ABSTRACT

GROELKE, BENJAMIN J. Inner-Loop Command Governors for Adaptive Cruise Control in Heavy-Duty Trucks. (Under the direction of Dr. Christopher Vermillion.)

In automobiles, cruise control is no longer a convenience feature aimed solely at improving driver comfort. Advances in Vehicle-to-Vehicle (V2V) and Infrastructure-to-Vehicle (I2V) communication technologies have pushed forward new predictive cruise control strategies that leverage knowledge of the driving scenario (such as route terrain and traffic) to improve fuel economy and safety. Predictive cruise control strategies have the potential to be especially beneficial for heavy-duty trucks, which consume large amounts of fuel and are capable of inflicting significant damage to surrounding vehicles if operated incorrectly. The research contained within this dissertation aims to fuse fuel efficiency with safe vehicle following for heavy-duty vehicles.

The research presented in this dissertation develops a longitudinal vehicle control framework where various upper-level driving controllers can be cascaded with a command governor (CG) to enforce vehicle-following constraints. Command governors are predictive control schemes used solely to enforce constraints and are computationally inexpensive in comparison to other predictive control techniques, such as model predictive control (MPC). This allows the upper-level controllers to focus on other objectives like maintaining a certain velocity or minimizing fuel consumption through optimal control techniques. This controller design is especially beneficial for heavy-duty trucks, which require large horizon lengths to engage in ecological driving, making it computationally burdensome to execute at a time step fast enough to effectively enforce vehicle-following constraints. The work presented in this dissertation developed various CG-based vehicle-following strategies and tests their efficacy in highway driving simulations using real-world trucking routes and a proprietary heavy-duty truck model provided by Volvo Group North America. A CG-based ecological and adaptive cruise controller (Eco-ACC) is developed by cascading an MPC-based ecological cruise controller (ECC) with a CG, and simulation results indicate up to a 6% reduction in fuel consumption in comparison to a Gipps’ car-following model. A CG-based ACC is designed that ensures rear-end collision avoidance through the use of various lead vehicle prediction techniques, while improving drivability and ecological performance when compared to a traditional PID-based ACC. Finally, the inner-loop CG is formulated such that recursive feasibility is ensured through the utilization of a controlled invariant set, addressing an open challenge in the CG literature.
DEDICATION

To my parents, whose love gave me everything.
BIOGRAPHY

After receiving his B.S. degree in physics at Appalachian State University in 2016, Ben pursued a M.S. degree in mechanical engineering at the University of North Carolina at Charlotte. During his time at the University of North Carolina at Charlotte, Ben joined the Control and Optimization for Renewables and Energy Efficiency (CORE) Lab under Dr. Chris Vermillion and worked on an ARPA-E NEXTCAR project until the summer of 2020. Specifically, Ben worked on the NEXTCAR project titled, "Maximizing Vehicle Fuel Economy through the Real-Time, Collaborative, and Predictive Co-Optimization of Routing, Speed, and Powertrain Control", as a part of a collaborative effort working toward using predictive optimal control to reduce fuel consumption in heavy-duty trucks by 20%. After receiving his M.S. degree from the University of North Carolina at Charlotte in 2018, Ben pursued his Ph.D. in mechanical engineering at North Carolina State University under Dr. Chris Vermillion. Ben’s research interests include command governors, online optimal control, and advanced driver-assistance systems.
ACKNOWLEDGEMENTS

I am forever grateful for my advisor, Dr. Chris Vermillion, whose passion for the success of my research became the catalyst for my development as a researcher. I have never met someone who is so dedicated to the development of their students. His dedication was evident in our weekly technical meetings, in which he often vetted new results or ideas I presented through impromptu quizzes and insisting that I give my answers in a lecture-style format. It is now apparent that his vetting process directly contributed to improving my ability to articulate and present technical ideas. My growth as a student researcher has progressed significantly over the course of my Ph.D., and this growth was largely made possible by the significant amounts of time and energy he spent advising me. I am also thankful for my committee members, Dr. Scott Ferguson, Dr. Gregory Buckner, and Dr. Mo-Yuen Chow, for their expertise and guidance throughout the preparation of my dissertation.

I appreciate all of my lab mates, both current (Mitchell, Joe, Christian, John, Blake, Ayaz, Ben #1 (I am Ben #2 for various reasons), James, Josh, and Kartik) and former (Nihar, Alireza, Shamir, and Ali) for their advice and assistance throughout my Ph.D. It has been a privilege to participate in our weekly meetings where one of us presents their latest work to the lab (sometimes referred to as a "roast"), and I am certain my presentation skills are better for it. I especially would like to thank John and Christian, who I have worked with closely on an ARPA-E NEXTCAR project for the last three years. I often benefited from their assistance and support throughout the course of this project.

Finally, I would like to thank my family and friends. My parents are a constant source of inspiration for me; I am amazed by their work ethic and their professional accomplishments. The support and encouragement they have given me throughout my education has been invaluable. My fiancée, Kristin, has been tremendously supportive of me, even though completing the requirements for my Ph.D. often meant spending many nights in. Luckily, she was able to attend the lab's Festivus party with me for three consecutive years (though, unfortunately, she never won the Feats of Strength, despite her rigorous training efforts). I would like to thank my friend, Jacob Fragnito, for his advice that I have leaned on throughout my Ph.D. Even though our relationship started off rocky (i.e., he was my bully in 7th grade), our passion for bricking three-pointers in cul-de-sac pickup basketball games led to the start of our friendship (i.e., I finally became taller than him) that has since blossomed. Finally, I would like to thank my dog, Lucy, for consistently taking me on walks throughout the duration of the stay-at-home order over the last few months. These walks allowed me to stretch my legs, which enabled me to spend the rest of my time at my desk to complete this dissertation and the work contained within.
# TABLE OF CONTENTS

LIST OF TABLES ................................................................. vii

LIST OF FIGURES ............................................................... viii

Chapter 1  Introduction ......................................................... 1
  1.1 Literature Survey and Key Gaps ............................................. 2
  1.1.1 Ecological Driving .................................................... 3
  1.1.2 Vehicle Following .................................................... 4
  1.1.3 Combined Ecological and Adaptive Cruise Control ................. 5
  1.1.4 Key Gaps .................................................................. 6
  1.2 Proposed Control Strategy and Key Challenges ......................... 7
  1.2.1 Key Challenges ....................................................... 9
  1.3 Dissertation Contributions ............................................... 9
  1.4 Dissertation Outline .................................................... 10

Chapter 2  Modeling and Performance Evaluation of Heavy-Duty Truck Driving .... 11
  2.1 Controller-Oriented Models ............................................... 11
  2.1.1 Nonlinear Model: Controlling Wheel Force ......................... 12
  2.1.2 Linear Model: Controlling Acceleration .......................... 13
  2.2 Performance Evaluation of Heavy-duty Trucks .......................... 13
  2.2.1 Stochastic Lead Vehicle Model ....................................... 13
  2.2.2 Gipps’ Car-Following Model ......................................... 14
  2.2.3 Ecological Performance ............................................. 15
  2.2.4 Drivability .......................................................... 15

Chapter 3  A Multi-Rate Eco-ACC for Heavy-Duty Trucks ....................... 17
  3.1 Overview of Proposed Eco-ACC .......................................... 17
  3.2 Controller-Oriented Models used by Upper-Level Controller ............ 19
  3.2.1 Energy-Based Vehicle Model ....................................... 19
  3.2.2 Fuel Consumption Model ............................................ 20
  3.3 MPC-based ECC Formulation .......................................... 20
  3.3.1 Initial Offline Optimization ......................................... 20
  3.3.2 MPC Formulation .................................................... 21
  3.4 Inner-Loop CG Formulation .............................................. 23
  3.5 Simulation Study of Eco-ACC ........................................... 24
  3.5.1 Simulation Setup .................................................... 24
  3.5.2 Eco-ACC Performance ............................................. 25
  3.5.3 Vehicle-Following Behavior of Eco-ACC ........................... 26

Chapter 4  A CG-Based Chance-Constrained ACC with Collision Avoidance ....... 30
  4.1 Chance-Constrained CG-based ACC Overview .......................... 31
  4.2 Characterizing Lead Vehicle Behavior ................................... 31
  4.2.1 Generating Synthetic Data for Simulations ......................... 32
  4.2.2 Deriving Probabilistic Bounds on Lead Vehicle Velocity ............ 33
  4.3 Chance-Constrained Inner-Loop CG ..................................... 36
  4.3.1 Ensuring Robust Vehicle Following Through Constraint Set Tightening .. 36
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.3.2</td>
<td>CG-Based Vehicle Following Strategy</td>
<td>37</td>
</tr>
<tr>
<td>4.3.3</td>
<td>Addressing Feasibility Issues with Chance Constraints</td>
<td>39</td>
</tr>
<tr>
<td>4.4</td>
<td>Simulation Study of CG-Based ACC with CA</td>
<td>40</td>
</tr>
<tr>
<td>4.4.1</td>
<td>Simulation Setup</td>
<td>40</td>
</tr>
<tr>
<td>4.4.2</td>
<td>Normal Driving Scenario</td>
<td>41</td>
</tr>
<tr>
<td>4.4.3</td>
<td>Emergency Stopping Scenario</td>
<td>46</td>
</tr>
<tr>
<td>Chapter 5</td>
<td>A Provably Recursively Feasible Command Governor for Vehicle Following</td>
<td>49</td>
</tr>
<tr>
<td>5.1</td>
<td>Proposed Rendezvous-based Inner-Loop CG for ACC</td>
<td>50</td>
</tr>
<tr>
<td>5.1.1</td>
<td>Background</td>
<td>50</td>
</tr>
<tr>
<td>5.1.2</td>
<td>Notation and Definitions</td>
<td>50</td>
</tr>
<tr>
<td>5.2</td>
<td>Rendezvous Control Formulation</td>
<td>51</td>
</tr>
<tr>
<td>5.2.1</td>
<td>Rendezvous Controller</td>
<td>51</td>
</tr>
<tr>
<td>5.2.2</td>
<td>Robust Controlled Invariant Set</td>
<td>53</td>
</tr>
<tr>
<td>5.3</td>
<td>Alternative Rendezvous Strategy via an Inner-Loop CG</td>
<td>58</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Inner-Loop CG for Rendezvous</td>
<td>58</td>
</tr>
<tr>
<td>5.3.2</td>
<td>Equivalence Between CG and Rendezvous Controller</td>
<td>59</td>
</tr>
<tr>
<td>5.4</td>
<td>Simulation Study of Rendezvous- and CG-based ACC</td>
<td>60</td>
</tr>
<tr>
<td>5.4.1</td>
<td>Simulation Setup</td>
<td>60</td>
</tr>
<tr>
<td>5.4.2</td>
<td>Performance</td>
<td>62</td>
</tr>
<tr>
<td>5.4.3</td>
<td>Vehicle Following Behavior</td>
<td>63</td>
</tr>
<tr>
<td>Chapter 6</td>
<td>Conclusions and Future Work</td>
<td>65</td>
</tr>
<tr>
<td>6.1</td>
<td>Summary of Contributions</td>
<td>65</td>
</tr>
<tr>
<td>6.2</td>
<td>Future Work</td>
<td>66</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>67</td>
<td></td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table 1.1  Overview of cruise control strategies referenced within the literature survey presented in this dissertation. .......................................................... 3

Table 2.1  Parameters for the Gipps’ car-following model used in this dissertation. .... 14

Table 5.1  Results are reported as a percent improvement from the baseline. A positive percent improvement for trip time, energy expended, and $J^2_{\text{ego}}$, corresponds to the ACC strategy having a faster trip time, expending less energy, and better drivability than the baseline, and vice versa. .......................... 62
**LIST OF FIGURES**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>In the vehicle-following scenario, the following vehicle is referred to as the &quot;ego&quot; vehicle and the preceding vehicle is referred to as the &quot;lead&quot; vehicle.</td>
</tr>
<tr>
<td>1.2</td>
<td>Block diagram of a traditional command governor.</td>
</tr>
<tr>
<td>1.3</td>
<td>Overall block diagram of the proposed ACC architecture. The term $x_{\text{lead}}$ is the state of the lead vehicle and $x_{\text{ego}}$ is the current state of the ego vehicle.</td>
</tr>
<tr>
<td>3.1</td>
<td>Conceptual diagram of the cascaded MPC and CG-based Eco-ACC strategy.</td>
</tr>
<tr>
<td>3.2</td>
<td>Block diagram for the overall Eco-ACC strategy.</td>
</tr>
<tr>
<td>3.3</td>
<td>Schematic of the inner-loop CG policy highlighting the receding horizon approach of set $O_T$ presented in this chapter.</td>
</tr>
<tr>
<td>3.4</td>
<td>Eco-ACC results for a fully-loaded trailer on HWY-1, HWY-2, and HWY-3 are indicated by &quot;◦&quot;, &quot;+&quot;, and &quot;◊&quot;, respectively.</td>
</tr>
<tr>
<td>3.5</td>
<td>Eco-ACC results for a half-loaded trailer on HWY-1, HWY-2, and HWY-3 are indicated by &quot;◦&quot;, &quot;+&quot;, and &quot;◊&quot;, respectively.</td>
</tr>
<tr>
<td>3.6</td>
<td>The vehicle-following scenario is represented by velocity, following distance, and wheel force profiles over a stretch of the 240 km route, HWY-3. Figures 3.6a and 3.6c show the velocity and following distance for each simulation. Additionally, Figure 3.6a shows the desired velocity prescribed by the offline DP solution and the route elevation. The cumulative braking energy expended, in Figure 3.6b, as well as vehicle jerk, in Figure 3.6d, are used to evaluate the ecological performance and drivability, respectively.</td>
</tr>
<tr>
<td>4.1</td>
<td>A PID-based cruise controller that tracks a desired velocity, $v_{\text{des}}$, is cascaded with a command governor that enforces vehicle-following constraints.</td>
</tr>
<tr>
<td>4.2</td>
<td>Velocity distribution of highway data acquired from [68]. The normal fit of the velocity data serves as the desired velocity distribution of the synthetic velocity data.</td>
</tr>
<tr>
<td>4.3</td>
<td>The FFT-based PSD of the synthetic and actual velocity data of the lead vehicle.</td>
</tr>
<tr>
<td>4.4</td>
<td>Probability densities for future velocity values for originating velocities of (a) 26 m s$^{-1}$, (b) 29.2 m s$^{-1}$, and (c) 32 m s$^{-1}$. Probability density values between 0 and 0.08 were shaded dark blue.</td>
</tr>
<tr>
<td>4.5</td>
<td>Velocity percentile bounds for $P = 50%$, $P = 25%$, $P = 12.5%$, and $P = 2.5%$.</td>
</tr>
<tr>
<td>4.6</td>
<td>Fitted and actual percentile velocity bounds for a horizon length of (a) 20 s and (b) 6 s. Both Figure 4.6a and Figure 4.6b contain fitted and actual percentile velocity bounds for a $v_{\text{origin}}$ of 26 m s$^{-1}$, 29.2 m s$^{-1}$, and 32 m s$^{-1}$.</td>
</tr>
<tr>
<td>4.7</td>
<td>An illustration of the two minimum following distance constraints that the CG enforces.</td>
</tr>
<tr>
<td>4.8</td>
<td>An illustration of the normal driving mode and the emergency stopping mode utilized by the CG.</td>
</tr>
<tr>
<td>4.9</td>
<td>A 400 s segment of the normal driving simulations for the highway scenario.</td>
</tr>
<tr>
<td>4.10</td>
<td>Fuel savings realized by the CG-ACC strategy when compared to (a) the Gipps’ car-following model and (b) the PID-ACC.</td>
</tr>
<tr>
<td>4.11</td>
<td>Percent reduction of total energy expended realized by the CG-ACC strategy when compared to (a) the Gipps’ car-following model and (b) the PID-ACC.</td>
</tr>
</tbody>
</table>
Figure 4.12  Percent reduction of total energy expended by the CG-ACC strategy due to the reduction of aerodynamic drag. Results compared to (a) the Gipps’ car-following model and (b) the PID-ACC. ................................. 45

Figure 4.13  Percent reduction of total energy expended by the CG-ACC strategy due to the reduction of braking. Results compared to (a) the Gipps’ car-following model and (b) the PID-ACC. ................................. 45

Figure 4.14  Percent reduction of $j_{ego}^2$ by the CG-ACC strategy when compared to (a) the Gipps’ car-following model and (b) the PID-ACC. Note that negative values correspond to a worsening of drivability by the CG-ACC. ................................. 46

Figure 4.15  Emergency stopping scenario of when a lead vehicle suddenly decelerates at $-4 \text{ m s}^{-2}$. Collision is avoided for the CG-ACC strategy. ................................. 47

Figure 4.16  Pareto front for PID-ACC simulations for various $t_b$ values. All CG-ACC simulations outperform the PID-ACC Pareto front of the PID-ACC simulations. 48

Figure 5.1  A block diagram of the proposed ACC, where the upper-level controller is a conventional cruise controller and the lower-level controller saturates the upper-level controller to ensure vehicle-following constraints. ................. 51

Figure 5.2  A schematic of the rendezvous controller as the lower-level controller. .......................... 53

Figure 5.3  The unbounded robust controlled invariant set produced by the rendezvous control law for specific values of $w_{r,\text{min}}, \xi_{2,\text{lim}}$, and $T_r$. The vector field shows the dynamics of (5.7) when $w_r = w_{r,\text{min}}$. ................................. 54

Figure 5.4  A schematic of the CG as the lower-level controller. ............................................. 58

Figure 5.5  The distribution of the lead vehicle acceleration for the entire route. .......................... 61

Figure 5.6  The robust controlled invariant set, $S_{CG}$, for various values of $T_{CG}$ used in simulation. .......................................................... 61

Figure 5.7  An aggressive lead vehicle cut-in scenario where the lead vehicle enters the lane and immediately begins decelerating to a stop. ................................. 64
There are significant environmental and safety concerns surrounding the use of heavy-duty trucks. Heavy-duty trucks are responsible for 23% of transportation-related CO₂ emissions in the U.S. (see [1]), while current projections forecast freight tonnage increasing 1.4% per year for the next few decades (see [2]). The need to reduce greenhouse gas emissions has motivated several industry- and government-lead efforts to increase the fuel economy of heavy-duty trucks (see [3]). Improving the fuel economy of heavy-duty trucks is potentially a lucrative endeavor as well, as the cost of fuel comprises 29% of the life-cycle cost of a heavy-duty truck (see [4]). Safety concerns are highlighted by the severe damage associated with on-road vehicle accidents involving heavy-duty trucks. Almost 46% of fatal rear-end collisions where the striking vehicle was a heavy-duty truck involved three or more vehicles (see [5]).

In order for heavy-duty trucks to achieve better fuel economy while meeting the increasing demand for freight transport, predictive vehicle control technologies offer a promising solution. Predictive vehicle control technologies can utilize upcoming road terrain information to optimize the vehicle speed trajectory to improve fuel economy (see [6]), a driving style referred to as ecological driving. Advances in Vehicle-to-Vehicle (V2V) and Infrastructure-to-Vehicle (I2V) communication technologies, in conjunction with velocity prediction techniques, make it likely that information regarding surrounding vehicles' driving behavior will be available to automated vehicles in the near future (see [7] and [8]). Predictive vehicle control technologies that engage in automatic vehicle following can utilize knowledge of the surrounding vehicles' braking behavior to ensure safety guarantees [9–11]. While fuel economy and safety considerations are both well-known for heavy-duty vehicles, the fusion of both considerations into a single control framework represents a formidable challenge, due to a significant disparity between the timescales associated with ecological driving.
In the vehicle-following scenario, the following vehicle is referred to as the "ego" vehicle and the preceding vehicle is referred to as the "lead" vehicle.

(see [12]) and safe vehicle following (see [13]). The overarching objective of the research explored in this dissertation is to develop predictive vehicle control strategies that fuse fuel efficiency with safety guarantees for heavy-duty vehicles.

1.1 Literature Survey and Key Gaps

A substantial body of literature has worked towards the development of vehicle control strategies that engage in either ecological driving, safe vehicle following, or a combination thereof. All three of the aforementioned control strategies are different forms of cruise control (CC), which is an advanced driver assistance system (ADAS) that relieves the driver from needing to adjust the throttle and/or brake to achieve some desired longitudinal vehicle motion. A cruise controller designed to engage in ecological driving is referred to as an ecological cruise controller (ECC). In order to optimally traverse terrain, ECCs infer road topology through the utilization of road elevation data along the route or via onboard measurement systems. Adaptive cruise control (ACC) is a type of CC that engages in automatic vehicle following. ACC-equipped vehicles employ onboard radar/lidar systems to measure the relative distance behind the preceding vehicle. In vehicle-following scenarios, the ego vehicle is the following vehicle that is being controlled, and the lead vehicle is the preceding vehicle, as shown in Figure 1.1. ACC systems are often designed to operate in non-cooperative vehicle-following scenarios, i.e., scenarios where the future velocity profile of the lead vehicle is unknown to the ego vehicle. An ACC that engages in ecological driving is referred to as an ecological and adaptive cruise controller (Eco-ACC).

The aforementioned cruise control strategies (i.e., ECC, ACC, and Eco-ACC) can be divided into two approaches: heuristic methods or optimization-based methods. Originally, cruise controllers were designed using heuristic methods that utilized static feedback control laws and various controller modes to achieve some desired longitudinal vehicle motion (see [14]). While initially introduced as a driver convenience mechanism (focused primarily on improving driver comfort), cruise control technologies have been re-tailored toward the goals of fuel economy and/or safe vehicle following. Optimization-based methods were introduced as the desired performance and safety of cruise control technologies increased and electronic control units (ECU) became more powerful. Optimization-based methods rely on explicit models of the vehicle dynamics to compute and then prescribe some control sequence that minimizes a specified objective function. Table 1.1 presents an overview of the cruise control strategies referenced within this literature survey, based
Table 1.1 Overview of cruise control strategies referenced within the literature survey presented in this dissertation.

<table>
<thead>
<tr>
<th>Heuristic</th>
<th>Ecological Driving</th>
<th>Vehicle Following</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimization Based</td>
<td>[14], [25], [26], [27], [28], [29], [32], [33], [34]</td>
<td>[35], [36], [40]</td>
<td>[42], [43]</td>
</tr>
</tbody>
</table>

1.1.1 Ecological Driving

The study of vehicle control strategies to reduce fuel consumption has been the focus of a wide array of literature. Optimization-based ecological driving was first explored in [15], where an optimal control algorithm is formulated to find an explicit solution for the optimal vehicle velocity over varying terrain to minimize fuel consumption. From the results in [15], ecological driving is characterized as a driving style that increases vehicle speed while traveling downhill and decreases vehicle speed while traveling uphill, thereby reducing the energy dissipated via braking to maintain the desired velocity when traveling downhill. However, nonlinearities in vehicle dynamics make explicit, continuous optimal control trajectories difficult to solve analytically or numerically (see [16]). Consequently, ECC strategies were initially designed using heuristic methods that modified a conventional proportional-integral-derivative (PID) cruise controller to allow the vehicle speed to decrease or increase when traveling uphill or downhill, respectively (see [14] and the references within). Most efforts to optimize control trajectories for fuel economy have focused on finite-dimensional optimization, relying on a discrete-time representation of the system dynamics and, in some cases, a quantized grid of admissible state variables. In the case of offline optimization, the discrete-time tool of choice is often dynamic programming (DP), whereas the tool of choice for most online implementations is model predictive control (MPC).

MPC has been widely used in online ECC strategies (see [17], [18]), due to MPC’s inherent ability to minimize a performance objective (such as fuel consumption) and enforce constraints (e.g., speed limit restrictions). Implementations of MPC where the performance objective is some measure of profitability (such as fuel consumed) are referred to as economic MPC. MPC repeatedly optimizes the control sequence over a receding horizon such that the performance objective is minimized, then implements the first element of the optimal control sequence at each time step. In [19], a simulation study concluded that MPC-based ECCs with longer horizons tend to save more fuel. MPC-based ECCs designed for passenger vehicles benefit substantially through horizons of hundreds of meters or less. On the other hand, due to their inertial properties, MPC-based ECC strategies for heavy-duty trucks require much longer horizons (see [20]). In [12], highway simulations revealed that heavy-duty trucks only begin to see near-full benefits of ECC under horizon lengths greater than 2000 m. Consequently, the most effective MPC-based ECCs designed for heavy-duty trucks utilize horizon
lengths of 2000 m or more (see [21] and [22]).

1.1.2 Vehicle Following

Passenger vehicles equipped with adaptive cruise control became commercially available worldwide in the year 2000 (see [23] and [24]). Shortly thereafter, ACC-equipped heavy-duty trucks were developed and tested in on-road environments (see [25–27]). Typical ACCs heuristically enforce a desired minimum following distance by saturating the output of a conventional cruise controller with the output of a proportional-derivative (PD) distance-keeping controller (see [28, 29]), but do not explicitly enforce vehicle-following constraints or ensure rear-end collision avoidance (see [30]).

To address this issue, collision avoidance (CA) systems have been used with ACC to ensure rear-end collision avoidance (see [31]). Typically, ACC with CA systems utilize different controller modes that are selected based on movements between the ego and lead vehicle (see [32–34]). In an ACC with CA system, there is typically a controller mode designed to ensure collision avoidance that is entered whenever the CA system observes the lead vehicle applying a sudden large braking effort or the headway has fallen below some minimum threshold. The work in [33] maps a warning index and the time-to-collision to a desired acceleration of the ego vehicle during an emergency stopping scenario. However, the map is obtained through a statistical classification, and safe vehicle following is not explicitly enforced. The work in [34] derives a safe ego vehicle acceleration as a function of time-to-collision to ensure collision avoidance. However, [34] considers the stopping profile of the lead vehicle to be known and assumes the stopping capabilities of the lead vehicle to match those of the ego vehicle. This is not applicable in the highway scenario with traffic, where the ego vehicle is a heavy-duty truck and the lead vehicle is a light-duty passenger vehicle.

ACC is an appealing application for MPC, as vehicle-following constraints can be explicitly enforced within the online optimization performed within the MPC. Recently, MPC-based ACC formulations have been proposed in [9–11] that robustly ensure safe vehicle following by designing the MPC to ensure recursive feasibility. Recursive feasibility is a property that ensures that constraints remain feasible from one time step to the next. In these formulations, recursive feasibility is achieved by assuming a known worst-case behavior of the lead vehicle over the prediction horizon of the MPC. However, to ensure real-time feasibility, the horizon lengths of MPC-based ACCs are required to be much shorter than the horizon length necessary to realize full ecological benefits. It is highly desirable to be able to look ahead and plan over a longer horizon, while simultaneously maintaining a small enough time step to react to sudden changes in lead vehicle behavior. Additionally, in the presence of a lead vehicle with some known worst-case behavior, it is very unlikely that the lead vehicle will engage in this worst-case behavior. In the non-cooperative vehicle-following scenario, deviations in the realized lead vehicle velocity from the expected lead vehicle velocity are often modeled as a stochastic process (see [35]). As a consequence, an MPC-based ACC that guarantees safe vehicle following by assuming the worst-case behavior of the lead vehicle will result in excessive conservatism (see [36]), thereby limiting the performance. As a result, instead of robustly satisfying constraints, the work in [36] explores the design of an MPC-based ACC that enforces the minimum
following distance as a chance constraint in the presence of a lead vehicle that is modeled as a stochastically varying disturbance, ensuring a prescribed probability of satisfying constraints. However, the MPC-based ACC in [36] does not ensure collision avoidance in the unlikely event that the lead vehicle does engage in the worst-case behavior.

One way to ensure robust safe vehicle following is to design a controller that results in a controlled invariant set within the state-space domain, where the states are the relative position and velocity between the ego and lead vehicle. Specifically, the works in [37–41] are unified through their use of controlled invariant sets to guarantee constraint satisfaction. In [37], a controlled invariant set is formulated by calculating a disturbance invariant Lyapunov level set for a user-defined Lyapunov function, where the desired following distance corresponds to the origin of the controlled invariant set. However, because the controlled invariant set is closed and compact, there is also an upper bound placed on the vehicle-following distance. In [38] and [39], a correct-by-construction approach is used for controller synthesis of an ACC system that utilizes an approximation of a controlled invariant set that is calculated offline using a simplified version of the actual vehicle model. However, the calculation of this approximated controlled invariant set is computationally expensive, and the computational cost is exponentially dependent on the size of the discrete grid. The work in [40] also utilizes an offline calculation to determine an approximation of a robust control invariant set, which can be computationally expensive. Additionally, in [40], the proposed ACC ensures constraint satisfaction by using MPC to prescribe a control signal that ensures that the vehicle remains within the controlled invariant set. In [41], the authors proposed using a barrier function to form a controlled invariant set. However, designing a valid barrier function is almost always not straightforward.

1.1.3 Combined Ecological and Adaptive Cruise Control

Ecological and adaptive cruise control has been widely studied in the literature. In [42] and [43], ACCs that heuristically enforce constraints improved fuel economy by exhibiting smooth acceleration response to lead vehicle cut-ins. However, ACCs using heuristic methods are not designed to engage in ecological driving. A simulation study in [44] concluded that MPC-based Eco-ACCs realized more fuel savings than ACCs designed using heuristic methods. For light-duty passenger vehicles, a large body of literature has studied MPC-based Eco-ACC. Furthermore, multiple papers have incorporated following distance considerations and penalties within MPC formulations, thereby achieving ecological vehicle following. The authors of [45] designed an Eco-ACC such that the MPC objective function approximated fuel consumption, while adding penalty terms to promote driver comfort and safety. Simulation results from [45] indicated that Eco-ACC improved fuel economy in comparison with typical ACC. In [46] and [47], the MPC objective function was designed to improve ecological driving by penalizing non-smooth acceleration profiles. Additionally, in [46] and [47], hard constraints on acceptable bounds for various desired vehicle following qualities (such as vehicle following distance, maximum jerk, and relative velocity) were incorporated and softened when necessary to ensure feasibility.

The aforementioned Eco-ACCs were designed for passenger vehicles, which benefit substantially
through MPC-based Eco-ACC strategies with horizons of hundreds of meters or less. Implementing MPC-based Eco-ACCs on heavy-duty trucks has recently been explored in [48] and [49]. Typically, MPC-based Eco-ACCs explicitly enforce vehicle-following constraints within the MPC, which is implemented at a time step of 100-200 ms to account sudden changes in the behavior of the lead vehicle [13]. Due to the limited computational resources on a typical ECU, an MPC-based Eco-ACC that explicitly enforces vehicle-following constraints implemented on a heavy-duty truck would require a shorter horizon (therefore sacrificing ecological performance, as highlighted in [20] and [12]) to ensure that the MPC is real-time feasible (see [50]). To avoid the above tradeoff between ecological performance and real-time feasibility, MPC-based Eco-ACCs in [48] and [49] implemented a PD distance-keeping controller that heuristically maintained safe vehicle following, in parallel with an MPC-based ECC designed for a heavy-duty truck. While solving all issues relating to computational complexity, the PD distance-keeping controller does not provide any guarantees of constraint satisfaction. Indeed, to the best of the author's knowledge, an MPC-based Eco-ACC designed for a heavy-duty truck that explicitly enforces vehicle-following constraints had yet to be investigated in the literature prior to the work presented in this dissertation.

1.1.4 Key Gaps

Gaps in the literature that the work in this dissertation aims to address are as follows:

- The design and evaluation of Eco-ACCs is well documented in the literature for light-duty vehicles. However, an Eco-ACC designed for a heavy-duty truck that explicitly enforces vehicle-following constraints has yet to be explored.

- In the literature, predictive ACCs that guarantee recursive feasibility do so by assuming the worst-case behavior of the lead vehicle over a receding horizon, which introduces excess conservatism, thereby limiting performance. Predictive ACCs that enforce chance constraints on the minimum following distance have proven to be less conservative and increase fuel efficiency. However, a chance-constrained ACC that can explicitly ensure safe vehicle following in the unlikely event that the lead vehicle engages in some known worst-case behavior has yet to be explored. Additionally, evaluating the performance of a chance constrained ACC for various levels of conservatism implemented in a heavy-duty truck has yet to be explored.

- In the literature, ACC formulations that ensure vehicle-following constraints through the utilization of controlled invariant sets have been explored. However, in the literature, these ACC formulations often have one of the following limitations:

  1. Unnecessary upper-bounds are placed on the vehicle following distance;
  2. Expensive calculations are required to determine an approximation of a controlled invariant set.
1.2 Proposed Control Strategy and Key Challenges

To address the aforementioned gaps in the literature, the research contained in this dissertation proposes the use of a command governor (CG) to enforce vehicle-following constraints. CGs and reference governors (RGs) are predictive control schemes used solely to enforce pointwise-in-time constraints. CGs and RGs are traditionally add-on schemes to closed-loop stable systems (linear or nonlinear) that operate in discrete-time and enforce constraints by adjusting the reference signal, \( r \), (see [51] and [52]) as shown in Figure 1.2. In comparison with MPC, CGs and RGs are computationally inexpensive (see [53]). A key characteristic of CGs and RGs is that they only adjust the applied reference, \( r_{adj} \), the minimum amount necessary to satisfy constraints. The distinction between CGs and RGs lies in the way \( r_{adj} \) is calculated; the adjusted reference applied at the current time step, \( t \), by a RG is bounded by the value of \( r_{adj} \) at the previous time step, \( t-1 \), whereas the adjustment applied by a CG is not. To ensure constraint satisfaction, RGs and CGs rely on a set referred to as the \textit{maximum output admissible set}, denoted as \( O_{\infty} \), which is the set of all constant references and originating states, \( x(t) \), such that constraints are satisfied for all time (see [54]). Typically, RGs and CGs are designed such that the adjusted reference and current state always remain within \( O_{\infty} \) (i.e., they are recursively feasible). However, this design often requires the constraints and reference signal, \( r \), to be constant (see [55]).

In this dissertation, a predictive ACC is proposed that decouples performance objectives (such as fuel economy and drivability) from vehicle-following constraints through the cascade of an upper-level cruise controller with a CG, as shown in Figure 1.3. The main focus of this work is the use of a CG that applies an adjusted control signal, \( u_{adj} \), to enforce vehicle-following constraints. In this work, the ego vehicle is controlled by either a propulsive force at the wheel or a longitudinal acceleration. The upper-level controller prescribes a control signal, \( u_{pre} \), based on some performance objective (e.g., ecological driving, or tracking a desired velocity setpoint). This research investigates the use of both an MPC-based ECC and a PID-based cruise controller (CC) as the upper-level controller. The works in [56] and [57] use a CG and RG, respectively, to enforce vehicle-following constraints within a coordinated platoon. In [56] and [57], a CG and RG are applied outside the loop, respectively, and rely on the communication of the preceding vehicle’s acceleration. The work in this dissertation explores the use of a CG inside the loop, adjusting the control signal to satisfy constraints for the non-cooperative vehicle-following scenario. Such a formulation, where an upper-level cruise controller
Figure 1.3 Overall block diagram of the proposed ACC architecture. The term \( x_{\text{lead}} \) is the state of the lead vehicle and \( x_{\text{ego}} \) is the current state of the ego vehicle.

is cascaded with the CG at the lower level, is new to the ACC and CG literature.

RGs and CGs are traditionally placed outside the loop as add-on schemes; however, they were initially implemented as continuous-time saturation laws (referred to as "error governors") inside the control loop, i.e., modifying the control signal fed into the plant in order to enforce constraints (see [58, 59]). However, the error governor could only ensure local stability and required the plant to be stable. Shortly thereafter, the traditional RG was developed in [60, 61], which could ensure global asymptotic stability for a closed-loop stable system (i.e., the plant was not required to be stable) and was computationally cheaper; thus, the error governor was largely abandoned. Recently, however, there has been a renewed interest in implementing RGs and CGs inside the loop (see [62–65]). Although theoretical properties for inner-loop RGs and CGs have only recently been studied for a specific set of system configurations [64, 65], experimental inner-loop RG results in [62, 63] were successful in enforcing compressor surge constraints in turbocharged gasoline engines. From a practical standpoint, inner-loop RGs and CGs offer more flexibility, as they can be implemented in systems where an external reference does not exist. One example is a human-in-the-loop system, where the reference setpoint is adjusted by the judgement of a human operator, effectively creating a feedback loop around the prescribed reference. Theoretical results in [65] designed an inner-loop RG for a human-in-the-loop system that preserved stability. In [64], an inner-loop RG was designed that ensured recursive feasibility and preserved stability in constrained systems that were passive. In this dissertation, the motivation for placing the CG inside the loop is due to the fact that predictive cruise controllers often do not track a reference, but instead control the vehicle such that some economic performance objective is minimized.

CGs are typically designed to utilize a maximal output admissible set, \( O_\infty \). In the proposed vehicle-following application, where the CG is applied inside the loop, the analogy of set \( O_\infty \) would be a set of constant control signals and originating states that satisfy constraints for all time. In realistic vehicle-following scenarios, however, there is no constant input that can be guaranteed with certainty to satisfy distance-keeping constraints over all time. This is the case for two reasons:
1. The future velocity profile of the lead vehicle is unknown; thus, the future constraints are unknown;

2. If the velocity profiles of the lead vehicles vary significantly (i.e., the disturbances are large), no constant input could be applied for the duration of the trip that upholds distance-keeping constraints and satisfies upper and lower speed bounds.

Due to the aforementioned limitations arising from uncertain future lead vehicle velocity, the CG strategy presented in this dissertation uses a truncated analogy of the maximal output admissible set, denoted as $O_T$, where $O_T$ is the set of all originating states, $x_{ego}(t)$, and constant inputs, $\bar{u}_{adj}$, such that constraints are satisfied over some finite prediction horizon (also referred to as the $i$-step output admissible set in [66]). The finite horizon is chosen to reasonably reflect the future horizon over which the lead vehicle's velocity profile can be predicted. As shown in Figure 1.3, the CG is downstream of the upper-level controller, receiving a prescribed control signal, $u_{pre}$, and applying an adjusted control signal, $u_{adj}$, such that vehicle-following constraints are satisfied over the CGs receding horizon. The CG strategy is formulated as follows:

$$u_{adj}(t) = \arg \min_{\bar{u}_{adj} \in U} \| \bar{u}_{adj} - u_{pre}(t) \|,$$  \quad (1.1a)

subject to: $\{x_{ego}(t), \bar{u}_{adj}\} \in O_T,$ \quad (1.1b)

where $U$ is the set of all possible control inputs. The CG adjusts $u_{adj}(t)$ only when it is necessary to satisfy constraints. If the prescribed control signal satisfies constraints over the CG horizon, $u_{adj}(t) = u_{pre}(t)$. In this dissertation, various representations of $O_T$ are explored and described in further detail in Chapters 3, 4, and 5.

1.2.1 Key Challenges

The proposed strategy presents the following key challenges that are addressed in this dissertation:

- Given that the control input is a propulsive wheel force, in a realistic non-cooperative vehicle-following scenario there is no constant input that guarantees constraint satisfaction for all time without being prohibitively conservative. Additionally, since the CG is implemented inside the loop, conventional tools for designing CGs are not applicable.

- General procedures for designing an inner-loop CG such that recursive feasibility is ensured have not been developed in the literature, either in a general inner-loop CG setting or the vehicle-specific setting considered in this work.

1.3 Dissertation Contributions

The goal of this dissertation is to develop new CG-based ACC strategies that are well-equipped to address the environmental and safety concerns surrounding heavy-duty trucks. The overarching
theme of the ACC strategies developed in this dissertation is the use of a CG inside the loop that enforces vehicle-following constraints through the adjustment of a prescribed control signal. The results presented in this dissertation will address each of the key gaps in the literature, along with the key challenges surrounding the proposed ACC strategy. Contributions of this dissertation relative to the ACC and CG literature include the following:

- **Development and validation of an Eco-ACC for heavy-duty trucks that explicitly enforces vehicle-following constraints with a long receding horizon** - By cascading an MPC-based ECC with a CG, this dissertation presents the design of an Eco-ACC that decouples performance objectives from constraint satisfaction. Since the CG, which is computationally inexpensive, enforces vehicle-following constraints, the MPC-based ECC can run at a much longer time step, and has real-time capabilities while utilizing a horizon length of 2000 m or more (see [48, 67]).

- **Development and validation of an ACC that enforces the desired minimum following distance as a chance constraint and ensures rear-end collision avoidance** - The velocity distribution of multiple lead vehicles entering and exiting the lane are stochastically characterized using on-road velocity data for a light-duty passenger vehicle. The stochastic characterization is used to form chance constraints on some desired minimum following distance that is enforced by the inner-loop CG. Additionally, by using a conservative estimate of the lead vehicle's maximum deceleration, a controller mode is designed that is able to handle emergency stopping scenarios.

- **Design and implementation of an inner-loop CG that enforces vehicle-following constraints and ensures recursive feasibility** - This dissertation presents a procedure for designing a feedback controller based on a rendezvous objective and analytically derives a controlled invariant set associated with this controller. Theoretical results demonstrate that the inner-loop CG policy is equivalent to the aforementioned rendezvous controller sampled in discrete-time, $t$, and is able to utilize the controlled invariant set to ensure recursive feasibility.

### 1.4 Dissertation Outline

To give the reader an understanding of how the proposed CG-based vehicle-following controllers were implemented and validated, the controller-oriented models, as well as the simulation setup, are presented in Chapter 2. Chapter 3 details a multi-rate Eco-ACC designed by cascading a long horizon MPC-based ECC with a short horizon CG inside-the-loop. Chapter 4 presents a CG-based ACC where the CG focuses on using prediction techniques to ensure collision avoidance based on some known worst-case scenario. In Chapter 5, a rendezvous-based ACC is designed where the inner-loop CG is formulated to utilize a controlled invariant set, ensuring recursive feasibility for the vehicle-following problem.
This chapter presents the reader with preliminary background information regarding how the heavy-duty truck was modeled and how performance was evaluated. A proprietary medium-fidelity heavy-duty truck model that was provided by Volvo Group North America was used to perform highway driving simulations. The CG used simplified controller-oriented models that consist of position and velocity states. The control signal that the CG adjusts is the propulsive wheel force, which can be mapped (given vehicle properties) to wheel torque or vehicle acceleration. The controller-oriented models used by the CG in this dissertation are detailed in Section 2.1. Performance metrics used throughout this dissertation are presented in Section 2.2.

### 2.1 Controller-Oriented Models

The controller-oriented models are used by the CG to calculate the predicted states of the ego vehicle over the CG horizon for a given control input. In Chapters 3 and 4, the CG adjusts a prescribed wheel force and uses the nonlinear controller-oriented model presented in Section 2.1.1. In Chapter 5, the CG adjusts a prescribed acceleration and uses the linear controller-oriented model presented in Section 2.1.2.
2.1.1 Nonlinear Model: Controlling Wheel Force

A second-order controller-oriented model of the longitudinal dynamics is used to characterize the ego vehicle, where the state variable, $x_{ego}$, is defined as

$$x_{ego} = [x_{ego} \ v_{ego}]^T,$$  \hspace{1cm} (2.1)

where $x_{ego}$ and $v_{ego}$ denote the position and velocity of the ego vehicle, respectively. The second-order model is given by Newton's second law:

$$\dot{x}_{ego} = v_{ego},$$  \hspace{1cm} (2.2a)

$$\dot{v}_{ego} = \frac{1}{m}(u - F_{drag} - F_{grav} - F_{roll}),$$  \hspace{1cm} (2.2b)

where $u$ is the propulsive force applied to the road through the wheel. The terms $F_{drag}$, $F_{grav}$, and $F_{roll}$ are external forces on the vehicle arising from aerodynamic drag, gravity, and rolling resistance, respectively. These external forces are defined as follows:

$$F_{drag} = \frac{1}{2} \rho C_d A_{ref} v_{ego}^2,$$  \hspace{1cm} (2.3)

$$F_{grav} = mg \sin(\theta(x_{ego})),$$  \hspace{1cm} (2.4)

$$F_{roll} = C_{rr} mg \cos(\theta(x_{ego})).$$  \hspace{1cm} (2.5)

Parameters $\rho$, $C_d$, $A_{ref}$, $m$, $g$, and $C_{rr}$ are defined as the air density, aerodynamic drag coefficient, reference area, ego vehicle mass, acceleration due to gravity, and rolling resistance coefficient. The variable $\theta(x_{ego})$ represents road grade as a function of the ego vehicle position along the route. The relative distance between the lead and ego vehicle is denoted as $s$, and is defined as

$$s = x_{lead} - x_{ego},$$  \hspace{1cm} (2.6)

where $x_{lead}$ is the position of the lead vehicle. In discrete-time, $t$, a forward Euler method is used to approximate (2.2)

$$x_{ego}(t + 1) = x_{ego}(t) + \Delta t \left(\frac{v_{ego}(t + 1) + v_{ego}(t)}{2}\right),$$  \hspace{1cm} (2.7a)

$$v_{ego}(t + 1) = v_{ego}(t) + \Delta t \left(\frac{u(t) - F_{drag}(t) - F_{grav}(t) - F_{roll}(t)}{m}\right),$$  \hspace{1cm} (2.7b)

where $\Delta t$ is the time between time steps $t$ and $t + 1$. When (2.7) is used by the CG to model the longitudinal dynamics, $\Delta t$ is set to the time step of the CG.
2.1.2 Linear Model: Controlling Acceleration

When controlling acceleration, the CG prescribes an acceleration based on the relative dynamics, \( x_r \), between the ego and lead vehicle. The vehicle-following scenario is modeled by the relative position, \( x_{1,r} \), and relative velocity, \( x_{2,r} \), of the ego and lead vehicle, defined as

\[
x_{1,r} = x_{\text{lead}} - x_{\text{ego}},
\]

\[
x_{2,r} = v_{\text{lead}} - v_{\text{ego}}.
\]

The relative dynamics between the lead and ego vehicle are given by

\[
\dot{x}_r = A x_r + B u_r + B_w w_r,
\]

where \( x_r^T = [x_{1,r} \ x_{2,r}] \), \( u_r \) is the prescribed relative acceleration, and \( w_r \) is the error of the predicted current relative acceleration. The acceleration of the ego vehicle, \( a_{\text{ego}} \), is assumed to be known. Therefore, \( w_r \) amounts to the error between the predicted acceleration, \( \hat{a}_{\text{lead}} \), and the actual acceleration, \( a_{\text{lead}} \) of the lead vehicle. The relative dynamics are modeled as a double integrator

\[
A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_w = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.
\]

The prescribed relative acceleration, \( u_r \), is related to the control input to the actual vehicle model by (2.2) and

\[
\dot{v}_{\text{ego}} = a_{\text{lead}} - u_r.
\]

2.2 Performance Evaluation of Heavy-duty Trucks

Performance of the proposed CG-based vehicle-following controllers were tested in simulation. A proprietary heavy-duty truck model and various real-world trucking routes were furnished by Volvo Group Trucks North America. Traffic was represented through a series of lead vehicles entering and exiting the lane along the simulated route. Lead vehicle entries and exits were modeled as a stochastic process, as detailed in this section, that was calibrated using on-road data collected from the Intelligent Vehicles Systems Group at Penn State University.

2.2.1 Stochastic Lead Vehicle Model

A stochastic lead vehicle traffic model, developed and calibrated using highway traffic data in [48], was used to provide multiple lead vehicle entries and exits over an entire route. Randomized entries and exits of lead vehicles were simulated by associating each lead vehicle with a random entry point, \( x_{\text{enter}} \), and random time duration, \( t_f \), which represents the amount of time the lead vehicle remained in front of the heavy-duty truck. Driving behavior of the lead vehicle was simulated by
Table 2.1 Parameters for the Gipps’ car-following model used in this dissertation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{\text{max,ego}}$</td>
<td>Desired maximum acceleration the ego realizes</td>
<td>$1.5 \text{ m s}^{-2}$</td>
</tr>
<tr>
<td>$b_{\text{max,ego}}$</td>
<td>Desired maximum deceleration the ego realizes</td>
<td>$-3.0 \text{ m s}^{-2}$</td>
</tr>
<tr>
<td>$\tau_{\text{react}}$</td>
<td>Driver reaction time</td>
<td>$1.0 \text{ s}$</td>
</tr>
<tr>
<td>$\hat{b}_{\text{max,lead}}$</td>
<td>Predicted desired maximum deceleration of lead vehicle</td>
<td>$-3.5 \text{ m s}^{-2}$</td>
</tr>
<tr>
<td>$s_{\text{des,stop}}$</td>
<td>Desired following distance when stopped</td>
<td>$6.5 \text{ m}$</td>
</tr>
</tbody>
</table>

assigning a random initial following distance, $\delta_0$, and random velocity profile, $v_{\text{lead}}(1, \ldots, N_{\text{lead}})$, to each lead vehicle. In summary, the lead vehicle behavior was quantified as

\[
\begin{align*}
  x_{\text{enter}} &= r_{\text{enter}}, \\
  t_f &= r_{\text{exit}}, \\
  \delta_0 &= r_{\text{gap}}, \\
  v_{\text{lead}}(i) &= r_{\text{vel}}(i), \forall i = 1, \ldots, N_{\text{lead}},
\end{align*}
\]

where $r_{\text{enter}}$, $r_{\text{exit}}$, $r_{\text{gap}}$, and $r_{\text{vel}}$ are Pearson random variables, calibrated using highway traffic data collected from the Intelligent Vehicles Systems Group at Penn State University. In Chapters 4 and 5, lead vehicle velocity was modeled as colored noise that was calibrated using velocity data from passenger vehicles on highway roads from [68] (available under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 license). In all simulation studies presented in this dissertation, lead vehicle entries and exits were modeled by the stochastic process defined in (2.13).

### 2.2.2 Gipps’ Car-Following Model

When evaluating the performance of proposed Eco-ACC and ACC formulations, a Gipps’ car-following model was used as a baseline. This is common practice in the Eco-ACC and ACC literature, as the Gipps’ car-following model represents typical vehicle-following behavior (see [69]). When the ego vehicle is driven by the baseline strategy, the Gipps’ car-following model prescribes a velocity based on

\[
v_{\text{ego}}(t + \tau_{\text{react}}) = \min \left( v_{\text{ego}}(t) + 2.5a_{\text{max,ego}}\tau_{\text{react}}\left( 1 - v_{\text{ego}}(t) / v_{\text{des,ego}} \right) \left( 0.025 + v_{\text{ego}}(t) / v_{\text{des,ego}} \right)^{1/2}, \right. \\
\left. b_{\text{max,ego}}\tau_{\text{react}} + \sqrt{b_{\text{max,ego}}^2\tau_{\text{react}}^2 - b_{\text{max,ego}}^2(2s_{\text{des,stop}} - v_{\text{ego}}(t)\tau_{\text{react}} - v_{\text{lead}}^2(t) / \hat{b}_{\text{max,lead}})}, \right)
\]

where $v_{\text{des,ego}}$ is the desired speed the ego vehicle wishes to travel (dictated by the speed limit of the route used in simulation). Parameter definitions and values of the Gipps’ car-following model used in this dissertation are shown in Table 2.1. Values were chosen based on typical heavy-duty truck driving behavioral data found in [70].
2.2.3 Ecological Performance

Ecological performance is quantified in terms of fuel consumed and energy expended. For a given simulation, the fuel consumed was calculated within the proprietary heavy-duty truck model provided by Volvo Group North America. In [20], an initial energy audit of heavy-duty trucks in typical highway driving scenarios found that the energy dissipated due to braking accounts for up to 20% of the total propulsive energy expended to traverse the route. Simulation results from [22] found that the majority of energy savings stemmed from a reduction in braking. In this dissertation, sources of energy savings are analyzed via three critical components of energy expenditure:

1. Energy expended at the wheel through the realized propulsive force, \( u_{wh} \), referred to as mechanical energy expended (i.e., work);
2. Energy dissipated by braking, either via brake pedal or engine braking;
3. Energy lost by the vehicle due to the forces of aerodynamic drag.

Energy loss due to rolling resistance and gravity (potential energy changes) is dependent on the route only (i.e., independent of driving style) and, thus, are not considered in the results. The total mechanical energy expended, energy dissipated via braking, and energy lost due to aerodynamic drag, were calculated for the entire route, and are denoted by \( E_{\text{mech}}^{\text{tot}} \), \( E_{\text{brake}}^{\text{tot}} \), and \( E_{\text{aero}}^{\text{tot}} \), respectively. These values were calculated post-simulation as

\[
E_{\text{mech}}^{\text{tot}} = \int_0^{x_{\text{dest}}} \max(0, u_{\text{wh}}) \, dx_{\text{ego}}, \quad (2.15)
\]

\[
E_{\text{brake}}^{\text{tot}} = \int_0^{x_{\text{dest}}} \min(0, u_{\text{wh}}) \, dx_{\text{ego}}, \quad (2.16)
\]

\[
E_{\text{drag}}^{\text{tot}} = \int_0^{x_{\text{dest}}} F_{\text{drag}} \, dx_{\text{ego}}, \quad (2.17)
\]

where \( x_{\text{dest}} \) is the position of the destination.

2.2.4 Drivability

Drivability refers to how smooth a vehicle’s response is and is often used to evaluate driver comfort. Vehicle jerk is commonly used to quantify drivability (see [71]), where greater jerk values correspond to worse drivability. In the results presented within this dissertation, drivability was quantified via the mean squared jerk, \( \bar{j}_{\text{ego}}^2 \), (see [72]) over the entire route, given by

\[
\bar{j}_{\text{ego}}^2 = \frac{1}{T_{\text{trip}}} \int_0^{T_{\text{trip}}} j_{\text{ego}}^2 \, d\tau, \quad (2.18)
\]
where $j_{ego}$ is the longitudinal jerk of the ego vehicle, $T_{trip}$ is the total trip time, and $\tau$ represents continuous-time.
This chapter is dedicated to the design and validation of an Eco-ACC that decouples ecological performance objectives from vehicle-following constraints. This decoupling addresses the tradeoff between ecological performance and real-time feasibility identified previously in the literature, specifically for heavy-duty trucks. The objectives of this chapter are:

1. Formulating an Eco-ACC that utilizes a CG to address the aforementioned tradeoff identified in the literature for Eco-ACC in heavy-duty trucks;

2. Validating the proposed Eco-ACC in simulation for various CG parameters in highway driving scenarios with traffic.

First, this chapter gives an overview of the Eco-ACC formulation and how it relates to previously designed MPC-based ECCs for heavy-duty trucks. Then, the formulation of the inner-loop CG within the Eco-ACC is detailed. A simulation study is then presented that validates the performance of the proposed Eco-ACC.

### 3.1 Overview of Proposed Eco-ACC

In this chapter, an Eco-ACC is presented, first introduced at a basic level in the author’s conference publication [73], that implements an MPC-based ECC that is cascaded with a command governor to enforce vehicle-following constraints. Since the CG enforces distance-keeping constraints, the MPC
can be implemented at a much longer time step, and was shown in [48] and [67] to be real-time feasible when using a horizon length of 2000 m or more. Such an inner-loop command governor, which is cascaded with an economic MPC, is new to the Eco-ACC literature (prior to the work in this dissertation). A conceptual diagram of this Eco-ACC strategy is shown in Figure 3.1. The MPC-based ECC (serving as the upper-level controller), which is fused with an offline dynamic programming (DP) optimization, was originally developed in [22]. The benefit of fusing an offline DP with online MPC is that the MPC is able to utilize global knowledge of the route that is accounted for in the offline DP solution. This is achieved by using the global and coarsely gridded DP solution as an initial guess for the MPC optimization, and by penalizing large deviations from the DP solution. The MPC in this work minimizes the energy expended as a surrogate for fuel consumption, as is done in [22] and [48], and utilizes an energy-based affine model of the longitudinal dynamics, rendering the optimization convex. The CG maintains distance-keeping constraints by adjusting the output of the upper-level MPC, thereby modulating the wheel force command passed to the lower-level powertrain controller. The block diagram of this control strategy is shown in Figure 3.2. The position, $x_{\text{lead}}$, and velocity, $v_{\text{lead}}$, of the lead vehicle are assumed to be provided via measurement through an onboard lidar/radar system.

![Figure 3.1 Conceptual diagram of the cascaded MPC and CG-based Eco-ACC strategy.](image)
3.2 Controller-Oriented Models used by Upper-Level Controller

3.2.1 Energy-Based Vehicle Model

The MPC uses a first-order energy-based model of the longitudinal dynamics of the ego vehicle. The dynamics are discretized in distance, where \( k \) corresponds to a discrete position along the route. The discrete energy-based model is given by

\[
E_{\text{kin}}(k+1) = E_{\text{kin}}(k) + E_{\text{mech}}(k) - E_{\text{brake}}(k) - E_{\text{env}}(k),
\]

where the state is the kinetic energy of the ego vehicle, \( E_{\text{kin}} \). The terms \( E_{\text{mech}} \) and \( E_{\text{brake}} \) represent the propulsive energy expended and the energy dissipated due to braking, respectively, over distance step \( k \). \( E_{\text{mech}} \) and \( E_{\text{brake}} \) are the control inputs to the model. The terms \( E_{\text{mech}} \) and \( E_{\text{brake}} \) are related to the propulsive force, \( u \), via

\[
E_{\text{mech}}(k) = \max(0,u(k))\Delta x,
\]

\[
E_{\text{brake}}(k) = \min(0,u(k))\Delta x.
\]

\( E_{\text{env}}(k) \) accounts for the total loss in kinetic energy over one distance step, \( \Delta x \), due to the combination of aerodynamic losses, rolling resistance losses, and increases in potential energy due to road grade. Thus, this quantity is expressed as

\[
E_{\text{env}}(k) = \frac{1}{2} C_d A_{\text{ref}} \rho E_{\text{kin}}(k) \Delta x + T_{\text{roll}}(k) \Delta x + F_{\text{grav}}(k) \Delta x.
\]
3.2.2 Fuel Consumption Model

The fuel consumption model presented here was used by the initial offline optimization (whereas in simulations, the proprietary heavy-duty truck model was used to characterize the fuel consumption rate). A brake-specific fuel consumption (BSFC) map of a heavy duty truck, was used to model the fuel consumption rate, \( R_{\text{fuel}} \), as follows:

\[
R_{\text{fuel}} = P_{\text{eng}} \cdot \text{BSFC}(\omega_{\text{eng}}, T_{\text{eng}}). \tag{3.5}
\]

The BSFC map is dependent on engine speed and engine torque, denoted as \( \omega_{\text{eng}} \) and \( T_{\text{eng}} \), respectively. \( P_{\text{eng}} \) refers to the engine power and was calculated using

\[
P_{\text{eng}} = \omega_{\text{eng}} T_{\text{eng}}, \tag{3.6}
\]

\[
\omega_{\text{eng}} = \frac{v_{\text{ego}} \tau_{\text{gr}} \tau_{\text{ax}}}{r_{\text{wh}}}, \tag{3.7}
\]

\[
T_{\text{eng}} = \frac{r_{\text{wh}} u}{\tau_{\text{gr}} \tau_{\text{ax}} \eta_{\text{dl}}}, \tag{3.8}
\]

where \( \eta_{\text{dl}} \), \( \tau_{\text{gr}} \), and \( \tau_{\text{ax}} \) are the driveline efficiency, gear ratio, and the drive axle gear ratio.

3.3 MPC-based ECC Formulation

3.3.1 Initial Offline Optimization

The initial offline optimization is performed using DP for a coarse discretization of position \( x_{\text{ego}} \). For a given destination, \( x_{\text{dest}} \), and desired trip time, \( T_{\text{des,trip}} \) (which can be prescribed by the driver or fleet operator, and is assumed to be feasible), the offline optimization finds the optimal control sequence, \( u^*_{\text{DP}} \), that minimizes fuel consumption. The optimal control sequence, \( u^*_{\text{DP}} \), is denoted by

\[
u^*_{\text{DP}} = [u^*_{\text{DP}}(0) \quad u^*_{\text{DP}}(1) \quad \ldots \quad u^*_{\text{DP}}(N_{\text{DP}} - 1)], \tag{3.9}
\]
where trip time is discretized into $N_{DP}$ time steps, each having length $\Delta t_{DP}$. The optimal sequence, $u^*_{DP}$, is calculated as follows:

$$u^*_{DP} = \arg\min_{u_{DP}} \sum_{i=0}^{N_{DP}-1} R_{\text{fuel}}(i) \Delta t_{DP},$$

subject to:

$$x(N_{DP}) = x_{\text{dest}},$$

$$v_{\text{min}}(x_{\text{ego}}(i)) \leq v_{\text{ego}}(i) \leq v_{\text{max}}(x_{\text{ego}}(i)),$$

$$u_{\text{min}} \leq u_{DP}(i) \leq u_{\text{max}}(v_{\text{ego}}(i)),$$

$$i = 0 \ldots N_{DP} - 1,$$

and the vehicle dynamics defined in (2.7). $R_{\text{fuel}}$ is the fuel consumption rate, which depends on the applied wheel force. The terms $v_{\text{min}}(x_{\text{ego}})$ and $v_{\text{max}}(x_{\text{ego}})$ are minimum and maximum velocity limits, respectively, where $v_{\text{max}}$ is determined by the speed limit for the given route at position $x_{\text{ego}}$, and $v_{\text{min}}$ was set to 70% of $v_{\text{max}}$. The terms $u_{\text{min}}$ and $u_{\text{max}}$ are the minimum and maximum limits of applied wheel force, respectively, and are set by the performance capabilities of the simulated heavy-duty truck. Given the longitudinal dynamics from (2.2), the offline optimal control sequence, $u^*_{DP}$, implicitly defines optimal position and velocity sequences, denoted by $x^*_{DP}$ and $v^*_{DP}$, respectively. The real-world highway routes that were used in simulation to validate the proposed Eco-ACC strategy contain no stops, thus, every element within the offline optimized velocity sequence, $v^*_{DP}$, is greater than zero. Consequently, every element in $u^*_{DP}$ and $v^*_{DP}$ can be indexed by a unique position, defined by sequence $x^*_{DP}$. Ultimately, the MPC-based ECC utilizes the offline optimal velocity sequence, $v^*_{DP}$, as a target velocity that is indexed by position.

### 3.3.2 MPC Formulation

The MPC formulation utilizes a receding distance horizon is discretized into $N_{MPC}$ distance steps, each of length $\Delta x_{MPC}$. The decision variables for the MPC strategy are the propulsive energy expended, $E_{\text{mech}}$, and the energy lost due to braking, $E_{\text{brake}}$, as defined in (3.2) and (3.3), respectively. The MPC calculates the optimal control sequences, $E^*_{\text{mech}}$, and, $E^*_{\text{brake}}$, such that the objective function is minimized over the receding horizon. The offline optimal position, $x^*_{DP}$, and velocity, $v^*_{DP}$, sequences are utilized by the MPC in two ways:

1. The offline optimal velocity sequence, $v^*_{DP}$, dictates a target velocity that is indexed by position along the route and is used to formulate tracking error penalties within the MPC optimization.

2. The offline optimal DP solution, $u^*_{DP}$, is indexed by position along the route and is referenced to form an initial guess for the MPC optimization.
The online optimization addressed by the MPC is formulated as

\[
\mathbf{u}_{\text{MPC}}^*(k) = \arg \min_{\mathbf{u}_{\text{mech}}(k), \mathbf{u}_{\text{brake}}(k)} \ J(\mathbf{E}_{\text{mech}}(k), \mathbf{E}_{\text{brake}}(k); \mathbf{E}_{\text{kin}}(k)),
\]

subject to:

\[
\begin{align*}
E_{\text{kin}}^{\text{min}}(i|k) & \leq E_{\text{kin}}(i|k) \leq E_{\text{kin}}^{\text{max}}(i|k), \\
0 & \leq E_{\text{mech}}(i|k) \leq E_{\text{mech}}^{\text{max}}, \\
0 & \leq E_{\text{brake}}(i|k) \leq E_{\text{brake}}^{\text{max}}, \\
i & = k \ldots k + N_{\text{MPC}} - 1,
\end{align*}
\]

Equation (3.1),

where

\[
\begin{align*}
\mathbf{E}_{\text{mech}}(k) & = [E_{\text{mech}}(k|k) \ E_{\text{mech}}(k + 1|k) \ldots E_{\text{mech}}(k + N_{\text{MPC}} - 1|k)], \\
\mathbf{E}_{\text{brake}}(k) & = [E_{\text{brake}}(k|k) \ E_{\text{brake}}(k + 1|k) \ldots E_{\text{brake}}(k + N_{\text{MPC}} - 1|k)], \\
\mathbf{u}_{\text{MPC}}(k) & = [(E_{\text{mech}}(k|k) + E_{\text{brake}}(k|k))/\Delta x_{\text{MPC}} \ (E_{\text{mech}}(k + 1|k) + E_{\text{brake}}(k + 1|k))/\Delta x_{\text{MPC}} \ldots \ (E_{\text{mech}}(k + N_{\text{MPC}} - 1|k) + E_{\text{brake}}(k + N_{\text{MPC}} - 1|k))/\Delta x_{\text{MPC}}].
\end{align*}
\]

The notation \((i|k)\) refers to the predicted value at the \(i\)th step within the prediction horizon for an optimization that is performed at step \(k\). The terms \(E_{\text{kin}}^{\text{min}}\) and \(E_{\text{kin}}^{\text{max}}\) in (3.11b) are the minimum and maximum admissible kinetic energies of the ego vehicle, as dictated by \(v_{\text{min}}(i|k)\) and \(v_{\text{max}}(i|k)\), respectively. The MPC prescribes a propulsive wheel force that is the net result of the optimized engine and braking effort, dictated by \(\mathbf{E}_{\text{mech}}^*\) and \(\mathbf{E}_{\text{brake}}^*\) respectively. As is standard for MPC, the first step of each optimized control sequence, \(\mathbf{u}_{\text{MPC}}^*(k)\), is implemented:

\[
\mathbf{u}_{\text{MPC}}^*(k|k) = [(E_{\text{mech}}^*(k|k) + E_{\text{brake}}^*(k|k))/\Delta x_{\text{MPC}},
\]

and the optimization is repeated over the receding time horizon at the next position step, \(k + 1\). The MPC cost function penalizes the tracking error between the predicted kinetic energy profile and the target kinetic energy profile, \(E_{\text{DP}}\), that is dictated by the optimal DP solutions \(\mathbf{x}_{\text{DP}}^*\) and \(\mathbf{v}_{\text{DP}}^*\) over the MPC horizon, resulting in the following MPC cost function:

\[
\begin{align*}
J(\mathbf{E}_{\text{mech}}(k), \mathbf{E}_{\text{brake}}(k); \mathbf{E}_{\text{kin}}(k)) = & \sum_{i=k}^{k+N_{\text{MPC}}-1} \left[ E_{\text{mech}}(i|k)^2 \right. \\
& + \lambda_b E_{\text{brake}}(i|k)^2 + \left. \lambda_d (E_{\text{kin}}(i|k) - E_{\text{DP}}(i|k))^2 \right],
\end{align*}
\]

where \(\lambda_b\) and \(\lambda_d\) are tuning weights.
3.4 Inner-Loop CG Formulation

The CG strategy is formulated as follows:

\[ u_{\text{CG}}(t) = \arg \min_{\bar{u}_{\text{CG}} \in U} \| \bar{u}_{\text{CG}} - u_{\text{MPC}}(t) \|, \]  
\[ \text{subj. to: } \{ x_{\text{ego}}(t), \bar{u}_{\text{CG}} \} \in O_T, \]

(3.15a, 3.15b)

where \( U \) is the set of all possible control inputs. The maximum correction the CG can apply is \( u_{\text{CG}}(t) = u_{\text{max,brk}} \), where \( u_{\text{max,brk}} \) is the maximum negative force that can be applied to the road through the wheel via braking. The CG adjusts \( u_{\text{CG}}(t) \) only when it is necessary to satisfy constraints.

If no constraint violation occurs over the CG horizon, \( u_{\text{CG}}(t) = u_{\text{MPC}}(t) \). If a constraint violation occurs over the CG horizon, the minimization in (3.15) is performed.

The CG is formulated to enforce two minimum following distance constraints:

1. A desired minimum following distance, \( s_{\text{large}} \), at the end of the CG horizon, \( T_{\text{large}} \).

2. A minimum safe following distance, \( s_{\text{small}} \), that must be enforced at every time step along some beginning portion of the CG horizon.

The reason for the incorporation of two following distance constraints in the CG formulation is driven by the fact that the lead vehicle’s velocity profile will often differ from what the CG assumes. Specifically, while the actual velocity profile of the lead vehicle is unknown, its velocity is assumed by the CG to be constant over the prediction horizon. If only the minimum following distance, \( s_{\text{small}} \), were enforced over the prediction horizon, then the CG would allow the ego vehicle to travel at a following distance near that minimum, under the assumption of a constant-speed lead vehicle. If the lead vehicle slowed down unexpectedly, instead of remaining at the assumed constant speed, then the CG would have no choice but to apply a large braking force at the subsequent time step. To mitigate this issue, the proposed algorithm introduces an additional terminal constraint at time \( T_{\text{large}} \), which requires a deceleration profile that forces the ego vehicle ultimately to back off to a larger following distance, \( s_{\text{large}} \). As illustrated in Figure 3.3, this results in a buffer between the ego vehicle’s actual following distance and the absolute minimum allowable following distance \( s_{\text{small}} \), thereby avoiding large braking events upon every unexpected deceleration of the lead vehicle. The minimum following distances, \( s_{\text{small}} \) and \( s_{\text{large}} \), are defined as

\[ s_{\text{small}}(t) = t_{\text{small}} v_{\text{lead}}(t), \]  
\[ s_{\text{large}}(t) = t_{\text{large}} v_{\text{lead}}(t), \]

(3.16, 3.17)

where \( t_{\text{small}} \) and \( t_{\text{large}} \) are constant headway times. Based on the constraints of Figure 3.3, the truncated output admissible set, \( O_T \), is defined as follows:

**Definition 3.4.1.** Truncated Output Admissible Set: \( O_T \) is the set of all initial state/control pairs, \( (x_{\text{ego}}(t), \bar{u}_{\text{CG}}) \) such that if \( u_{\text{CG}}(\bar{t}) = \bar{u}_{\text{CG}}, \forall \bar{t} \in [t \quad t + T_{\text{large}}] \), then:
\[ s(t + T_{\text{large}}|t) \geq s_{\text{large}}, \]
\[ s(\tilde{t}|t) \geq s_{\text{small}}, \forall \tilde{t} \in [t, t + T_{\text{small}}]. \]

The prediction horizon, \( T_{\text{large}} \), is discretized into \( N_{\text{large}} \) time steps, each having length \( \Delta t_{\text{CG}} \). The CG adjustment in (3.15) is performed at every time step, \( \Delta t_{\text{CG}} \). Whenever the current state, \( x_{\text{ego}}(t) \), and current MPC output, \( u_{\text{MPC}}(t) \), are elements of the set \( O_T \), (i.e., \( u_{\text{MPC}}(t) \) satisfies constraints), \( u_{\text{CG}}(t) = u_{\text{MPC}}(t) \). Otherwise, a bisection search, detailed in Algorithm 1, is used by the command governor to find the smallest adjustment required to satisfy the constraints specified in the definition of \( O_T \). In Algorithm 1, \( \epsilon \) is the convergence tolerance. The smaller receding horizon, \( T_{\text{small}} \), is discretized into \( N_{\text{small}} \) steps, each also having length \( \Delta t_{\text{CG}} \). Values for the parameters used in the CG formulation are given in Section 3.5.

### 3.5 Simulation Study of Eco-ACC

Simulations of the cascaded MPC and CG-based Eco-ACC strategy were performed on a heavy-duty truck over a highway route for various CG horizon lengths, \( T_{\text{large}} \). In this section, simulation results are presented to assess the impact of CG parameters on fuel economy and drivability.

#### 3.5.1 Simulation Setup

Distance and elevation data for three representative trucking routes furnished by Volvo Group North America, denoted by HWY-1, HWY-2, and HWY-3, with lengths of 180 km, 360 km, and 240 km, respectively, were used in simulations. Additionally, simulations were performed for a fully-loaded
Algorithm 1 Command Governor Implementation

1: \( u_{ub} = u_{MPC}, \ u_{lb} = u_{\text{max,brk}}, \ \text{FLAG} = 0 \)
2: if \( \{x_{ego}(t), u_{\text{max,brk}}\} \notin O_T \) then
3: \( \bar{u}_{CG} = u_{\text{max,brk}} \)
4: else
5: \( \text{while FLAG} = 0 \) do
6: \( u_m = \frac{u_{ub} + u_{lb}}{2} \)
7: if \( \{x_{ego}(t), u_m\} \in O_T \) and \( u_{ub} - u_{lb} < \epsilon \) then
8: \( \text{FLAG} = 1; \)
9: \( \bar{u}_{CG} = u_m \)
end
10: if \( \{x_{ego}(t), u_m\} \in O_T \) then
11: \( u_{ub} = u_m \)
12: else
13: \( u_{lb} = u_m \)
end
end

and a half-loaded trailer, resulting in total truck weights of 35 000 kg and 20 000 kg, respectively. A stochastic lead vehicle traffic model, developed and calibrated using highway traffic data in [48], was used to provide multiple lead vehicle velocity profiles over the entire route. Over the representative trucking routes HWY-1, HWY-2, and HWY-3, a total of 29, 32, and 40 lead vehicles were encountered, respectively. For the baseline strategy, a Gipps’ car-following model was used to determine the velocity setpoint, where the desired following distance was set to \( s_{\text{large}} \). In all of the simulations performed, \( t_{\text{large}} = 3 \text{ s}, \ t_{\text{small}} = 1 \text{ s}, \) and the CG time step, \( \Delta t_{CG} \), was set to 200 ms. \( T_{\text{small}} \) was held at a relatively short time horizon of 2 s to prevent unnecessary adjustments of applied wheel force, as the approximation of the lead vehicle’s future behavior had less accuracy for longer horizon lengths (see [8]). The MPC used a horizon length of 2000 m and a step size, \( \Delta x_{\text{MPC}} \), of 40 m. Weights \( \lambda_b \) and \( \lambda_d \) in (3.14) were set to 100 and 0.1, respectively. The MPC optimization was implemented every 1 s, which works out to every 20-30 m at the simulated highway speeds.

3.5.2 Eco-ACC Performance

Ecological performance and drivability are reported in Figures 3.4 and 3.5 for a fully-loaded and half-loaded trailer, respectively. Figures 3.4a and 3.5a show the total energy expended and fuel consumed for various values of \( T_{\text{large}} \). Fuel economy improves as \( T_{\text{large}} \) increases, and then plateaus for \( T_{\text{large}} \geq 25 \text{ s} \). In Figures 3.4b and 3.5b, the energy savings due to the reduction of braking and aerodynamic drag are compared to the baseline. The energy expended due to aerodynamic drag is largely dependent on trip time. As a result, negative energy savings due to drag indicate that the Eco-ACC strategy completed the trip faster than the baseline. The results in Figures 3.4b and 3.5b indicate that a reduction in braking was responsible for the vast majority of total energy savings.

Drivability results in Figures 3.4c and 3.5c indicate comparable drivability to the baseline strategy.
The best drivability results occurred when 10 s ≤ $T_{\text{large}}$ ≤ 30 s. The next section analyzes how $T_{\text{large}}$ affected vehicle-following behavior and how these behaviors impacted performance.

### 3.5.3 Vehicle-Following Behavior of Eco-ACC

In order to gain an understanding of how the Eco-ACC strategy modulates the velocity and following distance of the ego vehicle under various CG horizon lengths, $T_{\text{large}}$, a stretch of simulated highway driving where the heavy-duty truck engaged in vehicle following is examined. The vehicle-following scenario in Figure 3.6 occurred on HWY-3 for a fully-loaded trailer, during which a lead vehicle entered and exited the lane at around 146 km and 151 km, respectively. The lead vehicle entered the lane at an initial following distance near $s_{\text{small}}$, traveling around 1 m s$^{-1}$ slower than the ego vehicle. Consequently, upon the lead vehicle entering the lane, every Eco-ACC simulation resulted in a braking force.

Before the lead vehicle entered the lane, the Eco-ACC maintained a velocity near the offline optimized velocity trajectory, $v_{\text{DP}}^*$, which is expected due to the penalty term in (3.14). When the lead vehicle entered the lane, the CG began modulating the prescribed wheel torque by the MPC to enforce the predicted vehicle-following constraints. Additionally, the desired velocity, $v_{\text{DP}}^*$, for this vehicle-following scenario was greater than the lead vehicle’s velocity. Consequently, the realized velocity of the Eco-ACC simulations deviated from $v_{\text{DP}}^*$ while the lead vehicle was present. However, the MPC was still able to engage in ecological driving whenever the prescribed MPC wheel torque satisfied the vehicle-following constraints enforced by the CG. For instance, as the Eco-ACC simulations climbed a relatively steep hill, near 150 km, the MPC prescribed a reduced velocity for the truck, and the Eco-ACC simulations began to follow a similar velocity trend as the offline optimized velocity trajectory. After the lead vehicle exited the lane, the Eco-ACC maintained a slower velocity than the offline optimized solution, but followed a similar velocity trend, until utilizing a downhill portion of the route, starting at 152 km, to accelerate back to a velocity near $v_{\text{DP}}^*$.

To investigate reasons for the plateau in ecological performance for $T_{\text{large}}$ ≥ 25 s, the cumulative energy loss due to braking is shown in Figure 3.6b. The greatest energy losses due to braking occurred when the lead vehicle entered the lane. Eco-ACC simulations with smaller $T_{\text{large}}$ values expended more braking energy upon the initial entry of the lead vehicle, specifically when $T_{\text{large}} < 20$ s, due to the application of a larger braking effort to quickly achieve $s ≥ s_{\text{large}}$. However, Eco-ACC simulations where $T_{\text{large}}$ was longer had the tendency to stay near $s_{\text{small}}$ after the lead vehicle entered the lane. This required the CG to apply a large braking effort if the lead vehicle suddenly slowed down, an example instance of which is seen for $T_{\text{large}} = 150$ s around 147.5 km in Figure 3.6.

Drivability for the vehicle-following scenario of Figure 3.6 was evaluated using the cumulative sum of the vehicle jerk squared, shown in Figure 3.6d. Drivability was affected by $T_{\text{large}}$ similarly to the way ecological performance was affected by $T_{\text{large}}$. The large braking events that occurred for $T_{\text{large}} < 20$ s and $T_{\text{large}} > 52$ s had a negative effect on drivability.
Figure 3.4 Eco-ACC results for a fully-loaded trailer on HWY-1, HWY-2, and HWY-3 are indicated by “◦”, “+”, and “⋄”, respectively.
Figure 3.5 Eco-ACC results for a half-loaded trailer on HWY-1, HWY-2, and HWY-3 are indicated by “◦”, “+”, and “⋄”, respectively.
Figure 3.6 The vehicle-following scenario is represented by velocity, following distance, and wheel force profiles over a stretch of the 240 km route, HWY-3. Figures 3.6a and 3.6c show the velocity and following distance for each simulation. Additionally, Figure 3.6a shows the desired velocity prescribed by the offline DP solution and the route elevation. The cumulative braking energy expended, in Figure 3.6b, as well as vehicle jerk, in Figure 3.6d, are used to evaluate the ecological performance and drivability, respectively.
The previous chapter focused on improving ecological performance in typical highway driving scenarios for a heavy-duty truck. This chapter presents a CG-based ACC that builds upon the CG formulation from Chapter 3 by formulating the CG such that collision avoidance is guaranteed. The CG-based ACC presented in this chapter ensures rear-end collision avoidance by enforcing a minimum following distance constraint that is dependent on the distance-to-collision. Additionally, a statistical characterization of highway velocity data for a light-duty vehicle is used to model the lead vehicle’s velocity over the CG horizon. Using this statistical characterization, the CG formulates the minimum following distance constraint as a chance constraint. If the minimum following distance constraint is violated, a braking effort is applied that ensures that a rear-end collision is avoided. Additionally, using the identified stochastic properties of the lead vehicle, the conservatism for a given prediction of the lead vehicle’s velocity over the CG horizon is quantified. The main objectives presented in this chapter are given as follows:

1. The formulation of a CG-based ACC that utilizes known statistical properties of the lead vehicle’s velocity to enforce vehicle-following constraints as chance constraints;

2. Validating the ecological performance and safety of the proposed CG-based ACC in normal driving scenarios as well as emergency stopping scenarios;

3. Evaluating vehicle-following performance of the heavy-duty truck for various prediction assumptions, as quantified by levels of conservatism.
4.1 Chance-Constrained CG-based ACC Overview

In the presence of stochastic disturbances (such as the lead vehicle behavior), achieving a worst-case performance guarantee is often unrealistic and leads to excessive conservatism under nearly every other circumstance. To address this issue, instead of robustly ensuring constraint satisfaction, the RGs developed in [74], [75], and [76] focus on the enforcement of chance constraints in the presence of a stochastic disturbances, ensuring constraint satisfaction with prescribed probability. Additionally, a chance constrained ACC was developed in [36] that realized superior ecological performance and drivability than a traditional PID-based ACC. However, the chance-constrained ACC in [36] could not ensure rear-end collision avoidance if the worst-case lead vehicle behavior was realized. In this chapter, a chance-constrained CG-based ACC is formulated such that if the chance constraint is violated, a braking effort is applied that ensures that a rear-end collision is avoided.

In this work, historical velocity data of a light-duty passenger vehicle is used to perform a stochastic characterization of the lead vehicle driving behavior. Using this characterization, probable lower bounds on future lead vehicle velocities are calculated. The CG-ACC then enforces the predicted minimum following distance constraints over a receding horizon under the assumption that the lead vehicle travels at some prescribed probable lower bound, which amounts to the enforcement of chance constraints on vehicle following. A block diagram of this CG-based ACC is shown in Figure 4.1. The upper-level controller is a PID-based CC that prescribes a control signal, $u_{PID}$, to track a user-specified velocity, $v_{des}$. The CG applies an adjusted control input, $u_{CG}$, to satisfy chance constraints. The plant model in Figure 4.1 is a proprietary medium-fidelity model provided by Volvo Group Trucks North America, where the powertrain controller receives the adjusted input and outputs an engine torque, $T_{eng}$, and engine speed $\omega_{eng}$.

4.2 Characterizing Lead Vehicle Behavior

In order to prescribe probable bounds on the lead vehicle behavior, and for the purposes of running simulation studies (later in the chapter), it is necessary to perform a statistical characterization.
of the lead vehicle’s behavior. To do this, velocity data for an actual light-duty vehicle driving in a highway environment were acquired from [68], and are available under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 license. The data were used to generate aggregate statistics, in addition to inferring temporal behavior of the lead vehicle. Ultimately, this statistical characterization was essential to developing a model for the probabilistic lower bound on lead vehicle speed, which has in turn been used in the CG formulation. The statistical characterization is also used for the purposes of generating synthetic lead vehicle data for simulations.

4.2.1 Generating Synthetic Data for Simulations

To characterize the lead vehicle behavior, the velocity distribution of the actual velocity data was analyzed. Figure 4.2 shows the normalized histogram of the actual velocity data, along with a normal fit. Additionally, a Fast Fourier Transform (FFT) was used to identify critical frequencies associated with the data, which are important in characterizing the temporal correlation within the actual velocity data. The synthetic lead vehicle velocity data was generated by passing a continuous white noise signal of zero-mean, $W \sim \mathcal{N}(0, \sigma_W^2)$, sampled at a time step of $\Delta t_W$, through a low-pass filter. The signal variance, $\sigma_W^2$, is based on the histogram in Figure 4.2, whereas the filter time constant, $\tau_{c,\text{filt}}$, is based on the FFT. The parameter $\tau_{c,\text{filt}}$ was set heuristically such that the FFT-based power spectral density (PSD) of the synthetic lead vehicle velocity data was most similar to that of the actual velocity data, as shown in Figure 4.3. Thus, the synthetic lead vehicle velocity is modeled as a colored noise signal, $V \sim \mathcal{N}(0, \sigma_V^2)$. The discrete dynamics of the colored noise generation process is defined as:

$$V(t+1) = \left(1 - \frac{\Delta t_W}{\tau_{c,\text{filt}}}\right)V(t) + \frac{\Delta t_W}{\tau_{c,\text{filt}}} W(t),$$

(4.1)
Figure 4.3 The FFT-based PSD of the synthetic and actual velocity data of the lead vehicle.

where \( W(t) \) and \( V(t) \) are the values of random variables \( W \) and \( V \) at time step \( t \), respectively. Once the colored noise signal \( V \) is generated, it is then translated by the mean lead vehicle velocity from the actual velocity data, \( \mu_{v,\text{data}} \).

### 4.2.2 Deriving Probabilistic Bounds on Lead Vehicle Velocity

Using the statistics and time constant, \( \tau_{c,\text{filt}} \), from the lead vehicle data, the evolution over time of the probability density of the future lead vehicle velocities was calculated. Figure 4.4 shows this evolution for three different originating lead vehicle velocities, namely 26 m s\(^{-1} \), 29.2 m s\(^{-1} \), and 32 m s\(^{-1} \). The mean velocity of the actual velocity data, \( \mu_{v,\text{data}} \), was 29.23 m s\(^{-1} \). Using the probability density of future lead vehicle velocities, it is possible to compute probable lower bounds on the lead vehicle velocity, as functions of time. Representative probable lower bounds, corresponding to different likelihoods, are shown in Figure 4.5 for an originating lead vehicle velocity of 32 m s\(^{-1} \). Each probable lower bound corresponds to a particular level of likelihood, corresponding to a percentile given by \( P \). Over an \( N \)-step finite prediction horizon, the lower bound velocity corresponding to the \( P \)th percentile is given by an \( N \)-element vector and is denoted as:

\[
v_{P,LB} = \begin{bmatrix} v_{P,LB}(t+1) & v_{P,LB}(t+2) & \ldots & v_{P,LB}(t+N_{\text{Horiz}}) \end{bmatrix}.
\]  

(4.2)

Given \( i \in 1, 2, \ldots, N_{\text{Horiz}} \), \( v_{P,LB}(t+i) \) is the velocity for which only \( P \)% of transitions from the originating lead vehicle velocity at time \( t \) resulted in a lead vehicle velocity that was less than \( v_{P,LB}(t+i) \) at time \( t+i \).

To simplify controller implementation, a function, \( f_{P,LB} \) of continuous time, \( \tau \), was fitted to the

\[ \text{PSD of Synthetic and Actual Velocity Data} \]

[Graph showing FFT-based PSD of synthetic and actual velocity data]
Figure 4.4 Probability densities for future velocity values for originating velocities of (a) 26 m s$^{-1}$, (b) 29.2 m s$^{-1}$, and (c) 32 m s$^{-1}$. Probability density values between 0 and 0.08 were shaded dark blue.

Figure 4.5 Velocity percentile bounds for $P = 50\%$, $P = 25\%$, $P = 12.5\%$, and $P = 2.5\%$. 
velocity bound $v_{P,lb}$. The function $f_{P,lb}$ was defined as:

$$f_{P,lb}(\tau) = \left( \mu_{v,\text{data}} - v_{\text{origin}} \right) \left( 1 - e^{-\frac{\tau}{\tau_{\text{filt}}}} \right) + \left( v_{P,SS} - \mu_{v,\text{data}} \right) \left( 1 - e^{-\frac{\tau}{\tau_{P,lb}}} \right),$$

(4.3)

where $v_{P,SS}$ is the velocity value at which the integral of the normal fit shown in Figure 4.2 equals $P\%$, and $v_{\text{origin}}$ is the originating velocity. A unique fit was performed for each percentile of interest through a golden-section search to find a $\tau_{P,lb}$ that minimized the sum of the squared difference between $v_{P,lb}$ and $f_{P,lb}$. Figure 4.6 shows function $f_{P,lb}$ and vector $v_{P,lb}$ for the 2.5% percentile (i.e., $P=2.5$) for a $v_{\text{origin}}$ of 26 m s$^{-1}$, 29.2 m s$^{-1}$, and 32 m s$^{-1}$, when $T_{\text{Horiz}}$ equals 20 s and 6 s. Online, the CG used $f_{P,lb}$ as the predicted lead vehicle velocity over the CG horizon. To prioritize safe vehicle following, if the measured lead vehicle acceleration was less than the current predicted acceleration, the lead vehicle was assumed to travel at the measured acceleration over the $T_{\text{small}}$ horizon. The
current predicted lead vehicle acceleration, $\dot{a}_{\text{lead}}(t)$, was approximated as:

$$
\dot{a}_{\text{lead}}(t) = \frac{f_{P,\text{lb}}(t + 1) - v_{\text{lead}}(t)}{\Delta t_{\text{CG}}},
$$

where $\Delta t_{\text{CG}}$ is the time step of the command governor.

## 4.3 Chance-Constrained Inner-Loop CG

Optimization-based ACCs in the literature often ensure rear-end collision avoidance by predicting the lead vehicle to exhibit some known worst-case stopping behavior over the receding horizon (see [9–11]). In [9–11], since the lead vehicle is predicted to exhibit some known worst-case behavior, constraint satisfaction (and, thus, collision avoidance) can be ensured for any prescribed minimum following distance. However, predicting the worst-case behavior introduces excess conservatism, potentially causing the ego vehicle to brake unnecessarily, thereby sacrificing ecological performance. In order to ensure rear-end collision avoidance without making the vehicle-following strategy excessively conservative, the following two mechanisms are incorporated in the CG-based ACC strategy:

1. The CG predicts the lead vehicle velocity over the CG horizon to be a user-specified $P$ probable lower bound of future lead vehicle velocities, $f_{P,\text{lb}}$.
2. The CG enforces a minimum following distance constraint that is defined by some known stopping capabilities of the lead vehicle for the worst-case scenario, which is presented in Section 4.3.1;

The first mechanism enforces the vehicle-following constraints as chance constraints, reducing the conservatism introduced by the lead vehicle prediction method. The lead vehicle's velocity is stochastically varying (as defined in Section 4.2.1) and introduces a stochastic disturbance in the form of deviations from the expected future velocity profile, $v_{50,\text{lb}}$. Consequently, $f_{P,\text{lb}}$ is not an absolute lower bound on future lead vehicle velocities. Instead, the CG ensures that constraints will be satisfied with a prescribed probability from one time step to the next, as described in Section 4.3.3. The second mechanism ensures that rear-end collision is avoided by setting the minimum following distance constraint based on the relative distance encroached upon in the worst-case stopping scenario. Thus, the second mechanism ensures there always exists a braking effort that avoids rear-end collision.

### 4.3.1 Ensuring Robust Vehicle Following Through Constraint Set Tightening

To ensure rear-end collision avoidance, the CG enforces a minimum following distance constraint that is defined by the stopping distance in the worst case scenario, i.e., the scenario where the lead vehicle suddenly applies maximum braking effort. The minimum allowable following distance is
denoted as $s_{\text{min}}$, and is calculated as:

$$s_{\text{min}} = s_{\text{stop}} + s_{\text{DTC}}(v_{\text{ego}}, v_{\text{lead}}),$$  \tag{4.5}$$

where $s_{\text{stop}}$ is the desired minimum following distance when both the lead and ego vehicle come to a stop, and $s_{\text{DTC}}$ is the distance-to-collision. The term $s_{\text{DTC}}$ is defined as:

$$s_{\text{DTC}}(v_{\text{ego}}, v_{\text{lead}}) = \left( v_{\text{ego}} t_{\text{stop,ego}} - v_{\text{lead}} t_{\text{stop,lead}} \right) + \left( \frac{a_{\text{mb,ego}}}{2} t_{\text{stop,ego}}^2 - \frac{a_{\text{mb,lead}}}{2} t_{\text{stop,lead}}^2 \right),$$  \tag{4.6}$$

where $a_{\text{mb,ego}}$ and $a_{\text{mb,lead}}$ are the maximum deceleration values (i.e., the most negative acceleration) the ego and lead vehicle can attain, respectively. The scenario where the lead vehicle realizes a deceleration of $a_{\text{mb,lead}}$ corresponds to the worst-case disturbance, i.e., the scenario where the lead vehicle applies its maximum braking effort. The ego vehicle achieves a deceleration of $a_{\text{mb,ego}}$ when it applies its maximum, safe braking effort, i.e., the driver retains steering control. The terms $t_{\text{stop,ego}}$ and $t_{\text{stop,lead}}$ are defined as:

$$t_{\text{stop,ego}} = \frac{v_{\text{ego}}}{|a_{\text{mb,ego}}|},$$

$$t_{\text{stop,lead}} = \frac{v_{\text{lead}}}{|a_{\text{mb,lead}}|}.  \tag{4.7}$$

Since the vehicle-following scenario under consideration is one where the ego vehicle is a heavy-duty truck and the lead vehicle is a light-duty passenger vehicle (capable of larger deceleration than the heavy-duty truck), it is assumed that $-|a_{\text{mb,lead}}| \leq -|a_{\text{mb,ego}}|$. Given that the CG-based ACC is executed on the ego vehicle, $a_{\text{mb,ego}}$ is considered to be known. Since the vehicle following problem is for the non-cooperative scenario, $a_{\text{mb,lead}}$ is approximated with a conservative guess about the lead vehicle’s maximum braking capability.

### 4.3.2 CG-Based Vehicle Following Strategy

The CG strategy is formulated as follows:

$$u_{\text{CG}}(t) = \arg \min_{u_{\text{CG}} \in U} \| \bar{u}_{\text{CG}} - u_{\text{PID}}(t) \|,$$

$$\text{subj.to :} \{x_{\text{ego}}(t), \bar{u}_{\text{CG}} \} \in O_T,  \tag{4.8}$$

where $U$ is the set of all possible control inputs. The CG only adjusts the control signal if the originating state and control signal prescribed by the PID controller, $u_{\text{PID}}(t)$, do not satisfy constraints; otherwise, the CG sets $u_{\text{CG}}(t) = u_{\text{PID}}(t)$. The CG adjusts the control signal by solving (4.8) using Algorithm 1 from Chapter 3, where $u_{\text{ab}} = u_{\text{PID}}$. 

37
In accordance with the rationale for the CG presented in Chapter 3, the CG is formulated to enforce two minimum following distance constraints:

1. $s_{\text{min}}$ from Equation (4.5), enforced at every step along some beginning portion of the receding horizon, $T_{\text{small}}$;

2. A relatively large minimum following distance, denoted as $s_{\text{large}}$, enforced at the end of the CG horizon, $T_{\text{large}}$.

Similar to the CG formulation of Chapter 3, enforcing $s_{\text{large}}$ is driven by the fact that the future lead vehicle velocity is unknown; if the lead vehicle suddenly slows down and the ego vehicle's following distance is near $s_{\text{min}}$, the CG would have to apply a large braking correction to satisfy constraints.

Unlike the CG presented in Chapter 3, the CG is able to ensure safe vehicle following for some defined worst-case scenario by enforcing $s_{\text{min}}(t)$ at every time step along $T_{\text{small}}$. An illustration of these constraints over the CG horizon is shown in Figure 4.7. The current value of a variable at discrete time $t$ is used as the value at the first prediction step, $(t|t)$, along the receding horizon, e.g. $s(t|t) = s(t)$. The term $s_{\text{large}}$ is defined as the product of some relatively large headway, $t_{\text{large}}$ and the current lead vehicle velocity, $v_{\text{lead}}(t)$. The CG horizon, $T_{\text{large}}$, is discretized into $N_{\text{large}}$ number of time steps, each having length $\Delta t_{\text{CG}}$. Based on the constraints pictured in Figure 4.7, the set $O_T$ is defined at discrete time $t$ as follows:

**Definition 4.3.1.** Truncated Output Admissible Set: $O_T$ is the set of all initial state/control pairs, $(x_{\text{ego}}(t), \bar{u}_{\text{CG}})$ such that if $u_{\text{CG}}(t + \tilde{i}) = \bar{u}_{\text{CG}}, \forall \tilde{i} = 0, 1, \ldots, N_{\text{large}} - 1$, then:
4.3.3 Addressing Feasibility Issues with Chance Constraints

Since the CG is implemented in discrete time, the CG is not able to adjust $u_{\text{CG}}$ at the instant the minimum following distance constraint, $s_{\text{min}}$, is violated. Consequently, $s(t)$ may be less than $s_{\text{min}}(t)$ due to errors in the predicted future lead vehicle velocity at time $t - 1$, causing the constraints of (4.8) to be infeasible. To address this issue, the CG effectively has two controller modes: a normal driving mode when $s(t) \geq s_{\text{min}}(t)$, i.e., $(x_{\text{ego}}(t), u_{\text{max,brk}}) \in O_T$, and an emergency stopping mode when $s(t) < s_{\text{min}}(t)$, i.e., $(x_{\text{ego}}(t), u_{\text{max,brk}}) \notin O_T$ as shown in Figure 4.8. In the normal driving mode, the CG performs a bisection search to find the $\bar{u}_{\text{CG}}$ that solves Equation (4.8) (see lines 5-13 of Algorithm 1). In the emergency stopping mode, the CG simply applies $u_{\text{max,brk}}$ (see lines 2-4 of Algorithm 1). The emergency stopping mode ensures rear-end collision avoidance for appropriate values of $s_{\text{stop}}$ since $s(t)$ can only ever be marginally less than $s_{\text{min}}(t)$ for small values of $\Delta t_{\text{CG}}$. Additionally, in normal driving scenarios the emergency stopping mode causes the ego vehicle to retreat to a feasible following distance. The CG can enter the emergency stopping mode in a normal driving scenario whenever the predicted lead vehicle velocity is greater than the realized lead vehicle velocity.

The predicted following distance sequence $(s(t + i|t), \forall i = 1, 2, \ldots, N_{\text{CG}})$ and minimum following distance sequence $(s_{\text{min}}(t + i|t), \forall i = 1, 2, \ldots, N_{\text{small}})$ are calculated using the velocity percentile bound fit, $f_{p_{\text{lb}}}$, as the predicted lead vehicle velocity over the CG horizon. Based on Equation (4.5),
for a given ego vehicle velocity $s_{\text{min}}(t)$ will increase as $v_{\text{lead}}(t)$ decreases. Thus, if $v_{\text{lead}}(t + 1 | t) > v_{\text{lead}}(t + 1 | t + 1)$, then it is possible that $s(t + 1 | t + 1) < s_{\text{min}}(t + 1 | t + 1)$ causing the CG to enter the emergency stopping mode. Since the CG uses $f_{\text{Pib}}$ to predict the lead vehicle velocity, the probability of $v_{\text{lead}}(t + 1 | t) > v_{\text{lead}}(t + 1 | t + 1)$ is $P$. If the CG is in the normal driving mode at discrete time $t$, the probability of the CG entering the emergency stopping mode at the next time step, $t + 1$, is less than or equal to $P$. The percentile $P$ is considered to be the upper bound probability of entering the emergency stopping mode, as often the ego vehicle will drive at following distances much greater than $s_{\text{min}}(t | t)$. Thus, $s(t + 1 | t + 1)$ does not necessarily violate $s_{\text{min}}(t + 1 | t + 1)$ when $v_{\text{lead}}(t + 1 | t) > v_{\text{lead}}(t + 1 | t + 1)$.

### 4.4 Simulation Study of CG-Based ACC with CA

This section contains simulation results for a heavy-duty truck in the highway driving scenario using the proposed CG-based ACC (CG-ACC). Simulations were performed on a proprietary medium-fidelity model of a heavy-duty truck provided by Volvo Group Trucks North America. Performance of the CG-ACC, relative to the Gipps’ car-following model and a PID-based ACC (PID-ACC), is quantified using metrics for evaluating safety, drivability, and ecological performance.

In the simulations performed, the CG-ACC predicted the lead vehicle velocity based on some probable lower bound as described in Section 4.2.2, referred to hereafter as the $P$-prediction model. Different levels of conservatism were used for predicting the lead vehicle over the CG horizon, for each CG-ACC simulation performed. Conservatism levels were defined by percentiles, $P$, of future lead vehicle velocities. For example, if $P = 15\%$ then the CG predicted the future velocity of the lead vehicle over the CG horizon to be the $15\%$ percentile of future lead vehicle velocities. For the CG-ACC simulations performed, a decrease in $P$ corresponds to an increase of conservatism regarding the lead vehicle’s predicted velocity profile. Simulations were also performed for the formulation where the CG-ACC strategy predicted that the lead vehicle would travel at some constant deceleration, $a_{\text{lead,dec}}$, which was the prediction model used in [77], and is referred to hereafter as the $C$-prediction model. Results for the CG-ACC using the $C$-prediction model are included for comparison.

#### 4.4.1 Simulation Setup

A 280 km, relatively flat highway route was used to simulate highway driving for a heavy-duty truck pulling a half-loaded trailer. Lead vehicle entries and exits were modeled using the stochastic lead vehicle model from [48]. Lead vehicle velocity was generated using the synthetic lead vehicle data presented in Section 4.2.1. Lead vehicle entries, exits, and velocity were combined to form a single lead vehicle scenario. To ensure fairness in the performance evaluation of different vehicle following algorithms, all simulations were performed for the same lead vehicle scenario. Parameters of the PID-ACC were tuned heuristically to achieve typical vehicle following behavior. The desired minimum
following distance of the PID-ACC, $s_{\text{min,PID}}(t)$ was set according to:

$$s_{\text{min,PID}} = t_h \cdot v_{\text{lead}},$$  \hfill (4.9)

where $t_h$ is the desired minimum headway and $v_{\text{lead}}$ is the current velocity of the lead vehicle. The CG-ACC was implemented at a time step of $\Delta t_{\text{CG}} = 0.2$ s. Online, the CG-ACC calculated $s_{\text{min}}$ as defined in (4.5), using assumed maximum braking decelerations of $a_{\text{mb,ego}} = -3 \text{ m s}^{-2}$ and $a_{\text{mb,lead}} = -5 \text{ m s}^{-2}$, respectively, based on values from [78] and [79]. For all CG-ACC simulations, $T_{\text{large}}$ was set to 30 s and $t_{\text{large}}$ was set to 3 s, as these parameter values significantly improved ecological performance and drivability in the results from Chapter 3. Additionally, $T_{\text{small}}$ was varied from 1 s to 6 s. Emergency stopping scenarios were simulated by having a lead vehicle enter the lane at a prescribed relative distance, exhibit normal driving behavior for 10 s, and then suddenly decelerate at $-4 \text{ m s}^{-2}$ to a stop.

### 4.4.2 Normal Driving Scenario

The results presented in this section correspond to simulations over the entire 280 km route for the normal highway driving scenario of a heavy-duty truck. The velocity setpoint fed into the PID controller was set to be the average velocity of the lead vehicle velocity data, namely $29.23 \text{ m s}^{-1}$. The lead vehicle was simulated using synthetic lead vehicle velocity data presented in Section 4.2.1. The trip duration for the CG-ACC strategy was approximately 2.7hrs. A lead vehicle was present roughly 84% of the total trip duration. In normal driving conditions, the desired minimum headway, $t_h$, for the PID-ACC was set to 2.5 s.

A section of the highway route where the lead vehicle enters the lane for roughly 400 s and then exits is shown in Figure 4.9. All of the plots in Figure 4.9 contain simulation results of the CG-ACC strategy, where $P$ was varied from 45 % to $5 \times 10^{-4}$ %. As shown in Figure 4.9c, a more conservative prediction of the lead vehicle velocity required more braking effort, resulting in larger following distances. Additionally, a lack of conservatism caused the heavy-duty truck to tend to a following distance closer to $s_{\text{min}}$, causing larger braking corrections when the lead vehicle suddenly slowed down. This behavior is exhibited in Figures 4.9a and 4.9b at around 4670 s. Since more conservative $P$ values resulted in larger following distances, the total trip time increased for these simulations as well.

### 4.4.2.1 Ecological Driving Performance

Figure 4.10 shows the reduction of fuel consumption by the CG-ACC strategy when compared to the Gipps’ car-following model (Figure 4.10a) and the PID-ACC strategy (Figure 4.10b). In all simulations, the CG-ACC expended less fuel than the Gipps’ car-following model and the PID-ACC. Varying the level of conservatism within the lead vehicle prediction had a greater impact on fuel expenditure than varying $T_{\text{small}}$. The horizontal axes in Figure 4.10 represent various levels of conservatism used by the CG-ACC. This figure shows the results of representing conservatism in two different ways:
Figure 4.9 A 400 s segment of the normal driving simulations for the highway scenario.
1. Conservatism that is represented by percentile $P$, using the probable lower bounds formulated in Section 4.2.2;

2. Conservatism that is represented by a predicted constant lead vehicle deceleration $a_{\text{lead,dec}}$, which corresponds to the CG-ACC strategy used in the author’s previous conference publication [77].

These results are partitioned by a narrow vertical break in Figure 4.10, and in subsequent figures as well, based on which of the two prediction models were used by the CG-ACC. For reference, when the $P$-prediction model uses $P = 2.5\%$ and $P = 5 \times 10^{-5}\%$, the average accelerations of the predicted lead vehicle trajectory for the first 3 s over the CG horizon are $-0.3 \text{m/s}^2$ and $-0.73 \text{m/s}^2$, respectively, for $v_{\text{origin}} = 29.23 \text{m/s}^2$. The fuel consumption results in Figure 4.10 indicate that when the CG used the $P$-prediction model, fuel savings increased as conservatism increased (i.e., as $P$ increased). However, when the CG used the $C$-prediction model, fuel savings decreased as conservatism increased (i.e., as $a_{\text{lead,dec}}$ decreased). The remainder of this subsection examines the energy savings (and sources thereof) to investigate the aforementioned trends between conservatism and fuel savings.

The percent reduction of the total energy expended by the CG-ACC strategy is shown in Figure 4.11, where the same trend between conservatism and ecological performance is present as in Figure 4.10. Thus, when the CG-ACC uses the $P$-prediction model, more energy savings are realized as conservatism is increased. However, as is seen in Figure 4.9, more conservatism leads to larger...
following distances, thereby increasing trip trip time and reducing energy lost due to aerodynamic drag. Figure 4.12 shows the percent reduction of total energy expended due to the reduction in energy loss via aerodynamic drag. The aerodynamic drag results indicate that increasing conservatism increases trip time for both the $P$-prediction model and the $C$-prediction model. The relative increase in trip time due to an increase in conservatism is less for the $P$-prediction model than for the $C$-prediction model. The remainder of the energy savings/losses stem from the energy dissipated from braking to maintain vehicle-following constraints. Figure 4.13 shows the percent reduction in total energy expended due to the reduction in energy dissipated via braking. Figure 4.13 indicates that a range of conservatism for the $P$-prediction model results in the best ecological vehicle-following performance, specifically for $P$ values ranging from 10 % to 2.5 %. Results in Figure 4.13 are corroborated by the vehicle-following behaviors depicted in Figure 4.9, where high levels of conservatism resulted in greater braking to maintain relatively large following distances, and low levels of conservatism resulted in large braking corrections when the lead vehicle suddenly slowed down. Additionally, energy savings due to the reduction of braking were far greater when the CG used the $P$-prediction model instead of the $C$-prediction model for high levels of conservatism. The lack of energy savings due to the reduction of braking for the $C$-prediction model resulted in less total energy savings for high levels of conservatism.
Figure 4.12 Percent reduction of total energy expended by the CG-ACC strategy due to the reduction of aerodynamic drag. Results compared to (a) the Gipps’ car-following model and (b) the PID-ACC.

Figure 4.13 Percent reduction of total energy expended by the CG-ACC strategy due to the reduction of braking. Results compared to (a) the Gipps’ car-following model and (b) the PID-ACC.
### 4.4.2.2 Drivability

Drivability results are shown in Figure 4.14 as a percent reduction in $\bar{j}_\text{ego}^2$. The percent reduction values in $\bar{j}_\text{ego}^2$ shown in Figure 4.14a are negative because the $\bar{j}_\text{ego}^2$ was higher for the CG-ACC strategy than the Gipps’ car-following model. However, the CG-ACC did outperform the PID-ACC strategy, as shown in Figure 4.14b. When the CG-ACC used the $P$-prediction model, the lowest $\bar{j}_\text{ego}^2$ values were realized for a range of conservatism between $P$ values of 10% and 2.5%. This is supported by braking energy results in Figure 4.13 and the results from Chapter 3, as large braking efforts have similar effects on drivability and ecological performance.

### 4.4.3 Emergency Stopping Scenario

The CG-ACC developed in Section 4.3 was designed to ensure collision avoidance for the worst case scenario, when the lead vehicle suddenly applies a maximum braking effort. An emergency stopping scenario was simulated by having a lead vehicle enter the lane, drive normally for around 10 s, and then suddenly decelerate at $-4 \text{ m s}^{-2}$. The stopping behaviors of the CG-ACC and PID-ACC for this emergency stopping scenario are depicted in Figure 4.15. Multiple simulations of the emergency stopping scenario were performed for the PID-ACC for various headway values, $t_h$. Additionally, multiple simulations of the emergency stopping scenario were performed for the CG-ACC using the $P$-prediction model for various $P$ values. Figure 4.15a shows the velocity profiles of the PID-ACC.
Figure 4.15 Emergency stopping scenario of when a lead vehicle suddenly decelerates at $-4 \text{ m/s}^2$. Collision is avoided for the CG-ACC strategy.
Figure 4.16 Pareto front for PID-ACC simulations for various $t_h$ values. All CG-ACC simulations outperform the PID-ACC Pareto front of the PID-ACC simulations.

and CG-ACC simulations, as well as those of the lead vehicle. Collisions were avoided for all CG-ACC simulations, as shown in Figure 4.15c, where $s_{\text{stop}}$ was set to 10 m. From the PID-ACC simulations, collisions were avoided for $t_h \geq 2.75$ s.

The tradeoff between safety and drivability for the PID-ACC is shown in Figure 4.16. In Figure 4.16, the y-axis shows the final following distance from the emergency stopping simulation, and the x-axis contains $f_{ego}^2$ values from the normal driving simulations. Points for the PID-ACC simulations form a Pareto optimal front, where each point corresponds to a unique $t_h$ value between 2.25 s and 4 s. Points for the CG-ACC correspond to unique $P$ values from $P = 45\%$ to $P = 5 \times 10^{-6}\%$. The CG-ACC points in Figure 4.16 outperform the Pareto optimal front formed by the PID-ACC points. Figure 4.16 shows that for the CG-ACC strategy, drivability is not compromised by increasing conservatism.
In Chapters 3 and 4, CG-based vehicle-following strategies were presented that achieved significant improvements in ecological performance, drivability, and safety compared to baseline strategies and conventional PID-based ACCs. However, the inner-loop CGs of these CG-based vehicle-following strategies do not guarantee recursive feasibility, a property commonly ensured in the RG and CG literature. For vehicle following controllers that explicitly enforce vehicle constraints, recursive feasibility ensures robust constraint satisfaction. Although the CG-based ACC in Chapter 4 ensured rear-end collision avoidance, it relied on an emergency stopping mode for scenarios when constraints were violated. Entering this emergency stopping mode caused the ego vehicle to apply a sudden large braking effort, therefore sacrificing ecological performance and drivability. The chapter presents an inner-loop CG based on a rendezvous objective that ensures recursive feasibility through the utilization of a controlled invariant set, removing the need to utilize an emergency stopping mode. The main objectives presented within this chapter are as follows:

- The design of an inner-loop CG that can ensure recursive feasibility when enforcing vehicle-following constraints;
- The evaluation of the performance and robustness of the proposed CG-based ACC when implemented in simulation on a heavy-duty truck.
5.1 Proposed Rendezvous-based Inner-Loop CG for ACC

In the literature, ACC formulations that ensure satisfaction of vehicle-following constraints through the use of controlled invariant sets often require expensive calculations to determine the controlled invariant set or unnecessarily place upper-bounds on the vehicle following distance. To get around these limitations, the proposed ACC formulation uses analytical techniques from [80] and [81] that calculate controlled invariant sets formed within closed-loop systems. Specifically, in [80] and [81], controlled invariant sets are determined for continuous-time systems by showing that the time derivative of the closed-loop state dynamics along the invariant set boundary points inwards. In this chapter, a similar procedure is utilized to prove the existence of an unbounded robust controlled invariant set for a feedback controller designed for rendezvous that is easily adapted to satisfy vehicle-following constraints. The output of the rendezvous feedback controller is compared to the output of an upper-level cruise controller and saturates the upper-level control signal if necessary to satisfy vehicle-following constraints. The robust controlled invariant set is derived analytically and does not require expensive computations beforehand. Additionally, this work shows that the derived unbounded robust controlled invariant set also exists when the rendezvous feedback controller is replaced with an inner-loop command governor that enforces vehicle-following constraints.

5.1.1 Background

The proposed ACC utilizes an upper-level and lower-level controller, as shown in Figure 5.1. The focus of this work is the development of a lower-level controller that ensures constraint satisfaction through the formulation of a robust controlled invariant set. The upper-level controller is a conventional cruise controller that continuously tracks a desired velocity, $v_{\text{des}}$, specified by the driver. The PID-CC prescribes an acceleration, $u_{\text{up}}$, for the ego vehicle. This prescribed acceleration is then saturated by a lower-level controller, $u_{\text{sat}}$. The objective of the lower-level controller is to rendezvous with the lead vehicle at some prescribed following distance, $s_{\text{rend}}$. The lower-level controller is designed for the scenario where the current position, $x_{\text{lead}}$, and velocity, $v_{\text{lead}}$, of the lead vehicle are known by the ego vehicle. The position and velocity of the ego vehicle are denoted by $x_{\text{ego}}$ and $v_{\text{ego}}$, respectively. The predicted acceleration of the lead vehicle, $\hat{a}_{\text{lead}}$, is assumed to have an accuracy within some known bounds.

5.1.2 Notation and Definitions

In this chapter, $\mathbb{R}$ is the set of real numbers and $\mathbb{R}_{\geq 0} = \{ x \in \mathbb{R} : x \geq 0 \}$. For any vector $v \in \mathbb{R}^c$ for $c > 1$, $\|v\|$ denotes the 2-norm of $v$. The term $A^T$ is used to denote the transpose of matrix $A$. The vehicle dynamics are represented as a continuous system $\dot{x} = f(x, u, w)$ where $x \in \mathbb{R}^n$ is the state of the system, $u \in \mathbb{R}^m$ is the control input, and $w \in \mathcal{W} \subseteq \mathbb{R}^p$, is the disturbance that is bounded by set $\mathcal{W}$. The state trajectory is denoted as $x(\tau, x_0, u, w)$, and evolves in continuous-time $\tau$, from the initial state $x_0$, given control and disturbance inputs, $u$ and $w$, respectively. Given $n$ number of states (i.e., $x = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}^T$), the $x_i$ nullcline is the set of points where $\dot{x}_i = 0$ for $i \in \{1, 2, \ldots, n\}$. In discrete
time, \( t - 1 \) represents the previous time step, \( t \) represents the current time step, \( t + 1 \) represents the next time step, and so forth. The state trajectory at discrete time \( t + k \) is denoted as \( x(t + k, x_r, u, w) \), where \( x_r \) is the originating state at time \( t \), given control and disturbance inputs \( u \) and \( w \), respectively. Initial conditions of individual states in discrete time are referenced as \( x_1(t), x_2(t), \ldots, x_n(t) \). Given a set \( S \in \mathbb{R}^n \), the boundary of \( S \) is denoted as \( \partial S \). The set \( S \) is robustly controlled invariant if the closed-loop trajectories for every originating state within \( S \) remain in \( S \) for all future time. This is formalized mathematically through the following definition.

**Definition 5.1.1.** A set \( S \subset \mathbb{R}^n \) is said to be robustly controlled invariant if there exists a continuous function \( \mu : \mathbb{R}^n \to \mathbb{R}^m \) for all \( x \in S \), such that \( f(x, \mu(x), w) \in S \) for all \( w \in \mathcal{W} \) (see [82]).

### 5.2 Rendezvous Control Formulation

In this section, it is shown that by saturating the control signal from upper-level controller with a rendezvous controller, an unbounded robust controlled invariant set is formed. The set is unbounded in the positive \( x_{1,r} \)- and \( x_{2,r} \)-direction, allowing the ego vehicle to disengage vehicle following when desired.

#### 5.2.1 Rendezvous Controller

For the vehicle-following problem, the rendezvous controller is designed such that \( x_{2,r} \to 0 \) and \( x_{1,r} \to s_{rend} \), where \( s_{rend} \) is the desired rendezvous following distance. The relative dynamics offset by \( s_{rend} \) are defined through the state variables, \( \xi = [\xi_1 \quad \xi_2]^T = [x_{1,r} - s_{rend} \quad x_{2,r}]^T \). Thus,

\[
\dot{\xi} = A\xi + Bu_r + B_w w_r.
\]

In this section, a rendezvous controller is designed that forms a robust controlled invariant set to ensure safe vehicle following distance, \( s_{safe} \), is robustly satisfied. At the outset, the following

**Figure 5.1** A block diagram of the proposed ACC, where the upper-level controller is a conventional cruise controller and the lower-level controller saturates the upper-level controller to ensure vehicle-following constraints.
stabilizing feedback control law is examined:

\[ u_r = -\frac{2}{T_r} \xi_1 - \frac{2}{T_r} \xi_2. \]  

(5.2)

Such a control policy results in the closed loop relative dynamics,

\[ \dot{\xi} = A_{CL} \xi + B_w w_r, \]  

(5.3)

where

\[ A_{CL} = \begin{bmatrix} 0 & 1 \\ -\frac{2}{T_r} & -\frac{2}{T_r} \end{bmatrix}. \]  

(5.4)

The term \( T_r \) is a constant that equates to the time required for \( \xi_1 \) to approach zero if the control signal was held constant and \( w_r = 0 \). For \( T_r > 0 \), the eigenvalues of \( A_{CL} \) have non-zero imaginary parts and negative real values. Consequently, for the worst-case scenario where \( w_r = w_{r,\text{min}} \) for all time, the closed loop system exhibits a stable spiral about the equilibrium. As a result, there exist initial conditions where \( \xi_1(0, \xi_0, u_r, w_r) \gg 0 \) such that solution \( \xi_1(\tau, \xi_0, u_r, w_r) \) violates a user-specified safe minimum following distance, \( \xi_{1,\text{safe}} \), at some time \( \tau \), where

\[ \xi_{1,\text{safe}} = s_{\text{safe}} - s_{\text{rend}}. \]  

(5.5)

To address this issue, the rendezvous controller is reformulated by placing a saturation limit on \( \xi_1 \) in the control law

\[ u_r = -\frac{2}{T_r^2} \xi_1 \max(\xi_1 - \xi_{1,\text{null}}, 0) - \frac{2}{T_r} \xi_2, \]  

(5.6)

where \( \xi_{1,\text{null}} \) is the value of \( \xi_1 \) at the intersection of the \( \xi_2 \) nullcline and the user-specified relative speed limit, \( \xi_{2,\text{lim}} \), as detailed in Section 5.2.2.

The switched control law (5.6) results in an affine hybrid system of the form

\[ \dot{\xi} = A_{CL,i} \xi + b_i, \]  

(5.7)

where \( i \in \{1, 2\} \), and

\[ i = \begin{cases} 1 & \text{if } \xi_1 \leq \xi_{1,\text{null}}, \\ 2 & \text{if } \xi_1 > \xi_{1,\text{null}}. \end{cases} \]  

(5.8)
Figure 5.2 A schematic of the rendezvous controller as the lower-level controller.

The subsystem matrices of (5.7) are given by

\[
A_{CL,1} = \begin{bmatrix} 0 & 1 \\ -2/T_r & -2/T_r \end{bmatrix}, \quad b_1 = \begin{bmatrix} 0 \\ w_r \end{bmatrix},
\]

(5.9)

\[
A_{CL,2} = \begin{bmatrix} 0 & 1 \\ 0 & -2/T_r \end{bmatrix}, \quad b_2 = \begin{bmatrix} 0 \\ -(2/T_r^2) \xi_{1,\text{null}} + w_r \end{bmatrix}.
\]

(5.10)

Finally, when the rendezvous controller is used as the lower-level controller, the saturated control signal, \(u_{\text{sat}}\), is defined as

\[
u_{\text{sat}} = \min(\hat{a}_{\text{lead}} - u_r, u_{\text{up}}),
\]

(5.11)

as detailed in Figure 5.2.

5.2.2 Robust Controlled Invariant Set

In this section, it is shown that for all \(\xi_0 \in \partial S_{CG}\), \(\xi(\tau, \xi_0, u_r, w_r)\) will remain in \(S_{CG}\) for all \(\tau \geq 0\) and \(w_r \in \mathcal{W}_r\). In [80], controlled invariant polytopes in the state space domain were formulated for affine hybrid systems by designing feedback control laws that enabled and disabled state trajectories from exiting through the boundaries of the polytopes. A more generalized approach for calculating invariant polytopes by evaluating the time derivative of the closed-loop state dynamics along the invariant set boundary was done in [81]. To show that \(S_{CG}\) is robust control invariant, the following prerequisite result is needed that determines whether a set can be left through a specific boundary.

**Lemma 5.2.1.** Given an affine system \(Ax + b \in \mathbb{R}^n\), the boundary \(\partial S\) cannot be exited if and only if

\[
\forall x \in \partial S : \eta(x)^T (Ax + b) \leq 0,
\]

(5.12)

where \(\eta(x)\) is the outward unit normal vector to the boundary at location \(x\). In accordance with the naming convention from [80], a boundary that cannot be exited is referred to as a blocked boundary.
Lemma 5.2.1 is given in [81] and [83] for any continuous boundary in the state space domain, provided $f$ is Lipschitz continuous. Additionally, a set $S \subset \mathbb{R}^n$, is invariant if and only if $\partial S$ is a blocked boundary (see [83, pp. 198-200]). Lemma 5.2.1 is utilized to prove the rendezvous control law results in a robust controlled invariant set (see Lemma 5.2.2). Consider the unbounded set $S_{\text{CG}}$ shown in Figure 5.3. The boundary of set $S_{\text{CG}}$ is defined as the union of four boundaries,

$$\partial S_{\text{CG}} = \partial S_{\text{CG},a} \cup \partial S_{\text{CG},b} \cup \partial S_{\text{CG},c} \cup \partial S_{\text{CG},d},$$

(5.13)
where

\[
\partial S_{CG,a} = \{ x \in \mathbb{R}^2 : x = [\xi_{1,\text{safe}} \ x_2]^T, \ \forall x_2 \geq 0 \}, \tag{5.14}
\]

\[
\partial S_{CG,b} = \{ x \in \mathbb{R}^2 : x \in \{ \xi (-\tau, \xi_0, u_r, w_{r,\text{min}}) \}, \ \forall \tau \in \mathbb{R}_{\geq 0} \cap [\mathbb{R} \ \mathbb{R}_{\geq 0} - |\xi_{2,\text{lim}}|]^T \}, \tag{5.15}
\]

\[
\partial S_{CG,c} = \{ x \in \mathbb{R}^2 : x = [x_1 \ \xi_{2,\text{lim}}]^T, \xi_1 (-\tau_{\text{lim}}, \xi_0, u_r, w_{r,\text{min}}) \leq x_1 \leq \xi_{1,\text{null}} \}, \tag{5.16}
\]

\[
\partial S_{CG,d} = \{ x \in \mathbb{R}^2 : x = [x_1 \ \xi_{2,\text{lim}}]^T, \xi_{1,\text{null}} \leq x_1 \}. \tag{5.17}
\]

Given that \( \partial S_{CG,a} \cap \partial S_{CG,b} \neq \emptyset \), \( \partial S_{CG,b} \cap \partial S_{CG,c} \neq \emptyset \), and \( \partial S_{CG,c} \cap \partial S_{CG,d} \neq \emptyset \), it is trivial that \( \partial S_{CG} \) is a blocked boundary if and only if \( \partial S_{CG,a}, \partial S_{CG,b}, \partial S_{CG,c}, \) and \( \partial S_{CG,d} \) are blocked boundaries.

**Lemma 5.2.2.** Given the closed loop dynamics (5.7) and (5.8), the boundary \( \partial S_{CG} \) is a blocked boundary.

**Proof.** This proof shows that \( \partial S_{CG} \) is a blocked boundary by showing \( \partial S_{CG,a}, \partial S_{CG,b}, \partial S_{CG,c}, \) and \( \partial S_{CG,d} \) are blocked boundaries.

**Boundary \( \partial S_{CG,a} \):** The boundary \( \partial S_{CG,a} \) is set by the user-specified safe minimum following distance, \( \xi_{1,\text{safe}} \), which must be less than zero (i.e. \( s_{\text{safe}} < s_{\text{rend}} \)) and less than the value at which the \( \xi_2 \) nullcline for the \( i = 1 \) subsystem of (5.7) intersects the \( \xi_1 \) axis. According to switching law (5.8), when \( \xi \in \partial S_{CG,a}, i = 1 \). Additionally, since \( \partial S_{CG,a} \) is parallel to the \( \xi_1 = 0 \) axis (see (5.14)), \( \eta(\xi)^T = [-1 \ 0] \) for all \( \xi \in S_{CG,a} \). According to (5.12) and (5.14),

\[
\eta(\xi)^T (A_{CL,1} \xi + b_1) =
\]

\[
= [-1 \ 0] \left( \begin{bmatrix} 0 & 1 \\ -2/T_1^2 & -2/T_r \end{bmatrix} \xi_1 + \begin{bmatrix} 0 \\ w_r \end{bmatrix} \right)
\]

\[
= -\xi_2, \ \forall \xi \in \partial S_{CG,a}, \ \text{and} \ \forall w_r \geq w_{r,\text{min}}. \tag{5.18}
\]

The right hand side of (5.18) is independent of the disturbance \( w_r \) and, from (5.14) \( \xi_2 \in \mathbb{R}_{\geq 0} \), thus \( \eta(\xi)^T (A_{CL,1} \xi + b_1) \leq 0, \ \forall \xi \in \partial S_{CG,a} \).

**Boundary \( \partial S_{CG,b} \):** According to switching law (5.8), when \( \xi \in \partial S_{CG,b}, i = 1 \). Boundary \( \partial S_{CG,b} \) is the solution to (5.7) for \( i = 1 \) and \( w_r = w_{r,\text{min}} \) (the worst-case disturbance). The equilibrium point of this system is a stable spiral. Thus, the outward unit normal vector \( \eta(\xi) \) changes along the boundary. The time instant that \( \xi (-\tau, \xi_0, u_r, w_{r,\text{min}}) \) reaches the user-specified relative speed limit, \( \xi_{2,\text{lim}} \), is denoted as \( \tau_{\text{lim}} \). By definitions (5.15) and (5.16), \( \xi (-\tau_{\text{lim}}, \xi_0, u_r, w_{r,\text{min}}) = \partial S_{CG,b} \cap \partial S_{CG,c} \). Prior to showing that \( \partial S_{CG,b} \) is a blocked boundary, a continuous-time representation of
The rendezvous controller is designed such that \( \eta(\xi(-\tau, \xi_0, u_r, w_{r,\min})) \), \( \forall \tau \geq 0 \) needs to be defined. Given that \( \xi(-\tau, \xi_0, u_r, w_{r,\min}) \) is defined as

\[
\xi(-\tau, \xi_0, u_r, w_{r,\min}) = \begin{bmatrix}
\xi_1(-\tau, \xi_0, u_r, w_{r,\min}) \\
\xi_2(-\tau, \xi_0, u_r, w_{r,\min})
\end{bmatrix}, \tag{5.19}
\]

the vector tangent to \( \xi(-\tau, \xi_0, u_r, w_{r,\min}) \), denoted as \( \Phi(-\tau, \xi_0, u_r, w_{r,\min}) \), is defined as

\[
\Phi(-\tau, \xi_0, u_r, w_{r,\min}) = \frac{d}{d\tau} \xi(-\tau, \xi_0, u_r, w_{r,\min}) = \begin{bmatrix}
\Phi_1(-\tau, \xi_0, u_r, w_{r,\min}) \\
\Phi_2(-\tau, \xi_0, u_r, w_{r,\min})
\end{bmatrix}. \tag{5.20}
\]

The outward unit normal vector is orthogonal to the tangent vector, thus the continuous-time representation for \( \eta \) is

\[
\eta(\xi) = \begin{bmatrix}
\eta_1(-\tau, \xi_0, u_r, w_{r,\min}) \\
\eta_2(-\tau, \xi_0, u_r, w_{r,\min})
\end{bmatrix} = \begin{bmatrix}
-\Phi_2(-\tau, \xi_0, u_r, w_{r,\min})/\|\Phi(-\tau, \xi_0, u_r, w_{r,\min})\| \\
\Phi_1(-\tau, \xi_0, u_r, w_{r,\min})/\|\Phi(-\tau, \xi_0, u_r, w_{r,\min})\|
\end{bmatrix}, \tag{5.21}
\]

for all \( \xi \in \partial S_{CG,b} \). Given the initial condition \( \xi_0 = [\xi_{1,\text{safe}} \ 0]^T \) and the dynamics from (5.7), the blocked boundary rule (5.12) reduces to

\[
\eta(\xi)^T(A_{CL,1} \xi + b_1) =
= e^{\frac{\tau}{T_r}} (w_{r,\min} - w_r)(w_{r,\min} T_r^2 + 2 \xi_{1,\text{safe}}) \sin \frac{\tau}{T_r}
\leq 0, \forall \xi \in \partial S_{CG,b}, \text{ and } \forall w_r \geq w_{r,\min}. \tag{5.22}
\]

The rendezvous controller is designed such that \( \xi_{1,\text{safe}} < 0 \). The \( \frac{\tau}{T_r} \) term in (5.22) is due to the fact that the equilibrium point of (5.7) for \( i = 1 \) and \( w_r = w_{r,\min} \) is a stable spiral. The rendezvous controller is designed to ensure that \( \tau_{\lim} < \pi T_r \). Additionally, since a negative \( w_r \) corresponds the lead vehicle decelerating more than expected, it is not restrictive to require \( w_{r,\min} \leq 0 \).

**Boundary** \( \partial S_{CG,c} \): According to switching law (5.8), when \( \xi \in \partial S_{CG,b}, i = 1 \). Since the boundary \( S_{CG,c} \) runs parallel to the \( \xi_1 = 0 \) axis at the relative speed limit, \( \eta(\xi)^T = [0 \ -1] \), and \( \xi = \xi_{2,\lim}, \forall \xi \in \partial S_{CG,c} \). According to (5.12),

\[
\eta(\xi)^T(A_{CL,1} \xi + b_1) =
= 2 \frac{\xi_1}{T_r} + 2 \frac{\xi_{2,\lim}}{T_r} - w_r, \forall \xi \in \partial S_{CG,c}, \text{ and } \forall w_r \geq w_{r,\min}. \tag{5.23}
\]

From (5.16), \( \partial S_{CG,c} \) is a line bounded by two endpoints, \( \xi(-\tau_{\lim}, \xi_0, u_r, w_{r,\min}) \) and \( [\xi_{1,\text{null}} \xi_{2,\lim}]^T \).
The term, $\xi_{1,\text{null}}$, is defined as

$$\xi_{1,\text{null}} = -T_r \xi_{2,\text{lim}} + \frac{T_r^2}{2} w_{r,\text{min}},$$  \hspace{1cm} (5.24)$$

which is derived from the point where $\xi_{2,\text{lim}}$ intersects the $\xi_2$ nullcline for system (5.7) subject to the worst-case disturbance ($w_r = w_{r,\text{min}}$). The disturbance, $w_r$ can be rewritten as

$$w_r = w_{r,\text{min}} + c_w = \frac{2 \xi_{2,\text{lim}}}{T_r} + \frac{2 \xi_{1,\text{null}}}{T_r} + c_w,$$  \hspace{1cm} (5.25)$$

where $c_w \in \mathbb{R}_{\geq 0}$. By substituting (5.25) in (5.23),

$$\eta(\xi)^T \left( A_{CL,1} \xi + b_1 \right) = \frac{2 \xi_1}{T_r^2} - \frac{2 \xi_{1,\text{null}}}{T_r^2} - c_w,$$

$$\leq 0, \forall \xi \in \partial \mathcal{S}_{CG}, \text{ and } \forall w_r \geq w_{r,\text{min}}. \hspace{1cm} (5.26)$$

**Boundary $\partial \mathcal{S}_{CG,b}$:** According to switching law (5.8), when $\xi \in \partial \mathcal{S}_{CG,b}$, $i = 2$. Since the boundary $\mathcal{S}_{CG,d}$ runs parallel to the $\xi_1 = 0$ axis at the relative speed limit, $\eta(\xi)^T = \begin{bmatrix} 0 & -1 \end{bmatrix}$, and $\xi_2 = \xi_{2,\text{lim}}$, $\forall \xi \in \partial \mathcal{S}_{CG,b}$. According to (5.12),

$$\eta(\xi)^T \left( A_{CL,2} \xi + b_2 \right) =$$

$$= \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & -2/T_r \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} 0 \\ w_r - \frac{2}{T_r^2} \xi_{1,\text{null}} \end{bmatrix}$$

$$= \frac{2}{T_r^2} \xi_{1,\text{null}} + \frac{2 \xi_{2,\text{lim}}}{T_r} - w_r, \forall \xi \in \partial \mathcal{S}_{CG,c}, \text{ and } \forall w_r \geq w_{r,\text{min}}. \hspace{1cm} (5.27)$$

By substituting the alternate form (5.25) for the disturbance, $w_r$,

$$\eta(\xi)^T \left( A_{CL,2} \xi + b_2 \right) = -c_w,$$

$$\leq 0, \forall \xi \in \partial \mathcal{S}_{CG,c}, \text{ and } \forall w_r \geq w_{r,\text{min}}. \hspace{1cm} (5.28)$$

In the proof of Lemma 5.2.2, it was demonstrated that $\mathcal{S}_{CG}$ is robustly controlled invariant for all disturbances when using the rendezvous controller. Additionally, $\mathcal{S}_{CG}$ is robustly controlled invariant when the rendezvous control law is used as a comparison controller to saturate the control signal from the upper-level controller to some lower-bound. When used as a comparison controller, the lower-level rendezvous controller (5.6) is reformulated as

$$u_r = -\frac{2}{T_r^2} \left( \xi_1 - \max(\xi_1 - \xi_{1,\text{null}}, 0) \right) - \frac{2}{T_r} \xi_2 + c_u,$$  \hspace{1cm} (5.29)$$

where $c_u \in \mathbb{R}_{\geq 0}$ and $c_u = 0$ corresponds to the upper-level control signal being saturated by the
rendezvous controller. Thus, the dynamics from (5.7) are given by

\[
\begin{align*}
A_{CL,1} &= \begin{bmatrix}
0 & 1 \\
-2/T_r^2 & -2/T_r
\end{bmatrix}, \quad b_1 = \begin{bmatrix}
0 \\
w_r + c_u
\end{bmatrix}, \\
A_{CL,2} &= \begin{bmatrix}
0 & 1 \\
0 & -2/T_r
\end{bmatrix}, \quad b_2 = \begin{bmatrix}
0 \\
-(2/T_r^2)\xi_{1,\text{null}} + w_r + c_u
\end{bmatrix}.
\end{align*}
\tag{5.30}
\tag{5.31}
\]

The disturbance can be reformulated as \( w_r = w_{r,\text{min}} + c \), where \( c = c_w + c_u \). Based on the calculations from the proof of Lemma 5.2.2, \( S_{CG} \) is robustly controlled invariant \( \forall c \in \mathbb{R}_{\geq 0} \).

## 5.3 Alternative Rendezvous Strategy via an Inner-Loop CG

In this section, an inner-loop CG is formulated that is equivalent to the rendezvous control law saturation designed in Section 5.2.1. Thus, the inner-loop CG ensures recursive feasibility via the controlled invariant set. A block diagram of the inner-loop CG acting as the lower-level controller is shown in Figure 5.4.

### 5.3.1 Inner-Loop CG for Rendezvous

The inner-loop CG calculates \( u_r \), at each discrete time \( t \), as follows:

\[
\begin{align*}
u_r(t) = \arg\min_{\bar{u}_r \in \mathcal{U}_r} & \quad \bar{u}_r - (\hat{a}_{\text{lead}}(t) - u_{\text{up}}(t)) \\
\text{subj.to:} & \quad (\xi_r, \bar{u}_r) \in O_T,
\end{align*}
\tag{5.32}
\]

where \( \mathcal{U}_r \) is the set of all possible control inputs. The CG applies \( u_r(t) \) over time step, \( \Delta t_{CG} \), where \( u_{\text{sat}}(t) = \hat{a}_{\text{lead}}(t) - u_r(t) \). The CG only saturates the control signal if \( (\xi_r, \hat{a}_{\text{lead}}(t) - u_{\text{up}}(t)) \notin O_T \). When the CG must saturate the control signal, a bisection search is performed online to solve (5.32). The set \( O_T \) is defined at discrete time \( t \) as follows:

**Definition 5.3.1.** The truncated output admissible set, \( O_T \), is the set of all initial state/control
pairs, \((\xi_t, \bar{u}_r)\) such that if \(u_r(t+k|t) = \bar{u}_r, \forall k = 1, 2, \ldots, N_{CG} - 1\), then \(\xi_1(t + N_{CG}|t, \xi_t, \bar{u}_r, 0) \geq \max(\xi_1(t) - \xi_{1, null}, 0)\), where \(N_{CG} = \frac{T_{CG}}{\Delta t_{CG}}\) and \(t + N_{CG}|t\) denotes time step at the end of the CG prediction horizon at discrete time \(t\).

The disturbance input is set to zero because the trajectory \(\xi(t+k|t, \xi_t, \bar{u}_r, 0)\) is a predicted trajectory. It is important to distinguish that \(\xi_1(t + N_{CG}|t, \xi_t, \bar{u}_r, 0)\) is not equivalent to \(\xi_1(t + N_{CG}, \xi_t, \bar{u}_r, 0)\), as the former is the predicted \(t + N_{CG}\) step at time \(t\) and the latter is the realized trajectory at time \(t + N_{CG}\).

### 5.3.2 Equivalence Between CG and Rendezvous Controller

In this section, the invariance properties for the rendezvous controller, derived in Section 5.2.2, are shown to apply for the inner-loop CG as well by proving the CG is equivalent to the rendezvous control policy.

**Lemma 5.3.1.** The CG policy described in (5.32) is equivalent to the rendezvous control policy (5.6) sampled in discrete-time for \(T_r = T_{CG}\).

**Proof.** Given time interval, \(T_{CG}\), and constant relative acceleration, \(\bar{u}_r\), the change in the relative position, \(\Delta \xi_1 = \xi_1(t + N_{CG}, \xi_t, \bar{u}_r, 0) - \xi_1(t)\), is defined by the kinematic equation of motion,

\[
\Delta \xi_1 = \xi_2(t) T_{CG} + \frac{1}{2} \bar{u}_r T_{CG}^2.
\]  

(5.33)

If the constant acceleration drives \(\xi_1(t + N_{CG}, \xi_t, \bar{u}_r, 0) \rightarrow \max(\xi_1(t) - \xi_{1, null}, 0)\), then the kinematic equation becomes

\[
\Delta \xi_1 = \max(\xi_1(t) - \xi_{1, null}, 0) - \xi_1(t)
= \xi_2(t) T_{CG} + \frac{1}{2} \bar{u}_r T_{CG}^2.
\]  

(5.34)

The right hand side of (5.34) is an affine function of \(\bar{u}_r\). Since the CG applies the minimum adjustment necessary, whenever the CG is active (i.e. \(\xi_1(t + N_{CG}|t, \xi_t, \bar{a}_{lead}(t) - u_{up}(t), 0) < \max(\xi_1(t) - \xi_{1, null}, 0)\)) it calculates \(u_r(t)\) based on the kinematic equation (5.34) as

\[
u_r(t) = -\frac{2}{T_{CG}^2} \left(\xi_1(t) - \max(\xi_1(t) - \xi_{1, null}, 0)\right) - \frac{2}{T_{CG}} \xi_2(t),
\]  

(5.35)

which is equivalent to the rendezvous control law (5.6) for \(T_r = T_{CG}\). \(\Box\)

**Theorem 5.3.2.** The inner-loop CG in (5.32) results in a robust controlled invariant set, \(S_{CG}\) ensuring recursive feasibility for all \(\xi_t \in S_{CG}\).
Proof. According to Lemma 5.3.1, the CG saturates the upper-level controller based on the rendezvous control policy (5.6) for \( T_r = T_{\text{CG}} \), resulting in the closed-loop dynamics given by (5.7). Consequently, the boundary \( \partial S_{\text{CG}} \) is blocked according to (5.18), (5.22), (5.26), and (5.28). Therefore \( S_{\text{CG}} \) is invariant. \qed

5.4 Simulation Study of Rendezvous- and CG-based ACC

This section details the performance of the rendezvous-based ACC and the CG-based ACC in comparison to a baseline Gipps’ car-following model and a typical PID-based ACC. Simulations were performed for a typical highway scenario where the ego vehicle was a heavy-duty truck. Traffic was simulated as multiple lead vehicle entries and exits into the lane that were modeled using the stochastic lead vehicle model from [48]. The velocity profiles for the lead vehicle were generated via the statistical characterization from Section 4.2.1.

5.4.1 Simulation Setup

Highway driving was simulated on a flat, 280 km route for a heavy-duty truck. The same lead vehicle scenario was used to evaluate each vehicle-following controller. For all vehicle-following strategies, the desired velocity of the ego vehicle, \( v_{\text{des}} \), was set to the mean velocity of the lead vehicle data over the entire route, \( 29.43 \text{ m s}^{-1} \). The desired minimum following distance for the PID-ACC was defined according to equation (4.9). In all rendezvous- and CG-based ACC simulations, \( w_{r, \text{min}} \) was set based on the known minimum acceleration of the lead vehicle over the entire route. The rendezvous controller and the CG were simulated for various values of \( T_{\text{CG}} \), where \( T_r = T_{\text{CG}} \). The CG was implemented at a time step of \( \Delta t_{\text{CG}} = 0.1 \text{ s} \). The worst-case disturbance value, \( w_{r, \text{min}} \), was set based on the prediction method used and the acceleration data of the simulated lead vehicle. All controllers (i.e. the baseline, PID-ACC, rendezvous-based ACC, and CG-based ACC) were implemented at a frequency of 10 Hz.

Designing the rendezvous controller to form a robust controlled invariant set ensures constraint satisfaction for all time. However, the ecological performance of the rendezvous following strategy depends on the prediction method used. This is highlighted in the results from Chapter 4, as varying conservatism greatly affected performance for both the \( C \)-prediction \( P \)-prediction models. Since the rendezvous controller and CG both saturate a relative acceleration, the \( C \)-prediction model is used, where \( \dot{a}_{\text{lead}} = a_{\text{lead,dec}} \) for the entire trip. The term \( a_{\text{lead,dec}} \) was chosen based on the results from [77] and the known minimum acceleration of the lead vehicle over the entire route. The acceleration distribution is shown in Figure 5.5. The minimum acceleration realized by the lead vehicle over the entire route was \( -1.2 \text{ m s}^{-2} \). Consequently, in the simulations performed \( a_{\text{lead,dec}} = -0.6 \text{ m s}^{-2} \) and \( w_{r, \text{min}} = -0.6 \text{ m s}^{-2} \).

Using the prediction parameters \( a_{\text{lead,dec}} = -0.6 \text{ m s}^{-2} \) and \( w_{r, \text{min}} = -0.6 \text{ m s}^{-2} \), highway driving simulations were performed for various \( T_{\text{CG}} \) values. A corresponding \( S_{\text{CG}} \) set was calculated for each \( T_{\text{CG}} \) value, as shown in Figure 5.6. The safe-minimum following distance, \( s_{\text{safe}} \), was set to 5 m for all...
Figure 5.5 The distribution of the lead vehicle acceleration for the entire route.

Figure 5.6 The robust controlled invariant set, $S_{CG}$, for various values of $T_{CG}$ used in simulation.
Table 5.1 Results are reported as a percent improvement from the baseline. A positive percent improvement for trip time, energy expended, and $j_{ego}^2$, corresponds to the ACC strategy having a faster trip time, expending less energy, and better drivability than the baseline, and vice versa.

<table>
<thead>
<tr>
<th>$T_{CG}$</th>
<th>ACC Strat.</th>
<th>Percent Improvement from Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trip Time</td>
<td>Energy due to Braking</td>
</tr>
<tr>
<td>5 s</td>
<td>Rend.</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>CG</td>
<td>0.29</td>
</tr>
<tr>
<td>6 s</td>
<td>Rend.</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>CG</td>
<td>0.28</td>
</tr>
<tr>
<td>7 s</td>
<td>Rend.</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>CG</td>
<td>0.26</td>
</tr>
<tr>
<td>8 s</td>
<td>Rend.</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>CG</td>
<td>0.23</td>
</tr>
<tr>
<td>9 s</td>
<td>Rend.</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>CG</td>
<td>0.18</td>
</tr>
<tr>
<td>10 s</td>
<td>Rend.</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>CG</td>
<td>0.05</td>
</tr>
<tr>
<td>11 s</td>
<td>Rend.</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>CG</td>
<td>-0.03</td>
</tr>
<tr>
<td>12 s</td>
<td>Rend.</td>
<td>-0.45</td>
</tr>
<tr>
<td></td>
<td>CG</td>
<td>-0.45</td>
</tr>
<tr>
<td>N.A.</td>
<td>PID</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Simulations. For each $S_{CG}$, $\xi_{1,\text{safe}}$ is required to assume a value less than where the $\xi_2$ nullcline for the $i = 1$ subsystem $(5.7)$ intersects the $\xi_1$ axis. For $T_{CG} = 12$ s and $T_{CG} = 10$ s, $s_{rend}$ was set to 58 m and 43 m, respectively (i.e. $\xi_{1,\text{safe}} = -53$ m and $\xi_{1,\text{safe}} = -38$ m, respectively). For $T_{CG} = 8$ s and $T_{CG} = 5$ s, $s_{rend}$ was set to 30 m (i.e. $\xi_{1,\text{safe}} = -25$ m). The relative speed limit, $\xi_{2,\text{lim}}$, was set to $-7.25$ m s$^{-1}$ for all simulations.

5.4.2 Performance

The ecological performance and drivability results for the rendezvous-, CG-, and PID-based ACC strategies are shown in Table 5.1. Results are reported as percent improvements from the baseline. The ecological performance of the rendezvous and CG strategy was comparable to the baseline for $T_{CG} > 6$ s and superior to the PID-ACC in all simulations. The rendezvous- and CG-based ACC strategies had better drivability than the PID-based ACC and the baseline strategy for $T_{CG} < 11$ s. As shown in Chapters 3 and 4, improvements in drivability correspond to a reduction in the number of occurrences where the ego vehicle applies a sudden large braking effort. Thus, the CG-based ACC strategy (as well as the rendezvous-based ACC strategy) is well-suited to handle aggressive lead vehicle cut-ins, as it applies the minimum amount of braking necessary to satisfy vehicle-following constraints.
5.4.3 Vehicle Following Behavior

In this section, the vehicle following behavior is analyzed for all ACC strategies and the baseline for an aggressive lead vehicle cut-in scenario. The lead vehicle cut-in scenario, and the response of the ACC strategies, are shown in Figure 5.7. The lead vehicle cuts into the lane 40 m ahead of the ego vehicle, traveling 5.5 m s\(^{-1}\) slower. Upon entering the lane, the lead vehicle decelerates at the worst-case deceleration, \(-1.2 \text{ m s}^{-2}\) (i.e. \(w_r = w_{r,min}\)), until it reaches a stop. Figures 5.7a and 5.7b show the stopping profiles for all ACC strategies and the baseline. Since the rendezvous- and CG-based ACC strategies explicitly enforce the desired minimum following distance and \(\xi_0 \in S_{CG}\), the minimum safe following distance constraint \(x_{1,r} \geq s_{safe} = 5 \text{ m}\) remains satisfied. Both the PID-based ACC and the baseline strategy come to a stop at a following distance less than \(s_{safe}\). The \(\xi\) trajectories for the rendezvous- and CG-based ACC strategies are shown in Figures 5.7d, 5.7e, 5.7f, and 5.7g for a \(T_{CG}\) of 5 s, 8 s, 10 s, and 12 s, respectively. The magnitude of the jerk realized by the ego vehicle is shown in Figure 5.7c. Over the entire cut-in scenario, when the ego vehicle was driven by the baseline or PID-based ACC strategy, the ego vehicle experienced an average jerk magnitude of 0.214 m s\(^{-3}\) and 0.190 m s\(^{-3}\), respectively. The rendezvous- and CG-based ACC strategies had an average jerk magnitude of 0.128 m s\(^{-3}\).
Figure 5.7 An aggressive lead vehicle cut-in scenario where the lead vehicle enters the lane and immediately begins decelerating to a stop.
This dissertation developed and validated a new vehicle-following controller framework that cascades a command governor with an upper-level cruise controller to enforce vehicle-following constraints. Specifically, the vehicle-following performance of the proposed controller framework was validated when cascading the CG with an MPC-based ECC (forming an Eco-ACC), and a PID-based CC (forming an ACC). Apart from controller development and validation, various prediction methods were developed that quantified the conservatism of lead vehicle prediction assumptions. Performance was evaluated for various levels of conservatism in lead vehicle behavior predictions within the proposed controller framework. In addition to validating the performance, new theoretical results were presented in Chapter 5 that guaranteed recursive feasibility for the inner-loop CG.

6.1 Summary of Contributions

The contributions presented within this dissertation are revisited, and significant achievements are summarized:

- Development and validation of an Eco-ACC for heavy-duty trucks that explicitly enforces vehicle-following constraints with a long receding horizon - Using the proposed controller framework, an MPC-based ECC was cascaded with a CG to form an Eco-ACC. The CG was implemented at a time step of 0.2 s to enforce vehicle-following constraints, and the MPC-based ECC was implemented at a much longer time step of 1 s. This Eco-ACC formulation was validated in simulation for a heavy-duty truck and realized superior ecological performance and drivability when compared to a typical Gipps’ car-following model and a PID-based ACC.
• **Development and validation of an ACC that enforces the desired minimum following distance as a chance constraint and ensures rear-end collision avoidance** - A stochastic characterization of lead vehicle velocity was calibrated using on-road velocity data. This stochastic characterization was used to predict the lead vehicle's velocity over the CG horizon, which amounted to enforcing chance constraints. A desired minimum following distance was formulated such that rear-end collision avoidance could be ensured. This ACC formulation was validated in simulation for a heavy-duty truck and realized superior ecological performance and drivability when compared to a typical PID-based ACC in normal highway-driving scenarios as well as emergency stopping scenarios.

• **Design and implementation of an inner-loop CG that ensures recursive feasibility in various highway driving scenarios** - This dissertation implemented an inner-loop CG that could ensure recursive feasibility when implemented downstream of a PID-based CC. Simulations were performed for various highway-driving scenarios and indicated superior fuel economy and drivability when compared to a Gipps’ car-following model and a PID-based ACC. Simulations also illustrated robust constraint satisfaction under worst-case lead vehicle behavior.

### 6.2 Future Work

Results presented in this dissertation provide significant contributions to the CG and ACC literature. Moving forward, opportunities for future contributions will arise from these results. In Chapter 3, an MPC-based ECC was used as the upper-level controller and was executed at a relatively long time step, as the CG was responsible for enforcing vehicle-following constraints. Similarly, existing MPC-based Eco-ACC formulations could be adapted such that vehicle-following constraints are removed from the MPC formulation, and are instead enforced by cascading the MPC with a CG. It would be interesting to evaluate the performance of various adapted MPC-based Eco-ACCs cascaded with a CG (forming a CG-based Eco-ACC), and then compare the performances to their respective existing MPC-based Eco-ACCs. In Chapter 4, a general procedure for quantifying conservatism for various lead vehicle predictions was developed and could be applied to any MPC-based vehicle-following controller. In Chapter 5, an inner-loop CG was presented that can ensure recursive feasibility for the vehicle-following scenario. Since recursive feasibility for inner-loop CGs has only been guaranteed for a limited set of applications, it would be of great interest to generalize the tools used in Chapter 5 to guarantee recursive feasibility for other inner-loop CG formulations.
BIBLIOGRAPHY


