ABSTRACT

CUDD, MICHELE DENISE. An Exploratory Qualitative Study of Prospective Teachers' Support of Student Thinking During Rehearsals. (Under the direction of Dr. Karen Hollebrands).

This exploratory qualitative study examined prospective teachers' support of student thinking during rehearsals in a methods course for future middle school and secondary mathematics teachers. Advocates of ambitious teaching believe that prospective teachers (PTs) need to experience authentic practice that approximates the complexities of teaching so that they enter classrooms better prepared. Research also proposes that teachers' utilization of student thinking in classrooms is valuable. This study examined PTs' support of student thinking during rehearsals to learn more about what how PTs respond to student thinking, and why.

The study's findings suggest that PTs are capable of supporting student thinking in a variety of ways during their enacted rehearsals. The ways the PTs supported student thinking were analyzed to consider opportunities for framing mathematics and mathematical ability inclusively. As a whole, the PTs showed less evidence of inclusively framing mathematical ability than activity. Additionally, the moves the PTs utilized were further categorized through the lens of inclusive framing to consider how future PTs might more intentionally use talk moves to frame mathematics and mathematical ability more inclusively. Further examination of the PTs' intentions provided insights into why they were supporting student thinking the way that they were. The PTs' practices were concerned with facilitation, encouraging students, centering student thinking, guiding student thinking, and practices related to the standards of mathematical practice. Some of their dispositions had an additional focus on culture building. The talk moves and intentions in connection with how the PTs supported student thinking during rehearsals can be used to inform teacher preparation programs.

An Exploratory Qualitative Study of Prospective Teachers' Support of Student Thinking During Rehearsals

by
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BIOGRAPHY

Michele Denise Cudd was born in Jonesboro, Arkansas, but was moved to Hammond, Louisiana a year later. She was raised on the food and music culture of New Orleans and more importantly, the joie de vivre. She completed a B.S. in mathematics and a B.A. in French while attending Southeastern Louisiana University. While completing her undergraduate degrees, she was active in both theater and a local modern dance company. It was during a one-year break from her degree, working as an assistante d'anglais in France, that she started to fall in love with teaching.

After deciding to get her teaching certification, she accumulated additional teaching experience working her favorite job ever the spring of 2007 as a naturalist educator on a bay in Alabama. Teaching kids about nature in an outdoor environment is about as good as it gets for her. After completing her teaching certification, she taught high school mathematics for 5.5 years in North Carolina. She later was encouraged to become an instructional coach, which she did for one year before enrolling in the PhD program. Her passion for mathematics education lies in supporting students to feel successful in mathematics so that it is not a barrier.

Her passion for travel has brought her to 49 states and more than 10 countries. She plans to continue supporting students and future teachers to experience high quality math instruction in their classrooms while traveling and playing in nature in her spare time.

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Chapter 1: Introduction

The purpose of this *exploratory qualitative* study is to examine decisions prospective teachers' (PTs) make when supporting student thinking during rehearsals. Specifically, the goals of this study are to learn more about which teacher discourse moves PTs utilize during rehearsals to support student thinking and what intentions motivate their enacted teacher moves. Additionally, the author aspires to learn more about how inclusively PTs frame mathematical activity and ability while supporting student thinking.

Background of the Study

Depending on how one views learning, student thinking can be valued very differently in classroom settings. For example, if the goal of learning is behavior focused, then student thinking that does not align with the desired behavior may be viewed as undesirable (Skinner, 1950). Teachers might be inclined to immediately try to fix the behavior so that the product matches the desired goal. From a constructivist viewpoint, a student builds understanding through interacting with the world and student thinking that does not align with the desired conception can be viewed as opportunities to develop a student's knowledge scheme (Piaget & Duckworth, 1970). From a social-constructive perspective, the potential of interaction adds another lens of interpretation (Steffe & Gale, 1995). Social experiences related to shared student thinking can provide opportunities for more effective learning. For example, one student's thinking might accelerate the learning of their peer(s) as described by Vygotsky's (1980) Zone of Proximal Development. Another possibility is that the external experience of interacting with a classmate while discussing student thinking might be internalized, deepening an understanding. For the purpose of this study, the social-constructivist view of mistakes is embraced.

To be able to support student thinking, PTs must first notice an opportunity before they can make a decision on how to respond (van Es & Sherin, 2002). Noticing is a lens that supports educators to better tend to certain foci. For example, Leatham, Peterson, Stockero, & Van Zoest (2015) used noticing to create a framework whose focus is determining whether or not an instance of student mathematical thinking is worth highlighting in classroom discourse. The focus of noticing is not always just on the mathematics. Louie (2018) described *equitable* noticing as extending beyond a mathematics focus to include foci such as student participation or identities. McDuffie et al. (2014) used noticing to support prospective K-8 teachers to notice more equitable practices through video analysis. While studies have shown that more experienced teachers tend to notice more, research also supports that PTs' noticing skills can be improved (e.g., McDuffie et al., 2014; Star & Strickland, 2008). Research has done much to work towards improving educators' noticing skills through reflection, but there is limited research that analyzes PT decisions in a practice teaching environment.

The complexity of teaching makes it challenging to know how to prepare educators for the classroom. One pedagogical approach that aims to improve PTs' practice before entering the classroom is rehearsals. Rehearsals are an approximation of the practice of teaching as they are designed to resemble the challenges that a teacher might experience (Lampert et al., 2013). To disentangle some of the complexities of teaching, rehearsals allow for PTs to focus on specific practices in a controlled teaching environment. In rehearsals, not unlike the theater version of the word, a PT acts as the teacher amongst their peers, who are typically playing the role of students. During the rehearsal, the Teacher Educator (TE) might pause or interrupt with focused questions to guide the experience. Rehearsals typically consists of three phases (e.g., Kazemi, Ghousseini, Cunard, & Turrou, 2016; Lampert et al., 2013). In the first phase, the PT plans to

teach an Instructional Activity (IA) with particular goals to a group of students. Secondly, the PT acts out the role of teacher for this IA in a methods course amongst peers typically playing the role of students during which the TE interrupts the rehearsal from time to time with intentional questions to guide the PT. Lastly, the PT enacts the same IA with a group of students and records it so they can reflect and analyze it. This study focuses mostly on the second phase in which the teacher is practicing among peers.

There is currently limited research on how PTs might develop strategies to better support student thinking. McDuffie et al. (2014) studied how prospective K-8 teachers notice equitable practices through video analysis. Patterns of noticing shifted to higher levels over the course of a semester. Star and Strickland (2008) used video viewing and saw growth in secondary PTs' ability to notice different aspects of classroom practice. There has also been some work with PTs on how they support student misconceptions. For example, Son (2013) studied how PTs interpret and respond to one particular fictitious student's errors on a ratio and proportion problem. Discourse moves were not a part of the study as it focused more on the PTs' CK and PCK. Duncan (2018) studied two PTs' beliefs related to student mistakes in an elementary content course and saw that their beliefs shifted to valuing mistakes more. Research suggests that PTs are capable of noticing at higher levels, that rehearsals are a desirable platform to practice teacher moves, and that there is not enough known about how PTs respond to students' thinking.

Included in supporting student thinking is how educators respond to student misconceptions. While it is accepted that mistakes are valuable learning opportunities, there is speculation that concerns about students being embarrassed might influence teacher decisions (e.g., Santagata, 2004). There are a handful of studies that focus on how teachers respond to

student mistakes (e.g., Schleppenbach, Flevares, Sims, & Perry, 2007; Tulis, 2013). Schleppenbach et al. (2007) noted in a comparison of Chinese and American TIMSS videos, teachers from both countries reported valuing student mistakes in interviews, however while some Chinese teachers communicated that belief to students, there was no evidence that US teachers communicated that belief to students. Student mistakes are important learning opportunities and thus novice educators need more support in working towards further tapping into those opportunities.

This study will contribute to a gap in the literature on discourse practices of secondary mathematics PTs. The National Council of Teachers of Mathematics (NCTM) (2000) maintains in their process standards that students should be supported to problem-solve, reason, communicate, connect, and represent. Yet there is scarce literature on PTs' discourse practices supporting student thinking. As PTs are our future teaching workforce, it is integral that we better understand their current practices so that we can support them to be more effective inservice professionals. The focus of this study involves a deep analysis of rehearsals in which one PT at a time plays the role of teacher while facilitating discussion with a small group of students. For two of the rehearsals, the student roles were assigned barriers to understanding in order to create opportunities for the PTs to interact with unpolished student thinking. Attention will be given to both the in-the-moment choices they make but also retrospective interviews to see what they notice they could have done differently.

Research Questions

The research questions for this study focus on two lenses: enactment (what actually occurs) and intentionality (why the PTs made the choices they did).

RQ1: How do PTs support student thinking during rehearsals?

a. How inclusively do PTs frame mathematical activity and mathematical ability when supporting student thinking during rehearsals?

RQ2: What intentions do PTs have behind the decisions they make during rehearsals?

a. How inclusively do PTs frame mathematical activity and mathematical ability when reflecting on their intentions during rehearsals?

Theoretical Framework

This study aligns with social-constructivist theory, that knowledge is not a fixed entity that can be copied, but that it is built individually through experiences and interactions in the world. According to Piaget & Duckworth (1970), when a learner interacts with the world, schemes are constructed to make sense of experiences. If there is no conflict with existing schemes, then new information is assimilated and finds a way to fit into existing knowledge structures. However, if conflict arises because new information does not fit within existing knowledge structures, then accommodation occurs (Piaget, 1976; Von Glasersfeld, 1982). Piaget focused heavily on the learning of the individual, but he also saw social interaction as an opportunity for conflict to arise, for a child to encounter a different view than his own, and for the ensuing discussion to allow the child to "reach logical resolution of their internal cognitive conflict, leading to cognitive advances" (Tudge & Rogoff, 1999, p.35).

Vygotsky believed that learning occurs through social interactions and that those social interactions had potential to even accelerate learning. "Learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers" (Vygotsky, 1980, p.90). It was through his presentation of the Zone of Proximal Development (ZPD) that the idea that development might

be accelerated via opportunities to learn with a more knowing other. Thus, not only is learning active and social, there is potential added value when learners are interacting with other people. It is important for students to have opportunities to be exposed to different thinking from peers and for teachers to support maximizing learning in those situations. As pointed out by Tudge and Rogoff (1999), "evidence is mounting that a crucial factor is the extent to which partners enter into each other's frame of reference and attempt to arrive jointly at solutions to problems" (p. 44). Interactions with peers and their thinking contribute to the quality of a student's learning.

Overview of Methodology

This is an exploratory qualitative study that examines how PTs support student thinking during rehearsals. The study took place in a methods course for future middle school and secondary mathematics teachers during the fall of 2019. Three PTs participated in this study. Over the course of the semester, the PTs each had the opportunity to play the role of teacher in a rehearsal three times. Each rehearsal was visually recorded so that their facilitation of discourse could be used for analysis. The analysis included coding the PTs' talk moves during the rehearsals and using Louie's (2017) Ways of Framing Mathematics and Mathematical Ability to characterize how inclusively the PTs supported student thinking. Following each PT's turn as teacher, the ensuing class debrief of the rehearsal was also recorded.

Soon after each rehearsal, the PTs were interviewed using stimulated recall during which they were shown clips of their rehearsal and asked questions tailored to the choices they made when facilitating the discussion. The PTs' responses were then analyzed using the Hammerness et al. (2005) Learning to Teach in Community framework to analyze the PTs' intentions related to supporting student thinking. The Learning to Teach in Community framework suggests that

new teachers learn to teach as part of a community in which they develop a vision, understandings, dispositions, practices, and tools. The PTs' responses were further examined to characterize how inclusively the PTs supported student thinking with their intentions.

A variety of other data was collected to triangulate, but also to add depth to analysis. The PTs were given a questionnaire at the beginning and end of the course that collected information related to their beliefs about mathematics and student misconceptions. Class sessions were audio recorded in case there were any discussions in which PTs ended up discussing potential actions when supporting thinking. Field notes were also taken throughout the course.

The study's findings showed that there were a variety of ways the PTs supported student thinking. The PTs tended to support student thinking similarly overall, but there were nuanced differences among them. Analysis of how inclusively they framed mathematical activity and ability suggested that certain moves are more naturally disposed to more inclusive framing. Analysis of the PTs' intentions revealed similarities among them, but also some interesting cases meriting further discussion.

Chapter 2: Literature Review

The literature review is organized into three sections. In the first section, research related to mathematical discourse and what is known about how teachers currently respond to student thinking will be described. At the end of that section there will be a brief discussion of how teachers respond to student mistakes. Secondly, an overview of positioning and framing is presented along with an overview of Louie's (2017) framework and other research related to more equitable instruction. Lastly, there will be a discussion on how novice teachers learn to teach, including research on teacher noticing and rehearsals.

Mathematical Discourse

Over the last few decades, there has been a greater emphasis on mathematical discourse to enrich student learning of mathematics. National Council of Teachers of Mathematics (NCTM) has long advocated for classroom discourse. In the *Professional Standards for Teaching Mathematics,* classroom discourse is described in terms of the teacher's role, the students' role, and offers tools for enhancing discourse (National Council of Teachers of Mathematics, 1991). The Common Core State Standards Initiative's (2021) Standard's for Mathematical Practice include asking students to "construct viable arguments and critique the reasoning of others." And in their Principles to Actions, NCTM (2014) suggested Mathematics Teaching Practices should include facilitate meaningful mathematical discourse, pose purposeful questions, and elicit and use evidence of student thinking. There is a demand for discourse to be an integral part of mathematical learning.

Before presenting research on mathematical discourse, it might be helpful to define mathematical discourse. In general, the study of discourse is the study of communication. Mathematical discourse could then be described as communication embedded in mathematical

learning experiences. Older studies that named mathematical discourse might have focused on analyzing the language itself (e.g., Corcoran, 1970). Over the last couple of decades, research on mathematical discourse has broadened to include more than just language. In 2003, Cazden and Beck listed five features of discourse that have been studied by researchers: "speaking rights and listening responsibilities, teacher questions, teacher feedback, pace and sequence, classroom routines" (p. 176). More recently, mathematics discourse may still focus on language, but has broadened to include anything connected to communication such as body movements, speaker's histories, etc. (Herbel-Eisenmann, Meaney, Bishop, & Heyd-Metzuyanim, 2017; Ryve, 2011; Sfard, 2001). Gee differentiated Discourse from discourse by claiming that "Discourse is a socially accepted association among ways of using language, other symbolic expressions, and 'artifacts,' of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or 'social network,' or to signal (that one is playing) a socially meaningful role," whereas discourse is limited to the exchange of language only (p. 131, as cited in Moschkovich, 2007, p. 25). What then makes discourse mathematical? Moschkovich (2007) stated that "language, utterances, and meanings are not mathematical in themselves but as they are embedded in mathematical practices" (p. 25). Many studies on mathematical discourse do not define it (Herbel-Eisenmann et al., 2017). Soucy McCrone (2005) described mathematical discourse as "the exchange of mathematical thoughts and information, in a learning environment, using either formal or informal mathematical language (p. 112)" (as cited in Herbel-Eisenmann et al., 2017, p. 728). After synthesizing several definitions for a dissertation, Cayton (2012) claimed "mathematical discourse refers to verbal interactions that focus on mathematics and serve to clarify, challenge, expand, investigate, or

justify content, solution strategies, or contributions from self and others." (p. 23). Mathematical discourse involves communication, not necessarily verbal, while learning mathematics.

While its importance in practice is well established, the philosophical underpinnings of mathematical discourse continue to be debated. One of the ongoing discussions on mathematical discourse concerns theory related to the word dialogic. As discourse and dialogue can be synonymous, several recent articles debate interpretations of dialogue suggesting the consideration of the Bakhtian perspective (e.g., Bakker, Smit, & Wegerif, 2015; Barwell, 2016; Kazak, Wegerif, & Fujita, 2015). Bakhtin believed meaning happened within dialogic relationships, but only when an answer gave rise to a new question (Kazak et al., 2015). Kazak et al. (2015) claim that "existing theories of conceptual development derived from both Piagetian and Vygotskian perspectives fail to take into account the important role of dialogic relationships between voices in enabling moments of insight and the growth of understanding" (p. 106). The Vygotskian perspective, called dialectic for its focus on language as a tool, implies a teacher who is guiding students to an established mathematical language. Barwell (2016) argues that from the Bakhtian perspective that mathematical journey is less linear, that "more formal and less formal are not in opposition, but work *together* and in relation with other discourses" (p. 342, italics added for emphasis).

Mathematical Discourse in Practice

In alignment with reform thinking, the role of teacher is that of a facilitator of mathematical discourse (NCTM, 2014). Stein, Engle, Smith, and Hughes (2008) explicate that during discussion, the role of teacher is "to develop and then build on the personal and collective sensemaking of students" (p. 315). Boaler and Brodie (2004) suggested that learning opportunities can come in the form of how a teacher facilitates learning among students,

specifically that what is important is "how they [students] work in groups, what the teachers says and how the students respond" (p. 781). Teachers can better support student learning by facilitating discourse as students engage with the content.

Mathematical discourse is sociocultural in that students are communicating with each other. According to Truxaw & DeFranco (2007), participating in a mathematical community through discourse is an important step for learning mathematics and for conceptual understanding (Wachira, Pourdavood, & Skitzi, 2013). Cazden & Beck (2003) describe discourse as an important social process during which students achieve complex and communicative goals. Lampert, Rittenhouse, & Crumbaugh (1996) argue that "classrooms should be 'mathematical communities' rather than collections of individual learners. Such communities would establish norms and patterns of discourse, not only for how to talk but for *what counts as evidence and therefore as knowledge*" (p. 739, italics added). Hufferd-Ackles, Fuson, and Sherin (2004) presented a Math-Talk Learning Community framework in which they discuss the trajectory towards a more student-centered learning environment based upon four components: "(a) Questioning, (b) Explaining math thinking, (c) Source of mathematical ideas, and (d) Responsibility for learning" (p. 87). Within this framework, the discourse shifts away from a focus on answers and towards students' mathematical thinking. Additionally, at the highest level of the framework, students are initiating and taking ownership of their learning while the teacher operates more as a facilitator. Mathematical discourse provides opportunities for students and teachers to learn together as members of a collective.

Approaches to Discourse Patterns

Mathematical discourse has been studied for several decades now and has been analyzed in different ways. Back in 1979, Mehan presented a structure that is pervasive in US

mathematics classroom discourse: Initiate, Respond, and Evaluate (IRE). IRE is teacher-centric in that the teacher initiates a question, the student responds, which is followed by teacher evaluation of the student response. IRE is frowned upon for its lack of emphasis on student thinking (Franke, Kazemi, & Battey, 2007). In 2000, Brendefur and Frykholm proposed four levels of math talk, consisting of uni-directional, contributive, reflective, and instructive communication. The levels of math talk refer to a general discourse classroom practices in which the higher the level, the deeper the role the sharing of student thinking has in classroom discourse. Thus, discourse can be studied from the level of an exchange to more holistic levels.

Describing categories of teaching techniques to support student thinking is another approach used to examine discourse patterns. In 1999, Fraivillig, Murphy, and Fuson presented the Advancing Children's Thinking (ACT) framework to present the observed teaching techniques of 18 first-grade teachers, but focused on one particularly skillful teacher. Supporting student reasoning was presented as the intersection of eliciting, supporting, and extending student thinking. Ellis, Özgür, and Reiten (2019) presented the Teacher Moves for Supporting Student Reasoning (TMSSR) framework based on the analysis of four middle school teachers. TMSSR, influenced by Fraivillig et al.'s (1999) ACT framework, included eliciting and extending, but replaced supporting with two new categories: responding to and facilitating student reasoning. Under each larger category, teacher moves are presented on continuums that vary in potential for supporting student reasoning. Extending student thinking is a category in other research on supporting student thinking (e.g., Boaler & Brodie, 2004; Stockero & Van Zoest, 2013). Cengiz et al. (2011) described three instructional actions to extend student thinking during class discussions: encouraging mathematical reflection, going beyond initial solution methods, and encouraging mathematical reasoning.

Another way of focusing on discourse is by considering responsive teaching: how teachers make decisions in response to the substance of student thinking (Dyer & Sherin, 2016; Hammer et al. , 2012). In 2013, Herbel-Eisenmann, Steele, and Cirillo presented teacher discourse moves (TDMs) which consist of: waiting, inviting student participation, revoicing, asking students to revoice, probing a student's thinking, and creating opportunities to engage with another's reasoning. Drageset (2014) studied videos of five grade 5-7 teachers in Norway to create a framework for analyzing mathematical discourse in the classroom. His framework is teacher centric as it focuses on the three teacher actions of redirecting, progressing, and focusing. In one study, Dyer and Sherin (2016) decided to focus specifically on how teachers responded in-the-moment to student thinking. They noted the frequencies of certain teacher response moves: substantive probing of student idea, invitation for student comment, and teacher uptake of the idea. There are multiple ways to think about how teachers respond to student thinking during mathematical discourse. The below figure lists collectively some of the different ways that researchers have thought about and studied mathematical discourse.

Table 1 *Research on Supporting Student Thinking*

(Mehan, 1979)	Initiate, Respond, and Evaluate (IRE)
(Fraivillig et al., 1999)	Advancing Children's Thinking (ACT) framework: Eliciting, Supporting, and Extending
(Brendefur & Frykholm, 2000)	Uni-directional, Contributive, Reflective, and Instructive communication
(Boaler & Brodie, 2004)	Categories of teacher questions include: "(1) gathering information, leading students through a method; (2) inserting terminology; (3) exploring mathematical meanings and/or relationships; (4) probing, getting students to explain their thinking; (5) generating discussion; (6) linking and applying; (7) extending thinking; (8) orienting and focusing; and (9) establishing content" (p. 777)

Table 1 *(continued)*

(Hufferd-ackles et al., 2004)	Levels of Math-Talk Learning Community framework: (a) Questioning, (b) Explaining math thinking, (c) Source of mathematical ideas, and (d) Responsibility for learning
(Herbel-Eisenmann et al., 2013)	(TDMs): waiting, inviting student participation, revoicing, asking students to revoice, probing a student's thinking, and creating opportunities to engage with another's reasoning.
(Stockero & Van Zoest, 2013)	5 Pivotal Teaching Moment types: extending, incorrect mathematics, sense-making, contradiction, and confusion 5 teacher decision actions: "(a) extends mathematics and/or makes connections; (b) pursues student thinking; (c) emphasizes meaning of the mathematics; (d) acknowledges, but continues as planned; and (e) ignores or dismisses." (p. 131)
(National Council of Teachers of Mathematics, 2014)	A Framework for Types of Questions Used in Mathematics Teaching: Gathering information, Probing thinking, Making the mathematics visible, Encouraging reflection and justification
(Drageset, 2014)	Redirecting, Progressing, and Focusing
(Freeburn, 2015)	Assessing questions, Advancing Questions from (Smith, Bill, and Hughes 2008) & Judicious telling (from Lobato, Clarke, and Ellis 2005)
(Kazemi & Cunard, 2016)	Observed moves (not a framework): Setting expectations, eliciting and recording student ideas, orienting students to one another and to the mathematics, content assistance
(Ellis et al., 2019)	Teacher Moves for Supporting Student Reasoning framework: Eliciting student reasoning, Responding to student reasoning, Facilitating student reasoning, Extending student reasoning

Within mathematical discourse, some research has focused specifically on the teacher's role in supporting student engagement with other students' mathematical ideas. Fraivillig et al. (1999) addressed this when describing instructional strategies that support the describer's and listeners' thinking. Two examples included students' different solution methods on the board and asking a student to explain a peer's method. Franke et al. (2015) analyzed discussions in 12 elementary schools for the level students engaged with each other's mathematical ideas and also

teacher moves – how they invited and followed up on students engaging in each other's thinking. Teachers' inviting moves consisted of explain someone else's solution, discuss differences between solutions, make a suggestion to another student about his or her idea, connect their ideas to other students' ideas, create a solution together with other students, and use a solution that was shared by another student. Teachers' follow-up moves included probing, scaffolding, and positioning. Kazemi and Cunard (2016) noted moves such as "asking other students to repeat or revoice a classmate's thinking or strategy, asking other students to try a classmate's strategy, turning students' attention to recording, introducing a group-worthy question, and asking students to make connections across or compare strategies" (p. 293). An important subset of supporting student thinking is supporting students to consider each other's thinking.

While Table 1 above describes general teacher moves that support student thinking, some research focuses on responding to specific instances of student thinking. For example, (Stein et al., 2015) focused on mathematical discourse related to student tasks. They presented five practices for helping teachers move beyond show and tell that emphasizes not only selecting student work that is correct but also student work that has misconceptions. In 2013, Stockero and Van Zoest studied video of novice secondary mathematics teachers to characterize Pivotal Teaching Moments (PTMs), more opportune instances of student thinking in which the teacher could engage. Five PTMs (extending, incorrect mathematics, sense-making, contradiction, and confusion) were identified along with five potential teacher responses: "(a) extends mathematics and/or makes connections; (b) pursues student thinking; (c) emphasizes meaning of the mathematics; (d) acknowledges, but continues as planned; and (e) ignores or dismisses" (p. 131). Results indicated that novice teachers missed a number of opportunities and that some teacher actions to PTMs could have more positive impacts on student learning than others. Cayton et al.

(2017) utilized PTMs to analyze teacher-student interactions in Geometry classrooms and noticed that different teachers played different roles to support student thinking in their classrooms. The Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOSTs) framework presented by Leatham et al. (2015) guides teachers in understanding whether an instance of student thinking might be worth highlighting in classroom discourse. In their framework, the power of whether to engage in the instance of student thinking is relative to other factors such as context and mathematical connections. MOSTs shift the focus away from how teachers respond to student thinking and instead analyses student thinking for certain characteristics/attributes. The five practices, PTMs, and MOSTs focus teacher responses to student thinking, albeit in different ways.

PTs and Responding to Student Thinking in Mathematical Discourse

There is not much known specifically about PTs and discourse moves in response to student thinking. Ghousseini, Beasley, and Lord (2015) studied the rehearsals of 25 primary PTs, who used a very specific list of four questions to guide facilitation of mathematical discourse. The PTs struggled with adapting the wording and the ordering of those four questions. Additionally, they struggled with responding to multiple student solutions and representing students' strategies in order to connect the goal of the lesson. Freeburn (2015) studied four PT's conceptions related to three specific discourse practices: assessing questions, advancing questions, and judicious telling. While there was no reporting on their implementation, he did discuss that the PTs' understandings of the three Types of Teacher Talk (TTT): "over the course of the semester, constructed conceptions of TTT that were oriented towards focusing on and promoting students' mathematical thinking" (p. iii). Kazemi & Cunard (2016) studied both the PTs' and the TE's moves in a methods course and out of a list of orienting moves, they noted the

only orienting moves the PTs implemented were turning students' attention to recording and introducing a group-worthy question. Shaughnessy and Boerst (2018) studied 47 students in an elementary education course and assessed them on four core components: "(a) eliciting the student's process, (b) probing the student's understanding of key mathematical ideas, (c) attending to the student's ideas, and (d) deploying other moves that support learning about student thinking" (p. 45). In their results they suggested that there are "(a) moves that require new learning, (b) moves that can be built upon, and (c) moves that may require unlearning" (p. 51). There is evidence that PTs can implement some moves to respond to student thinking.

Responding to Mistakes/Misconceptions During Mathematical Discourse

Teacher responses to mistakes, when student thinking may be unpolished, are important instances of responding to student thinking. There is a current trend in mathematics education research to be more intentional in emphasizing the valuing of student mistakes (e.g., Boaler, 2015; Horn, 2012; Stein et al., 2008). Student mistakes that are a result of cognitive struggle have been found to result in the brain growing. This shift towards thinking of mistakes as positive can have a liberating effect on students (Boaler, 2015). Instead of brushing aside mistakes, there is a push to have class discussions in which correct logic of student thinking is unveiled and valued (Boaler, 2015; Jansen, Cooper, Vascellaro, & Wandless, 2017). Ames (1992) encouraged viewing mistakes as part of learning when suggesting teacher practices that would support students working towards mastery learning. Stockero and Van Zoest (2013) named "Incorrect mathematics" as one of five PTMs, but also elucidated that not all mistakes are equally worth engaging with in that sometimes an error would not qualify as a PTM when "it is based on an incorrect calculation or something else that is not likely to interfere with or provide

opportunities to improve students' mathematical understanding" (p. 135). Mistakes have potential as valuable instances of student thinking to include in discourse.

Although discussing student mistakes can contribute to rich mathematical discourse, there is evidence to suggest that teachers engage in practices that avoid dealing with errors. Teachers, because they want students to feel successful, have been known to lower the cognitive demand of mathematical tasks (Doyle, 1983, as cited in Henningsen & Stein, 1997). Boaler (2015) mentioned that "many classrooms are designed to give students work they will get correct" (p. 12). Based on TIMSS videos analysis, Stigler, Gonzales, Kawanaka, Knoll, and Serrano (1999) characterized U.S. 8th grade mathematics classrooms as focused on the learning of skills, a focus which suggests that teachers might clearly demonstrate procedures in an effort to minimize confusion. Tulis (2013) shared previous German research which suggested that teachers "may follow an error prevention approach instead of viewing errors as learning opportunities" (p. 57). Hence there is this idea that teachers may design instructional experiences for students so that they are as error-free as possible, which may also be depriving students of opportunities to make sense of the mathematics they are learning. It is worth mentioning that in order for misconceptions to surface in mathematical discourse, there must be opportunities for students to have them in the first place.

Internationally, studies have shown teachers respond to student mistakes in different ways. After studying TIMSS videos of Italian and U.S. 8th grade mathematics courses, Santagata (2004) noted that mistakes were discussed twice as often in Italy than the U.S. Additionally, it was noted that U.S. teachers tend to first compliment a student for having tried before correcting the student's mistake. Unlike the Italian teachers, who mitigated their responses to students' mistakes on average 6% per lesson, U.S. teachers mitigated 38% of their public responses to

students' mistakes per lesson. Santagata goes further and hypothesizes that this concern for student self-esteem might play a role in teacher decisions "to reduce the complexity of the problems as they guide students through their solution." (p. 160). In analyzing the responses of teachers during whole class discussion in videos of Chinese and US teaching in elementary grades on specific math topics, Schleppenbach et al. (2007) found that Chinese teachers responded to mistakes with questions significantly more frequently.

There are generally four different ways a teacher can elect to engage with student thinking once a student has spoken up about a misunderstanding or mistake: ignore, correct, let the student correct, or ask a peer to correct. Some studies have noted that ignoring incorrect student thinking occurs (Kaldrimidou, Sakonidis, & Tzekaki, 2008; Tulis, 2013). For example, in advocating for teachers using more than just correct student work, Stein et al. (2008) shared the example of Mr. Crane, a teacher who intentionally avoided calling on students with incorrect solutions to present. Santagata (2005) analyzed TIMSS video and observed that U.S. teachers responded by correcting students' mistakes themselves 25% of the time. While it may be tough for teachers to resist the temptation to correct students' answers (Stein et al., 2008), when the teacher does the cognitive lifting and corrects the error themselves, students are no longer positioned as sense makers. Another way a teacher might respond is to ask another student. Oser & Spychiger (2005) named this the "Bermuda triangle of error correction," described as a teacher redirecting an incorrectly answered question to another student. The teacher response of shifting to another student has the potential to devalue the initially called upon student's thinking. Providing students the opportunity to make sense of their own misunderstanding is the better way teachers can respond (Oser & Spychiger, 2005; Schleppenbach et al., 2007). Schleppenbach et al. (2007) described this in a teacher-belief theme observed as "the importance

of asking students to think through their own errors" (p. 140). While teachers respond to student thinking in a variety of ways, the most supportive move is to allow the student to have the opportunity to be the sense-maker, to correct the error.

There have been a couple of studies in which TEs have studied how PTs responded to student mistakes (Baldinger & Campbell, 2019; Campbell, Baldinger, Freeland, Graif, & Karr, 2019). Baldinger and Campbell (2019) studied PTs' responses to student errors in two different approximations of practice: a rehearsal and two scripted tasks. In the rehearsal, there was a 'planted error' and the scenario for the scripted task included a student error. The scripted task is a scenario presented to the PT in which they must write a dialogue and include their rationale. In general, the PTs' developing visions across the three data sources were discussed. Campbell et al. (2019) focused only on the PTs' responses to scripted tasks and focused mostly on teacher moves as evidence of their developing tools and practices. Teacher revoicing, tabling ideas, and orienting a student to one another were all moves they noticed the PTs' developing.

Mathematical Discourse and this Study

This study captured evidence related to how a teacher supports student thinking. In the context of role-playing the teacher while interacting with a small group of students, the question is how did the PT support student thinking during discussions. Starting with Herbel-Eisenmann et al.'s (2013) TDMs as a base, the specific talk moves the PT utilized were documented to note how the PTs responded to students. Shaughnessy and Boerst (2018) suggested that PTs already have some skills related to student teaching and thus one goal is to learn more about what skills they have. As this study examines PTs verbally interacting with students to support their thinking, mathematical discourse is the larger category within which to study their actionable moves.

Working toward Inclusive Practices

In working toward inclusive practices, mathematical discourse can play a role in how a student identifies as a doer of mathematics. In connection with statements made in class, how students identify with what is being said can be described as positioning. For example, a student might perceive they are positioned to be a stronger or less strong math student in connection with something said during class. Framing, which can be accidental or intentional, is how the substance of an activity is described. One important way teachers can attempt to alter students' perceptions is by re-framing, shifting language in connection with a specific activity to be more intentional in how the substance of the activity is communicated. Teachers can work toward framing mathematics and mathematical ability more inclusively to create a more inclusive mathematical learning environment for students. Below both positioning and framing will be further discussed before introducing Louie's (2017) framework along with deeper discussion of both inclusive and exclusionary framing mathematical activity and ability.

Positioning involves an individual's identity in connection with discourse. Davies and Harré (1990) describe positioning as the "discursive production of selves." The three elements of positioning include categories, story lines, and speech acts. A person can identify with belonging to a certain category, e.g., daughter, and be experiencing a story line, e.g., forgot to take out trash, and then related speech acts between a parent and the daughter may or may not challenge her perception of herself as a daughter. The philosophy of positioning is based on immanentist views in which "there are only actual conversations, past and present" (p. 2). These conversations are processed at the individual level in such a way that various identities can be reinforced or challenged by how they connect to previous constructions. Herbel-Eisenmann, Wagner, Johnson, Suh, and Figueras (2015) present a reconfigured relationship among

communication acts and interpretive frames as a more fluid cycle of communication acts, discourse choices, positioning, and positioning formats/discourse choices. As Davies and Harré (1990) noted, "[positioning] is a resource through which speakers and hearers can negotiate new positions" (p. 26). In classrooms, positioning has the potential to shift how students have constructed themselves as students of mathematics.

Framing more inclusively creates opportunities for students to identify more positively with an activity. Framing has been studied as one approach to shift toward more equitable instruction by "re-framing" what it means to learn and do mathematics. "A frame defines the meaning of a situation for participants interacting within it; it renders a context meaningful such that individuals can give a response to the question: 'What is it that is going on here?' [Goffman, 1974]" (Hand, Penuel, & Gutiérrez, 2012, p. 252). Framing, like positioning, can occur quite frequently without anyone's conscious awareness (Louie, 2017). However, framing can be quite deliberate as an approach to "alter the culture of exclusion by actively 'reframing' mathematics teaching and learning in ways that expand students' opportunities to learn" (Hand et al., 2012, p. 265). Framing and positioning similarly having the potential to address issues of power related to social activity and what it means to engage in that activity.

Teacher beliefs related to the frames do not always connect to the teacher enactment (Louie, 2017). For example, Louie (2017) shares the case study of Ryan who worked hard towards being an inclusive teacher but spent only 48% of the time in inclusionary framing. In another case, the teacher had not shifted her personal beliefs to align with the re-framing, but enacted practices in alignment with her desired beliefs which resulted in positive outcomes (Louie, 2018). And as Skott (2001) suggested in describing the multiple and sometimes conflicting educational priorities juggled by teachers, sometimes certain concerns can outweigh

others which might appear as a contradiction to an outsider (as cited in Philipp, 2007). Furthermore, revisiting the idea that much framing happens unintentionally, Stein et al. (2008) present the example that teachers who support the discussion of incorrect strategies may unintentionally assign value to student thinking through facial expressions or other signals, potentially devaluing what they might have intended to value. Louie (2017) posits that efforts to disrupt dominant frames take persistence.

Furthermore, even when teachers succeed in inclusive framing, students still may not truly hear the re-framed message as intended. Louie (2017) gave a counter example of a teacher telling students to "work in your groups and help each other out" which could be perceived by some as inclusive mathematical ability positioning students as resources but by others as exclusive mathematical ability if the student felt the teacher intended the group because the student was not competent alone. The teacher can work towards inclusivity, but students are not blank slates, they will be influenced by their past experiences.

Research is limited on teachers and their efforts to implement equitable frames in mathematics. Jilk (2016) worked on increasing teacher capacity related to inclusivity in that she studied a video club that focused specifically on supporting teacher noticing of students' mathematical strengths. She found that a focus on the intellectual strengths behind student thinking was important for disrupting teachers' deficit views of students. In a study that studied student perceptions, Heinze and Reiss (2007) shared the results of their quasi-experimental study in which two groups of grade 7-8 teachers were given training on teacher reasoning and proof, but one group also received training on mistake-handling. The results suggest that the teacher training was successful in that students perceived positive affective teacher behavior in relation to student mistakes. Additionally, students in the experimental group also performed

significantly better on the geometry content in a post-test. Evidence suggests teachers can be supported to shift toward more equitable framing and that those results may benefit students.

Inclusive and Exclusive Framing Mathematical Activity and Ability

Explicitly re-framing mathematics and mathematical ability inclusively has potential to communicate a greater worth of student thinking. As Horn (2012) puts it "if, instead, we focus on creating a classroom environment that fosters a sense of belonging and normalizes confusion, we, as teachers, have something to do" (p. 16). This is not an easy shift as "we are most often immersed in a culture that attends to students' shortcomings" (Jilk, 2016, p. 189). This re-framing is important so that students hear teacher messages suggesting their students are capable doers of mathematics.

Louie (2017) introduced a framework juxtaposing exclusionary and inclusive ways of framing mathematics and mathematical ability. Exclusionary framing of mathematical activity was described as the rote practice frame whereas inclusive framing of mathematical activity was composed of both the sense-making frame and the multidimensional frame. Mathematical ability framing was contrasted with the hierarchical ability frame describing exclusionary framing and the multidimensional ability frame describing inclusive framing. Louie was clear that not all framing should fall into exclusionary nor inclusive categories, but that an abundance of exclusionary framing could communicate unintended negative messages to students. Below her framework (see Figure 1) is a deeper discussion of exclusionary and inclusive framing of mathematics and mathematical ability.

Figure 1 *Ways of Framing Mathematics and Mathematical Ability (Louie, 2017, p. 496)*

Ways of Framing Mathematics and Mathematical Ability

	Exclusionary	Inclusive
The nature of mathematical activity	***The rote practice frame*** *Mathematics is a fixed body of knowledge to be absorbed and practiced. Correctness is paramount.* • Presenting standard formulas, algorithms, and so forth • Assigning routine tasks requiring only the application of previously demonstrated algorithms • Asking closed questions in conversation with students • Explicitly stating the importance of repetitive practice • Focusing discussion exclusively on answers	***The sense-making frame*** *Mathematics is about making sense of ideas and understanding connections.* • Assigning open-ended, nonroutine tasks • Asking open-ended questions and pressing for meaning in conversation with students • Explicitly stating the importance of sense making ***The multidimensional math frame*** *Mathematics includes activities such as collaboration, experimentation, and argumentation, not just rote practice.* • Assigning open-ended, nonroutine tasks • Explicitly naming skills that have not traditionally been seen as mathematical as mathematically important
The nature of mathematical ability	***The hierarchical ability frame*** *Mathematical ability is distributed along a linear continuum. Some people have a lot; others have very little.* • Explicitly valorizing speed and correctness • Positioning some students as helpers and others as in need of help	***The multidimensional ability frame*** *Everyone has both intellectual strengths and areas for growth that are relevant to mathematics learning.* • Valorizing skills that have not traditionally been seen as mathematical • Naming a variety of students as resources for their peers' learning • Making statements about mutual dependence (everyone contributes, everyone learns together)

Mathematical Activity. Exclusionary framing of mathematical activity can be described as framing mathematical activity with the rote practice frame (Louie, 2017). In alignment with the rote practice frame, "mathematics is a fixed body of knowledge to be absorbed and practiced. Correctness is paramount" (p. 496). Opportunities to learn math include being given standard algorithms and being assigned routine tasks (Louie, 2017). Students feel they must perform finished products in mathematics class instead of critically analyzing their own thinking (Jansen et al., 2017). Louie (2017) also described one aspect of this framing as teachers "asking closed

questions in conversation with students" (p. 496). Horn (2012) addressed a similar issue when she noted that correct answers may be valued because they support the flow of IRE dialogue, "often at the expense of valuing student's mathematical thinking" (p. 5). "Eliciting mathematical ideas from students who are perceived as always giving the right answer positions correctness as more valuable than mathematical thinking" (NCTM, 2018, p. 31). In the exclusionary framing of mathematical activity, discussions are focused exclusively on answers. Exclusionary framing of mathematical activity focuses more on the product than the process, tremendously reducing the value placed on students engaging in reasoning.

Inclusive mathematical activity inclusively means framing mathematical activity using the sense-making frame and the multi-dimensional mathematics frame (Louie, 2017). In describing mathematics as sense-makingLouie (2017) stressed that "mathematics is about making sense of ideas and understanding connections" (p. 496). Presenting students to the opportunity to engage in less formulaic mathematics such as in open-ended, nonroutine tasks, is one way to shift what is understood to qualify as mathematical activity. Mathematics as sense-making has previously been advocated for by others. Boaler (2015) communicated that assignments need to be an opportunity for sense making, not an exercise in filling a page with correct answers. The multi-dimensional math frame asserts that "mathematics includes activities such as collaboration, experimentation, and argumentation" (Louie, 2017, p. 496). "Adopting a broader perspective on 'mathematics' could have implications for students who have been historically marginalized by school mathematics and could contribute to making mathematics education more equitable" (Herbel-Eisenmann et al., 2017, p. 734). Teachers need to support students in sharing in-progress thinking so that students feel their thinking is valued (Jansen et al., 2017) as mathematical activity. Focusing on valuing student thinking over correct answers is

important when framing mathematics inclusively. Inclusive mathematical activity means reframing what qualifies as the "product" and as "actions" when doing mathematics.

Mathematical Ability. Louie (2017) described exclusive framing of mathematical ability as the hierarchical ability frame – some have more ability than others. This aligns with the traditional notion of smartness that promotes the idea that only a select few students' thinking is worth engaging in Horn (2012). The exclusive framing of mathematical ability also materializes when some students are positioned as helpers and others as in need of help. Speed and correctness are explicitly valued (Louie, 2017). Ames (1992) pointed out that when the focus is on performing, then students' self-concept of ability mediates affective variables, but not when the focus is on trying and learning. Framing mathematical ability exclusively limits how many students might identify as mathematically able.

Louie (2017) described inclusive framing of mathematical ability as the multidimensional ability frame: "everyone has both intellectual strengths and areas for growth that are relevant to mathematics learning" (p. 496). One way to support this frame is to "valorize skills that have not been traditionally seen as mathematical" (p. 496). For example, Cohen, Lotan, Scarloss, and Arellano (1999) named the approach of explicitly valuing a variety of mathematical activities at the onset of a task. The teacher specifically states the different abilities needed reinforced by communication that each student will perceive himself strong in some and weak in others. Louie (2017) "naming a variety of students as resources for their peers' learning" as another way to frame mathematical ability inclusively. Horn (2012) suggested that if thinking is valued over performance, more students will have valuable contributions and those need to be assigned competence. Framing mathematical ability inclusively means working towards equal status amongst students in that none should be singled out as smarter or less knowledgeable than others

as they each bring value. Furthermore, they are each enriching each other's learning. Louie (2017) described "making statements about mutual dependence (everyone contributes, everyone learns together)" (p. 496) as another way to frame mathematical ability inclusively.

Mistakes and Inclusive/Exclusive Framing

Different ways of framing student mistakes/misconceptions exist on a continuum from deficiencies of students to avoid, to informing teachers of learning gaps, to being learning opportunities for students (Borasi, 1994). In a performance culture, when students make mistakes, they believe it means they are not a math person because those mistakes are not being valued (Boaler, 2015). Exclusive framing of mistakes communicates them as having no value. Borasi (1994), when studying errors as a springboard for inquiry, suggested that while mistakes were being valued as indicators of what students are not yet understanding, they were not yet being framed as learning opportunities *for students*. NCTM (2018) further explicated the teaching practice of eliciting and using evidence of student thinking to "promote a classroom culture in which mistakes and errors are viewed as important reasoning opportunities, to encourage a wider range of students to engage in mathematical discussions with their peers and the teacher" (p. 34). Framing mistakes inclusively means framing mistakes as valuable to student learning, as an important part of sense-making. Louie (2017) did not explicitly say student mistakes/misconceptions should be valued as learning opportunities, but that notion lies opposite the "correctness is paramount" description provided in connection with exclusive framing of mathematical activity. Thus, inclusive framing of mistakes/misconceptions would frame them as valuable opportunities for students' sense-making.

Framing and this Study

Framing is one approach teachers may embrace to work towards shifting how mathematics learning is perceived in their classrooms. "The teacher plays a central role in establishing the mathematical quality of the classroom environment and in establishing norms for mathematical aspects of students' activity" (Yackel & Cobb, 1996, p. 475). Lampert et al. (1996), in their article on argumentation in discourse, mentioned that "the teacher must take on the task of modeling a different set of social norms and offering students safe mechanisms for expressing their thinking when it is different from that of their peers" (p. 760). This study looked at not only how teachers supported student thinking, but also used the Louie (2017) framework to examine the PTs' inclusive framing.

Supporting Teachers to Develop Their Practice

PTs are being prepared to enter dynamic classrooms and methods courses are working towards preparing PTs to teach more ambitiously than those before them. Noticing is an important aspect of PTs learning to have more ambitious practice. PTs cannot respond effectively to instances of student thinking if they do not yet see the opportunities to engage in student thinking. University preparation of teachers have been on the receiving end of criticism for its lack of approximating the complexities of teacher practice (e.g., Clift & Brady, 2005; Hammerness et al., 2005). Lampert et al. (2013) qualified preparing teachers as consisting of "two challenges: preparing beginning teachers to actually be able to do teaching when they get into classrooms, and preparing them to do teaching that is more socially and intellectually ambitious than the current norm" (p. 226). Rehearsals, approximations of the practice of teaching, are one approach to providing PTs experience to responding to student thinking. Teacher noticing and rehearsals are discussed below in more detail.

Teacher Noticing

Teacher noticing is a construct that suggests heightened attention to certain foci has potential to improve instruction. Noticing has been defined in several ways (e.g., Jacobs et al., 2010; Mason, 2011; van Es & Sherin, 2002) always consisting of three aspects. Sherin et al. (2011) point to researchers of noticing as asking the "primal questions of teaching: Where do teachers look, what do they see, and what sense do they make of what they see" (p. 3)? While there are parallels between everyday noticing and teacher noticing – they both involve selective attention, teacher noticing relates to improving the ability to see some important details amongst the fuzziness and complexities of teaching. Sherin & van Es (2009) connect teacher noticing back to Goodwin's (1994) concept of *professional vision* which involves "ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group (p. 606)." Essentially, Goodwin's concept of professional vision grounds noticing in that it is a way of seeing occurrences related to some focus, making sense of why they matter, and deciding how to respond in a way that is accountable to ambitious teaching.

In general, noticing studies work towards noticing at higher levels, but the target of what and how they notice varies. Research studies often claimed that teachers noticed at higher levels (e.g., McDuffie et al., 2014; Star & Strickland, 2008; van Es & Sherin, 2008), but individual paths to increased noticing are not necessarily linear (van Es & Sherin, 2008). Van Es & Sherin (2002) focused on teacher noticing of student thinking, teacher's role, and discourse. More recently, there has been a shift to broaden understandings of professional vision to include more equitable noticing (Hand, 2012; Jilk, 2016; Louie, 2018). One of the four lenses McDuffie et al. (2014) studied was "power and participation" and all four lenses were explicitly focused on students' multiple mathematical knowledge bases, which included students' resources such as

family, community, etc. Jilk (2016) studied a video club of inservice teachers who responded to prompts which included what are students understanding, what are the norms for participation, and what did students do or say that was mathematically smart. Jilk (2016) conjectured that tending to the first two prompts helped support responses for how the students are mathematically smart. Wager (2014) studied what elementary teachers in a professional development course noticed about student's participation in mathematics classrooms. Noticing has been used to shift teachers to notice a variety of different things.

Novice teachers miss opportunities compared to more experienced teachers (Berliner, 2001), but studies support the notion that PTs can show growth in noticing (e.g., Amador et al., 2016; Roth McDuffie et al., 2014; Star & Strickland, 2008). McDuffie et al. (2014) studied 73 K-8 PTs in methods courses at four different universities as they noticed equitable practices (explicit focus on students' resources) through repeated video analysis focused on teaching, learning, task, or power and participation. Star and Strickland (2008) used TIMSS video with secondary mathematics PTs in a methods course to study their noticing of different aspects of classroom practice (classroom environment, classroom management, mathematical content, tasks, and communication). Amador et al. (2016) studied noticing with elementary PTs in the context of Japanese lesson study instead of video. The PTs took turns teaching in an elementary class followed by a debrief of the lesson with peers, the host teacher, and the TE. The three studies with PTs provide evidence that they too can improve their ability to notice with intentional guidance.

A common characteristic in noticing studies is the learning and growing of participants as a community of learners. The video clubs were a social event in which video was watched and discussed as a group (e.g., Jilk, 2016; Louie, 2018; M. G. Sherin & van Es, 2005, 2009; van Es &

Sherin, 2008). "As Video Club (VC) facilitators emphasized, hearing colleagues articulate students' strengths also supported everyone present to learn to respond, giving them words to use in their classrooms to authentically and convincingly assign competence to students with diverse strengths" (p. 60, Louie, 2018). A Japanese lesson study context for noticing also was a community learning experience as the PTs, the TE, and the class teacher met to debrief after each lesson (Amador et al., 2016). And the studies that took place in methods courses with PTs also were a community learning experience in that they were watching videos and debriefing together as a class (e.g., McDuffie et al., 2014; Star & Strickland, 2008). Across a variety of settings, noticing skills might be developing individually in one's mind, but the *learning* of noticing is a communal experience.

Rehearsals

In an effort to address the call for improved teacher preparation, many are pushing for PTs to have more practitioner-like experiences (e.g., Ball & Forzani, 2009; Lampert et al., 2013). One approach to providing PTs experience managing the complexities of teaching, including ambitious practice, is through scenarios in which the PT plays the role of teacher for a particular instructional activity. Rehearsals are a learning experience incorporated into teacher education in response to the call for a more practice-based teacher education. The rehearsal itself is one aspect in which the PT is role-playing the teacher, peer PTs are playing the role of students or observing, and the TE interjects from time to time for a variety of reasons. The rehearsal has been described as focusing on an Instructional Activity (IA) which could be anything from launching a task to reviewing homework. Grossman et al. (2009) breaks down pedagogies of practice into three components: decomposition of, representations of, and approximations of. Rehearsals are considered an approximation of practice as they resemble teaching practice.

Hunter and Anthony (2012) further describe the value of these approximations of practice as supporting PTs to learn "to do" interactive mathematics teaching.

Rehearsals are typically just one phase of a larger learning cycle. Lampert et al. (2013) described a cycle of enactment and investigation (CEI) consisting of 6 phases: (1) observation, (2) collective analysis, (3) preparation, (4) rehearsal, (5) classroom enactment, and (6) collective analysis. Arbaugh et al. (2019) used an adapted model in which the rehearsal and classroom enactment phases were replaced with one rehearsal with simulated students. Their CEI consisted of "(1) working through a mathematical task, (2) individual analysis of a representation of practice, (3) collective analysis of the representation of practice, (4) small-group planning for the enactment of a task, (5) rehearsal of a teaching task, and (6) collective analysis of the rehearsal." Rehearsals, the enactment phase in which the PT plays the role of teacher amongst peers, are typically part of a larger investigative cycle.

Rehearsals are a collective and social learning experience in which all participating parties learn. The PT will receive feedback on the rehearsal from at least two sources – their peers and the TEs who are supporting the rehearsal (Lampert et al., 2013). Baldinger and Campbell (2019) refer to these as "coached rehearsals." Additionally, Pfaff (2017) posited that there are a number of learning opportunities for the peers role-playing the students. Playing the role of students allows the peer PTs to connect more with how students might think. Furthermore, the interjections of the TE are not just for the PT playing the role of teacher. One of Kazemi et al.'s (2016) insights regarding rehearsals is that they require supporting a culture of making one's practice public. Rehearsals are an opportunity for everyone in the community to learn by experiencing and then discussing the PT's choices as teacher from their respective roles and observations.

Research on Rehearsals

The exchanges between TEs and PTs are one aspect of rehearsals that can provide insights regarding the PTs' experience. Lampert et al. (2013) studied the interactional exchanges, such as the substance and frequency, between the TEs and PTs at three different teacher education programs focused on preparing elementary PTs. They noted the percentage of exchanges falling into four different categories: directive feedback (60.85%), evaluative feedback (28.14%), scaffolds enactment (21.09%), and facilitates discussion (17.29%). Ghousseini et al. (2015) also focused on the exchanges but shifted to notice what problems of practice arose. There were two problems of practice noted in relation to PTs adapting the wording and order of the questions and there were also problems of practice noted in relation to responding to student solutions. Ghousseini (2017) still studied the TE/PT exchanges during rehearsals, but also analyzed connections to the elementary teachers' mathematical knowledge for teaching (MKT).

Another approach to studying the development of PTs' learning in connection with rehearsals is to analyze artifacts and other evidence of PTs' understandings beyond the rehearsal itself. Freeburn (2015) used rehearsals and other activities in a methods course to examine four secondary PTs' conceptions of three different types of teacher talk and noted that while the PTs arrived at similar conceptions, evidence supporting their conceptions was visible at different times through different course activities. Arbaugh et al. (2019) analyzed reported data of secondary PTs in conjunction with rehearsals to note the different dimensions of learning experienced at different phases of the learning cycle. In New Zealand, Hunter and Anthony (2012) studied PTs' journal entries as part of a methods course in which there was one rehearsal cycle and perceived that PTs progressed to becoming more confident to notice and respond.

Rehearsals have also been utilized as professional growth for teachers. Pfaff (2017) studied rehearsals during professional development in two different contexts (Masters class; experiment with school districts) with middle school teachers in her dissertation focusing on the role of teacher rehearsal in classroom mathematics discourse. Her conclusions on the value of rehearsals include that rehearsals help teachers rehearse routines, learning opportunities exist for more than just the teacher during role-play, and rehearsals provide generative learning opportunities as an element of the cycle of enactment and investigation. During a summer PD for inservice teachers, Munson and Baldinger (2019) studied teacher noticing of teachers playing the role of student during rehearsal debriefs. Using Jacobs et al.'s (2010) noticing framework, they analyzed 560 of 762 talk turn segments, of which 34.3% were attending, 45.7% interpreting, and 20% implicating, paying attention to whose perspective (teacher or student) they noticed from. They found that "non-rehearsing teachers primarily attended to and interpreted their experiences from the position of student, but made connections beyond the rehearsal from the position of teacher" (p. 520). Rehearsals are not just a learning experience for novice teachers.

Learning to Teach in Community Framework

One way of describing the complexities of learning to teach is the Learning to Teach in Community framework (Hammerness et al., 2005). The framework suggests that new teachers learn "in a community that enables them to develop a *vision* for their practice; a set of *understandings* about teaching, learning, and children; *dispositions* about how to use this knowledge; *practices* that allow them to act on their intentions and beliefs; and *tools* to support their efforts" (p. 385, italics in original). Tools can be of a more abstract nature (conceptual tools) and include theories or be of a more concrete nature (practical tools) and include specific approaches and strategies (Grossman, Smagorinsky, & Valencia, 1999). Practices are the

developing instructional activities, such as leading a discussion, within a teacher's repertoire. Dispositions are described as "habits of thinking and action--about teaching, children, and the role of the teacher" (Hammerness et al., 2005, p. 387). Understandings are the "deep knowledge of content, pedagogy, students, and social contexts." And vision is the image of what is possible as novice teachers grow in their role as an educator. Vision, understanding, dispositions, practices, and tools are all developing as teachers participate in their learning community.

Figure 2 *Reproduced from Hammerness et al. (2005) "Learning to Teach in Community" Framework*

Summary

The main focus of this study analyzes how PTs support students in their mathematical learning. One of the main ways to study how teachers support students is through mathematical discourse. There is a cornucopia of research on mathematical discourse, yet there is very little research on PTs' facilitation of discourse. This study has an added important layer of a focus on inclusive practices. Within mathematical discourse, what are the PTs communicating to their students regarding what qualifies as mathematics and mathematical ability? The PTs were part of a learning community in a methods course focused on equitable teaching practices. This study examined how PTs supported student thinking as they engaged in the complexities of authentic teaching practice during rehearsals.

Chapter 3: Methods

The purpose of this exploratory qualitative study was to learn more about decisions that PTs made when supporting student thinking during rehearsals. In particular, this study examines PTs' enactment of teacher talk moves aligning with inclusive ways of framing mathematical activity and mathematical ability. Research suggests that PTs working toward responding inclusively to student thinking is in alignment with what others call "ambitious teaching." The research questions focused on two lenses: enactment (responses to student thinking), and intentionality (why the PTs made the decisions they did). The research questions are:

1. How do PTs support student thinking during rehearsals?
    a. How inclusively do PTs frame mathematical activity and mathematical ability when supporting student thinking during rehearsals?
2. What intentions do PTs have behind the decisions they make during rehearsals?
    a. How inclusively do PTs frame mathematical activity and mathematical ability when reflecting on their intentions during rehearsals?

Rationale for Qualitative Research Design

A qualitative research approach was appropriate for this study as the researcher was exploring one particular phenomenon intimately. Merriam and Tisdell (2016) described basic qualitative research as having the goal of extending knowledge by learning more about a phenomenon. As this focus on teacher responses to student thinking involved intricate interactions, qualitative research was necessary to uncover how teachers chose to respond and to understand why the PTs elected to respond in the way that they did. Based on Creswell's (2013) list of philosophical assumptions, this study fell into the Pragmatism category as learning more about the practices of current PTs can inform the larger problem of how to prepare future

teachers to enact more equitable practices in mathematics classrooms. This study explored the experience of PTs responding to student thinking during rehearsals, an approximation of the complex practice of teaching.

Participants

The participants were all students in a senior level methods course for middle school and high school mathematics teachers. On the first day of class the students were given an overview of the study and asked if they would participate, and all 11 agreed to participate. The 11 students consisted of five females and six males. For the most part the class was predominantly white, but there was one Asian male and one Middle Eastern female. Of the 11, six were intentionally selected to be the focus of the study.

Purposeful Selection of Participants

The participants are both a purposeful and convenient sample. The sample was convenient in that the participants were students in a class within which the researcher was a Teaching Assistant (TA). Purposeful sampling is a non-probability sampling technique in which participants are selected based on specific criteria. The criteria for participating consisted of being a student in a particular senior level methods course at an Eastern U.S. doctoral granting public institution. Purposeful sampling was utilized in this study so that there were "information-rich cases related to the phenomenon of interest" (Palinkas et al., 2015, p. 533).

The PTs responses to the questionnaire at the beginning of the semester were the main filter used to narrow the group to six participants. Their responses to questions about their perceptions of mathematical activity and ability were printed and then labeled on the back with the question number and another number representing the participant who had responded. Two were intentionally removed before the process began. As there was one lone graduate student in

the class and one repeater, it was decided to focus just on undergraduates who were taking the course for the first time. The key for the 11 students was set aside before cutting began, although #4 and #8 were noted to be removed as they were the two participants who were cut before this filtering process began. The responses of the nine remaining participants were spread out and examined for key words that might indicate inclusivity/exclusivity mindsets. Those words were highlighted and then those responses were sorted into like categories. If more than one theme was in a response, the response could be in more than one category.

An Excel spreadsheet was created to manage the sorting of their responses. Across the top were the questions the participants had been asked and then underneath each cell contained themes and the numbers (of participants) that were categorized in that theme. For example, under the question "What does it mean to be smart in mathematics?" the themes noted were: performing focus (5,7), extrapolate (2, 3, 10), different ways (11), understand (1, 6), and growth mindset (1, 11), problem solver (2, 3, 9, 10), persevere through "wrong" (3, 9). In addition to their coded number, a snippet of their response was included.

From the onset, the goal was to identify two highly inclusive, two mediumly inclusive, and two less inclusive. However, it became evident that selecting participants who were in the middle was no easy task. So, three highly inclusive and three less inclusive were selected. The reason for selecting the two opposite ends of the spectrum was to capture what may be more representative of experiences of a variety of students. The first two selected were #3 and #9 because in describing their "best day teaching math" they valued discussion. Their other responses were examined, and they were both observed to consistently have responses in alignment with inclusive views.

The next trend noticed was three participants (#1, 5, and 7) who viewed mathematics from a more traditional teacher-centered perspective. In reviewing their responses, they were less consistently exclusive across questions. For example, #5 had the most student-centered response about having students explain when work is correct in that he communicated the value to the *student* (as opposed to informing the teacher). But, overall, those three participants described mathematical activity with a focus on performance and correctness. This left one more participant to group with the more inclusive group. While there was a participant selected, that person ended up being swapped out so that the final six consisted of three females and three males.

While data was being analyzed, the six participants were further reduced to three. One of the six decided to withdraw from the program and it was considered that her participation might not have been authentic since she may have had one foot out the door in her mind during the rehearsals. Another was excluded because after examining the data, the participant's responses suggested he was humoring everything at the surface level. And the third was later excluded because her data was not as representative of the work of the class.

After data was collected, the participants were reduced to three. Each participant was given a pseudonym to maintain confidentiality. The three participants are:

- Jackson (#1): Low inclusivity; Described teaching math as "to communicate these rules"
- Willow (#3): High inclusivity; Best day teaching would "include a lot of discussion among my students and using their own ways of doing math that they have developed"
- Harry (#5): Low inclusivity; When student incorrect: "offer them to consider a different method and that they may find more success that way"

Context

The course syllabus described the class as focusing on "purposes, methods, curricula, and evaluation practices for teaching mathematics in the middle and high school." While in this course, the PTs were simultaneously in another course which placed them in the field one day each week, so they had the opportunity to see a teacher in practice regularly. In the methods course, in addition to other assignments, the PTs had several opportunities to "practice" being a teacher through role play. The PTs engaged in this practice through a virtual simulation (Mursion) and through rehearsals. The opportunity to see the PTs practicing being a teacher in rehearsals was one of the main reasons this context was chosen. The other was the course's focus on equitable teaching practices. One of the stated learning outcomes for the course included applying and reflecting on equitable and student-centered teaching practices. Additionally, the researcher had been the TA in this course the previous fall semester and was already familiar with the content and structure of the course. Thus, the context of this particular methods course was selected because of the opportunities to practice being a teacher combined with the intentional focus towards shifting towards more equitable teaching practices.

In this methods course, there was a focus to support PTs in actions and beliefs that align with ambitious teaching practices. The first two days of class, the participants read the first couple of chapters of Mathematical Mindsets (Boaler, 2008) and Strength in Numbers (Horn, 2012). Both chapters in both books work toward shifting their interpretations towards students to be more inclusive in general. The second chapter in each book presents equitable teaching practices. As one approach to changing behavior is that one's beliefs must be changed first because they act as filters to what one sees (Pajares, 1992, as cited by Philipp, 2007), the

readings early on work towards shifting beliefs. Below is an overview of weekly topics across the course of the semester.

Table 2 *Methods Course Outline*

Week	Topic
1	Funds of Knowledge/ Growth Mindset
2	Equitable Teaching Practices
3	Classroom Management: Setting Norms, Rules, and Procedures Social-Emotional Learning
4	Classroom Procedures, Scenarios
5	Mursion teaching practice, Classroom Environment Philosophies
6	Unit planning, language demands (edTPA)
7	Group roles, planning a learning segment
8	Supporting students with special needs/Emerging bilinguals, Language supports
9	A framework for instruction: Launch/Explore/Discuss and the Five Practices
10	Rehearsals: Launching a high demand task
11	Rehearsals: Monitoring Topic: Monitoring, questioning, and talk moves
12	Teacher Discourse Moves (finishing Monitoring rehearsals)
13	Rehearsals: Facilitating Whole Group Discussions
14	Assessment
15	Professional Growth and PS 2 Information

The undergraduate mathematics education program focuses on a combination of mathematics courses, education courses, and mathematics education courses with field experiences increasing in duration as students near graduation. The PTs complete a minimum of 40 hours of advanced mathematics courses. There are also three general education courses, one

in each of their first three years. The program includes five courses focused specifically on mathematics education, four of which occur in the last two years. They complete 65 hours of field experiences as part of their coursework before their student teaching experience in their final semester. Figure 3 below shows the suggested timeline of core courses and their connected field experiences.

Figure 3 *Timeline of Core Courses*

	Fall	Spring
Freshman	ED 100: Intro to Education	
Sophomore	EMS 204: Intro to Mathematics Education *[class includes a 10-hour field placement]* ED 204: Intro to 21st Century Teaching	
Junior	ED 311: Classroom Assessment Principles & Practices ED 312: Assess. Principles & Practices: Prof. Learning Lab *[class includes a 10-hour field placement]* ELP 344: School and Society	EMS 472: Teaching Math Topics in High School *[class includes a 15-hour field placement]* EMS 480: Teaching Math with Technology ECI 416: Teaching Exceptional Students
Senior	**EMS 470: Methods & Materials for Teaching Math** *[class includes a 30-hour field placement]* EMS 490: School Math from an Adv. Perspective	EMS 471: Student Teaching EMS 495: Seminar

A rehearsal is an approximation of the practice of teaching in that it is a small, focused amount of time during which a PT plays the role of teacher focusing on a specific IA (Lampert et al., 2013). In this methods course, the three rehearsal the PTs utilizes for practice focuses on the IA of launching, monitoring, and facilitating whole group discussion of the task. The task is

loosely referred to as the zip line problem as the problem is optimizing where to locate the end of a zip line on an island that starts at two towers on land on either side of the island. During the rehearsal, one PT will act out the role of the teacher launching the task while four other PTs will play the role of students. The "students" are randomly handed a role to play in response to the task. Specifically, each student role has a barrier to the problem and a method to start the problem that might be consistent with what a teacher might experience in a classroom. The zip line task is available in Appendix E. Student profiles for the Launch rehearsal are in Appendix F, for the Monitoring rehearsal in Appendix G, and for the Discussion rehearsal in Appendix H.

Methods

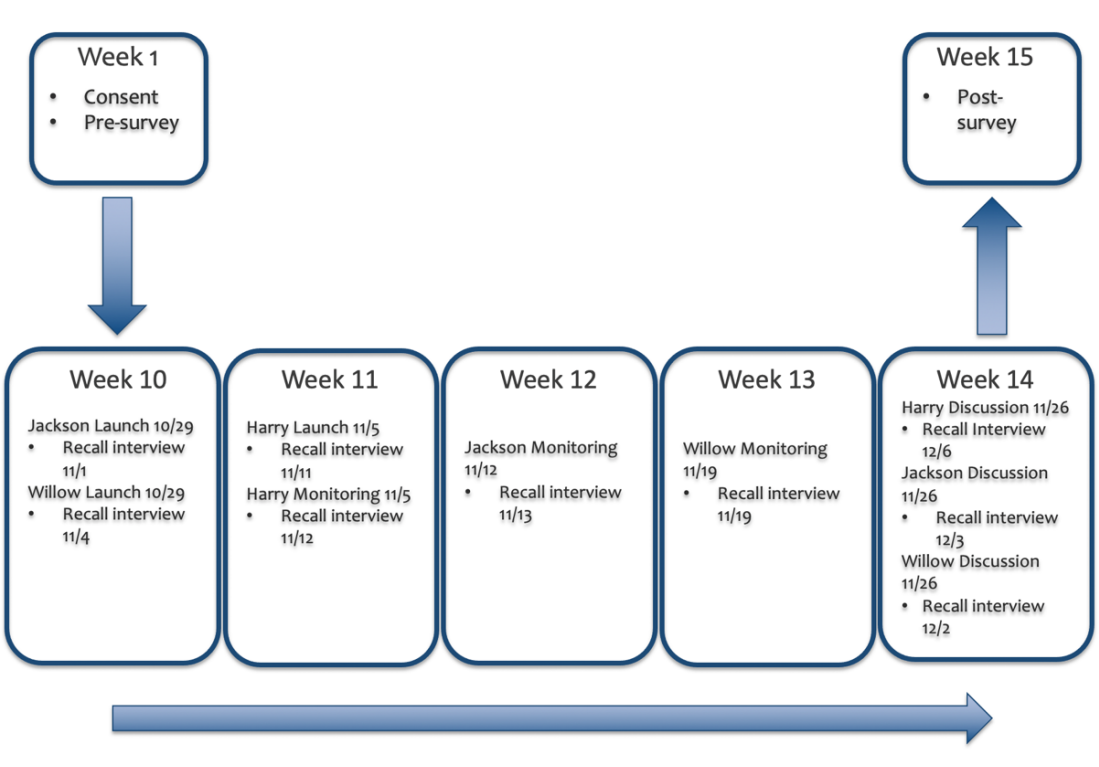
The design of this study focused on PT interactions with student thinking during rehearsals. The rehearsals were part of the methods course in which there are readings and other activities to encourage the PTs to consider broader definitions of mathematics and what it means for students to be smart. To gain insights on how the PTs' perspectives may have shifted across the course, there was a questionnaire at the beginning and end of the course that asked them to share their opinions on mathematical activity and ability. There were three rehearsals in which each PT played the role of teacher once, while observing peers or playing the role of a student when it was not their turn to teach. Following the rehearsal, the whole class debriefed the PT's facilitation of the rehearsal. Typically, this was started by the TE asking the PT what they thought about their rehearsal to allow them to reflect before others spoke up. Soon after the PT plays the role of teacher, the author met with the PT for a stimulated recall interview. The primary sources of data for this study are the pre/post class questionnaire, transcripts of videos, and transcripts of interviews while secondary sources included audio recordings of the rehearsals.

Initially the plan was for there to be three days of rehearsals, one day for each type (Launch, Monitor, Discussion) of rehearsal. However, the first day there was not enough time for everyone to have their turn playing the role of teacher, which resulted in Harry doing his Launch rehearsal and Monitoring rehearsal on the same day. Willow was sick one day and so her Monitoring rehearsal was done a week later in connection with her not feeling well enough to play the role of teacher the week she should have otherwise had a turn.

The goal with stimulated recall interviews was to do them as soon as possible after the corresponding rehearsal so that the rehearsal was still somewhat fresh in their memories.

Below is a summary of data collection timeline and procedures.

Figure 4 *Data Collection Timeline*



At the beginning of the course and at the end of the course, the students responded to a Google Form to learn more about their perceptions related to teaching mathematics. Questions were created using Louie's (2017) ways of framing mathematics and mathematical ability as a

guide. The questions asked them six questions to learn more about their professional views before and after the course. Their responses were compared and presented at the beginning of each case to add depth to their cases. The questions asked in the questionnaire are available in Appendix A.

On days the methods class was doing rehearsals, two video recording devices were set up and the record button was pushed before the researcher and the other half of the class left to do rehearsals elsewhere. There were two video cameras used for each rehearsal. One was placed approximately six feet behind the students on top of a student desk that included a view of the students, then teacher and the board. Another camera was placed by the board and was focused on the students who might be looking in the direction of the teacher.

The recordings of the rehearsals captured how the PTs responded to student thinking during the rehearsal, but they also recorded the discussions during TE pauses and the debriefs after each rehearsal. The rehearsal and related discussions were recorded and transcribed. Most of the analysis of how PTs responded to student thinking was based on the transcripts of the rehearsals. The video aided in capturing what was written on the board by the teacher and students and contributed to the analysis in connection with how student thinking was supported in connection with modeling with mathematics.

Another source of data was the stimulated recall interviews. The corresponding rehearsal was transcribed, and a list of questions related to what the PT had done was created before going into this interview. Both the researcher and the PT had a copy of the transcribed rehearsal to review in addition to the video playback. These interviews involved sitting one on one with the PT who played the role of teacher and looking back at specific moments of their recorded rehearsal to ask what their motivation was when they asked a particular student a question or

responded to a student in a particular way. The videos of the interviews were also transcribed for analysis. As action and intent were not always perfectly aligned, the goal was to gain deeper understandings of what they were working towards, particularly in relation to equitable practices. The researcher also hoped that whatever inhibitions participants might have regarding engaging student thinking would be exposed through these reflective interviews to learn more about how to support them in future practice.

Audio recordings were also collected of each rehearsal. Before class started an audio recorder was placed in the middle of the desks of the students. The audio recordings were not a main data source but were used to aid with transcriptions when the video failed to clearly capture what participants had said.

Data Analysis by Research Question

RQ1: How do PTs support student thinking during rehearsals?

a. How inclusively do PTs frame mathematical activity and mathematical ability when supporting student thinking during rehearsals?

First the rehearsals were transcribed by the researcher before the stimulated recall interview took place. Before coding began, those transcriptions were reviewed a couple more times just to ensure the rehearsal had been properly captured in the transcription. The three PTs' rehearsals were coded using ATLAS.ti to establish the different ways the PTs supported student thinking during rehearsals. The a priori codes consisted of Herbel-Eisenmann et al.'s (2013) TDMs (waiting, inviting student participation, revoicing, asking students to revoice, probing a student's thinking, and creating opportunities to engage with another's reasoning) since the PTs had recently learned about them and it was expected they would implement some of them. Emergent coding was also used to note other moves that did not fit into that original list of

TDMs. Additionally, early on it became apparent that inviting student participation was too broad of a category and so that category was further broken down into three separate inviting moves: inviting student thinking (asking an open-ended question), inviting to do (asking student to read, draw, etc.), and inviting identify (asking a closed-ended question. This was especially necessary given the nature of the frameworks that would be used in connection with RQ1a since inviting a student to respond to a closed question falls into a different level than inviting students to respond to open-ended questions on Louie's (2017) framework.

After one PT had been coded for three rehearsals, the researcher sought out the assistance of a colleague to co-code so that the results would more accurately reflect the data. After attaining IRB approval, a codebook and a brief overview of the study were sent to the co-coder in advance of a meeting in which they coded two rehearsals together. During their first meeting, one additional code was added: affirmed correctness. They then separated and each individually coded the rest of the rehearsals. They met and discussed their codes for the different PTs and their rehearsals. Whenever there was disagreement, they discussed until agreement was met.

To answer the research question, an Excel spreadsheet was created that listed the different moves in the rows. One X was marked in each column as the moves were used over time in the rehearsals. These Xs were then tallied using Excel's count function to create tables with total moves per rehearsal. The different moves were presented in charts to address the relative distribution of the moves by each PT. The original Excel spreadsheet was also analyzed for patterns of moves. Since the Xs showed the order of the moves over time, patterns of combinations of talk moves could also be analyzed. For deeper discussion of each rehearsal, the different talk moves were color-coded and put into a table to see holistically how the PT used

talk moves throughout the rehearsal. Additionally, there was a deeper analysis of instances when the PTs responded to instances when students spoke up with more than a recall response.

This analysis was done in connection with the deeper analysis of instances when the PT responded to student thinking. The specifics of which moves the PT used in addition to whether they were valuing correctness or thinking and whether they were positioning students to learn from each other were analyzed and discussed.

RQ2: What intentions do PTs have behind the decisions they make during rehearsals?

a. How inclusively do PTs frame mathematical activity and mathematical ability when reflecting on their intentions during rehearsals?

The second research question was mostly addressed through analysis of the transcribed stimulated recall interviews. The interviews were transcribed using a transcription service, Temi. After those initial transcriptions were edited to ensure they accurately represented the PTs' responses, the transcripts were coded using ATLAS.ti. Stockero and Van Zoest (2013), when reflecting on PTMs, suggested that "additional research should include interviews with teachers that will provide more complete information about not only whether they noticed a PTM during instruction, but the thinking behind their decisions to either act upon or ignore PTMs" (p. 145). The purpose of the stimulated recall interviews was to learn more about why PTs make the decisions to respond in specific ways.

Emergent coding was used to note themes for why the PTs responded the way that they did. After the first cycle of coding of one participant, Harry, it became apparent that the codes needed some sort of organization that was not obvious. Some examples of codes included influenced by another rehearsal, intent learn student approach, desired math outcome, contextual goal, and barrier focused. Further research was done on how others had interpreted PTs in

rehearsals and the Hammerness et al. (2005) Learning to Teach in Community framework was found to be a good fit for organizing the themes that had been coded. The codes from the first cycle were then organized into categories that aligned with their categories of dispositions, practices, tools, understanding, and vision.

After having organized many of the codes using the framework, a new file was created in ATLAS.ti to code Harry freshly using the new approach to coding. As a focus on those particular categories had not been used initially, it was important to give Harry's responses a fresh eye through those lenses to notice what additional codes emerged. His Launch interview was coded first. After new codes from the Monitoring interview were added, the previous interview was revisited for those new codes. This process was repeated for the Discussion interview. Jackson was coded next with the same iterative process. Once Jackson's three interviews were coded, Harry's three interviews were revisited for those additional codes. This process was repeated once again with Willow's three interviews. The Practices and Dispositions were organized into themed categories to better describe why the PTs responded to student thinking the way that they did.

This analysis was done based on the interviews. In addition to the theme codes in connection with the Hammerness et al. (2005) framework, there were codes that connected to inclusive/exclusive framing. Those codes were answer/solution focus, attention to misconception, doing mathematics, mistakes desirable, students explaining (for their sake), valuing correctness, and more knowledgeable other. In addition to the coded practices and dispositions, the inclusive/exclusive coded instances were reviewed when reflecting on the inclusivity of the PTs' practices and dispositions.

Cross-case Analysis

Since each of the PTs implemented different quantities of talk moves, the relative frequency of their total implemented moves was compared. A chart was made with color coding to highlight their three most and least frequently used moves. Additionally, their individual quirks in connection to the different talk moves were analyzed to present a qualitative description of the PTs' implementation of the moves. Lastly, their patterns of talk moves were examined to note patterns that were common to all of them and patterns that were unique.

A combination of the definitions of different talk moves along with analysis of their implemented moves were used to decide where the PTs' implementation of the talk moves fell on Louie's (2017) framework. Some were easier to place than others. For example, student revoicing was decided to be inclusive ability since the nature of the move positions a student to learn from another. The PTs instances of student revoicing were still glanced at to confirm, but some moves align directly with the framework. Another example that was relatively easy to categorize was inviting thinking moves. Inviting thinking moves frequently either consisted of "how" questions, "why" questions, or a request for an explanation, which aligns well with "asking open-ended questions and pressing for meaning" (p. 496) from the sense-making frame. However, the code positive evaluation was less clear at a glance, so a list of all their responses was analyzed to make sense of how clear the PTs were about what they were evaluating. The PTs were not that clear and so those moves were thought of as slightly inclusive because while their focus tended to be more on ideas and questions than answers, the focus of the framing was unclear.

To examine commonalities and differences across the three PTs, Venn diagrams were made of both their practices and dispositions. Categories within the practices and dispositions were color coded to aid in seeing trends in connection with the different types.

Similar to the analysis of talk moves in research question, certain practices and dispositions more naturally aligned with framing within Louie's (2017) framework. Again, analysis included comparing the nature of the practice or disposition compared to Louie's framework compared to how the PT spoke about the practice or disposition. For example, the practice of pursuing student thinking was easier to categorize as more inclusive since it positions students as capable sense-makers and thus frames mathematics and mathematical ability inclusively. However, other practices were less clear. For example, the PTs all checked in with their students frequently and so how they talked about checking in was further analyzed to categorize the level of inclusivity of the framing.

Subjectivity Statement

Before starting on the doctoral program, I worked one year as an instructional coach in which researched best practices became a new normal in my world. The teacher support work opened my eyes to the qualities of teaching that served to empower students mathematically, but also made me more aware of the disparities across classrooms and schools. Part of my work as an instructional coach was supporting mathematics teachers to experience student-centered instruction through the facilitation of tasks from the Mathematics Assessment Project. The power of hearing students making sense of mathematics influenced my beliefs in such a way that I went into my doctoral studies convinced students need to *experience* mathematics by doing, discovering, and discussing.

The other aspect of my positionality is the focus on being more inclusive. Even within one classroom, there are huge disparities among which students are valued for contributing to math discussions. If the shift to valuing student thinking as a quality contribution can be more pervasive, then more students can be positioned as quality contributors to the discussion. While idealistic, inclusive framing while supporting student thinking is one path that has potential to break down math as a gate keeper class and shift it toward being a steppingstone to opportunities.

During the collection of data, the researcher worked with participants as their TA in the methods course. Due to the size of the class, the students were split in half for rehearsals and the group not participating in the study did their rehearsals with the researcher. So, the researcher was not in the classroom experiencing or influencing anything in connection with the recorded rehearsals. There is the argument that the PTs may have enacted their rehearsals in a way to please their professor and the TA. However, since the rehearsals were to collect evidence of what is in their capacity to implement, their motivation relative to the course had no bearing on the data collected.

Another possibility is that the PTs may have responded during the interviews in a way that they believed was in alignment with the expectations of the course. This is certainly a possibility, but as the interview had both video and a transcription, the PTs had to respond to the reality of what happened and so there was no room for contrived memories. And as there were many questions asked during the interview to learn about their intentions, it is unlikely that their responses were contrived to align with perceived “right answers.”

While analyzing the data during the spring semester, the researcher ended up being assigned to be the university supervisor for two of the participants. This resulted in biases

growing in different directions about the practices of these two PTs as the researcher watched them grow in their teaching practice in classrooms. For this reason, the researcher was diligent in always referring back to the collected data and codes so that analysis was not influenced by more recent memories of their teaching.

Ethical Considerations

The researcher presented an overview of the study with straightforward descriptions regarding participation, including positives and negatives. As the researcher does play a role as an instructor in the methods course in which the study is took place, it was stressed that participation was completely optional. Furthermore, as rehearsals were part of the norm of the class, it was communicated that videos of rehearsals would only be used for the purpose of research, not as a tool to better assess their "performance." The participants were also given copies of the consent form both via email and through their LMS so that they could revisit it at their convenience. The researcher communicated that they could withdraw at any point in time. Most data collected was digital and thus directly put into a password protected cloud folder that only the researcher had access to. The one access exception was the sharing of the transcriptions (using pseudonyms) of the rehearsals that were shared with the colleague for co-coding. The few paper copies of data were filed in a locked cabinet. Regarding protecting their identities, pseudonyms were used on collected data.

Issues of Trustworthiness

The researcher's trustworthiness and credibility will be established through a variety of measures. One type of validation includes triangulation, the use of multiple data sources, so that the researcher can look "for recurring behaviors or actions and considers disconfirming evidence and contrary interpretations" (Creswell, 2007, p. 204). The recorded rehearsals, post-rehearsal

debriefs, and stimulated recall interviews provided three different opportunities to review PTs' actions and intentions related to supporting student thinking.

Limitations and Delimitations of the Study

This study focuses on the responses of future teachers in a contrived environment, and thus is not representative of what they might do in a classroom full of students. Furthermore, the PTs were responding to students in the context of task facilitation, which created better opportunities for engaging with student thinking than the everyday classroom might. This study is proof that some PTs have the capacity to incorporate inclusive support of student thinking given certain conditions. Additionally, this study is not generalizable other than the insights related to their experiences that might aide teacher preparation programs.

Additionally, while research suggests that working toward inclusive framing creates better opportunities for students, there is limited research that includes the experience of students. Studies have shown that teachers can improve noticing skills through reflecting on video, but less is known about how that connects to classroom practices.

Chapter 4: Analysis of Harry

Harry will first be presented in relation to his responses to questions concerning students' mathematical activity, mathematical ability, and student thinking to present a foundation of Harry's perceptions of teaching and learning mathematics. To address how Harry supported student thinking during rehearsals, an overview of Harry's experience through the three rehearsals will be described, including summative data describing his talk moves. Then for each rehearsal, Harry's moves in response to students will be analyzed followed by a more in-depth analysis of how he responded to student thinking and a discussion of how his moves connected to the two frames. To address the second research question of why Harry does what he does, themes in connection with the Hammerness et al. (2005) framework will be presented. At the end, there is a discussion of Harry's themes in connection with his framing of mathematics and mathematical ability.

Harry's Perspective on Mathematics and Mathematics Teaching

To understand some about the PT's views on mathematics and teaching mathematics, his responses to a questionnaire were considered with how they align with Louie's (2017) "Ways of Framing Mathematics and Mathematical Ability." Since later his interactions with students will be discussed, it is important to understand what he says about mathematical activity, mathematical ability, and how to support student thinking.

Harry and Mathematical Activity

Harry's perspectives on what counts as mathematical activity toggled across both the exclusive and inclusive frames for the nature of mathematical activity. In general, his focus on numbers and operations suggests that Harry falls into Louie's exclusionary rote-practice frame that describes mathematics as "a fixed body of knowledge to be absorbed and practiced."

However, there seems to be some leaning towards inclusive thinking in that Harry perceives mathematics as a describer of the world or environment. Twice he utilized the word "describe" in connection to the world/environment/events suggesting he perceives mathematics as a contextual tool to explain the world. As the sense-making frame includes "understanding connections," some of Harry's statements suggest that he sees mathematics as the connection between numbers and their environment, but his descriptions do not focus enough on sense-making to be sure that is what he means by describing. So, Harry does not think of mathematical activity as focused only on rote practice nor is there evidence to suggest that he has embraced mathematics as sense-making.

Harry and Mathematical Ability

Harry was less clear when describing his thoughts on mathematical ability. Before the methods course, he described being smart in mathematics as being able "to logically think about numbers in context and successfully perform operations." The first half of his response sounds like sense-making and thus leans towards inclusive mathematical ability. However, a focus on successfully performing operations leans towards the exclusive side of mathematical ability as he is explicitly valuing correctness of procedures. After the methods course, he stated that being smart in mathematics is "to understand the relationships between numbers, logic, and operations such that they can apply the knowledge to the world around them." While his later description still mentions operations, the focus seems to be more on understanding connections and applying those understandings. His heavier focus on sense-making instead of successfully doing operations suggests he is shifting toward inclusive thinking regarding mathematical ability.

Harry and Responding to Student Thinking

Harry had varied ways of responding to students based on whether they presented a correct response or not. Both before and after the course, he averred that it is important for students to provide mathematical explanations when they have answered a problem correctly. His later response went into further detail about how he values the explanation. "It is important to provide explanations so that they can reflect on their process, provide me with information about potential misconceptions, and it is a way for students to check themselves." His viewpoint on how to respond to student's incorrect responses progressed. Before the methods course, in response to how to handle a student's incorrect response, he said to "respectfully offer them to consider a different method and that they may find more success that way." However, after the methods course, Harry stated that he "would look at it positively and explain the importance that incorrect answers can be used as a means of growth." Instead of redirecting he suggested valuing the thinking of the student and viewed the mistake as valuable to student learning in general. Across the semester, Harry's valuing of all instances of student thinking was shifting toward being more inclusive.

Moves Implemented by Harry to Support Student Thinking During the Three Rehearsals

One way of making sense of how Harry supported student thinking during the rehearsals was to analyze his talk moves. The transcript of the rehearsal was coded to see what exactly Harry was doing when he was speaking to his students. First, there is a brief description of when Harry played the role of teacher in connection with other rehearsals. Then there is an overview of Harry's talk moves across all three rehearsals. Then each rehearsal is described followed by in-depth analysis and descriptions of the moves utilized by Harry.

Table 3 *Harry's Rehearsal Structure*

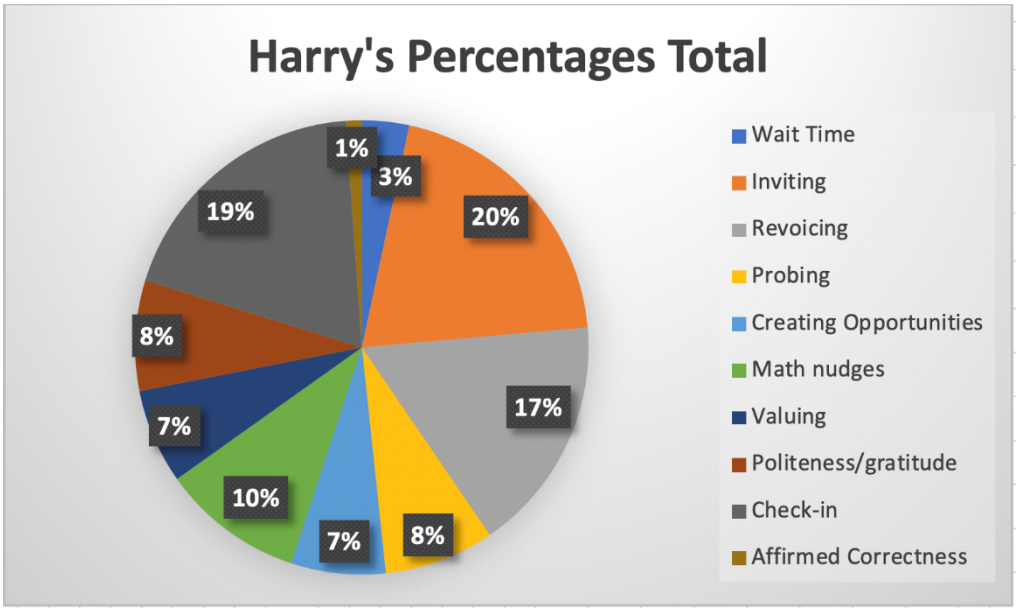
Order	Date	Teacher	Student	Student	Student	Observer	Observer
Launching 1	10/29/2019	Jackson	Harry	Willow	Wesley	Stacy	Kayla
Launching 2	10/29/2019	Willow	Jackson	Kayla	Stacy	Wesley	Harry
Launching 3	10/29/2019	Stacy	Jackson	Harry	Wesley	Stacy	Kayla
Launching 4	10/29/2019	Kayla	Harry	Willow	Stacy	Wesley	Jackson
Launching 5	11/5/2019	Harry	Kayla	Willow	Wesley	Stacy	Jackson
Launching 6	11/5/2019	Wesley	Kayla	Jackson	Stacy	Willow	Harry
Monitoring 1	11/5/2019	Kayla	Harry	Willow	Jackson	Stacy	Wesley
Monitoring 2	11/5/2019	Harry	Stacy	Wesley	Jackson	Willow	Kayla
Monitoring 3	11/5/2019	Wesley	Kayla	Harry	Jackson	Stacy	Willow
Monitoring 4	11/12/2019	Stacy	Kayla	Welsey	Harry	Jackson	Willow (partially there)
Monitoring 5	11/12/2019	Jackson	Wesley	Stacy	Kayla	Harry	
Monitoring 6	11/19/2019	Willow	Stacy	Wesley	Kayla	Jackson	Harry
Discussion 1	11/19/2019	Wesley	Harry	Willow	Kayla	Jackson	Stacy (co-teacher)
Discussion 2	11/19/2019	Stacy	Harry	Willow	Kayla	Jackson	Wesley (co-teacher)
Discussion 3	11/26/2019	Harry	Stacy	Willow	Wesley	Kayla	Jackson (co-teacher)
Discussion 4	11/26/2019	Jackson	Stacy	Willow	Wesley	Kayla	Harry (co-teacher)
Discussion 5	11/26/2019	Willow	Jackson	Harry	Stacy	Wesley	Kayla (co-teacher)
Discussion 6	11/26/2019	Kayla	Jackson	Harry	Stacy	Wesley	Willow (co-teacher)

Rehearsal Structure

Harry was the fifth PT to play the role of teacher in the launching task. Ideally, all six students would have done the launching task on the same day, but they ran out of time. So, while Harry was fifth to go, he was the first to play the role of teacher on the day that he had his turn. In an effort to mix up the order, he was scheduled to go second for monitoring rehearsals. This resulted in Harry playing the role of teacher for both launching and monitoring rehearsals on the same day. For the discussion rehearsal, he was in the second pair of students and again that ended up meaning that they went first on a different day. Within his pair, he was the first to be the teacher. While he was not technically first to play the role of teacher for any of the rehearsals, the fact that he went first on two of the days and second overall for the monitoring rehearsal suggests that he was not able to implement any insights from observing peers that went before him.

Talk Moves Across All Three Rehearsals

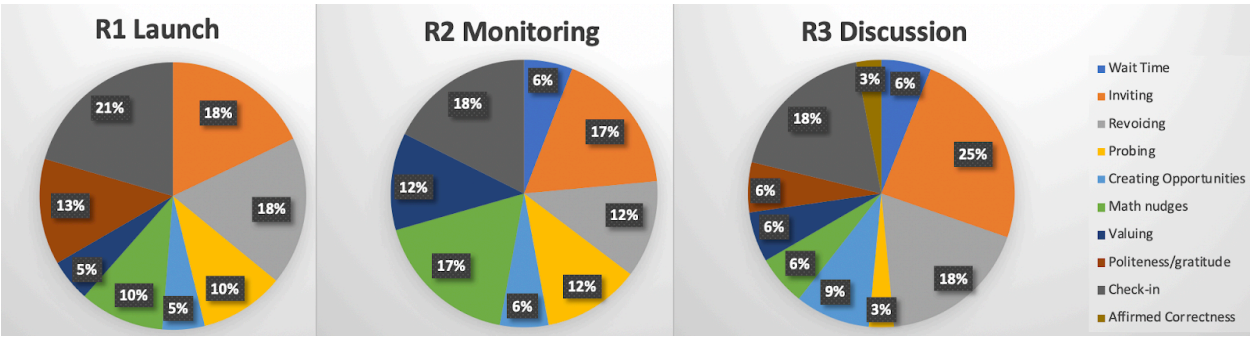
Figure 5 *Harry's Percentage of Talk Moves Across All Three Rehearsals*



In total, Harry implemented 89 moves to support student thinking across the three rehearsals. Harry implemented 39 moves to support student thinking during the first rehearsal (launching the task), 17 moves during the second rehearsal (monitoring), and 33 moves in the third rehearsal (class discussion). Harry used inviting (20%), checking-in (19%), or revoicing (17%) for more than half of his total moves to support student thinking. Math nudges were used for 10% of his moves. Harry was explicitly polite as frequently as he probed student thinking (8%). He created opportunities for students to engage with each other's thinking as frequently as he explicitly valued student thinking (7%). Harry exercised wait time for 3% of his moves and affirmed correctness 1%.

Comparing Relative Usage Per Rehearsal

Figure 6 *Harry's Relative Usage of Talk Moves Per Rehearsal*

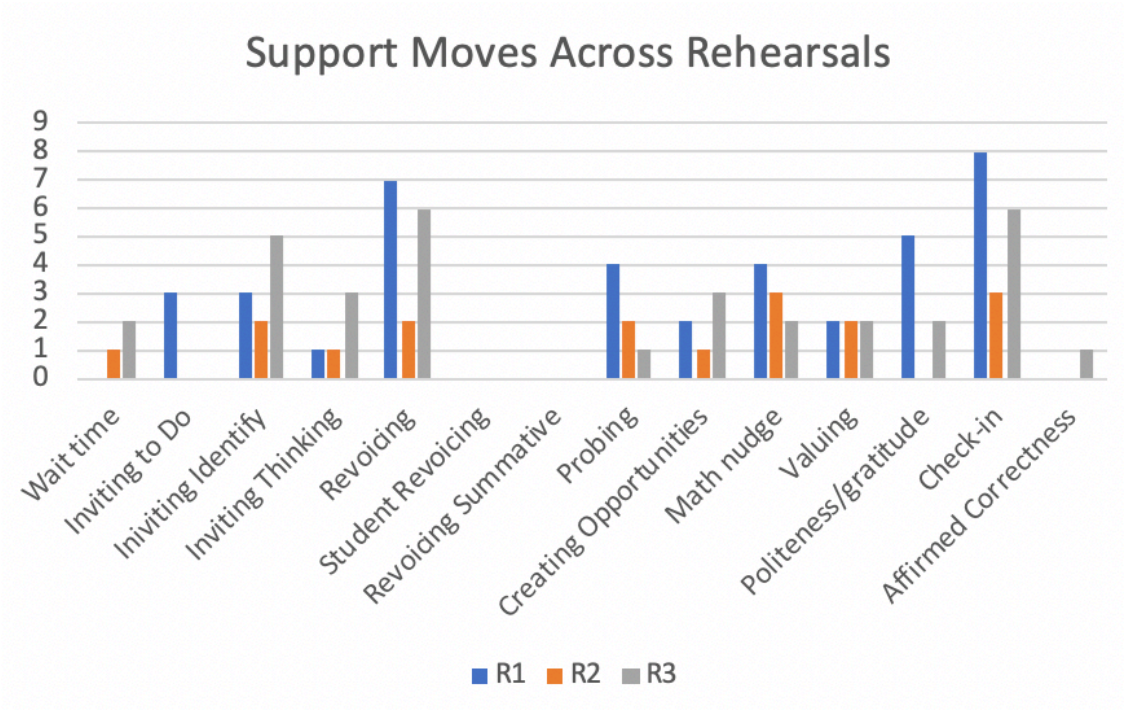


As each rehearsal was for a different amount of time, one approach to analyzing his implementations is to notice how he used moves relative to other moves within the same rehearsal. In all three rehearsals, the combination of inviting, revoicing, and checking-in make up more than 50% of the total moves during the rehearsal. At a glance, there were large variations among his usage of probing questions and math nudges.

Analysis Across Rehearsals

In the graph below the different rehearsals are represented by different colors for each talk move to note the variation among the talk moves among rehearsals. Inviting was further broken down into three different scenarios: inviting students to do something (e.g., go to the board, read the task) which was coded as Inviting to Do, inviting students to identify (e.g., what do these numbers mean) which was coded as Inviting Identify, and inviting students to share their thinking (e.g., asking for an explanation) which was coded as Inviting Thinking. Then below there is a brief discussion of which talk moves Harry used with an increase in frequency, inconsistent frequency, and a decrease in frequency. Then there is a discussion of Harry's patterns of usage of these various talk moves.

Figure 7 *Harry's Talk Moves Across Rehearsals*



In the graph above the different rehearsals are represented by different colors for each talk move to note the variation among the number of talk moves among rehearsals. Harry's use of wait time increased across rehearsals in that there were no instances in the Launch, one in Monitoring, and two in the Discussion rehearsal. Harry's use of probing decreased across rehearsals as did his use of math nudges. Some moves were uniquely used in one particular rehearsal. Harry used Inviting to Do only during the Launch and Affirmed Correctness only during the Discussion. Harry was consistent in valuing student thinking; he did so twice during each rehearsal. There were no instances of Harry doing any summative revoicing or asking a student to revoice. Harry invited student thinking once during each of the first two rehearsals, but three times during the Discussion.

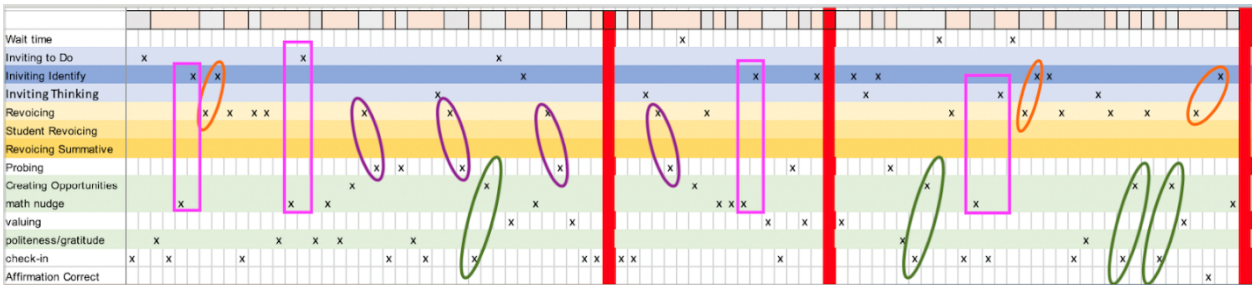
There were several moves that Harry implemented with a frequency that corresponded to the lengths of the respective rehearsals. Harry spent the most amount of time interacting with students during the Launch rehearsal (8 minutes and 46 seconds), least during Monitoring (4

minutes and 12 seconds), and in between during the Discussion (6 minutes and 20 seconds). This was true for Harry's implementation of revoicing, politeness, and checking-in. However, there were two moves whose implementation in the Discussion was greater frequency than in the Launch. Harry invited students to identify five times during the Discussion as compared to three in the Launch and two in Monitoring. Harry also created opportunities for students to engage in each other's thinking with greater frequency during the Discussion. He created opportunities three times during the Discussion, twice during the Launch, and once during Monitoring.

Patterns of Talk Moves

After looking across time, the researcher became curious to see in what patterns the moves might be executed. Sometimes several of these talk moves occurred back-to-back whereas others stood alone. The red vertical lines separate the three rehearsals. The color-coding at the top of the graphic separates teacher discourse chunks from when students had an opportunity to say something. Very specifically, the cells at the top change color each time a student says something. The goal was to see if there were any trends with how Harry used the talk moves rhythmically.

Figure 8 *Harry's Patterns of Talk Moves Across Rehearsals*



There were a few interesting trends. In more than half of the instances that Harry probed, he first revoiced student thinking before probing. They appear above with the purple oval

around the paired moves. For example, if a student said, "in the middle," he would respond with "In the middle, why in the middle?" Harry also had a tendency to check-in with student thinking just before creating opportunities to engage with another's thinking. For example, in the Discussion he asked, "Does everyone see how she did that?" before continuing with "so since she started out with 300 and 300, where would the island be in the lagoon if both distances was 300 meters away from the bank?" These are highlighted by the green ovals. Harry also invited students to identify on three occasions after revoicing as shown by the orange ovals. After nearly half of his math nudges, he invited his students to participate in some way, marked above with the pink rectangles. Harry's largest rhythmic trend was to pair revoicing with some sort of invitational move (as probing is also an invitation to share thinking).

Analyzing Harry's Talk Moves Per Rehearsal

While it is worthwhile to look at patterns of Harry's talk moves, some of the trends might be explained due to the nature of the different types of rehearsals. For example, it makes sense that there are fewer math nudges during the third rehearsal as that is when the students are sharing their completed progress on the task. To understand more about what Harry actually does, it is worth analyzing each rehearsal separately and taking a more scrutinous look at how he supports instances when a student engaged in the discussion by sharing their thinking (as opposed to just all discourse).

First Rehearsal: Launching

The chart below shows the different talk moves that Harry utilized while playing the role of teacher. Each line represents the moves made by Harry when it was his turn speaking. The numbered lines are later referenced in connection with how the PT responds to certain instances

of student thinking. The TE pauses are noted on the left. Moves of the same kind were color-coded to aid in the sense-making of this data.

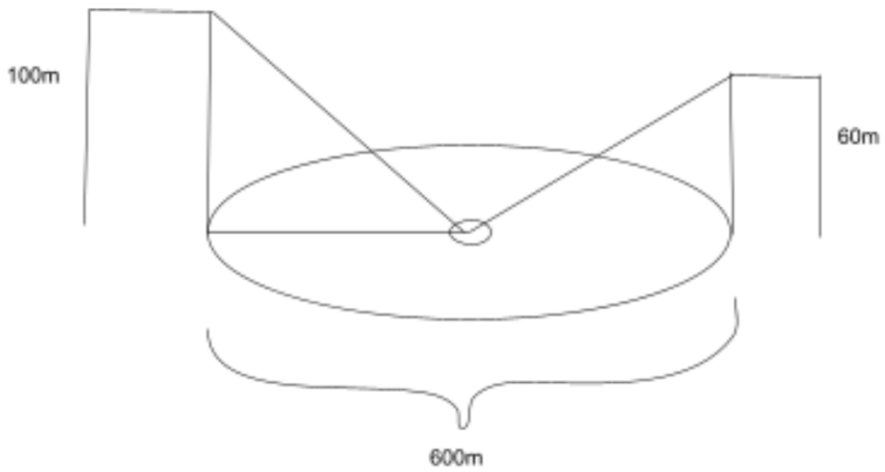
Table 4 *Harry's Talk Moves During Launch*

Start time: 3:48	1	check-in	inviting DO		
	2	polite	check-in	math nudge	invite Identify
	3	Revoice	invite Identify		
	4	Revoice	check-in		
	5	Revoice			
	6	Revoice	polite	math nudge	inviting DO
	7	polite			
(TE: 7:23 - 8:08)					
	8	math nudge	polite	create opportunities	
	9	Revoice	probing		
	10	check-in			
	11	probing			
	12	polite	check-in	inviting thinking	
	13	revoice	probing		
	14	check-in			
(TE: 11:09 - 11:53)					
	15	create opportunities	inviting DO		
	16	valuing	invite Identify		
	17	math nudge			
	18	revoice	probing		
	19	valuing	check-in		
	20	check-in			
End time: 14:03					

The launching rehearsal lasted a total of ten minutes and 55 seconds. During that time, Harry was paused twice by the TE for about 45 seconds. Initially, he started off by checking-in on their zip line knowledge and inviting students to read the task (Inviting to Do). He then gave them a math nudge to focus on the numbers as they identify key information (Invite Identify). Over the next three talk turns, Harry revoiced the student's shared key information and then typically asked another question related to identifying more key information. On the sixth talk turn, Harry revoiced once more before nudging them toward drawing a model. Harry then asked if a student wanted to come up and draw a model (Inviting to Do). At this point, the TE paused to ask about his expectations of the drawing and what he might do if the student's picture was not what it should be. After resuming the role of teacher, Harry created an opportunity for students to engage in the reasoning of another by asking the students whether or

not they agreed with the drawn image. After a student had drawn the below picture on the board, Harry asked "so does everyone agree with this picture?"

Figure 9 *Student Drawing Reconstructed for Clarity*



Following that, he probed students' thinking three times. First, in response to a student communicating that she did not agree with the picture, he asked that student "Why do you not agree with this picture?" Soon after he probed further into the thinking of the student who drew the picture by asking, "You drew the island kind of close to the middle. Do you think that the island is going to be in the exact middle or do you think it's gonna be further over here or further over there or?" And soon after a student raised her hand and stated that she thought the island should be in the middle, Harry responded with both a revoicing and a probe, "You think it should be in the middle. Why do you think it should be in the middle"?

The TE paused him a second time to ask him how confident he was that his students understood what the question was asking, to which he responded that he was not all that confident. After the pause, he started off by asking the students to reread the problem and then having the students talk to each other about strategies (Creating opportunities). He then valued a student's response by stating "I heard a lot of good discussion about how you would go about

solving the task" and invited more thinking by asking them to share some of the solutions they had come up with in the next talk move. He then nudged the students mathematically highlighting that the island doesn't have to be exactly in the middle. When a student suggested that it doesn't really matter, he revoiced again followed immediately by probing, "It doesn't matter. Why doesn't it matter?" The rehearsal then had two more talk moves in which he checked in with Willow and Kayla who each nodded in response to having said similar things (to Wesley). The TE then ended the rehearsal.

Harry's Support of Student Thinking during Launching the Task. The chart of the different talk moves suggests that Harry is adept at utilizing several of the talk moves and thus, facilitating mathematical discourse. However, without further examining exactly how Harry supports instances of student thinking and why he does what he does, the above is not adequately tending to how he used these moves to *support* student thinking. For the purpose of this analysis, student thinking was defined as a contribution from a student that consisted of more than a recall response. During the first rehearsal, Harry was exposed to five instances of student thinking, forcing him to have to make those on-the-fly decisions about how to respond. Below each of those instances is discussed in detail.

Kayla shook her head no [9]. After Wesley drew his model, Harry asked his students if they agreed with it. Kayla shook her head no. Harry initially attended to her disagreement by asking a probing question - why she doesn't agree with it. She responded to his probe by asking a question about the tower placement which he answered by referencing the task description.

Wesley said, "it doesn't matter" [12]. Harry responded with positive reinforcement because he believed Wesley was suggesting an idea that supported them exploring different options for the island's location. Harry lamented "I wish I would have asked him why does it

not matter." Later it became evident that Harry misinterpreted Wesley's statement, but in the moment, he responded with an "awesome. Does everybody get that?" So, he was polite and then checked in with his students in response to Wesley's suggestion.

Kayla said the island should be in the middle [13]. Kayla had raised her hand with a question to which Harry responded with a "yeah?," inviting her to share her thinking. She then volunteered that she thought the island should be in the middle. In response, he did his classic pairing of revoicing followed by a probing question. He responded to her suggestion with, "You think it should be in the middle. Why do you think it should be in the middle?" She went on to say she thought that because the towers are on opposite sides to which Harry eventually ended up pointing out that the island is "likely not going to be in the middle."

Students discussed amongst themselves [16]. Harry had his students talk amongst themselves for a few moments. During the discussion, Kayla mentioned using triangles and Wesley said something about the island being in the middle. When Harry heard that suggestion of putting the island at 300 meters (the middle of the lagoon), he interrupted with "I heard a lot of good discussion about how you would go about solving the task. Um, could you share some of the solutions that you came up with?" So, he valued the discussion that had been occurring and then invited his students to identify what they had come up with.

Wesley contributed "the island is in the middle" [17]. Wesley was the first to respond and suggested "it should be 300 on both sides." In response to Wesley's statement, Harry did a math nudge. Harry asked, "Is it in the middle or is it just somewhere in between?" Wesley then made another statement - "in-between, middle, same thing. It doesn't matter." Harry then went on to revoice and probe further asking why it does not matter.

Discussion of Harry's Support of Student Thinking During Launch.

Of the five opportunities to interact with student thinking, Harry mostly responded with a question to hear more student thinking. During two of the above instances, [9] and [13], Harry initially asked Kayla a probing question to learn more about her thinking but then "told" her how to think. In [9] he pointed out the problem description of the towers and in [13] he ended up telling her that it was unlikely that the island would be in the middle. During [12], Harry assumed student thinking and thus responded with feedback that was supportive, but likely did not move students forward since he misunderstood Wesley's thinking. As the rehearsal progressed, Harry started to shift away from telling. When Harry had the students chat with each other in [16], Harry listened to their exchanges of ideas, explicitly valued their thinking when he pulled them back together as a class, and then invited them to share solutions they had come up with. Harry responded most inclusively when students shared their thinking with each other. When a student shared a misconception, Harry did a math nudge in [17] which consisted of an attempt to redirect in the form of a question. Instead of responding by saying the island is not in the middle, Harry asked "Is it in the middle or is it just somewhere in between?" After this nudge, Harry ended up revoicing and probing in response to Wesley's next response that further elaborated on the island being in the middle. While Harry initially did the math nudge, his revoice and probe in response to a student's misconception is exactly how he responded to Kayla in both [9] and [13]. However, it is important to note that in this instance Harry did not directly right Wesley's misconception. Harry was trying to redirect him without directly telling him although Wesley still believed the island was in the middle when the rehearsal ended.

Harry's Support of Student Thinking and the Two Frames During Launch.

In general, Harry responded to student thinking initially in an inclusive way in the sense that he asked to learn more about their thinking. So, on the surface level, he can be interpreted as

leaning towards Louie's sense-making frame, viewing mathematical activity inclusively, in that he asked students to elaborate on their understandings. Harry is hearing students when they communicate an initial misunderstanding and interacts to learn more. His nudges to correct individual thinking suggests he was viewing a student's communicated misconception as something that needs to be tended to and fixed instead of an opportunity for sense-making. There also appears to be a consistent rhythm of revoicing in the form of a question when there is a misconception. Three of the four probing questions posed by Harry during this rehearsal were in response to student misconceptions. This is in alignment with what others have noted about how students perceive their answer is wrong when a teacher asks a question. And this is in alignment with what Harry said in the questionnaire about how he would respond to a student misconception - that he would respectfully suggest something different.

What is perhaps interesting in this rehearsal is that the exchange that Harry responded the most inclusively to was when he was not responding to a specific instance of student thinking, but the discussion of the students as a small group. When he had the students chat as a group, he listened to their ideas and then explicitly valued their ideas "I heard a lot of good discussion about how you would go about solving the task." This valuing of students engaging with each other aligns with Louie's inclusive multi-dimensional ability frame in that Harry is valuing students' engagement with the task with each other. While he does not explicitly state that students are resources for each other, his comment suggests that they were benefiting from each other.

Second Rehearsal: Monitoring

Table 5 *Harry's Talk Moves During Monitoring*

Start time: 72:50	21	check-in			
	22	check-in			
	23	inviting thinking			
(TE: 74:02 - 74:27)					
	24	Revoice	probing	wait time	creating opportunities
	25	Revoice			
	26	math nudge			
	27	math nudge			
	28	math nudge	invite identify		
	29	valuing	check-in		
	30	probing			
	31	valuing			
(TE: 76:58 - 80:17)					
	32	invite Identify			
End time: 80:46					

The monitoring rehearsal lasted a total of seven minutes and 56 seconds. After a couple of check-ins, Harry invited student thinking early on in the rehearsal by asking Stacy "What was your question again?" in response to her asking "do we all have the same equation?" The TE paused him shortly afterward for 25 seconds to clarify that all the students had the same work on their boards. Once that was cleared up, Stacy mentioned "So we all have the same equation…" to which Harry responded, "So you all have the same equation and what is your equation trying to find?" After at least six seconds of wait time, he then created an opportunity for them to engage with each other's thinking by having them talk to each other in an attempt to answer his probing question. One student responded "yes", and the students started to chat among themselves.

One student said he was having trouble with the two variables, and before there were student responses, Harry jumped back in and revoiced that struggle, "It sounds like you are struggling with the two different variables that you have." Once a student agreed with that statement, his next 2 talk moves were math nudges guiding them towards using one variable. First, he stated "So what you're saying is that we need to put them in one variable or it

would be easier to solve this equation if we only had one variable?" When a student asked what he meant by that Harry asked, "is there a way that we could write what you have now in terms of just one variable?" And when a student, Wesley, responded with "like with the x and the z?," Harry went on to state the two expressions the students had on their boards and then nudged once more with "Based on what we have in the 600-meter length of the lagoon, is there a way that we can get Z in terms of X?" Harry then invited students to identify by asking "What would be the length of Z if we were gonna write it in terms of X?" Once a student volunteered the desired expression, he valued it by calling it "a good idea." Harry then probed the student for more reasoning behind the expression when he asked, "Do you know why it would be 600 minus x?" He valued the student's response with "that's a really good idea."

At this point, the TE pauses Harry and asks a series of questions about where he thinks the group's thinking is going and how he might support them. This discussion lasted for three minutes and 19 seconds. Somewhere near the end of that Harry started to resume the rehearsal saying, "So, using your computer or your graphing calculator…" when the TE paused him again. During this pause, she reminded him that he is trying to figure out their thinking. When the rehearsal resumes again, Harry instead invited the students to identify "appropriate strategies now that you have your question in terms of one variable?" The rehearsal ended shortly after that.

Harry's Support of Student Thinking during Monitoring the Task. In general, Harry's interaction with student thinking during this rehearsal consisted of two instances when a student communicated a struggle they were having with their progress and one instance when a student responded to a question he had posed. Once Harry recognized what he perceived as their

main struggle, he focused on guiding them towards writing their equation in terms of one variable.

Stacy shared group progress and her concerns [24]. Stacy told Harry that they used the Pythagorean theorem and came up with an equation for the hypotenuse of each triangle. After a brief pause by the TE to clarify that students in a group had the same work, Stacy stated again that she was uncertain about her equation. Harry revoiced her concern and asked a probing question to the group about what their equation is trying to find. When no one responded after some wait time, Harry asked if they wanted to talk among themselves, so he created an opportunity for his students to engage in each other's reasoning.

Wesley communicated the issue is two variables [25]. When the students were talking as a group, Wesley stated "so we're trying to find these lengths right here. But we have 2 variables, 2 different variables. So. Um. I don't know where to go from that." Harry revoiced saying "Okay, so it sounds like you are struggling with the two different variables that you have." Stacy agreed with Harry's statement and then Harry went on to say "So what you're saying is that we need to put them in one variable or it would be easier to solve this equation if we only had one variable?" Harry initially responded to student thinking with a revoice, but following that revoicing Harry lobbed three math nudges that directed students to put their equation in terms of one variable.

Jackson's correct response [29]. Harry had asked his students "what would be the length of Z if we were gonna write it in terms of X?" to which Jackson responded, "600 minus X?" Harry responded by valuing that statement with "that's a good idea" and then checked-in with his students to see if everyone heard that. This exchange continued with Harry asking the probing question of "do you know why it would be 600 minus X?" Jackson responded with

"This whole length is 600 and so then if this is X from this length to this length, then this length is left over and then it's 600 minus X." Harry then responded with "That's a really good idea." Harry valued the student's initial response and delved further into the response by asking why it would be that.

Discussion of Harry's Support of Student Thinking During Monitoring.

There were three instances when Harry had the opportunity to respond to student thinking. When Stacy communicated confusion [24] about her equation, Harry revoiced and probed and eventually positioned the students to engage with each other to make sense of the problem. However, when Harry had them chat with each other, he really only let them talk long enough to jump in once he heard thinking he could direct which means only one student shared any thinking. When Wesley mentioned the two variables were an issue [25], Harry spoke up and revoiced to confirm the issue, but then followed that with three math nudges. The second of those math nudges explicitly suggested what to do even though it was in the form of a question. In the last instance of student thinking communicated, Jackson responded to Harry's question about the new expression after putting their equation in one variable [29]. Harry responded by valuing Jackson's initial response then checked to see if his peers had heard him. Harry then went on to probe his contribution and then valued that as well.

Harry's Support of Student Thinking and the Two Frames During Monitoring.

Harry's responses to student thinking tend towards inclusive ways of framing mathematics and mathematical ability. Once Harry understood that Stacy's problem was also the group's problem, he responded inclusively by asking "what is your equation trying to find?" Pressing for meaning falls on the inclusive side of mathematical activity. Later, after Jackson shared a correct mathematical expression, Harry asked *why* it was what it was. This is

consistent with what Harry said previously during the questionnaire about valuing explanations from students who provide correct responses. This also is inclusive in that he is explicitly asking for meaning and including justifications as part of mathematical activity.

Harry was less inclusive when students struggled with their approach. In responding to what Wesley said, Harry first revoiced but then did math nudges back-to-back to try to get the students to see that they just needed to put their equations in terms of one variable. By telling his students what to think by strong suggestion in his questions, he did not position his students to be sense-makers of their struggle. So, the exceptions of Harry's inclusive framing were in connection with how to respond to student thinking when his instinct was to help.

In thinking of what exactly counts as mathematical ability, Harry asked if they wanted to talk to each other to make sense of his question when no student immediately answered. By asking them to talk to each other to work towards a response, he is positioning them as resources for each other and framing mathematical ability inclusively. While Harry did mostly respond to student thinking inclusively, he did not hold his own actions accountable in that he never did return to the question about what the equation was trying to find, nor did he let the students have a conversation that developed since he immediately responded to Wesley once the issue of two variables was mentioned.

Third Rehearsal: Discussion

Table 6 *Harry's Talk Moves During Discussion*

Start time: 2:16	33	valuing	invite Identify		
	34	inviting thinking			
	35	invite Identify			
	36	probing			
	37	polite	check-in	create opportunities	wait time
	38	Revoice	check-in		
	39	math nudge	check-in		
	40	inviting thinking	wait time		
	41	revoice	invite identify		
	42	invite Identify			
	43	revoice	check-in	polite	inviting thinking
	44	revoice			
	45	check-in			
	46	create opportunities			
	47	revoice			
	48	check-in			
	49	create opportunities			
	50	valuing	revoice	Affirmation of correct	invite Identify
	51	math nudge			
(TE: 8:22 - 8:36)					
End time: 8:36					

Harry's discussion rehearsal lasted a total of six minutes and 20 seconds. The TE paused near the end during which there was a little back and forth about whether or not he was actually done, and it was decided that he was. The rehearsal started off with a talk move by Harry valuing student thinking by stating "so we're going to regroup and talk about some of the different solutions that we saw around the classroom." He then invited student thinking by asking a student to walk through how her group came up with their approach (table of values) and what each number means. Twice while she was explaining he invited her to identify aspects of her group's approach. He asked her "What is that part called?" After a little more explanation, he probed by asking "what do those numbers tell you?" After she was done explaining, he thanked her and then created an opportunity for students to engage in her thinking by asking "...so since she started out with 300 and 300. Where would the island be in the lagoon

if both distances was 300 meters away from the bank?" Following this question, there was no immediate answer and Harry exercised at least three seconds of wait time.

When a student finally responded with "in the middle" Harry revoiced that student statement. In his next talk turn, he gave a math nudge by stating "We know that placing it in the middle is not going to minimize the total amount of zip line needed to get to the island, correct?" He then posed a question inviting student thinking, but after 11 seconds of wait time no one had responded prompting the TE to encourage them to respond with a "Students!" Harry involved students in discourse the next couple of talk moves by asking them to identify the distance the island was from each tower. After checking that his students understood the two values added together was the total zipline, he drew attention to Willow's equation inviting her thinking on how she got that. She shared that they had used the Pythagorean theorem and Harry revoiced her.

At that point in time, Stacy volunteered that her group had created two equations but didn't know what to do next. Harry used that to create an opportunity to engage in another's thinking by asking "so how does this table and this equation compare to one another?" In response to Stacy saying that Willow's group used a table but that her group had an equation and didn't know what to do with it, Harry responded with a full revoicing by asking "so what you're saying is that you use both parts of these equations just like Willow's group did, correct?" The student responded with a yes, but when Harry further checked in on her thinking, the student corrected him saying that they had trouble figuring out how to do the minimization. He used that to create another opportunity to engage in another's thinking by asking "so based on what you just saw with Willow's solution, do you think that that is a good way of going about finding some values that could eventually lead to finding the minimal length of zip line needed for

this?" After Stacy responded, Harry used several moves in his next talk turn. He valued her thinking saying "that's a great point. Even though it's a table, (it) will give us a minimal value." He affirmed correctness while revoicing her thinking, "you're right we are going to have to plug in a lot of values to get the minimal value." And then he ended that talk turn with an invitation for students to identify by asking "do you know another way to do that?" He nudged Stacy just a little by directing her attention to the table. It was soon after this that the TE paused him to remind him that he was only supposed to do the first two approaches and the rehearsal ended soon after this as there was no further interaction with students.

Harry's Perception of Student Thinking During the Discussion. Harry supported student thinking during the discussion in a different way than during the other rehearsals. He had the first student explain her group's work and as she explained he asked questions to help highlight aspects of her communication he felt was important for the other students.

Willow explains her group's work [35]. Harry had invited Willow to "walk us through how you came up with this and what each of these numbers mean." She started off with "So these numbers, in that spot, represent that part," so Harry responded by asking her to identify, "Yeah, what is that part called?" Willow explained some more including that they started off with 300 and 300 before trying different numbers. Harry responded by probing further into what she was saying when he asked, "okay and what do those numbers tell you?" She responded saying they were the total amount of zip line wire. He responded politely with an "awesome" and checked in to see if everyone understood her.

Willow Responding to Harry's questions [38]. After Willow explained her work, Harry asked "where would the island be in the lagoon if both distances was 300 meters away from the bank?" Willow answered, "in the middle" to which he revoiced "it'd be in the middle,

right?" After describing some of the number combinations, Harry invited students to identify again, "so what does that tell us about the length of which the island needs to be from the 100-meter tower and the 60-meter tower?" There was a long pause after which the TE nudged them to respond, and Willow responded with "closer to the 60-meter." Harry revoiced what she said and stated the other answer he had hoped to hear "okay, so it's gonna be closer to the 60-meter tower and further from the 100-meter tower." So, Harry revoiced Willow's responses both times she responded correctly to his questions.

Stacy expresses her group's struggle [46]. Stacy stated, "we created the two equations but we did not know what we should do next." In response to her, Harry mentioned "I was gonna use that as a transition to this right here" but ultimately created an opportunity for students to engage with the thinking of another when he asked, "so how does this table and this equation compare to one another?" She responded, "they're the same thing except their group is just plugging in values for x." After Harry said "ok," she went on to say, "they're using a table, but we had an equation and we did not know what to do with it." Harry then revoiced her assessment when he said, "okay, so what you're saying is that you use both parts of these equations just like Willow's group did, correct?" Stacy said yes. And then Harry checked in with her by asking, "And then your trouble was getting it into a form where you could actually see like an end value of what your length to the island was going to be, correct?" To which Stacy responded, "uh, we had trouble trying to figure out how to do the minimization." In trying to address her struggle, he created another opportunity for her to engage in another student's reasoning by nudging her to consider Willow's approach as a way to find the minimum. So, while Harry initially responded by helping Stacy see connections, Harry did not actually tend to her group's work nor to her communicated frustration about the minimum.

Discussion of Harry's Support of Student Thinking During the Discussion.

There were three instances Harry had opportunities to respond to student thinking. The first instance was Willow sharing her group's work [35]. Harry allowed her to "go up there" after she asked and then in general supported her explanations with questions that invited her to be more specific and tended to the context of the problem. He generally supported her thinking by helping elucidate her explanations through questions. When she was done, he thanked her. Harry asked two general comprehension questions that Willow ended up answering [38]. He responded with a revoice and a check-in to her first response and a revoice and an invitation to identify for the second one. Lastly, Harry responded to Stacy's struggle [46] by creating an opportunity to engage in the reasoning of another since he responded by asking her to compare. And then a revoicing when she responded with how Willow's compared. Stacy never really explained her group's thinking or approach with the exception of their struggle.

Harry's Support of Student Thinking and the Two Frames During Discussion.

Harry's actions framed mathematics inclusively with the exception of responding to Stacy. He let Willow present her group's work and supported her work by asking mostly questions that nudged her to be more specific. He attended to sense making by asking her one question related to the context of the problem. After she finished, he asked his students two questions that pressed for understanding the context. Again, it is important to note that he pressed for meaning and explanation with Willow's work which happened to have a correct answer, but he did not have Stacy explain the work that her group did. When Stacy shared that her group did not know what to do next, he responded by asking Stacy to compare her approach to Willow's. In asking Stacy to compare and later give her opinion of Willow's approach, Harry added comparison and analysis to his framing of what counts as mathematical activity. While

one of the main goals of the discussion rehearsal is making connections among approaches, Stacy's mathematical work was sort of ignored once her lack of solution was communicated. So, the sense-making mathematical activity seemed to focus mostly on an approach that had a correct answer. Harry also framed mathematical ability inclusively by encouraging students to present to each other. Allowing Willow to present upfront supports the idea of students learning from each other. In conclusion, Harry framed mathematics inclusively but struggled to maintain that framing when a student did not have a correct answer.

Why Does Harry Do What He Does?

In trying to make sense of why Harry interacts with students the way that she does, the Hammerness et al. (2005) Learning to Teach in Community framework was used.

Learning Community

As there is evidence of Harry being influenced by the community even across these rehearsals, the takeaways are discussed. Harry valued feedback from the Learning Community and worked towards improving his practice based on their input. During the Launch interview, Harry expressed appreciation in relation to the TE stopping him because "it was a chance for me to evaluate what I had done so far." During the Monitoring rehearsal, Harry implemented a strategy they had read about in class. He had read that responding to a question with a question as a "good thing to do whenever you're trying to get students, or whenever you're trying to scaffold students or get them to explain more of their decisions and ideas." During the Discussion interview, Harry spoke of intentional changes based on feedback from his previous rehearsal. One of his classmates had noted that his wait time was "a bit fast" and so he shared that he was working on his wait time. Harry also internalized their compliments in that he shared "they also complimented my questions last time, so I wanted to keep on asking questions." And

he remembered Jackson complimenting his transition from the group who used a table to the group who used an equation. As a PT, Harry is eager to grow by trying practices he has read about and by attempting to implement feedback.

Conceptual Tools

Harry's conceptual practices include facilitation practices such as fun/engaging, checking if "on the same page," and transitions; encouraging students practices through positive evaluation and validating student thinking; centering student thinking through students as deciders, eliciting student thinking, and not forcing the conversation; guiding students' thinking through directing attention to key information, clarifying and nudging, managing misconceptions, connecting context, and connecting student work; practices related to standards for mathematical practice such as attending to precision and model with mathematics.

Facilitation Practices. Facilitation practices include fun/engaging, checking if "on the same page," and transitions.

Fun/Engaging. During the Launch, Harry intentionally tried to make the learning experience more engaging. Early on Harry asked the students who had been on a zipline. Harry justified his question saying "it seemed like a fun question to ask. Maybe get them a little more excited." Harry elaborated on his attempt towards buy-in when he said I was "just trying to get them interested in the activity. So, ziplining seems fun. It seems like an adventure-seeking type of activity. So, I wanted to… get them interested in what we were about to do." Soon after the initial zipline questions, Harry asked for a volunteer to read the problem because he "thought that it'd be nice for them to listen to a peer read it." When listing out key information on the board, he named this list "fun facts." He claimed, "I think listing out fun facts is just better than listing

out ideas." From trying to get his students excited about zip lines to naming his list something less mundane, Harry was trying to spice up the experience some.

Checking if "On the Same Page." Harry checked in with his students some across all three rehearsals. During the Launch, Harry explained that he asked everyone if they agreed with Wesley's drawn model "just to make sure everyone was on the same page and if someone thought that it should be drawn differently, to get that idea out." Later, in response to a student's comment about the location of the island, Harry asked the students, "does everybody get that?" Harry explained he asked that "just to see if everyone was on the same page." During the Monitoring rehearsal, Harry asked his students if everyone had heard what one particular student had said "just to make sure that everyone understood where that logic was coming from." During the Discussion rehearsal, after a student explained her group's approach and solution, Harry asked "does everybody see how she did that?" "just to make sure that everyone was on the same page, to make sure that everyone knew what was going on." Sometimes these check-ins were to make sure other students were understanding another student's contribution and sometimes they were just verifying that students were following the course of the discussion.

Transitions. Harry explicitly mentioned a focus on transitions during the Discussion interview. Before the interview started, Harry shared that one of the ways he thought the rehearsal had gone well was that he had some nice transitions. Harry pointed out the equation that a group used "as more of a transition because another group used an equation." Within the rehearsal itself, when Stacy shared her group's frustration, Harry stated "so I was gonna use that as a transition to this right here" before asking her a question. In the interview, Harry did share that he wished he could have gotten Willow to say more about the Pythagorean theorem before transitioning, but that Stacy sort of forced his hand when she piped up about their struggling to

find a minimum. Even near the end he asked his students if they knew another way to get the minimum value to start the transition into talking about graphs.

Encouraging Students Practices. Encouraging students practices include positive reinforcement and validating student thinking,

Positive Reinforcement. Harry frequently responded to students using positive words. During the Launch he responded with comments such as "awesome," "very nice," "good start," and "I heard a lot of good discussion." He responded positively because of reasons such as students sharing ideas, good labeling on a drawing, and that their ideas were good. During the Monitoring rehearsal, positive reinforcement was not used with as much frequency, but he did respond to a student's suggested correct expression of "600 minus X" with "that's a good idea." Harry said "awesome" a total of seven times across the three rehearsals. Harry intentionally and frequently would respond to students with positive words.

Validating Student Thinking. Harry made a point to validate student thinking during rehearsals. During the Launch rehearsal, Harry responded to a student's suggestion that the island be in the middle with "that makes sense." In the interview, Harry elaborated that her misunderstanding made sense since the group had not discussed minimization yet. During the interview he went on to say, "I'm not saying, yeah you're right it [the island] is in the middle. I'm saying I mean it would make sense that it *could* be in the middle." During the Monitoring rehearsal, Harry asked, "Did everyone hear that?" because he wanted to ensure others had heard a student's idea. During the Discussion rehearsal, when Stacy mentioned that the table approach was less efficient, Harry stated that her point was great "just so everyone would know that what she said was valid." Shortly afterwards, he also validated Willow's approach saying that the table does work. As he put it, "just because it's a table and it's not

efficient doesn't mean that it's wrong. Like it will give us a minimum value." When probed a little more about his choice to make that statement then, Harry explained that he had realized that he had not yet said Willow's idea was correct. Harry intentionally made statements during rehearsals to validate student ideas.

Centering Student Thinking Practices. Centering student thinking practices includes students as deciders, eliciting student thinking, and not forcing the conversation.

Students as Deciders. Harry wanted his students to be active in their learning. During the Launch this included having his students read the problem out loud, draw a model on the board, and discuss among themselves. Harry explained he wanted a volunteer to come draw to "get students involved in the decision-making process so I'm not the only one up there doing something." During Monitoring, Harry had them discuss among themselves again when they were unable to respond to a question. He also was intentional in mentioning "appropriate strategies" instead of explicitly stating approaches such as table or graph because he did not want to influence his students' choice. During the Discussion, when Willow asked if she could go to the board, he supported her in doing so.

Eliciting Student Thinking. Across all three rehearsals, Harry elicited student thinking. For example, during the Launch rehearsal, when a student shook her head no in response to her agreement on the drawing, Harry asked her why she didn't agree with it. He wanted "to hear her idea of how she thought the picture should be drawn." Later, after his students chatted among themselves, Harry asked them to share their ideas because he "wanted to hear what they were talking about." During the Monitoring interview, when a student suggested a particular algebraic expression, Harry asked the student if he knew why it would be that expression. There was also a moment during the Monitoring rehearsal that Harry responded to a

student's question with a question. During the Discussion interview, Harry claimed that one way he was supporting student thinking was by "asking questions for clarification" while students presented their group's approach. Harry used questioning to learn more about what students were thinking.

Not Forcing the Conversation. During the Discussion rehearsal, Harry explained that he held his tongue and did not interrupt or redirect students at times when he was inclined to do so. For example, there was a moment when a student used vague descriptors of "and then those are that" and Harry shared in the interview that he wanted to ask, "What are those and that's, but then she kept on talking and I didn't want to interrupt what she was going to say because I wanted her to get those separate thoughts out." Another instance was when Stacey jumped in to mention her group's issue and Harry shared that "I kind of wish Willow would've kept on talking about the Pythagorean theorem, but then Stacy stepped in and then I just transitioned." At a different point Harry said "okay" a few times in a row. When asked what that was about, he shared that he "was getting ready to say more and she just started talking…. And so, since she started talking, I didn't want to just interrupt her again, so I wanted to hear out what she had to say." Harry is valuing student talk to the point of being considerate and not redirecting them to fit his notion of what had to be stated.

Guiding Students' Thinking Practices. Guiding students' thinking practices includes directing attention to key information, clarifying and nudging students' thinking, managing misconceptions, connecting context, and connecting student work.

Directing Attention to Key Information. There were a few exchanges across rehearsals during which Harry brought key details to students' attention. During the Launch interview, Harry directed Kayla to the part of the problem that states the towers are on opposite sides of the

lagoon when she asked why they are on opposite sides. After a pause during his Monitoring rehearsal, Harry explained that he suggested for his students to use a computer once it was suggested that they graph to find the minimum. During the Discussion interview, Harry's explanations portrayed him facilitating in a narrator sort of way. When Willow started, Harry said "it looks like you've created a table of values" to help other students make sense of what they were looking at. Harry also explicitly stated, "we know that since this was 622.14, like that's the total length of zip line wire" because he wanted to confirm a student understood what the number represented in the context of the problem.

Clarifying and Nudging Students' Thinking. During rehearsals, Harry focused students' attention to key ideas. During the Launch, the focus had to do with the location of the island. He explained that he told his students that "the island is going to be variable, subject to change" because "what we're trying to figure out is like where in the lagoon the island should be in order for like the sum of those two zipline lengths is the shortest." During Monitoring, Harry was focused on "trying to get them to see how they could write in terms of one variable." He used some intentional repetition to draw attention to that idea. Harry stated that the reason he basically said the same thing twice during the rehearsal was because "I just wanted to um, to like really strongly hint that that's probably what we needed to do." Harry used some more intentional repetition during the Discussion rehearsal to highlight the answer and to note the relationship of the distances from each tower to the island. For example, in response to why he asked, "how far away from the hundred-meter tower?" he said, "just to keep on reiterating it. Like that was one of the main objectives in this whole part was so that they could see that it is not 300 and 300 is not 350 and 250, it's 375 and 225 that produces the lowest amount of zip line wire." He explained that he also was nudging his students to see the minimum in the table - that

"it [the table] was like high 630 something and it was like starting to work its way down to like this lowest value, the 620.96 and then it started getting higher. So, I wanted them to see that."

Managing Misconceptions. Harry was focused on tending to barriers across rehearsals. During the Launch interview, Harry explicitly claimed that one of his goals had been "trying to figure out their barriers." He shared that one of the reasons he had a student draw the model was because the model would show him if there's any misconceptions before starting the task. In his efforts to address misconceptions, Harry addressed student confusion about tower location, addressed a "just in case" misconception that had not been mentioned by a student, and was trying to attend to students' suggestion that the island was in the middle. During the Monitoring rehearsal he also was fixated on resolving misconceptions. He started off by asking students how they were finding the task in case some struggles were presented then. Then later there was the exchange when he nudged his students toward putting the equations in terms of one variable. Harry explained that "was me just saying since you're struggling with two variables, what way would make it easier? Like how can we get it into a, or how can we make this equation look like something you've already solved before?" During the Discussion rehearsal, Harry asked about distances from the island to the towers because that was "one of the big misconceptions that I saw a lot of students had, so I just wanted to really get that in there." Harry was aware his students might have misconceptions and was attempting to address those in a variety of ways.

Connecting Context. Across all three rehearsals, Harry communicated intentions of attending to the context of the task. During the Launch interview, Harry explained he facilitated the making of a list of "fun facts" because "making sure that they know what those numbers represent is pretty important so that they can begin to formulate ideas and begin to put things in

formulas and tables and whatnot." An example during the Monitoring rehearsal included Harry asking his students what their equation was trying to find because he "was making sure that they understood that they were trying to find the lengths of the zip lines." During the Discussion interview, Harry explained that the reason he asked Willow to "walk us through how they came up with it and what each of the numbers mean" was because "it's important to know that like these numbers aren't just arbitrary numbers… that those numbers were going to add up to be 600 but also affected the total zip line wire." Harry consistently had intentions of tending to the context of the task.

Connecting Student Work. During the Discussion, Harry made connections across student work. After a certain point, Harry drew attention to Willow's equation because he realized they had been focusing only on the numbers so far, but the equation worked "as more of a transition because another group used an equation." When Stacy jumped into the conversation about her equation, Harry asked his students how the table and equation compared, thus asking his students to make connections. Another way he asked his students to make connections was by asking if they thought Willow's way was a good way of finding minimum length. Harry made connections between the two approaches by drawing attention to the similarities between their equations and by asking his students to compare and critique.

Practices Related to Standards for Mathematical Practice. Practices related to standards for mathematical practice include attending to precision and model with mathematics.

Attending to Precision. During the Discussion rehearsal, there were two instances when Harry asked a student to be more precise. When Willow was explaining her group's approach, he asked her "what is that part called" to push her "to be more precise." There was another moment when she was presenting that Harry was about to jump in and ask her to be precise

again, but her explanation went on to include a key point that he did not want to interrupt. When she mentioned different numbers, he asked "and what do those numbers tell you?" So, he was holding her accountable during her explanations to the class.

Model with Mathematics. During two of the rehearsals, Launch and Monitoring, Harry facilitated modeling. During the Launch rehearsal he mentioned to his students that "a good approach is to get a picture down." Harry then had a student draw and label a model on the board. In the interview, he explained that the picture is "just like laying the groundwork of how we want it to be represented visually to solve the task," that the "whole point of drawing that picture is to put everything in context with numbers." During the Monitoring rehearsal, after some nudging from the TE, Harry was working towards getting his students to graph their equation using Desmos or their calculator so that they could physically see the minimum.

Dispositions

Dispositions are described as "habits of thinking and action--about teaching, children, and the role of the teacher" (Hammerness et al., 2005, p. 387). Statements made by the PT that communicated how he positioned himself in connection to his students' learning, not necessarily in connection with a specific teacher move, were coded as dispositions. In the framework, Dispositions are described as "Habits of thinking and action regarding teaching and children" and this guided the organization of this section. I started noticing which of my codes better described habits of thinking or action and whether the focus was on teaching or on children, in this case the students. Most of the codes fell into categories related to teaching, but a few were directed towards students. Harry's *habits of thinking regarding students* included considering student thinking and competent students. His *habits of thinking regarding teaching* include reflection upon practice, goal/success, and valuing multiple approaches. His *habits of action*

regarding teaching were further categorized into focusing on culture building, student thinking, and facilitation. Culture building dispositions include student authority and positive reinforcement. Focusing on student thinking dispositions include valuing student talk and valuing student understanding. Facilitation dispositions include desire to maintain high cognitive demand, problem solving, desire to have a smooth discussion, the importance of clearer communication, and noticing a shift needed in student thinking.

Habits of Thinking Regarding Students. Habits of thinking regarding students include relating empathetically with students, considering students' thinking, and competent students.

Relating Empathetically with Students. Harry was considerate of the student experience. In connection with his fun/engaging practice, Harry wants to create a learning experience that is more than mundane for his students. He mentioned wanting to "incorporate a fun aspect as much as I can into a lesson to keep people interested." So he is concerned with holding students' interest. He had a student read the task initially because he thought it would be "nice for them to listen to a peer read it." Harry aims to construct a learning experience where his students are interested.

Considering Students' Thinking. During the three rehearsals, Harry repeatedly considered students' sense-making from their perspective. During the interviews, he would reason why they responded the way that they did. When a student did not catch on to the location of the towers, Harry suggested that perhaps she read the task too fast and skipped over that statement. Harry also expressed understanding why a student was thinking that the island would be in the middle since they had not yet talked about minimization. In the Discussion interview, he explained that he was happy that Willow had eventually mentioned that the two columns gave the total length of the zip line wire. Harry explained that, "if someone who didn't

use this approach were to be looking at all of those numbers, they would probably be asking themselves what are those numbers? Like why is that important? Why is this one high and this one low? um, like what is special about this?" Harry was connecting with what it might be like for students who are seeing this information for the first time. Across the interviews, it became apparent that Harry had a level of sensitivity toward his students and their sense-making.

Competent Students. Harry mostly viewed his students as competent, but there was a little inconsistency. During the Launch interview he described in detail what was nice about Wesley's drawing. Harry viewed his students as competent also in the sense that he misinterpreted Wesley's comment about it not mattering where the island is. However, there was one instance during which he was describing what he might do to better meet the needs of a student who had not spoken much during the Launch and he mentioned that he would have "circled back around and seen what she was doing during the Monitoring phase, if anything." That "if anything" at the end suggests that he was perceiving her silence as indicating she was not working on the task. During the Monitoring interview, Harry was discussing how to end the rehearsal and mentioned the idea of leaving them one or two steps away from getting an answer. In how he talked about his students, his belief that they are capable was clear - that with just "a little bit more time and collaboration, hypothetically, they could get it." During the Discussion interview, Harry described his students' responses as intuitive. There is multiple evidence to suggest that Harry mostly views his students as capable but evidence also exists that suggests he perceives silence as non-engagement.

Habits of Thinking Regarding Teaching. Habits of thinking regarding teaching include reflection upon practice, goals/success, and valuing multiple approaches.

Reflection upon practice. Harry shared thoughts on how he might have done things differently in hindsight. During the Launch interview, Harry explained that he probably should not have named a potential misconception that had not yet been vocalized by students because "if a student isn't showing that they have that misunderstanding, then bringing it up could do more harm than good in some cases." Harry also shared that he wished he would have asked Wesley why he thought it did not really matter where the island was. Near the end of the interview, Harry proposed that if he had a student as quiet as Willow in a real classroom setting, that he probably would have launched the task and then checked on her during the monitoring phase of the task. During the Monitoring interview, Harry mentioned that he maybe should not have used the word "easier" because "it's just seeing it in a different way. So for some people it might be easier to see it in this way and some people it might be easier to see it that way." And in connection with feedback he received from his peer Stacy, Harry shared, "I wish I would have not asked so many questions in there and then if I did ask a question, let them answer it before asking another one." During the Discussion interview, when Harry saw that he stated the answer and its contextual meaning, he volunteered that he should have asked his student what 622.14 is instead of stating it himself. Harry also just stated in general that "it's easy to think of all the questions you should've asked after you look at it," which implies he may be reflecting on much more than was shared during the interview.

Goals/Success. Goals of different varieties seemed to matter to Harry's perception of his success. During the Launch interview, Harry shared that he felt there had been a lack of attention to an important mathematical concept, minimization. Before any video of the rehearsal was watched, when asked how the rehearsal went, Harry responded with "I don't think I ever got to the point where I was explaining minimization or maximization." During the Monitoring

rehearsal Harry fixated on moving his students beyond their barriers. Harry "thought that since I had gotten them to the point of figuring out the equation in just terms of one variable that they could then do whatever they wanted to after that. And that was the only thing that I had to do." Later, during a pause and nudge from the TE, Harry expressed confusion about what to do next because "I thought that like my job was to get them to put it in terms of one variable. I didn't know like what else to do." During the Discussion interview, before any reviewing of the video occurred, Harry thought the rehearsal went pretty well because "I think I was able to connect the first two ways students had solved the problems pretty well. I thought I had some nice transitions and, yeah. I think I addressed everything that needed to be addressed from what I can remember." Goals, albeit very different from each other, guided Harry's assessment of his performance.

Valuing Multiple Approaches. During interviews, Harry made comments that suggest that he values multiple approaches. In the Monitoring interview, Harry expressed that he told a student his idea was good because it was correct, but also acknowledged that "there are other ways to solve it." During the Discussion interview, while reflecting on a student's response saying a table is inefficient, Harry explained that "you can't just count out a table because you don't think it's the most efficient way. Because for other people, they might think that that's the best way to do it because that makes the most sense to them." Harry went on to say that "it [table] might not be the fastest and most efficient way, but I mean, it is effective. Like you can still get a value from it. Like you can still get the right answer." The loudest way he communicated valuing multiple approaches was in response to the question about how he thought he made his students feel smart. He responded with "connecting Willow's and Stacy's ideas as both being correct even though they disagreed with each other."

Habits of Action Regarding Teaching. Habits of action regarding teaching include culture building dispositions, focusing on student thinking dispositions, and facilitation dispositions.

Culture Building Dispositions. Culture building dispositions include student authority, and positive reinforcement.

Student Authority. Harry wanted his students to have input in their learning. During Launch, Harry felt that he made his students feel smart by "letting them have control of identifying key aspects about the problem and letting them draw and then letting them talk with each other." During the Monitoring rehearsal, he was ready to let his students continue working on their own because he "thought that since I had gotten them to the point of figuring out the equation in just terms of one variable that they could then do whatever they wanted to after that." Then later Harry explicitly communicated that he "wanted them to get to a solution based on a lot of the information that they had came up with themselves." So he believed in his students moving themselves forward.

Positive Reinforcement. Harry consistently communicated his beliefs about positive reinforcement. When explaining why he says "awesome" so much, Harry said, "as much positive reinforcement as I can work into a classroom, I try it. So I think if I try to keep my attitude positive and keep them, I guess, engaged by using positive reinforcement, I think that's good for class morale… it makes me more approachable as a teacher." During the Discussion interview, Harry stated "so it's always nice to have some positive reinforcement. Like even if it's wrong, I'm still gonna say awesome probably. Say 'awesome. All right, well that's a good idea but'." He also expounded on what he says awesome in response to - "I mean awesome whenever they identify something correctly, like whenever they're working through a thought

process or, um, you know they have enough courage to answer a question out loud in front of their classmates." Harry posited that he used positive reinforcement to support student thinking and to make his students feel smart. He values positive reinforcement as a rapport builder, as a way to support students feeling smart, and as a way to support student thinking in general.

Focusing on Student Thinking Dispositions. Focusing on student thinking dispositions include valuing student talk, and valuing student understanding.

Valuing Student Talk. Harry valued students talking during the rehearsals both as individuals and to each other. During the Launch rehearsal, Harry asked his students to reread the problem and talk to each other about how to go about finding solutions. And in describing the mathematics his students had the opportunity to do during the Launch, he reflected "I think they did a lot of conceptual, more than procedural, mathematics just because they were talking about it." During the Monitoring interview, Harry responded to a student with "a probing question, just to make sure that I can get some more responses from them." Later, he again gave his students the opportunity to chat among themselves. During the Discussion rehearsal, Harry had a student explain her approach to her classmates. In describing why he felt the Discussion rehearsal went well overall, one of the descriptors he shared was "my students had the opportunity to talk." For more than one rehearsal, letting students talk to each other was a way that Harry felt he made them feel smart. Harry sees value in students talking to each other as possibly a way to move each other forward, as a way for his students to feel smart, and as a way for his students to experience mathematics more conceptually.

Valuing Student Understanding. Harry wanted his students to understand certain concepts as they progressed through the task. During the Launch interview, Harry asked a student to explain why she thought the island should be in the middle because he wanted to know

her understanding of where the island should go. During the Monitoring interview, Harry elaborated that he asked his students what their equation was trying to find "to make sure that they had an equation that would make sense to them." During the Discussion interview, there were several concepts that Harry wanted his students to understand. He wanted his students to understand that the two distances from the island were related to each other. He wanted his students to notice how the minimum stood out among the other values that were larger in the table Willow presented. He also wanted his students to understand that the island's location mattered in connection with minimization, that it needed to be closer to the shorter tower. So in earlier rehearsals, he was valuing students' interpretations of the task so far and later he was valuing their understanding of key ideas in connection with the task.

Facilitation Dispositions. Facilitation dispositions include desire to maintain high cognitive demand, problem solving, desire to have a smooth discussion, and importance of clearer communication.

Desire to Maintain High Cognitive Demand. Harry shared his internal struggle to not tell his students too much and explicitly communicated wanting to maintain a high cognitive demand. When a student suggested that the island should be in the middle of the lagoon, Harry struggled with how to nudge his students toward the idea of minimization. During the Launch interview, Harry shared that he "wanted to keep it at a high cognitive demand, so trying to keep that high cognitive demand without telling them too much and without telling them too little was a lot harder than I thought it was going to be." He then elaborated "cause you don't want to tell them what to do but then you also have to find a happy medium between not telling them anything and then telling them what to do." There was also the exchange in the Monitoring rehearsal when Harry suggested his students graph and the TE paused him. During the pause,

Harry stated "I don't want to tell them just to graph it" even though he had just done exactly that. In the interview he shared that telling them "would take away from the cognitive demand of the task. I didn't want to just tell them to graph it because then they would have exactly done that." This is slightly confusing since he is struggling to act on his stated desire not to lower the cognitive demand, but it is important to note that he wanted to maintain a high cognitive demand even if he was struggling at times to do so.

Problem Solving. During his facilitation of the Launch rehearsal, Harry suggested approaches that might could be applied to other problems. For example, Harry stated that the "numbers are going to be pretty significant to how this task may be worked out, so I think it would be a good idea to write those down on our fun facts list." In the interview he elaborated that "making sure that they know what those numbers represent is pretty important so that they can begin to formulate ideas and begin to put things in formulas and tables and whatnot." Soon after they created their list, Harry shared that he thinks it is a good approach to get a picture down. In the interview, he explained that he did this "to point them in a direction that I guess I would take to solve it, and a way that a lot of other people would use to solve it and I think getting a visual would probably help everyone in this task." Harry shared two specific approaches to guide students processing problems such as this - listing key information and drawing a model.

Desire to Have a Smooth Discussion. Harry made a few comments during interviews that suggested that he would prefer for the discussions to be homogenous. During the Launch interview, Harry admitted hoping that Wesley had no barrier because "if I could figure out if one of my students didn't have a barrier then that could make like a group discussion go by really smoothly." More specifically, when Harry thought Wesley had no barrier, he hoped during the

time that he had the students chat with each other that "Wesley would've just told them because he was the one without a barrier." At the beginning of the Monitoring rehearsal, when Harry discovered that his three students had the same work so far on the task, he felt relieved because "with everyone having the same work, it would be easier to address some different misconceptions or barriers that people may have about that one particular solution strategy." Later in the rehearsal, Harry asked students a question which included a suggestion for what to do next. When asked why, he reflected "I think that was me just saying since you're struggling with two variables, what way would make it easier?" When reflecting on what he would do differently, he focused on what sounds like a "smoother" discussion in that he said "I probably would have went straight from doing, uh, getting it in terms of one variable X and saying, okay, now that you have it in terms of one variable, what's your next step? And then ask them for that. And then get a response. And then honestly, I feel like that would've been like almost over." Harry seems to be viewing the facilitation of Monitoring very systematically.

Importance of Clearer Communication. There was not strong evidence of this disposition throughout, and yet it was evident enough to describe. During the Launch rehearsal, Harry asked the question "what does that tell us about the length which the island needs to be from the 100-meter tower and the 60-meter tower?" After that question Harry provided eleven seconds of wait time before the TE verbally encouraged students to respond. During the interview Harry mentioned, "I don't know why it took them so long. My questioning might've just been bad." Later he referenced that again saying that he thought it was a pretty straight forward question, but then said, "I could have worded it odd." At the end of the interview, in addition to mentioning he wished he would have asked more questions, he added on "rephrase questions differently" too. And at another point during the Launch interview, Harry was

reflecting on his communication about it making sense that the island could be in the middle when he shared "I know that I could've said it better, but it's practice." So, Harry was noticing that how the wording he selected to communicate might influence students' processing of content.

Understandings

As discussed above, the researcher has elected to only consider the knowledge aspect of the PTs' understandings from the Hammerness et al. (2005) framework. Below Harry's experience solving the task before participating in rehearsals, his familiarity with different approaches, and key math ideas are presented.

Harry's Experience Solving the Task. During the Launch interview, Harry shared that he thought students heading towards using triangles was good because it was similar to "the way that I solved it whenever I did it was using the Pythagorean theorem and finding the hypotenuse." During the Monitoring interview, Harry shared his experience solving the task. Harry explained that "Whenever I initially did it, it was Jackson and I working on it together and we solved it in a way that was just using the equation. But then right afterwards, we used a table because, um, we heard people say that, uh, you could use a table to do this. And then we just started plugging in values, like after we had done it and we were like, Oh yeah, we could've just done that." What is unclear is what he and Jackson did with their equation to solve the task. Regardless, Harry had familiarity with equations constructed using the Pythagorean theorem and using tables to solve the task.

Harry's Thoughts on the Different Approaches. Harry did not communicate an attachment to one particular approach. In response to the idea of a table with the two lengths of zip line, Harry stated "I felt that that was a good way to do it cause I remember us going over

that." During the Monitoring rehearsal, he acknowledged recognizing the way he solved it the first time, but then he went on to say, "But then there's different things you can do, you know, you can graph, like they said, there's a table method that I was eventually thinking of." So, Harry expressed some comfort with several approaches. He also acknowledged some of the trade-offs of different approaches in that during the Monitoring interview, Harry acknowledged that using a table to find the minimum could take some time. Regarding the table approach, he shared "I don't want to just sit there while they plug in every value for X or sit there while they're typing in their values, their things, while they're typing in their equation into their calculator." But he also recognized that Stacy's group's equation did not have what he called an "end value." So, while he viewed the table approach as less efficient, he did see its value in providing a solution.

Key Math Ideas. Harry was aware of two key ideas related to solving the task: minimization of the zip line wire and deciding on the location of the island that accomplished that. During the Launching interview, he repeatedly referenced not addressing the key mathematical concept of minimization. He understood the minimization aspect of the problem in that he stated that the reason he said that "it's [island] gonna be variable, subject to change" was "because depending on where it is in the lagoon, we may be using more zipline than we actually want." After discussing some other details, he stated "what we're trying to figure out is like where in the lagoon the island should be in order for like the sum of those two zipline lengths is the shortest." Furthermore, in response to a student mentioning that you "just need to get the wires to the island," Harry shared that "you do need to get the wires to the island but the position of the island is pretty important." Harry knew that the solution to the task was when the island was 225 feet from the shorter tower and 375 feet from the taller tower. During the Discussion,

he was pushing the understanding to his students about the island needing to be closer to the shorter tower. Harry understood the bigger picture mathematical ideas in connection with the task.

Vision

Harry aspires to involve his students in such a way that they are eager to learn mathematics in his classroom. He sees himself as a mathematics educator who aims to make the mathematics classroom experience more engaging and fun. Since Harry wants his students to feel supported and encouraged, positive reinforcement is very important to him. Harry further believes in supporting students by not interrupting them when they are speaking, even if the student is pursuing an unexpected direction. Harry aims to be considerate of students in that he wants students to be active in their learning through modeling, speaking, and participating in class. He also aims to be considerate of his students' learning journey in that when his students are confused, it is important that he make sense of why they might have a certain misconception. In addition to being very positive and empathetic, maintaining a high cognitive demand is important to him.

Revisiting the Research Question - Why does Harry do what he does?

This section addresses Harry's thoughts in connection with the research question: *How inclusively do PTs frame mathematical activity and mathematical ability when reflecting on their intentions during rehearsals?*

Harry's Whys in connection with the Two Frames

While the Louie (2017) framework is intended to analyze enacted practices, the intentions of the PTs were also of interest to the author and so below Harry's actions along with intent are considered in connection with the three frames used in this study.

Mathematical Activity. Harry's framing of mathematical activity was described as a conflicting internal struggle and procedure/answer focused.

Conflicting Internal Struggle. According to Louie, mathematical activity involves sense-making. Harry struggled with supporting authentic sense-making perhaps because he both wanted to maintain a high cognitive demand while simultaneously experiencing a smooth discussion. For example, after having experienced a different rehearsal in which one student had no barrier, Harry found himself hoping that his student Wesley had also been given the role of a student with no barrier. However, Wesley had a barrier that did not become evident until later in the rehearsal because Harry assumed he understood his thinking instead of probing. When asked what he might do differently, Harry responded that "I definitely would have addressed minimizing earlier and I would have gone over like what the task was actually trying to find." So, while earlier Harry had stated he would've wanted to hear more of Wesley's thinking, his hindsight focused more on teacher-centric helping as opposed to supporting students in sense-making. Another instance in which Harry struggled was in the Monitoring rehearsal. Harry sees himself as someone who encourages idea sharing, thus he supports the airing of misconceptions. But, when his students communicated that they were struggling because they had equations with too many variables, Harry asked the question "So what you're saying is that we need to put them in one variable or it would be easier to solve this equation if we only had one variable?" He essentially suggested what it was they should do, robbing them

of the cognitive demand. While his instinct was to help him so that the discussion is smoother, he was unaware that he had fed them that idea. During the interview he credited his students with coming up with that idea.

Procedure/Answer-Focused. Harry did seem to value sense-making at times, but he also was overly focused on mathematics as procedural in other ways. During the Monitoring interview, Harry communicated a focus more on the students' output than on their thinking. Harry was talking about helping students get their equations in terms of one variable and said, "And that was the only thing that I had to do, so then I would come back later to see if, if they had taken steps to get the right answer or to solve the task." Harry did not continue on to mention anything about checking understandings. This solution focus was reinforced by Harry's statement at the beginning of the Monitoring interview when he shared that "the goal of monitoring was to get to a solution and I think I achieved that." Additionally, during the Discussion rehearsal, Harry saw Stacy's lack of answer as deficient and focused the discussion on nudging her to consider an alternative method that could result in an answer. His concern for a smooth discussion and his assumptions of student thinking suggest that he was not valuing sense-making enough.

Mathematical Ability. Harry leans towards a slightly exclusive framing of mathematical ability in his intentions. Harry did not explicitly say he was positioning Willow in need of help during the Launch rehearsal, but he shared during the interview that he was hoping that the group would move Willow forward since she had been quiet. While they were discussing as a group, Harry stated that he "was kind of focused on like listening to their ideas and then seeing if Willow was grasping what they were saying to point to." So, he perceived Willow's silence as a deficiency and perceived her to be in need of help. During the Discussion

interview, Harry shared that he intentionally named both students' approaches as valid. However, he also shared, "I believe the biggest misconception was in the second group because they didn't have an answer. So they didn't know how to take their equation and put it into either a table or a graph. And I believe I was able to use Willow's group's solution to identify a potential way that this group may want to do a problem similar to this in the future." Harry appeared to view Stacy's group's work as deficient because of its lack of answer. During the rehearsal, when Stacy mentioned that her group had struggled to find a minimum, Stacy was not given an opportunity to share her group's thinking. Harry did not value her work for what it was nor did he address her communication about struggling to find a minimum - he focused on her lack of answer instead of supporting their approach to the task. In both of these cases, Harry positioned a student as in need of help instead of viewing them as a resource for their peers.

Harry's Awareness of His Framing

There were two instances where Harry communicated being aware of how he framed his responses to students. During the Monitoring interview, Harry shared that he positively reinforced a student's suggestion of "600 minus X." Harry explained, "I said that that's a really good idea because I didn't want to tell him that that was right." While it is unclear why he did not want to label the response as correct, he was aware of his avoidance of labeling a student's response as correct. Harry also became increasingly aware of his framing of approaches as "easy." During a rehearsal, Harry had asked what would be easier, but then he recanted during the interview stating that "for some people it might be easier to see it in this way and some people it might be easier to see it that way." When asked to expand on his thinking that he should not have used the word "easier," he responded with "I guess because it's not really making it easier, it's just seeing it in a different way. So, for some people it might be easier to

see it in this way and some people it might be easier to see it that way. So, um, yeah. I probably shouldn't just generalize the word easy because there's a couple of different routes they can take." Harry is aware of the impact that labels such as "correct" and "easy" might have on student perception.

Summary

Harry seemed more comfortable supporting student thinking during the Launch (as there were specific math things that had to happen) and the Discussion (finalized work being projected), however the Monitoring of student thinking seemed particularly challenging for him. At one point during the Monitoring rehearsal the TE paused him and stated, "remember, you're trying to figure out their thinking." Perhaps this was in connection with a lack of preparedness/confidence for anticipating student thinking. He knew to revoice and that his students should be talking, but he still did not really seem to know how to engage in their thinking. He seemed to be parroting what he heard enough to ask more about it, but it was not always clear that he really understood what his students were struggling with. When his students did communicate a struggle, his instinct was to help and how he helps is varied. He was guilty of asking loaded questions and not realizing they were loaded.

Harry is generally responding to student thinking inclusively and he is sensitive to how certain language choices might frame mathematical ability as exclusive. He understands questioning is important and aims to ask good questions. Also, not unlike other research has suggested, his inclusive framing of ability is in spite of his mind still thinking somewhat exclusively. He also seems to treat having his students chat together as a "move," which means he is open to letting students learn from each other. And yet it is also important that he not view that as a way for some students to be helpers and others to be helped. During the Launch

rehearsal, it was noted that Harry responded most inclusively to students chatting among themselves.

Chapter 5: Analysis of Willow

Willow will first be presented in relation to her responses to questions concerning students' mathematical activity, mathematical ability, and misconceptions to present a description of Willow's perceptions of teaching and learning mathematics. To address how Willow supported student thinking and misconceptions during rehearsals, an overview of Willow's experience through the three rehearsals will be described, including summative data describing her talk moves. Then, for each rehearsal, Willow's moves in response to students will be analyzed followed by a more in-depth analysis of how she responded to student thinking and a discussion of how her moves connected to the three frames. To address the second research question of why Willow does what she does, the Hammerness et al. (2005) framework will be used to organize her shared intentions. At the end, there will be a discussion of Willow's themes in connection with the three frames.

Willow's Perspective on Mathematics and Mathematics Teaching

To understand some about the PT's views on mathematics and teaching mathematics, Willow's responses to a questionnaire were considered with how they align with Louie's (2017) Ways of Framing Mathematics and Mathematical Ability and how she perceives handling student misconceptions. Since later her interactions with students will be discussed, understanding how she thinks about what qualifies as mathematical activity and ability and how she feels about student mistakes merits description.

Willow and Mathematical Activity

Both before and after the methods course, Willow described mathematics as the study of numbers and operations used to solve problems. She described learning mathematics as *understanding* why those numbers and operations work to solve problems. At the beginning of

the semester, Willow described teaching mathematics as presenting strategies and helping students gain skills. She elaborated to state that "you should allow students to discover the math they are learning." Later she did not mention the "discover" idea again but described teaching mathematics as helping explain and supporting students in how to use numbers and operations to solve problems.

Willow and Mathematical Ability

Willow's responses described the ability to apply skills to solve problems. In her earlier answer, she elaborated to include the idea of perseverance - "if you get an answer wrong, you can think about the problem in a new way and you're able to work towards the right answer."

Willow and Supporting Student Thinking

Willow's responses regarding how to respond to student thinking was consistent in wanting to elicit an explanation. In response to whether or not students should explain correct answers, she felt students should explain so that they understand "the process for when the context or the numbers change." In addressing how to respond to an incorrect answer, she described asking "why the student got their incorrect answer would allow the student to reflect on their answer and realize where they went wrong on their own." She also described more of a discovery approach in that she suggested the idea of having the student apply that same reasoning to another problem and see if it worked. Her later response focused on student reasoning but mentioned more teacher guiding than student discovery. "I would ask them to walk me through their reasoning and ask them guiding questions to point them in the direction of the correct answer." Willow values students sharing their reasoning in connection with correct and incorrect responses.

Moves Implemented by Willow to Support Student Thinking During the Three Rehearsals

Table 7 *Willow's Rehearsal Structure*

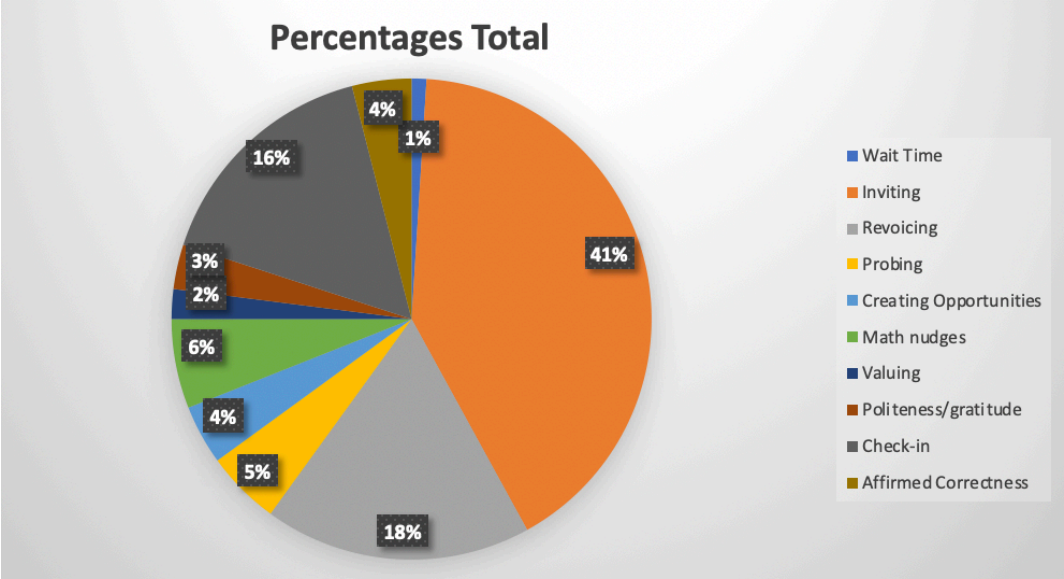
Order	Date	Teacher	Student	Student	Student	Observer	Observer
Launching 1	10/29/2019	Jackson	Harry	Willow	Wesley	Stacy	Kayla
Launching 2	10/29/2019	Willow	Jackson	Kayla	Stacy	Wesley	Harry
Launching 3	10/29/2019	Stacy	Jackson	Harry	Wesley	Stacy	Kayla
Launching 4	10/29/2019	Kayla	Harry	Willow	Stacy	Wesley	Jackson
Launching 5	11/5/2019	Harry	Kayla	Willow	Wesley	Stacy	Jackson
Launching 6	11/5/2019	Wesley	Kayla	Jackson	Stacy	Willow	Harry
Monitoring 1	11/5/2019	Kayla	Harry	Willow	Jackson	Stacy	Wesley
Monitoring 2	11/5/2019	Harry	Stacy	Wesley	Jackson	Willow	Kayla
Monitoring 3	11/5/2019	Wesley	Kayla	Harry	Jackson	Stacy	Willow
Monitoring 4	11/12/2019	Stacy	Kayla	Welsey	Harry	Jackson	Willow (partially there)
Monitoring 5	11/12/2019	Jackson	Wesley	Stacy	Kayla	Harry	
Monitoring 6	11/19/2019	Willow	Stacy	Wesley	Kayla	Jackson	Harry
Discussion 1	11/19/2019	Wesley	Harry	Willow	Kayla	Jackson	Stacy (co-teacher)
Discussion 2	11/19/2019	Stacy	Harry	Willow	Kayla	Jackson	Wesley (co-teacher)
Discussion 3	11/26/2019	Harry	Stacy	Willow	Wesley	Kayla	Jackson (co-teacher)
Discussion 4	11/26/2019	Jackson	Stacy	Willow	Wesley	Kayla	Harry (co-teacher)
Discussion 5	11/26/2019	Willow	Jackson	Harry	Stacy	Wesley	Kayla (co-teacher)
Discussion 6	11/26/2019	Kayla	Jackson	Harry	Stacy	Wesley	Willow (co-teacher)

Rehearsal Structure

Willow was the second student to play the role of teacher during the Launch rehearsals. Willow was the last to do the Monitoring rehearsal and did so on a later date than planned as she ended up going home early on 11/12 because she was ill. So on 11/19, Willow was the first student to play the role of teacher that day as she was making up her time as a teacher from leaving early the previous class. It is worth noting that because she left early on 11/12, she missed out on observing Jackson's enactment and much of Stacy's as well. For the Discussion rehearsal, Willow was partnered with Kayla. Within that pair, she was the first to play the role of teacher. She and Kayla were the last of the three pairs to facilitate the Discussion rehearsals.

Talk Moves Across All Three Rehearsals

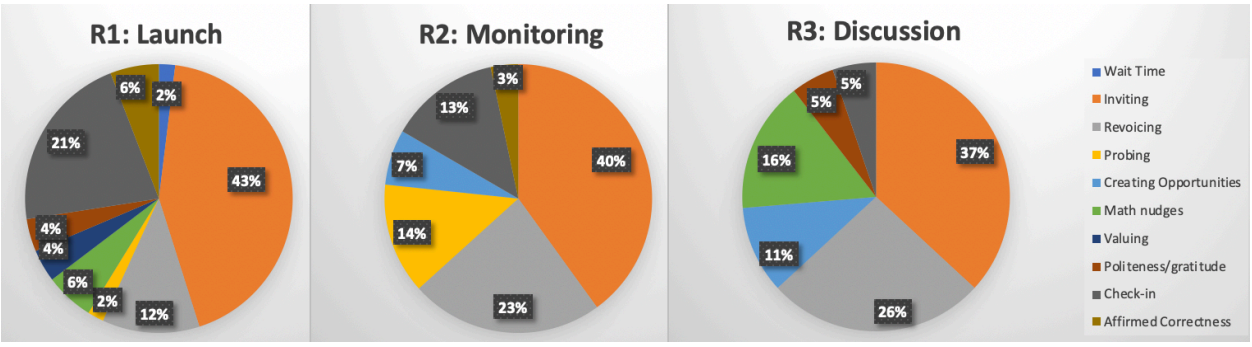
Figure 10 *Willow's Percentage of Talk Moves Across All Three Rehearsals*



In total, Willow implemented 51 moves to support student thinking during the first rehearsal (Launching the task), 30 moves during the second rehearsal (Monitoring), and 19 during the third rehearsal (Class Discussion). Willow used inviting moves (41%) the most. Revoicing (18%) was her second most frequent talk move. Willow checked in (16%) with her students with some frequency. The rest of the talk moves were implemented with less frequency. Math nudges accounted for 6% of her moves and probing for 5%. Creating opportunities for students to engage in each other's thinking and affirming correctness each accounted for 4% of her moves. Willow responded with politeness (3%) and valued student responses (2%). Wait time was utilized the least at 1%.

Comparing Relative Usage Per Rehearsal

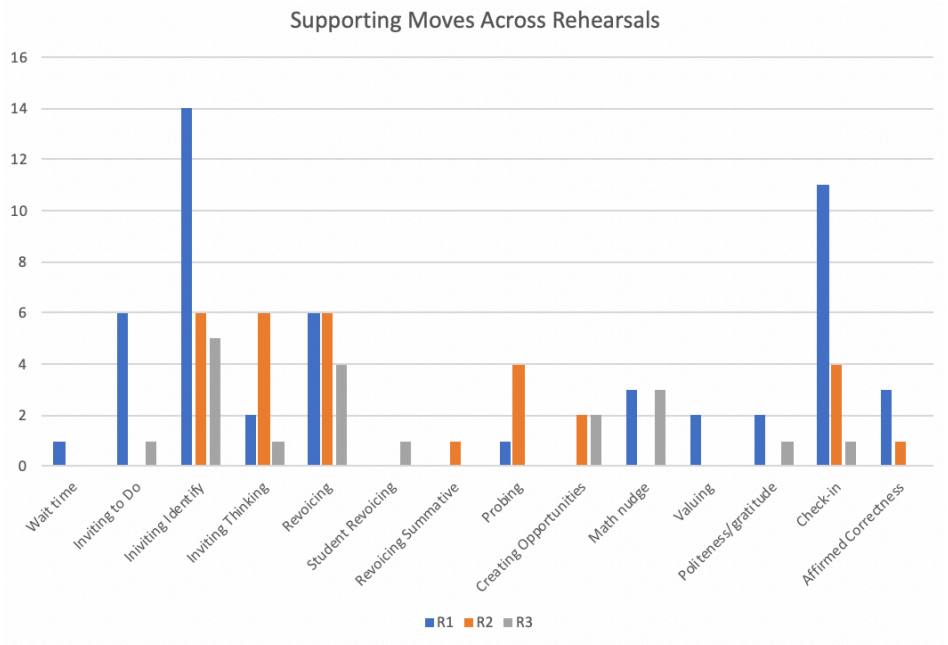
Figure 11 *Willow's Relative Usage of Talk Moves Per Rehearsal*



As each rehearsal was for a different amount of time, one approach is to notice how she used moves relative to other moves within the same rehearsal. In all three rehearsals, the combination of inviting and revoicing moves makes up more than 50% of total moves during the rehearsal, so those function as the connectors in dialogue when she is leading her students.

Analysis Across Rehearsals

Figure 12 *Willow's Talk Moves Per Rehearsal*

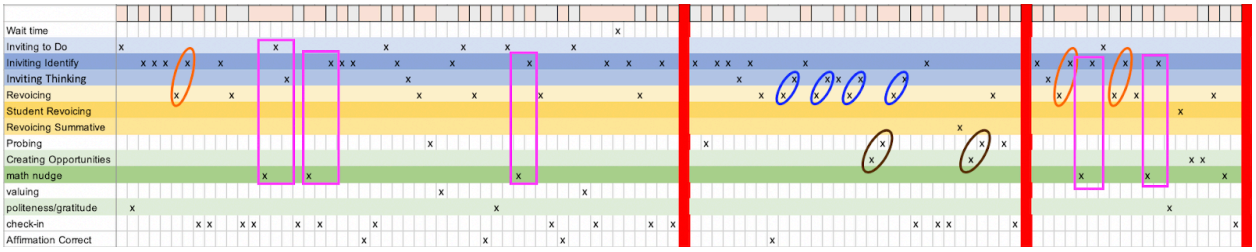


In the graph above the different rehearsals are represented by different colors for each talk move to note the variation among the number of talk moves among rehearsals. During the Launch was when the one occurrence of wait time and the two occurrences of valuing student thinking occurred. Inviting to do, math nudges, and politeness/gratitude were used during the Launch, not at all during Monitoring, and the same quantity or fewer times during the Discussion. Willow invited students to identify and checked in with them during the Launch more than double any of the other rehearsals. There was a steady decrease in Willow's affirmation of correctness across the three rehearsals. During the Discussion rehearsal, Willow asked students to revoice once.

Willow had some notable differences in the talk moves for the Monitoring rehearsal. While used not once during the Launch, Willow created opportunities for students to engage with each other's thinking twice during both the Monitoring and the Discussion rehearsals. She invited student thinking three times as many times as during the Launch or Discussion. She also revoiced as many times during the Monitoring rehearsal as the Launch, which is remarkable since there were many more moves during the Launch. Her only instance of summative revoicing occurred during the Monitoring rehearsal. Willow probed student thinking the most during the Monitoring rehearsal.

Patterns of Talk Moves

Figure 13 *Willow's Patterns of Talk Moves Across Rehearsals*



There were a few trends noted while analyzing Willow's patterns. In general, if she did a math nudge (pink rectangles), it was followed by an inviting move. During the Monitoring rehearsal, there was a pattern in that four of her revoicings were immediately followed by an invitation to share student thinking (blue ovals). Also, her only two instances of creating opportunities for students to engage with each other's thinking during the Monitoring rehearsal were immediately followed by a probing question (brown ovals). There was also a little bit of a trend for Willow to invite students to identify after revoicing (orange ovals). Lastly, Willow finished each rehearsal by checking in with her students.

Analyzing Willow's Talk Moves Per Rehearsal

Below each rehearsal is described in more detail so that there is some context presented in how Willow implemented the different talk moves. Afterwards, how Willow responds to instances of student thinking are examined in closer detail. Instances of student thinking were identified by first looking at responses to her inviting thinking and probing moves. Additionally, any instance when a student volunteered a question or comment were also treated as instances of student thinking for Willow to respond to.

First Rehearsal: Launching

The chart below shows the different talk moves that Willow utilized while playing the role of teacher. Each line represents the moves made by Willow when it was her turn speaking. The TE pauses are noted on the left. Moves of the same kind were color-coded to aid in sense-making of the data.

Table 8 *Willow's Talk Moves During Launch*

Start time: 24:00	1	Inviting Do				
	2	Polite				
	3	invite Identify				
	4	invite Identify				
	5	invite Identify				
	6	Revoice	invite Identify			
	7	Check-in				
(TE: 25:32 - 26:03)						
	8	Check-in				
	9	invite Identify				
	10	Revoice	Check-in			
	11	Check-in	Math Nudge	Inviting Do	inviting thinking	
	12	Check-in				
	13	Math Nudge	Check-in			
	14	invite Identify				
	15	invite Identify				
	16	invite Identify				
	17	Affirmation of correctness	Check-in			
	18	Inviting Do				
	19	invite Identify				
	20	inviting thinking				
	21	Revoice	Probing			
	22	Valuing	invite Identify			
	23	Inviting Do				
	24	Revoice				
	25	Affirmation of correctness				
	26	Polite				
	27	Inviting Do				
	28	Math Nudge				
(TE: 30:54 - 31:23)						
	29	invite Identify				
	30	Revoice	Check-in			
	31	Affirmation of correctness				
	32	Inviting Do				
	33	Valuing	Check-in	invite Identify	wait time	invite Identify
	34	Revoice				
	35	Check-in				
(TE: 32:45 - 38:30)						
	36	invite Identify				
	37	Check-in				
End time: 39:00						

The Launching rehearsal lasted a total of fifteen minutes. During that time Willow was paused by the TE three times. The first time was briefer for 31 seconds to discuss students' familiarity with the state fair to connect to their understanding of what an attraction is. The second interruption was also brief and lasted for 29 seconds to discuss how to respond to Jackson's question about the bridge. The third interruption was a longer one at five minutes and 45 seconds. The TE paused in connection with her asking a question about an assumption not pointed out by students which turned into a long discussion about where she might go next. Initially Willow asked for someone to read the task (Invite to Do) and then thanked the student for volunteering. After the problem was read she asked her students if there was

anything that jumped out to them (Invite Identify). A student responded asking what an attraction is, and in an effort to have other students share their thoughts on the definition, Willow asked students to identify three more times before providing some additional info on what an attraction is and then checking in with them to see if they have any questions about the task. At this point the TE paused.

After the pause, Willow resumed by checking in to see if they had any more questions about the task. When a student asked what "minimize" means, Willow sought input from the other students and then revoiced what the student said before checking in to make sure that the explanation made sense. Willow asked once more if there were questions before suggesting they approach the task sentence by sentence. This evolved into her asking a student to draw one sentence. When the student was drawing without speaking, Willow then invited the student to tell everyone what she was thinking. After the student finished, Willow started to ask for questions when a student interrupted to ask why the towers were on opposite sides. She then referenced the problem to point out where it said that and checked in to make sure that made sense to him.

In processing the next sentence in the task, Willow asked students to identify three times in connection with the drawn diagram. She then affirmed a student's response as correct and then checked in to see if everyone agreed. Then the next sentence explaining the setup of the ziplines to the island resulted in her inviting another student to draw on the board. She then invited him to identify what he drew, then explain his thinking. And when he responded with a "just cause," she then revoiced as she probed to learn more. When he did respond, she valued his response by saying "like it" before moving on to the next sentence. As the next sentence included the goal of the task, she asked a student to explain what it is we're trying to find. When

the student started to explain she was motioning in the air, so Willow invited her to come up to the board. While supporting this student's explanations, Willow revoiced, affirmed correctness, and then thanked her.

A student, Jackson, then asked about the bridge missing from their diagram to which she responded by inviting him to come draw it. When he explained he did not know where it would be, she referenced the problem again when the TE paused the rehearsal. Willow resumed by asking students to identify where the riders exit the ride. She then revoiced the student's response and checked in on a related understanding. Another student volunteered that the bridge could not be under the zip line or else people would get kicked and Willow affirmed that was true. When Jackson suggested the bridge go on top or bottom, she invited him to come draw it. She responded to his drawing by saying she liked it and checked in to see if his peers also liked it.

Willow then asked her students to identify what else they knew about the zip line wire. When nobody immediately responded, she exercised wait time and then asked a different question. She asked a specific question about where the zip lines would meet on the island. When a student responded, Willow revoiced her initial response and then checked in with the others to make sure the decision about the wires meeting at a same point was okay. The TE then paused her and asked why she was addressing that particular assumption. This was a longer pause during which Willow said she was unsure how to move the group forward to see the larger goal. The TE suggested asking "what is it that we're trying to find?" and that is exactly how Willow resumed the rehearsal by inviting students to identify. When a student responded, Willow did one last check in by asking "did that make sense to everybody?"

Willow's Support of Student Thinking during Launching the Task. The chart above indicates that Willow is adept at utilizing different talk moves but does not adequately examine how she uses those to support instances of responding to student thinking. Below, specific instances of student thinking combined with Willow's response are discussed.

Jackson asked, "what's an attraction?" [4]. In response to Jackson asking this, Willow invited students to identify when she immediately responded by asking the students "Does anybody else know what an attraction is?" Stacy responded with the question "a tourist attraction?" and Kayla suggested "a park." After those two students responded, she described it as a place that attracts people.

Stacy asked about "minimize" [10]. In response to Willow asking if there were any more questions about the task, Stacy asked "so what does minimize mean?" In similar fashion as the previous student question, Willow immediately responded by asking the other two students if they had any ideas on what minimize means. Kayla suggested it meant small, which Willow revoiced and then checked back in with Stacy to see if that made sense.

Willow asked Kayla to draw and vocalize her thinking [12]. Willow had asked for a volunteer to draw and when Kayla volunteered and started drawing, she was not speaking. So Willow asked her "can you tell us what you are thinking while you are drawing." Kayla responded sharing the label of each part of her drawing. Willow responded with "ok" and then checked in with her classmates.

Jackson asked why towers were on left and right side [13]. Willow responded with a math nudge by pointing out that the problem states "two towers on opposite sides of a man-made lagoon." And then explained how if one tower is here (on the right), then opposite of that would be on the other side (left). She then checked in with him before moving on.

Willow asked why Jackson drew the island where he did [21]. Jackson had drawn the zip lines connecting to an island. After Jackson identified what he had drawn, Willow asked him "so why did you choose to put the island there?" to which he responded with a "just cause." Willow then responded by probing to get more out of him. He ended up sharing it was because "it said it was in the lagoon."

Jackson asked, "where is our bridge?" [27]. When Jackson asked, "where is our bridge?," Willow responded by inviting him to come draw it. When he responded by saying he didn't know where it would be, Willow responded with another math nudge directing attention to the task description where it said that "island riders will exit the ride by walking across the long bridge." This is when the TE paused her for a moment to see where she was going with this. When Willow resumed, she asked her students to identify where the riders are when they exit the ride. Jackson volunteered that they are on the island. Willow revoiced that and checked in with the class that the bridge would have to start at the island.

Kayla and people getting kicked [31]. When trying to figure out where to put the bridge, Kayla volunteered that the bridge "can't be under the zip line or those people will get kicked." Willow responded by confirming her idea with a "that's true." Jackson then volunteered that the bridge "has to go top or bottom" and Willow responded by asking him again if he wanted to come draw it. He went up and drew the bridge.

Discussion of Willow's Support of Student Thinking During Launch

In response to the two questions posed by students [4, 10] in connection with vocabulary in the task description, Willow responded by positioning peers as resources by immediately getting their input on their understanding of the words. There were two instances when a student was drawing, and Willow asked them to share what they were thinking. The first time [12], she

responded to the student's shared description with an okay. The second time [21], she ended up probing when the student did not initially share real reasoning. There are also two occasions [13, 27] when a student, Jackson, had a question in connection with the task's setup. In each instance, she redirected attention back to the task description to make sense of the question. Lastly, during the discussion about the bridge, Kayla volunteered an idea [31] and Willow responded by affirming her logic.

Willow's Support of Student Thinking and the Two Frames

Willow framed mathematical ability inclusively by positioning students as resources for each other. On at least two occasions her initial response to a student question was to redirect it back to the student's peers. Her framing of mathematical activity was leaning towards inclusive in that she was asking students to share their thinking out loud. When a student was unsure about the meaning of a word, she asked the class to provide their understandings to help make sense. When Jackson was unsure about where to draw the bridge, Willow again redirected the class back to the task description to help him make sense of where the bridge would go. This resulted in Jackson later drawing the bridge on their model. And on the occasion when a student gave a response of "just cause" she pressed for more by asking a probing question. Additionally, when there were questions about the setup of the task, instead of feeding the students an answer, she redirected them back to the wording of the task to help them with sense making.

Second Rehearsal: Monitoring

Table 9 *Willow's Talk Moves During Monitoring*

Start time: 3:03				
	38	invite Identify		
	39	Probing		
	40	invite Identify		
	41	invite Identify		
	42	inviting thinking		
	43	invite Identify	Revoice	
	44	Affirmation of correctness	Revoice	inviting thinking
(TE: 4:55 - 7:03)				
	45	invite Identify		
	46	Revoice	inviting thinking	
	47	inviting thinking		
	48	Revoice		
	49	inviting thinking		
	50	Create opportunities		
	51	Probing	Revoice	
	52	inviting thinking		
	53	Check-in		
	54	invite Identify		
	55	Check-in		
	56	Check-in	Revoice Summative	Create opportunities
	57	Probing		
	58	Revoice		
	59	Probing		
	60	Check-in		
End time: 11:44				

The Monitoring rehearsal lasted a total of eight minutes and 41 seconds. During that time there was one TE pause that lasted two minutes and eight seconds. During the pause, Willow shared that she was struggling with how to move her students forward without doing the thinking for them. At the beginning of the rehearsal, Willow asked her students what they had so far. The next four exchanges were Willow asking questions to hear from different members of the group about the work they had so far in connection to the context. When Stacy volunteered that they did not know what to do next even though they had an equation, Willow invited them to identify the goal of the task. She then revoiced what a student said about finding a total to ask "what are we doing with the total?" Wesley responded saying they are minimizing the total length of wire. Willow affirmed that he was correct, revoiced what he said, and then asked if

they had any ideas on how to minimize their equation. Shortly after two students said "no," the TE paused the rehearsal.

When the rehearsal resumed, Willow asked her students to identify the part of the equation they wanted to minimize. Kayla responded with "this side." Willow revoiced and then asked how one might go about that. When Kayla stalled on responding, Willow then invited the other two students to share their ideas. No one responded to her question, but Wesley volunteered that he was confused by all the square roots and that he was unsure about their equation. Willow revoiced his concern as a question and then asked him to walk her through how they set up their equation. After Wesley explained some more, he identified another gap in his understanding - he did not understand why they were adding up square roots. Willow responded by asking the other two students to engage in his thinking by asking if they knew how to explain that. Stacy shared that she thought they could distribute, so Willow revoiced while asking Stacy a probing question to share why she thought she could distribute. Stacy responded but trailed off into an "I don't know." Willow then invited Kayla to share her ideas. After Kayla explained about the square roots in the equation, Willow checked back in with Wesley who then wanted to know where the "600 minus x" came from. Willow directed his question to Stacy to invite her to answer. After a few "oo-ooh"s from Wesley in response to Stacy, Willow checked back in to see if what Stacy had said made sense.

After those exchanges, Willow asked if there were more questions. She then revoiced summatively what Kayla and others had said much earlier in the rehearsal followed by creating an opportunity for them to engage in each other's thinking, "Kayla wants to start with the right side of the equation and we're trying to minimize it. So, how should we go about that?" In response to a student suggesting a table, Willow probed to learn more, then revoiced the

student's idea to put positive values in for x. Willow then probed again asking what she would get when she put in values for x. After the student responded, Willow checked back in with Wesley and the TE ended it.

Willow's Support of Student Thinking during Monitoring the Task. To address this, the codes of invite thinking and probing were used to initially identify instances of student thinking. Then the rehearsal was read through again to note any instances when a student volunteered an idea or question. Then there was one last read through to see if anything was overlooked.

Kayla responds to how they set up the equation [40]. In response to Willow's question about how they set up their equation, Kayla explained that they used the Pythagorean theorem. Willow did not address her response directly but asked a classmate to identify an aspect of their drawing afterwards.

Wesley responded to why they are adding hypotenuses [43]. In response to Willow's question about why they're adding them together, Wesley stated "because whenever you add them together, that's when you get the total amount of wire. And we're trying to find the total amount." Willow responded to this with an "okay" and then Stacy jumped in with their frustration.

Stacy stated they have an equation but do not know what to do next [43]. Willow responded by inviting the students to share the goal of the task. She revoiced Wesley's statement about finding the total, but then asked them what it was that they were doing with the total.

Kayla was unsure [46]. In response to how they might go about minimizing "this side," Kayla suggested that the equation looks a little more complicated than what she is used to. Willow responded by inviting her peers to share their ideas.

Wesley stated he was confused about square roots [47]. After he shared his confusion, Willow responded by revoicing his dilemma in the form of a question. After he said a little bit more on the issue, she invited him to walk her through how they set up the equation. He explained what he could and then stated he was confused about "why we're adding up the square roots and stuff." Willow responded to this question by asking his peers if they know how to explain that.

Stacy thought that you could distribute [51]. Stacy shared that she did not think that "the x-squared is a problem inside of the square root." She elaborated that she thought you could distribute the exponent. Willow responded by asking a probing question - "why do you think you can distribute it?" Stacy then responded saying she was not sure ending her statement with an "I don't know." Willow then asked Kayla if she had an idea.

Kayla shares the Pythagorean theorem [53]. Kayla points out they used the Pythagorean theorem for one triangle and that they would just do the same for the other one, and then add them. Willow responded to her idea by checking with Wesley to see if that made sense.

Wesley asked where the 600 minus x came from [54]. After Wesley acknowledged that he better understood, he asked where the 600 minus x came from. Willow responded by asking Stacy if she knew. After Stacy explained, Willow checked back in with Wesley to see if her explanation made sense. He said, "that makes more sense."

Stacy suggested doing a table [57]. Near the end, when Willow asked how they should go about trying to minimize, Stacy suggested "I think maybe we should create a table." Willow then responded with the probing question – "what would you put in your table?" After she responded saying she would put positive values, Willow revoiced her idea as a question which the student confirmed. Then Willow probed again asking "what would you get when you put

values in for x"? Then in response to Stacy's explanation, Willow asked Wesley if what she said made sense.

Discussion of Willow's Support of Student Thinking During Monitoring

When students responded to questions posed by Willow, she typically either asked another question to the same student or asked a question to another student. Early on as Willow was learning about the progress this group had made [40, 43], Willow did not engage with the response the previous student had made. In one case she asked another student a different question and in the latter case she responded with an "ok" before a different student made a comment. When a student communicated an issue, Willow almost always first tried to pull more information from the other two students. For example, when Stacy volunteered that she did not know what to do next [43], Willow posed a question about the goal to her peers. Willow also responded by posing the question to peers when Kayla was unsure [46] and when Wesley was confused about square roots [54]. Sometimes Willow would just dig deeper with the student who asked the question. When Wesley was confused about the square roots in their equation [47], Willow asked him a question to clarify and then asked him to explain how they got their equation. When Stacy volunteered that you could not distribute [51], Willow initially responded by asking her to say more about why she thought she could not distribute. However, when Stacy got tongue-twisted in sharing her thinking, Willow then asked Kayla to contribute. Near the end, in response to the idea of using a table, Willow responded by asking more questions. For example, [57] she responded by probing, revoicing, and then probing again.

Willow's Support of Student Thinking and the Two Frames During Monitoring

Willow's framing of mathematical activity was inclusive in that she asked many questions and pressed for meaning with students. During this rehearsal, Willow's valuing of

sense making was evidenced by her inviting students to share their thinking six times and asking probing questions four times. In the way that Willow was consistently trying to position students to learn from and with each other, she was nudging her students towards collaboration which also aligns with inclusive mathematical activity. She also framed mathematical ability inclusively by asking students questions throughout. Every student was positioned to be intellectually able to contribute.

Willow supported misconceptions in that she treated every student statement, whether providing a correct response or stating a gap in understanding, as valuable to the conversation. When a student presented a gap in understanding, she generally posed questions to others to have information presented to bridge the gap. Willow then would frequently check back in with the student who had the misunderstanding to make sure they understood. She was inclusive in her framing of misconceptions in that she saw every verbal contribution related to the task as a conception, and since those questions were part of the discussion that moved the students forward, they were desirable.

Third Rehearsal: Discussion

Table 10 *Willow's Talk Moves During Discussion*

Start time: 28:32				
	61	invite Identify		
	62	inviting thinking		
	63	Revoice	invite Identify	Math Nudge
(TE: 29:51 - 31:34)				
	64	invite Identify		
	65	Inviting Do		
	66	Revoice		
	67	invite Identify		
	68	Revoice		
	69	Math Nudge		
	70	invite Identify		
	71	Polite	Revoice Student	Create opportunities
	72	Create opportunities		
	73	Check-in	Polite	invite Identify
	74	inviting thinking		
	75	Revoice		
	76	Revoice		
	77	Create opportunities		
	78	Create opportunities		
	79	Revoice		
	80	Math Nudge		
	81	Check-in		
End time: 36:55				

The Discussion rehearsal lasted a total of eight minutes and 23 seconds. There was one TE break that lasted one minute and 43 seconds during which there was a discussion around how to respond to a student's suggestion that 225 and 375 are both X. At the beginning of the rehearsal, Willow stated they were going to go over everyone's solution and asked who had the solution being displayed. She then invited Stacy to explain her thinking. Stacy explained their table approach and stated, "we came to the conclusion that by plugging in 375 and 225 for X that we would get 62.967, which is the smallest number." Willow then responded by asking "so you're saying that the 225 and the 375 are both X? Or is there one in particular?" When Stacy responded saying they are both X, the TE paused the rehearsal.

The rehearsal resumed with Willow inviting Stacy to identify her Xs in the diagram. When Stacy started to draw in the air, Willow invited her to come up to the

board. Stacy explained the drawing and that X represented one distance and "600 minus X" represented another. Willow revoiced and then asked Stacy to identify which one is X in their equation. Stacy provided a long explanation suggesting that there were still two different Xs. Willow revoiced Stacy's previous statement that X represented one distance and "600 minus X" represented another as a question. Stacy agreed. Then Willow provided a math nudge when she asked, "so this can't just be X right?" Stacy agreed and Willow thanked her and then asked the other students if they could revoice what she was saying. Willow created an opportunity for them to engage with another's reasoning when she asked them to address "how she went through and how she figured out like her values"?

After Harry responded with much of what Stacy had said, Willow again pushed for more engagement with Stacy's reasoning by asking Jackson to explain how she got these values. Willow then checked back in with Harry to make sure that Jackson's response made sense. She thanked them and then asked which group was next. She immediately asked them to explain their thinking. After Jackson explained, she revoiced one part of it and Jackson agreed. Then she revoiced another part and Jackson clarified. She then created an opportunity to engage with the thinking of others by asking Harry to explain how the equation related to the table. She then asked Stacy if she saw the similarities between the equation and her group's work.

Willow then revoiced Jackson's previously stated struggle about getting to the next step with their equation as a question. After Jackson agreed, and then Willow asked "do you see how you could maybe use a table to find the minimum?" Jackson agreed that they could have. Willow checked in to make sure they understood that it was the minimum that Stacy found. The TE ended the rehearsal here.

Willow's Support of Student Thinking During the Discussion. Below are the instances of student thinking that Willow had the opportunity to respond to during the Discussion rehearsal.

Stacy shared her group's thinking [63]. Willow had asked Stacy to "go through and kind of explain your thinking." Stacy shared their setup of the drawing and that they had plugged in values and figured out that the smallest was 375 and 225. Willow responded by revoicing and an invitation to identify followed by a math nudge hinting at the idea that only one could be X.

Stacy clarified about her Xs [68]. Willow had asked her to clarify which was X in the equation. Stacy responded explaining much of the setup over again and near the end stated "right here we plugged in two different values for X. But the X right here represents a different distance than the X right here. So the X right here has a different meaning than the X right here." Willow responded by revoicing in question form what Stacy had previously said about one part being X and the other part "600 minus X." Stacy agreed that she had said that. Then Willow provided a math nudge suggesting that the "600 minus X" could not just be X. Stacy agreed.

Jackson shared his group's thinking [75]. Jackson shared about the equation his group had created using the Pythagorean theorem. Willow revoiced key points from what he had said the next two exchanges as Jackson either agreed or provided clarification.

Discussion of Willow's Support of Student Thinking During the Discussion

Willow generally supported student thinking by responding to what the student was saying. When Stacy was claiming that there were two different Xs [63], initially, through revoicing, Willow attempted to use Stacy's words to try to nudge her toward seeing that only one

could be X. After the TE pause [68], Willow again ended up addressing Stacy's two Xs and this time she was more successful in using Stacy's words to help her see that she was contradicting herself. When Jackson presented his group's work, Willow revoiced his statements twice. Thus, in general, Willow is supporting student thinking by restating their thoughts to the class.

Willow's Support of Student Thinking and the Two Frames During Discussion

Willow leaned towards inclusive framing of mathematical activity in that she was asking questions to press for meaning. Specifically in the case of Stacy and her misunderstanding about two different numbers both representing X, Willow pressed to help Stacy make sense of her own words. She also worked towards sense-making by having the students consider how the two approaches related to each other. Willow also leaned towards an inclusive framing of mathematical activity in that she had students explain frequently during her rehearsal, positioning students to learn from each other. She also was framing mathematical ability inclusively by creating opportunities for students to engage in the reasoning of others.

Willow was perhaps less consistent regarding misconceptions. When Stacy was confused about what X was, Willow did start off by responding inclusively in that she initially revoiced and asked a question. Then in her second question she did a math nudge that was of a more "fixing" nature. After the TE break when Willow had another opportunity to respond to Stacy's misconception, she was more inclusive in that she used Stacy's own explanation of the drawing to lead her to understanding that both cannot be X. Another instance when Willow's framing of student struggles was visible has to do with how she responded to Jackson. His group had created an equation but did not know how to find the minimum. After both groups had shared, Willow asked Jackson "do you see how you could maybe use a table to find the

minimum?" This is using his frustration as a learning opportunity, but she is not allowing him the opportunity to make sense of it himself first.

Why Does Willow Do What She Does?

In trying to make sense of why Willow interacts with students the way that she does, the Hammerness et al. (2005) Learning to Teach in Community framework was used.

Learning Community

During the Launch interview, Willow valued the pauses by the TE. She shared "I felt more comfortable because I knew like if something, like if I got stuck, the TE would be there to like say 'pause' and like help me get back on track. So that helped a lot." For example, when she was struggling on how to clarify what an attraction is, the TE nudged her towards mentioning the state fair and that excited Willow because she felt "like all students would know about the fair." She was struggling with a particular exchange and valued the TE pausing to make things clearer. Willow did not explicitly mention her peers, but during a TE pause in the Discussion rehearsal, Willow was confused about how to respond to a student. In fact, when the TE paused her, she asked if they could just pretend that she had not asked a particular question. Willow explained that she "felt like trying to explain it when I didn't like see a clear way to explain it would be more confusing than just like ignoring it and like going on with it I guess." But during the pause options were discussed and when the rehearsal resumed, she took the advice of a peer and asked the students where the Xs were in the diagram, which resulted in Willow then being able to revoice the student's own words to help the student realize that she was contradicting herself. Willow was appreciating the support of the TE and learning from the community.

Conceptual Tools Practiced

Willow's conceptual practices include leaving no student behind through inclusion, checking in, and positioning students as resources for each other; supporting student contributions through positively responding; centering student thinking through students as deciders, eliciting student thinking, and pursuing student thinking; guiding students' thinking through directing attention to key information, clarifying and nudging student thinking, managing misconceptions, connecting context, connecting student work, and transitions; and standards for mathematical practice practices such as attending to precision and model with mathematics.

No Student Left Behind Practices. No student left behind practices include inclusion, checking agreement, and positioning students as resources for each other.

Inclusion. Willow worked towards including all students. During the Launch interview, this was evidenced by her commitment to tend to each student's question - she did not want anything preventing them from moving forward with the task. Early in the Launch interview, Willow explained that part of why she had asked a student to explain her thinking while she was drawing was so that others could follow along. Willow noted that in general, she was calling on people in such a way to "make sure each student participated." During the next two interviews, in response to a question about which students she noticed participating, she pointed out that she was calling on students intentionally to include them, so there were not many opportunities to volunteer. For example, during the Discussion interview, she explained that she intentionally had two students revoice Stacy's approach with the goal of engaging them in the discussion. Later in the rehearsal, Willow asked Harry to explain how the equation related to the table because neither of those had been his work and so she was intentional to include him. In

two of the interviews, Willow communicated that one of the ways she felt she made her students feel smart was by letting all of them contribute.

Checking Agreement. Willow checked for agreement with her students across all three rehearsals to make sure everyone was on the same page. During the Launch interview, Willow explained that she had asked if there were questions after Kayla had finished her drawing so that Kayla could stay and explain to her peers if there were questions. At one point Willow asked, "does everyone agree with that?" just to make sure that she "wasn't just going along with one student's answer." During the Discussion interview, Willow explained how she was checking for agreement with one particular student who had said both 225 and 375 were X. After clearing up that confusion, Willow checked for final agreement when she asked, "so this can't just be X, right?" Across the three rehearsals, Willow asked her students if something "made sense" seven times. Willow also asked if there were any more questions five times across the three rehearsals. During the Monitoring interview she explained that she was "making sure there were no more confusing parts and like that they were all on the same page at that point." Willow wanted to make sure that ideas made sense to her students.

Positioning Students as Resources for Each Other. Willow consistently positioned students to learn from each other. During the Launch, when students had questions about terminology, Willow initially asked other students to contribute their understanding of the words. During the Monitoring rehearsal, when Wesley had a question related to his group's equation, Willow asked Kayla and Stacy if they could explain since the three of them had set the equation up together. Since Willow knew one of them understood, she "just wanted them to like try and explain it to him in a way that might make more sense to him." After the students did share their ideas, Willow checked back in with Wesley to see if what they had said made sense,

reinforcing that what his peers had said could have moved him forward. In the Discussion rehearsal, Willow asked Stacy to explain her thinking "so the other students could like see how she went through and made the table." Willow frequently positioned students to learn from each other during her rehearsals.

Supporting Student Contributions Practices. Willow's only supporting student contributions practice was positively responding.

Positively Responding. Willow was intentional in positively responding to students. During the Launch interview, Willow explained that she responded with "like it" to a student's drawing because the student did what she had requested she do so she was "saying thanks for contributing." Another time she responded with an "I like it" because a student "made his interpretation of where the bridge would go." During the Monitoring interview, Willow explained about how she bit her tongue on a question that would have implied she thought a student did not know how to set up the equation. She was intentional in changing the wording of her question to allow him to share his thinking instead of potentially setting him up to feel deficient. She ended up responding with "'walk me through it' instead of 'do you know how?'" Willow responded graciously to students when they participated and also made efforts to use non-deficit language with students.

Centering Student Thinking Practices. Centering Student Thinking Practices include students as deciders, eliciting student thinking, and pursuing student thinking.

Students as Deciders. Willow supported students to make decisions. During the Launch she had different students take turns contributing to a diagram on the board, deciding how the model should be drawn. During the Launch rehearsal, Willow had her students decide whether the zip lines met at the same point on the island or whether there was distance between. During

both the Launch and the Discussion, Willow invited a student to go to the board if she wanted to make explaining easier. During the Monitoring interview, Willow shared that she was fine with a student's suggestion about minimizing one side of their equation. "I was like, it's fine if we like use that. So I guess like I had to kind of like think about it a little bit to like explain like how much to minimize that whole side rather than just the Y value I guess." Willow supported her students in making decisions.

Eliciting Student Thinking. Willow asked for students to share their thinking across all three rehearsals. When Kayla was drawing, Willow asked her to tell the class what she was thinking as she drew so that the class could follow along. During the Launch interview, Willow also admitted to using revoicing to get a student to say more about what he was thinking. She started off the Monitoring rehearsal by asking her students to "explain what they were thinking about the problem without me asking like a specific question about it." Willow asked questions to see what a student was thinking about when she set up an equation, to have a student participate who had not spoken yet, and to encourage peer students to help explain when their classmate was unclear on something. During the Discussion interview, Willow had one student explain her thinking on their table and then later had another student explain his equation. At least twice during the interview, Willow mentioned intentionally wanting to hear student thinking so that she could figure out how to relate it to another student's thinking. Another way she elicited student thinking was by asking for questions. Across the rehearsals she asked for questions from students five times.

Pursuing Student Thinking. Willow pursued student thinking in the sense that she used their stated questions to progress them forward. During the Launch interview, Willow shared that she had wanted to draw a bridge because that mattered to a student. She said she did not

think about how it was not really important, but "since it popped out to him, like I wanted to still address it." During the Monitoring interview, Willow described going along with the suggestion of minimizing one side of the equation even though she was somewhat unsure by that suggestion. There were several instances when Willow asked questions to dig deeper into what students were suggesting - Wesley and the two square roots, Stacy thinking she could distribute, and Kayla's drawing. During the Discussion interview, Willow pursued Stacy's suggestion that both 225 and 375 were X with clarifying questions to help her see that she was contradicting herself. Near the end of this interview, Willow shared that one of the ways she felt she made them feel smart was by "just going along with their explanations… going along and like trying to actually understand what they were saying."

Guiding Student Thinking Practices. Guiding Student Thinking Practices include directing attention to key information, clarifying and nudging students' thinking, managing misconceptions, connecting context, and connecting student work.

Directing Attention to Key Information. During the Launch interview, she explained how they were "listing out all the key ideas and then seeing where we could go from that." After they had a diagram, Willow asked her students about the placement of the island because that was "one of the main parts of the problem." When Jackson was not sure where to draw the bridge, Willow reread the original problem to help him make sense about what it said in connection with the bridge, that "once they get to the island they have to exit off of the bridge." During the Monitoring interview, Willow explained that she had asked her students what they were doing with the total to direct attention to the idea that "they weren't supposed to be just like finding the total amount of zip line wire. They were supposed to be trying to

minimize that." During the Discussion interview, Willow discussed how she directed Stacy towards her own descriptions of the model to help her see that she was contradicting herself.

Clarifying and Nudging Students' Thinking. Willow guided students towards clarity across all three rehearsals. During the Launching interview, Willow shared that she ended up having to clarify terminology such as attraction and minimize with her students. When Jackson was confused about where the towers should be, Willow used the diagram in addition to the language from the task to clarify. And when Jackson was unsure about the location of the bridge, she had them reread the task and asked where the riders are when they exit to "give them a place to start with the placement of the bridge." She also asked her students a few identifying questions just to make sure they were clear about the task. During the Monitoring interview, Willow shared early on that she used revoicing as a means of clarifying. During the Discussion rehearsal, Willow asked Stacy to clarify what she said about 225 and 375 both being X so that other students weren't confused. When Stacy did not catch on right away, Willow asked her where the Xs were in the diagram to help her see that the two lengths were not the same.

Managing Misconceptions. Willow made efforts to clear up student confusion. During the Launch rehearsal there was some confusion about vocabulary within the task and about the setup of the diagram. Willow worked towards clearing up their confusion by involving other students to clear up vocabulary and incorporating the diagram and language from the task to clear up confusion related to the diagram. Early in the Monitoring interview, Willow shared one of her goals was "trying to figure out their barriers I guess and addressing those." In general, Willow responded to student confusion with questions to either have the student say more or see if a peer could contribute. During the Monitoring rehearsal this included Wesley's confusion about their equation and Stacy's suggestion to distribute. During the Discussion interview,

Willow tended to a misconception by a student that two different values were X by questioning her and then using her own words to highlight the contradiction. Later in the interview, Willow shared that she said, "do you see how you could maybe use a table to find the minimum?" so that Jackson's group could be aware of an approach that would work with the equation they constructed.

Connecting Context. Willow attended to connecting to the context of the task. For example, in the Launch interview, she explained that she had asked her students "which part is 600 meters wide in Kayla's diagram?" to "relate it back to the task." During the Monitoring interview Willow explained that she asked about the hypotenuse in terms of the problem, why they were adding the two hypotenuses together, and what they were doing with that total to connect them back "again to the minimizing the zip line wire." While she did facilitate connecting to the context during the first two rehearsals, during the Discussion interview she shared that even though her students had made connections to the context when they shared their approaches, she wished she had done more connecting herself.

Connecting Student Work. During the Discussion interview, Willow described instances when she intentionally was connecting student work. At the beginning of the interview, Willow confessed that she felt that she was "so focused on making the connections that I didn't really talk about each solution like by itself as much as I maybe should have." Willow shared that she requested students to explain their thinking so that she could "figure out how to relate it to the next one." In response to an explanation from Jackson, Willow stated "so you're saying that this part and the other square root are the same as her square roots" to revoice his thinking while "making the connection between the two solutions." She later asked Harry to explain how the equation related to the table since neither were his group's approach and she wanted to make

sure he could see connections between the two. Near the end, Willow brought up Jackson's struggle with finding the minimum to "relate it to the other, to Stacy's solution." Willow rarely pointed out connections herself, but guided students towards making connections among the two approaches presented.

Transitions. Willow was intentional in framing and transitioning sometimes. During the Launch interview, after responding to a student who was confused by the word "attraction," Willow mentioned not knowing how to transition back to talking about the task, so she ended up asking if there were any more questions about the task. During the Monitoring rehearsal, there was a moment when Willow stated "so Kayla wants to start with the right side of the equation and we're trying to minimize it" which was an intentional "recap" before planning to ask them how they were going to minimize it. During the Discussion interview, Willow started off by stating we are going over everyone's solutions "to introduce what we're doing." Later she shared how she said "thank you" to a student to not only thank him for explaining, but it "was also like a segue kind of like thanks, like we're done with that part. Now we're moving on."

Practices Related to Standards for Mathematical Practice. Practices related to standards for mathematical practice include attending to precision and model with mathematics.

Attending to Precision. There were a few moments when Willow attended to precision during her facilitation of the rehearsals. In the Launch rehearsal, Willow had her students clarify whether or not the bottom included the towers "just to be able to label it correctly." Later when a student responded to where the island is placed with "in the lagoon," she agreed that what he said "did highlight the fact that like it didn't say a specific placement of the island." In the Discussion interview, Willow explained how she worked towards helping Stacy recognize that 225 and 375 could not both be X as she had suggested. It was in tending to the different labels

on the diagram that Willow helped her see that her solution was not precise - her suggested two solutions represented the two distances on either side of the island, but only one could be X.

Model with Mathematics. Modeling played an important role in all three of Willow's rehearsals. During the Launch interview, Willow explained how she had students take turns contributing to a drawing of their understanding of the problem. When Jackson was unclear about the towers being on opposite sides, Willow was able to use Kayla's drawing to help him. "I think just like giving like the visual of like being able to explain it in terms of the lagoon in between them like made it easier for him to like actually see that they were on opposite sides." When Jackson wanted to know where the bridge would be, she nudged him towards making sense of that and then had him come up and add a bridge to their drawing. During the Modeling interview, Willow shared her reasoning by asking different questions about different aspects of the drawing (e.g. the hypotenuse) her students had on their whiteboards to make sense of where her students were struggling to make progress. During the Discussion interview, Willow explained how she used the model drawn by a student when there was confusion about what exactly was X. After a TE pause, Willow ended up asking Stacy where the Xs were in the diagram "so that she could kind of see that they both aren't X. Cause they wouldn't be the same lengths in the diagram." Willow facilitated the use of the diagram by inviting Stacy to go up and point on the diagram.

Dispositions

In the framework, Dispositions are described as "Habits of thinking and action regarding teaching and children" and this guided my organization of this section. I started noticing which of my codes better described habits of thinking or action and whether the focus was on teaching or on children. Most of the codes fell into categories related to teaching, but a few were directed

towards students. Willow's *habits of thinking regarding students* included relating empathetically with students, making sense of student thinking, and positive response. Her *habits of thinking regarding teaching* include reflection upon practice, goal/success, and time management awareness. Her *habits of action regarding teaching* were further categorized into culture building, focusing on student thinking, and facilitation. Culture building dispositions included collectively moving forward and student authority. Focusing on student thinking dispositions include valuing student talk, valuing student understanding, and valuing student discovery. Facilitation dispositions include maintaining a high cognitive demand, and problem solving.

Habits of Thinking Regarding Students. Habits of Thinking Regarding Students include relating empathetically with students, considering student thinking, and positive response.

Relating Empathetically with Students. Willow considered the learning experience from the students' point of view. During the Launch interview, Willow explained she had one of her students read the task because "I think students might pay attention more if their classmate was reading it." Willow sought the assistance of other students when there were questions about vocabulary because she thought students would better understand their peers. Willow pursued ideas of students that may have been tangential in nature because, "if a student has a question that's just like not addressed or like just like kind of blown over, like they'll just be like stuck on that and won't be able to like move on in the task." In response to Stacy trying to point at the board from her seat, Willow invited her to go to the board if she wanted, thinking that being able to actually point to the diagram at the board might be easier for her. During the Monitoring interview, Willow was thinking out loud through different struggles her students might have

been having and concluded "that would make sense." During the Discussion interview, she explained she had Stacy clarify because what she had said "might be something confusing for like Jackson and Harry." Willow consistently thought about the student experience both in connection to general strategies and in connection with the details occurring in her rehearsals.

Considering Students' Thinking. During the interviews, Willow shared her thoughts on trying to understand what was going on in her students' heads. During the Monitoring interview, Willow shared how she had noticed that her students did not have a solution or approach, so she was "just going to try and like break it down in terms of the task and like relate it back to the minimizing zip line more." Later when it was clear her students did not know how to minimize, she reflected that she "wasn't sure if like they were struggling with the fact that they had to minimize it or like that they couldn't figure out how to minimize it. I guess because if they were just finding the totals then like that would make sense that they didn't know where to go next because like what numbers do you plug in? But if you're like minimizing it, I thought that they might like throw something out there. Like we could try some different values or something like that." Later with Wesley and his confusion about square roots, she initially "thought he meant like why there was a square root plus a square root - so like the two separate square roots," but then after he responded, she realized that his issue was in the equation's connection to the Pythagorean theorem. During the Discussion interview, Willow shared that she thought Stacy "was thinking is like they're two separate equations there. And I was just going to try and relate it to the total column to show her that they're - it's the same equation." Willow was trying to make sense of student thinking as she responded.

Positive Response. There were not many times during the interviews that Willow discussed responding positively to students. However, during the Discussion interview, Willow

suggested her beliefs align with responding to students graciously. In response to why she told a student 'thank you,' she reflected, "I don't know why I do that because sometimes when they respond I'm like, 'okay. Good job. Thank you.' and other times I just like, I'm like, move on. But I mean to do that like every time." She is of a mindset to respond in a positive way with consistency.

Habits of Thinking Regarding Teaching. Habits of thinking regarding teaching include reflection upon practice, goals/success, and time management awareness.

Reflection upon practice. Willow shared reflections on her implementation. During the Launch interview, Willow shared that she wanted to address student desires such as drawing a bridge even though they were not important to the problem, but also confessed that she had not thought about how it would affect the rest of the problem. During the Monitoring interview, Willow wished she could have figured out Wesley's confusion to address it sooner, but also acknowledged that because she did not know what he was struggling with she had no way of knowing to ask him sooner. In sharing her thoughts on Stacy and her confusion around distributing, Willow did wish she had at the very least said something like "'I'll get back to that'… not just like leaving her hanging I guess." During the Discussion interview, Willow reflected on things she could have done better such as talking about the solution and relating it to the context more. Across all three interviews, Willow repeatedly expressed struggling with what to say. During the Launch interview she mentioned not wanting to put words in her students' mouths. During the Monitoring interview, she shared not wanting to overly influence their approach to the problem. "It was hard to not just say 'why don't you try graphing it or why don't you try putting it in a table?'" During the Discussion interview, she also communicated that same struggle, but then also shared her realization that she just needed to trust herself more

to be able to respond in the moment. As she put it, "I think that when I just trusted myself and like went with it, I was able to do well." Willow was noticing aspects of her practice that could have been improved, but she was also learning to trust herself.

Goals/Success. Willow had goals she was working towards during the rehearsals. During the Launch interview, Willow shared that because of her experience playing the role of a student in a different rehearsal, she "wanted to make sure that I addressed that and like all the like little things that students might get caught up on so that they like would be able to like move through the task." And yet in her effort to address student questions, she shared, "I didn't really know how to like transition back into like talking about the tasks and I wanted to get it more like focused on the task rather than like keep going off on like attractions and stuff like that." So, during the Launch in working towards one goal, she felt she shorted herself in another. During the Monitoring interview, Willow shared that her personal goals towards her enactment included incorporating revoicing and "just trying to figure out their barriers, I guess, and addressing those." During the Discussion interview, Willow mentioned that she was focusing on connecting student work and stating the solution. Near the end of the interview, Willow shared that she felt she had done well with connecting student work, but felt she needed to "talk about the solution and kind of relate it back to context more." During the Discussion interview, Willow shared not feeling so successful with tending to unanticipated responses from students.

Time Management Awareness. Willow was acutely aware of how the rehearsal was going in connection with time. During the Launch interview, Willow mentioned several times that she felt they were spending too much time on explanations and she was wanting "to like get to the actual task too." She even intentionally tried to speed things up. "I knew I was taking too long on the first two explanations, so I wanted to like try and do this one a little bit

faster." During the interview she mentioned that her response to Jackson "was kind of a little bit rushed just because we had been, like we hadn't done anything with the problems still." When asked what she might do differently, Willow stated "I would definitely try and like speed up or like, not really pay as much attention to the arbitrary parts of the problem." During the Monitoring interview, Willow again communicated concern about time management and making progress on the task. She explained that she revoiced Kayla's idea to minimize the right side of the equation to recap but also "wanted to get them back, like to the problem and like how to keep going." In describing what she would do differently, she mentioned wanting to "find a way to get Wesley's thing out earlier, like how he didn't know how to set up the equation." During the Discussion interview, she talked about one of the ways she was using 'thank you' was to communicate that they were moving on.

Habits of action regarding teaching. Habits of action regarding teaching include culture building dispositions, focusing on student thinking dispositions, and facilitation dispositions.

Culture building dispositions. Culture building dispositions include collectively move forward, and student authority.

Collectively Move Forward. Willow thought of her class as a collective in which students participated. During the Launch interview, Willow explained that she frequently asked if there were any more questions "because I felt like before we could move on with the problem, I wanted to address anything that we needed to." Willow explained she checked in with the other students at one point "just to make sure that I wasn't going along with one student's answer and like not explaining it if I needed to." Near the end of the interview, she mentioned she had been "making sure that they were like with me on it. Like each step, like being able to go

forward knowing that each of them didn't like have any more questions about what we'd just done." During the Monitoring interview, Willow explained that when Wesley was confused about his group's work, she worked to have Stacy and Kayla help him better understand as they were a group and had been a part of the initial decisions about their work. During the interview, she also explained that she wanted to attend to questions so "that I could keep them moving forward all together." Willow wanted to make sure all in the collective understood before moving on.

Student Authority. Willow valued her students as thinkers and doers in the classroom. During the Launch interview, Willow explained how she "wanted the other students to try and help me" in making sense of vocabulary that other students struggled with. In response to two different questions about vocabulary, Willow immediately asked the other students for input on the meaning of the word. When Kayla was done drawing, Willow asked if there were any questions so that Kayla could stay up and explain to her peers if there were. Willow had a student read the task initially and had students take turns contributing to the diagram. Willow asked them about the placement of the island because she wanted them to suggest a spot for the island in the diagram on the board. During a TE pause in the Monitoring rehearsal, Willow expressed that she "wanted them to like explore and find it on their own." In describing how she felt she made her students feel smart, Willow stated that she "tried to make sure that each of them was contributing something to the group." During both the Launch and the Discussion interviews, Willow explained that she invited Stacy (she had been pointing to the diagram from her seat) to go up to the board so that she could more easily explain her thinking.

Focusing on Student Thinking dispositions. Focusing on Student Thinking dispositions include valuing student talk, valuing student understanding, and valuing student discovery.

Valuing Student Talk. Willow consistently valued students participating vocally. During the Launch interview, Willow described revoicing a student because she "wanted him to like expand on it," she wanted to hear him talk more about it. She surmised that one of the ways she made her students feel smart was in "going based off of like their interpretation of the problem." At the beginning of the Monitoring interview, Willow was suggesting that revoicing is the easiest talk move to incorporate because "like you can revoice it yourself or get a student to and that incorporates them into the discussion." She was intentional in having students not only talk themselves, but potentially even echo what a peer might say. She started off the rehearsal by asking them what they were working on because she "wanted the students to like explain like what they were thinking about the problem." During the Discussion interview, Willow shared that she "just like wanted them to kind of each explain like how they came up with their solutions." Willow valued students participating in discussions.

Valuing Student Understanding. Willow was consistently passionate about wanting her students to understand. There were several times during the Launch rehearsal that Willow explained she checked in with students because she wanted to see if they understood or "if I needed to like go into more detail or try to explain it in a different way." Later she asked students several questions in a row "so they could show that they understood the problem." Her deep-rooted desire for valuing student understanding was evident in a statement she made early on in the Monitoring interview when she stated, "I think like when you discuss things with students, like you already have to kind of clarify and like just make sure that everyone gets it." During this rehearsal she again asked if things made sense to students after they had questions before moving on. Near the end, in response to how she felt she made her students feel smart, she stated she was trying to ensure "that they each knew how to like do every part of the

task and that they all knew like where they could go next too." During the Discussion interview, Willow attended to correcting a student misunderstanding when it presented itself. As this was the rehearsal in which different groups were sharing their approaches, Willow was intentional in asking students to explain each other's work "to make sure that they were like following along and still understood what was going on."

Valuing Student Discovery. Willow wanted students to uncover ideas on their own. During the Launch interview, Willow shared that she struggled with not telling her students insights about the island, but in her opinion "that's like the whole purpose of the problem, is to get like get students to know that without saying it directly, like that would have taken like half the problem away I guess." During the Monitoring interview, Willow again shared how she was struggling not to lead her students towards a certain approach, that she "wanted them to like explore and find it on their own, I guess, like think of it on their own." During the Discussion interview, Willow discussed managing Stacy's confusion over two values being X. She used Stacy's own thinking to help her discover her own contradiction.

Facilitation dispositions. Facilitation dispositions include maintain high cognitive demand, and problem solving.

Maintain High Cognitive Demand. Willow was concerned with maintaining a high cognitive demand during her rehearsals. During the Launch interview she described feeling stuck at one point because she was struggling with "not just like stating that the island would not be in that specific spot." She explicitly stated that she "didn't want to like lessen the cognitive demand of the problem, but like I wanted them to like give it to me." During the Monitoring rehearsal, Willow explained during a TE pause that she did not want to lead her students to something, but for them to find it on their own. At the end of the interview, she revisited this

idea by saying that she felt she had supported student thinking in that she "didn't really like lower the level of the task, I guess. Even though I wanted to like really badly. Like I didn't lead them to anything and I didn't really like give them more than what the task did." While she did not address maintaining a high cognitive demand during the Discussion interview, her choice to have students describe similarities among approaches is consistent with maintaining a high cognitive demand.

Problem Solving. During the Launch interview, Willow elaborated on her approach to the task with her students. Early on, she asked her students if there was anything that jumped out to them - "key ideas, like somewhere to start basically." She elaborated that she "just wanted like somewhere like to, for each of them to have like a starting point. So like, uh, listing out all the key ideas and then seeing where we could go from that." Later in the interview, Willow explained that the reason she wanted to break the task down by sentences was "I think just so like we wouldn't just like skip over things and we could address each part of the problem." When a student did not know where to put the bridge on their drawing, Willow was intentional in having the student reread the problem to try to make sense of where the bridge might go.

Understanding

Willow's Experience Solving the Task. During the Monitoring rehearsal, Willow shared that when she solved the task, she "put in random values for X." During the Launch interview, she referenced that the idea of having the zip lines meet in the same spot on the island came from her experience solving the problem initially. She thought it would be easier for her students to be able to use the whole 600 meters instead of having to adjust for a different setup in

which the zip lines did not meet at the same point on the island. As she put it, "I think I just like kind of used what I knew."

Anticipating Student Responses. Willow repeatedly shared that she was uncertain about how to respond to students. She knew that questions about assumptions might arise since that was her barrier when playing the role of student. So she wanted to ensure they were addressed so that students did not get caught up on those. However, during the Launch interview, Willow shared that she was "taken off guard" by several questions. She was asked to clarify what an attraction is, what it means to minimize, why the towers were drawn on opposite sides, and where the bridge would go in their diagram. During the Monitoring rehearsal, Willow asked what part of the equation do you want to minimize and Kayla responded with "this side." During the interview, Willow explained that her response "wasn't like exactly what I was hoping for her to say cause I wanted her to say the Y value." During the Discussion rehearsal, Willow was also caught off guard when Stacy suggested two different numbers were X. Willow initially responded to Stacy with a question, but when the student did not respond how she expected and still thought two different numbers were X, Willow admitted she struggled and was unsure how to recover. Willow struggled with how to respond when caught off guard by unanticipated student responses.

Her Thoughts on Different Approaches. Willow was not explicit in favoring any approach in any way. During the Monitoring interview, at one point she said "I was just trying to remember like how I did this problem when I solved it. Um and like I was trying to think of ways that I could get them to start thinking in the same kind of ways without like leading them to it." During the same interview she also disclosed her own challenge in not leading them to a particular approach. She shared, "I had solutions in my head that I knew about, but I didn't want

to just like, I didn't really know how to just like say something to get them to start thinking about a graph or get them to start thinking about a table." She did point out to Jackson that he "could've used a table to find the minimum," but there was no clear communication on one approach being better than another. Thus, she acknowledged that the table was the approach during the Discussion that provided a solution.

Key Math Ideas. During the Launch interview, Willow asked her students about the placement of the island "because I knew that that was like one of the main parts of the problem that we didn't know that that's where the island goes." During a pause, Willow responded to a TE question with, "so the question is asking how far from the bank of the lagoon the island should be in order to minimize the length of zipline wire." During the Monitoring interview, Willow shared that she understood her students were supposed to be trying to minimize the total amount of zip line wire. During the Discussion interview, Willow explained how she knew that the two numbers suggested by Stacy could not both be X because they represented different lengths in the diagram. So Willow understood the goal of the task, the role that minimizing played in making sense of the task, and that the students' diagram represented two different lengths.

Vision

What has been learned from above about Willow is that she is passionate about her students driving the discussion and advancing their understanding through questions. She initially described her best day of teaching mathematics as including "a lot of discussion among my students and using their own ways of doing math that they have developed," and later as "seeing the change in a student, from not knowing or not liking some part of math to knowing or liking it." Her focus on student discussion is already visible in the practices that center student

thinking and in her disposition that values student talk. Her desire to maintain a high cognitive demand and not tell them too much while also valuing student discovery supports the idea of using students' ways of doing math that they have developed. Her later ideal focuses more on student understanding - and that is also visible in her conceptual practices (leaving no student behind through inclusion, checking in, and positioning students as resources for each other) and in dispositions such as collectively moving forward and valuing student understanding. Willow sees her future teacher self as someone who questions her students toward sense making.

Revisiting the Research Question - Why does Willow do what she does?

This section addresses Willow's thoughts in connection with the second research question: *How inclusively do PTs frame mathematical activity and mathematical ability when reflecting on their intentions during rehearsals?*

Willow's Whys in connection with the Two Frames

The intentions of the PTs were also of interest to the author and so below Willow's actions along with intent are considered in connection with the three frames used in this study.

Mathematical Activity. Willow's framing of mathematical activity was described in relation to the sense-making frame and the multidimensional ability frame.

Sense-making Frame. Willow was more inclusive in her framing of mathematical activity in that she wanted her students to do the thinking. During the Monitoring interview, Willow expressed remorse over leaving Stacy's confusion related to distributing the radical unresolved. Her frustration over leaving a student's confusion unaddressed suggests that she values sense-making over answers. She did not present any formula to her students. Furthermore, she tried to help her students make more sense of their thinking through questioning. For example, during the Monitoring rehearsal when a student suggested that they

minimize one side of the equation, which represented a side of a triangle, Willow asked her "so how do you think you could go about that?" And during the Discussion rehearsal, when Stacy suggested that two different numbers represented the same distance, Willow asked her questions to help her make sense of the contradiction that she was saying. To shed more light on how Willow valued connections, she admitted during the Discussion interview that she "was so focused on making the connections that I didn't really, um, talk about each solution like by itself as much as I maybe should have." Willow strongly viewed mathematical learning as sense-making.

Multidimensional Ability Frame. During the interviews, Willow discussed the mathematics that she felt her students did during the task. During the Launch interview, Willow recounted her students labeled the diagram. During the Monitoring interview, Willow mentioned that she got them to "think about minimizing a function" and that a table is a valid approach to do that. During the Discussion interview, she reported that her students "got to interpret parts of the equation in terms of the context of the problem." So, while these concepts of what counts as mathematics may not have been explicitly stated to the students, she is viewing mathematics to be much more than rote practice.

Mathematical Ability. Willow's framing of mathematical ability was mostly in alignment with the multiple ability frame (inclusive). It was common for Willow to not immediately answer a student's question, but to restate it and ask it to other students so that they could provide their input. Hence, she was positioning students as resources for their peers' learning. During the Monitoring interview, Willow shared her desire to adjust the language of a question to be less deficit-sounding. She realized that Wesley probably did not know how his group set up their equation and started to ask, "Do you know how" but then cut herself off and

said instead “Walk me through how you set up the equation.” This shift in wording went from suggesting he did not have a skill to letting him share the skills he had - which went from positioning Wesley as a student in need to a competent student. While playing the role of teacher, she made no explicit statements about mutual dependence, but she was constantly asking a variety of students to participate and asking questions to check in on their collective understanding. Additionally, Willow was not explicitly valuing correctness. Across the three rehearsals, she rarely made comments about what was correct. In response to student statements, three times she basically said “okay, right” more of as casual acknowledgement and once she said “that’s true” in response to a student’s comment about exiting riders getting kicked in the head by those on the zipline if the bridge went under the zipline. Thus, her combination of not valorizing correctness while also positioning students to learn from each other implies she was framing mathematical activity inclusively.

Summary

Willow was very responsive to her students and whatever needs they had in the moment and across rehearsals. During the Launch she did more checking in to make sure students were in agreement as they were drawing the diagram. During the Monitoring rehearsal, probing questions were used with much higher frequency. And during the Discussion, math nudges and creating opportunities to engage with each other’s thinking were used with heightened frequency compared to other rehearsals. The created opportunities had to do with true questions asking students to make sense of each other’s thinking. Two of the three math nudges she did were in connection with trying to help Stacy realize her misstatement. She asked many questions and those types of questions varied depending on the learning situation - whether the students were

making sense of the task, needing help progressing with their own thinking, or sharing their progress with peers.

Willow values sense making but does not press for student understanding. When Kayla suggested minimizing a side, Willow asked her questions but never pressed for the meaning in connection with why that would not make sense. She allowed Kayla to follow that idea and when she got stuck, other students were brought into the conversation, but the fact that minimizing a side of a triangle would mean the side would not exist was never addressed. And with Stacy, she did not help her make sense of *why* she thought there were two values for X. Willow helped her see that there was only one X based on her drawing, but she did not help her understand why she thought there were two.

Willow strongly views her role as teacher as a facilitator of student learning. She values questioning students to help them make progress as a group. When one student has a question, she frequently tries to bring other students into that conversation so that multiple students are sharing their thinking so that they can learn from each other. She definitely has that "guide on the side" mindset about teaching which allows her to successfully center student thinking.

Chapter 6: Analysis of Jackson

Jackson will first be presented in relation to his responses to questions concerning students' mathematical activity, mathematical ability, and misconceptions to present a foundation of Jackson's perceptions of teaching and learning mathematics. To address how Jackson supported student thinking and misconceptions during rehearsals, an overview of Jackson's experience through the three rehearsals will be described, including summative data describing his talk moves. Then, for each rehearsal, Jackson's moves in response to students will be analyzed followed by a more in-depth analysis of how he responded to student thinking and a discussion of how his moves connected to the three frames. To address the second research question of why Jackson does what he does, the Hammerness et al. (2005) framework will be used to organize his shared intentions. At the end, there will be a discussion of Jackson's themes in connection with the three frames.

Jackson's Perspective on Mathematics and Mathematics Teaching

Jackson and Mathematical Activity

At the beginning of the semester, Jackson described mathematics as being more focused on rules and measurements. After the course, his description of mathematics focused more on seeing and studying patterns. In describing what it means to learn mathematics, he answered "it means to learn the fundamentals of problems and being able to apply those fundamentals to any problem." So, Jackson moved to applications and the idea of transfer. In describing his post-semester understanding of what it means to teach mathematics, he shared "it means to teach the 'eye' of seeing those patterns in mathematics."

Jackson and Mathematical Ability

Jackson's responses were consistent regarding what it means to be smart in that he claimed that "anyone can be smart at math."

Jackson and Responding to Student Thinking

Jackson's responses regarding how to respond to student thinking showed development across the semester. His view of responding to an incorrect student response was more consistent in that he wanted to learn more about their thinking. He referred to an incorrect response as "still valuable knowledge." Initially he did not support students explaining correct answers suggesting that explaining every step took away from "the universal language of math." However, after the course, his mind had changed and he stated, "explanations and reasonings are the backbone of all mathematical thinking and logic."

Moves Implemented by Jackson to Support Student Thinking During the Three Rehearsals

Table 11 *Jackson's Rehearsal Structure*

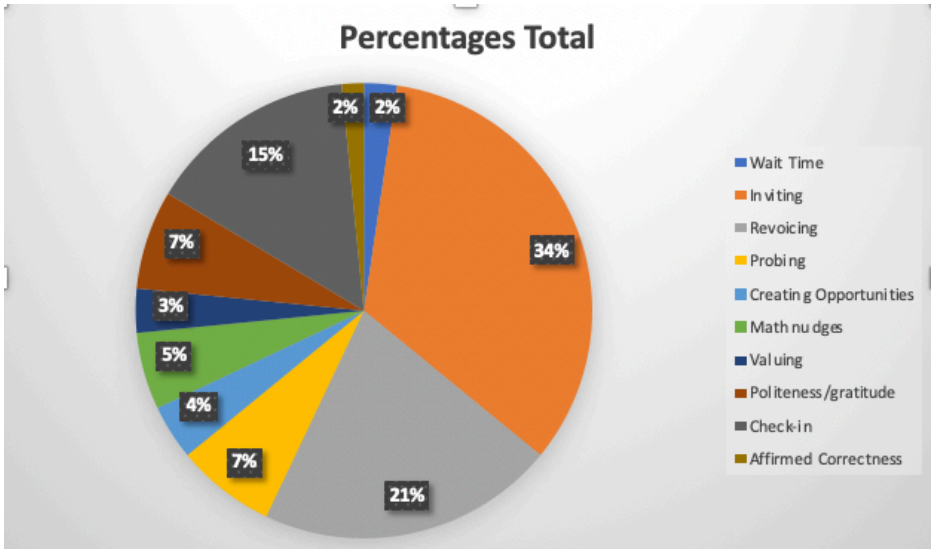
Order	Date	Teacher	Student	Student	Student	Observer	Observer
Launching 1	10/29/2019	Jackson	Harry	Willow	Wesley	Stacy	Kayla
Launching 2	10/29/2019	Willow	Jackson	Kayla	Stacy	Wesley	Harry
Launching 3	10/29/2019	Stacy	Jackson	Harry	Wesley	Stacy	Kayla
Launching 4	10/29/2019	Kayla	Harry	Willow	Stacy	Wesley	Jackson
Launching 5	11/5/2019	Harry	Kayla	Willow	Wesley	Stacy	Jackson
Launching 6	11/5/2019	Wesley	Kayla	Jackson	Stacy	Willow	Harry
Monitoring 1	11/5/2019	Kayla	Harry	Willow	Jackson	Stacy	Wesley
Monitoring 2	11/5/2019	Harry	Stacy	Wesley	Jackson	Willow	Kayla
Monitoring 3	11/5/2019	Wesley	Kayla	Harry	Jackson	Stacy	Willow
Monitoring 4	11/12/2019	Stacy	Kayla	Welsey	Harry	Jackson	Willow?
Monitoring 5	11/12/2019	Jackson	Wesley	Stacy	Kayla	Harry	Willow?
Monitoring 6	11/19/2019	Willow	Stacy	Wesley	Kayla	Jackson	Harry
Discussion 1	11/19/2019	Wesley	Harry	Willow	Kayla	Jackson	Stacy (co-teacher)
Discussion 2	11/19/2019	Stacy	Harry	Willow	Kayla	Jackson	Wesley (co-teacher)
Discussion 3	11/26/2019	Harry	Stacy	Willow	Wesley	Kayla	Jackson (co-teacher)
Discussion 4	11/26/2019	Jackson	Stacy	Willow	Wesley	Kayla	Harry (co-teacher)
Discussion 5	11/26/2019	Willow	Jackson	Harry	Stacy	Wesley	Kayla (co-teacher_
Discussion 6	11/26/2019	Kayla	Jackson	Harry	Stacy	Wesley	Willow (co-teacher)

Rehearsal Structure

Jackson was the very first to play the role of teacher during the Launch rehearsals. For the Monitoring rehearsal, Jackson was the fifth of the group to play the role of teacher, but second on the day he went. For the Discussion rehearsal he was in a pair that went second overall, but they were the first to go on the day they went. Within his pair, Jackson was the second to be the teacher. While Jackson was the very first PT to assume the role of teacher, he had the opportunity to view someone before him during later rehearsals.

Talk Moves Across All Three Rehearsals

Figure 14 *Jackson's Percentages of Talk Moves Across All Three Rehearsals*

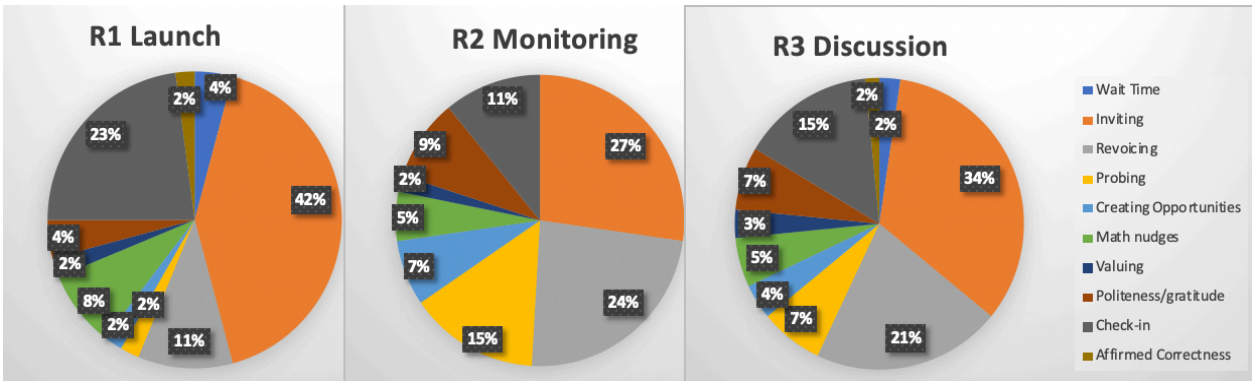


In total, Jackson implemented 48 moves to support student thinking during the first rehearsal (Launching the task), 55 moves during the second rehearsal (Monitoring), and 25 during the third rehearsal (Class Discussion). Jackson used inviting moves (34%) most for around a third of his total moves. Revoicing (21%) was his second most frequent talk move. Jackson checked in (15%) with his students with some frequency. Jackson was explicitly polite for 7% of his moves, gave math nudges for 5%, probed student thinking for 7%, created

opportunities for students to engage in each other's thinking for 4%, and explicitly valued student thinking in 3% of his moves. Affirming correctness (2%) and wait time (2%) were both tied in utilization.

Comparing Relative Usage Per Rehearsal

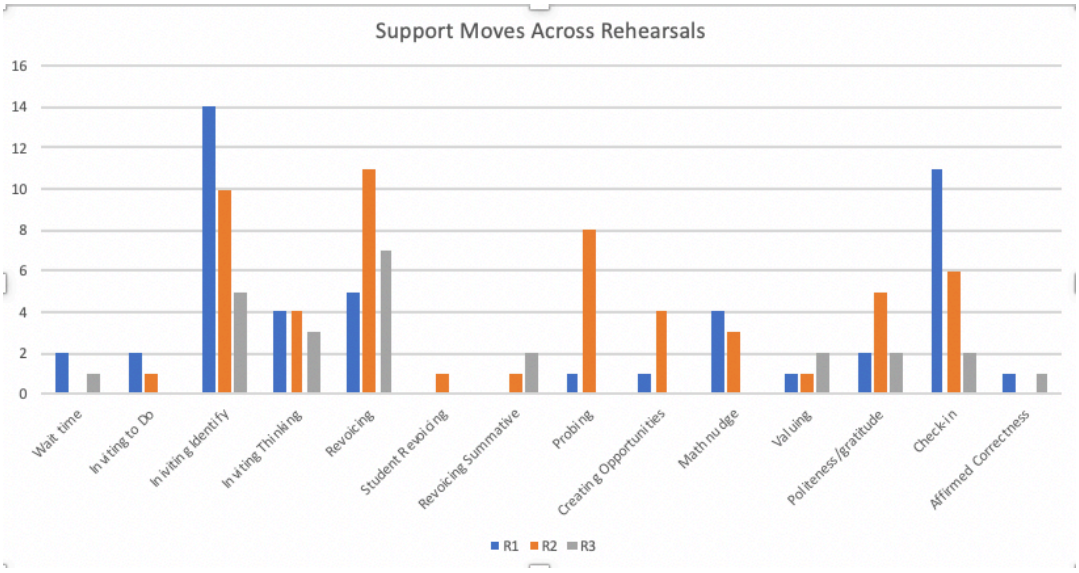
Figure 15 *Jackson's Talk Moves Across Rehearsals*



As each rehearsal was for a different amount of time, one approach to analyzing his implementations is to notice how he used moves relative to other moves within the same rehearsal. In all three rehearsals, the combination of inviting and revoicing make up more than 50% of total moves during the rehearsal, so those function as the connectors in dialogue when he is leading his students.

Analysis Across Rehearsals

Figure 16 *Jackson's Talk Moves Per Rehearsal*

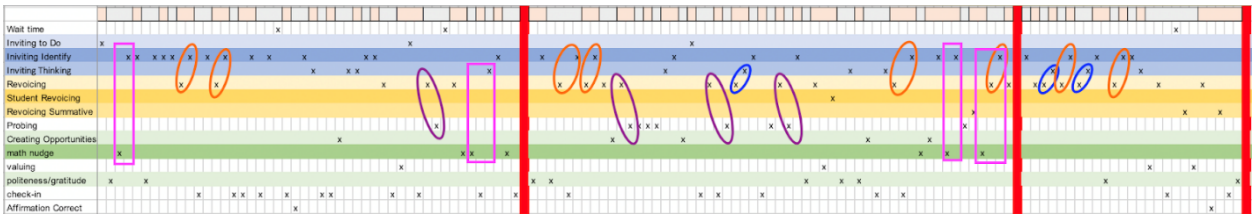


In the graph above the different rehearsals are represented by different colors for each talk move to note the variation among the number of talk moves among rehearsals. Jackson did not exercise wait time with much consistency in that there were two instances of wait time during the Launch, none during Monitoring, and one during the Discussion. Inviting to do, inviting identify, math nudges, and check-ins decreased across rehearsals. Jackson was fairly consistent with inviting thinking, valuing student thinking, and affirmed correctness. Jackson worked in a student revoicing only once during the Monitoring rehearsal. Jackson increased his summative revoicing across rehearsals in that he did so zero times during the Launch, once during Monitoring, and twice during the Discussion. Revoicing, politeness, probing, and creating opportunities were used almost twice as often during the Monitoring rehearsal than other rehearsals. Jackson revoiced 14 times during Monitoring as compared to five during the Launch and seven during the Discussion. He used polite language five times during Monitoring as compared to twice during each of the other two rehearsals. Jackson probed student thinking

eight times during Monitoring, but only once during the Launch and not at all during the Discussion. He created opportunities to engage in other's thinking four times during Monitoring, but only once during the Launch and not at all during the Discussion.

Patterns of Talk Moves

Figure 17 *Jackson's Patterns of Talk Moves Across Rehearsals*



In analyzing Jackson's patterns, there were a few trends noted. Most of the times he revoiced it was paired with an invitation to identify (orange oval), an invitation to share thinking (blue oval) or a probing question (purple oval). Jackson almost always revoiced before asking a probing question. During the Monitoring rehearsal, he asked four probing questions in a row. If he provides math nudges, they tend to be near the end of the rehearsal. He followed up about half of his math nudges with an invitation for students to participate (pink rectangles).

Analyzing Jackson's Talk Moves Per Rehearsal

Below each rehearsal is described in more detail so that there is some context presented in how Jackson implemented the different talk moves. Afterwards, how Jackson responds to instances of student thinking are examined in closer detail. Instances of student thinking were identified by first looking at responses to his inviting thinking and probing moves. If a student volunteered a question or comment, those were also treated as instances of student thinking for Jackson to respond to.

First Rehearsal: Launching

The chart below shows the different talk moves that Jackson utilized while playing the role of teacher. Each line represents the moves made by Jackson when it was his turn speaking. The TE pauses are noted on the left. Moves of the same kind were color-coded to aid in sense-making of this data.

Table 12 *Jackson's Talk Moves During Launch*

Start time: 4:37	1	Inviting Do		
	2	Polite	Math Nudge	invite Identify
	3	invite Identify		
	4	Polite	invite Identify	
	5	invite Identify		
	6	invite Identify		
	7	Revoice	invite Identify	
	8	Check-in	invite Identify	
	9	Revoice	invite Identify	
	10	Check-in		
	11	Check-in	invite Identify	
	12	Check-in	invite Identify	wait time
	13	Check-in		
	14	Affirmation of correctness	invite Identify	
	16	inviting thinking		
	17	Check-in		
	18	Check-in		
(TE: 8:00 - 8:23)				
	19	Create opportunities	inviting thinking	
	20	inviting thinking		
	21	invite Identify		
	22	invite Identify		
	23	Revoice	Check-in	
	24	Valuing	Inviting Do	Check-in
(TE: 10:50 - 12:10)				
	26	Revoice	Probing	wait time
	27	Revoice	Math Nudge	
	28	Math Nudge		
	29	Check-in		
	30	inviting thinking		
	31	invite Identify		
	32	Math Nudge	Check-in	
End time: 15:03				

The Launching rehearsal lasted a total of ten minutes and 26 seconds. During that time Jackson was interrupted twice by the TE. The first time was briefer for 23 seconds. The second interruption lasted for a minute and 20 seconds. Initially Jackson asked a student to read the task (Invite to Do). Then Jackson suggested approaching the task by getting numbers on the board

(Math Nudge). This prompted him to invite students to identify information six times in a row. There was some added politeness during one of the exchanges and in another he repeated what the student said before asking students to identify more information.

Jackson then shifted to drawing a model of the problem using student input, which resulted in his checking-in with them seven times, asking them to identify five more times, inviting their thinking once, revoicing once, affirming correctness once, and one instance of wait time. His checking in was asking students if his drawn representation was accurately portraying what they had communicated when they had identified. There was a moment when Jackson confirmed a student's thinking was correct. A student said, "I don't think so" to which Jackson responded, affirming his thinking, with a "nope." The TE interrupted Jackson to ask where he was going with this and Jackson explained that he had wanted to get a picture on the board that they all could use that was based on their thinking.

Jackson continued by creating an opportunity to engage in each other's thinking by having them discuss ideas towards solving the problem. After the chat, he initially invited them to share their thinking, but when no one responded he asked them to identify twice before revoicing a student's suggested assumption and checking in with the class on that idea. Then a student asked a question about the location of the island and Jackson initially valued the student's question by stating "ooooh, that's a good question." He suggested they redraw the picture to test the student's question, but then realized he did not really know where he was going with this and asked the TE if he could pause. She nudged him to bring it back to Wesley's stated thinking earlier about minimization.

Jackson started back up with revoicing Wesley's thinking followed by a probing question and then wait time. Jackson then revoiced one more time before doing two math nudges in a row

to try to get Wesley to understand that the different heights of the towers is what is preventing the island from being in the center of the lagoon. Jackson checked in with Wesley to see if that explanation answered his question. Then in response to Harry's confusion, Jackson invited student thinking by asking him to identify. He then did a math nudge followed by a check-in with Harry to see if that answered his question.

Jackson's Support of Student Thinking during Launching the Task. Table 12 indicates that Jackson is adept at utilizing different talk moves but does not adequately examine how he uses those to support instances of responding to student thinking. Below, specific instances of student thinking combined with Jackson's response are discussed.

Harry suggested how the wires should be drawn [17]. Jackson was still working towards getting a model of the task on the board using input from students. He asked how the wires should be drawn and Harry suggested that he "draw them going to the island." Jackson drew them and then checked in with Harry "ok. So like that?"

Students discussed among themselves [20]. Jackson had asked his students to discuss any options they had to solve the problem. After his students had chatted for a few seconds, Jackson pulled them back together and asked, "What were some of the options you guys came up with to solve the task?" So, he immediately dove into wanting to hear their thinking.

Willow spoke up with a concern [22]. In response to Jackson asking if they had any questions while they were discussion in their group, Willow said that she hadn't mentioned this to her group, but "how would it be possible to solve it because the lines… zip line isn't like strait down… like, it has a curve in it." Jackson responded enthusiastically with an "oooh" and then invited the students to identify what assumptions they want to make about the problem - should they assume the line is taut or curved.

Wesley clarified he does not get it [24]. After it was agreed the wires were taut, Jackson checks in with a "Does that work?" to which Wesley jumped in saying "I just do not understand what it's asking. Like can't it be anywhere and we'll have the same amount of wire?" Jackson responded enthusiastically with another "oooh" and valued his question by stating "that's a good question." Jackson started to suggest redrawing the picture on the board to test something when he asked the TE if he could pause.

Wesley still did not understand [27]. Jackson revoiced Wesley's confusion and asked how the class could talk him through it, to which Wesley responded, "I just don't see how it matters where the island is." Jackson then revoiced what Wesley said as a question and then gave him a math nudge saying that the placement matters "cause of the heights of the towers."

Harry did not get "minimize" [31]. Jackson had just gotten done trying to help Wesley understand when Harry raised his hand and stated "so I don't know what you mean by minimize. The island should just be in the middle of the lagoon, right?" Jackson then invited him to rephrase his question to which Harry asked if it is about "minimizing the shapes or the lines" to which Jackson responded with another of his "oooh"s combined with a math nudge that addressed his concern.

Discussion of Jackson's Support of Student Thinking During Launch

The first instance of student thinking [17] was a suggestion on how to draw the zip lines, to which Jackson complied and then checked in with the student. After Jackson positioned his students to chat together [20], his first question afterwards asked for them to share from their discussion. When Willow shared her concern [22], Jackson put the decision about the assumption back on his students. When Wesley initially communicated that he didn't understand [24], Jackson explicitly valued his question before seeking help from the TE. The last two

instances of student thinking communicated involved issues related to the placement of the island. In both cases, [27] and [31], Jackson addressed the student's misconception with a math nudge.

Jackson's Support of Student Thinking and the Two Frames During Launch

In general, Jackson tended to be more inclusive in his framing during the beginning of the rehearsal and shifted to more exclusive framing towards the end of the rehearsal. Jackson framed mathematical ability inclusively by positioning students as resources for each other. By positioning the students as decision-makers in response to Willow's question and by asking the students to chat together, he was positioning them as resources for each other. However, his math nudges did not create opportunities for his students to show off intellectual strengths.

Jackson framed mathematical activity inclusively in that his asking to hear the results of their discussion suggests he was valuing their sense-making. However, with the two misconceptions communicated by Wesley and Harry near the end relating to the location of the island, Jackson shifted to more exclusive framing. Initially Jackson wanted to try to help Wesley make sense of his misconception. He suggested that the group redraw the model on the board to test Wesley's misunderstanding. However, Jackson was unsure of where he was going with that and ended up pausing the rehearsal. When the group returned to playing their roles, Jackson revoiced Wesley's thinking and asked how the class could help him, and when Wesley again stated that he did not understand, Jackson directly addressed his misconception with a math nudge. With Harry, Jackson did initially press the student for more information, but ended up telling the student why. By "fixing" student thinking via math nudges, Jackson did the sense-making for his students, framing mathematical activity less inclusively.

Second Rehearsal: Monitoring

Table 13 *Jackson's Talk Moves During Monitoring*

Start time: 23:32					
	35	Polite	invite Identify		
	36	Polite	Revoice	Check-in	invite Identify
	37	Revoice	invite Identify		
	38	Revoice	Create opportunities		
	39	Revoice	Probing		
(TE: 25:23 - 26:34)					
	40	Probing			
	41	Probing			
	42	Probing			
	43	inviting thinking	invite Identify		
	44	Create opportunities			
	45	Inviting Do			
(TE: 27:26 - 27:45)					
	46	Check-in	Revoice		
	47	Check-in			
	48	Probing			
	49	Revoice	inviting thinking		
	50	invite Identify			
(TE: 28:36 - 30:24)					
	51	Check-in			
	52	invite Identify			
	53	Revoice			
	54	Probing			
	55	invite Identify			
	56	Polite			
	57	Revoice	Valuing	Revoice Student	
	58	Polite	inviting thinking		
	59	Polite			
	60	Create opportunities	Check-in	inviting thinking	
	61	Revoice	Check-in	invite Identify	
	62	Math Nudge			
	63	Create opportunities	invite Identify	Math Nudge	invite Identify
	64	Probing			
	65	Revoice Summative	Math Nudge		
	66	Revoice	invite Identify		
	67	Revoice			
End time: 35:20					

The Monitoring rehearsal lasted a total of eleven minutes and 48 seconds. During that time there were three breaks to discuss with the TE. The first pause lasted one minute and eleven seconds, the second pause lasted just under 20 seconds, the third pause was a minute and 48 seconds. The rehearsal started off with Jackson asking his students if they minded if he sat with them; he stayed in that spot next to one student and across from the other two for the duration of the rehearsal. After asking the group for some information on their progress, the next few teacher turns involved Jackson revoicing what had been shared with him by a student, then posing another question to learn more about their progress. After he probed a student's idea of using proportions, Jackson asked for a time out to verify that proportions were a usable strategy for this task.

The TE coached him briefly on proportions before he resumed asking Stacy three more probing questions consecutively. During these probing questions he was pushing Stacy on how she knows she had similar triangles. After the last probing question of "how do you know that?" and she could not immediately respond, he told her to "think on that for a second" and resumed discourse with Wesley and Kayla. He asked Kayla to engage with Wesley's thinking by asking her if she agrees with his table approach. Jackson was interrupted by the TE asking him where he was going with this just as he had given Kayla and Wesley loose instructions to continue pursuing their thinking. Jackson resumed by checking in with Stacy and revoicing her approach. Then he checked in with Kayla as she was also pursuing thinking involving similar triangles. He probed Kayla's thinking by asking her about her similar angles. He then revoiced what she said and invited her to clarify what else she needed to show triangle similarity. This ended with Jackson asking what she was writing down before the TE jumped in to remind everyone different ways to show triangle similarity.

Jackson resumed the rehearsal by spending the next few exchanges learning what progress Wesley had made using his table approach. Jackson then revoiced Wesley's discovery that as X is getting larger, the wire is being minimized. He explicitly valued Wesley's idea and then asked Kayla and Stacy if they could revoice what Wesley just said. As a result of Stacy attempting to do so, Wesley ended up sharing his equation and so Jackson asked how he got the equation. The next few exchanges were between Stacy and Wesley as she made sense of his table and equation. Jackson then checked in with Kayla to see if she was understanding what her peers were saying to which she responded that she still did not understand why the triangles were similar. Jackson revoiced her choice to stay with similarity, but then started asking Wesley more

questions about his table. Over the next couple of exchanges, Wesley concurred that it's tough to get an exact answer from his table.

Jackson then used that agreed upon limitation of tables to bring Wesley into Kayla and Stacy's similarity approach. He reminded his students with a math nudge that they needed to find another set of similar angles. Just as he was doing so, Jackson noticed Kayla writing, so he asked her about what she was writing. In the next exchange he probed further into her thinking that was considering looking at the problem as if they were 45-degree right triangles. He revoiced her whole approach before using a math nudge to explain why that approach does not work in this case. The last few exchanges were Jackson asking Kayla some questions and revoicing to help her label her triangles to further pursue her similarity approach.

Jackson's Support of Student Thinking during Monitoring the Task. To address this, the codes of invite thinking and probing were used to initially identify instances of student thinking. Then the rehearsal was read through again to note any instances when a student volunteered an idea or question. Then there was one last read through to see if anything was overlooked.

Wesley stated he was unsure about using proportions [38]. Jackson had asked the group what they had written, and Wesley responded claiming that they wrote a proportion. Then Wesley immediately said that he was not sure that it was what they should do. He then furthered that thought saying they should probably go somewhere else with it. Jackson responded by revoicing Wesley's concern that proportions were maybe not the best idea and then created an opportunity for his peers to engage in his thinking by asking if they agreed or not.

Stacy said solving for x is okay [39]. In response to Jackson's question about whether they agree or not, Stacy stated that she does believe that they can solve for x (using

proportions). Jackson responded to Stacy by revoicing her claim and then probed deeper by asking her to "walk me through why do you think proportions are good in this case." She further explained that they have a proportion, and they can easily solve for x. This was when Jackson did a time out for help on the discussion of proportions.

Stacy defended her claim to use proportions [41]. After the break, Jackson resumed by probing Stacy's thinking by asking her why she can use proportions. She stated that she could because she has similar triangles. Jackson probed again asking her how she knew they were similar. She mentioned that they have the same angles. He probed again with a "how do you know that?" after which she gave him a "mmm" and he then told her "Think about that for a second."

Wesley and Kayla had different ideas [44.] Jackson asked what they wanted to try since they didn't want to do proportions. Wesley suggested making a table. Jackson then created an opportunity for Kayla to engage in Wesley's thinking by asking if she agreed or if she wanted to try something else. Kayla responded saying she wanted to try to see why the triangles are similar. Jackson then directed each of them to pursue their line of thinking.

Kayla and similarity [50]. Jackson had asked about Kayla's progress on similarity to which she shared that she only had two similar right angles. Jackson invited her to explain what else she would need to show that the two triangles are similar to which she responded "either similar sides or similar angles. I think both of those work." In response to that, Jackson asked her what she was writing down to which she responded, "triangle congruency?" and looked questioningly at the TE. Jackson shared she had written SAS, SSS and so the TE paused the rehearsal to address everyone's understandings of triangle similarity.

Wesley shares his table approach and findings [52]. Jackson checked in on Wesley's table approach and Wesley explained that he had started with a few points. Jackson then asked a probing question to learn what he had found with his points. Wesley went on to share his approach of starting in the middle and trying a point on either side to see a pattern in response to a revoicing and a probe by Jackson. Near the end of this exchange, Jackson pushed him to identify what numbers he was finding to which Wesley shared that he was noticing as it was getting closer to 225, it was getting maximized. In response to this larger idea, Jackson revoiced and then asked the others if they heard that and then asked for a student to revoice Wesley's thinking. In Stacy attempting to revoice, she and Wesley ended up discussing his approach as she asked clarifying questions to understand what he did to arrive at his conclusion.

Kayla still did not know why they were similar [61]. In response to Jackson asking if there are any questions, Kayla communicated that she still did not know why the triangles were similar. Jackson responded by revoicing her continued pursuit of similarity and then shifted back to checking in with Wesley.

Kayla and the 45-45-90 idea [64]. After establishing that Wesley's table approach was not quickly arriving at a solution, Jackson brought the students back together considering similarity when he asked Kayla what she was writing. Kayla shared that her writing reflects her thinking, considering a case when one side was 100. Jackson probed further into her proposed direction of thinking. She then responded with the conclusion that her case does not show similarity. Jackson revoiced her approach of assuming the triangles are 45-45-90s, and then revoiced her conclusion that that assumption is not true for this problem. He then provided a math nudge in connection with her idea clarifying why it does not work great and then suggested they list out the angles that are related to each other.

Discussion of Jackson's Support of Student Thinking During Monitoring

In response to Wesley's confusion about their approach [38], Jackson shifted the response on the approach to his peers. Jackson mostly probed to learn more about Stacy's thinking both when she initially suggested they could solve for X [39] and again when she believed she had similar triangles [41]. When Wesley and Kayla wanted to try different approaches [44], Jackson encouraged them each to pursue their line of thinking. When Kayla communicated struggling to find similarity, Jackson initially asked questions [50] to learn more about where she was in her thinking. Later when he checked in on her, he acknowledged she was still pursuing similarity [61] and then near the end he considered her approach and included the whole class in her thinking by revoicing her considered example [64]. With Wesley, Jackson asked about his progress through a series of questions which led to a conclusion [52] that was shared with his classmates and asked to be revoiced.

Jackson's Support of Student Thinking and the Two Frames During Monitoring

Jackson's framing was mostly inclusive in nature. Regarding mathematical activity, Jackson pressed for meaning frequently. When Stacy was assuming the triangles were similar, Jackson asked her three questions pushing her to justify that assumption ending with "How do you know that?" to which she could not really respond, and he told her to "Think about that for a second." Jackson supported student misconceptions inclusively in that in many cases he supported them in figuring them out. When Wesley was unsure about his approach, Jackson allowed him to pursue his idea of considering another approach. When Kayla had decided to pursue the assumption that the triangles were 45-45-90 because of familiarity with their properties, Jackson explicitly valued her sense-making approach. After a few questions from Jackson to learn what she did, he stated to her peers why she chose that approach and confirmed

that assumption cannot not work after she also had noted that approach would not work. Jackson also supported sense-making among his students by not interfering with Stacy while she asked Wesley three questions as she made sense of his approach when she engaged in trying to revoice his thinking.

Regarding mathematical ability, there were a few instances during the rehearsal when Jackson had students consider each other as resources even though he allowed them to pursue different approaches. Early on, Jackson had them engage with each other's approaches enough to ascertain whether or not they wanted to pursue that approach as well. Later, Jackson drew attention to Wesley's finding and asked his peers to revoice it. Jackson used Wesley's admittance that it was tough to get an exact solution using his approach to connect him with Stacy and Kayla's approaches.

Third Rehearsal: Discussion

Table 14 *Jackson's Talk Moves During Discussion*

Start time: 16:56							
	68	invite Identify					
	69	Revoice					
	70	Revoice					
	71	inviting thinking					
	72	Revoice					
	73	invite Identify					
	74	Revoice	invite Identify				
	75	invite Identify	Polite				
	76	Revoice					
	77	invite Identify					
	78	invite Identify					
	79	inviting thinking					
	80	Valuing	Revoice	Check-in	wait time	Revoice Summative	Valuing
	81	Revoice	Affirmation of correctn	Revoice Summative	Check-in	Polite	
End time: 20:49)							

The Discussion rehearsal lasted a total of three minutes and 53 seconds. There were zero TE breaks. As Jackson was continuing a discussion during which two students had already presented their work, his first move was to invite Stacy to share both her approach and where she got stuck. He revoiced parts of her explanation for his next two talk turns. Jackson then invited Wesley to share his group's work. While Wesley shared his graphing strategy, Jackson did a

combination of revoicing or asking clarifying questions during the next seven talk moves. Near the end, Jackson invited the thinking of Wesley's two classmates by asking what his solution means in the context of the problem. Then the last two talk moves were mostly one larger summative statement by Jackson that was interrupted by Willow defending her group's work because he did not give them credit for having the solution. During those two talk moves, Jackson revoiced each of the three approaches giving credit to the students who presented on each and concluded with stating the solution followed by asking if they had any questions.

Jackson's Support of Student Thinking During the Discussion. Below are the instances of student thinking that Jackson had the opportunity to respond to during the Discussion rehearsal.

Stacy shared her group's struggle [69]. Jackson started off the rehearsal by asking Stacy to reiterate her group's strategy and stopping point. She shared that they used the Pythagorean theorem for each triangle and came up with an equation, but struggled with how to minimize. Jackson responded by revoicing her twice with her confirming once in between. He used her stopping point as a segue to introduce Wesley's group.

Wesley shared his group's work [72]. Wesley was asked to walk the class through his work that was projected. Wesley explained the equation and graph his group used to arrive at a solution and Jackson invited him to identify in the form of clarifying questions while also revoicing at times.

Stacy responded to a contextual question [80]. After Wesley presented, Jackson asked the other students what the hypotenuse tells us in the context of the problem and Stacy responded stating that "it tells the length of the zipline wire." Jackson responded positively with "good" and then revoiced her response.

Willow defended her solution [81]. Jackson was doing some summative revoicing of the different work of his students when Willow interrupted him. Jackson had said that Willow's group "worked their way through a table but couldn't quite come to an exact answer," at which point Willow quickly jumped in to defend her work and said, "Well we did come to an exact answer." Jackson responded by affirming her statement and revoicing that she did come to an exact answer.

Discussion of Jackson's Support of Student Thinking During Discussion

Jackson mostly let other students speak for themselves. He let Stacy review her group's approach a little bit and then revoiced her thinking. He had Wesley share his group's work and Jackson asked questions and revoiced his approach. Near the end Jackson valued Stacy's response to his question and revoiced her answer to his contextual question. Then when Willow corrected him on his overlooking the fact that she had the solution, he responded by confirming that she did have the solution, but also noted that while their answer was correct that they did not know it was correct since they had not tested all points.

Jackson's Support of Student Thinking and the Two Frames During Discussion

Overall, Jackson was mostly inclusive in his framing. One could argue that at the surface level, Jackson mostly asked identifying questions that could be answered by pointing to the graph or responding with one word. However, as he was asking these questions to make sense of Wesley's approach for the rest of the class, he was framing mathematical activity inclusively. Jackson was also asking questions to tie in the context of the problem which falls under the inclusive sense-making frame. By having his students present to each other, he was positioning them as resources for each other and thus framing mathematical ability inclusively. Earlier in the discussion when Stacy shared that her group struggled with

minimization, Jackson explicitly mentioned that Wesley's group had been able to answer that - thus positioning Wesley's group's work as a resource for Stacy.

Addressing Why Jackson Does What He Does

In trying to make sense of why Jackson interacts with students the way that he does, the Hammerness et al. (2005) Learning to Teach in Community framework was used.

Learning Community

Jackson valued learning from the rehearsals within the learning community. As he shared during the Launch interview, "I remember it was really enlightening to be able to stop and pause the task, and talk to *the TE* and say like, you know, 'I'm kind of stuck here. Can you help me out?' Like, where should I go from here?" Jackson also described another situation where he did not know how to move a student forward and the TE pointed out that Wesley had said he knew how to minimize the wire, giving Jackson "a point to then go off of." During the Monitoring interview, Jackson shared that he got the feedback from his peers that he had addressed all of their barriers during the rehearsal. So Jackson was valuing the TE's facilitation support and feedback from his peers.

Conceptual Tools Practiced

Jackson's conceptual practices include fostering a learning community through inclusion, checking in, and positioning students as resources for each other; supporting student contributions through positive evaluation, and validating student thinking; centering student thinking through students as deciders, eliciting student thinking, and pursuing student thinking; guiding students' thinking through directing attention to key information, clarifying and nudging student thinking, managing misconceptions, connecting context, and connecting student work;

and standards for mathematical practice practices such as attending to precision and model with mathematics.

Fostering a Learning Community Practices. Fostering a learning community practices include inclusion, checking in, and positioning students as resources for each other.

Inclusion. Jackson took action to either include students or prevent them from feeling left out. During the Launch interview, Jackson shared that he tacked on the word "anyone?" to a question because he wanted students other than the one who had posed the question to answer. During the Monitoring interview, Jackson explained that he told a student to "think about it for a second" because he did not want to "get too involved with a single student going back and forth asking questions." He challenged her to think about it for a second so that the other students wouldn't "zone out." Later Jackson ended a productive conversation between two students because he noticed Kayla was by herself and he wanted to bring her back into the conversation. During the Discussion interview, Jackson explained that he addressed Willow and Stacy when asking a question because he "was trying to get them back into the conversation. They had been quiet for a while."

Checking In. Jackson's desire to be inclusive was also evident in how he checked in with the class on certain ideas. During the Launch, a student suggested assuming the ziplines are straight because that would be easier, Jackson first checked to see if that worked for everybody before moving forward with that idea. During the Monitoring interview, Jackson "wanted to see what the rest of the group thought first" after hearing from Wesley that he was not confident about pursuing proportions. During the Discussion, once Wesley was done presenting Jackson asked if there were questions because "someone might not understand and maybe we need to go

over a little more and make sure everybody's on the same page." Jackson frequently checked in with the students to see if they were on board with an idea.

Positioning Students as Resources for Each Other. During the Launch interview, Jackson described his goal in having students talk to each other as an approach "to generate more student thinking about the task itself." He also suggested that in a real class, groups other than Wesley's might also be able to "help clarify Wesley's questions about what they're asking us to do." So, he's positioning his three students as resources for each other, but also within a larger pretend classroom. During the Monitoring interview, Jackson shared that he asked questions so that he could "bounce those [student ideas] off the other two people in the group." He intentionally was engaging students with each other's thinking. Jackson also asked his students to revoice what Wesley had said at one point, positioning Wesley as a resource in that moment. In the Discussion, Jackson positioned students as resources by having them present their work to each other.

Supporting Student Contributions Practices. Supporting student contributions practices include positive evaluation [the E in IRE], and validating student thinking.

Positive Evaluation [the E in IRE]. Jackson frequently responded positively to students, but not with the expressed intent of encouragement but more a valuation of their statements. During the Launch when asked why he told his students they did a very good job, he responded saying "They did a very nice job." When asked why he responded to a student's question with "Ooh, that's a good question," Jackson responded saying, "cause that was a good question" and went on to explain (he thought it was good because) it was relevant to the task. During the Monitoring rehearsal, when asked why he told Wesley "that's a good conceptual thought," Jackson responded explaining that Wesley didn't have a specific answer

but described the pattern he was noticing and so "that's a good thought conceptually… he's kind of putting into a real-world sense what the proportionality is doing." During the Discussion interview, Jackson explained that he qualified Wesley's work with "fortunately" because "if we can't figure out the minimum, then we're just stuck. So thankfully, fortunately, you know, Wesley came up with a solution." So, Jackson frequently positively evaluated student responses.

Validating Student Thinking. Jackson did not explicitly claim to validate student thinking, but several times he did exactly that. During the Monitoring rehearsal, Jackson chose to share Kayla's exploration of the 45-45-90 triangle with the class because "it has some nice properties, but we can't use it because of these reasons." Near the end of the Discussion rehearsal, Jackson was revisiting the different approaches and stated, "Willow's group came up with the Pythagorean theorem, which I thought was a great solution." He claimed he did so because they came up with the minimum even though they did not know it was exactly the minimum. When Willow corrected him on having come to an exact answer, Jackson validated her statement by saying "that's true, Willow, you did come to an exact answer." And at the end of the rehearsal, in response to how he made his students feel smart, Jackson mentioned that "I gave them credit for each of their solutions and didn't discredit any other solutions."

Centering Student Thinking Practices. Centering student thinking practices include students as deciders, eliciting student thinking, and pursuing student thinking.

Students as Deciders. Jackson communicated wanting his students to have a say in their learning. Early on in the Launch interview, Jackson communicated that he wanted "to have a student-led discussion." During a pause in the rehearsal, he explained that he had his students *tell* him what to draw on the board so that it was not him "drawing on the board, here's X, here's 600, solve." When there was a question about an assumption regarding whether the ziplines

should be taut or curved, Jackson redirected the question back to his students to decide. During the Launch, Jackson provided opportunities for students to make decisions about aspects of the task.

Eliciting Student Thinking. Jackson valued having students' thinking vocalized. During the Launch interview, Jackson shared instances when he asked his students questions to hear about approaches to the task or possible questions about the task. There was a point when Jackson explained that he had a student rephrase - "instead of trying to interpret what he meant, I just made him say it out loud." Jackson also shared his desire for student questions as an expression of student thinking. He clarified that when he asked his students to talk to each other that "really what I wanted is questions." Jackson started off the Monitoring rehearsal by saying "someone talk to me." Jackson wanted them to explain what they had so far so that he could hear what was in their minds. Then when the students pursued different approaches, Jackson consistently asked questions about their approaches. During the Discussion, Jackson specifically asked to hear a student's thinking to revisit a previously presented student solution and to draw attention to one group's struggle in addition to asking clarifying questions to the student presenting. Jackson consistently elicited student thinking for a variety of intentions.

Pursuing Student Thinking. Jackson's actions frequently were influenced by his desire to let his students *pursue* their thinking. During the Monitoring interview, Jackson shared that he wanted his students to be bought into their approach. As he put it, "I just kind of rolled with whatever they wanted to do." This resulted in Wesley working with a table because he doubted proportions, Stacy working with proportions, and Kayla focusing on the similarity of triangles. In her pursuit of similar triangles, Kayla wanted to assume the triangles were 45-45-90, and Jackson responded with "ooh, okay, that's a really cool idea." He supported her in

pursuing that direction so that she could figure out for herself whether or not that would work. As he put it, "it's not me telling them anything. It's them coming up with their answers and their reasons." During the Monitoring rehearsal, Jackson focused much more on pursuing different ideas students had.

Guiding Students' Thinking Practices. Guiding students' thinking practices include directing attention to key info, clarifying and nudging students' thinking, managing misconceptions, connecting context, and connecting student work.

Directing Attention to Key info. Jackson pointed out key information that was more general in nature instead of trying to shape their thinking. During the Launch interview, Jackson shared that he wanted the numbers from the problem on the board so that there is "a reference to like all of our information on the board." Jackson asked "what are we trying to find in the task" to bring attention to the island to get it added to their drawing. During the Discussion interview, Jackson explained that he asked his students to state the length from the island to the other tower just to point out the exact placement of the island "no matter which bank you look at."

Clarifying and Nudging Students' Thinking. There were moments during the Launch and Monitoring rehearsals when Jackson tried to shift students' thinking some by drawing attention to important ideas. In response to Wesley not understanding why the island wasn't just in the middle, Jackson attempted to point out that the different heights of the towers is why the island cannot be in the middle by drawing a second model on the board. "I was trying to have some sort of visual representation of the two comparisons of the wires. And see that one is different depending on where you place the island." And when Harry mentioned he was unclear on what it was being minimized - shapes or wires, Jackson tried to clear that up by responding that it was the wires, and again addressed the heights of the towers in connection with the

placement of the island previously mentioned with Wesley. During the Monitoring task, Jackson was nudging Stacy to think about the different ways that triangles can be similar. As he put it, he was asking her "what else do you need to prove similarity?" Later he nudged Wesley away from using a table because of him not being able to make more progress with it and wanting to reconnect him to his groupmates. What is interesting is that twice during this rehearsal, Jackson highlighted one student's realization and made sure the peers heard it. When Wesley noticed a trend about as X is getting larger, the wire is being minimized, Jackson had Wesley share with the group and asked someone to revoice it. When Kayla explored assuming the triangles were 45-45-90, Jackson shared her idea and discovery with the class. In both of the rehearsals Jackson was nudging his students toward particular ideas, but there was a shift in his approach in that during the Launch he was explaining to his students whereas during Monitoring he was asking his students questions.

Managing Misconceptions. Jackson gave attention to students when there was a spoken misunderstanding, such as the examples mentioned above, but he also was thinking through other ways to tend to possible misunderstandings. During the interview, Jackson explained that he viewed the picture his students guided him to draw as a way to expose possible misconceptions even though no students communicated an issue related to the initial drawing. Related to misconceptions with Wesley, Jackson put the students in a group thinking they'd be able to make progress that way. During the Monitoring interview, Jackson shared that during the debrief he had received feedback that all barriers had been addressed, but that he "didn't even notice throughout. I was just kind of rolling with what they were thinking on face value." Jackson had several strategies in mind to expose student thinking.

Connecting Context. Jackson mostly gave this explicit attention during the Discussion rehearsal and interview. Jackson shared that he asked Stacy about the hypotenuse equation in her triangle with the intent of "relating it back to the context of the problem." Jackson asked Wesley about the minimum he had found with the intent of Wesley addressing "what does it mean in the context of the problem." At another point in the interview, Jackson explained that he was wanting them to state what X and Y meant in the context when he asked, "what do the two parts of the minimum tell us?" Jackson also made sure his students understood how far the island would be from the other tower.

Connecting Student Work. During the Discussion interview, Jackson described ways he drew attention to ideas while connecting student work. Jackson explained how he intentionally asked Stacy to share about how her group struggled to find the minimum to segue to Wesley's solution, since his group was able to find the minimum. Later Jackson asked Wesley to point out how his work connected to Stacy's and when Wesley said, "this is a graph of that previous equation that Stacy had," Jackson noted "so the same equation." Then near the end, Jackson connected the three approaches again by referencing Willow's table as being points on the graph, the fact that Stacy's equation and Wesley's equation both used the Pythagorean theorem, and that the minimum on Wesley's graph was the same solution Willow's group had previously found using a table.

Practices Related to Standards for Mathematical Practice. Practices related to standards for mathematical practice include attending to precision and model with mathematics.

Attending to Precision. During the Monitoring and Discussion interviews, Jackson shared instances where his goal was pushing his students to be more precise. During the Monitoring interview, Jackson shared that he had asked his students to put the congruency marks

on the angles in their model because he wanted his students to be more specific about which angles they were looking at. During the Discussion interview, Jackson shared that he asked a student "the total what?" because the student "wasn't specific enough." It was also during the Discussion interview that Jackson shared that even though Willow's group had presented a correct solution, he did not view it as a precise answer in that he felt that their approach lacked the certainty that their solution was indeed the minimum.

Model with Mathematics. During the Launch interview, Jackson explained that he wanted a drawing on the board because "having a basis of what we should be looking at is imperative to like understanding the question." When there was a student misconception, Jackson attempted to address it with "some sort of visual representation of the two comparisons of the wires." And then even after Jackson got stuck in his model comparison attempt, he revisited the original model to try to make sense of Wesley's misunderstanding. During Monitoring, Jackson shared that he thought his students' model was "spot on" and later when his students were exploring similarity of triangles as an approach, he was pressing them with questions because he "wanted them to start marking up their board. Like showing these angles are congruent." During the Discussion interview, Jackson asked Stacy about her the hypotenuses in her model to "relate it back to the context of the problem." Jackson described the mathematics his students got to do during the Discussion rehearsal as relating models and comparing and contrasting.

Dispositions

In the framework, Dispositions are described as "Habits of thinking and action regarding teaching and children" and this guided my organization of this section. I started noticing which codes better described habits of thinking or action and whether the focus was on teaching or on

children. Most of the codes fell into categories related to teaching, but a few were directed towards students. Jackson's *habits of thinking regarding students* included competent students and considering the student experience. His *habits of thinking regarding teaching* include reflection upon practice, goal/success, and valuing progress. His *habits of action regarding teaching* were further categorized into culture building, focusing on student thinking, and facilitation. Culture building dispositions included "class as we" and student authority. Focusing on student thinking dispositions include valuing student discovery ("taking an inquiry stance"?), valuing student understanding, and valuing student talk. Facilitation dispositions include maintaining a high cognitive demand, problem solving, and the importance of teacher language.

Habits of thinking regarding students. Habits of thinking regarding students include competent students and relating empathetically to students.

Competent Students. Throughout the interviews, Jackson consistently talked about his students from an asset-perspective as opposed to a deficit-perspective. During the Launch interview, Jackson described his logic in redirecting a question asked to him back to his students because "someone else might have a great idea or a great explanation of, you know, and of the questions that I ask." At the beginning of the Monitoring interview, Jackson explained that when it started, he felt his students had "a good roll with their picture." When a student suggested pursuing similarity, Jackson shared that he was delighted because that leads to proportionality. In at least two other cases when students' approaches were not fully formed or perfect, Jackson saw value. He felt Wesley was "putting into a real-world sense what the proportionality is doing" and he felt Kayla's exploration of the 45-45-90 triangle was a quality idea because "if it was two 45-degree triangles, that would make this problem a lot

easier." During the Discussion interview, Jackson continued to speak of his students as competent by sharing that he felt the hardest part of the task was "knowing you have to use the Pythagorean theorem and then coming up with a conjoined equation between the two triangles that you're working with." Jackson consistently sees his students through the lens of their accomplishments.

Relating Empathetically with Students. Jackson shared a few different beliefs about the student experience in classrooms across his interviews. During the Launch interview, Jackson shared that he chose to have another student read the task so that there was "a different voice in the room, which automatically I think makes people listen a little better." That "in the classroom when the teacher's up at the board saying everything, it kind of drones on and on and you just kind of start blocking it out." As previously mentioned under the idea of student ownership, Jackson stated he believes that students are "more pleased with themselves when they get to an answer because it's their answer." During the Monitoring interview, Jackson shared that he wanted to avoid spending too much time with one particular student "because as a student, sometimes when a teacher gets too involved with a single student going back and forth asking questions, I lose interest."

Habits of Thinking Regarding Teaching. Habits of thinking regarding teaching include reflection upon practice, goal/success, and valuing progress.

Reflection Upon Practice. Jackson mostly vocalized reflections on his practice during the Launch interview. During the Launch interview he shared that he drew the model due to convenience, but he also mentioned that a better way to do that would be to have a student representative draw while other students provide input. Another example involved Jackson sharing that he had felt stuck because he had "committed to a method and then couldn't finish

that method." He then shared that the TE had later spoken with him about how it would be "okay to say 'sorry, this isn't going to make sense' or 'let me redraw this' or like 'let me rethink this idea and come back to you.'" After one positive experience redirecting a question back to his students, Jackson shared one thing he would aspire to do differently would be to "definitely lob more questions back" to students. Jackson struggled with Wesley's confusion. He noted that he felt like he couldn't help Wesley because he didn't know his actual problem with the task. Later in the interview Jackson further elaborated on his frustrations in stating, "Wesley's thing is just he didn't understand anything in the task and I was just like, okay, how do I fix that?" After a TE pause, Jackson understood to question the student by building from what he did understand - "kind of using what he already knew to then go forward of like where, what's your question?" When reflecting on Wesley suggesting the island should be in the middle of the lagoon, Jackson lamented, "well I should've just asked him. I should've been 'ok, what do you mean? Like are you confused about blah, blah, blah or are you confused about this instead? You know, I didn't, but it's a learning process." Jackson viewed the rehearsals as a learning process and his reflections often included how he might improve his practice.

Goal/Success. Across the rehearsals, Jackson referenced different goals during the interviews. During the Launch interview, Jackson spoke of wanting to have a "student-led discussion" of the task. Jackson also disclosed his struggle in helping students move beyond focusing on minimizing the zipline wires and "instead focus on the distance of the island to the bank." Another small goal of his was getting students to create questions while chatting to each other. Jackson felt his Monitoring rehearsal went well because he had used almost all of the discourse moves and had addressed student barriers. At the beginning of his Monitoring interview, Jackson also shared that he "wanted to teach the way I would actually teach a

group." And as he reflected at the end that he would do his facilitation exactly the same, he felt positive about his facilitation.

Valuing Progress. Across all three interviews, Jackson described the rehearsals with a focus on progress/momentum. During the Launch interview, Jackson shared that when Willow suggested the island be drawn in the middle, he was just "happy that we had an island on the board." And later with Wesley's misunderstanding, Jackson's approach was "kind of using what he already knew to then go forward… how do we keep going?" In the Monitoring interview, Jackson shared that because his students had a good picture that was just "what strategy are you going to use going forward from your picture to solve for the minimum." When Kayla said she thought the triangles were similar, Jackson shared in the interview that he was thinking "that's a good step" because it leads to proportionality. And near the end of the interview, in response to what he might do differently, he said "I would hopefully get to that point a little earlier." Then in the Discussion interview Jackson referenced another rehearsal when saying "they still worked on it and still tried, you know, still got pretty far I think." Jackson shared one of his classroom environment philosophies is that "it's a whole learning process throughout class," so any progress was valued.

Habits of Action Regarding Teaching. Habits of action regarding teaching include culture building dispositions, focus on student thinking dispositions, and facilitation dispositions.

Culture Building Dispositions. Cultural building dispositions include class as "we" and student authority dispositions.

Class as "We." Jackson consistently spoke of wanting his students' learning experience to be more of a collaborative experience. During the Launch interview, Jackson shared "I should not be the only one teaching the class. It should be other people as well, like helping out. It's a

collaborative effort." In reflecting about what he might do differently, Jackson shared that he wanted to work toward redirecting more questions back to students. During the Monitoring interview, Jackson explained that early on he asked his students if they minded if he sat with him because in his opinion, "Students to teachers is almost on the same level… So I sat down with the group and was actually working together collaboratively, the four of us." Several times when talking about this rehearsal, he used "we" instead of they. For example, in response to a question about Wesley not thinking proportions were the best idea, Jackson mentioned the question "where do we go forward with this?" Jackson also shared intentionally bringing students into each other's work throughout the rehearsal. When Wesley got stuck in his approach, Jackson described bringing him back into the group conversation as "you know, you are now back part of this team. Let's work together to figure this out." During the Discussion interview, Jackson was reflecting on why he felt his students had done a "good job." Jackson explained that he was pleased because "even though the groups work independently, they all feed into each other to come up with a classroom consensus like this is our answer… this is what we came up with." Jackson strongly believes in collaborative learning in his classroom and that he is a part of that collaboration.

Student Authority. Jackson believes students should have input in their learning. During the Launch interview, Jackson explained that he wanted his students to tell him how to draw the picture because "I didn't want to give them the information." Jackson elaborated "I think that if students take ownership of anything that they're doing they (1) try harder and (2)... they're more pleased with themselves when they get to answer because it's their answer, not my answer as the teacher, not the answer I want them to have, you know." He shared that had there been a disagreement about the tautness of the zip lines, he would've given his students the authority to

figure out what might work for them. In reflecting on the Launch, Jackson shared, "I tried to make everything very like the students were leading the conversation, like they were leading how to draw the picture and they were trying to lead about like what are the methods you guys would use to solve this task? Or what are your questions about the task?"

Focusing on Student Thinking Dispositions. Dispositions that focus on student thinking include valuing student understanding, valuing student discovery, and valuing student talk.

Valuing Student Understanding. During the Monitoring and Discussion interviews, Jackson's responses to questions communicated that he valued students understanding. Early on in the Monitoring rehearsal, Jackson was not satisfied hearing from just one student about their picture. He asked other students in the group to explain other aspects of their drawing to make sure everybody had "a good foundation of like what actually is in their pictures." He pushed a student to explain why she could use proportions because he felt it was important that she understood that similarity had to be established before just randomly setting up a proportion. Jackson also pushed Wesley to share more about his table - "what do these points in the table mean?" Jackson also found it exciting that when Kayla explored the idea of assuming the triangles were 45-45-90 and came to the realization that "it doesn't work." During the Discussion interview, Jackson shared that he had pressed Stacy about her hypotenuse to ensure she understood the meaning of it. Near the end, Jackson explained that one of the ways he supported student thinking was by asking questions that pressed for meaning - "let's be clear, like what exactly do you mean here?"

Valuing Student Discovery. Jackson valued students making realizations on their own. During the Launch interview, Jackson explained that had his students disagreed about the ziplines being taut or curved, he would have encouraged them to "try it out, see if it works." In

response to a student question, Jackson attempted to draw a second image so that his students could compare to "discover" that the lengths of zipline would be different if the island were in different places in the lagoon. During the Monitoring rehearsal Jackson supported students trying different approaches and at the end he felt that worked well. In response to a question about handling a student being unsure about using proportions, Jackson shared "so my position is if a student thinks they want to do something, just let them try it." He wants them to discover "what's working, what's not working."

Valuing Student Talk. Jackson valued students speaking during his rehearsals. During the Launch interview, Jackson shared that he likes when students ask questions. He also noted that redirecting a student question back to his students "gives me kind of this cover of well, maybe somebody else already knows how to answer this question, or if they don't, it gives me time to think about it myself." So, in addition to giving students an opportunity to respond, he sees the additional value of giving the teacher more time to think. During the Monitoring interview he shared one of the reasons he thought it went well was because there was good discussion within the group. Throughout this rehearsal he asked students questions so that they were explaining where they were in their particular approach. He explicitly valued that two of his students ended up in an on-topic side conversation. "Stacy just went on a little teaching Wesley rant… and I was like, all right, cool. Just, you know, Wesley, answer these questions that Stacy has for you. You know, explain your thinking." During the Discussion, Jackson centered student voices by having Wesley present and by having Willow re-explain her group's work. Jackson consistently values students verbally participating in class.

Facilitation Dispositions. Facilitation dispositions include maintaining a high cognitive demand, problem solving, and the importance of teacher language.

Maintain High Cognitive Demand. During the Launch interview, Jackson brought up a couple of moments during the rehearsal where he wanted to maintain the demand of the task. Regarding his students having input on the drawing of a model, Jackson shared "it kind of limits the critical thinking of the whole task if you just give them the picture and like exactly say like this bottom part is X." When discussing the student question about whether to assume the ziplines are taut or curved, Jackson said he would let them pursue whichever way they wanted because "if they have to go and figure out that it has to be straight, it actually helps them learn about the task." Later on, Jackson was thinking out loud about describing aspects of their drawing, that the total is 600m across the lagoon, but caught himself when considering mentioning a variable. "Not like with the X variable.... cause that's changing the demand of the task." Jackson communicated wanting to hold his students accountable to the thinking behind the task.

Problem Solving. In the Launch, Jackson discussed having a picture on the board so that there was one common visual aid that all groups could use - "like having that basis on the board for the whole class to see." During a pause with the TE, Jackson explained that after the picture was drawn, he intended to start asking questions such as "what are we trying to find?" Also in the Launch interview, Jackson mentioned trying to use the approach of comparing two different models with a student to help him understand that it does matter where the island is in the lagoon. During the Monitoring interview, Jackson valued that his students had a good model with their triangles properly labeled so that they could just focus on a strategy to move them forward. During the Discussion interview, Jackson was reflecting on the work his students had done with the Pythagorean theorem to make the progress that they had. He shared his opinion on

how he thought the toughest part of the task is coming up with a joined equation using the Pythagorean theorem.

Importance of Teacher Language. There were a few times during interviews that Jackson reflected on his choice of wording not being the best. During the Launch interview, Jackson spent some time reflecting on his poor choice of words in saying "there's an island in the middle." Even before we paused the video, when he heard himself say it while the video was playing, he muttered "which is not true." He shared, "I should've said the island is in the lagoon, where would we like to place the island? Cause then, yeah, 'middle' gives a wrong, gives the students the wrong idea about the island." During the Discussion interview, there were several references by Jackson about Willow correcting his statement about her group not having an exact answer. During the interview Jackson shared that he "didn't mean to say it was wrong, but I, but I said it was wrong." Jackson's awareness suggests he understands the potential impact of his words.

Understandings

Jackson's Experience Solving the Task. During the interviews, Jackson shared that when they originally did this task in class, he had been paired with Harry. Their approach had been to start making squares, but then did something with triangles to arrive at "an answer that was very close to the answer." He also shared that he was familiar with the Pythagorean approach because another classmate had been in the final group to explain their work and so "that was like considered the quote unquote right answer." So, he was comfortable with the Discussion rehearsal because the progression among the three approaches was "very much based on the Pythagorean theorem." He also shared that he thought the hardest part was "knowing that

you have to use the Pythagorean theorem and then coming up with a conjoined equation between the two triangles that you're working with."

While the class had reviewed the approach of using proportions to solve the problem, he and his partner had not used proportions. So, during his Monitoring rehearsal, when his students started pursuing that approach, he admitted that he had completely forgotten about it. He paused the rehearsal to get clarification from the TE on whether or not proportions could be a fruitful approach. As the students were also struggling playing their parts, the TE stopped the rehearsal and reminded them all of the different ways to show similarity. During the interview he shared that initially he did not think he needed the review of similarity, but then once the TE started, he was grateful for the review.

Anticipating Student Responses. Jackson shared that he was not as aware of potential student responses as he might want to be. When discussing a moment during the Launch in which he felt stuck, he shared "I thought it was difficult in terms of anticipation because like, even though we had done the task before, like all of the students and their slips kinda had different questions than what we had questions for." He shared that it was "difficult to then wrap my head around, okay, like what are they actually asking?" Connected to that, Jackson felt prepared for Willow's question about the zipline being curved or straight because his class had previously discussed that. During the Monitoring rehearsal itself, Jackson called a time out because he was not as familiar as he wanted to be about using proportions, the approach a student was pursuing. "You know, it was weeks ago at this point, but I don't remember using proportions in my group… I just wanted to time out to make sure that proportions actually did work here."

His Thoughts on Different Approaches. While Jackson valued pursuing different approaches, he viewed tables as "less than" in their ability to provide a solution. Jackson talked about tables as if they could not yield an exact answer. During the Monitoring rehearsal he redirected Wesley because a table "was a good method to start out with, but you can't get to an exact answer with what you have right now." This perception of a table not providing a good enough answer carried forth into the Discussion rehearsal when he stated that Willow's group "worked their way through a table but couldn't quite come to an exact answer." He explained "unless you do every single point in the infinite amount of numbers, you don't know if it's the minimum or not. It could be the minimum value of the points you checked." During the interview, he explained that he explicitly stated the solution of 375 meters "because that's the answer that Wesley found and that's the answer that Willow found. So that was our class answer of the day." So even though Jackson would recognize the answer obtained using a table as correct, he did not think of it as "exact."

Key Math Ideas. During the Launch interview, Jackson did make it clear that he understood that the goal of the task was to "find the distance of the island to the bank" as opposed to finding "the minimum amount of wire." His student, Wesley, had claimed it was the latter and Jackson shared that he struggled with how to get students to shift their focus from the wires to the island. That was a common point of confusion for students. To help his students understand why the island could not just be in the middle, Jackson was trying to help them see that the different heights of the towers were influencing the placement of the island. He was "thinking like if you had it in GeoGebra and you moved it left and right" they would see how the total length of zipline wire is influenced by the placement of the island.

Vision

What has been learned about Jackson from above is that he strives to build classroom community by being inclusive, checking in on students, and positioning them as resources for each other. Those practices align with his conviction that class be collaborative. Part of that collaboration involves students being supported to share ideas, pursue their thinking, and make decisions. He views his students as competent and responds to their contributions positively by affirming correct answers and validating their thinking. Jackson views his role as that of a facilitator who is there to guide his students thinking while maintaining a high cognitive demand. He shared that one of his classroom environment philosophies is that "it's a whole learning process throughout the whole class." And he himself described his own experience teaching in rehearsals as a learning process. Jackson aspires to have a positive, collaborative classroom while centering student thinking as his students explore different ideas as part of the learning process.

Revisiting the Research Question - Why does Jackson do what he does?

This section addresses Jackson's thoughts in connection with the second research question: *How inclusively do PTs frame mathematical activity and mathematical ability when reflecting on their intentions during rehearsals?*

Jackson's Whys in Connection with the Two Frames

Mathematical Activity. Jackson's framing of mathematical activity was described in relation to the sense-making frame and the multidimensional ability frame.

Sense-making Frame. Jackson's approach was inclusive in that he strongly valued students making sense of their thinking. This was most evident during the Monitoring

rehearsal. For example, there was an exchange of Jackson asking Stacy three probing questions in a row as he pressed her to explain why it was that she could use proportions. In general, he valued students making sense of their thinking - "even if you're wrong, try it and see." His embracement of it being okay to be wrong further reinforces Jackson's inclusive framing of mathematical activity.

Multidimensional Ability Frame. Jackson may not explicitly have viewed mathematics as including collaboration, experimentation, and argumentation, but his beliefs and choices aligned with this inclusive framing of mathematical activity. Jackson described the mathematics that his students had the opportunity to do as larger than focusing on formulas. During the Launch interview, Jackson described his students *thinking*. He shared that his students "thought about math concepts with minimization… were starting to think about triangles… so not a ton of doing math, but maybe some conceptual math understanding." During the Discussion interview, Jackson shared that his students did "comparing and contrasting mediums of displaying information."

Mathematical Ability. Jackson consistently treated all his students as if they were valued in the classroom because they were contributing to the community. When students presented their ideas, Jackson typically responded with enthusiasm regardless of whether or not the idea was "correct." So, he interacts with his students as if they each have something to contribute. During the Monitoring interview, Jackson was particularly excited about a conversation between Stacy and Wesley in which one was asking the other probing questions to make sense of the other's approach. Jackson made no explicit statement to his students about mutual dependence, but there's evidence of that mindset from the interviews in which he proudly described his students working together. The only potential exclusive move Jackson made had

to do with Stacy during the Discussion rehearsal when he pointed out her groups' inability to find the minimum to segue to another group's solution. Jackson did not explicitly position Stacy as in need of help, but it is possible that students may perceive that attention to their struggle negatively.

Summary

During the Launch rehearsal, Jackson responded frequently by trying to do the sense-making for his students - particularly in the cases of Wesley and Harry. During the Monitoring rehearsal, Jackson seemed to have figured out how he wanted to teach, and he did exactly that and was so pleased with himself that he said he would do the same thing again - he would change nothing if given another chance. Jackson shifted the responsibility for sorting out understandings to the students.

In general, Jackson responded to potential student misunderstandings with positive enthusiasm. Specifically, he valued that the student said something because he viewed the student's uncertainty about the problem as an opportunity for sense-making. He consistently viewed student misconceptions as opportunities for the class to further examine and make sense of the problem. The way that he has shifted to support student thinking aligns his teaching practice with inclusive framing. Even the idea of a misconception almost vanishes in that if student thinking is being valued for what it is, then it is worth engaging in as part of the sense making process. Specifically, not unlike Benny from the case of Benny (Erlwanger, 1973/2004), there is no "miss" to the conception, because the goal is to engage in thinking and push for understanding.

Chapter 7: Cross Case Analysis

Each of the research questions will be examined below by looking across the cases. The findings are organized by research questions. Quotes from the rehearsals and interviews with the PTs are included as evidence. First, the talk moves utilized by the PTs are presented with a discussion of their implementation. Next, a brief analysis of their talk move patterns is presented to note any moves that might be used together. Following that analysis, the PTs' talk moves are analyzed using the Louie (2017) framework for framing mathematical ability and activity to address.

To address the second research question, the PTs' conceptual practices and dispositions will be presented. First, their conceptual practices will be presented. Following that their conceptual practices are analyzed using the same Louie (2017) framework. Then the dispositions will be presented followed by an analysis using the same framework. At the end, there is a synopsis looking at three cases holistically to see the commonalities and differences among the research questions.

Question 1: How are PTs supporting student thinking during rehearsals?

The PTs supported student thinking in a variety of ways during rehearsals. As they had been taught Herbal-Eisenmann's TDMs, it was expected to see the PTs practice wait time, inviting moves, revoicing, student revoicing, probing, and creating opportunities. As previously mentioned, inviting moves and revoicing moves were further broken down into more moves. Additional moves observed included math nudges, valuing, politeness/gratitude, checking-in, and affirmed correctness. Below the frequency and quality of usage of the different moves among the PTs are discussed.

Total Moves Across All Three Rehearsals

First, it is important to note that each PT had a different experience during rehearsals regarding the amount of time they were actively teaching during the rehearsal. The total time each PT spent teaching during the rehearsal was calculated by subtracting out the TE pauses and placed in Table 15 below. Willow and Jackson each had two more minutes of practice time than Harry. Also, there were larger differences in the amount of time the PTs spent acting out the role of teacher in the latter two rehearsals.

Table 15 *Length of Each Rehearsal for Each PT*

	Harry	**Willow**	**Jackson**
Launch	8m46s	8m15s	8m43s
Monitoring	4m12s	6m33s	8m30s
Discussion	6m6s	6m40s	3m53s
Total	**19m4s**	**21m28s**	**21m6s**

Table 16 below contain each of the three PTs enacted practices across the three rehearsals. The two charts are the same aside from using rainbow color order as a heat map to highlight the three (left) most frequently used and three (right) least frequently used moves. The chart on the left offers a heat map of the PTs' three most frequently used moves and the chart on the right highlights the PT's three least frequently used moves.

Table 16 *Total Moves Across PTs Across Rehearsals*

	Harry	Jackson	Willow
Wait Time	3.37%	2.33%	1.00%
Inviting Thinking	5.62%	8.59%	9.00%
Inviting to Do	3.37%	2.34%	7.00%
Inviting Identify	11.24%	22.66%	25.00%
Revoicing	16.85%	17.97%	16.00%
Student Revoicing	0.00%	0.78%	1.00%
Revoicing Summative	0.00%	2.34%	1.00%
Probing	7.87%	6.98%	5.00%
Creating Opportunities	6.74%	3.88%	4.00%
Math nudges	10.11%	5.43%	6.00%
Valuing	6.74%	3.10%	2.00%
Politeness/gratitude	7.87%	6.98%	3.00%
Check-in	19.10%	14.73%	16.00%
Affirmed Correctness	1.12%	1.55%	4.00%

	Harry	Jackson	Willow
Wait Time	3.37%	2.34%	1.00%
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Student Revoicing	0.00%	0.78%	1.00%
Revoicing Summative	0.00%	2.34%	1.00%
Probing	7.87%	6.98%	5.00%
Creating Opportunities	6.74%	3.88%	4.00%
Math nudges	10.11%	5.43%	6.00%
Valuing	6.74%	3.10%	2.00%
Politeness/gratitude	7.87%	6.98%	3.00%
Check-in	19.10%	14.73%	16.00%
Affirmed Correctness	1.12%	1.55%	4.00%

By looking at the charts above, it is evident that the three PTs used the same three moves the most. Inviting identify, revoicing, and checking-in were the three moves utilized the most by the PTs. Willow and Jackson are more similar with the frequency of those moves. Harry used the same top three moves, but he checked-in more than revoicing, and inviting students to identify was third for him whereas it was the most used move by the other two PTs.

It is also interesting to note which moves were least used. Student revoicing was the least used by all three PTs. Harry and Jackson are more similar regarding their least used moves. Their least used moves included wait time, inviting to do, student revoicing, revoicing summative, and affirmed correctness. Willow had some overlap with wait time, student revoicing, and revoicing summative, but her other two least moves were unique to her: valuing and politeness/gratitude.

There is evidence that the PTs are adept at utilizing many of the moves. There is no perfect formula for the rates a teacher should be using these moves, and yet why some might matter more than others in connection with equitable teaching will be discussed in more depth below.

Talk Moves Discussion

Each of the moves is discussed below noting the different ways the PTs utilized the different moves.

Wait Time

All three of the PTs demonstrated waiting at least three seconds for students to respond. Both Harry and Jackson each had to wait twice for students to respond. Another time when his students were unable to quickly respond, Harry asked them to chat among themselves about the question. The one time Willow used wait time was by pausing after asking "So what else do we know about zip lines?" When no one responded, she asked a more specific question. Jackson also had an instance where he ended up rephrasing his question when no one responded initially.

Inviting

There were three different inviting moves coded: inviting to do, inviting identify, and inviting thinking.

Inviting to Do. All three PTs asked for a student to read the task at the beginning of the Launch. Harry and Willow asked for a student to come up and draw a model. Harry asked his students to "reread the problem real quick." Since Willow approached the task sentence by sentence, there were more opportunities for students to draw on the board during the Launch. In other rehearsals, Willow invited a student to "come up" twice when the student was pointing to something on the board. Willow also invited the same student twice to come up and draw a bridge - once before he understood where it went and again after he did understand. During the Launch, Jackson stated "let's redraw our picture really quick" in response to a student

question. Jackson specifically invited students to work on their stated preferred method during the Monitoring rehearsal. There were some strong similarities in their inviting students to read the task and draw, but there were some interesting differences as well.

Inviting Identify. All three PTs asked some binary questions. For example, Willow asked her students if they had an idea of what two different contextual words meant. All three PTs asked questions asking students to identify aspects of the drawing they were putting on the board. For example, Harry asked, "there's two of them, so how big is the other one?" They all asked what it is they were trying to find in the task. During the Monitoring rehearsals, Willow and Jackson asked students to relay what the numbers in their diagram meant.

Inviting Thinking. The three PTs invited students to share their thinking. All three of the PTs asked their students at times to "walk us through" what they had done. Harry and Jackson each had at least one "what does that tell us" in connection with the context of the problem. Harry and Jackson also asked some "how did you get that" questions. These invitations also included inviting students to pose a question. Harry and Jackson each responded to a student gesture to invite a particular student to ask a question. Willow was unique in explicitly asking students to explain their thinking three times. Willow also asked two "why" questions: "Why did you choose to put the island there?" and "why are y'all adding them together"? Willow and Jackson also asked for some general ideas on how to solve the problem. Jackson was unique in telling a student to "think about that for a second." There were more similarities among Harry and Jackson asking "how" questions while Willow asked "why" questions and asked for explanations.

Revoicing Moves

All of the PTs revoiced students for at least 15% of their total moves. As Willow put it, revoicing is "the easiest to integrate." Much of the revoicing consisted of repeating or rephrasing something a student said followed by a question. Full revoicing is revoicing combined with an explicit check-in with a student on understanding (O'Connor, 2009, as cited in Herbal-Eisenmann, 2013). There was one clear instance of full revoicing when Harry asked, "so what you're saying is that you use both parts of these equations just like Monica's group did, correct?" Harry also made a couple of other statements that were similar but missing an actual question. For example, he revoiced saying, "so it sounds like you are struggling with the two different variables that you have." Another time Harry revoiced when affirming correctness, "you're right. We are gonna have to plug in a lot of values to get the minimal value." Harry always revoiced to whom he was talking, whether it was the whole group or an individual whereas Jackson and Willow did some revoicing to highlight one student's thinking to other students. For example, Willow revoiced, "he said find the total, but what are we doing with the total?" On at least four occasions Jackson also did this, but interestingly he included the student's name. For example, "so Kayla's saying that she has two right angles." Willow and Jackson each asked a student to revoice once. Jackson asked, "can any of you revoice what Wesley just said?" During her Discussion rehearsal, Willow asked, "so Jackson or Harry, can you revoice what Stacy was saying? Like how she went through and how she figured out her values?" There was also an instance when Jackson asked a student to rephrase his question, "Harry, can you rephrase what you just asked?"

There was another type of revoicing coded called summative revoicing. Summative revoicing is a specific summarizing of a student's approach that assigns credit to the

student. Willow did this once during the Monitoring rehearsal when she said, "so Kayla wants to start with this right side of the equation and we're trying to minimize it." Willow stated this approximately 10 talk turns after Kayla originally suggested starting with the right side. Jackson did this three times, usually near the end of the rehearsal. In the Discussion, he said "Willow's group came up with the Pythagorean theorem…" Summative revoicing is essentially revoicing in a summative way, but the revoicing occurs later in the rehearsal - not right after a student has said something.

Probing Moves

The three PTs probed further into student thinking. A common way they probed student thinking was to revoice something a student averred and ask why. There were also a few "what" questions such as "what do those numbers tell you?" or "What are you finding from that?" Jackson had a few pressing questions of "how do you know?" In response to a student saying, "I've got an equation and an alligator in my lagoon," Willow asked, "how did you set up the equation?" Jackson and Harry each had an exchange where they were probing into why a student did not agree with a drawn representation on the board. Most of the probing moves were implemented to have a student explain more, but there were a few pressing for justification.

Creating Opportunities

The PTs all created opportunities to engage with the thinking of others by asking students to discuss among themselves or by considering another student's reasoning. One way they did this was by asking their students to chat with each other about a specific question. There were three different ways the PTs engaged other students with one particular student's thinking: asking a question specifically in connection with that student's idea, evaluating whether it was a good approach, and seeing if they agreed with the student's reasoning. Harry and Willow also

asked comparison questions that asked students to explain connections between the equation and the table (two different approaches presented by different students).

Math Nudges

One of the codes that emerged was math nudge. Math nudges are suggestions or hints that might guide student thinking. In further examining the types of statements that were coded as math nudges, different types of math nudges were communicated to the students. The different math nudges were further categorized below as: general hints, revisiting language of the task, cueing, are you sure?, redirection, providing a summary explanation, presenting a connection, and providing a procedural explanation. Examples of the math nudges that helped construct this list are in the table below.

Table 17 *Examples of Math Nudges Observed by PTs*

	Harry	Willow	Jackson
Providing Guidance	Launch: • "Now I think a good approach to this is to get a picture down on our sheets of paper…"	Launch: • "Ok. So let's break down the task by sentences. So the first sentence says a new amusement park is building…."	Launch: • "So the first thing I want to do is, can we go ahead and write all the numbers that are in this task up on the board and on your paper so we can get a good idea of what we're looking at"
Revisiting the language of the task	Launch: • "We know that the islands *(motions to the towers)* are going to be on opposite sides also because it says it in the very beginning of the problem."	Launch: • "So it says, in the problem, two towers on opposite sides of a man made lagoon. So the lagoon is right here and if this tower is over here, the opposite side would be over here."	
Cueing	Monitoring: • "So what you're saying is that we need to put them in one variable or it would be easier to solve this equation if we only had one variable?" (in response to them saying struggling with 2 variables)		Launch: • "So we want to minimize this length of wire A and this length of wire B. ok. And the way we do that is by finding out how far we have to go from this island to this tower."

Table 17 *continued*

Are you sure that's what you meant?	Launch: • "Is it in the middle or just somewhere in between?" (in response to a student who suggested 300 on both sides)	Discussion: • "So you are saying that the 225 and the 375 are both X? Or is there one in particular?" (Stacy said 225 and 375 were both X)	
Redirection			Monitoring: • "So maybe conceptually that worked, but maybe not the best method to figure out an exact length of the wire" (Wesley sharing hard to get exact answer with table)
Providing a summary explanation	Discussion: • "We know that placing it in the middle is not going to minimize the total amount of zip line needed to get to the island, correct?"		
Presenting a connection	Discussion: • "Would you know another way of getting, of not using a table to get the minimal value?" ...After looking at the table?"	Discussion: • "And so do you see how you could maybe use a table to find the minimum"	
Providing procedural explanation			Monitoring: • "And we know similar triangles can be proven with two angles. Ok. So we have to find another angle that are the same."

General hints were general hints in connection with approaching any task, not necessarily this one. Revisiting the language of the task was when the PT directed students back to the task description in response to a question. Telling key ideas (cueing under Ellis et al.) were when the PT told students key ideas that could be viewed as lowering the cognitive demand by doing the sense-making for them. "Are you sure that's what you meant?" nudges were responses to a student's comment where the comment was referenced, and an alternative idea suggested in the form of a question. Redirection was only done by Jackson during Monitoring when he explicitly shifted students who were pursuing alternative approaches back to a group approach. Providing

a summary explanation was used by Harry to highlight a key idea during the Discussion. Presenting a connection was used by both Harry and Monica to point out an aspect of one approach to their students. Providing a procedural explanation was only used once by Jackson when he was aiding his student who was pursuing similar triangles as an approach to solving the problem.

Valuing Student Thinking

The three PTs were each coded for valuing student thinking, but none of their responses were specific enough to qualify as assigning competence (Cohen, 1994). Harry and Jackson each made comments like "that's a good idea" or "that's a good start," but neither named what about the idea was good or what about the start was good. So, the students received positive encouragement, but not clear support for the mathematics of what they were doing. Willow was even less specific in that her valuing move was to respond with "I like it." And, like her peers, she was not explicit about what she liked. The PTs are responding positively to students, but certainly have some growth in connection with highlighting the mathematics/thinking they are valuing.

Politeness/Gratitude

Each of the different PTs responded with basic politeness and gratitude. Harry frequently would respond to students with "awesome," but also had a "very nice" and a "thank you." Willow responded with "thank you" a few times. In response to his students, Jackson frequently would say "okay, cool." He also asked permission from them with questions such as "May I sit here?" and "can I pause you?"

Check-in

The three PTs checked in with their students for about 15% of their moves on average. Each of the PTs did a revoicing followed by checking in with a "Right?" or a "Correct?" at least once. Harry's check-ins focused slightly more on expected comprehension. For example, a couple of times Harry asked, "Does everyone see how…?" He also asked, "Does everyone get that?" and "that makes sense, right?" The one time he invited questions it was very specific, "does anyone have any questions on what numbers are in here versus in here?" Willow and Jackson checked in with their students in ways that were more inquisitive of student comfort levels with task progression and also were more inviting of questions. Willow's signature way of checking in was to ask, "does that make sense?" She did some version of that seven times. Other check-in questions of hers included "does everyone agree with that?", "does everyone like that?" and "does that work for you?" Both Willow and Jackson asked their students a few times if they had any questions. Jackson also seemed to want his students to be comfortable with what was being said. He also asked his students a couple of times "does that make sense?" Additionally, he asked, "is everybody okay with…?" and "does that work?" Twice Jackson checked in with a particular student to confirm their need had been addressed with "did that answer your question?"

Affirmed Correctness

The three PTs all affirmed correctness on occasion by responding with a short comment. Harry responded with "you're right" in response to a student saying that they were going to have to plug in a lot of values to find the minimal value using a table. Jackson affirmed Willow's statement that she did, in fact, have an exact answer with a "you did." He also affirmed Wesley's "I don't think so" in response to whether they knew where the island was with

a “nope.” Willow responded with “right” on a couple of occasions and “that’s true” on another. None of the PTs are providing long-winded statements about what is correct, just briefly provided some quick feedback to students during the rehearsals.

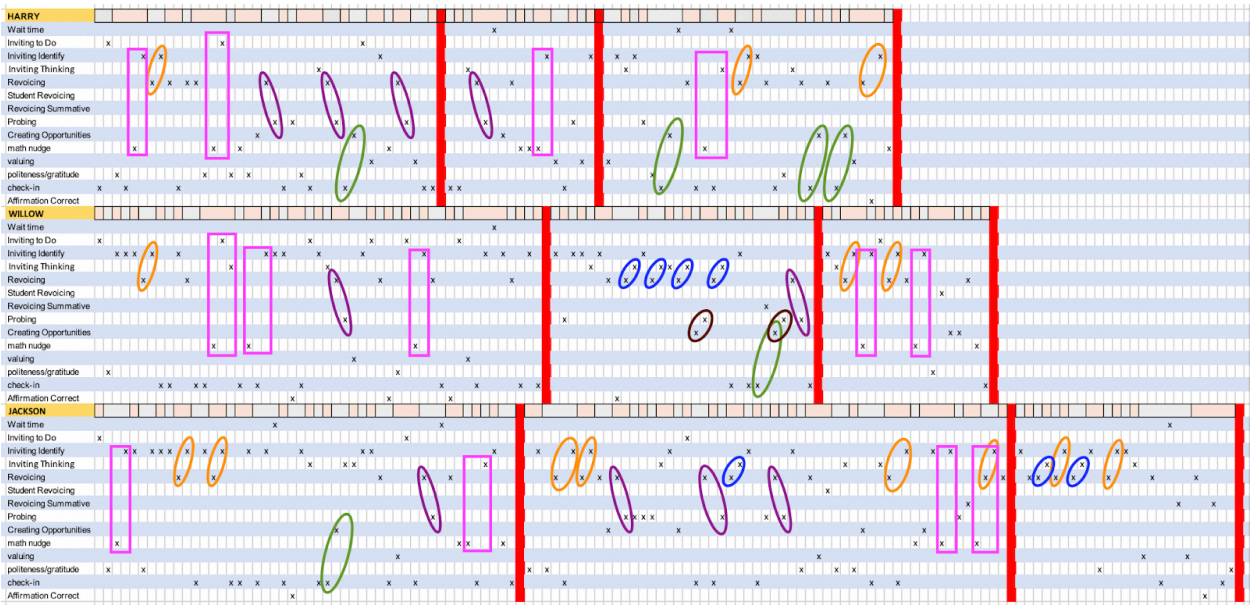
Summary

The PTs supported student thinking in a variety of ways during the rehearsals. Many of the talk moves observed are well established in previous literature as ways of supporting classroom discourse. All three had been exposed to TDMs before the rehearsals and utilized most of them. Inviting moves and revoicing were used regularly throughout by all three PTs. Some of the PTs implemented less commonly used moves such as student revoicing. A potentially new move, summative revoicing, was noted. Some of the more interesting moves were the math nudges. When the PTs did value student contributions, the valuing was typically a simple vague statement such as “good job.” The PTs utilized a variety of moves to support students during rehearsals.

Patterns of Moves across the PTs

While supporting student thinking with talk moves, the question arose of whether there were differences in how the PTs used the moves together. Below the three images of the PTs’ patterns of moves across rehearsals are displayed on one grid so that trends can be discussed. The red vertical line separates the different rehearsals for each of the PTs.

Figure 18 *Patterns of Talk Moves Across PTs*



The combinations noted above include six pairings: creating opportunities: probing (brown oval), revoicing: inviting thinking (blue oval), revoicing: probing (purple oval), revoicing: invite identify (orange oval), check-in: creating opportunities (green oval), and math nudge: inviting moves (pink rectangle). All of the PTs used the math nudge: inviting move combination, but it was strongest with Willow in that five of the six times she nudged her students mathematically, she followed that with an inviting move of some kind. Another pairing frequently used by all three PTs was revoicing followed by inviting identify (orange ovals). As one of Jackson's signature moves, of the 21 times he revoiced, he followed with an invitation to identify eight times. Revoicing was also followed by probing questions (purple ovals) some for all three PTs. Willow and Jackson also paired revoicing with inviting thinking (blue ovals). Harry was unique in repeatedly following a check-in with creating opportunities (green ovals). The pattern of pairing creating opportunities with probing (brown ovals) was only utilized by Willow. There were pairings favored by some and absent in others.

Table 18 *Frequency of Patterns of Talk Moves*

	Harry	Willow	Jackson
Revoicing → Invite identify (orange)	3	3	8
Revoicing → Inviting thinking (blue)	0	4	3
Revoicing → Probing (purple)	4	2	4
Creating opportunities → Probing (brown)	0	2	0
Math nudge → Inviting moves (pink)	4	5	4
Check-in → Creating opportunities (green)	4	1	1

There are some clear patterns of usage in the table. For all three PTs, revoicing was often paired with another move. It is logical for revoicing to be paired since revoicing in and of itself does not merit a response from students whereas many of the other moves do. Certain paired moves were heavy in usage for some PTs. Jackson's go-to move was revoicing followed by inviting to identify. Harry's preferred move seemed to be checking-in followed by creating opportunities. There are other paired moves that stand out as being used less frequently by others. It is curious that Harry never revoiced before asking students to share their thinking when that was a common move by his peers. And Willow was alone in her asking a probing question after creating opportunities, which is a strong pairing of moves. Neither Harry nor Jackson asked a probing question after creating opportunities.

Summary

There were patterns of moves used by the PTs. Some of the patterns may be due to the nature of the move itself, as in the case of revoicing since revoicing does not necessarily merit a response from students. And yet each PT had their own pattern of paired talk moves that supported student thinking during rehearsals.

Question 1a: Inclusivity of Talk Moves

Thinking about supporting student thinking through the lens of Louie's (2017) ways for framing mathematical activity and ability, where were there inclusively framed opportunities for students to participate? Where were there opportunities for students to engage in sense-making? Where were there opportunities for students to engage in a multi-dimensional math frame? And where were there opportunities for students to be positioned as valuable to the learning experience - as a resource for others? In an effort to analyze how the PTs framed mathematics ability and activity in their moves, the moves were organized into fewer categories in connection with the opportunities the move offers the student. The categories are inviting thinking, wait time, valuing thinking, other inviting, checking in, and guiding thinking.

Mathematical Activity

One of the descriptors under the sense-making frame and the multi-dimensional math frame is "assigning open-ended, non-routine tasks." As this study observed the launch, monitoring, and discussion phases of a task, the PTs were already teaching in a learning environment that was set up to frame mathematical activity inclusively. Within the task facilitation environment, what opportunities were there for students to experience inclusive framing of mathematical activity?

Sense-making. The most inclusive mathematical activity moves were the ones that invited students to share their thinking. Inviting a student to share their thinking (probing, inviting thinking, creating opportunities) are all moves that encourage sense-making. Probing qualifies as sense-making as it involves digging deeper into an already communicated student idea. Inviting thinking typically included questions in which students had to explain their reasoning. And creating opportunities positioned students to make sense of other students'

thinking. When wait time was observed, it was in conjunction with one of those three thinking moves about fifty percent of the time. So, wait time can be viewed as an "assist" to inviting thinking in the sense that by itself it does not necessarily encourage sense-making, but students do need to be provided adequate time to respond, especially when being asked tougher questions.

Multi-dimensional Math Frame. Louie (2017) asserts that "mathematics includes activities such as collaboration, experimentation, and argumentation, not just rote practice." As the students were already in a group to work on the task, they were already structured to collaborate. The PTs further communicated expectations in connection with collaboration in their moves that created opportunities to engage with the thinking of another. Jackson and Harry explicitly asked their students to talk with each other to discuss options to solve the problem. Willow never asked her students to talk with each other in that same way, but she positioned her students to work together to make sense of the mathematics. When Wesley had a question about adding square roots, Willow responded by asking his peers, "Kayla and Stacy, do you know how to explain that?"

The moves that supported argumentation were also the inviting thinking moves: probing, inviting thinking, and creating opportunities. For example, Jackson pressed his student for justification when he probed her thinking with "how do you know?" By repeatedly pushing for her to justify why the triangles were similar, he was pushing for argumentation. Harry and Willow framed mathematical activity to include argumentation when they asked students to compare and explain connections between the equation and the table. The PT moves that frame math as multi-dimensional were the inviting thinking moves.

Mathematical Ability

Louie (2017) describes framing mathematical ability inclusively as "everyone has both intellectual strengths and areas for growth that are relevant to mathematics learning." So framing ability inclusively means "valorizing skills that have not traditionally been seen as mathematical, naming a variety of students as resources for their peers' learning, and making statements about mutual dependence (everyone contributes, everyone learns together)." How did the PTs frame mathematical ability?

The PTs framed mathematical ability inclusively by creating opportunities, summative revoicing, and student revoicing. Skills that at least one PT valorized included drawing and exploring an idea that did not yield a correct result. Two of the PTs invited students to draw aspects of a model on the board. Jackson did a summative revoicing of Kayla's exploration of an approach that proved to not be fruitful. Valorizing approaches that do not yield correct answers is less traditional. There were a few different ways the PTs named a variety of students as resources for their peers' learning. Both Harry and Willow asked their students to chat with each other in connection with a question asked. By asking students to discuss possibilities with each other, they are positioned as resources to each other. All the ways the PTs asked students to engage in each other's reasoning also positioned students as resources for their peers' learning. Additionally, all of the summary revoicing specifically named a student and their mathematical progress, thus naming that student's work as a resource for their peers. Student revoicing, while only practiced twice, is another way to position a student as a resource for a peer.

Talk Moves Outside the Inclusive Mathematics and Mathematical Ability

For many of the moves, it depends on who is being credited and what they are being credited for.

Valuing Moves. Valuing moves have the potential to be inclusive regarding mathematical ability in that who and what is being valued connects back to what is valued as ability and activity. For example, if a teacher values a student for thinking about the problem in a different way, then the teacher has positioned that student as competent by valorizing that skill and has broadened what counts as mathematics by explicitly valuing the thinking over the answer. It has already been discussed how revoicing summative and student revoicing support inclusive ways of framing mathematical ability. In addition to those moves, the PTs also responded positively to students with less clear intentions about what they were valuing. For example, Harry responded to a student's correct algebraic expression with "that's a good idea." So, was the fact that it was correct what was good? Or was it that the student knew how to use inverse operations to isolate the variable or something else entirely. So, in that case, the student may feel valued, but may not know what mathematical activity or ability is being valued. This is also true for statements made by Jackson and Willow's "I like it". It could be argued that because there was some focus on ideas and starts instead of answers and finishes that even in their less specific statements the PTs were leaning towards being inclusive. But let us not forget Harry's "idea" that might have been masquerading for "correct answer." And general politeness may be encouraging to students but responding with "thank you" or "awesome" also is not clear enough. In order for the valuing of students' ability and activity to be truly inclusive, the PTs would need to be more specific about what it is they are valuing.

Revoicing. Revoicing was another valuing move used habitually by the PTs. In general, just repeating or rephrasing what a student says without any additional insights does not frame mathematics activity or ability as inclusive or exclusive. By repeating or rephrasing what a student says, their thinking is being heard again to the class, which has potential to inclusively frame mathematics activity and ability since sense-making by students is being repeated. The revoicing done by Jackson that included naming the student along with what they said is framing mathematical ability inclusively in that he is naming peers as people to learn from. Revoicing operates similarly to checking-in since it is mildly valuing student thinking by repeating it, but in general is not truly framing anything inclusively.

Checking in. Checking-in was viewed as framing mathematical activity or ability *slightly* inclusively. As the exclusionary frame of mathematics connects to behaviorism with its focus on rote practice, the choice by PTs to ask their students "does that make sense?" is one example that positions mathematical activity inclusively. None of the check-ins by the PTs had to do with a correct answer, they were typically about agreement in connection with decisions or assumptions while making sense of the task. Checking-in on understanding or agreement also frames mathematical activity to include listening as passively participating in sense-making. Additionally, checking in on students by asking if they have questions suggests they are inviting students to be sense-makers. Granted, like wait time, checking in is more of an assist to the other inclusive moves. Since the PTs in this study were checking in more on students in connection with sense-making and understanding, checking-in was being framed slightly inclusively.

Inviting Other. Other inviting moves include opportunities for students to participate that do not include verbally expressing their mathematical thinking. Within this category, the

common ways the PTs invited other opportunities had to do with drawing, reading the task, and responding to recall questions. Drawing a diagram was considered inclusive as the student was sharing their thinking pictorially and that was broadening what counts as mathematics. Harry and Willow's invitations to students to draw on the board were viewed as inclusive mathematical activity and ability. Other inviting moves that invited students to participate by responding to a recall question did not afford students the opportunity to respond open-endedly. Because the inviting other moves include a variety of opportunities, the PTs also varied in how inclusively they framed mathematics when using them.

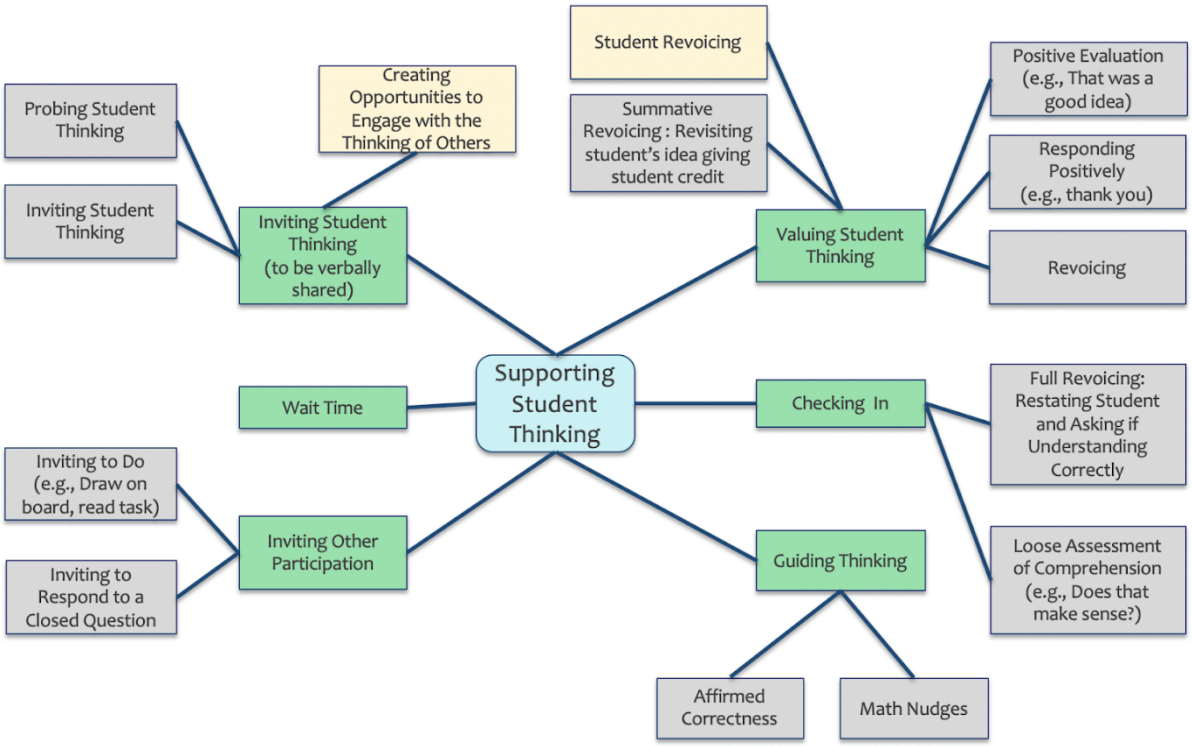
Guiding Moves. In general, guiding student thinking moves are less inclusive because they shape student thinking, suggesting students are on the receiving end of sense-making. For example, affirmed correctness lets students know that something was correct. Students certainly benefit from knowing what is mathematically permissible. Math nudges can vary tremendously in how they present opportunities for sense-making to the student. On one end, the PTs might be guiding student thinking in such a way that they are providing structure to their sense-making, as with the *general hints*. And on another end, the teacher may be hinting quite strongly for students to pursue a particular path or procedure, as was the case with the *telling key ideas*. In general, the PTs' guiding moves were slightly exclusive in that the trend was for the teacher to do the sense-making for the students.

Visualizing the Moves with Inclusivity

The below graphic shows the different moves discussed and how they were grouped for the purpose of this study. The PTs framed mathematical activity most inclusively by using the inviting thinking moves: inviting student thinking, probing student thinking, and creating opportunities. Those three moves all fell under the larger group of inviting thinking moves

which are in the top left corner of the map. The PTs framed mathematical ability most inclusively by using creating opportunities (an inviting student thinking move) and student revoicing and summative revoicing (valuing student thinking moves). While those three moves stem from two different larger groups, they are visually top and center on the map. The two moves that are coded in yellow represent moves that specifically ask students to respond to students. Generally, the categories are loosely organized in such a way that the left side has more of a focus on mathematical activity and the right side more of a focus on mathematical ability.

Figure 19 *Talk Moves for Inclusivity Analysis*



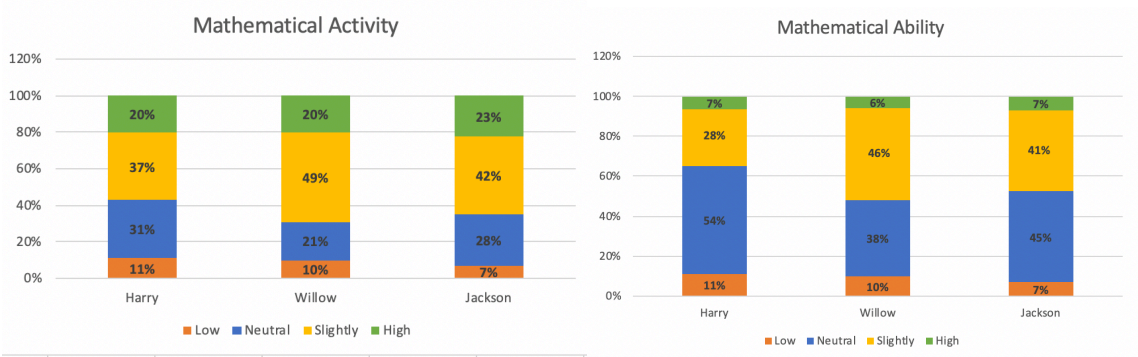
While the map above did help make sense of the different moves used by the PTs and how they work with Louie's (2017) framework, in truth the respective talk moves are not

necessarily so clearly delineated. There can be much overlap among the different moves. For example, restating or rephrasing student thinking is at the very least valuing that student's thinking by repeating it. However, the PT may have repeated the student's thinking intentionally to also guide others.

Framed Mathematical Activity and Mathematical Ability of the Three PTs

Using the above analysis to categorize the different talk moves, a chart was created to briefly look at an overview of the three PTs and how they framed mathematical activity and mathematical ability holistically. At a glance, Harry was the least inclusive in that he had more moves categorized as low and neutral in both charts. Jackson seems to be the most inclusive in that he had the highest moves in both charts. Willow had more moves categorized as slightly in both categories. There are some similarities among the three PTs. Based on these categorizations, the PTs are much stronger at framing mathematical activity as inclusive than they are mathematical ability.

Figure 20 *Inclusive Mathematics and Mathematical Activity in Talk Moves Per PT*



Summary

The PTs are supporting student thinking using a variety of different talk moves. The most used moves are revoicing, inviting identify, and checking-in. As the PTs were less certain

about how to respond to students when they students were struggling, the different math nudges capture their variety of attempts to guide student thinking. There were some patterns of moves shared by all the PTs such as revoicing followed by probing, but each also had their unique paired moves too.

How inclusive the PTs framing was had less to do with the move itself, but certain moves do frame mathematical ability and activity more inclusively than others. The PTs are showing evidence of inclusivity in their moves, but there are certainly also missed opportunities. For example, by only stating "good job" they are not framing mathematical activity or ability clearly enough for students to receive the message of what activity or ability might be "good." There is also overlap with the talk moves in the sense that what is high for activity is correlated with what is high for ability. For example, if a student is being valued for a skill that is less traditionally valued in mathematics classrooms, then while naming that student as a resource the teacher is also broadening what counts as mathematics. While many of the moves are dependent upon what mathematics is being given attention to, certain participation moves frame mathematical activity more inclusively because they structure sense-making to be done by the students.

Question 2: Conceptual Practices and Dispositions

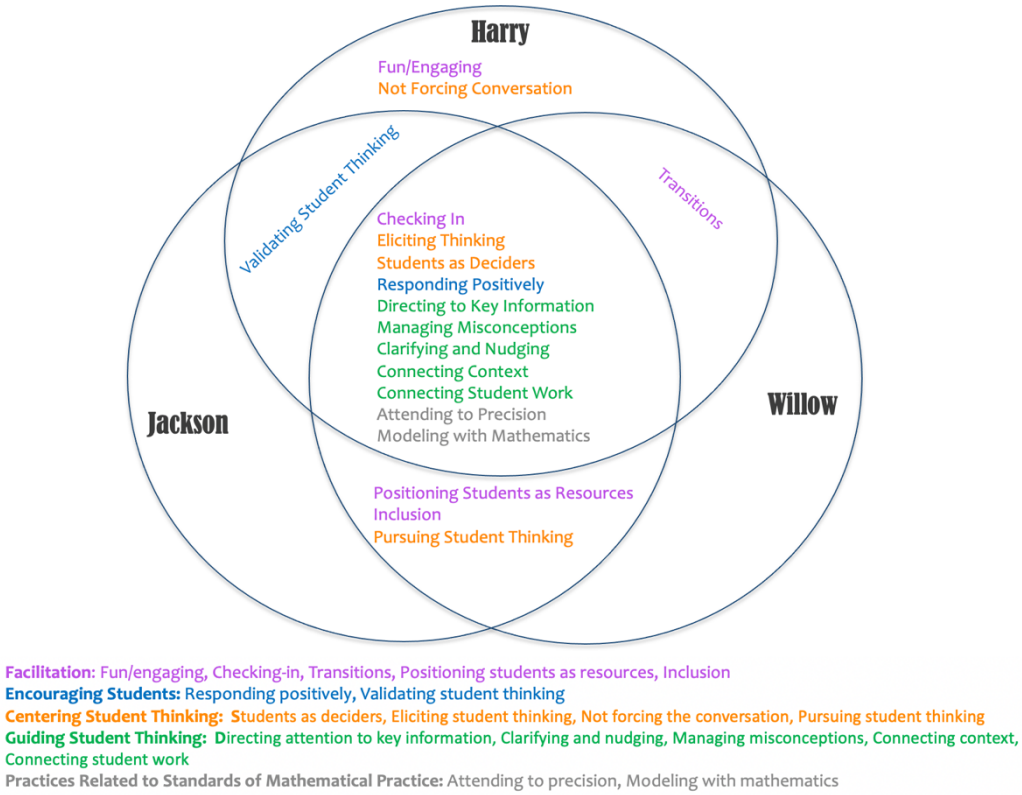
The second question under investigation is "What conceptual practices and dispositions inform PTs' instructional decisions during rehearsals?" The Hammerness et al. (2005) Learning to Teach in Community framework was used to make sense of PT's intentions behind decision-making that supports student thinking during rehearsals. The two aspects of that framework that can best answer the research question are the PTs' practices (the intentions and beliefs behind what they do) and their dispositions (habits of thinking and action). First conceptual practices

will be discussed followed by an analysis using the Louie (2017) framework. Then dispositions will be discussed followed by an analysis using the same framework.

Conceptual Practices Used in All Three Cases

In the Venn diagram below, each of the three larger circles represents each PT and their conceptual practices. In the center are the practices utilized by all three. The colors represent the different larger categories that were used to describe groups of practices. At the bottom is the color-coded key for those categories and associated practices.

Figure 21 *Venn Diagram of PTs' Conceptual Practices*



There were several conceptual practices that all three of the PTs had in common. They all regularly checked in with the students to ensure the students understood or were okay with what was being said. The PTs intentionally elicited student explanations and wanted students to make some decisions. Guiding student thinking practices such as managing misconceptions,

directing attention to key information, clarifying and nudging, connecting context, and connecting student work were practiced by all three PTs. Additionally, all three were attending to precision and modeling with mathematics.

All three responded positively to students, but the nuances among why they were responding positively are interesting. Harry was explicit in his intention to positively reinforce. He wanted his students to experience positive responses. In the instances Willow responded positively, it was more in connection with her being grateful for students responding to a particular request. Jackson responded positively in an evaluative sense. When asked why he responded with “good question” he replied, “'cause that was a good question.” So, their intentions varied slightly when responding positively to students.

Conceptual Practices Used in Some Cases

There were also conceptual practices that not everyone had in common. Harry shared intentions toward having a fun/engaging class and not forcing the conversation. Harry and Willow both were intentional about having transitions. Harry and Jackson both were explicit in wanting to validate student thinking. Willow and Jackson had three practical tools in common: positioning students as resources, inclusion, and pursuing student thinking. They were very similar in how they positioned students as resources for each other. Willow and Jackson were both inclusive by inviting students into the discussion. Jackson additionally took steps such as not spending too much time with one student so that others did not feel left out. While pursuing student thinking, Willow pursued student thinking in the sense that she would pursue directions they suggested such as Kayla suggesting they minimize one side. During the Monitoring rehearsal, Jackson let his students each pursue their own ideas.

The Conceptual Practices using the Louie framework

The categorization is based mostly on how the PTs in this study used them. It would be inaccurate to label practices and dispositions as inclusive or exclusive as each one could be either at times.

More Inclusive Framing Practiced by All Three PTs

The practices of eliciting student thinking, letting students decide, modeling with mathematics, attending to precision, connecting student work, and connecting context were inclusive in nature. Centering student thinking practices such as eliciting student thinking and letting students decide both can be viewed as aspects of sense-making. Eliciting student thinking is requesting students to share their thinking to be considered for sense-making by others. Allowing students to decide is giving them some control as they participate in sense-making. The two practices related to standards for mathematical practice were also inclusive in that modeling with mathematics was mostly in connection with using a drawn model to make sense of the task and attending to precision was inclusive in that being more precise with their labels and verbal descriptions is part of communicating in connection with sense-making. Guiding practices such as connecting student work and connecting context have the potential to be very inclusive in that they are part of sense-making. One could argue that if only the teacher is doing the connecting, then perhaps it is less inclusive, but for all three of the PTs in this study, the students were involved in the connecting of student work and the connecting context.

More Inclusive Framing Practiced by Some of the PTs

Not forcing the conversation, validating student thinking, pursuing student thinking, positioning students as resources, and inclusion were inclusive in nature. Harry's practice of not forcing the conversation suggests he is letting the students be the sense-makers since he did not interrupt them when he wanted to jump into the conversation. Harry and Jackson both were intentional in validating student thinking. By acknowledging the perspective of students, they are framing mathematical ability and activity inclusively. The three practices shared by Willow and Jackson also are all mostly inclusive. The centering student thinking practice of pursuing student thinking is inclusive in nature in that by pursuing student thinking the students are being positioned as the sense-makers. The practice of positioning students as resources is inclusive in "naming a variety of students as resources for their peers' learning" is one of the ways Louie (2017) describes inclusive mathematical ability. Thinking of student idea sharing as collaboration in sense-making, it could also be argued that positioning students as resources is also framing mathematical activity inclusively. And the other practice both Willow and Jackson had in common was inclusion. They were inclusive in their mathematical activity in that they intentionally invited students to participate in sense-making who maybe had not spoken in a while. This also could be viewed as inclusive ability since by including others, they are valuing thinking from a variety of students.

Less Consistently Inclusive Practiced by All Three PTs

Responding positively and checking-in were common practice among all three PTs that were less clearly inclusive. The three PTs responded positively, but their intentions did not seem to align with inclusive framing. Their brief comments such as "good idea" and "great point" do slightly suggest that they are valuing sense-making, but they are not specific enough. Checking-

in with students was inconsistently inclusive. When the checking in was done to clarify the PT's understanding of something said by a student, that leans inclusive in that the student sharing their thinking is sense-making. However, checking-in by asking the students "does that make sense?" does not really do anything for mathematical activity or ability.

There were several guiding student thinking practices that were less clear on how they connect to inclusive ways of framing mathematical activity and ability. Directing students to key information is showing students where to look, which puts some of the thinking on the students since they have to then also do the looking. Pointing students in a direction allows them to do more sense-making than just telling them, and yet it is not clearly valuing sense-making either. Clarifying and nudging students as a practice could be inclusive or exclusive. It depends on who is doing the sense-making with the nudge and whether the nudge is in connection with student insights or if it is the teacher's nudge in a predetermined direction. Harry's clarifying and nudging was mostly exclusive in nature since he was doing the sense-making for the students in his nudges. Willow did more clarifying than nudging and so in bringing clarity to terminology, her students were engaging in sense-making. Jackson initially was doing more of the sense-making for students, but then shifted to asking his students questions to nudge them. Managing misconceptions is another practice that is less clear. The PTs wanted to address any learning barriers their students might have. So not unlike clarifying and nudging, which has some overlap with this practice, how misconceptions are managed can be exclusive or inclusive.

Less Consistently Inclusive Practiced by Some

Both the focus on being fun/engaging and the focus on transitions were less clear. In an effort to be engaging, Harry asked his students about their own zip line experiences which could be viewed as an approach to help them make sense of the task by tapping into their own

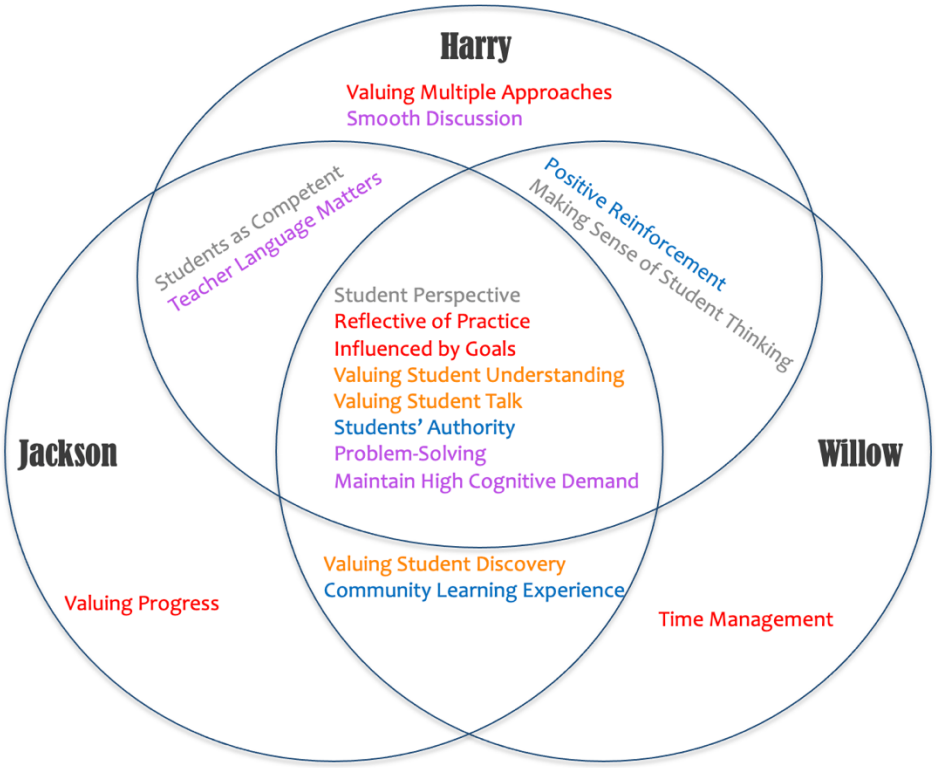
experiences. Yet this was not enough to support the idea that he is being inclusive by asking about their experiences. So, his focus on being fun and engaging was inconsistently inclusive. Harry and Willow both had a focus on transitions. Harry focused on transitions during the Discussion interview in a way that suggested he was intentional in trying to bridge from one student's work to another. Willow mentioned wanting to introduce something, recap something, and so on. Since the focus of their transitions was largely on connections, then this practice probably leans toward framing mathematical activity inclusively and yet transitions have little to do with what counts as mathematical activity. They are a facilitation move that may or may not aid in students' sense-making.

Dispositions in All Three Cases

The PTs were credited only for having dispositions that they communicated. So, some of the dispositions that were only credited to one PT may actually be true of all, or that disposition may be unique to that person. The Hammerness et al. (2005) framework was used to group some of the dispositions and there were so many habits of action regarding teaching that those were further grouped.

In the Venn diagram below, each of the three larger circles represents each PT and their dispositions. In the center are the dispositions the three PTs showed evidence of having. The colors represent the different larger categories that were used to describe groups of dispositions. At the bottom is the color-coded key for those categories and associated dispositions.

Figure 22 *Venn Diagram of PTs' Dispositions*



Habits of Thinking regarding students: Student perspective, Making sense of student thinking, Viewing students as competent
Habits of Thinking regarding Teaching: Reflective of practice, Influenced by goals, Valuing multiple approaches, Time management, Valuing progress
Habits of Action Regarding Teaching:
Focusing on Student Thinking Dispositions: Valuing student talk, Valuing student understanding, Valuing students discovering
Culture Building Dispositions: Students' authority, Community learning experience, Positive reinforcement
Facilitation Dispositions: Maintain high cognitive demand, Problem solving, Teacher language matters, Smooth discussion

Habits of Thinking Regarding Students and Teaching in All Three Cases

There were several dispositions that the PTs had in common. In thinking about students, they all considered the learning experience from the student perspective. The PTs shared that certain decisions were made because they believe students wanted to hear other students' voices and explanations. They also had commonalities in how they think about teaching in that they were all reflective of their practice and influenced by goals. They all reflected on their practice and mentioned moments they wished they had done something differently. While all three mentioned a corrective action, Jackson shared nuggets of wisdom that he generalized for future

situations such as asking questions to build off of what a student does understand when they are completely lost. Goals were influencing their decisions whether they were in connection with mathematical concepts like minimization or experimenting with facilitation goals like focusing on revoicing.

Habits of Action Regarding Teaching in All Three Cases

The three PTs all exhibited the dispositions of valuing student understanding, valuing student talk, students' authority, problem-solving, and maintaining a high cognitive demand. The PTs worked toward ensuring student understanding by using approaches such as checking in with their students and eliciting explanations of their idea or another's. All three of the PTs repeatedly expressed wanting student talk to be ubiquitous in their rehearsals, but their valuing of student talk varied. Harry valued students talking at more of a surface level in that he asked a probing question "just to make sure that I can get some more responses from them," whereas Willow and Jackson explicitly valued student talk as a way for students to communicate explanations. Students' authority was a culture-building disposition shared by all. The three PTs each were wanting students to take part in some decision-making. All three shared the facilitation dispositions of problem-solving and maintaining a high cognitive demand. They all believed in setting students up with strategies such as visual aids to be more successful in attempting the task. All three were very explicit in not wanting to lower the cognitive demand of the task while sharing their struggles to not provide too much guidance.

Habits of Thinking Regarding Students and Teaching in Some Cases

There were also dispositions that not everyone had in common. Other dispositions related to habits of thinking regarding teaching that were unique to particular PTs include Harry's valuing multiple approaches, Willow's time management awareness and Jackson's

valuing progress. During interviews, Harry explained that even though one student thought using a table was an inefficient approach, for another student that method might make more sense to them. Willow had concerns during her rehearsal in connection with time management. She was noticing that certain things were taking more time than expected and so she was sensing a need to return to the task. Jackson's thoughts were more focused on a sense of momentum and progress. For example, in the Discussion interview, he mentioned his students "still got pretty far I think." Other dispositions related to habits of thinking regarding students that were shared among two PTs include considering student thinking during rehearsals and students as competent. Harry and Willow both shared their thoughts during interviews about how they were considering student thinking during their rehearsals. One way they considered student thinking was by reasoning out loud about possible student interpretations and making connections with why a student's response was what it was given their progression in the task at that moment in time. Jackson and Harry generally spoke of their students as competent. Harry did have one moment of inconsistency when he suggested perceiving silence as the student not working on the task whereas Jackson always talked about his students from an asset perspective.

Habits of Action Regarding Teaching in Some Cases

There were also dispositions that were habits of action regarding teaching that not all had in common. Harry was the only PT who expressed the facilitation disposition of desiring a smooth discussion. In reflecting on his facilitation of the Monitoring rehearsal, he wanted to improve it by quickly addressing the math issue and then saying, "now that you have it in terms of one variable, what's your next step?" Harry and Willow both shared the cultural building disposition of positive reinforcement. Harry felt positive reinforcement was good for class morale and made him more approachable. Willow wanted to respond to students with "thank

you" and "good job." Willow and Jackson shared the student thinking disposition of valuing student discovery and the culture-building disposition of community learning. Willow wanted her students to discover their approach on their own sans her biases and Jackson had a "try it out, see if it works" mindset for his students. Willow and Jackson both also had strong dispositions about their students' learning as part of a community. Willow had a strong focus on her students "moving forward together" and Jackson had a focus on class as "we." Harry and Jackson shared the facilitation disposition that teacher language matters. They both divulged their realization that how they said things as a teacher impacted their students - from odd wording to unintentionally misleading terminology.

The Dispositions using the Louie framework

More Inclusive Habits of Thinking held by All Three PTs

Being reflective of their practice was one inclusive disposition they all shared. It is important to note that the questions in this interview were nudging them to be reflective and that they were likely influenced by the stated goals in class for the respective rehearsals. All three of the PTs were reflective of their practice in ways that focused on being more responsive to student needs such as providing more space for them to respond, better supporting their struggles, asking why when there is ambiguity in the student statement, and shifting more questions back to students. In general, the PTs' desires to better respond to students' thinking inclusively supports a mathematical activity as sense-making.

More Inclusive Habits of Thinking held by Some of the PTs

Inclusive dispositions held by some include valuing multiple approaches, valuing progress, students as competent, and making sense of student thinking. Harry's valuing multiple

approaches is inclusive in that he is valuing more than one way of approaching the task, so what counts as mathematics is more than the answer. Jackson's valuing progress was curiously inclusive in that his view that class is a learning process meant he viewed anything his students did as progress. Because Harry and Jackson were describing what their students did well instead of getting correct answers, their viewing students as competent suggests inclusive framing. Harry's one inconsistency does suggest that he might be overlooking or undervaluing silent ways of engaging. As Willow and Harry were thinking about the sense-making that their students were doing, this could be viewed as inclusive activity in that by considering the different ways their students were processing the task, they were honoring student sense-making.

Less Inclusive Habits of Thinking held by All Three PTs

Less inclusive dispositions held by the three PTs included being influenced by goals and considering the student perspective. Their goals/successes were not strongly connected to either of the ways of framing mathematical activity or ability. When considering the student perspective, the PTs varied in what they were considering. Harry's consideration of the student learning experience was focused less on the mathematical aspect of learning and so does not fall on the framework. Willow was definitively inclusive with her consideration of the student learning experience in that her utilization of peer explanations for vocabulary positions students as resources supports the multidimensional ability frame and her desire to respond to student tangential questions suggests she supports student sense-making. Jackson was less clear in his inclusivity. His belief that students would be happier coming up with their own answer suggests that sense-making needs to happen on their end and thus leans towards being inclusive mathematical activity.

Less Inclusive Habits of Thinking held by Some of the PTs

The one disposition that was less clear held by one of the PTs was Willow's time management awareness. This disposition was inconsistent in that part of why she was concerned with time was because she was spending time on sense-making with students, which she does value, but also values making progress on the task. So, in being concerned with time management her larger focus was unclear.

More Inclusive Habits of Action held by All PTs

The dispositions that were more inclusive were valuing student understanding, and valuing student talk, students' authority, and maintaining a high cognitive demand. All three PTs valued student talk and student understanding, but there were nuances among those dispositions. Harry was less explicit in what he wanted from the talk, but in valuing talk he was valuing opportunities for students to do sense-making. Willow and Jackson explicitly valued talk in the form of explanations, suggesting they were inclusively valuing student talk as opportunities for sense-making. Regarding valuing student understanding, Harry wanted his students to understand, and yet his focus on valuing understanding was sometimes in connection with ideas he presented to his students. Willow wanted her students to understand and frequently checked in with them, but there was a heavier emphasis on their understanding of each other's thinking than some predetermined mathematical ideas. Jackson also valued students understanding the contextual connections of their work and the reasoning behind certain approaches. By wanting to give students some authority, all three of the PTs were inclusive in their attitudes towards opportunities to participate in mathematics. The students need to have some authority in order to have the opportunity to do some sense-making and to engage in

mathematics in non-traditional ways. The disposition to maintain a high cognitive demand is inclusive in that all of the PTs were wanting their students to be the sense-makers of the task.

More Inclusive Habits of Action held by Some PTs

The more inclusive dispositions held by some PTs are valuing student discovery which is a focus on student thinking disposition and community learning experience which is a community-building disposition. Willow and Jackson both valued student discovery. Willow wanted her students to discover their own approach in the sense that she did not want to bias or influence their thinking. And Jackson valued student discovery in a more exploratory way - he wanted students to consider options and discover what will work for their goals and what will not. Willow and Jackson also both nurtured a community learning experience. Willow generally worked towards no student being left out by positioning students as resources for each other. Jackson explicitly valued collaboration.

Less Inclusive Habits of Action held by All PTs

The one disposition shared by all the PTs that was less inclusive in nature was the problem-solving disposition. The problem-solving disposition is less clearly inclusive in that by guiding with approaches, some of the sense-making is being done by the teacher and yet, from another perspective, by using suggested problem-solving techniques, students might better be able to do some sense-making.

Less Inclusive Habits of Action held by Some PTs

Less inclusive dispositions include the culture-building disposition of positive reinforcement and the facilitation dispositions of smother discussion and teacher language matters. Harry and Willow both expressed intentions to regularly respond positively to

students. As discussed previously, the lack of clarity on what they would be positively reinforcing is why this disposition is less clearly inclusive. Harry and Jackson's disposition involving clarity with teacher language is perhaps neither and again would matter more in connection with what it is they need to be clearer about. Harry's disposition in favor of a smoother discussion suggests he is leaning towards an exclusionary framing of mathematical activity in that he was viewing his role as a facilitator as more of a performer than of a sense-maker.

Summary

The PTs in this study had several practices in common. The PTs had all of the guiding student thinking practices in common: directing attention to key information, managing misconceptions, clarifying and nudging, connecting context, and connecting student work. They also all practiced the two practices related to the standards of mathematical practice. Other practices the PTs had in common include checking in, eliciting thinking, students as deciders, and responding positively. Practices not shared by all three PTs include validating student thinking, fun/engaging, not forcing the conversation, transitions, positioning students as resources, inclusion, and pursuing student thinking.

The PTs in this study had several dispositions in common. They all valued the student perspective. They were reflective of their practice and influenced by goals. In their reflections, they were valuing student understanding and valuing student talk. They felt students should have some authority in the classroom. Maintaining a high cognitive demand and a focus on problem-solving were also shared dispositions. Dispositions that were not shared by the three PTs include: valuing progress, teacher language matters, students as competent, valuing multiple

approaches, smooth discussion, positive reinforcement, making sense of student thinking, and time management.

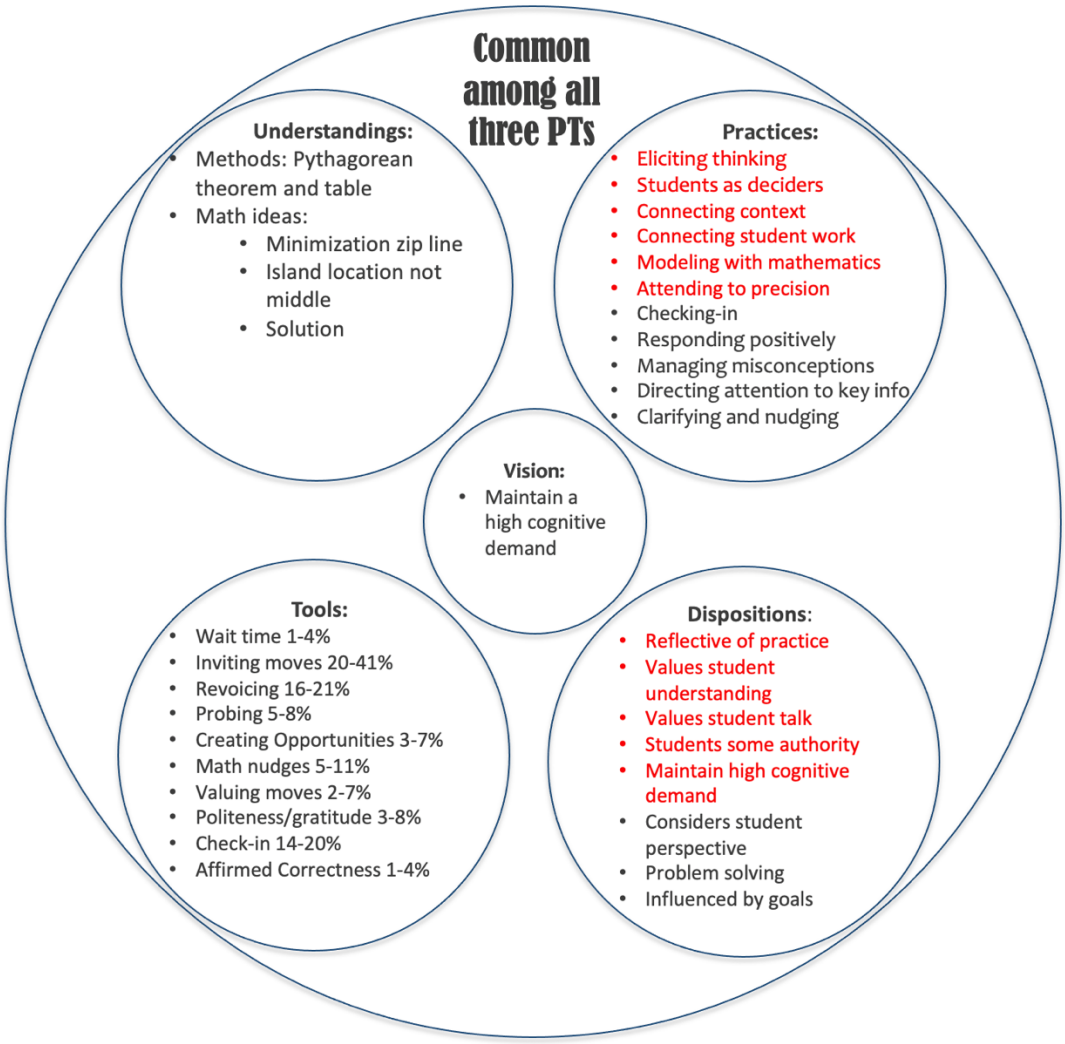
The PTs hold a variety of dispositions as they are developing their practice. Within those, some are more supportive of inclusive framing of mathematical activity and ability. The only group of dispositions that was entirely inclusive was the focus on student thinking dispositions (valuing student talk, valuing student understanding, and valuing students discovering). As expected, each of the PTs holds a variety of dispositions, including some that are more inclusive.

Final Thoughts on the Cross-case Analysis

Before discussing the similarities and differences among the cases, it is worth revisiting the fact that each rehearsal was very different. The Launch and Discussion rehearsals had more similarities, but during Monitoring, the students could have approached the task in a variety of ways with different levels of progress as well. So aside from the professional differences among the PTs, there were also many other factors that might have played a role in their facilitation of rehearsals.

To connect all the ideas above, the Hammerness et al. (2005) framework was used below to house the similarities among the three PTs. Below the three PTs' shared aspects of the learning to teach in community framework are listed. The practices and dispositions that were considered more inclusive are in red.

Figure 23 *PTs' Commonalities in the Hammerness et al. (2005) Framework*



In addition to the practices and dispositions listed above that they all had in common, the PTs had differences that are noted in the table below. Some practices and dispositions were shared by two of the PTs and those are thus listed under each corresponding PT. The practices and dispositions that were considered more inclusive are in red.

Table 19 *Practices and Dispositions Across PTs*

	Practices	Dispositions
Harry	• Validating student thinking • Not forcing the conversation • Fun/engaging • Transitions	• Students as competent • Valuing multiple approaches • Making sense of student thinking • Teacher language matters • Smooth discussion • Positive reinforcement
Willow	• Positioning students as resources • Inclusion • Pursuing student thinking • Transitions	• Making sense of student thinking • Valuing student discovery • Community learning experience • Positive reinforcement • Time management
Jackson	• Validating student thinking • Positioning students as resources • Inclusion • Pursuing student thinking	• Students as competent • Valuing progress • Valuing student discovery • Community learning experience • Teacher language matters

Inclusivity and the Cross-case Analysis

By examining these PTs through a microscope, the goal was to learn more about what they are currently capable of in a simulated teaching environment and how inclusively they are framing mathematical activity and ability. They all have many practices and dispositions in common that are both inclusive and less clearly inclusive. When glancing at the more holistic view of each of the PTs, they each have different strengths. Harry values being positive and encouraging. Willow wants her students to understand through explaining and questioning. Jackson wants his students to discover while he facilitates in a hands-off way. Each PT is supporting student thinking in a variety of ways and showing evidence of framing mathematical activity and ability inclusively.

In connection with responses on what qualifies as mathematical activity and ability, the PTs in this study were selected to represent a diverse group of future teachers. Two of the PTs had more exclusionary responses in general and one had more inclusive responses. While the PT who had the most exclusionary responses stayed less inclusive and the PT who had the most

inclusive responses stayed more inclusive, there was one PT who had a major shift over the course of this study. Jackson initially had more exclusionary responses but his actions across the rehearsals showed growth in that he exhibited more inclusive framing than perhaps even Willow by the end of the rehearsals. While it is possible that Jackson had what some might call a growth spurt, it is also possible that his responses to the initial questionnaire were not genuinely representative of his beliefs. It is worth noting Jackson's shift in that his growth spurt could be very encouraging for teacher education programs.

Chapter 8: Discussion

The purpose of this study was to examine PTs' support of student thinking during rehearsals. Specifically, this study examined what PTs did during rehearsals to support student thinking, what their in-the-moment intentions were, and what larger intentions drove their choices. After establishing those actions and intentions, the second goal of this study was to examine how inclusively the PTs were framing mathematical activity and ability. As mathematics is known as a gatekeeper (e.g., Stinson, 2004), creating a mathematical learning environment in which more students might feel empowered is important.

In this chapter, first there is a brief discussion describing the PTs' talk moves followed by an analysis of their paired talk moves. The PTs' talk moves are then discussed through the lens of Louie's (2017) framework regarding their inclusive framing of mathematics and mathematical ability. Implications related to the PTs' talk moves are then presented. Next, there is a discussion of the practices and dispositions observed by the PTs followed by another analysis using Louie's (2017) framework regarding their inclusive framing of mathematics and mathematical ability. Implications related to the PTs' practices and dispositions are presented followed by limitations of the study and suggestions for future research.

Teacher Talk Moves Observed

The categories of teacher talk moves observed in this study include inviting student thinking, wait time, inviting other participation, valuing student thinking, checking in, and guiding student thinking. Herbel-Eisenmann et al.'s (2013) TDMs are included within these categories as the PTs in this study had been exposed to them in their methods course. For this study, inviting participation moves were further categorized into inviting to do, inviting identify, and inviting thinking. Other moves noted include math nudges, valuing student thinking,

politeness/gratitude, check-in, and affirmed correctness. These moves were grouped into categories: inviting student thinking, wait time, inviting other participation, valuing student thinking, checking in, and guiding student thinking moves. Each of these categories is discussed in further detail below.

The inviting thinking moves included inviting thinking, probing, and creating opportunities to engage with the reasoning of another. Inviting thinking moves were classic eliciting student thinking moves when the PT asked for a student's thoughts. Probing moves were follow-up questions about a student's thinking. Creating opportunities to engage with the reasoning of another involved students being positioned to engage with each other's thinking. All three of these moves typically involved a question that pushed for students to share their thinking.

Wait time was not used that often but was used by each of the PTs at least once. Wait time works in conjunction with inviting moves in that if wait time is never observed then perhaps questions posed were not challenging enough. Fraivillig et al. (1999) described the teacher technique of "allows additional time for student thinking" at the intersection of eliciting and supporting student reasoning. Waiting is also one of Herbal-Eisenmann's (2013) TDMs. Yet, waiting operates differently from the rest of the moves because it makes space for students to respond.

Valuing student thinking moves were moves that had the potential to make a student's thinking feel valued. The moves in this study were student revoicing, summative revoicing, positive evaluation, responding positively, and revoicing. Student revoicing, summative revoicing, and revoicing all include repeating content that a student said. By repeating a student's thinking, that student was positioned as a resource for others. Positive evaluation and

responding positively were encouraging ways to respond to students. Positive evaluation gave students slightly more direction about what was being responded to, but this was typically "good job" whereas responding positively was responding with kindness. Assigning competence would be included in this category, but there were no observed instances of assigning competence. All these moves had the potential to communicate to students that their ideas were of value.

One talk move that merits further discussion is revoicing. Revoicing was used with great frequency by all three PTs. Revoicing is perhaps the most interesting of the TDMs because of its versatility. In revoicing, the teacher can draw attention to different types of mathematical activity. And in revoicing, the teacher can draw attention to different types of mathematical ability. Herbel-Eisenmann et al. (2013) described revoicing as a way to check in on understanding with students, particularly in the case of full revoicing. Revoicing can also be used to highlight key ideas for the sake of students hearing the idea. Revoicing can also be used to shift status with students by revoicing the thinking of a student who may not speak up all that often. As a result, revoicing can be used exclusively or inclusively for both mathematical activity and ability. It is also unique in being the only TDM (aside from waiting for a response) that does not request an action from students. Revoicing functions as a way for the teacher to assign value to ideas and people.

Checking-in moves included checking for agreement, checking on progress, and checking for understanding. In a way, checking-in operated similarly to wait time in that it provided a moment for students to speak up if there was an issue. Checking on agreement suggested that students were active thinkers collaborating on the task. Some checking-in had to do with gauging students in connection with progress - "how is it going so far?" But what was curious was that checking for understanding remained at a surface level. After asking "does that make

sense?" or "does everybody get that?," the PTs never pressed for evidence of understanding. Had the PTs pressed for evidence of what the students were understanding, then checking for understanding could be considered much more inclusive than it was in this study. Any checking-in done in this study occurred at a surface level where a student might be able to get away with just nodding in response.

Guiding student thinking moves were moves in which the PT was guiding student thinking. In this study, the guiding student thinking moves included the various math nudges and affirmations of correctness. What also stands out above guiding moves is that the teacher is communicating without an expected response from students - math nudges do not require a question to be asked. The different types of math nudges observed in this study included: providing guidance, revisiting the language of the task, cueing, asking "are you sure?," redirection, providing a summary explanation, presenting a connection, and providing a procedural explanation. Some of these same moves were noted by Ellis et al. (2019). Guiding moves vary in how much they support student reasoning in that these moves vary from general guidance to very specific information provided by the teacher. If the teacher is providing or funneling students to think a certain way, then student thinking is not being supported.

Patterns of Talk Moves

Most of this study focused on individual talk moves, but there was some attention given to patterns of talk moves used together by the PTs. The observed combinations of moves used by the PTs included: creating opportunities: probing, revoicing: inviting thinking, revoicing: probing, revoicing: invite identify, check-in: creating opportunities, and math nudge: inviting moves. Some of the patterns may be due to the nature of the move itself, as in the case of revoicing since revoicing does not necessarily merit a response from students. Each of the PTs

had what could be considered a signature pairing of moves – two moves used together more frequently than by their peers. For Jackson it was revoicing: invite identify, for Harry it was check-in: creating opportunities, and for Willow it was creating opportunities: probing. Each PT had their own pattern of paired talk moves that supported student thinking during rehearsals.

Talk Moves and the Framing of Mathematical Activity and Ability

Another important aspect of this study involved examining teacher talk moves for their potential to support student thinking from an inclusive perspective. Ellis et al. (2019) also considered teacher moves for their potential for supporting student reasoning by presenting a model placing the moves on continuums that vary in their potential for supporting student reasoning. In this study, the potential of teacher talk moves were examined through the lens of the inclusive framing of mathematical activity and ability.

The moves that most inclusively framed mathematical activity in this study were the moves that presented opportunities for students to share their thinking. Those moves were inviting thinking, asking probing questions, and creating opportunities to engage with the thinking of another. These three moves have the common function of eliciting student thinking. Inviting thinking moves are the most general form of inviting thinking as this is simply an open-ended question that provides an opportunity for students to share their reasoning. Probing shifts the focus of the thinking to one particular student's idea (or group's idea) that has been communicated. Fraivillig et al. (1999) categorized probing as a teaching technique that lies at the intersection of eliciting and supporting. The teacher is eliciting student reasoning while simultaneously supporting the initial student's thinking by deeming it worth delving deeper into. And creating opportunities involves asking others to engage with a student's communicated idea. Hence, these three moves invite students to share their mathematical reasoning.

The moves noted that framed mathematical ability inclusively in this study were the moves that positioned a student as a resource for their peers. These moves were student revoicing, creating opportunities, and summative revoicing. Student revoicing positions a student to make sense of another student's thinking and explain their thinking out loud (Ellis et al., 2019; Fraivillig et al., 1999; Herbel-Eisenmann et al., 2013). So, by asking for a student's thinking to be revoiced, that student is being named as a resource for their peers. Creating opportunities included both instances when the PT asked students to chat among themselves about a specific question and instances when students were asked to engage with the specific thinking of another student. As both are positioning students to learn from and with each other, creating opportunities to engage with another's reasoning was considered to promote the idea of all students having intellectual contributions. Since summative revoicing is a newly documented move, it is discussed below in more detail.

Summative revoicing was a move that *assigned credit* and was implemented by two of the PTs. What made this move stand out from revoicing was that the initial student thinking that was being revoiced occurred several talk turns prior. It was not an immediate revoicing. Additionally, what made this move special was that the student was named while being given credit for their thinking. This strongly connects to what Fraivillig et al. (1999) named "assigning ownership." They observed one teacher who "employed a teaching technique that promoted the elicitation process by giving individual students credit for ownership of their ideas" (p. 165). What is key about summative revoicing is that the student is named in connection with their problem-solving approach and the combined revoicing and naming is not immediate. Summative revoicing is public and specific, which sounds similar to assigning competence but without any evaluation. Cohen & Lotan (2014) elucidated that in assigning competence,

"evaluations are specific and refer to particular intellectual abilities or skills" (p. 158). So, assigning ownership is slightly different from assigning competence in that the statement can be purely descriptive, and the statement does not have to be about a skill or ability. Furthermore, the delayed nature of summative revoicing also separates it from assigning competence and assigning ownership. There is understandably more weight given to an idea that a teacher restates again later in a discussion. In summative revoicing, the student's idea is summarized, and the student is given credit for the idea in a delayed revoicing. In the rehearsals, this was typically done near the end of a rehearsal as a way of revisiting ideas mentioned during the learning activity. And in focusing specifically on assigning ownership, the teacher is naming the student as a resource for their peers, which is one of the main descriptors in framing mathematical ability inclusively (Louie, 2017). Summative revoicing positions the student as an intellectual contributor by naming the student and the contribution.

Patterns of Talk Moves and Inclusive Framing

In general, the paired talk moves used by the PTs probably do not provide insights related to the inclusive framing practices of the PTs but highlighted the idea that individual moves do not tell the whole picture. For example, Willow was the only PT that asked a probing question immediately after creating opportunities for students to engage with the thinking of another. Creating opportunities for students to engage with the reasoning of another was categorized as inclusive framing of mathematical activity and ability. In the analysis of individual moves, the probing move would not have been credited for also framing ability inclusively and yet because the probing question was about another student's work, certainly that particular combination of moves is more powerful at framing mathematical activity and ability inclusively than it was originally coded.

Summary

The PTs' talk moves in rehearsals were analyzed to make sense of how novice educators might support student thinking. They supported student thinking by eliciting student thinking, allowing time for students to respond, inviting students to participate in other ways, valuing student thinking, checking-in on student thinking, and guiding student thinking. Their implemented moves were then analyzed using Louie's (2017) framework to gain insights on how they are framing mathematical activity and ability. Each PT's utilization of certain moves was considered in creating the framework that considers how the talk moves have the potential to nurture more inclusive learning environments.

Talk moves are challenging to analyze on their own in the sense that intention can vary. In inviting student participation, the goal could be to hear mathematical thinking or the goal could be more social to include someone or even more critical by working to shift status. "Probing may stem from a teacher's genuine desire to know more about the student's thinking, or it could be used to make a student's thinking explicit for the benefit of other students" (Herbel-Eisenmann et al., 2013, p. 183). The moves to support student thinking can be used to highlight mathematical ideas, but they can also be used for social purposes to position students in different ways.

Louie (2017) mentioned that the exclusionary framing of mathematical activity and ability were often linked, and as mentioned above, there is an observable overlap in the moves that frame mathematical activity and ability inclusively. For example, the move that fell into both categories was creating opportunities to engage with the reasoning of another. By asking other students to engage with the reasoning of another, that "another" student is being named as a resource and the students are being asked to make sense of that student's thinking. Creating

opportunities is typically more inclusive in the sense that with student revoicing, only one student is revoicing whereas with creating opportunities, many students may be invited to engage in the reasoning of the other student.

Masqueraded Moves

In studying the PTs' talk moves along with their recorded rehearsals and interviews with the PTs, some intentions were unmasked that are worth discussing. Louie (2017) pointed out that there must be a culture established for teacher moves to be effective. If the classroom culture does not support inclusive framing, teachers implementing moves designed to elicit student thinking might be met with one-word responses from students. In this study, the talk moves were coded for how they were presented by the teacher, but there were a couple of interesting moves where there was an observed mismatch of intention.

During interviews with Harry, his responses suggested that he was avoiding explicitly stating that a student was correct. Harry specifically disclosed that he "said that that's a really good idea because I didn't want to tell him that that was right." And near the end of the Discussion rehearsal, Harry explained that he told his students "I'd like to say that the table does work" because he realized that he "never said that Willow was correct. So, I wanted to leave it as open-ended as possible and I wanted to also tie in together that Willow's idea is correct." While Louie's framework does point out that an over-attentiveness to answers being correct can be experienced as exclusionary by students, it was almost as if he was avoiding stating anything was correct. By working so hard to not overvalue correctness, it was as if he was hesitant to say anything was correct at all.

There was another incident with Harry that was particularly interesting in that during the rehearsal, he told his students what to do in the form of a question. Teachers wanting to help

their students by telling is a common struggle (e.g., Chazan & Ball, 1999; Philipp, 2007; Stein et al., 2008). After initially revoicing his students' communication about struggling with two variables, Harry then asked, "so what you're saying is that we need to put them in one variable or it would be easier to solve this equation if we only had one variable?" So, he masqueraded that it was *his* idea by putting the suggestion of using one variable in question form. Later it became apparent that Harry was unaware that he did this. During the post-rehearsal debrief, a peer asked, "is it okay to just say you need one variable? Like that would be easier." Harry responded by claiming that his students told him that they needed one variable. And then elaborated that he "probably wouldn't have just gone out and said that they need one variable." Harry's leading question tricked everyone but his one peer who asked if that was an okay thing to do. Yet it is important to recognize that a suggestion in the form of a question does not make it any less of a suggestion.

Implications from Talk Moves

The PTs most used moves were inviting identify, revoicing, and checking-in. These were also the moves that were considered slightly inclusive in their use by the PTs. Inviting identify moves include closed-ended questions and thus do not leave much room to broaden what counts as mathematical activity. Revoicing values what is said, but it depends on what is said and whether the student is being credited for that thinking. Checking-in was done at a surface level. These three moves could be paraphrased by function to be similar to: what are you seeing?, here is what I am hearing, are you understanding? There is no ideal formula for how students should be participating in a mathematics classroom, and yet there certainly is potential to frame mathematical activity and ability more inclusively.

The masqueraded talk moves were presented to highlight potential issues. Harry had some deep-rooted understanding that he needed to not focus on correctness, which is wonderful, and yet there certainly needs to be balanced. Students do sometimes need to know when they are correct. From another perspective, there is also potential for students to still pick up on the valuing of correctness if Harry were to always respond to a correctly solved for expression with "good idea." In another masqueraded move, Harry was unaware that he had presented his own idea in the form of a question. That occurrence is worth noting since that is exactly the type of talk move we want to help future teachers to avoid. Research already suggests that students do not always hear things how teachers intend them to be heard (Davies & Harré, 1990; Louie, 2017). It is important to be aware of what might be PTs' natural inclinations to disguise moves so that TEs are prepared to address those inclinations.

Considering ways to reinforce and encourage desirable PT moves is even more important than being aware of masqueraded moves. Overall, the PTs in this group were more successful at inclusive framing of mathematical activity than ability. However, encouraging PTs to hold students more accountable to justifying would improve the PTs' inclusive framing of mathematical activity. Ellis et al. (2019) named pressing for justification as one of the ways to extend student reasoning. During the rehearsals, PTs appeared satisfied with students nodding their heads or asking a question in response to a checking-in move. To frame mathematical activity more inclusively, hearing student justifications in connection with what made sense would better support student thinking. Pressing for understanding would push students to engage in more authentic sense-making.

Encouraging more assigning credit is one way that PTs may be more successful in inclusively framing students' mathematical ability. There were two ways the PTs assigned credit

to students during rehearsals, through summative revoicing and by revoicing while also naming the student. Publicly naming a student's contribution has been documented to address status imbalances within the class (Cohen et al., 1999) and serves to acknowledge the different strengths of students. Two of the three PTs in this study executed summative revoicing moves at least once, suggesting that summative revoicing might be easier for novice teachers to execute than other moves that assign value to specific student thinking. The PTs had learned about assigning competence and yet none of the PTs were coded to have assigned competence in this study.

Another way the PTs assigned credit was by doing a combination of revoicing and naming the student. Out of the 72 times PTs revoiced, they also named the student they were revoicing four of those times. Three of those were Jackson saying statements like "Stacy thinks that…" and Willow did so once by naming her student who had communicated trouble finding the minimum. The idea of assigning credit is not new, (Fraivillig et al., 1999) observed one teacher who would name an approach after the student who suggested it and refer to it as that student's method. The idea of assigning credit in this study does not go quite that far but consists of publicly acknowledging the thought and the student. Granted, it certainly would be silly and unnecessary to say a student's name every time a teacher revoiced. Since some of the PTs in this study executed moves such as summative revoicing and revoicing while naming the student, encouraging PTs to assign credit seems likely to help PTs be more successful at framing ability inclusively earlier in their career.

Conceptual Practices and Dispositions

There were many conceptual practices and dispositions discernable among the three PTs. The conceptual practices were organized into categories in connection with facilitation,

encouraging students, centering student thinking, guiding student thinking, and practices related to standards of mathematical practice. Dispositions included habits of thinking regarding students, habits of thinking regarding teaching, and habits of action regarding teaching that were further divided into a focus on student thinking, culture-building, and facilitation. By examining all the practices and dispositions communicated, more can be understood about how the PTs are developing in their roles as educators. Below the categories of practices and dispositions related to each other are discussed.

Facilitation practices and dispositions were concerned with management of students and content. The conceptual practices that fell under this category were fun/engaging, checking-in, transitions, positioning students as resources, and inclusion. The facilitation dispositions included maintaining a high cognitive demand, problem-solving, teacher language matters, and smooth discussion. Additionally, some of the habits of thinking dispositions such as student perspective, time management, and valuing progress also align with others in this category. Harry wanted to be fun/engaging. All the PTs were intentional in checking in with their students to formatively assess where they were in their thinking or if they were understanding. The PTs in this study who expressed concern over transitions and teacher language were realizing that better transitions and clearer language can facilitate rather than hinder student thinking. Willow and Jackson were intentional in positioning students to learn from each other. Inclusion fell into this category in that both Willow and Harry would intentionally call on students to bring them into the conversation. Maintaining a high cognitive demand was important to all three PTs although they all expressed frustration in their efforts to do so. All three PTs wanted to support their students by providing guidance with problem-solving. Harry shared that he preferred to have a smooth discussion. Considering the student perspective connected to facilitation

decisions such as having a student read or letting a student go to the board. Willow's concern with time management also has more to do with facilitation.

Encouraging student practices and culture-building dispositions were about creating a supportive learning environment. The practices that were encouraging for students were responding positively and validating student thinking. The culture-building dispositions included students' authority, community learning experience, and positive reinforcement. Habits of thinking dispositions related to supporting student thinking included making sense of student thinking, viewing students as competent, valuing multiple approaches, and valuing progress. The PTs' intention to respond positively in the moment generally connected to a larger disposition toward positive reinforcement. In addition to validating correct thinking, the PTs' validating student thinking included acknowledging student thinking that would be viewed as incorrect from an outsider, thinking that was not fully developed. The dispositions such as making sense of student thinking and viewing students as competent seemed to play a role in validating student thinking since the PTs saw their students as capable learners and put themselves in their learning shoes to understand why they might think the way that they did. Jackson's valuing progress has some overlap with viewing students as competent in the sense that he valued any progress his students made as progress in a positive direction. The disposition of valuing multiple approaches supported multiple ways of thinking about the same task. With the disposition of students' authority, the PTs were intentional in letting students have input in intellectual discussions. Willow and Jackson were ardent valuers of learning as part of a community. The PTs exhibited evidence of several supportive practices and dispositions.

Centering student thinking practices and dispositions focused attention on student thinking. The centering student thinking practices included students as deciders, eliciting student thinking, not forcing the conversation, and pursuing student thinking. The dispositions that center student thinking included valuing student talk, valuing students understanding, and valuing students discovering. All three PTs were intentional in eliciting student thinking and relatedly they all valued student talk. The focus on sharing student thinking aligns with Fraivillig et al.'s (1999) teacher technique of using students' explanations for lesson's content. Harry's not forcing the conversation sounds similar to Fraivillig et al.'s (1999) "waits for and listens to students' descriptions of solution methods." Willow and Jackson were intentional in letting students pursue their thinking which connects to their disposition of valuing students' discovering. All three PTs also communicated valuing students' understanding.

The PTs exhibited practices and dispositions that were related to guiding student thinking. Guiding student thinking practices included directing attention to key information, clarifying and nudging, managing misconceptions, connecting context, and connecting student work. Dispositions included being reflective of practice and influenced by goals. All three PTs claimed ownership of all these intentions and dispositions. They wanted to direct attention to key information. There were ideas they wanted to clarify or nudge a student toward. Clearing up misconceptions was important to each of them. Under the larger category of extending student reasoning, Fraivillig et al. (1999) further described encouraging reflection as "encourages students to consider and discuss interrelationships among concepts" and "encourages students to analyze, compare, and generalize mathematical concepts" (p. 155). So, connecting context and connecting student work were encouraging reflection. The PTs were often reflective of their

practice in connection with how they guided student thinking, particularly regarding misconceptions. The PTs were all reflecting on their facilitation regarding goals.

The PTs communicated a more mathematical focus in their work by practicing the standards of mathematical practice. Attending to precision and modeling with mathematics were in a separate category to highlight their connections to the Standards for Mathematical Practice, they can also be thought of as guiding practices. Modeling with mathematics supports students to make connections among representations and thus can be thought of as guiding their thinking. Pressing for precision was noted by Ellis et al. (2019) as a low potential teacher move used to extend student reasoning. However, because the PTs pressed for precision in connection with student explanations, their attending to precision was more inclusive because they were supporting students to better communicate their explanations.

Practices and Dispositions Framed Inclusively

The conceptual practices that provided opportunities for students to participate in sense-making included eliciting student thinking, not forcing the conversation, pursuing student thinking, and validating student thinking. These encouraged students to share their thinking, to move forward with a student's idea, and to have their thinking corroborated. Dispositions that supported sense-making included maintaining a high cognitive demand, valuing student talk, students' authority, valuing discovery, making sense of student thinking, and being reflective of their practice. For students to discover, some authority has to be given to them, so the practice of students as deciders combined with the students' authority disposition puts students in a position to take risks and make choices. The PTs were reflective of their practice in ways that generally related to supporting student sense-making.

The practices and dispositions that encouraged a multidimensional math frame included modeling with mathematics, valuing multiple approaches, connecting student work, connecting context, and attending to precision. Modeling with mathematics can be considered another dimension of mathematics on its own. Valuing multiple approaches communicates more dimensionality to mathematics since there is more than one way to think about the task. Connecting student work and connecting context were opportunities given to the students to make draw connections among approaches and concepts. The PTs attended to precision by supporting students sharing their thinking to be more precise in what they were communicating.

There were several practices and dispositions that aligned with framing mathematical ability inclusively. The practices that oriented students to learn from each other included positioning students as resources, and inclusion. Dispositions included community-learning experience, valuing progress, and students as competent. Positioning students as resources orients students to learn from each other. Inclusion involved bringing students into discussions who had not participated recently, thus naming more students as resources for each other. The community learning experience disposition described students as a collaborative community in which they each can contribute. Within that community, student progress was valued and students were viewed as competent.

Summary

The intentions of the PTs were considered to learn more about how they were supporting student thinking. Practices and dispositions related to facilitating instruction and guiding student thinking varied in their opportunities for students. The practices and dispositions that centered student thinking were considered to be inclusive since they positioned student explanations as content to learn from. The practices and dispositions affiliated with responding to student

thinking had more potential to position students as resources for others to learn from, but inclusivity varied in that some just focused on responding with general positivity which does not communicate what is being valued.

Dispositions Meriting Further Discussion

The idea of viewing all student thinking as conceptions instead of labeling some as 'mis'conceptions was one that arose with one PT during this study. Jackson's disposition about students as competent and his disposition about managing students' misconceptions revealed that he did not view his students' thinking as missing a target. During his Monitoring interview, he explained that he had not noticed he had tended to their misconceptions in the moment, that he "was just kind of rolling with what they were thinking on face value." This connects to previous literature that suggests misconceptions are conceptions, constructed ideas, such as is described in Leatham and Winiecke's (2014) discussion of the case of Benny. In a classroom in which mathematical activity is framed inclusively as sense-making, then all student thinking is seen as valuable as part of that sense-making. Viewing student thinking from this perspective frames mathematical activity and ability inclusively.

Harry's desire to have a smooth discussion was the only disposition that appeared exclusionary in nature. During the Launch interview, he shared that he had been hopeful one of his students had no barrier. He was planning to let that student help their peers so that the group discussion would be smooth. During the Monitoring interview, he explained that his idea of how it could be improved would be to shorten it and make it more formulaic. Harry's desire to have a smooth discussion conflicts with the integral role that mathematical argumentation plays in learning (Marchant et al., 2021; NCTM, 2014) and runs contrary to Chazan and Ball's (1999) assertion that the teacher should be supporting an "atmosphere of intellectual ferment." While it

may be that Harry was wanting to avoid argumentation because his own content foundations might not have been flexible enough to respond to varied student responses (Stein et al., 2015), his desire to avoid mathematical argumentation is of concern. Harry seemed to have more of a "doing school" perspective in connection with rehearsals in that his focus seemed to be more on performing than on supporting student thinking, so his actions and dispositions may not be representative of what would occur with actual students. Yet there is concern that in valuing a smooth discussion he may not genuinely value student explanations, particularly ones that might contradict each other and create discussion dissonance.

All three of the PTs desired to maintain a high cognitive demand, but some struggled with how to do this. During the Monitoring rehearsal, Harry admitted that "trying to keep that high cognitive demand without telling them too much and without telling them too little was a lot harder than I thought it was going to be." Many teachers struggle with how to help their students see connections without telling them what to see (e.g., Baxter & Williams, 2010; Chazan & Ball, 1999). Willow also wanted to maintain a high cognitive demand and shared that she sometimes struggled with what to say. Jackson talked about not lowering the demand in connection with his not giving them too much information detail-wise. He shared the temptation to throw variables onto their model during the Launch. All three had the desire to hold back on communicating too much to students so that students could do their own sense-making, but also shared that holding back was more challenging than expected.

Implications

In preparing future teachers, being aware of the types of practices and dispositions novice educators bring into their practice can inform teacher educators of what potential is possible and perhaps highlights other practices and dispositions that might need further or different

attention. It is encouraging that one of the PTs viewed his students' thinking as conceptions as opposed to seeing some as 'mis'conceptions. Thus, there is evidence that future teachers can make progress early in their career toward that asset-focused view of students' thinking. The desire of one of the PTs to have a smooth discussion suggests that the PT was missing the importance of students making sense of each other's thinking. Since one had this disposition, it is quite likely others might as well. So, supporting PTs to value student thinking over their own possible discomfort in connection with the uncertainty of student responses is important. The internal struggle of the PTs to maintain a high cognitive demand suggests that PTs might need more support with how to maintain a high cognitive demand when presented with situations in which they are tempted to provide more assistance than they should. Spending some time specifically on how to respond to student struggles either by discussing vignettes or by acting out specific scenarios might broaden their types of responses and build their confidence in responding.

Limitations

The rehearsals were of the launching, monitoring, and discussion of a task. According to the Louie (2017) framework, "assigning open-ended, nonroutine tasks" is framing mathematical activity inclusively, included in both the sense-making frame and the multidimensional math frame. It makes sense then that the PTs are set up to be more inclusive than if they were teaching the first day of a new unit in a secondary mathematics course. Thus, these representations of their practice are not necessarily representative of what might occur in an everyday math classroom. However, it does also provide a fertile environment for the PTs to practice moves that do support student thinking. So, while these moves are not necessarily

representative of what might occur in the everyday classroom, they are evidence of what these PTs are capable of before entering the classroom as a student teacher.

A limitation of the stimulated recall interviews was that there was a time gap between the rehearsal and the interview, thus the interviews are comprised of whatever the PT thought to say that day in response to the questions. The video does aid the discussion of what happened, but the actual in-the-moment why may not be accurately recalled. It is also possible that important intentions might have been forgotten and instead the PT shared the intention that came to mind while reviewing the video. The practices and dispositions that were derived from the stimulated recall interviews are representative of the PTs, but they likely do not paint the whole picture.

The situated nature of the study also is a limitation of the data collected. The PTs were in a methods course in which the researcher was a graduate assistant. While it is unlikely that their actual talk moves were impacted by that relationship, during interviews their responses were possibly influenced by the responses that would be more in alignment with the expectations related to the methods course.

Suggestions for Future Research

Suggestions for future research involves the idea of presenting talk moves with those two foci of framing mathematical activity and ability inclusively to PTs and observing if that makes a difference. One idea would be to consider the student perspective. As much of the research has to do with teacher moves, it would certainly be beneficial to learn more about how students perceive these same moves. Doing research in which students can reflect on how they experience teacher moves that frame mathematical activity and ability inclusively could help the field understand the importance of these shifts to instruction.

Another suggestion for future research involves doing a focused intervention with PTs. After examining how different moves have different potential to frame mathematical activity and ability, it would be worthwhile to see if PTs could implement more inclusive framing. Ideally after given some tailored instruction related to more intentional naming of students in connection with ideas and holding students more accountable for what it is they claim to understand, there would be more evidence of those moves in action. And yet it is also possible that PTs are only capable of certain leaps this early on in their professional journey. So one suggestion is to study PTs facilitation after presenting certain moves as being stronger at broadening students' understanding of what counts as mathematics and ability to see if it would help PTs be more intentional with how they use particular talk moves.

Closing Remarks

When thinking about how to shift the classroom environment to be more inclusive, working towards the moves that better support opportunities for students to participate in inclusive framing of mathematical activity and ability is important. In connection with the reorganization of talk moves in Figure 17, math teacher educators could highlight the capacity of different talk moves to frame mathematics and mathematical ability inclusively to support PTs to be more intentional in their re-framing. Another approach would be to work towards increasing the noticing of student mathematical competence. Like Jilk's 2016 study, spending some time focusing on noticing students' mathematical strengths could support PTs to respond to student thinking using the multidimensional ability frame.

As approximations of practice, rehearsals are an excellent tool for PTs to refine their practice within their community of practice. Additionally, the iterations of rehearsals allowed the PTs to learn and adjust. For the two PTs who implemented the more inclusive ability moves

of student revoicing and summative revoicing, none of those moves occurred until the second or third rehearsal. Granted, that could be due to the nature of the rehearsal, but it also might just be their practice progressing.

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APPENDICES

Appendix A: Pre/post Questionnaire

What is your name?

What is today's date?

What is mathematics?

What does it mean to learn mathematics?

What does it mean to teach mathematics?

Imagine your best day of teaching mathematics. Describe what it makes it the best day.

What does it mean to be smart in mathematics?

How do you respond to a student when an answer he/she has provided is incorrect?

Do you believe it is important for students to provide mathematical explanations when they have answered a problem correctly? Explain.

Appendix B: Stimulated Recall Interview

Thank you for choosing to participate in this study. This interview is one of three that I would like to use to learn about your experience playing the role of teacher during the in-class rehearsal. As you read in the informed consent that you signed, if at any time you would like to stop, you may do so. Also, your identity will be kept confidential in any reports prepared related to this study.

This is a stimulated recall interview. This means that I will play video clips from the recording of the in-class rehearsal. I will pause the video to ask questions or discuss what we watched in the video with you.

Do you have any questions for me before we begin?

First, I'm going to ask you a couple of questions about what you recall without using any video.

1. What do you remember from the rehearsal?
2. How did you feel about the rehearsal overall?

Now I'm going to ask you about specific instances in the video.

<u>Potential stimulated-recall questions:</u>

1. *Why did you ask that student _____ ?*
2. *What prompted you to choose to _____ ?*
3. *What were you thinking when _____?*
4. *How did you feel when _____?*

Do you have any questions or further thoughts you'd like to share?

Thank you for your participation!

Appendix C: Codebook for Talk Moves

Code family	Code	Code definition	Sample Quote
Waiting	Waiting	• After posing a question or after a student response	• [3 seconds+ of silence]
Inviting	Inviting Do	• A student is invited to actively do something (e.g., Go to the board, etc.)	• "Can someone read the task out loud?"
	Inviting Identify	• A student is invited to identify something (typically a recall question) or to respond to a binary question	• "So, if you see a number on the paper could you tell me what that number is and what it represents?"
	Inviting Think	• Asking a student to explain their thinking	• So, another thing that I want to point out is this equation that your group used. How did you get that?
Creating Opportunities to Engage with the Thinking of Others	Creating Opportunities	• Students asked to consider the reasoning of another student, during the discussion this frequently involved asking students to compare ○ *Note: This included moments when the teacher asked students to discuss unrefined thinking among themselves*	• "Can you explain how this equation would relate to the table?"
Probing	Probing	• Asking a question about a student's presented idea or question ○ Note: It is *in response* to a student's share	• "Why do you think it should be in the middle?" *(in response to a student saying they believe it's in the middle)*
Valuing	Valuing Student Thinking	• Verbal encouragement that is more than "good job" (qualifies a what)	• "So, that's a great point"
	Assigning competence	• Evaluative, specific, and public response	*(no codes)*

	Affirmation of Correctness	• Noting the correctness of a student's response	• "That's true"
Checking In	Checking In	• Typically this is a question that could have a "nod" response, but a question that loosely asks if they are understanding or if they have questions, etc.	• "Does everyone see how she did that? Does anyone have any questions on what numbers are in here versus in here?"
Math Nudge	Math Nudge	• Suggestions/hints - can be in form of question or statement	• "Is there a way that we could write what you have now in terms of just one variable?"
Polite	Politeness/Gratitude	• General pleasantries that have nothing to do with quality of student's contributions	• "Awesome. Thank you."
Revoicing	Revoicing	• Restating or rephrasing student contribution	• "You think it should be in the middle." in response to a student saying, "I think it should be in the middle."
	Student Revoice (Asking students to revoice)	• A student is asked to restate or rephrase	• "Can any of you revoice what Wesley just said?"
	Revoice Summative	• Restates typically not *right* after student contributes… Gives student credit	• "Willow's group came up with the Pythagorean theorem, which I thought was a great solution, um, worked their way through a table but couldn't quite come to an exact answer, which is okay. Then Stacy's group"

Appendix D: Codebook for Research Question 2

Learning Community		
Code	**Definition**	**Example**
Implementing Research	Mentions incorporating (or trying to) strategies learned previously in class	"I remember talking about it in the EdTPA task. Um, that like answering questions with questions is a good thing to do whenever you're trying to get students, or whenever you're trying to scaffold students or get them to explain more of their decisions and ideas."
Peer Feedback	Comments made reflecting on what others had said during the debriefs	"They told me I handled them, all of them. Um, I didn't even notice throughout, I was just kind of rolling with what they were thinking on face value."
Other	Any other reference to the learning community, usually in connection with the TE's pauses	"I felt more comfortable because I knew like if something, like if I got stuck, Dr Edgington would be there to like, say pause, and like help me get back on track. So that helped a lot."
Practices		
Checking Agreement	Checking to see if everyone is "on same page," if they have questions about something, or if the PT understood what the student was saying (e.g. revoicing for clarification)	"Just to make sure everyone was on the same page and if someone thought that it should be drawn differently…"
Connecting Context	Intentionally guiding students to make connections with the context [zip line, island, etc.] of the task	"So that was the whole point of drawing that picture is to put everything in contact in context with numbers"
Eliciting student thinking	Stated intent of hearing student share out loud	"just to hear her idea of how she thought the picture should be drawn."
Fun/engage	Intentions to increase engagement or fun	"I was just trying to, I guess, get them interested in the activity. So, ziplining seems fun."
Guiding Connecting Student work	Helping students see connections among their representations (during the Discussion rehearsal only)	"So how do we get from Stacy's work to Wesley's group work? And we say, okay, Wesley used the equation that Stacy's group came up with, you know, independently, but it's the same equation. Then it kind of relates all the answers together, or at least those two answers."

Code	Definition	Example
Guiding Clarification	Responding to students with a statement/question with the intent of clarifying	"Just to reiterate. Okay, if this side's 375 and this side's 225, that's the exact placement of the island no matter which bank you look at"
Guiding general approach	Sharing strategies with students that were not specific to this problem	"I think getting a visual would probably help everyone in this task. This is a very hard task to try to think about in your head without drawing a picture."
Inclusion expectations	Explicit intention of including another student(s)	"I was trying to get them back into the conversation. They had been quiet for a while. "
Managing Misconceptions	Statements regarding working toward eliminating barriers or addressing vocalized student misconceptions	"...so I was trying to get to the point where I could address his barrier."
Modeling	Using representations/ diagrams to aid in sense making	"So, you know, we were trying to figure out, okay, where is your X in your diagram? How does that relate to how or what you're saying?"
Positioning students as resources for each other	Stated intent of orienting students to each other (work in group or present strategy) or supporting an instance when students naturally oriented to each other	"So Stacy goes on a roll here and starts like actually just teaching Wesley. It's funny"
Positively responding/reacting	Stated use of positive reinforcement (or similar verbiage)	"That was an "ooh" as in "That's a good question, Willow" Ooh."
Pursuing Student Thinking	Following student line of thought via probing or supporting them to discover on their own	"So, Wesley says he doesn't think it's the best idea. So, I go "alright." You know, I'm going to probably let Wesley do his own thing. "
Students as Deciders	Intention communicated for students to have some authority to make decisions	"It's not me telling them anything. It's them all coming up with their answers and their reasons."
Tending to Precision	Pushing students to be more precise	I: Why did you ask him the total what? PT: "He wasn't specific enough."
Validating student thinking	Stating that a student's response is "valid"	I: "What was it that you were saying makes sense?" PT: "So, the island's gonna be in between the two towers - that makes sense."
Transitions	Intentional attention to connecting sections of class or different student works.	"… find equation but didn't have a minimum. She kept saying that over and over. And so then, I knew in Wesley's solution, they did find a minimum. So, I wanted it to kind of segue that with, okay, tell us about the problem. What problem did you have?"
Dispositions		

Code	Definition	Example
Habits of Thinking Regarding Students		
Competent students	Talking about students from an "asset" viewpoint (as opposed to deficit)	"It's okay that they didn't come up with an exact answer because they still came up with a good method of solving the task"
Making sense of student thinking	Working towards gathering information from students with the intent of trying to understand student's thinking	I: why repeat what he said? TC: "Because I was trying to figure out what he meant by it doesn't matter."
Relating Empathetically with Students	Empathizing with the perspective of a student	"Because as a student, sometimes when a teacher gets too involved with a single student going back and forth asking questions, I lose interest."
Habits of Thinking Regarding Teaching		
Reflective of Practice	Statements where the TC mentions wishing they had done something differently or what they might do next time.	"Well I should've just asked him. I should've just been ok, what do you mean? Like, are you confused about blah blah, blah or are you confused about this instead?"
Valuing Progress	Statements valuing student progress of any nature (some progress valued included exploring options that did not work)	"I was like, Oh sweet, someone thinks it's similar triangles that leads to proportionality. I was like, cool, that's a, that's a good step. They can make that connection."
Valuing Multiple Approaches	Statements mentioning the PT's opinion of the varied approaches students can take to solve the task	" And so like, it might not be the most, like it might not be the fastest and most efficient way, but I mean, it is effective. Like you can still get a value from it. Like you can still get the right answer. But yeah, she was right in saying that it isn't the most efficient because it's not."
Time management awareness	Concern regarding time in connection with the facilitation of the task	I guess I also kind of felt like we had been like going over this kind of stuff for a while. So I kinda just wanted to like get to the actual task too.
Goal/Success	Statements that describe what they were aiming to do or what they did that they felt good about	"in the launch, I didn't feel like I had gotten students to where they needed to be to solve the problem using minimization and maximization concepts. So this time I wanted to make sure students would be able to complete the task."
Habits of Action Regarding Teaching		
Focusing on Student Thinking Dispositions		
Valuing Student Talk	The PT mentioning wanting to hear from students - this included students talking to each other.	"then I wanted to start getting a group conversation about like, well Wesley doesn't think proportions are the best way."
Code	**Definition**	**Example**

Valuing Student Understanding	Communicating that an answer alone is not enough… that his intent in pressing for more is about students understanding	"so Stacy kind of explained the 600 minus X a little bit, but I wanted everybody to have a good foundation of like what actually is in their pictures. And I asked Kayla. Okay, I see there are other numbers in your picture. Can you explain that to me? You know, what do these other numbers mean?"
Valuing Student Discovery	Communicates desire for students to explore to build understandings	"So my position is if a student thinks they want to do something, just let them try it. "
Culture Building Dispositions		
Collaborative/class is "we"	Emphasis on students as a collective whether the focus is specifically on collaborating or just the class as "we"	"So, like I should not be the only one teaching the class. It should be other people as well, like helping out. It's a collaborative effort."
Student Authority	Valuing students having some ownership.	"If they had gone with the curve, I probably would have let them do it. And then just see like, all right, if it's curved, go with it and just see if you can answer the problem or not. "
Positive Reinforcement (Importance of)	Stated beliefs in relation to positive reinforcement	"As much positive reinforcement as I can work into a classroom, I try it. So I think if I try to keep my attitude positive and and keep them, I guess, engaged by using positive reinforcement, I think that's good for class morale"
Facilitation Dispositions		
Maintain high cognitive demand	Not wanting to lessen the mathematical potential of the task. This included statements about not providing too much help to students.	"I didn't want to give them the information. And.. It kind of limits the critical thinking of the whole task if you just give them the picture and like exactly say like this bottom part is X"
Problem Solving	Statements about problem solving that were not specific to their students' approach and not in connection with a specific action. This included discussion of model drawing, listing key info, and contextual connections.	"I found that, especially when we did the exercise first, having a basis of what we should be looking at is imperative to like understanding the question. 'Cause like originally at first, I know like some groups had the picture drawn, not incorrectly per se, but drawn in a different way that would've gotten them to a different answer than the task was supposed to focus on. Um, so, like having that basis on the board for the whole class to see kind of would help remediate that problem situation."
Smooth Discussion	Mentions the wish for a less messy teaching experience	"I was hoping that if they would've began asking questions about why do you think this, why do you think that, and Wesley would've just told them because he was the one without a barrier."
Code	**Definition**	**Example**

Teacher Language Matters	Communicates wanting to have used different wording to improve clarity for students	"I should've said the island is in the lagoon, where would we like to place the island? Cause then, yeah, 'middle' gives a wrong, gives the students the wrong idea about the island."
Understanding		
Content	Comments that connect to the mathematical content or the context (e.g. minimization, zip lines, island placement, etc.)	I was trying to see who knew in a zip line was so that they could like start to form a shape.
Misconceptions	Comments suggesting an awareness of misconceptions related to this problem (e.g. island in middle)	So I think one of the biggest misconceptions is that it is in the middle.
Similar to My Way	Statements referencing how the PT solved the problem when they initially did the problem in the class	We solved it in a way that was just using the equation. But then right afterwards, we used a table because, um, we heard people say that, uh, you could use a table to do this.
Facilitation_ Anticipating	Statements related to anticipating student thinking	"And I was prepared for that question cause we had talked about that in the launch, in the anticipation of doing the task already"
Framework (Louie, 2017)		
Answer/ solution focus	Statements related to concern about the answer/solution to the task	"So then I would come back later to see if they had taken steps to get the right answer or to solve the task."
Attention to misconception	Statements related to student misconceptions	"Like the first question that I asked after like Stacy's first misconception, I thought was like a good question. But when she didn't really like get the answer that I wanted her to, um, then I kind of struggled"
Doing Mathematics	Statements about the math their students did. During the interviews this was prompted by the question that asked them to describe the math their students got to do	"I guess I kind of got them to think about minimizing a function. And like, uh, the table is a good way to go about that. And that they can/can't do that to minimize a function, but other than that, I don't really know."
Students explaining	PT shares intent related to students explaining	"I asked her to do that because it's important for her to know like what exactly she wrote down on her paper, but it's also going to be beneficial for the class to see a different way of solving something"
Valuing Correctness	Statements mentioning attention to the correctness of student ideas	"So I said that that's a really good idea because I didn't want to tell him that that was right."

Appendix E: Zip Line Task created by Tim Hendrix

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Appendix F: Student Profiles for the Launch Rehearsal

Profiles created by Cyndi Edgington

Student A- Launch Rehearsal

A new amusement park is building a zip line attraction. **The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide.** One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers **to an island in the lagoon**. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Barrier: Student misunderstands the placement of the towers and island in relation to the lagoon.
Strategy: Student draws towers side by side on one side of the lagoon and places the island in the center of the lagoon.

Student B- Launch Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! **How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?**

Barrier: I don't know how to get started. (The student is confused and doesn't know where to even begin)

Strategy: None.

Student C- Launch Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! **How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?**

Barrier: Student doesn't understand the problem situation or the idea of minimization.

Strategy: The island must be in the middle of the lagoon, duh.

Student D- Launch Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. **There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon.** Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is

expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Barrier: Student doesn't understand the problem situation.

Strategy: The zip line should extend from the top of higher tower to the lower tower.

Student E- Launch Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. **But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?**

Barrier: Student doesn't understand what the problem is asking (i.e. understands the context and can re-voice, but doesn't understand the question being asked).

Strategy: Create a table to compute the amount of zip line for each tower depending on where the island is located.

Student F- Launch Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from

each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Barrier: Student is confused about necessary assumptions (i.e. The towers must be perpendicular to the ground; The zip line isn't actually a straight line- there is bend in the wire, so it's impossible to figure out)

Strategy: none

Student G- Launch Rehearsal

A new amusement park is building a **zip line attraction**. The attraction will have two towers on opposite sides of a **man-made lagoon full of alligators**. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by **walking across a long bridge**. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Barrier: The student is confused about the context and is distracted by the non-mathematical aspects of the task- what is an attraction? Why are there alligators?

Strategy: None

Student H- Launch Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. **But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?**

Barrier: The student doesn't understand what minimize means.

Strategy: Find an equation.

Student I- Launch Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Barrier: None. Student understands the context and what the problem is asking.

Strategy: Make a table.

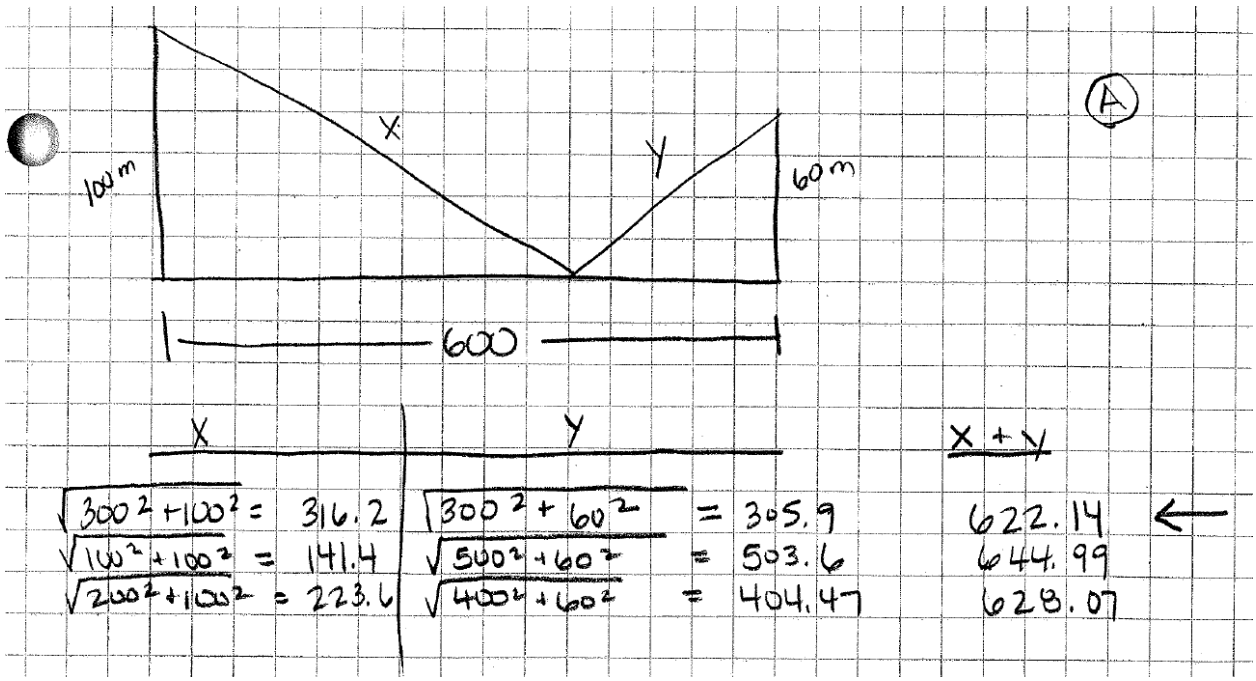
Appendix G: Student Profiles for the Monitoring Rehearsal

Profiles created by Cyndi Edgington

Group A Student A- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Group Strategy: Use Pythagorean Theorem and organize information in a table.
Barrier: Student suggested starting in the middle where x=300 and y=300. Since the middle is the lowest of the three, this must be the minimizing point.

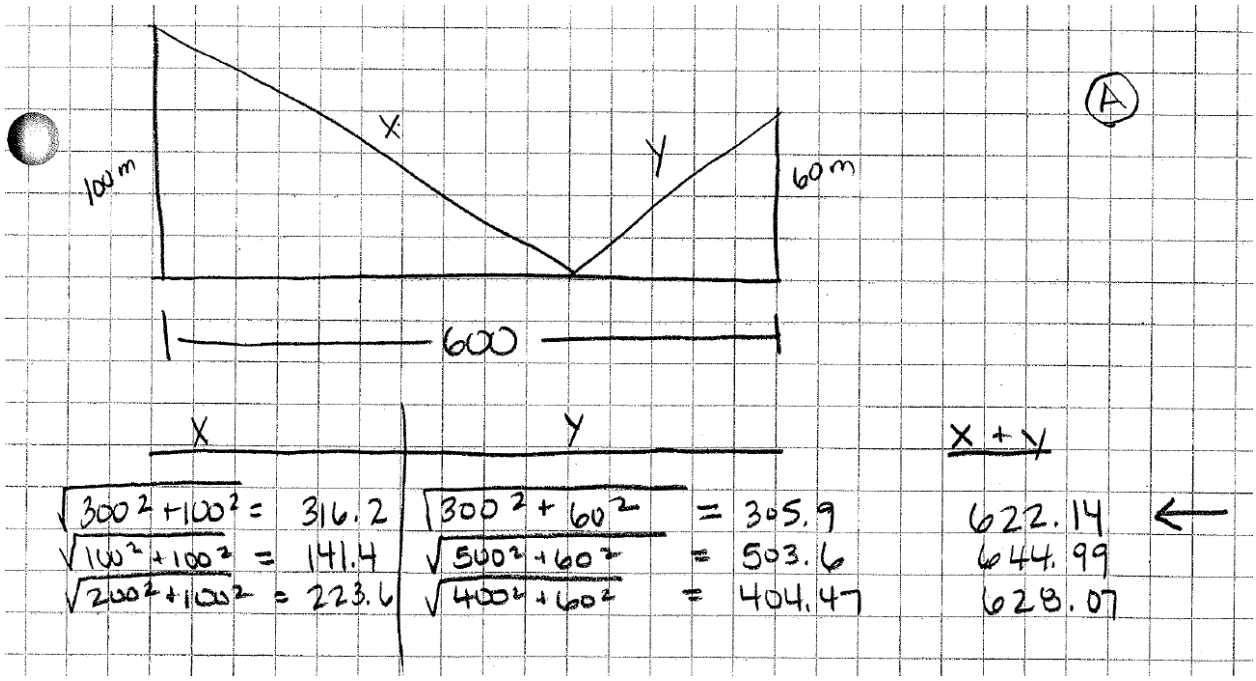


Group A Student B- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from

each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

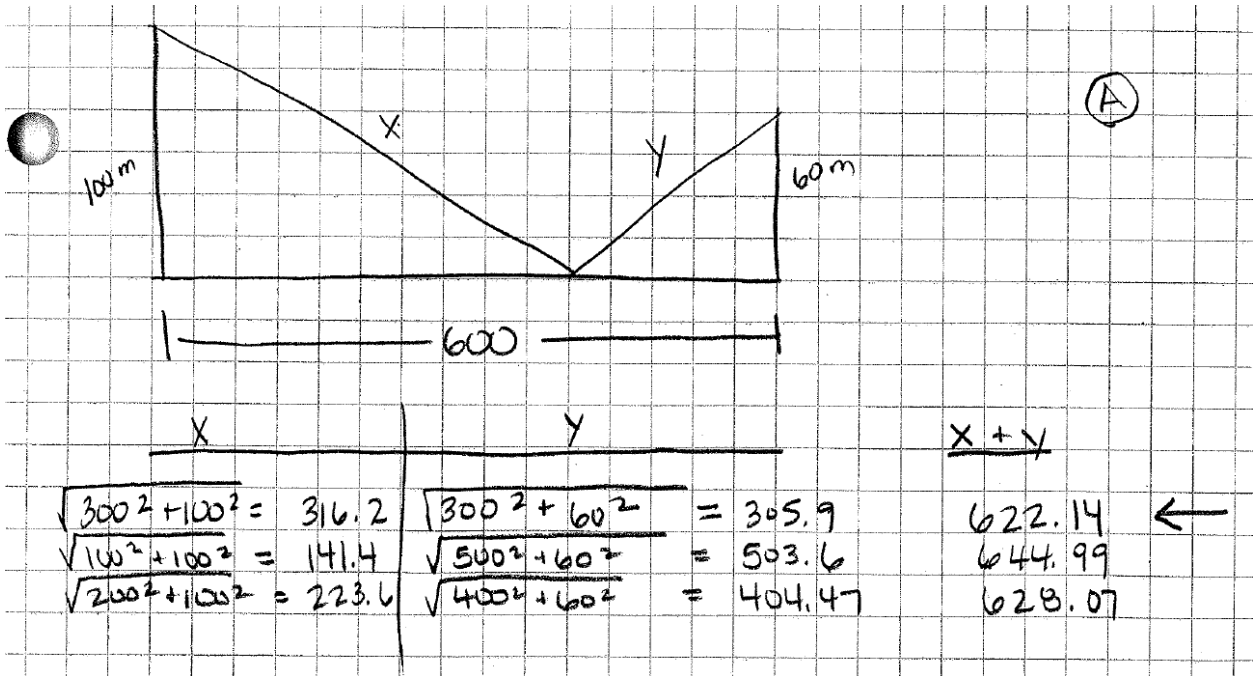
Group Strategy: Use Pythagorean Theorem and organize information in a table.
Barrier: Student understands the guess and check method, but is not content to only use 3 guesses.



Group A Student C- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

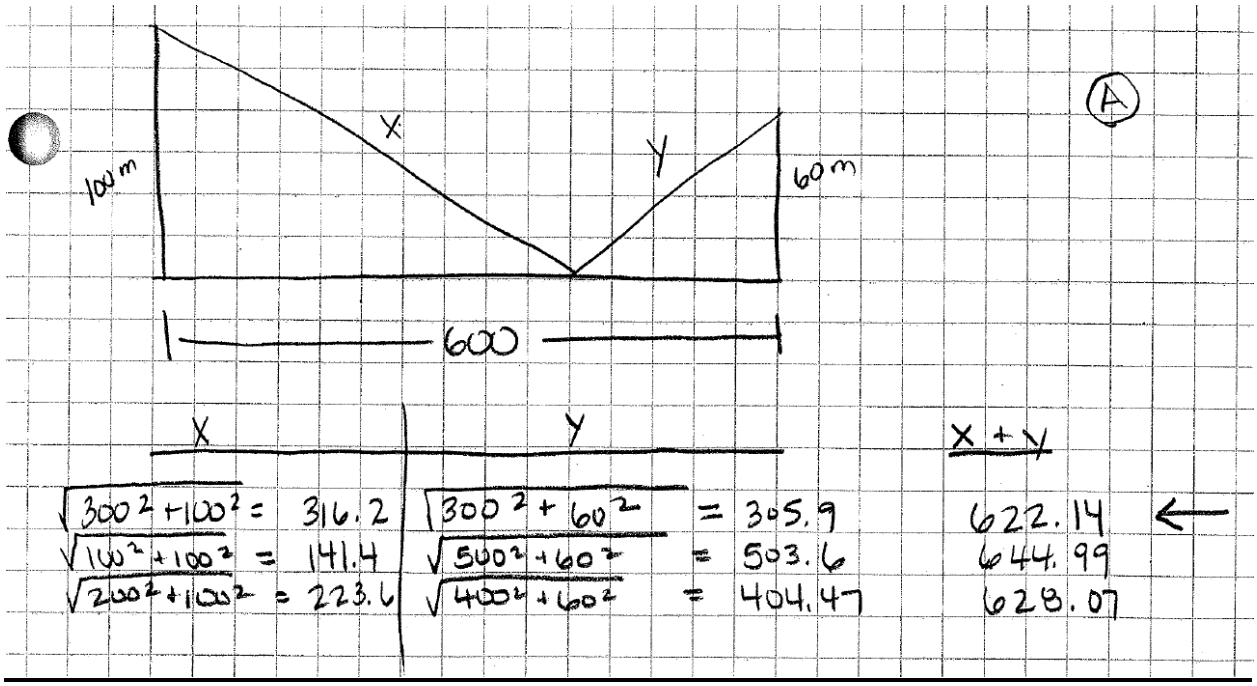
Group Strategy: Use Pythagorean Theorem and organize information in a table.
Barrier: Student understands the guess and check method, but thinks they might be able to determine an equation that will be efficient.



Group A Student D- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Group Strategy: Use Pythagorean Theorem and organize information in a table.
Barrier: Student is unsure of the process used to complete the table.



Group B Student A- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Strategy: None.
Barrier: I don't know how to get started. (The student is confused and doesn't know where to even begin, even though creating a table was mentioned in the Launch)

Group B Student B- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Strategy: None.
Barrier: The student thinks they should use the Pythagorean Theorem, but isn't sure so is afraid to speak up unless prompted by the teacher.

Group B Student C- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Strategy: None.
Barrier: Student has an idea for a diagram, but isn't sure if it's correct. Student won't produce anything unless prompted by the teacher.

Group B Student D- Monitoring Rehearsal

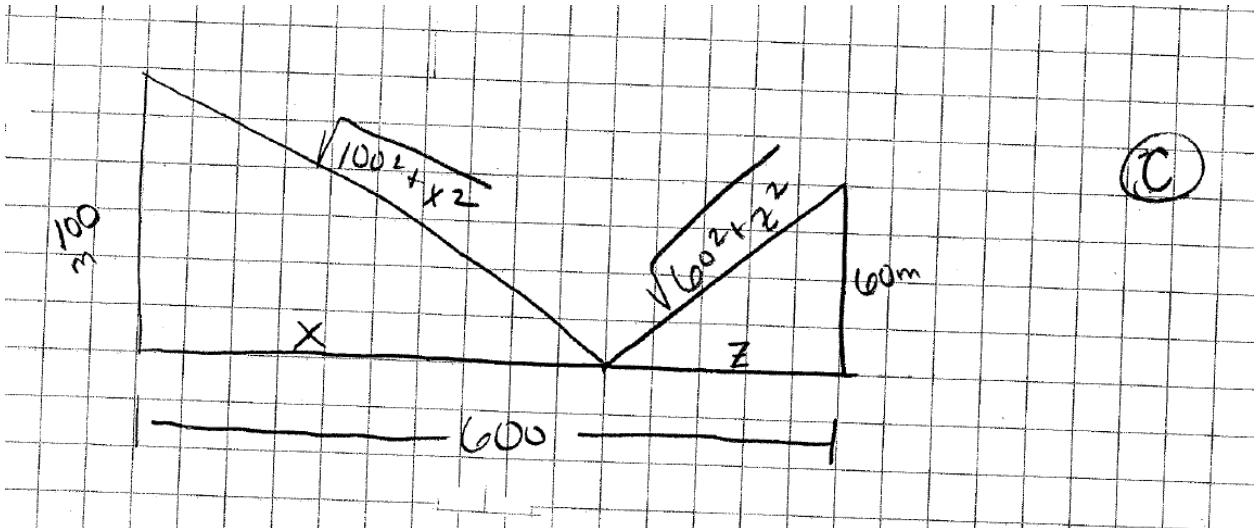
A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Strategy: None.
Barrier: Student has an idea for a diagram, but isn't sure if it's correct. Student won't produce anything unless prompted by the teacher.

Group C Student A- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

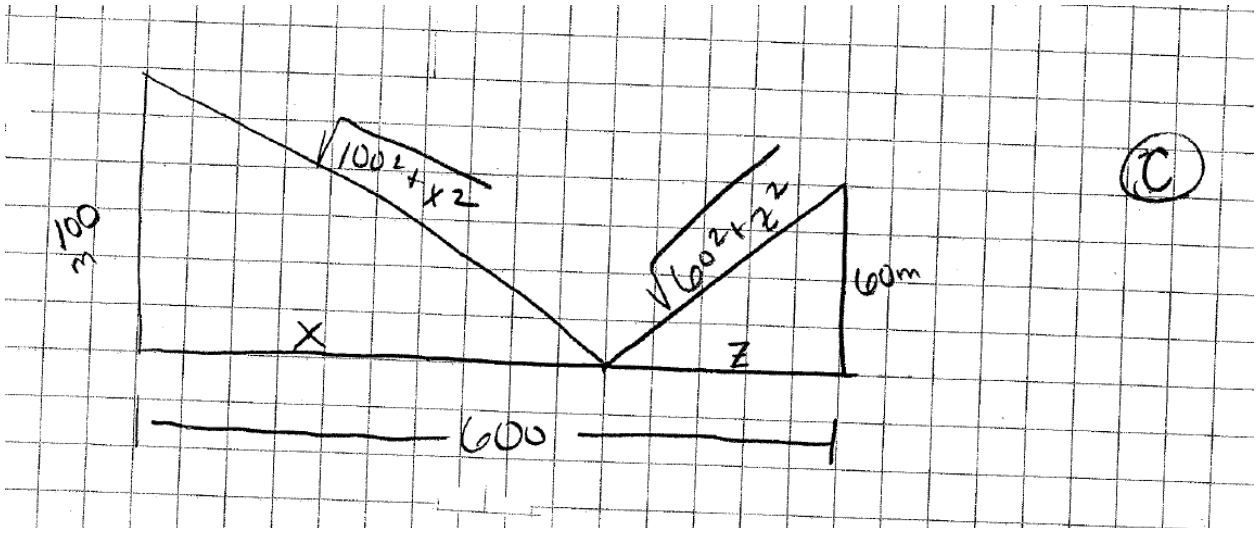
Group Strategy: Use Pythagorean Theorem to write an equation: $y = \sqrt{100^2 + x^2} + \sqrt{60^2 + z^2}$
Barrier: The student knows s/he can write an equation, but doesn't know an expression for each segment along the lagoon.



Group C Student B- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

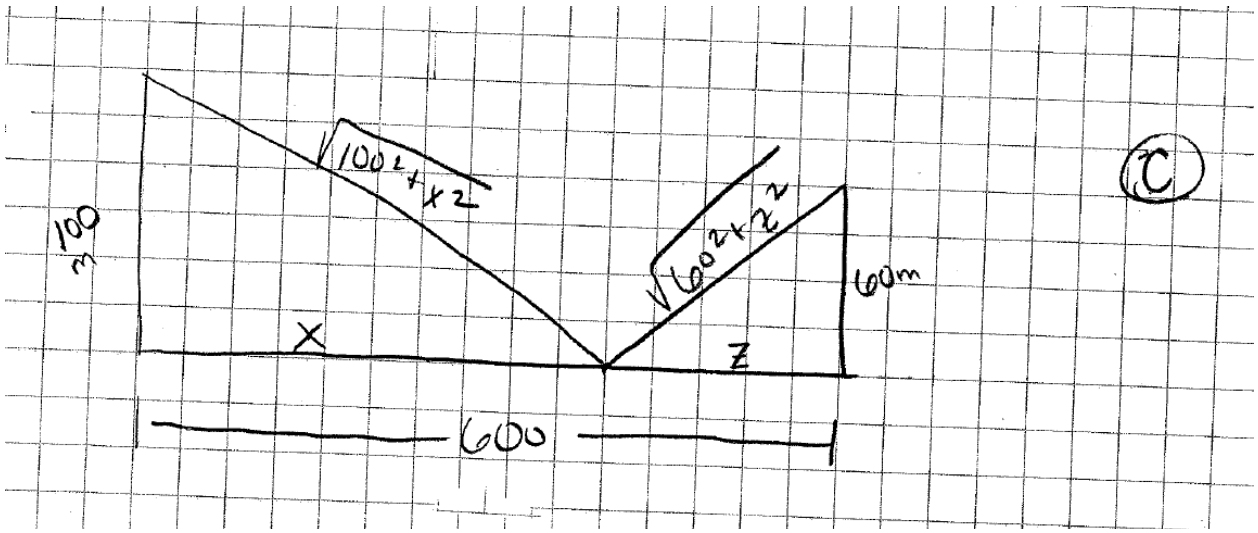
Group Strategy: Use Pythagorean Theorem to write an equation: $y = \sqrt{100^2 + x^2} + \sqrt{60^2 + z^2}$
Barrier: Student thinks they should graph it to find the minimum point, but isn't sure about the equation.



Group C Student C- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

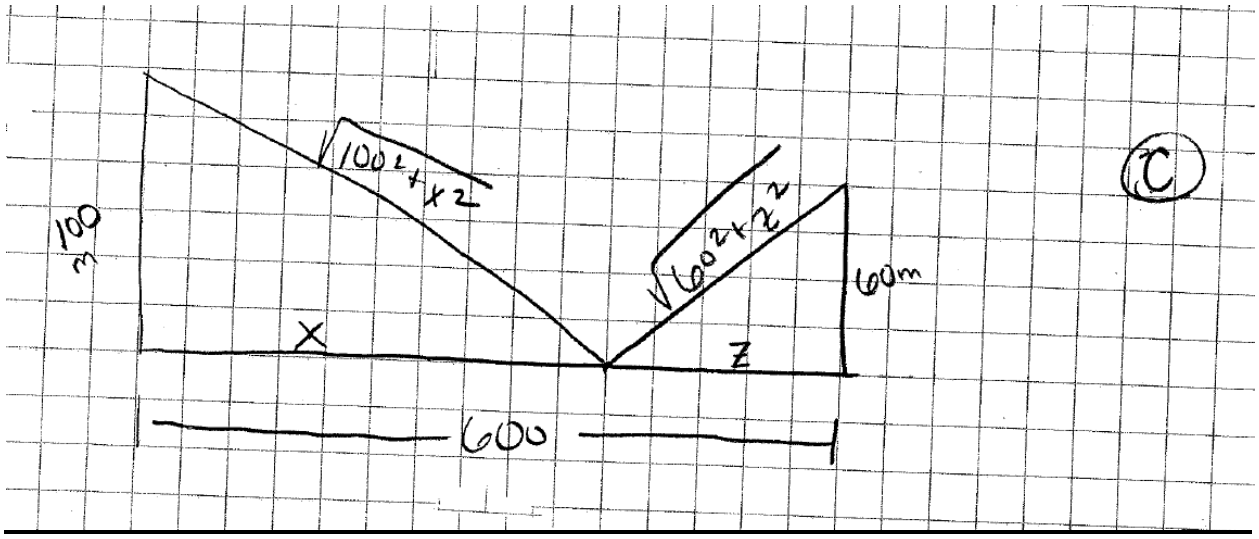
Group Strategy: Use Pythagorean Theorem to write an equation: $y = \sqrt{100^2 + x^2} + \sqrt{60^2 + z^2}$
Barrier: Student doesn't understand how they determined the equation, but is afraid to admit that to the group/teacher. If the teacher asks, student struggles to explain where each part of the equation comes from.



Group C Student D- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

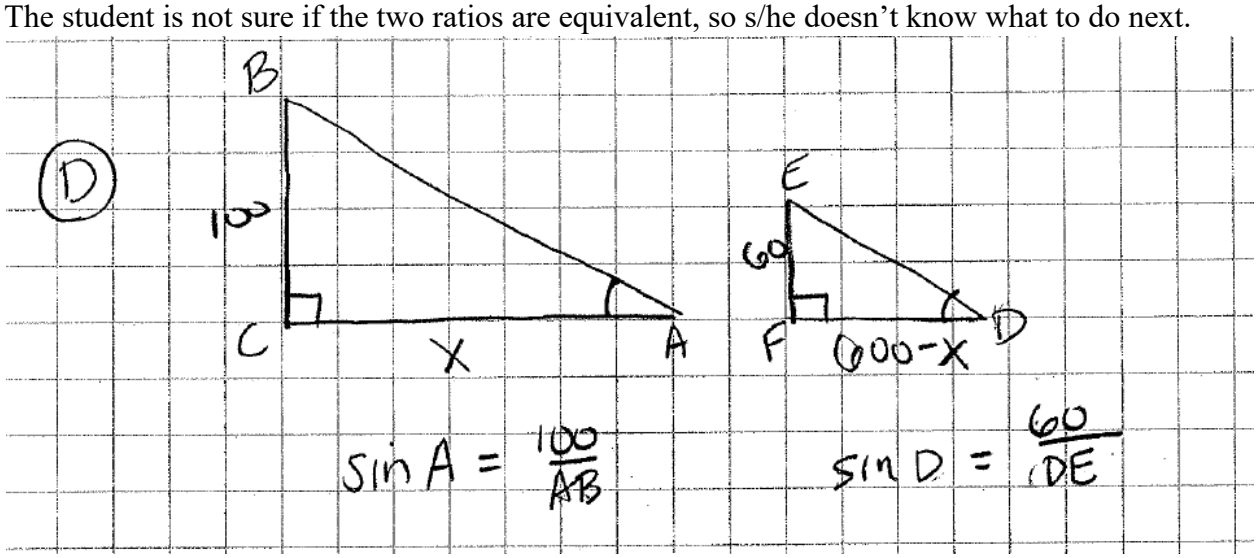
Group Strategy: Use Pythagorean Theorem to write an equation: $y = \sqrt{100^2 + x^2} + \sqrt{60^2 + z^2}$
Barrier: The student knows s/he can write an equation, but doesn't know an expression for each segment along the lagoon.



Group D Student A- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

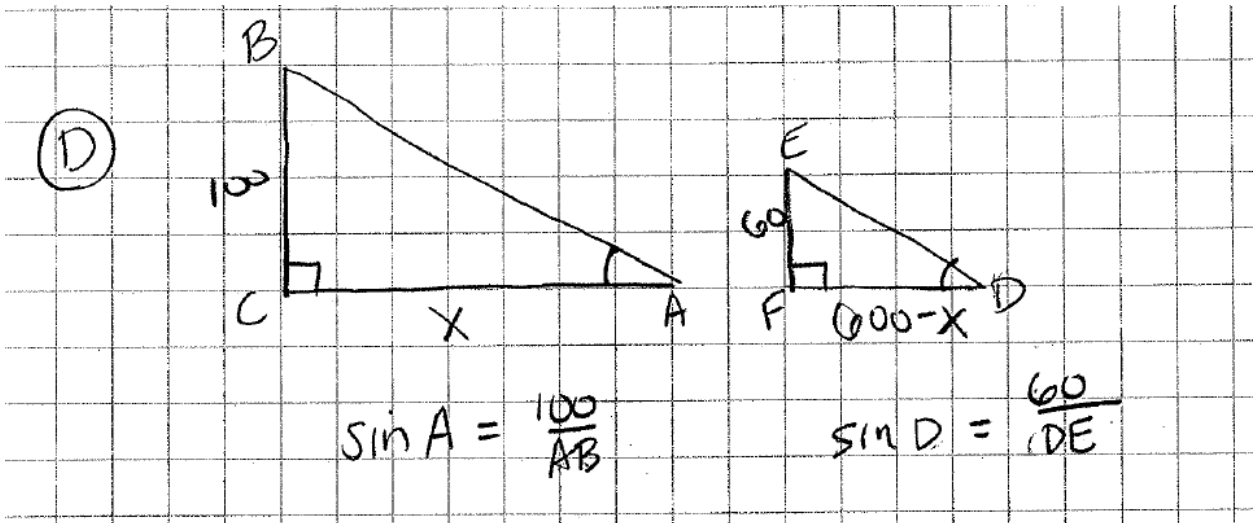
Strategy: Students uses trigonometric ratios.
Barrier: Student sees right triangles, so uses the sine ratio (uses incorrect trigonometric ratio). The student is not sure if the two ratios are equivalent, so s/he doesn't know what to do next.



Group D Student B- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

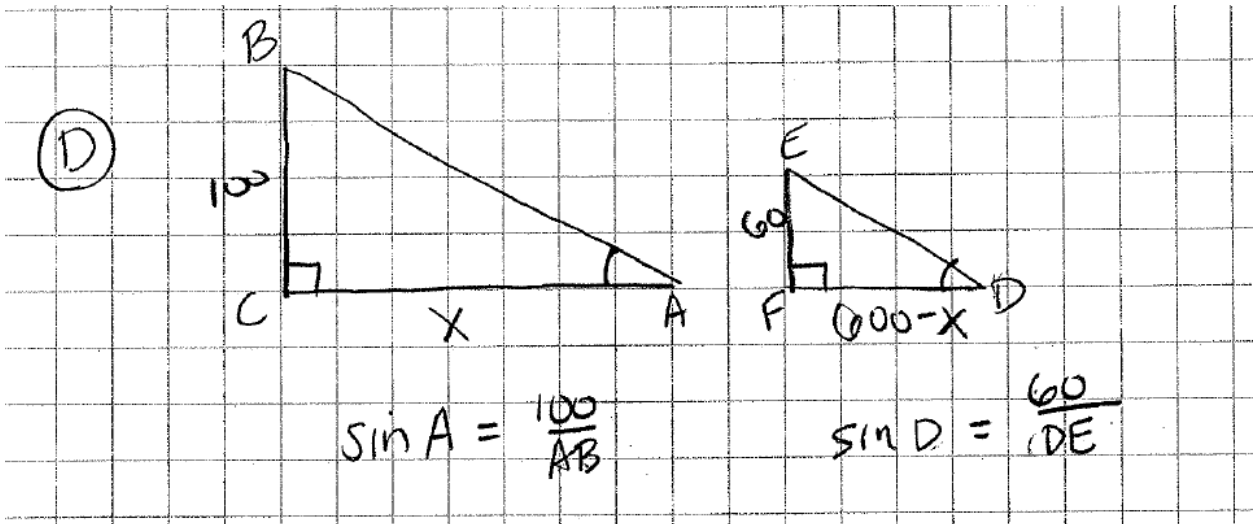
Strategy: Students uses trigonometric ratios.
Barrier: Student sees right triangles but thinks they shouldn't be using the sine ratio.



Group D Student C- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Strategy: Students uses trigonometric ratios.
Barrier: Student sees right triangles, but isn't convinced the angles are equivalent, so student is not sure they can use trig ratios. "Wouldn't we have to prove those angles are the same first?"



Group D Student D- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Strategy: Students uses trigonometric ratios.
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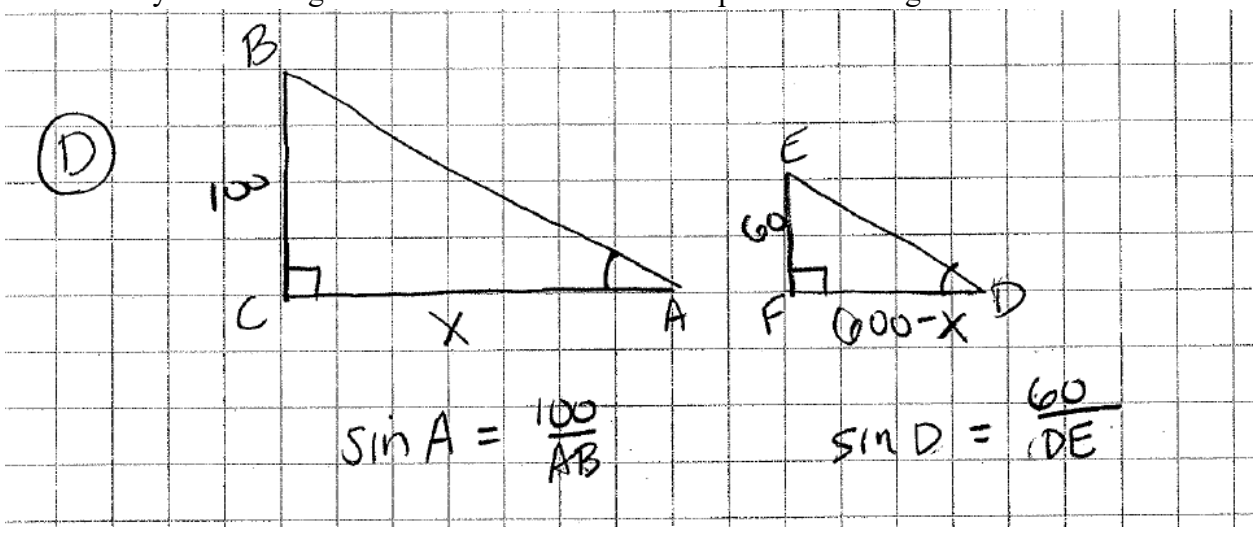
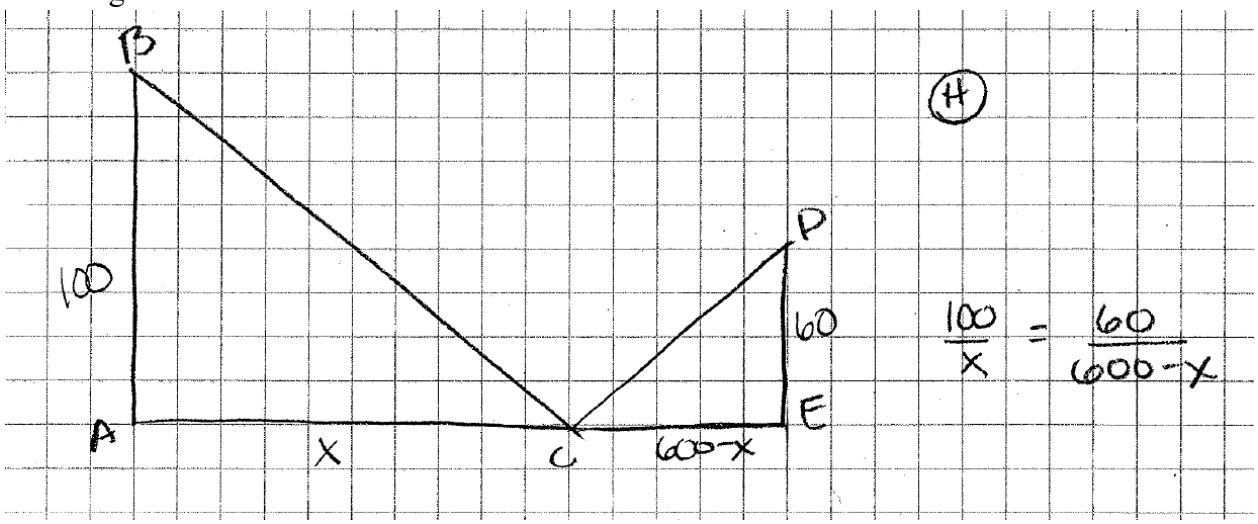


Group E Student A- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

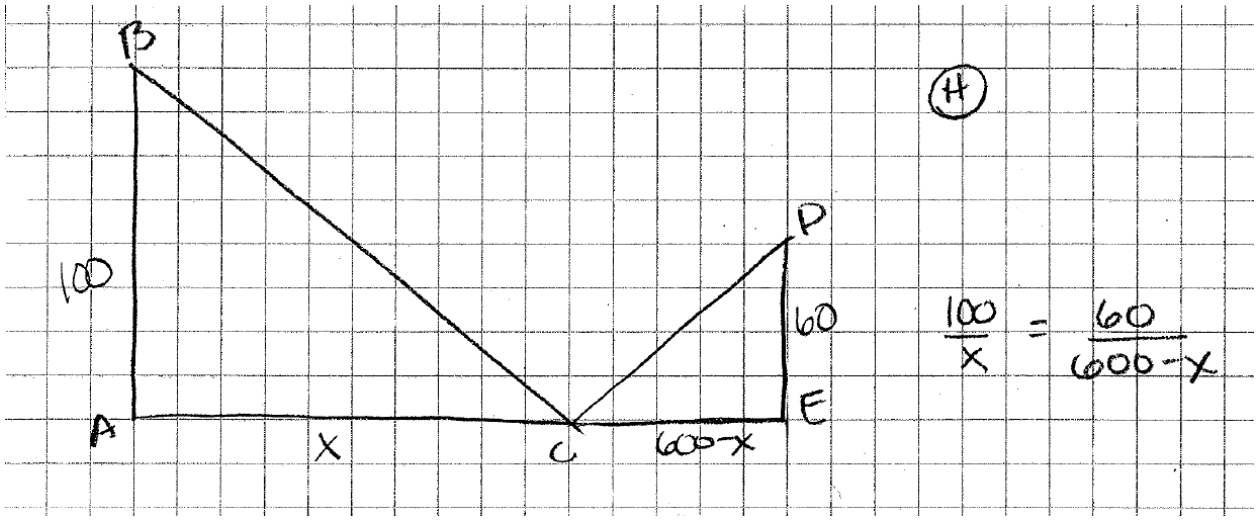
Strategy: The student writes a proportion to solve for the missing side lengths of the triangles.
Barrier: The student assumed the triangles are similar, but doesn't recognize the need to prove the triangles are similar first.



Group E Student B- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

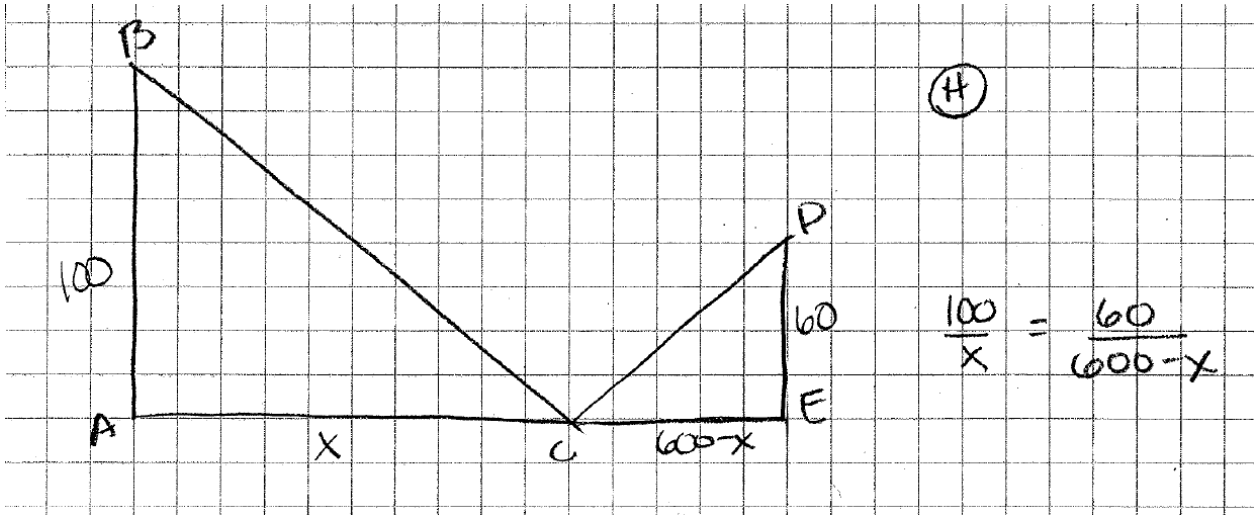
Strategy: The student writes a proportion to solve for the missing side lengths of the triangles.
Barrier: The student isn't convinced the triangles are similar and thinks in order to use a proportion, we have to prove they are similar first.



Group E Student C- Monitoring Rehearsal

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Strategy: The student writes a proportion to solve for the missing side lengths of the triangles.
Barrier: The student assumes the triangles are similar, but doesn't understand how to write the proportion. Is that the only proportion we can write?



Group E Student C- Monitoring Rehearsal

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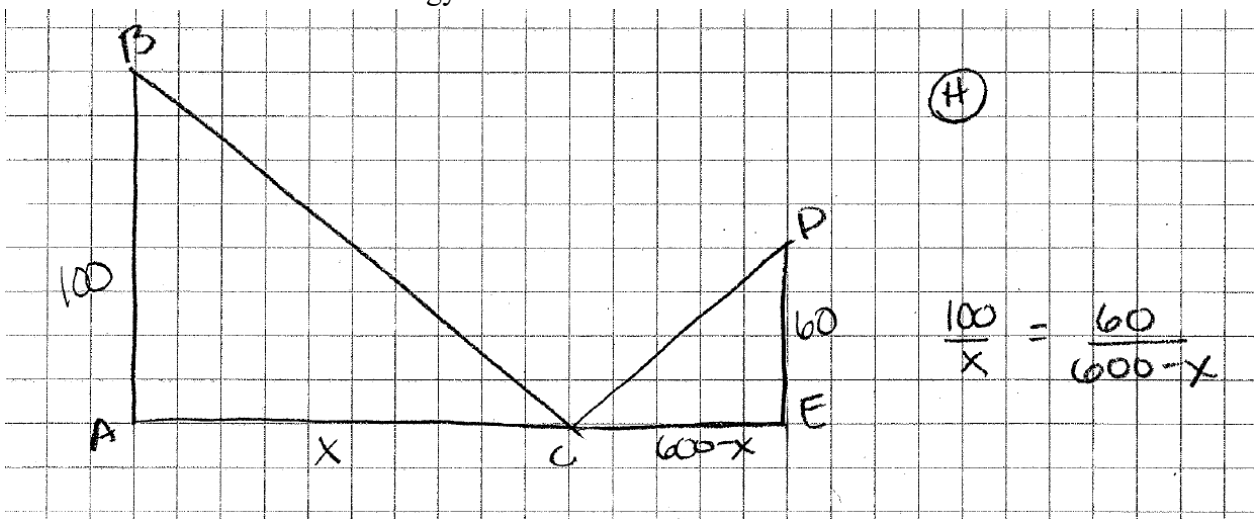
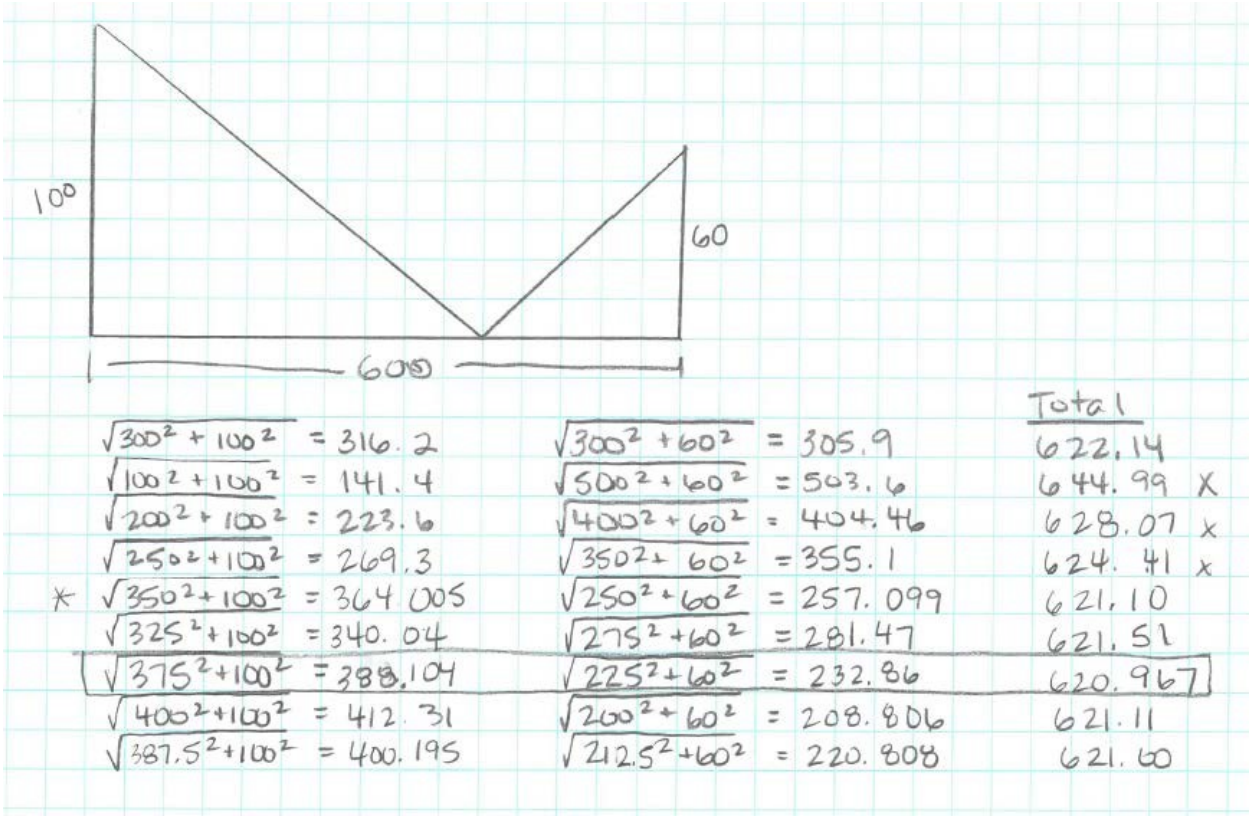
Strategy: The student writes a proportion to solve for the missing side lengths of the triangles.
Barrier: The student doesn't think the triangles are similar, so we shouldn't write a proportion. We need to find a different strategy.



Appendix H: Student Profiles for the Discussion Rehearsal

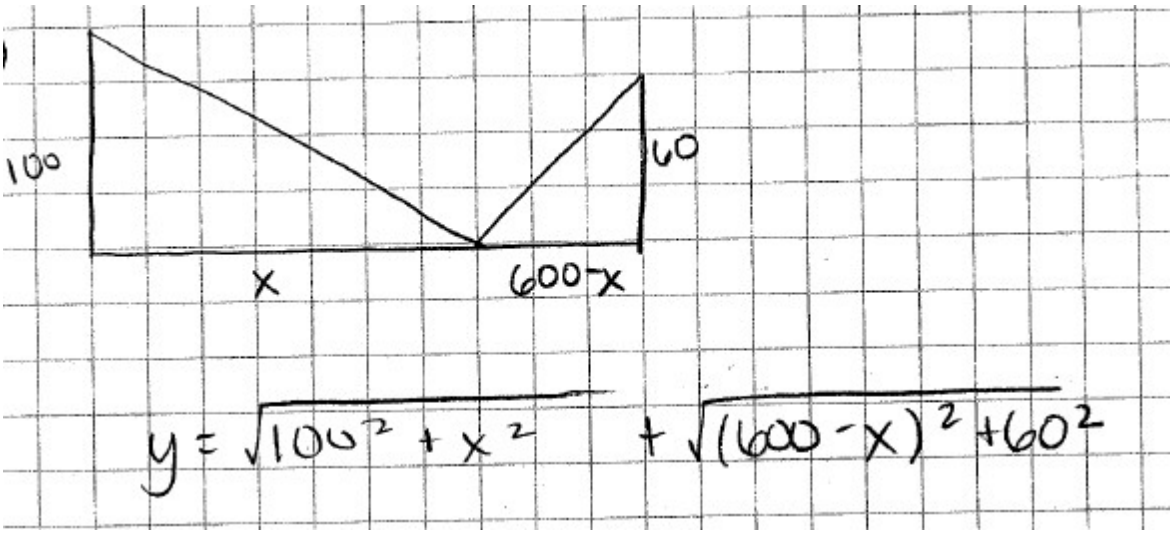
Group 1- Whole Group Discussion Rehearsal
How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Group Strategy: Make a Table
This group was able to find the location of the island and minimum amount of zip line by creating a table using the Pythagorean Theorem. The group used a guess and check method and noticed the amount of total zip line got smaller as the island moved from the middle of the lagoon more towards the 60 m tall tower. They think the island should be placed 225 m from the 60 m tall tower and the minimum amount of zip line is about 620.967 m.



Group 2 – Whole Group Discussion Rehearsal
How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

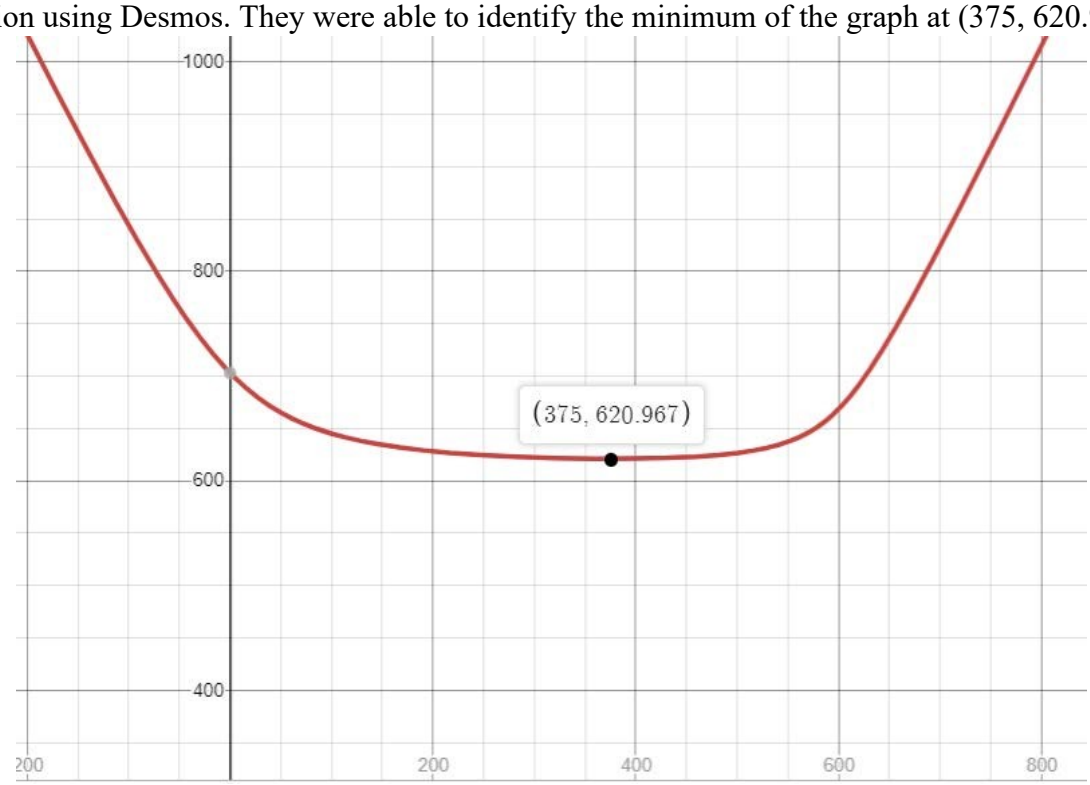
Group Strategy: Write an equation
This group was able to write an equation by using the Pythagorean Theorem twice, but couldn't quite get to the point of finding the minimum value from the graph.



Group 3 – Whole Group Discussion Rehearsal
How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

Strategy: Write an equation and graph it.

This group wrote the equation $y = \sqrt{(100^2 + x^2)} + \sqrt{(60^2 + (600 - x)^2)}$ and graphed the equation using Desmos. They were able to identify the minimum of the graph at (375, 620.967).



Group 4 – Whole Group Discussion Rehearsal

Strategy: Use similar triangles and solve a proportion.

This group was able to set up a proportion using similar triangles. While they were not able to prove the triangles are similar, they got the same answer as other groups, so they think they are on the right track.