This thesis aims to present a low-order method for the prediction of propeller interaction effects on a wing regarding the total lift and lift distributions. A wing model based on the Weissinger method is modified to predict the effect of a propeller upstream of the wing. Two different propeller models are used to determine the propeller induced velocities within the slipstream. The first model is a vortex ring method, which predicts the propeller axial induced velocity. The second model is a blade performance theory, which utilizes the induced velocity calculated by the vortex ring method and predicts the propeller tangential velocity along with a more detailed axial induced velocity. However, with limited propeller geometry, the vortex ring axial induced velocity is used over the blade performance theory results because it remains more consistent for a variety of propeller types and configurations. Utilizing these methods minimizes the number of inputs required for the model to predict the interaction effects as well as keeping the computational cost very low compared to more detailed solvers. The low computational cost allows for rapid testing of various propeller-wing configurations to narrow down design choices. Overall, the lift distributions show good agreement with numerical and experimental data for a variety of different cases. Only major discrepancies occurred near and beyond stall, where the inviscid nature of the model is unable to make accurate predictions. General trends in overall wing lift are also captured by the model even in difficult cases such as low aspect ratios and low Reynolds numbers.
Low-Order Modeling of Propeller-Wing Interaction Using a Modified Weissinger Method

by

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DEDICATION

I dedicate this thesis to both my parents and grandparents for their unconditional love and support. Without their help it would not have been possible for me to attend a university, much less continue on for a master’s degree. I am very thankful for everything they have done for me up to this point in my life. I am also thankful to my girlfriend Katy for her support and motivation during the completion of my degree.
BIOGRAPHY

Brendan D’Angelo was born on February 22nd, 1997 in Charlotte, North Carolina. He received his Bachelor’s degree in Aerospace Engineering in the spring of 2019 from North Carolina State University. He then decided to continue his education at North Carolina State University and was admitted in the fall of 2019 to pursue a Master’s degree in Aerospace Engineering. During that fall semester he joined the Applied Aerodynamics Group, which was led by Dr. Ashok Gopalarathnam. Upon obtaining his Master’s degree, Brendan hopes to become an aerospace related engineer in the private sector and possibly returning for his PhD after working for a few years.
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NOMENCLATURE

$\alpha_i$ Induced Angle of Attack
$\alpha_w$ Wing Angle of Attack
$\alpha_{0l}$ Zero Lift Angle of Attack
$\alpha_{eff}$ Effective Angle of Attack
$\alpha_g$ Geometric Angle of Attack
$\alpha$ Angle of Attack
$\beta$ Blade Section Pitch Angle
$F$ Aerodynamic Influence Coefficient Matrix
$\epsilon$ Wing Section Twist
$\Gamma_{ring}$ Ring Circulation Strength
$\Gamma$ Circulation
$\omega$ Propeller Angular Velocity
$\phi$ Angle of Propeller Trailing Vortices
$\rho$ Fluid Density
$\sigma$ Rotor Solidity
$\theta$ Dihedral Angle
$A_0$ Propeller Disc Area
$a_0$ 2D Lift-Curve Slope
$A_{eff}$ Effective Propeller Disc Area
$B$ Number of Blades

$b$ Wingspan

$c$ Chord

$C_L$ Wing Lift Coefficient

$C_l$ Lift Coefficient

$C_t$ Sectional Lift Coefficient

$C_T$ Thrust Coefficient

$C_{D_i}$ Wing Induced Drag Coefficient

$ds$ Wing Section Length

$F$ Prandtl’s Tip Loss Factor

$R$ Propeller Radius

$R_c$ Propeller Root Cutout

$R_P$ Radial Distance to Point

$R_{local}$ Local Slipstream Radius

$r_{prop}$ Slipstream Radial Location at the Propeller Plane

$r_{slip}$ Slipstream Radial Location Downstream

$S$ Wing Reference Area

$T$ Thrust

$V_\infty$ Freestream Velocity

$V_I$ Propeller Central Induced Velocity at the Propeller Plane
\[ V_R \quad \text{Resultant Velocity} \]
\[ V_{tip} \quad \text{Blade Tip Velocity} \]
\[ W \quad \text{Propeller Total Induced Velocity} \]
\[ w \quad \text{Induced Downwash Velocity} \]
\[ W_a \quad \text{Propeller Axial Induced Velocity} \]
\[ W_t \quad \text{Propeller Tangential Induced Velocity} \]
\[ Z_P \quad \text{Axial Distance to Point} \]
Chapter 1

Introduction

1.1 Background

As electric technologies such as batteries and propulsion continue to advance, it is becoming more attractive to explore hybrid and electric transportation both on the ground and in the air. Due to this increase in popularity, the Scalable Convergent Electric Propulsion Technology Operations Research (SCEPTOR) program is being led by NASA to showcase the use of electric propulsion distributed along a wing [1]. This program has led to the designing and building of the X-57 Maxwell, which uses 12 propellers oriented in front of the wing leading edge as well as two larger propellers mounted on the wing tips, allowing for improvements in cruise efficiency according to Cole et. al. [2]. In addition to the X-57, NASA is also investigating a tilt-rotor heavy lift aircraft for both civil and military applications [3]. Propeller-wing interaction for tilt-wing aircraft is especially important in hover and can have significant effects on the maximum payload weight [4].

Understanding and integrating the concepts of propeller-wing interaction early in the design cycle is important for many propeller-driven aircraft. However, it is also challenging due to the complexity involved in incorporating this concept with the overall design of the aircraft. Most development of conventional aircraft keep the design of the propulsion system separate from
that of the wing early in the design phases [5]. This heavily limits the flexibility for adjusting the propeller or wing further down the line when the two systems are studied together in-depth. By integrating the two systems earlier, designs can more fully utilize the propeller-wing interaction to benefit aircraft performance.

It has been shown both historically by Snyder [6] and recently by Sinnige [7] that wingtip-mounted propellers can offer increases in lift and decreases in drag when in an inboard-up configuration. However, this may not always be the case as shown by Cole et. al. [2]. In the investigation, the X-57 was used to show that a wingtip-mounted inboard-up propeller is not necessarily the most efficient design in terms of the lift over drag ratio (L/D). Instead, propeller-wing interaction is more complex than a simple, narrow statement, and each configuration should be studied to determine the more detailed effects [2].

The spanwise position of the propeller and the rotation direction are not the only aspects that need to be considered when evaluating the propeller-wing interaction. The vertical position of the propeller also plays a large role in the effect on lift and drag of a wing [8]. The axial position of the propeller as well as the configuration of the propeller, tractor or pusher, may influence performance [9].

The complexity of the problem is a reason why there is a lack of empirical models that have been developed to calculate the effects that a propeller and wing have on each other. By simplifying this problem and developing modeling tools, aircraft designers may be able to better work around the propeller-wing effects.

1.2 Propeller and Wing Models

There have been many low-order models developed to capture the effects of both propellers and wings, each with their own advantages and disadvantages. A few of these that are closely related to the models used in this thesis will be briefly described here to give insight into why the models used were chosen. Many of these models have been modified for various types of studies so each will be presented along with papers that used the model.
Lifting-line theory (LLT) provides a basis for many low-order wing methods. This theory uses the Kutta–Joukowski theorem along with the concept of circulation to transform the problem of finding the lift distribution into that of finding the circulation distribution. To solve for the circulation distribution, the wing is divided into panels in the spanwise direction. Each of these panels is assumed to have constant circulation, but the change in circulation between panels is modeled as a trailing vortex filament stretching downstream. The trailing vortices induce a downwash or upwash on each of the panels, which relates the circulation distribution to the downwash allowing for both to be solved. The advantage of this method is that it has a very low computational cost. However, this method also has a disadvantage of being unable to accurately model a swept wing due to the required bend in the lifting line at the center of the wing. Although lifting-line theory has been around for some time, modern adaptations continue to be developed such as that from Philips [10], which is based on a full three-dimensional vortex lifting law instead of the classical two-dimensional one developed by Prandtl.

The Weissinger method is a modified lifting-line theory that addresses the issue of swept wings. In order to accomplish this the Weissinger model adds a control point at the spanwise center of each section along the three-quarter chord location. The downwash is then measured at this point as opposed to directly on the lifting line. This model is still computationally inexpensive, but, due to having only a single chordwise section, the accuracy of the results can vary depending on the wing geometry. Similar to lifting-line theory, the Weissinger method continues to be used in research to continue to expand upon its capabilities.

The Weissinger method is actually a simple version of a vortex lattice method (VLM). However, most VLMs contain multiple chordwise panels, which allow for the chordwise vorticity to vary and making small aspect ratio wing results more valid. Each panel contains its own control point and circulation, which normally leads to more accurate results, but also increases the computational time required compared to the standard Weissinger method. There are also 3-dimensional VLMs that model the camber line of the wing, but again increases the computational cost. Out of the previous low-order wing models, the vortex lattice method is
used the most extensively because of the versatility of the model to be applied to many different wing configurations successfully.

In addition to the low-order wing models, there are also propeller models that have been used to predict the forces and/or slipstream, some of which use methods similar to the wing models that were previously discussed. For the modeling of the propeller itself, any of the wing models listed above, LLT, Weissinger or VLM, can be used. However, the propeller wakes are modeled differently from that of a wing using these methods. There are also methods that further simplify the propeller into a disk.

Helical wake modeling is a propeller wake model that uses trailing edge vortices similar to the wing models, however, in this case the propeller blades act as wings and so while they spin, the trailing edge vortices form a helical shape. The authors of [11, 12, 13] all use a VLM modeled propeller with a helical wake model. To simplify the model, Duarte [14] broke down the propeller wake into a region where each blade's trailing vortices begin to roll up into a single helical vortex and another region where the vorticity of each blade is concentrated on this single vortex. A diagram of this model is presented in figure 1.1. These models can use either a prescribed propeller wake or a free wake method, which differ in how the wake geometry is obtained. Robison [15] goes into more detail on each of these wake models, from which a brief summary is presented here. For a prescribed wake, the geometry of the wake is described using functions that are based on the results at the propeller plane. This differs from a free wake model, which uses the self-induced velocities of the wake itself to generate the wake geometry.
The vortex ring method is a simplification of the helical wake model where, instead of the vortices forming a helix, they are instead distributed as rings traveling downstream. This simplification takes away the ability to model swirl using only the rings, but allows the axial induced velocity to be calculated much more easily. This model is also useful for modeling of a steady-state propeller since the spacing between rings can be made small so that the induced effects vary only slightly between rings. Because of this, the rings can instead be modeled as stationary as opposed to moving downstream, further simplifying the model. A version of this model is discussed by Dong [16] in a wind turbine application, which shows that the method continues to remain relevant and useful in modern day analysis.
Blade element theory breaks down a propeller’s blades into sections in the radial direction and then determines the forces on each of these sections. The forces from the sections are then added together to find the total propeller forces and moments. The way that the propeller is divided in blade element theory is shown in figure 1.2. Blade element momentum theory (BEMT) combines blade element theory with actuator disc theory to obtain the relationship between the blade sections and the flow at the propeller. Now, this means that the effect of the propeller on the fluid is taken into account so that the velocities at the propeller plane are able to be calculated. This gives a more detailed picture of a propeller than is given by blade element theory alone. Froude [17] developed this idea of determining the propeller forces by dividing the propeller up into sections while also considering the angular momentum of the fluid. This method does require lift and drag data for each of the blade sections in order to determine the induced velocities.
Blade performance theory is a modified version of blade element momentum theory and so the two model the propeller in the same way. The analysis is done at the same level as BEMT, but this modified theory predicts the induced velocity from the propeller at the propeller plane as well as in the wake. This theory does not look to determine the thrust generated by the propeller but assumes this is a given. Instead, the theory uses each blade section to determine the induced velocity provided by the section. This allows for the induced velocity over the entire propeller plane to be calculated. Then the geometry of the contracting slipstream is used to determine how these velocities change downstream.

On top of the low-order models presented here, computational fluid dynamics (CFD) is also
used to study the propeller-wing interaction. If used correctly, CFD can give accurate results but comes at a large computational cost as well as requiring detailed models. This is why CFD is normally reserved for later on in the design cycle, when there are less changes being made to the overall design of the aircraft. However, this is also the phase in the design cycle where major changes become very costly, so only minor changes are usually made by this stage.

Use of these propeller methods along with the previously discussed wing methods have been used successfully in predicting the effects of propeller-wing interaction on lift and drag for a wide range of cases recently by Bohari [18]. This gives confidence that a low-order method, which incorporates some of these models together will be able to show general trends in the interaction so that the performance of a wing-propeller configuration can be estimated and then used to refine early aircraft designs or further studies on the interaction.

1.3 Motivation

With the ever-increasing push to reduce global emissions, technology in many sectors has been advancing toward a more eco-friendly state. Transportation is a notable example of this with a heavy focus of research being conducted on hybrid and electric transportation technologies. As these technologies have evolved, the applications that they can be used for have grown, with concepts for new electric aircraft increasing with the growth in popularity of urban air mobility.

Propeller-wing interaction is becoming a significant research topic due to the increased interest in electric powered aircraft and urban air mobility. Much of this research has been focused on CFD because of the complex nature of the interaction between a wing and propeller, such as the recent study by Chauhan [19] who studied the interaction using a Reynolds-averaged Navier-Stokes CFD simulation with an actuator-disk to model the propeller. It is valuable to study this interaction since it may provide benefits such as increased lift and reduced drag if implemented in a design properly. However, there are few low-order models that accurately predict the effects of propellers on a wing even for standard propeller-wing cases.
1.4 Objectives

The objective of this thesis is to present an implementation of a low-order model that incorporates propeller effects into a wing model and to show where this new combined model either succeeds and fails to give accurate results. To accomplish this, a model was developed in Matlab that is based on a modified Weissinger method and uses the change in velocity due to a propeller slipstream to determine the lift distribution over a wing. To calculate the slipstream velocity, two different methods were used. The first is a vortex ring method (VRM), which is used to calculate the propeller axial induced velocity. The next is a Blade performance theory, which is a modified blade element momentum theory (BEMT) and is used to calculate the propeller tangential induced velocity.

In this thesis, the modified Weissinger method is used for various configurations, such as propeller position along the wingspan, propeller vertical position, changes in propeller advance ratio, changes in wing planform, and changes in wing angle of attack. These different configurations are used to determine how closely the modified Weissinger method predicts the effects of propeller-wing interaction for each circumstance. Based on the accuracy of the results, there is a future possibility of using this combined method to offer aircraft designers a technique to design around the propeller-wing interaction in the early design phases.

1.5 Outline

In Chapter 1 this thesis covers some background information on propeller-wing interaction by going over some relevant studies and aircraft that have utilized the interaction. A brief overview of some propeller and wing models is also presented in this Chapter along with the motivation and objectives regarding the research conducted. Chapter 2 provides a more in-depth look into the methods used to develop the propeller-wing interaction model. The chapter also covers some necessary background knowledge for each of the methods and sources for further information on each method.
Chapter 3 presents results for each of the models used along with the combined model. Each model is validated against experimental data and numerical methods from several sources for various configurations. Overall, the results match up well with the experimental and numerical findings from other papers.

Lastly, Chapter 4 discusses the conclusions which have been drawn from the previous chapter. The discussion includes how well the objectives for the research have been met. This chapter also provides insight into possible future work and improvements that can be made to the model to attempt to guide future research looking to build on the current model.
Chapter 2

Theory

The low-order propeller-wing interaction model that is presented in this thesis uses a vortex ring method with a prescribed wake along with blade performance theory as the model for a propeller, while using a Weissinger method for the wing. This chapter will provide an overview for the theory for each method as well as how they were implemented together. However, not all of the detailed information on each theory is presented and so each section contains references for additional information.

2.1 Wing Method

The basis of the wing model is centered around extended lifting line theory, which is also referred to as Weissinger’s method, and so the basics of lifting line theory will be presented in this section along with the extensions for the Weissinger method. For a more in-depth study on the topic, refer to Weissinger’s fundamental work, which is presented by Spalart [20] in a more modern form. However, before diving into the theory of the wing model, some fundamentals on vorticity and circulation will be discussed first.
2.1.1 Vorticity and Circulation

Out of the different types of two-dimensional fundamental flow fields, the point vortex is the one that applies to the theory behind the wing model used in this thesis. A point vortex, shown in figure 2.1, is a point which causes the fluid around the point to flow in a circular pattern with that point being the center. Although the flow is moving in a circular pattern, it is irrotational because the fluid particles themselves do not rotate, but instead are only moving in the circular path.

Circulation ($\Gamma$) is the line integral of the velocity vector taken along a closed loop as is shown in figure 2.1.
in equation 2.1. By convention, circulation is taken positive in the counterclockwise direction. Simplifying equation 2.1 for a point vortex leads to equation 2.2.

\[ \Gamma = \oint V ds \] (2.1)

\[ \Gamma = 2\pi r V \] (2.2)

From this as well as the knowledge that point vortex flow does not contain any radial velocity, the flow field velocity at any point in radial coordinates can be seen in equations 2.3 and 2.4.

\[ V_\theta = \frac{\Gamma}{2\pi r} \] (2.3)

\[ V_r = 0 \] (2.4)

By applying this concept of a point vortex in a three-dimensional space, a line vortex can be created. A line vortex is a filament containing cross sections resembling a point vortex. The shape and circulation strength, which is defined for the entire vortex, are the two defining features of the line vortex. A line vortex induces velocities on points similar to how a point vortex does, however, now that the vortex is being looked at in three-dimensional space and is not all located at a single point, the equations to find the velocities become more complex.

To aid in solving for the induced velocities from a line vortex at a point, the Biot-Savart law can be used. This law is shown in equation 2.5 and figure 2.2 and can be used to find the induced velocity, \( d\vec{V} \), by a small vortex line segment of length \( d\vec{l} \) and strength \( \Gamma \) at point \( P \), with a radial vector \( \vec{r} \) pointing from \( d\vec{l} \) to \( P \).
This equation can be expanded to find the total induced velocity from a filament $\vec{V}$, which is given in equation 2.6.

$$
\vec{V} = \int_{-\infty}^{\infty} d\vec{V} = \int_{-\infty}^{\infty} \frac{\Gamma}{4\pi} \frac{\vec{d}l \times \vec{r}}{|\vec{r}|^3}
$$

This equation can be simplified in various cases where the line vortex may be a simple line segment or in the form of a circle. Two of these cases can be used in the wing models to be
reviewed and so will be discussed here. A third simplification of this law applies only to the propeller model and so will be discussed in a later section. The first of these cases is for a straight semi-infinite vortex filament with a point that lies in the plane normal to the segment at the starting point for the segment. Equation 2.7 shows the magnitude of the induced velocity on a point a distance of $h$ from the filament starting point.

$$|\vec{V}_P| = \frac{\Gamma}{4\pi h}$$ (2.7)

For a finite-length straight segment, where the filament stretches from point A to B, equation 2.8 is used. The angle $\beta_1$ is the angle between the line from A to B and the segment AP, where P is the point of the induced velocity calculation. $\beta_2$ is the angle between the line A to B and the segment BP. $h$ is simply the perpendicular distance from the line segment to the point P.

$$|\vec{V}| = \frac{\Gamma}{4\pi h} (\cos \beta_1 - \cos \beta_2)$$ (2.8)

These two simplified cases of the Biot-Savart law provide a way to calculate the induced velocities for a wing modeled by straight vortex filaments. These filaments are used extensively in lifting line theory and in Weissinger’s modifications to the theory. The specific applications will be discussed in their respective sections.

### 2.1.2 Lifting Line Theory

Lifting line theory (LLT) is derived from the idea that lift is generated by vorticity along a lifting surface. The relationship between the lift and vorticity is shown by the Kutta-Joukowski theorem for a lifting section in equation 2.10. This vorticity is distributed across the entire surface; however, an approximation can be made to model this as a single line vortex along the span of the wing, called a bound vortex. In order to comply with Kelvin’s circulation theorem, trailing vortices must extend downstream from the ends of the spanwise line vortex, which is at the wing tips for a wing model, and then these trailing vortices are closed far downstream by
the starting vortex. This starting vortex can be assumed at infinity for a steady-state case and so it has no effect around the wing itself. The combination of the bound vortex and trailing vortices is known as a horseshoe vortex. Using the simplified Biot-Savart law given in equations 2.7 and 2.8, the induced velocity, $w(y)$, along a wing modeled with span $b$ and located on the bound vortex is shown in equation 2.9 with upwash being positive. Since a line vortex cannot induced velocity directly on itself, the bound vortex does not induce any velocity on the given spanwise points.

$$w(y) = -\frac{\Gamma}{4\pi} \left[ \frac{b}{(\frac{b}{2})^2 - y^2} \right]$$  \quad (2.9)
When modeled with only one pair of trailing vortices, as shown in figure 2.3, the lift distribution along the wing is not captured, instead it is constant along the wing. To remedy this, Prandtl [21] recommended to allow for the vorticity strength to change along the span of the wing. To continue to comply with Kelvin’s theorem, trailing vortices must be shed at each point that the circulation along the span changes as shown in figure 2.4. By having wingspan sections that have a trailing vortex on each end of the wing sections, the wing model essentially becomes one where there are multiple horseshoe vortices, with the largest spanning from one wingtip to the other and each subsequent horseshoe vortex spanning one less wing section on each end of the wing. This model now allows for the lift distribution along the span of the wing.
to be captured since the circulation along the span is now modeled.

![Figure 2.4: Multiple Trailing Vortex Lifting Line Theory Diagram](image)

\[ L' = \rho_\infty V_\infty \Gamma \]  

(2.10)

For a finite wing divided into segments of constant circulation, equation 2.10 becomes equation 2.11 where \( L' \) is the aerodynamic lift per unit span generated by that section and \( V \) is the local freestream velocity of that section.
\[ \vec{L}' = \rho(\vec{V} \times \vec{\Gamma}) \] (2.11)

To obtain the total induced velocity at a point along the span, the effects of each of the trailing vortices must be taken into account. If the points are being calculated along the bound vortex, then equation 2.7 can be used for each of the trailing vortices. The induced velocity from the trailing vortices causes an induced angle of attack, \( \alpha_i \), at each point, \( y_0 \), along the span. This induced angle of attack is given by equation 2.12 and can be estimated as equation 2.13 for small angles.

\[ \alpha_i(y_0) = \tan^{-1}\left(\frac{-w(y_0)}{V_\infty}\right) \] (2.12)

\[ \approx -\frac{w(y_0)}{V_\infty} \] (2.13)

Now, the effective angle of attack, \( \alpha_{eff} \), can be calculated, which takes into account both the induced angle of attack and the angle of attack of the section, \( \alpha \). The starting equation for the effective angle of attack is shown in equation 2.14 and then the resulting equation after substitution is shown in equation 2.15.

\[ \alpha_{eff}(y_0) = \alpha(y_0) - \alpha_i(y_0) \] (2.14)

\[ \alpha_{eff}(y_0) = \frac{2\Gamma(y_0)}{a_0(y_0)V_\infty c(y_0)} + \alpha_{0l}(y_0) \] (2.15)

Where \( c(y_0) \) is the section chord length, \( a_0(y_0) \) is the two-dimensional lift-curve slope of the airfoil and \( \alpha_{0l} \) is the angle of attack for zero lift at the section. This equation leads directly into the fundamental equation for lifting line theory given in equation 2.16, which solves for the geometric angle of attack as a sum of the effective, induced and zero-lift angles of attack.
\[ \alpha(y_0) = \frac{2}{a_0(y_0)c(y_0)} \frac{\Gamma}{V_\infty}(y_0) + \alpha_0(y_0) + \frac{1}{4\pi V_\infty} \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{d\Gamma}{dy} dy \] (2.16)

The methods used to formulate lifting-line theory bring about some limitations. The first limitation is that this theory is not valid for small aspect ratio wings due to there being no variation along the chord of the wing. A more realistic model for a wing has vorticity smeared along the surface of the entire wing. As the aspect ratio decreases, the effects of these chordwise changes in vorticity become more prevalent and at some point cannot be ignored. The second limitation is that this theory does not work well for swept wings or wings with winglets. This is due to the requirement of a bend in the bound vortex at the center of the wing, which causes high induced velocities to be present at the center of the wing.

2.1.3 Weissinger Modifications to LLT

Weissinger made a few modifications to the lifting-line theory to get around some of the limitations that the theory has. The most notable modifications are the addition of control points for each span-wise section of the wing and a change in how the line vortices are modeled. These changes allow for the model to predict the lift distribution on swept wings, which greatly extends the cases where the model can be used. A diagram of Weissinger’s model is shown in figure 2.5.
In order to agree with thin airfoil theory (TAFT) results, which derived the lift-curve slope of an airfoil as $2\pi$ per radian, the location of the bound vortex should be on the quarter chord line and the control points should be placed along the three-quarter chord line for each section. The control points are used to enforce a zero normal flow boundary condition. The two contributions to this normal flow are the freestream velocity, $(V_{\infty,n})_i$ and the induced velocity from the horseshoe vortices, $V_{ni}$. Setting the sum of these two normal contributions as shown in equation 2.17 will allow the boundary condition to be enforced at each control point.

$$V_{ni} + (V_{\infty,n})_i = 0$$  \hspace{1cm} (2.17)
The induced velocity from the horseshoe vortices is calculated the same as before except that the points being evaluated are not along the bound vortices and so the effect of these vortices needs to be taken into account. The freestream contribution is dependent of the angle of attack of the wing and the geometry of the wing. The equation for this contribution is given in 2.18 where $\epsilon_i$ is the twist of the section and $\alpha_{gi}$ is the geometric angle of attack of the section shown in equation 2.19 and contains $\alpha_w$, which is the wings angle of attack.

\[(V_{\infty-n})_i = V_{\infty} \sin (\alpha_{gi} + \epsilon_i - \alpha_{0li}) \quad (2.18)\]

\[\alpha_{gi} = \sin^{-1} \left[ \sin (\alpha_w) \cos (\theta_i) \right] \quad (2.19)\]

Next, the aerodynamic influence coefficient matrix (AIC), denoted by $[F]$, is used to develop a set of linear equations that will be used to solve for the circulation distribution over the lifting surface. The AIC matrix is a matrix created from the induced velocities of each of the vortex segments on the control points. This matrix is of size NxN where N is the number of horseshoe vortices or sections in the model. Equation 2.20 shows the system of linear equations that needs to be solved for $\Gamma$, where $V_n$ and $\Gamma$ are Nx1 vectors that define the distribution along the span.

\[[F]\{\Gamma\} = \{V_n\} \quad (2.20)\]

Once the circulation of each section has been found the elemental lift, $dL_i$ can be found using the Kutta-Joukowski theorem and the section length, $ds_i$. Equation 2.21 shows this theory being used with the assumption of a negligible component of the induced velocity in the freestream direction.

\[dL_i = \rho V_{\infty} \Gamma_i ds_i \quad (2.21)\]
\[ C_{l_i} = \frac{2}{c_i} \left( \frac{\Gamma}{V_\infty} \right) \]  

(2.22)

\[ C_L = \frac{2}{S_{ref}} \sum_{i=1}^{N} \left( \frac{\Gamma}{V_\infty} \right) \cos (\theta_i) ds_i \]  

(2.23)

Equation 2.22 then shows the sectional lift coefficient from the sectional lift and then equation 2.23 gives the total wing lift coefficient. The induced drag can also be found as seen in equation 2.24 and uses the normal wash, \( w_i \), which is the downwash normal to the wing as viewed from downstream.

\[ C_{D_i} = \frac{2}{S_{ref}} \sum_{i=1}^{N} \left( \frac{w_i}{V_\infty} \right) \left( \frac{\Gamma_i}{V_\infty} \right) ds_i \]  

(2.24)

### 2.2 Propeller Methods

There are two propeller methods that were incorporated to add propeller effects into the Weissinger wing model. These two models are the vortex ring method and blade performance theory. Further details on either of these methods can be found in [22], where the models were derived and compared to wind tunnel tests. The reasoning behind choosing these models as well as a basic overview of a few different types of propeller models are discussed in 1.2.

#### 2.2.1 Vortex Ring Method

The vortex ring model (VRM) is one where the propeller wake is modeled by a system of ring vortices. This model is derived from the same idea as lifting-line theory where a vortex system is used to represent a finite wing. However, instead of a wing, the propeller blades are the lifting surfaces. This means the bound vortex is along the propeller blade and there is a shed vortex both at the tip and at the hub of each propeller blade. These shed vortices are used to model the slipstream of the propeller. Since the blades of the propeller are rotating, the trailing vortices follow a spiral path as they move downstream. The vortices following this type of path is both
difficult to analyze and is also not ideal for steady-state analysis, especially for analysis near the vortices since they are constantly moving. To remedy this, only the circumferential portion of the vortices are modeled, while the axial component is neglected. This transforms the spiral vortices into rings that are evenly spaced apart in the propeller’s axial direction as shown in figure 2.6. For a steady case, the rings in the theory stretch downstream toward infinity, however, when designing the model the rings only stretch a distance of $100R$, where $R$ is the propeller radius. This gives results that are accurate until far downstream when nearing the end of the rings. The model can be further simplified if the rings are assumed very close together to conform into a cylinder of vorticity. This assumption was not made in the model presented here to allow for the size of the slipstream to change based on the radius of each ring and its axial location instead of deriving the induced velocity for a shrinking cylinder. Also, by using the vortex rings it allows the model to be adapted in the future to include rotation of the rings based on changes in shape of the slipstream.
In order to determine the strength of each ring vortex, the induced velocity at some point within the slipstream must be calculated. Only one point is needed because it is assumed that all the rings have the same vortex strength due to the size of the rings deviating only slightly and the propeller being assumed to operate at a steady-state condition. The induced velocity that is used in this case is the average velocity of the rotor, which is also shown to be the induced velocity at the center of the rotor by Heyson [23]. Equations 2.25 through 2.29 are used to calculate this induced velocity.

\[
V_I = \frac{C_{T1} V_{tip} A_{eff}}{2 \sqrt{\mu^2 + \lambda^2}}
\]  

(2.25)
\[ C_T^1 = \frac{4C_T}{\pi^3} \] (2.26)

\[ \lambda = \frac{V_\infty \cos (\alpha) + V_I}{V_{tip}} \] (2.27)

\[ \mu = \frac{V_\infty \cos (\alpha)}{V_{tip}} \] (2.28)

\[ A_{eff} = \frac{1}{1 - R_c^2} \] (2.29)

Where \( V_I \) is the induced velocity at the center of the propeller on the propeller plane, \( C_T \) is the thrust coefficient of the propeller, \( R_c \) is the root cutout of the propeller, \( V_{tip} \) is the blade tip velocity, and \( \alpha \) is the angle of attack of the propeller relative to the freestream. The induced velocity must be solved iteratively with an initial guess recommended by Chandrasekaran [22] given in equation 2.30.

\[ V_{Iinit} = \frac{1}{2} \left( -V_\infty \cos (\alpha) + \sqrt{(V_\infty \cos \alpha)^2 + \frac{2T}{\rho A_0}} \right) \] (2.30)

This equation uses the thrust \( T \) of the propeller along with the area \( A_0 \) of the rotor disc. This initial guess for the induced velocity is plugged in and a new induced velocity is found. This process is repeated until the value of the induced velocity has converged. The vortex strength of all rings can now be found by using equation 2.31, for a point on the ring’s axis, or equation 2.32, for all other points, which is derived by Castles [24]. The equations for \( A, B, C, D \) and \( F \) are functions of the radial, \( d_r \), and axial, \( d_a \), distances from the center of the ring to the point, the minimum, \( d_{min} \), and maximum, \( d_{max} \), distances from the point to the edges of the ring, as well as the complete elliptic integrals of the first and second kind. A diagram showing the geometrical relationship used to calculate these distances is given in figure 2.7.
\[ V_I = \sum_{i=1}^{M} \frac{\Gamma_{ring_i}}{2R_i} \frac{1}{(1 + d_i^2)^{\frac{3}{2}}} \]  

(2.31)

\[ V_I = \sum_{i=1}^{M} \frac{\Gamma_{ring_i}}{2\pi R_i} (A_iB_i + C_iD_iF_i) \]  

(2.32)

This equation is applied by setting \( V_I \) as the induced velocity due to all of the rings at the center of the propeller plane. To achieve this, each of the ring induced velocity to this point
must be calculated and summed to the total velocity. For this equation, \( M \) is the total number of rings, \( R_P \) is the radius of each ring and \( \Gamma_{ring_i} \) is the vortex strength of each ring, which is assumed to be the same for all rings. Each of \( A, B, C, D \) and \( F \) must be calculated for each ring since these are dependent on the location of the point with respect to the ring.

\[
W_a(x) = V_I \left( 1 + \frac{x}{\pi R} \sqrt{1 + \left(\frac{x}{\pi R}\right)^2} \right) \tag{2.33}
\]

In order to include the effects of a contracting wake, the variation in the axial velocity must be found. The relation between the axial induced velocity at the center of the propeller, \( W_a \), the propeller radius, \( R \), and the axial distance downstream, \( x \), is given by equation 2.33, which is derived in [25, 26]. Now, the continuity equation shown in equation 2.34 is used by relating the flow at the propeller plane to the flow in a plane downstream, while assuming that there is no radial variation in induced velocity. Lastly, the radius of the slipstream at an axial location, \( r(x) \), is solved for in equation 2.35.

\[
(V_\infty \cos(\alpha) + V_I) \pi R^2 \rho = (V_\infty \cos(\alpha) + W_a(x)) \pi r^2(x) \rho \tag{2.34}
\]

\[
r(x) = R \sqrt{\frac{V_\infty \cos(\alpha) + V_I}{V_\infty \cos(\alpha) + W_a(x)}} \tag{2.35}
\]

Now that the radius of the slipstream at any axial location has been determined, the radius of the vortex rings are set to match this new slipstream shape. A new vortex strength for each of the rings can now be calculated based on the changed ring geometry. In most cases the change in strength of the rings caused by the changes in their radii is small because the rings closest to the propeller have a small variation compared to those rings further downstream due to the slipstream contraction.

Although the vortex rings are used to determine the axial velocity within the propeller wake, due to the simplifications that have been made they do not induce any circumferential, or
tangential, velocity. In order to model this tangential velocity, the helical shape of the vortices would need to be reinstated. Instead of this, a blade performance theory was employed to model the induced tangential velocity of the propeller.

### 2.2.2 Blade Performance Theory

Blade performance theory looks to model the induced velocities over the propeller and how they change in the radial direction. The following analysis is done by McCormick [26] and so more detail on the equations and model of this theory can be found in the reference. Compared to the vortex ring method discussed in the previous section, blade performance theory requires more detailed information about the propeller so that the sections can be analyzed individually. This information includes the number of blades as well as the chord, twist, zero-lift angle of attack and lift curve slope distributions along the blade radial direction. Some of this information can be difficult to extract, namely the twist and zero-lift angle of attack, and so in many cases these must be assumed as being that of a similar propeller where the information is available. The lift-curve slope for all sections was assumed as being $2\pi$ per radian as is derived in thin airfoil theory.

The theory divides the propeller blades into radial sections where the properties of each blade section is used to calculate the induced velocities generated by the entire propeller. This is essentially an identical model to blade element momentum theory (BEMT), however, the difference lies in the output quantities of interest. BEMT focuses on finding forces such as lift and drag on each section so that thrust and efficiency can be determined. On the other hand, blade performance theory focuses on finding the induced velocities in the rotor plane and the wake. Since the formulation of the propeller-wing model requires the induced velocities, a blade performance theory was chosen in order to calculate these velocities. To begin the calculation of the induced velocities, a few parameters based on the propeller blade must first be defined.

$$\sigma = \frac{Bc(r)}{\pi R}$$

(2.36)
\[ F = \frac{2}{\pi} \cos^{-1} \left( e^{\frac{-B(R-r)}{2r \sin(\phi_{tip})}} \right) \]  

(2.37)

Where \( \sigma \) is the solidity of the rotor at a radius \( r \), \( B \) is the number of propeller blades, \( c(r) \) is the blade chord at a radius \( r \), \( \phi_{tip} \) is the angle of the tip trailing vortices as they move downstream. \( F \) is Prandtl’s tip loss correction, which was introduced by Prandtl in reference [21] to account for the model’s over prediction of the forces on the propeller. Next, the change in angle of attack caused by the induced velocities at the propeller plane is calculated using equation 2.38, which is derived from the geometry of the propeller section shown in figure 2.8 with consideration for tip loss effects.

\[ \alpha_i = \frac{1}{2} \left( -G + \sqrt{G^2 + 4H} \right) \]  

(2.38)

\[ G = \tan(\phi) \frac{\sigma}{8 \frac{r}{R} F \cos(\phi)} \]  

(2.39)

\[ H = \frac{\sigma a_0(\beta - \phi)}{8 \frac{r}{R} F \cos(\phi)} \]  

(2.40)
The velocity resulting from the freestream and blade rotation is then calculated for a section using equation 2.41 where $\omega$ is the angular velocity of the propeller.

$$V_R = \sqrt{(V_\infty \cos(\alpha))^2 + (\omega r)^2} \quad (2.41)$$

$$W_{t_{init}} = V_R \alpha_i \sin(\phi + \alpha_i) \quad (2.42)$$

Next, an initial guess for the tangential induced velocity for the section at the propeller plane is given by equation 2.42. Then equations 2.43 through 2.46 can be calculated iteratively.
until the solutions for the axial and tangential induced velocities converge. These equations come from the geometry of the propeller section and velocity vectors where $V_e$ is the actual velocity seen by the section with the freestream, propeller rotation, and induced velocities taken into account.

\[
\frac{W_a}{V_{tip}} = \frac{1}{2} \left[ -\frac{V_\infty \cos(\alpha)}{V_{tip}} + \sqrt{\left(\frac{V_\infty \cos(\alpha)}{V_{tip}}\right)^2 + 4 \frac{W_I}{V_{tip}} \left(\frac{r}{R} - \frac{W_t}{V_{tip}}\right)} \right] \tag{2.43}
\]

\[
C_l = a_0 \left( \beta - \tan^{-1}\left(\frac{W_I}{W_a}\right) \right) \tag{2.44}
\]

\[
\frac{V_e}{V_{tip}} = \sqrt{\left(\frac{V_\infty \cos(\alpha)}{V_{tip}} + \frac{W_a}{V_{tip}}\right)^2 + \left(\frac{r}{R} - \frac{W_t}{V_{tip}}\right)^2} \tag{2.45}
\]

\[
\frac{W_t}{V_{tip}} = \frac{\sigma C_l}{8 \pi F} \frac{V_e}{V_{tip}} \tag{2.46}
\]

Once the equations have converged to a solution within a tolerance, then the tangential and axial induced velocities are known at the propeller plane. However, when analysis is done downstream it is necessary to know the induced velocities in the slipstream of the propeller, $W_a(x)$. For the axial induced velocity, equation 2.33 is used again, but modified to become equation 2.47, with $V_I$ instead becoming the induced axial velocity at the propeller plane along the same streamline as that for the desired downstream location, $W_{a_{prop}}$.

\[
W_a(x) = W_{a_{prop}} \left( 1 + \frac{x}{R} \right) \tag{2.47}
\]

The location on the propeller plane is found using equation 2.48, which is just a relation of the ratio of the local radius location and the radius of the point for the propeller plane and the plane downstream. In this equation, $R$ is simply the radius of the propeller, $r_{slip}$ is the radius of the desired point from the propeller axis line, $R_{local}$ is the local slipstream radius at the desired
point, and \( r_{prop} \) is the slipstream radial location at the propeller plane. A diagram illustrating these four radial distances is given in figure 2.9

![Diagram showing relationship of local radius to propeller radius.](image)

Figure 2.9: Relationship of Local Radius to Propeller Radius

\[
r_{prop} = R \frac{r_{slip}}{R_{local}} \tag{2.48}
\]

Finally, for the tangential induced velocity in the slipstream, the method of conservation of the circulation is employed as described in reference [25] and shown in equation 2.49.
$$W_t(x) = \frac{R}{R_{local}} W_{prop} F$$ \hspace{1cm} (2.49)

Now that the axial and tangential induced velocities can be found anywhere in the propeller slipstream, the propeller model can be integrated with the wing to determine the effects that the propeller slipstream has on the wing aerodynamics.

### 2.3 Propeller-Wing Model

The wing and propeller methods that have been discussed in this chapter were integrated to be used as a low-order propeller-wing interaction model. The implementation of a propeller in the Weissinger method required some changes to be made to accommodate for the non-uniform velocity now seen by the wing. This section will discuss the changes made and the reasoning behind each of the changes. The parts of the theory that are not mentioned here are consistent with those discussed in section 2.1.

As a starting point, the induced velocities of the propeller need to be calculated at each section’s control point. To do this, first, the axial induced velocity of the propeller is found using the vortex ring method. This means that some geometric parameters need to be found so that the VRM receives all the required inputs. The control point’s axial distance downstream and radial distance from the axis are determined by using the following equations that are developed based on vector algebra.

\[
P_{axis} = P_{\text{prop}} + \bar{a}X \tag{2.50}
\]

\[
\bar{b} = P_c - P_{axis} \tag{2.51}
\]

In equation 2.50, $P_{axis}$ represents any point along the propeller axis by relating it to the central point in the propeller plane, $\bar{P}_{prop}$ and the vector $\bar{a}$, which is the unit vector pointing
along the propeller axis. Now $X$ is allowed to be any number so that any point along the propeller axis can be found using this equation. Next, the vector $\vec{b}$ is used to represent the vector pointing from a point on the propeller axis to the control point, $\vec{P}_c$, and is given by equation 2.51. Now, the value for $X$ is found for when the two vectors, $\vec{a}$ and $\vec{b}$, are perpendicular by setting the dot product between the two equal to zero.

$$\vec{a} \cdot \vec{b} = 0 \quad (2.52)$$

The value for $X$ where this is true is now called $X_{sol}$ and, equation 2.53 gives the location on the propeller axis that lies closest to the control point, $\vec{P}_a$.

$$P_a = P_{prop} + \vec{a}X_{sol} \quad (2.53)$$

Next, the values for the axial distance, $Z_P$, and radial distance, $R_P$, can be found using equations 2.54 and 2.55 respectively. Figure 2.10 gives a visual representation of what each of these vectors represent and how the calculation was formulated.

$$Z_P = |P_a - P_{prop}| \quad (2.54)$$

$$R_P = |P_c - P_a| \quad (2.55)$$
These values are then used with the vortex ring method, described in section 2.2.1, to calculate the axial velocity of the propeller plane ring on the control point. The value for $R_P$ remains the same for all rings downstream, however, $Z_P$ changes since the axial position of the rings is changing. The $Z_P$ value for each of the rings can be determined by simply finding the difference between the ring’s axial location relative to the propeller plane, $Z_{ring}$, and the propeller plane’s value for $Z_P$.

Using some of the information from the VRM along with a few additional inputs, blade performance theory can now be used to determine the axial induced velocity as it changes over the radius as well as the swirl velocity. Since the slipstream and control point locations are
determined in the vortex ring method, the blade performance method is only used for points that are within the slipstream. It is assumed that all points outside of the slipstream have zero axial and tangential induced velocity from the propeller.

The most fundamental change required to implement the propeller effects is the addition of the propeller normal contributions at the control points. The standard Weissinger method only contains contributions from the freestream and the induced velocities of the horseshoe vortices. Now, the propeller slipstream must also be accounted for when enforcing the zero-normal flow boundary condition at the control point. As a result, equation 2.17 becomes equation 2.56 where $V_{prop}$ is the normal induced velocity by the propeller, $V_{wing}$ is the normal induced velocity by the wing horseshoe vortices and subscript $i$ represents which wing section is being calculated.

\[(V_{wing})_i + (V_{prop})_i + (V_{\infty,n})_i = 0\] (2.56)

Both the wing and freestream components of the normal velocity are already calculated in a standard Weissinger method. However, the propeller component needs to be calculated based on the combination of the axial and tangential propeller induced velocities. In many cases, only components of these induced velocities act normal to the panel at the control point. Trigonometry is used to find the section normal component of the total propeller induced velocity at the control point similar to how the freestream normal component is found in the standard Weissinger method. The normal velocity components for the freestream and the propeller are added together for each section to become the new value for $V_n$ found in equation 2.20. In this equation, $F$ is calculated the same as it is for a wing alone so that $\Gamma$ becomes the only unknown. This means the $\Gamma$ distribution along the wing can now be solved for.

Next, the total velocity vector at the section midpoints along the quarter chord location must be calculated. The propeller axial and tangential induced velocities for each section are calculated in the same way as was done at the control point. In the propeller coordinate system the axial velocity acts along the $x$-axis and the tangential velocity acts in the $y - z$ plane. The sum of these two vectors is found in terms of this coordinate system. This vector is then rotated
along the y-axis by the angle defined by the propeller axis to the wing coordinate’s x-axis so that it is expressed in the wing coordinate system. Only this single rotation is needed because the y-axis in propeller and wing coordinates point in the same direction.

Figures 2.11 and 2.12 show how the induced velocities from a propeller effects the lift distribution over a generic wing. It is shown that, on one side of the propeller, the flow is pushed upward which causes an increase in the effective angle of attack for the wing within this section. It is also shown that, on the other side of the propeller, the flow is instead pushed downward causing the effective angle of attack to decrease and even has the possibility of causing the lift to act downward for low angles of attack. The effects on the lift distribution for inboard up and outboard up cases are shown in figure 2.11 where the lift decreases and increases within the downward and upward spinning regions respectively. The reason for this effect is given in figure 2.12 where the local flow in the two regions within the slipstream is shown to influence the effective angle of attack.
Figure 2.11: Propeller-Wing Interaction Lift Distribution Diagram
The original method uses a simplification that assumes the only velocity seen by the quarter-chord midpoint is from the freestream, however, this is no longer the case when the induced velocity from a propeller is introduced. The assumption resulted in equation 2.21 previously, but with the addition of induced velocity which does not point along the freestream direction, the force generated can no longer be considered entirely lift. Instead, there is also a component of the force generated that is along the freestream and is therefore considered drag. The more generalized form to account for the new induced velocity is shown in equation 2.57.

\[
\mathbf{dF}_i = \rho \left( \mathbf{V}_i \times \mathbf{ds}_i \right) \Gamma_i \quad (2.57)
\]
Where $d\vec{F}_i$ is the total force vector generated by the section, $\vec{V}_i$ is the velocity seen by the quarter-chord midpoint, and $\hat{ds}_i$ is the unit vector point along the quarter-chord line for the section. The previous equation gives the incremental force, which contains both the lift and induced drag of the section. The incremental lift and drag can be found by taking the component of this force in the directions perpendicular and parallel to the freestream velocity, respectively. Then the sectional lift coefficients can be found using equation 2.58 with $c_i$ being the section chord measured at the midpoint.

$$C_{l_i} = \frac{d\ell_i}{\frac{1}{2} \rho V^2_\infty c_i}$$

(2.58)

Lastly, the total wing lift coefficient can be found by first finding the total lift of the wing, which is given by equation 2.59, and then by converting the total lift into nondimensionalized form as shown in equation 2.60. The dihedral angle of each section, $\theta_i$, as well as the length of each section, $ds$, must be considered to calculate the wing lift correctly. In this thesis, the lift coefficient is nondimensionalized by the wing’s total planform area, $S$. The total induced drag and coefficient are found similarly with the only difference being the removal of the dihedral angle.

$$L_{tot} = \sum_{i=1}^{N} dL_i \cos (\theta_i) ds_i$$

(2.59)

$$C_L = \frac{L_{tot}}{\frac{1}{2} \rho V^2_\infty S}$$

(2.60)

In order to show this modified method in a more simplified form, the flowchart given in figure 2.13 was created. The flowchart presents the steps and logic behind the combined propeller-wing method so that a better understanding of the model can be reached.
Input: Wing and propeller geometry and properties

Process geometry into sections and compute section properties

Use vortex ring method to calculate $W_a$ at section midpoint and control point

Point within slipstream?

Use blade performance theory to calculate $W_t$

Set $W_t = 0$

Last section?

Calculate $F$ matrix based on wing geometry

Calculate rhs based on angle of attack and induced velocities

Compute section and total wing force coefficients

Last $n$?

End

Figure 2.13: Combined Model Flowchart
2.3.1 Model Capabilities

The modified propeller-wing interaction model has a wide range of capabilities as well as limitations, which will be described in this section. Each one of the capabilities is then demonstrated in the next chapter to validate whether the results obtained hold up to what has been found in previous studies.

The most basic capability of the model is that it can be used as a standard Weissinger method without the presence of a propeller. This allows for wings to be modeled and evaluated in the same way as is expected in a low-order wing model. The particular model supports wings containing multiple patches that have linearly varying chord, twist and $\alpha_0$. Each wing patch is defined by the starting and ending point of either the quarter chord line or the leading edge, which allows for wings with sweep and dihedral. The patches can also represent separate lifting surfaces such as a combination of a wing and tail. Also, the number of sections desired is given individually for each patch so that smaller patches can be modeled with fewer sections. An input file must supply a freestream velocity and can either evaluate a single angle of attack or a range of angles of attack.

Much like the wing, the propeller location is defined in the same coordinate system and represents the propeller center point. This allows for a propeller to be placed anywhere in relation to the wing. However, if the propeller is placed downstream of the wing, effects will not be seen, since only the effects of the wake of the propeller are modeled.

The propeller is also described by its radius, thrust coefficient and RPM. These three values are required as the main performance parameters that define the propeller. These values allow for the axial induced velocity to be calculated at any point in the propeller slipstream and so are necessary when including a propeller in the model. The model can be used to include the effects with or without swirl so that the effects of a simple jet can also be studied using the model.

For a more detailed propeller model that includes the swirl velocity induced by the propeller, more parameters must be given to the model. These parameters include the spin direction of the propeller and the number of blades. The geometry of the propeller must also be known for
the blade performance calculations, which requires the pitch and chord distributions over the blades, the root cutout of the propeller hub, and the lift-curve slope of the 2D propeller airfoil. However, these four parameters are difficult to find or determine in many studies, so reasonable values were used in the comparisons contained in the next chapter where these parameters could not be determined.

The model makes an effort to introduce the effects of a contracting slipstream by having the induced velocity to be calculated by contracting rings. It also sets the swirl velocity equal to zero outside of the contracted slipstream radius at a certain downstream location and to employ methods to determine the downstream tangential velocity as the slipstream contracts. The addition of this gives the ability to study propeller effects further downstream such as on a horizontal or vertical tail.

On top of this, the model also supports multiple propellers with varying parameters. This is not limited to symmetrical configurations and so counter-rotating propellers can be modeled with opposite spin on different sides of the wing as is the case in some aircraft. This means that cases where one propeller is operating at a reduced thrust coefficient can be analyzed. The propellers can also vary in regard to their radius and number of blades, which allows for cases where multiple sizes and types of propellers are used in a single configuration.

Multiple overlapping propellers can also be handled by the model. Although, the overlapping sections are treated separately until the induced velocity of the overlapping propellers are added together at the end, so the results are likely to deviate from experimental results or more detailed models. This can still give reasonable results for propellers with small overlap as is demonstrated in the next chapter.

Also, propellers can be set at an angle of attack relative to the wing coordinate x-axis. However, this is only valid for wings close to the propeller if the propeller axis and freestream velocity are not aligned. The reason for this is that the slipstream does not curve downstream due to the freestream. This can cause the slipstream to be in a much different position far downstream than is expected and shown in experiments or CFD.
The model still contains the inherent limitations of the Weissinger method. The first of these limitations is that the method is not valid for small aspect ratio wings, which is usually considered to start at an aspect ratio of five. This is due to the method not predicting chordwise effects, which become prominent in low aspect ratio wings. This brings us to the second limitation, which is the inability to model chordwise changes in the wing. The model only incorporates a single chordwise section and so to get around these two limitations a VLM with more sections along the chord is required. The third limitation is one that is inherent in any inviscid solver, which is the inability to model viscous effects. Situations that have prominent viscous effects such as stall must be corrected for in order to get reasonable results. Finally, the model made for this thesis does not incorporate yaw into either the propeller models or the wing model. Again, introducing yaw would require a more detailed VLM to be used so that chordwise distributions can be calculated.
Chapter 3

Results

The results presented in this thesis are validated against multiple sources including experiments and other propeller and/or wing models.

3.1 Validation Sources

Veldhuis [12] used various methods to study the aerodynamic interference between a wing and a tractor propeller. The first of these methods was the use of a VLM code that incorporated a BEMT model, which is similar to the model presented here so it acts as a good source of validation. A Navier-Stokes code from Fluent 6.1 is also used with the propeller modeled as an actuator disk. This actuator disk model allows for asymmetry in the propeller loading by making the pressure and velocity dependent on both the radial and azimuthal position in the disk. Three different wind tunnel models were also used in the study. The first is denoted as PROWIM and consists of a rectangular wing with aspect ratio of 5.33 and no twist. The airfoil used is a NACA 64-A015 and the wing has a half-span of 0.64m. A nacelle is located 0.3m from the center of the wing, which holds the propeller approximately 0.2m in front of the leading edge of the wing. The axis of rotation for the propeller lies along the MAC-line and the propeller itself has a diameter of 0.236m. The second model, denoted by APROPOS, is very similar to PROWIM except in that the nacelle is a separate structure from the wing. The nacelle containing the
propeller is instead held by a separate strut out in front of the wing. The dimensions of the wing itself remain the same as the previous model. A representation for both the PROWIM model and APROPOS model is shown in figure 3.1. The final wind tunnel model is a 1:20 scale of the prototype Fokker F27 aircraft. Lastly, data was gathered from flight tests of a Fokker 50 aircraft where pressure measurements were made along the wing at two different flight conditions: one as a high thrust case and the other as a low thrust case.

Ananda et. al. [9] performed experiments in the low turbulence subsonic wind tunnel at the University of Illinois at Urbana-Champaign (UIUC). A wing and propeller setup was tested
in this wind tunnel at low Reynolds numbers to explore the propeller-wing interaction effects. The wing was situated on a platform force balance, which was used to measure the wing’s aerodynamic loads. The wing itself was rectangular with an aspect ratio of 4, a chord length of 3.5 inches and used a Wortmann FX 63-137 airfoil. The propeller was situated upstream of the wing on a separate fairing that ran perpendicular to the span of the wing. The propeller used was a GWS 5x4.3 micro propeller, which has a 5 in diameter and 4.3 in pitch. Multiple propeller positions were tested where the propeller was moved in either the propeller’s axial direction or in the direction perpendicular to the wing span. The RPM of the propeller was measured for each test and the thrust coefficient is given by performance data. A representation for the propeller-wing wind tunnel setup is given in figure 3.2.
Lepicovsky and Bell [27] performed propeller experiments and measured the velocity components at multiple locations downstream of the propeller. A laser velocimeter was used to gather the velocity component data over a period of time and then the mean value was taken for each location. All three components, axial, radial and tangential, of velocity were measured by the device along both axial and radial directions. The tested propeller was a two-bladed, 330.2 mm diameter, wooden propeller. All experiments were conducted with the propeller at an RPM of 4250. Thrust coefficient was not measured in the experiments, but, for the sake of comparing the current work, the thrust coefficient was estimated at 0.05. The propeller was operated in stationary air, which is within the capabilities of the propeller model that is used in the current work.
work.

Hunsaker et. al. [28] developed a lifting-line model which uses 2D airfoil data and the measured upstream velocity profile to predict the lift on a wing. This model is then compared with experiments done by Stuper [29] on a rectangular wing with large circular end caps with a Gottingen 409 airfoil at a aspect ratio of four and a Reynolds number of around 406,000. Stuper performed wind tunnel experiments with the wing and a jet, which created a near constant increase in velocity within the jet’s slipstream. Cases of the wing with no jet, 18%, and 36% velocity increases were performed and the lift distribution over the wing was measured using pressure taps. In order to model the end caps, Hunsaker used vertical rectangular sections at the ends of the wing model. Hunsaker also used prescribed velocity fields from experiments in his model to get the effects of the jet on the wing. The 2D airfoil lift behavior was given to the model from an incompressible Reynolds-Averaged Navier-Stokes (RANS) solution. Hunsaker also developed a propeller model which was compared with experimental data from Lepicovsky [27].

In the following sections, comparisons are made with the sources listed above so that the validity of the models can be determined at various conditions. The first section looks into the Weissinger model alone, the second reviews the results calculated by the propeller models alone and the final section validates the combined propeller-wing model.

3.2 Weissinger Model Validation

The Weissinger model has been widely used and verified so only a brief validation is presented in this section.

The Stuper [29] wing is used as validation of the current model’s wing method in figure 3.3 since baseline data without the jet is available. To model the wing end caps that are used in the experiment, vertical rectangular, winglet like, sections are placed at the wing tips. This causes the wing to behave more similarly to an infinite wing than if the winglets are left out. The figure shows that the addition of these winglets is a good model for the end caps at lower
angles of attack, but begins to breakdown at higher angles. Overall, the lift distribution matches well with the experimental data and Hunsaker’s lifting-line model. At 12 degrees, the wing is near stall so the model predicts a higher lift than what is measured in experiments. Hunsaker also uses 2D RANS data as an input to his model so stall effects are captured better than the purely inviscid model developed in this thesis. The current model predicts lower lift coefficients near the wing tips, which is partially due to the end caps not being accurately modeled. This same reason accounts for the increased lift in the center of the wing especially at higher angles. The 2D lift-curve slope is inherently set to the same as that predicted by thing airfoil theory from the placement of the lifting line and control points. However, the actual lift-curve slope of this airfoil is actually smaller than this theoretical value which also accounts for part of the increased lift seen at higher angles of attack.
3.3 Propeller Model Validation

The models that have been used in the numerical formulation of a propeller are validated in this section. Both the vortex ring method and blade performance models are compared to experiments and models developed in other papers. The comparisons look at how well the results match for axial and tangential induced velocities estimated at the propeller plane as well as the development of these velocities downstream. Also the shape of the slipstream itself is studied and compared with other numerical results.
3.3.1 Propeller Axial Induced Velocity

The following plots give a comparison for the induced axial velocity calculated by the model with experimental data from Lepicovsky [27] and another numerical method from Hunsaker [28]. Figure 3.4 shows the calculated and measured axial velocity along the radius near the propeller plane at a axial distance of $X/D = 0.009$ downstream. Detailed information on the propeller geometry was not found and so a linear twist distribution that passes through 24 degrees at the three-quarter radial location was assumed. The chord of the propeller blades was assumed constant at 17% of the radius, which appears to resemble provided sketches except at the blade tips. The figure shows that blade performance considerably overpredicts the induced velocities compared to the experimental and numerical data. However, the overall shape of the curve matches closely with the two comparisons. Major differences at the root of the propeller are shown, which are due to the inability for the model to accurately predict the flow near the root cut-out location. Instead, a velocity of zero is assumed at the center and a linear distribution is assumed until the first converged point. More detailed geometry of the propeller blades would likely cause blade performance theory to more closely match the experimental and numerical results. Vortex ring method predicts a near constant velocity profile, which is expected due to the nature of this model. The VRM predicts the average velocity over the propeller plane, which appears to be reasonable compared to the experimental and numerical data in the figure.
Figure 3.4: Nondimensionalized axial induced velocity over propeller radius with $V_\infty = 0$, $C_T = 0.05$, RPM = 4250 and diameter of 0.3302 m at a location of $X/D = 0.009$ downstream.

Figure 3.5 gives the axial induced velocity as measured at various points downstream of the propeller. The radial location of measurement is kept constant at a value of $r/R = 0.693$. This location was chosen by Lepicovsky because it is the location where the axial velocity was measured at a maximum in the near field of the propeller. Both the vortex ring and blade performance methods are not formulated to predict the induced velocity upstream of the propeller, so this data was not included. It is shown that as the flow moves downstream the models predict that the axial flow will increase. This is caused by the slipstream contraction, which is used in the two models. In reality viscosity will begin to slow down the flow at some point as is shown by the experimental data. As shown before blade performance overestimates the axial velocity at an increasing amount as the flow moves downstream. The vortex ring model
underestimates the axial velocity because this is taken at the maximum axial velocity in the propeller plane. The vortex ring method is modeling the average velocity over the propeller plane and so the value is significantly lower. However, as viscosity effects become more prevalent in the flow downstream, the experimentally measured flow appears to approach the average velocity as calculated by the vortex ring model. This means that the vortex ring model can be used to predict the axial flow velocity downstream with a reasonable amount of accuracy.

Figure 3.5: Nondimensionalized axial induced velocity stretching downstream of the propeller with $V_\infty = 0$, $C_T = 0.05$, RPM = 4250 and diameter of 0.3302 m at a radial location $r/R = 0.693$
3.3.2 Propeller Tangential Induced Velocity

Now, comparisons are made for the tangential induced velocity as measured in experiments and by another numerical model. Figure 3.6 shows the tangential velocity comparison at the near propeller plane location. The vortex ring method is not able to predict the tangential velocity and so only blade performance theory is being compared. The overall trends over the propeller radius appear to match with the numerical results. The tangential velocity is calculated as being high near the hub and decreases non-linearly until reaching zero at the edge of the slipstream. The experimental results show a more constant tangential velocity throughout the center of the propeller radius. The differences in the results are likely in part from the assumed propeller geometry, which Hunsaker also had to make. However, even with the assumed geometry, the velocity values over the propeller remain relatively close to the numerical and experimental values throughout the propeller radius.
Figure 3.6: Nondimensionalized tangential induced velocity over propeller radius with $V_\infty = 0$, $C_T = 0.05$, RPM = 4250 and diameter of 0.3302 m at a location of $X/D = 0.009$ downstream.

The tangential velocity measured along the axial direction is given in figure 3.7. All measurements are taken at the same radial location of $r/R = 0.693$, which was the experimentally measured maximum axial velocity location. The figure shows that near the propeller plane the tangential velocity at this location compares closely with the experimental values. Blade performance theory predicts that the tangential velocity will increase as the slipstream contracts as shown in the figure. However, experimental values show that the tangential velocity stays relatively the same, which is likely due to viscous effects counteracting the increased velocity that would be seen from the contraction. Values from blade performance still match closely up to a point downstream, which reiterates the limitation of not modeling the viscous effects on the tangential velocity far downstream.
Figure 3.7: Nondimensionalized tangential induced velocity stretching downstream of the propeller with $V_\infty = 0$, $C_T = 0.05$, RPM = 4250 and diameter of 0.3302 m at a radial location $r/R = 0.693$

3.3.3 Propeller Slipstream Contraction

The predicted slipstream radius downstream of the propeller is compared with that from Hunsaker’s model [28] in figure 3.8. It is difficult to measure the slipstream radius experimentally, so this comparison is only made with another numerical model. The two models are very similar except that the Hunsaker model contracts at a much higher rate near the propeller plane. This seems to be an inherent difference in the two models since the rapid contraction is unable to be reproduced by the current model when changing the propeller parameters. The two models begin to predict a near constant slipstream radius of approximately the same value. The far downstream radius is estimated to be a 25% decrease from the original propeller radius for both
models in this case. The current model assumes that the center induced velocity is the same along the entire radius to calculate the slipstream radius downstream. This assumption appears to overestimate the size of the slipstream, but reaches a similar value far downstream. Lifting this assumption and using the induced velocity distribution along the radius would likely give results that more closely match Hunsaker and more detailed computer models.

![Graph showing slipstream radius stretching downstream of a propeller](image)

Figure 3.8: Slipstream radius stretching downstream of a propeller with $V_\infty = 0$, $C_T = 0.05$, RPM = 4250 and radius of about 0.09 m

### 3.4 Combined Model Validation

The combined propeller-wing model is validated in this section to determine the accuracy and capabilities of the model. To validate the model, comparisons are made with other models and experiments for a wide range of studies including propeller rotation direction, spanwise position,
axial position, vertical position and performance characteristics. Each of these is compared to
determine areas where the model succeeds and fails to make accurate predictions. First, the lift
distribution predicted by the model is studied to show the general lift effects of a propeller on a
wing.

Figure 3.9 gives a comparison to results from a VLM and from experiments developed and
conducted by Veldhuis [12]. A prediction for the wing with no propeller using the current model
is shown as another point of comparison. In the figure the current model predicts the sectional
lift coefficient at zero since the wing is symmetric and at a zero angle of attack. The experimental
data is from the PROWIM experiment and the VLM was modeled to replicate the experimental
characteristics. The rectangular wing in tandem with a comparatively large propeller offers a
good study of comparison to show the accuracy of the developed model. The figure gives the
lift distribution over the wing for an angle of attack of zero and a propeller operating at a
steady-state condition. The results match closely with the VLM and experimental data over
the entire span of the wing. It can be seen that the positive and negative lift coefficient peaks
predicted by the current model differ slightly from both the experimental data and the VLM.
This is likely because of a combination of the inaccuracies of calculating the swirl velocity as
well as the axial induced velocity being assumed to be constant over the propeller radius. This
means that hub effects are not modeled very accurately and cause the lift coefficient peaks to
shift toward the center of the propeller.
Figure 3.9: Spanwise lift distribution comparison from the PROWIM model with propeller rotating inboard up used by Veldhuis. $J = 0.85$; $T_c = 0.1268$; $\alpha_p = 0^\circ$; $\alpha = 0^\circ$

Figure 3.10 gives a comparison for the same wing used in the previous figure, but instead at an angle of attack of four degrees. Once again the model shows close agreement with Veldhuis’ experimental data and VLM prediction. The peaks of the lift coefficient are slightly higher, which is likely because the current model is not intended for modeling a propeller at an angle of attack. Instead, the propeller was kept at the same angle of attack to avoid the effects caused by the propeller not being aligned with the freestream velocity. The PROWIM model has the propeller attached to a nacelle on the wing so a rotation of the wing also causes a rotation of the propeller by the same angle. Sharp corners in the plot can be seen at the edges of the propeller slipstream. These sharp edges are partially due to the finite number of wing sections, but also because the induced tangential velocity is abruptly set to zero anywhere outside the slipstream.
in the model. Once again it is shown that the lift coefficient peaks are closer to the center of the propeller, which, as stated earlier, is likely from the propeller induced velocity model used.

Figure 3.10: Spanwise lift distribution Comparison from the PROWIM model with propeller rotating inboard up used by Veldhuis. $J = 0.85; T_c = 0.1268; \alpha_p = 0^\circ; \alpha = 4^\circ$

### 3.4.1 Propeller Rotation Direction

The rotation of the propeller has an effect on the lift distribution and overall lift of the wing. The model is able to capture these effects by prescribing the direction of the tangential velocity that is found from blade performance theory. Based on the location of the propeller along the wing, the rotation direction can be either defined as inboard up, with the inboard side of the propeller moving vertically upward in relation to the wing, or outboard up for the opposite spin direction. Comparisons are made in this subsection to determine if the general trends that are
seen by the two different spin directions are captured in the model.

Figure 3.11 gives a comparison for the total wing lift coefficient at various angles of attack at the two different spin conditions. The comparison in this figure is made with experimental results from the PROWIM model. The predicted results for the wing with no propeller from the model is given as the baseline lift curve. It is shown that the model deviates from the experimental wing lift as the angle of attack is increased. However, the trend of the inboard up configuration having a marginally higher lift coefficient is maintained by the model. Also, both the model and experimental results show an overall increase in lift for both inboard and outboard up configurations as compared to the baseline wing. The lift-curve slope of the experimental results is higher than that predicted by the model for both cases. This may be because the airfoil used has a max thickness of 15, which may cause it to deviate from the assumed lift-curve slope predicted by TAFT.
3.4.2 Propeller Spanwise Position

The validity of the model in predicting the effects of the spanwise position of the propeller are tested in this subsection. Both the wing overall lift and the wing lift distribution are tested to determine the accuracy of the results. Figure 3.12 gives the wing lift coefficient for varying propeller positions along the span of the wing. The angle of attack, advance ratio, and thrust coefficient are held constant and the propeller is aligned with the freestream since the APROPOS wing model is now being used. It should be noted that the changes in lift coefficient are magnified here to better see the effects. The baseline wing with no propeller is also presented to give a reference point. The propeller is operating in a inboard up configuration for all positions in this figure. It can be seen that the presence of the propeller in an inboard up configuration causes
the lift coefficient to increase relative to a standard wing in both the experiment and model. Overall, the model and experiment show similar trends with the lift coefficient increasing as the propeller is moved toward the wing tip. This is believed to be the case due to the wing tip vortices being increasingly counteracted by the propeller as it moves closer to the wing tip. The model and experiment both demonstrate that the relationship between the propeller spanwise position and the lift coefficient of the wing is not linear. Instead larger increases in lift are made as the propeller moves closer to the wing tip. It is also shown that the model predicts a higher lift coefficient for most spanwise propeller positions except close to the wing tip.

Figure 3.12: Wing lift coefficient vs propeller spanwise location for the APROPOS wing in Veldhuis’ work. $J = 0.92; T_c = 0.127; \alpha_p = 0^\circ; \alpha = 4.2^\circ$

Figure 3.13 gives the spanwise loading distribution for three different propeller positions
with a Fokker 50-like configuration. The data that the current model is compared to is a VLM developed by Veldhuis, which was made to model propeller effects on wings. The wing being modeled is similar to a Fokker 50 where it is both tapered and swept, but does not contain twist. Veldhuis models the fuselage and nacelle effects, but these are neglected since the wing method used does not support these effects. Instead, the chord is assumed constant and unswept through the area where the fuselage resides. Propeller details beyond the number of blades and diameter are not immediately available and so assumptions have been made on the twist and chord distributions of the blades to calculate the tangential induced velocity produced by the blades. The model matches closely with the results from Veldhuis for most of the span. There is a large discrepancy at the center of wing, which is due to the absence of fuselage in the model. Also, it appears that the inboard up side (right) of the wing matches more closely in the propeller slipstream area than the outboard up side (left). The outboard up propeller predicts a lower loading peak than what is predicted by the VLM. The inboard side, however, predicts a similar loading peak as is given by the VLM. The reason for the difference between the outboard and inboard sides is not immediately known.
Figure 3.13: Spanwise loading distribution for multiple propeller positions for Veldhuis’ high speed case. $J = 1.63; T_c = 0.046; \alpha_p = 0^\circ; \alpha = 4.2^\circ$

### 3.4.3 Propeller Axial Position

The effects of varying the propeller position along the x-axis of the wing predicted by the LOM was compared to experimental results from Ananda et. al. in figure 3.14. The presented data shows the results of the total wing lift coefficient versus the wing angle of attack for multiple propeller axial positions. For this figure, the propeller is operated at 7,000 RPM with a Reynolds number of 70,000. The propeller axis for each position is aligned with the quarter chord of the wing. The wing is then rotated about this quarter-chord location to vary the angle of attack. Position 1, 2 and 3 are situated at a distance of 0.725D, 0.475D and 0.265D respectively upstream of the wing quarter-chord location, with D being the propeller diameter.
This case is a difficult one for the model due to the influential effects of a laminar separation bubble at this low Reynolds number for the given airfoil. Also, the low aspect ratio of four that was used for the experiments is at the boundary of what the Weissinger method can accurately predict. In order to try to implement a solution to the large disparity in the results, a correction was made to the LOM predictions. This correction was made by using the 2D airfoil data gathered in XFOIL for inviscid, tripped, and free transition cases. The lift coefficient of the wing is then matched with that of the 2D inviscid solution to determine the corresponding angle of attack. The deviation between the 2D inviscid and free transition lift coefficients are found for the determined angle of attack. The same procedure is then done for the 2D inviscid and tripped data. The difference between the 2D inviscid and free transition case is meant to represent the effect of the portion of the wing which contains laminar flow due to it residing outside of the turbulent propeller slipstream. Additionally, the difference between the 2D inviscid and tripped data represents the effect of the wing that resides in the propeller slipstream. A weighted average based on the amount of the wing that is within the propeller slipstream is then used to correct the total wing lift coefficient.
Although the direct results differ substantially from the experiments in figure 3.14, the corrected results make a good prediction compared to the experimental data. It can be seen by the comparison that the predicted lift-curve slope of the wing differs from the experimental data. This is likely due to the wing being of a low aspect ratio causing the Weissinger method to over predict the increase in lift with angle of attack. As discussed previously, this is due to the chordwise distribution in the wing method being neglected by using only a single chordwise panel. Both the experimental results and the model demonstrate that the variation of the axial distance of the propeller at the three positions has little effect on the total lift of the wing. The reason for this is likely caused by the slipstream having only a small deviation at each analyzed point. The model predicts that the majority of the slipstream axial velocity increase occurs close
to the propeller plane and then occurs much slowly further downstream. The positions studied here are likely outside of the region where rapid axial velocity changes occur.

3.4.4 Propeller Vertical Position

The ability for the model to predict the effects of changing the propeller vertical position relative to the wing is also demonstrated through comparison with other studies. Ananda et. al. [9] performed propeller-wing interaction experiments where the propeller was placed at three different locations while the wing was kept stationary. The three locations include position 1 where the propeller is in line with the quarter chord of the wing, position 5 where the propeller is located 0.25D above the quarter chord, and position 6 where the propeller is located 0.25D below the quarter chord. The axis of the propeller remains aligned with the freestream in all three locations.

In figure 3.15 the experimental results are compared with the model predictions. The lift coefficient from the model was corrected based on the same technique as described previously to account for some of the 2D viscous effects. The comparison shows fairly good agreement between the corrected model and the experiment at low angles of attack. The model does not capture the effects as the wing approaches stall, which is expected considering the model is inviscid. The experimental results demonstrate that the vertical position of the propeller has little effect on the overall lift coefficient of the wing at low angles of attack. The model shows this same trend with the lift coefficients for the three positions being relatively close together at these same angles. As was seen previously for the same wing, the lift curve slope of the wing is not accurately predicted likely due to the low aspect ratio and complex viscous effects. It is interesting to note that the model predicts identical results for positions 5 and 6, which means that positive and negative vertical propeller positions will provide the same results.
Figure 3.15: Lift coefficient versus angle of attack for varying propeller vertical positions of a rectangular wing with a Wortmann FX 63-137 airfoil, AR of 4, Re = 70,000, RPM = 7,000, J = 0.62 in Ananda’s work. 100 spanwise lattices used.

3.4.5 Slipstream Performance Effects

Figure 3.16 gives a comparison between the model, corrected for viscous effects, and Ananda’s experiments, where the propeller advance ratio has been varied. The experimental results are given for a propeller aligned with the quarter-chord line of the wing at two different RPM settings, 5000 and 7000, along with a baseline containing no propeller. The experimental results show slight increases in the lift coefficient of the wing as the RPM of the propeller increases. The model predicts this trend as well at small angles of attack, although the difference is smaller than that seen for the experiments. Once again the lift curve slope is not predicted correctly as is discussed previously. Also, the difference between the model and experiment increases as the
wing approaches stall. Although this experiment lies outside of the models capabilities, it was included to see if the inclusion of a correction factor would allow the model to still be sued for such cases.

Figure 3.16: Lift coefficient vs. angle of attack for varying propeller RPMs and advance ratios of a rectangular wing with a Wortmann FX 63-137 airfoil, \( A \) of 4, Re = 60,000 in Ananda’s work. 100 spanwise lattices used.

Further comparisons are made with experiments by Stuper [29] and predictions by a lifting line model from Hunsaker [28]. Data for the wing alone is given in figure 3.3, but a jet is introduced in the experiment and model that were used. The jet produces a near constant velocity profile, which is similar to the profile that is produced by the ring vortex method. At the edges of the jet there is a velocity deficit which is lower than the freestream and likely due to the boundary layer that is formed around the nozzle of the jet. There is no induced tangential
velocity on the flow caused by the jet, so all induced flow acts in the axial direction. Figure 3.17 shows the results of the propeller wing model with the experimental and other models results for an average jet velocity 18% higher than the freestream. The data is given for three different angles of attack being 4, 8 and 12 degrees. Only the axial velocity from the vortex ring method is used in the current model for this prediction.

It appears that the model compares closely with both experimental data and Hunsaker’s model at lower angles of attack. The effect of the velocity deficit seems to be more apparent in Hunsaker’s model as the angle of attack is increased, but that is not modeled in the vortex ring method used. The disparities seen at 12 degrees are likely from a combination of the wing nearing stall and the jet velocity deficit effects. Also, the overall shape of the lift distribution begins to change from the experimental data and the model as the angle is increased. Other than the previously mentioned complications, this is partially caused by the difficulties in simulating the wing end caps in the current model. Rectangular winglets were used to try to simulate the effects, but as the angle of attack increases the results begin to show dissimilarity.
Figure 3.17: Spanwise lift distribution for varying angles of attack of a rectangular wing with circular end caps and a Gottingen 409 airfoil. Jet velocity of 18% increase over freestream, \( AR \) of 4 and \( Re = 406,000 \) with experimental results from Stuper and numerical results from Hunsaker. 100 spanwise lattices used.

Figure 3.18 gives a comparison for a higher velocity jet at 36% more than the freestream velocity and other characteristics remaining the same as the previous plot. The same disparities can be seen at higher angles of attack as was shown before. However, at small angles of attack, the results still match closely with the experimental and numerical results. The increased jet velocity is predicted to cause a larger increase in lift at the location of the jet. It is likely that the results would more closely match if the model could better simulate the wing end caps and if the velocity deficit seen at the edges of the slipstream could be modeled.
The overall wing lift coefficient is given in figure 3.19 for the previous two jet velocity cases along with a no jet case. The current model compares quite well with the experimental results and underpredicts the lift-curve slope from the numerical results. Since the model does not try to predict stall, results past 12 degrees do not match the experimental values. The trend of increasing jet velocity resulting in increased lift-curve slope is shown for all three sets of results.
To look more closely at the changes in the total wing lift, it is helpful to see the change in lift coefficient from the no jet baseline wing. The results for this are given in figure 3.20 for Hunsaker’s model and Stuper’s experiments. The experimental results needed to be interpolated since the angles of attack for the measurements did not entirely line up and some angles did not contain jet data. This figure better shows the differences in lift of the wings with jets from the wing alone. Each of the lift changes are measured from their respective no jet case. It is shown that the model coincides with the numerical predictions made by Hunsaker’s closely for both jet cases. The model slightly underpredicts the change in lift at lower angles of attack by a small margin and a larger margin for experimental values. The general trends remain present with the
difference in lift between the two jets growing as the angle of attack increases. This same trend is shown by the numerical model and the experimental data, although less pronounced. There is no stall prediction in the current model so the reduction of the change in lift that is seen in the numerical model is not shown in the current work. It is interesting to note that there is a $\Delta C_L$ peak between 6 and 8 degrees in the experimental data. This seems to arise from a jump in overall lift for the airfoil at this angle of attack, which was not reproduced by Hunsaker in his model, and can be partially the cause for the consistently higher experimental $\Delta C_L$ values around this angle.

Figure 3.20: Change in lift from no jet for a rectangular wing with circular end caps and a Gottingen 409 airfoil. Jet velocities of 18% and 36% over the freestream, $AR$ of 4 and $Re = 406,000$ with experimental results from Stuper and numerical results from Hunsaker. 100 spanwise lattices used.
Chapter 4

Conclusions

The study of propeller-wing interaction is important for future aeronautical advancements in taking advantage of the increasing usage of propeller driven flight in both large and small scale aircraft. Computational studies and experimental work have shown that propeller-wing advantages can be harnessed under certain designs. However, these methods are inherently time-consuming and costly, which provides a reason to develop low-order models that can accurately predict these effects at a fraction of the time and cost for use in early engineering design phases. Without such models it is difficult to incorporate the complexities of propeller-wing interaction further down in the design process.

The objective of this research is to develop a low-order propeller-wing interaction model that is capable of quickly predicting the lift distribution and total wing lift of a wing in the slipstream of a propeller. The model is based on a Weissinger method incorporating the propeller induced velocity fields when calculating the circulation over the wing. The propeller-induced velocities are calculated using a vortex ring method and blade performance theory, which keep the details and number of inputs required to a minimum. This approach allows for many different configurations to be tested in quick succession and compared so that a design can be optimized to meet design requirements.

Overall, the lift distributions agree well with experimental and other numerical results with
only major discrepancies occurring near stall. The trends in total wing lift for various propeller positions and performance are also captured by the model, but with small differences seen in wing lift-curve slope from experimental values. The results have shown that good predictions can be made even for difficult cases with the inclusion of corrections. The difficult cases that have been studied here include low Reynolds numbers with large laminar separation bubbles as well as low aspect ratio wings.

4.1 Future Improvements

Due to the nature of the models, there are many complex effects that are not taken into account. However, there are improvements that can be made to the model to make it able to apply to a broader range of cases and configurations. One of these is the inclusion of a more accurate induced propeller velocity model that does not require the additional details of blade performance theory. This could be done by incorporating a vortex lattice method with spiraling trailing vortices from the propeller blades. A model using this technique would be able to predict both the axial and tangential velocities within the slipstream over time as the propeller is rotating, which means wing lift could also be predicted over the time period.

The addition of a numerical model to mimic the curve of the propeller slipstream if the propeller and freestream are not aligned would allow for more accurate results far downstream when the propeller is at and angle of attack. The inclusion of this into the model will likely result in better prediction for the induced velocities for surfaces such as horizontal and vertical tails that reside further downstream of the propeller plane. Cases where the propeller axial direction is not aligned with the freestream are common in actual flight conditions and may have their own benefits or detriments that could be studied with a model accounting for the effects.

The current model does not predict the effects of a pusher propeller or a propeller downstream of a lifting surface at this time. By adding a model which predicts the effects of a downstream propeller, further aircraft configurations can be studied using the model. Conversion of the wing model to a vortex lattice method would also give an opportunity for the model to study the
chordwise effects of the propeller on the wing instead of just the spanwise effects as presented in this thesis. A VLM would also be more accurate in the overall prediction of wing especially when using a detailed 3D VLM. However, this will also have the effect of increasing the computational time of the model. Each of these improvements focus on the model’s ability to predict the propeller-wing interaction effects for a large variety of cases with the minimum number of necessary inputs.
REFERENCES


