ABSTRACT

KARAMARKOVICH, SARAH MARINA KESSLER. Trends and Patterns in Mathematics Motivation During Elementary School: Combining Variable-Centered and Person-Centered Approaches. (Under the direction of Drs. Teomara Rutherford and DeLeon Gray).

Motivation is a complex and dynamic phenomenon that is crucial for students' engagement and achievement. It is well known that motivation, especially mathematics motivation, declines over time, often starting in middle childhood. This dissertation is a collection of three studies that examine the trends and patterns of motivation within and across school years during middle childhood.

In the first study, I examined linear and unstructured trends of motivation in third to fifth graders. The unstructured model, which allowed motivation to fluctuate without constraint, provided better fit. These models found a dip in motivation in the middle of the school year, but motivation often rebounded either at the end of the school year or beginning of the next school year. However, motivation still decreased over time such that older students consistently had lower motivation than younger students.

In the second study, I combined person-centered and variable-centered approaches to identify profiles of emotions in fourth and fifth graders and examined how those profiles mediated the relation between motivation and achievement. Three of the four profiles identified primarily either positive (two profiles) or negative (one profile) emotions; however, about half of the students had mixed emotions, reporting both positive and negative emotions (one profile). The emotion profiles generally mediated the association between motivation and achievement. Students with higher math motivation were more likely to be in positive profiles and, in turn, students with positive emotion profiles were higher achievers, whereas students with lower math motivation were more likely to be in the negative and mixed emotion profiles and those students had lower math

achievement. The one exception is that the mixed emotion profile did not mediate the relation between math value and achievement.

In the final study, I pioneered a new analysis to profile the change in motivation within one school year. With this new analysis, longitudinal latent profile analysis (LPA-long), I found that some students have consistent motivation during the school year, whereas others have motivation that fluctuates, dipping in either the middle or end of the school year. Overall, students who had high motivation year-round were the highest achievers. This result was not corroborated when comparing the LPA-long profiles to a traditional latent transition analysis; however, this is likely due to measurement issues with the latent transition analysis. The LPA-long was easier to interpret, especially for identifying how motivation changed throughout the school year.

The context-dependent nature and complexity of SEVT has posed challenges in examining trends and patterns. My dissertation addresses that complexity by exploring different combinations of person-centered and variable-centered analyses. Overall, the three studies in my dissertation show that students sometimes have mixed feelings about mathematics and that mathematics motivation does not linearly change over time.

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Trends and Patterns in Mathematics Motivation During Elementary School: Combining Variable-Centered and Person-Centered Approaches

by
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DEDICATION

This dissertation is dedicated to my husband, Ian—thank you for your endless support, love, and willingness to make me coffee—and our two dogs, Freyja and Opal—thank you for giving me a reason to take a break and get out of the house every day.

BIOGRAPHY

Sarah Marina Kessler Karamarkovich is an educational psychologist and statistician. She has always had a passion for mathematics and education, fostered by her mother (a scientist) and her father (a teacher). While earning her bachelors' in psychology and mathematics from Longwood University, she realized she wanted to direct that passion into studying how learning occurs and what influences learning. After encouragement from her professors, especially Dr. Stephanie Buchert and Dr. Thomas Wears, Sarah decided to pursue her doctorate in Educational Psychology and Masters of Statistics at North Carolina State University.

During her graduate career, Sarah has worked as a lab manager, online course instructor, quantitative methods teaching assistant, and math interventionist. Along with those positions, Sarah's research was funded through the National Science Foundation's Graduate Research Fellowship Program. With the support and guidance of her mentor, Dr. Teomara Rutherford, Sarah presented 21 posters/presentations at various conferences and published four manuscripts, with three more either under review or in preparation. Through her various positions, courses, and research, Sarah developed her identity as an educational and developmental psychologist who focuses on mathematics motivation and learning. Her aim as a researcher is to study how mathematics motivation and learning change throughout the school year and develop over time with the overall goal of understanding what influences achievement by looking at the multifaceted aspects of student characteristics.

Sarah has accepted a job as an educational data analyst at Petersburg City Public Schools in Virginia. With family in Virginia, she is excited to take this next step surrounded by those she loves and looks forward to improving students' educational outcomes and equity through research and data-based decisions.

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CHAPTER ONE: INTRODUCTION

Mathematics skills are important for success in school, daily life, and careers in science, technology, engineering, and mathematics (STEM). The number of U.S. STEM positions has been on the rise (U.S. Department of Commerce, 2011) and the U.S. Bureau of Labor Statistics (2020) projects the number of STEM occupations will continue to grow and at a faster rate than other occupations. However, students are not doing well in math (e.g., Lee, 2012) and there is a shortage in the STEM workforce (U.S. Congress Joint Economic Committee, 2012). Understanding and intervening in the development of students' mathematics motivation may be one way to address this gap, as motivation is associated with career choice, even when accounting for achievement (Lauermann et al., 2017; Wang et al., 2013).

Motivation is also critical for success in learning and performing in mathematics—it influences students' engagement and achievement (Eccles & Wigfield, 2020; Wigfield & Eccles, 2000). When students value a subject and expect to do well, they tend to be more engaged and perform better than those who do not (e.g., Cole et al., 2008; Hulleman et al., 2008; Wang & Eccles, 2013). These are the core tenets of Eccles' and Wigfield's (2020) Situated Expectancy–Value Theory (SEVT) of motivation, my theoretical framework in this dissertation. In SEVT, Eccles and Wigfield (2020) emphasize that motivation is not stable, it fluctuates based on environment and development. However, prior research has often studied motivation by aggregating measures and examining mean level trends, operationalizing motivation in a way that is not aligned with its complex, dynamic, and context-dependent nature (e.g., Jacobs et al., 2002; Wigfield et al., 1997). For example, by examining differences in average levels of motivation across ages, researchers have often concluded that motivation for certain subjects, including mathematics, declines in middle childhood (Jacobs et al., 2002; Wigfield, 1994). These mean level

trends indicate a general decline in expectancies and values but may not capture the actual patterns and trends in motivation (see Musu-Gillette et al., 2015). For example, some students' motivation declines from elementary through middle school but rebounds in high school (Archambault et al., 2010). Instead, approaches that allow for motivation to follow any pattern or trend, not just linear trends and averages, may be more aligned with the dynamic and context-dependent nature of motivation.

Due to the complexity and dynamic nature of motivation, researchers have used many techniques to try to capture that complexity (e.g., Guo et al., 2017; Jacobs et al., 2002; Lazarides et al., 2020). Most common are variable-centered approaches that look at antecedents, outcomes, and changes in motivation (e.g., Guo et al., 2015a; Meece et al., 1990; Musu-Gillette et al., 2015; Trautwein et al., 2012). Rising in popularity are person-centered approaches that identify patterns of responses (e.g., Anderson & Cross, 2014; Dietrich & Lazarides, 2019; Magnusson, 2003; Umarji et al., 2018). In contrast to the variable-centered approaches described above, these analyses assume heterogeneity in responses across people and look at the complex interactions between components of motivation. Although there are guidelines on how to conduct analyses consistent with each approach, there is no consensus on how to best examine motivation overall. At a recent American Educational Research Association meeting (2019), a symposium hosted by leading motivation researchers (Matthew Bernacki, Avi Kaplan, and Lisa Linnenbrink-Garcia) discussed this issue. During the symposium, *Embracing and Modeling the Complex Dynamics of Motivation and Engagement: Contextual, Temporal, Dynamic, and Systematic Symposium*, researchers presented on the wide variety of data collection and analytical approaches used to describe the development, fluctuation, and impact of motivation. By understanding the complexity of motivation and how it changes, researchers will be better able to design and implement

interventions to improve motivation and thus achievement. This dissertation comprises three studies that each apply a different method to capture the complexity and development of motivation during middle to late elementary school; a critical point in development when motivation—as viewed "overall"—often declines.

Structure

This dissertation proposal will have five chapters—this one (the introduction and literature review), a chapter for each study, and a discussion chapter. In the literature review, I provide an overview of my theoretical framework (Situated Expectancy–Value), its ties to mathematics achievement, and the many ways it has been studied. The chapters for each study (two, three, and four) will have their own specific literature review, research questions, methods, and analysis sections. Lastly, the discussion will tie together the results and conclusions from all three studies.

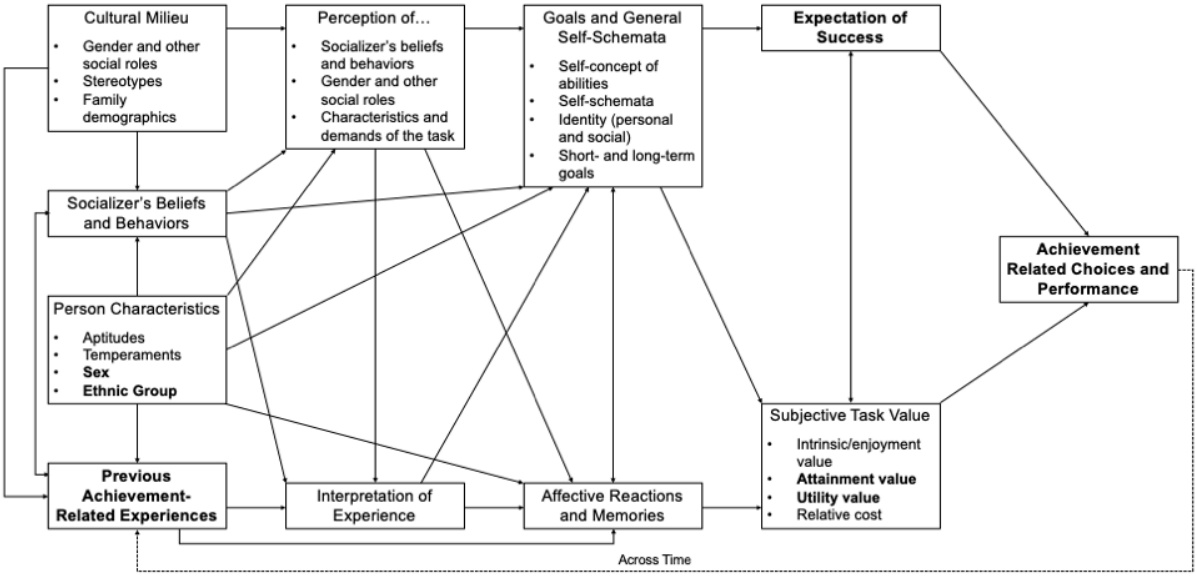
Literature Review

Theoretical Framework: Situated Expectancy–Value Theory

Eccles' and Wigfield (2020) Situated Expectancy–Value Theory (SEVT) of motivation states that a person's expectation of success and subjective task value influence their choices. In brief, if a person expects to do well and values the task, they are more likely to pursue it. This theory stems from Atkinson's (1957) work regarding motive, probability for success, and incentive. Atkinson (1957) proposed that there are three key variables influencing motivation—(1) *motive*: a disposition or stable characteristic regarding a person's capacity for satisfaction; (2) *expectancy*: the perceived probability of success; and (3) *incentive*: the reason to reach, or not reach, a certain goal (e.g., the reward or punishment for doing something). The relative strengths of each of these three variables are what motivates a person to achieve or avoid failure: Motivation = Motive $\times$ Expectancy $\times$ Incentive.

Figure 1.1

Situated Expectancy–Value Theory Adapted from Eccles & Wigfield (2020)



Note. Bolded terms identify components examined in these three dissertation studies.

In 1983, Eccles and colleagues expanded Atkinson's (1957) theory of motivation. Eccles' Expectancy–Value Theory (EVT) added mediators and determinants of a person's expectancies, values, and behavior, grounded in social, developmental, and sociocultural theories with the goal of explaining gender differences in mathematics (Eccles (Parsons) et al., 1983). In contrast to Atinkson's model, this model focused on students' perceptions of reality, rather than reality itself. Additionally, it included predictors and mediators of success, such as socializers and past experiences (Eccles (Parsons) et al., 1983). The overall goal of EVT was to explain student academic choices and achievement, especially with respect to girls/women in mathematics.

In 2020, Eccles and Wigfield relabeled Expectancy–Value Theory as Situated Expectancy–Value Theory. This relabeling draws attention to the continual influence of context, both in how students respond to and co-create their environments (Eccles & Wigfield, 2020). Eccles and Wigfield (2020) posit that situational and contextual factors

> influence all aspects of a growing individual's life space, as well as each individual's interpretation of their experiences. It is important to note that these influences occur all throughout the model; that is, we do not view them as endogenous influences that only have impact at the outset. Rather, they are infused [throughout]. (p.10)

Although this dissertation is framed with SEVT, the literature review largely relies on research conducted with EVT due to its very recent relabeling. The following sections will describe in detail the different components of SEVT (see Figure 1.1; bolded terms identify components examined in these three studies).

Components of SEVT

Expectancy. Expectancies for success are an "individuals' beliefs about how well they will do on an upcoming task" (Eccles & Wigfield, 2020). Similar measures are included in most motivational theories, such as self-efficacy in Social Cognitive Theory (Bandura, 1986), need for competence in Self-Determination Theory (Ryan & Deci, 2017), and control in Attribution Theory (Weiner, 1985) and Control-Value Theory (Pekrun, 2006). In the past, measures of expectancies and academic self-concepts have been combined or used interchangeably due to the considerable overlap in these measures (see Anderman, 2020; Eccles et al., 1993; Hattie et al., 2020); however, they are theoretically distinct—academic self-concepts are more stable and general, whereas expectancies for success are context and task-specific and take into account perceived task difficulty (Eccles & Wigfield, 2020).

Subjective Task Value. There are four main constructs within subjective task value—intrinsic value, attainment value, utility value, and cost (Eccles & Wigfield, 2020; Wigfield & Eccles, 2020; Wigfield et al., 2016). Intrinsic value is the anticipated joy a person expects to get from participating in a task (Eccles & Wigfield, 2020). Intrinsic value is similar to interest and

intrinsic motivation, in that all concepts emphasize enjoyment and engaging in an activity of a person's own volition (Eccles & Wigfield, 2020; Wigfield et al., 2016). Unlike intrinsic value, utility value focuses on the usefulness of a task and has been conceptualized as a means to an end (Eccles & Wigfield, 2020). When a task is closely tied to important, personal goals, the usefulness of the task (utility value) is strongly related to attainment value. Attainment value is how important a task is to a person's identity—"the extent to which tasks do or not allow persons to manifest those behaviors that they view as central to their own core sense of themselves or allow them to express or confirm important aspects of their central selves" (Eccles & Wigfield, 2020, p. 5).

These aspects of subjective task value (intrinsic, utility, and attainment value) are all theorized to be positively related to achievement-related choices and performance (i.e., the more value a student has in a task, the more likely they are to engage in it and do well; Eccles (Parsons) et al., 1983; Eccles & Wigfield, 2020). In contrast, *cost* captures the negative aspects of the task (i.e., reasons to *not* engage in the task). The original EVT model proposed three different types of cost—effort cost: how much effort is needed to complete the task; opportunity cost: the potential missed opportunities by spending time on the task; and emotional cost: the negative emotions, such as anxiety, from participating in and/or failing the task (Eccles (Parsons) et al., 1983). Recent research has suggested a fourth type of cost, outside effort cost (see Flake et al., 2015), where outside opportunities and responsibilities make it difficult to participate in the given task. However, Eccles and Wigfield (2020; Wigfield & Eccles, 2020) maintain the original three component structure due to the overlap between the types of cost (see Part et al., 2020 for further support of the three cost components).

Changes in Expectancies and Value Over Time

Children as young as two can understand what it means to succeed or fail (Butler, 2005;

Heckhausen, 1987), a crucial precursor to developing an expectation for success. Additionally, preschoolers are generally able to estimate others' ability based on how quickly a task was completed and how difficult the task was (Heyman & Compton, 2006; Heyman et al., 2003). It should be noted, however, that these students also related intelligence to being nice (i.e., nice people are smart; Heyman et al., 2003). It is not until middle childhood (around ages seven to nine) that students develop more calibrated expectancies for their own performance based on attributions of past successes/failures, rather than optimism (e.g., Muenks et al., 2018; Nicholls, 1979; Normandeau & Gobeil, 1998; Parson & Ruble, 1977; Wigfield et al., 2015). The theory that students start with overly optimistic expectancies that become more realistic as they get older is widely posited (see Eccles & Wigfield, 2020; Muenks et al., 2018). This is largely based on the finding that students' expectancy beliefs decline starting in elementary school (e.g., Butler, 2005; Jacobs et al., 2002; Wigfield et al., 2015), but become more stable as students get older (Pomerantz & Saxon, 2001; Wigfield et al., 1997).

Although expectancy and value are intertwined (e.g., Spinath & Steinmayr, 2008), students can generally distinguish between the two as young as first grade (Eccles et al., 1993; Wigfield & Eccles, 2020). The individual components of subject task value, however, are generally difficult for students to differentiate until they are older (Wigfield, 1994). It is theorized that intrinsic value, in the form of interest, likely develops first, with attainment and utility value developing in middle childhood and throughout adolescence as "children mature and develop more conscious and reasoned personal identities, goals, and worldviews" (Wigfield & Eccles, 2020, p. 173; see also Eccles, 2009; Oyserman, 2014). Much like expectancy, prior research using linear trends has found that subjective task values tend to decline throughout elementary school and into high school (Jacobs et al., 2002; Wigfield & Eccles, 2020).

The majority of research on changes in expectancies and values motivation over time has used mean-level differences (see Butler, 2005 for review); however, more modern research has used the person-centered approach of growth mixture modeling to identify different trajectories (Archambault et al., 2010; Gaspard et al., 2020; Musu-Gillette et al., 2015). By using growth mixture modeling, Archambault et al. (2010), Gaspard et al. (2020), and Musu-Gillette et al. (2015) were able to identify group students that have similar, potentially nonlinear, trends in aspects of motivation. In each study, students reported their expectancy and value (an aggregate measure of utility and attainment value) for math (Musu-Gillette et al., 2015), literacy/language arts (Archambault et al., 2010), or both (Gaspard et al., 2020). They found that expectancy and value generally declined over time; however, the rate of change and potential rebound varied (Archambault et al., 2010; Gaspard et al., 2020; Musu-Gillette et al., 2015). For example, Musu-Gillette and colleagues (2015) found a profile wherein math value decreased from fourth to tenth grade but remained stable for the remainder of high school. Similarly, Archambault and colleagues (2010) found multiple profiles that had a parabolic shape: literacy expectancy and value each declined in elementary school, was lowest in middle school, but increased throughout high school. These studies support the notion that motivation is complex and dynamic and, thus, researchers should use analyses that do not constrain trends and patterns of motivation to mean-level differences.

What Influences Expectancy and Value

One of the major differences between Atkinson's (1957) expectancy×value theory and modern SEVT is the inclusion of additional predictors of expectancies and values (Eccles & Wigfield, 2020). For example, personal characteristics, such as gender and race, influence how students are treated and can impact their achievement-related experiences. These in turn influence

the students' interpretations of their environment and goals, all impacting expectations and values. Although the paths appear to be static, Eccles and Wigfield (2020) emphasize the continual fluctuation of each component and its impact on achievement based on situational factors, such as classroom characteristics, peers, and development.

Socializers, such as parents and teachers, play a key role in developing students' expectancies and values. Namely, socializers communicate their own beliefs about students' ability and what is valuable through their praise for the student (Henderlong Corpus & Lepper, 2007) and differentiational treatment between students or groups of students (Jurrism et al., 2009). Peers can also influence students' expectancies and values, primarily by providing a source of external comparison (Goetz et al., 2008). Peers and teachers function both as socializers and as part of the larger school context. Wang and Eccles (2013) found that key aspects of school characteristics (school structure, teaching for relevance, teacher emotional support, and peer emotional support; Midgley et al., 1998) were positively related to expectancy and value.

Expectancies and values are influenced by the individual as well. Prior achievement and feedback influences students' expectancy for success and goals (Eccles & Wigfield, 2020; Nagy et al., 2006), both within and between domains (Goetz et al., 2008; March, 2007; Möller et al., 2009). Research on between domain differences has largely focused on math and language, such that a students' achievement in one domain has a negative association with their self-concept in the other domain (Marsh, 2007). For example, students who do well in language courses tend to have lower mathematics self-concept, after controlling for prior mathematics achievement, due to the internal comparison of self-concept between the two subjects (Eccles, 2009; Goetz et al., 2008; Guo et al., 2015b; Marsh, 2007).

What Expectancy and Value Influence

The vast majority of research on SEVT focuses on the outcomes of expectancy and value: achievement-related choices and performance (Eccles & Wigfield, 2020). A substantial body of research has supported the theoretical ties between expectancy/value and behavior. Researchers have found that higher expectancy and value can lead to more engagement (e.g., Wang & Eccles, 2013), course enrollment (e.g., Durik et al., 2006; Simpkins et al., 2006), higher achievement (e.g., Cole et al., 2008; Hulleman et al., 2008), and career goals (e.g., Wang et al., 2013). For example, Wang and colleagues (2013) found that mathematics expectancy in high school predicted having a STEM career 15 years later, even after accounting for the association between math achievement and STEM career.

Expectancies and values can also act as mediators, explaining some of the effect of a predictor on achievement/engagement. For example, Wang and Eccles (2013) found that the influence of school characteristics (e.g., support, autonomy, and teaching) on students' behavioral, emotional, and cognitive engagement was mediated by motivational beliefs (i.e., expectancy and value). Additionally, expectancies and values have been shown to mediate some of the relation between student background, such as gender, and achievement (e.g., Eccles et al., 1999; Guo et al., 2015a, 2015b; Nagy et al., 2008).

Researchers have also conducted interventions to improve SEVT (Harackiewicz & Priniski, 2018). These studies often impact students' utility value by relating the course/subject to the students' life (e.g., Harackiewicz et al., 2014) or community goals (e.g., Brown et al., 2015). With largely positive results (see Harackiewicz & Priniski, 2018; Rosenzweig & Wigfield, 2016 for reviews), these interventions further support the causal link between expectancy/value and achievement-related choices/performance.

Academic Emotions and SEVT

Academic emotions are inherently tied to SEVT—intrinsic value is a measurement of enjoyment and emotional cost represents the negative emotions felt towards a task (Eccles & Wigfield, 2020). Pekrun (2006) ties aspects of Eccles' and colleagues' Expectancy–Value Theory and Weiner's (1985) attributional theory to achievement emotions in Control-Value Theory (CVT).

Control–Value Theory

In Control-Value Theory (CVT), Pekrun (2006) theorizes that students' motivation for an activity can influence their emotions, which in turn influences achievement (see also Pekrun et al., 2007, 2017). There is substantial overlap in CVT and SEVT, especially in regard to control and expectancy, and with values comprising a named component in each. One component of *control* is how well a person believes they will do on a task, i.e., their expectancy (Eccles & Wigfield, 2020; Pekrun, 2006). The second component of control is their attributions (what the student believes influences their performance; Pekrun, 2006; Pekrun & Perry, 2014; Pekrun & Stephens, 2010; Weiner, 1985). Pekrun's (2006) value has two components—*intrinsic value* and *extrinsic value*. These definitions are identical to those of intrinsic and utility value in SEVT: intrinsic enjoyment of and engagement in a task of a person's own volition and extrinsic value is the instrumental usefulness of the task in reaching other goals (Eccles & Wigfield, 2020; Pekrun, 2006).

Different combinations of control and value can impact students' academic emotions (Pekrun, 2006). For example, if a student has high value for a task but a low level of control, they will likely feel frustrated while completing the task and, if they did poorly, disappointed (Pekrun, 2006). Academic emotions have consistently been tied to achievement: generally, positive emotions were related to higher achievement, whereas negative emotions were related to lower

achievement (e.g., Burić & Sorić, 2012; Roos et al., 2020; Villavicencio & Bernardo, 2016; *cf.* Pekrun & Linnenbrink-Garcia, 2014). Although CVT describes achievement emotions as discrete, an increasing number of motivation researchers have used person-centered analyses to explore the possibility of mixed emotions (e.g., Ganotice et al., 2016; Jarrell et al., 2016, 2017; Raccanello et al.,2018; Robinson et al., 2017, 2020).

Modeling SEVT

Studies on motivation typically use variable-centered approaches to examine differences in motivation by age (e.g., Jacobs et al., 2002; Spinath & Spinath, 2005), as well as the antecedents (e.g., Simpkins et al., 2012) and outcomes of motivational components (e.g., Durik et al., 2006). More recent studies have examined the interaction between expectancy and values, which harkens back to Atkinson's (1957) original expectancy×value theory (e.g., Guo et al., 2017; Nagengast et al., 2011, 2013; Trautwein et al., 2012). For example, Trautwein and colleagues (2012) found statistically significant interactions between math expectancy and each type of value when predicting mathematics achievement: students with high expectancy and value had the highest achievement, whereas students with high value but low expectancy had the lowest achievement.

There has also been an influx of person-centered analyses, which can capture varying interactions with multiple variables (Magnusson, 1998). Person-centered analyses reject the assumption that motivation functions the same way for all students, and instead examines patterns in levels and/or trends of SEVT (e.g., Archambault et al., 2010; Wang et al., 2013). These analyses have been used to describe motivation trajectories over time (as discussed previously), as well as patterns of motivation at specific timepoints. Motivation researchers have used a variety of person-centered approaches, such as cluster analysis (e.g., Umarji et al., 2018) and latent profile analysis (e.g., Gaspard et al., 2019) for a single timepoint or I-States (e.g., (Lazarides et al., 2019) and latent

transition analysis (e.g., Dietrich & Lazarides, 2019) for two or more timepoints. These studies have identified high/low profiles and profiles with divergent expectancies and values (e.g., high expectancy but low value, Andersen & Cross, 2014).

Motivation has consistently been tied to students' engagement and learning (e.g., Durik et al., 2006; Cole et al., 2008; Hulleman et al., 2008; Simpkins et al., 2006; Wang & Eccles, 2013; Wang et al., 2013). Because of motivation's importance, researchers have studied how motivation changes over time and across ages (e.g., Butler, 2005; Muenks et al., 2018; Wigfield & Eccles, 2020), as well as patterns of motivation (e.g., Andersen & Cross, 2014; Dietrich & Lazarides, 2019; Umarji et al., 2019). In this dissertation, I extend this research by using a combination of both traditional variable-centered and person-centered approaches to better understand how SEVT and academic emotions function and fluctuate within and between school years.

Current Studies

This dissertation has three studies that all revolve around the patterns and trends in mathematics motivation (specifically expectancy, importance, and usefulness) in mid to late elementary students. All studies use data from motivation surveys embedded within a digital mathematics curriculum as part of a broader NSF-funded study (Evaluation for Actionable Change: A Data-Driven Approach; NSF Grant Number 1544273). These surveys were given three times a school year (beginning, middle, and end of the school year) and measured mathematics expectancy, usefulness, and importance (utility and attainment value, respectively). Students also reported their self-concept, enjoyment, and academic emotions for multiple subjects.

In the first study, *Models of Growth*, my goal was to understand how mathematics expectancy and value change within and across school years. Prior studies have primarily examined change between school years (e.g., Jacobs et al., 2002); however, few have considered

changes within the school year (*cf.* Maulana et al., 2016). Yet with school context being a major influence on motivation (Eccles & Wigfield, 2020; Muenks et al., 2018; Wigfield & Eccles, 2020), it is likely that motivation fluctuates within a school year. Examining both within- and between-school year fluctuation provides a more in-depth examination of the change in expectancies and values across the critical middle childhood time-period. This study used traditional variable-centered analyses to examine trends. Specifically, I asked the following questions:

2a. What are trends in expectancies, current value, and future value across two years?

2b. Do these trends differ by age/cohort (i.e., third-fourth graders vs fourth-fifth graders)?

2c. Which model (linear or nonlinear) best fits the data?

By using timepoint as either a continuous or factor variable, I can determine if trends in expectancies and value are linear or non-linear within and between the school year.

I frame the second study, *Mixed Feelings*, within Control-Value Theory (Pekrun 2006), which combines aspects of Situated Expectancy–Value Theory and academic emotions. Research examining patterns of emotions is rare, yet there is evidence that emotions may co-occur (e.g., Larsen & McGraw, 2011; Tulis & Ainley, 2011). In this study, I combined a single-timepoint person-centered analysis with a variable-centered analysis to test multiple aspects of the CVT framework and expand upon the current methodologies used to describe motivation. My research questions were:

3a. What profiles of mathematics emotions are exhibited by fourth and fifth graders? Do these patterns differ between fourth and fifth graders?

3b. Do these profiles mediate the relation between control-value measures and achievement?

To answer these questions, I examined how patterns of emotions mediate the relation between expectancy/value and achievement. The proposed model included select aspects from each

component of the CVT framework within a path analysis to identify relations. By examining these associations with elementary students, I contribute to the scant existing knowledge on elementary students' achievement emotions and provide evidence as to how early patterns of emotions for mathematics may influence academic achievement and choices.

In *Models of Growth*, I considered the change in motivation over time and, in *Mixed Feelings*, I identified a single-timepoint profiles of emotion. For the third study, *Profiling Change*, I combined these two goals and explored how to profile the change in motivation within one school year. To profile change, I compared patterns of motivation using two different approaches: a latent profile analysis using all timepoints and a traditional latent transition analysis. To this end, I asked the following questions:

4a. What profiles of motivation are present throughout the school year?

- 4i. What motivation profiles emerge when considering all time points?
- 4ii. What motivation profiles emerge when considering each time point separately?
- 4iii. When looking at each time point separately, how do students move between profiles at each time point?

4b. How are these profiles related to achievement?

I chose to do two analyses due to the push for person-centered approaches (Bergman & El-Khouri, 2003; Magnusson, 2003) for studying motivation but lack of consensus on how to approach longitudinal person-centered analyses. With each analysis having its own advantages and disadvantages, it is important to be able to determine which technique is most appropriate for motivation. I compared these two approaches to illuminate the relative strengths of each.

The context-dependent nature and complexity of SEVT has posed challenges in examining trends and patterns. My dissertation addresses that complexity by exploring different combinations

of person-centered and variable-centered analyses in three different studies. These studies demonstrate how student motivation develops in late childhood, a critical period when motivation often declines (e.g., Butler, 2005; Jacobs et al., 2002; Wigfield et al., 2015).

References

Anderman, E. (2020). Achievement motivation theory: Balancing precision and utility. *Contemporary Educational Psychology*, 61, 101864. https://doi.org/10.1016/j.cedpsych.2020.101864.

Andersen, L., & Cross, T. L. (2014). Are students with high ability in math more motivated in math and science than other students? *Roeper Review, 36*(4), 221–234. https://doi.org/10.1080/02783193.2014.945221

Archambault, I., Eccles, J. S., & Vida, M. N. (2010). Ability self-concepts and subjective value in literacy: Joint trajectories from grades 1-12. *Journal of Educational Psychology, 102*(4), 804–816. https://doi.org/10.1037/a0021075.

Atkinson, J. W. (1957). Motivational determinants of risk-taking behavior. *Psychological Review, 64*(6, Pt.1), 359–372. https://doi.org/10.1037/h0043445

Bandura, A. (1986). *Social foundations of thought and action*. Prentice Hall.

Bergman, L. R., & El-Khouri, B. M. (2003). A person-oriented approach: Methods for today and methods for tomorrow. N*ew directions for child and adolescent development, 2003*(101), 25-38. https://doi.org/10.1002/cd.80

Bernacki, M., Kaplan, A., & Linnenbrink-Garcia, L. (2019, April 5-9). *Embracing and Modeling the Complex Dynamics of Motivation and Engagement: Contextual, Temporal, Dynamic, and Systematic Symposium*. [Symposium] American Educational Research Association 2019 Meeting, Toronto, CA.

Brown, E. R., Smith, J. L., Thoman, D. B., Allen, J. M., & Muragishi, G. (2015). From bench to bedside: A communal utility value intervention to enhance students' biomedical science

motivation. *Journal of Educational Psychology, 107*(4), 1116–1135. doi:10.1037/edu0000033

Burić, I., & Sorić, I. (2012). The role of test hope and hopelessness in self-regulated learning: Relations between volitional strategies, cognitive appraisals and academic achievement. *Learning and Individual Differences, 22*(4), 523–529. https://doi.org/10.1016/j.lindif.2012.03.011

Butler, R. A. (2005). Competence assessment, competence, and motivation between early and middle childhood. In A. J. Elliot, & C. S. Dweck (Eds.). *Handbook of competence and motivation* (pp. 202–221). New York, NY: Guilford Press.

Cole, J. S., Bergin, D. A., & Whittaker, T. A. (2008). Predicting student achievement for low stakes tests with effort and task value. *Contemporary Educational Psychology, 33*(4), 609–624. https://doi.org/10.1016/j.cedpsych.2007.10.002

Dietrich, J., & Lazarides, R. (2019). Gendered development of motivational belief patterns in mathematics across a school year and career plans in math-related fields. *Frontiers in Psychology, 10*, 1472. https://doi.org/10.3389/fpsyg.2019.01472

Durik, A. M., Vida, M., & Eccles, J. S. (2006). Task values and ability beliefs as predictors of high school literacy choices: A developmental analysis. *Journal of Educational Psychology, 98*(2), 382–393. https://doi.org/10.1037/0022-0663.98.2.382

Eccles, J. S. (2009). Who am I and what am I going to do with my life? Personal and collective identities as motivators of action. *Educational Psychologist, 44*(2), 78–89. https://doi.org/10.1080/00461520902832368.

Eccles, J. S., & Wigfield, A. (2020). From expectancy-value theory to situated expectancy-value theory: A developmental, social cognitive, and sociocultural perspective on motivation.

Contemporary Educational Psychology, 61, 101859. https://doi.org/10.1016/j.cedpsych.2020.101859

Eccles, J., Wigfield, A., Harold, R. D., & Blumenfeld, P. (1993). Age and gender differences in children's self-and task perceptions during elementary school. *Child Development, 64*(3), 830-847. https://doi.org/10.2307/1131221

Eccles-Parsons, J. S., Adler, T. F., Futterman, R., Goff, S. B., Kaczala, C. M., Meece, J. L., et al. (1983). Expectancies, values, and academic behaviors. In J. T. Spence (Ed.), *Achievement and achievement motivation* (pp. 75–146). W. H. Freeman.

Flake, J. K., Barron, K. E., Hulleman, C., McCoach, B. D., & Welsh, M. E. (2015). Measuring cost: The forgotten component of expectancy-value theory. *Contemporary Educational Psychology, 41*, 232–244. https://doi.org/10.1016/j.cedpsych.2015.03.002

Ganotice, F. A., Datu, J. A. D., & King, R. B. (2016). Which emotional profiles exhibit the best learning outcomes? A person-centered analysis of students' academic emotions. *School Psychology International, 37*(5), 498–518. https://doi.org/10.1177/0143034316660147

Gaspard, H., Wille, E., Wormington, S. V., & Hulleman, C. S. (2019). How are upper secondary school students' expectancy-value profiles associated with achievement and university STEM major? A cross-domain comparison. *Contemporary Educational Psychology, 58*, 149-162. https://doi.org/10.1016/j.cedpsych.2019.02.005

Goetz, T., Frenzel, A. C., Hall, N. C., & Pekrun, R. (2008). Antecedents of academic emotions: Testing the internal/external frame of reference model for academic enjoyment. *Contemporary Educational Psychology, 33*(1), 9-33. https://doi.org/10.1016/j.cedpsych.2006.12.002

Guo, J., Marsh, H. W., Parker, P. D., Morin, A. J. S., & Dicke, T. (2017). Extending expectancy-value theory predictions of achievement and aspirations in science: Dimensional comparison processes and expectancy-by-value interactions. *Learning and Instruction, 49*, 81–91. https://doi.org/10.1016/j.learninstruc.2016.12.007

Guo, J., Marsh, H. W., Parker, P. D., Morin, A. J., & Yeung, A. S. (2015a). Expectancy-value in mathematics, gender and socioeconomic background as predictors of achievement and aspirations: A multi-cohort study. *Learning and individual differences, 37*, 161-168. https://doi.org/10.1016/j.lindif.2015.01.008

Guo, J., Parker, P. D., Marsh, H. W., & Morin, A. J. S. (2015b). Achievement, motivation, and educational choices: A longitudinal study of expectancy and value using a multiplicative perspective. *Developmental Psychology, 51*(8), 1163–1176. https://doi.org/10.1037/a0039440

Harackiewicz, J. M., & Priniski, S. J. (2018). Improving student outcomes in higher education: The science of targeted intervention. *Annual Review of Psychology, 69*, 409-435. https://doi.org/10.1146/annurev-psych-122216-011725

Harackiewicz, J. M., Canning, E. A., Tibbetts, Y., Giffen, C. J., Blair, S. S., Rouse, D. I., & Hyde, J. S. (2014). Closing the social class achievement gap for first-generation students in undergraduate biology. *Journal of Educational Psychology, 106*(2), 375-389. https://doi.org/10.1037/a0034679

Hattie, J., Hodus, F., & Kang, S., H. K. (2020). Theories of motivation: Integration and ways forward. *Contemporary Educational Psychology, 61*, 101865. https://doi.org/10.1016/j.cedpsych.2020.101865.

Heckhausen, H. (1987). Emotional components of action: Their ontogeny as reflected in achievement behavior. In D. Gîrlitz, & J. F. Wohlwill (Eds.). *Curiosity, imagination, and play.* Hillsdale, NJ: Erlbaum.

Henderlong Corpus, J., & Lepper, M. R. (2007). The effects of person versus performance praise on children's motivation: Gender and age as moderating factors. *Educational Psychology, 27*, 487–508. https://doi.org/10.1080/01443410601159852

Heyman, G. D., & Compton, B. J. (2006). Context sensitivity in children's reasoning about ability across the elementary school years. *Developmental Science, 9*, 616–627. https://doi.org/10.1111/j.1467-7687.2006.00540.x

Heyman, G. D., Gee, C. L., & Giles, J. W. (2003). Preschool children's reasoning about ability. *Child Development, 74*, 516–534. https://doi.org/10.1111/1467-8624.7402013

Hulleman, C. S., Durik, A. M., Schweigert, S. A., & Harackiewicz, J. M. (2008). Task values, achievement goals, and interest: An integrative analysis. *Journal of Educational Psychology, 100*(2), 398–416. https://doi.org/10.1037/0022-0663.100.2.398

Jacobs, J., Lanza, S., Osgood, D. W., Eccles, J. S., & Wigfield, A. (2002). Ontogeny of children's self-beliefs: Gender and domain differences across grades one through 12. *Child Development, 73*, 509–527. https://doi.org/10.1111/1467-8624.00421

Jarrell, A., Harley, J. M., & LaJoie, S. P. (2016). The link between achievement emotions, appraisals, and task performance: Pedagogical considerations for emotions in CBLEs. *Journal of Computers in Education, 3*, 289–307. http://dx.doi.org/10.1007/s40692-016-0064-3

Jarrell, A., Harley, J. M., LaJoie, S. P., & Naismith, L. (2017). Success, failure and emotions: Examining the relationship between performance feedback and emotions in diagnostic

reasoning. *Educational Technology Research and Development, 65*, 1263–1284. http://dx.doi.org/10.1007/s11423-017-9521-6

Jussim, L., Robustelli, S. L., & Cain, T. R. (2009). Teacher expectations as self-fulfilling prophecies. In K. R. Wentzel, & A. Wigfield (Eds.). *Handbook of motivation at school* (pp. 381–410). New York, NY: Routledge.

Larsen, J. T., & McGraw, A. P. (2011). Further evidence for mixed emotions. *Journal of Personality and Social Psychology, 100*(6), 1095-1110. https://doi.org/10.1037/a0021846

Lauermann, F., Tsai, Y.-M., & Eccles, J. S. (2017). Math-related career aspirations and choices within Eccles et al.'s expectancy–value theory of achievement-related behaviors. *Developmental Psychology, 53*(8), 1540–1559. https://doi.org/10.1037/dev0000367

Lazarides, R., Dicke, A.-L., Rubach, C., & Eccles, J. S. (2020). Profiles of motivational beliefs in math: Exploring their development, relations to student-perceived classroom characteristics, and impact on future career aspirations and choices. *Journal of Educational Psychology, 112*(1), 70–92. https://doi.org/10.1037/edu0000368

Lazarides, R., Dietrich, J., & Taskinen, P. H. (2019). Stability and change in students' motivational profiles in mathematics classrooms: The role of perceived teaching. *Teaching and Teacher Education, 79*, 164-175. https://doi.org/10.1016/j.tate.2018.12.016

Lee, J. (2012). College for all: Gaps between desirable and actual P-12 math achievement trajectories for college readiness. *Educational Researcher, 41*(2), 43-55. https://doi.org/10.3102/0013189X11432746

Magnusson, D. (1998). *The logic and implications of a person-oriented approach*. Sage Publications, Inc.

Magnusson, D. (2003). The person approach: Concepts, measurement models, and research strategy. New Directions for Child and Adolescent Development, 2003(101), 3–23. https://doi.org/10.1002/cd.79

Marsh, H. W. (2007). *Self-concept theory, measurement and research into practice: The role of self-concept in educational psychology*. Leicester, England: British Psychological Society.

Meece, J. L., Wigfield, A., & Eccles, J. S. (1990). Predictors of math anxiety and its consequences for young adolescents' course enrollment intentions and performances in mathematics. *Journal of Educational Psychology, 82*, 60–70. https://doi.org/10.1037/0022-0663.82.1.60.

Midgley, C., Feldlaufer, H., & Eccles, J. S. (1989). Student/teacher relations and attitudes toward mathematics before and after the transition to junior high school. *Child Development, 60*, 981–992. https://doi.org/10.2307/1131038

MIND Research Institute. (2017). Mind Research Institute: Learn math visually. Retrieved from http://mindresearch.org

Möller, J., Pohlmann, B., Köller, O., & Marsh, H. W. (2009). A meta-analytic path analysis of the internal/external frame of reference model of academic achievement and academic self-concept. *Review of Educational Research, 79*(3), 1129–1167. https://doi.org/10.3102/0034654309337522

Muenks, K., Wigfield, A., & Eccles, J. S. (2018). I can do this! The development and calibration of children's expectations for success and competence beliefs. *Developmental Review, 48*, 24-39. https://doi.org/10.1016/j.dr.2018.04.001

Musu-Gillette, L. E., Wigfield, A., Harring, J. R., & Eccles, J. S. (2015). Trajectories of change in students' self-concepts of ability and values in math and college major choice. *Educational Research and Evaluation, 21*(4), 343-370. https://doi.org/10.1080/13803611.2015.1057161

Nagengast, B., Marsh, H. W., Scalas, L. F., Xu, M. K., Hau, K.-T., & Trautwein, U. (2011). Who took the "×" out of Expectancy–Value Theory? A psychological mystery, a substantive-methodological synergy, and a cross-national generalization. *Psychological Science, 22*(8), 1058-1066. https://doi.org/ 10.1177/0956797611415540

Nagengast, B., Trautwein, U., Kelava, A., & Lüdtke, O. (2013). Synergistic effects of expectancy and value on homework engagement: The case for a within-person perspective. *Multivariate Behavioral Research, 48*(3), 428-460. https://doi.org/10.1080/00273171.2013.775060

Nagy, G., Trautwein, U., Baumert, J., Köller, O., & Garrett, J. (2006). Gender and course selection in upper secondary education: Effects of academic self-concept and intrinsic value. *Educational Research and Evaluation, 12*, 323–345. http://doi.org/10.1080/13803610600765687

Nicholls, J. G. (1978). The development of the concepts of effort and ability, perceptions of academic attainment, and the understanding that difficult tasks require more ability. *Child Development, 49*, 800–814. https://doi.org/10.2307/1128250

Normandeau, S., & Gobeil, A. (1998). A developmental perspective on children's understanding of causal attributions in achievement-related situations. *International Journal of Behavioral Development, 22*, 611–632.

Oyserman, D. (2014). Identity-based motivation: Core processes and intervention examples. In S. Karabenick & T. Urdan (Eds.), *Motivational interventions* (Vol. 18: Advances in motivation and achievement, pp. 213–242). Emerald

Parsons, J. E., & Ruble, D. N. (1977). The development of achievement-related expectancies. *Child Development, 48*, 1075–1079. https://doi.org/10.2307/1128364

Part, R., Perera, H. N., Marchand, G. C., & Bernacki, M. L. (2020). Revisiting the dimensionality of subjective task value: Towards clarification of competing perspectives. *Contemporary Educational Psychology, 62*, 101875. https://doi.org/10.1016/j.cedpsych.2020.101875

Pekrun, R. (2006). The Control-Value Theory of Achievement Emotions: Assumptions, corollaries, and implications for educational research and practice. *Educational Psychology Review, 18*(4), 315–341. https://doi.org/10.1007/s10648-006-9029-9

Pekrun, R., Frenzel, A. C., Goetz, T., & Perry, R. P. (2007). The control-value theory of achievement emotions. In P. A. Schutz & R. Pekrun (Eds.), *Emotion in Education: A Volume in the Educational Psychology Series*, 13–36. Elsevier Inc. https://doi.org/10.1016/B978-012372545-5/50003-4

Pekrun, R., Lichtenfeld, S., Marsh, H. W., Murayama, K., & Goetz, T. (2017). Achievement emotions and academic performance: Longitudinal models of reciprocal effects. *Child Development, 88*(5), 1653–1670. https://doi.org/10.1111/cdev.12704

Pekrun, R., & Perry, R. P. (2014). Control-value theory of achievement emotions. In R. Pekrun & L. Linnenbrink-Garcia (Eds.) *International handbook of emotions in education* (pp. 120-141). Routledge.

Pekrun, R., & Stephens, E. J. (2010). Achievement emotions: A Control-Value approach. *Social and Personality Psychology Compass, 4*(4), 238–255. https://doi.org/10.1111/j.1751-9004.2010.00259.x

Pomerantz, E. M., & Saxon, J. L. (2001). Conceptions of ability as stable and self-evaluative processes: A longitudinal examination. *Child Development, 72*, 152–173. https://doi.org/10.1111/1467-8624.00271

Raccanello, D., Hall, R., & Burro, R. (2018). Salience of primary and secondary school students' achievement emotions and perceived antecedents: Interviews on literacy and mathematics domains. *Learning and Individual Differences, 65*, 65–79. http://doi.org/10.1016/j.lindif.2018.05.015

Robinson, K. A., Beymer, P. N., Ranellucci, J., & Schmidt, J. A. (2020, February 27). Momentary emotion profiles in high school science and their relations to control, value, achievement, and science career intentions. *Motivation Science.* https://doi.org/10.1037/mot0000174

Robinson, K. A., Ranellucci, J., Lee, Y., Wormington, S. V., Roseth, C. J., & Linnenbrink-Garcia, L. (2017). Affective profiles and academic success in a college science course. *Contemporary Educational Psychology, 51*, 209–221. http://doi.org/10.1016/j.cedpsych.2017.08.004

Roos, A.-L., Goetz, T., Voracek, M., Krannich, M., Bieg, M., Jarrell, A., & Pekrun, R. (2020). Test anxiety and physiological arousal: A systematic review and meta-analysis. *Educational Psychology Review*. https://doi.org/10.1007/s10648-020-09543-z

Rosenzweig, E. Q., & Wigfield, A. (2016). STEM motivation interventions for adolescents: A promising start, but further to go. *Educational Psychologist, 51*(2), 146-163. https://doi.org/10.1080/00461520.2016.1154792

Ryan, R. M., & Deci, E. L. (2017). *Self-determination theory: Basic psychological needs in motivation, development, and wellness*. Guilford Publications.

Simpkins, S. D., Davis-Kean, P. E., & Eccles, J. S. (2006). Math and science motivation: A longitudinal examination of the links between choices and beliefs. *Developmental Psychology, 42*(1), 70–83. https://doi.org/10.1037/0012-1649.42.1.70.

Spinath, B., & Spinath, F. M. (2005). Longitudinal analysis of the link between learning motivation and competence beliefs among elementary school children. *Learning and Instruction, 15*(2), 87-102. https://doi.org/10.1016/j.learninstruc.2005.04.008

Spinath, B., & Steinmayr, R. (2008). Longitudinal analysis of intrinsic motivation and competence beliefs: Is there a relation over time? *Child Development, 79*, 1555–1569. https://doi.org/10.1111/j.1467-8624.2008.01205.x

Trautwein, U., Marsh, H. W., Nagengast, B., Ludtke, O., Nagy, G., & Jonkmann, K. (2012). Probing for the multiplicative term in modern expectancy-value theory: A latent interaction modeling study. *Journal of Educational Psychology, 104*, 763–777. https://doi.org/10.1037/a0027470.

Tulis, M., & Ainley, M. (2011). Interest, enjoyment and pride after failure experiences? Predictors of students' state-emotions after success and failure during learning in mathematics. *Educational Psychology, 31*(7), 779-807. https://doi.org/10.1080/01443410.2011.608524

Umarji, O., McPartlan, P., & Eccles, J. (2018). Patterns of math and English self-concepts as motivation for college major selection. *Contemporary Educational Psychology, 53*, 146–158. https://doi.org/10.1016/j.cedpsych.2018.03.004

U.S. Bureau of Labor Statistics (2020). *Employment in STEM occupations.* https://www.bls.gov/emp/tables/stem-employment.htm

U.S. Congress Joint Economic Committee (2012). *STEM education: Preparing for jobs of the future.* https://www.jec.senate.gov/public/_cache/files/6aaa7e1f-9586-47be-82e7-326f47658320/stem-education---preparing-for-the-jobs-of-the-future-.pdf

U.S. Department of Commerce (2011). *STEM: Good jobs now and for the future.* https://www.commerce.gov/sites/default/files/migrated/reports/stemfinalyjuly14_1.pdf

Villavicencio, F. T., Bernardo, A. B., & I. (2016). Beyond math anxiety: Positive emotions predict mathematics achievement, self-regulation, and self-efficacy. *The Asia- Pacific Education Researcher; Manila, 25*(3), 415–422. http://doi.org/10.1007/s40299-015-0251-4

Wang, M.-T., & Eccles, J. S. (2013). School context, achievement motivation, and academic engagement: A longitudinal study of school engagement using a multidimensional perspective. *Learning and Instruction, 28*, 12–23. https://doi.org/10.1016/j.learninstruc.2013.04.002

Wang, M.-T., Eccles, J. S., & Kenny, S. (2013). Not lack of ability but more choice: Individual and gender differences in choice of careers in science, technology, engineering, and mathematics, *Psychological Science, 24*(5), 770–775. https://doi.org/10.1177/0956797612458937

Weiner, B. (1985). An attributional theory of achievement motivation and emotion. *Psychological Review, 92*(4), 548–573. https://doi.org/10.1037/0033-295x.92.4.548

Wigfield, A. (1994). Expectancy-value theory of achievement motivation: A developmental perspective. *Educational Psychology Review, 6*(1), 49–78. https://doi.org/10.1007/bf02209024

Wigfield, A., & Eccles, J. S. (2000). Expectancy–Value Theory of Achievement Motivation. *Contemporary Educational Psychology, 25*(1), 68–81. https://doi.org/10.1006/ceps.1999.1015

Wigfield, A., & Eccles, J. S. (2020). 35 Years of research on students' subjective task values and motivation: A look back and a look Forward. In A. Elliot (Vol. Ed.), *Advances in motivation science: Vol. 7,* (pp. 162–193). Elsevier.

Wigfield, A, Eccles, J. S., Fredricks, J., Simpkins, Roeser, R., & Schiefele, U. (2015). Development of achievement motivation and engagement. In R. Lerner (Series ed.) and M. Lamb & C. Garcia Coll (Vol. Eds.), *Handbook of child psychology and developmental science* (7th ed., pp. 657–700). New York, NY: Wiley.

Wigfield, A., Eccles, J. S., Yoon, K. S., Harold, R. D., Arbreton, A., Freedman-Doan, C., et al. (1997). Changes in children's competence beliefs and subjective task values across the elementary school years: A three-year study. *Journal of Educational Psychology, 89*, 451–469. https://doi.org/10.1037/0022-0663.89.3.451

Wigfield, A., Tonks, S. M., & Klauda, S. L. (2016). Expectancy-value theory. In K. R. Wentzel, & D. B. Miele (Eds.). *Handbook of motivation at school* (pp. 55–74). (2nd ed.). Routledge

CHAPTER TWO: STUDY ONE

Abstract

Motivation is key to students' engagement and achievement, yet it is known to decline, especially in mid to late elementary school. Research on how motivation changes across school years is ubiquitous, yet little research examines changes in motivation within the school year, even though that is when motivation is most likely to change because of school context, teachers, and peers. In this study I model how elementary students' mathematics motivation changes over two years using six timepoints (beginning, middle, and end of the two school years). I compared a linear model to an unstructured model that allowed for the trends in motivation to take any shape. On average, expectancy and value both decreased linearly over the two years; however, the unstructured models revealed within-year variation, with motivation often peaking at the beginning of the school year and dipping in the middle. The unstructured model had better fit than the linear model. Although motivation declined over the two years, it remained relatively high—the average never dropping below four on a five-point scale.

Keywords: Situated Expectancy–Value Theory, mathematics, longitudinal, multilevel models

Models of growth: Longitudinal Expectancy–Value in Elementary Mathematics

Motivation is critical for success in mathematics—it influences students' engagement and achievement, which can influence future success in mathematics and pursual of a STEM career (Eccles & Wigfield, 2020; Singh et al., 2002; Wigfield & Eccles, 2000). It is a common finding that motivation for certain subjects, including mathematics, declines late in middle childhood (e.g., Wigfield, 1994; Wigfield & Cambria, 2010). However, this decline may not be linear nor monolithic; student motivation depends on context (Eccles & Wigfield, 2020) and can fluctuate, increasing and decreasing, both across and within school years (e.g., Archambault et al., 2010; Musu-Gillette et al., 2015). For example, teachers' communications of expectations, teaching style, and differential treatment of students all happen within a school year and impact motivation (e.g., Bartholomew et al., 2018; Eccles, 2012; Maulana et al., 2016; Pitzer & Skinner, 2017; Wigfield et al., 2015). Examining motivation within Eccles' and Wigfield's (2020) Situated Expectancy–Value Theory, I investigate how students' expectancy and value (utility/importance) for mathematics changes throughout two years in middle childhood, measured at six timepoints. Using multilevel mixed-effects linear regression, I compare a model that assumes motivation follows a linear trend across time to one that does not constrain the data to a singular shape.

Theoretical Framework: Situated Expectancy–Value Theory

Recently, Eccles and Wigfield (2020) relabeled Expectancy–Value Theory (Eccles (Parsons) et al., 1983) as Situated Expectancy–Value Theory (SEVT). This transition draws attention to the dynamic and developmental nature of expectancies for success and subjective task values. *Expectancy* represents whether a person believes they can successfully complete a task, and subjective task value encompasses how useful, important, and interesting a task is to the individual, as well as the extent to which participating in the task brings a cost (e.g., effort,

emotional cost; Eccles & Wigfield, 2020; Eccles (Parson) et al., 1983). When a person believes they can do well on a task and when they value the task, they are more likely to engage with the task and perform well (Eccles & Wigfield, 2020; Nolen, 2020).

In considering three components of subjective task value, *utility value* is the usefulness of the task to reach a person's goal or plan (Eccles & Wigfield, 2020). For example, a student may value math because they want to become a scientist and doing well in math will help them achieve that goal. Utility value often resembles extrinsic motivation (i.e., completing a task as a means to an end rather than for personal enjoyment/interest; Ryan & Deci, 2016); however, Eccles and Wigfield (2020) propose that utility value may be deeper than extrinsic motivation, because it can be tied to individually determined goals and a sense of self, rather than just an outcome. *Attainment value* is how well the task aligns with a person's sense of sense (i.e., the task is important to their identity; Eccles & Wigfield, 2020). In contrast to the example of utility value, a student may value math because they see themself as a mathematician. *Intrinsic value* is the enjoyment of and interest in the given task; this component of subjective task value resembles intrinsic motivation and interest (Eccles & Wigfield, 2020). These three components of subjective task values are closely tied to one another, especially if a task is central to a person's goals and identity—a mathematician will likely have high utility, attainment, and intrinsic value for a math task because it (1) may push them further in the field or get them closer to publishing a paper, (2) is important to their identity as a mathematician, and (3) they have a sustained interest and enjoyment of math. Contrary to these components, *cost* is the negative aspect of subjective task values representing the effort needed to complete the task (effort cost), what a person may miss because they are doing this task (opportunity cost), and the emotional toll of the task (e.g., anxiety, frustration, disappointment; Eccles & Wigfield, 2020; Flake et al., 2015). Students' expectancies and values, as well as the

relative weight of each of these components, vary across time and contexts (Eccles & Wigfield, 2020).

Development of Expectancies and Values

Research on the development of SEVT has focused on how students distinguish their expectancies and values within and between domains (e.g., Gaspard et al., 2020; Wigfield et al., 2020), as well as how these expectancies and values change over time (e.g., Butler, 2005; Jacobs et al., 2002; Wigfield, 1994). By studying the differences in expectancies and values between and across ages (i.e., cross sectional and longitudinal data, respectively), researchers can gain a better understanding of how SEVT develops.

Differentiating SEVT

Components of SEVT. In order to understand how SEVT develops, researchers must also know how and if students differentiate between the different components of SEVT. From a young age (as early as first grade), students are able to differentiate between their competence beliefs (expectancies) and subjective task values (Eccles et al., 1993). However, Wigfield (1994) proposed that developing and differentiating the different components of subjective task value may not happen as early, suggesting that interest develops first, likely due to socialization with toys and activities, whereas attainment value and utility value develop in middle childhood and adolescence. Using factor analyses, Eccles et al. (1993) found that utility, attainment, and intrinsic value formed three distinct but related factors in fifth graders and beyond (cost was not measured). However, in early elementary school students, only two factors emerged—interest (intrinsic value) was distinct, but utility and importance (attainment value) loaded onto the same factor (Wigfield et al., 1997). The distinction between usefulness and importance is likely difficult for children in

early elementary school because much of identity formation—critical for attainment value—does not take place until adolescence (Eccles, 2009; Meeus et al., 2010; Oyserman, 2014).

The lack of differentiation in attainment and utility value has also been corroborated using cognitive interviews on measures specifically focused on the importance and usefulness of math (Rutherford et al., 2019b). When asked to define usefulness and importance in the context of math, many children (ages seven to 12) struggled, with some saying they are the same thing: "important is if it is useful" (Rutherford et al., 2019b). Karamarkovich and Rutherford (2018, 2019) also found that elementary students had similar patterns of future utility and attainment value that differed from the patterns of current utility and attainment value, suggesting that time differentiated these measures, rather than the type of value (i.e., future usefulness and future importance were more similar than current usefulness and future usefulness). Due to the substantial overlap in utility and attainment value, it is common for researchers to combine them in analyses (e.g., Archambault et al., 2010; Durik et al., 2006; Musu-Gillette et al., 2015; Simpkins et al., 2006). In addition to examining how the components of SEVT are different from each other, researchers have also examined how SEVT is different between domains.

Dimensional Comparison of SEVT. From a young age, students can also differentiate their expectancies and value between domains—they can make dimensional comparisons (Marsh et al., 2002). Dimensional comparisons—"individuals' comparison of how they do on one set of activities (e.g., their math performance) with how they do on another set (e.g., verbal performance)" (Wigfield et al., 2020)—are sources of information that help people develop their SEVT (Eccles & Wigfield, 2020). Eccles et al. (1993) found that students in first grade had distinct factors of value and expectancy for different subjects. Additionally, Marsh and colleagues (2002) found that children as young as four years old were able to distinguish their self-concept across

multiple domains (verbal, math, and other non-academic self-concepts). However, the correlation between these young children's math and verbal self-concepts were far larger than those of children in early elementary school and beyond (Marsh et al., 2002). This suggests that effects of dimensional comparison may be stronger as students get older. Different components of subjective task value may also be more susceptible to dimensional comparisons than others, especially those tied to ability (Gaspard et al., 2018). For example, Gaspard and colleagues (2018) found that the motivational components that had the strongest relation to self-concept, such as effort cost and intrinsic value, also had the highest levels of domain specificity.

SEVT Between and Across Ages

One long-standing accepted truism among motivation research is that academic motivation, including motivation for mathematics, declines in middle childhood (Wigfield & Eccles, 2020). Early studies posited that expectancies develop through children's understanding of the cause and effect of ability and effort on performance (Nicholls, 1978; Nicholls & Miller, 1980). More modern theories suggest that as students develop, they become better calibrated with their expectancies (i.e., there is little difference between expected and actual performance; Alexander, 2013). Thus, students may have a less optimistic and more realistic expectancy for achievement (Muenks et al., 2018; Wigfield et al., 2015).

The concept of developing a more "realistic" self-image is also posited in Eccles and Wigfield's (2020) rebranding of SEVT, where they state that "children’s ASCs [academic self-concepts] and STVs [subjective task values] likely become much more sophisticated, conscious, and stable as children mature and develop more conscious and reasoned personal identities, goals, and worldviews" (p. 7). Examining subjective task values between grades (fifth through 12th graders), Gaspard and colleagues (2017) found that the older students consistently had lower value

for mathematics, as well as other domains. These findings are replicated using longitudinal data as well—Jacobs and colleagues (2002) found a decline in math expectancies and values from first to 12th grade using cross-sequential data.

Recent research has also used person-centered analyses to examine different patterns of growth, rather than traditional linear and mean-level analyses, generally using growth mixture modeling (e.g., Archambault et al., 2010; Musu-Gillette et al., 2015). Two studies—Archambault et al.'s (2010) study on literacy motivation and Musu-Gillette et al.'s (2015) on mathematics motivation—used growth mixture modeling to find patterns in both self-concept of ability and subjective task value from elementary to high school. Both studies found some trajectories that appeared linear, whereas others were curved. For example, Archambault and colleagues (2010) found a parabolic trajectory that quickly declined until high school and then rebounded slightly and another that remained stable in elementary school but declined in middle school. By using growth mixture modeling, researchers have found the change in motivation across years is not linear nor monolithic.

School Context and Motivation

Although there is little research on how motivation fluctuates during the school year, there is strong theoretical grounding to suggest it would. SEVT posits that school environment plays a crucial role in students' motivation and is infused throughout the model, not just as a predictor of motivation (Eccles & Wigfield, 2020). A primary socializer for students and students' beliefs are their teachers. In particular, teachers communicate their expectancies for their students, often through differential treatment, and these expectations are a direct influence on students' own expectations for their success (see Muenks et al., 2018).

Additionally, teaching style and instructional behaviors are related to expectancies, subjective task values, engagement, and motivational resilience (e.g., Bartholomew et al., 2018; Eccles, 2012; Maulana et al., 2016; Pitzer & Skinner, 2017; Wigfield et al., 2015). In one of the few studies that used more than one timepoint within a school year, Maulana and colleagues (2016) examined how adolescent students (ages 11-13) perceived teachers behavior was related to their motivation throughout the school year. At five timepoints throughout the school year, they measured how students perceived their math and English as a foreign language teachers' clarity of instruction, classroom management, strong teacher control, shared control, and loose teacher control, as well as students' self-efficacy, intrinsic value, and test anxiety. Students' intrinsic value largely decreased during the year but peaked in the middle of the school year, whereas self-efficacy linearly decreased throughout the school year. Students' test anxiety spiked in the second month of the school year then continued to increase. Teachers' instructional behaviors were perceived to decrease throughout the year, although strong and shared control peaked in the fourth month before continuing to decline. All five of the perceived teachers' instructional behaviors positively predicted intrinsic value, and all but strong control was positively related to students' self-efficacy. Only clarity of instruction and shared control were related to test anxiety, such that when students did not perceive that the instructions were clear and felt they had some control, they experienced more test anxiety (Maulana et al., 2016).

Beyond teachers, there are other school contexts that can influence motivation, such as peers (Eccles & Wigfield, 2020; Ladd et al., 2009; Wentzel et al., 2010) and high-stake testing (Amrein & Berliner, 2003; Roderick & Engel, 2001; Ryan & Weinstein, 2009). In Florida, where this study is situated, high stakes end-of-grade summative testing begins at third grade (Florida Standards Assessments; Florida Department of Education, 2018, 2019). My sample consists of

two cohorts: one that begins in third grade and transitions to fourth and one that begins in fourth grade and transitions to fifth; therefore, it is possible that end-of-year motivation is associated with high stakes test-taking. Other elements of the school year may also play a role: special events, school holidays, and changes in instructional focus may be linked with time-points throughout the year.

Prior research has focused almost exclusively on the change in motivation by grade-level or on a yearly basis (*cf.* Maulana et al., 2016). Although this provides important information on general trends and long-term trajectories, it ignores how motivation may change within the school year. Given the shifting achievement-related context even within the academic year, it is likely that motivation fluctuates both between (i.e., by grade-level/age) and within school years. To examine this fluctuation, I utilize both unstructured and linear frameworks to model change in motivation at six time-points across two years.

Current Study

Examining motivation within Eccles' and Wigfield's (2020) SEVT, I investigate how expectancy and value (herein utility/importance) for mathematics changes throughout two years, measured at six timepoints, using multilevel mixed-effects regression. I compare two models: a linear model and one where time is "unstructured," allowing trends in expectancies and value to take any shape. I specifically ask:

2a. What are the trends in mathematics expectancies, current value, and future value across two years?

2b. Do these trends differ by age/cohort (i.e.., third-fourth graders vs fourth-fifth graders)?

2c. Which model (linear or unstructured) best fits the data?

Method

Context

This study is part of a larger NSF-funded project using embedded assessments and data-mining techniques to understand student and teacher use of Spatial Temporal (ST) Math, a digital interactive mathematics software created by MIND Research Institute. ST Math is currently used in 48 states with over 1.2 million students and is designed to align to those states' standards (MIND Research Institute, 2018). Prior research has found that ST Math has a small effect on mathematics achievement and improves student mathematics self-beliefs, especially for lower-performing students (Rutherford et al., 2014, 2019a; Schenke et al., 2014). As part of the current project, motivation survey questions were designed by MIND in consultation with the project researchers and were embedded within ST Math. It is these surveys that are the subject of the current study.

Participants

Participants were third through fifth grade elementary students from a school district in Florida that used ST Math as part of regular mathematics instruction (N = 14,818). I used a cross-sequential design with two cohorts: cohort one was students who were in third graders in the first year (2017-2018) and fourth graders in the second year (2018-2019); cohort two was students who were in fourth graders in the first year (2017-2018) and fifth graders in the second year (2018-2019). This sample was limited to students who had complete data—6,898 were excluded because they did not respond to all six surveys across the two years (N = 7,920, 53%) and an additional 32 were excluded because they did not have demographic information (N = 7,888, 43%). Of those 6,898 excluded for missing survey data, 1,205 did not have any survey data, 63 responded to one survey, 286 responded to two surveys, 1,269 responded to three surveys, 1,133 responded to four surveys, and 2,942 responded to five surveys. Excluding those students dropped the sample size

to 4,270 in cohort one (third-fourth graders; 57% of the total cohort one) and 3,618 in cohort two (fourth-fifth graders; 49% of the total cohort two). Overall, *t*-tests comparing the analysis sample to the overall sample indicated that the analysis sample had more students in cohort one, girls, and English Language Learners than those excluded from the sample. Additionally, there were more White students and fewer Black students in the analysis sample. The analysis sample was approximately half girls and the majority of students in the sample were White (52%) and qualified for free or reduced lunch (75%), a proxy measure of socio-economic status. See Table 2.1 for sample demographics and comparison of analysis and excluded sample by grade.

Table 2.1

Demographics of Total and Analysis Sample

	Cohort One Third-Fourth Graders			Cohort Two Fourth-Fifth Graders		
	Sample	Total	*p*-value	Sample	Total	*p*-value
Whether a Boy	51%	51%	.2195	50%	52%	**.0011**
Disability	16%	16%	.7358	14%	13%	.3384
English Language Learner	14%	13%	**.0003**	11%	12%	.3050
Free/Reduced Lunch	75%	76%	.0759	74%	74%	.7682
Race						
Black	18%	22%	**< .0001**	18%	20%	**< .0001**
Hispanic	19%	18%	.0525	17%	18%	.1228
White	54%	51%	**< .0001**	55%	52%	**< .0001**
Other	9%	9%	.3368	10%	10%	.9854
N	4,270	6,937		3,618	6,961	

Note. *p*-value from *t*-test for proportion comparing those included in the sample to those excluded. Statistically significant *p*-values bolded ($p < .05$).

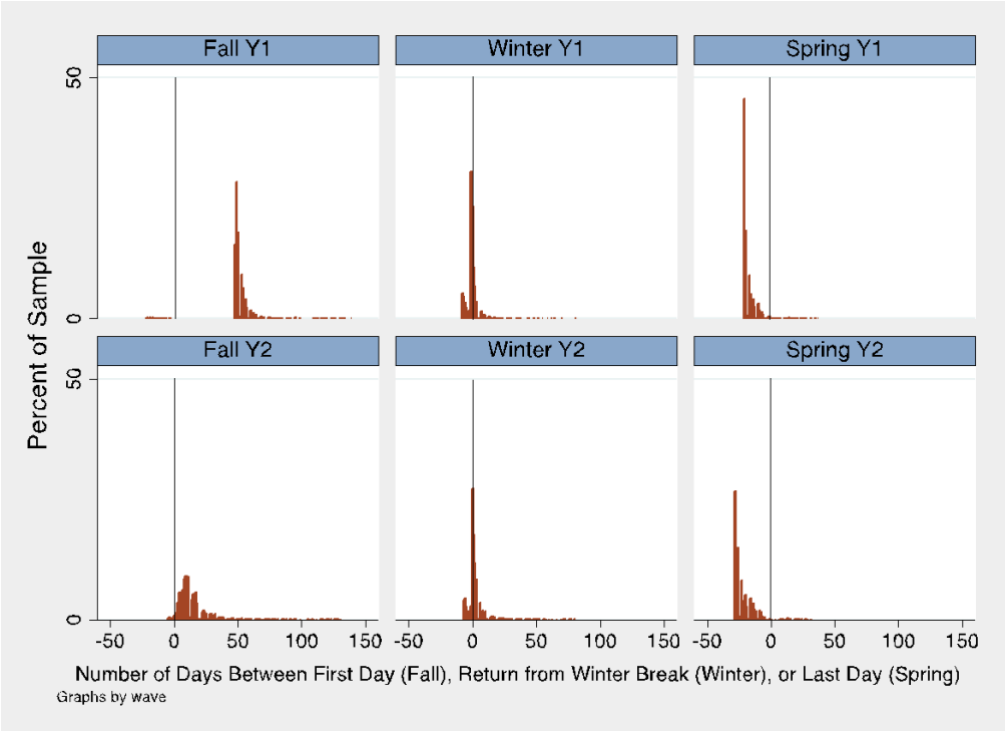
Measures

Surveys were distributed through ST Math three times during the school year for two years (2017-2018 and 2018-2019). Students responded to the survey the first time they logged onto ST Math at the beginning of the school year and the first time they logged on after winter break. The students received the final survey of the year approximately one month before the end of the school year. See Figure 2.1 for the distribution of when the survey was taken, as compared to the critical

days (i.e., the first day of school, returning from winter break, and the last day of school for fall, winter, and spring, respectively). The survey was designed with the theoretical framework of SEVT (Eccles & Wigfield, 2020) and modeled after existing surveys for elementary students, such as those in Fredricks and Eccles (2002). The measured a variety of outcomes, including self-concept for academic subjects, subject favoritism, and expectancies and values for mathematics. For this study, only expectancies and values are used.

Figure 2.1

Number of Days Between First Day (Fall), Return from Winter Break (Winter), or Last Day (Spring)



Wave	Days between when the survey taken and…	Mean	Median	SD	Min	Max
Fall Y1	first day of school 2017	51.23	50	13.21	-22	139
Winter Y1	return from winter break 2018	-0.32	-1	5.39	-8	81
Spring Y1	last day of school 2018	-17.54	-20	6.51	-21	37
Fall Y2	first day of school 2018	15.62	11	15.72	-5	131
Winter Y2	return from winter break 2019	2.34	1	8.47	-7	80
Spring Y2	last day of school 2019	-22.25	-26	7.77	-28	32

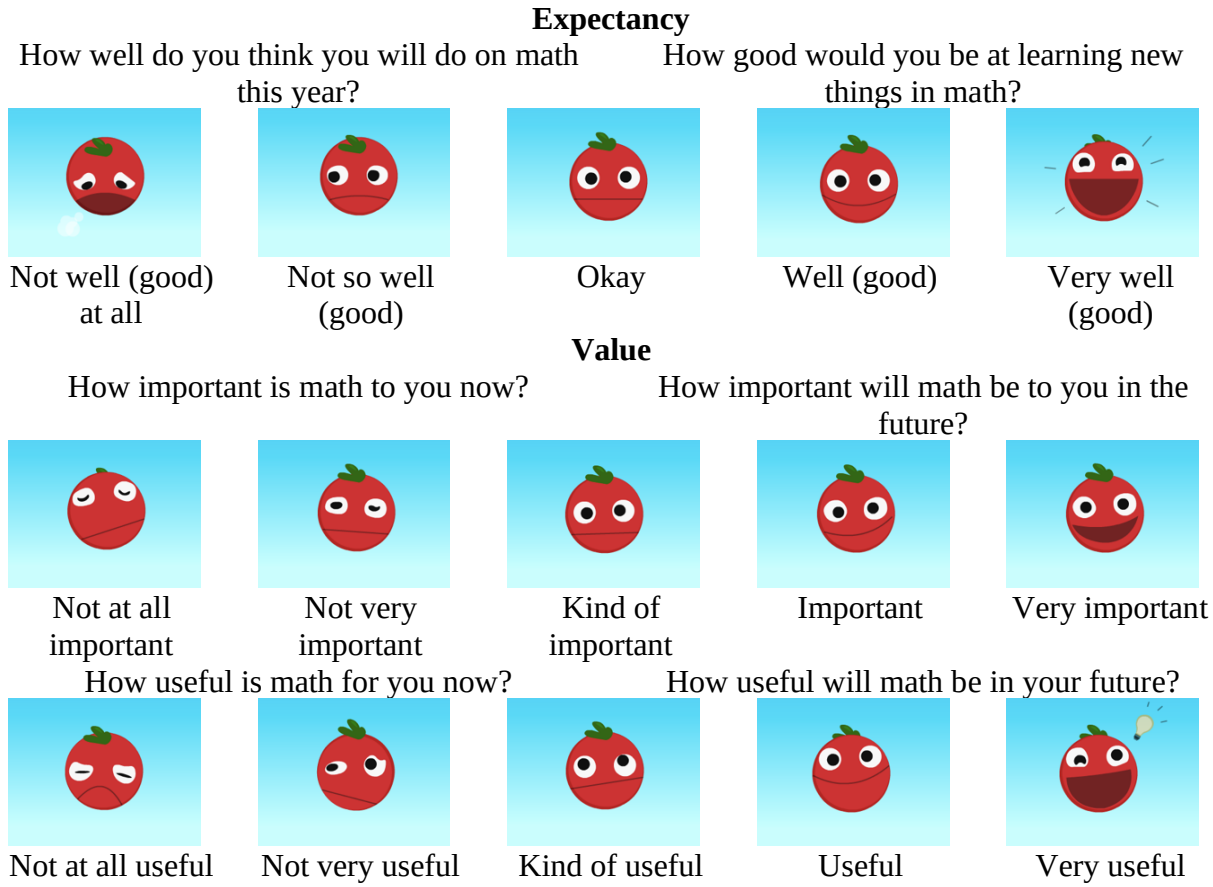
Note. Vertical line represents the first day of the school year for Fall waves, return from winter break for Winter waves, and last day of school for Spring waves.

Expectancy

Students' mathematics expectancy was measured using two questions: "How well do you think you will do on math this year?" and "How good would you be at learning new things in math?" Both questions were presented with Likert-like scale options of one to five, with one being "Not well [good] at all" and five being "Very well [good]." Emoji-like tomatoes were also displayed with faces that changed based on students' answers (see Figure 2.2). Expectancy for each timepoint was created by taking the mean of responses to both expectancy questions (Cronbach's α = .74; Ordinal α = .81, Gadermann et al., 2012).

Figure 2.2

Expectancy and Value Questions with Corresponding Tomojis



Note.
Expectancy Ordinal α = 0.81; Current Value Ordinal α = 0.82; Future Value Ordinal α = 0.89
Importance Ordinal α = 0.79; Usefulness Ordinal α = 0.77 (see Gadermann et al., 2012)

Value

Mathematics value was measured with questions about usefulness and importance. These questions followed a similar format to the expectancy questions—five-point Likert-like scales with tomato emojis. Students in elementary school often have difficulty distinguishing between utility value and attainment value (Wigfield & Cambria, 2010); however, using these measures students have instead displayed different patterns of current and future value (see Karamarkovich & Rutherford, 2018, 2019). For these reasons, I created current value and future value variables rather than importance and usefulness variables.

The current value variable was an average of responses to “How useful is math for you now?” and “How important is math to you now?” (Cronbach's α = .76; Ordinal α = .82). Similarly, I created a future value by averaging responses to “How useful will math be in your future?” and “How important will math be to you in the future?” (Cronbach's α = .82; Ordinal α = .89). See Figure 2.2 for questions.

Analyses

Preliminary Analyses

Preliminary analyses included (1) summary statistics by grade and timepoint, (2) zero-order correlations between expectancy, current value, future value, and demographics, and (3) estimation of a null model to determine variation at the within- and between-person levels. I ran a null model for each component of motivation (i.e., expectancy, current value, and future value), using motivation as the dependent variable:

Within-Person: $\text{Motivation}_{it} = \beta_{0it} + r_{it}$
Between-Person: $\beta_{0i} = \gamma_{00} + u_{0i}$

In these models, β_{0it} is average motivation for each student, r_{it} is the variability of motivation within each student, γ_{00} is the average motivation across all students and timepoints, and u_{0i} is the average variability of motivation between students. All analyses detailed below were conducted using Stata 14 (StataCorp, 2015).

Research Question 2A

In this study, the overarching research question is: *What are the trends in expectancies, current value, and future value across two years?* I examined these measures at six timepoints to understand how motivation changes both within and between school years. To answer this question, I ran two sets of multilevel random slopes regressions: one with wave as a continuous variable (to force a linear pattern) and one with wave as a factor variable (unconstrained to pattern). In the first set of models, I treated wave as a continuous variable to identify linear trends:

Within-Person: $\text{Motivation}_{it} = \beta_{0it} + \beta_{1it}(\text{Wave}) + r_{it}$

Between-Person: $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{Cohort}) + \gamma_{02\text{-}07}(\text{Demographics}) + u_{0i}$

$\beta_{1i} = \gamma_{10}$

At the within-person level, β_{0it} is average motivation for each student at the first timepoint, β_{1it} is the relation between wave and motivation, and r_{it} is the residual variability of motivation within each student after accounting for wave. The first between-person equation represents the average motivation across students: γ_{00} is the average motivation across all students at the first timepoint and u_{0i} is the average variability of motivation between students. Average motivation was also adjusted for cohort (i.e., third-fourth grade compared to fourth-fifth grade; γ_{01}) and demographics (γ_{02}-γ_{07}). Demographics included qualification for free/reduced lunch (a pseudo measure for socio-economic status), status as an English Language Learner, if the student had a disability, if they were a boy, and race/ethnicity (Black, Hispanic, White, or Other). The second between-person

equation represents the average slope (i.e., relation between wave and motivation) across students: γ_{10}.

In the second set of models, I treated wave as a factor variable to allow the trend to take any shape. This unstructured format identified individual slopes for each timepoint:

Within-Person: $\text{Motivation}_{it} = \beta_{0it} + \beta_{1it}(\text{Wave 2}) + \beta_{2it}(\text{Wave 3}) + \beta_{3it}(\text{Wave 4}) + \beta_{4it}(\text{Wave 5}) + \beta_{5it}(\text{Wave 6}) + r_{it}$

Between-Person: $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{Cohort}) + \gamma_{02\text{-}07}(\text{Demographics}) + u_{0i}$

$\beta_{1i} = \gamma_{10}$

$\vdots$

$\beta_{5i} = \gamma_{50}$

At the within-person level, β_{0it} is average motivation for each student at the first timepoint; β_{1it}, β_{2it}, β_{3it}, β_{4it}, and β_{5it} are the relation between each wave and motivation (waves two through six respectively); and r_{it} is the residual variability of motivation within each student after accounting for wave. The first between-person equation represents the average motivation across students: γ_{00} is the average motivation across all students at the first timepoint, and u_{0i} is the average variability of motivation between students. As with the previous equations, γ_{01}-γ_{07} represent the average difference in motivation by cohort and demographics. The second through sixth between-person equations represents the average slope for each wave (i.e., the relation between wave two and motivation, relation between wave three and motivation, etc.) across students.

Research Question 2B

Motivation is known to decline but recent research suggests that what this trend looks like may depend on age (e.g., Archambault et al., 2010; Musu-Gillette et al., 2015). To determine if trends in motivation differed by age/cohort, cross-level interactions were added to the above equations. For linear wave, the equations were as follows:

Within-Person: $\text{Motivation}_{it} = \beta_{0it} + \beta_{1it}(\text{Wave}) + r_{it}$

Between-Person: $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{Cohort}) + \gamma_{02\text{-}07}(\text{Demographics}) + u_{0i}$

$$\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{Cohort})$$

The additional term of γ_{11} estimates the effect of cohort on the slope of wave (i.e., do third-fourth have a different linear trend in motivation compared to fourth-fifth graders). Similar cross-level interaction terms were added to the unstructured model:

Within-Person: $\text{Motivation}_{it} = \beta_{0it} + \beta_{1it}(\text{Wave 2}) + \beta_{2it}(\text{Wave 3}) + \beta_{3it}(\text{Wave 4}) + \beta_{4it}(\text{Wave 5}) + \beta_{5it}(\text{Wave 6}) + r_{it}$

Between-Person:

$$\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{Cohort}) + \gamma_{02\text{-}07}(\text{Demographics}) + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{Cohort})$$
$$\vdots$$
$$\beta_{5i} = \gamma_{50} + \gamma_{51}(\text{Cohort})$$

Like the linear models, γ_{11}- γ_{51} estimates the effect of cohort on each of the individual slopes. To determine if the interaction statistically significantly improved the model, I tested for differences in log likelihood (a measure of model fit) using χ^2 tests with parameter difference as the degrees of freedom (e.g., adding the interacting added one additional parameter to the linear model and six to the unstructured model). To better understand these differences, I also calculated percent reduction in residual variance from the null model.

Research Question 2C

Delving deeper into the trends themselves, I further asked: *Which model (linear or unstructured) best fits the data?* To determine which model best fit the data, I tested for statistically significant differences in log likelihood between the linear and unstructured models both with and without the interaction term. I also calculated percent reduction in residual variance from the null model.

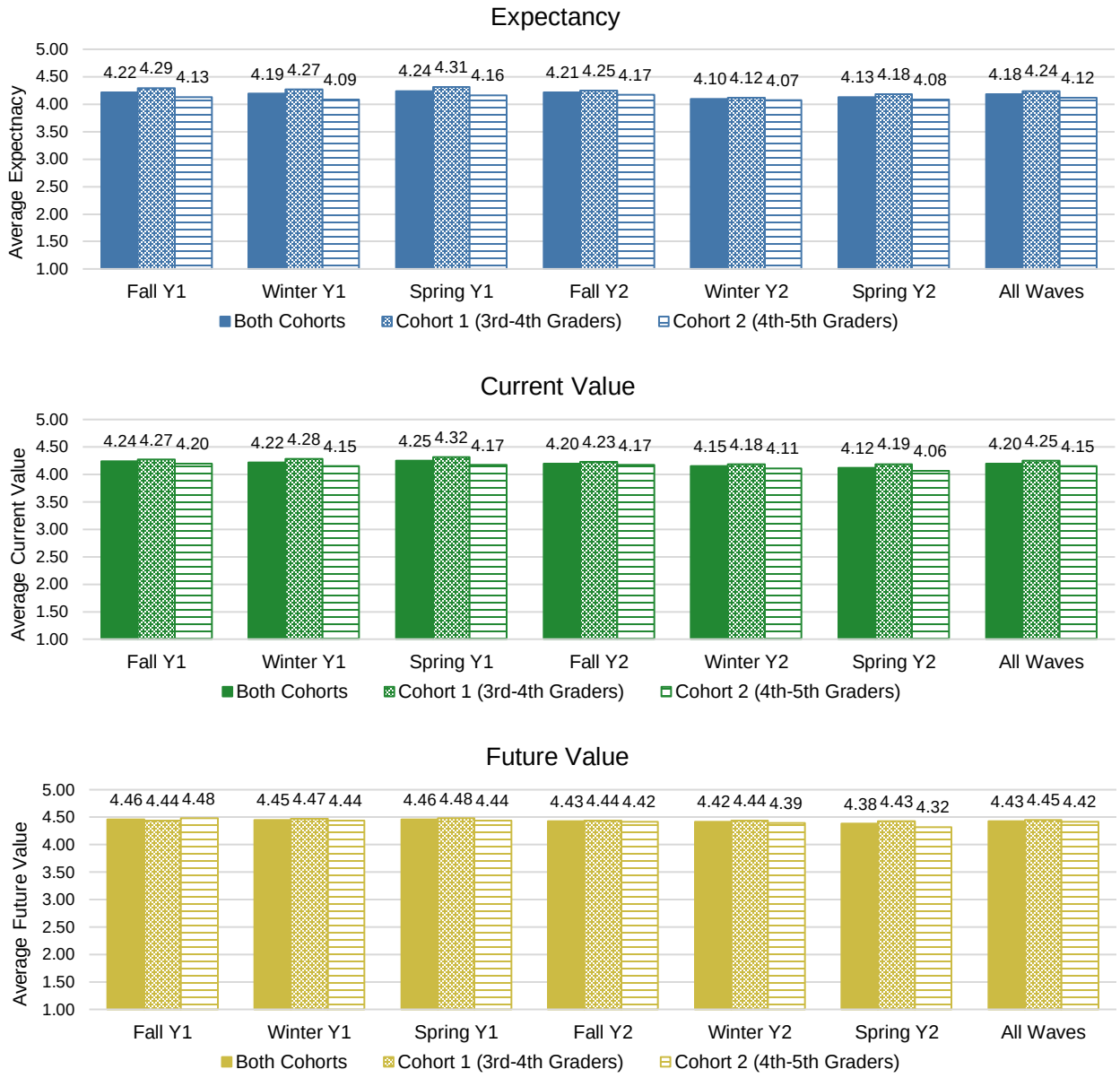
Results

Preliminary Results

When looking at the average across all timepoints, the older cohort (those who started in fourth grade, followed to fifth grade) had lower mathematics expectancy (t = -15.52, p < .001),

Figure 2.3

Average Math Expectancy, Current Value, and Future Value Across Timepoints



current math value (t = -11.80, p < .001), and future math value (t = -4.03, p < .001) than the younger cohort (those who started in third grade, followed to fourth grade). Figure 2.3 displays average motivation by grade across all timepoints. Intraclass correlations (ICC; derived from the null models) indicated that between 36% and 43% of variance in mathematics motivation was

between students. The majority of the variance was attributable to within student differences (i.e., changes in motivation over time).

Correlations largely followed the expected pattern (e.g., components of motivation—expectancy, current value, and future value—were positively correlated with each other). Student demographics were correlated with current math value (nine of the nine demographics were statistically significantly correlated with current value), as well as expectancy and future value (six out of nine demographics were statistically significantly correlated with both expectancy and future value). See Table 2.2 for full correlation results.

Table 2.2

Zero-Order Correlations of Expectancy and Value with Demographics

	Expectancy	Current Value	Future Value
Expectancy	M = 4.182 SD = 0.848		
Current Value	.5957[c]	M = 4.200 SD = 0.920	
Future Value	.4344[c]	.6321[c]	M = 4.434 SD = 0.858
Older Cohort	-.0711[c]	-.0541[c]	-.0185[c]
Whether a Boy	.0552[c]	-.0093[a]	-.0459[c]
SWD	-.0195[c]	-.0148[b]	-.0495[c]
FRL	-.0084	.0583[c]	-.0021
ELL	-.0008	.0371[c]	-.0072
Black	.0562[c]	.0766[c]	.0348[c]
Hispanic	-.0126[b]	.0227[c]	-.0012
White	-.0340[c]	-.0828[c]	-.0320[c]
Other	.0003	.0100[a]	.0101[a]

Note. Overall means and standard deviation of expectancy in value in diagonal.
[a] $p < .05$, [b] $p < .01$, [c] $p < .001$

Linear Models

Research Question 2A: Full Model

On average, math expectancy, current math value, and future math value all decreased over time (β_{Wave}'s ranging from -0.021 to -0.031, p's < .001). Additionally, the older cohort (fourth-fifth

graders) had lower math expectancy (β = -0.143, $p < .001$), current math value (β = -0.107, $p < .001$), and future math value (β = -0.041, $p = .007$) than younger cohort (third-fourth graders).

Research Question 2B: Full Model with Interaction

When examining the cross-level interaction of cohort and time (wave), results varied by motivation component. The interaction was statistically significant for math expectancy (β = 0.026, $p < .001$): math expectancy was lower on average for the older cohort (β = -0.233, $p < .001$)

Table 2.3

Linear Mixed Effects Models of Motivation on Wave and Grade (Unstandardized Coefficients)

	Expectancy		Current Value		Future Value	
	Full Model	With Interaction	Full Model	With Interaction	Full Model	With Interaction
Fixed Effects						
Wave	-0.026[c]	-0.036[c]	-0.029[c]	-0.028[c]	-0.018[c]	-0.004
Older Cohort	-0.121[c]	-0.198[c]	-0.099[c]	-0.095[c]	-0.035[b]	0.046[a]
Wave × Cohort		0.022[c]		-0.001		-0.023[c]
Boy	0.100[c]	0.100[c]	-0.011	-0.011	-0.067[c]	-0.067[c]
FRL	-0.047[b]	-0.047[b]	0.063[b]	0.063[b]	-0.022	-0.022
Disability	-0.073[c]	-0.073[c]	-0.054[a]	-0.054[a]	-0.107[c]	-0.107[c]
ELL	0.027	0.027	0.090[c]	0.090[c]	-0.010	-0.010
Days Diff.	-0.001[c]	-0.001[c]	-0.001[c]	-0.001[c]	< -0.001[a]	< -0.001
Race						
Black	0.149[c]	0.149[c]	0.203[c]	0.203[c]	0.098[c]	0.098[c]
Hispanic	0.005	0.005	0.057[a]	0.057[a]	0.032	0.032
Other	0.029	0.029	0.077[b]	0.077[b]	0.050[a]	0.050[a]
Intercept	4.293[c]	4.329[c]	4.250[c]	4.248[c]	4.553[c]	4.516[c]
Random Effects						
Intercept	0.29075	0.29082	0.35642	0.35642	0.25878	0.25886
Residual	0.41797	0.41756	0.47749	0.47749	0.47273	0.47228
Test of Statistical Significance for Model Changes						
Deviance	105990	105951	112742	112741	110329	110291
Improvement		Yes		No		Yes

Note. Unstandardized coefficients presented here. In-text coefficients are partially standardized (i.e., β is the standard deviation change in motivation for each unit change in fixed effect variables).
Days Diff: The number of days between when the survey was taken and the first day of the school year for Fall waves, return from winter break for Winter waves, and the last day of school for Spring waves. FRL: Whether the student qualifies for free or reduced-price lunch. ELL: If the student is an English Language Learner
[a] $p < .05$, [b] $p < .01$, [c] $p < .001$

and declined over time for all students ($\beta = -0.043$, $p < .001$); however, expectancy declined more drastically for the younger cohort ($\beta_{younger} = -0.043$, $p < .001$; $\beta_{older} = -0.017$, $p < .001$). The interaction was also statistically significant for future math value ($\beta = -0.026$, $p < .001$): although the older cohort had higher future math value on average ($\beta = 0.052$, $p = .015$) it declined more drastically for the older cohort than the younger cohort ($\beta_{younger} = -0.008$, $p = .016$; $\beta_{older} = -0.035$, $p < .001$). Current math value declined at a similar rate for all students ($\beta = -0.031$, $p < .001$) and was, on average, lower for the older cohort ($\beta = -0.103$, $p < .001$). The interaction was not statistically significant ($\beta = -0.001$, $p = .782$) and addition of the interaction term did not improve the linear current math value model. See Table 2.3 for all linear model results.

Unstructured Models

Research Question 2A: Full Model

The unstructured models compare all waves to the first fall wave. Math expectancy dipped in both winter waves ($\beta_{WinterY1} = -0.075$, $\beta_{WinterY2} = -0.178$, p's $\leq .001$) and the second spring wave ($\beta_{SpringY2} = -0.153$, $p < .001$). Expectancy remained similar to the first fall wave in the first spring wave ($\beta_{SpringY1} = -0.020$, $p = .486$) and the second fall wave ($\beta_{FallY2} = -0.029$, $p = .108$). Current and future math value followed the same general pattern: value decreased in the second year (β's ranging from -0.052 to -0.169, p's $\leq .007$), with students reporting the lowest value at the end of the second school year (Spring Y2). During the first year, value was similar to the Fall Y1 during the first spring wave ($\beta_{SpringY1} = -0.038$ for current value and $\beta_{SpringY1} = -0.042$ for future value, p's $\geq .136$). Current math value dipped in the first winter wave ($\beta_{WinterY1} = -0.058$, $p = .011$) but future math value did not ($\beta_{WinterY1} = -0.042$, $p = .112$). Similar to the linear models, the older cohort had lower motivation on average (b's ranging from -0.036 to -0.121, p's $\leq .007$). See Table 2.4 for all unstructured model results.

Table 2.4

Unstructured Mixed Effects Models of Motivation on Wave and Grade (Unstandardized Coefficients)

	Expectancy		Current Value		Future Value	
	Full Model	With Interaction	Full Model	With Interaction	Full Model	With Interaction
Fixed Effects						
Wave						
Winter Y1	-0.063[b]	-0.052[a]	-0.053[a]	-0.020	-0.033	0.007
Spring Y1	-0.017	-0.024	-0.035	0.008	-0.036	0.011
Fall Y2	-0.025	-0.059[b]	-0.063[c]	-0.065[b]	-0.044[b]	-0.010
Winter Y2	-0.151[c]	-0.203[c]	-0.128[c]	-0.121[c]	-0.067[b]	-0.017
Spring Y3	-0.130[c]	-0.159[c]	-0.155[c]	-0.118[c]	-0.118[c]	-0.036
Older Cohort	-0.121[c]	-0.159[c]	-0.099[c]	-0.070[b]	-0.036[b]	0.038
Wave × Cohort						
Winter Y1		-0.028		-0.058[b]		-0.066[b]
Spring Y1		0.012		-0.070[b]		-0.075[b]
Fall Y2		0.072[c]		0.016		-0.060[b]
Winter Y2		0.110[c]		-0.001		-0.090[c]
Spring Y2		0.058[b]		-0.059[b]		-0.149[c]
Boy	0.100[c]	0.100[c]	-0.011	-0.012	-0.067[c]	-0.067[c]
FRL	-0.047[b]	-0.047[b]	0.063[b]	0.063[b]	-0.022	-0.022
Disability	-0.073[c]	-0.073[c]	-0.054[a]	-0.055[a]	-0.107[c]	-0.107[c]
ELL	0.027	0.027	0.090[c]	0.091[c]	-0.010	-0.010
Days Diff.	-0.001[a]	-0.001[a]	-0.001	-0.001	-0.001	< -0.001
Race						
Black	0.149[c]	0.149[c]	0.203[c]	0.203[c]	0.099[c]	0.098[c]
Hispanic	0.005	0.005	0.057[a]	0.057[a]	0.032	0.032
Other	0.029	0.029	0.077[b]	0.077[b]	0.050[a]	0.050[a]
Intercept	4.266[c]	4.285	4.221[c]	4.201[c]	4.542[c]	4.499[c]
Random Effects						
Intercept	0.29100	0.29117	0.35648	0.35656	0.25876	0.25889
Residual	0.41645	0.41582	0.47710	0.47674	0.47260	0.47202
Test of Statistical Significance for Model Changes: Full Model v. With Interaction						
Deviance	105845	105783	112710	112681	110316	110269
Improvement		Yes		Yes		Yes

Note. Unstandardized coefficients presented here. In-text coefficients are partially standardized (i.e., β is the standard deviation change in motivation for each unit change in fixed effect variables).

Days Diff: The number of days between when the survey was taken and the first day of the school year for Fall waves, return from winter break for Winter waves, and the last day of school for Spring waves. FRL: Whether the student qualifies for free or reduced-price lunch. ELL: If the student is an English Language Learner

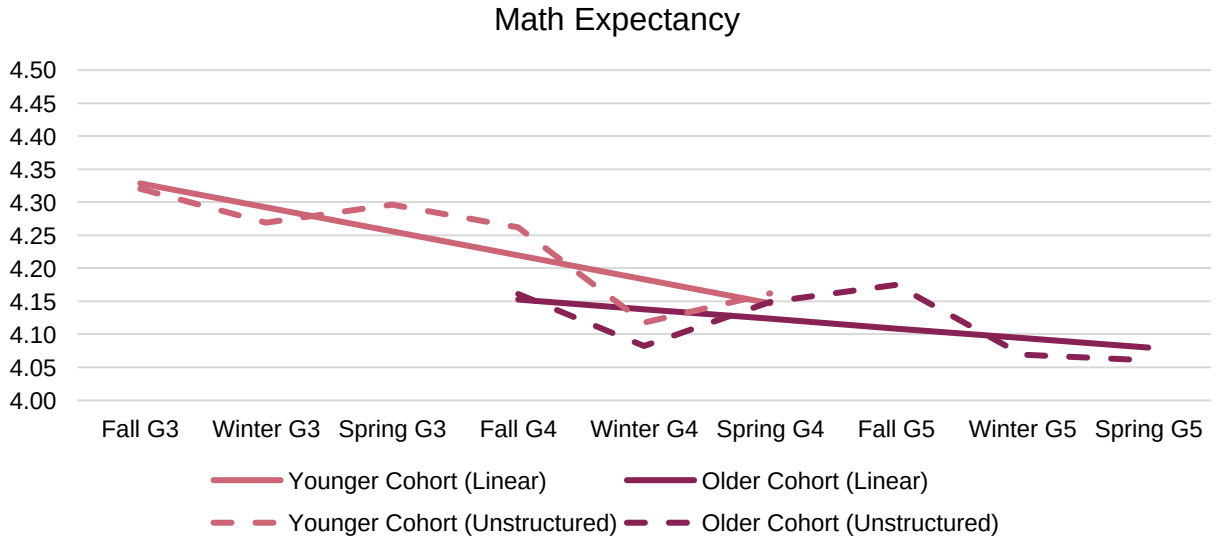
[a] $p < .05$, [b] $p < .01$, [c] $p < .001$

Research Question 2B: Full Model with Interaction

Interactions varied by both wave and motivation component. When the interaction terms were included in the expectancy model, the main effects for wave were largely the same as the prior model, with the exception of expectancy now being statistically significantly lower at Fall Y2 than Fall Y1 (β_{FallY2} = -0.069, p = .001). Students in the older cohort had lower expectancy on average (β_{older} = -0.187, p < .001). During the second year, expectancy was lower than at Fall Y1 (β_{Wave}'s ranging from -0.069 to -0.239, p's ≤ .001) but differed by cohort ($b_{Wave \times Cohort}$'s ranging from 0.068 to 0.130, p's < .01). At Fall Y2, average math expectancy was not statistically significantly different than Fall Y1 for the older cohort ($\beta_{olderF2}$ = 0.016, p = .0478) but statistically significantly lower than at Fall Y1 for the younger cohort ($\beta_{youngerF2}$ = -0.069, p = .001). At both Winter Y2 and Spring Y2, math expectancy was lower than at Fall Y1 but more so for the younger cohort ($\beta_{youngerW2}$ = -0.239, $\beta_{youngerS2}$ = -0.187, p's < .001) than the older cohort ($\beta_{olderW2}$ = -0.109, $\beta_{olderS2}$ = -0.119, p's < .001). Figure 2.4 displays regression adjusted expectancy for both the linear and unstructured models.

Figure 2.4

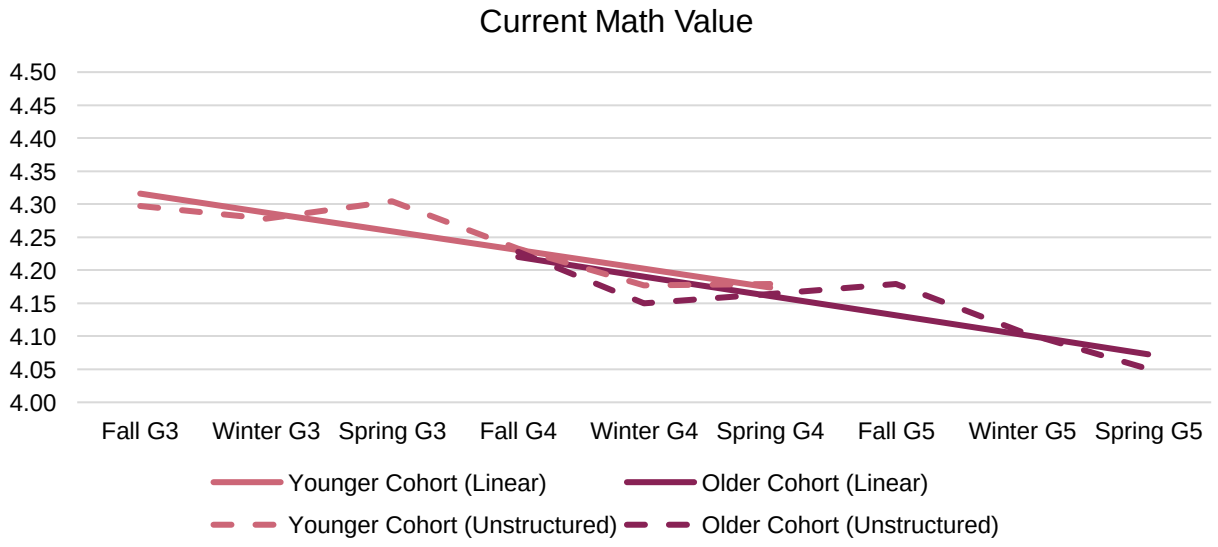
Regression-Adjusted Math Expectancy Average by Grade and Model



When the interaction terms were added to the current value model, there remained a main effect for cohort, such that the older cohort had, on average, statistically significantly lower current value than the younger cohort (β_{Cohort} = -0.076, p = .001). The main effects for wave on current value were not statistically significant for the first year ($\beta_{WinterY1}$ = -0.021, $\beta_{Spring\ Y2}$ = 0.007, p's ≥ .410) but the interactions for cohort and wave during the first year were statistically significant ($\beta_{WY1\times Cohort}$ = -0.063, $\beta_{SY1\times Cohort}$ = -0.076, p's ≤ .009). Conversely, the main effects for wave on current value were statistically significant for the second year (β's ranging from -0.071 to -0.131, p's ≤ .001) but the interaction for cohort and wave was only statistically significant for the Spring of Y2 ($\beta_{SY2\times Cohort}$ = -0.064, p = .007). The younger cohort's current math value remained stable in the first year ($\beta_{youngerW1}$ = -0.021, $\beta_{youngerS1}$ = 0.007, p's ≥ .410), whereas the older cohort's current value was lower in both Winter and Spring Y1 than Fall Y1 ($\beta_{olderW1}$ = -0.084, $\beta_{olderS1}$ = -0.069, p's ≤ .024). In Spring Y2, current value was lower than Fall Y1 for both cohorts, but more so for the older cohort ($\beta_{olderS2}$ = -0.193, p < .001) than the younger cohort ($\beta_{youngerS2}$ = -0.128, p < .001). Figure 2.5 displays regression adjusted current math value.

Figure 2.5

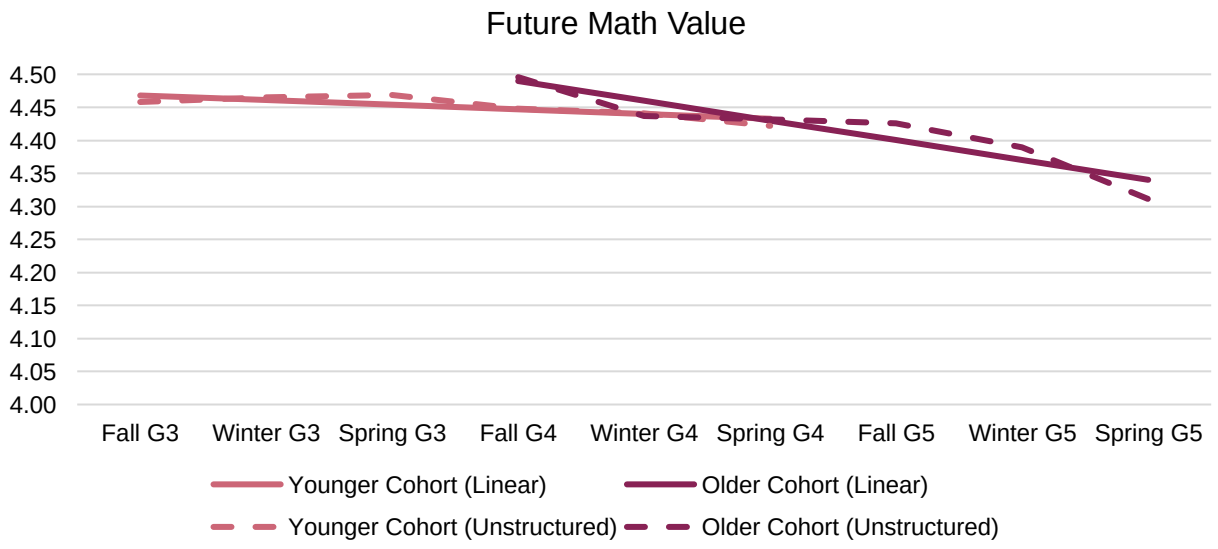
Regression-Adjusted Current Math Value Average by Grade and Model



When the interaction terms were added to the unstructured future math value model, the main effect of wave was no longer statistically significant (β_{Wave}'s ranging from -0.042 to 0.013, p's $\geq$ .228); however, the interactions were statistically significant for all waves ($\beta_{Wave \times Cohort}$'s ranging from -0.070 to -0.173, p's $\leq$ .006). The younger cohort's future math value remained the same throughout both years (β's ranging from -0.042 to 0.013, p's $\geq$ .228). Conversely, the older cohort's future math value was consistently statistically significantly lower than Fall Y1 (β's ranging from -0.069 to -0.215, p's $\leq$ .022), with older students reporting the lowest future math value being at the end of the second year ($b_{olderS2}$ = -0.185, p < .001). Figure 2.6 displays regression adjusted future math value.

Figure 2.6

Regression-Adjusted Future Math Value Average by Grade and Model



Comparing Fourth Graders

Both the younger and older cohorts have data for fourth grade—for the younger cohort fourth grade is the second year (Fall Y2, Winter Y2, Spring Y2) and for the older cohort fourth grade is the first year (Fall Y1, Winter Y1, Spring Y1). To better understand if the trends were dependent on cohort or age, I compared how fourth graders reported motivation across cohorts.

Table 2.5

Comparison of Fourth Graders Between Cohorts

Math Expectancy						
	Linear			Unstructured		
	Younger Cohort	Older Cohort	95% CI of Difference	Younger Cohort	Older Cohort	95% CI of Difference
Fall G4	**4.22**	**4.15**	**[0.015, 0.119]**	**4.26**	**4.16**	**[0.027, 0.173]**
Winter G4	4.18	4.14	[-0.006, 0.096]	4.12	4.08	[-0.028, 0.100]
Spring G4	4.15	4.12	[-0.029, 0.075]	4.16	4.15	[-0.052, 0.077]
Current Math Value						
	Linear			Unstructured		
	Younger Cohort	Older Cohort	95% CI of Difference	Younger Cohort	Older Cohort	95% CI of Difference
Fall G4	4.23	4.22	[-0.047, 0.068]	4.23	4.23	[-0.074, 0.084]
Winter G4	4.20	4.19	[-0.044, 0.067]	4.18	4.15	[-0.043, 0.096]
Spring G4	4.17	4.16	[-0.044, 0.069]	4.18	4.16	[-0.055, 0.085]
Future Math Value						
	Linear			Unstructured		
	Younger Cohort	Older Cohort	95% CI of Difference	Younger Cohort	Older Cohort	95% CI of Difference
Fall G4	4.45	4.49	[-0.095, 0.009]	4.45	4.50	[-0.123, 0.027]
Winter G4	4.44	4.46	[-0.080, 0.030]	4.44	4.44	[-0.061, 0.070]
Spring G4	4.43	4.43	[-0.049, 0.054]	4.42	4.43	[-0.075, 0.056]

Note. Bolded values are statistically significantly different. The 95% confidence intervals are Bonferroni-corrected for multiple comparisons. CI = Confidence Interval

The results were the same for both linear and unstructured models (see Table 2.5). Fourth graders' value did not differ by cohort (contrasts ranging from -0.048 to 0.027, Bonferroni-adjusted *p*'s ≥ .346). Fourth graders winter and spring math expectancy was not statistically significant different by cohort (contrasts ranging from 0.013 to 0.045, Bonferroni-adjusted *p*'s ≥ .182); however, the younger cohort had higher average expectancy in the fall of fourth grade than the older cohort ($contrast_{linear}$ = 0.067, $contrast_{unstrustured}$ = 0.100, Bonferroni-adjusted *p*'s ≤ .001).

Comparison of Linear and Unstructured Models

In the third research question, I asked which models (linear or unstructured) best fit the data. By comparing the difference in log likelihoods between the linear and unstructured models,

Table 2.6

Comparison of Linear and Unstructured Models

	Null	Full Model		With Interaction	
		Linear	Unstructured	Linear	Unstructured
Math Expectancy					
Percent of Residual Variance by Level					
Between Student	41.67%	41.02%	41.13%	41.04%	41.18%
Within Student	58.33%	58.98%	58.87%	58.96%	58.82%
Percent Reduction of Variance from Prior Model					
Between Student			-0.09%		-0.10%
Within Student			0.37%		0.10%
Overall			0.18%		0.05%
Percent Reduction of Variance from Null Model					
Overall		1.48%	1.66%	1.50%	1.72%
Test of Statistical Significance for Model Changes: Linear v. Unstructured					
Deviance	106343	105990	105845	105951	105783
Improvement			Yes		Yes
Current Math Value					
Percent of Residual Variance by Level					
Between Student	43.39%	42.74%	42.76%	42.74%	42.79%
Within Student	56.61%	57.26%	57.24%	57.26%	57.21%
Percent Reduction of Variance from Prior Model					
Between Student			-0.02%		-0.04%
Within Student			0.08%		0.16%
Overall			0.04%		0.07%
Percent Reduction of Variance from Null Model					
Overall		1.56%	1.60%	1.56%	1.63%
Test of Statistical Significance for Model Changes: Linear v. Unstructured					
Deviance	113117	112742	112710	112742	112681
Improvement			Yes		Yes
Future Math Value					
Percent of Residual Variance by Level					
Between Student	36.59%	36.38%	36.45%	36.40%	36.49%
Within Student	65.83%	66.45%	66.56%	66.42%	66.53%
Percent Reduction of Variance from Prior Model					
Between Student			0.01%		-0.01%
Within Student			0.03%		0.06%
Overall			0.20%		0.23%
Percent Reduction of Variance from Null Model					
Overall		1.11%	1.31%	1.16%	1.38%
Test of Statistical Significance for Model Changes: Linear v. Unstructured					
Deviance	110503	110329	110316	110291	110269
Improvement			Yes		Yes

I found that the unstructured models provided statistically significant better model fit (i.e., less deviance) than the linear models for math expectancy, current math value, and future math value. The unstructured models explained more variance than the linear models both with and without the interaction terms (ranging from 0.04% to 0.23% overall). Most of the additional variance explained came from within students (ranging from 0.03% to 0.37% additional within student variation explained). This suggests that motivation does fluctuate within and between school years and that linear functions do not accurately capture students' change in motivation. See Table 2.6 for full description of variance reduction by model.

Discussion

In this study, I used a cross-sequential design and multilevel mixed-effects regressions to model motivation change over two years. By comparing a model that assumes motivation follows a linear trend across time to one that does not constrain the data to a singular shape, I was able to examine how motivation may fluctuate within and between school years. This contrasts with the majority of previous research on motivation, which focuses almost exclusively on changes between grades/ages. By knowing when and how motivation fluctuates, educators and motivation researchers can better plan when to intervene with students' motivation.

In line with prior research (e.g., Eccles & Wigfield, 2020; Wigfield & Cambria, 2010), when using a linear model, I found a decline in mathematics expectancy, current mathematics value, and future mathematics value over the two years (*research question 2a*). Expectancy and current value declined at a similar rate but future value was more stable. Additionally, the older cohort had lower expectancy and current/future value on average compared to the younger cohort. However, the rate of decline differed by cohort and motivation component (*research question 2b*).

Although the models examined cohort difference, not grade, the similarity between fourth graders in both cohorts suggests that the cohort effects may be due to age differences.

The linear models were a statistically significant improvement over the null models; however, the unstructured models fit the data better than the linear models (*research question 2c*). These models consistently showed a dip in motivation in the middle of the school year but a rebound either in the spring or at the beginning of the next school year (*cf.* future value for the younger cohort did not fluctuate over time in this manner). There are many possible explanations for this pattern. Opposite to the "summer slide" of achievement (Alexander et al., 2016), students likely enter a new school year with optimism and enthusiasm. Although the idea of "optimism early and realism later" is generally associated with age differences (see Muenks et al., 2018), it is possible that students experience this on a smaller scale within a school year as they are given more feedback and can better calibrate their expectancies. It is also possible that at the beginning of the school year, students have not had little time for their teachers' teaching style, instructional behaviors, and expectancies to impact their own motivation. Most research regarding the impact of teachers on student motivation happens in the middle and end of the school year (e.g., Bartholomew et al., 2018; Eccles, 2012; Pitzer & Skinner, 2017; Wigfield et al., 2015), so little is known about quickly teachers' behaviors influence student motivation, especially at the beginning of the school year.

Students’ motivation often dipped in the middle of the school year. This may be due to the “realism” of the school year, as described above. It is also possible that returning from winter break may impact students’ motivation: the return to school after a major holiday season without the promise of something new, like at the beginning of the school year, may negatively impact motivation.

Although motivation consistently dipped in the winter, the spring had more variability—sometimes motivation rebounded, whereas other times it declined, especially for fifth graders. This may be due in part to the high stakes testing at the end of the school year. When students took the survey in relation to when they took the Florida Standards Assessment (FSA) may influence their calibration and/or emotions towards math, and thus their expectancies and values. For example, if a student had not yet taken the FSA, they may feel nervous and have lower expectancy. Conversely, they may feel optimistic and have higher expectancy. On the other hand, if the student had already taken the FSA, they may feel that mathematics is no longer valuable to them this year (current value) because they had largely completed their course material. However, they may also feel relief or buoyed general feelings that may lead to higher motivation.

With motivation dipping in the middle of the school year, educators and motivation researchers should plan to intervene right before the middle of the school year. This could mitigate the drop in the winter and keep motivation high year-round. Expectancy and current value declined more than future value, so these interventions should focus on tying mathematics to students' daily life (e.g., Harackiewicz et al., 2014) and fostering mathematics expectancy (e.g., Marsh et al., 2017). Any interventions at the beginning of the school year may want to focus on calibration (e.g., Hattie, 2013; Labuhn et al., 2010) to counteract any negative effects from the optimism mentioned previously.

Limitations and Future Directions

One limitation to this study is the lack of context surrounding the changes in motivation. Although I can postulate why there is a decline in motivation in the middle of the school year, which may or may not remain at the end of the school year, I cannot be sure if those explanations are accurate. Future research can collect data on teacher practices or more detailed classroom

schedules to reveal contextual factors that may drive changes in student motivation (e.g., timing of end-of-grade high stakes testing on motivation). Additionally, future research should examine how these trends may or may not differ in year-round classrooms. These students may not have a pronounced beginning of the school year bump in motivation and have many more breaks than just the winter holiday. Differences in trends by school type would provide more context around what is influencing motivation.

The cross-sequential design with multiple within school year timepoints allowed us to model how motivation changed within a school year. Future research extending these results should include more timepoints within and/or across school years to better capture the "situated" in SEVT. Additionally, going beyond variable-centered approaches and using person-centered approaches (e.g., growth mixture modeling) could paint a more accurate picture of how motivation interacts with time.

Conclusion

I present a nuanced examination of change in expectancies and values across the middle childhood time-period, when motivation often declines, by comparing linear models to those that allow for investigation of unconstrained associations between school year timing and motivation. Both models offer valuable insights—the linear models corroborate prior research that find a decline in motivation over time (e.g., Archambault et al., 2010; Jacobs et al., 2002; Musu-Gillette et al., 2015; Wigfield, 1994) and the unstructured models provide evidence for non-linear fluctuation in motivation over the school year By knowing how and when motivation changes, educators and motivation researchers have a better of motivation and can better design and implement motivation-enhancing interventions.

References

Alexander, P. A. (2013). Calibration: What is it and why it matters? An introduction to the special issue on calibrating calibration. *Learning and Instruction, 24*, 1–3. https://doi.org/10.1016/j.learninstruc.2012.10.003

Amrein, A. L. & Berliner, D. C. (2003). The effects of high-stakes testing on student motivation and learning. *Educational Leadership, 60*(5), 32-38.

Archambault, I., Eccles, J. S., & Vida, M. N. (2010). Ability self-concepts and subjective value in literacy: Joint trajectories from grades 1 through 12. *Journal of Educational Psychology, 102*(4), 804–816. https://doi.org/10.1037/a0021075

Bartholomew, K. J., Ntoumanis, N., Mouratidis, A. Katartzi, E., Thogersen-Ntoumani, C., & Vlachopoulos, S. (2018). Beware of your teaching style: A school-year long investigation of controlling teaching and student motivational experiences. *Learning and Instruction, 53*, 50-63. https://doi.org/10.1016/j.learninstruc.2017.07.006

Eccles, J. S. (2009). Who am I and what am I going to do with my life? Personal and collective identities as motivators of action. *Educational Psychologist, 44*(2), 78–89. https://doi.org/10.1080/00461520902832368.

Eccles, J. S. (2012). Supporting America's children and adolescents. *Macalester International, 29*, 1–23. : http://digitalcommons.macalester.edu/macintl/vol29/iss1/4

Eccles-Parsons, J. S., Adler, T. F., Futterman, R., Goff, S. B., Kaczala, C. M., Meece, J. L., et al. (1983). Expectancies, values, and academic behaviors. In J. T. Spence (Ed.), *Achievement and achievement motivation* (pp. 75–146). W. H. Freeman.

Eccles, J. S., Midgley, C., Wigfield, A., Reuman, D., Mac Iver, D., & Feldlaufer, H. (1993). Negative effects of traditional middle schools on students' motivation. *Elementary School Journal, 93*, 553–574 https://doi.org/0013-5984/93/9305-0008$01.00.

Eccles, J. S., & Wigfield, A. (2020). From expectancy-value theory to situated expectancy-value theory: A developmental, social cognitive, and sociocultural perspective on motivation. *Contemporary Educational Psychology, 61*, 101859. https://doi.org/10.1016/j.cedpsych.2020.101859

Flake, J. K., Barron, K. E., Hulleman, C., McCoach, B. D., & Welsh, M. E. (2015). Measuring cost: The forgotten component of expectancy-value theory. *Contemporary Educational Psychology, 41*, 232–244. https://doi.org/10.1016/j.cedpsych.2015.03.002

Florida Department of Education (2018). *Florida Standards Assessments 2017–2018 volume 4: Evidence of reliability and validity.* http://www.fldoe.org/core/fileparse.php/5663/urlt/V4-FSA-1718-TechRpt.pdf

Florida Department of Education (2019). *Florida Standards Assessments 2018–2019 volume 4: Evidence of reliability and validity.* http://www.fldoe.org/core/fileparse.php/5663/urlt/V4-FSATechReportYear1819.pdf

Gaspard, H., Häfner, I., Parrisius, C., Trautwein, U., & Nagengast, B. (2017). Assessing task values in five subjects during secondary school: Measurement structure and mean level differences across grade level, gender, and academic subject. *Contemporary Educational Psychology, 48*, 67–84. https://doi.org/10.1016/j.cedpsych.2016.09.003.

Gaspard, H., Wigfield, A., Jiang, Y., Nagengast, B., Trautwein, U., & Marsh, H. W. (2018). Dimensional comparisons: How academic track students' achievements are related to

their expectancy and value beliefs across multiple domains. *Contemporary Educational Psychology, 52*, 1–14. https://doi.org/10.1016/j.cedpsych.2017.10.003

Gaspard, H., Wille, E., Wormington, S. V., & Hulleman, C. S. (2020). How are upper secondary school students' expectancy-value profiles associated with achievement and university STEM major? A cross-domain comparison. Contemporary Educational Psychology, 58, 149-162. https://doi.org/10.1016/j.cedpsych.2019.02.005

Jacobs, J., Lanza, S., Osgood, D. W., Eccles, J. S., & Wigfield, A. (2002). Ontogeny of children's self-beliefs: Gender and domain differences across grades one through 12. *Child Development, 73*, 509–527. https://doi.org/10.1111/1467-8624.00421

Karamarkovich, S. M., & Rutherford T. (2018, August 9-12). *Differences in profiles of motivation for mathematics across grades and districts* [Paper presentation]. American Psychological Association 2018 Convention, San Francisco, CA.

Karamarkovich S. M., & Rutherford, T. (2019, April 5-9). *Profiling change: Methods for comparing patterns in mathematics expectancies and values across time* [Poster presentation in Embracing and Modeling the Complex Dynamics of Motivation and Engagement: Contextual, Temporal, Dynamic, and Systematic Symposium]. American Educational Research Association 2019 Meeting, Toronto, CA.

Ladd, G. W., Herald-Brown, S. L., & Kochel, K. P. (2009). Peers and motivation. In K. R. Wenzel & A. Wigfield (Eds.), *Educational psychology handbook series. Handbook of motivation at school* (p. 323–348). Routledge/Taylor & Francis Group.

Marsh, H. W., Ellis, L. A., & Craven, R. G. (2002). How do preschool children feel about themselves? Unraveling measurement and multidimensional self-concept structure. *Developmental Psychology, 38*(3), 376–393. https://doi.org/10.1037/0012-1649.38.3.376

Maulana, R., Opdenakker, M.-C., & Bosker, R. (2016). Teachers' instructional behaviors as important predictors of academic motivation: Changes and links across the school year. *Learning and Individual Differences, 50*, 147-156. https://doi.org/10.1016/j.lindif.2016.07.019

Meeus, W., Van De Schoot, R., Keijsers, L., Schwartz, S. J., & Branje, S. (2010). On the progression and stability of adolescent identity formation: A five-wave longitudinal study in early-to-middle and middle-to-late adolescence. *Child Development, 81*(5), 1565-1581. https://doi.org/10.1111/j.1467-8624.2010.01492.x

MIND Research Institute. (2017). Mind Research Institute: Learn math visually. Retrieved from http://mindresearch.org

Muenks, K., Wigfield, A., & Eccles, J. S. (2018). I can do this! The development and calibration of children's expectations for success and competence beliefs. *Developmental Review, 48*, 24-39. https://doi.org/10.1016/j.dr.2018.04.001

Musu-Gillette, L. E., Wigfield, A., Harring, J. R., & Eccles, J. S. (2015). Trajectories of change in students' self-concepts of ability and values in math and college major choice. *Educational Research and Evaluation, 21*(4), 343-370. https://doi.org/10.1080/13803611.2015.1057161

Nicholls, J. G. (1978). The development of the concepts of effort and ability, perceptions of academic attainment, and the understanding that difficult tasks require more ability. *Child Development, 49*, 800–814. https://doi.org/10.2307/1128250

Nicholls, J. G., & Miller, A. T. (1984). The differentiation of the concepts of difficulty and ability. *Child Development, 54*, 951–959.

Nolen, S. B. (2020). A situative turn in the conversation on motivation theories. *Contemporary Educational Psychology, 61*, 101866. https://doi-org.prox.lib.ncsu.edu/10.1016/j.cedpsych.2020.101866

Oyserman, D. (2014). Identity-based motivation: Core processes and intervention examples. In S. Karabenick & T. Urdan (Eds.), *Motivational interventions* (Vol. 18: Advances in motivation and achievement, pp. 213–242). Emerald

Pitzer, J. & Skinner, E. (2017). Predictors of changes in students; motivational resilience over the school year: The role of teacher support, self-appraisals, and emotional reactivity. *International Journal of Behavioral Development, 41*(1), 15-29. https://doi.org/10.1177/0165025416642051

Roderick, M. & Engel, M. (2001). The grasshopper and the ant: motivational responses of low-achieving students to high-stakes testing. *Educational Evaluation and Policy Analysis, 23*(3), 197-227. https://doi.org/10.3102%2F01623737023003197

Rutherford, T., Farkas, G., Duncan, G., Burchinal, M., Kibrick, M., Graham, J., Richland, L., Tran, N., Schneider, S., Duran, L., & Martinez, M. E. (2014). A randomized trial of an elementary school mathematics software intervention: Spatial-Temporal Math. *Journal of Research on Educational Effectiveness, 7*(4), 358-383. https://doi.org/10.1080/19345747.2013.856978

Rutherford, T., Liu, A. S., Lam, A. C., Schenke, K. (2020). Impact on mathematics self-beliefs from a mastery-based mathematics software. *Journal of Research on Technology in Education, 52*(1), 79-94. https://doi.org/10.1080/15391523.2019.1689210

Rutherford, T., Liu, A., Logan, W., Karamarkovich, S. M. , & Wagemaker, M. (2019a, August 8-11). "I chose math because…" Cognitive interviews of a mathematics motivation

measure [Poster presentation]. American Psychological Association 2019 Convention, Chicago, IL.

Ryan, R. M., & Deci, E. L. (2017). *Self-determination theory: Basic psychological needs in motivation, development, and wellness.* Guilford Publications.

Ryan, R. M., & Weinstein, N. (2009). Undermining quality teaching and learning: A self-determination theory perspective on high-stakes testing. *Theory and Research in Education, 7*(2), 224-233. https://doi.org/10.1177%2F1477878509104327

Singh, K., Granville, M., & Dika, S. (2002). Mathematics and science achievement: Effects of motivation, interest, and academic engagement. *The Journal of Educational Research, 95*(6), 323-332. https://doi.org/10.1080/00220670209596607

Wentzel, K. R., Battle, A., Russell, S. L., & Looney, L. B. (2010). Social supports from teachers and peers as predictors of academic and social motivation. Contemporary educational psychology, 35(3), 193-202. https://doi.org/10.1016/j.cedpsych.2010.03.002

Wigfield, A. (1994). Expectancy-value theory of achievement motivation: A developmental perspective. *Educational Psychology Review, 6*(1), 49-78. https://doi.org/10.1007/BF02209024

Wigfield, A., & Cambria, J. (2010). Students' achievement values, goal orientations, and interest: Definitions, development, and relations to achievement outcomes. *Developmental Review, 30*(1), 1–35. https://doi.org/10.1016/j.dr.2009.12.001

Wigfield, A., & Eccles, J. S. (2000). Expectancy–Value Theory of Achievement Motivation. *Contemporary Educational Psychology, 25*(1), 68–81. https://doi.org/10.1006/ceps.1999.1015

Wigfield, A, Eccles, J. S., Fredricks, J., Simpkins, Roeser, R., & Schiefele, U. (2015). Development of achievement motivation and engagement. In R. Lerner (Series ed.) and M. Lamb & C. Garcia Coll (Eds.), *Handbook of child psychology and developmental science* (7th ed., pp. 657–700). Wiley.

Wigfield, A., Eccles, J. S., & Möller, J. (2020). How dimensional comparisons help to understand linkages between expectancies, values, performance, and choice. *Educational Psychology Review, 32*(3), 657–680. https://doi.org/10.1007/s10648-020-09524-2

Wigfield, A., Eccles, J. S., Yoon, K. S., Harold, R. D., Arbreton, A., Freedman-Doan, C., et al. (1997). Changes in children's competence beliefs and subjective task values across the elementary school years: A three-year study. *Journal of Educational Psychology, 89*, 451–469. https://doi.org/10.1037/0022-0663.89.3.451.

CHAPTER THREE: STUDY TWO

Abstract

Mixed feelings happen in and outside of the classroom; yet prior research has focused on discrete emotions, essentially ignoring the interaction between emotions. I extend prior person-centered studies of achievement emotions by placing emotions within the Control-Value Theory framework to examine how patterns of emotions mediate the relation between motivation and achievement. I found four profiles of emotion in both fourth (n = 5,228) and fifth graders (n = 5,299)—two positive profiles, a negative profile, and a mixed emotions profile where frustrated and challenged were the primary emotions. All profiles mediated the relationship between math expectancy and achievement. However, the mediation of math value and achievement varied by profile.

Keywords: control-value theory, emotions, mathematics, latent class analysis

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Mixed Feelings: Profiles of Emotions Among Elementary Mathematics Students and How They Function Within a Control-Value Framework

Classrooms are not emotionless places. This is true even for STEM (science, technology, engineering, and mathematics) classrooms, where it may seem like the subjects are detached from emotions (e.g., Williams-Johnson et al., 2008). Given the prevalence of student emotions in the classroom (e.g., Lichtenfeld et al., 2012), it is important to understand the role in learning and achievement that they may play. These emotions are not mutually exclusive; we can have mixed feelings. Although emotional theorists acknowledge multiple emotions can happen simultaneously (Larsen & McGraw, 2011), most research on academic emotions focuses on discrete emotions, ignoring the possibility of co-occurrence (e.g., Lichtenfeld et al., 2012). By only focusing on discrete emotions, researchers do not consider what patterns of emotions may exist and how these patterns may be related to academic achievement. Understanding how emotions may co-occur can provide more in-depth information on the role emotions play in academic settings. In this study, I use a person-centered approach to report on these co-occurrences in examining patterns of emotions, their motivational antecedents, and their associations with later achievement with the Control-Value Theory framework.

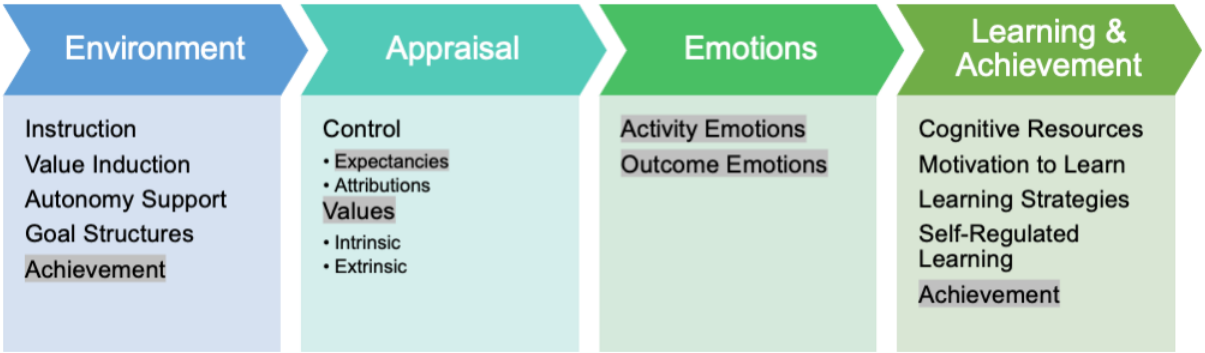
Theoretical Framework: Control-Value Theory

I frame this work with Control-Value Theory of Achievement Emotions (CVT; Pekrun, 2006). Researchers use CVT to attempt to explain how students are motivated and how they perform based on how they feel, how they value the task, and the context of a particular academic situation (e.g., Goetz et al., 2010; Hanin & Van Nieuwenhoven, 2016; Pekrun et al., 2017; Villavicencio & Bernardo, 2016). Prior studies have examined how academic emotions are formed (e.g., Frenzel et al., 2007; Goetz et al., 2010; Tulis & Ainley, 2011), as well as how emotions

influence a variety of outcomes (e.g., value and self-concept; Hanin & Van Nieuwenhoven, 2016; achievement; Peixoto et al., 2017; achievement and self-efficacy; Villavicencio & Bernardo, 2016). In particular, students' motivation for an activity can influence their emotions, which in turn influences achievement (Pekrun et al., 2017). For example, if a student does not think an activity is important, they will likely feel bored and unengaged, leading to poorer performance on the activity (Pekrun et al., 2007; Pekrun et al., 2014). CVT addresses each of these topics and comprises four parts—environment, appraisal, emotion, and learning and achievement (Figure 3.1; Pekrun, 2006).

Figure 3.1

Control-Value Framework Adapted from Pekrun (2006)



Note. Highlighted areas indicate aspects of the theory that are the focus of the current study.

Environment

The environment includes a variety of context-related variables. Imagine a mathematics classroom. In it, there are students from a range of backgrounds with different goals. Although these students are in the same classroom, their environments, as conceptualized by CVT, nevertheless differ (Pekrun, 2006; Pekrun et al., 2007). For some students the instruction/task(s) may be too difficult, others too easy (*instruction*). Some students may have been told that

mathematics has value, whereas others may not have heard this before (*value induction*). Some students may be allowed to choose how they complete a task (*autonomy support*). For some, learning will be their primary goal (i.e., *mastery goals*; Ames, 1992; Elliot, 1999) but others may be more focused on the grade they receive, regardless of what they have actually learned (i.e., *performance goals*; Ames, 1992; Elliot, 1999). Lastly, students will differ on how they have performed on previous activities and the feedback they have been given; prior performance is related to both future performance (Pekrun et al., 2017) and to emotional and motivational components within CVT (Pekrun, 2006; Pekrun et al., 2007).

Appraisal

The appraisal component is where Control-Value gets its namesake—students appraise the task/situation and determine their beliefs about their *control* over their success and how much they *value* said success; in this way, assumptions from Expectancy-Value theory (Wigfield & Eccles, 2000) are integrated into CVT (Pekrun et al., 2007; Wigfield & Cambria, 2010). Control is determined by how well a student expects to perform (expectancies; Pekrun, 2006) and what the student believes influences their performance (e.g., attributions; Pekrun, 2006; Pekrun & Perry, 2014; Pekrun & Stephens, 2010; Weiner, 1985). For example, a student may believe that they will score highly on their next test because they studied a lot. On the other end of the spectrum, a student may believe they will fail their next test because they think their teacher does not like them. The first would lead to a determination of higher control than the second.

The second component of appraisal comprises the student's values, which can be further broken down into intrinsic and extrinsic values (Pekrun, 2006; Pekrun & Perry, 2014; Pekrun & Stephens, 2010). Intrinsic value is the inherent/personal value of an activity or outcome (e.g., a student may want to participate in a mathematics competition because they are interested in math).

Extrinsic value is the instrumental usefulness of actions or outcomes for the attainment of other goals (e.g., a student may want to participate in a mathematics competition because it will look good on their resume and help them get into college). Succinctly, at this second stage of CVT, a student's beliefs about their control and their value for a task or subject influence their emotions surrounding the task or subject.

Emotions

Achievement emotions are a subset of emotions that are tied directly to achievement activities (i.e., tasks on which we perform, such as tests or homework) or achievement outcomes (i.e., how we actually perform on the task, such as failing a test; Pekrun et al., 2007). A three-dimensional taxonomy is often used to examine achievement emotions (Pekrun et al., 2007). One dimension is the focus of the emotion—students have emotions when working on an activity, as well as when they complete it and/or receive feedback. For example, a student may feel frustration on a test (*activity focus*) and then anger when they receive their grade (*outcome focus*; Pekrun et al., 2007). Another dimension is valence of the emotion—is the emotion positive and pleasant or negative and unpleasant? Examples of *positive* emotions are enjoyment and pride, and examples of *negative* emotions are boredom and disappointment. How the emotion is *activating* or *deactivating* is the third dimension of Pekrun's (2006) emotional taxonomy, which encompasses the physiological response to the emotion. Activation can moderate the effect of the valence of an emotion on engagement. For example, boredom is a negative, deactivating emotion, which often leads to the student disengaging from the activity (Pekrun et al., 2007). On the other end of the spectrum, enjoyment is a positive, activating emotion, that increases engagement (Pekrun et al., 2007). However, deactivating positive emotions (e.g., relaxation) and activating negative emotions (e.g., anger and anxiety) have more complex relations with engagement (Pekrun, 2006). Pekrun

(2006) suggests that activating negative emotions, particularly anxiety, can increase extrinsic motivation while also decreasing intrinsic motivation.

Emotions Relevant to this Study. This study focuses on seven emotions—bored, frustrated, hopeful, nervous, happy, excited, and challenged. Three of these emotions are explicitly mentioned in CVT (bored, frustrated, and hopeful), three are theoretically grounded in CVT (nervous, happy, excited), and one was not founded in CVT but has been researched in other emotional contexts (challenged).

Bored. Boredom is one of the most commonly experienced emotions in mathematics classrooms—almost half of students report feeling bored in math class (grades 5-10 in Daschmann et al., 2011; undergraduates in Pekrun et al., 2010). As mentioned above, boredom is considered a negative, deactivating emotion and is often tied to disengagement (Pekrun et al., 2011), poor self-regulation (Di Leo et al., 2019), and lower achievement (Pekrun et al., 2010). In Daschmann et al.'s (2011) study, boredom was predominantly related to students disliking the teaching and not feeling involved in the classroom. Putwain et al. (2018) found that students with low mathematics intrinsic value experience boredom more often than students with higher mathematics intrinsic value. This was especially true for students with high perceived control.

Frustrated. Frustration is a negative activating emotion (Pekrun et al., 2007) and has been tied to lower levels of perceived control but has a non-statistically significant relation with value (Di Leo et al., 2019; Pekrun, 2006). Frustration is commonly reported when students are faced with a difficult problem (Di Leo et al., 2019; Op't Eynde et al., 2007). For example, Di Leo and colleagues conducted think-aloud interviews with fifth graders as they solved a multi-step mathematics problem. Frustration was the most commonly reported emotion (followed by confusion) and often led to anger, hopelessness, and confusion (Di Leo et al., 2019). High

frustration tolerance is predictive of academic outcomes, such as GPA and test scores (Meindl et al., 2019).

Hopeful. Hope is a positive activating emotion (Pekrun, 2006); however its ties to control and value are not as clear cut as other positive activating emotions. Pekrun (2006) posited that hope and anxiety function similarly: both are theorized to be associated with a lack of control with difference occurring based on achievement goals. For example, students would feel hopeful when they focus on success (performance-approach) but anxious when they focus on failure (performance-avoidance). Hope has been tied to higher levels of intrinsic motivation, effort, and achievement (Pekrun et al., 2011), as well as interest (Ainley, 2018). In contrast to hope, hopelessness is a negative deactivating emotion (Pekrun, 2006). Lack of control is also a predictor of hopelessness; however, value is negatively related to hopelessness (Peixoto et al., 2017), unlike hope (Shao et al., 2020). Additionally, hopelessness is related to lower achievement (Peixoto et al., 2017).

Nervous. Nervousness is the affective component of anxiety (Pekrun, 2006; Pekrun & Linnenbrink-Garcia, 2012) and is often used as a measure of anxiety, for example the Achievement Emotions Questionnaire (Pekrun et al., 2011) has the item "I got tense and nervous while studying." Nervous has been used to measure negative activated affect, especially with elementary students (e.g., Ganley & McGraw, 2016; Linnenbrink-Garcia et al., 2011; Ramirez et al., 2013). Although few studies look specifically at nervousness, anxiety is one of the most researched academic emotions, especially within mathematics (Villavicencio & Bernardo, 2016). Students with mathematics anxiety tend to have lower control/self-concept (Ahmed et al., 2012; Ganley & McGraw, 2016; Jameson, 2014; Peixoto et al., 2017). Anxiety has been used to predict a variety of outcomes; however, results are inconsistent, especially regarding the relation between anxiety

and mathematics achievement (Hanin & Van Nieuwenhoven, 2016; Ma & Xu, 2004; Peixoto et al., 2016; Villavicencio & Bernardo, 2016). This is likely due to the activating aspect of anxiety—although anxiety is negative, it may support engagement; however, it is more common for anxiety to decrease engagement (Pekrun & Linnenbrink-Garcia, 2012).

Happy. Happiness is a positive emotion and has either a neutral (Feldman Barrett & Russell, 1998; Linnenbrink-Garcia et al., 2011) or activated valence (Ben-Eliyahu & Linnenbrink-Garcia, 2013; Fong Lam et al., 2015). Within CVT, happy is most aligned with enjoyment (Di Leo et al., 2019; Pekrun & Meier, 2011) and has been used as a measure of enjoyment in the Epistemic Emotion Scales (Pekrun & Meier, 2011). Students that are happy tend to have mastery-approach goals (Linnenbrink & Pintrich, 2003) and feel more accepted/connected to their schools (Fong Lam et al., 2015). Enjoyment is also related to motivation and achievement (Litchenfeld et al., 2012; Pekrun et al., 2011).

Excited. Excited is also a positive emotion that is aligned with enjoyment (Di Leo et al., 2019; Pekrun & Meier, 2011). Unlike happiness, excitement is consistently considered an activating emotion (e.g., Ben-Eliyahu & Linnenbrink-Garcia, 2013; Linnenbrink-Garcia et al., 2011). Positive activating emotions, like excitement, are related to students' engagement (Pekrun & Linnenbrink-Garcia, 2012) and use of metacognitive strategies (Linnenbrink et al., 2005). However, it is also possible for students to become overexcited and then distracted (Pekrun & Linnenbrink-Garcia, 2012)

Challenged. Challenged is not theoretically grounded in CVT; however, there has been research on challenged as an emotion for nearly 40 years (e.g., Smith & Ellsworth, 1985) and across many settings, such as with physical (e.g., Skinner & Brewer, 2004) and academic (Yih et al., 2013) activities. Smith and Ellsworth (1985) examined the dimensional structure of challenged,

along with 14 other emotions, and found that undergraduates considered challenged was associated with slightly positive with a need for high effort and attention. Smith and colleagues (1993) combined challenged and hope into one emotion with the core relational theme being effortful optimism and a potential for success. Other research has tied challenged to boredom, such that when students feel challenged, they are less likely to feel boredom (van Tilburg & Igou, 2012). With challenged being positively related to hope and negatively related to boredom, it seems that challenged is a positive emotion (Kirby et al., 2014; Smith & Ellsworth, 1985; Smith et al., 1993); however, it can also be tied to negative emotions, such as anxiety, anger, and dejection (Kirby et al., 2014). Although the valence is not as clear cut, challenged is an activated emotion: in a chapter dedicated to challenged as an emotion, Kirby and colleagues (2014) use words/phrases such as "energized state," "eagerness," "agency and optimism," and "elevated problem-focus" to describe the subjective feeling of challenged.

Link Between Appraisal and Emotion. Pekrun (2006) detailed how the combination of value and control theoretically influences emotion. He further broke down the outcome dimension of emotion into prospective and retrospective. Take a test for example. If the test has not happened yet, the student would have a *prospective* outcome focus. With a prospective outcome focus, control is defined by expectancies (expected performance is high, medium, or low; Pekrun 2006). If the student values the outcome of the test and expects they can control their grade and do well, they may have anticipatory joy. On the other hand, if the student values the outcome of the test but does not feel they have control over the outcome, they will feel hopelessness (Pekrun, 2006). If the test has happened, the student would have a *retrospective* outcome focus. With a retrospective outcome focus, control is defined by attributions (performance was controlled by the self, other, or is irrelevant; Pekrun, 2006). However, there are some cases when it may not matter how the

student views their control/attribution (see *attribution-independent emotions*; Weiner, 1985). If the student values the outcome, did well, and attributed the outcome to themselves, they will feel pride and joy. However, if they did not do well, and they attributed the failure to themselves, they would feel shame and sadness.

Learning and Achievement

The last component of CVT is learning and achievement. This includes the amount of energy and/or resources the student will expend on the activity, their achievement goals (i.e., *learning* or grade; *mastery or performance goals*, respectively; see Ames, 1992; Elliot, 1999), their learning strategies (such as study habits), as well as how the student will plan, monitor, and modify their learning strategies (i.e., *self-regulated learning*; see Zimmerman & Schunk, 2011). These three sub-components of resources, goals, and strategies all influence the student's performance, often measured as achievement.

Link Between Appraisal and Learning/Achievement. When students value a subject and expect to do well (i.e., how they appraisal a subject/task), they tend to be more engaged and perform better than those who do not (e.g., Cole et al., 2008; Hulleman et al., 2008; Wang & Eccles, 2013). A substantial body of research has supported the theoretical ties between expectancy/value and behavior. Researchers have found that higher expectancy and value can lead to more engagement (e.g., Wang & Eccles, 2013), course enrollment (e.g., Durik et al., 2006; Simpkins et al., 2006), higher achievement (e.g., Cole et al., 2008; Hulleman et al., 2008), and career goals (e.g., Wang et al., 2013).

Link Between Emotion and Learning/Achievement. Pekrun and Stephens (2010) posited that both the valence and activation of an emotion influence learning and achievement. Positive activating emotions are expected to lead to better learning and achievement because they

promote concentration, interest, the use of flexible cognitive strategies and self-regulation, which positively affect performance (Pekrun et al., 2002; Zimmerman & Schunk, 1990). On the other hand, deactivating positive emotions likely reduce task attention, undermining motivation, and thus performance (Pekrun et al., 2002).

It should be noted that not all positive emotions are good and not all negative emotions are bad (Pekrun & Stephens, 2010). Shame, a negative activating emotion, has been shown to increase motivation in those that retain high value and control for the activity (e.g., Turner & Scallert, 2001). Research on anger has had mixed results—some studies have suggested anger is negatively related to self-efficacy, interest, and achievement (Boekaerts, 1993; Pekrun et al., 2004); others have suggested it can improve motivation and performance, as long as the student does not have a depressed mood (Lane et al., 2005). Anxiety, the most researched achievement emotion (e.g., Roos et al., 2020) is also a negative activating emotion, but has been consistently linked to poorer performance (Zeidner, 2007).

Emotion as a Mediator between Appraisal and Learning/Achievement. Although the CVT model suggests the presence of indirect effects—appraisal influences emotions and, in turn, emotions influence learning with appraisal also influencing learning—only a few studies have tested how emotions may mediate the relation between appraisal and learning/achievement. Luo and colleagues (2016) examined how math enjoyment, pride, boredom, and anxiety (measured with the AEQ-Math, Pekrun et al., 2007) mediated the relation between math appraisal (self-efficacy and value) and homework behaviors (homework effort and distraction) in eighth grade students. They found that enjoyment, boredom, and anxiety mediated the effect of both self-efficacy and value on homework effort, whereas pride only mediated the effect of self-efficacy on effort. Additionally, through math enjoyment, boredom, and anxiety, math self-efficacy and value

had a negative relation with homework distraction (Luo et al., 2016).

Shao, Pekrun, Marsh, and Loderer (2020) modeled emotions as mediating the effect of control, value, and the interaction between the two on performance. They found that half of the emotions (enjoyment, hope, pride, and hopelessness) mediated the effect of control on foreign language performance; however, the other four emotions (anger, anxiety, shame, and boredom) did not. Value was neither directly nor indirectly statistically significantly related to performance. The relation between control×value and performance was mediation by enjoyment, hope, pride, and hopelessness, such that students with high value for foreign languages had a stronger relation between control and emotions and thus higher performance (Shao et al., 2020).

Control-Value and Development

Research on elementary students' academic emotions almost exclusively focuses on emotion regulation (e.g., Graziano et al., 2007). These studies examine how emotion regulation is associated with academic readiness and success (e.g., Blair, 2002; Graziano et al., 2007; Valiente et al., 2012). The bulk of research on achievement emotions within the Control-Value framework has focused on adolescents, with research on elementary students being far rarer (see Lichtenfeld et al., 2012). Of the papers cited in the previous section, only one focuses solely on elementary students (Frenzel et al., 2007), and three include elementary and older students (up to tenth grade in Daschmann et al., 2011; Di Leo et al., 2019; Pekrun et al., 2017). Even in these studies, the youngest students are in fifth grade. Yet, achievement emotions are felt by students at all ages (Lichtenfeld et al., 2012); the emotions of younger students are equally worthy of exploration.

Beyond the ubiquity of emotions across all ages, elementary school is when students start to develop expectancies, attributions, and values (Pekrun, 2006; Wigfield, 1994; Wigfield & Cambria, 2010). Students first have global understandings of competencies that become more

differentiated (i.e., subject-specific) over time (Wigfield et al., 2020). In a meta-analysis of self-concept and achievement, Möller and colleagues (2009) found as students get older, they are more able to differentiate competence beliefs values between domains. As an example, Wigfield and Eccles (1994) followed first, second, and fourth graders over three years and measured their self-esteem and subject-specific competence beliefs, interest, and values. Although students' self-esteem and mathematics interest were stable throughout the three years, students' mathematics competence beliefs and values decreased (Wigfield & Eccles, 1994). Additionally, on average, boys were more confident in mathematics than girls but were similar in their ratings of mathematics value. This may be because of mathematics-gender stereotypes, which have been found in students as young as six (Cvencek et al., 2011).

With the development of subject-specific appraisals (i.e., expectancies and values) in elementary school, it follows CVT that students' academic emotions will develop similarly. This makes late elementary school a critical time to assess the implications of CVT, especially given research that shows motivation for mathematics as early as elementary school can predict college major selection (Musu-Gillette et al., 2015).

Person-Centered Approaches to Studying Motivation and Emotions

A tradition of variable-centered approaches in motivation research has resulted in descriptions of the associations of individual variables, holding constant other variables—a method unlikely to describe relations between variables as they exist in actual people (Bergman & El-Khouri, 2003). Person-centered approaches allow researchers to examine the patterns of responses that account for multiple combinations of variables beyond traditional mean centered approaches (Wormington & Linnenbrink-Garcia, 2017). This provides a deeper understanding of motivation beyond high or low motivation (Laursen & Hoff, 2006); which is why they have been

increasingly used by motivation researchers to describe constructs such as intrinsic/extrinsic motivation, expectancies and values, and goal orientations (Bergman & El-Khouri, 2003; Hayenga & Corpus, 2010; Howard & Hoffman, 2017; Linnenbrink-Garcia et al., 2018). Despite this increasing popularity, there has been scant research that uses person-centered approaches to study patterns of emotions. In the case of academic emotions, using a person-centered approach, specifically cluster analyses, can account for "mixed emotions" (Larsen & McGraw, 2011), one of my goals in this study.

Person-Centered Approaches in CVT

Research using person-centered approaches to study academic emotions is new (i.e., primarily since 2016) and, therefore, scarce. The few studies that examine emotion profiles tend to have older participants (i.e., high school or older; Ganotice et al., 2016; Jarrell et al., 2016; 2017; Robinson et al., 2017, 2020). The majority of these studies use a version of the Academic Emotions Questionnaire (AEQ; King, 2011; Pekrun et al., 2002, 2005); however, a few use alternative methods, such as interviews (Raccanello et al., 2018) or single item experience sampling measures (Robinson et al., 2020). Additionally, the majority of the studies use some form of cluster analysis to determine emotion profiles (Ganotice et al., 2016; Jarrell et al., 2016, 2017; Raccanello et al., 2018; Robinson et al., 2017). See Appendix 3.A for more information about measures and analyses for the studies reviewed below.

Although the number of emotion profiles differs by study (three or four clusters), a generally positive and a generally negative profile are consistently found. For example, Ganotice et al.'s (2016) study with adolescents (using the AEQ) found an "adaptive" profile, in which students reported high levels of positive emotions (enjoyment, hope, and pride) and low levels of negative emotions (anger, anxiety, shame, hopelessness, and boredom). Additionally, all studies

found a profile that reported both positive and negative emotions—Ganotice et al.'s (2016) found a "moderate" profile where students reported mid levels of both positive and negative emotions; Jarrell et al. (2016, 2017; using the AEQ) identified a "low" profile with below average reporting of both positive and negative emotions; Robinson et al. (2017) found a "moderate-low" profile where undergraduates reported average or below average levels of positive and negative affect (both activated and deactivated; using a measure adapted from Ben-Eliyahu & Linnenbrink-Garcia, 2013); and Robinson et al. (2020) found a "moderate-high all" profile where undergraduates reported above average levels of feeling excited, happy, frustrated, and bored (using single-item measures for each emotion). Although these profiles may suggest issues in measurement, such as survey inattention, they also suggest the co-occurrence of positive and negative emotions, i.e., mixed feelings.

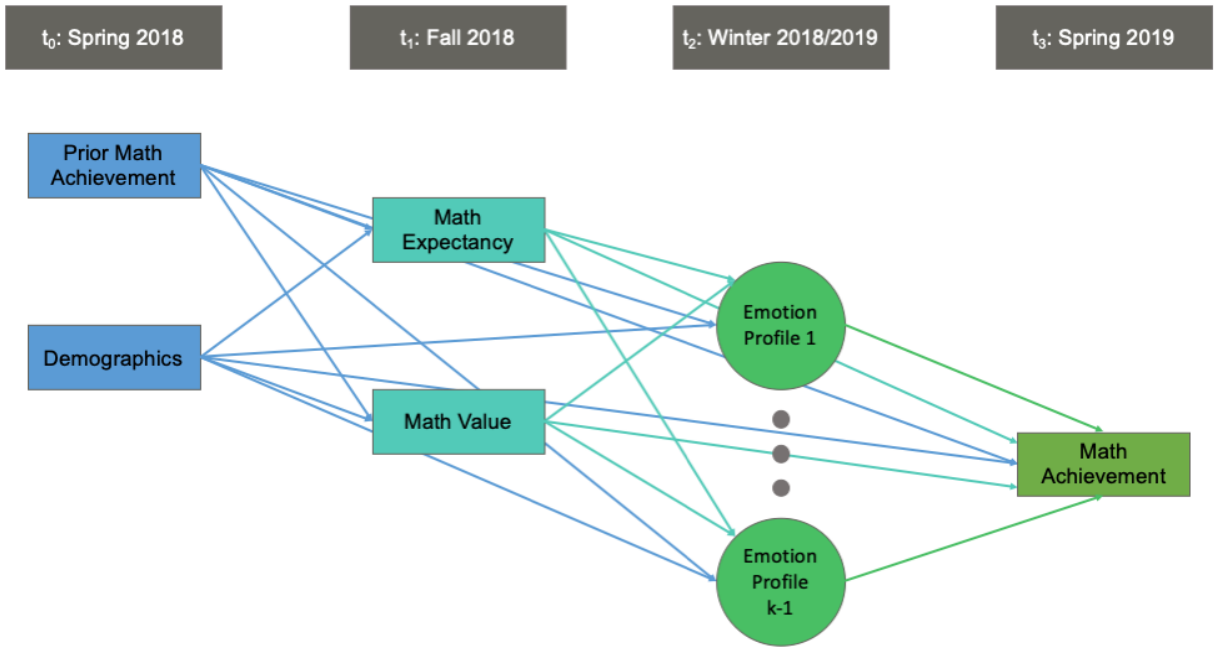
Only two previous studies have used person-centered approaches to examine patterns of emotions in elementary students (Hanin & Van Nieuwenhoven, 2019; Raccanello et al., 2018). Across both studies and similar to studies with older students, a generally positive and negative profile was found, although the negative profile varied on the types of emotion present (Hanin & Van Nieuwenhoven, 2019; Raccanello et al., 2018). Hanin and Van Nieuwenhoven (2019; single-item measures for enjoyment, hope, pride, nervousness, shame, hopelessness, boredom, sadness, fear, and worry) identified three profiles of negative emotion—bored, anxious, and resigned, whereas Raccanello et al. (2018; interviews coded for enjoyment, hope, pride, relief, relaxation, anger, anxiety, shame, boredom, and sadness) had a profile with students that just reported being sad.

Current Study

Overall, research examining patterns of emotions is rare, yet there is evidence that emotions

Figure 3.2

Proposed Multinomial Logistic Model



may co-occur (e.g., Larsen & McGraw, 2011; Tulis & Ainley, 2011). I build upon these prior person-centered studies of achievement emotions to place emotions within the framework of CVT to examine how patterns of emotions mediate the relation between motivation (specifically expectancy and value) and achievement. The proposed model includes select aspects from each component of the CVT framework within a path analysis to identify relations between these aspects (see Figure 3.2). Further, by examining these associations with elementary students, I contribute to the scant existing knowledge on elementary students' achievement emotions and provide evidence as to how early patterns of emotions for mathematics may influence academic achievement and choices. I specifically ask the following questions:

3a. What profiles of mathematics emotions are exhibited by fourth and fifth graders? Do these patterns differ between fourth and fifth graders?

3b. Do these profiles mediate the relation between control-value measures and achievement?

Method

Participants and Context

Data were collected from 6,392 fourth graders and 6,698 fifth graders through the online mathematics learning platform, Spatial Temporal Mathematics (ST Math). ST Math, created by MIND Research Institute, is an interactive mathematics instructional software for computers and tablets that is based on visual instruction (MIND Research, 2017). ST Math is based on theory that suggests that the ability to visualize mathematics concepts leads to better conceptual knowledge and performance (see Geary, 1995; National Research Council, 2005; Shaw & Peterson, 2000) and has been previously shown to result in small improvements in math achievement (Wendt et al., 2014), especially on topics involving number sense (Rutherford et al., 2014; Schenke et al., 2014), as well as improvements to mathematics self-efficacy (Rutherford et al., 2020). ST Math is currently used in 45 states and aligns with both Common Core and relevant state standards. It is intended to be supplemental to the schools' mathematics curriculum and is intended to be used twice a week for 45 minutes. This study is part of a larger NSF-funded project using embedded assessments and data-mining techniques to understand student and teacher use of ST Math. As part of the project, motivation survey questions were designed by MIND in consultation with the project researchers and were embedded within ST Math. It is these surveys that are the subject of the current study.

Participants are fourth and fifth graders from a school district in Florida who use ST Math as part of their regular instruction. This sample was limited to students who had complete data (i.e., prior and current mathematics achievement and survey measures at all time points)—5,228 fourth graders (82% of the total) and 5,299 fifth graders (79% of the total). Fourth graders were

nested within 283 mathematics teachers and fifth graders were nested within 276 mathematics teachers. Both grades had approximately half girls and half boys (51% in both grades). The majority of students in the sample were White (54% in both grades) and qualified for free or reduced lunch (73% in both grades), a proxy measure of socio-economic status. See Table 3.1 for sample demographics and how they compare to the total sample.

Table 3.1

Demographics of Total and Analysis Sample

	Fourth grade			Fifth grade		
	Sample	Total	*p*-value	Sample	Total	*p*-value
Whether a Boy	51%	51%	.8466	51%	52%	**.0454**
Disability	14%	14%	**.0250**	13%	13%	**.0020**
English Language Learner	13%	12%	.1174	12%	12%	.4208
Free/Reduced Lunch	73%	75%	**< .0001**	73%	75%	**< .0001**
Race						
Black	18%	21%	**< .0001**	18%	20%	**< .0001**
Hispanic	18%	18%	.3810	18%	18%	.5600
White	54%	52%	**< .0001**	54%	52%	**< .0001**
Other	9%	9%	.5775	10%	10%	.3452
N	5,228	6,392		5,299	6,698	

Note. *p*-value from *t*-test for proportion comparing those included in the sample to those excluded. Statistically significant *p*-values bolded ($p < .05$).

Measures

Surveys were distributed through ST Math to fourth and fifth graders at the beginning, middle, and end of the 2018-2019 school year. The survey was designed to measure a variety of outcomes, including self-efficacy for academic subjects, subject favoritism, and expectancies and values for mathematics. This survey was developed by MIND Research Institute (the creators of ST Math) in collaboration with the researchers and modeled after existing surveys, such as those in Fredricks and Eccles (2002). Although the survey was not designed specifically as measuring concepts purely defined within the CVT framework, there is substantial overlap between survey items and the framework. For example, expectancy is often used as a measure of control (Pekrun

et al., 2007). To better understand how children interpreted and responded to the survey, Rutherford et al. (2019) conducted cognitive interviews with children (ages seven to 12) and coded each question for item interpretation, cognitive elaboration, and congruent answer choice (see Karabenic et al., 2007).

Expectancy

In this survey, mathematics expectancy was measured by two questions targeting math expectancy for that year and math learning expectancy—"How well do you think you will do on math this year?" and "How good would you be at learning new things in math?" Students responded using a five-point Likert-like scale. Each point was associated with a specific tomato emoji ("tomijis"; see Figure 3.3). The use of pictorial facial expressions is a common practice when surveying children (e.g., Hanin & Van Niewenhoven, 2019) and can benefit low-literacy respondents (Stange et al., 2018). For these measures, as well as the value measures, students generally found the tomojis engaging and enjoyable (Rutherford et al., 2019).

Overall, the cognitive validity for expectancy was high—on a scale of zero to four, average cognitive validity scores were 3.73 and 3.63 for math expectancy for this year and math learning expectancy, respectively (Rutherford et al., 2019). Over 90% of students who were interviewed accurately interpreted and congruently responded to both measures. Although the vast majority of students (95%) could coherently elaborate on their response for math expectancy for that year, fewer (83%) could coherently elaborate on their response for math learning (Rutherford et al., 2019). This study used an average of the two expectancy questions (Cronbach's a = 0.70; Ordinal a = 0.82, Gadermann et al., 2012).

Value

Usefulness, a measure of extrinsic value, was also measured using an emoji-based five-

point Likert-like scale. Both current and future usefulness were measured with the following questions: “How useful is math for you now?” and “How useful will math be in your future?” Similar questions were used to assess importance, a measure of intrinsic value. I used an average of all four value questions (i.e., current usefulness, future usefulness, current importance, and future importance; Cronbach’s α = .85; Ordinal α = .89, Gadermann et al., 2012). Like expectancy, the value measures had high cognitive validity—students interviewed generally interpreted the value measures correctly (90% to 95%) and had congruent answer choices (88% to 100%) but

Figure 3.3

Expectancy and Value Questions with Corresponding Tomojis

Expectancy

How well do you think you will do on math this year? | How good would you be at learning new things in math?

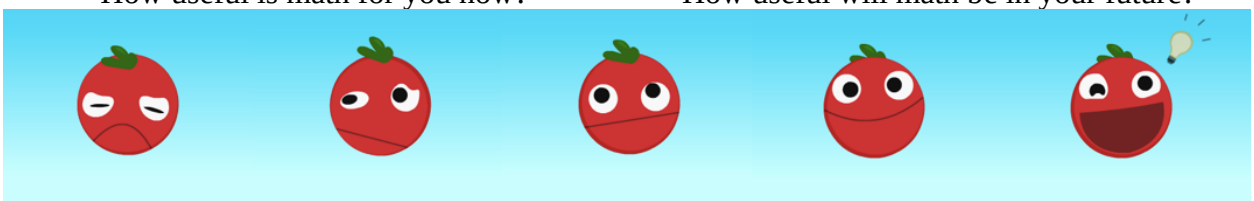
Not well (good) at all	Not so well (good)	Okay	Well (good)	Very well (good)

Value

How important is math to you now? | How important will math be to you in the future?

Not at all important	Not very important	Kind of important	Important	Very important

How useful is math for you now? | How useful will math be in your future?



Not at all useful	Not very useful	Kind of useful	Useful	Very useful

Note. Expectancy ordinal A = 0.816; Value ordinal α = 0.893 (see Gadermann et al., 2012). Value is the aggregate of all importance and usefulness measures.

coherent elaboration was lower for all value measures (83% to 93%). Average cognitive validity ranged from 3.32 (current importance) to 3.58 (current usefulness; Rutherford et al., 2019).

Although importance and usefulness are theoretically separate (i.e., intrinsic and extrinsic value), I combine them into a general value measure. Prior research has found that younger students tend to have difficulty disentangling these measures (e.g., Wigfield & Cambria, 2010; Wigfield & Eccles, 2010), which was corroborated by the cognitive interviews. As one student said, "important is if it is useful" (Rutherford et al., 2019). Additionally, the measures have high internal reliability (at least .80). For my models, I use the surveys on students' math expectancies and values that were collected at the beginning of the 2018-2019 school year (t_1: Fall 2018). See Figure 3.3 for value questions and corresponding emojis.

Emotions

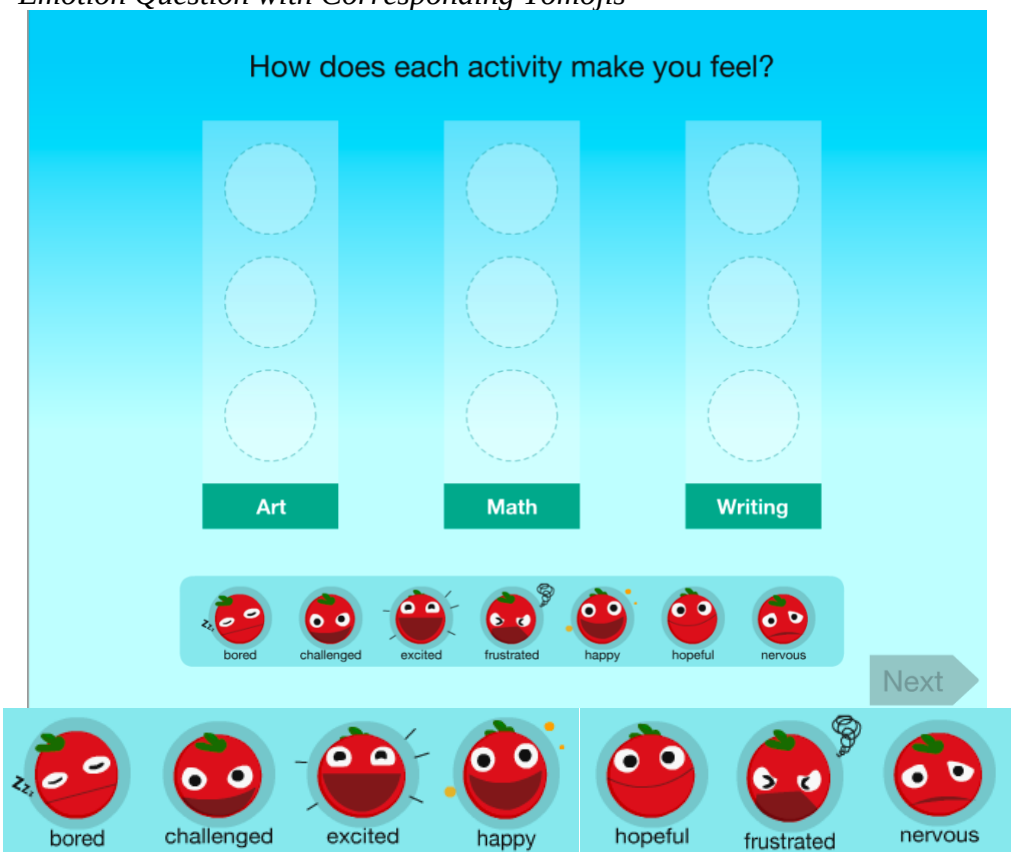
To measure emotions, students were asked “How does each activity make you feel?” for math, their favorite subject, and least favorite subject (this study only uses mathematics emotions). Students could select one to three emotions out of the following: bored, challenged, excited, frustrated, happy, hopeful, nervous (Figure 3.4). Students were not able to proceed to the next question until they selected at least one emotion for each subject; they were only allowed to select each emotion once for each subject. The majority of students in the sample selected three mathematics emotions (93% of fourth graders and 92% of fifth graders), but some did select only one (4% of both fourth and fifth graders) or two emotions (3% and 4% of fourth and fifth graders, respectively). All students in the cognitive interviewing study correctly interpreted the measure, had congruent responses, and coherent elaboration; average cognitive validity was 3.69 (Rutherford et al., 2019).

Six of the seven emotions were chosen to represent both positive and negative valence with

a variety of activations (Pekrun et al., 2002). Challenged, although not theoretically grounded in CVT research, was included in the survey because it was a critical element to MIND's conceptualization of emotions within ST Math and may be especially relevant given the context as a digital learning environment (Eligio et al., 2014; Smith & Ellsworth, 1985). I retained challenged in the analysis because it was reported by nearly 70% of students and was the only emotion with grade-level differences (fifth graders reported feeling challenged more often than fourth graders, see Table 3.2 on page 93). Additionally, removing challenged from the analysis may inaccurately portray the emotions and emotion profiles of the students. Data on student math emotions were collected during the middle of the 2018-2019 school year (t_2: Winter 2018/2019). Distribution of emotions reported are described in Appendix 3.B.

Figure 3.4

Emotion Question with Corresponding Tomojis



Demographics and Achievement

Achievement and demographic information were provided to MIND by the school district. MIND de-identified and matched these district data with survey data before providing both to the researchers. Achievement data come from the Florida Standards Assessments tests. Florida Standards Assessments (FSAs) are end of the grade, summative tests based on Florida's education standards. All measures of internal reliability (Cronbach's α, stratified α, and Feldt-Raju coefficient) were at least 0.93 for fourth grade mathematics FSAs from the 2017-2018 school year and 0.94 from the 2018-2019 school year (Florida Department of Education, 2018, 2019). The mathematics scale scores were collected for both prior mathematics achievement (t_0: Spring 2018) and current mathematics achievement (t_3: Spring 2019).

Demographic data was reported by the district to MIND following the same procedure as achievement data. Demographics for current analyses were those reported at the same time as the 2018-2019 FSA (t_0: Spring 2018). Information on students' disability, status as an English Language Learner, and eligibility for free/reduced lunch were coded as binary variables (e.g., did the student have a disability). The district coded gender as male or female; I recoded into a binary to represent whether the student was a boy. Students' race/ethnicity was coded in the district data as Asian or Pacific Islander, Black (non-Hispanic), Hispanic, American Indian or Alaskan Native, Multiracial, or White (non-Hispanic). For this study, I combined Asian/Pacific Islander, American Indian/Alaskan Native, and Multiracial into one category due to relatively low numbers of students identifying as these races (Asian: 4.30%, American Indian: 0.15%, and Multi-Racial: 5.07%). In Florida schools, race and gender are both identified by each student's parent (Florida Department of Education, 2016); however, there is variability in how these data are collected (e.g., verbal questions, written forms, leading statements; see Campbell-Montalvo, 2020), so I cannot say that

the coding accurately reflects a student's or parent's racial, ethnic, or gender identity. Although race was not a primary focus of this study, students' math emotions, expectancies, and values, as well as the relation between these and math achievement, will likely differ based on their backgrounds and experiences with institutionalized white supremacy (DeCuir-Gunby & Schutz, 2014). See Table 3.1 on page 85 for information on student demographics.

Analysis

Latent Class Analysis

Latent class analysis (LCA) is a mixture modeling method used with categorical variables to identify patterns in responses, in this case, emotions. For these analyses, I kept emotions as dichotomous variables (i.e., did they report the emotion or not). Models with k versus $k + 1$ classes were tested iteratively to determine the most appropriate number of classes. To determine best fit, sample size adjusted Bayesian Information Criterion, Entropy, and the bootstrap likelihood ratio test (BLRT) were used (Nylund et al., 2007). For profiles with similar fits, interpretability was used to select the best model. These models were run separately for fourth and fifth grade students to determine if patterns differed between the grades. Models were run in *Mplus* (version 8; Muthén & Muthén, 1998-2017) and Appendix 3.C has sample *Mplus* code.

Path Analysis

Multinomial logistic regressions of emotion profile membership on motivation variables, demographics, and prior math achievement were estimated to determine which students were in each class. Achievement was regressed on emotion profile, motivation variables, prior achievement, and demographics (see Figure 3.2 on page 83 for proposed model).

Mediation Models

Indirect effects of motivation variables and prior achievement on achievement through

emotion profile were tested to determine if emotion profiles mediated these relationships. To do this, I ran separate path analyses for each profile. The models were identical to the one described above but used profile membership as a binary variable (i.e., was the student in the profile or not) instead of a multinomial profile variable, which compares each profile to a reference profile. Then, I used an equation generalized for categorical mediators (Iacobucci, 2012) to determine if the mediation was statistically significant:

$$z_{mediation} = \frac{z_a \times z_b}{\sqrt{z_a^2 + z_b^2 + 1}}, \text{ where } z_a = \frac{a}{s_a} \text{ and } z_b = \frac{b}{s_b}.$$

In the equation, z_a represents the association between motivation and emotion profile and z_b represents the association between emotion profile and achievement, while controlling for motivation, as well as demographics and prior math achievement (s_a and s_b are their respective standard errors; Iacobucci, 2012).

Results

Preliminary Analyses

Fourth and fifth graders reported similar levels of math value (t = 1.71, p = .088). Although, on average, fourth graders had higher math expectancy (t = 4.58, p < .001), fifth graders had higher math achievement scale scores (t_0: t = -31.24, t_3: t = -16.14, p < .001). Intraclass correlations (ICCs) indicated that between 18% and 24% of variance in mathematics achievement was between teacher (i.e., the majority of the variance was attributable to the students, not the teachers). The ICC for math expectancy and value was much lower—no more than 2.5% of variance was attributable to between-teacher differences.

Students did not differ in how often they reported each emotion by grade, with one exception—fifth graders reported feeling challenged in mathematics more often than fourth graders (t = -3.47, p < .001). Challenged was the emotion reported most often (68% of fourth

Table 3.2

Summary Statistics of Achievement, Motivation, and Emotions

		Fourth Grade				Fifth grade				*p*
		Mean	SD	Min, Max	ICC	Mean	SD	Min, Max	ICC	
t_0	Prior Math Achievement	303.14	19.19	240, 360	18.71%	315.81	22.27	251, 376	22.89%	**< .001**
t_1	Expectancy	4.27	0.80	1, 5	2.06%	4.19	0.80	1, 5	1.95%	**< .001**
	Value	4.34	0.79	1, 5	1.13%	4.31	0.78	1, 5	2.35%	.088
t_2	Excited	0.42	0.49	0, 1	1.60%	0.41	0.49	0, 1	2.23%	.074
	Happy	0.45	0.49	0, 1	< 0.01%	0.44	0.50	0, 1	2.32%	.561
	Hopeful	0.40	0.49	0, 1	1.25%	0.39	0.49	0, 1	0.83%	.486
	Bored	0.23	0.42	0, 1	1.61%	0.23	0.42	0, 1	5.69%	.841
	Frustrated	0.42	0.49	0, 1	1.08%	0.43	0.50	0, 1	2.60%	.210
	Nervous	0.28	0.45	0, 1	1.38%	0.26	0.44	0, 1	< 0.01%	.086
	Challenged	0.68	0.47	0, 1	0.91%	0.71	0.45	0, 1	1.52%	**.001**
t_3	Math Achievement	318.14	22.23	251, 376	22.90%	323.93	23.07	256, 388	24.34%	**< .001**
	N	5,228				5,299				

Note. Intraclass correlation (ICC) is the amount of variance between teachers. The *p*-value is from *t*-tests of differences comparing the fourth and fifth graders. Statistically significant *p*-values bolded ($p < .05$).

graders and 71% of fifth graders), whereas bored was reported least often (23% of fourth and fifth graders). Between-teacher variance in how students reported math emotion was very small—the ICC was less than 0.01% for fourth grade happiness and fifth grade nervousness; the highest ICC was 5.69% for fifth grade boredom.

Correlations followed the expected pattern (e.g., prior math achievement was positively correlated with expectancy, value, positive emotions, and math achievement). See Table 3.2 for summary statistics of mathematics expectancy, value, emotion, and achievement (Appendix 3.D displays correlations).

Latent Class Analysis

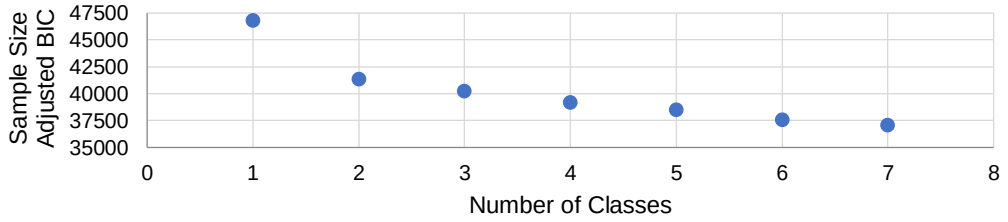
For both fourth and fifth graders, four profiles provided the best fit. Table 3.3 contains fit indexes for both grades. The bootstrap likelihood ratio test was statistically significant ($p < .001$)

for the four class models, suggesting that these models were better than the three class models (Nylund et al., 2007). Additionally, using the sample-size adjusted BIC, the four class models are past the "elbow," which indicates that the any further decrease in the sample-size adjusted BIC are past the "point of 'diminishing returns' in model fit" (Nylund-Gibson & Choi, 2018, p. 443). Lastly, the entropy for the four class models were above 0.80, indicating good classification (Masyn, 2013).

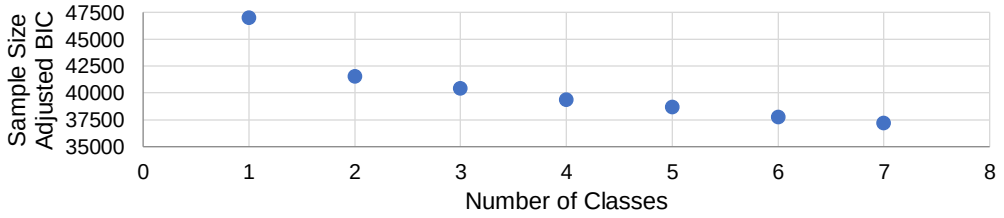
Table 3.3

Latent Class Analysis Fit Indexes

	Class	# of Parameters	Sample-size Adjusted BIC	Entropy	Bootstrap Likelihood Ratio Test	
					BLRT	*p*-value
Fourth Graders	1	7	46826.288			
	2	15	41376.764	0.819	5492.596	< .001
	3	23	40267.495	0.850	1152.342	< .001
	4	31	39225.138	0.890	1085.430	< .001
	5	39	38503.804	0.935	764.406	< .001
	6	47	37573.332	0.965	978.609	< .001
	7	55	37099.259	0.975	516.372*	< .001



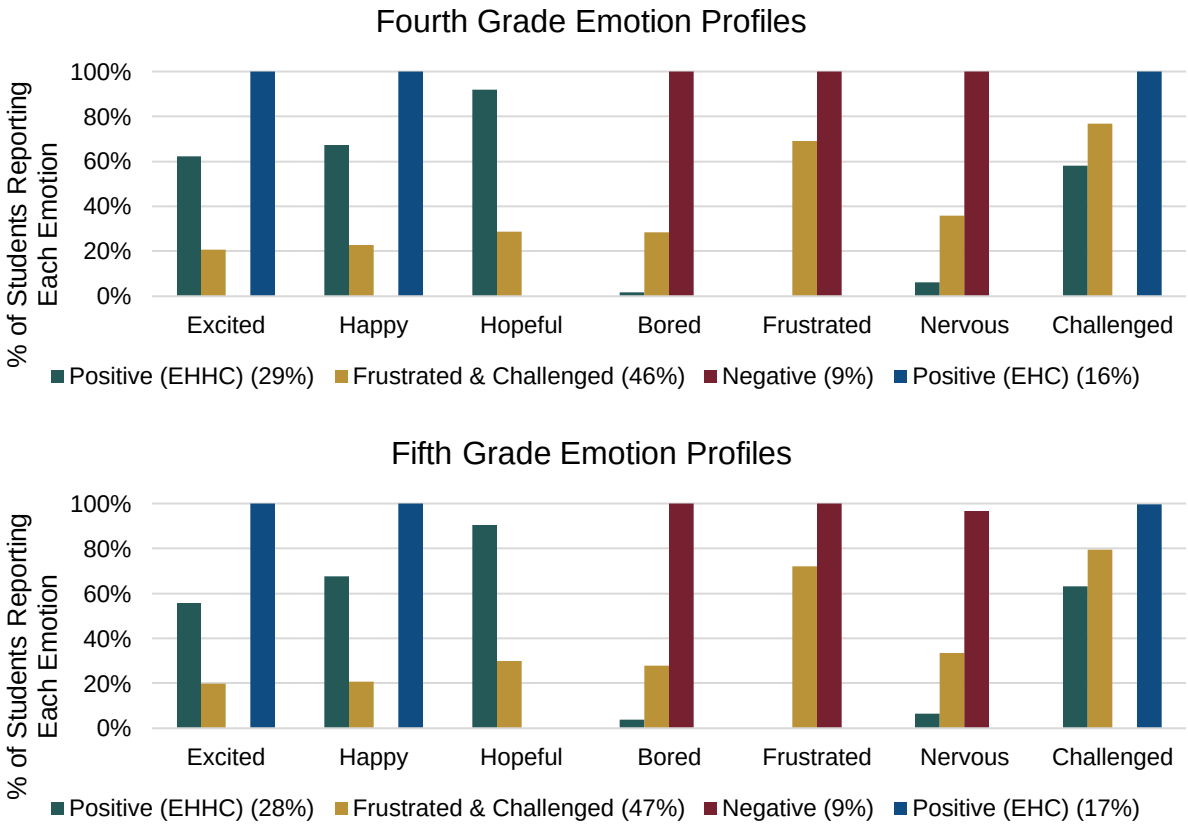
Fifth Graders	1	7	46980.311			
	2	15	41541.734	0.817	5481.758	< .001
	3	23	40416.078	0.855	1168.837	< .001
	4	31	39344.735	0.882	1114.524	< .001
	5	39	38685.607	0.935	702.308	< .001
	6	47	37744.997	0.951	958.360*	< .001
	7	55	37157.974	0.961	630.204*	< .001



Note. The highlight solution was selected.
*The best loglikelihood value was not replicated; therefore, the solution is not trustworthy.

Figure 3.5

Latent Class Analysis Profiles of Emotions



The LCAs identified similar patterns of emotion for both grades. The largest profile was the *Frustrated & Challenged* profile (46% of fourth graders; 47% of fifth graders). As the name suggests, the most commonly reported emotions in this profile were frustrated and challenged—at least 70% of students reported frustrated and challenged—and all other emotions, including positive emotions, were reported by 20%-35% of students in this profile.

There were two positive profiles—*Positive (EHHC)* and *Positive (EHC)*. Students in the *Positive (EHHC)* profile reported some combination of the positive emotions (excited, happy, and hopeful), as well as challenged. This profile made up 28% of fourth graders and 29% of fifth graders. All students in the *Positive (EHC)* profile exclusively reported feeling excited, happy, and challenged in math. This profile was smaller than the other positive profile, making up 16% of

fourth graders and 17% of fifth graders.

The last and smallest profile was the *Negative* profile. Students in this profile only reported feeling negative emotions towards math—bored, frustrated, and nervous. Only 9% of both fourth and fifth graders were in this profile. See Figure 3.5 for fourth and fifth emotion profiles.

Path Analysis

I used both fourth and fifth graders in the path analysis, rather than estimating separate models for each grade, because the emotion profiles were so similar (Appendix 3.E visually compares profiles across grades). To account for grade-wise differences, grade was used as a dummy variable in the path analysis (0 = fourth grader, 1 = fifth grader). With the combination of multinomial (predicting emotion profile) and identity pathways (predicting appraisal and achievement), it is impossible to get standardized coefficients across all paths. *Z*-scores for model coefficients are presented in Table 3.4 (see Appendix 3.F for unstandardized coefficients/log odds, standard errors, and confidence intervals).

Emotion

Students with higher prior math achievement and math expectancy were more likely to be in the Positive (EHHC) profile than any other profile (z's ranging from -2.03 to -17.40, p's < .05). Using Wald's χ^2 post-hoc tests, I further determined that, on average, the Negative profile had the lowest prior math achievement and expectancy (z's ranging from -6.24 to 15.08, p's < .001), followed by students in the Frustrated & Challenged profile (z's = 12.14 and 13.71, p < .001), then the Positive (EHC) profile, with students in the Positive (EHHC) profile having the highest prior math achievement and expectancy. See Figure 3.6 for post-hoc results.

Additionally, students in the Frustrated & Challenged and the Negative profiles reported lower math value than students in both the Positive profiles (z's = -6.91 and -11.95, p's < .001,

respectively), with students in the Negative profile having lower math value, on average, than those in the Frustrated and Challenged profile (z = -9.11, p < .001).

Table 3.4

Z-Scores for Path Analysis Coefficients

	Appraisal		Emotion			Math Achievement
	Expectancy	Value	Frustrated & Challenged	Negative	Positive (EHC)	
Demographics						
Fifth Grader	**-11.47**[c]	**-4.10**[c]	**3.97**[c]	**3.76**[c]	-0.10	**-12.80**[c]
Disability	1.95	-0.18	**-3.04**[b]	**-3.10**[b]	1.33	**-3.46**[b]
FRL	1.32	**2.06**[a]	-0.53	-1.79	0.03	**-10.87**[c]
ELL	1.59	**2.48**[b]	**-5.79**[c]	**-5.98**[c]	-0.31	-0.23
Boy	**4.02**[c]	**-3.33**[b]	**-7.54**[b]	**-4.86**[c]	-0.26	**-1.54**[a]
Race						
Black	**12.05**[c]	**9.89**[c]	**-5.19**[c]	**-5.59**[c]	**-2.71**[b]	**-12.59**[c]
Hispanic	**2.65**[b]	**2.50**[a]	0.41	-0.32	-0.24	-1.41
Other	0.72	0.92	-1.70	**-2.56**[a]	-1.11	**2.04**[a]
Environment						
Prior Math Achievement	**21.85**[c]	**7.91**[c]	**-15.33**[c]	**-17.40**[c]	**-5.33**[b]	**118.56**[c]
Appraisal						
Expectancy			**-12.62**[c]	**-14.54**[c]	**-2.03**[a]	**2.46**[a]
Value			**-6.91**[c]	**-11.95**[c]	-1.54	**-2.18**[a]
Emotion						
Frustrated & Challenged						**-7.33**[c]
Negative						**-10.92**[c]
Positive (EHC)						-1.33
Intercept	10.87[c]	25.00[c]	24.01[c]	25.01[c]	7.38[c]	28.60[c]
Error Variance	0.60	0.61				162.10

Note. Positive (Excited, Happy, Hopeful, Challenged) was used as the reference profile. The demographic reference group are girls who are White, in fourth grade, who are not classified as having a disability, eligible for free/reduced lunch, or as English Language Learners. FRL=Free/Reduced Lunch; ELL=English Language Learner; EHC=Excited Happy Challenged
[a] p < .05, [b] p < .01, [c] p < .001; Statistically significant coefficients also bolded

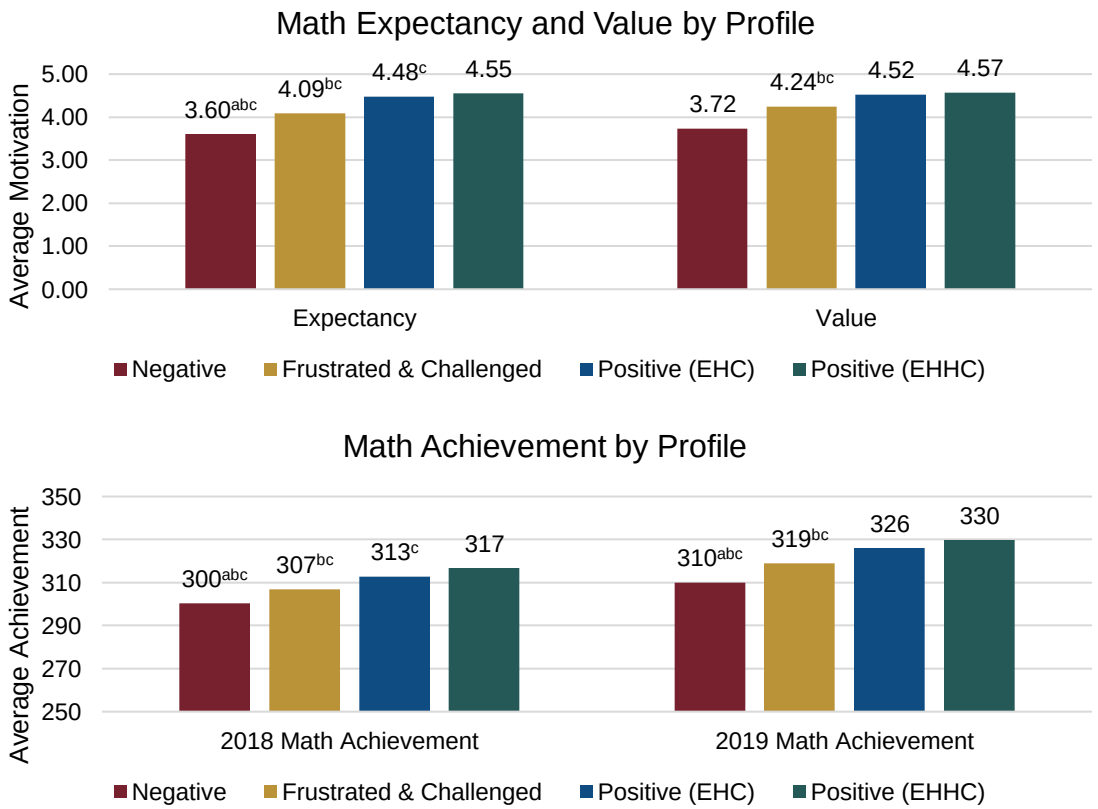
Math Achievement

On average, students in the Negative profile had the lowest math achievement, followed by students in the Frustrated & Challenged profile (post-hoc comparing Negative and Frustrated

& Challenged: z = -7.17, p < .001). Students in these profiles had statistically significantly lower math achievement than those in the Positive (EHHC) profile (post-hoc's comparing Positive (EHHC) to Frustrated and Challenged and Negative: z's = -7.33 and -10.92, p's < .001, respectively), as well as the Positive (EHC) profiles (z = 7.22 and 10.89, p's < .001). Students in the Positive (EHC) did not statistically significantly differ from those in the Positive (EHHC) profile (z = -1.33, p = .184). On average, students with higher math expectancy had higher math achievement. After accounting for all other variables (i.e., expectancy, prior achievement, emotion, and demographics), students that had higher value for mathematics had lower scores.

Figure 3.6

Comparison of Motivation and Achievement by Profile



Note. Difference from Positive (EHHC) determined by path analysis; difference from Frustrated & Challenged and Positive (EHC) determined by Wald's χ^2 post-hoc test
[a] Statistically significantly different from Frustrated & Challenged; [b] Statistically significantly different from Positive (EHC); [c] Statistically significantly different from Positive (EHHC)

Post-hoc results displayed in Figure 3.6 (Wald's χ^2 results in Appendix 3.G).

Mediation Models

Indirect effects were calculated using Iacobucci's (2012) test for categorical mediators. Table 3.5 presents direct and indirect effects of expectancy and value on achievement and Appendix 3.H displays mediation models. Membership to both Positive profiles (EHHC and EHC) mediated the relation between appraisal and achievement. Students with higher math expectancy and value were more likely to be in the Positive profiles (z's ranging from 5.98 to 11.82, p's < .001), and, in turn, membership to the Positive profiles were related to higher math achievement (Positive (EHHC) z = 5.72, Positive (EHC) z = 6.08, p's < .001). The indirect effects of expectancy on achievement through positive emotion profile were statistically significant (Positive (EHHC) $z_{indirect}$ = 4.95, Positive (EHC) $z_{indirect}$ = 5.39, p's < .001) and explained about half of the total effect (55.43% and 49.88%, respectively). The direct effect of expectancy on achievement also remained statistically significant (Positive (EHHC) z_{direct} = 4.34, 44.57% of total effect; Positive (EHC) z_{direct} = 4.17, 50.12% of total effect; p's < .001). The indirect effects of value on achievement through positive emotion profile were also statistically significant (Positive (EHHC) $z_{indirect}$ = 4.10, Positive (EHC) $z_{indirect}$ = 5.394, p's < .001) and explained over three-quarters of the total effect (82.48% and 75.51%, respectively). Conversely, the direct effect of value on achievement was not statistically significant (Positive (EHHC) z_{direct} = -0.68, 17.52% of total effect; Positive (EHC) z_{direct} = -0.73, 24.49% of total effect; p's ≥ .464).

Both the Negative and Frustrated & Challenged profiles also mediated the relation between math expectancy and achievement—students with higher math expectancy were less likely to be in the Negative and Frustrated & Challenged profiles (z = -10.11, z = -10.43, p's < .001, respectively), and, in turn, membership to those profiles were related to lower math achievement

(z = -8.97, z = -4.65, p's < .001, respectively). Similar to the positive profiles, the indirect effects of expectancy on achievement through emotion were statistically significant (Negative $z_{indirect}$ = 6.69, 72.07% of total effect; Frustrated & Challenged $z_{indirect}$ = 4.23, 31.52% of total effect; , p's < .001) and the direct effect of expectancy on achievement also remained statistically significant (Negative z_{direct} = 3.85, 27.93% of total effect; Frustrated & Challenged z_{direct} = 4.37, 68.48% of total effect; p's < .001).

Table 3.5

Iacobucci Test of Indirect Effects of Motivation on Math Achievement Through Emotion Profile

		Direct Effects		**Indirect Effects**			
Emotion Profile	Appraisal	App → Ach	% of Total Effect	App → Emo	Emo → Ach	App → Emo → Ach	% of Total Effect
Frustrated & Challenged	Expectancy	4.37[c]	68.48%	-10.43[c]	-4.65[c]	4.23[c]	31.52%
	Value	-0.46	55.54%	-1.92	-4.65[c]	1.74	44.46%
Negative	Expectancy	3.85[c]	27.93%	-10.11[c]	-8.97[c]	6.69[c]	72.07%
	Value	-1.53	12.68%	-11.23[c]	-8.97[c]	6.99[c]	87.32%
Positive (EHC)	Expectancy	4.17[c]	50.12%	11.82[c]	6.08[c]	5.39[c]	49.88%
	Value	-0.73	24.49%	6.45[c]	6.08[c]	4.40[c]	75.51%
Positive (EHHC)	Expectancy	4.34[c]	44.57%	10.02[c]	5.72[c]	4.95[c]	55.43%
	Value	-0.68	17.52%	5.98[c]	5.72[c]	4.10[c]	82.48%

Note. Positive (EHHC) = Positive (Excited, Happy, Hopeful, Challenged); Positive (EHC) = Positive (Excited, Happy, Challenged)
App = Appraisal; Emo = Emotion Profile; Ach = Math Achievement
[a] p < .05, [b] p < .01, [c] p < .001

As with expectancy, students with higher math value were less likely to be in the Negative profile (z = -11.23, p < .001) and, in turn, those students had lower math achievement. $z_{indirect}$ = 6.987, p < .001; z_{direct} = -1.534, p = .125). The effect of value on math achievement was mediated by membership to the negative profile, with indirect effects accounting for 87.32% of the total effects ($z_{indirect}$ = 6.99, p < .001; z_{direct} = -1.53, 12.68% of total effect, p = .125). Membership in the Frustrated & Challenged profile did not mediate the relation between math value and achievement; neither direct (z_{direct} = -0.46, p = .645) nor indirect effects ($z_{indirect}$ = 1.74, p = .080) were statistically

significant.

The mediation models were also replicated with the discrete emotions. The effect of math expectancy and value on achievement was mediated by each emotion ($z_{indirect}$'s ranging from 1.96 to 8.16, 11.09-80.48% of total effects, $p < .05$). Appraisal was positively related to positive emotions (excited, happy, hopeful; z's ranging from 2.11 to 13.95, $p < .05$), as well as challenged, and those emotions were, in turn, positively related to achievement (z's ranging from 4.97 to 8.12, $p < .001$). Conversely, appraisal was negatively related to negative emotions (bored, frustrated, nervous; z's ranging from -6.35 to -15.80, $p < .001$) and those emotions were, in turn, negatively related to achievement (z's ranging from -8.54 to -9.55, $p < .001$). The direct effect of expectancy on achievement was statistically significant across all models (z_{direct}'s ranging from 3.32 to 4.75, 32.46-88.91% of total effects, $p < .001$) but the direct effect of value on achievement was not (z_{direct}'s ranging from -0.86 to -1.79, 19.52-33.20% of total effects, $p \geq .073$). See Appendix 3.I for mediation results.

Discussion

Research Question 3A: Emotion Profiles

With my first research question, I aimed to identify and compare emotion profiles in fourth and fifth graders. The emotion profiles I found were nearly identical across these grades. Although this may be a function of survey structure (there are only so many emotion combinations), Raccanello et al. (2018) also found that multiple age groups demonstrated similar profiles (fourth, seventh, and eleventh graders).

Like previous studies on emotional profiles, I found at least one positive profile and a negative profile (Ganotice et al., 2016; Hanin & Van Nieuwenhoven, 2019; Jarrell et al., 2016; 2017; Raccanello et al., 2018; Robinson et al., 2017, 2020). Additionally, I found one "mixed

emotion" profile—students in this profile mainly reported feeling frustrated and challenged but all other emotions, including positive emotions, were also reported by some students. This embodies the notion of mixed feelings, as students primarily reported a negative emotion (frustrated) but also may have reported a positive emotion.

Not only was a mixed emotions profile present but it was also the largest profile, making up nearly half of the sample (46% and 47% of the fourth and fifth grade sample, respectively). The large proportion of students reporting feeling frustrated and challenged may be due to the context the survey was given: a digital mathematics learning environment. Within this context, students are faced with different games and mathematics material that are designed to challenge students, which may turn to frustration (see Kirby et al., 2014 and discussion below). Additionally, in this study, emotions were measured in the middle of the school year. In the previous study, I found that motivation is often lowest in the middle of the school year. This may also contribute to the large percentage of students reporting mixed emotions, especially with frustrated being one of the most commonly reported emotions in the profile.

The Role of "Challenged"

Challenged is not theoretically grounded within CVT, but it appeared in three of the four profiles—both positive profiles and a mixed emotion (Frustrated & Challenged) profile. This is likely due to differences in how students perceive and respond to difficult situations. For example, in cognitive interviews with elementary students (Rutherford et al., 2019), one child explained that challenged was a positive emotion because:

> there should always be challenges cause if there’s not really challenges, you’re not gonna grow at all, because if there was no challenges and you already know how to do this, school isn’t teaching you anything, and your brain’s not gonna grow at all.

Conversely, another child gave challenged a negative valence, explaining that:

> I feel challenged in math because I don't really get a lot about math and like when I don't get it, I'm like oh my gosh umm... umm... and then like I don't usually answer a lot of questions and so and I pay attention and then like the day of the math test I'm like oh my gosh what, what do we do—this is so frustrating!

Because of this, it is difficult for us to label the Frustrated and Challenged profile as more negative than positive. The prevalence of challenged in this profile, along with some representation among the other positive emotions, makes interpretation of this profile less than clear-cut. This also applies to the Positive profiles, both of which included challenged. Although challenged may muddy the waters of the emotion profiles a bit, MIND included this variable as an important element of their theory of change, and given the structure of the survey, excluding challenged from analysis might result in missing data and/or unfairly represent students' actual reporting of emotions.

Antecedents and Outcomes of Emotion Profiles

In line with prior research, the negative profile was related to lower motivation and achievement (e.g., Ganotice et al., 2016; Robinson et al., 2017). The two positive profiles differed in the prevalence of students reporting feeling hopeful. Although many of the results were similar across these two positive profiles, students that did not report feeling hopeful towards math—those in the Positive (EHC) profiles—tended to have lower prior math achievement and expectancy. However, there was no difference between these two profiles in predicting the mathematics achievement outcome. This may be due to how hopeful was interpreted—some students described hopeful as wanting to participate in the subject more, while others felt hopeful that they would do better in a subject they had previously struggled with (Rutherford et al., 2019). Either interpretation

is in line with CVT: students who have lower prior achievement may feel less control (Pekrun, 2006; Pekrun et al., 2011), a theorized predictor of feeling hopeful which aligns with the latter description. At the same time, hopeful is a positive, activating emotion which is more likely to be felt when students are interested (Ainley, 2018), the former interpretation, and thus tied to later achievement (Pekrun et al., 2011).

Examining these clusters of emotion profiles provides insight that a variable-centered approach may not have provided. Although frustrated is a negative emotion, the mixed emotional profile of students in Frustrated and Challenged might have a protective effect—this profile was associated with more positive antecedents and outcomes than the purely negative profile. On the other hand, prior research has indicated that seemingly negative emotions, such as frustration, can nevertheless have positive associations with learning and achievement, especially when momentarily occurring (e.g., Liu et al., 2013). In my study, although students were reporting emotions for mathematics overall, the existence of frustration in a profile with more typical positive emotions may indicate that for these students, frustration itself is positive.

Research Question 3B: Mediation within the CVT Framework

My second research question examined the role of emotions within the CVT framework and how emotional profiles may mediate the relation between appraisal (i.e., expectancy and values) and achievement. Taken as a whole, my modeled results were consistent with all pathways within the CVT framework (see Figure 3.1 on page 70). As an element of environment, prior achievement was predictive of all downstream elements of the model: appraisal, emotions, and achievement outcomes. The link from achievement to these downstream elements and from appraisal and emotions to achievement, support a theorized reciprocal relation between achievement and emotions and motivation (Pekrun et al., 2014, 2017). As for appraisal, expectancy

distinguished between all emotion profiles, but value was unable to distinguish between the two positive profiles. This may be partially due to my operationalization of value as including both utility and importance—this may be out of line with the external/internal distinctions in CVT. Alternatively, value may have less salience in achievement-related situations than expectancy; prior research has found expectancies to be the stronger predictor of performance, whereas value is more predictive of intentions and decisions, such as course enrollment (see Wigfield & Cambria, 2010).

In considering the mediation question specifically, I found that all emotional profiles did in fact mediate the relation between math expectancy and achievement, with indirect effects accounting for 32-72% of the effect of expectancy on achievement. Additionally, three of the four profiles mediated the association between math value and math achievement, such that the indirect effects accounted for 76-87% of the total effect and the direct effects were not statistically significant. This meditation of the relation between value and achievement by emotion profiles is more in line with CVT but appears contrary to the direct-effect-only regression results in Table 3.4 (page 97), which display a puzzling negative association between value and achievement after accounting for all other variables.

Limitations and Future Directions

One major limitation of this study was how emotions were operationalized. Although the simplified measure of emotion likely reduced cognitive load, it was impossible to tell how strongly students' felt the emotion(s) they reported. It is possible, especially with the mixed emotion profile, that students felt emotions more or less strongly, such as the moderate or low profiles found in prior studies (e.g., Ganotice et al., 2016; Hanin & Van Nieuwenhoven, 2019; Jarrell et al., 2016; 2017; Robinson et al., 2017, 2020). Similarly, the constructs were not measured with as many

items as in prior research or with items, such as specific emotions, that are not exactly in line with CVT. This is especially true for challenged. Removing challenged from analyses would cause measurement issues, as students who reported challenged would only be "reporting" two emotions in the analyses, which is not accurate. However, keeping challenged in the analyses also creates some measurement issues because it is impossible to know how students are interpreting challenged. More validation work is needed to relate this emotion scale to more traditional measures, such as the AEQ. Additionally, further validation of the expectancy and value measures would be beneficial. Future research should include a measure of negative value, which could potentially explain the negative association between value and achievement.

This imprecision in measurement operates within the ecology of the study along with the context—by partnering with a commercial software developer, I was able to include large numbers of students with a measure that detracted minimally from their school day. By using secondary data, I am able to collect a wide range of data "in the wild" using measures that are already embedded by developers. Additionally, I am able to collect data from a large and diverse sample that may be more generalizable than the traditionally white, middle class samples in education research (Henrich et al., 2010; Nielsen et al., 2017; Shadish et al., 2001). Limitations in measurement should be considered in light of these benefits.

Further, although my measures are ordered in time, the analyses do not present a longitudinal view of the relations between these variables, nor am I able to estimate causality with my methods. I present a snapshot of how emotions may function within a CVT framework for elementary-aged children as an important first step toward examining emotion and motivation development; longitudinal investigations of these relations will allow us to form a clearer picture of their reciprocal nature and other developmental relations. Similarly, CVT, as most education-

related theories, finds its greatest utility in relating educational practice and experiences with motivation, emotions, and subsequent achievement. By fleshing out the *Environment* aspects of CVT within models such as this one, we can better understand where educators can intervene to foster positive motivational and emotional states for students.

Conclusion

My study is the first to test membership of emotion profiles within a CVT framework with elementary students. My results largely support the CVT model, with each part of the model feeding into the next. Importantly, higher expectancies and values were associated with more positive profiles of mathematics emotions. Environment, appraisal, and emotions all predicted achievement as expected, with the exception of a negative association between value and achievement in the final model. Although emotion profiles tended to split among positive/negative lines, the Frustrated & Challenged profile suggests the presence of mixed emotions, as students reported both positive and negative emotions for mathematics within this profile. A better understanding of the circumstances that give rise to such mixed emotions can help researchers and educators ensure more positive emotional pathways toward student mathematics achievement.

References

Ahmed, W., Minnaert, A., Kuyper, H., & van der Werf, G. (2012). Reciprocal relationships between math self-concept and math anxiety. *Learning and Individual Differences, 22*(3), 385-389. https://doi.org/10.1016/j.lindif.2011.12.004

Ainley, M. (2018). Students' interest and engagement in classroom activities. In S. L. Christenson, A. L. Reschly, & C. Wylie (Eds.) *Handbook of Research on Student Engagement* (pp. 283-302). Springer. https://doi.org/10.1007/978-1-4614-2018-7

Ainley, M., & Ainley, J. (2011). Student engagement with science in early adolescence: The contribution of enjoyment to students' continuing interest in learning about science. *Contemporary Educational Psychology, 36*(1), 4–12. https://doi.org/10.1016/j.cedpsych.2010.08.001

Ames, C. (1992). Classrooms: Goals, structures, and student motivation. *Journal of Educational Psychology, 84*(3), 261–271. https://doi.org/10.1037/0022-0663.84.3.261

Ben-Eliyahu, A., Linnenbrink-Garcia, L. Extending self-regulated learning to include self-regulated emotion strategies. *Motivation and Emotion, 37*, 558–573 (2013). https://doi.org/10.1007/s11031-012-9332-3

Bergman, L. R., & El-Khouri, B. M. (2003). A person-oriented approach: Methods for today and methods for tomorrow. *New directions for child and adolescent development, 2003*(101), 25-38. https://doi.org/10.1002/cd.80

Blair, C. (2002). School readiness: Integrating cognition and emotion in a neurobiological conceptualization of children's functioning at school entry. *American Psychologist, 57*(2), 111–127. https://doi.org/10.1037//0003-066X.57.2.111

Boekaerts, M. (1993). Being concerned with well-being and with learning. *Educational Psychologist*, *28*(2), 149–167. https://doi.org/10.1207/s15326985ep2802_4

Buff, A. (2014). Enjoyment of learning and its personal antecedents: Testing the change–change assumption of the control-value theory of achievement emotions. *Learning and Individual Differences*, *31*, 21–29. https://doi.org/10.1016/j.lindif.2013.12.007

Burić, I., & Sorić, I. (2012). The role of test hope and hopelessness in self-regulated learning: Relations between volitional strategies, cognitive appraisals and academic achievement. *Learning and Individual Differences*, *22*(4), 523–529. https://doi.org/10.1016/j.lindif.2012.03.011

Campbell-Montalvo, R. A. (2020). Being QuantCritical of US K-12 demographic data: Using and reporting race/ethnicity in Florida Heartland Schools. *Race Ethnicity and Education*, *23*(2), 180-199. https://doi.org/10.1080/13613324.2019.1679748

Cvencek, D., Meltzoff, A. N., & Greenwald, A. G. (2011). Math–gender stereotypes in elementary school children. *Child Development*, *82*(3), 766–779. https://doi.org/10.1111/j.1467-8624.2010.01529.x

Daniels, L. M., Stupnisky, R. H., Pekrun, R., Haynes, T. L., Perry, R. P., & Newall, N. E. (2009). A longitudinal analysis of achievement goals: From affective antecedents to emotional effects and achievement outcomes. *Journal of Educational Psychology*, *101*(4), 948–963. https://doi.org/10.1037/a0016096

Daschmann, E. C., Goetz, T., & Stupnisky, R. H. (2011). Testing the predictors of boredom at school: Development and validation of the precursors to boredom scales. *British Journal of Educational Psychology*, *81*(3), 421–440. https://doi.org/10.1348/000709910X526038

DeCuir-Gunby, J. T., & Schutz, P. A. (2014). Researching race within educational psychology contexts. Educational Psychologist, 49(4), 244-260. https://doi.org/10.1080/00461520.2014.957828

Dettmers, S., Trautwein, U., Lüdtke, O., Goetz, T., Frenzel, A. C., & Pekrun, R. (2011). Students' emotions during homework in mathematics: Testing a theoretical model of antecedents and achievement outcomes. *Contemporary Educational Psychology*, *36*(1), 25–35. https://doi.org/10.1016/j.cedpsych.2010.10.001

Di Leo, I., Muis, K. R., Singh, C. A., & Psaradellis, C. (2019). Curiosity… Confusion? Frustration! The role and sequencing of emotions during mathematics problem solving. *Contemporary Educational Psychology, 58*, 121–137. https://doi.org/10.1016/j.cedpsych.2019.03.001

Elliot, A. J. (1999). Approach and avoidance motivation and achievement goals. *Educational Psychologist, 34*(3), 169–189. https://doi.org/10.1207/s15326985ep3403_3

Feldman Barrett, L., & Russell, J. A. (1998). Independence and bipolarity in the structure of current affect. *Journal of Personality and Social Psychology, 74*(4), 967–984. https://doi.org/10.1037/0022-3514.74.4.967

Florida Department of Education (2016). *Collecting and reporting race and ethnicity data: For PK-12 students and staff*. http://www.fldoe.org/core/fileparse.php/7574/urlt/0101164-race.pdf

Florida Department of Education (2018). *Florida Standards Assessments 2017–2018 volume 4: Evidence of reliability and validity.* http://www.fldoe.org/core/fileparse.php/5663/urlt/V4-FSA-1718-TechRpt.pdf

Florida Department of Education (2019). *Florida Standards Assessments 2018–2019 volume 4: Evidence of reliability and validity.* http://www.fldoe.org/core/fileparse.php/5663/urlt/V4-FSATechReportYear1819.pdf

Fredricks, J. A., & Eccles, J. S. (2002). Children's competence and value beliefs from childhood through adolescence: growth trajectories in two male-sex-typed domains. *Developmental Psychology, 38*(4), 519-533. https://doi.org/10.1037/0012-1649.38.4.519

Frenzel, A. C., Pekrun, R., & Goetz, T. (2007). Girls and mathematics —A "hopeless" issue? A control-value approach to gender differences in emotions towards mathematics. *European Journal of Psychology of Education*, *22*(4), 497–514. https://doi.org/10.1007/BF03173468

Fong Lam, U., Chen, W.-W., Zhang, J., & Liang, T. (2015). It feels good to learn where I belong: School belonging, academic emotions, and academic achievement in adolescents. *School Psychology International, 36*(4), 393–409. https://doi.org/10.1177/0143034315589649

Gadermann, A. M., Guhn, M., & Zumbo, B. D. (2012). Estimating ordinal reliability for Likert-type and ordinal item response data: A conceptual, empirical, and practical guide. *Practical Assessment, Research, and Evaluation, 17*(3). https://doi.org/10.7275/n560-j767

Ganley, C. M. & McGraw, A. L. (2016). The development and validation of a revised version of the math anxiety scale for young children. Frontiers in Psychology, 7(1181). https://doi.org/10.3389/fpsyg.2016.01181

Ganotice, F. A., Datu, J. A. D., & King, R. B. (2016). Which emotional profiles exhibit the best learning outcomes? A person-centered analysis of students' academic emotions. *School Psychology International*, *37*(5), 498–518. https://doi.org/10.1177/0143034316660147

Geary, D. C. (1995). Reflections of evolution and culture in children's cognition: Implications for mathematical development and instruction. The American Psychologist, 50, 24-37. https://doi.org/10.1037/0003-066X.50.1.24

Goetz, T., Cronjaeger, H., Frenzel, A. C., Lüdtke, O., & Hall, N. C. (2010). Academic self-concept and emotion relations: Domain specificity and age effects. *Contemporary Educational Psychology*, *35*(1), 44–58. https://doi.org/10.1016/j.cedpsych.2009.10.001

Goldin, G. A. (2014). Perspectives on emotion in mathematical engagement, learning, and problem solving. In R. Pekrun & L. Linnenbrink-Garcia (Eds.) *International handbook of emotions in education* (pp. 391-414). Routledge.

Graziano, P. A., Reavis, R. D., Keane, S. P., & Calkins, S. D. (2007). The role of emotion regulation and children's early academic success. *Journal of School Psychology*, *45*(1), 3–19. https://doi.org/10.1016/j.jsp.2006.09.002

Hanin, V., & Van Nieuwenhoven, C. (2019). The influence of motivational and emotional factors in mathematical learning in secondary education. *European Review of Applied Psychology*, *66*(3), 127–138. https://doi.org/10.1016/j.erap.2016.04.006

Hayenga, A. O., & Corpus, J. H. (2010). Profiles of intrinsic and extrinsic motivations: A person-centered approach to motivation and achievement in middle school. *Motivation and Emotion*, *34*(4), 371-383. https://doi.org/10.1007/s11031-010-9181-x

Howard, M. C., & Hoffman, M. E. (2018). Variable-centered, person-centered, and person-specific approaches: where theory meets the method. *Organizational Research Methods*, *21*(4), 846-876. https://doi.org/10.1177/1094428117744021

Iacobucci, D. (2012). Mediation analysis and categorical variables: The final frontier. *Journal of Consumer Psychology*, *22*(4), 582–594. https://doi.org/10.1016/j.jcps.2012.03.006

Jameson, M. M. (2014). Contextual factors related to math anxiety in second-grade children. The Journal of Experimental Education, 82(4), 518-536. https://doi.org/10.1080/00220973.2013.813367

Jarrell, A., Harley, J. M., & LaJoie, S. P. (2016). The link between achievement emotions, appraisals, and task performance: Pedagogical considerations for emotions in CBLEs. Journal of Computers in Education, 3, 289–307. http://dx.doi.org/10.1007/s40692-016-0064-3

Jarrell, A., Harley, J. M., LaJoie, S. P., & Naismith, L. (2017). Success, failure and emotions: Examining the relationship between performance feedback and emotions in diagnostic reasoning. Educational Technology Research and Development, 65, 1263–1284. http://dx.doi.org/10.1007/s11423-017-9521-6

Karabenick, S. A., Woolley, M. E., Friedel, J. M., Ammon, B. V., Blazevski, J., Bonney, C. R., de Groot, E., Gilbert, M. C., Musu, L., Kempler, T. M., & Kelly, K. L. (2007). Cognitive processing of self-report items in educational research: Do they think what we mean? *Educational Psychologist*, *42*(3), 139-151. https://doi.org/10.1080/00461520701416231

King, R. B., McInerney, D. M., Ganotice, F. A., & Villarosa, J. B. (2015). Positive affect catalyzes academic engagement: Cross-sectional, longitudinal, and experimental

evidence. *Learning and Individual Differences, 39*, 64–72. https://doi.org/10.1016/j.lindif.2015.03.005

Kirby, L. D., Morrow, J., & Yih, J. (2014). The challenge of challenge: Pursuing determination as an emotion. In M. M. Tugade, M. N. Shiota, & L. D. Kirby (Eds.), *Handbook of Positive Emotion*s (p. 378–395). The Guilford Press.

Lane, A. M., Whyte, G. P., Terry, P. C., & Nevill, A. M. (2005). Mood, self-set goals and examination performance: the moderating effect of depressed mood. *Personality and Individual Differences, 39*(1), 143–153. https://doi.org/10.1016/j.paid.2004.12.015

Larsen, J. T., & McGraw, A. P. (2011). Further evidence for mixed emotions. *Journal of Personality and Social Psychology, 100*(6), 1095-1110. https://doi.org/10.1037/a0021846

Laursen, B., & Hoff, E. (2006). Person-centered and variable-centered approaches to longitudinal data. *Merrill-Palmer Quarterly (1982-)*, 377-389. https://doi.org/10.1353/mpq.2006.0029

Lichtenfeld, S., Pekrun, R., Stupnisky, R. H., Reiss, K., & Murayama, K. (2012). Measuring students' emotions in the early years: the achievement emotions questionnaire-elementary school (AEQ-ES). *Learning and Individual Differences, 22*(2), 190-201. https://doi.org/10.1016/j.lindif.2011.04.009

Linnenbrink, E. A., & Pintrich, P. R. (2003). The role of self-efficacy beliefs in student engagement and learning in the classroom. *Reading & Writing Quarterly, 19*(2), 119-137. https://doi.org/10.1080/10573560308223

Linnenbrink-Garcia, L., Rogat, T. K., Koskey, K. L. K. (2011). Affect and engagement during small group instruction. Contemporary Educational Psychology, 36(1), 13-24. https://doi-org.prox.lib.ncsu.edu/10.1016/j.cedpsych.2010.09.001

Linnenbrink-Garcia, L., Wormington, S. V., Snyder, K. E., Riggsbee, J., Perez, T., Ben-Eliyahu, A., & Hill, N. E. (2018). Multiple pathways to success: An examination of integrative motivational profiles among upper elementary and college students. *Journal of Educational Psychology, 110*(7), 1026–1048. doi:10.1037/edu0000245

Liu, Z., Pataranutaporn, V., Ocumpaugh, J., & Baker, R. (2013). Sequences of frustration and confusion, and learning. In S. K. D'Mello, R. A. Calvo, & A. Olney (Eds.) *Proceedings of the 6th International Conference on Educational Data Mining (EDM 2013)*. International Educational Data Mining Society. https://www.educationaldatamining.org/EDM2013/proceedings/EDM2013Proceedings.pdf

Masyn, K. E. (2013). Latent class analysis and finite mixture modeling. In T. D. Little (Ed.), *The Oxford handbook of quantitative methods: Vol. 2. Statistical analysis* (pp. 551– 611). Oxford University Press. http://dx.doi.org/10.1093/oxfordhb/9780199934898.013.0025

Meindl, P., Yu, A., Galla, B. M., Quirk, A., Haeck, C., Goyer, J. P., Lejuez, C. W., D'Mello, S. K., & Duckworth, A. L. (2019). A brief behavioral measure of frustration tolerance predicts academic achievement immediately and two years later. *Emotion, 19*(6), 1081. https://psycnet.apa.org/doi/10.1037/emo0000492

MIND Research Institute. (2017). Mind Research Institute: Learn math visually. Retrieved from http://mindresearch.org

Möller, J., Pohlmann, B., Köller, O., & Marsh, H. W. (2009). A meta-analytic path analysis of the internal/external frame of reference model of academic achievement and academic self-concept. *Review of Educational Research, 79*(3), 1129–1167. https://doi.org/10.3102/0034654309337522

Musu-Gillette, L. E., Wigfield, A., Harring, J. R., & Eccles, J. S. (2015). Trajectories of change in students' self-concepts of ability and values in math and college major choice. *Educational Research and Evaluation*, *21*(4), 343-370. https://doi.org/10.1080/13803611.2015.1057161

Muthén, L.K. and Muthén, B.O. (1998-2017). *Mplus User's Guide. Eighth Edition*. Muthén & Muthén

National Research Council. (2005). How students learn: Mathematics in the classroom. In M. S. Donovan & J. D. Bransford (Eds.*), Committee on how people learn: A targeted report for teachers* (Division of Behavioral and Social Sciences and Education). The National Academies Press.

Nylund, K. L., Asparouhov, T., & Muthén, B. O. (2007). Deciding on the number of classes in latent class analysis and growth mixture modeling: A Monte Carlo simulation study. *Structural Equation Modeling*, *14*(4), 535-569. https://doi.org/10.1080/10705510701575396

Nylund-Gibson, K., & Choi, A. Y. (2018). Ten frequently asked questions about latent class analysis. *Translational Issues in Psychological Science*, *4*(4), 440–461. https://doi.org/10.1037/tps0000176

Op't Eynde, P., de Corte, E., & Verschaffel, L. (2007). Students emotions: A key component of self-regulated learning. In G. D. Phye, P. Schults, & R. Pekrun (Eds.) Emotion in Education (pp. 185-204). Elsevier Inc. https://doi.org/10.1016/B978-012372545-5/50012-5

Peixoto, F., Sanches, C., Mata, L., & Monteiro, V. (2017). "How do you feel about math?": Relationships between competence and value appraisals, achievement emotions and

academic achievement. *European Journal of Psychology of Education*, *32*(3), 385–405. https://doi.org/10.1007/s10212-016-0299-4

Pekrun, R. (2006). The Control-Value Theory of Achievement Emotions: Assumptions, corollaries, and implications for educational research and practice. *Educational Psychology Review*, *18*(4), 315–341. https://doi.org/10.1007/s10648-006-9029-9

Pekrun, R., Frenzel, A. C., Goetz, T., & Perry, R. P. (2007). The control-value theory of achievement emotions. In P. A. Schutz & R. Pekrun (Eds.), *Emotion in Education: A Volume in the Educational Psychology Series*, 13–36. Elsevier Inc. https://doi.org/10.1016/B978-012372545-5/50003-4

Pekrun, R., Goetz, T., Frenzel, A. C., Barchfeld, P., Perry, R. P. (2011). Measuring emotions in students' learning and performance: The Achievement Emotions Questionnaire (AEQ). Contemporary Educational Psychology, 36(1), 36-48. https://doi.org/10.1016/j.cedpsych.2010.10.002

Pekrun, R., Hall, N. C., Goetz, T., & Perry, R. P. (2014). Boredom and academic achievement: Testing a model of reciprocal causation. *Journal of Educational Psychology*, *106*(3), 696-710. https://doi.org/10.1037/a0036006

Pekrun, R., Lichtenfeld, S., Marsh, H. W., Murayama, K., & Goetz, T. (2017). Achievement emotions and academic performance: Longitudinal models of reciprocal effects. *Child Development*, *88*(5), 1653–1670. https://doi.org/10.1111/cdev.12704

Pekrun, R., & Linnenbrink-Garcia, L. (Eds.) (2014). *International Handbook of Emotions in Education*. Routledge.

Pekrun, R., & Linnenbrink-Garcia, L. (2012). Academic emotions and student engagement. In S. L. Christenson, A. L. Reschly, & C. Wylie (Eds.) Handbook of Research on Student Engagement (pp. 259-282). Springer. https://doi.org/10.1007/978-1-4614-2018-7

Pekrun, R., & Meier, E. (2011). Epistemic emotion scales (EES) (Unpublished thesis). Department of Psychology, University of Munich, Munich, Germany.

Pekrun, R., & Perry, R. P. (2014). Control-value theory of achievement emotions. In R. Pekrun & L. Linnenbrink-Garcia (Eds.) *International handbook of emotions in education* (pp. 120-141). Routledge.

Pekrun, R., & Stephens, E. J. (2010). Achievement emotions: A Control-Value approach. *Social and Personality Psychology Compass*, *4*(4), 238–255. https://doi.org/10.1111/j.1751-9004.2010.00259.x

Putwain, D. W., Becker, S., Symes, W., & Pekrun, R. (2018). Reciprocal relations between students' academic enjoyment, boredom, and achievement over time. *Learning and Instruction*, *54*, 73–81. https://doi.org/10.1016/j.learninstruc.2017.08.004

Raccanello, D., Hall, R., & Burro, R. (2018). Salience of primary and secondary school students' achievement emotions and perceived antecedents: Interviews on literacy and mathematics domains. *Learning and Individual Differences*, *65*, 65–79. http://doi.org/10.1016/j.lindif.2018.05.015

Ramirez, G., Gunderson, E. A., Levine, S. C., & Beilock, S. L. (2013). Math anxiety, working memory, and math achievement in early elementary school. Journal of Cognition and Development, 14(2), 187-202. https://doi.org/10.1080/15248372.2012.664593

Robinson, K. A., Beymer, P. N., Ranellucci, J., & Schmidt, J. A. (2020). Momentary emotion profiles in high school science and their relations to control, value, achievement, and

science career intentions. *Motivation Science, 6*(4), 401-412. https://doi.org/10.1037/mot0000174

Robinson, K. A., Ranellucci, J., Lee, Y., Wormington, S. V., Roseth, C. J., & Linnenbrink-Garcia, L. (2017). Affective profiles and academic success in a college science course. Contemporary Educational Psychology, 51, 209–221. http://doi.org/10.1016/j.cedpsych.2017.08.004

Roos, A.-L., Goetz, T., Voracek, M., Krannich, M., Bieg, M., Jarrell, A., & Pekrun, R. (2020). Test anxiety and physiological arousal: A systematic review and meta-analysis. *Educational Psychology Review*. https://doi.org/10.1007/s10648-020-09543-z

Rutherford, T., Farkas, G., Duncan, G., Burchinal, M., Kibrick, M., Graham, J., Richland, L., Tran, N., Schneider, S., Duran, L., & Martinez, M. E. (2014). A randomized trial of an elementary school mathematics software intervention: Spatial-Temporal Math. *Journal of Research on Educational Effectiveness, 7*(4), 358-383. https://doi.org/10.1080/19345747.2013.856978

Rutherford, T., Liu, A. S., Lam, A. C., Schenke, K. (2020). Impact on mathematics self-beliefs from a mastery-based mathematics software. *Journal of Research on Technology in Education, 52*(1), 79-94. https://doi.org/10.1080/15391523.2019.1689210

Rutherford, T., Liu, A., Logan, W., Karamarkovich, S. M., & Wagemaker, M. (2019, August 8-11). *"I chose math because…" Cognitive interviews of a mathematics motivation measure* [Poster presentation]. American Psychological Association 2019 Convention, Chicago, IL.

Schenke, K., Rutherford, T., & Farkas, G. (2014). Alignment of game design features and state mathematics standards: Do results reflect intentions? *Computers & Education, 76*, 215-224. https://doi.org/10.1016/j.compedu.2014.03.019

Shaw, G., & Peterson, M. (2000*). Keeping Mozart in mind*. The Academic Press.

Shao, K., Pekrun, P., Marsh, H. W., & Loderer, K. (2020). Control-value appraisals, achievement emotions, and foreign language performance: A latent interaction analysis. Learning and Instruction, 69. https://doi.org/10.1016/j.learninstruc.2020.101356

Skinner, N., & Brewer, N. (2004). Adaptive approaches to competition: Challenge appraisals and positive emotion. *Journal of Sport and Exercise Psychology, 26*(2), 283-305. https://doi.org/10.1123/jsep.26.2.283

Smith, C. A., & Ellsworth, P. C. (1985). Patterns of cognitive appraisal in emotion. *Journal of Personality and Social Psychology, 48*(4), 813-838. https://doi.org/10.1037/0022-3514.48.4.813

Smith, C. A., Haynes, K. N., Lazarus, R. S., & Pope, L. K. (1993). In search of the" hot" cognitions: Attributions, appraisals, and their relation to emotion. *Journal of Personality and Social Psychology, 65*(5), 916 –929. https://doi.org/10.1037/0022-3514.65.5.916

Stange, M., Barry, A., Smyth, J., & Olson, K. (2016). Effects of smiley face scales on visual processing of satisfaction questions in web surveys. *Social Science Computer Review, 36*(6), 756–766. doi:10.1177/0894439316674166

Tulis, M., & Ainley, M. (2011). Interest, enjoyment and pride after failure experiences? Predictors of students' state-emotions after success and failure during learning in mathematics. *Educational Psychology, 31*(7), 779-807. https://doi.org/10.1080/01443410.2011.608524

Turner, J. E., & Schallert, D. L. (2001). Expectancy–value relationships of shame reactions and shame resiliency. *Journal of Educational Psychology, 93*(2), 320–329. https://doi.org/10.1037/0022-0663.93.2.320

Weiner, B. (1985). An attributional theory of achievement motivation and emotion. *Psychological Review, 92*(4), 548–573. https://doi.org/10.1037/0033-295x.92.4.548

Wigfield, A. (1994). Expectancy-value theory of achievement motivation: A developmental perspective. *Educational Psychology Review, 6*(1), 49–78. https://doi.org/10.1007/bf02209024

Wigfield, A., & Cambria, J. (2010). Students' achievement values, goal orientations, and interest: Definitions, development, and relations to achievement outcomes. *Developmental Review, 30*(1), 1–35. https://doi.org/10.1016/j.dr.2009.12.001

Wigfield, A., & Eccles, J. S. (1994). Children's competence beliefs, achievement values, and general self-esteem: Change across elementary and middle school. *The Journal of Early Adolescence, 14*(2), 107-138. https://doi.org/10.1177/027243169401400203

Wigfield, A., & Eccles, J. S. (2000). Expectancy–Value Theory of Achievement Motivation. *Contemporary Educational Psychology, 25*(1), 68–81. https://doi.org/10.1006/ceps.1999.1015

Wigfield, A., Eccles, J. S., & Möller, J. (2020). How dimensional comparisons help to understand linkages between expectancies, values, performance, and choice. *Educational Psychology Review, 32*(3), 657–680. https://doi.org/10.1007/s10648-020-09524-2

Williams-Johnson, M., Cross, D., Hong, J., Aultman, L., Osbon, J., & Schutz, P. (2008). "There are no emotions in math": How teachers approach emotions in the classroom. *Teachers College Record, 110*(8), 1574-1610.

Wormington, S. V., & Linnenbrink-Garcia, L. (2017). A new look at multiple goal pursuit: The promise of a person-centered approach. *Educational Psychology Review*, *29*(3), 407-445. https://doi.org/10.1007/s10648-016-9358-2

Valiente, C., Swanson, J., & Eisenberg, N. (2012). Linking students' emotions and academic achievement: When and why emotions matter. *Child Development Perspectives*, *6*(2), 129–135. https://doi.org/10.1111/j.1750-8606.2011.00192.x

van Tilburg, W.A.P., Igou, E.R. On boredom: Lack of challenge and meaning as distinct boredom experiences. *Motivation and Emotion, 36***,** 181–194. https://doi.org/10.1007/s11031-011-9234-9

Villavicencio, F. T., Bernardo, A. B., & I. (2016). Beyond math anxiety: Positive emotions predict mathematics achievement, self-regulation, and self-efficacy. *The Asia- Pacific Education Researcher; Manila*, *25*(3), 415–422. http://doi.org/10.1007/s40299-015-0251-4

Yih, J., Christian, F. L., Kirby, L. D., & Smith, C. A. (2013). The effects of challenge and pride on perseverance. Paper presented at the 14th Annual Meeting of the Society for Personality and Social Psychology, New Orleans, LA.

Zeidner, M. (2007). Test anxiety in educational contexts. In P. Schutz & R. Pekrun (Eds.) *Emotion in Education* (pp.165–184). Routledge. https://doi.org/10.1016/b978-012372545-5/50011-3

Zimmerman, B. J., & Schunk, D. H. (Eds.) (2011). *Handbook of self-regulation of learning and performance*. Routledge.

CHAPTER FOUR: STUDY THREE

Abstract

Motivation, which is an important predictor of success, fluctuates over time and context. As such, motivation researchers have used a combination of longitudinal and person-centered approaches to capture the complexity of motivation. In this study, I pioneered a new method to profile change—a latent profile analysis with multiple timepoints (LPA-long)—and compared it to a traditional latent transition analysis. My sample was fourth graders who took three motivation surveys within one school year (n = 5,365). I found five LPA-long profiles: three that had stable motivation (high, low, or high future value) and two that demonstrated a change in motivation (dip in winter/spring). The stable motivation profiles were also present in each of the LTA fall, winter, and spring profiles. There were differences between analyses in how motivation profiles were related to achievement. Overall, the traditional LTA provided detailed information on transitions and complex patterns of motivation, whereas the novel LPA-long provides a simplicity that allowed for greater interpretability.

Keywords: Situated Expectancy–Value Theory, mathematics, latent transition analysis

Profiling Change: Methods for Comparing Patterns in Mathematics Expectancies and Values Across Time

Motivation is a key influencer of students' academic behaviors and achievement (Singh et al., 2002; Wigfield & Eccles, 2000). A student's motivation, including their expectation for success and how much they value the task, is dynamic and fluctuates based on environment (Eccles (Parsons) et al., 1983; Wigfield & Eccles, 2000). Methodological advances in motivation have provided a richer understanding of this fluctuating nature, including how motivation may dynamically change across time and context (Eccles & Wigfield, 2020; Wigfield & Cambria, 2010; Wigfield & Eccles, 2000) and how motivational realistically vests within individuals—using, for example, person-centered approaches (Magnusson, 1998). However, different methods have been developed to separately answer dynamic vs. person-centered questions, making elusive a clear-cut method for analyzing dynamic and person-centered motivation questions together. Within this paper, I address this issue in two ways. First, I review different person-centered approaches for analyzing data at single and multiple timepoints. Second, I pioneer a new approach to longitudinal person-centered analysis—a latent profile analysis with multiple timepoints.

Theoretical Framework

Eccles and Wigfield's (2020) Situated Expectancy–Value Theory (SEVT) of motivation states that a person's expectation of success and subjective task values influence their choices. Previously known as Expectancy–Value Theory (Eccles (Parson) et al., 1983), this model posits that if a person expects to do well and values a task, they are more likely to pursue it. When re-defining the theory as SEVT, Eccles and Wigfield (2020) emphasize how the relations between expectancy, value, and achievement are not static and vary both across and within individuals; they note that this variance is related to social and developmental influences. In SEVT, Eccles and

Wigfield (2020) define expectancy using efficacy expectation—a person's belief about how well they will be able to complete a certain task. Subjective task value can be broken down into smaller components—attainment value, utility value, intrinsic value, and cost (Eccles & Wigfield, 2020). Attainment value is how *important* a task is to a person's identity. Utility value is how *useful* a task is. Intrinsic value is the person's *interest* or *enjoyment* in a task. Lastly, perceived cost is the *negative* aspect of a task that may decrease a person's pursual (e.g., time and effort; Eccles & Wigfield, 2020).

Each of these components have been tied to academic outcomes. For example, students who report higher levels of importance (attainment value), usefulness (utility value), and expectancies tend to be higher achievers and have more improvement in their skills over time (e.g., Guo et al., 2015; Meece et al., 1990; Meyer et al., 2019; Trautwein et al., 2012; Viljaranta et al., 2016).

Modeling Expectancy–Value Theory

Although the early models considered expectancy and value to be multiplicative (i.e., expectancy × value; Atkinson, 1957), early analyses using Eccles' Expectancy–Value theory focused more on the unique variance of the individual components—measuring associations between one component and outcomes at mean levels of other components (Eccles (Parsons) et al., 1983). With methodological advances, researchers were better able to examine the interaction between these latent constructs (e.g., Nagengast et al., 2013). For example, when Trautwein and colleagues (2012) predicted mathematics achievement, they found statistically significant interactions between expectancy and values (i.e., attainment, intrinsic, utility, and cost value). All four interactions had similar patterns—mathematics achievement was highest for students with high expectancy/high value and lowest for students with high value/low expectancy. This

highlights the importance of value for achievement—when value is low, expectancy is not enough to influence achievement/aspirations and when value is high, low expectancy can be even more detrimental (Guo et al., 2017; Nagengast et al., 2011, 2013; Trautwein et al., 2012).

Person-Centered Approaches

Although examining interactions between variables may reveal more of how expectancies and values operate in combinations, they may not capture the multiple combinations of variables and potential interactions between them that occur in real life; to do this, many researchers turn to person-centered approaches (e.g., Andersen & Cross, 2014; Dietrich & Lazarides, 2019; Magnusson, 1998; Umarji et al., 2018). Results from person centered analyses provide a deeper understanding of motivation beyond high or low motivation (Laursen & Hoff, 2006), which can explain why they have been increasingly used by motivation researchers (e.g., Bergman & El-Khouri, 2003; Chen, 2012; Hayenga & Corpus, 2010; Howard & Hoffman, 2017; Linnenbrink-Garcia et al., 2018). Person-centered approaches allow researchers to look beyond the association of variables to identify how these associations may differ between groups in the population. These analyses:

> (1) reject the assumption that the entire population is homogeneous with respect to how variables influence each other and (2) search for categories of individuals characterized by patterns of association among variables that are similar within groups and different between groups. (Laursen & Hoff, 2006, pp. 379-280).

The most common person-centered analyses are cluster analysis and latent profile/class analysis (Laursen & Hoff, 2006).

Cluster Analysis. Cluster analyses have been used to examine different types of motivation and different subjects across multiple age groups in primary and secondary education

(e.g., Buehl & Alexander, 2005; Conley, 2012; Karamarkovich & Rutherford, 2018; Umarji et al., 2018). Results often show high, medium, and low clusters of motivation with little differentiation across types (e.g., Hayenga & Corpus, 2010; Jang & Liu, 2012; Umarji et al., 2018), leaving questions about the added value of the cluster analysis beyond variable-centered interaction approaches. However, some studies using cluster analyses have found qualitatively different clusters that go beyond high/medium/low (e.g., Conley, 2012; Karamarkovich & Rutherford, 2018). For example, Conley (2012) found a cluster where students reported high levels of interest, attainment/utility value, and competence beliefs but also high cost, which is typically negatively associated with other measures of motivation (Flake et al., 2015; Gaspard et al., 2015). Karamarkovich and Rutherford (2018) also found a cluster with high value but average expectancy, as well as a cluster with low value but average expectancy. Cluster analyses can also be used to find groups combining data across qualitatively different outcomes. For example, when Umarji and colleagues (2018) conducted cluster analyses on mathematics and English self-concept together, they found clusters that distinguished mathematics and English motivation (e.g., high math, low English).

Latent Profile Analysis. Latent profile analysis (LPA), also known as latent class cluster analysis (Vermunt & Magidson, 2002), is a finite mixture model (McLachlan & Peel, 2000) similar to cluster analysis in that it groups people who have similar responses to a measure (Collins & Lanza, 2010; Goodman, 2002; Nylund et al., 2007). In a LPA, the profile a person belongs to is the latent variable, with profiles consisting of people who have similar patterns of responses. The latent profile model represents the distributions of each item of the measure (or indicator) as a function of profile membership:

$$f(\theta) = \sum_{1}^{k} \pi_k f_k(\theta_k)$$

where k is the latent profile, π_k is the probability of belonging to the latent profile k, and x_i is the observed score for each of the i indicators. LPA assumes that each indicator is normally distributed; although most measures of motivation use Likert-like scales (which are not continuous; Wu & Leung, 2017), motivation researchers typically still use LPA instead of its categorical counterpart latent class analysis.

Like cluster analyses, latent profile analyses have been conducted across motivational theories (e.g., Gaspard et al., 2019; Luo et al., 2011), subjects (e.g., Dietrich et al., 2019; Robinson et al., 2020) and ages (e.g., Andersen & Cross, 2014; Robinson et al., 2020), largely finding high and low profiles (e.g., Andersen & Cross, 2014; Gaspard et al., 2019; Wang et al., 2013). Andersen and Cross (2014) also found a profile where students had high mathematics expectancy but low values. Similar to Umarji and colleagues' (2018) study, Gaspard et al. (2019) and Wang et al. (2013) looked at the combination of mathematics and language self-concept and value. They both found similar patterns that distinguished between mathematics and language motivation. Wang and colleagues (2013) also examined how profiles of motivation in 12th grade were related to career choices when participants were 33, 15 years later. They found that the majority of participants in STEM careers were in high math/moderate language and high math/high language profiles.

Longitudinal Expectancy–Value

Students do not often maintain the same levels of expectancies and values across time (Eccles & Wigfield, 2020; Wigfield & Eccles, 2000). Expectancy and value beliefs develop and change via experiences of mastery, failure, and feedback in school and at home (Eccles & Wigfield, 2020; Wigfield & Cambria, 2010; Wigfield & Eccles, 2020). Results from variable-centered longitudinal approaches have found that, on average, motivation often declines over time

as students receive more feedback and become better calibrated (Wigfield & Cambria, 2010; Wigfield & Eccles, 2000). However, these changes may not be the same across students; researchers have used a variety of analysis techniques to determine the patterns in these changes in motivation.

Growth Mixture Modeling. Growth mixture modeling has been used to identify how individuals can be grouped based on trends of a variable over time (i.e., trajectories; Archambault et al., 2010; Musu-Gillette et al., 2015). Growth mixture modeling is also a person-centered analysis; however, rather than identifying patterns in multiple variables, like a cluster or latent profile analysis, growth mixture modeling identifies patterns using only one variable. For example, Archambault et al.'s (2010) study on literacy motivation and Musu-Gillette et al.'s (2015) on math motivation measured self-concept of ability and subjective task value from elementary to high school. Although separate models were run for each motivation component, three common trajectories were found: high trajectory where student motivation did not decline as quickly or substantially as the other students; a moderate trajectory where student motivation declines more drastically than the high trajectory but not as drastically as the fast decline trajectory; and the fast decline trajectory (Archambault et al., 2010; Musu-Gillette et al., 2015). Archambault and colleagues (2010) found an additional four trajectories—two of which were parabolic with different levels of motivation, one that quickly declined until high school and then rebounded slightly, and one that remained stable in elementary school but declined in middle school. By using growth mixture modeling, researchers have found the change in motivation over time is not linear nor monolithic. These profiles can paint the picture of how individual variables change over time, yet they do not consider how multiple variables may change together. To address this issue, researchers turn to other person-centered approaches that include time, such as I-States as objects

of analysis or latent transition analysis.

I-States. I-States as objects of analysis (often called ISOA or I-States) was first proposed by Bergman and El-Khouri (1999). To conduct this type of analysis, researchers use responses across all timepoints (Bergman & El-Khouri, 1999). For example, if a researcher has 100 participants and collects data at three timepoints—they would have 300 i-states. These i-states are then used in a single cluster analysis or a latent profile analysis. Once profiles are determined, researchers then reorganize the data to reassign each i-state to its timepoint (i.e., participants have a profile membership for each timepoint). I-States assumes the number and structure of profiles remain the same at all timepoints, which is good for short-term changes in profile membership and improves power but does not account for situations in which profiles exist only at certain timepoints (Bergman & El-Khouri, 1999; Nurmi & Aunola, 2005). Although the profile structures remain the same across timepoints, profile membership can vary, which allows researchers to determine the stability of profiles across time.

Studies using I-States to examine mathematics motivation in adolescents have also found an overall high motivation profile (Lazarides et al., 2016, 2019; Viljaranta et al., 2016), low motivation profile (Lazarides et al., 2016, 2019; Viljaranta et al., 2016), and moderate motivation profile (Lazarides et al., 2019). For example, Lazarides and colleagues (2020) used I-States to examine changes and stability in profiles of mathematics expectancy and values across four timepoints when students were in seventh (two timepoints), tenth, and eleventh grades. They found four profiles—low motivation beliefs, low intrinsic value (with medium expectancy and importance value), high motivational beliefs, and medium motivational beliefs. These profiles were relatively stable across the time-periods measured, with changes primarily between similar profiles (e.g., from low intrinsic value to low motivation; Lazarides et al., 2016, 2019, 2020).

Latent Transition Analysis. Latent transition analysis (LTA) is the longitudinal counterpart to latent profile analysis. It determines both the number of profiles at two or more timepoints and how these profiles, including profile membership, may change between timepoints. This method differs from I-States in that each timepoint is considered separately, allowing for the possibility of new and different profiles between timepoints. Although each timepoint is analyzed separately (as if conducting a latent profile analysis), a latent transition analysis controls for the previous profiles by regressing the profile variable on the profiles from the previous timepoint. This controls for previous profiles and identifies how students move between profiles (i.e., transition).

Although LTA allows for changes in profile (i.e., different profiles may be present at different timepoints), it is possible that the profiles will remain the same. Dietrich and Lazarides (2019) found this stagnancy when examining expectancies and values in ninth and tenth graders at the beginning and end of the school year. At both timepoints, they found four profiles—high motivation, above average motivation, average motivation, and low motivation. These profiles were highly stable with less than 25% of their sample moving between profiles across time (Dietrich & Lazarides, 2019).

Synthesizing Results Across Analysis Type

There are many similarities across these person-centered approaches—overall high motivation and low motivation are present in all studies reviewed above (e.g., Andersen & Cross, 2014; Dietrich & Lazarides, 2019; Karamarkovich & Rutherford, 2018; Lazarides et al., 2020). However, there were also profiles that had divergent expectancies and values (e.g., high motivation and high cost, Conley, 2012; high expectancy but low value, Andersen & Cross, 2014), as well as domain-specific motivations (e.g., Gaspard et al., 2019; Umarji et al., 2018; Wang et al., 2013).

When considering multiple timepoints, students' motivation generally declined over time (e.g., Archambault et al., 2010; Musu-Gillette et al., 2015). However, when examining patterns of motivation across time, students largely stayed in the same profile, and if they moved, it rarely resulted in a large change in profile (e.g., high to low or vice versa; Dietrich & Lazarides, 2019; Lazarides et al., 2020). Although this suggests some stability over time, this may be an artifact of most research only using two timepoints (e.g., Dietrich & Lazarides, 2019; Lazarides et al., 2016, 2019; Viljaranta et al., 2016). Only one study reviewed here, conducted by Lazarides and colleagues (2020) had more than two timepoints; however, the use of I-States prevents the possibility of new profiles, which may influence the perceived stability.

Strengths and Weaknesses in Existing Approaches

When deciding which person-centered analysis to use, there are few guidelines. Latent profile analyses are modeled using the distribution of the data—"individuals belonging to the same class are similar to one another such that their observed scores on a set of indicators are assumed to come from the same probability distributions" (Tein et al., 2013; p. 641). This provides more information on model fit and probability of profile membership (Magidson & Vermunt, 2002; Tein et al., 2013). Major strengths of using latent profile analysis are the ability for researchers to compare models with different numbers of profiles using statistical tests, such as likelihood ratio tests, and the ability to account for uncertainty in profile membership (Collins & Lanza, 2010). However, latent profile analyses may not provide the most distinct profiles—when comparing latent profile analysis, cluster analysis, and Kohonen maps (not discussed here), Eshghi and colleagues (2011) found that cluster analyses provided the most homogeneous clusters with the most distinction between clusters. Although this provides a major advantage for cluster analyses over LPAs, the lack of model fit comparisons provided by cluster analyses makes determining the

optimal number of clusters difficult (Eshghi et al., 2011).

When considering how patterns of responses change over time, researchers are faced with even fewer guidelines. Growth mixture modeling is often used with longitudinal data but does not consider how variables may interact with each other over time, focusing solely on patterns of change within one variable. I-States as objects of analysis uses all data from all timepoints; this increases the amount of statistical power but limits the possibility of fluctuation in both number and changes in profiles (Bergman & El-Khouri, 1999; Nurmi & Aunola, 2005). LTA allows for variation in profiles across timepoints but does not model growth in variables in the way that growth models do. In addition, LTA has not commonly been used to study SEVT (cf. Dietrich & Lazarides, 2019).

Although there has been a major response to the push for person-centered approaches for studying motivation, there is a lack of consensus on how to approach longitudinal person-centered analyses. With each analysis having its own advantages and disadvantages, researchers need more information in order to determine which technique is most appropriate for their data and variables of interest. To address this concern, I compare two different approaches to model change in expectancies and values: a latent profile analysis using all timepoints and a latent transition analysis.

Current Study

In this study, I examine how fourth graders' patterns of expectancies and values change throughout the school year. Prior research has examined profiles of responses over time (latent transition analysis) and profiles of trends for individual variables (growth mixture modeling); however, neither of these approaches captures how multiple variables together might create patterns of change itself. In this study, I propose a novel approach to examining profiles of

expectancies and values over time by using what has been a traditionally single-point analysis, Latent Profile Analysis (LPA), with all timepoints, which I term Longitudinal LPA (LPA-Long). By using all data points within one LPA, I can examine patterns of both responses and trends simultaneously, thus *profiling change*. Due to its novel nature, I compare LPA-Long to a more traditional LTA and illuminate the relative strengths of each. To this end, I ask the following questions:

4a. What profiles of motivation are present throughout the school year?

- 4i. What motivation profiles emerge when considering all timepoints (novel LPA-long)?
- 4ii. What motivation profiles emerge when considering each timepoint separately (LTA)?
- 4iii. When looking at each timepoint separately (LTA), how do students move between profiles at each timepoint?

4b. How are these profiles related to achievement?

Method

Context

This study is part of a larger NSF-funded project using embedded assessments and data-mining techniques to understand student and teacher use of Spatial Temporal Math (ST Math), a digital interactive mathematics software created by MIND Research Institute. ST Math is currently being used in 48 states with over 1.2 million students and is designed to align to those states' standards (MIND Research Institute, 2018). Prior research from a randomized study found that ST Math had a small effect on mathematics achievement and improved student mathematics self-beliefs, especially for lower-performing students (Rutherford et al., 2014, 2019a; Schenke et al., 2014). As part of the current project, motivation survey questions were designed by MIND in

consultation with the project researchers and were embedded within ST Math. It is these surveys that are the subject of the current study.

Participants

Although the overall project includes five school districts, the current study focuses on fourth grade students from one participating district—the only district who had provided student achievement data at the time of this study. Data were collected on over 6,000 fourth graders, but this sample is limited to those who completed the survey at all three timepoints and had all demographic and achievement data (N=5,365, 78%). The sample was evenly split between boys and girls; 63% of the sample qualified for free or reduced lunch. The limited sample differed from those within this district but not in the sample on many fronts—primarily, there were fewer males, fewer students who were classified as gifted, and fewer students that identified as a race other than Black, Hispanic/Latino, or White; however, all differences were under two percentage points, with most under one percentage point. See Table 4.1 for a summary of all demographics.

Table 4.1

Summary of Demographics for the Sample

	Sample	Total	*p*
Male	50.68%	51.72%	**.001**
Free/Reduced Lunch	63.32%	63.19%	.688
Disability	13.08%	13.12%	.867
Gifted	10.38%	11.94%	**<.001**
English Language Learner	11.30%	11.17%	.548
Race			
Black	20.67%	20.22%	.077
Hispanic/Latino	17.71%	17.91%	.397
White	52.10%	51.96%	.672
Other	9.52%	9.91%	**.045**
N	5,365	6,882	

Note. Chi squared tests were run to determine differences between students in the sample and those excluded from the sample. Statistically significant *p*-values are bolded.

Measures

Survey

The motivation survey was designed by MIND Research Institute (the creators of ST Math) largely based on the expectancy–value theory of motivation (Wigfield & Eccles, 2000) and theories of academic emotions (Pekrun, 2006). The survey was distributed to students through the ST Math platform and was given at the beginning (Fall), middle (Winter), and end (Spring) of the 2017-2018 school year—a student's first log-in after each survey activation would trigger a survey pop-up window. Within this study, I focus on the expectancy-value questions: six questions regarding expectancy, importance, and usefulness of mathematics for both now and in the future (see Table 4.2). Questions were based on the expectancy–value survey from Wigfield and Eccles (2000). Students were presented with questions one at a time and provided their answers using a five-option Likert-like scale. Scale points were labeled (e.g., "Not well at all," Not so well,"…."Very well"). When students clicked on each label, a tomato cartoon would display a unique facial expression to emphasize the label meaning (see Figure 4.1).

Table 4.2

Survey Questions

Question	Measure
How **well** do you think you will do on math **this year?**	Current Expectancy
How good would you be at learning new things in math?	Expectancy for Learning
How **useful** is math for you **now**?	Current Usefulness
How **useful** will math be **in your future**?	Future Usefulness
How **important** is math to you **now**?	Current Importance
How **important** will math be **in the future**?	Future Importance

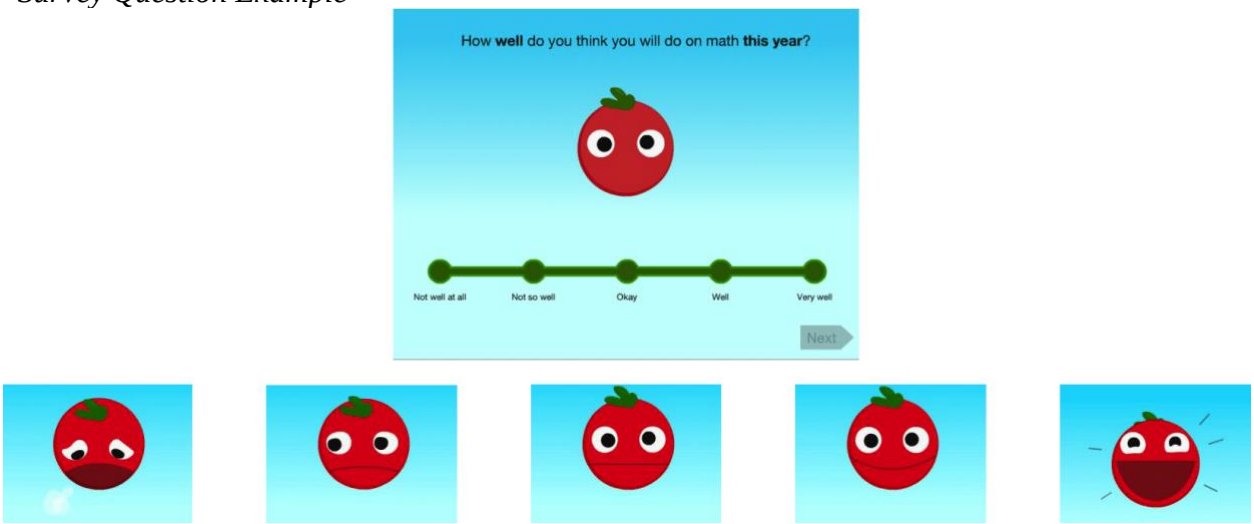
Achievement and Demographics

Achievement and demographic information were provided to MIND by the school district. MIND de-identified and matched these district data with survey data before providing both to the researchers. Achievement data come from the Florida Standard Assessments tests (Florida

Department of Education, 2017). Florida Standards Assessments (FSAs) are end of the grade, summative tests based on Florida's education standards. All measures of internal reliability from the 2016-2017 school year were at least 0.89 for fourth grade mathematics FSAs (Florida Department of Education, 2017). The mathematics scale scores were collected for both the study year (2017-2018) and the year prior (2016-2017); scale scores allow for comparison between years and across grades. Compared to the total sample, the reduced sample scored lower on both the 2017 and 2018 FSAs.

Figure 4.1

Survey Question Example



Note. Example of a survey item. This question measures current expectancy (for this year), which I designate as "Expectancy Now."

Demographics—gender, race, whether the student qualified for free/reduced price lunch, disability status, classification as gifted, and if the student was an English Language Learner—used in analyses were those reported with the prior mathematics achievement score (2016-2017); however, if these values were missing, demographics from the study year (2017-2018) were used. See Table 4.1 for demographics and Table 4.3 for descriptive statistics for achievement and survey questions.

Table 4.3

Descriptive Statistics

	Sample		Total			
	Mean	SD	Mean	SD	Min, Max	*p* of difference
2017 Math Score	301.89	18.88	302.53	19..08	240, 360	**< .001**
N		5,365		6,265		
Fall 2017						
Expectancy Now	4.18	0.91	4.19	0.91	1, 5	.059
Expectancy for Learning	4.13	0.89	4.13	0.89	1, 5	.510
Usefulness Now	4.18	0.95	4.18	0.95	1, 5	.897
Usefulness Future	4.50	0.85	4.50	0.86	1, 5	.873
Importance Now	4.27	0.95	4.27	0.94	1, 5	.583
Importance Future	4.50	0.85	4.51	0.84	1, 5	.414
N		5,365		6,683		
Winter 2017/2018						
Expectancy Now	4.13	0.97	4.14	0.96	1, 5	**.027**
Expectancy for Learning	4.09	0.94	4.09	0.94	1, 5	.274
Usefulness Now	4.14	1.04	4.14	1.04	1, 5	.987
Usefulness Future	4.46	0.92	4.46	0.92	1, 5	.657
Importance Now	4.20	1.02	4.20	1.02	1, 5	.253
Importance Future	4.46	0.92	4.46	0.92	1, 5	.990
N		5,365		6,779		
Spring 2018						
Expectancy Now	4.20	0.97	4.19	0.98	1, 5	.143
Expectancy for Learning	4.15	0.96	4.16	0.95	1, 5	.308
Usefulness Now	4.19	1.03	4.19	1.03	1, 5	.976
Usefulness Future	4.46	0.94	4.46	0.94	1, 5	.957
Importance Now	4.20	1.04	4.20	1.04	1, 5	.307
Importance Future	4.45	0.93	4.45	0.93	1, 5	.729
N		5,365		6,002		
2018 Math Score	313.99	22.30	314.31	23.02	251, 376	**.030**
N		5,365		6,882		

Note. To compare the total sample to reduced sample I used independent sample *t*-tests. Significant *p*-values bolded.

Analyses

We conducted the analyses in three main steps. First, I conducted the two types of person-centered analyses to answer the first research question (*what profiles of motivation are present throughout the school year*). Second, I predicted profile membership using students' prior mathematics achievement and demographics. Lastly, I predicted end-of-year mathematics

achievement using students' motivation profile, prior achievement, and demographics. Examining the relation between prior math achievement and profile membership, as well as the relation between profile membership and math achievement answered the second research question (*how are these profiles related to achievement*).

Person-Centered Analyses

We used latent profile and transition analyses for a variety of reasons. First, I wanted to examine patterns within and between variables (growth mixture modeling can only examine variables independently). Latent profile and transition analyses are also inherently related, whereas I-States can be conducted with either cluster analysis or LPA. Lastly, I wanted to account for the possibility of different profiles emerging as the school year continues, which I-States does not allow.

Latent Profile Analysis. I conducted a single LPA, using data from all three timepoints in the analysis, resulting in profiles that described patterns of motivation over time (LPA-long). Therefore, profiles were based on 18 data points from each student (e.g., three expectancies for now, three expectancies for learning, three usefulness for now). To determine the number of profiles, models were compared using fit criteria—namely sample-size adjusted Bayesian Information Criterion (adjusted BIC), the Vuong-Lo-Mendell-Rubin Likelihood Ratio Test, and entropy (Chen et al., 2017; Tein et al., 2013). The adjusted BIC was determined using the following equation: $-2 \log L + p \log$, where L is the maximized likelihood of the fitted model, p is the number of free parameters (degrees of freedom) in the model, and N is the sample size. This adjusted BIC takes into account the likelihood that the model is the most parsimonious and corrects for the sample size. A lower adjusted BIC indicates a better fit (Chen et al., 2017; Tein et al., 2013). The Vuong-Lo-Mendell-Rubin (VLMR) Likelihood Ratio Test compares the current models with the

previous model to determine if the addition of a profile statistically significantly improves model fit (Chen et al., 2017). For example, when running a model with three profiles, the VLMR test compares it to the model with two profiles. If the test is significant (i.e., *p*-value less than 0.05), the three profile model is an improvement over the two profile model. Entropy measures the distinction between the profiles by aggregating the posterior probabilities of a person correctly belonging to that profile (Tein et al., 2013). Normalized entropy ranges from zero to one, with values closer to one being better—Muthén and Muthén (2017) recommend having entropy greater than 0.80. Model selection was based on minimizing adjusted BIC and maximizing entropy, in addition to qualitative determination that profiles aligned distinctly with existing motivational theory (Collier & Leite, 2017; Tein et al., 2013).

Latent Transition Analysis. The second set of analyses in this study used latent transition analysis (LTA). The main distinction between this analysis and the one described above is that each timepoint was analyzed separately, which allowed us to analyze how students moved between profiles (Rindskopf, 2010). This analysis followed three steps. First, a latent profile analysis is estimated using only the first timepoint (Fall; see above for details about latent profile analyses). Then, a latent transition analysis is conducted with the second timepoint (Winter) to determine Winter profiles and transitions from Fall to Winter. Lastly, a latent transition analysis is conducted with the third timepoint (Spring) to determine Spring profiles and transitions from Winter to Spring. This analysis provided three sets of profiles and two sets of transitions.

Predicting Profile Membership

To examine variables that predicted profile membership, I used multinomial logistic regressions with standard errors adjusted for nesting within schools. The categorical latent variable of profile membership served as the outcome and was regressed on students' prior achievement

and demographics, nesting on school. I ran one model for the LPA-long and three models—one for each timepoint—for the LTA.

Predicting Achievement

To predict achievement from profiles, I ran Ordinary Least Squares regressions. To account for nesting of students within schools, standard errors were adjusted using the Huber-White cluster adjustment. End-of-grade mathematics scale scores were regressed on prior mathematics achievement (previous year scale score) and dummy coded-profile membership, controlling for demographics and nesting on school. For both the LPA-long and the LTA, I ran one model each. To account for LTA profiles at the different timepoints, I created a conglomerate profile of profiles to account for profile membership at each timepoint (see Robinson et al., 2020 for a similar method). For example, if there were three profiles representing high, medium, and low motivation, a conglomerate profile could be High-High-High, High-Medium-High, High-Medium-Low, etc.

Results

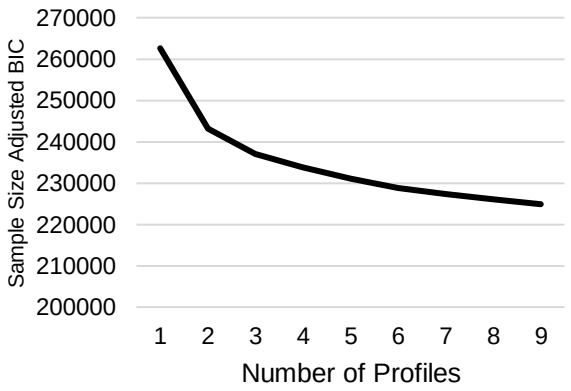
Latent Profile Analysis (LPA-long)

Five motivation profiles presented the best solution based on a small adjusted BIC and high entropy (see Table 4.4). Additionally, the five profiles were all distinct from each other. Each profile included six variables across three timepoints (18 total variables). Profiles are illustrated in Figure 4.2; vertical lines have been added to the figures to indicate different timepoints.

The most prevalent profile was the *high motivation* profile. As the name suggests, students in this profile had consistently high measures of mathematics motivation throughout the school year, ranging from 0.30 to 0.49 standard deviations above the average of all motivation measures. About 57% of the sample were in the high motivation profile. The second largest profile, with nearly a quarter of the sample (23%), was the *high future* profile. The defining feature of this

Table 4.4

LPA-long Fit Statistics

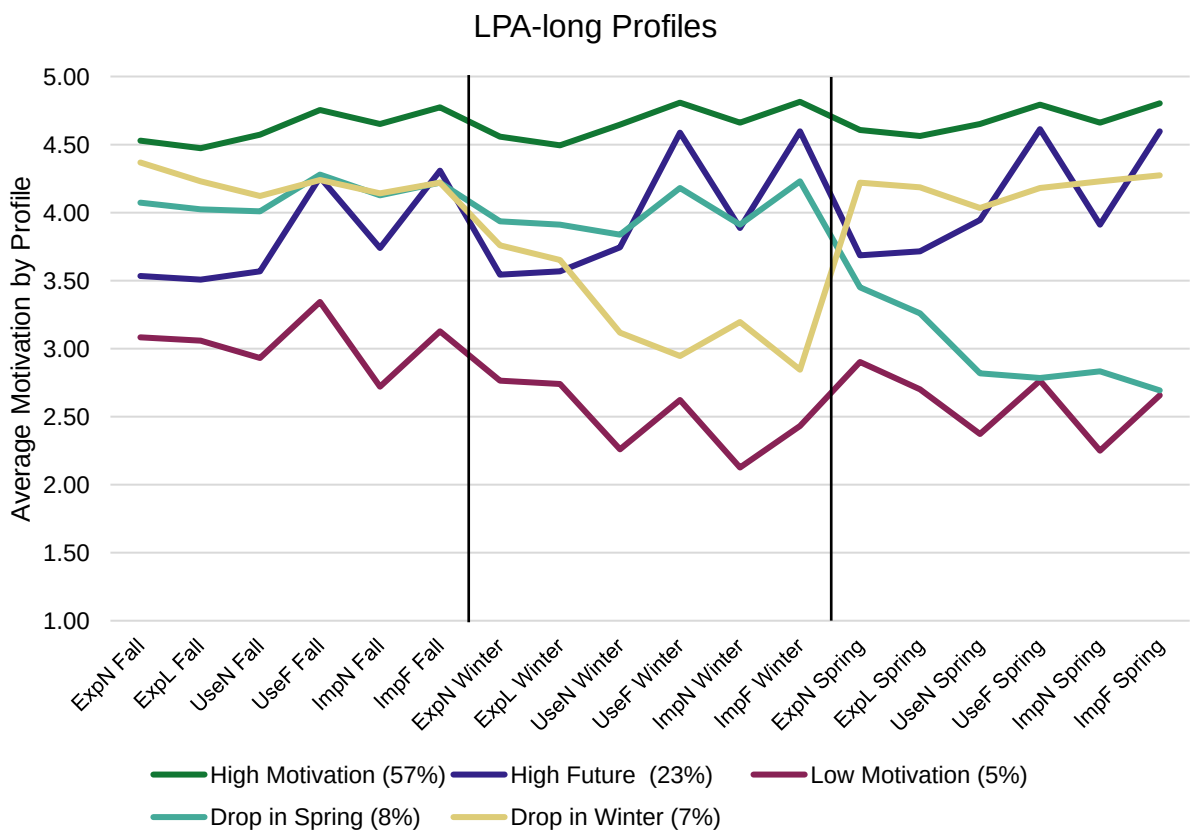
Profiles	Parameters	Entropy	Vuong-Lo-Mendell-Rubin Likelihood Ratio Test			Sample Size Adjusted BIC
			LL Diff.	*p*-value	Value	Plot
1	36				262606	
2	55	0.932	20547	< .0001	243162	
3	74	0.887	6255	< .0001	237039	
4	93	0.907	3315	< .0001	233827	
5	112	0.912	2785	.0003	231144	
6	131	0.902	2429	.4525	228818	
7	150	0.907	19	.3430	227396	
8	169	0.907	19	.7032	226033	
9	188	0.909	19	.2971	224885	

profile was the relatively high value of the future variables (future usefulness and future importance). These students reported future usefulness and future importance of mathematics higher than they rated expectancies, as well as current usefulness and importance (future value was 0.30 to 0.77 standard deviations above expectancy and current value). This profile was also consistent across the school year. Students in the *drop in spring* profile had lower motivation at the spring timepoint (end of the school year) than at both the fall and winter timepoints, especially for value. Spring motivation was 0.65 to 1.58 standard deviations lower than motivation in the previous two timepoints. The drop in spring profile included 8% of students. Similar to the drop in spring profile, students in the *drop in winter* profile had approximately average motivation for two of the three timepoints. However, students who were in this profile had lower motivation in the winter (middle of the school year, immediately after winter break. Winter motivation was 0.42 to 1.61 standard deviations lower than fall and spring motivation. Additionally, at the winter timepoint, students rated future usefulness and importance lower than current usefulness and

importance, which was opposite the pattern in all other profiles. Seven percent of fourth graders were in this profile. Students in the *low motivation* profile consistently reported low motivation throughout the school year. At each timepoint, all components of students' motivation were lower than the other profiles, ranging from 1.20 to 2.22 standard deviations below the average of all motivation measures. Five percent of participants were in the low motivation profile.

Figure 4.2

LPA-long Profiles



Note. Vertical lines added to differentiate the timepoints.

Latent Profile and Transition Analysis (LTA)

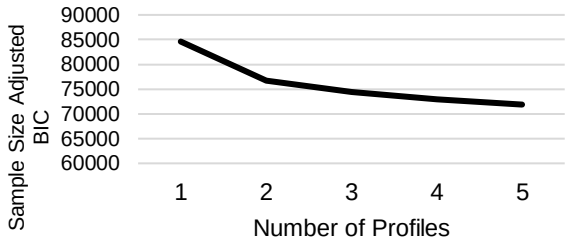
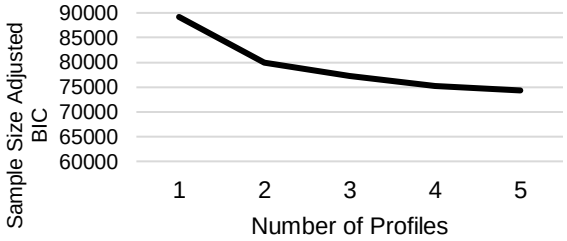
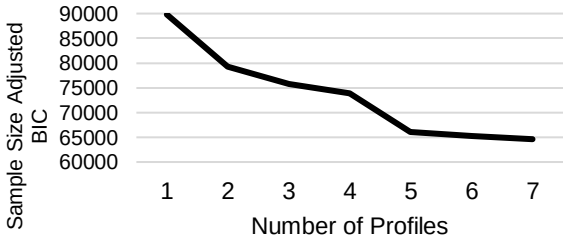
Profiles

Like the LPA-long, five motivation profiles presented the best solution for all three timepoints. For the fall and winter, models did not converge for more than five profiles.

Additionally, the five profile solutions had small adjusted BIC, high entropy, and profiles within were theoretically distinct. See Table 4.5 for fit statistics.

Table 4.5

LPTA Fit Statistics

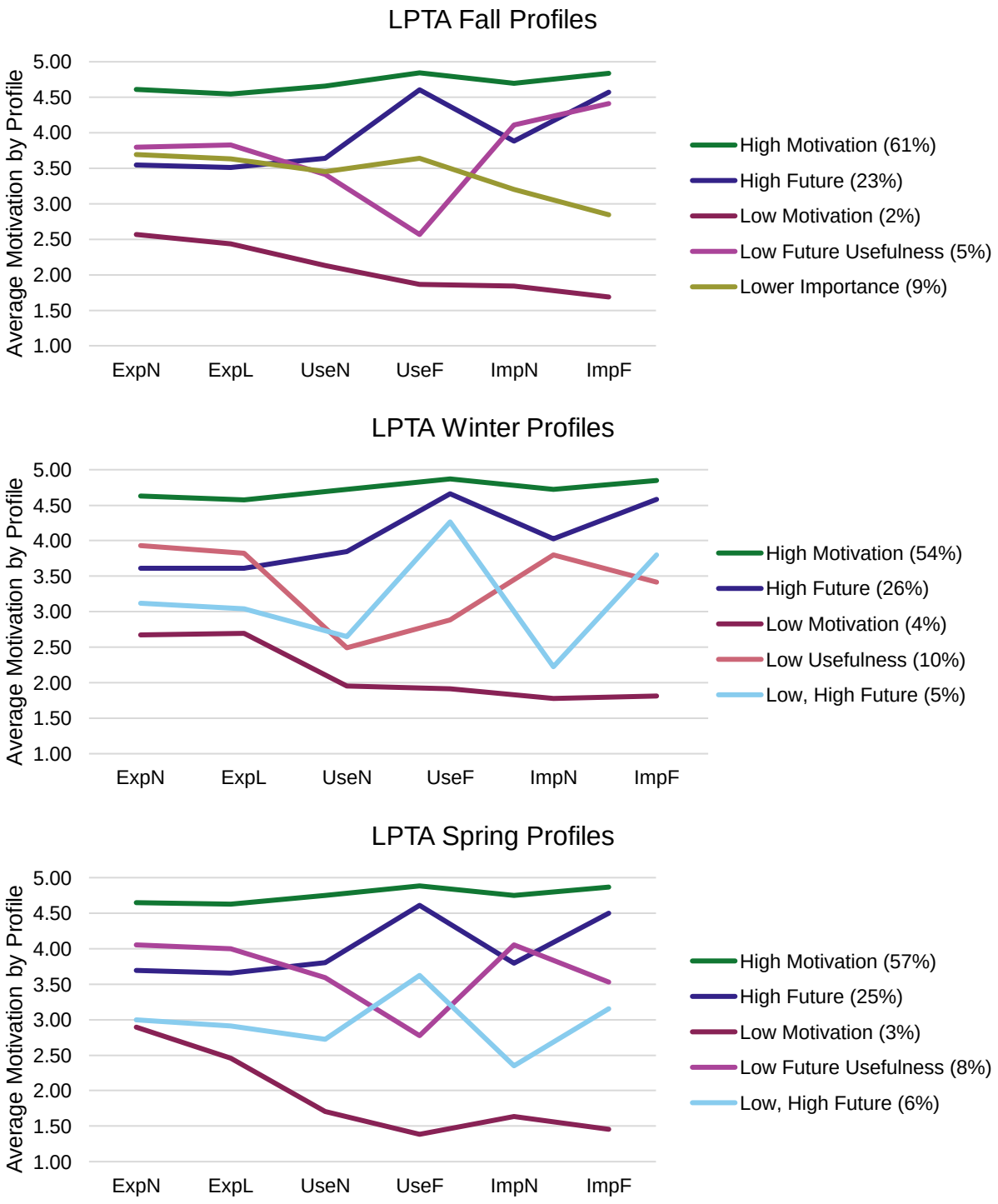
	Profiles	Parameters	Entropy	Vuong-Lo-Mendell-Rubin Likelihood Ratio Test		Sample Size Adjusted BIC	
				LL Diff.	*p*-value	Value	Plot
Fall	1	12				84612	
	2	19	0.920	7957	< .0001	76693	
	3	26	0.818	2326	.0305	74405	
	4	33	0.865	1561	.0039	72881	
	5	40	0.866	414	.0098	71877	
	6			no convergence			
Winter	1	12				89184	
	2	19	0.935	9329	< .0001	79893	
	3	26	0.885	2721	.0732	77210	
	4	33	0.874	1962	.0088	75285	
	5	40	0.893	997	.0002	74326	
	6			no convergence			
Spring	1	12				89810	
	2	19	0.947	10645	< .0001	79204	
	3	26	0.918	3487	< .0001	75755	
	4	33	0.884	1907	.1941	73886	
	5	40	0.953	1871	< .0001	66046	
	6	47	0.940	765	< .0001	65319	
	7	54	0.941	746	< .0001	64611	
	8			no convergence			

There were a total of seven profiles found across all three timepoints with three similar profiles found at all three timepoints. Two of the other profiles were present in two of the three timepoints: one in both fall and spring and another in both winter and spring. Common profiles

will be discussed first. See Figure 4.3 for profiles in the fall, winter, and spring, respectively.

Figure 4.3

LPTA Profiles by Wave



Similar to the *high motivation* profiles in the LPA-long, the high motivation profile consisted of students who reported high levels of each motivation component (ranging from 0.40 to 0.56 standard deviations above the average of all measures). The high motivation profile was consistently the largest profile at all timepoints (54-61% of students). Students in the *high future* profile rated future usefulness and future importance higher than other motivation components. Future value ranged from 0.29 to 0.84 standard deviations higher than expectancy and current value. The high future profile contained 23% of students in the fall, 26% of students in the winter, and 25% of students in spring. The *low motivation* profile was consistently the smallest profile at all three timepoints (ranging from 2%-4% of students). Although there were slight variations in the pattern of motivation components at each timepoint, the low motivation profile always had lower motivation components than other profiles (ranging from 1.34 to 3.35 standard deviations below the average of all measures).

The *low future usefulness* profile was found at the fall and spring timepoints. This profile's unique feature was its lower future usefulness (0.85 to 2.16 standard deviations lower) as compared to the other motivation components. Although the spring profile also displayed lower future importance than current importance, low reporting for this variable was not as drastic as that for usefulness (future importance was 0.98 standard deviations below the mean, whereas future usefulness was 1.83 standard deviations below the mean). Five percent of students were in the low future usefulness profile in the fall and 8% were in the profile in the spring. The *low, high future* profile was found at both the winter and spring timepoints. It followed a similar pattern as the high future profile—the main distinction being how low the other motivation components were. Future usefulness and future importance were much higher than current usefulness, current importance, and expectancies. In the winter, students rated expectancies and current values 1.08 to 1.97

standard deviations lower than average, whereas future usefulness and importance were only 0.20 and 0.72 standard deviations below the average, respectively. In the spring, the low, high future profiles had lower future usefulness and future importance, as well as lower expectancies and current values (ranging from 0.91 to 1.79 standard deviations below average). However, the future usefulness and importance were still higher than the other motivation components. The low, high future profile contained about 5% of students in the winter and 6% of students in the spring.

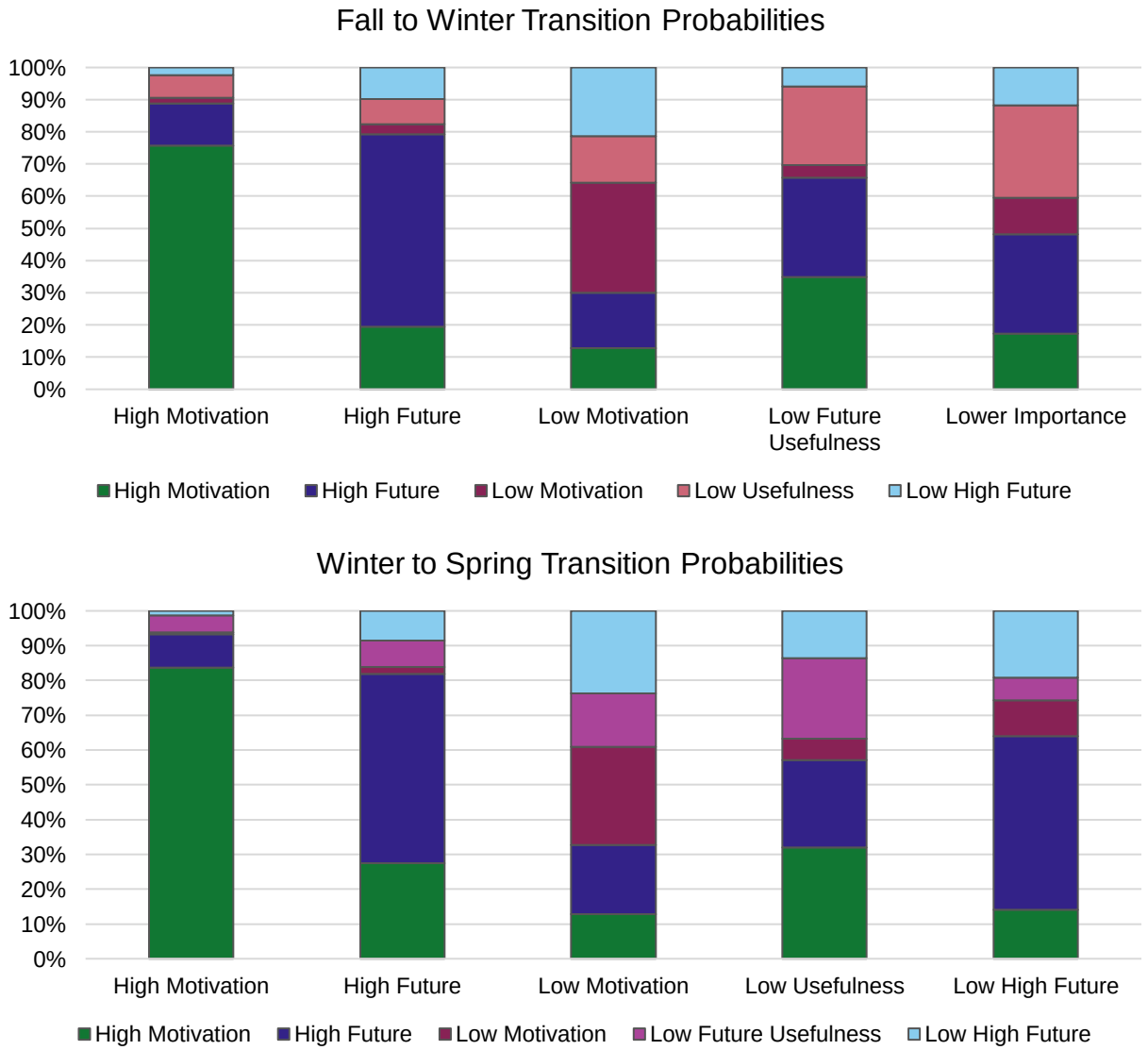
The *lower importance* profile appeared only in the fall. Students in this profile reported similar levels of expectancies and usefulness (0.53 to 0.99 standard deviations below the averages) but lower levels of importance, especially future importance (1.13 and 2.04 standard deviations lower than average). Nine percent of the sample were in the lower importance profile in the fall. The last profile was the *low usefulness* profile. Students in the low usefulness profile had lower levels of usefulness than expectancies and importance. This profile only appeared in the winter and consisted of 10% of students.

Transitions

Fall to Winter. Three profiles were the same in the fall and winter—high motivation, high future, and low motivation. The majority of students who were in these profiles in the fall, remained in these profiles in the winter, with 76% of students staying in the high motivation profile, 60% of students staying in the high future profile, and 34% of students staying in the low motivation profile. Although 34% seems to be a relatively small percentage remaining in the low motivation profile compared to the percentage of students remaining in the other two profiles, students in this profile dispersed among the profiles, and transitions into other profiles were even lower—15% moved to the low usefulness profile, 17% to the high future profile, 13% to the high motivation profile, and 21% to the low, high future profile. Students who started in the lower

Figure 4.4

LPTA Transitions



importance profile mostly moved to the high future profile (31%), followed by the low usefulness profile (29%). Students in the low future usefulness fall profile mostly moved to the high motivation profile (35%) and the high future profile (31%).

Winter to Spring. Similar to the fall to winter transitions, 84% of students stayed in the high motivation profile, 55% of students stayed in the high future profile, and 28% stayed in the low motivation profile (i.e., the majority of students stayed in the same profile from winter to

spring rather than move to any other given profile). Students in the low usefulness profile in the winter mostly moved to the high motivation (32%), high future (25%), and low future usefulness (23%) profiles in the spring. Additionally, the majority of students in the low, high future profile moved to the high future profile (50%) in the spring. See Figure 4.4 both sets of transitions.

Predicting Profiles

To examine variables that predicted profile membership, I used multinomial logistic regressions. The high motivation profile was used as the reference group because it was the largest profile across analyses and timepoints. To further examine which students were in each profile, I conducted Wald's χ^2 post-hoc tests to compare coefficients across the regressions.

LPA-long

Compared to students in the high motivation profile, students in all other profiles had lower prior mathematics achievement (relative risk ratios ranging from 0.976 to 0.988, *p*'s < .001). Wald's χ^2 post-hoc tests further indicated that the effect of prior math achievement on profile membership for drop in winter was statistically significantly different from the other three profiles (χ^2 ranging from 4.41 to 13.92, *p*'s < .05). With a relative risk ratio closest to one, this indicates that students in the drop in winter profile (rrr = 0.988) had prior mathematics scores that were *lower* than students in the high motivation profile but *higher* than students in the high future (rrr = 0.976, χ^2 = 8.95, *p* = .003), low motivation (rrr = 0.968, χ^2 = 13.92, *p* < .001) and drop in spring (r = 0.978, χ^2 = 4.41, *p* = .036) profiles. There was also a statistically significant difference in the effect of prior math achievement on low motivation and high future profile membership (χ^2 = 4.00, *p* = .045)—on average students with lower prior math achievement were more likely to be in the low motivation profile than the high future profile. See Table 4.6 for full model results and Figure 4.5 for each profile's average difference of prior math achievement scores from the overall average.

Table 4.6

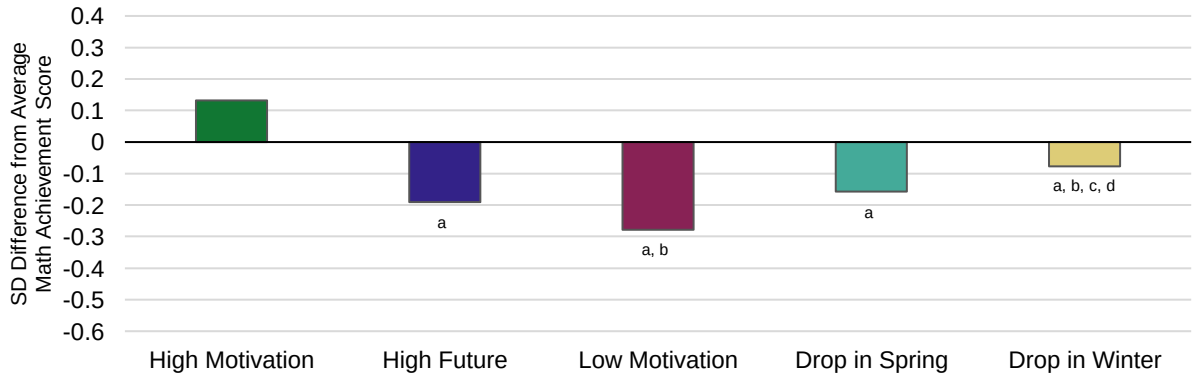
Multinomial Logistic Regressions Predicting LPA-long Profile Membership (Relative Risk Ratio)

	High Future	**Low Motivation**	**Drop in Spring**	**Drop in Winter**
Prior Math Achievement	0.976[c]	0.968[c]	0.978[c]	0.988[c]
	[0.971, 0.981]	[0.959, 0.977]	[0.970, 0.986]	[0.982, 0.994]
Boy	0.895	1.153	1.149	1.323[b]
	[0.792, 1.012]	[0.875, 1.519]	[0.926, 1.425]	[1.084, 1.616]
Free/Reduced Lunch	0.852[a]	0.776	0.853	0.843
	[0.734, 0.989]	[0.576, 1.045]	[0.683, 1.064]	[0.662, 1.073]
Disability	1.075	1.130	1.268	1.316
	[0.865, 1.336]	[0.770, 1.656]	[0.915, 1.756]	[0.961, 1.802]
Gifted	1.242	1.389	1.387	1.174
	[0.990, 1.559]	[0.893, 2.160]	[0.916, 2.099]	[0.831, 1.657]
English Language Learner	0.831	0.575[a]	0.753	1.191
	[0.628, 1.099]	[0.346, 0.955]	[0.499, 1.136]	[0.804, 1.766]
Race/Ethnicity				
Black	0.472[c]	0.336[c]	0.520[c]	0.984
	[0.377, 0.591]	[0.210, 0.538]	[0.383, 0.707]	[0.739, 1.309]
Hispanic	0.892	0.849	0.937	0.847
	[0.716, 1.112]	[0.536, 1.343]	[0.655, 1.341]	[0.609, 1.180]
Other	0.861	0.937	1.109	0.833
	[0.661, 1.121]	[0.599, 1.462]	[0.733, 1.680]	[0.597, 1.163]
Constant	868.819	1985.184	130.521	4.947
	[193.4, 3903]	[124.3, 31697]	[11.54, 1475]	[0.717, 34.14]
Pseudo R^2	0.0225			

Note. N = 5,365
[a] $p < .05$, [b] $p < .01$, [c] $p < .001$

Figure 4.5

Average SD Difference of Prior Math Scores (2017) from Overall Mean by LPA-long Profile



Note. Multinomial logistic regression coefficient is statistically significantly different from: [a] High Motivation, [b] High Future, [c] Low Motivation, [d] Drip in Spring.

LTA

Similar to the LPA-long profiles, in the fall, students with higher average prior math achievement were most likely to belong to the high motivation profile (relative risk ratios ranging from 0.958 to 0.984, *p*'s < .001). Using Wald's χ^2 post-hoc tests, I found that the effect of prior math achievement on profile membership for drop in winter was statistically significantly different from the other three profiles (χ^2 ranging 14.67 to 37.11, *p*'s < .001). With the relative risk ratio closest to zero, this indicates that students in the low motivation profile (rrr = 0.958) had the lowest prior mathematics achievement. The effect of prior math achievement on profile membership was similar for the high future and lower importance profiles (rrr's = 0.984, χ^2 = 0.01, *p* = .931).

Table 4.7

Multinomial Logistic Regressions Predicting LPTA Fall Profile Membership

	High Future	**Low Motivation**	**Low Future Usefulness**	**Lower Importance**
Prior Math Achievement	0.984[c] [0.980, 0.988]	0.958[c] [0.949, 0.967]	0.977[c] [0.971, 0.983]	0.984[c] [0.979, 0.990]
Boy	0.836[b] [0.744, 0.941]	1.183 [0.756, 1.850]	1.034 [0.851, 1.256]	1.154 [0.956, 1.394]
Free/Reduced Lunch	0.813[b] [0.701, 0.942]	0.651[a] [0.430, 0.984]	1.014 [0.742, 1.386]	0.826 [0.661, 1.033]
Disability	1.138 [0.917, 1.411]	1.054 [0.626, 1.774]	1.641 [1.128, 2.386]	1.656[b] [1.193, 2.298]
Gifted	1.111 [0.874, 1.412]	1.148 [0.572, 2.304]	0.887 [0.471, 1.667]	1.140 [0.828, 1.569]
English Language Learner	0.817 [0.658, 1.015]	0.635 [0.325, 1.239]	1.293 [0.836, 2.000]	1.217 [0.857, 1.730]
Race/Ethnicity				
Black	0.510[c] [0.412, 0.631]	0.358[c] [0.202, 0.635]	0.730 [0.510, 1.045]	0.635[b] [0.478, 0.845]
Hispanic	0.932 [0.777, 1.118]	0.658 [0.390, 1.111]	0.967 [0.653, 1.430]	1.019 [0.729, 1.425]
Other	0.954 [0.722, 1.260]	0.720 [0.377, 1.372]	0.962 [0.610, 1.517]	1.025 [0.693, 1.517]
Constant	65.681 [18.04, 239.2]	21063 [1163, 381581]	88.450 [11.53, 678.8]	15.846 [2.617, 95.93]
Pseudo R^2	0.0222			

Note. N = 5,365. Relative risk ratios reported.
[a] *p* < .05, [b] *p* < .01, [c] *p* < .001

However, only the high future profile (rrr = 0.984) was statistically significantly different from the low future usefulness profile (rrr = 0.977)—students who were in the high future profile had higher prior math achievement, on average, than students who were in the low future usefulness profile (χ^2 = 4.12, p = .042). Full fall model results are presented in Table 4.7.

Students in the winter high motivation profile also had higher prior math achievement than all other profiles (relative risk ratios ranging from 0.970 to 0.986, p's < .001). Wald's χ^2 post-hoc tests indicated that the next highest achievers were students in the low usefulness profile—students in the low usefulness winter profile had prior mathematics scores that were lower, on average, than students in the high motivation profile (rrr = 0.986, p < .001) but higher, on average, than students

Table 4.8

Multinomial Logistic Regressions Predicting LPTA Winter Profile Membership

	High Future	**Low Motivation**	**Low Usefulness**	**Low, High Future**
Prior Math Achievement	0.980[c]	0.972[c]	0.986[c]	0.970[c]
	[0.976, 0.985]	[0.964, 0.981]	[0.981, 0.992]	[0.962, 0.977]
Boy	0.975	1.191	1.285[a]	1.277[a]
	[0.744, 0.941]	[0.882, 1.610]	[1.042, 1.586]	[1.007, 1.619]
Free/Reduced Lunch	0.821[b]	0.893	0.848	0.808
	[0.711, 0.947]	[0.655, 1.217]	[0.661, 1.089]	[0.594, 1.100]
Disability	0.950	1.075	1.493[b]	1.122
	[0.742, 1.215]	[0.626, 1.774]	[1.185, 1.881]	[0.796, 1.581]
Gifted	1.034	1.219	1.148	1.457
	[0.828, 1.292]	[0.764, 1.945]	[0.871, 1.513]	[0.972, 2.183]
English Language Learner	0.805	0.647	1.121	0.521[a]
	[0.634, 1.022]	[0.385, 1.086]	[0.768, 1.637]	[0.315, 0.862]
Race/Ethnicity				
Black	0.490[c]	0.433[b]	0.771[a]	0.398[c]
	[0.402, 0.597]	[0.262, 0.714]	[0.601, 0.990]	[0.271, 0.586]
Hispanic	1.075	0.841	0.843	0.913
	[0.892, 1.295]	[0.525, 1.346]	[0.601, 1.181]	[0.612, 1.360]
Other	0.936	0.688	0.992	0.677
	[0.712, 1.231]	[0.382, 1.237]	[0.712, 1.382]	[0.410, 1.118]
Constant	270.51	431.06	11.408	1375.9
	[68.90, 1062]	[30.15, 6163]	[2.269, 57.35]	[109.8, 17241]
Pseudo R^2		0.0208		

Note. N = 5,365. Relative risk ratios reported.
[a] p < .05, [b] p < .01, [c] p < .001

in the high future (rrr = 0.980, χ^2 = 4.16, p = .041), low motivation (rrr = 0.972, χ^2 = 3.87, p = .049), and low, high future (r = 0.970, χ^2 = 7.69, p = .006) profiles. The effect of prior math achievement on profile membership was similar for the low motivation and low, high future profiles (χ^2 = 0.25, p = .620) and both profiles had lower prior math achievement than the high future profile (low motivation: χ^2 = 3.87, p = .049; low, high future: χ^2 = 7.69, p = .006). Full winter model results are in Table 4.8.

As with all previous timepoints and models, in the spring, students with higher average prior math achievement were most likely to belong to the high motivation profile (relative risk ratios ranging from 0.973 to 0.987, p's < .001). Students in both the high future (rrr = 0.981) and

Table 4.9

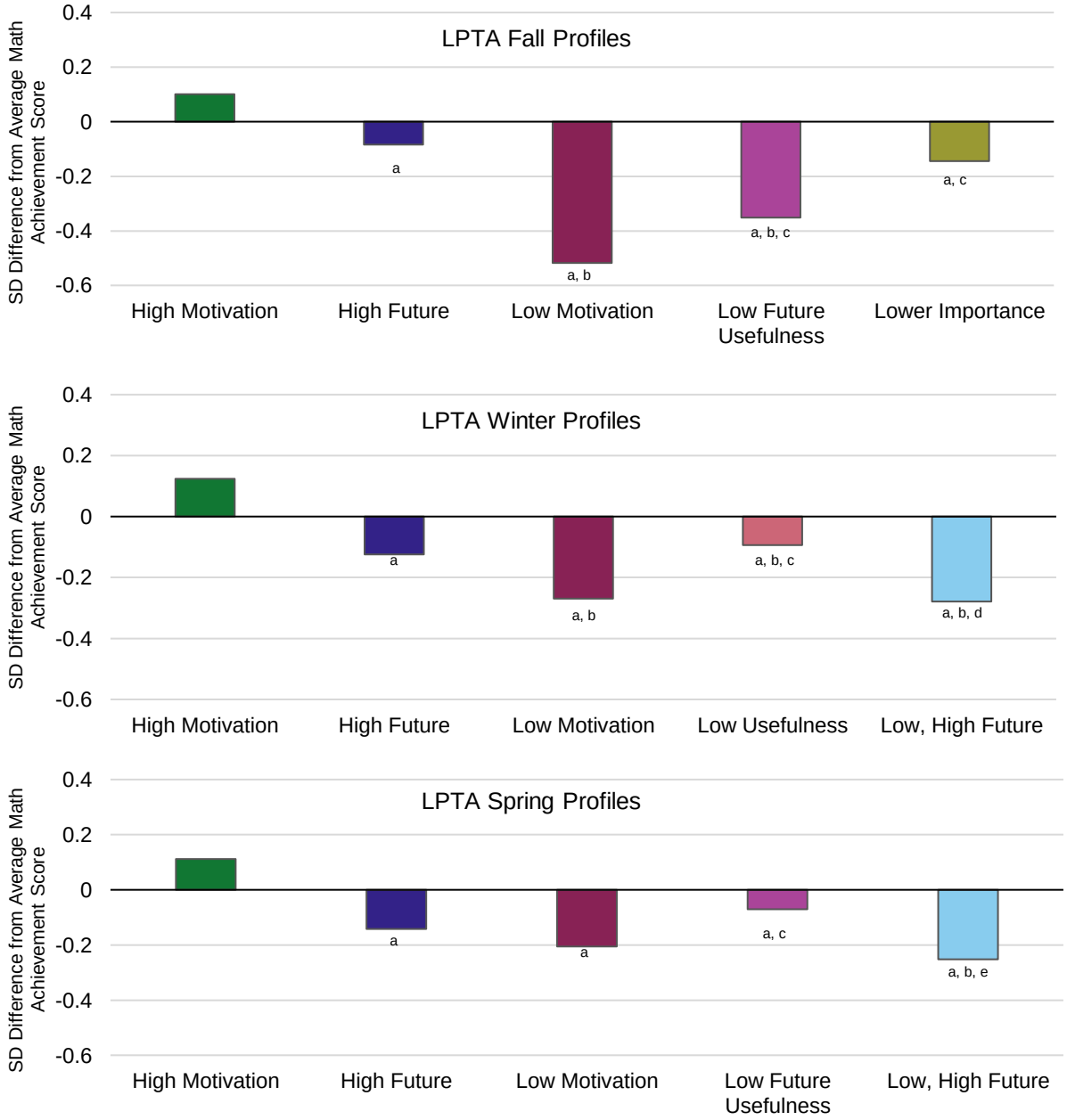
Multinomial Logistic Regressions Predicting LPTA Spring Profile Membership

	High Future	**Low Motivation**	**Low Future Usefulness**	**Low, High Future**
Prior Math Achievement	0.981[c]	0.974[c]	0.987[c]	0.973[c]
	[0.977, 0.985]	[0.964, 0.984]	[0.981, 0.993]	[0.966, 0.980]
Boy	0.844[b]	1.827[b]	0.895	1.014
	[0.759, 0.939]	[1.234, 2.706]	[0.728, 1.100]	[0.840, 1.225]
Free/Reduced Lunch	0.893	1.020	0.771[a]	0.786[a]
	[0.794, 1.006]	[0.710, 1.466]	[0.597, 0.995]	[0.618, 0.999]
Disability	1.145	0.755	1.245	1.293
	[0.941, 1.393]	[0.413, 1.380]	[0.960, 1.614]	[0.896, 1.868]
Gifted	1.185	1.404	1.166	1.293
	[0.949, 1.480]	[0.815, 2.418]	[0.762, 1.786]	[0.896, 1.868]
English Language Learner	0.752[a]	0.502	0.892	0.554[b]
	[0.590, 0.959]	[0.247, 1.017]	[0.572, 1.389]	[0.362, 0.848]
Race/Ethnicity				
Black	0.540[c]	0.404[c]	0.901	0.419[c]
	[0.441, 0.661]	[0.256, 0.636]	[0.692, 1.174]	[0.294, 0.599]
Hispanic	0.991	0.562[a]	0.933	1.026
	[0.812, 1.209]	[0.322, 0.983]	[0.646, 1.348]	[0.715, 1.474]
Other	0.856	0.561	1.148	1.014
	[0.673, 1.090]	[0.277, 1.134]	[0.792, 1.664]	[0.658, 1.561]
Constant	180.40	148.55	8.269	602.35
	[55.23, 589.3]	[5.625, 3923]	[1.230, 55.58]	[59.53, 6094]
Pseudo R^2	0.0200			

Note. N = 5,365. Relative risk ratios reported.
[a] p < .05, [b] p < .01, [c] p < .001

Figure 4.6

Average SD Difference of Prior Math Scores (2017) from Overall Mean by LPTA Profile



Note. Multinomial logistic regression coefficient is statistically significantly different from: [a] High Motivation, [b] High Future, [c] Low Motivation, [d] Low Usefulness, [e] Low Future Usefulness.

low future usefulness (rrr = 0.987) profiles had higher prior math achievement than students who were in the low, high future profile (rrr = 0.973; χ^2 = 4.99, p = .026; χ^2 = 11.52, p = .001, respectively). Additionally, students with higher prior math achievement were more likely to be in

the low future usefulness profile than the low motivation profile (rrr = 0.974, χ^2 = 5.49, p = .019). See Table 4.9 for full spring model results and Figure 4.6 for each LTA profile's average standard deviation difference of prior math achievement scores from the overall average.

Predicting Achievement

To predict achievement from profiles, I ran Ordinary Least Squares regressions with standard errors adjusted for nesting within schools one regression for the LPA-long profiles and one for the LTA profiles. To further examine how profiles differed in predicting math achievement, I conducted Wald's *F* post-hoc tests. Each model controlled for prior achievement and demographics.

Table 4.10

Regression of Math Achievement on Demographics and LPA-long Profiles (Unstandardized Coefficients)

	Demographics Model		**Model with Profiles**	
	b	C.I.	*b*	C.I.
Profile				
High Future			-2.203[c]	[-3.192, -1.394]
Low Motivation			-2.365[b]	[-3.849, -0.880]
Drop in Spring			-2.149[b]	[-3.649, -0.649]
Drop in Winter			-1.842[a]	[-3.411, -0.272]
Demographics				
Prior Math Achievement	0.862[c]	[0.832, 0.891]	0.849[c]	[0.820, 0.879]
Boy	1.349[b]	[0.564, 2.135]	1.355[b]	[0.572, 1.138]
Free/Reduced Lunch	-2.275[c]	[-3.243, -1.308]	-2.364[c]	[-3.319, -1.408]
Disability	-4.978[c]	[-6.478, -3.479]	-4.905[c]	[-6.422, -3.388]
Gifted	3.909[c]	[2.397, 5.420]	4.031[c]	[2.515, 5.548]
English Language Learner	-1.908[b]	[-3.286, -0.529]	-2.010[b]	[-3.417, -0.603]
Race/Ethnicity				
Black	-5.396[c]	[-6.870, -3.922]	-5.738[c]	[-7.214, -4.262]
Hispanic	-1.051	[-2.112, 0.011]	-1.112	[-2.184, -0.041]
Other	0.561	[-0.790, 1.912]	0.510	[-0.824, 1.845]
Constant	56.385	[47.084, 65.686]	61.100	[51.804, 70.397]
R^2	0.6788		0.6812	

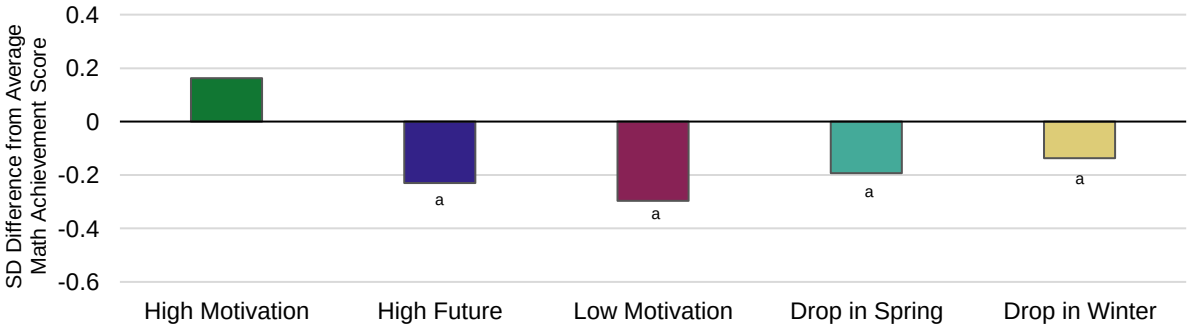
Note. N = 5,365
[a] $p < .05$, [b] $p < .01$, [c] $p < .001$

LPA-long

A model with just prior achievement and demographics explained 67.88% of the variance in math achievement. The addition of the LPA-long profiles increased the amount of math achievement variance explained to 68.12% (an additional 0.24%). The high motivation profile was again used as the reference group. When compared to students in the high motivation profile, students in all other LPA-long profiles had significantly lower mathematics achievement scores (*b*'s ranging from -1.84 to -2.37, *p*'s < .05). Additional post-hoc tests did not indicate any statistically significant differences between the other profiles—students in the high future (*b* = -2.29), low motivation (*b* = -2.36), drop in spring (*b* = -2.15), and drop in winter (*b* = -1.84) profiles performed similarly (*F*'s ranging from 0.01 to 0.28, *p*'s > 0.50). See Table 4.10 for full model results. Figure 4.7 displays each LPA-long profile's average standard deviation difference of regression adjusted math achievement scores and indicates differences from the post-hoc tests.

Figure 4.7

Average SD Difference of Regression Adjusted Math Scores (2018) from Overall Mean by LPA-long Profile



Note. [a] OLS regression coefficient is statistically significantly different from High Motivation.

LTA

In order to run a model analogous to the LPA-long model, I created a conglomerate profile of profiles to account for profile membership at each timepoint. I limited the model to conglomerate profiles with at least 50 students, 13 profiles out of 122. The 13 profiles consisted

of 75% of the sample—high motivation at all three timepoints was the largest conglomerate with 40% of the sample and was used as the reference group. See Table 4.11 for description of the conglomerate profiles.

Table 4.11

Description of LPTA Conglomerate Profiles

Fall	Winter	Spring	Code	% of Sample
High Motivation	High Motivation	High Motivation	HM_HM_HM	40.04%
High Motivation	High Motivation	Low Usefulness	HM_HM_LU	2.14%
High Motivation	High Future	High Motivation	HM_HF_HM	3.15%
High Motivation	High Future	High Future	HM_HF_HF	3.69%
High Motivation	Low Usefulness	High Motivation	HM_LU_HM	2.27%
High Future	High Motivation	High Motivation	HF_HM_HM	3.69%
High Future	High Future	High Motivation	HF_HF_HM	2.70%
High Future	High Future	High Future	HF_HF_HF	8.69%
High Future	High Future	Low Usefulness	HF_HF_LU	0.95%
High Future	High Future	Low, High Future	HF_HF_LHF	1.14%
High Future	Low, High Future	High Future	HF_LHF_HF	1.29%
Low, High Future	High Motivation	High Motivation	LFU_HM_HM	1.08%

A model with just prior achievement and demographics explained 67.91% of the variance in math achievement. Unlike the LPA-long model, the addition of the LTA profiles did not increase the amount of math achievement variance explained (R^2 = 0.6761). Compared to students who remained in the high motivation profile at all three timepoints, half of the profiles had lower mathematics achievement (*b*'s ranging from -2.54 to -2.37, *p*'s < .041). However, the students in the other half of the profiles did not perform statistically significantly differently than the high motivation students (*b*'s ranging from -02.794 to 0.737, *p*'s > .077). Using Wald's *F* post-hoc tests, I found that students who were in the high motivation profile in the fall/winter and high future profile in the spring had statistically significantly lower math achievement than six profiles: HM, HM, HM (*b* = -3.67, *p* < .001), HM, HM, LU (*b* = -0.05, *F* = 5.44, *p* = .022); HM, HF, HF (*b* = -1.82, *F* = 4.12, *p* = .046); HF, HM, HM (*b* = 0.22, *F* = 14.21, *p* < .001); HF, HF, HM (*b* = -0.50,

Table 4.12

Regression of Math Achievement on Demographics and LPTA Profiles

	Demographics Model		Model with Profiles	
	b	C.I.	*b*	C.I.
Profile				
HM, HM, HF			-3.667[c]	[-5.471, -1.863]
HM, HM, LU			-0.048	[-2.751, 2.655]
HM, HF, HM			-2.381[a]	[-4.383, -0.380]
HM, HF, HF			-1.823[a]	[-3.557, -0.089]
HM, LU, HM			-2.538[a]	[-4.972, -0.104]
HF, HM, HM			0.221	[-1.505, 1.948]
HF, HF, HM			-0.496	[-2.353, 1.361]
HF, HF, HF			-1.900[b]	[-3.090, -0.710]
HF, HF, LU			0.737	[-2.407, 3.881]
HF, HF, LHF			-2.794	[-5.899, 0.311]
HF, LHF, HF			-3.711[b]	[-6.212, -1.209]
LFU, HM, HM			-0.732	[-4.619, 3.146]
Demographics				
Prior Math Achievement	0.862[c]	[0.832, 0.892]	0.831[c]	[0.796, 0.866]
Boy	1.359[b]	[0.576, 2.143]	0.905[a]	[0.039, 1.771]
Free/Reduced Lunch	-2.278[c]	[-3.245, -1.310]	-2.169[c]	[-3.309, -1.028]
Disability	-4.968[c]	[-6.470, -3.467]	-4.805[c]	[-6.688, -2.923]
Gifted	3.901[c]	[2.389, 5.413]	3.994[c]	[2.382, 5.607]
English Language Learner	-1.961[b]	[-3.340, -0.583]	-2.059[a]	[-3.758, -0.361]
Race/Ethnicity				
Black	-5.378[c]	[-6.852, -3.904]	-6.139[c]	[-7.722, -4.556]
Hispanic	-1.061	[-2.128, 0.006]	-1.233	[-2.540, 0.075]
Other	0.571	[-0.776, 1.918]	0.682	[-0.765, 2.128]
Constant	56.233	[46.961, 65.505]	67.165	[56.393, 77.937]
R^2	0.6791		0.6761	

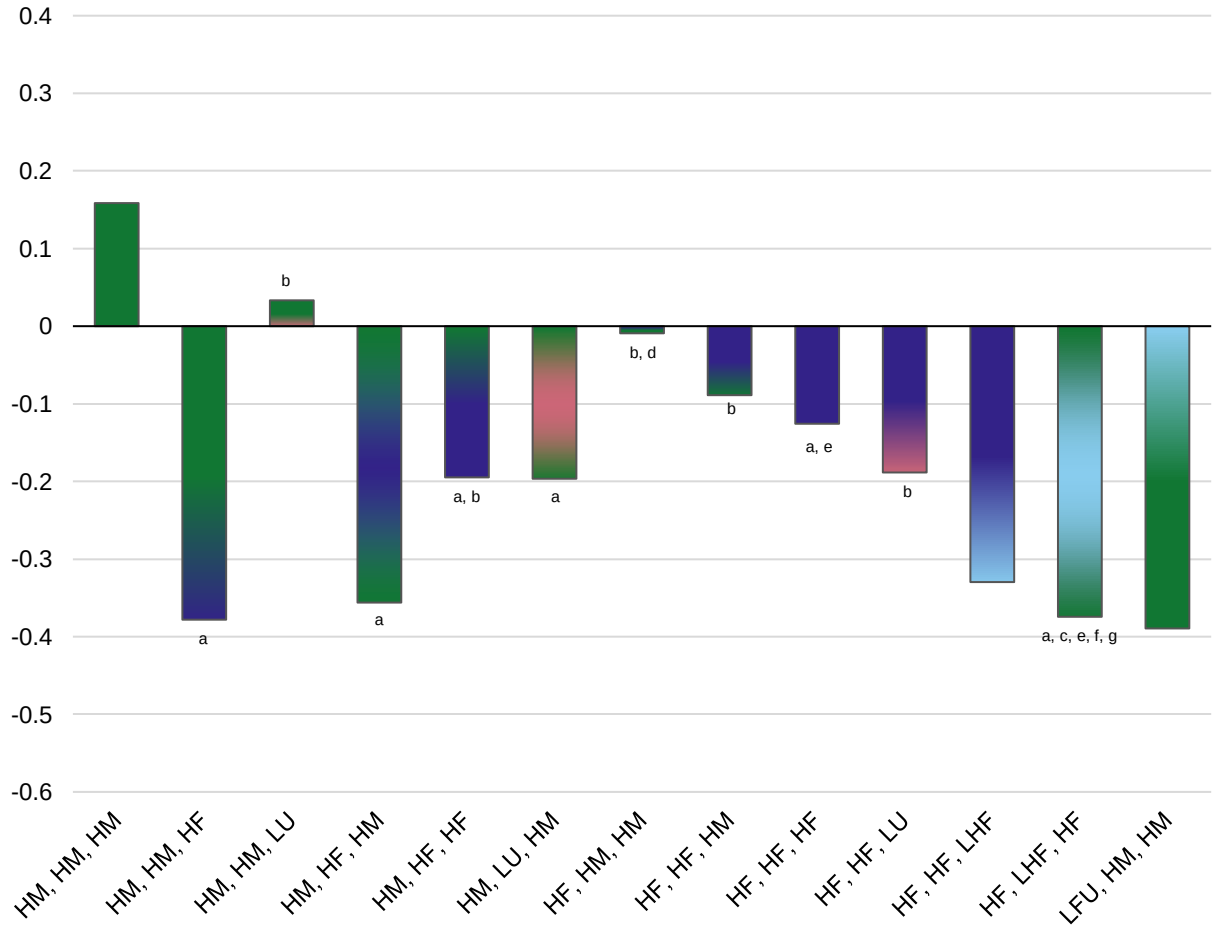
Note. N = 3,986
[a] $p < .05$, [b] $p < .01$, [c] $p < .001$

$F = 7.71$, $p = .007$); and HF, HF, LU ($b = 0.74$, $F = 6.32$, $p = .014$). Similarly, students were in the high future profile in the fall/spring and low, high future profile in the winter had statistically significantly lower math achievement than five profiles: HM, HM, HM ($b = -3.71$, $p = .004$); HM, HM, LU ($b = -0.05$, $F = 4.91$, $p = .030$); HF, HM, HM ($b = 0.22$, $F = 7.18$, $p = .009$); HF, HF, HM ($b = -0.50$, $F = 4.42$, $p = .039$); and HF, HF, LU ($b = 0.737$, $F = 4.95$, $p = .029$). See Table 4.12 for full model results. Figure 4.8 displays the conglomerate LTA profile's average standard

deviation difference of regression adjusted math achievement scores and indicates differences from the post-hoc tests.

Figure 4.8

Average SD Difference of Regression Adjusted Math Scores (2018) from Overall Mean by LPTA Profile



Note. OLS regression coefficient is statistically significantly different from:
[a] HM, HM, HM; [b] HM, HM, HF; [c] HM, HM, LU; [d] HM, HF, HF; [e] HF, HM, HM;
[f] HF, HF, HM; [g] HF, HF, LU

The differences in profiles may be from the reduction in sample, as well as size differences between the profiles. For example, students who were in the fall low future usefulness and winter/spring high motivation profiles had math achievement similar to students in the fall/winter high motivation and spring high future profiles: respectively, average math achievement was

309.05 and 309.25 with standard deviations of 19.09 and 21.15. The HM, HM, HF profile, which was over three times the size of the LFU, HM, HM profile ($n_{HM, HM, HF}$ = 186, $n_{LFU, HM, HM}$ = 58),

Table 4.13

Regression of Math Achievement on LPA-long Profiles with Full and LPTA Reduced Sample

	Model with Full Sample		**Model with LPTA Sample**	
	b	*b*	*b*	C.I.
Profile				
High Future	-2.203[c]	-2.203[c]	-1.835[b]	[-2.851, -0.818]
Low Motivation	**-2.365[b]**	**-2.365[b]**	**6.427**	**[-4.981, 17.835]**
Drop in Spring	**-2.149[b]**	**-2.149[b]**	**0.140**	**[-2.416, 2.696]**
Drop in Winter	-1.842[a]	-1.842[a]	-4.452[b]	[-7.100, -1.803]
Demographics				
Prior Math Achievement	0.849[c]	0.849[c]	0.835[c]	[0.801, 0.869]
Boy	1.355[b]	1.355[b]	0.874[a]	[0.014, 1.734]
Free/Reduced Lunch	-2.364[c]	-2.364[c]	-2.205[c]	[-3.324, -1.087]
Disability	-4.905[c]	-4.905[c]	-4.872[c]	[-6.745, -2.999]
Gifted	4.031[c]	4.031[c]	4.040[c]	[2.450, 5.629]
English Language Learner	-2.010[b]	-2.010[b]	-1.920[a]	[-3.629, -0.210]
Race/Ethnicity				
Black	-5.738[c]	-5.738[c]	-6.010[c]	[-7.566, -4.454]
Hispanic	-1.112	-1.112	-1.285	[-2.594, 0.024]
Other	0.510	0.510	0.672	[-0.786, 2.131]
Constant	61.100	61.100	65.798	[55.357, 76.238]
R^2	0.6812		0.6745	
N	5,365		3,986	

Note. Bolded values indicated difference between models.
[a] $p < .05$, [b] $p < .01$, [c] $p < .001$

was statistically significantly different from the high motivation all year profile (b = -3.67, p < .001) but the LFU, HM, HM profile was not (b = -0.73, p = .709). To further explore the impact of the reduced sample, I re-ran the LPA-long model with sample reduction. This sample reduction primarily impacted the low motivation profile, dropping the number of students from 252 to five. The high motivation profile was reduced the least—there were 2,924 students in the high motivation reduced LTA sample, which was a loss of 131 students (4.29%). In this model, students in the low motivation (b = 6.43, p = .265) and drop in spring profiles (b = 0.14, p = .913) no longer had statistically significantly lower math performance than students in the high motivation profile.

This suggests that the sample reduction influenced the results for the LTA model and these two models (LPA-long and LTA) are not directly comparable. See Table 4.13 for a comparison of LPA-long models with full and LTA reduced sample.

Discussion

Research Question 4A: Motivation Profiles

For my first research question, I asked what profiles of motivation existed throughout the school year and if these profiles vary by analysis type (i.e., unconventional LPA-long compared to traditional LTA). I found a total of nine profiles—three that were consistent across all analyses and timepoints, two unique to the LPA-long, and four unique to the LTA. Like all prior research, I found overall high motivation and low motivation profiles (e.g., Andersen & Cross, 2014; Dietrich & Lazarides, 2019; Karamarkovich & Rutherford, 2018; Lazarides et al., 2020). Additionally, I found novel profiles: some of which displayed divergent expectancies and values (e.g., low usefulness or lower importance), whereas others differed by timeframe (i.e., current versus future value, such as high future).

Research Question 4B: Profiles and Achievement

To answer the second research question about how profiles are related to achievement, I examined both how achievement predicts profile membership and how profile membership predicts achievement. As expected, students with higher prior math achievement were most likely to be in the high motivation profile across all analyses and timepoints. Generally, students with the lowest prior math achievement were more likely to be in the low motivation and high future profiles. This is in line with many previous studies, both person-centered (Hayenga & Corpus, 2010) and variable-centered (Guo et al., 2015).

Although only one was presented here, multiple attempts were made at predicting achievement with the LTA profiles, such as (1) using just the spring profile, however, that did not account for the fall and winter profiles, defeating the purpose of the LTA; (2) including dummy coded profiles for all timepoints, however, interpreting the reference group was difficult; and (3) creating a profile of profiles to account for profile membership at each timepoint, which I decided was the most comparable to the LPA-long, even though I limited to the number groups, which reduced the sample size by 25%.

How the profiles are related to achievement differed by analysis: although students in the High Motivation LPA-long profile performed better than other profiles, the LTA high motivation all year group was not the highest performing. Seven (out of 12) conglomerate profiles were not statistically significantly different from the LTA high motivation. Although this may in part be due to the limited sample, this contrasts the findings from the LPA-long analysis, and prior research (e.g., Gaspard et al., 2019), that suggest the highest achievers are those students with high motivation year-round. Students who were not in the LPA-long high motivation profile all performed similarly, suggesting that the presence of lower motivation at any time (e.g., year-round in the low motivation profile or dropping during a specific time in the year) or for any component (i.e., expectancy and current value) is related to lower performance. Based on this finding and from the findings in the first study, future interventions that attempt to improve motivation should be conducted closer to the middle of the school year to avoid potential dips and focus primarily on current value and expectancies.

Longitudinal Latent Profile Analysis versus Traditional Latent Transition Analysis

The LPA-long analysis provided a holistic interpretation of motivation throughout the school year. Profiling change using the LPA-long clearly demonstrated if/when motivation

dropped during the school year. However, it may be limited in demonstrating how students move across profiles. For example, fewer than 0.5% of students stayed in the Low Motivation profiles at all three timepoints in the LTA but 5% of students were in the Low Motivation profile in the LPA-long model (n = 19 and n = 252, respectively). This may be in part due to the number of profiles identified with the LTA, as many of the profiles found in the LTA were not present in the LPA-long. This lack of precision in identifying profiles with the LPA-long likely goes hand in hand with the increase in datapoints. One advantage of the LPA-long is that certain changes were captured in the LPA-long model but do not have a direct comparison to the transitions in the LTA model (i.e., the profiles of change that identified drops in motivation in the winter/spring).

The traditional LTA provided a detailed view of how students moved between profiles and allowed for different profiles at each of the timepoints. This detailed approach increased the number of profiles found (the LTA found four unique profiles not in the LPA-long) and allows for transitions between profiles (a total of 50 possible transition). However, this additional detail made further analyses, such as predicting transitions, profile membership, and achievement, far more complex and difficult. The most comparable analysis, creating a conglomerate profile, resulted in a reduced, and likely biased, sample.

Overall, each analysis has its strengths and weakness and future researchers should consider their goal when deciding which method to use. If the research objective is to capture the complexity and multiple subgroups of motivation, an LTA would be more appropriate. On the other hand, for an efficient snapshot of profiles of change and how those profiles relate to future achievement, researchers should use analyses like the LPA-long. The ease of communicating the LPA-long results may be especially beneficial when teaching about motivation and how it changes to students or practitioners.

Limitations and Future Directions

One major limitation of this study is the number of data points, and thus sample size required, for the LPA-long. By profiling change in this way, a large sample size is needed to account for all possible interactions between the variables without convergence issues. This sample size seems to be sufficient but this may not be the case for other studies, making it difficult for other researchers to use this method. Additionally, I attempted to run parallel models for both sets of analyses; however, the conglomerate LTA profiles reduced the sample size and may not represent profiles of change like the LPA-long profiles.

Although the aim of this study was to examine profiles of change during the school year, future research profiling change using LPA-long over many years would provide more insight into how motivation fluctuates as students develop, beyond single variable analyses. I chose to use latent profile analysis because of the direct comparison to latent transition analysis; however, future research could use a similar approach with different person-centered analyses (e.g., longitudinal cluster analysis).

Conclusion

In this study I pioneered a new method to profile change: a latent profile analysis with multiple timepoints. This approach goes beyond traditional person-centered and longitudinal designs by modeling how multiple variables interact with each other over time. When compared to a traditional latent profile and transition analysis, I found that although the LTA provided detailed information on transitions and complex patterns of motivation, the LPA-long provided a simplicity that allows for greater interpretability. By knowing when and how motivation dips, researchers and educators can better target key aspects of motivation at times when students need motivating the most, leading to better engagement and achievement year round.

References

Andersen, L., & Cross, T. L. (2014). Are students with high ability in math more motivated in math and science than other students? *Roeper Review, 36*(4), 221–234. https://doi.org/10.1080/02783193.2014.945221

Archambault, I., Eccles, J. S., & Vida, M. N. (2010). Ability self-concepts and subjective value in literacy: Joint trajectories from grades 1 through 12. *Journal of Educational Psychology, 102*(4), 804–816. https://doi.org/10.1037/a0021075

Atkinson, J. W. (1957). Motivational determinants of risk-taking behavior. *Psychological Review, 64*(6, Pt.1), 359–372. https://doi.org/10.1037/h0043445

Bergman, L. R., & El-Khouri, B. M. (1999). Studying individual patterns of development using I-states as objects analysis (ISOA). *Biometrical Journal: Journal of Mathematical Methods in Biosciences, 41*(6), 753-770. https://doi.org/10.1002/(SICI)1521-4036(199910)41:6%3C753::AID-BIMJ753%3E3.0.CO;2-K

Bergman, L. R., & El-Khouri, B. M. (2003). A person-oriented approach: Methods for today and methods for tomorrow. *New directions for child and adolescent development, 2003*(101), 25-38. https://doi.org/10.1002/cd.80

Buehl, M. M., & Alexander, P. A. (2005). Motivation and performance differences in students' domain-specific epistemological belief profiles. American Educational Research Journal, 42(4), 697-726. https://doi.org/10.3102%2F00028312042004697

Chen, J. A. (2012). Implicit theories, epistemic beliefs, and science motivation: A person-centered approach. *Learning and Individual Differences, 22*(6), 724-735. https://doi.org/10.1016/j.lindif.2012.07.013

Chen, Q., Luo, W., Palardy, G. J., Glaman, R., & McEnturff, A. (2017). The efficacy of common fit indices for enumerating classes in growth mixture models when nested data structure is ignored: A Monte Carlo study. Sage Open, 7(1), 1-19. https://doi.org/10.1177/2158244017700459

Collier, Z. K., & Leite, W. L. (2017). A comparison of three-step approaches for auxiliary variables in latent class and latent profile analysis. *Structural Equation Modeling: A Multidisciplinary Journal, 24*(6), 819-830. https://doi.org/10.1080/10705511.2017.1365304

Collins, L. M. & Lanza, S. T. (2010) *Latent class and latent transition analysis: With applications in the social, behavioral, and healthy sciences.* Wiley.

Conley, A. M. (2012). Patterns of motivation beliefs: Combining achievement goal and expectancy-value perspectives. *Journal of Educational Psychology, 104*(1), 32-47. https://psycnet.apa.org/doi/10.1037/a0026042

Dietrich, J., & Lazarides, R. (2019). Gendered development of motivational belief patterns in mathematics across a school year and career plans in math-related fields. F*rontiers in Psychology, 10*, 1472. https://doi.org/10.3389/fpsyg.2019.01472

Dietrich, J., Moeller, J., Guo, J., Viljaranta, J., & Kracke, B. (2019). In-the-moment profiles of expectancies, task values, and costs. *Frontiers in Psychology, 10.* https://doi.org/10.3389/fpsyg.2019.01662

Eccles-Parsons, J. S., Adler, T. F., Futterman, R., Goff, S. B., Kaczala, C. M., Meece, J. L., et al. (1983). Expectancies, values, and academic behaviors. In J. T. Spence (Ed.), *Achievement and achievement motivation* (pp. 75–146). W. H. Freeman.

Eccles, J. S., & Wigfield, A. (2020). From expectancy-value theory to situated expectancy-value theory: A developmental, social cognitive, and sociocultural perspective on motivation. *Contemporary Educational Psychology, 61*, 101859. https://doi.org/10.1016/j.cedpsych.2020.101859

Eshghi, A., Haughton, D., Legrand, P., Skaletsky, M., & Woolford, S. (2011). Identifying groups: A comparison of methodologies. *Journal of Data Science, 9*(2), 271-291.

Flake, J. K., Barron, K. E., Hulleman, C., McCoach, B. D., & Welsh, M. E. (2015). Measuring cost: The forgotten component of expectancy-value theory. *Contemporary Educational Psychology, 41*, 232–244. https://doi.org/10.1016/j.cedpsych.2015.03.002

Florida Department of Education (2017). *Florida Standards Assessments 2016–2017 volume 4: Evidence of reliability and validity.* http://www.fldoe.org/core/fileparse.php/5663/urlt/V4FSA1617TechRpt.pdf

Florida Department of Education (2018). *Florida Standards Assessments 2017–2018 volume 4: Evidence of reliability and validity.* http://www.fldoe.org/core/fileparse.php/5663/urlt/V4-FSA-1718-TechRpt.pdf

Gaspard, H., Dicke, A. L., Flunger, B., Schreier, B., Häfner, I., Trautwein, U., & Nagengast, B. (2015). More value through greater differentiation: Gender differences in value beliefs about math. *Journal of Educational Psychology, 107*(3), 663. https://psycnet.apa.org/doi/10.1037/edu0000003

Gaspard, H., Wille, E., Wormington, S. V., & Hulleman, C. S. (2019). How are upper secondary school students' expectancy-value profiles associated with achievement and university STEM major? A cross-domain comparison. *Contemporary Educational Psychology, 58*, 149-162. https://doi.org/10.1016/j.cedpsych.2019.02.005

Goodman, L. A. (2002). Latent class analysis: The empirical study of latent types, latent variables, and latent structures, and some notes on the history of this subject. In J. A. Hagenaars & A. L. McCutcheon (Eds.), *Applied latent class analysis* (pp. 3–55). Cambridge University Press.

Guo, J., Parker, P. D., Marsh, H. W., & Morin, A. J. S. (2015). Achievement, motivation, and educational choices: A longitudinal study of expectancy and value using a multiplicative perspective. *Developmental Psychology, 51*(8), 1163–1176. https://doi.org/10.1037/a0039440

Hayenga, A. O., & Corpus, J. H. (2010). Profiles of intrinsic and extrinsic motivations: A person-centered approach to motivation and achievement in middle school. *Motivation and Emotion*, *34*(4), 371-383. https://doi.org/10.1007/s11031-010-9181-x

Howard, M. C., & Hoffman, M. E. (2018). Variable-centered, person-centered, and person-specific approaches: where theory meets the method. *Organizational Research Methods*, *21*(4), 846-876. https://doi.org/10.1177/1094428117744021

Jang, L. Y., & Liu, W. C. (2012). 2× 2 Achievement goals and achievement emotions: a cluster analysis of students' motivation. *European Journal of Psychology of Education*, *27*(1), 59-76. https://doi.org/10.1007/s10212-011-0066-5

Karamarkovich, S. M., & Rutherford T. (2018, August 9-12). *Differences in profiles of motivation for mathematics across grades and districts* [Paper presentation]. American Psychological Association 2018 Convention, San Francisco, CA.

Laursen, B., & Hoff, E. (2006). Person-centered and variable-centered approaches to longitudinal data. *Merrill-Palmer Quarterly (1982-)*, 377-389. https://doi.org/10.1353/mpq.2006.0029

Lazarides, R., Dicke, A.-L., Rubach, C., & Eccles, J. S. (2020). Profiles of motivational beliefs in math: Exploring their development, relations to student-perceived classroom characteristics, and impact on future career aspirations and choices. *Journal of Educational Psychology, 112*(1), 70–92. https://doi.org/10.1037/edu0000368

Lazarides, R., Dietrich, J., & Taskinen, P. H. (2019). Stability and change in students' motivational profiles in mathematics classrooms: The role of perceived teaching. *Teaching and Teacher Education, 79*, 164-175. https://doi.org/10.1016/j.tate.2018.12.016

Lazarides, R., Viljaranta, J., Aunola, K., Pesu, L., & Nurmi, J. E. (2016). The role of parental expectations and students' motivational profiles for educational aspirations. Learning and Individual Differences, 51, 29-36. https://doi.org/10.1016/j.lindif.2016.08.024

Linnenbrink-Garcia, L., Wormington, S. V., Snyder, K. E., Riggsbee, J., Perez, T., Ben-Eliyahu, A., & Hill, N. E. (2018). Multiple pathways to success: An examination of integrative motivational profiles among upper elementary and college students. *Journal of Educational Psychology, 110*(7), 1026–1048. https://doi.org/10.1037/edu0000245

Luo, W., Paris, S. G., Hogan, D., & Luo, Z. (2011). Do performance goals promote learning? A pattern analysis of Singapore students' achievement goals. C*ontemporary Educational Psychology, 36*(2), 165-176. https://doi.org/10.1016/j.cedpsych.2011.02.003

Magidson, J., & Vermunt, J. (2002). Latent class models for clustering: A comparison with K-means. *Canadian Journal of Marketing Research, 20*(1), 36-43.

Magnusson, D. (2003). The person approach: Concepts, measurement models, and research strategy. *New Directions for Child and Adolescent Development, 2003*(101), 3–23. https://doi.org/10.1002/cd.79

McLachlan, G., & Peel, D. (2000). Mixtures of factor analyzers. In *Proceedings of the Seventeenth International Conference on Machine Learning* (pp. 599-606).

Meece, J. L., Wigfield, A., & Eccles, J. S. (1990). Predictors of math anxiety and its consequences for young adolescents' course enrollment intentions and performances in mathematics. *Journal of Educational Psychology, 82*, 60–70. https://doi.org/10.1037/0022-0663.82.1.60.

Meyer, J., Fleckenstein, J., & Köller, O. (2019). Expectancy value interactions and academic achievement: Differential relationships with achievement measures. *Contemporary Educational Psychology, 58*, 58-74. https://doi.org/10.1016/j.cedpsych.2019.01.006

MIND Research Institute. (2018). Mind Research Institute: Learn math visually. Retrieved from http://mindresearch.org

Musu-Gillette, L. E., Wigfield, A., Harring, J. R., & Eccles, J. S. (2015). Trajectories of change in students' self-concepts of ability and values in math and college major choice. *Educational Research and Evaluation, 21*(4), 343-370. https://doi.org/10.1080/13803611.2015.1057161

Muthén, L.K. and Muthén, B.O. (2017). *Mplus User's Guide. Eighth Edition*. Muthén & Muthén

Nagengast, B., Trautwein, U., Kelava, A., & Lüdtke, O. (2013). Synergistic effects of expectancy and value on homework engagement: The case for a within-person perspective. *Multivariate Behavioral Research, 48*(3), 428-460. https://doi.org/10.1080/00273171.2013.775060

Nurmi, J. E., & Aunola, K. (2005). Task-motivation during the first school years: A person-oriented approach to longitudinal data. *Learning and Instruction, 15*(2), 103-122. https://doi.org/10.1016/j.learninstruc.2005.04.009

Nylund, K. L., Asparouhov, T., & Muthén, B. O. (2007). Deciding on the number of classes in latent class analysis and growth mixture modeling: A Monte Carlo simulation study. *Structural Equation Modeling, 14*(4), 535-569. https://doi.org/10.1080/10705510701575396

Pekrun, R. (2006). The Control-Value Theory of Achievement Emotions: Assumptions, corollaries, and implications for educational research and practice. *Educational Psychology Review*, *18*(4), 315–341. https://doi.org/10.1007/s10648-006-9029-9

Rindskopf, D. (2010). Latent transition analysis. In G. R. Hancock, R. O. Mueller, & L. M. Stapleton (Eds.), *The reviewer's guide to quantitative methods in the social sciences* (pp. 109-207). Routledge

Robinson, K. A., Beymer, P. N., Ranellucci, J., & Schmidt, J. A. (2020). Momentary emotion profiles in high school science and their relations to control, value, achievement, and science career intentions. *Motivation Science*. https://doi.org/10.1037/mot0000174

Robinson, K. A., Ranellucci, J., Lee, Y., Wormington, S. V., Roseth, C. J., & Linnenbrink-Garcia, L. (2017). Affective profiles and academic success in a college science course. Contemporary Educational Psychology, 51, 209–221. http://doi.org/10.1016/j.cedpsych.2017.08.004

Rutherford, T., Farkas, G., Duncan, G., Burchinal, M., Kibrick, M., Graham, J., … Martinez, M. E. (2014). A randomized trial of an elementary school mathematics software intervention: Spatial-Temporal Math. *Journal of Research on Educational Effectiveness, 7*(4), 358–383. https://doi.org/10.1080/19345747.2013.856978

Rutherford, T., Liu, A. S., Lam, A. S., & Schenke, K. (2019). Impact on mathematics self-beliefs from a mastery-based mathematics software. *Journal of Research on Technology in Education, 52*(1), 79–94. https://doi.org/10.1080/15391523.2019.1689210

Rutherford, T., Liu, A., Logan, W., Karamarkovich, S. M. , & Wagemaker, M. (2019b, August 8-11). "I chose math because…" Cognitive interviews of a mathematics motivation measure [Poster presentation]. American Psychological Association 2019 Convention, Chicago, IL.

Schenke, K., Rutherford, T., & Farkas, G. (2014). Alignment of game design features and state mathematics standards: Do results reflect intentions? *Computers & Education, 76*, 215-224. https://doi.org/10.1016/j.compedu.2014.03.019

Singh, K., Granville, M., & Dika, S. (2002). Mathematics and science achievement: Effects of motivation, interest, and academic engagement. *The Journal of Educational Research, 95*(6), 323-332. https://doi.org/10.1080/00220670209596607

Tein, J. Y., Coxe, S., & Cham, H. (2013). Statistical power to detect the correct number of classes in latent profile analysis. *Structural Equation Modeling: A Multidisciplinary Journal, 20*(4), 640-657. https://doi.org/10.1080/10705511.2013.824781

Trautwein, U., Marsh, H. W., Nagengast, B., Ludtke, O., Nagy, G., & Jonkmann, K. (2012). Probing for the multiplicative term in modern expectancy-value theory: A latent interaction modeling study. *Journal of Educational Psychology, 104*, 763–777. https://doi.org/10.1037/a0027470.

Umarji, O., McPartlan, P., & Eccles, J. (2018). Patterns of math and English self-concepts as motivation for college major selection. Contemporary Educational Psychology, 53, 146–158. https://doi.org/10.1016/j.cedpsych.2018.03.004

Vermunt, J. K., & Magidson, J. (2002). Latent class cluster analysis. In J. A. Hagenaars & A. L. McCutcheon (Eds.) A*pplied latent class analysis* (pp. 89-106). Cambridge University Press

Viljaranta, J., Aunola, K., & Hirvonen, R. (2016). Motivation and academic performance among first graders: A person-oriented approach. *Learning and Individual Differences, 49*, 366-372. https://doi.org/10.1016/j.lindif.2016.06.002

Wang, M.-T., Eccles, J. S., & Kenny, S. (2013). Not lack of ability but more choice: Individual and gender differences in choice of careers in science, technology, engineering, and mathematics, *Psychological Science, 24*(5), 770–775. https://doi.org/10.1177/0956797612458937

Weidinger, A. F., Spinath, B., & Steinmayr, R. (2020). The value of valuing math: Longitudinal links between students' intrinsic, attainment, and utility values and grades in math. Motivation Science, 6(4), 413–422. https://doi.org/10.1037/mot0000179

Wigfield, A., & Cambria, J. (2010). Students' achievement values, goal orientations, and interest: Definitions, development, and relations to achievement outcomes. *Developmental Review, 30*(1), 1–35. https://doi.org/10.1016/j.dr.2009.12.001

Wigfield, A., & Eccles, J. S. (2000). Expectancy–Value Theory of Achievement Motivation. *Contemporary Educational Psychology, 25*(1), 68–81. https://doi.org/10.1006/ceps.1999.1015

Wigfield, A., & Eccles, J. S. (2020). 35 Years of research on students' subjective task values and motivation: A look back and a look Forward. In A. Elliot (Vol. Ed.), *Advances in motivation science: Vol. 7*, (pp. 162–193). Elsevier.

CHAPTER FIVE: CONCLUSION

Mathematics skills developed in elementary are crucial for future achievement in mathematics (e.g., Bailey et al., 2014; Geary et al., 2013) and has even been tied to future socio-economic status over thirty years later (Ritchie & Bates, 2013). Due to the importance of mathematics, especially early on, there has been a lot of research on how to improve mathematics in elementary school (see Pellegrini et al., 2021). In a meta-analysis of 87 experimental studies, Pellegrini and colleagues (2021) found that programs that focus on enhancing mathematics motivation and active learning are often some of the most successful in improving students' mathematics achievement.

Students tend to be more successful when they believe they will do well and value what they are doing, i.e., when they are motivated (Cole et al., 2008; Eccles & Wigfield, 2020; Hulleman et al., 2008; Wang & Eccles, 2013). However, motivation, especially mathematics motivation, tends to decline in middle childhood when students become more calibrated in their expectancies (Muenks et al., 2018) and more developed in their personal identities and goals (Wigfield & Eccles, 2020). Prior research has primarily focused on changes in motivation over multiple years (e.g., Musu-Gillette et al., 2015), without considering how changes may occur during the school year (*cf.* Maulana et al., 2016). However, school context, such as teachers and peers, are one of the biggest influences on motivation (Eccles & Wigfield, 2020; Muenks et al., 2018; Wigfield & Eccles, 2020); therefore more research needs to be done on the trends of motivation within and between school years. The studies within this dissertation use a variety of analyses, both variable-centered and person-centered, to better understand the trends and patterns of motivation during those critical years of motivation development.

Summary of Findings

The goal of *Models of Growth* was to examine trends in motivation over two years in middle to late elementary school. I used a cross-sequential design and compared linear and unstructured multilevel mixed-effects regressions to model how motivation fluctuates over time. When using linear models, the results align with previous research, both mean-centered (e.g., Wigfield & Cambria, 2010) and person-centered trajectories (e.g., Gaspard et al., 2020): expectancies and values declined over time from third to fifth grade. However, the unstructured models, which did not constrain change over time to be linear, fit the data better. This suggests that mathematics motivation, especially expectancy and current value, is situated (i.e., it varies by context; Eccles & Wigfield, 2020). Future value did it fluctuate as greatly during the school year as both current value and expectancy, nor did it decline as much. This suggests that students' long-term goals, such as careers, may be more stable than school year-specific expectations and values.

The unstructured models also demonstrated a dip in motivation in the middle of the school year, with motivation often rebounding at the beginning of the next school year. This embodies a common motivation theory that motivation declines because students are optimistic when they are younger and realistic when they are older (see Muenks et al., 2018); however, on a much smaller scale of being optimistic at the beginning of the school year and more realistic at the middle/end.

In *Mixed Feelings*, I combined person-centered and variable-centered approaches to identify profiles of emotions in fourth and fifth graders and how those profiles mediated the relation between motivation and achievement. I found two positive profiles: one where students reported a combination of all the positive emotions and another where students exclusively reported being happy and excited but not hopeful. Students in both positive profiles reported feeling challenged, which is not theoretically ground in Control-Value Theory but represents a

commonly felt emotion (Kirby et al., 2014; van Tilburg & Igou, 2012). I also found a mixed emotions profile where students reported challenged along with both positive and negative emotions. Lastly, a small percentage of students were in a negative profile where students exclusively reported feeling bored, frustrated, and nervous.

These profiles all mediated the relation between math expectancy and achievement, such that students with higher math expectancy were more likely to be in positive profiles and, in turn, students with positive emotion profiles were higher achievers. Conversely, students with lower math expectancy were more likely to be in the negative and mixed emotion profiles and those students had lower math achievement. A similar pattern was found for profiles mediating the relation between math value and achievement with the exception of mixed emotion profile, which was not a statistically significant mediator. The profiles and mediation largely fit into the Control-Value theoretical model, with the key exception being the mixed emotion profiles. Understanding how and why students experience mixed emotions, especially during a period when motivation declines, can help researchers and educators create positive emotional and motivational states for students.

The third study, *Profiling Change*, combines two goals from the previous studies: identifying profiles of motivation and examining trends of motivation over time. In order to profile change, I pioneered a new analysis: a latent profile analysis with multiple timepoints, termed Longitudinal LPA or LPA-long. This analysis examines patterns between and within variables over time, unlike traditional methods that identify profiles for each timepoint or look at how individual variables change. I found five profiles using this method—high motivation all year, high future value, low motivation all year, drop in motivation during the middle of the school year (winter), and drop in motivation at the end of the school year (spring). Three of these profiles (high

motivation, high future, and low motivation) were also found at each timepoint when conducting a traditional latent transition analysis. The other LTA profiles were low future usefulness (fall and spring), low, high future (winter and spring), lower importance (fall), and low usefulness (winter).

Students in the high motivation all year profile had the highest achievement compared to students in the other LPA-long profiles. However, this did not align with the results from the latent transition analysis. To run a comparable model with the latent transition analysis, profiles at all timepoints were condensed into a conglomerate profile (e.g., if a student was in the high motivation profile at all three timepoints, their conglomerate profile would be HM, HM, HM). Unlike the LPA-long model, students in the high motivation all year conglomerate profile had similar math achievement as nearly half of the other conglomerate profiles. However, this appears to be due to the reduction in sample size and may not accurately reflect how LTA profiles are related to achievement. Overall, both the traditional LTA and the innovative LPA-long capture the complexity of motivation. However, the LPA-long allowed for a greater interpretability in profiling change, especially for identifying how and when motivation dips during the school year.

Themes

Each study within this dissertation attempted to capture the complexity of motivation at a critical age in development using a combination of both variable-centered and person-centered approaches. Although each study has its own set of question, taken together, the three studies have a common goal using similar data to provide insight into the trends and patterns of motivation. Below I discuss two themes that reach across the studies.

Drop in Motivation During the Middle and End of the School Year

Models of Growth and *Profiling Change* (studies one and three respectively) both examined change in motivation within at least one school year. Different analyses were used in

each—*Models of Growth* used a mean-level regression and *Profiling Change* used a profile analysis—yet they both demonstrated a similar trend of students' motivation dropping in either the middle and/or end of the school year. However, how motivation dipped varied by study. In *Models of Growth*, students' expectancy and current value dipped more than their future value; whereas, in *Profiling Change*, students who were in the drop in winter/spring profiles reported lower future value than both expectancies and current value. Another difference between the studies is that in *Profiling Change*, the drop in winter/spring profiles only made up 15% of the sample, whereas the variable-centered models in *Models of Growth* showed an overall dip on average. This may be due to the degree of change—in the drop in winter and drop in spring profiles, motivation in the winter/spring was 0.37 to 1.77 standard deviations lower than average, whereas in *Models of Growth*, winter/spring motivation was only 0.02 to 0.22 standard deviations lower than the motivation at the first wave. Although the way motivation dropped differed, both studies suggest that the middle of the school year is a critical time for students' motivation. This may be the most impactful time to intervene to improve motivation, potentially mitigating the drop in the winter and preventing the drop in the spring.

The "Best" Isn't Always the Best: Profiles and Achievement

In *Mixed Feelings* (study two) and *Profiling Change* (study three), I identified profiles of emotions and motivation. Both studies had profiles that represented what would be the ideal or best motivation/emotion pattern. For *Mixed Feelings*, it was the profile with all the positive emotions—students reported feeling a combination of excited, happy, hopeful, and challenged (see Smith & Ellsworth, 1985 for explanation of challenged)—and for *Profiling Change* it was when students reported high motivation all year, either as a profile in the LPA-long analyses or the conglomerate HM, HM, HM LTA profile. Students in these profiles tended to be high achievers,

however, they were not always the highest achievers. For example, students who only reported two of the three positive emotions (excited and happy) and challenged had similar achievement, on average, as the students who reported a combination of all positive emotions. Similarly, students in the high motivation all year LTA profile did not have statistically significantly higher performance than over half of the other profiles.

Although students in these "best" profiles had higher achievement than students in negative profiles (e.g., the negative emotion profile in *Mixed Feelings* or the low motivation profiles in *Profiling Change*), they were not necessarily the highest achievers. This begs the question that if students do not need to have the "best" emotions/motivation, what is the threshold for motivation having a positive impact on engagement and learning? The idea of a threshold or point at which motivation is good enough, is especially critical for motivation interventionists and educators who are attempting to increase students' motivation.

Significance

The studies within this dissertation, both individually and together, make significant contributions to practice, theory, and methodology.

Implications for Practice: When to Intervene

The major implication for practice is knowing when and how motivation researchers and educators should intervene to improve emotion. As discussed previously, the best time for a motivation intervention may be slightly before the middle of the school year to mitigate any drops in motivation at that time and possibly prevent a drop at the end of the school year as well. These interventions should target students' current math value, connecting what they are learning to their short-term goals (Harackiewicz et al., 2014), and/or their expectancies (Marsh et al., 2017). The majority of SEVT interventions focus on utility value (Harackiewicz & Priniski, 2018), which

traditionally considers future goals as well (Eccles & Wigfield, 2020); however, due to the relative stability of future value compared to current value and expectancy, intervening with future value may not be as impactful.

Contrasting the dip in motivation in the middle of the school year, two of the studies indicate that students may have higher motivation at the beginning of the school year. Although higher expectancy is tied to better performance (e.g., Schunk & Pajares, 2009), the spike in motivation at the beginning of the year suggests that students may not be well calibrated at that time. Therefore, the beginning of the school year would be a good time to conduct calibration interventions, with continual feedback throughout the year to help students remain calibrated (see Hattie, 2013; Labuhn et al., 2010).

These "interventions" do not necessarily need to be conducted by researchers—teachers' practices are also key to students' motivation. For example, Wang and Eccles (2013) found that teacher emotional support was positively related to students' behavioral and emotional engagement and that teaching for relevance was positively related to students' emotional and cognitive engagement. Additionally, both teacher practices were tied to students' academic self-concept and subjective task value (Wang & Eccles, 2013). With motivation dipping in the middle of the school year, teachers may want to increase the number of activities that support student autonomy, such as providing students opportunities to make decisions (e.g., (Katz & Assor, 2006; Reeve et al., 1999) and relating material to students' interests (e.g., Wigfield et al., 2006).

Implications for Theory: Construct vs Timeframe

The other theoretical implication of this dissertation is the treatment of value. The profiles in *Profiling Change* often demonstrated that students reported similar levels of future importance and future usefulness, as well as current importance and current usefulness. This was especially

apparent in the high future profile, where students reported high future usefulness and future importance but lower current usefulness and current importance. Additionally, current value and future value had higher reliability scores (α = 0.75 and 0.82 respectively) than importance and usefulness (α = 0.70 and 0.72 respectively). These findings were used to support aggregating current and future value in *Models of Growth*, rather than aggregating importance and usefulness.

My findings corroborate prior research that has found that younger students may have difficulty disentangling usefulness and importance (e.g., Rutherford et al., 2019b; Wigfield & Cambria, 2010; Wigfield & Eccles, 2010). The close connection between utility and attainment value is grounded in the SEVT literature (Eccles & Wigfield, 2020; Wigfield & Eccles, 2020) and prior studies have also combined importance and usefulness (e.g., Archambault et al., 2010; Durik et al., 2006; Musu-Gillette et al., 2015; Simpkins et al., 2006). However, these studies do not contextualize importance in the "future," as the future has only been used to define utility value—"how well a particular task fits into an individual's present or future plans" (Eccles & Wigfield, 2020, p. 5). This calls into question how "the future" is incorporated into measuring value. Yet the similarity in how students responded to the future usefulness and future importance questions suggest that both utility and attainment value may depend on one's short- and long-term goals. Take for example a student who wants to become a mathematician. Math would be important and useful currently; however, it would be even more useful to them during their future career as a mathematician and important to their identity as a mathematician in the future. Therefore, a task or subject can have "relative personal/identity-based importance" (Eccles & Wigfield, 2020) and it is likely inaccurate to consider how math fits into one's future goals as solely utility value.

These studies suggest that utility value and attainment value, though theoretically distinct, may not be practically distinct at this age. It may be more meaningful to combine utility and

attainment value into one measure and, instead, examine the difference in utility/attainment value for short-term and long-term goals (i.e., "how important/useful is math to you **now**" versus "how important/useful is math to you in the **future**"). By understanding how value may differ by short-term and long-term goals, researchers and educators can identify what components of value are more malleable, and thus easier to enhance through instruction and interventions (see Harackiewicz & Priniski, 2018; Rosenzweig & Wigfield, 2016).

Although the measures in the survey provide a unique understanding of current versus future value, one issue with how students conceptualize the "future." Rutherford and colleagues (2019) conducted cognitive interviews with children (ages seven to 12) and found that some students consider the future to be only in a couple years, whereas others think of the future in terms of when they are working adults. This variation is likely related to their reporting of future value. Future research should tie students' timeframe for the future to how they respond to the future value questions. This can be done through qualitative and/or quantitative approaches. For example, researchers could take a qualitative approach and interview students about what they think of when responding to the "future" questions. On the other hand, researchers could take a quantitative approach and add additional "future" questions with specific timeframes, such as "when you think of yourself as a high school" or "when you imagine working in your ideal career." Future research should use either approach with younger and older students to determine if there are differences by age as students develop a more nuanced understanding of their identity and, potentially, the difference between usefulness and importance (Wigfield & Eccles, 2020).

Methodology Advances

Each study used analyses that were either a combination or a comparison of traditional and innovative approaches. The first study (*Models of Growth*) compared a more traditional model,

where change over time was constrained to be linear, to a model without such constraints. This differs from prior research that has looked at either non-linear trends and/or growth mixture models (e.g., Gaspard et al., 2020) to examine motivation. By allowing the data to drive the shape of the model, I was able to get a more realistic depiction of how motivation fluctuates.

The second study (*Mixed Feelings*) uses person-centered analyses to identify profiles of motivation. Although there has been an influx of person-centered studies in motivation (e.g., Bergman & El-Khouri, 2003; Chen, 2012; Hayenga & Corpus, 2010; Howard & Hoffman, 2017; Linnenbrink-Garcia et al., 2018), person-centered approaches have only recently been used with achievement emotions (i.e., primarily in the last five years; Ganotice et al., 2016; Jarrell et al., 2016, 2017; Raccanello et al., 2018; Robinson et al., 2017, 2020). Additionally, prior research has looked at how motivation/emotions act as a mediator of the effect of predictors on achievement (e.g., the effect of gender on math achievement is mediated by motivation: Eccles et al., 1999; Guo et al., 2015a, 2015b; Nagy et al., 2008), yet none have combined person-centered analyses with mediation. In this study I attempted a multilevel multinomial mediation model, yet due to convergence issues, mediation models with each profile were run separately. Although this poses some issues, the method of using profiles as a mediator provides an innovative way to look at the role of profiles in achievement.

Lastly, the third study (*Profiling Change*) creates a new analysis to profile change: a latent profile analysis with all timepoints. It combines the aspects of profile analysis, I-States, and growth mixture modeling to examine patterns between variables and within variables over time. This technique may be best suited for a few timepoints and variables, as each additional timepoint and variable makes the model exponentially more complicated. However, in *Profiling Change* the LPA-long was easier to interpret and use in analyses, such as tying the profiles to future

achievement, than the more traditional latent transition analysis. Overall, each study had some form of new or innovative way to model motivation in middle childhood and, often, these new approaches (e.g., the unstructured model and the LPA-long) provided a more accurate depiction of motivation.

Future Research

A major contribution of these studies is examining through multiple analyses how motivation may fluctuate during the school year. Given this fluctuation in motivation, future research should consider when is the best time to measure motivation for accurate relations with engagement and achievement. For example, is motivation at the beginning of the school year more or less predictive of achievement than motivation at the middle of the school year?

Although these studies demonstrate *how* motivation changes during the school year, more work needs to be done looking at *why* motivation changes within the school year. Prior work on how school context influences motivation largely relies on just one or two timepoints (e.g., Eccles, 2012; Pitzer & Skinner, 2017; Wigfield et al., 2015; *cf.* Maulana et al., 2016). To truly understand the "situated" aspect of SEVT, more timepoints within and across years should be explored, especially using analyses that do not constrain trends to linear models.

Additionally, future research should use mixed methods and/or more measures throughout the SEVT model to better understand the context surrounding each timepoint and what caused motivation to fluctuate (e.g., calibration, teaching style, peers, parents). By collecting more measures, both quantitative and qualitative, future research can better capture how each student is nested within the classroom, school, community, and society as a whole. This would help situate the students' experiences within the larger system of their school and community and potentially

identify practices within and outside of the classroom that may influence motivation and achievement.

Conclusion

Motivation is critical to success but also is a complex process that is constantly changing based on context and development. My dissertation addresses that complexity by using a variety of methods during a critical time in motivation development (middle childhood). Exploring the trends and patterns in motivation can help us, as educators and motivation researchers, create a supportive and positive environment to keep students engaged and learning.

References

Archambault, I., Eccles, J. S., & Vida, M. N. (2010). Ability self-concepts and subjective value in literacy: Joint trajectories from grades 1-12. *Journal of Educational Psychology, 102*(4), 804–816. https://doi.org/10.1037/a0021075.

Bailey, D. H., Siegler, R. S., Geary, D. C. (2014). Early predictors of middle school fraction knowledge. *Developmental Science, 17*(5), 775-85. doi:10.1111/desc.12155

Bergman, L. R., & El-Khouri, B. M. (2003). A person-oriented approach: Methods for today and methods for tomorrow. N*ew directions for child and adolescent development, 2003*(101), 25-38. https://doi.org/10.1002/cd.80

Chen, J. A. (2012). Implicit theories, epistemic beliefs, and science motivation: A person-centered approach. *Learning and Individual Differences, 22*(6), 724-735. https://doi.org/10.1016/j.lindif.2012.07.013

Cole, J. S., Bergin, D. A., & Whittaker, T. A. (2008). Predicting student achievement for low stakes tests with effort and task value. *Contemporary Educational Psychology, 33*(4), 609–624. https://doi.org/10.1016/j.cedpsych.2007.10.002

Durik, A. M., Vida, M., & Eccles, J. S. (2006). Task values and ability beliefs as predictors of high school literacy choices: A developmental analysis. *Journal of Educational Psychology, 98*(2), 382–393. https://doi.org/10.1037/0022-0663.98.2.382

Eccles, J. S. (2012). Supporting America's children and adolescents. *Macalester International, 29*, 1–23. http://digitalcommons.macalester.edu/macintl/vol29/iss1/4

Eccles, J. S., Barber, B., & Jozefowicz, D. (1999). Linking gender to educational, occupational, and recreational choices: Applying the Eccles et al. model of achievement-related choices. In W. B. Swann, Jr., J. H. Langlois, & L. A. Gilbert (Eds.), *Sexism and*

stereotypes in modern society: The gender science of Janet Taylor Spence (p. 153–192). American Psychological Association. https://doi.org/10.1037/10277-007

Eccles, J. S., & Wigfield, A. (2020). From expectancy-value theory to situated expectancy-value theory: A developmental, social cognitive, and sociocultural perspective on motivation. *Contemporary Educational Psychology, 61*, 101859. https://doi.org/10.1016/j.cedpsych.2020.101859

Ganotice, F. A., Datu, J. A. D., & King, R. B. (2016). Which emotional profiles exhibit the best learning outcomes? A person-centered analysis of students' academic emotions. *School Psychology International, 37*(5), 498–518. https://doi.org/10.1177/0143034316660147

Gaspard, H., Wille, E., Wormington, S. V., & Hulleman, C. S. (2019). How are upper secondary school students' expectancy-value profiles associated with achievement and university STEM major? A cross-domain comparison. *Contemporary Educational Psychology, 58*, 149-162. https://doi.org/10.1016/j.cedpsych.2019.02.005

Geary, D. C., Hoard, M. K., Nugent, L., Bailey, D. H. (2013). Adolescents' functional numeracy is predicted by their school entry number system knowledge. *PLoS ONE, 8*. doi:10.1371/journal.pone.0054651

Guo, J., Marsh, H. W., Parker, P. D., Morin, A. J., & Yeung, A. S. (2015a). Expectancy-value in mathematics, gender and socioeconomic background as predictors of achievement and aspirations: A multi-cohort study. *Learning and individual differences, 37*, 161-168. https://doi.org/10.1016/j.lindif.2015.01.008

Guo, J., Parker, P. D., Marsh, H. W., & Morin, A. J. S. (2015b). Achievement, motivation, and educational choices: A longitudinal study of expectancy and value using a multiplicative

perspective. *Developmental Psychology, 51*(8), 1163–1176. https://doi.org/10.1037/a0039440

Harackiewicz, J. M., & Priniski, S. J. (2018). Improving student outcomes in higher education: The science of targeted intervention. *Annual Review of Psychology, 69*, 409-435. https://doi.org/10.1146/annurev-psych-122216-011725

Harackiewicz, J. M., Canning, E. A., Tibbetts, Y., Giffen, C. J., Blair, S. S., Rouse, D. I., & Hyde, J. S. (2014). Closing the social class achievement gap for first-generation students in undergraduate biology. *Journal of Educational Psychology, 106*(2), 375-389. https://doi.org/10.1037/a0034679

Hattie, J. (2013). Calibration and confidence: where to next? *Learning and Instruction, 24*, 62–66. https://doi.org/10.1016/j.learninstruc.2012.05.009

Hayenga, A. O., & Corpus, J. H. (2010). Profiles of intrinsic and extrinsic motivations: A person-centered approach to motivation and achievement in middle school. *Motivation and Emotion, 34*(4), 371-383. https://doi.org/10.1007/s11031-010-9181-x

Howard, M. C., & Hoffman, M. E. (2018). Variable-centered, person-centered, and person specific approaches: where theory meets the method. *Organizational Research Methods, 21*(4), 846-876. https://doi.org/10.1177/1094428117744021

Hulleman, C. S., Durik, A. M., Schweigert, S. A., & Harackiewicz, J. M. (2008). Task values, achievement goals, and interest: An integrative analysis. *Journal of Educational Psychology, 100*(2), 398–416. https://doi.org/10.1037/0022-0663.100.2.398

Jarrell, A., Harley, J. M., & LaJoie, S. P. (2016). The link between achievement emotions, appraisals, and task performance: Pedagogical considerations for emotions in CBLEs.

Journal of Computers in Education, 3, 289–307. http://dx.doi.org/10.1007/s40692-016-0064-3

Jarrell, A., Harley, J. M., LaJoie, S. P., & Naismith, L. (2017). Success, failure and emotions: Examining the relationship between performance feedback and emotions in diagnostic reasoning. *Educational Technology Research and Development, 65*, 1263–1284. http://dx.doi.org/10.1007/s11423-017-9521-6

Katz, I., & Assor, A. (2006). When choice motivates and when it does not. *Educational Psychology Review, 19*, 429-442. http://doi.org/10.1007/s10648-006-9027-y.

Kirby, L. D., Morrow, J., & Yih, J. (2014). The challenge of challenge: Pursuing determination as an emotion. In M. M. Tugade, M. N. Shiota, & L. D. Kirby (Eds.), *Handbook of Positive Emotions* (p. 378–395). The Guilford Press.

Labuhn, A. S., Zimmerman, B. J., & Hasselhorn, M. (2010). Enhancing students' self-regulation and mathematics performance: The influence of feedback and self-evaluative standards. *Metacognition and Learning, 5*, 173–194. https://doi.org/10.1007/s11409-010-9056-2

Linnenbrink-Garcia, L., Wormington, S. V., Snyder, K. E., Riggsbee, J., Perez, T., Ben-Eliyahu, A., & Hill, N. E. (2018). Multiple pathways to success: An examination of integrative motivational profiles among upper elementary and college students. *Journal of Educational Psychology, 110*(7), 1026–1048. https://doi.org/10.1037/edu0000245

Marsh, H. W., Martin, & Yeung, A. S., & Craven, R. G. (2017). Competence self-perceptions: A cornerstone of achievement motivation and the positive psychology movement. In A. Elliot (Ed.), *Handbook of competence and motivation* (Vol. 2). New York, NY: Guilford.

Maulana, R., Opdenakker, M.-C., & Bosker, R. (2016). Teachers' instructional behaviors as important predictors of academic motivation: Changes and links across the school year.

Learning and Individual Differences, 50, 147-156. https://doi.org/10.1016/j.lindif.2016.07.019

Muenks, K., Wigfield, A., & Eccles, J. S. (2018). I can do this! The development and calibration of children's expectations for success and competence beliefs. *Developmental Review, 48*, 24-39. https://doi.org/10.1016/j.dr.2018.04.001

Musu-Gillette, L. E., Wigfield, A., Harring, J. R., & Eccles, J. S. (2015). Trajectories of change in students' self-concepts of ability and values in math and college major choice. *Educational Research and Evaluation, 21*(4), 343-370. https://doi.org/10.1080/13803611.2015.1057161

Nagy, G., Trautwein, U., Baumert, J., Köller, O., & Garrett, J. (2006). Gender and course selection in upper secondary education: Effects of academic self-concept and intrinsic value. *Educational Research and Evaluation, 12*, 323–345. http://doi.org/10.1080/13803610600765687

Pekrun, R. (2006). The Control-Value Theory of Achievement Emotions: Assumptions, corollaries, and implications for educational research and practice. *Educational Psychology Review, 18*(4), 315–341. https://doi.org/10.1007/s10648-006-9029-9

Pitzer, J. & Skinner, E. (2017). Predictors of changes in students; motivational resilience over the school year: The role of teacher support, self-appraisals, and emotional reactivity. *International Journal of Behavioral Development, 41*(1), 15-29. https://doi.org/10.1177/0165025416642051

Raccanello, D., Hall, R., & Burro, R. (2018). Salience of primary and secondary school students' achievement emotions and perceived antecedents: Interviews on literacy and mathematics

domains. *Learning and Individual Differences, 65*, 65–79. http://doi.org/10.1016/j.lindif.2018.05.015

Reeve, J., Bolt, E., & Cai, Y. (1999). Autonomy supportive teachers: how they teach and motivate students. *Journal of Educational Psychology, 91*, 537-548. http://doi.org/10.1037/0022-0663.91.3.537.

Ritchie, S. J., Bates, T. C. (2013). Enduring Links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological Science, 24*(7), 1301–1308. doi:10.1177/0956797612466268

Robinson, K. A., Beymer, P. N., Ranellucci, J., & Schmidt, J. A. (2020, February 27). Momentary emotion profiles in high school science and their relations to control, value, achievement, and science career intentions. *Motivation Science.* https://doi.org/10.1037/mot0000174

Robinson, K. A., Ranellucci, J., Lee, Y., Wormington, S. V., Roseth, C. J., & Linnenbrink-Garcia, L. (2017). Affective profiles and academic success in a college science course. *Contemporary Educational Psychology, 51*, 209–221. http://doi.org/10.1016/j.cedpsych.2017.08.004

Rutherford, T., Liu, A., Logan, W., Karamarkovich, S. M. , & Wagemaker, M. (2019a, August 8-11). "I chose math because…" Cognitive interviews of a mathematics motivation measure [Poster presentation]. American Psychological Association 2019 Convention, Chicago, IL.

Schunk, D. H., & Pajares, F. (2009). Self-efficacy theory. In K. R. Wentzel, D. B. Miele (Eds.) *Handbook of motivation at school* (pp. 35-54). Routledge.

Simpkins, S. D., Davis-Kean, P. E., & Eccles, J. S. (2006). Math and science motivation: A longitudinal examination of the links between choices and beliefs. *Developmental Psychology, 42*(1), 70–83. https://doi.org/10.1037/0012-1649.42.1.70.

Smith, C. A., & Ellsworth, P. C. (1985). Patterns of cognitive appraisal in emotion. Journal of *Personality and Social Psychology, 48*(4), 813-838. https://doi.org/10.1037/0022-3514.48.4.813

van Tilburg, W.A.P., Igou, E.R. On boredom: Lack of challenge and meaning as distinct boredom experiences. *Motivation and Emotion, 36*, 181–194. https://doi.org/10.1007/s11031-011-9234-9

Wang, M.-T., & Eccles, J. S. (2013). School context, achievement motivation, and academic engagement: A longitudinal study of school engagement using a multidimensional perspective. *Learning and Instruction, 28*, 12–23. https://doi.org/10.1016/j.learninstruc.2013.04.002

Wigfield, A., & Cambria, J. (2010). Students' achievement values, goal orientations, and interest: Definitions, development, and relations to achievement outcomes. *Developmental Review, 30*(1), 1–35. https://doi.org/10.1016/j.dr.2009.12.001

Wigfield, A., & Eccles, J. S. (2020). 35 Years of research on students' subjective task values and motivation: A look back and a look Forward. In A. Elliot (Vol. Ed.), *Advances in motivation science: Vol. 7,* (pp. 162–193). Elsevier.

Wigfield, A., Eccles, J. S., Davis-Kean, P., Roeser, R., & Scheifele, U. (2006). Motivation to succeed. In N. Eisenberg (Series Ed.) & W. Damon (Vol. Ed.) (6th ed.), *Handbook of child psychology: Vol. 3. Social, emotional, and personality development* (pp. 933-1002). Wiley.

Wigfield, A, Eccles, J. S., Fredricks, J., Simpkins, Roeser, R., & Schiefele, U. (2015). Development of achievement motivation and engagement. In R. Lerner (Series ed.) and M. Lamb & C. Garcia Coll (Vol. Eds.), *Handbook of child psychology and developmental science* (7th ed., pp. 657–700). New York, NY: Wiley.

APPENDICES

Appendix 3.A

Studies using Person-Centered Approaches to Examine Academic Emotions

Citation	Sample	Emotion Measure	Analysis	Profiles
Ganotice et al. (2016) *Study 1*	Adolescents (N=1,147)	Short version of the AEQ (King, 2010): enjoyment, hope, pride, anger, anxiety, shame, hopelessness, boredom	Ward's hierarchical & k-means clustering	● Adaptive shame ● Moderate ● Maladaptive ● Adaptive
Ganotice et al. (2016) *Study 2*	Adolescents (N=341)	AEQ—Math (Pekrun et al., 2005): enjoyment, hope, pride, anger, anxiety, shame, hopelessness, boredom		
Hanin & Van Niewenhoven (2019)	Elementary students (N=354)	Single item Likert-like scale rating (1-5) of emotions with pictorial facial expressions: enjoyment, hope, pride, nervousness, shame, hopelessness, boredom, sadness, fear, and worry	Ward's hierarchical & k-means clustering	● Positive ● Bored ● Anxious ● Resigned
Jarrell et al. (2017)	Medical students (N=30)	AEQ (Pekrun et al., 2002). Retrospective outcome emotions only: joy, pride, relief, anger, and shame	k-means clustering	● Low affect ● Negative affect ● Positive affect
Jarrell et al. (2016)	Medical students (N=26)	AEQ (Pekrun et al., 2002). Activity focus only: enjoyment, hope, pride, anger, anxiety, shame, and hopelessness	k-means clustering	● Positive emotion ● Negative emotion ● Low emotion
Raccanello et al. (2018)	Fourth, seventh, and 11th graders (N=149)	Interviews coded for achievement emotions: enjoyment, hope, pride, relief, relaxation, anger, anxiety, shame, boredom, and sadness	model-based cluster analysis	● Happy ● Relaxed ● Depressed
Robinson et al. (2017)	Undergraduates (N=278)	Adapted from Ben-Eliyahu & Linnenbrink-Garcia (2013): positive activated, positive deactivated, negative activated, and negative deactivated	Ward's hierarchical & k-means clustering	● Positive ● Deactivated ● Negative ● Moderate-Low
Robinson et al. (2020)	High schoolers (N = 244)	Likert-like scale (0-3) rating of four emotions: happy, excited, frustrated, and bored	Latent profile analysis	● Moderate-low deactivated ● Negative ● Positive ● Moderate-high all

Note. AEQ = Academic Emotions Questionnaire

Appendix 3.B

Appendix 3.B.1

Emotions Reported

	Favorite	**Mathematics**	**Least Favorite**	**Avg. Times Reported**
Excited	90.65%	41.45%	8.32%	1.40 (0.64)
Happy	82.80%	44.59%	14.42%	1.42 (0.73)
Hopeful	45.79%	39.39%	22.82%	1.08 (0.82)
Bored	3.50%	23.00%	76.13%	1.03 (0.64)
Frustrated	5.91%	42.79%	68.98%	1.18 (0.73)
Nervous	4.67%	26.97%	60.34%	0.92 (0.71)
Challenged	56.83%	69.89%	28.32%	1.55 (0.83)

Note. N = 10,527. Each emotion can be reported zero to three times. Standard deviation in parentheses

Appendix 3.B.2

Emotions Reported When Students Put Math as One of Their Top Two Favorite Subjects

	Favorite (1st)	Math (2nd)	Repetition	Favorite (2nd)	Math (1st)	Repetition	Favorite (Combined)	Math (Combined)	Repetition
Excited	91.58%	61.87%	57.74%	78.93%	85.26%	68.43%	86.47%	71.32%	62.06%
Happy	81.53%	64.96%	55.73%	78.13%	71.15%	57.93%	80.16%	67.47%	56.62%
Hopeful	40.58%	46.88%	20.97%	55.05%	36.38%	22.52%	46.42%	42.64%	21.59%
Bored	2.50%	4.83%	0.49%	5.93%	3.29%	0.72%	3.88%	4.21%	0.58%
Frustrated	6.08%	23.90%	2.88%	10.02%	10.98%	3.13%	7.67%	18.68%	2.98%
Nervous	4.45%	11.79%	1.47%	8.65%	5.37%	0.96%	6.15%	9.19%	1.26%
Challenged	61.87%	73.22%	48.72%	52.16%	79.25%	44.63%	57.95%	75.66%	47.07%
N		1,841			1,248			3,089	

Note. Repetition defined as reporting the same emotion for both their favorite emotion and math.

Appendix 3.B.3

Emotions Reported When Students Put Math as One of Their Bottom Two Favorite Subjects

	Least Favorite (9th)	Math (8th)	Repetition	Least Favorite (8th)	Math (9th)	Repetition	Least Favorite (Combined)	Math (Combined)	Repetition
Excited	6.92%	9.34%	1.21%	18.14%	2.53%	1.32%	13.99%	5.05%	1.28%
Happy	12.63%	12.28%	3.29%	24.32%	4.86%	2.43%	20.00%	7.60%	2.75%
Hopeful	22.32%	25.09%	8.30%	33.64%	12.97%	5.57%	29.46%	17.44%	6.58%
Bored	76.64%	50.35%	42.04%	65.45%	70.01%	49.75%	69.58%	62.75%	46.90%
Frustrated	69.38%	75.61%	54.67%	52.68%	87.84%	49.34%	58.85%	83.32%	51.31%
Nervous	58.13%	51.04%	31.83%	48.73%	69.71%	37.49%	52.20%	62.81%	35.40%
Challenged	32.70%	63.49%	24.22%	41.74%	40.32%	21.18%	38.40%	48.88%	22.30%
N		578			987			1,565	

Note. Repetition defined as reporting the same emotion for both their least favorite emotion and math.

Appendix 3.C

Sample Mplus Code

```
TITLE:
  Fourth Grade Emotion LCA; Four Classes

DATA:
  File = Fourth Grade_7.2.2020.txt;

VARIABLE:
  Name = ID PriorMath UseNow UseFut MathAch Expect
       Bored Chall Excited Frust Happy Hopeful Nervous
       teachID;
  Usevariables = Bored Chall Excited Frust Happy Hopeful Nervous;
  Categorical = Bored Chall Excited Frust Happy Hopeful Nervous;
  Idvriable = ID;
  Missing = all(999);
  Classes = c(4);

ANALYSIS:
  Estimator = MLR;
  Type = mixture;
  Starts = 100 50;
  Processors = 4;
  LRTstarts = 10 2 100 50;
  LRTbootstrap = 50;

MODEL:

OUTPUT:
  Tech1 Tech7 Tech10 Tech11 Tech14
  residual patterns svalues;

SAVEDATA:
 File = G4_4classes.txt;
 Save = cprob;
 Format = free;
```

Appendix 3.D

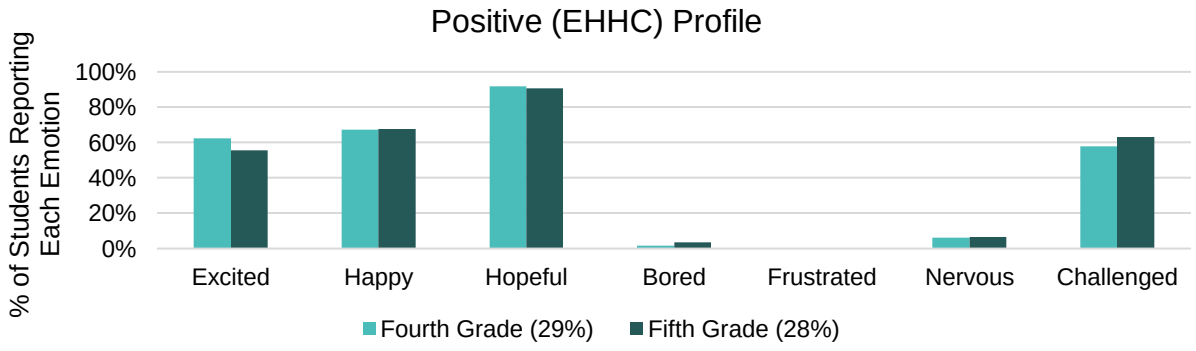
Correlations Between Mathematics Expectancy, Value, Emotion, and Achievement

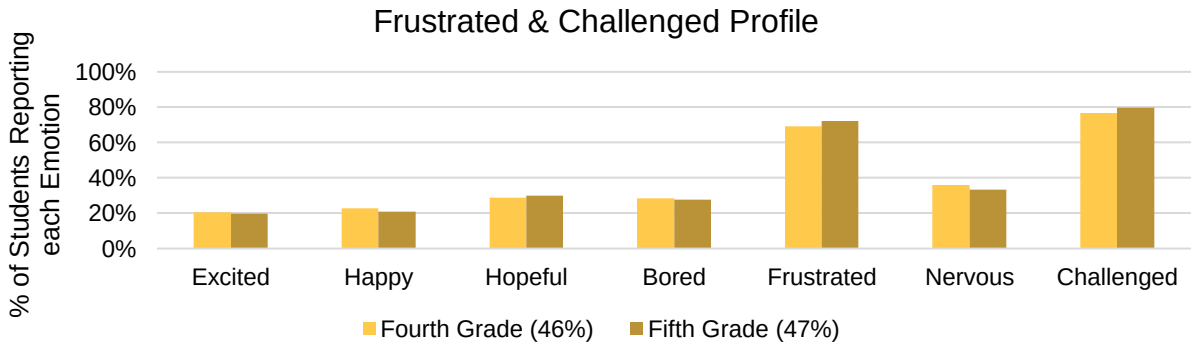
		t_0			t_1			t_2			t_3	
		1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
t_0	1. Prior Math Achievement		0.143[c]	0.037[b]	0.124[c]	0.128[c]	0.080[c]	-0.116[c]	-0.160[c]	-0.197[c]	0.182[c]	0.807[c]
t_1	2. Expectancy	0.227[c]		0.580[c]	0.241[c]	0.222[c]	0.107[c]	-0.248[c]	-0.252[c]	-0.210[c]	0.084[c]	0.128[c]
	3.Value	0.052[c]	0.567[c]		0.206[c]	0.177[c]	0.117[c]	-0.270[c]	-0.186[c]	-0.163[c]	0.101[c]	0.025
t_2	4. Excited	0.156[c]	0.287[c]	0.247[c]		0.281[c]	0.004	-0.402[c]	-0.477[c]	-0.433[c]	0.031[a]	0.123[c]
	5. Happy	0.185[c]	0.272[c]	0.195[c]	0.294[c]		0.063[c]	-0.411[c]	-0.503[c]	-0.391[c]	-0.035[a]	0.137[c]
	6. Hopeful	0.094[c]	0.124[c]	0.100[c]	-0.047[c]	0.026		-0.297[c]	-0.359[c]	-0.243[c]	-0.103[c]	0.096[c]
	7. Bored	-0.125[c]	-0.285[c]	-0.291[c]	-0.394[c]	-0.398[c]	-0.280[c]		0.361[c]	0.242[c]	-0.293[c]	-0.128[c]
	8. Frustrated	-0.237[c]	-0.298[c]	-0.212[c]	-0.486[c]	-0.522[c]	-0.320[c]	0.341[c]		0.294[c]	-0.135[c]	-0.170[c]
	9. Nervous	-0.212[c]	-0.285[c]	-0.194[c]	-0.383[c]	-0.410[c]	-0.237[c]	0.201[c]	0.317[c]		-0.247[c]	-0.189[c]
	10. Challenged	0.154[c]	0.130[c]	0.142[c]	0.038[b]	0.013	-0.072[c]	-0.335[c]	-0.159[c]	-0.256[c]		0.183[c]
t_3	11. Math Achievement	0.838[c]	0.217[c]	0.061[c]	0.177[c]	0.204[c]	0.108[c]	-0.156[c]	-0.254[c]	-0.230[c]	0.162[c]	

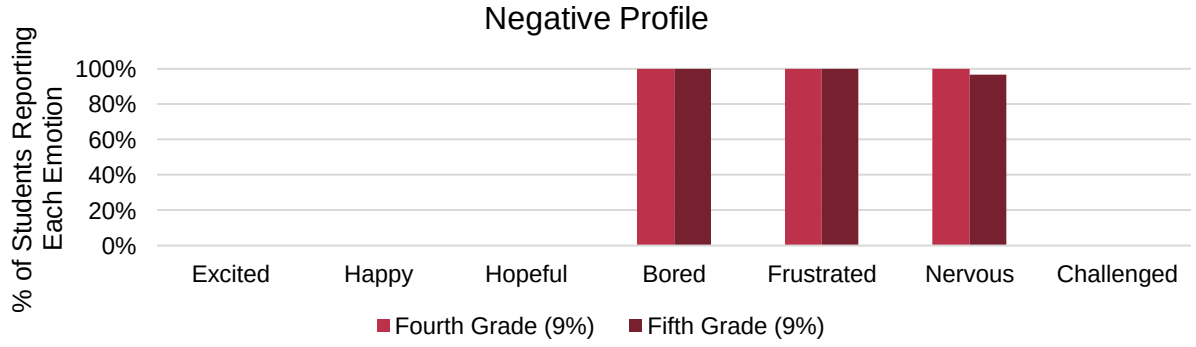
Note. Correlations for fourth graders above the diagonal and fifth graders below the diagonal.

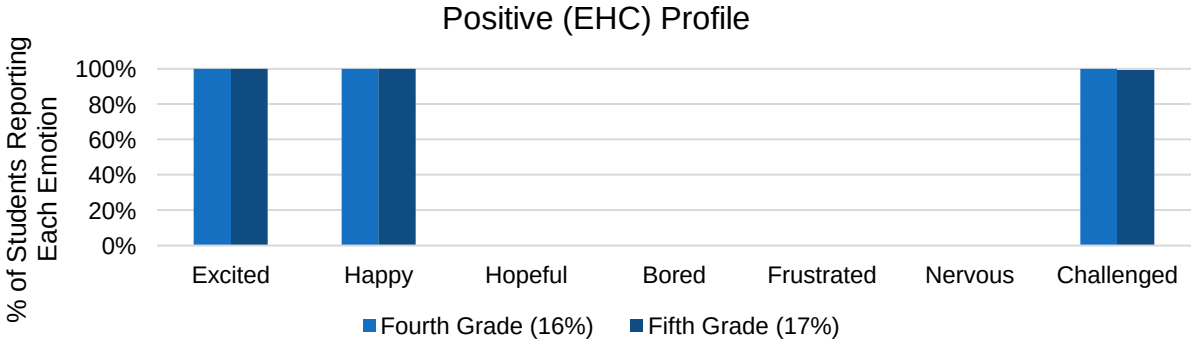
Appendix 3.E

Comparison of Profiles Across Grades









Appendix 3.F

Unstandardized Coefficients with Standard Errors and Confidence Intervals from Path Analysis (Demographics)

	Appraisal		Emotion			Math Achievement
	Expectancy	Value	Frustrated & Challenged	Negative	Positive (EHC)	
	b (se) [95% CI]		log odds *(se)* [95% CI]			*b (se)* [95% CI]
Demographics						
Fifth Grader	**-0.183**[c] *(0.016)* [-0.214, -0.151]	**-0.066**[c] *(0.016)* [-0.097, -0.034]	**0.252**[c] *(0.002)* [0.126, 0.377]	**0.348**[c] *(0.092)* [0.167, 0.529]	-0.007 *(0.066)* [-0.137, 0.123]	**-3.391**[c] *(0.265)* [-3.910, -2.872]
Disability	0.046 *(0.023)* [-0.0002, 0.0914]	-0.004 *(0.023)* [-0.050, 0.042]	**-0.290**[b] *(0.095)* [-0.476. -0.103]	**-0.418**[b] *(0.135)* [-0.683, -0.153]	0.128 *(0.096)* [-0.060, 0.316]	**-1.337**[c] *(0.386)* [-2.094, -0.580]
FRL	0.025 *(0.019)* [-0.012, 0.062]	**0.039**[a] *(0.019)* [0.002, 0.076]	-0.039 *(0.073)* [-0.182, 0.105]	-0.193 *(0.108)* [-0.406, 0.019]	0.003 *(0.076)* [-0.147, 0.152]	**-3.377**[c] *(0.311)* [-3.986, -2.768]
ELL	0.042 *(0.027)* [-0.010, 0.095]	**0.066**[a] *(0.027)* [0.014, 0.119]	**-0.595**[c] *(0.103)* [-0.797, -0.394]	**-0.994**[c] *(0.166)* [-1.320, -0.669]	-0.032 *(0.103)* [-0.234, 0.171]	-0.103 *(0.441)* [-0.967, 0.761]
Boy	**0.061**[c] *(0.015)* [0.031, 0.091]	**-0.051**[b] *(0.015)* [-0.081 -0.021]	**-0.452**[c] *(0.060)* [-0.570, -0.335]	**-0.433**[c] *(0.089)* [-0.608, -0.258]	-0.017 *(0.063)* [-0.140, 0.106]	**-0.391**[a] *(0.254)* [-0.889, 0.107]
Race						
Black	**0.268**[c] *(0.022)* [0.0224,0.311]	**0.221**[a] *(0.022)* [0.177, 0.265]	**-0.452**[c] *(0.087)* [-0.623, -0.281]	**-0.727**[c] *(0.130)* [-0.983, -0.472]	**-0.246**[b] *(0.091)* [-0.423, -0.067]	**-4.656**[c] *(0.370)* [-5.381, -3.931]
Hispanic	**0.062**[b] *(0.024)* [0.016, 0.108]	**0.059**[a] *(0.024)* [0.013, 0.105]	0.038 *(0.093)* [-0.145, 0.221]	-0.043 *(0.137)* [-0.313, 0.226]	-0.023 *(0.097)* [-0.213, 0.167]	-0.548 *(0.388)* [-1.308, 0.212]
Other	0.019 *(0.027)* [-0.034, 0.072]	0.025 *(0.027)* [-0.028, 0.078]	-0.173 *(0.102)* [-0.373, 0.027]	**-0.420**[a] *(0.164)* [-0.741, -0.098]	-0.117 *(0.105)* [-0.323, 0.089]	**0.906**[a] *(0.444)* [0.035, 1.777]

Appendix 3.F (cont'd)

Unstandardized Coefficients with Standard Errors and Confidence Intervals from Path Analysis (Environment, Appraisal, and Emotion)

	Appraisal		Emotion			Math Achievement
	Expectancy	Value	Frustrated & Challenged	Negative	Positive (EHC)	
	b (se) [95% CI]		log odds *(se)* [95% CI]			*b (se)* [95% CI]
Environment						
Prior Math Achievement	**0.009**[c] *(0.0004)* [0.008, 0.010]	**0.003**[c] *(0.0004)* [0.002, 0.004]	**-0.026**[c] *(0.002)* [-0.030, -0.023]	**-0.044**[c] *(0.003)* [-0.049, -0.039]	**-0.009**[c] *(0.002)* [-0.013, -0.006]	**0.840**[c] *(0.007)* [0.826, 0.854]
Appraisal						
Expectancy			**-0.706**[c] *(0.056)* [-0.815, -0.596]	**-1.005**[c] *(0.069)* [-1.141, -0.870]	**-0.123**[a] *(0.061)* [-0.242, -0.004]	**0.490**[a] *(0.200)* [0.099, 0.881]
Value			**-0.379**[c] *(0.055)* [-0.487, -0.272]	**-0.795**[c] *(0.066)* [-0.925, -0.664]	-0.091 *(0.059)* [-0.207, 0.025]	**-0.430**[a] *(0.197)* [-0.817, -0.044]
Emotion						
Frustrated & Challenged						**-2.747**[c] *(0.375)* [-3.481, -2.012]
Negative						**-6.092**[c] *(0.558)* [-7.185, -4.999]
Positive (EHC)						-0.519 *(0.391)* [-1.283, 0.247]
Intercept	1.421[c]	3.283[c]	14.359[c]	21.039[c]	4.536[c]	69.060[c]
Error Variance	0.600	0.61				163.11

Note. Standard errors in parentheses and italicized; 95% confidence intervals in brackets.
Positive (Excited, Happy, Hopeful, Challenged) was used as the reference profile. The demographic reference group are girls who are White, in fourth grade, who are not classified as having a disability, eligible for free/reduced lunch, or as English Language Learners.
FRL=Free/Reduced Lunch; ELL=English Language Learner; EHC=Excited Happy Challenged
[a] $p < .05$, [b] $p < .01$, [c] $p < .001$; Statistically significant coefficients also bolded

Appendix 3.G

Post-Hoc Tests of Differences between Profiles' Motivation and Achievement

	Profile	Positive (EHHC)	Frustrated & Challenged	Negative
			Wald's χ^2 Post-hoc test	
Profile on Expectancy	Frustrated & Challenged	$z = -12.62^c$		
	Negative	$z = -14.54^c$	$\chi^2 = 38.93^c$ ($z = -6.24$)	
	Positive (EHC)	$z = -2.03^c$	$\chi^2 = 187.93^c$ ($z = 13.71$)	$\chi^2 = 227.54^c$ ($z = 15.08$)
Profile on Value	Frustrated & Challenged	$z = -6.91^c$		
	Negative	$z = -11.95^c$	$\chi^2 = 82.95^c$ ($z = -9.11$)	
	Positive (EHC)	$z = -1.54$	$\chi^2 = 46.72^c$ ($z = 6.84$)	$\chi^2 = 157.22^c$ ($z = 12.54$)
Profile on Prior Math Achievement	Frustrated & Challenged	$z = -15.33^c$		
	Negative	$z = -17.40^c$	$\chi^2 = 70.72^c$ ($z = -8.41$)	
	Positive (EHC)	$z = -5.33^c$	$\chi^2 = 147.35^c$ ($z = 12.14$)	$\chi^2 = 224.82^c$ ($z = 14.99$)
Math Achievement on Profile	Frustrated & Challenged	$z = -7.33^c$		
	Negative	$z = -10.92^c$	$\chi^2 = 51.43^c$ ($z = -7.17$)	
	Positive (EHC)	$z = -1.33$	$\chi^2 = 52.06^c$ ($z = 7.22$)	$\chi^2 = 118.56^c$ ($z = 10.89$)

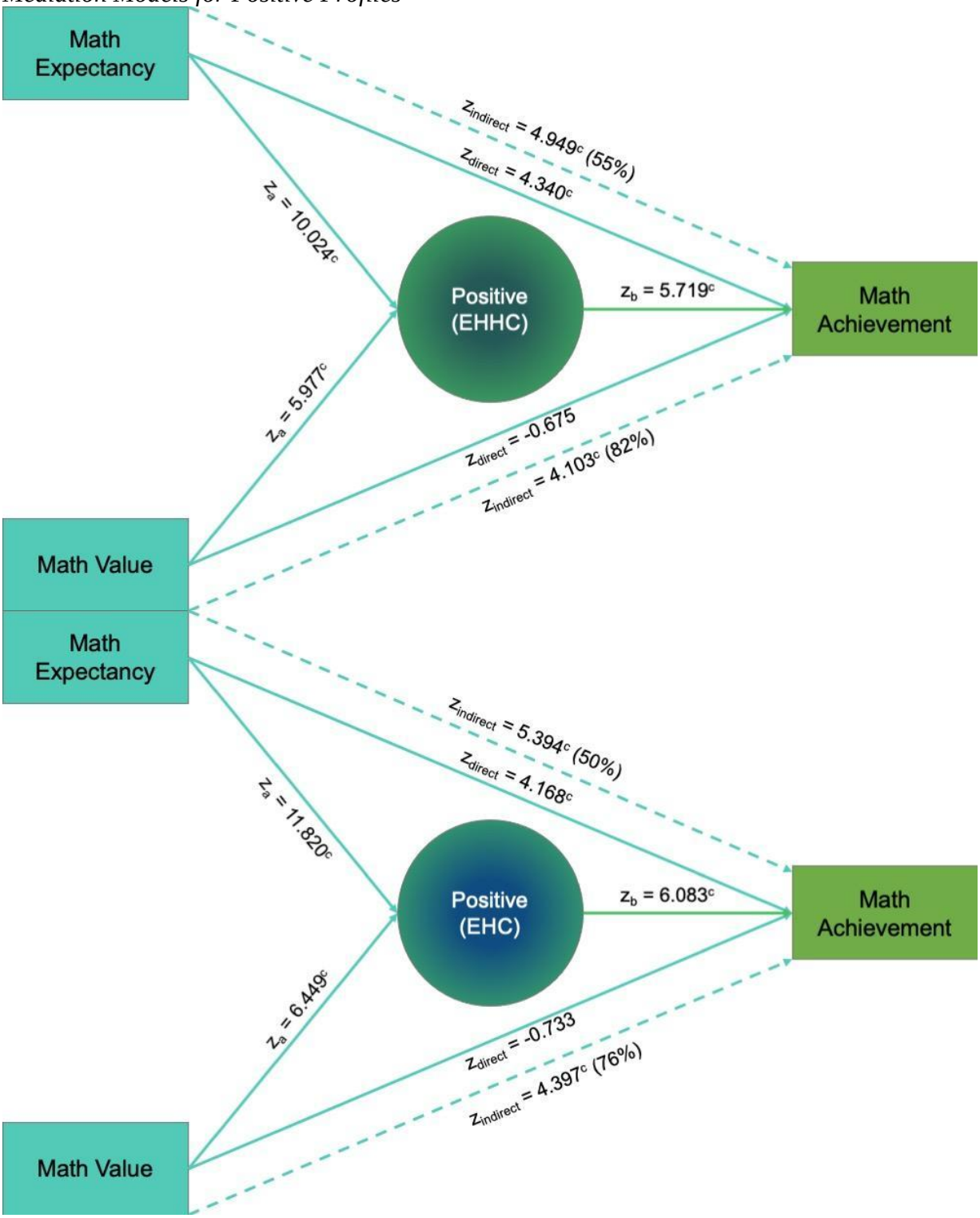
Note. z and χ^2 values compare the profile in the row to the profile in the column. For example, students in the Frustrated & Challenged profile had lower math achievement than students in the Positive (EHHC) profile, as indicated by $z = -7.33$. To compare students' math achievement between students in the Negative profile and the Frustrated & Challenged profile, move down one and right one; students in the Negative profile had lower math achievement than students in the Frustrated & Challenged profile ($\chi^2 = 51.43$, $z = -7.17$).

[a] $p < .05$, [b] $p < .01$, [c] $p < .001$

Appendix 3.H

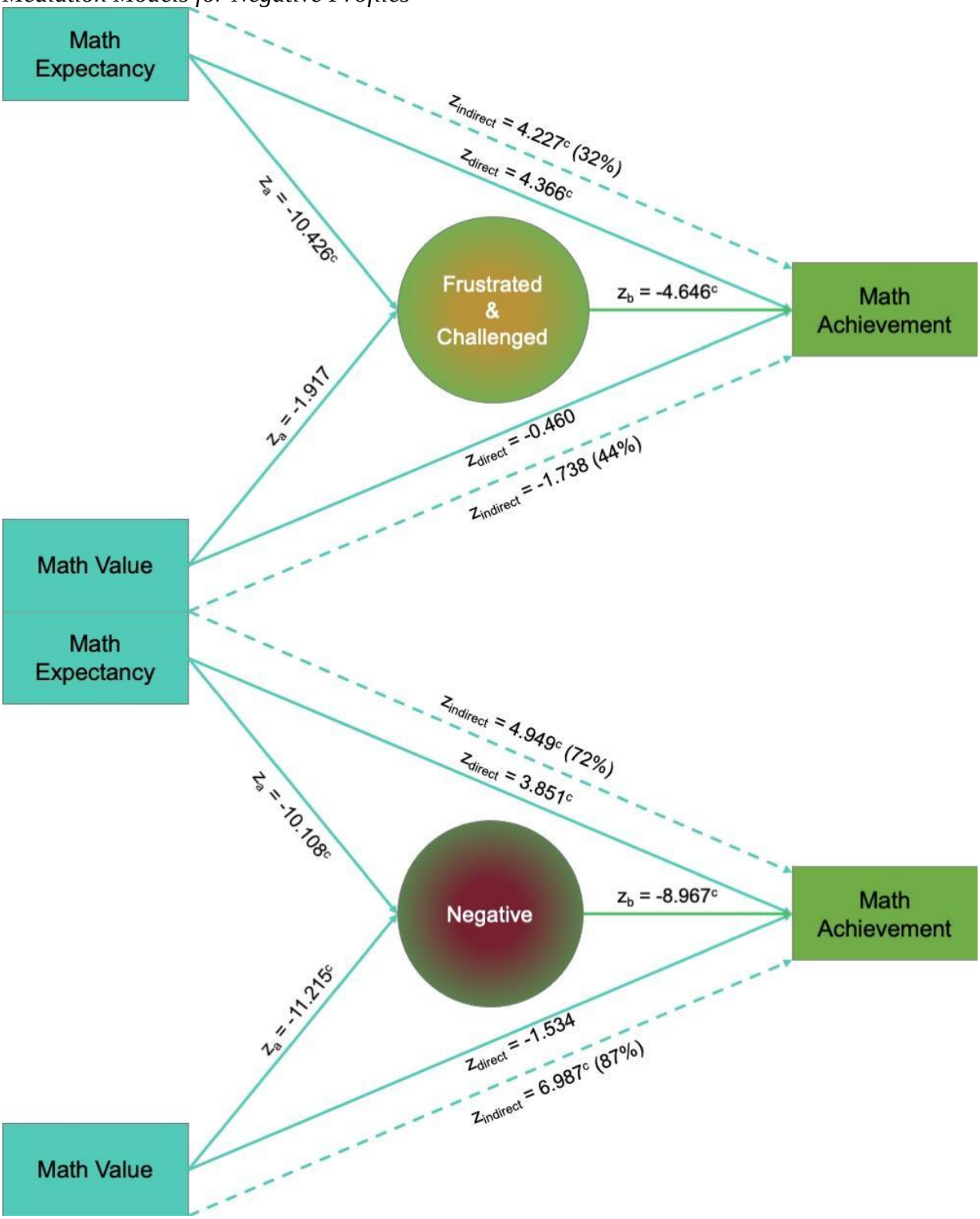
Appendix 3.H.1

Mediation Models for Positive Profiles



Appendix 3.H.2

Mediation Models for Negative Profiles



Note.
*Statistically significant effect

Appendix 3.I

Iacobucci Test of Indirect Effects of Motivation on Math Achievement Through Discrete Emotions

		Direct Effects		Indirect Effects			
Emotion	Appraisal	Appraisal → Achievement	% of Total Effect	Appraisal → Emotion	Emotion → Achievement	Appraisal → Emotion → Achievement	% of Total Effect
Excited	Expectancy	3.82[c]	42.52%	13.95[c]	7.59[c]	6.65[c]	57.48%
	Value	-1.18	21.56%	11.32[c]	7.59[c]	6.29[c]	78.44%
Happy	Expectancy	3.70[c]	40.72%	14.43[c]	8.12[c]	7.06[c]	59.28%
	Value	-0.94	25.29%	7.49	8.12[c]	5.48[c]	74.71%
Hopeful	Expectancy	4.62[c]	80.74%	6.20[c]	4.97[c]	3.85[c]	19.26%
	Value	-0.67	33.20%	5.13[c]	4.97[c]	3.54[c]	66.80%
Bored	Expectancy	3.91[c]	42.78%	-10.77[c]	-8.59[c]	6.70[c]	57.22%
	Value	-1.79	19.52%	-15.58[c]	-8.59[c]	7.51[c]	80.48%
Frustrated	Expectancy	3.32[c]	32.46%	-15.80[c]	-9.55[c]	8.16[c]	67.54%
	Value	-0.21	24.59%	-7.40[c]	-9.55[c]	5.83[c]	75.41%
Nervous	Expectancy	3.68[c]	39.03%	-13.06[c]	-8.54[c]	7.13[c]	60.97%
	Value	-0.92	24.65%	-6.35[c]	-8.54[c]	5.07[c]	75.35%
Challenged	Expectancy	4.75[c]	88.91%	2.11[a]	5.95[c]	1.96[a]	11.09%
	Value	-0.86	27.70%	8.08[c]	5.95[c]	4.77[c]	72.30%

Note.
[a] $p < .05$, [b] $p < .01$, [c] $p < .001$