
This dissertation presents economically meaningful trend-cycle decompositions of macroeconomic data. Chapter 1 introduces the common themes and goals of the dissertation.

Chapter 2 estimates unobserved components (UC) models with real and financial trends and business and credit cycles to assess different measures of the credit cycle used by policymakers. The permanent components of the real and financial sectors are a Beveridge-Nelson and local linear trend, respectively. The business and credit cycles evolve jointly as a second-order vector autoregression. Bootstrap methods are applied to UC model estimates retrieved from classical optimization of the predictive likelihood of the Kalman filter. Results indicate the slope of the financial trend better predicts the credit to GDP ratio in the United States than the estimated business and credit cycles and the Basel gap. This suggests policymakers should focus on permanent shocks to the financial sector to gauge the state financial stability.

Chapter 3 estimates new Keynesian unobserved components (NK-UC) models. The NK-UC models feature a consumption generating equation with habit formation, a hybrid-new Keynesian Phillips curve (NKPC), the Fisher equation, and a monetary policy rule in the real interest rate. These equations are informed by the decomposition of consumption, real aggregate activity, inflation, and the nominal policy rate into trends and gaps. I assume the permanent components of real aggregate activity and inflation are Beveridge and Nelson (1981) trends. Including the Fisher equation in the NK-UC models forces realized inflation to share its trend with the nominal policy rate, which defines the unobserved real rate. The NK-UC models are estimated with different observed measures of real aggregate activity on quarterly U.S. data from 1960 to 2018. Bootstrap likelihood ratio tests indicate the addition of a serially correlated markup shock in the hybrid-NKPC improves the fit of the NK-UC models.
DEDICATION

To my wife, Christie.
BIOGRAPHY

Andrew Hessler was raised in Montauk, New York. Andrew graduated from Phillips Exeter Academy as a member of the Class of 2010. He received bachelors degrees in Economics and Mathematics from SUNY Geneseo in 2014. Andrew began his doctoral study in Economics at North Carolina State University in 2014. His graduate research is in applied macroeconomics. During his time in Raleigh, he worked as an instructor in the Poole College of Management. Andrew will join the Department of Economics at Williams College as a visiting assistant professor starting Fall 2022.
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The dynamics of the business cycle are central to macroeconomics. An challenge facing policymakers and practitioners is the nonstationarity of observed macroeconomic aggregates. Examining the comovement of economic variables across the business cycle necessitates observed data to be rendered stationary. The measurement of unobserved trends and cycles continues to be an important task for economists.

The trend-cycle decomposition of macroeconomic aggregates by atheoretic detrending methods remains prevalent despite well-known criticisms. The Hodrick-Prescott filter is a prominent example. Harvey and Jaeger (1993) and Cogley and Nason (1995a) demonstrate how atheoretic methods like the Hodrick-Prescott filter lead to spurious results for I(1) variables. Atheoretic detrending methods impose a priori assumptions on the data and provide no cross-equation restrictions between economic variables. The resulting estimates cannot be given a structural interpretation. As pointed out by Koopmans (1947), an economically meaningful cycle cannot exist without theory.

This dissertation focuses on imposing economic theory on unobserved components (UC) models to conduct the trend-cycle decomposition of macroeconomic aggregates. Unobserved components models are state space models with unobserved states. In Chapter 2, cross equation restrictions are imposed on the real and financial sectors to estimate
trends and cycles. The new Keynesian framework informs the decomposition of real activity, inflation, and interest rates into trends and cycles in Chapter 3.

The observed variables considered in this dissertation are assumed to have unit roots. Trends capture long-run relationships between observed variables as implied by economic theory. The cycles of the UC models evolve jointly as vector autoregressions (VARs). This specification allows for the examination of the cyclical dynamics between sectors. The flexibility of UC models permits a variety of decompositions to be considered. While remaining flexible, UC models impose fewer cross-equation restrictions than a fully fledged linearized dynamic stochastic general equilibrium model.

The UC models in this dissertation are linear. The Kalman filter is applied to these linear state space models to compute the predictive log likelihood of the UC models given initial states and an initial parameter vector. Classical optimization maximizes the log likelihood to obtain estimates of the parameters and states of the models. The cycles of the UC models are highly persistent. This creates an issue using asymptotic theory to analyze the estimates. When AR parameters are near the boundary of the parameter space, the sampling distribution is non-normal; see Morley et al. (2003).

To overcome this issue, I bootstrap empirical distributions of the parameters and states following Stoffer and Wall (2004). This helps avoid issues with using large sample theory in presence of small samples and some parameters on the edge of the parameter space. Additionally, bootstrap likelihood ratio tests are employed to compare competing UC models as in Morley et al. (2016).

In Chapter 2, I estimate unobserved components models to examine credit cycle estimation. A small new Keynesian UC model is presented and is used to examine key frictions in the NK framework in Chapter 3.
2.1 Introduction

Since the global financial crisis of 2007-2009, economists and policymakers have renewed their focus on financial markets. Their motivation is to better understand whether macroprudential policy should be employed to stabilize fluctuations in output and inflation in response to shocks in financial markets. For example, the countercyclical capital buffer (CCyB), which is a macroprudential policy tool introduced in the Basel III Agreement, asks regulators to increase (decrease) bank capital buffers when financial markets experience
an expansionary (contractionary) episode.\footnote{The Basel Committee on Banking Supervision (2017a) define a capital buffer as the ratio of high quality liquid assets held by banks to the individual bank's risk weighted assets.} The purpose of this macroprudential policy tool is to smooth business cycle fluctuations by reducing instabilities in financial markets.

The effectiveness of the CCyB depends on accurate measurement of the business cycle on the real side of an economy and similar transitory fluctuations in its financial markets. The credit cycle is the label often given to the hidden state variable that captures the response of financial markets to transitory disturbances. The Basel Committee on Banking Supervision (2010) recommends measuring the credit cycle by applying a one-sided Hodrick-Prescott (HP) filter to the quarterly credit to GDP ratio.

The choice by the Basel Committee on Banking Supervision (BCBS) to use the HP filter has drawn criticism. Along with the well known critiques of the HP filter by Harvey and Jaeger (1993), King and Rebelo (1993), Cogley and Nason (1995b), Canova (1998), and Hamilton (2018) among others, there is an extensive literature showing the BCBS-HP filtered credit cycle can exhibit excess volatility and persistence and its conditional mean can be biased. A subset of this literature includes Edge and Meisenzahl (2011), Alessandri et al. (2015), Barrell et al. (2018), Darraçq Pariès et al. (2019), Jokipii et al. (2020), and Alessandri et al. (2021). An interesting aspect of these critiques is provided by Galán and Mencía (2021), Schüler (2020), and Jylha and Lof (2021). They draw attention to the BCBS (2010) suggesting the HP smoothing parameter be set to $4 \times 10^6$ instead of the conventional value of 1600 for quarterly data.

Despite these issues, the Basel Committee on Banking Supervision (BCBS) reports that its HP filtered credit cycle, which is also known as the Basel gap, is widely used by national financial market regulators. Support for the Basel gap is found in Drehmann et al. (2010), Drehmann and Juselius (2014), and Borio et al. (2016). Drehmann et al. (2010) contend the Basel gap is a leading indicator of financial distress. Drehmann and Juselius (2014) support this finding over long horizons, while an alternative measure, the debt service ratio, performs better over shorter horizons. Borio et al. (2016) find incorporating the Basel gap into estimations of potential output improves the precision and robustness of real time output gap estimates.

This paper presents an alternative approach to estimating and testing the credit cycle motivated by the lack of consensus about the Basel gap. I obtain estimates of the credit cycle by imposing restrictions associated with the permanent income hypothesis (PIH) and a macro-finance theory of leverage on unobserved components (UC) models. The PIH predicts a decomposition of consumption and income into the common PI trend and
business cycle. I extend ideas of Brunnermeier and Sannikov (2014) to place restrictions on the financial sector. Brunnermeier and Sannikov (2014) construct a macro-finance model in which the demand for credit originates in the optimal choice of leverage by borrowers. This choice predicts a long-run equilibrium that jointly restricts movements in capital and the level of debt held by the productive sector. Similar to the PIH, the long-run relationship in the stock of nonfinancial assets and credit supply predicts a permanent-transitory decomposition, which I refer to as the financial trend and the credit cycle.

The UC models embed the trend-cycle restrictions of the PIH and the Brunnermeier and Sannikov theory of leverage in the measurement equations. The measurement equations are grounded in a vector of constant dollar observables consisting of non-durable goods and services consumption expenditures, disposable income, nonfinancial credit, and nonfinancial assets. These variables are sufficient to recover the state variables, which are the PI and financial trends and business and credit cycles, given an appropriate specification of the system of state equations. I assume the PI trend evolves as a random walk with drift while a local linear trend produces the permanent financial component. As a result, the PI trend is interpreted as the permanent component of the Beveridge and Nelson (1981) decomposition while the permanent component of the financial sector consists of the levels trend and its time-varying I(1) slope.

An unrestricted second-order bivariate autoregression generates the business and credit cycles. This is my baseline UC model. I create five additional UC models by placing restrictions on the lag coefficients of the business and credit cycles or covariance matrix of the errors of the reduced-form VAR(2). The UC models are estimated using the Kalman filter and its predictive likelihood and classical optimization methods on a quarterly U.S. sample from 1960 to 2018. However, bootstrapped methods are employed to construct the empirical sampling distributions of the UC model parameters, state variables, and test statistics.

Estimating the UC models yield five main contributions. First, bootstrapped likelihood ratio (LR) tests favor the baseline UC model that lacks exclusion restrictions on the lags of the other cycle in the two regressions of the reduced-form VAR(2). Nonetheless, joint tests of these lag coefficients suggest Granger causality does not run in either direction between the business and credit cycles.

Second, estimates of the slope of the financial trend and the business cycle display troughs during almost every NBER dated recession between 1960 and 2018. The credit cycle has three troughs. The first two troughs are in the mid 1960s and mid 1990s, but the third occurs at the end of the 2007-2009 recession and financial crises. My estimated credit cycle
is more persistent and volatile than the business cycle, but less volatile and smoother than the Basel gap.

Third, mapping the reduced-form VAR(2) into a structural VAR (SVAR) yields impulse response functions (IRFs) for the business and credit cycles with respect to their shocks. When the business cycle is ordered after the credit cycle, its IRF to a credit cycle shock is humped-shaped. However, the uncertainty bands around this IRF include zero at every forecast horizon except between the 1- and 2-year horizons. Reversing the order of the SVAR results in substantial uncertainty surrounding the equivalent IRF of the business cycle to a credit cycle shock.

Fourth, I report predictive regressions of the \( h \)-quarter ahead bootstrapped business cycles on the bootstrapped credit cycle. Tests show the credit cycle is a useful predictor of the \( h \)-step ahead business cycle only at horizons longer than two years. In contrast, the business cycle has predictive power for the credit cycle at every forecast horizon from one quarter to four years.

Fifth, regressing the \( h \)-quarter ahead growth rate of the credit to GDP ratio on the Basel gap results in serially correlated residuals at all forecast horizons. This reinforces results in Alessandri et al. (2021), Galán and Mencía (2021), and Schüler et al. (2020) that the Basel gap is a weak predictor of the future path of the credit to GDP ratio. Adding the bootstrapped business and credit cycles to the regression yields serially uncorrelated residuals at low-order forecast horizons. Interestingly, my estimate of the slope of the financial trend is the best predictor of the growth of the credit to GDP ratio considered. Regressing the \( h \)-quarter ahead growth rate of the credit to GDP ratio on the bootstrapped slope of the financial trend produces serially uncorrelated residuals at forecast horizons of up to one year. This finding suggests it is permanent shocks to the financial sector which matter for financial stability.

My approach to studying the credit cycle within a structural time series model is closest to Galati et al. (2016) and Rünstler and Vlekke (2018). They also estimate UC models to recover estimates of the credit cycle. However, their interest is only in estimating the credit cycle. I estimate UC models on real and financial variables to generate estimates of the PI and financial trends and business and credit cycles. Loading additional observable information into UC models that restrict the joint process generating the real and financial sides of the U.S. economy gives estimates of the credit cycle that are more efficient and economically interesting. Further, estimates of the slope of the financial trend and business and credit cycles are used to assess their and the Basel gap’s predictive content. The predictive regressions indicate my estimates of the business and credit cycles and especially the slope
of the financial trend provide better signals of the state of the financial markets than the Basel gap. Hence, my results lend support to a growing literature that argues policymakers exercise caution if using the Basel gap to assess the state of the financial markets for which they are responsible.

Section 2 lays out the UC models. Section 3 describes the data. My estimation methods are discussed in Section 4. Section 5 presents the estimates of the PI and financial trends and business and credit cycles, estimates of the SVARs of these cycles, the IRFs and forecast error variance decompositions (FEVDs), and predictive regressions. Section 6 concludes.

2.2 The UC Models

I estimate PI and financial trends and business and credit cycles using UC models. The UC models are described by measurement and state transition equations. The measurement vector, $Y_t$, contains the $n$ observed variables in the model. The system of measurement equations is

$$Y_t = CX_t + De_t.$$  \hspace{1cm} (2.1)

In equation (2.1), the measurement error, $e_t$, is a white noise process with $\text{var}(e_t) = I_n$. The states are placed in the $k$-dimensional vector $X_t$, which evolves as the system of state transition equations

$$X_t = AX_{t-1} + H + B\epsilon_t.$$ \hspace{1cm} (2.2)

Static drift parameters are stored in the vector $H$, which also contains zeros. In equation (2.2), the state transition error is a white noise process with $\text{var}(\epsilon_t) = I_m$, $m \leq k$.

2.2.1 The PIH and the Business Cycle

The PIH identifies the common trend of the consumption-income pair. Households consume their PI level which is their current expected discounted level of future income. By assuming a random walk with drift drives the PI trend, it is identified with the Beveridge and Nelson (1981) trend as does Morley (2007). The consumption-income pair yield the business cycle as the common transitory component that remains after removing the common PI trend.
2.2.2 Leverage, the Financial Trend, and the Credit Cycle

Much of the literature measures credit cycles from the perspective of firms’ and households’ ability to pay their debt obligations. For example, Drehmann et al. (2012) calculate a credit cycle using a band-pass filter on the ratio of credit to GDP. In this interpretation, the credit cycle is in an expansionary phase when credit growth outpaces income growth. Their story is increasing debt, relative to income, increases default risk in financial markets and the likelihood of a credit contraction in the future.

My models innovate by identifying a long-run relationship between credit supply and nonfinancial assets. This long-run relationship is motivated by the macro-finance theory of Brunnermeier and Sannikov (2014). Their model begins with productive agents borrowing from non-productive agents to purchase physical capital. Productive agents seek to maximize growth in net worth by targeting a level of leverage. Leverage is defined as the percentage of net worth borrowed to fund physical capital expenditures. Leverage is stationary in this model, which predicts there is a long run relationship between debt and physical capital. Similar to Brunnermeier and Sannikov (2014), deviations from this long-run relationship are identified as the credit cycle.

The permanent financial component is a local linear trend. This specification implies the level of the trend and its slope are I(1) processes. A local linear trend is consistent with the HP and Baxter-King filters, as discussed by Harvey and Trimbur (2003), among others. This assumption also makes for straightforward comparisons with studies using these filters to estimate credit cycles as, for example, by Borio et al. (2018).

2.2.3 The Measurement Equations

Restrictions on the real and financial sectors of Model 1 are embedded in the system of measurement equations

\[
\begin{bmatrix}
    c_{on_t} \\
    inc_t \\
    nfc_t \\
    nfa_t \\
\end{bmatrix}
= \begin{bmatrix}
    1 & 0 & 0 & \kappa & 0 & 0 & 0 \\
    \alpha & 0 & 0 & 1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 & 0 & \lambda & 0 \\
    0 & \beta & 0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
    \tau_t \\
    \psi_t \\
    \xi_t \\
    \delta_t \\
\end{bmatrix}
+ De_t, \tag{2.3}
\]
where $e_t = [e_{\text{con},t} \ e_{\text{inc},t} \ e_{\text{nfc},t} \ e_{\text{nfa},t}] \sim N(0_{4\times1}, I_{4\times4})$, and $D$ is a square matrix with the volatility of measurement errors $\sigma_{\text{con},t}, \sigma_{\text{inc},t}, \sigma_{\text{nfc},t},$ and $\sigma_{\text{nfa},t}$ on the diagonal and zeros elsewhere. The real sector, consumption and income, is composed of the PI trend, $\tau_t$, and the business cycle, $\delta_t$. Similar to Morley (2007), I normalize the response of consumption, $\text{con}_t$, to the PI trend. The factor loading of income, $\text{inc}_t$, on the business cycle is also normalized to one. In equation (2.3), $\alpha_t$ is the factor loading of income on the PI trend, and $\kappa_t$ is the factor loading of consumption on the business cycle. Further, I normalize the response of credit supply, $\text{nfc}_t$, to the financial trend, $\psi_t$, and the response of nonfinancial assets, $\text{nfa}_t$, to the credit cycle, $\phi_t$. The response of nonfinancial assets to the financial trend is described by $\beta$. The response of the supply of credit to the credit cycle is measured by $\lambda$.

### 2.2.4 The State Equations

The trends and cycles of the UC models make up the state vector. The PI trend is a random walk with drift $\mu$, which is consistent with a Beveridge-Nelson trend. As already mentioned, the financial trend evolves as a local linear trend.\(^2\) The level of the financial trend is $\psi_t$ and $\xi_t$ is its slope. The business and credit cycles, $\delta_t$ and $\phi_t$, are a reduced-form VAR(2). This structure is summarized in the system of state transition equations of Model 1

\[
\begin{pmatrix}
\tau_{t+1} \\
\psi_{t+1} \\
\xi_{t+1} \\
\delta_{t+1} \\
\delta_t \\
\phi_{t+1} \\
\phi_t
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\tau_t \\
\psi_t \\
\xi_t \\
\delta_t \\
\delta_{t-1} \\
\phi_t \\
\phi_{t-1}
\end{pmatrix} +
\begin{pmatrix}
\mu \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix} + B e_t,
\]

where $e_t = [e_{\text{con},t} \ e_{\text{inc},t} \ e_{\text{nfc},t} \ e_{\text{nfa},t}] \sim N(0_{4\times1}, I_{4\times4})$, and $D$ is a square matrix with the volatility of measurement errors $\sigma_{\text{con},t}, \sigma_{\text{inc},t}, \sigma_{\text{nfc},t},$ and $\sigma_{\text{nfa},t}$ on the diagonal and zeros elsewhere. The real sector, consumption and income, is composed of the PI trend, $\tau_t$, and the business cycle, $\delta_t$. Similar to Morley (2007), I normalize the response of consumption, $\text{con}_t$, to the PI trend. The factor loading of income, $\text{inc}_t$, on the business cycle is also normalized to one. In equation (2.3), $\alpha_t$ is the factor loading of income on the PI trend, and $\kappa_t$ is the factor loading of consumption on the business cycle. Further, I normalize the response of credit supply, $\text{nfc}_t$, to the financial trend, $\psi_t$, and the response of nonfinancial assets, $\text{nfa}_t$, to the credit cycle, $\phi_t$. The response of nonfinancial assets to the financial trend is described by $\beta$. The response of the supply of credit to the credit cycle is measured by $\lambda$.

\(^2\)Attempts to model the financial trend as a random walk with drift were not supported by the data.
Table 2.1: Summary of Model Restrictions

### Reduced-form VAR specifications

<table>
<thead>
<tr>
<th>Model</th>
<th>Cycle Description</th>
<th>Restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>Granger causality runs in both directions.</td>
<td>none</td>
</tr>
<tr>
<td>Model 2</td>
<td>Credit cycle Granger causes business cycle.</td>
<td>$\zeta_1 = \zeta_2 = 0$</td>
</tr>
<tr>
<td>Model 3</td>
<td>Business cycle Granger causes credit cycle.</td>
<td>$\theta_1 = \theta_2 = 0$</td>
</tr>
<tr>
<td>Model 4</td>
<td>Cycles not Granger cause each other.</td>
<td>$\zeta_1 = \zeta_2 = \theta_1 = \theta_2 = 0$</td>
</tr>
</tbody>
</table>

### SVAR specifications

<table>
<thead>
<tr>
<th>Model</th>
<th>Cycle Description</th>
<th>Restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 5</td>
<td>At impact the business cycle responds to the credit cycle.</td>
<td>$\zeta_0^* = \sigma_{\delta,\phi} = 0$</td>
</tr>
<tr>
<td>Model 6</td>
<td>At impact the credit cycle responds to the business cycle.</td>
<td>$\theta_0^* = \sigma_{\delta,\phi} = 0$</td>
</tr>
</tbody>
</table>

Where $\varepsilon_t = \begin{bmatrix} \varepsilon_{\tau,t} \\ \varepsilon_{\psi,t} \\ \varepsilon_{\xi,t} \\ \varepsilon_{\delta,t} \\ \varepsilon_{\phi,t} \end{bmatrix} \sim N(0_{5 \times 1}, I_{5 \times 5})$, and $BB' = \begin{bmatrix} \sigma_\tau^2 & 0 & 0 & \sigma_{\tau,\delta} & 0 & 0 & 0 \\ 0 & \sigma_\psi^2 & 0 & 0 & 0 & \sigma_{\psi,\phi} & 0 \\ 0 & 0 & \sigma_\delta^2 & 0 & 0 & 0 & 0 \\ \sigma_{\tau,\delta} & 0 & 0 & \sigma_\delta^2 & 0 & \sigma_{\delta,\phi} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{\psi,\phi} & 0 & \sigma_{\delta,\phi} & 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

In the system of state equations (2.4), innovations to the trend and cycle within a sector are correlated. The PI trend and the financial trend are independent. In Model 1, the real and financial sectors are connected by the VAR(2) specification of the business and credit cycles. Similar restrictions on the state equations are found in Lee and Nelson (2007).

I estimate three alternative UC models by placing exclusion restrictions on the reduced-form VAR(2) of the state system of equations (2.2). Model 2 sets the response of the credit cycle to lags of the business cycle, $\zeta_1$ and $\zeta_2$, to zero. The credit cycle Granger causes the business cycle under these restrictions, which are motivated by Borio et al. (2018). Next, Model 3 assumes Granger causality runs in the opposite direction by restricting $\theta_1 = \theta_2 = 0$. In this case, there is reduced-form predictability from lags of the business cycle to the
credit cycle. The final model, Model 4, imposes zero restrictions on the off diagonals, \( \zeta_1 = \zeta_2 = \theta_1 = \theta_2 = 0 \), of the reduced-form VAR(2). The dynamics of the business cycle and credit cycle are separate in Model 4. The top panel of Table 2.1 summarizes Models 1, 2, 3, and 4.

2.2.5 Using SVARs to Generate Business and Credit Cycles

Models 1, 2, 3, and 4 have reduced-form VARs that can be mapped into structural VARs. The structural VAR is

\[
\Theta_0 \begin{bmatrix} \delta_t \\ \phi_t \end{bmatrix} = \Theta_1 \begin{bmatrix} \delta_{t-1} \\ \phi_{t-1} \end{bmatrix} + \Theta_2 \begin{bmatrix} \delta_{t-2} \\ \phi_{t-2} \end{bmatrix} + B^*_c \epsilon_{c,t},
\]

(2.5)

where \( \epsilon_{c,t} \sim N(0_{2 \times 1}, I_{2 \times 2}) \), \( B^*_c B^*_c = \begin{bmatrix} \sigma^*_\delta^2 & 0 \\ 0 & \sigma^*_\phi^2 \end{bmatrix} \), and \( B^*_c = \Theta_0 B_c \) is the submatrix of \( B \) corresponding to \( \delta_t \) and \( \phi_t \). The first step in mapping from the reduced-form VARs of Models 1, 2, 3, or 4 to the structural VAR of (2.5) involves pre-multiplying (2.5) by \( \Theta^{-1}_0 \)

\[
\begin{bmatrix} \delta_t \\ \phi_t \end{bmatrix} = \Theta^{-1}_0 \Theta_1 \begin{bmatrix} \delta_{t-1} \\ \phi_{t-1} \end{bmatrix} + \Theta^{-1}_0 \Theta_2 \begin{bmatrix} \delta_{t-2} \\ \phi_{t-2} \end{bmatrix} + \Theta^{-1}_0 B^*_c \epsilon_{c,t},
\]

where \( \Theta^{-1}_0 B^*_c \epsilon_{c,t} \sim N(0_{2 \times 1}, \Theta^{-1}_0 B^*_c B^*_c \Theta^{-1}_0) \). Next, the impact matrix, \( \Theta_0 \), of the structural VAR is recovered using one of two orderings of the business and credit cycles. In the first structural VAR, which is labeled Model 5, the credit cycle is structurally causally prior to the business cycle

\[
\Theta_{0, CB} = \begin{bmatrix} 1 & -\theta^*_0 \\ 0 & 1 \end{bmatrix}.
\]

Take the upper Cholesky decomposition of the covariance matrix of the business and credit cycles of the reduced-form VAR, \( \Theta^{-1}_0 B^*_c B^*_c \Theta_{0, CB}^{-1} \), which requires solving the bivariate system

\[
\left[ \Theta_{0, CB}^{-1} B^*_c B^*_c \Theta_{0, CB}^{-1} \right]^{1/2} = \begin{bmatrix} 1 & -\theta^*_0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sigma^*_\delta^2 & 0 \\ 0 & \sigma^*_\phi^2 \end{bmatrix}.
\]

The structural VAR is found by pre-multiplying the lag coefficient matrices of the reduced-form VAR by \( \Theta_{0, CB} \) to produce Model 5.
Model 6 reverses the structural ordering to place the business cycle before the credit cycle

\[ \Theta_{0,BC} = \begin{bmatrix} 1 & 0 \\ -\zeta_0^* & 1 \end{bmatrix}. \]

A similar process recovers this impact matrix

\[
\begin{bmatrix} \Theta_{0,BC} & B_c^* B_c^\prime \Theta_{0,BC}^{-1} \end{bmatrix}^{1/2} = \begin{bmatrix} 1 & 0 \\ -\zeta_0^* & 1 \end{bmatrix}^{-1} \begin{bmatrix} \sigma_{\phi}^* & 0 \\ 0 & \sigma_{\phi}^* \end{bmatrix},
\]

but in this case a lower Cholesky decomposition of the covariance matrix of the reduced-form VAR innovations is computed. The coefficient matrices of the structural VAR, Model 6, are recovered by pre-multiplying the reduced-form VAR by \( \Theta_{0,BC} \).

### 2.3 Data

Data on consumption and income in the U.S. measures activity in the real sector. The financial sector is measured by data on credit supply and nonfinancial assets. The data are in constant dollars, per capita, logged and multiplied by 400.\(^3\) The quarterly sample runs from 1960Q1 to 2018Q4.

#### 2.3.1 The Real Sector

Consumption is equated to aggregate personal consumption expenditures on nondurable goods and services. Tests of the PIH most often measure consumption as its flow from nondurable goods and services. For example, at least since Hall (1978), consumer durable goods expenditures are excluded to avoid issues with imputing the value of the service flow from the stock of these goods. I use an ideal Fisher index to construct constant dollar nondurable goods and services consumption as discussed in Whelan (2002). Income is measured by real personal income excluding transfer payments.\(^4\)

#### 2.3.2 The Financial Sector

Data for the financial sector comes from the Financial Accounts of the United States that is published by the Board of Governors of the Federal Reserve System. Credit supply is the

---

\(^3\)Details about the construction of the data are given in Appendix B.2.

\(^4\)Consumption and income data are retrieved from FRED at the Federal Reserve Bank of St. Louis.
**Figure 2.1:** Data in Log Levels, 1960Q1-2018Q4

Notes: The gray bars represent NBER recession dates. Details of data construction are found in Section B.2.
**Figure 2.2:** Growth Rates, 1960Q1-2018Q4

Notes: The dotted lines represent the mean of the growth rates over the sample period. Otherwise, see the notes to Figure 2.1.
sum of debt securities and loans of nonfinancial corporate businesses, households and nonprofit organizations, as well as loans of nonfinancial noncorporate businesses. This measure of credit is used by Borio (2014) and Drehmann et al. (2010) in their construction of the credit to GDP ratio. Aggregate nonfinancial assets of the private nonfinancial sector are held by nonfinancial corporate businesses, nonfinancial noncorporate businesses, and households and nonprofit organizations.

2.3.3 Describing the Data

Figures 2.1 and 2.2 plot the data in log levels and growth rates. Consumption and income appear to comove throughout the sample. Income is more volatile than consumption and business cycle movements are more pronounced. Consumption growth is below its sample mean and income declines during each NBER dated recession. Contractions in these series are most pronounced in the 1973-1975 and 2007-2009 recessions. Both recessions are of similar duration and severity in the real sector.

Credit supply and nonfinancial assets often contract during NBER dated recessions with the exception of the 2001 recession. Moreover, the financial series seem to have a prolonged period of negative growth on either side of the 1991 recession. This episode was followed by more than a decade of above average growth in credit supply and nonfinancial assets leading up to steep declines during the financial crisis. Credit supply growth remained well below its sample mean for several years following the most recent financial crisis.

2.4 Econometric Methods

The innovations form of the Kalman filter is used to compute the log likelihood of Models 1, 2, 3, and 4, given initial state conditions, $X_{0|0}$, and an initial parameter vector, $\Gamma_{0}$.

The log likelihood is maximized, via classical optimization, to obtain estimates of the parameters and states of the UC models.

I adapt the bootstrap algorithm of Stoffer and Wall (2004) to produce the small sample distributions of model parameters and the states. Bootstrapped empirical distributions of the maximum likelihood (ML) estimates overcome problems created by reduced rank Hessian matrices and applying asymptotic theory in the presence of small sample sizes; see

---

5A detailed discussion of the ML estimation and the innovations form of the Kalman filter is given in Appendix A.2. The initialization of the innovations form of the Kalman filter is discussed in B.3.2. I apply the bootstrap procedure of Stoffer and Wall (2004), which is described in Appendix B.4.
Angelini et al. (2021), Stoffer and Wall (2004), and Ansley and Newbold (1980). Another issue with using asymptotic theory is the autoregressive parameters are near the boundary of the parameter space when cyclical components are highly persistent; see Morley et al. (2003). The bootstrap algorithm first resamples with replacement the standardized errors from the Kalman filter of the ML estimates. These resampled standardized errors are used to back out a synthetic sample using the state space representation of a UC model. Next, the UC model is estimated on the bootstrap sample and the results are recorded. One thousand artificial samples are produced to create bootstrap distributions of UC model parameters, the covariance matrix of the parameters, and likelihood ratio statistics.6

Empirical distributions of likelihood ratio (LR) statistics are used to evaluate which UC model best fits the data. The LR tests give evidence about whether the credit cycle Granger causes the business cycle. The likelihood of the UC model under the null corresponds to Model 1. The null is compared with Model 2, Model 3, and Model 4. Bootstrap methods described by Morley et al. (2016) produce the empirical distributions of the LR statistics. The LR statistics are computed at the ML estimates of the alternative and null UC models

\[ LR = -2(l h(\hat{\Gamma}_i) - l h(\hat{\Gamma})) \], \( i \in [2, 3, 4] \),

where \( l h \) denotes the UC model likelihood.

There is a five step algorithm to compute bootstrap p-values of the LR statistics. The steps are

i. generate 1000 bootstrap samples under the null of Model 1,

ii. estimate the UC models on the 1000 bootstrap samples,

iii. calculate 1000 bootstrap log likelihoods for the UC models,

iv. construct 1000 LR statistics for UC Models 2, 3, and 4 against the null of UC Model 1,

v. count the number of LR statistics greater than its sample counterpart for the three UC model comparisons.

The p-values equal the counts obtained in step (v) of the algorithm divided by 1000.

---

6Julia 1.3.1 is used to estimate the UC models and generate the bootstrap samples. Code is available upon request.
2.5 Results

Section 2.5.1 reviews estimates of Models 1, 2, 3, and 4. The fit of the UC models and the results of the bootstrap LR tests are in Section 2.5.2. Section 2.5.3 discusses the estimates of the trends and cycles across the sample period. I report the IRFs and FEVDs of Models 5 and 6 in Section 2.5.4. Section 2.5.5 explores the reduced form predictive content of the credit cycle for the business cycle. Finally, Section 2.5.6 tests whether my estimated credit cycle predicts growth of the credit to GDP ratio better $h$-quarters ahead compared with the Basel gap.

2.5.1 UC Model Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model 1</th>
<th>Model 2 $\zeta_1, \zeta_2 = 0$</th>
<th>Model 3 $\theta_1, \theta_2 = 0$</th>
<th>Model 4 $\theta_1, \theta_2, \zeta_1, \zeta_2, = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>1.02 (&lt;0.01)</td>
<td>1.02 (&lt;0.01)</td>
<td>1.02 (&lt;0.01)</td>
<td>1.02 (&lt;0.01)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.62 (&lt;0.01)</td>
<td>0.62 (&lt;0.01)</td>
<td>0.62 (&lt;0.01)</td>
<td>0.62 (&lt;0.01)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.21 (0.09)</td>
<td>0.21 (0.09)</td>
<td>0.22 (0.09)</td>
<td>0.22 (0.09)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.22 (0.07)</td>
<td>0.21 (0.07)</td>
<td>0.22 (0.07)</td>
<td>0.21 (0.07)</td>
</tr>
<tr>
<td>$\sigma_{con}$</td>
<td>0.15 (0.18)</td>
<td>0.15 (0.18)</td>
<td>0.15 (0.18)</td>
<td>0.15 (0.18)</td>
</tr>
<tr>
<td>$\sigma_{inc}$</td>
<td>1.63 (0.26)</td>
<td>1.63 (0.26)</td>
<td>1.67 (0.21)</td>
<td>1.67 (0.21)</td>
</tr>
<tr>
<td>$\sigma_{nfc}$</td>
<td>0.55 (0.23)</td>
<td>0.56 (0.23)</td>
<td>0.57 (0.22)</td>
<td>0.58 (0.21)</td>
</tr>
<tr>
<td>$\sigma_{nfa}$</td>
<td>0.69 (0.20)</td>
<td>0.71 (0.19)</td>
<td>0.67 (0.22)</td>
<td>0.69 (0.21)</td>
</tr>
</tbody>
</table>

Notes: Bootstrap standard errors are calculated as $\sqrt{\frac{\sum_{b=1}^{B} (\hat{\theta}_b - \bar{\theta})^2}{B-1}}$ and based on $B = 1000$ bootstrap samples.

Table 2.2 reports estimates of the factor loadings, $\alpha$, $\beta$, $\lambda$, and $\kappa$ on the states. The
Table 2.3: ML Estimate of UC Model State Equations, 1960Q1-2018Q4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate (s.e.)</td>
<td>Estimate (s.e.)</td>
<td>Estimate (s.e.)</td>
<td>Estimate (s.e.)</td>
</tr>
<tr>
<td>$\mu_\tau$</td>
<td>1.52 (0.12)</td>
<td>1.52 (0.12)</td>
<td>1.52 (0.12)</td>
<td>1.52 (0.12)</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>1.62 (0.15)</td>
<td>1.62 (0.15)</td>
<td>1.68 (0.11)</td>
<td>1.68 (0.11)</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>-0.66 (0.15)</td>
<td>-0.66 (0.15)</td>
<td>-0.71 (0.11)</td>
<td>-0.72 (0.11)</td>
</tr>
<tr>
<td>$\vartheta_1$</td>
<td>0.07 (0.05)</td>
<td>0.07 (0.05)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\vartheta_2$</td>
<td>-0.08 (0.05)</td>
<td>-0.08 (0.05)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\zeta_1$</td>
<td>0.02 (0.11)</td>
<td>-</td>
<td>0.02 (0.11)</td>
<td>-</td>
</tr>
<tr>
<td>$\zeta_2$</td>
<td>-0.01 (0.11)</td>
<td>-</td>
<td>-0.00 (0.11)</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>1.71 (0.07)</td>
<td>1.73 (0.06)</td>
<td>1.69 (0.07)</td>
<td>1.71 (0.06)</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>-0.72 (0.07)</td>
<td>-0.75 (0.06)</td>
<td>-0.70 (0.07)</td>
<td>-0.72 (0.06)</td>
</tr>
<tr>
<td>$\sigma_\tau$</td>
<td>1.81 (0.26)</td>
<td>1.80 (0.26)</td>
<td>1.80 (0.34)</td>
<td>1.80 (0.31)</td>
</tr>
<tr>
<td>$\sigma_\psi$</td>
<td>1.18 (0.35)</td>
<td>1.15 (0.34)</td>
<td>1.14 (0.36)</td>
<td>1.12 (0.35)</td>
</tr>
<tr>
<td>$\sigma_\xi$</td>
<td>1.21 (0.14)</td>
<td>1.21 (0.14)</td>
<td>1.21 (0.14)</td>
<td>1.21 (0.14)</td>
</tr>
<tr>
<td>$\sigma_\delta$</td>
<td>1.71 (0.64)</td>
<td>1.71 (0.63)</td>
<td>1.69 (0.62)</td>
<td>1.69 (0.60)</td>
</tr>
<tr>
<td>$\sigma_\phi$</td>
<td>2.66 (0.36)</td>
<td>2.60 (0.34)</td>
<td>2.70 (0.37)</td>
<td>2.65 (0.34)</td>
</tr>
<tr>
<td>$\varrho_{\tau,\delta}$</td>
<td>-0.02 (0.06)</td>
<td>-0.02 (0.06)</td>
<td>-0.01 (0.06)</td>
<td>-0.01 (0.06)</td>
</tr>
<tr>
<td>$\varrho_{\psi,\phi}$</td>
<td>-0.15 (0.40)</td>
<td>-0.15 (0.48)</td>
<td>-0.15 (0.54)</td>
<td>-0.15 (0.69)</td>
</tr>
<tr>
<td>$\varrho_{\delta,\phi}$</td>
<td>0.05 (0.04)</td>
<td>0.05 (0.04)</td>
<td>0.07 (0.04)</td>
<td>0.08 (0.04)</td>
</tr>
</tbody>
</table>

Notes: See the notes to Table 2.2.
cointegrating vector of consumption and income is approximately \([1, -1]\) according to the estimates of \(\alpha\). This supports the PIH. The estimate of \(\kappa\) indicate movements in consumption are dominated by the PI trend rather than business cycle fluctuations. Credit supply grows at a slower rate than nonfinancial assets because the point estimates of \(\beta\) are nearer a half than one. Similar to the relationship of consumption and the PI trend, estimates of \(\lambda\) show the supply of credit responds far more to changes in the level of the financial trend compared with the credit cycle. Estimates of the factor loadings are consistent across the UC models implying differences in the models are not reflected in the measurement equations.

Measurement error in income displays the greatest volatility in the measurement equations. Estimates of \(\sigma_{inc}\) are more than ten times larger than estimates of \(\sigma_{con}\) and two to three times the size of estimates of \(\sigma_{nfa}\) and \(\sigma_{nfc}\). Comparing the latter two standard deviations show the volatility of the measurement errors of credit supply and nonfinancial assets have similar magnitudes.

Table 2.3 contains parameter estimates and associated bootstrap standard errors of the state equations of Models 1, 2, 3, and 4. Estimates of the drift in the PI trend, \(\mu\), are nearly identical across the models. The responses of the business cycle and credit cycle to their own lags, \(\theta_i\) and \(\gamma_i\) respectively, for \(i = 1, 2\), indicate that the cycles are highly persistent. Both pairs of parameters sum to close to one across all four models. This finding is further verified by the eigenvalues of the VAR(2). The eigenvalues of the VAR(2) in Model 1 are complex and indicate a high degree of persistence. A shock to the largest eigenvalue \((0.912 \pm 0.037i)\) has a half life of nearly two years. The largest eigenvalue of Model 2 is 0.938 while for Models 3 and 4 it is 0.954, which yield half-lives of about 3 years for Model 2 and almost 4 years for Models 3 and 4.

The off-diagonal elements of the VAR estimates, \(\vartheta_1\), \(\vartheta_2\), \(\zeta_1\), and \(\zeta_2\), capture the importance of lags in the credit cycle for the business cycle and lags of the business cycle for the credit cycle, respectively. These parameters are small and statistically insignificant for Models 1 through 3. This indicates there is little information contained in the business cycle for the credit cycle. The converse is also true.

Volatility of the shock innovation of the PI trend, \(\sigma_\tau\), is estimated to exceed that of the business cycle, \(\sigma_\delta\). The local linear trend of the financial sector produces estimates of the volatility of innovations to the slope of the financial trend, \(\sigma_\xi\), that are greater than the estimate of the volatility of innovations to the financial trend level, \(\sigma_\psi\). However, these components are less than half the size of the volatility of innovations to the credit cycle, \(\sigma_\phi\). The trend-cycle within sector correlations, \(\rho_{\tau,\delta}\) and \(\rho_{\psi,\phi}\), are negative but small and...
The correlation between cycles, ρδ,φ, is small across the models and statistically insignificant in Models 1 and 2, but has a t-ratio of about two in Models 3 and 4.

### Table 2.4: Bootstrap Likelihood Ratio Test Results

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<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
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<td>-2142.61</td>
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<tr>
<td>(boot. se)</td>
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<td>(35.82)</td>
<td>(35.75)</td>
<td>(11.28)</td>
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<tr>
<td>p-val</td>
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<td>0.201</td>
<td>0.266</td>
<td>0.154</td>
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</table>

Notes: The test statistic is $LR = -2(\ln(\hat{\Gamma}_i) - \ln(\hat{\Gamma}_j))$, $i \in [2, 3, 4]$, where $\Gamma_i$ is the parameter vector for model $i$. The p-values are computed as the percentage of bootstrap estimates that have a larger test statistic than the true value of the test statistic.

### 2.5.2 Fit of the UC Models

The results of the bootstrap likelihood ratio tests are summarized in Table 2.4. Model 1 is assumed to be the null model and is compared to the Models 2, 3, and 4 which are the alternative models. The null hypotheses fail to be rejected across the three tests. The data fits best to Model 1 with business and credit cycles that evolve jointly as a reduced-form VAR(2) relative to Models 2, 3, and 4. Thus, there is no evidence supporting Granger causality running from the credit cycle to the business cycle or the converse. This finding contradicts Borio et al. (2018). They present evidence of predictive causality running from the credit cycle to the business cycle. Placing exclusion restrictions on the business and credit cycles is at odds with the U.S. data. The rest of the paper focuses on the estimates produced by Model 1 as it is the model with the best fit to the data.

### 2.5.3 Estimates of the Trends and Cycles

Figures 2.3, 2.4, and 2.5 plot the median estimates of the bootstrap trends and cycles for the real and the financial sectors of Model 1 along with sup-t uncertainty bands of Olea and Plagborg-Møller (2018). The sup-t uncertainty bands yield simultaneous coverage.
Figure 2.3: Bootstrap Estimates of the Permanent Income and Financial Trends, 1960Q1 to 2018Q4

Notes: The gray bars represent NBER recession dates. The blue shaded areas are the 90% sup-t uncertainty bands.

Figure 2.4: Bootstrap Estimates of the Slope of the Financial Trend, 1960Q1 to 2018Q4

Notes: The blue shaded areas are the 68% sup-t uncertainty bands. Otherwise see notes to figure 3.
Figure 2.5: Bootstrap Estimates of the Business and Credit Cycles, 1960Q1 to 2018Q4

Notes: The red line is the Basel gap scaled by 500 to help draw comparisons. The blue shaded areas are the 68% sup-t uncertainty bands. Otherwise see notes to figure 3.

probability equal to the given confidence level. The significance level is 0.1 to achieve 90% uncertainty bands for plots of the PI trend and level of the financial trend. Plots of the slope of the financial trend and business and credit cycles are surrounded by 68% uncertainty bands implying a significance level of 0.32. Wider bands for the permanent components aid in visualization.

The PI trend and financial trend are similar from 1960 until after the “double-dip” recession. Both trends feature pronounced downward movements around the 1973-1975 recession and the “double-dip” recession as seen in the top and bottom panels of Figure 2.3. The PI trend differs from the financial trend in the subsequent period from 1983 to 2007. The PI trend grows steadily, while the financial trend exhibits large movements throughout the 1990s. Both trends contract during the 2007-2009 recession. During this period, movement in the PI trend is not as pronounced as the contractions of the 1973-1975 and “double-dip” recessions. However, there is substantial uncertainty surrounding the PI trend during these recessions. The financial trend has a sharp contraction from a peak in 2009 until 2013. The financial trend has not reached its pre-2007 level by the end of the sample in contrast to the PI trend.

There is also substantial uncertainty around the estimate of the slope of the financial
trend. Figure 2.4 shows the slope of the financial trend contracts during each NBER dated recession with the exception of the 1960-1961 and 2001 recessions. During the latter recession the slope actually increases. The most severe contractions in the slope occur during the 1973-1975 and 2007-2009 recessions.

The top and bottom panels of Figure 2.5 display the business and credit cycles. The former cycle has troughs at or after NBER dates. The credit cycle features long swings. The first credit cycle peak lines up with the double-dip recession. After this however, the credit cycle does not match up with NBER dates. The credit cycle bottoms out in the mid 1990s and peaks for a second time in 2005, two years before the most recent financial crisis. The credit cycle troughs in 2010 following the 2007-2009 recession. The estimate of the slope of the financial trend appears to move with the business cycle. This removes some of the business cycle comovement and less persistent movements from the credit cycle.

The business and credit cycles differ both quantitatively and qualitatively over the time period. The volatility of the credit cycle is much larger than the business cycle. The bootstrap median standard deviation of the business cycle is 11.08 with 5% and 95% quantiles of 9.22 and 13.55. These values for the credit cycle are 33.74, 30.66, and 36.62 respectively. These observations are in line with Borio (2014).

The Basel gap, plotted in Figure 2.5, is at odds with the estimated credit cycle. From 1960 until about 1983, the Basel gap is muted relative to the credit cycle. The Basel gap behaves much differently after 1983 with two long swings. Borio (2014) claims the shift in the behavior of the credit to GDP ratio in the mid-1980s reflects increasing financial liberalization and globalization which loosened financial constraints. The first major peak in the Basel gap occurs in 1986 several years after the estimated credit cycle, which peaks during the double-dip recession. Both series decline and experience a protracted trough in the mid-1990s followed by a steady climb into the 2000s. The estimated credit cycle peaks in 2005 and its 68% uncertainty bands do not cover the peak in the Basel gap in 2007. Its trough occurs in 2012 while the estimated about three years earlier. Hence, my estimate's indicate expansion in the U.S. financial markets ended two years or more before the start of that financial crisis and recession, but recovery was under way by the beginning of 2010.

The credit cycle is more persistent than the business cycle. The spectral densities of the business and credit cycles and Basel gap are plotted in the top and bottom panels of Figure 2.6. The top panel shows the spectral density of the business cycle achieves maximum power at 7.5 years per cycle. In contrast, Morley et al. (2003) find the business cycle has a period of 2.5 years. This discrepancy results from the business and credit cycles being a reduced form VAR(2). The maximum power is 10 years per cycle for the estimated credit
Figure 2.6: Bootstrapped Spectral Densities of the Business and Credit Cycles, 1960Q1 to 2018Q4

Notes: Plots display median bootstrap estimates of the spectral densities using a smoothed periodogram with Bartlett window of length 7. The red line is the spectral density of the Basel gap scaled by 24000 to help draw comparisons. The blue shaded areas are the 68% sup-t uncertainty bands.
cycle and for the Basel gap. These results contrast with those of Drehmann et al. (2012) who find the length of their average credit cycle to be around 16 years.

### 2.5.4 Structural VAR Results

Table 2.5 reports parameter estimates of the structural VARs, Model 5 and Model 6. The business cycle responds to the credit cycle on impact in Model 5. The impact response is reversed in Model 6. The business cycle responds negatively on impact to the credit cycle in Model 5, although the estimate of $\varphi_0^*$ is statistically insignificant. In Model 6, the credit cycle responds negatively to the business cycle on impact, as shown by $\zeta_0^*$. Once again, the estimated impact coefficient is statistically insignificant.

Estimates of the lag coefficients of the structural VAR(2)s are similar across Model 5 and Model 6. The own lag coefficients, $\theta_1^*$, $\theta_2^*$, $\gamma_1^*$ and $\gamma_2^*$, shown in Table 2.5 differ only marginally from the estimates of the own reduced-form lags of Table 2.3. Whether structural or reduced-form, these estimates always have large t-ratios (in absolute value). This is not true of the estimates of the off-diagonal lag coefficients, $\vartheta_i^*$ and $\zeta_i^*$ for $i = 1, 2$. These estimates are insignificant with t-ratios less than 2 (in absolute value). The inference is there is little support the business and credit cycles have a structural causal relationship in the short-run.

I compute IRFs to explore the structural responses of shocks to the business and credit cycles. Figures 2.7 and 2.8 display median IRFs and sup-t confidence bands for Model 5 and Model 6 in response to one standard deviation business and credit cycle shocks. The only statistically significant and economically meaningful IRFs are with respect to own shocks, as shown by Figures 2.7 and 2.8. The median IRF of the business cycle has a hump shape in response to its own shock, which peaks at 4 quarters. This IRF reverts to steady state in about four to five years. The credit cycle IRF also features a hump shape in response to its own shock, which peaks at 6 quarters. The median response takes between six to ten years to revert to zero. In response to a business cycle shock, the credit cycle exhibits little in the way of an economically interesting response in Models 5 and 6.

The business cycle features a hump shape in response to a credit cycle shock, which peaks around 6 quarters in Models 5 and 6. The responses have 90% uncertainty bands that are strictly positive only at the 4- to 8-quarter horizon in the top right panel of Figure 2.7. Hence, Figure 2.7 depicts the business cycle having statistically and economically meaningful responses to the credit cycle shock for one to two years assuming this shock affects the business cycle at impact. When the direction of this structural impact causality is
Table 2.5: Estimates of Structural VARs of the Business and Credit Cycles, 1960Q1 to 2018Q4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate (s.e.)</th>
<th>Model 5 $\zeta_0^* = 0$</th>
<th>Estimate (s.e.)</th>
<th>Model 6 $\theta^*_0 = 0$</th>
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<td>1.62 (0.15)</td>
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<td>-0.66 (0.15)</td>
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<td>$\varphi_1^*$</td>
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<td>-0.08 (0.05)</td>
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Notes: The table reports estimates of $\zeta_0^*$ and $\theta^*_0$ that are multiplied by negative one to be consistent with the construction of the SVAR impact matrices in Section 2.2.5. Otherwise, see the notes to Table 2.2.
Figure 2.7: IRFs of Model 5

Notes: The blue line is the bootstrap median IRF. The blue shaded areas are 90% sup-t uncertainty bands. The shocks are one standard deviation shocks.

Figure 2.8: IRFs of Model 6

Notes: See notes to Figure 2.7.
reversed, the IRF is muted and the 90% uncertainty bands cover zero quarter by quarter from impact to the 10-year horizon as depicted in the top right panel of Figure 2.8. Comparing the IRFs of the business cycle to the credit cycle shock reveals the sensitivity of the results to the identification scheme.

The mean bootstrap FEVDs for Model 5 are reported in Table 2.6.\textsuperscript{8} Remember that in Model 5, the credit cycle is assumed to structurally cause the business cycle at impact. The FEVD for the PI trend indicates that 92% of the variation is explained by its own shock across all horizons. Variation in the level of the financial trend is evenly split between its own shock and the credit cycle from impact to the 1-year horizon. However, beginning with the 1-year horizon the shock to the slope of the financial trend comes to dominate movements in the financial trend. This dynamic only increases with the forecast horizon. Fluctuations in the slope of the financial trend are driven only by its own shock.

This is in contrast with the FEVDs of the business cycle. After one year, 75% of the variation in the business cycle is explained by its own-shock, but this drops to 62% by the 10-year horizon. At this horizon, the credit cycle shock is responsible for about a quarter of the variation in the business cycle. The credit cycle is economically meaningful as a driver of business cycle fluctuations when the business cycle responds at impact to the credit cycle.

The business cycle is not important for explaining fluctuations in the credit cycle under this identification. About two-thirds of variation in the credit cycle comes from its own shock at all forecast horizons, while about one quarter comes from the level of the financial trend.

Table 2.7 reports the mean bootstrap FEVDs for Model 6 in which the credit cycle responds to the business cycle on impact. The FEVDs for the PI trend and the level and slope of the financial trend are similar to the results in Table 2.6 for Model 5. The credit cycle ceases to be an important driver of business cycle fluctuations when causality runs from the business cycle to the credit cycle. Just under 90% of the business cycle variation comes from its own shock after one year. About three quarters of this variation comes from its own shock after 10 years and just over 10% comes from the credit cycle. The business cycle does not drive fluctuations in the credit cycle in Model 6. After 10 years over 60% of credit cycle variation comes from its own shocks and about 25% comes from the level of the financial trend.

\textsuperscript{8}I report the mean FEVDs to ensure the estimates sum to one at each horizon.
Table 2.6: FEVDs of Model 5

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<th>Shock Horizon</th>
<th>$\tau_t$</th>
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<th>$\psi_t$</th>
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<td>0.70</td>
</tr>
<tr>
<td>1</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.70</td>
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<tr>
<td>2</td>
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<td>0.00</td>
<td>0.00</td>
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</tr>
<tr>
<td>4</td>
<td>0.00</td>
<td>0.30</td>
<td>0.00</td>
<td>0.01</td>
<td>0.69</td>
</tr>
<tr>
<td>8</td>
<td>0.00</td>
<td>0.29</td>
<td>0.00</td>
<td>0.02</td>
<td>0.69</td>
</tr>
<tr>
<td>16</td>
<td>0.00</td>
<td>0.28</td>
<td>0.00</td>
<td>0.04</td>
<td>0.68</td>
</tr>
<tr>
<td>24</td>
<td>0.01</td>
<td>0.26</td>
<td>0.00</td>
<td>0.06</td>
<td>0.67</td>
</tr>
<tr>
<td>32</td>
<td>0.01</td>
<td>0.26</td>
<td>0.00</td>
<td>0.07</td>
<td>0.66</td>
</tr>
<tr>
<td>40</td>
<td>0.01</td>
<td>0.26</td>
<td>0.00</td>
<td>0.07</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Notes: Each table reports the bootstrap mean FEVD for one standard deviation shocks to the permanent income trend ($\tau_t$), the financial trend ($\psi_t$), financial trend drift ($\xi_t$), the business cycle ($\delta_t$), and the credit cycle ($\phi_t$).
Table 2.7: FEVDs of Model 6

<table>
<thead>
<tr>
<th>Shock Horizon</th>
<th>Permanent Income Trend</th>
<th>Financial Trend</th>
<th>Business Cycle</th>
<th>Slope of Financial Trend</th>
<th>Credit Cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau_t$ $\psi_t$ $\xi_t$ $\delta_t$ $\phi_t$</td>
<td>$\tau_t$ $\psi_t$ $\xi_t$ $\delta_t$ $\phi_t$</td>
<td>$\tau_t$ $\psi_t$ $\xi_t$ $\delta_t$ $\phi_t$</td>
<td>$\tau_t$ $\psi_t$ $\xi_t$ $\delta_t$ $\phi_t$</td>
<td>$\tau_t$ $\psi_t$ $\xi_t$ $\delta_t$ $\phi_t$</td>
</tr>
<tr>
<td>0</td>
<td>0.92 0.00 0.00 0.08 0.00</td>
<td>0.00 0.49 0.00 0.00 0.51</td>
<td>0.09 0.00 0.00 0.91 0.00</td>
<td>0.00 0.28 0.00 0.05 0.67</td>
<td>0.00 0.25 0.00 0.13 0.61</td>
</tr>
<tr>
<td>1</td>
<td>0.92 0.00 0.00 0.08 0.00</td>
<td>0.00 0.39 0.19 0.00 0.42</td>
<td>0.09 0.00 0.00 0.91 0.00</td>
<td>0.00 0.28 0.00 0.05 0.67</td>
<td>0.00 0.25 0.00 0.13 0.61</td>
</tr>
<tr>
<td>2</td>
<td>0.92 0.00 0.00 0.08 0.00</td>
<td>0.00 0.28 0.40 0.00 0.32</td>
<td>0.08 0.01 0.97 0.00 0.02</td>
<td>0.00 0.28 0.00 0.05 0.67</td>
<td>0.00 0.25 0.00 0.13 0.61</td>
</tr>
<tr>
<td>4</td>
<td>0.92 0.00 0.00 0.08 0.00</td>
<td>0.00 0.14 0.69 0.00 0.17</td>
<td>0.08 0.01 0.98 0.00 0.01</td>
<td>0.00 0.28 0.00 0.05 0.67</td>
<td>0.00 0.25 0.00 0.13 0.61</td>
</tr>
<tr>
<td>8</td>
<td>0.92 0.00 0.00 0.08 0.00</td>
<td>0.00 0.05 0.89 0.00 0.06</td>
<td>0.08 0.01 0.98 0.00 0.01</td>
<td>0.00 0.28 0.00 0.05 0.67</td>
<td>0.00 0.25 0.00 0.13 0.61</td>
</tr>
<tr>
<td>16</td>
<td>0.92 0.00 0.00 0.08 0.00</td>
<td>0.00 0.01 0.97 0.00 0.02</td>
<td>0.08 0.01 0.98 0.00 0.01</td>
<td>0.00 0.28 0.00 0.05 0.67</td>
<td>0.00 0.25 0.00 0.13 0.61</td>
</tr>
<tr>
<td>24</td>
<td>0.92 0.00 0.00 0.08 0.00</td>
<td>0.00 0.01 0.98 0.00 0.01</td>
<td>0.08 0.01 0.98 0.00 0.01</td>
<td>0.00 0.28 0.00 0.05 0.67</td>
<td>0.00 0.25 0.00 0.13 0.61</td>
</tr>
<tr>
<td>32</td>
<td>0.92 0.00 0.00 0.08 0.00</td>
<td>0.00 0.01 1.00 0.00 0.00</td>
<td>0.08 0.01 0.98 0.00 0.01</td>
<td>0.00 0.28 0.00 0.05 0.67</td>
<td>0.00 0.25 0.00 0.13 0.61</td>
</tr>
<tr>
<td>40</td>
<td>0.92 0.00 0.00 0.08 0.00</td>
<td>0.00 0.00 1.00 0.00 0.00</td>
<td>0.08 0.01 0.98 0.00 0.01</td>
<td>0.00 0.28 0.00 0.05 0.67</td>
<td>0.00 0.25 0.00 0.13 0.61</td>
</tr>
</tbody>
</table>

Notes: See notes to Table 2.6.
2.5.5 The Predictability of Business and Credit Cycles

This section reexamines claims made by Schularick and Taylor (2012) and Borio et al. (2018). They, among others, report the credit cycle and Basel gap have short-term predictive power for the business cycle. Bootstrap t-statistics and p-values are computed to evaluate the significance of the credit cycle and the first difference of the PI trend for predicting the $h$-quarter ahead business cycle, $h \in [1, 2, 4, 8, 12, 16]$. I also test the significance of the business cycle, the second difference of the level of the financial trend, and the first difference of the slope of the financial trend for predicting the credit cycle $h$-quarters ahead.\(^9\)

Table 2.8 reports the regression equation considered and the bootstrap mean estimates of these regressions. The estimates of the first regression imply there is no predictive power in the credit cycle for the business cycle over the 1-year horizon. The estimated coefficient on the credit cycle, $\phi_t$, for predicting the business cycle, $\delta_t$, are all small and insignificant at the 5% level for 1 to 4 quarters ahead. These results are a challenge for Borio et al. (2018). They claim their estimated credit cycle is a significant predictor of recessions at the 1 year horizon. My results do, however, lend evidence to Borio et al. (2018)’s claim that the credit cycle is a significant predictor of the business cycle over 2- and 3-year horizons. The estimated coefficients on the credit cycle are negative at all horizons indicating that a credit cycle expansion predicts a business cycle contraction.

The second regression tests the implications of the Beveridge-Nelson decomposition. This decomposition implies that the growth rate of the trend is orthogonal to the cycle. As expected, the estimated coefficient on the first differences of the PI trend are negative across all horizons. However, these estimates are insignificant at the 5% and 10% level across all horizons.

The third regression examines the predictive content of the business cycle for the credit cycle. The estimated coefficients on the business cycle are positive across all horizons indicating greater transitory real economic activity anticipates temporary increases in credit activity. The estimates are significant at the 5% level. In contrast, Section 5.4 provided evidence the business cycle does not structurally cause the credit cycle. Additionally, there is some evidence the credit cycle structurally causes the business cycle over the 1- to 2-year horizon. These results serve as a caution against equating statistical predictability and structural causality.

The final two regressions assess the predictive content of the second difference of the level of the financial trend and the first difference of the slope of the financial trend for the

---

\(^9\)The first two lags of the dependent variable are included in the regressions to eliminate own predictability.
Table 2.8: Tests of Business Cycle and Credit Cycle Predictive Content, 1960Q1 to 2018Q4

Regression: \( (cycle)_{t+h} = \alpha + \beta (predictor)_t + \gamma(L)(cycle)_t + e_t \)

<table>
<thead>
<tr>
<th>Cycle</th>
<th>Predictor</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_t )</td>
<td>( \phi_t )</td>
<td>( \beta )</td>
<td>-0.005</td>
<td>-0.013</td>
<td>-0.032</td>
<td>-0.088</td>
<td>-0.137</td>
</tr>
<tr>
<td></td>
<td></td>
<td>se</td>
<td>0.011</td>
<td>0.016</td>
<td>0.023</td>
<td>0.028</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td></td>
<td>t-stat</td>
<td>-0.451</td>
<td>-0.781</td>
<td>-1.415</td>
<td>-3.212</td>
<td>-4.862</td>
</tr>
<tr>
<td></td>
<td></td>
<td>p-val</td>
<td>0.329</td>
<td>0.223</td>
<td>0.085</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>( \delta_t )</td>
<td>( \Delta \tau_t )</td>
<td>( \beta )</td>
<td>-0.359</td>
<td>-0.318</td>
<td>-0.146</td>
<td>-0.194</td>
<td>-0.153</td>
</tr>
<tr>
<td></td>
<td></td>
<td>se</td>
<td>0.204</td>
<td>0.287</td>
<td>0.412</td>
<td>0.522</td>
<td>0.564</td>
</tr>
<tr>
<td></td>
<td></td>
<td>t-stat</td>
<td>-1.724</td>
<td>-1.064</td>
<td>-0.305</td>
<td>-0.340</td>
<td>-0.245</td>
</tr>
<tr>
<td></td>
<td></td>
<td>p-val</td>
<td>0.116</td>
<td>0.157</td>
<td>0.228</td>
<td>0.288</td>
<td>0.318</td>
</tr>
<tr>
<td>( \phi_t )</td>
<td>( \delta_t )</td>
<td>( \beta )</td>
<td>0.130</td>
<td>0.178</td>
<td>0.247</td>
<td>0.418</td>
<td>0.633</td>
</tr>
<tr>
<td></td>
<td></td>
<td>se</td>
<td>0.060</td>
<td>0.084</td>
<td>0.124</td>
<td>0.195</td>
<td>0.242</td>
</tr>
<tr>
<td></td>
<td></td>
<td>t-stat</td>
<td>2.180</td>
<td>2.122</td>
<td>2.000</td>
<td>2.145</td>
<td>2.612</td>
</tr>
<tr>
<td></td>
<td></td>
<td>p-val</td>
<td>0.017</td>
<td>0.019</td>
<td>0.026</td>
<td>0.018</td>
<td>0.005</td>
</tr>
<tr>
<td>( \phi_t )</td>
<td>( \Delta^2 \psi_t )</td>
<td>( \beta )</td>
<td>0.133</td>
<td>0.061</td>
<td>0.528</td>
<td>0.387</td>
<td>0.281</td>
</tr>
<tr>
<td></td>
<td></td>
<td>se</td>
<td>0.158</td>
<td>0.223</td>
<td>0.325</td>
<td>0.520</td>
<td>0.652</td>
</tr>
<tr>
<td></td>
<td></td>
<td>t-stat</td>
<td>0.848</td>
<td>0.268</td>
<td>1.616</td>
<td>0.739</td>
<td>0.422</td>
</tr>
<tr>
<td></td>
<td></td>
<td>p-val</td>
<td>0.216</td>
<td>0.390</td>
<td>0.056</td>
<td>0.232</td>
<td>0.338</td>
</tr>
<tr>
<td>( \phi_t )</td>
<td>( \Delta \xi_t )</td>
<td>( \beta )</td>
<td>0.387</td>
<td>0.232</td>
<td>1.848</td>
<td>2.216</td>
<td>2.643</td>
</tr>
<tr>
<td></td>
<td></td>
<td>se</td>
<td>0.525</td>
<td>0.743</td>
<td>1.072</td>
<td>1.705</td>
<td>2.139</td>
</tr>
<tr>
<td></td>
<td></td>
<td>t-stat</td>
<td>0.768</td>
<td>0.329</td>
<td>1.731</td>
<td>1.289</td>
<td>1.219</td>
</tr>
<tr>
<td></td>
<td></td>
<td>p-val</td>
<td>0.236</td>
<td>0.335</td>
<td>0.047</td>
<td>0.102</td>
<td>0.117</td>
</tr>
</tbody>
</table>

Notes: The coefficients, standard errors, and t-values are the mean values of the regressions from 1000 bootstrap resamples. The standard errors are Newey-West corrected. The term \( \gamma(L) \) is a second order lag polynomial.
credit cycle. The estimated coefficients on the second differences of the level of the financial
trend are positive, but insignificant at the 5% level across all horizons. The estimated
coefficients on the first difference of the slope of the financial trend are also positive across
all horizons. However, the estimates are insignificant at the 5% level with the exception of
the one year horizon.

2.5.6 Predictive Regressions for the Growth Rate of credit to GDP

This section constructs bootstrapped Breusch-Godfrey tests to investigate the ability of the
Basel gap, the estimated business and credit cycles, and the estimated slope of the financial
trend to predict the growth rate of the credit to GDP ratio. The Breusch-Godfrey test estimated
here has two steps. The first step regresses the $h$-step ahead growth rate of the credit
to GDP ratio on an intercept and predictor variables. Next, the residuals from the first step
are regressed on its own lagged value and the explanatory variables of the first regression.
The test statistic is the Lagrangian multiplier statistic that equals $T$ times the $R^2$
of the second step regression, where $T$ is the number of observations. The test statistic
follows a chi-squared distribution with one degree of freedom. The null hypothesis of the
Breusch-Godfrey test is the residuals of the first regression are serially uncorrelated. Serial
correlation indicates predictability in the error terms. Hence, unaccounted for information
exists in the dependent variable of the first-step regression. Nelson (2008) runs similar
regressions to examine whether the HP-filtered measure of the output gap contributes to
the ability of the Beveridge-Nelson trend to predict output growth.

Table 2.9a shows the $h$-quarter ahead growth rate of the credit-GDP ratio regressed on
an intercept and the Basel gap, where $h \in [1, 2, 4, 8, 12, 16]$. The estimated coefficients on
the Basel gap, $\beta_1$, in Table 2.9a are positive and significant at the 5% level over the first
year. The coefficient approaches zero at the 8-quarter horizon before turning negative.
The $R^2$ peaks at 8.8% at $h = 1$, is 6.9% at $h = 2$, and is under 3% at all other horizons. The
Breusch-Godfrey test shows there is serial correlation in the residuals of this regression.
The null of no serial correlation is rejected at the 5% level across all horizons indicating
information in the dependent variable is left unexplained.

Table 2.9b adds the estimated credit cycle to the previous regression. The estimated
coefficient on the Basel gap, $\beta_1$, is once again positive from the 1- to the 4-quarter ahead
forecast horizons, but is insignificant at the 5% level beyond the 1-year ahead forecast. The

---

$^{10}$Augmented Dickey Fuller tests reject the null of a unit root at the 1% level for the growth rate of the credit
to GDP ratio. The null fails to be rejected for the log level of the credit to GDP ratio.
Table 2.9a: Predictive Regressions for the Growth Rate of credit to GDP, 1960Q1 to 2018Q4

Regression: \[ \Delta \left( \frac{\text{Credit}}{\text{GDP}} \right)_{t+h} = \beta_0 + \beta_1 (\text{Basel Gap})_t + e_t \]

<table>
<thead>
<tr>
<th>Number of Quarters Ahead</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_1 ) coef</td>
<td>0.051</td>
<td>0.045</td>
<td>0.028</td>
<td>0.002</td>
<td>-0.016</td>
<td>-0.024</td>
</tr>
<tr>
<td>se</td>
<td>0.011</td>
<td>0.012</td>
<td>0.012</td>
<td>0.013</td>
<td>0.013</td>
<td>0.014</td>
</tr>
<tr>
<td>t-stat</td>
<td>4.461</td>
<td>3.801</td>
<td>2.261</td>
<td>0.122</td>
<td>-1.239</td>
<td>-1.770</td>
</tr>
<tr>
<td>p-val</td>
<td>0.000</td>
<td>0.000</td>
<td>0.012</td>
<td>0.452</td>
<td>0.108</td>
<td>0.039</td>
</tr>
<tr>
<td>( R^2 ) value</td>
<td>0.088</td>
<td>0.069</td>
<td>0.027</td>
<td>0.000</td>
<td>0.009</td>
<td>0.039</td>
</tr>
<tr>
<td>Breusch-Godfrey value</td>
<td>5.037</td>
<td>10.002</td>
<td>14.027</td>
<td>15.781</td>
<td>15.775</td>
<td>15.946</td>
</tr>
<tr>
<td>p-val</td>
<td>0.025</td>
<td>0.002</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: The null hypothesis of the Breusch-Godfrey test is that the regression errors are not serially correlated.

Table 2.9b: Predictive Regressions for the Growth Rate of credit to GDP, 1960Q1 to 2018Q4

Regression: \[ \Delta \left( \frac{\text{Credit}}{\text{GDP}} \right)_{t+h} = \beta_0 + \beta_1 (\text{Basel Gap})_t + \beta_2 \phi_t + e_t \]

<table>
<thead>
<tr>
<th>Number of Quarters Ahead</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_1 ) coef</td>
<td>0.036</td>
<td>0.029</td>
<td>0.010</td>
<td>-0.015</td>
<td>-0.025</td>
<td>-0.026</td>
</tr>
<tr>
<td>se</td>
<td>0.012</td>
<td>0.012</td>
<td>0.013</td>
<td>0.013</td>
<td>0.014</td>
<td>0.015</td>
</tr>
<tr>
<td>t-stat</td>
<td>3.075</td>
<td>2.425</td>
<td>0.798</td>
<td>-1.115</td>
<td>-1.781</td>
<td>-1.704</td>
</tr>
<tr>
<td>p-val</td>
<td>0.001</td>
<td>0.008</td>
<td>0.213</td>
<td>0.133</td>
<td>0.038</td>
<td>0.045</td>
</tr>
<tr>
<td>( \beta_2 ) coef</td>
<td>0.008</td>
<td>0.009</td>
<td>0.009</td>
<td>0.008</td>
<td>0.004</td>
<td>0.001</td>
</tr>
<tr>
<td>se</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>t-stat</td>
<td>3.625</td>
<td>3.642</td>
<td>3.798</td>
<td>3.082</td>
<td>1.531</td>
<td>0.274</td>
</tr>
<tr>
<td>p-val</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.064</td>
<td>0.392</td>
</tr>
<tr>
<td>( R^2 ) value</td>
<td>0.141</td>
<td>0.126</td>
<td>0.094</td>
<td>0.049</td>
<td>0.021</td>
<td>0.018</td>
</tr>
<tr>
<td>p-val</td>
<td>0.096</td>
<td>0.019</td>
<td>0.004</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: The coefficients, standard errors, and t-values are the mean values of the regressions from 1000 bootstrap resamples. The null hypothesis of the Breusch-Godfrey tests is that the regression errors are not serially correlated.
Table 2.9c: Predictive Regressions for the Growth Rate of credit to GDP, 1960Q1 to 2018Q4

Regression: \[ \Delta \left( \frac{\text{Credit}}{\text{GDP}} \right)_{t+h} = \beta_0 + \beta_1 (\text{Basel Gap})_t + \beta_2 \phi_t + \beta_3 \delta_t + \epsilon_t \]

<table>
<thead>
<tr>
<th>Number of Quarters Ahead</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>coef</td>
<td>0.039</td>
<td>0.032</td>
<td>0.011</td>
<td>-0.015</td>
<td>-0.025</td>
<td>-0.026</td>
</tr>
<tr>
<td>se</td>
<td>0.011</td>
<td>0.011</td>
<td>0.012</td>
<td>0.013</td>
<td>0.014</td>
<td>0.015</td>
</tr>
<tr>
<td>t-stat</td>
<td>3.592</td>
<td>2.842</td>
<td>0.917</td>
<td>-1.110</td>
<td>-1.780</td>
<td>-1.721</td>
</tr>
<tr>
<td>p-val</td>
<td>0.000</td>
<td>0.003</td>
<td>0.180</td>
<td>0.134</td>
<td>0.038</td>
<td>0.043</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>coef</td>
<td>0.006</td>
<td>0.007</td>
<td>0.008</td>
<td>0.008</td>
<td>0.004</td>
<td>0.001</td>
</tr>
<tr>
<td>se</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>t-stat</td>
<td>2.913</td>
<td>2.938</td>
<td>3.367</td>
<td>2.995</td>
<td>1.513</td>
<td>0.192</td>
</tr>
<tr>
<td>p-val</td>
<td>0.002</td>
<td>0.002</td>
<td>0.000</td>
<td>0.002</td>
<td>0.066</td>
<td>0.424</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>coef</td>
<td>0.029</td>
<td>0.029</td>
<td>0.018</td>
<td>0.002</td>
<td>-0.000</td>
<td>0.004</td>
</tr>
<tr>
<td>se</td>
<td>0.006</td>
<td>0.006</td>
<td>0.007</td>
<td>0.008</td>
<td>0.008</td>
<td>0.008</td>
</tr>
<tr>
<td>t-stat</td>
<td>4.609</td>
<td>4.572</td>
<td>2.613</td>
<td>0.265</td>
<td>-0.015</td>
<td>0.562</td>
</tr>
<tr>
<td>p-val</td>
<td>0.000</td>
<td>0.000</td>
<td>0.005</td>
<td>0.395</td>
<td>0.473</td>
<td>0.288</td>
</tr>
<tr>
<td>( R^2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>value</td>
<td>0.214</td>
<td>0.204</td>
<td>0.124</td>
<td>0.049</td>
<td>0.021</td>
<td>0.020</td>
</tr>
<tr>
<td>Breusch-Godfrey</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>value</td>
<td>0.145</td>
<td>1.755</td>
<td>5.574</td>
<td>11.525</td>
<td>14.520</td>
<td>15.704</td>
</tr>
<tr>
<td>p-val</td>
<td>0.709</td>
<td>0.187</td>
<td>0.018</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: See notes to Table 2.9b.
estimate for $\beta_1$ turns negative at longer horizons, but has a bootstrapped p-value less than 4% at the 4-year forecast horizon. This implies the Basel gap predicts mean reversion in the credit to GDP ratio at longer horizons. Mean reversion suggests the Basel gap is forecasting financial stability in the longer run when the estimated credit cycle is taken into account. Thus, the Basel gap may not be suited to drawing conclusions about financial instabilities.

The bootstrap mean estimate of the coefficient on the credit cycle, $\beta_2$, is positive, but smaller in magnitude than $\beta_1$, across all horizons. These estimates are significant at the 5% level across the first eight quarters. The $R^2$ peaks at 14.1% for the 1-quarter ahead forecast. The Breusch-Godfrey test indicates that adding the credit cycle removes autocorrelation in the residuals only at the 1-quarter ahead forecast.

Table 2.9c adds the estimated business cycle to the regression in Table 2.9b. The coefficients on the Basel gap and the credit cycle, $\beta_1$ and $\beta_2$, are consistent with the estimates in Table 2.9b. The bootstrap mean estimate of the coefficient on the business cycle, $\beta_3$, is positive and significant at the 5% level from 1- to 4-quarter ahead forecasts. The coefficients are larger in magnitude than the coefficient on the credit cycle over these forecast horizons. At horizons longer than one year, $\beta_3$ is not significantly different from zero at the 5% level. The $R^2$ peaks at 21.4% at the 1-quarter ahead forecast, but falls to about 3% beyond a 2-year forecast horizon. The Breusch-Godfrey tests indicate the estimated business cycle improves the prediction of growth in the credit to GDP ratio after accounting for the Basel gap and the estimated credit cycle. The null hypothesis of no serial correlation in the residuals is rejected at better than the 18% level over 1- and 2-quarter ahead forecast horizons.

Table 2.10: Predictive Regressions for the Growth Rate of credit to GDP, 1960Q1 to 2018Q4

<table>
<thead>
<tr>
<th>Number of Quarters Ahead</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_4$</td>
<td>coef</td>
<td>0.177</td>
<td>0.189</td>
<td>0.160</td>
<td>0.077</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>se</td>
<td>0.023</td>
<td>0.023</td>
<td>0.026</td>
<td>0.034</td>
<td>0.036</td>
</tr>
<tr>
<td></td>
<td>t-stat</td>
<td>7.712</td>
<td>8.304</td>
<td>6.157</td>
<td>2.241</td>
<td>0.336</td>
</tr>
<tr>
<td></td>
<td>p-val</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.016</td>
<td>0.369</td>
</tr>
<tr>
<td>$R^2$</td>
<td>value</td>
<td>0.248</td>
<td>0.286</td>
<td>0.207</td>
<td>0.049</td>
<td>0.002</td>
</tr>
<tr>
<td>Breusch-Godfrey</td>
<td>value</td>
<td>1.448</td>
<td>0.174</td>
<td>0.755</td>
<td>11.347</td>
<td>16.739</td>
</tr>
<tr>
<td></td>
<td>p-val</td>
<td>0.267</td>
<td>0.744</td>
<td>0.469</td>
<td>0.001</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: See notes to Table 2.9a.
I also consider the ability of the estimated slope of the financial trend to predict growth in the credit to GDP ratio. Table 2.10 reports on bootstrapped regressions of this ratio on the estimated slope of the financial trend. The bootstrap mean estimate of the coefficient on the slope of the financial trend, $\beta_4$, is positive from the 1- to the 12-quarter ahead forecast horizons, but is insignificant at the 5% level beyond the 3-year ahead forecast. This coefficient is larger in magnitude than those of the business and credit cycles in Table 2.9c in all but the 3-year horizon. The $R^2$ is above 20% from the 1- to 4-quarter horizon and peaks at 28.6% at the 2-quarter horizon. The $R^2$ falls below 5% at longer horizons.

The Breusch-Godfrey tests indicate the estimated financial trend slope has more predictive power for growth in the credit to GDP ratio than the other predictors considered. The null hypothesis of no serial correlation in the residuals is rejected at better than the 25% level over 1- and 4-quarter ahead forecast horizons. Hence, the direction of the financial trend is important for predicting the growth in the credit to GDP ratio. This suggests it is permanent shocks rather than transitory movements which matters for gauging the state of financial stability.

### 2.6 Conclusion

This paper estimates UC models to examine the usefulness of macroprudential policy and present new estimates of the credit cycle. Income and consumption share a common Beveridge-Nelson trend as implied by the permanent income hypothesis. The macrofinance model of leverage in Brunnermeier and Sannikov (2014) is used to place parameter restrictions on credit supply and nonfinancial assets. The common permanent component of credit supply and nonfinancial assets is a local linear trend. The business and credit cycles form a VAR(2). Estimation of the UC models is done via classical optimization of the predictive likelihood of the Kalman filter on a quarterly U.S. sample from 1960 to 2018. The UC models are bootstrapped to construct the empirical sampling distributions of the model parameters, state variables, and test statistics.

There are five key contributions of this paper. First, my estimates support modeling the credit cycle jointly with the business cycle as a reduced-form VAR(2). Second, the estimated credit cycle features two peaks of similar magnitude, with the latter being two years prior to the financial crisis in contrast to the Basel gap. Third, recursive structural VARs lend support for causality running from the credit cycle to the business cycle over the 1- to 2-year horizon.

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11Inclusion of the slope of the financial trend into the regression in Table 2.9c does not alter the results in Table 2.10. These results are available on request.
horizon. Third, I find no evidence of reduced-form predictability of the credit cycle for the business cycle at the 1-, 2-, and 4-quarter horizons. Interestingly, the business cycle is a good predictor of the credit cycle from the 1-quarter to 4-year horizon. Fourth, the Basel gap is a poor predictor of the growth of the credit to GDP ratio at short, medium, and long forecast horizons. At 1- and 2-quarter horizons, the estimated credit and business cycles predict the growth rate of the credit to GDP ratio. However, the slope of the financial trend has forecasting power at the 1- to 4-quarter horizons for the growth of the credit to GDP ratio. Hence, my results caution against the use of the Basel gap as a signal of the underlying state of the financial markets. Evidence in this paper indicates that it is permanent shocks to the financial sector that matter for financial stability.

Future work should focus on utilizing theories from the financial frictions literature to restrict the UC model. These theories can be used to address whether trends in the real and financial sectors are independent. If this assumption is not supported by the data, the implications for aggregate fluctuations should be of interest to economists and policymakers.
CHAPTER 3

UNOBSERVED COMPONENTS MODEL
ESTIMATES OF CREDIT CYCLES: TESTS AND PREDICTIONS

3.1 Introduction

The new Keynesian (NK) model pervades modern macroeconomics. The NK Phillips curve (PC) separates this class of models from other monetary models. Along with the NKPC, many NK models share a consumption generating equation and an interest rate rule that governs monetary policy.\(^1\) Despite its popularity, estimation of NK models often lacks proper treatment of nonstationarities in aggregate data. For example, NK models typically ignore trend inflation. This stands in contrast to Ascari and Sbordone (2014). They argue

\(^1\)See Christiano et al. (2005), Smets and Wouters (2007), and Del Negro and Schorfheide (2008) for examples of the canonical NK model.
trend inflation is a key dynamic in aggregate data.

The dynamic stochastic general equilibrium (DSGE) version of NK models is a collection of at least one nominal friction, several real frictions, and not a few exogenous shocks. The nominal friction is most often Calvo staggered price setting while the most prominent real friction is habit in preferences over consumption. Leading exogenous shocks are shocks to aggregate supply and demand and markup and monetary policy shocks.

Unobserved components (UC) models provide an approach to estimate NK models in the presence of sample data driven by stochastic trends. Doménech and Gómez (2006) construct an UC model with a hybrid-NKPC, Okun's law, and a dynamic investment-savings equation. They estimate a common trend in output and investment, which yields a real activity gap, and trends in inflation and unemployment. Basistha and Nelson (2007) build on the trend-cycle decomposition of Watson (1986) and Morley et al. (2003) by incorporating a hybrid-NKPC to investigate the impact of these restrictions on estimates of trend and gap output. Lee and Nelson (2007) point out that estimates of the slope of the NKPC depend on the model specification of the forecast horizon. The authors build an UC model with a structural VAR in inflation and real activity gaps. This structure yields statistically and economically meaningful estimates of the slope of the NKPC.

This paper estimates several NK-UC models. The measurement system of the NK-UC models includes a consumption generating equation, a hybrid-NKPC, and the Fisher equation. To this system, I add an auxiliary equation which decomposes real aggregate activity into a permanent income (PI) trend and real activity gap shared with consumption. These measurement equations place restrictions on the observables of consumption, real aggregate activity, inflation, and the policy rate. The restrictions decompose the sample data into PI and inflation trends, real activity and inflation gaps, and the real rate. The real rate evolves according to a real interest rate rule governing monetary policy.

The use of NK-UC models allows for the examination of key frictions without specifying a fully fledged NK-DSGE model. At the same time, the NK-UC models estimate trends and gaps in the real and nominal sectors. My NK-UC models feature one nominal and one real friction. Calvo staggered price setting, which gives rise to the NKPC, is the nominal friction. The real friction is habit formation in consumption. This paper compares a baseline NK-UC model with four alternatives to draw conclusions about the fit of the NK framework to the data.

An innovation of this paper is the use of the Fisher equation to handle the apparent common trend in inflation and the nominal policy rate. The inclusion of the Fisher equation is

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\(^2\)Leu and Sheen (2011), Holston et al. (2017), and Manopimoke (2017) estimate a real rate gap which is the
a departure from the canonical NK model as in Goodfriend and King (2013). Coupled with
trend inflation, the Fisher equation gives a stationary real rate. The real rate becomes the
focus of monetary policy in the NK-UC model in accordance with the Taylor principle.

The NK-UC models assume the real rate is the intermediate target of monetary policy.
The final targets are the real activity and inflation gaps, as is standard in NK literature since
Taylor (1993) and Taylor (1999). An implication is monetary policy operates through the real
rate. The real interest rate monetary policy rule is smoothed by its own lag and responds
systematically to the gaps and unsystematically to the monetary policy shock.

Monetary policy is tied to the real sector in the NK-UC models through the consump-
tion generating equation of Kano and Nason (2014) which features habit formation in
consumption. Habit formation is a real friction used in NK models to better match the
observed dynamics of consumption and real aggregate activity to a transitory shock. The
consumption generating equation decomposes consumption into the PI trend, the real
activity gap, and the expected future path of real interest rates. The permanent income
hypothesis (PIH) identifies the common PI trend of the consumption-real activity pair. With
habit formation, consumption no longer solely depends on the level of the real activity gap,
but also its rate of change. The first alternative NK-UC model considered sets the habit
formation parameter to zero.

In the NK-UC models, the hybrid-NKPC relates inflation to its trend, the real activity
and inflation gaps, and a markup shock. I extend the NKPC of Kim et al. (2014) to include
lagged inflation as in Gali and Gertler (1999). The purely forward-looking NKPC describes
the optimizing behavior of a monopolistic firm facing a Calvo staggered pricing mechanism
as developed by Yun (1996). The hybrid-NKPC arises when firms unable to reoptimize their
prices in date $t$, index their prices to lagged inflation; see Ascari et al. (2011).

Additional alternative NK-UC models place restrictions on the hybrid-NKPC. A vertical
long-run hybrid-NKPC defines the second alternative NK-UC model. This restricts the
backward- and forward-looking coefficients in the hybrid-NKPC to sum to one. The next
alternative NK-UC model features the purely forward-looking NKPC. The final alternative
model introduces serial correlation in the markup shock in the hybrid-NKPC.

The state variables of the NK-UC models are the PI and inflation trends, the real activity
and inflation gaps, and the real rate. The trends evolve as independent random walks.
Consequently, these permanent components of the NK-UC models are interpreted as
Beveridge and Nelson (1981) trends; see Watson (1986) and Morley et al. (2003). I assume
 deviation of the real rate from its estimated natural rate in an UC model.
real activity and inflation gaps are driven by a bivariate autoregression (AR). When combined with the interest rate rule of the real rate, the bivariate AR becomes part of a structural VAR.\(^3\)

I use classical optimization of the predictive likelihood of the Kalman filter to estimate the state-space representation of the baseline and alternative NK-UC models. The NK-UC models are estimated on a quarterly U.S. sample of consumption, real aggregate activity, inflation, and the federal funds rate from 1960 to 2018. I consider two measures of real aggregate activity, real disposable income and real GDP. Tests of the PIH are often conducted on consumption and disposable income, while real GDP is the standard measure of real aggregate activity in NK models.

As in Hessler (2021), the bootstrap algorithm of Stoffer and Wall (2004) produces the small sample distributions of model parameters and the states. Bootstrap p-values of likelihood ratio tests are computed following Morley et al. (2016) to evaluate the fit of the alternative NK-UC models relative to the baseline NK-UC model.

My estimates yield two contributions to the literature. First, evidence is presented that serial correlation in the markup shock improves the fit of the NK-UC model to the sample data. Bootstrap likelihood ratio tests reject the null of the baseline NK-UC model in favor of the alternative with a serially correlated markup shock. This finding is robust to both measures of real aggregate activity.

Second, I find the inflation gap has little persistence compared to the real activity gap when estimated jointly with trend inflation. Low persistence in gap inflation is in line with the findings of Ascari and Sbordone (2014) and Lee and Nelson (2007). Observed inflation moves closely with its trend, or long-run inflation expectations, over the sample. Additionally, the slope of the hybrid-NKPC is larger in magnitude than the forward- and backward-looking coefficients on gap inflation across almost all NK-UC models.

Section 3.2 presents the NK-UC models. The data used is discussed in Section 3.3. Section 3.4 reports the parameter estimates as well as the trends and gaps of the NK-UC models. Section 3.5 concludes.

### 3.2 The NK-UC Models

This section describes the baseline and alternative NK-UC models. The measurement equations of the NK-UC models are the consumption generating equation, a hybrid-NKPC, the Fisher equation, and an auxiliary equation for real aggregate activity. These equations

\(^{3}\)In a future draft, a set of identifications for the structural VAR will be considered to compute impulse response functions and forecast error variance decompositions.
inform the decompositions of consumption, real aggregate activity, inflation, and the nominal policy rate into trends and gaps. Consumption and real aggregate activity share the PI trend, while the inflation trend is common to the nominal policy rate and inflation. The gap variables, coupled with a interest rate rule in the real rate, form a structural VAR. The trends and structural VAR form the state system of the NK-UC models.

### 3.2.1 The Dynamic IS Equation

The consumption generating equation of the NK-UC model is adapted from Kano and Nason (2014). Internal habit formation is assumed in consumption preferences of the representative household. Household utility over uncertain consumption, $c_t$, is

$$U(c_t, c_{t-1}) = \ln(c_t - hc_{t-1}),$$

where $h \in [0, 1]$ is the habit parameter. Households face the budget constraint

$$\frac{B_{t+1}}{P_t} + c_t + T_t = y_t + R_t \frac{B_t}{P_t},$$

where $B_{t+1}$ is the stock of government bonds held from date $t$ to date $t+1$, $R_t$ is the nominal return on bonds, $T_t$ is a lump-sum tax on households, and $y_t$ is exogenous and stochastic labor income. A unit root productivity shock drives labor income. This unit root process is interpreted as the PI trend.

Households choose consumption and a level of one-period government bonds subject to their utility function and budget constraint. The first-order necessary conditions yield a consumption-bond Euler equation relating consumption to its previous period value and the expectation of its value one-step ahead as well as the real interest rate.

The log-linearized consumption-bond Euler equation is solved around the stochastically detrended steady state. Real variables are rendered stationary by dividing by the PI trend. Consumption is formulated in terms of deviations from its steady state. Unwinding the log-linearized consumption-bond Euler equation to include the PI trend results in the consumption generating equation

$$c_t = \tau_t + a h e_{c,t} + \left(1 + \frac{h}{\alpha}\right) \delta_{t-1} - \frac{h}{\alpha} \delta_t - 2$$

$$+ (\alpha - \beta h) (\alpha - h) \sum_{j=0}^{\infty} \left(\frac{\beta h}{\alpha}\right)^j E_t \{ r_{t+j} \} + \sigma c e_{c,t}, \ e_{c,t} \sim \mathcal{N}(0, 1),$$

(3.1)
where \( \tau_t \) is the PI trend and \( \epsilon_{\tau,t} \) is its associated error, \( \alpha \) is deterministic PI trend growth, \( \delta_t \) is the real activity gap, \( \beta \) is the households discount factor, \( r_t \) is the real interest rate, and \( e_{c,t} \) is consumption measurement error.\(^4\) Equation (3.1) shows consumption moves one-for-one with the PI trend. Habit formation introduces serial correlation in consumption. The consumption generating function is forward-looking in the real rate which ties consumption to monetary policy.

Assume the PI trend is a random walk with drift

\[
\tau_t = \mu + \tau_{t-1} + \sigma_{\tau} \epsilon_{\tau,t}, \quad \epsilon_{\tau,t} \sim N(0, 1),
\]

where \( \mu (= \ln(\alpha)) \) is the constant drift parameter and \( \sigma_{\tau} \) is the scale volatility on the innovation, \( \epsilon_{\tau,t} \), to \( \tau_t \). Hence, consumption and labor income have the PI trend in common as the PIH predicts.

An auxiliary relation defining labor income, or real aggregate activity, is added to the NK-UC models. The sum of the PI trend, real activity gap, and a measurement error gives the measurement equation for real aggregate activity

\[
y_t = \tau_t + \upsilon \delta_t + \sigma_y e_{y,t}, \quad e_{y,t} \sim N(0, 1).
\]

I experiment with the notion of the real activity gap, \( \delta_t \), by changing the observed measure of real aggregate activity.

### 3.2.2 The Hybrid-New Keynesian Phillips Curve

Beginning with the NKPC of Kim et al. (2014), lagged inflation is included to obtain the hybrid-NKPC

\[
\pi_t - \tilde{\pi}_t = \gamma_f E_t \{ \pi_{t+1} - \tilde{\pi}_{t+1} \} + \gamma_b (\pi_{t-1} - \tilde{\pi}_{t-1}) + \kappa \delta_t + \sigma_\eta \eta_t, \quad \eta_t \sim N(0, 1),
\]

where \( \pi_t \) is inflation, \( \tilde{\pi}_t \) is its trend level, and \( \eta_t \) is the markup shock. Gali and Gertler (1999) and Ascari et al. (2011) motivate the hybrid-NKPC by giving price setters unable to update their prices optimally a rule that adjusts prices to lagged prices or inflation. Equation (3.4) postulates the NKPC in terms of inflation gaps, or deviations of observed inflation from its stochastic trend, and the real activity gap. The NKPC is along the lines of Woodford (2008) and similar to the all-gap PC of McNeil and Smith (2021). Trend inflation is a driftless

\(^4\)See Appendix A and Kano and Nason (2014) for the derivation of the consumption generating equation.
random walk
\[ \tilde{\pi}_t = \tilde{\pi}_{t-1} + \sigma_{\tilde{\pi}} \epsilon_{\tilde{\pi},t}, \epsilon_{\tilde{\pi},t} \sim N(0,1). \] (3.5)

as suggested by Ascari and Sbordone (2014), among others.

The measurement equation for inflation is constructed from the hybrid-NKPC of equation (3.4). Rearranging it and defining the inflation gap as \( \sigma_t = \pi_t - \tilde{\pi}_t \) gives

\[ \pi_t = \pi_{t-1} + \gamma_f E_t \{ \sigma_{t+1} \} + \gamma_b \{ \sigma_{t-1} \} + \kappa \delta_t + \sigma_{\pi} \epsilon_{\pi,t}, \] (3.6)

where the markup shock is summarized in the observation error, \( \epsilon_{\pi,t} \). Equation (3.6) summarizes the dynamics of observed inflation according to NK theory. Current inflation responds to its trend, expected one step ahead and lagged gap inflation, the real activity gap, and the markup shock.

3.2.3 The Fisher Equation and Monetary Policy

The Fisher equation connects the nominal policy rate, \( R_t \), to expected inflation

\[ R_t = E_t \{ \tilde{\pi}_{t+1} \} + r_t. \] (3.7)

Substitute for expected inflation in equation (3.7) using the definition of gap inflation and take expectations in period \( t + 1 \) to find

\[ R_t = \tilde{\pi}_t + E_t \{ \sigma_{t+1} \} + r_t. \] (3.8)

Equation (3.8) shows the nominal policy and observed inflation share the inflation trend, \( \tilde{\pi}_t \).

The nominal policy rate is driven by the real rate according to the Fisher equation. I equate the real rate with Fed monetary policy. The Fed adjusts the real rate systematically in response to movements in the inflation gap, \( \sigma_t \), and the real activity gap, \( \delta_t \), while smoothing with its own lag. The real interest rate rule

\[ r_t = \theta_{\sigma} \sigma_t + \theta_{\delta} \delta_t + \theta_r r_{t-1} + \sigma_r \epsilon_{r,t}, \epsilon_{r,t} \sim N(0,1), \] (3.9)

summarizes the systematic components, interest rate smoothing, and an unsystematic component, which is the monetary policy shock, \( \epsilon_{r,t} \). A real interest rate target is in line with the Federal Open Market Committee as revealed in their policy discussions which
reports real interest rate estimates and discusses the Taylor principle (Federal Open Market Committee (2015)). The Fisher equation shows that inflation moves one-for-one with its trend. As long as the coefficient on gap inflation in the real interest rate rule, $\theta$, is positive, the Taylor principle is satisfied in the NK-UC models.

### 3.2.4 Trends of the NK-UC Model

The PI trend, $\tau_t$, is a random walk with drift, as shown in equation (3.2). The random walk assumption gives the PI trend a Beveridge and Nelson (1981) trend interpretation. Another implication of equation (3.2) is the long-run restriction

$$\lim_{k \to \infty} E[c_{t+k} - k\mu | I_t] = \tau_t,$$

where $I_t$ is all data available to the econometrician prior to date $t$ and $\mu = E[\Delta c_t]$ is the deterministic drift. The date $t$ forecast of the log level of consumption at the infinite horizon is the current realization of the PI trend.

Similarly, a unit root in trend inflation, equation (3.5), identifies it with a Beveridge-Nelson trend, where

$$\lim_{k \to \infty} E[\pi_{t+k} | I_t] = \bar{\pi}_t.$$

Thus, the current long-run expectation of the level of inflation equals date $t$ trend inflation. Finally, the PI trend and trend inflation are independent random walks and are the nonstationary components of the NK-UC models.

### 3.2.5 Structural VAR of the NK-UC Model

The three stationary state variables of the NK-UC models are the real activity and inflation gaps and the real rate. I assume the real activity and inflation gaps are driven by a bivariate AR

$$\begin{bmatrix} \delta_t \\ \sigma_t \end{bmatrix} = \begin{bmatrix} \phi_{\delta,1} & \phi_{\sigma,1} \\ \varphi_{\delta,1} & \varphi_{\sigma,1} \end{bmatrix} \begin{bmatrix} \delta_{t-1} \\ \sigma_{t-1} \end{bmatrix} + \begin{bmatrix} \phi_{\delta,2} & \phi_{\sigma,2} \\ \varphi_{\delta,2} & \varphi_{\sigma,2} \end{bmatrix} \begin{bmatrix} \delta_{t-2} \\ \sigma_{t-2} \end{bmatrix} + B_{\delta} \epsilon_{\delta},$$

where $\epsilon_{\delta,t} = \begin{bmatrix} \epsilon_{\delta,t} \\ \epsilon_{\sigma,t} \end{bmatrix} \sim N(0_{2 \times 1}, I_{2 \times 2})$, and $B_{\delta} B_{\delta}' = \begin{bmatrix} \sigma_{\delta}^2 & \sigma_{\delta,\sigma} \\ \sigma_{\delta,\sigma} & \sigma_{\sigma}^2 \end{bmatrix}$. 

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Combined with the real rate, equation (3.9), the stationary states form a structural VAR,

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
-\theta_\delta & -\theta_\sigma & 1
\end{bmatrix}
\begin{bmatrix}
\delta_t \\
\sigma_t \\
r_t
\end{bmatrix}
= 
\begin{bmatrix}
\phi_{\delta,1} & \phi_{\sigma,1} & 0 \\
\varphi_{\delta,1} & \varphi_{\sigma,1} & 0 \\
0 & 0 & \theta_r
\end{bmatrix}
\begin{bmatrix}
\delta_{t-1} \\
\sigma_{t-1} \\
r_{t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
\phi_{\delta,2} & \phi_{\sigma,2} & 0 \\
\varphi_{\delta,2} & \varphi_{\sigma,2} & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_{t-2} \\
\sigma_{t-2} \\
r_{t-2}
\end{bmatrix}
+ B\Xi \epsilon_{\Xi,t},
\]

where \(\epsilon_{\Xi,t} = \begin{bmatrix} \epsilon_{\delta,t} \\ \epsilon_{\sigma,t} \\ \epsilon_{r,t} \end{bmatrix} \sim N(0_{3\times1}, I_{3\times3})\), and \(B\Xi B' = \begin{bmatrix} \sigma_\delta^2 & \sigma_{\delta,\sigma} & 0 \\ \sigma_{\delta,\sigma} & \sigma_\sigma^2 & 0 \\ 0 & 0 & \sigma_r^2 \end{bmatrix}\).

The structural VAR cannot be estimated in an UC model. Pre-multiplying the structural VAR by the inverse of the impact matrix yields the reduced-form VAR of the stationary states

\[
\begin{bmatrix}
\delta_t \\
\sigma_t \\
r_t
\end{bmatrix}
= 
\begin{bmatrix}
\phi_{\delta,1} & \phi_{\sigma,1} & 0 \\
\varphi_{\delta,1} & \varphi_{\sigma,1} & 0 \\
\theta_\delta & \theta_\sigma & \theta_r
\end{bmatrix}
\begin{bmatrix}
\delta_{t-1} \\
\sigma_{t-1} \\
r_{t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
\phi_{\delta,2} & \phi_{\sigma,2} & 0 \\
\varphi_{\delta,2} & \varphi_{\sigma,2} & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_{t-2} \\
\sigma_{t-2} \\
r_{t-2}
\end{bmatrix}
+ B_T \epsilon_{T,t},
\]

where \(\epsilon_{T,t} = \begin{bmatrix} \epsilon_{\delta,t} \\ \epsilon_{\sigma,t} \\ \epsilon_{r,t} \end{bmatrix} \sim N(0_{3\times1}, I_{3\times3})\), and \(B_T B'_T = \begin{bmatrix} \sigma_\delta^2 & \sigma_{\delta,\sigma} & \sigma_{\delta,r} \\ \sigma_{\delta,\sigma} & \sigma_\sigma^2 & \sigma_{\sigma,r} \\ \sigma_{\delta,r} & \sigma_{\sigma,r} & \sigma_r^2 \end{bmatrix}\).

where \(\theta_{\delta,i} = \theta_\delta \phi_{\delta,i} + \theta_\sigma \phi_{\sigma,i}\) for \(i = 1, 2\) and \(\theta_{\sigma,i} = \theta_\delta \varphi_{\delta,i} + \theta_\sigma \varphi_{\sigma,i}\) for \(i = 1, 2\). The real interest rate rule, equation (3.9), imposes cross-equation restrictions on the lags and covariance matrix of the errors of the reduced form VAR.

I consider several identification schemes to recover real and nominal demand shocks and the monetary policy shock from the reduced form VAR. These identification schemes will be discussed and used to compute impulse response functions (IRFs) and forecast error variance decompositions (FEVDs) in future work.

### 3.2.6 System of State Equations

The trends described in Section 3.2.4 are stacked on top of the reduced-form VAR from Section 3.2.5 to form the system of state equations in the NK-UC models.
\[ \begin{bmatrix} \tau_t \\ \bar{\pi}_t \\ \delta_t \\ \delta_{t-1} \\ \delta_{t-2} \\ \sigma_t \\ \sigma_{t-1} \\ r_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_{\delta,1} & \phi_{\delta,2} & 0 & \phi_{\sigma,1} & \phi_{\sigma,2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \varphi_{\delta,1} & \varphi_{\delta,2} & 0 & \varphi_{\sigma,1} & \varphi_{\sigma,2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \theta_{\delta,1} & \theta_{\delta,2} & 0 & \theta_{\sigma,1} & \theta_{\sigma,2} & \theta_r \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ \bar{\pi}_{t-1} \\ \delta_{t-1} \\ \delta_{t-2} \\ \sigma_{t-1} \\ \sigma_{t-2} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \mu \\ \mu \end{bmatrix} + B \varepsilon_t, \quad (3.10) \]

where \( \varepsilon_t = \begin{bmatrix} \varepsilon_{\tau,t} \\ \varepsilon_{\bar{\pi},t} \\ \varepsilon_{\delta,t} \\ \varepsilon_{\sigma,t} \\ \varepsilon_{r,t} \end{bmatrix} \sim N(0_{5 \times 1}, I_{5 \times 5}) \), and \( BB' = \begin{bmatrix} \sigma_{\tau}^2 & 0 & \sigma_{\tau,\delta} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{\bar{\pi}}^2 & 0 & 0 & 0 & \sigma_{\bar{\pi},\sigma} & 0 & 0 \\ \sigma_{\tau,\delta} & 0 & \sigma_{\delta}^2 & 0 & 0 & \sigma_{\delta,\sigma} & 0 & \sigma_{\delta,r} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\bar{\pi},\sigma} & \sigma_{\delta,\sigma} & 0 & 0 & \sigma_{\sigma,\sigma}^2 & 0 \\ 0 & 0 & \sigma_{\bar{\pi},\delta} & \sigma_{\delta,\sigma} & 0 & 0 & 0 & \sigma_{\sigma,r} \\ 0 & 0 & \sigma_{\delta,\sigma} & \sigma_{\sigma,\sigma} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\delta,r} & \sigma_{\sigma,r} & 0 & 0 & \sigma_{\sigma,r}^2 \end{bmatrix} \).

The covariance matrix of the errors of the reduced form VAR is restricted by correlation between innovations to the PI trend and real activity gap and between trend and gap inflation. There is no correlation among the PI and inflation trends and monetary policy shock and innovations to the trends.
3.2.7 System of Measurement Equations

The measurement equations (3.1), (3.3), (3.6), and (3.8) are summarized in the system of equations

\[
\begin{bmatrix}
\begin{array}{cccc}
1 & 0 & 0 & 1 + \frac{b}{a} \\
\xi & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}
\end{bmatrix}
\begin{bmatrix}
\begin{array}{c}
c_t \\
y_t \\
\pi_t \\
R_t
\end{array}
\end{bmatrix}
\begin{bmatrix}
\begin{array}{c}
\bar{c} \\
\bar{y} \\
\bar{\pi} \\
\bar{R}
\end{array}
\end{bmatrix}
\begin{bmatrix}
\begin{array}{c}
e_{c,t} \\
e_{y,t} \\
e_{\pi,t}
\end{array}
\end{bmatrix}
\sim N(0_{3\times1}, I_{3\times3}), \quad \text{and} \quad DD' =
\begin{bmatrix}
\begin{array}{cccc}
\sigma^2_c & 0 & 0 & 0 \\
0 & \sigma_y^2 & 0 & 0 \\
0 & 0 & \sigma^2_{\pi} & 0 \\
0 & 0 & 0 & 0
\end{array}
\end{bmatrix}.
\]

In equation (3.11), \( \chi = \frac{(a - \beta h)(a - h)}{a^2} \), \( \theta_r = [0_7 \ 1] \), and \( \theta_{\sigma} = [0_5 \ 1 \ 0_2] \). The loading matrix \( \Sigma \) is the sum of the contemporaneous and forward-looking impacts of the states on the observed variables. In the latter, \( \chi \theta_r \left[ I - \left( \frac{\beta h}{a} \right) A \right]^{-1} \) is the solution of the expected discounted forward sum in the consumption generating equation (1). The terms \( \gamma_f \theta_{\sigma} A \) and \( \theta_{\sigma} A \) give the one-step ahead expectations of the inflation gap which appear in the hybrid-NKPC and Fisher equations.

Innovations to the PI trend appear in the consumption generating equation (3.1) and the system of state equations (3.10). Hence, the measurement and state error terms are correlated. The covariance matrix of these error terms is

\[
E[\varepsilon_t \varepsilon_s'] = \begin{cases} 
\Gamma, & \text{for } t = s, \\
0, & \text{for } t \neq s,
\end{cases}
\]

where \( \Gamma \) is a 5 \times 4 matrix. The (1,1) element of \( \Gamma \) is \( E[e_{c,t} e_{\pi,t}] = \sigma_{c,\pi} \). The remaining entries of \( \Gamma \) are zero. Correlation of the observation and state errors is addressed by modifying the Kalman filter predictive and updating equations. Details are in Appendix B.3 and section 3.2.4 of Harvey (1990).
3.2.8 The Baseline and Alternative NK-UC Models

I estimate four alternatives to the baseline NK-UC model, labelled NK-UC-B. The alternative NK-UC models are summarized in Table 3.1. Habit in consumption is eliminated in the first alternative NK-UC model, NK-UC-A1. When \( h = 0 \), the linearized bond Euler equation equates consumption to the PI trend plus measurement error. This is the textbook PIH generating equation for consumption. Comparing the NK-UC-B and NK-UC-A1 models is a test of the impact of habit in consumption on the fit of the NK-UC model to the data.

Table 3.1: Summary of the NK-UC Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
<th>Restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>NK-UC-B</td>
<td>Baseline</td>
<td>none</td>
</tr>
<tr>
<td>NK-UC-A1</td>
<td>No consumption habit</td>
<td>( h = 0 )</td>
</tr>
<tr>
<td>NK-UC-A2</td>
<td>Vertical long-run hybrid-NKPC</td>
<td>( \gamma_b + \gamma_f = 1 )</td>
</tr>
<tr>
<td>NK-UC-A3</td>
<td>Purely forward-looking NKPC</td>
<td>( \gamma_b = 0 )</td>
</tr>
<tr>
<td>NK-UC-A4</td>
<td>Persistent markup shock</td>
<td>( \eta_t \sim \text{AR}(1) )</td>
</tr>
</tbody>
</table>

In the next alternative model, NK-UC-A2, the coefficients on the backward- and forward-looking components of the hybrid-NKPC, \( \gamma_b \) and \( \gamma_f \), are restricted to sum to one. In this case, the hybrid-NKPC is vertical in the long-run. This is consistent with long-run monetary policy neutrality which is a standard assumption in NK-DSGE models; see Christiano et al. (2005).

Ascari and Sbordone (2014) contend that the NKPC is purely forward-looking in the presence of trend inflation. I explore this hypothesis using the third alternative NK-UC model, NK-UC-A3. This alternative model restricts the coefficient on the backward-looking component of the hybrid-NKPC, \( \gamma_b \), to be zero.

The final alternative model considered, NK-UC-A4, endows the markup shock, \( \eta_t \), with persistence. The persistence is produced by a AR(1), \( \eta_t = \nu \eta_{t-1} + \sigma \varepsilon_{\eta,t}, |\rho_\eta| < 1 \), and \( \varepsilon_{\eta,t} \sim N(0,1) \).\(^5\) Similar to Kim et al. (2014), I adopt this specification to remain agnostic about the source(s) of persistence in realized inflation.

\(^5\)The AR(1) process for the markup shock is appended to the system of state equations (3.10) in the NK-UC-A4 model.
3.3 Data

This section describes the sample data on consumption, real aggregate activity, inflation, and the nominal policy rate. The discussion covers the two measures of real aggregate activity on which each of the NK-UC models are estimated. The consumption and real aggregate activity data are per capita, logged, and multiplied by 400. Inflation and the nominal policy rate are reported in annualized percentage terms. The quarterly sample runs from 1960Q1 to 2018Q4.

3.3.1 Consumption and Real Aggregate Activity

The real sector in the NK-UC model is summarized by consumption, $c_t$, and a real aggregate activity variable, $y_t$. Consumption is constant dollar personal consumption expenditures (PCE) on nondurable goods and services. This measure of consumption is consistent with the literature that tests the PIH; see West (1988), Campbell (1987), Stock and West (1988), Morley (2007), and also Hessler (2021). The price deflators for PCE of nondurable goods and services are chain-type indices. As a result, I follow Whelan (2002) to construct real aggregate nondurable goods and services consumption and its price deflator using the ideal Fisher index; see Appendix B.2.

Figure 3.1: Log Levels of Consumption and the Real Aggregate Activity, 1960Q1-2018Q4

Notes: The gray bars represent NBER recession dates.
Two different definitions of real aggregate activity are employed to estimate each of the NK-UC models. The first definition of income sets $y_t = \ln DI_t$, where $DI$ denotes real disposable income.\(^6\) The empirical literature testing the PIH often employs data on nondurable goods and services consumption and disposable income. Leading examples are Hall (1978), Nelson (1987), and Kiley (2010).

I replace disposable income with real GDP as the second specification of real aggregate activity. Real GDP measures income in the NK literature; see Christiano et al. (2005), Smets and Wouters (2007), and Del Negro and Schorfheide (2008). This measure of income is also used in NK-UC models. For example, see Doménech and Gómez (2006), Basistha and Nelson (2007), Lee and Nelson (2007), and Kim et al. (2014).

Figures 3.1 and 3.2 plot consumption, disposable income, and real GDP in log levels and differences of the log levels, respectively. Disposable income and real GDP decline during recessions in the sample period as expected. Consumption growth rates are also negative or near zero in recessions. Consumption, disposable income, and real GDP experience higher than average growth in the 1960s and second half of the 1990s. The lowest growth rates of these real series occur with the 2007-2009 recession. This trough is followed by below

---

\(^{6}\)Real disposable income is real personal income excluding transfer receipts (FRED\(^\circled{\mathcal{O}}\) ID: PIECTR).
average growth through the end of the sample.

### 3.3.2 Inflation and the Federal Funds Rate

The observables of the nominal side of the NK-UC models are inflation, $\pi_t$, and the nominal policy rate, $R_t$. Inflation is the growth rate of the nondurable goods and services consumption deflator to be consistent with constant dollar PCE on nondurable goods and services, $c_t$. The effective federal funds rate measures the nominal policy rate, $R_t$.

![Figure 3.3: Inflation and the Nominal Policy Rate, 1960Q1-2018Q4](image)

**Notes:** See the notes to Figure 3.1.

Figure 3.3 plots the levels of these two series. Nondurable goods and services inflation rises in the 1960s but remains below the federal funds rate. Stop and go inflation begins in 1967 and continues through the 1970s. Inflation and the federal funds rate peak above 10% during the 1973-1975 recession. The double-dip recession coincides with the beginning of the decline in the overall level of inflation. This reduction in inflation is accelerated during the Volcker disinflation of 1982-1986.

The Federal Reserve pushed interest rates lower throughout the savings and loans crisis of the late 1980s and early 1990s. The federal funds rate remained above inflation over this period. Throughout the rest of the 1990s, the Federal Reserve under Alan Greenspan raised interest rates to around 5% as the economy expanded at an above average rate. Inflation
remained stable during this time.

After the 2001 recession, the federal funds rate was held at a low level for a considerable period before the Federal Reserve raised the policy rate in 2004 peaking in 2005. The 2007-2009 recession resulted in a short-lived deflation. In response to the recession and sluggish growth in its aftermath, the Fed lowered the federal funds rate to near zero for more than six years. Inflation averaged less than 2% from 2009 to the end of the sample.

3.4 Results

This section presents evidence of the fit of the NK-UC models to the data and estimates of the parameters, trends, and gaps of these models. The NK-UC models are estimated using classical optimization methods. These methods maximize the predictive log likelihood retrieved from the generalized innovations form of the Kalman filter given an initial condition of the states, $X_{0|0}$, and an initial parameter vector, $\Psi_{0,i} = [\Psi_{0,i}^{State} \Psi_{0,i}^{Meas}]$, where $i = B, A1, A2, A3, A4$, and

$$\Psi_{0,i}^{Meas} = [h \gamma_f \gamma_b \kappa \xi \nu \sigma_c \sigma_y \sigma_\pi]$$
$$\Psi_{0,i}^{State} = [\phi_{j,k} \varphi_{j,k} \theta_{\delta} \theta_{\sigma} \theta_r \nu \sigma_{\tau} \sigma_{\pi} \sigma_{\sigma} \sigma_{\sigma} \sigma_{\eta} \sigma_{\tau,\pi} \sigma_{\tau,\sigma} \sigma_{\delta,\sigma}]$$

for $j = \delta, \sigma$, and $k = 1, 2$. I employ the bootstrap algorithm of Stoffer and Wall (2004) to obtain the small sample distribution of the parameter estimates, $\hat{\Psi}_i$.

In estimation, deterministic PI trend growth, $\alpha$, is set to 1.00385 giving a value of 1.54 for the constant drift term of the PI trend, $\mu$. This value of drift is equal to the sample annualized growth rate of consumption. I also set the households discount factor, $\beta$, equal to 0.992 which implies a quarterly discount factor of 0.8%, similar to Christiano et al. (2005) who set this value to 0.74%.

3.4.1 Fit of the NK-UC Models

Table 3.2 reports the maximum likelihood estimates (MLEs) of the log likelihood and the results of the bootstrap likelihood ratio tests for the NK-UC models estimated with both measures of real aggregate activity.

---

7See Appendix B.3 for more details. Codes for estimation are written in Julia 1.5.4 and are available upon request.

To see this note that $\mu = \log(\alpha) * 400$. 

---
Table 3.2: Tests of NK-UC Models

Real Aggregate Activity = Real Disposable Income, 1960Q1 to 2018Q4

<table>
<thead>
<tr>
<th></th>
<th>NK-UC-B</th>
<th>NK-UC-A1 $h = 0$</th>
<th>NK-UC-A2 $\gamma_b + \gamma_f = 1$</th>
<th>NK-UC-A3 $\gamma_b = 0$</th>
<th>NK-UC-A4 $\nu \in (0, 1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log L. boot s.e.</td>
<td>-1758.40 (18.26)</td>
<td>-1756.71 (17.50)</td>
<td>-1758.86 (19.81)</td>
<td>-1826.51 (19.07)</td>
<td>-1752.24 (16.72)</td>
</tr>
<tr>
<td>p-val</td>
<td>-</td>
<td>0.58</td>
<td>0.50</td>
<td>1.00</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Real Aggregate Activity = Real GDP, 1960Q1 to 2018Q4

<table>
<thead>
<tr>
<th></th>
<th>NK-UC-B</th>
<th>NK-UC-A1 $h = 0$</th>
<th>NK-UC-A2 $\gamma_b + \gamma_f = 1$</th>
<th>NK-UC-A3 $\gamma_b = 0$</th>
<th>NK-UC-A4 $\nu \in (0, 1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log L. boot s.e.</td>
<td>-1734.28 (13.88)</td>
<td>-1737.15 (12.44)</td>
<td>-1735.27 (13.94)</td>
<td>-1711.63 (10.87)</td>
<td>-1726.74 (13.77)</td>
</tr>
<tr>
<td>p-val</td>
<td>-</td>
<td>0.40</td>
<td>0.50</td>
<td>0.14</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Notes: The test statistic is $LR = -2(lh(\hat{\Psi}_B) - lh(\hat{\Psi}_i))$, $i \in [A1, A2, A3, A4]$, where $\Psi_i$ is the parameter vector for model NK-UC-$i$. The p-values are computed as the percentage of bootstrap estimates that have a larger test statistic than the true value of the test statistic.

The null hypothesis of the baseline NK-UC model fails to be rejected at the 10% significance level for the NK-UC model which restricts habit formation to be zero, NK-UC-A1. This result holds for both measures of real aggregate activity considered, suggesting habit formation is an important friction in the NK-UC model in accordance with the canonical NK-DSGE model. Morley (2007) examines the PIH in the presence of a real trend in an UC model and also finds support for the inclusion of habit formation.

The NK-UC model with a vertical long-run hybrid-NKPC, NK-UC-A2, fails to reject the null hypothesis at greater than the 10% significance level when real disposable income measures real aggregate activity. The null also fails to be rejected at the 10% significance level when real GDP replaces real disposable income in estimation. In much of the NK literature, a vertical long-run hybrid-NKPC is assumed which ensures long-run monetary neutrality. My results suggest modelers should remain agnostic as to the long-run interaction between monetary policy and the real sector in the NK model.

When the backward-looking component of the NK-UC model is shut down, I fail to reject the null hypothesis of the baseline NK-UC at the 10% significance level. It is worth noting that when real aggregate activity is measured by real disposable income, model NK-UC-A3 is rejected for every bootstrap resample. The lack of support for a purely-forward looking NKPC in the presence of an inflation trend is in contrast to the findings of Ascari.
and Sbordone (2014).

The best fitting model is the NK-UC model with serial correlation in the markup shock, NK-UC-A4. The null of the baseline NK-UC model is rejected at the 5% level for model NK-UC-A4. This result is robust to both measures of real aggregate activity used. Thus, addition of serial correlation in the markup shock improves the fit of the NK-UC model. This is support for the UC model of Kim et al. (2014). Serial correlation in the markup shock is used in the NK-DSGE literature to slow the speed of adjustment to the optimal markup in the hybrid-NKPC; see Smets and Wouters (2007).

3.4.2 NK-UC Model Parameter Estimates

This section reports the parameter estimates of the NK-UC models estimated with real disposable income as the measure of real aggregate activity. The measurement errors of the NK-UC models are similar across all observables and are discussed in Appendix B.5.1 to save space. The parameter estimates using real GDP are reported and discussed in Appendix B.5.2.

Table 3.3 reports the parameters of the system of measurement equations (3.11) for the NK-UC models. In the NK-UC model with a purely forward-looking NKPC, the habit formation parameter $h$ is 0.65, which is in line with NK literature; see Kano and Nason (2014). However, it is important to note the NK-UC-A3 model is a poor fit to the data as discussed above. The habit formation parameter is significant and has a value of 0.39, 0.37, and 0.29 in models NK-UC-B, -A2, and -A4, respectively. Habit formation introduces intertemporal complementarity into consumption. This generates hump-shaped IRFs for consumption in response to transitory shocks. Kano and Nason (2014) point out that the lower the estimated values for the habit formation parameter, the sooner the peak and the faster the decay of the IRF of consumption to a transitory shock.

The coefficients on the hybrid-NKPC are similar for the baseline and the NK-UC model without habit formation. In these models, the coefficients on the expectation of the inflation gap and lagged inflation gaps, $\gamma_f$ and $\gamma_b$, are less than half of magnitude of the slope of the hybrid-NKPC, $\kappa$. This value for $\kappa$ is similar in the NK-UC model with a long-run vertical hybrid-NKPC, NK-UC-A2. The coefficient on the expectation of inflation gap is 0.93 in the NK-UC-A2 model. When the backward-looking component of the hybrid-NKPC is shut down in NK-UC-A3, $\gamma_f$ approaches one and the slope of the hybrid-NKPC is not significantly different from zero. This is consistent with Lee and Nelson (2007). Their measure of the slope of the NKPC is procyclical but insignificant after accounting for trend inflation.
### Table 3.3: MLEs of NK-UC Model Measurement Equations

Real Aggregate Activity = Real Disposable Income, 1960Q1 to 2018Q4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NK-UC-B</th>
<th>NK-UC-A1</th>
<th>NK-UC-A2</th>
<th>NK-UC-A3</th>
<th>NK-UC-A4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate (s.e.)</td>
<td>Estimate (s.e.)</td>
<td>Estimate (s.e.)</td>
<td>Estimate (s.e.)</td>
<td>Estimate (s.e.)</td>
</tr>
<tr>
<td>$h$</td>
<td>0.39 (0.15)</td>
<td>-</td>
<td>0.37 (0.14)</td>
<td>0.65 (0.20)</td>
<td>0.29 (0.14)</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>0.30 (0.21)</td>
<td>0.20 (0.07)</td>
<td>0.93 (0.29)</td>
<td>0.99 (0.29)</td>
<td>0.61 (0.29)</td>
</tr>
<tr>
<td>$\gamma_b$</td>
<td>0.24 (0.14)</td>
<td>0.15 (0.04)</td>
<td>0.07 (0.29)</td>
<td>-</td>
<td>0.30 (0.21)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.61 (0.36)</td>
<td>0.53 (0.31)</td>
<td>0.52 (0.34)</td>
<td>0.02 (0.43)</td>
<td>0.62 (0.35)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>12.75 (1.06)</td>
<td>12.90 (1.47)</td>
<td>12.34 (1.65)</td>
<td>12.00 (0.96)</td>
<td>12.41 (1.66)</td>
</tr>
</tbody>
</table>

Notes: Bootstrap standard errors are calculated as $\sqrt{\sum_{b=1}^{B} (\hat{\theta}_{i,b} - \hat{\theta}_i)^2}$, where $\hat{\theta}_{i,b}$ is the bootstrap parameter estimate, $\hat{\theta}_i$ is the ML parameter estimate, and $B = 1000$ bootstrap samples. The factor loading for income, $\zeta$, is 1.04 in all NK-UC models and omitted from the table to save space.

NK-UC model with a serially correlated markup shock, NK-UC-A4, has values of 0.61 and 0.30 for $\gamma_f$ and $\gamma_b$, respectively. In model NK-UC-A4, the slope of the hybrid-NKPC is 0.62. These are reasonable values and imply the real activity gap plays a meaningful part in explaining observed inflation.

The factor loading of real aggregate activity on the permanent income trend, $\zeta$, is close to one in all NK-UC models which is support for the PIH. A large value for $\nu$ shows real aggregate activity has a larger response to movements in its gap compared with consumption. This indicates a significant role for consumption smoothing with respect to the transitory real activity gap.

Table 3.4 contains the estimates of the parameters of the state equations. The coefficients on the own lags of the real activity gap, $\phi_{\delta,1}$ and $\phi_{\delta,2}$, sum to greater than 0.96 across all models and have opposite signs. A high degree of persistence in the real activity gap is in line with related literature on the PIH; see Morley (2007). The corresponding coefficients for the inflation gap, $\varphi_{\sigma,1}$ and $\varphi_{\sigma,2}$, are small and imply low persistence. Persistence in inflation is primarily driven by its trend, or long-term inflation expectations, as pointed out by Ascari and Sbordone (2014).

The response of the real activity gap to lags of gap inflation is small relative to its response to own-lags as revealed by estimates of $\phi_{\sigma,1}$ and $\phi_{\sigma,2}$. In contrast, the inflation gap
Table 3.4: MLEs of NK-UC Model State Equations
Real Aggregate Activity = Real Disposable Income, 1960Q1 to 2018Q4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NK-UC-B</th>
<th>NK-UC-A1</th>
<th>NK-UC-A2</th>
<th>NK-UC-A3</th>
<th>NK-UC-A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi \delta,1 )</td>
<td>1.92 (0.10)</td>
<td>1.96 (0.06)</td>
<td>1.90 (0.11)</td>
<td>1.17 (0.22)</td>
<td>1.90 (0.13)</td>
</tr>
<tr>
<td>( \phi \delta,2 )</td>
<td>-0.94 (0.10)</td>
<td>-0.99 (0.06)</td>
<td>-0.92 (0.11)</td>
<td>-0.21 (0.20)</td>
<td>-0.92 (0.12)</td>
</tr>
<tr>
<td>( \phi \sigma,1 )</td>
<td>-0.12 (0.14)</td>
<td>-0.02 (0.10)</td>
<td>-0.10 (0.12)</td>
<td>-0.05 (0.10)</td>
<td>-0.11 (0.13)</td>
</tr>
<tr>
<td>( \phi \sigma,2 )</td>
<td>0.27 (0.12)</td>
<td>0.25 (0.07)</td>
<td>0.25 (0.11)</td>
<td>-0.05 (0.10)</td>
<td>0.26 (0.11)</td>
</tr>
<tr>
<td>( \varphi \delta,1 )</td>
<td>-0.94 (0.10)</td>
<td>-0.98 (0.06)</td>
<td>-0.93 (0.11)</td>
<td>-0.20 (0.24)</td>
<td>-0.92 (0.13)</td>
</tr>
<tr>
<td>( \varphi \delta,2 )</td>
<td>0.96 (0.13)</td>
<td>1.00 (0.08)</td>
<td>0.96 (0.14)</td>
<td>0.17 (0.25)</td>
<td>0.93 (0.15)</td>
</tr>
<tr>
<td>( \varphi \sigma,1 )</td>
<td>0.12 (0.12)</td>
<td>&lt;0.01 (0.08)</td>
<td>0.13 (0.11)</td>
<td>0.24 (0.08)</td>
<td>0.12 (0.11)</td>
</tr>
<tr>
<td>( \varphi \sigma,2 )</td>
<td>-0.28 (0.18)</td>
<td>-0.23 (0.13)</td>
<td>-0.27 (0.16)</td>
<td>0.06 (0.13)</td>
<td>-0.28 (0.17)</td>
</tr>
<tr>
<td>( \theta \delta )</td>
<td>-2.61 (1.00)</td>
<td>-2.95 (0.86)</td>
<td>-2.40 (0.94)</td>
<td>-3.63 (13.47)</td>
<td>-2.60 (0.95)</td>
</tr>
<tr>
<td>( \theta \sigma )</td>
<td>0.43 (0.24)</td>
<td>0.71 (0.31)</td>
<td>0.38 (0.21)</td>
<td>-0.38 (0.37)</td>
<td>0.42 (0.24)</td>
</tr>
<tr>
<td>( \theta r )</td>
<td>0.97 (0.03)</td>
<td>0.98 (0.02)</td>
<td>0.97 (0.03)</td>
<td>0.99 (0.02)</td>
<td>0.97 (0.03)</td>
</tr>
<tr>
<td>( \nu )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.33 (0.09)</td>
</tr>
</tbody>
</table>

Notes: See notes to Table 3.3.
Table 3.5: MLEs of NK-UC Model State Errors
Real Aggregate Activity = Real Disposable Income, 1960Q1 to 2018Q4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NK-UC-B</th>
<th>NK-UC-A1</th>
<th>NK-UC-A2</th>
<th>NK-UC-A3</th>
<th>NK-UC-A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\tau}$</td>
<td>2.01 (0.15)</td>
<td>2.03 (0.16)</td>
<td>2.00 (0.15)</td>
<td>2.19 (0.17)</td>
<td>2.02 (0.16)</td>
</tr>
<tr>
<td>$\sigma_{\hat{\pi}}$</td>
<td>0.53 (0.08)</td>
<td>0.54 (0.08)</td>
<td>0.52 (0.08)</td>
<td>0.52 (0.08)</td>
<td>0.47 (0.09)</td>
</tr>
<tr>
<td>$\sigma_{\delta}$</td>
<td>0.09 (0.02)</td>
<td>0.10 (0.02)</td>
<td>0.10 (0.03)</td>
<td>0.30 (0.05)</td>
<td>0.10 (0.03)</td>
</tr>
<tr>
<td>$\sigma_{\sigma}$</td>
<td>0.27 (0.41)</td>
<td>0.21 (0.20)</td>
<td>0.30 (0.54)</td>
<td>0.24 (1.06)</td>
<td>0.28 (0.35)</td>
</tr>
<tr>
<td>$\sigma_{r}$</td>
<td>0.71 (0.12)</td>
<td>0.64 (0.08)</td>
<td>0.72 (0.12)</td>
<td>1.01 (0.16)</td>
<td>0.73 (0.13)</td>
</tr>
<tr>
<td>$\sigma_{y}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.86 (0.13)</td>
</tr>
<tr>
<td>$\rho_{\tau,c}$</td>
<td>-0.78 (0.09)</td>
<td>-0.87 (0.08)</td>
<td>-0.77 (0.10)</td>
<td>-0.75 (0.08)</td>
<td>-0.79 (0.09)</td>
</tr>
<tr>
<td>$\rho_{\tau,\delta}$</td>
<td>0.22 (0.15)</td>
<td>0.22 (0.13)</td>
<td>0.21 (0.14)</td>
<td>-0.18 (0.16)</td>
<td>0.21 (0.15)</td>
</tr>
<tr>
<td>$\rho_{\pi,\sigma}$</td>
<td>-0.09 (0.22)</td>
<td>-0.10 (0.21)</td>
<td>-0.14 (0.20)</td>
<td>-0.19 (0.32)</td>
<td>-0.08 (0.21)</td>
</tr>
<tr>
<td>$\rho_{\delta,\sigma}$</td>
<td>-0.65 (0.26)</td>
<td>-0.60 (0.20)</td>
<td>-0.66 (0.22)</td>
<td>-0.81 (0.24)</td>
<td>-0.64 (0.23)</td>
</tr>
</tbody>
</table>

Notes: See notes to Table 3.3.
coefficients on the lags of the real activity gap, $\varphi_{\delta,1}$ and $\varphi_{\delta,2}$, are more than three times the magnitude of its own-lag coefficients. This is true across the NK-UC models with the exception of the purely forward-looking NK-UC model. Thus, the inflation gap responds strongly to lags of the real activity gap consistent with the predictive Phillips curve of Stock and Watson (2009).

The impact coefficient of the real activity gap on the real rate, $\theta_{\delta}$, is less than -2.0 across all NK-UC models. A negative value for $\theta_{\delta}$ implies the real rate increases in response to a decline in the real activity gap. The impact coefficient on gap inflation, $\theta_{\varpi}$, is positive and less than one, across all models with the exception of the NK-UC model with a purely forward-looking NKPC. A positive value for $\theta_{\varpi}$ satisfies the Taylor principle as can be seen from the Fisher equation. A value greater than 0.97 for $\theta_r$ across all NK-UC models represents a high degree of smoothing in the interest rate rule.

The AR(1) coefficient of the markup shock, $\nu$, in model NK-UC-A4 is estimated to be 0.33 and is significant at the 95% level. The volatility of the markup shock is reported in Table 3.5 and takes a value of 0.86. The estimate of the markup shock process shows that it is a significant shock in the NK-UC model.

Table 3.5 also reports the remaining estimates of the state errors. The scale volatility of the PI trend, $\sigma_{\tau}$, is nearly four times that of the inflation trend, $\sigma_{\bar{\pi}}$, in the NK-UC models. With the exception of the NK-UC model with a purely forward-looking NKPC, the volatility of the real activity gap, $\sigma_{\delta}$, is around 0.10 and less than half that of the inflation gap, $\sigma_{\varpi}$. The scale volatility of the real rate, $\sigma_r$, is larger than the corresponding value for the real activity and inflation gaps.

The correlation between consumption and the PI trend, $\rho_{c,\tau}$, is negative in all NK-UC models. Therefore, the consumption generating equation (3.1) implies consumption partially adjusts to a positive PI trend innovation in the short run as is found in Morley (2007). The negative correlation of consumption and the PI trend may explain the positive correlation found between the PI trend and the real activity gap, $\rho_{\tau,\delta}$. A positive correlation coefficient has real aggregate activity responding more than one-for-one with a PI trend shock. In contrast, I find insignificant negative correlation between the inflation gap and its trend, $\rho_{\varpi,\bar{\pi}}$. The correlation between the real activity and inflation gaps, $\rho_{\delta,\varpi}$, is also negative, in line with the UC model estimates of Basistha and Nelson (2007).
3.4.3 Estimates of the Trends and Gaps

This subsection reports the trends and gaps of the best fitting NK-UC model estimated with real disposable income as the real aggregate activity measure, model NK-UC-A4. This model features serial correlation in the markup shock of the NKPC. The corresponding estimates of model NK-UC-A4 with real GDP are reported and discussed in Appendix B.5.3. Figure 3.4 plots the estimate of the permanent income and inflation trends of NK-UC-A4. The gaps of this model are plotted in Figure 3.5.

![Figure 3.4: Estimates of NK-UC-A4 Trends](image)

Real Aggregate Activity = Real Disposable Income, 1960Q1 to 2018Q4

Notes: Real disposable income is translated by -65 to aid in visualization. The gray bars represent NBER recession dates. The blue shaded areas are the 95% sup-t uncertainty bands computed from 1000 bootstrap samples as in Olea and Plagborg-Møller (2018).

The PI trend closely tracks consumption, but is considerably smoother than real aggregate activity, throughout the sample period. From the 1960s up until the 1973-1975
recession, real disposable income grew faster than the PI trend. This is reflected by a long period of a positive real activity gap and consumption deviating slightly above the PI trend. Growth rates of disposable income are similar to the PI trend from the end of the 1973-1975 recession until the late 1980s. In the aftermath of the 1991 recession, the PI trend grows at a faster rate than real disposable income. As a consequence, the real activity gap is negative until the late 1990s. The largest deviation of consumption and real aggregate activity from the PI trend occurs during the 2007-2009 recession. Real disposable income contracts sharply deviating far below the PI trend. The real activity gap captures this moment as its deepest trough in the sample. Similarly, consumption remains below the PI trend until 2012.

The inflation trend deviates below observed inflation from the middle of the 1960s until 1973-1975 recession. This coincides with a prolonged period of a positive real activity gap which enters the hybrid-NKPC with a positive coefficient. The inflation gap does not differ significantly from zero until the 1973-1975 recession. Observed inflation hits its sample peak, which is well above trend inflation, at the start of the 1973-1975 recession. This can be explained by an inflation gap which increases above zero at this time and a peak in the real activity gap. Observed and trend inflation remain high throughout the remainder of the 1970s before another peak during the double dip recessions of the early 1980s. This is the highest value for trend inflation in the sample.

The Volcker disinflation of the 1980s leads to a downward shift in inflation and its trend. The inflation trend closely tracks observed inflation from this time until the end of the sample. An exception is the mid-1990s when the real activity gap drags observed inflation below its trend. Trend and observed inflation contract during the 2007-2009 recession with observed inflation briefly turning negative. Subsequently, inflation remains muted and its trend reaches low levels not seen outside of a recession since the 1960s.

The real activity gap declines with each recession in the sample as expected. The real activity gap reaches its highest level prior to the 1973-1975. Following the 1973-1975 recession until the late 1980s, the real activity gap is stable relative to the rest of the sample. The PI trend moves closely with real aggregate activity over this period. Brief contractions during the double dip recession of the early 1980s are an exception. As noted above, the PI trend outstrips growth in disposable income in the 1990s which corresponds to a long contraction in the real activity gap. The real activity gap declines steadily from a peak in the late 1990s to trough in 2005, but does not show a strong downward movement during the 2001 recession. It reaches its lowest point after the 2007-2009 recession, but returns to positive territory in early 2012.
Figure 3.5: Estimates of NK-UC-A4 Gaps
Real Aggregate Activity = Real Disposable Income, 1960Q1 to 2018Q4

Notes: See the notes to Figure 3.4.

The inflation gap is not as persistent as the real activity gap and is characterized by short-lived expansions and contractions. The sup-t uncertainty bands cover zero for large portions of the sample. Thus, the dynamics of inflation are dominated by its trend and not intrinsic gap inflation, as argued by Ascari and Sbordone (2014).

The most distinguishable movements in the inflation gap occur around recessions. The inflation gap peaks with each recession. Gap inflation contracts following recessions. These movements reflect the strong negative response of gap inflation to lags of the real activity gap.

The real interest rate is positive and increases in the 1960s and early 1970s coinciding with observed inflation increasing above its trend and a positive real activity gap as explained above. The real rate shifts downwards in the 1969-1970 recession and becomes negative following the 1973-1975 recession. This episode precedes the double dip recession which exhibits the highest rates of inflation and the nominal policy rate in the sample.

The Volcker disinflation, beginning in the early 1980s, produced the highest real rates of the sample. The inflation trend declines while the real activity and inflation gaps remain steady throughout the 1980s. This suggests contractionary monetary policy is the key driver
behind declining inflation expectations during this period. A close relationship between monetary policy and inflation expectations can also be seen in the late 1990s and mid-2000s. In the former period, a contractionary policy rate reduced inflation expectations. Conversely, the real rate becomes negative from 2002 until 2005 coinciding with an increase in the inflation trend.

In response to the 2007-2009 recession, the Federal Reserve pushed the nominal policy rate to near zero from until 2016. The real rate turns negative from 2007 until the end of the sample. However, there is a high degree of uncertainty around the estimate of the real rate. Despite this expansionary monetary policy, observed inflation and its trend remain at low levels and actually decline in the last years of the sample.

### 3.5 Conclusion

This paper presents NK-UC model estimates which employ restrictions from the NK framework. The NK-UC models feature a consumption generating equation, a hybrid-NKPC, the Fisher equation, and an interest rate rule in the real rate. These equations inform the decomposition of consumption, real aggregate activity, inflation, and the nominal policy rate into trends and gaps. The NK-UC models are pared down versions of the canonical NK model which allows for the examination of key frictions in the NK framework.

Baseline and alternative NK-UC models focus on habit formation in consumption and different specifications of the hybrid-NKPC. Bootstrap likelihood ratio tests show the best fitting NK-UC model has serial correlation of the markup shock in the hybrid-NKPC. At the same time, the best fitting model does not restrict the long-run hybrid-NKPC curve to be vertical. Practitioners working with NK models should carefully consider the treatment of nonstationarities in observed data. The omission of trends in the NK model may lead to misspecification.

Trends and gaps of the best fitting NK-UC model are presented. Observed inflation is closely tracked by its estimated trend. Additionally, the inflation gap is estimated to have low persistence as in Ascari and Sbordone (2014). Therefore, inflation is found to be primarily driven by long-term inflation expectations as opposed to an intrinsic process of gap inflation.

I plan to consider four identification schemes for the structural VAR in future work. The goal is to compute IRFs and FEVDs with respect to real aggregate demand, nominal aggregate demand, and monetary policy shocks. Future work also includes incorporating
structural breaks into the NK-UC models. Kim et al. (2014) estimate an UC model with serial correlation in the markup shock of the NKPC that has two structural breaks on a quarterly U.S. sample. Finally, I plan to conduct Angelini et al. (2022) misspecification tests for the estimation of the NK-UC models.
REFERENCES


APPENDICES
A.1 Data Construction

This section describes the construction of the data used in estimation. The quarterly sample runs from 1960Q1 to 2018Q4. The data are per capita, logged, and multiplied by 400. Population is equated to the civilian noninstitutional population (FRED ID: CNP16OV).

A.1.1 Real Consumption and Real Income per capita

Consumption is equated to real nondurable and services consumption (ND&S) per capita. I use an ideal Fisher index to construct constant dollar nondurable goods and services consumption as discussed in Whelan (2002). Personal Consumption Expenditures: Non-durable Goods (FRED ID: PCND) measures nominal nondurable consumption and Personal Consumption Expenditures: Services (FRED ID: PCESV) measures nominal services consumption. These variables are deflated by the associated chain-type price indices for
nondurable consumption and services consumption (FRED IDs: DNDGRG3Q086SBEA and DSERRG3Q086SBEA respectively) to produce constant dollar versions.

Whelan (2002) shows that an ideal Fisher index is the correct way to aggregate chain-weighted NIPA data. The first step constructs the growth rate of aggregating nondurables goods and services:

\[
\Delta Q = \frac{Q(t)}{Q(t-1)} = \sqrt{\frac{\sum_{i=1}^{n} P_i(t)Q_i(t)}{\sum_{i=1}^{n} P_i(t)Q_i(t-1)} \times \frac{\sum_{i=1}^{n} P_i(t-1)Q_i(t)}{\sum_{i=1}^{n} P_i(t-1)Q_i(t-1)}}
\]

where \( P_i \) and \( Q_i \) are the price indices and constant dollar quantities of nondurable and services consumption. The growth rates of the combined real series, \( \Delta Q \), are used to compute the real consumption series. The nominal values of nondurable goods and services are added in the base period to find the base quarter to generate real consumption in this quarter. The current base quarter is 2012Q3 in the U.S. NIPAs. Using the base period real value I compute the rest of the series by “chaining” the series forwards and backwards using the growth rates, \( \Delta Q \). Finally, dividing by population produces real nondurable goods and services consumption per capita, \( c o n_t \).

Real personal income excluding current transfer receipts (FRED ID: W875RX1) measures income. This series is personal income (excluding government transfer receipts) minus personal current taxes and contributions for government social insurance. To find real income per capita, \( y_t \), income is divided by population.

### A.1.2 Financial Sector Data

The data for the financial sector comes from the “Financial Accounts of the United States.” These data are published by the Board of Governors of the Federal Reserve System. Each of the financial sector data series are per capita and deflated by the ND&S PCE deflator.

Credit to private nonfinancial sector is computed as the sum of “Nonfinancial corporate business; debt securities and loans; liability” (FRB ID: FL104104005.Q), “Nonfinancial
noncorporate business; loans; liability" (FRB ID: FL114123005.Q), and “Households and nonprofit organizations; debt securities and loans; liability" (FRB ID: FL154104005.Q). This measure equals the supply of credit to private nonfinancial sector as reported by the BIS.\(^1\)

Aggregate nonfinancial assets of private nonfinancial sector is computed as the sum of “Nonfinancial corporate business; nonfinancial assets" (FRB ID: FL102010005.Q), “Nonfinancial noncorporate business; nonfinancial assets" (FRB ID: FL11201005.Q), and “Households and nonprofit organizations; nonfinancial assets" (FRB ID: FL15201005.Q).

These financial sector data series are not seasonally adjusted as opposed to the data on the real sector. In order to remove any excess seasonality in the series I seasonal adjust the financial data using X-13ARIMA-SEATS, which is provided by the U.S. Census Bureau.\(^2\)

### A.1.3 Engle-Granger Regressions Of the Financial Sector

The levels of both series fail the Augmented Dickey-Fuller (ADF) test, therefore, the series are at least integrated of order 1. In order to confirm that credit and nonfinancial assets are cointegrated, I employ Engle and Granger (1987). The Engle-Granger test involves regressing the level of nonfinancial assets on credit and an intercept and testing whether the residuals are stationary. The lags for the second stage of the Engle-Granger test are chosen by estimating an autoregression of lag length 12 and then reducing the number of lags until the last autoregression coefficient is significant. The number of lags chosen for the Engle-Granger test on the residuals is 5 and the ADF statistic is -3.39 with the 1% critical value of -2.58. This indicates that the null of a unit root in the residuals is rejected with a p-value of 0.0007.\(^3\)

---

\(^1\) The BIS measure is entitled “Total Credit to Private Non-Financial Sector, Adjusted for Breaks, for United States” (FRED ID: QUSPAMUSDA). There are, however, no adjustments for breaks in the US data provided by the BIS; see Dembiermont et al. (2013).


\(^3\) The financial variables are modeled as I(2) variables in the paper as attempts to model them as I(1) were rejected by the data. The Engle-Granger test indicates only that the resulting residuals are of lower order than the original series.
A.1.4 Engle-Kocizki Regression

In order to confirm that credit and nonfinancial assets share a common cycle, I conduct an Engle-Kozicki test. Engle and Kozicki (1993) demonstrate that two series have a common feature if there is some linear combination of the series that is unpredictable. In this paper, I am interested in testing whether or not there is a linear combination of the first differences of credit and nonfinancial assets produces a series which is unpredictable. Vahid and Engle (1993) extend the Engle-Kozicki test to a multivariate system when the variables cointegrate.

The test relies on a two-step procedure. The first stage is accomplished by regressing a set of instruments on the growth rates of nonfinancial assets. The instruments are one lag of the growth rates of credit and nonfinancial assets and the first lag of the residuals from the first-stage Engle-Granger regression. The second stage of the regression involves regressing the growth rates of credit on the predicted values from the first-stage regression and three lags of the residuals from the first stage of the regression. The test statistic is $0.625 \sim \chi^2(2)$, where degrees of freedom equal to the number of instruments minus one. This fails to reject the null hypothesis at the 5% significance level, so there is a common feature in credit and nonfinancial assets.

A.2 Maximum likelihood (ML) Estimation of UC Models

A.2.1 Innovations Form of the Kalman Filter

The loading matrices and covariance structure of the UC model are assumed to be uniquely parameterized by a vector of coefficients $\Gamma$. The innovations form of the Kalman filter approach to ML estimation is used to estimate $\Gamma$. Given an initial guess for the state vector, $X_{0|0}$, and the covariance matrix of the prediction error, $P_{0|0}$, the innovations form of the Kalman filter is described by:
\[
\epsilon_t = Y - CX_{t-1}, \\
\Omega_t = CP_{t-1}C' + DD', \tag{A.1}
\]

\[
\Omega_t = CP_{t-1}C' + DD', \tag{A.2}
\]

**Prediction**

\[
X_{t+1|t} = AX_{t-1|t} + HZ_t + K \epsilon_t, \tag{A.3}
\]

\[
P_{t+1|t} = AP_{t|t-1}A' + BB' - K_t \Omega_t K'_t, \tag{A.4}
\]

and

\[
K_t = AP_{t|t-1}C' \Omega^{-1}_t. \tag{A.5}
\]

Conditional on \(\Gamma\), the loglikelihood for the model is computed as

\[
llh = 0.5 \ast (n \ast \log(2\pi)) + \sum_{t=1}^{n} \log(|\Omega_t|) + \epsilon'_t \Omega^{-1}_t \epsilon_t, \tag{A.6}
\]

where \(T\) is the number of observations. In equation (B.14), \(\epsilon_t\) is the innovation error and \(\Omega_t\) is the conditional variance of the prediction error taken. The log likelihood of (B.14) is optimized with respect to \(\Gamma\) to produce ML estimates of the UC models.

### A.2.2 Initial Parameter Guesses

The parameter vector, \(\Gamma\), needs an initial guess, \(\Gamma_0\), to estimate the UC models by ML. I use information from the data to inform most of the UC model parameters. For \(C\), I conduct Engle-Granger regressions to make initial guesses for the cointegrating vector between variables. I guess that each variable responds to only one trend and subtract the constant from the Engle-Granger regression from the loading variable of interest before estimation to avoid issues with initial values.\(^\dagger\) I initialize the real variables responses to the level of the financial trend, \(\psi_t\), to zero and the financial variables response to the real trend, \(\tau_t\), to zero. The errors from each Engle-Granger regression are retained and used in Engle-Kocizki regressions to estimate the cofeature vector between the same variables.

\(^\dagger\)To make this more clear, income is assumed to be cointegrated with consumption. So, I estimate \(inc_t = \alpha + a\ con_t + err_t\) and set \(\alpha\) to its estimated value and subtract \(\alpha\) from \(inc_t\) to demean the series.
In the state transition equation, the constant drift parameter, \( \mu_\tau \), is set to the sample average growth rate of consumption. I set innovations to the real trend, \( \sigma_\tau \), and the level of the financial trend, \( \sigma_\psi \), to the standard deviation of the log differences of consumption and the log differences of credit respectively. Innovations to the business cycle, \( \sigma_\delta \), and the financial cycle, \( \sigma_\phi \), are set to the standard deviations of the log differences of income and the log differences of nonfinancial assets respectively. The covariance between the real trend and business cycle, \( \sigma_{\tau,\delta} \), is set to the covariance of the errors of the Engle-Granger regression of income on consumption and the Engle-Kocizki regression of income on consumption. I use this same logic to set the covariance to the financial trend and cycle, \( \sigma_{\psi,\phi} \).

Finally, some variables are set in an ad hoc manner. For example, the AR(2) parameters are set to 0.85 and −0.05 respectively, to target complex roots. The classical measurement errors in the matrix \( DD' \) are all set to be one in the bivariate models. Additionally, the innovation to the slope of the financial trend, \( \sigma_\xi \), is set to one.

### A.3 The Bootstrap Algorithm

There are problems with ML estimation when the sample size is small, the underlying processes are persistent, and/or the sample is non-Gaussian. One way to deal with these issues is to bootstrap the UC model and use the sample statistics to assess its fit to the data. Stoffer and Wall (2004) demonstrate that bootstrap resampling of the innovations form of the Kalman filter overcomes these issues. The bootstrap algorithm of Stoffer and Wall (2004) is described in the following steps:

1. Construct the standardized innovations, \( e_t \), from the Kalman filter,

\[
e_t(\hat{\Gamma}) = \Omega_t^{-1/2}(\hat{\Gamma})e_t(\hat{\Gamma}). \tag{A.7}
\]
Here, \( \hat{\Gamma} \) is the maximum likelihood estimate of the parameters and \( \Omega_t \) is defined in equation (B.10).

2. Sample, with replacement, \( T \) times from the set \{\( e_1(\hat{\Gamma}) \), ..., \( e_T(\hat{\Gamma}) \)\} to obtain a bootstrap sample of innovations, \{\( e^*_1, e^*_2, ..., e^*_T \)\}.

3. Construct a bootstrap data set \{\( y^*_1, ..., y^*_T \)\}, by solving

\[
\begin{bmatrix}
X^*_t \\
Y^*_t
\end{bmatrix} = \begin{bmatrix}
A(\hat{\Gamma}) & 0 \\
C(\hat{\Gamma}) & 0
\end{bmatrix} \begin{bmatrix}
X^*_{t-1} \\
Y^*_{t-1}
\end{bmatrix} + \begin{bmatrix}
P(\hat{\Gamma})_{t|t-1} C(\hat{\Gamma})' \Omega(\hat{\Gamma})^{-1/2} \\
\Omega(\hat{\Gamma})^{-1/2}
\end{bmatrix} \begin{bmatrix}
e^*_t
\end{bmatrix},
\]

for \( t = 1, ..., T \). The initial conditions of the Kalman filter remain fixed and the parameter vector is held fixed at \( \hat{\Gamma} \). In order to eliminate the effects of the initial conditions, set the first four values of the constructed data set equal to the first four values of the observed data, \( y_t \), as recommended by Stoffer and Wall (2004).

4. Using the bootstrap data set, \{\( y^*_t; t = 1, ..., T \)\}, construct the likelihood, \( L_{y^*}(\Gamma) \), and obtain the maximum likelihood estimate of \( \Gamma \), \( \hat{\Gamma}^* \).

5. Repeat steps 2 through 4 a large number of times, \( B \), obtaining a bootstrap sample of parameter estimates, \{\( \hat{\Gamma}^*_b; b = 1, ..., B \)\}. For this paper, I set \( B = 1000 \). The finite sample distribution of \( (\hat{\Gamma} - \Gamma)^2 \) may be approximated by the bootstrapped distribution of \( (\hat{\Gamma}^*_b - \hat{\Gamma})^2 \), for \( b = 1, ..., B \). The standard errors for \( \hat{\Gamma}_i \), \( i = 1, ..., k \) where \( k \) is the number of parameters, are computed as \( \sqrt{\frac{\sum_{b=1}^{B} (\hat{\Gamma}^*_b - \hat{\Gamma}_i)^2}{B-1}} \).
A.4 Bootstrapped Sup-t Uncertainty Bands

I construct sup-t uncertainty bands for the estimated trends and cycles following Olea and Plagborg-Møller (2018). The algorithm used to compute the plug-in sup-t confidence band used in the trend and cycle plots is described in the following steps:

1. Using the bootstrap estimates from step 4 of the bootstrap algorithm in Section B.4, obtain the prediction mean square error, $\hat{P}_{t|t-1}$, from the Kalman filter.

2. Compute the mean and the variance of the bootstrap states at each date $t$. Compute $\text{var}(\hat{\Gamma}) = \hat{\Sigma}$ as the sum of the variance of bootstrap states and the diagonal prediction mean square error matrix.

3. Draw a large number $S$ of i.i.d. normal vectors $\hat{V}(\ell) \sim N_k(0_k, \hat{\Sigma})$, $\ell = 1, \ldots, S$, and $k$ is the dimension of the parameter vector.

4. Find the sup-t critical value, $\hat{q}_{1-\alpha}$, as the $1-\alpha$ quantile of $\max_j |\hat{\Sigma}^{-1/2}_{jj} \hat{V}(\ell)|$ for $\ell = 1, \ldots, S$, $S$ is set to 100,000.

5. Compute the sup-t confidence band as the pair $(\hat{\Gamma}_j - \text{var}(\hat{\Gamma}_j)\hat{q}_{1-\alpha}, \hat{\Gamma}_j - \text{var}(\hat{\Gamma}_j)\hat{q}_{1-\alpha})$ for $j = 1, \ldots, T$.

The following algorithm computes the bootstrap sup-t confidence band used in the IRF:

1. Compute the IRF of interest for each of the $B = 1000$ estimates across the horizon $h$.

2. Find the empirical quantiles $\hat{\xi}$ and $1 - \hat{\xi}$ of the bootstrap estimates which ensures that $1 - \alpha$ of the observations across the entire horizon are included.

3. Compute the confidence band by reporting the quantiles $\hat{\xi}$ and $1 - \hat{\xi}$ of the bootstrap estimates at each horizon, $h = 1, \ldots, H$. 
A.5 Bivariate UC Models

A.5.1 Bivariate UC Model of the Real Sector

The structure of the UC model yields a Beveridge and Nelson (1981) decomposition. The Beveridge-Nelson decomposition has a trend that is a random walk (with drift). The cycle is the residual of the observed data minus the trend. Assuming the sample data are I(1), makes the cycle stationary. This decomposition has been used to estimate trends and cycles in the real sector by Vahid and Engle (1993) and Morley et al. (2003).

These restrictions can be seen in the measurement equations,

\[
\begin{bmatrix}
    \text{con}_t \\
    \text{inc}_t
\end{bmatrix} =
\begin{bmatrix}
    1 & \kappa & 0 \\
    \alpha & 1 & 0
\end{bmatrix}
\begin{bmatrix}
    \tau_t \\
    \delta_t \\
    \delta_{t-1}
\end{bmatrix} + D e_t,
\]

(A.9)

where \( e_t = \begin{bmatrix} e_{\text{con},t} \\ e_{\text{inc},t} \end{bmatrix} \sim N(0_{2\times1}, I_2) \), and \( DD' = \begin{bmatrix} \sigma^2_{\text{con}} & 0 \\ 0 & \sigma^2_{\text{inc}} \end{bmatrix} \).

The permanent income trend, \( \tau_t \), is a random walk with drift and the business cycle, \( \delta_t \), is a second order autoregression (AR). The laws of motion of these unobserved components are summarized by the state transition equations,

\[
\begin{bmatrix}
    \tau_{t+1} \\
    \delta_{t+1} \\
    \delta_t
\end{bmatrix} =
\begin{bmatrix}
    \mu & 1 & 0 \\
    0 & 0 & \theta_1 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    \tau_t \\
    \delta_t \\
    \delta_{t-1}
\end{bmatrix} + B e_t,
\]

(A.10)

where \( e_t = \begin{bmatrix} e_{\tau_t} \\ e_{\delta_t} \end{bmatrix} \sim N(0_{2\times1}, I_2) \), and \( BB' = \begin{bmatrix} \sigma^2_{\tau} & \sigma_{\tau,\delta} \\ \sigma_{\tau,\delta} & \sigma^2_{\delta} \end{bmatrix} \).
A.5.2 Bivariate UC Model of the Financial Sector

Much of the literature measures credit cycles from the perspective of firms and households ability to pay their debt obligations. For example, Drehmann et al. (2012) calculate a credit cycle by using a band-pass filter on the ratio of credit to GDP. In this interpretation, when credit growth outpaces income growth, the credit cycle is in an expansion. This paper differs from the literature by interpreting credit cycles from a different point of view. The viewpoint I take is that firms and households demand credit to target a level of leverage in order to increase their future net worth as in Brunnermeier and Sannikov (2014).  

I test for cointegration and a common cycle in the financial sector following Vahid and Engle (1993). First I test for cointegration between the supply of credit and aggregate nonfinancial assets. Using the Engle-Granger two-step procedure to test for cointegration between credit supply and credit demand.

The results are summarized in Table A.1. The two series are cointegrated at the 5% level. A test for a common cycle is developed by Engle and Kozicki (1993). The Engle-Kozicki regression tests for whether a linear combination of the first differences of the two variables has a lower degree of serial correlation than the individual first differenced variables. The null hypothesis assumes a common cycle exists and the data fail to reject the null at the 5% significance level.

The restrictions on credit to the private nonfinancial sector, \( n f c_t \), and aggregate nonfinancial assets of the private nonfinancial sector, \( n f a_t \), respectively can be see in the

---

5I also considered aggregate net worth of the private nonfinancial sector to measure net worth. The residuals of the two-step Engle Granger regression of Aggregate Net worth and Credit Supply are stationary. However, the residuals appear to be more nonstationary than the residuals using nonfinancial assets in place of aggregate net worth.

6Prior to cointegration testing, the data for credit and nonfinancial assets were seasonally adjusted, deflated by the PCE deflator, and transformed into logged per capita variables as described in Section B.2
Table A.1: Engle-Granger and Engle-Kocizki Regression Results, 1960Q1 to 2018Q4

<table>
<thead>
<tr>
<th></th>
<th>Coefficient (Asymptotic se)</th>
<th>Null Hypothesis</th>
<th>Test Statistic (5% Critical Value)</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Engle-Granger Regression</strong></td>
<td>1.53 (0.12)</td>
<td>No Common Trend</td>
<td>-3.39 (-1.94)</td>
<td>Reject Null</td>
</tr>
<tr>
<td><strong>Engle-Kocizki Regression</strong></td>
<td>0.43 (&lt;0.01)</td>
<td>Common Cycle</td>
<td>0.63 (5.99)</td>
<td>Fail to Reject Null</td>
</tr>
</tbody>
</table>

Notes: Details for the Engle-Granger and Engle-Kocizki Regressions are located in Section B.2.

measurement equations,

\[
\begin{bmatrix}
  c_{o}n_{t} \\
  i_{n}c_{t}
\end{bmatrix} =
\begin{bmatrix}
  1 & 0 & \lambda & 0 \\
  \beta & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
  \psi_{t} \\
  \xi_{t} \\
  \phi_{t} \\
  \phi_{t-1}
\end{bmatrix} + D e_{t}, \quad (A.11)
\]

where \( e_{t} = \begin{bmatrix} e_{nf,c,t} \\ e_{nf,a,t} \end{bmatrix} \sim N(0_{2 \times 1}, I_{2}) \), and \( DD' = \begin{bmatrix} \sigma_{nfc}^2 & 0 \\ 0 & \sigma_{nfa}^2 \end{bmatrix} \).

In (A.11), the financial trend is a local linear trend with level \( \psi_{t} \) and with stochastic drift term \( \xi_{t} \), and the credit cycle, \( \phi_{t} \) is a AR(2). Initially, I estimate a bivariate UC model in which the nonstationary component is a random walk with drift. Figure A.1 plots the autocorrelation functions (ACFs) of the innovations to the random walk and local linear financial trends recovered from the Kalman filter. The ACF of the random walk financial trend features a high degree of serial correlation, which suggests a random walk financial trend is misspecified. Introducing a stochastic drift term to create a local linear trend specification succeeds in removing this serial correlation as displayed by the ACF in the right side panel of Figure A.1.
Figure A.1: ACFs of Innovations to Financial Trend and Cycle

Notes: The state innovation errors come from the diagonal of the prediction mean squared error of the Kalman filter.

The financial trend and credit cycle are summarized by the state transition equations,

\[
\begin{bmatrix}
\psi_{t+1} \\
\xi_{t+1} \\
\phi_{t+1} \\
\phi_t
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \gamma_1 & \gamma_2 \\
0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
\psi_t \\
\xi_t \\
\phi_t \\
\phi_{t-1}
\end{bmatrix} + B\epsilon_t \tag{A.12}
\]

where \( \epsilon_t = \begin{bmatrix}
\epsilon_{\psi t} \\
\epsilon_{\xi t} \\
\epsilon_{\phi t}
\end{bmatrix} \sim N(0_{3 \times 1}, I_3) \), and \( BB' = \begin{bmatrix}
\sigma^2_{\psi} & 0 & \sigma_{\psi,\phi} \\
0 & \sigma^2_{\xi} & 0 \\
\sigma_{\psi,\phi} & 0 & \sigma_{\phi}
\end{bmatrix} \).
A.6 Predictive Regressions

A.6.1 Bruesch-Godfrey Tests of second difference of credit to GDP

This section conducts Bruesch-Godfrey tests to investigate the ability of the Basel gap, the estimated business and credit cycles, and the estimated slope of the financial trend to predict the second difference of the credit to GDP ratio compared to the business and credit cycle and slope of the financial trend. Table 2 summarizes the results for these regressions. Each regression has the $h$-step ahead second difference of the credit to GDP ratio as the dependent variable. The sole independent variable is the Basel gap in Table A.2a, the credit cycle in Table A.2b, the business cycle in Table A.2c, and the slope of the financial trend in Table A.2d. The null hypothesis of each Bruesch-Godfrey test at each horizon is rejected at less than 1% significance level implying that none of the predictors considered is a good forecaster of the second difference of the credit to GDP ratio.

Table A.2a: Predictive Regressions for the second difference of credit to GDP, 1960Q1 to 2018Q4

Regression: \[ \Delta^2 \left( \frac{Credit}{GDP} \right)_{t+h} = \beta_0 + \beta_1 (Basel \ Gap)_t + e_t \]

<table>
<thead>
<tr>
<th>Number of Quarters Ahead</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>coef</td>
<td>-0.001</td>
<td>-0.000</td>
<td>-0.000</td>
<td>-0.001</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>se</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>t-stat</td>
<td>0.321</td>
<td>-0.112</td>
<td>-0.185</td>
<td>-0.445</td>
<td>-0.698</td>
</tr>
<tr>
<td></td>
<td>p-val</td>
<td>0.374</td>
<td>0.456</td>
<td>0.427</td>
<td>0.328</td>
<td>0.243</td>
</tr>
<tr>
<td>$R^2$</td>
<td>value</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>Breusch-Godfrey</td>
<td>value</td>
<td>104.840</td>
<td>103.3643</td>
<td>100.822</td>
<td>99.535</td>
<td>96.364</td>
</tr>
<tr>
<td></td>
<td>p-val</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: The null hypothesis of the Breusch-Godfrey test is that the regression errors are not serially correlated.
**Table A.2b:** Predictive Regressions for the second difference of credit to GDP, 1960Q1 to 2018Q4

Regression:

\[ \Delta^2 \left( \frac{Credit}{GDP} \right)_{t+h} = \beta_0 + \beta_2 \phi_t + e_t \]

<table>
<thead>
<tr>
<th>Number of Quarters Ahead</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_2 ) coef</td>
<td>-0.024</td>
<td>-0.006</td>
<td>-0.006</td>
<td>-0.002</td>
<td>-0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>se</td>
<td>0.008</td>
<td>0.008</td>
<td>0.008</td>
<td>0.008</td>
<td>0.008</td>
<td>0.009</td>
</tr>
<tr>
<td>t-stat</td>
<td>-2.994</td>
<td>-0.804</td>
<td>-0.706</td>
<td>-0.306</td>
<td>-0.174</td>
<td>0.107</td>
</tr>
<tr>
<td>p-val</td>
<td>0.002</td>
<td>0.211</td>
<td>0.241</td>
<td>0.380</td>
<td>0.431</td>
<td>0.457</td>
</tr>
<tr>
<td>R(^2) value</td>
<td>0.013</td>
<td>0.001</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Breusch-Godfrey value</td>
<td>106.075</td>
<td>105.883</td>
<td>101.692</td>
<td>99.575</td>
<td>96.100</td>
<td>91.554</td>
</tr>
<tr>
<td>p-val</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: See notes to Table A.2a.

**Table A.2c:** Predictive Regressions for the second difference of credit to GDP, 1960Q1 to 2018Q4

Regression:

\[ \Delta^2 \left( \frac{Credit}{GDP} \right)_{t+h} = \beta_0 + \beta_3 \delta_t + e_t \]

<table>
<thead>
<tr>
<th>Number of Quarters Ahead</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_3 ) coef</td>
<td>0.003</td>
<td>0.001</td>
<td>-0.005</td>
<td>-0.004</td>
<td>-0.000</td>
<td>0.002</td>
</tr>
<tr>
<td>se</td>
<td>0.003</td>
<td>0.003</td>
<td>0.004</td>
<td>0.003</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>t-stat</td>
<td>0.857</td>
<td>0.255</td>
<td>-1.557</td>
<td>-1.314</td>
<td>-0.048</td>
<td>0.406</td>
</tr>
<tr>
<td>p-val</td>
<td>0.199</td>
<td>0.393</td>
<td>0.061</td>
<td>0.096</td>
<td>0.470</td>
<td>0.343</td>
</tr>
<tr>
<td>R(^2) value</td>
<td>0.000</td>
<td>0.000</td>
<td>0.002</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Breusch-Godfrey value</td>
<td>105.188</td>
<td>103.597</td>
<td>101.538</td>
<td>99.804</td>
<td>95.977</td>
<td>91.648</td>
</tr>
<tr>
<td>p-val</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: See notes to Table A.2a.
Table A.2d: Predictive Regressions for the second difference of credit to GDP, 1960Q1 to 2018Q4

Regression: \[ \Delta^2 \left( \frac{Credit}{GDP} \right)_{t+h} = \beta_0 + \beta_4 (Slope of the Financial Trend)_{t} + e_t \]

<table>
<thead>
<tr>
<th>Number of Quarters Ahead</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_4 )</td>
<td>coef</td>
<td>-0.018</td>
<td>0.012</td>
<td>-0.014</td>
<td>-0.021</td>
<td>-0.013</td>
</tr>
<tr>
<td></td>
<td>se</td>
<td>0.011</td>
<td>0.012</td>
<td>0.011</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>t-stat</td>
<td>-1.550</td>
<td>0.965</td>
<td>-1.306</td>
<td>-2.214</td>
<td>-1.283</td>
</tr>
<tr>
<td></td>
<td>p-val</td>
<td>0.064</td>
<td>0.175</td>
<td>0.100</td>
<td>0.015</td>
<td>0.101</td>
</tr>
<tr>
<td>R²</td>
<td>value</td>
<td>0.002</td>
<td>0.001</td>
<td>0.001</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>Breusch-Godfrey</td>
<td>value</td>
<td>104.508</td>
<td>103.346</td>
<td>101.362</td>
<td>100.663</td>
<td>96.705</td>
</tr>
<tr>
<td></td>
<td>p-val</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: See notes to Table A.2a.
A.7 Multivariate UC Model Plots

Figure A.2: Estimated Permanent Income Trends, 1960Q1 to 2018Q4

Notes: The gray bars represent NBER recession dates. The dotted lines are sup-t uncertainty bands at the 5% significance level.
Figure A.3: Estimated Financial Trend Levels, 1960Q1 to 2018Q4

Notes: See notes to Figure A.2.
Figure A.4: Estimated Financial Trend Slopes, 1960Q1 to 2018Q4

Notes: See notes to Figure A.2.
Figure A.5: Estimated Business Cycles, 1960Q1 to 2018Q4

Notes: See notes to Figure A.2.
Figure A.6: Estimated Credit Cycles, 1960Q1 to 2018Q4

Notes: See notes to Figure A.2.
B.1 Linearized Consumption-Bond Euler Equation

The household's problem summarized in Section 2.1 yields the following first order necessary conditions (FONCs)

\[ \lambda_t = \frac{1}{c_t - h c_{t-1}} + E_t \left\{ \frac{\beta h}{c_{t+1} - h c_t} \right\} \]  

(B.1)

and

\[ \lambda_t = E_t \left\{ \beta \frac{R_{t+1}}{P_{t+1} / P_t} \lambda_{t+1} \right\}. \]  

(B.2)
Equation (B.1) is the household’s marginal utility of consumption and equation (B.2) is the bond Euler equation. Labor income, is a random walk with drift, as summarized by $\tau_t$ in equation (2) in the paper. Consumption inherits the unit root in labor income. In order to find the linearized consumption-bond Euler equation the FONCs must first be stochastically detrended.

Define stochastically detrended consumption as $\tilde{c}_t = c_t / \tau_t$. Also define $\tilde{\lambda} = \lambda \tau_t$ and $\alpha_t = \tau_t / \tau_{t-1} = \exp(\mu \epsilon_{t,t})$. The stochastically detrended FONCs are

$$\tilde{\lambda}_t = \frac{\alpha_t}{\alpha_t \tilde{c}_t - h \tilde{c}_{t-1}} - \beta h E_t \left\{ \frac{1}{\alpha_{t+1} \tilde{c}_{t+1} - h \tilde{c}_t} \right\},$$

(B.3)

and

$$\tilde{\lambda}_t = \beta E_t \left\{ \frac{\tilde{\lambda}_{t+1} R_{t+1}}{\alpha_{t+1} (1 + \pi_{t+1})} \right\},$$

(B.4)

where $(1 + \pi_{t+1}) = P_{t+1}/P_t$ is the inflation rate. Log-linearization of both FONCs and combining the resulting equations results in the second-order stochastic difference equation

$$\alpha \beta h E_t \{ \Delta \hat{c}_{t+2} + \epsilon_{\tau,t+2} \} - (\beta h^2 + \alpha^2) E_t \{ \Delta \hat{c}_{t+1} + \epsilon_{\tau,t+1} \}$$

$$+ a h(\Delta \hat{c}_t + \epsilon_{\tau,t}) = -(\alpha - \beta h)(\alpha - h) E_t \{ r_{t+1} \}.$$  

(B.5)

In equation (B.5), variables without a time-subscript are the steady state values, $\hat{c}_t = \ln \tilde{c}_t - \ln c$, $\alpha = \exp(\mu)$, and $r_{t+1}$ is the real rate.

Since $E_t \{ \epsilon_{\tau,t+j} \} = 0$ for $j \geq 0$, equation (B.5) becomes

$$\alpha \beta h E_t \{ \Delta \hat{c}_{t+2} \} - (\beta h^2 + \alpha^2) E_t \{ \Delta \hat{c}_{t+1} \}$$

$$+ a h(\Delta \hat{c}_t) = -(\alpha - \beta h)(\alpha - h) E_t \{ r_{t+1} \}.$$  

(B.6)
The solution to this second-order difference equation is

\[
\left(1 - \frac{h}{\alpha}L\right)\Delta \hat{c}_t = a h \varepsilon_{\tau,t} + \frac{(a - \beta h) (a - h)}{\alpha^2} \sum_{j=0}^{\infty} \left(\frac{\beta h}{\alpha}\right)^j E_t\{r_{t+j}\}. \tag{B.7}
\]

Using the definition of \(\hat{c}_t\) and \(\hat{c}_t\) and rearranging yields equation (1) in the paper which is repeated here,

\[
c_t = \tau_t + a h \varepsilon_{\tau,t} + \left(1 + \frac{h}{\alpha}\right) \delta_{t-1} - \frac{h}{\alpha} \delta_{t-2} + \frac{(a - \beta h) (a - h)}{\alpha^2} \sum_{j=0}^{\infty} \left(\frac{\beta h}{\alpha}\right)^j E_t\{r_{t+j}\} + \sigma_c e_{c,t}, \quad e_{c,t} \sim N(0, 1), \tag{B.8}
\]

where the constant \(\ln c\) is ignored and observation error \(e_{c,t}\) is introduced.

\section*{B.2 Data}

This section describes the construction of the data used in estimation of the NK-UC models. The quarterly sample runs from 1960Q1 to 2018Q4. The data on consumption and real aggregate activity are per capita, logged, and multiplied by 400. Inflation and the federal funds rate are in annualized percentage terms. Population is equated to the civilian noninstitutional population (FRED ID: CNP16OV).

Consumption is real nondurable and services consumption per capita. The series used to construct this measure are Personal Consumption Expenditures: Nondurable Goods (FRED ID: PCND), Personal Consumption Expenditures: Services (FRED ID: PCESV), and the chain-type price indices for nondurable goods and services (FRED IDs: DNDGRG3Q086SBEA and DSERRG3Q086SBEA). Chain-type price indices necessitate the construction of an ideal Fisher index as in Whelan (2002) in order to compute real nondurable and services consumption.
I repeat the discussion of the ideal Fisher index from Appendix A of Hessler (2021) here for convenience. Whelan (2002) shows that an ideal Fisher index is the correct way to aggregate chain-weighted NIPA data. The first step constructs the growth rate of aggregating nondurables goods and services:

$$\Delta Q = \frac{Q(t)}{Q(t-1)} = \sqrt{\frac{\sum_{i=1}^{n} P_i(t)Q_i(t)}{\sum_{i=1}^{n} P_i(t)Q_i(t-1)} \times \frac{\sum_{i=1}^{n} P_i(t-1)Q_i(t)}{\sum_{i=1}^{n} P_i(t-1)Q_i(t-1)}}$$

where $P_i$ and $Q_i$ are the price indices and constant dollar quantities of nondurable and services consumption. The growth rates of the combined real series, $\Delta Q$, are used to compute the real consumption series. The nominal values of nondurable goods and services are added in the base period to find the base quarter to generate real consumption in this quarter. The current base quarter is 2012Q3 in the U.S. NIPAs. Using the base period real value I compute the rest of the series by “chaining” the series forwards and backwards using the growth rates, $\Delta Q$. Finally, dividing by population produces real nondurable goods and services consumption per capita, $c_t$.

Inflation is computed as the log difference of the nondurable and services consumption deflator produced as a byproduct of the ideal Fisher index construction. Disposable income is measured by Real Personal Income Excluding Transfer Receipts (FRED ID: PIECTR). Real GDP is Real Gross Domestic Product (FRED ID: GDPC1). The nominal interest rate is the Effective Federal Funds Rate (FRED ID: FEDFUNDS).

**B.3 Estimation Methods**

**B.3.1 Generalized Innovations Form of the Kalman Filter**

The loading matrices and covariance structure of the NK-UC model are assumed to be uniquely parameterized by a vector of coefficients, $\Psi$. The generalized innovations form
of the Kalman filter approach to maximum likelihood (ML) estimation is used to estimate \( \Psi \). The generalized innovations form of the Kalman filter differs from its standard formulation by allowing the measurement and state equations to covary. The covariance of the measurement and state equations are captured in the matrix \( \Gamma \). Given an initial guess for the state vector, \( X_{0|0} \), and the covariance matrix of the prediction error, \( P_{0|0} \), the generalized innovations form of the Kalman filter is described by:

**Innovations**

\[
\begin{align*}
\epsilon_t &= Z_t - CX_{t|t-1}, \quad (B.9) \\
\Omega_t &= CP_{t|t-1}C' + C\Gamma + \Gamma'C' + DD', \quad (B.10)
\end{align*}
\]

**Prediction**

\[
\begin{align*}
X_{t+1|t} &= AX_{t-1|t} + HG_t + K\epsilon_t, \quad (B.11) \\
P_{t+1|t} &= AP_{t|t-1}A' + BB' - K_t\Omega_tK_t', \quad (B.12)
\end{align*}
\]

and

\[
K_t = (AP_{t|t-1}C' + \Gamma)\Omega_t^{-1}. \quad (B.13)
\]

Conditional on, \( \Psi \), the loglikelihood for the model is computed as

\[
ll_h = 0.5(n \star \log(2\pi)) + \sum_{t=1}^{n} \log(|\Omega_t|) + \epsilon_t'\Omega_t^{-1}\epsilon_t, \quad (B.14)
\]

where \( T \) is the number of observations. In equation (B.14), \( \epsilon_t \) is the innovation error and \( \Omega_t \) is the conditional variance of the prediction error taken. The log likelihood of (B.14) is optimized with respect to \( \Psi \) to produce ML estimates of the NK-UC models.

**B.3.2 Initial Parameter Guesses**

The parameter vector, \( \Psi \), needs initial guesses \( \Psi_{0,i} \), for \( i = B, A1, A2, A3, \) and \( A4 \), to estimate the NK-UC models by ML. I use information from the data and new Keynesian (NK) literature, as well as ad hoc assumptions, to inform the initial guesses of the NK-UC model parameters.

The habit formation parameter is set to \( h = 0.65 \) which is standard in NK models; see
Kano and Nason (2014). The slope of the hybrid-NKPC, $\kappa$, is set to 0.29 as in Basistha and Nelson (2007) who estimate a UC model with a NKPC and real activity gap measured with real GDP. The initial guesses of the forward- and backward-looking coefficients of the hybrid-NKPC are $\gamma_f = 0.6$ and $\gamma_b = 0.4$, respectively.

For the loading factor of real aggregate activity on the PI trend, $\zeta$, I conduct Engle and Granger (1987) regressions to make initial guesses for the cointegrating vector between consumption and the measures of real aggregate activity. Each variable responds to only one trend and I subtract the constant of the Engle-Granger regression from the loading variable of interest before estimation to avoid issues with initial values.\(^1\) I set the loading factor of real aggregate activity to the real activity gap, $\nu$, to 12.0. The classical measurement errors in the matrix $DD'$ are all set to be one.

In the state transition equation, I set innovations to the real trend, $\sigma_\tau$, and the inflation trend, $\sigma_\bar{\pi}$, to the standard deviation of the log differences of consumption and the differences of inflation respectively. Innovations to the business cycle, $\sigma_\delta$, and gap inflation, $\sigma_{\bar{\pi}}$, are set to the standard deviations of the log differences of real aggregate activity and the differences of inflation, respectively. The scale volatility of the real rate, $\sigma_r$, is set to the standard deviation of ex post real rate computed as the difference between the nominal policy rate and inflation.

The covariance between the real trend and business cycle is set to target a correlation coefficient of $\rho_{\tau,\delta} = -0.5$. Similarly, the covariance between the inflation trend and gap targets a correlation coefficient of $\rho_{\bar{\pi},\sigma} = -0.5$. These correlations reflect the fact that I expect the gaps to be negatively correlated to their respective Beveridge and Nelson (1981) trends. I use initial guesses of zero for the correlation between the real activity and inflation gaps, $\rho_{\delta,\sigma}$, and between the PI trend and the observation error for consumption, $\rho_{\tau,c}$.

The coefficients on the own-lags of the real activity gaps are set to 0.85 and $-0.05$.

\(^1\)To make this more clear, real aggregate activity is assumed to be cointegrated with consumption. So, I estimate $y_t = a + \zeta c_t + err_t$ and set $\zeta$ to its estimated value and subtract $a$ from $y_t$ to demean the series.
respectively, to target complex roots in the VAR processes. I use the same starting parameters for the own-lags of the inflation gap. Off-diagonal elements of the VAR are initialized to zero. The impact coefficient of the real activity gap on the real interest rate is set to $\theta_\delta = -0.8$, while the corresponding coefficient for the inflation gap is $\theta_\varpi = 0.8$. The initial guess for the smoothing coefficient of the real interest rate rule is $\theta_r = 0.9$. The AR(1) coefficient of the markup shock is set to $\nu = 0.1$.

### B.4 The Bootstrap Algorithm

I use the same bootstrap algorithm as described in Appendix C of Hessler (2021) which is reproduced here for convenience. There are problems with ML estimation when the sample size is small, the underlying processes are persistent, and/or the sample is non-Gaussian. One way to deal with these issues is to bootstrap the NK-UC model and use the sample statistics to assess its fit to the data. Stoffer and Wall (2004) demonstrate that bootstrap resampling of the innovations form of the Kalman filter overcomes these issues. The bootstrap algorithm of Stoffer and Wall (2004) is described in the following steps:

1. Construct the standardized innovations, $e_t$, from the Kalman filter,

$$e_t(\hat{\Psi}) = \Omega_t^{-1/2}(\hat{\Psi})e_{\text{t}}(\hat{\Psi}).$$  \hspace{1cm} (B.15)

Here, $\hat{\Psi}$ is the maximum likelihood estimate of the parameters and $\Omega_t$ is defined in equation (B.10).

2. Sample, with replacement, $T$ times from the set $\{e_1(\hat{\Psi}), \ldots, e_T(\hat{\Psi})\}$ to obtain a bootstrap sample of innovations, $\{e_1^*, e_2^*, \ldots, e_T^*\}$. 
3. Construct a bootstrap data set \( \{Z_1^*,...,Z_T^*\} \), by solving

\[
\begin{bmatrix}
X_t^* \\
Z_t^*
\end{bmatrix} = \begin{bmatrix} A(\hat{\Psi}) & 0 \\
C(\hat{\Psi}) & 0
\end{bmatrix} \begin{bmatrix} X_{t-1}^* \\
Z_{t-1}^*
\end{bmatrix} + \begin{bmatrix} P(\hat{\Psi})_{t|t-1} C(\hat{\Psi})' \Omega(\hat{\Psi})_{t}^{-1/2} \\
\Omega(\hat{\Psi})_{t}^{-1/2}
\end{bmatrix} \begin{bmatrix} e_t^* \\
\end{bmatrix},
\]

for \( t = 1,\ldots,T \). The initial conditions of the Kalman filter remain fixed and the parameter vector is held fixed at \( \hat{\Psi} \). In order to eliminate the effects of the initial conditions, set the first four values of the constructed data set equal to the first four values of the observed data, \( Z_t \), as recommended by Stoffer and Wall (2004).

4. Using the bootstrap data set, \( \{Z_t^*; t = 1,\ldots,T\} \), construct the likelihood, \( L_{Z_t}(\Psi) \), and obtain the maximum likelihood estimate of \( \Psi \), \( \hat{\Psi}^* \).

5. Repeat steps 2 through 4 a large number of times, \( B \), obtaining a bootstrap sample of parameter estimates, \( \{\hat{\Psi}_b^*; b = 1,\ldots,B\} \). For this paper, I set \( B = 1000 \). The finite sample distribution of \( (\hat{\Psi} - \Psi) \) may be approximated by the bootstrapped distribution of \( (\hat{\Psi}_b^* - \hat{\Psi})^2 \), for \( b = 1,\ldots,B \). The standard errors for \( \hat{\Psi}_i \), \( i = 1,\ldots,k \) where \( k \) is the number of parameters, are computed as \( \sqrt{\frac{\sum_{b=1}^{B}(\hat{\Psi}_b^* - \hat{\Psi})^2}{B-1}} \).

B.5 Parameter Estimates

B.5.1 Measurement Errors of the NK-UC Models: Real Aggregate Activity = Real Disposable Income

The scale volatilities of the measurement equations of the NK-UC models estimated with real disposable income as the measure of real aggregate activity are reported in Table B.1. The scale volatilities are similar in magnitude and significant at the 95% significance level. These results are economically uninteresting and so are omitted from the paper.
Table B.1: MLEs of NK-UC Model Measurement Errors  
Real Aggregate Activity = Real Disposable Income, 1960Q1 to 2018Q4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NK-UC-B</th>
<th>NK-UC-A1</th>
<th>NK-UC-A2</th>
<th>NK-UC-A3</th>
<th>NK-UC-A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_c$</td>
<td>1.26 (0.31)</td>
<td>1.28 (0.31)</td>
<td>1.22 (0.32)</td>
<td>1.46 (0.49)</td>
<td>1.29 (0.31)</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>1.73 (0.51)</td>
<td>1.62 (0.50)</td>
<td>1.71 (0.51)</td>
<td>0.97 (0.54)</td>
<td>1.70 (0.53)</td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>1.51 (0.34)</td>
<td>1.55 (0.34)</td>
<td>1.52 (0.34)</td>
<td>1.50 (0.39)</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: Bootstrap standard errors are calculated as $\sqrt{\frac{\sum_{b=1}^{B}(\hat{\theta}_{i,b} - \hat{\theta}_i)^2}{B-1}}$ where $\hat{\theta}_{i,b}$ is the bootstrap parameter estimate, $\hat{\theta}_i$ is the ML parameter estimate, and $B = 1000$ bootstrap samples.

B.5.2 NK-UC Model Parameter Estimates: Real Aggregate Activity = Real GDP

Table B.2 reports the measurement system parameter estimates for the NK-UC models using real GDP as the measure of real aggregate activity. The estimate for the habit formation parameter is larger than the corresponding estimates which have disposable income as the measure of real aggregate activity. This is true across all NK-UC models with the exception of the model with a purely forward-looking NKPC. Larger values for the habit formation parameter push the peak of the response of consumption to a transitory shock back and is more in line with the canonical NK-DSGE model.

The slope of the hybrid-NKPC, $\kappa$, is larger when the NK-UC models are estimated with real GDP. The coefficients on the forward- and backward-looking terms of the hybrid-NKPC, $\gamma_f$ and $\gamma_b$, sum to less than one in all NK-UC models except for the NK-UC model with a vertical hybrid-NKPC curve, NK-UC-A2. The estimate of $\gamma_f$ is smaller in model NK-UC-A4 at 0.28 than its estimate with real aggregate activity measured by real disposable income, 0.61. This indicates that forward expectations in the inflation gap play a smaller role in the hybrid-NKPC of the best fitting NK-UC model. The loading factor of real aggregate activity on the PI trend, $\zeta$, and on the real activity gap, $\nu$, are similar to those reported in Section 4.2.
Table B.2: MLEs of NK-UC Model Measurement Equations
Real Aggregate Activity = Real GDP, 1960Q1 to 2018Q4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NK-UC-B</th>
<th>NK-UC-A1</th>
<th>NK-UC-A2</th>
<th>NK-UC-A3</th>
<th>NK-UC-A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>0.49</td>
<td>-</td>
<td>0.52</td>
<td>0.29</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td></td>
<td>(0.17)</td>
<td>(0.11)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>0.23</td>
<td>0.35</td>
<td>0.60</td>
<td>0.15</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.14)</td>
<td>(0.25)</td>
<td>(0.07)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>$\gamma_b$</td>
<td>0.26</td>
<td>0.36</td>
<td>0.40</td>
<td>-</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.18)</td>
<td>(0.25)</td>
<td></td>
<td>(0.03)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>1.63</td>
<td>1.24</td>
<td>1.47</td>
<td>1.12</td>
<td>1.76</td>
</tr>
<tr>
<td></td>
<td>(0.44)</td>
<td>(0.37)</td>
<td>(0.40)</td>
<td>(0.48)</td>
<td>(0.58)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>13.99</td>
<td>12.78</td>
<td>12.48</td>
<td>14.92</td>
<td>15.12</td>
</tr>
<tr>
<td></td>
<td>(0.47)</td>
<td>(0.81)</td>
<td>(0.59)</td>
<td>(1.24)</td>
<td>(0.32)</td>
</tr>
</tbody>
</table>

Notes: See notes to Table B.3. The factor loading for income, $\zeta$ is 1.05 in all NK-UC models and omitted from the table to save space.

indicating these estimates are not sensitive to the measure of real aggregate activity used.

Table B.3 records the scale volatilities of the measurement errors of the NK-UC models. These parameter estimates are near those of Table B.1. Hence, the measurement error estimates are another dimension on which the measure of real aggregate activity does not matter.

Table B.3: MLEs of NK-UC Model Measurement Errors
Real Aggregate Activity = Real GDP, 1960Q1 to 2018Q4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NK-UC-B</th>
<th>NK-UC-A1</th>
<th>NK-UC-A2</th>
<th>NK-UC-A3</th>
<th>NK-UC-A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_c$</td>
<td>1.26</td>
<td>0.98</td>
<td>1.11</td>
<td>0.32</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(0.29)</td>
<td>(0.28)</td>
<td>(0.15)</td>
<td>(0.28)</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>1.02</td>
<td>&lt;0.01</td>
<td>0.95</td>
<td>0.69</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>(0.38)</td>
<td>(0.27)</td>
<td>(0.40)</td>
<td>(0.38)</td>
<td>(0.38)</td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>1.55</td>
<td>1.55</td>
<td>1.55</td>
<td>1.47</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.33)</td>
<td>(0.35)</td>
<td>(0.34)</td>
<td>(0.31)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: See notes to Table B.3.
Table B.4 contain estimates of the state parameters of the NK-UC models estimated with real GDP. In the NK-UC-B, -A1, -A2, and -A4 models, the VAR estimates are smaller in magnitude but preserve the same signs and dynamics as the NK-UC models estimated with real disposable income. The VAR estimates for the NK-UC model with a purely forward-looking NKPC are different than the other models. The coefficients on the own-lags of the real activity gap are both positive and sum to 0.77. Additionally, the estimates of the coefficients of the inflation gap on the lags of the real activity gap, $\varphi_{\delta,1}$ and $\varphi_{\delta,2}$, have similar magnitudes to the corresponding estimates in all other models, but the signs are switched.

Table B.5 reports the scale volatilities of the state errors in the NK-UC models estimated with real GDP. The correlation between the permanent income trend and real activity gap is negative in NK-UC-A2, and A4. This indicates that real aggregate activity adjusts slowly to the PI trend. The scale volatilities of the PI and inflation trends, real activity and inflation gaps, real rate, and markup shock are robust to the choice of the real aggregate activity variable. This is also true of the correlations between the PI trend and consumption, the inflation trend and its gap, and the inflation and real activity gaps.

### B.5.3 Estimates of the Trends and Gaps: Real Aggregate Activity = Real GDP

This subsection reports the trends and gaps of the best fitting NK-UC model, NK-UC-A4, estimated with real aggregate activity measured by real GDP. Figure B.1 plots the estimate of the permanent income and inflation trends and Figure B.2 plots the gaps of NK-UC-A4 estimated on real GDP.

The PI trend closely tracks consumption throughout the sample period. Real GDP is less smooth than the PI trend during the sample. Deviations in the growth rate of real GDP from that of the PI trend are reflected by movements in the real activity gap.

At the beginning of the sample, the inflation trend deviates well below observed inflation until just after the 1969-1970 recession. Observed inflation hits its highest peak in the sample
Table B.4: MLEs of NK-UC Model State Equations
Real Aggregate Activity = Real GDP, 1960Q1 to 2018Q4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NK-UC-B</th>
<th>NK-UC-A1</th>
<th>NK-UC-A2</th>
<th>NK-UC-A3</th>
<th>NK-UC-A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{\delta,1}$</td>
<td>1.56 (0.13)</td>
<td>1.19 (0.13)</td>
<td>1.47 (0.14)</td>
<td>0.46 (0.11)</td>
<td>1.45 (0.13)</td>
</tr>
<tr>
<td>$\phi_{\delta,2}$</td>
<td>-0.63 (0.12)</td>
<td>-0.28 (0.12)</td>
<td>-0.54 (0.13)</td>
<td>0.31 (0.10)</td>
<td>-0.53 (0.12)</td>
</tr>
<tr>
<td>$\phi_{\omega,1}$</td>
<td>-0.10 (0.07)</td>
<td>-0.32 (0.17)</td>
<td>-0.18 (0.08)</td>
<td>-0.02 (0.01)</td>
<td>-0.14 (0.08)</td>
</tr>
<tr>
<td>$\phi_{\omega,2}$</td>
<td>0.28 (0.09)</td>
<td>0.33 (0.11)</td>
<td>0.30 (0.08)</td>
<td>-0.02 (0.01)</td>
<td>0.27 (0.09)</td>
</tr>
<tr>
<td>$\varphi_{\delta,1}$</td>
<td>-0.57 (0.13)</td>
<td>-0.20 (0.13)</td>
<td>-0.47 (0.14)</td>
<td>0.54 (0.11)</td>
<td>-0.46 (0.13)</td>
</tr>
<tr>
<td>$\varphi_{\delta,2}$</td>
<td>0.60 (0.12)</td>
<td>0.24 (0.14)</td>
<td>0.51 (0.13)</td>
<td>-0.34 (0.10)</td>
<td>0.49 (0.13)</td>
</tr>
<tr>
<td>$\varphi_{\omega,1}$</td>
<td>0.08 (0.04)</td>
<td>0.21 (0.10)</td>
<td>0.12 (0.04)</td>
<td>0.14 (0.05)</td>
<td>0.10 (0.04)</td>
</tr>
<tr>
<td>$\varphi_{\omega,2}$</td>
<td>-0.24 (0.12)</td>
<td>-0.37 (0.16)</td>
<td>-0.29 (0.12)</td>
<td>0.12 (0.06)</td>
<td>-0.26 (0.12)</td>
</tr>
<tr>
<td>$\theta_{\delta}$</td>
<td>-3.40 (0.95)</td>
<td>-3.70 (1.05)</td>
<td>-3.23 (0.82)</td>
<td>-0.07 (0.03)</td>
<td>-3.94 (1.12)</td>
</tr>
<tr>
<td>$\theta_{\omega}$</td>
<td>0.84 (0.26)</td>
<td>0.17 (0.51)</td>
<td>0.59 (0.25)</td>
<td>-0.14 (0.15)</td>
<td>0.81 (0.30)</td>
</tr>
<tr>
<td>$\theta_{r}$</td>
<td>0.97 (0.02)</td>
<td>0.97 (0.02)</td>
<td>0.97 (0.02)</td>
<td>0.95 (0.02)</td>
<td>0.96 (0.02)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.34 (0.08)</td>
</tr>
</tbody>
</table>

Notes: See notes to Table B.3.
Table B.5: MLEs of NK-UC Model State Shocks
Real Aggregate Activity = Real GDP, 1960Q1 to 2018Q4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NK-UC-B</th>
<th>NK-UC-A1</th>
<th>NK-UC-A2</th>
<th>NK-UC-A3</th>
<th>NK-UC-A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_\tau$</td>
<td>1.94 (0.14)</td>
<td>2.08 (0.17)</td>
<td>1.93 (0.14)</td>
<td>2.08 (0.14)</td>
<td>1.94 (0.15)</td>
</tr>
<tr>
<td>$\sigma_{\bar{\pi}}$</td>
<td>0.46 (0.07)</td>
<td>0.48 (0.07)</td>
<td>0.46 (0.07)</td>
<td>0.58 (0.10)</td>
<td>0.39 (0.08)</td>
</tr>
<tr>
<td>$\sigma_\delta$</td>
<td>0.12 (0.02)</td>
<td>0.20 (0.03)</td>
<td>0.14 (0.03)</td>
<td>0.09 (0.02)</td>
<td>0.12 (0.02)</td>
</tr>
<tr>
<td>$\sigma_{\pi}$</td>
<td>0.20 (0.15)</td>
<td>0.17 (0.13)</td>
<td>0.21 (0.15)</td>
<td>6.17 (1.08)</td>
<td>0.18 (0.14)</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.70 (0.11)</td>
<td>0.63 (0.07)</td>
<td>0.71 (0.11)</td>
<td>0.75 (0.13)</td>
<td>0.73 (0.12)</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.86 (0.13)</td>
</tr>
<tr>
<td>$\rho_{\tau,c}$</td>
<td>-0.73 (0.08)</td>
<td>-0.81 (0.07)</td>
<td>-0.69 (0.09)</td>
<td>-0.94 (0.03)</td>
<td>-0.70 (0.08)</td>
</tr>
<tr>
<td>$\rho_{\tau,\delta}$</td>
<td>0.12 (0.13)</td>
<td>-0.13 (0.10)</td>
<td>0.08 (0.11)</td>
<td>-0.37 (0.09)</td>
<td>0.07 (0.12)</td>
</tr>
<tr>
<td>$\rho_{\pi,\sigma}$</td>
<td>-0.27 (0.28)</td>
<td>0.13 (0.27)</td>
<td>-0.30 (0.28)</td>
<td>-0.58 (0.11)</td>
<td>-0.27 (0.31)</td>
</tr>
<tr>
<td>$\rho_{\delta,\sigma}$</td>
<td>-0.40 (0.19)</td>
<td>-0.34 (0.22)</td>
<td>-0.36 (0.17)</td>
<td>-0.72 (0.10)</td>
<td>-0.37 (0.18)</td>
</tr>
</tbody>
</table>

Notes: See notes to Table B.3.
**Figure B.1:** Estimates of NK-UC-A4 Trends
Real Aggregate Activity = Real GDP, 1960Q1 to 2018Q4

Notes: Real GDP is translated by -190 to aid in visualization. The gray bars represent NBER recession dates. The blue shaded areas are the 95% sup-t uncertainty bands computed from 1000 bootstrap samples as in Olea and Plagborg-Møller (2018).
at the start of the 1973-1975 recession. Trend inflation does not peak as high as observed inflation and lags behind, peaking during the recession. As observed inflation declines until the mid-1980s it remains below trend inflation. This is true once again in the mid-1990s. In the 2000s, inflation and its trend track each other closely. The 2007-2009 recession results in a sharp contraction of inflation and its trend. These series remain low and stable until the end of the period.

Real GDP, inflation, and the federal funds rate contract during the 1960-1961 recession while consumption grows steadily. The real activity gap estimated with real GDP contracts sharply relative to that estimated with real disposable income as reported in the Section 4.3. The real rate estimated with real GDP is negative during the 1960-1961 recession.

**Figure B.2: Estimates of NK-UC-A4 Gaps**
Real Aggregate Activity = Real GDP, 1960Q1 to 2018Q4

Notes: See the notes to Figure B.1.
a more rapid rate than consumption. During this time of expansion, the inflation gap is characterized by short and small fluctuations around zero. The real rate expands more rapidly than its corresponding estimate with real disposable income as the measure of real aggregate activity. This results in inflation trend contraction in model NK-UC-A4 estimated with real GDP.

The real activity gap contracts sharply in the 1969-1970 recessions and the 1973-1975 recessions. In the aftermath the 1973-1975 recession, the real activity gap measured remains negative for a couple of years. The inflation gap peaks just prior to the 1969-1970 and declines during the subsequent recession. The real rate declines with these recessions and becomes negative following the 1973-1975 recession. The real rate remains negative from the end of this recession until 1978.

The real activity gap declines sharply and reaches negative territory during the double dip recession. The end of this period marks a low point for the real activity gap. Throughout the mid-1980s, the nominal policy rate declines but remains above inflation, resulting in high real rates. The real activity and inflation gaps remain muted throughout the 1980s.

The real rate moves downwards in the savings and loans crisis of the late 1980s and early 1990s. The real activity gap features a long negative swing which troughs at the same time as the real rate. In the middle of the 1990s, the nominal policy rate is lifted until the end of the decade. This results in a rebound of the real interest rate occurring before the real activity gaps recover to positive territory. This combination of events results in a decline in observed inflation and its trend. The inflation gap features short lived expansions and contractions from the middle of the 1980s until the 2001 recession. This indicates that the inflation trend is accounting for most of the movement in inflation over the same time frame.

The inflation gap and real activity gaps decrease during the 2001 recession. The real rate declines steadily from 2000 until a trough in negative territory prior to 2005. During
the 2007-2009 recession the real activity gap and real rate decline while the inflation gap increases. The real activity gap spends an extended period of time in negative territory and recovers in 2012. The real rate is negative after the 2007-2009 recession until the end of the sample.