team Interview Protocols

Day 1

1. Can you describe, in your own words, the challenge you are working on today?
2. Can you describe your team's algorithm? How does it work? What variables are you thinking of using in your algorithm?
3. What is an algorithm?
4. Do you think this a realistic challenge? Why? What makes it feel realistic?
5. What is an entrepreneur?
6. Do you think entrepreneurship is important?
7. What are some of the challenges you are having in building your algorithm?
8. How did the challenge videos affect your thinking about the challenge?

Day 2

1. Can you tell me about your algorithm? What does it do? How does it do it?
2. How did the Key Business Proposition influence your algorithm and your company? What do you think the importance is of a key business proposition?
3. Who are your customers?
4. What is your business model?
5. How did seeing the Inlucvie website influence how you think about the challenge? Did it make it feel like this activity was more realistic, less realistic, or did it not affect it? Why?
6. Did seeing the Inlucvie website influence how you are thinking about your company? Do you think Inlucvie will be able to make money? Why?

Day 3

1. How would you explain your algorithm to a friend?
2. What variables are you using in your algorithm? How did you choose those variables? What is the relationship between the variables and the rating?
3. How did the spreadsheet resource influence your thinking about your algorithm?
4. How did the Technical Brief influence your thinking?
5. Did you include weighted categories in your algorithm? How?
6. Do you think your algorithm could work as a real business?
7. How are you feeling about the pitch competition on Friday? Why?

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Appendix M

Design & Pitch Challenges in STEM Focus Group Protocol

Student IDs: S2_____ and S2_____

Thank you for agreeing to participate in this study and to answer a few questions about your participation in the Design & Pitch challenge. This focus group is part of a research study at North Carolina State University. The answers you provide will help improve how we design and run the challenges. Your answers will not be shared with your teacher and all feedback will remain anonymous.

I am a part of the team that wrote the challenges, but your statements will be anonymous, and the team is committed to improvement. We welcome any and all feedback.

If at any time you want to stop participating in the focus group, just let me know and we can stop.

Do you have any questions before we begin?

Questions about the Engagement during the Challenge:

1. What did you think of the challenge you participated in? What did you like about it? What didn't you like about it?
2. What about the challenge context (algorithms for ratings and rankings) did you like? What about it did you not like?
3. How was the challenge similar to a typical math class? How was it different?
4. What, if anything would you change about your algorithm? Why?
5. How did each person in your team contribute during the challenge?
6. How did the competition aspect of the challenge affect your experience?
7. How did the expert judges affect your experience with the challenge?

Questions about students' STEM career interest:

1. How interested are you in someday working in a STEM career? Why?
2. How likely are you to someday use STEM in your career? Why?
3. How did the challenge affect your interest in a STEM-related career?

Questions about students' perceptions of their math learning:

1. How did you use math in developing your solution?
2. What role, if any, did the technical brief have in developing your solution?
3. During the challenge, what did you do when you came across a concept that you didn't understand? What did you do to learn that concept?
4. How did you work with your partners to learn about the supporting resources, like the spreadsheet and the sample algorithms?

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Appendix N

The *Building Algorithms* Challenge Task-Based Interview

A company uses an algorithm to compare soccer players. Each player is assigned a score in three different skills: speed, dribbling, and agility. These scores are then combined to calculate an overall score for the player.

The following algebraic expression represents the algorithm the company uses to combine the ratings, where x = speed score, y = dribbling score, and z = agility score.

$$\text{Overall Score} = 1.5x + 4y + 2z$$

1. Describe how the company calculates a soccer player's overall score.

2. What does the "1.5" represent in the algorithm?

3. Which skill does the company think is most important for a soccer player to have? How can you tell from the algorithm?

4. Find the overall score of a soccer player who received the following:

$$\begin{aligned} \text{Speed Score} &= 6 \\ \text{Dribbling Score} &= 10 \\ \text{Agility Score} &= 5 \end{aligned}$$

5. How would the player's overall score change if the company used the following algorithm instead, where x = speed score, y = dribbling score, and z = agility score?

$$\text{Overall Score} = 1.5x + 4y + 2z$$

6. How could you change the algorithm so that a player's agility score is four times as important as her dribbling score?
7. The algorithm shown in the spreadsheet below was created by a team of students to rate restaurants.

Restaurant Algorithm:

	A	B	C	D	E
1	Restaurant	Cleanliness	Speed	Menu	Overall Score
2	McDonald's	4	8	7	$=3*B2+0.5*C2+5*D2$
3	Panera	4	5	9	$=3*B3+0.5*C3+5*D3$
4	Wendy's	6	9	5	$=3*B4+0.5*C4+5*D4$

- Describe how the algorithm calculates a restaurant's overall score.
- What is the overall score for Panera?
- Write the algorithm as an algebraic expression.

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