ABSTRACT

SHAHRIAR, AZMAYEEN RAFAT. Framework for a Reliability-Based Approach for Analyses of Bridge Pier and Embankment Scour. (Under the direction of Dr. Mohammed A. Gabr, and Dr. Brina M. Montoya).

Scour, defined by the loss of geomaterials surrounding a foundation support system, is a primary cause of bridge failure in the United States and worldwide. Current practice of bridge scour prediction is mostly based on the use of deterministic models. At present, there are more than 20 pier-scour deterministic models available in the literature. Some models are derived based on laboratory scale data, while others are derived from scour measurements taken in field conditions. In addition, there are models that consider both the laboratory scale data and field scale data. It is well established that pier scour depth is influenced by multiple factors; some factors define the structure and geometric scale of the pier flow field, while some factors characterize the scour depth sensitivities within the geometric scale limit. The choice of scour-influencing factors that are included in a given deterministic model leads to estimating different scour depth magnitudes under similar set of hydraulic, structural, and geotechnical attributes. With no prior knowledge of the extent of bias yielded by a given deterministic model, one cannot discern whether these predictions are conservative or unconservative, let alone the degree of conservatism/unconservatism in view of the application realm. Even with the general knowledge that a given deterministic model is conservative, it is important to quantify the extent of such conservatism in terms of percent probability the expected measured scour depth magnitude will be exceeded.

Work herein presents statistical models that extend four deterministic approaches reported in literature to predict the expected scour depth while quantifying inherent model bias and uncertainty in view of data scatter. Clear water and live bed scour databases are used herein, and the analyses quantify model scatter by comparatively assessing the computed scour depth versus measured data reported in the database. A relationship between Probability of Deceeedance (POD) associated with the predicted scour depth and a modification factor is devised. POD is the probability that the predicted scour depth will be less than the measured scour depth. The modification factor allows for the use of the scour magnitude computed from the deterministic models while quantifying the probability of a computed scour depth being less than or more than a most likely value.
Work herein is further focused on developing a framework for reliability-based pier scour assessment methodology and demonstrate its integration with the concept of Load and Resistance Factor Design (LRFD) approach. Uncertainty of the parameters affecting the scour depth estimation is considered. The methodology to develop a reliability index based on scour estimation entails consideration of statistical attributes (mean, coefficient of variation, probabilistic distribution) of the input parameters in scour analyses (e.g., approach flow velocity, approach flow depth, median grain size of the bed material, Manning’s roughness coefficient, channel bed slope) to analyze the limit state function that defines the margin of safety. Monte Carlo simulation approach was considered to compute the probability of exceedance of the limit state function followed by development of the relationship among scour factors, reliability level ($\beta$) and the associated POD. Example applications of axially and laterally loaded pile design approach considering scour factor in the LRFD framework has been demonstrated.

The proposed framework will help mitigate the inconsistency posed by adopting a $\beta$-based approach for the design of various superstructure bridge components as opposed to a deterministic approach for assessing the scour magnitude at the foundation system. Having a uniform level in the reliability index of the bridge components will facilitate the development of integral risk-based design approach for bridge structures.
Framework for a Reliability-Based Approach for Analyses of Bridge Pier and Embankment Scour

by
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DEDICATION

I dedicate this work to my late father, Kalam, and my mother, Rina for their unwavering support in my life.
BIOGRAPHY

Azmayeen earned his BS in Civil Engineering from Bangladesh University of Engineering and Technology (BUET) in 2016. That year he joined as a faculty member at BUET where he taught Civil engineering courses to undergraduate students and was involved in several geotechnical, and structural engineering projects. He received his MS degree in Geotechnical Engineering from BUET in 2018. He started his Ph.D. at North Carolina State University in August 2018 under the direction of Dr. Mohammed A. Gabr, and Dr. Brina M. Montoya.
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Chapter 1: Introduction

Motivation

Scour around existing bridge foundations have been a primary cause of bridge collapse in the United States and worldwide (Melville and Coleman, 2000; Liang et al. 2015; Qi et al. 2016). Records in National Bridge Inventory (NBI) suggests that more than 80% of the total 583,000 bridges in the United States are built over waterways. Support system of structures crossing waterways are subjected to scour during their design life owing to the flowing water-induced bed shear stresses. Scour is amplified by extreme flooding due to increased baseflow and associate velocities; approximately 85% of the bridges fail due to externally triggered events like flood, collision, overload etc. (Wardhana and Hadipriono 2003). Johnson et al. (2015) noted that for the bridges under federal aid system, the average annual flood damage repair cost could be estimated at $50 million.

Traditional practice of bridge scour estimation relies upon the use of deterministic models. Although many pier scour prediction models are available in literature, the Hydraulic Engineering Circular No. 18 (Arneson et al. 2012) is one of the most commonly used in the Unites States (Johnson et al. 2015). There is a consensus that the HEC 18 (Arneson et al. 2012) model often provides a conservative estimate of scour depth (Briaud et al. 2014; Yao 2013). However, the degree of inherent conservatism embedded in such models is unknown. A knowledge of the level of biasness of the scour prediction models is necessary to discern the extent of conservatism/un-conservatism in view of a given model’s application to a wide range of site conditions.

Investigation into the scour prediction models suggests that pier scour estimation is predominantly influenced by flow depth to pier width ratio, pier width to median grain size ratio, pier face shape, pier aspect ratio, skew angle of pier, and upstream mean flow velocity to critical velocity ratio (Melville and Coleman 2000; Ettema et al. 2011). The majority of the input parameters related to flow and streambed are not deterministic in nature and can be random variables with a particular statistical distribution. This raises the necessity to assess input parameter uncertainty, along with the inherent biasness of the results from the deterministic model.

Multiple advancements have been made in probabilistic scour assessment front. Johnson (1992) developed a mathematical expression between probability of bridge failure and safety factors using data from Chee (1982) and Chiew (1984). A best-fit model to represent the databases used was considered as the basic deterministic model, thereby model uncertainty was not
considered. Briaud et al. (2007) developed a probabilistic bridge scour model that considered the uncertainty of the hydrologic loading conditions. Bolduc et al. (2008) introduced a probabilistic model based on the bias in the HEC 18 (2012) model using three databases, which are Gudavalli database (Gudavalli 1997), Landers-Mueller database (Landers and Mueller 1996), and Kwak database (Kwak 2000). Bolduc et al. (2008) model was not effective in removing heteroscedasticity and non-normal error inherent in the measured scour depth. Johnson et al. (2015) examined the hydraulic and hydrologic uncertainties associated with HEC-18 and Florida Department of Transportation (FDOT, 2011) scour prediction models. A large number of simulations (10,000) in HEC RAS were considered to assess the hydraulic conditions at a bridge, given the hydrologic input variable uncertainties. Three levels of hydrologic uncertainties were considered, viz. low, medium, and high uncertainty. Briaud et al. (2014) developed a risk-based pier scour model through quantification of the “risk associated with failure”, where risk was defined as the product of the probability of occurrence times the value of consequence. In summary, probabilistic aspects of scour estimation have been advanced to some extent to make a designer aware of hydraulic, and hydrologic uncertainties associated the model’s estimate and the associated risk. However, there is a need to understand inherent model uncertainty and bias, and its impact on a target probability of deceedance. In addition, the effect of input parameter uncertainty on the target probability of deceedance needs investigation.

The use of the reliability theory, in terms of reliability index, has been introduced through the concept of Load and Resistance Factor Design (LRFD) guidelines in the United States (Nowak 1999). Reliability index is a measure of the number of standard deviations that the mean margin of safety falls on the “safe” side (Paikowsky 2004), where the margin of safety is dependent on the limit state function representing the safety of a system. The calibration procedure of LRFD code is well documented in Nowak (1999) and Kulicki et al. (2007). Lagasse et al. (2013) described that a commonly used calibration procedure is to achieve a consistent reliability level for the super- and sub-structure in a system. The concept of LRFD is yet to be applied to the realm of scour assessment. AASHTO (2007) states that scour design for the design flood must satisfy the requirement that the factored foundation resistance after scour is greater than the factored load determined with the scoured soil removed. The resistance factors will be those used in the Strength Limit State, without scour. Important to note that the resistance factors vary depending on the target reliability index. AASHTO (2007) also suggests that the scour analyses should be conducted
in accordance with HEC 18 manual. Therefore, it appears that the structural design is dependent on the target reliability index, while scour design is deterministic. In addition, there are numerous deterministic scour prediction models that yield different magnitudes of scour estimates when the hydraulic, structural, and geotechnical input parameters are identical for each model. This indicates that the scour magnitude assessed during the design phase will depend on the specific model used for such assessment. Furthermore, the relationship between reliability index and corresponding resistance factors needs to be used for scour design needs exploration for successful incorporation of the concept of LRFD in bridge design considering scour effects.

Objectives
To address the aforementioned research needs the following objectives are set:

1. Quantify the range of errors and degree of conservatism associated with deterministic pier and embankment scour depth prediction models
2. Quantify inherent model bias and uncertainty associated with deterministic pier scour depth prediction models
3. Develop a framework to comprehend scour modification factor necessary to achieve a target probability of deceedance
4. Develop scour factors corresponding to reliability index to be used in LRFD bridge foundation design considering scour effects
5. Demonstrate incorporation of reliability-based scour assessment in the AASHTO LRFD framework in pile capacity analyses

Dissertation Organization
The thesis consists of eight chapters. A summary of the chapters is provided herein.

Chapter 2 provides a detailed overview of the current state of knowledge on erodibility, geotechnical aspects of erodibility, factors influencing and complicating estimation of pier scour, and databases available on erodibility and pier scour measurements. A summary of the deterministic pier scour models developed since 1990, and the extent to which such models consider factors affecting pier scour, are presented. In addition, developments in probabilistic pier scour analyses, and observation-based models are summarized.

Chapter 3 presents statistical models that extend five deterministic approaches reported in literature to predict the expected scour depth while quantifying inherent model bias and uncertainty
in view of data scatter. Clear water scour database is used. A relationship between probability of deceedance associated with the predicted scour depth and a modification factor (that is applied into the deterministic prediction) is devised.

Chapter 4 investigates four bridge scour prediction models in terms of two statistical parameters, termed herein Mean Absolute Percentage Error (MAPE, as a measure of accuracy of the prediction), and Level of conservatism (as a measure of conservativeness of the model). Live bed laboratory and field scour databases are used in analyses to quantify model scatter by comparatively assessing the computed scour depth versus measured data. Statistical models are applied to ascertain the biasness of a given deterministic model. Accuracy and conservatism of a given model are consequently adjusted through proposed modification factors.

Chapter 5 focuses on developing a reliability-based pier scour assessment methodology and demonstrating the implications on the response of a pile group supporting a bridge pier. Four commonly used pier scour prediction models are considered. Responses for narrow pier, intermediate pier, and wide pier, which is used to describe different pier flow field are examined separately. To facilitate the analyses, parameters from the Woodrow Wilson bridge pier were considered. A relationship between the probability of exceedance over the design life and the scour depth is presented to demonstrate the risk associated with the use of scour depth assessed using Hydraulic Engineering Circular (HEC) 18 approach. The estimated scour depth using both HEC-18 and the $\beta$-based approaches was applied to the modeled bridge pile foundation system and the response of the pile foundation under AASHTO loading scenarios were studied.

Chapter 6 presents the incorporation of the reliability index-based scour assessment protocol in the design of pile foundation in clay. Three example applications of axially and laterally loaded pile design approach considering scour factor in the LRFD framework has been demonstrated. For axial pile capacity problem, the method proposed by American Petroleum Institute (API) was considered, while for the lateral pile capacity problem, the method proposed by Matlock (1970), commonly known as the p-y method was adopted.

Chapter 7 focuses on assessing the performance of abutment scour prediction methodology by considering three abutment scour prediction models suggested by Hydraulic Engineering Circular No. 18. An abutment scour database is utilized to quantify the predicted and measured scour depth relationship. Abutment scour prediction models are assessed in terms of two statistical
parameters, termed herein Mean Absolute Percentage Error (as a measure of accuracy of the prediction), and Level of conservatism, percentage of cases the predicted scour exceeded the measured scour (as a measure of conservativeness of the model). For vertical wall and spill through abutments, responses for long abutment, and intermediate abutment are examined separately.

Chapter 8 summarizes the essential contributions of the research conducted herein and suggests some future work direction.
2. **Chapter 2: A review on the bridge pier scour**

This chapter was previously published as:

Abstract

Scour, defined by the loss of geomaterials surrounding a foundation support system, is a primary cause of bridge failure in the United States and worldwide. Work herein presents a comprehensive review of the current state of knowledge on geotechnical aspects of erodibility, factors influencing pier scour, factors complicating pier scour assessment, and databases available on erodibility and pier scour. A summary of deterministic pier scour models, developed since 1990, is presented in view of the factors affecting scour rate and equilibrium magnitude. The study discusses challenges in the predictive approaches reviewed in the paper. In addition, advancements in probabilistic pier scour models, and observation-based models are summarized. Four pier scour models, namely Wilson (1995) model, Melville (1997) model, Hydraulic Engineering Circular No. 18 (2012) model, and Briaud (2014a) model are comparatively applied to data from laboratory pier scour database. Error statistics and accuracy, precision, and probabilistic distribution of predictions from these models are presented and discussed.

Keywords: Bridge scour; Erodibility; Geotechnical aspects; Critical shear stress; Deterministic models; HEC-18 model.
Introduction

Scour of foundation systems is the primary cause of bridge collapse in the United States (Melville and Coleman, 2000; Briaud et al. 2001, 2005). Scour leads to damage of the bridge foundation and abutment, causing major operational disruption and financial losses (Shirole and Holt, 1991; Briaud et al. 1999). Murillo (1987), based on the investigation of bridge failures between 1961-1974, observed that among 86 bridge failures, 46 can be attributed to scour. Lagasse et al. (1997) indicated the average cost for flood damage repair of highways in the United States was estimated at $50 million per year. This cost only considers the damage to infrastructures; however, the additional cost to the afflicted population could be as much as five times the repair costs (Rhodes and Trent, 1993). Mueller (2000) and Fisher et al. (2013) reported that more than 2500 bridges were damaged due to flooding-related scour during the period 1980 to 1990 in the Northeastern and Midwestern USA. Wardhana and Hadipriono (2003), based on their investigation of five hundred bridge failures between 1989 and 2000 across the United States, described that 53% of the recorded failures was due to flooding and scour related complications. Lin et al. (2006) reported that during the period of 1996 to 2001, 68 bridge failures in the United States were related to scour. Hunt (2009) noted that up until 2009, there were 20,904 scour-critical bridges in the United States and additional 80,000 bridges were scour susceptible. During a single flood event in the upstream and downstream Missouri river basins, which occurred in 1993, at least 22 of the 28 bridges on the waterway experienced some form of distress due to scour (Prendergast and Gavin, 2014). The associated repair costs were more than $8 million as reported in Kamojjala et al. (1994). With extreme weather events becoming frequent (McKenna et al. 2020), the potential economic, financial, and human life loss due to scour-induced failures signify the importance of identifying the principal causes of scour and taking necessary mitigation actions prior to damage being incurred.

Many empirical models have been developed to estimate the equilibrium scour depth at bridge piers. The development for these equations incorporates data from laboratory tests, field tests, or both. Also, pier scour is a complex process being affected by many factors including, for example, flow depth to pier width ratio, pier face shape, pier aspect ratio, and skew angle of pier. Not all these factors are considered while developing the empirical models. In addition, the model equations are commonly based on simplified assumptions, while many practical factors such as pier proximity to abutment, debris accumulation, and flood plain vegetation further complicate the
assessment of scour. Subsequently, under identical hydraulic and geometric conditions, different models yield vastly different scour estimates; these can be conservative or unconservative. Accordingly, the conclusion about the scour criticality of a given bridge pier is dependent on the model being used in the analysis. In the design stage, the use of a model yielding unconservative results may lead to bridge failure, whereas selection of a highly conservative model will result in an adverse economic outcome. Therefore, understanding and documentation of the pier scour estimation models in relation to the factors influencing and complicating pier scour estimation are necessary.

Developments in the understanding of scour mechanism have been made in hydraulic and geotechnical engineering. The hydraulic, geotechnical, and structural impact of scour has been studied in their respective fields. Attempts have been made to relate ‘erodibility magnitude and rate’ to the properties of soil (plasticity index, liquidity index, water content, median grain size, etc.). The review presented herein is focused on summarizing the current state of knowledge on erodibility, geotechnical aspects of erodibility, factors influencing and complicating estimation of pier scour, and databases available on erodibility and pier scour measurements. A summary of the deterministic pier scour models developed since 1990, and the extent to which such models consider factors affecting pier scour, are presented. In addition, developments in probabilistic pier scour analyses, and observation-based models are summarized. Four fundamentally different and widely used pier scour models, namely Wilson (1995) model, Melville (1997) model, Hydraulic Engineering Circular (HEC) No. 18 (Arneson et al. 2012) model, and Briaud et al (2014a) model are chosen for comparative analyses. These four models were developed each relying on a different method of collecting data. The laboratory live bed and clear water data, as summarized by Benedict and Caldwell (2014), were used in the analyses herein. Error statistics were computed and included in the discussion on accuracy, precision, and probabilistic distribution of predictions from these four models. Conclusions were drawn based on the synthesis of literature and critical analyses presented.

Scour

Macro perspective

Scour can be defined as the removal of materials from bed and banks of streams, around the piers and abutment of waterway bridges or other hydraulic structures. Bed materials may consist of granular or cohesive deposits or a combination of the two. Under similar hydraulic conditions,
loose granular soil will scour at a rate different from cohesive deposits. Information in the literature suggests that the detachment of particles from the bed can be related to the flow velocity and scour can be classified as occurring under clear-water or live-bed conditions. If the flow velocity is such that there is no transport of bed material from upstream of the bridge crossing, the condition is considered *clear water*. In contrast, if the transport of bed material occurs from the upstream area, the condition is *live bed*.

It is important to note that there are three main sources of scour, which contributes to the total scour estimate. These are, i) Long-term aggradation and degradation, ii) Contraction scour, and iii) Local scour. Long-term aggradation involves the deposition of material eroded from the upstream of the bridge, whereas degradation is a type of scour that occurs due to a deficit in sediment supply from upstream over relatively long reaches as occurring in clear-water conditions (Arneson et al. 2012). Contraction scour occurs due to the geometric contraction of flow resulting in increased flow velocities, turbulence, and therefore bed shear stress driving removal of materials from the bed. The difference between long-term degradation and contraction scour is the latter is associated with any constriction of flow. Local scour takes place when the flow is obstructed by a structure such as pier, abutment, spurs, or embankment. Arneson et al. (2012) considered local scour as the combined effect of flow acceleration and the vortices induced by the obstruction. Three principal flow features were described in this case: i) down flow at the face of pier, ii) the horseshoe vortex at the base of the pier, and iii) the wake vortices formed at the downstream direction of pier (Arneson et al. 2012, Melville and Coleman 2000). The down flow at the pier face erodes the bed material, which are then transported past the pier by the horseshoe vortex and wake vortices thereafter. The mechanism is presented schematically in Figure 2-1. Deng and Cai (2010) reported that the strength of the horseshoe vortex is reduced as the depth of scour increases. Thereby, with time, reducing the transport rate of bed material past the bridge pier. For uniform sediments, under clear water conditions, the scour progresses asymptotically towards an equilibrium depth, while, for live bed conditions, the attainment of peak scour depth is relatively faster followed by a fluctuation of bed form (Raudkivi and Ettema 1983; Chiew 1984; Ettema et al. 2011). Chee (1982), Chiew (1984), Melville (1984), Melville and Sutherland (1988) suggest that for uniform sediments, the scour depth prediction under live bed condition depends significantly on the size and steepness of the bed features at a certain flow velocity. If the bed form is higher, then the observed scour depth is lower. Ettema et al. (2011), Oliveto and Hager (2014)
illustrated that sediment deposition bar (dune) at the pier tail water alters the flow field, thereby affecting the scour hole development. They also observed that the time required for the dune-crest to attain its maximum height is nearly constant (slightly dependent on sediment uniformity); while the magnitude of maximum dune-crest height is on average 0.25 times the downstream flow depth. As the scouring processes cease, a new equilibrium is established between bed material inflow and outflow.

**Meso to micro perspective**

Soil erodes if the applied hydraulic shear stress exceeds the critical shear stress. The key forces acting on a soil particle are: weight of the particles, electrical forces between the particles, forces at contact points of the particles, and water pressure around the particles. Hofland et al. (2005), through an experimental investigation on granular beds, showed that the forces exerted on a soil particle due to turbulent stress fluctuation at the bed level is sufficient to dislodge it. The turbulent stress fluctuations acting on the soil particles surrounding an obstruction may result in suction pressures, which affect the erosion of the bed material. Briaud (2008) noted that flow of water exerts a pressure around the particles, and the normal pressure imposed at the base of the particle will be higher than that acting at the top of the particle, resulting in a buoyant force. As conventionally, the shear stress acting on the soil-water interface is considered the parameter most influencing erodibility, Briaud (2008) suggested that the constitutive law of the erosion process can be expressed as in Eq. (1).

\[ \dot{z} = f(\tau) \]  

(1)

Where, \( \dot{z} \) is erosion rate and \( \tau \) is hydraulic shear stress at the interface of soil and water when it exceeds the critical shear stress. To normalize Eq. (1), Shafii et al. (2016) proposed Eq. (2), which can be expressed as-

\[ \frac{\dot{z}}{v_c} = \alpha' \left( \frac{\tau - \tau_c}{\tau_c} \right)^m \]  

(2)

Where, \( \alpha' \) and \( m \) are unitless erosion model parameters depending on soil properties. \( v_c \) and \( \tau_c \) are critical velocity and critical shear stress respectively (below which no erosion occurs). Shafii et al. (2019), through a hydrodynamic analysis to discern the forces acting on the gravel particles, identified that critical normal stress on the soil particle as another important parameter, which influences erosion rate. The developed model is presented in Eq. (3),
\[
\frac{\dot{z}}{0.1} = \left(\frac{\tau}{\tau_c}\right)^\alpha \left(\frac{\sigma}{\sigma_c}\right)^\beta
\]

Where, \( \dot{z} \) is erosion rate (mm/hr), \( \tau_c \) and \( \sigma_c \) are critical shear stress (Pa) and critical normal stress associated with an erosion rate of 0.1 mm/hr respectively, \( \sigma \) is the normal stress (Pa) acting on the particle at bed level, \( \alpha \) and \( \beta \) are unitless erosion model parameters. Briaud et al. (2019) indicated that parameters \( \alpha \) and \( \beta \) cannot be determined definitively with the existing state of knowledge, and it would be cumbersome to estimate the parameters on a site-specific basis.

Numerical analyses are also used to understand the interaction between flow and obstacles and the impact on soil erosion. Zhang et al. (2016) conducted a Molecular Dynamics (MD) simulation, a tool to comprehend physical mechanisms at the molecular scale, to explain the interaction occurring at soil-water interface. Joseph and Hunt (2004), Kloss et al. (2012), Ni et al. (2015), Huang et al. (2014), Chang et al. (2016), Kawano et al. (2017), Tao and Tao (2017) introduced the approach of coupled Computational Fluid-Dynamics-Discrete Element Modeling (CFD-DEM) to investigate various hydro-mechanical problems associated with granular media. CFD was used to model the interstitial fluid flow and DEM was used to simulate the particles assembly. Their introduction of the CFD-DEM technique led researchers, including Guo and Yu (2017) and Guo et al. (2018), to incorporate the CFD-DEM approach into investigation soil erosion. Although soil erosion was modeled in those studies, none have focused on erosion rate. Consequently, there is a need to better understand the progressive development of scour considering the coupled action of shear stress and the turbulent layer generated when the flow experiences obstructions.

**Critical shear stress and critical velocity**

Critical shear stress and critical velocity are two parameters with paramount importance in scour studies. Several attempts have been made to relate erodibility to geotechnical properties. Smerdon and Beasley (1959) developed a relation between critical shear stress with soil plasticity index and the dispersion ratio (Table 2-1). Grissinger (1966) noted the influence of type and amount of clay minerals, mineral orientation, bulk density, temperature, and antecedent water on erodibility. Parchure and Mehta (1986) observed a log-linear relationship between erosion rate and shear stress in excess of bed shear strength. Their study was focused on erosion rate models for kaolinite and lake mud composed of montmorillonite, illite, kaolinite and quartz. Hanson and
Simon (2001) developed a relationship between critical shear stress and “linear slope” of erosion rate versus shear stress for soils with 50-80% silt size materials and relatively low dry unit weight (ranging from 11-15 kN/m³). Julian and Torres (2006) suggested a model to estimate the critical shear stress based on the percent silt and clay content in the material. The minimum critical shear stress was considered 0.1 Pa, based on Shields (1936). It was also noted that the presence of vegetation increases the critical shear stress, and a multiplication factor based on the type of vegetation was introduced. Thoman and Niezgoda (2008) proposed a multilinear regression equation relating soil activity, dispersion ratio, specific weight, pH of the eroding fluid and moisture content to estimate the critical shear stress \( \tau_c \). Their reported critical shear stress ranged between 0.11-15.35 Pa. Both Shields (1936) and Briaud (2008) concluded that \( \tau_c \) for coarse grained soils depends on median grain size, \( d_{50} \); for fine grained soil \( d_{50} < 0.075 \text{ mm} \) however, the relationship between \( \tau_c \) and \( d_{50} \) is significantly scattered. Briaud (2008) suggested an expression for critical shear stress for soils with \( d_{50} > 0.1 \text{ mm} \), which is listed in Table 2-1.

Shafii et al. (2016) suggested a set of equations to predict the critical shear stress of coarse-grained soils based on \( d_{50} \), moisture content, and percent of fines. In the case of fine-grained soil, plasticity index, undrained shear strength, \( d_{50} \), and moisture content are considered as primary variables affecting critical shear stress. Briaud et al. (2017) performed a multilinear regression analysis of the testing data obtained from Erosion Function Apparatus (EFA) and proposed a set of equations to predict critical shear stress and velocity. Table 2-1 presents a summary of the predictive equations and number of tests considered to develop the equations for both coarse grained and fine-grained soil. Briaud et al. (2019) documented the parameters that affect the erodibility of soil and classified these into two categories: viz-a-viz more typically obtained parameters and less typically obtained parameters. A set of equations was suggested to estimate the critical shear stress obtained from EFA (Table 2-1). For fine grained soils, the independent variables are unit weight, activity, moisture content, percent fines, \( d_{50} \), and undrained shear strength. For coarse grained soils, coefficient of uniformity, unit weight and \( d_{50} \) are the independent variables. Briaud et al. (2019) also provided equations to estimate critical shear stress obtained from Jet Erosion Test (JET) and Hole Erosion Test (HET). Nevertheless, as the equations in Table 2-1 for assessing the critical stress were developed based on different data sets, there is a need to demonstrate the magnitude of the critical stress estimated using these different methods for a same dataset.
Briaud et al. (2008) developed a chart for different geomaterials, relating erodibility with the soil classification (according to the Unified Soil Classification System). A geomaterial can be classified into one of six categories as presented in Figure 2-2. It is important to note that a soil class falls across two different categories. As previously discussed, erodibility may not just be a function of the shear stress exerted on the soil particles, as the stress fluctuations due to turbulent boundary layer might also contribute to soil erodibility. Future consideration of this effect might lead to narrowing the bandwidth for a given soil class shown in Figure 2-2.

Similar to the case of critical shear stress, there are a number of equations available to estimate the critical velocity, which is defined as the flow velocity at, or above which particles dislodge. This critical velocity also determines the live bed or clear water conditions. Therefore, its inaccurate estimation will result in mischaracterization of the state of upstream sediment transport condition and in turn, the scour assessment. Collection of equations to predict critical velocity can be found in Henderson (1966), Melville and Sutherland (1988), Briaud et al. (2019).

**Pier Scour**

Ettema et al. (2011) classified factors influencing pier scour into two broad categories, a) primary factors and b) secondary factors. It was suggested that primary factors are those parameters defining the structure and geometric scale of the pier flow field, thus, influencing the maximum scour depth. Parameters included in the primary factors’ category are flow depth to pier width ratio, pier width to \(d_{50}\) ratio, pier face shape, pier aspect ratio, and skew angle of pier. In contrast, secondary factors are those which influence the computed scour depth sensitivity within a specific geometric scale limit. These include Flow intensity, Euler number or Reynolds number, sediment non-uniformity, and the temporal rate of scour. Ettema et al. (2011) suggested that consideration of secondary factors will lead to a scour depth estimation less than the potential maximum scour obtained from only considering the primary parameters, thus reducing the conservativeness of the estimate. Figure 2-3 presents a conceptual sketch of the primary and secondary factors influencing pier scour magnitude.

**Primary factors**

Melville and Coleman (2000) and Sheppard and Melville (2011) identified the water depth to pier width ratio \((y/b)\), also termed as the geometric scale of the pier flow field, as one of the most important parameters driving the maximum scour depth. Based on experimental and numerical studies, Melville and Coleman (2000) classified the pier scour flow field into three
categories. Table 2-2 shows such classification based on $y/b$ and summarizes the type of turbulent structures developed due to the presence of obstacles for the three categories of piers.

Sheppard et al. (2004) and Sheppard and Miller (2006) observed that scour depth is dependent on the ratio of pier width to sediment size, $b/d_{50}$. Lee and Sturm (2009) experimentally observed that smaller values of $b/d_{50}$ can impede the scour progression and subsequently confine the scour depth. They further suggested that scour depth to pier width ratio increases logarithmically in the range $b/d_{50} \leq 25$, while it decreases for $b/d_{50} > 25$, and eventually remains constant if $b/d_{50}$ exceeds 400. Ettema et al. (2011) suggested that if $b/d_{50} \leq 8$, individual particles are large relative to the groove excavated by the down flow and erosion is impeded because the rough and porous bed dissipates some of the down flow energy.

In general, piers are constructed in a variety of shapes that include cylindrical, oblong or rectangular configurations. As pier shape influences the flow field surrounding a pier, the magnitude of scour is also affected in the process. The deterministic pier scour models by Melville (1997), HEC 18 (2012), and Briaud (2014a) adopt a multiplication factor to consider pier shape varieties. From an experimental investigation, Mostafa (1994) observed a variation of scour depth as a function of the magnitude of the aspect ratios; such variation was attributed to the change in formation and spacing of turbulence structures at different aspect ratios. Pier skewness to flow leads to altering the effective pier width and influencing the flow field, and consequently the scour depth. Yang et al. (2017) observed that when skewness exceeds 30°, irrespective of the undisturbed bed level, the bottom of the equilibrium scour hole reaches the same depth below the pile cap in clear water conditions. Nevertheless, it is a common practice to combine the effects of pier skewness, shape, and pier width in terms of expressing an effective pier width (Mostafa 1994, Ettema et al. 1998, and Arneson et al. 2012).

**Secondary factors**

Sediment’s non-uniformity, presented in Eq. (4) is one of the secondary factors influencing bridge pier scour.

$$\sigma_g = \sqrt{\frac{d_{84}}{d_{16}}}$$  \hspace{1cm} (4)
Where, \(d_{84}\) and \(d_{16}\) are the particle diameters corresponding to 84% and 16% finer. Until recently, the effect of sediment non-uniformity was omitted or not considered. For example, in the Pier Scour Database, PSDb (Benedict and Caldwell, 2014), only in 30% of the cases was the sediment non-uniformity reported. However, in most of the cases (96%), the median particle size was reported. For clear water condition, the scour depth decreases with the increase in sediment non-uniformity (Ettema 1980; Melville 1997; Raikar and Dey 2005). In case of non-uniform sediments (\(\sigma_g > 1.3\)), it is possible to form an armor layer surrounding the pier. The armor layer resists the subsequent development of the scour depth due to the progressively increasing critical shear stress with the presence of the larger grain sizes.

The energy associated with turbulence structures is determined in terms of the pier Euler number \(\left(\frac{v^2}{gb}\right)\); where \(v\) is the upstream flow velocity, \(g\) is the gravitational acceleration, and \(b\) is the pier width) and pier Reynolds number \(\left(\frac{\rho v b}{\mu}\right)\); where \(\rho\) and \(\mu\) are the density and viscosity of water respectively). Ettema et al. (1998) and Ettema et al. (2006) observed that scour depth increases significantly as the pier Euler number increases, suggesting the consideration of a correction factor accounting for Euler number impacts. It is interesting to note that most of the scour prediction equations (Wilson 1995; Melville 1997; HEC 18 2012) available in the literature do not consider the effect of the power of turbulence structures.

Pier scour is also affected by the flow intensity, expressed as a ratio of upstream flow velocity, \(v\) to the sediment critical velocity, \(v_c\). The effect of uniformity of sediments, size and steepness of bed features in relation to \(v/v_c\) has been studied extensively in the literature (Chee 1982; Chiew 1984; Melville 1984; Melville and Sutherland 1988). Temporal evolution of scour has been studied by Shen et al. (1969), Ettema (1980), Melville and Chiew (1999), Oliveto and Hager (2002, 2005), Miller (2003), and Kothyari et al. (2007). However, Ettema et al. (2010) pointed out that data on the time development of local scour at wide piers or piers with complex geometry is scarce; therefore, the validity of the temporal evolution theory when applied to wide piers or long skewed piers is questionable.

**Databases**

The review of databases is classified into two categories, i) system-level scour observations, and ii) material-level erodibility observations.
**System-level scour observations**
Benedict and Caldwell (2014) performed an extensive literature review to identify the potential sources of pier scour data and collected 2,427 laboratory and field scour datasets. The data encompass a wide range of laboratory and field conditions. Of the 2427 data, 1858 are field measurements, and the rest are laboratory measurements. The field data were collected from 23 states within the United States and 6 other countries. The collected data include a wide range of stream gradients, drainage areas, sediment sizes, flow depth, flow velocity, and pier sizes. On the other hand, the laboratory data encompasses 569 measurements under a wide range of conditions. The data were initially compiled by Sheppard et al. (2011) from 17 former investigations. While the laboratory data in the database were classified into clear-water and live-bed categories, the field data were not. Nevertheless, the database is the largest collection of pier scour data available in the United States, with a complete set of parameters to allow for comparative analysis of models that require different input parameters.

**Material-level erodibility observations**
Over the past 25 years, various testing approaches for assessing the erodibility of soil have been reported in literature. Some of these tests have laboratory applicability, whereas some have field applicability. Table 2-3 presents a summary of erosion evaluation tests. For all testing approaches listed in Table 2-3, factors considered to influence erosion were critical velocity, critical shear stress, slope of erosion rate versus velocity curve and initial slope of erosion rate versus shear stress curve (Briaud et al. 2019). Review of Table 2-3 suggests that erosion tests, for which “undisturbed samples” can be retrieved for laboratory testing, were mostly focused on clays and silts; testing approaches for coarser particles are still lacking due to the need to mainly perform testing in-situ.

Briaud et al. (2019) conducted an extensive numerical, and laboratory testing using data from EFA, JET, and HET to observe the differences among the results from different tests. CHEN4D coding (Chen et al. 1990) was used to perform CFD simulations to obtain the hydraulic shear stress on the soil bed prior to erosion. The shear stress at the interface was determined with Moody charts (Moody 1944) as well. The Moody chart is the basis for a critical shear stress estimation using EFA test. It was observed that Moody charts generally overestimated critical shear stress compared to the numerical simulation. The maximum discrepancy was reported for a coarse sand, where the critical shear stress was 25 Pa (from numerical simulation) and using
Moody chart the critical shear stress was found to be 50 Pa. Briaud et al. (2019) also mentioned that such discrepancy is more pronounced at higher shear stresses. Comparing with the site-specific cases, the JET test has more applicability when agricultural erosion or levee erosion due to overtopping is of interest. The HET test is suggested to have more suitability in assessing suffusion, or internal erosion of earth embankments. However, for other erosion tests, no explicit comments were made by Briaud et al. (2019) regarding applicability. A database of 950 erosion tests performed all over the world was compiled by Briaud et al. (2019). Geotechnical properties of soil and the erosion test response were reported in the database. This is one of the first attempts to compile comparative data on from various erosion tests to assist in understanding the applicability of different testing approaches.

**Scour Prediction Models**

The available scour prediction studies can be broadly classified into three different categories. These are, i) Deterministic, ii) Probabilistic, and iii) Observation-based.

**Deterministic models**

Ettema et al. (2011) and Sheppard et al. (2011) compiled the pier-scour prediction models developed since 1949. At present, there are more than 20 pier-scour models available in the literature. These models are developed in most cases based on either extensive laboratory testing data, from field data, or from both. However, as a part of the review herein the scour prediction models developed since 1990 are presented in Table 2-4, albeit this list is not exhaustive. Gao et al. (1990) model was developed from local scour data collected in China and consisting of 137 live bed and 115 clear water data points. Wilson (1995) model is based on field data collected from 22 bridges in Mississippi. Melville (1997) model is based on extensive laboratory and field data collected over a period of 25 years. Sheppard and Miller (2006) model was developed primarily based on laboratory data and few field observations. Oliveto and Hager (2002, 2005), Kothyari et al. (2007) model is based on experiments performed on pier width ranging from 0.02-0.50 m, \(d_{50}\) ranging from 0.55-5.3 mm. The latest update of HEC-18 model by Arneson et al. (2012) is the most widely used pier-scour prediction model. It is developed based on data from laboratory testing on sediments with median grain size, \(d_{50}\) ranging from 0.24-0.52 mm. Briaud (2014a) model is based on data from large scale laboratory flume testing (\(d_{50}\) ranged from 0.10-0.60 mm).
To provide a comparative assessment of the influencing parameters on computing scour magnitude, Table 2-5 is provided, listing the parameters considered in each of the models in Table 2-4. Sheppard et al. (2011) considered 23 different scour prediction models and investigated the performance of the models when compared to 928 field and 569 laboratory data. Their conclusion suggested that results from Sheppard and Miller (2006) and Melville (1997) models provided reasonable values when compared with field data. Careful observation of Table 2-5 suggests that these two models incorporated all the primary variables, in contrast to the remaining listed models in Table 2-5. Sheppard et al. (2011, 2014) melded and empirically modified Sheppard and Miller (2006) and Melville (1997) models to obtain a refined estimate of scour for wide piers, as the existing deterministic models cannot predict the scour depth for wide piers with accuracy (compared to narrow, and intermediate piers).

Proportional approach

Although there seems to be a significant number of studies that were focused on developing deterministic pier scour equations, focus on probabilistic approaches is limited. To the authors’ knowledge, the first probability-based pier scour model was introduced by Johnson (1992). The Johnson (1992) study was focused on the relationship between probability of bridge failure and safety factors. Johnson and Dock (1998), using a Monte-Carlo simulation technique, developed probabilistic bridge scour depth estimates, considering hydraulic and geometrical parameter uncertainty. Probabilistic framework was also developed for assessing the likelihood of achieving a specific scour depth, probability of exceedance, as well as adequate pile depth. Briaud et al. (2007) developed a probabilistic bridge scour model that considered the uncertainty of hydrologic conditions; however, uncertainties associated with input parameters (geometry of obstacle, soil erodibility) were not considered. Bolduc et al. (2008) introduced a probabilistic model based on the bias and scatter around the mean. Bias was defined as the ratio between the mean-measured value to the mean-predicted value. Johnson et al. (2014) examined the overall uncertainty in local pier or abutment scour in combination with contraction scour. As a part of the study, reliability-based scour design factors were suggested. Briaud et al. (2014) developed a reliability-based pier scour model focusing on the risk associated with failure of shallow and deep foundations subjected to scour. It was demonstrated that the scour depth prediction from HEC-18 should be multiplied by the factor 2.05 to ensure a probability of exceedance of 0.001 for shallow foundations in sand subjected to scour. While the HEC-18 scour prediction model is considered to be highly
conservative, yet per Briaud et al. (2014), the scour predictions from HEC-18 did not correspond to sufficiently low probability of failure.

**Observation-based approach**

The designation of a bridge pier as “scour critical” highly depends on the method used to estimate the scour. Most scour prediction models do not take into account the key physical and engineering parameters of soils being eroded, such as plasticity, density and shear strength. Although scour prediction models such as Briaud et al. (2008) and Briaud (2014b) are based on using erodibility parameters, the use of such models necessitates performing site-specific erosion testing. To address this issue, Govindasamy et al. (2013) developed a model that is referred to as the Observation Method for Scour (OMS). One point of subjectivity in the OMS technique is if one soil classification falls into two different categories, the same structure can be classified as scour-critical or non-critical based on the erosion category that was assigned to the soil. Nevertheless, Govindasamy (2009) demonstrated successful application of OMS to 16 Texas bridges. Results however revealed that some of the scour-critical bridges are not scour critical based on the OMS analyses. The OMS technique has been automated for the state of Texas and is used for first order approximation of scour criticality (Govindasamy et al., 2013).

**Factors Complicating Pier Scour Estimation**

The descriptions herein focus on the effect of several factors that have the tendency to complicate scour estimation for bridge piers. Some of the factors affect the flow field surrounding piers, whereas others affect the erodibility of the sediment around the foundation.

**Pier proximity to abutment**

Comparatively short first deck span is not uncommon in bridge design due to the construction economy advantages in having the pier close to the abutment (Sturm et al., 2011). However, such placement highly influences the flow field around the abutment. Consequently, pier scour occurs in the zone of the abutment flow field. Ettema et al. (2011) suggested that scour depth at the pier in such cases is dominated by abutment scour. Nevertheless, the potential severity of pier placement near the abutment toe has received less attention. Hong et al. (2005) observed that the location of deepest scour changes due to the placement of pier near the abutment toe. However, no scour depth prediction model was suggested by Hong et al. (2005). Croad (1989) suggested that if a pier falls in abutment scour region, the scour depth at the pier can be estimated as equal to 0.9 times the abutment scour. Ettema et al. (2011) inferred that the conventional
equations for pier scour estimation are not applicable if the pier is located in the abutment scour region.

**Debris accumulation**

Waterborne debris, composed primarily of tree trunks and limbs, often accumulates at bridges during high flow events (Lagasse et al. 2010), leading to obstruction or constriction of flow. This might cause damage to the bridge foundation and/or increased scour. The size of debris may vary from a small cluster of tree branches to a near complete blockage of flow by large tree trunks or other debris. The impact of debris accumulation on scour estimation has been studied by Chang and Shen (1979), Diehl (1997), and Parola et al. (2010). None of the studies has however resulted in a complete analysis for debris-loaded pier. Dongol (1989) and Melville and Dongol (1992), based on 17 laboratory investigation in clear water conditions, provided an expression for assessing effective pier width, with the impact of the floating debris.

Lagasse et al. (2010) reported that the photographic archive of debris at bridges across the United States suggests that the geometry of the debris accumulated at the bridges can be classified into a finite number of common shapes. Based on their findings on mobility and transport mechanism of debris, geometry of the debris, Lagasse et al. (2010) suggested a methodology to estimate the effective pier width when debris accumulation is expected, and such width can be used in Richardson and Davis (2001) equation to estimate the magnitude of scour.

The effect of the geomorphic characteristics of rivers, physiographic regions, and types of vegetation present are some of the factors that contribute to quantifying the extent of debris surrounding bridges. Based on the measurements of spring-dominated streams, Manga and Kirchner (2000) showed that although Large Woody Debris (LWD) cover less than 2% of the surface area of the stream, the debris existence contributes to half of the total flow resistance. It was reported that addition of LWD to a stream increases the total shear stress but reduces the shear stress borne by streambed. Buffington and Montgomery (1999) have also reached the same conclusion. Pagliara and Carnacina (2011) identified that cylindrical shapes can be one of the debris shapes, and the effect of downstream extension of debris accumulation is also an important parameter influencing flow features. Panici and de Almeida (2018) concluded that for a given debris length, accumulation pattern varies with the flow velocity. At low flow velocity, accumulations are wide and shallow, whereas with increased flow velocity, accumulations tend to
be narrow and deep. Schalko et al. (2018) reported the backwater rise due to large debris accumulations leads to flood hazard, which can intensify scour critically. Nevertheless, such effects, to the authors’ knowledge are yet to be formulated into a comprehensive model for analyses of pier scour loaded with debris. Lagasse et al. (2010) noted that the paucity of data on debris accumulation is a primary factor constraining the development of such a model.

**Vegetation within the floodplain**

The flood plain vegetation affects the flow field surrounding the abutment, which influences the scour magnitude (Sturm et al. 2011). Vegetated surface may extend the clear water-scour beyond the critical entrainment condition for the foundation soil or sediment beneath (Ettema et al. 2011). Vegetation in the streambed exerts additional drag which eventually reduces the mean flow within vegetated regions (Shi et al. 1995). This reduced velocity promotes sediment accumulation as near-bed stresses are reduced (Leonard and Luther 1995). Guardo and Tomasello (1995) used a modified Manning’s equation to model the induced drag. Nepf (1999) noted that modified Manning’s model cannot accurately represent the regions of emergent vegetation. Subsequently, Nepf (1999) developed a model describing drag, turbulence, and diffusion for flow through emergent vegetation. Jordanova and James (2003), and Kothyari et al. (2009) suggested that sediment transport rates in vegetated channels are related to the bed shear stress in a way similar to bare channel flows. However, Nepf (2012a), Nepf (2012b), Tanino and Nepf (2008), Biron et al. (2004), Nepf and Vivoni (2000) noted that the typical methods to estimate the bed shear stress are not appropriate in vegetated channels for several reasons including that vegetation-induced turbulence can inhibit sediment deposition (Yang and Nepf, 2018). To predict the bed shear stress in emergent vegetation, Rowinski and Kubrak (2002) developed a one-dimensional model, which is based on modified mixing length concept. An idealized case with equidistant arrangement of vegetation in longitudinal and transverse direction was assumed; transverse momentum transport was not considered in the model. Yang et al. (2015) developed a model to estimate the bed shear stress in vegetated channel within a viscous sub-layer at the bed. Yang et al. (2015) model considers emergent vegetation and channels with smooth and impermeable bed. Yang and Nepf (2018) based on laboratory experiments with model vegetation showed that existing sediment transport models underestimate the sediment transported by an order of magnitude, as the models do not account for the effect of vegetation. A Turbulent Kinetic Energy (TKE) based model was proposed by Yang and Nepf (2018) to estimate bed-load transport rate.
that is applicable for both bare and vegetated channels. While possible effects of vegetation on the bed shear stress is studied in relation to sediment transport under wide ranging flow conditions, integration of this understanding in bridge pier scour estimation models remains to be developed.

**Layered sediments**

The subsurface soil profile surrounding a pier foundation may consist of multiple subsurface layers with varying level of resistance to scour. If the upper bed soil layer is sufficiently deep to accommodate the maximum scour depth, then the conventional approaches available for uniform deposits can be used. In contrast, if scour reaches an underlying layer with different soil type once a top layer is eroded, then scour rate and magnitude will vary. Ettema et al. (1980), Breusers and Raudkivi (1991), and Melville and Coleman (2000) identified that the scour depth in the latter case depends on the disintegration of the layer in upstream direction, downstream direction, or both. Lagasse et al. (2007) and Mashahir et al. (2010) identified the necessity of sediment layering considerations in riprap protection of piers. In addition to several available laboratory testing techniques, the in-situ erosion evaluation probe introduced by Gabr et al. (2013), and Kayser and Gabr (2013) provides the means for assessing the erodibility parameters with depth in the field. The Federal Highway Administration recently developed an in-situ scour and erosion testing device (ISTD) (Bergendahl and Kerenyi, 2016), which measures the scour potential of the sediment based on the reduction in erosion rate due to water flow with the depth. Briaud et al. (2019) noted that ISTD is applicable for soils that have a maximum SPT N-value of 30. In addition, the applicability is limited to soils that are below the groundwater table.

**Vertical contraction of flow**

When the bridge deck is submerged, as in the case of severe flood storms for example, an additional scour-inducing process, namely vertical contraction of flow is introduced. This process can result in a significantly larger scour depth, exceeding the estimates from the conventional scour prediction models. Deck submergence is studied by Richardson and Davis (1995), Arneson and Abt (1998), Lyn (2008), and Guo et al. (2009). However, in most cases clear water situations were considered. Guo et al. (2009) developed an expression for maximum scour depth in pressure flow situations under clear water conditions, assuming a rectangular girder, uniform bed material, and no pier present to simplify the analyses. A model assessing scour with the presence of a pier in live bed conditions, or with non-rectangular girders, or non-uniform bed materials with the vertical contraction of flow is lacking in literature.
Applicability of Four Models

Data for analyses

Database reported in Benedict and Caldwell (2014), also referred to as PSDb (2014), were used herein. The database encompasses 569 measurements with a wide range of laboratory conditions. The dataset was initially classified into two categories: clear-water and live-bed. It is noted that laboratory live-bed tests are mostly conducted using uniform sediments. Similarly, only 27 out of 340 reported clear water laboratory tests were performed on non-uniform sediments.

In most cases, the deterministic pier-scour models do not differentiate their application in terms of clear-water versus live-bed conditions. However, the performance of a model may vary depending on comparing its output using the clear-water versus live-bed scour data. Under this assumption, the ranges of selected variables for live-bed and clear water conditions are presented in Table 2-6 and Table 2-7, respectively. It is important to note that the minimum, median, and maximum values reported in Tables 2-6 and 2-7 reflect the range for a particular variable but do not correspond to the minimum and maximum values in the other column (i.e. data across rows are not from the same test). Accordingly, the data in Tables 2-6 and 2-7 are not correlated to each other across each row. A total of 229 live-bed scour measurements were compiled in Benedict and Caldwell (2014), with the rest being clear water measurements.

Analyses

The focus herein was on four pier-scour prediction models, namely: Wilson (1995) model, Melville (1997) model, HEC-18 (2012), and Briaud (2014a) model. These four models are chosen due to the differences in the groups of data upon which these models were developed. Details on the group of data for each of the models are presented in the Deterministic models’ section. The model equations are provided in Table 2-4. Error in models’ estimation was considered as the metric describing the performance of a model. Error is defined as the difference between the predicted and measured scour depth, expressed as a percentage of measured scour depth, as shown in Eq. (5):

$$\text{Error, } \% = \frac{y_s(\text{predicted}) - y_s(\text{measured})}{y_s(\text{measured})} \times 100\% \tag{5}$$

Where, $y_s(\text{predicted})$ and $y_s(\text{measured})$ are the predicted and measured scour depths respectively. A positive error magnitude indicates that the pier-scour model is over-predicting the “measured” value and conversely, a negative error is indicative of under-prediction.
**Error-scour depth relation**

Error-normalized scour depth relation for the four models, using live bed and clear water datasets, are presented in Figures 2-4 and 2-5, respectively. Analyses of the data in Figure 2-4 show that the four predictive models have large error margins (-61% to 467%). For Melville (1997) and HEC 18 (2012) models, the scatter in data seems to be reduced as the magnitude of the normalized scour depth ($y_s/b$) is increased. The error in the Melville (1997) model decreases as $y_s/b$ increases from 1.0-2.5. In the case of HEC 18 (2012) model, the error consistently reduces as $y_s/b$ increases from 1.5-2.5. The error in the scour estimates from Briaud (2014) model did not yield a discernable trend. In the live-bed dataset of PSDb (2014), there are only five data points corresponding to the wide pier category (Table 2-2). These five data points provide $y_s/b$ values that are $< 1$. At $y_s/b < 1$, and in a number of occasions, error exceeding 100% (Eq. 5) was observed. Measurements’ inaccuracy might be partially contributing to such high error (>100%) at the $y_s/b < 1$. The lack of achievement of scour depth equilibrium during testing can also be an issue contributing to the estimated error, as the flow duration was not reported in 15 out of 18 occasions in which $y_s/b < 1$ was observed. Data analyses also suggest sediment non-uniformity ($\sigma_g$) might contribute to the error when $y_s/b < 1.3$. However, it is evident that there is a lack of experimental data with large $\sigma_g$ and $y_s/b < 1.3$. As mentioned earlier, Melville (1997) suggested that in case of non-uniform sediments, it is possible that fine particles are transported away, and an armor layer of the remaining coarse material is formed at the pier.

Data in Figure 2-5 suggest that the error associated with clear water condition varies in the range of minus 69.5% to plus 2240%, which is significantly higher than the range utilizing the live bed dataset. For all the four models, and at $y_s/b < 1$ the error tended to exceed 500%. The error scatter from HEC 18 (2012) and Melville (1997) models seems to be less dispersed as compared to the results from the other two models. The laboratory test corresponding to the maximum error in all four models was performed for 200 minutes, thereby raising the concern of whether the results are representing the full evolution of scour depth.

**Error-sediment size relation**

Figures 2-6 and 2-7 show the relation between the % error and $b/d_{50}$. Data in Figure 2-6 suggest that the error in Briaud (2014a) model decreases as the $b/d_{50}$ is increased. Briaud (2014a)
model was developed based on flume testing results on clays \((d_{50} < 0.075 \text{ mm})\) and sands \((d_{50} = 0.1 - 0.6 \text{ mm})\). The model does not explicitly consider the effect of \(b/d_{50}\) as noted in Table 2-5. Although Lee and Sturm (2009) suggested that at \(b/d_{50} \leq 25\), the scour progression is impeded, implying the predicted scour depth should exceed the measured scour depth, such a trend cannot be discerned from Figures 2-6 and 2-7. However, in the range of \(b/d_{50} =40-80\), and for all four models’ predictions for live bed data, six measurements are noted to provide anomalous error compared to the response obtained utilizing the rest of the dataset (refer to Figure 2-6). The parameters associated with these six measurements are presented in Table 2-8. All these measurements are associated with narrow pier category (Table 2-2), and the sediments are coarse grained \((d_{50} > 0.6 \text{ mm})\). The concern seems to be the sediment non-uniformity (reported to be 2.8 and 5.5,.) being quite high compared to the demarcation value of 1.3 (Melville 1997). As previously noted, high \(\sigma_g\) is suggestive of potential armor layer formation, which can effectively reduce the progression of scour depth. For live bed conditions \(b/d_{50}\) was observed to vary from 10-1989; whereas, for clear water conditions the range is 3.67-4160, indicating a wider range of laboratory test data coverage for clear water conditions.

**Error distribution and statistics**

The relationship between the predicted and measured normalized scour depth is presented in Figures 2-8 and 2-9. It appears that on average, Melville (1997), HEC-18 (2012), and Briaud (2014) models provide a conservative estimate of scour under live bed conditions. However, strong correlation between predicted and measured \(y_s/b\) (as indicated by \(R^2\) value) was not obtained for the four models considered herein. The degree of conservatism of Melville (1997) and HEC-18 (2012) models tends to be of similar magnitude (42%). Results show that use of Wilson (1995) model would require increasing the predicted normalized scour depth by a magnitude of 2 standard deviations (0.82 in this study) to ensure a conservative estimate.

For clear water conditions, and as presented in Figure 2-9, the four models provide conservative estimates. The degree of conservativeness being highest (34%) for the Wilson (1995) and Melville (1997) models. Predicted and measured normalized scour depths are not strongly correlated in the four models, as indicated by the \(R^2\). Melville (1997) model can be considered as the most conservative model among the four models as Melville (1997) model underpredicted measured data in least number of occasions (n = 11 out of 340 total cases).
The distributions of error, in terms of the cumulative density function, for the four models used herein are presented in Figure 2-10. The error distributions for live bed and clear water conditions in all four models are normally distributed with a positive skew. Figure 2-10(a) shows that the probability to obtain an underprediction using Wilson (1995) model is 72%, whereas, for Briaud (2014a) model, the probability is 23%. While using Melville (1997) and HEC 18 (2012) models, the probability of obtaining an underprediction is 4%, and 9% respectively.

The probability, for example, to obtain an error $<150\%$ is 98%, 96%, 98%, and 95% for Wilson (1995), Melville (1997), HEC 18 (2012), and Briaud (2014a) models, respectively. The probability of obtaining an error for a given range of underestimating or overestimating scour magnitude is summarized in Table 2-9. Analyses of data in Figure 2-10(b) suggest that the probability of an unconservative estimate from Wilson (1995), Melville (1997), HEC 18 (2012), and Briaud (2014a) models are 43%, 11%, 18%, and 42% respectively. These values are higher compared to the probability corresponding to the estimates for live bed condition. Although the error-normalized scour depth relation for clear water case in Figure 2-5 showed occurrences of error exceeding 500%, data in Figure 2-10(b) suggests that for all the four models, the probability to obtain an error $>500\%$ is $\leq 4\%$. Evaluation of probabilities corresponding to live bed and clear water conditions in Table 2-9 indicates that for Melville (1997), HEC 18 (2012) and Briaud (2014a) models, the probability of obtaining an unconservative estimate of scour is higher (by 7%, 10%, and 19% respectively) for clear water conditions. It also seems that obtaining an error $>150\%$ is more probable for clear water conditions compared to the live bed conditions.

However, the choice of a model should not only depend on the conservativeness of the predictions, but also on the consistency of estimation, i.e., its precision and spread of error. The statistical parameters computed based on the results of the four models and including average error, standard deviation, and variance are shown in Tables 2-10 and 2-11. Although the average error from Melville (1997), HEC-18 (2012), and Briaud (2014) models ranges from 43-60% for live bed conditions, the standard deviation and variance of Briaud (2014a) model are 0.64 and 0.42 respectively, which is relatively higher compared to the standard deviation and variance of the other two models. For clear water condition, maximum standard deviation and variance for Wilson (1995) model are 1.13 and 1.27, respectively, whereas the minimums were obtained for Melville (1997) model (which are 0.41 and 0.17, respectively).
Summary and Conclusions

A review of aspects related to scour and soil erodibility has been presented covering details on erodibility parameters and analyses models. The review also included common factors influencing pier scour and demonstrated performance and error statistics of four pier scour models. Based on the review and analyses presented herein, the following conclusions are made:

i. Data in literature indicate that the stress fluctuations due to turbulent boundary layer contribute to soil erodibility. Future consideration of this effect can lead to narrowing the current erodibility criteria for a given soil class and thereby reducing the possibility to classify a soil across two different erodibility categories.

ii. Similar to the observation of Melville and Coleman (2000), Ettema et al. (2017), laboratory data on wide pier studies are scarce. There is a need for research to develop the reliability of pier scour estimates for wide piers. Sheppard et al. (2011, 2014) provided the only complete analysis for wide piers focusing on a parametric response.

iii. Until recently the effect of sediment non-uniformity was not considered in the soil erosion analyses. Laboratory studies, as reported in PSDb (2014), were mainly focused on flume testing with uniform deposits, whereas non-uniform deposits may lead to an armor layer formation at the pier. Such formation can progressively reduce the boundary layer velocity, leading to reduction in the scour magnitude. Further laboratory studies on non-uniform sediments are needed to discern such phenomenon for both live bed and clear water conditions.

iv. Although a significant number of studies were focused on developing deterministic pier scour equations, the focus on probabilistic approach is limited. A probabilistic approach can also have different connotation; it may account for uncertainties in hydraulic loading, geometrical site conditions including flood plain, sediments spatial and depth variabilities, or failure mechanism assumed for the bridge elements. Studies suggest that although in the common literature HEC-18 scour prediction model is considered to be highly conservative, it is yet not sufficient to ensure a low probability of failure. Hence, the use of a reliability-based approach in design should be considered in practice.
v. To mitigate the cost concerns associated with site-specific erosion testing, Observation Method for Scour (OMS) was developed by Govindasamy et al. (2013). This approach can be used for first order scour criticality assessment although sufficient level of site data is needed with subjectivity of the approach embedded in the a’priori assumption of the type of bed material and erosion category.

vi. A rigorous study is required to develop a clear demarcation between abutment scour region and contraction scour region and possible relationship to scour depth formulae. Further studies are also needed to systematically integrate the effect of vegetation on bridge pier scour onto estimation models. In view of the frequent occurrence of hurricanes and severe rainstorms, studies are required for pressure flow situations in live-bed conditions with non-rectangular girders, and non-uniform bed materials to complete the research gap.

vii. For both live bed and clear water conditions, and at a normalized scour depth \( y_s/b < 1 \), the four pier scour prediction models considered herein yielded significant error (for live bed conditions, >100%; for clear water conditions, >500%) when comparison with measured data was made. Measurement inaccuracy, sediment non-uniformity, non-equilibrium scour depth during measurements, and/or pier category difference might be the reasons for such error level.

viii. On average, for live bed condition, Melville (1997), HEC-18 (2012), and Briaud (2014) models provide a conservative estimate of pier scour. Both Melville (1997) and HEC-18 (2012) models are conservative to a similar extent (42%). In comparison, for clear water condition, all the four models provide a conservative estimate, with the degree of conservatism being highest (34%) for Wilson (1995) and Melville (1997) models. However, the “average” measure of error needs to be presented in the context of the precision of prediction (i.e., being consistently conservative for example).

ix. For live bed condition, Briaud (2014a) model yielded the highest standard deviation and variance, whereas, for clear water condition, Wilson (1995) model yielded the highest standard deviation and variance. For live bed condition, the probability to obtain an underprediction of scour magnitude using in Wilson (1995) model is 72%,
whereas, for Melville (1997) model, such probability is 4%. For clear water condition, the probability to obtain an underprediction using Wilson (1995) model is 43%; whereas, for Melville (1997) model, such probability is 11%.

In general, the shear stress acting at the soil-water interface layer is considered as a key parameter influencing erodibility. Briaud et al. (2019) observed that Moody charts generally overestimated critical shear stress compared to the CFD modeling, with the discrepancy more pronounced at higher shear stress magnitudes. With the advancement of numerical analysis techniques, such as coupled CFD-DEM or coupled CFD-MD, understanding of the initiation and progression of particles mobility and transport, and considering the coupled action of shear stress and the turbulent interface layer, is needed especially in cases when flow experiencing obstructions.

**Acknowledgements**
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**Data Availability**
Datasets for this research is available in the in-text data citation reference: Benedict and Caldwell (2014).
Table 2-1. Models to estimate critical shear stress based on geotechnical properties.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Model expression</th>
<th>Number of data</th>
<th>Test type</th>
<th>Soil type</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shields (1936)</td>
<td>$\tau_c(Pa) = 0.63d_{50}(mm)$</td>
<td>--</td>
<td>--</td>
<td>Coarse</td>
<td>--</td>
</tr>
<tr>
<td>Smerdon and Beasley (1959)</td>
<td>$\tau_c(ps f) = 0.0034PI(%)^{0.84}$</td>
<td>11</td>
<td>Flume</td>
<td>Fine</td>
<td>0.80</td>
</tr>
<tr>
<td>Smerdon and Beasley (1959)</td>
<td>$\tau_c(ps f) = 0.213DR(%)^{-0.63}$</td>
<td>11</td>
<td>Flume</td>
<td>Fine</td>
<td>0.80</td>
</tr>
<tr>
<td>Parchure and Mehta (1986)</td>
<td>$\ln\frac{\varepsilon}{\varepsilon_f} = \alpha(\tau_b - \tau_s)^{1/2}$</td>
<td>47*</td>
<td>Flume</td>
<td>Fine</td>
<td>--</td>
</tr>
<tr>
<td>Hanson and Simon (2001)</td>
<td>$\varepsilon(m/s) = 0.2\tau_c(Pa)^{-0.5}\tau_e - \tau_c(Pa)$</td>
<td>--</td>
<td>JET</td>
<td>Coarse and fine</td>
<td>0.64</td>
</tr>
<tr>
<td>Julian and Torres (2006)</td>
<td>$\tau_c(Pa) = 0.1 + 0.1779SC(%) + 0.0028SC(%)^2 - 2.34E - 5.0SC(%)^3$</td>
<td>16</td>
<td>--</td>
<td>Fine</td>
<td>0.91</td>
</tr>
<tr>
<td>Thoman and Niezgoda (2008)</td>
<td>$\tau_c(Pa) = 77.28 + 2.2A + 0.26DR - 13.49SG - 6.4pH + 0.12WC(%)$</td>
<td>25</td>
<td>JET</td>
<td>Fine</td>
<td>0.72</td>
</tr>
<tr>
<td>Briaud (2008)</td>
<td>$\tau_c(Pa) = d_{50}(mm)$</td>
<td>--</td>
<td>EFA</td>
<td>Coarse</td>
<td>--</td>
</tr>
<tr>
<td>Shafii et al. (2016)</td>
<td>$\tau_c(Pa) = 0.165 \times d_{50}(mm)^{0.529} \times WC(%)^{0.788} \times PF(%)^{-0.23}$</td>
<td>180</td>
<td>EFA</td>
<td>Coarse</td>
<td>0.79</td>
</tr>
<tr>
<td>Shafii et al. (2016)</td>
<td>$\tau_c(Pa) = 0.005 \times PI^{0.44} \times S_u(kPa)^{0.83} \times WC(%)^{1.03} \times d_{50}(mm)^{0.29}$</td>
<td>180</td>
<td>EFA</td>
<td>Fine</td>
<td>0.52</td>
</tr>
<tr>
<td>Briaud et al. (2017)</td>
<td>$\tau_c(Pa) = 2.507 \times 10^{-12} \times y(kN/m^3)^{3.931} \times PF(%)^{4.382}$</td>
<td>13</td>
<td>EFA</td>
<td>Coarse</td>
<td>0.5</td>
</tr>
<tr>
<td>Briaud et al. (2017)</td>
<td>$\tau_c(Pa) = 0.978 \times WC(%)^{-2.306} \times PF(%)^{1.991}$</td>
<td>13</td>
<td>EFA</td>
<td>Coarse</td>
<td>0.7</td>
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<tr>
<td>Briaud et al. (2017)</td>
<td>$\tau_c(Pa) = 3.347 \times 10^{-10} \times PI(%)^{-1.855} \times d_{50}(mm)^{-1.05} \times WC(%)^{-6.707}$</td>
<td>17</td>
<td>EFA</td>
<td>Fine</td>
<td>0.72</td>
</tr>
<tr>
<td>Briaud et al. (2017)</td>
<td>$\tau_c(Pa) = 2.28 \times 10^{-15} \times PI(%)^{-1.732} \times WC(%)^{3.106} \times PF(%)^{6.412}$</td>
<td>55</td>
<td>EFA</td>
<td>Fine</td>
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Table 2-1 (continued).

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<tr>
<th>Briaud et al. (2017)</th>
<th>( \tau_c(Pa) = 1.354 \times 10^{-7} \times PL(%)^{0.666} \times d_{50}(mm)^{-0.189} \times WC(%)^{0.046} )</th>
<th>17</th>
<th>EFA</th>
<th>Fine</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Briaud et al. (2019)</td>
<td>( \tau_c(Pa) = 158.06 \times \gamma^5 \times A^{-0.46} \times WC(%)^{10.03} \times S_u(kPa)^{1.83} \times PF(%)^{-18.28} \times d_{50}(mm)^{-4.21} )</td>
<td>44</td>
<td>EFA</td>
<td>Fine</td>
<td>0.94</td>
</tr>
<tr>
<td>Briaud et al. (2019)</td>
<td>( \tau_c(Pa) = 1.58 \times C_u^{-0.04} \times \gamma(kN/m^3)^{0.02} \times d_{50}(mm)^{0.77} )</td>
<td>28</td>
<td>EFA</td>
<td>Coarse</td>
<td>0.93</td>
</tr>
<tr>
<td>Briaud et al. (2019)</td>
<td>( \tau_c(Pa) = -0.248 \times PC(%) - 1.23 \times \gamma + 0.21 \times WC(%) + 0.07 \times S_u(kPa) - 36.89 \times d_{50}(mm) + 31.82 )</td>
<td>28</td>
<td>JET</td>
<td>Coarse and fine</td>
<td>0.50</td>
</tr>
<tr>
<td>Briaud et al. (2019)</td>
<td>( \tau_c(Pa) = 25.07 \times PI^{0.27} \times S_u(kPa)^{0.55} \times d_{50}(mm)^{0.50} )</td>
<td>21</td>
<td>HET</td>
<td>Coarse and fine</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Note: \( \tau_c \) = critical shear stress, \( d_{50} \) = median grain size, \( PI \) = plasticity index, \( DR \) = dispersion ratio, \( \varepsilon \) = erosion rate, \( \varepsilon_f \) = floc erosion rate (erosion rate when \( \tau_b - \tau_s = 0 \)), \( \alpha =18.4 \, m/N^{1/2} \), \( \tau_b \) = time-mean bed shear stress, \( \tau_s \) = bed shear strength, \( \varepsilon \) = erosion rate, \( \tau_e \) = effective bed shear stress, \( SC \) = silt content, \( E \) = lateral erosion rate, \( A \) = activity, \( SG \) = specific gravity, \( WC \) = water content, \( S_u \) = undrained shear strength, \( PF \) = percent finer than sieve #200, \( \gamma \) = wet unit weight, \( PL \) = plastic limit, \( C_u \) = coefficient of uniformity, \( PC \) = percent clay

*Counted from the figure presented in Parchure and Mehta (1986)
Table 2-2. Classification of pier based on $y/b$ (summarized from Melville and Coleman 2000; Ettema et al. 2011).

<table>
<thead>
<tr>
<th>Pier class</th>
<th>Range of $y/b$</th>
<th>Remarks*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Narrow pier</td>
<td>$\frac{y}{b} &gt; 1.4$</td>
<td>The vertical component of flow at the pier’s leading face causes the deepest scour to occur at the pier face. Flow contraction occurs as the stream passes the pier’s sides, leading to scour in the trailing face of pier as well. The deterministic pier scour models can reasonably predict the scour for narrow piers.</td>
</tr>
<tr>
<td>Intermediate pier</td>
<td>$0.2 \leq \frac{y}{b} \leq 1.4$</td>
<td>The main flow field feature is similar to narrow pier, however, as the flow depth decreases in relation to pier width, the main flow features become disrupted leading to a reduced capacity of flow to erode bed material. The deterministic pier scour models can reasonably predict the scour for intermediate piers.</td>
</tr>
<tr>
<td>Wide pier</td>
<td>$\frac{y}{b} &lt; 0.2$</td>
<td>The down flow development is disrupted owing to the lateral movement of the flow along the pier’s leading face and causes the deepest scour to occur at the pier flanks. Laboratory data on wide piers are fairly scarce, a modicum of research is necessary to assess the reliability in pier scour estimates.</td>
</tr>
</tbody>
</table>

*Ettema et al. (2017) presented a comprehensive summary of the complex flow field evolution for the three categories of pier.
Table 2-3. Erosion tests available in the literature with the applicable soil types.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Erosion test</th>
<th>Applicable soil type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moore and Masch (1962)</td>
<td>Rotating Cylinder Test (RCT)</td>
<td>Cohesive</td>
</tr>
<tr>
<td>Sherard et al. (1976)</td>
<td>Pinhole Erosion Test (PET)</td>
<td>Cohesive</td>
</tr>
<tr>
<td>Lefebvre et al. (1985)</td>
<td>Drill Hole Test (DHT)</td>
<td>Cohesive</td>
</tr>
<tr>
<td>Hanson (1990), Hanson and Cook (2004)</td>
<td>Jet Erosion Test (JET)</td>
<td>Cohesive, cohesionless</td>
</tr>
<tr>
<td>Briaud et al. (2001)</td>
<td>Erosion Function Apparatus (EFA)</td>
<td>Cohesive, cohesionless</td>
</tr>
<tr>
<td>Bloomquist et al. (2012)</td>
<td>Rotating Erosion Testing Apparatus (RETA)</td>
<td>Stiff clays, hard rock</td>
</tr>
<tr>
<td>Gabr et al. (2013)</td>
<td>In Situ Erosion Evaluation Probe (ISEEP)</td>
<td>Cohesionless</td>
</tr>
<tr>
<td>Briaud et al. (2017)</td>
<td>Borehole Erosion Test (BET)</td>
<td>Cohesive, cohesionless</td>
</tr>
</tbody>
</table>
Table 2-4. List of deterministic pier scour models developed since 1990.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Model</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gao et al. (1993)</td>
<td>$y_s = 0.46K_\xi b^{0.6}y^{0.15}d_{50}^{-0.07}(\frac{v - v_c'}{v_c - v_c'})^n$</td>
<td>$y_s$ = scour depth&lt;br&gt;$K_\xi$ = pier shape factor&lt;br&gt;$b$ = pier width&lt;br&gt;$y$ = flow depth upstream of pier&lt;br&gt;$d_{50}$ = median grain size&lt;br&gt;$v$ = approach flow velocity&lt;br&gt;$v_c$ = sediment critical velocity&lt;br&gt;$v_c'$ = initial velocity of local scour</td>
</tr>
<tr>
<td>Wilson (1995)</td>
<td>$\frac{y_s}{b'} = 0.9(\frac{y}{b'})^{0.4}$</td>
<td>$b'$ = projected pier width</td>
</tr>
<tr>
<td>Melville (1997)</td>
<td>$y_s = K_{yb}K_IK_dK_sK_\theta K_G$</td>
<td>$K_{yb}$ = pier depth-size factor&lt;br&gt;$K_I$ = flow intensity factor&lt;br&gt;$K_d$ = sediment size factor&lt;br&gt;$K_s$ = pier nose shape factor&lt;br&gt;$K_\theta$ = pier alignment factor&lt;br&gt;$K_G$ = channel geometry factor (= 1 for pier)</td>
</tr>
<tr>
<td>Kothyari et al. (2007)</td>
<td>$y_s (yD)^{1/3} = 0.068N\left(\frac{d_{84}}{d_{16}}\right)^{-0.25}P^{1.5}\log\left(\frac{t}{t_R}\right)$</td>
<td>$D$ = pier diameter&lt;br&gt;$N$ = shape factor&lt;br&gt;$P$ = Froude number&lt;br&gt;$g$ = gravitational acceleration&lt;br&gt;$d_{84}$ = diameter corresponding to 84% finer&lt;br&gt;$d_{16}$ = diameter corresponding to 16% finer</td>
</tr>
<tr>
<td>Oliveto and Hager</td>
<td>$t_R = \frac{\left(\frac{d_{84}}{d_{16}}\right)^{1/3}(gd_{50})^{0.5}}{(yD)^{1/3}}$</td>
<td></td>
</tr>
</tbody>
</table>

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Table 2-4 (continued).

<table>
<thead>
<tr>
<th>Source</th>
<th>Equation</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sheppard and Miller (2006)</td>
<td>$\frac{y_s}{b'<em>{c}} = 2.5f_1\left(\frac{y}{b'}\right)f_2\left(\frac{b'}{d</em>{50}}\right) \left[1 - 1.75\left(\ln\left(\frac{v}{v_{c}}\right)\right)^2\right]$</td>
<td>$v_{lp}$ = live bed peak scour velocity</td>
</tr>
<tr>
<td></td>
<td>$\frac{y_s}{b'} = f_1\left(\frac{y}{b'}\right)\left[2.2\left(\frac{v}{v_{c}} - 1\right) + 2.5f_2\left(\frac{b'}{d_{50}}\right)\left(\frac{v_{lp}}{v_{c}} - 1\right)\right]$</td>
<td>$v_{lp}$ = live bed peak scour velocity</td>
</tr>
<tr>
<td></td>
<td>$f_1\left(\frac{y}{b'}\right) = 2.2$</td>
<td>$v_{lp}$ = live bed peak scour velocity</td>
</tr>
<tr>
<td></td>
<td>$f_2\left(\frac{b'}{d_{50}}\right) = \frac{0.4\left(\frac{b'}{d_{50}}\right)^{1.2} + 10.6\left(\frac{b'}{d_{50}}\right)^{-0.13}}{0.4\left(\frac{b'}{d_{50}}\right)^{1.2} + 10.6\left(\frac{b'}{d_{50}}\right)^{-0.13}}$</td>
<td>$v_{lp}$ = live bed peak scour velocity</td>
</tr>
<tr>
<td>Arneson et al. (2012)</td>
<td>$\frac{y_s}{b} = 2K_1K_2K_3\left(\frac{y}{b}\right)^{0.35}F^{0.43}$</td>
<td>$K_1$ = pier nose shape factor $K_2$ = pier alignment factor $K_3$ = bed condition factor</td>
</tr>
<tr>
<td>Briaud (2014a)</td>
<td>$\frac{y_s}{b'} = 2.2K_{pw}K_{psh}K_{pa}K_{psp}(2.6F_{pier} - F_{c(pier)})^{0.7}$</td>
<td>$K_{pw}$ = water depth influence factor $K_{psh}$ = pier shape influence factor $K_{pa}$ = aspect ratio influence factor $K_{psp}$ = pier spacing influence factor $F_{pier}$ = pier Froude number $F_{c(pier)}$ = critical pier Froude number</td>
</tr>
</tbody>
</table>
Table 2-5. Consideration of pier scour influencing parameters in different models developed since 1990.

<table>
<thead>
<tr>
<th>Reference</th>
<th>$y/b$</th>
<th>$b/d_{50}$</th>
<th>Pier face shape</th>
<th>Aspect ratio</th>
<th>Skew angle</th>
<th>$v/v_c$</th>
<th>Sediment non-uniformity</th>
<th>Euler/Reynolds number</th>
<th>Temporal rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gao et al. (1993)</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>Wilson (1995)</td>
<td>C</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>Melville (1997)</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>Kothyari et al. (2007)</td>
<td>C</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>C</td>
<td>C</td>
<td>NC</td>
<td>C</td>
</tr>
<tr>
<td>Arneson et al. (2012)</td>
<td>C</td>
<td>NC</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>Briaud (2014a)</td>
<td>C</td>
<td>NC</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>NC</td>
<td>C</td>
<td>C</td>
</tr>
</tbody>
</table>

Note: C means considered in the respective model; NC means not considered in the respective model

Table 2-6. Range of some selected variables associated with live-bed laboratory data in Benedict and Caldwell (2014).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pier width normal to flow, $b'$ (ft)</th>
<th>Approach velocity, $v$ (ft/s)</th>
<th>Approach depth, $y$ (ft)</th>
<th>Flow depth, $d_{50}$ (mm)</th>
<th>Median grain size, $y_s$ (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>0.094</td>
<td>0.76</td>
<td>0.164</td>
<td>0.24</td>
<td>0.049</td>
</tr>
<tr>
<td>Median</td>
<td>0.164</td>
<td>2.03</td>
<td>0.558</td>
<td>0.6</td>
<td>0.253</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.5</td>
<td>5.28</td>
<td>1.969</td>
<td>3.2</td>
<td>0.492</td>
</tr>
</tbody>
</table>
Table 2-7. Range of some selected variables associated with clear-water laboratory data in Benedict and Caldwell (2014).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pier width normal to flow, $b'$ (ft)</th>
<th>Approach flow velocity, $v$ (ft/s)</th>
<th>Approach depth, $y$ (ft)</th>
<th>Median grain size, $d_{50}$ (mm)</th>
<th>Measured pier scour depth, $y_s$ (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>0.052</td>
<td>0.49</td>
<td>0.066</td>
<td>0.22</td>
<td>0.013</td>
</tr>
<tr>
<td>Median</td>
<td>0.246</td>
<td>0.98</td>
<td>0.656</td>
<td>0.96</td>
<td>0.327</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.002</td>
<td>3.96</td>
<td>6.234</td>
<td>7.80</td>
<td>4.626</td>
</tr>
</tbody>
</table>

Table 2-8. Data associated with high error for live bed condition in the range of $40 \leq b/d_{50} \leq 80$.

<table>
<thead>
<tr>
<th>Data source</th>
<th>Pier width normal to flow, $b'$ (ft)</th>
<th>Median grain size, $d_{50}$ (mm)</th>
<th>Sediment non-uniformity, $\sigma_g$</th>
<th>Sediment size scaling factor, $b/d_{50}$</th>
<th>Flow to pier width ratio, $y/b$</th>
<th>Froude number, $F_r$</th>
<th>Pier scour depth, $y_s$ (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chiew (1984)</td>
<td>0.105</td>
<td>0.6</td>
<td>5.5</td>
<td>53.35</td>
<td>5.31</td>
<td>0.29</td>
<td>0.049</td>
</tr>
<tr>
<td>Chiew (1984)</td>
<td>0.105</td>
<td>0.6</td>
<td>5.5</td>
<td>53.35</td>
<td>5.31</td>
<td>0.39</td>
<td>0.089</td>
</tr>
<tr>
<td>Chiew (1984)</td>
<td>0.105</td>
<td>0.6</td>
<td>5.5</td>
<td>53.35</td>
<td>5.31</td>
<td>0.61</td>
<td>0.102</td>
</tr>
<tr>
<td>Chiew (1984)</td>
<td>0.148</td>
<td>0.8</td>
<td>2.8</td>
<td>56.40</td>
<td>3.77</td>
<td>0.29</td>
<td>0.102</td>
</tr>
<tr>
<td>Chiew (1984)</td>
<td>0.148</td>
<td>0.6</td>
<td>5.5</td>
<td>75.20</td>
<td>3.77</td>
<td>0.37</td>
<td>0.128</td>
</tr>
</tbody>
</table>
Table 2-9. Probabilistic error analyses using live bed and clear water dataset.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Live bed</td>
<td>Clear water</td>
<td>Live bed</td>
<td>Clear water</td>
</tr>
<tr>
<td>&lt;-50%</td>
<td>0.04</td>
<td>0.08</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>-50%-0%</td>
<td>0.68</td>
<td>0.35</td>
<td>0.11</td>
<td>0.09</td>
</tr>
<tr>
<td>0%-50%</td>
<td>0.23</td>
<td>0.29</td>
<td>0.46</td>
<td>0.54</td>
</tr>
<tr>
<td>50%-100%</td>
<td>0.02</td>
<td>0.11</td>
<td>0.38</td>
<td>0.30</td>
</tr>
<tr>
<td>100%-150%</td>
<td>0.01</td>
<td>0.08</td>
<td>0.09</td>
<td>0.08</td>
</tr>
<tr>
<td>&gt;150%</td>
<td>0.02</td>
<td>0.09</td>
<td>0.04</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 2-10. Live bed error statistics corresponding to the four models considered in the study.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average error (%)</td>
<td>-7.63</td>
<td>59.37</td>
<td>52.10</td>
<td>43.51</td>
</tr>
<tr>
<td>Minimum error (%)</td>
<td>-61.44</td>
<td>-22.18</td>
<td>-27.77</td>
<td>-41.39</td>
</tr>
<tr>
<td>Maximum error (%)</td>
<td>276.20</td>
<td>414.29</td>
<td>414.29</td>
<td>466.86</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.41</td>
<td>0.36</td>
<td>0.37</td>
<td>0.65</td>
</tr>
<tr>
<td>Variance</td>
<td>0.17</td>
<td>0.13</td>
<td>0.14</td>
<td>0.42</td>
</tr>
</tbody>
</table>
Table 2-11. Clear water error statistics corresponding to the four models considered in the study.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average error (%)</td>
<td>71.75</td>
<td>74.38</td>
<td>67.07</td>
<td>51.15</td>
</tr>
<tr>
<td>Minimum error (%)</td>
<td>-69.49</td>
<td>-20.83</td>
<td>-31.32</td>
<td>-61.95</td>
</tr>
<tr>
<td>Maximum error (%)</td>
<td>2239.94</td>
<td>1339.04</td>
<td>2099.83</td>
<td>1334.02</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.13</td>
<td>0.41</td>
<td>0.56</td>
<td>0.86</td>
</tr>
<tr>
<td>Variance</td>
<td>1.27</td>
<td>0.17</td>
<td>0.32</td>
<td>0.74</td>
</tr>
</tbody>
</table>
Figure 2-1. Illustration of the flow around a circular pier in a scour hole (from Arneson et al. 2012).
Figure 2-2. Erosion category chart with USCS symbols (from Briaud 2013).
Figure 2-3. A conceptual sketch showing the primary and secondary factors influencing pier scour.
Figure 2-6. Error- \( b/d_{50} \) relation in the range for (a) Wilson (1995) model, (b) Melville (1997) model, (c) HEC-18 (2012) model, and (d) Briaud (2014a) model using live bed dataset. Six measurements that provided anomalous error are identified using a dashed oval boundary.
Figure 2-7. Error- b/d$_{50}$ relation in the range for (a) Wilson (1995) model, (b) Melville (1997) model, (c) HEC-18 (2012) model, and (d) Briaud (2014a) model using clear water dataset.
Figure 2-10. For the four models considered in the study cumulative density function for (a) live-bed conditions, and (b) clear water conditions.
Chapter 3: Estimating clear water local scour considering model bias and uncertainty

This chapter was previously published as:

Abstract

Local bridge scour, which is defined as the loss of soil particles/mass surrounding a pier foundation due to the flowing water-induced shear stresses, is a primary cause of bridge failure in the United States and worldwide. Current practice of bridge scour prediction is mostly based on the use of deterministic models. Work herein presents statistical models that extend five deterministic approaches reported in literature to predict the expected scour depth while quantifying inherent model bias and uncertainty in view of data scatter. Clear water scour database is used herein, and the analyses quantify model scatter by comparatively assessing the computed scour depth versus measured data reported in the database. A relationship between probability of deceedance associated with the predicted scour depth and a modification factor (that is applied into the deterministic prediction) is devised. The modification factor allows for the use of the scour magnitude computed from the deterministic models while quantifying the probability of a computed scour depth being less than or more than a most likely value (per measurements reported in the database). The application of the proposed model is demonstrated with an example and the results are discussed.

Keywords: Scour; Bridge pier; Probability of deceedance; Clear-water; Statistical models
Introduction

Local bridge scour is the loss of soil particles/mass surrounding a pier foundation due to the flowing water-induced shear stresses, which leads to mobilization and transport of the bed material. This process of local scouring is the primary cause of bridge collapse in the United States and worldwide (Melville and Coleman, 2000; Liang et al. 2015; Qi et al. 2016). Based on the investigation of 86 bridge failures that occurred from 1961 to 1976, Murillo (1987) concluded that 46 cases could be attributed to erosion of streambed caused by running water. In general, flood is the main source of the increased flow in the stream that induces development of scour holes surrounding a pier. Wardhana and Hadipriono (2003) analyzed 503 failures of bridge structures that happened in the United States and concluded that 11% of the bridge failure can be attributed to design, detailing, construction, and maintenance reasons, while 85% of the bridge failure is related to externally-triggered events (like flood, collision, overload etc.). It was also found that the principal cause (243 cases) of bridge failure is flood induced scour, while the rest can be related to collision, overload, deterioration, and miscellaneous (like fire, ice etc.). Even scouring phenomena, which do not lead to failure, cause major operational disruption and financial losses (Shirole and Holt, 1991; Briaud et al. 1999; Azamatullah et al. 2013). The average annual cost for flood damage and scour repair of highways in the United States is estimated to be $50 million (Lagasse et al. 1995; 2012). Rhodes and Trent (1993) reported that the additional cost to the afflicted population can be as much as five times the repair cost. Lin et al. (2006) reported that during the period of 1996 to 2001, 68 bridge failures that occurred in the United States are related to scour. Hunt (2009) noted that there are 20904 scour-critical bridges in the United States and approximately 80000 bridges that are deemed scour susceptible. With climate change impacts, like increase in heavy precipitation, and severe flooding coupled with scour is the leading cause of bridge damage (Liang et al. 2015), which eventually demonstrates the importance of properly estimating the scour magnitude beforehand and thoroughly incorporating in design, and countermeasures.

The available scour prediction models commonly in use can be classified into three categories: i) Deterministic, ii) Probabilistic, and iii) Observation-based. Current practice of bridge scour prediction is mostly based on the use of deterministic models. At present, there are more than 20 pier-scour deterministic models available in the literature. Some models are derived based on laboratory scale data (e.g., Arneson et al. 2012; Briaud 2014), while others are derived from
scour measurements taken in field conditions (e.g., Wilson 1995). In addition, there are models that consider both the laboratory scale data and field scale data (e.g., Melville 1997; Sheppard et al. 2014). Ettema et al. (2017) and Link et al. (2019) suggested that the incapability of the laboratory scale models to maintain a geometric similitude among the three length scales (pier, flow depth, and bed sediment size) results in differences in scour estimations from laboratory test based models and field observation based models. The quantitative estimation of the scale effects, to date, is a challenge for physical model test researchers (Heller 2011, 2017; Kiraga et al. 2020). Although, in Froude modeling the concepts of self-similitude and Reynolds number invariance have been proposed, such models do not simulate viscous forces, which are necessary to simulate sediment erosion. Recently, Link et al. (2019) suggested that to exclude scale effects in laboratory-scale scour modeling, appropriate selection of the dimensionless effective flow work and dimensionless grain diameter is necessary.

Literature suggests that pier scour is influenced by flow depth to pier width ratio, pier width to median grain size ratio, pier face shape, pier aspect ratio, skew angle of pier, flow intensity, Euler number or Reynolds number, sediment non-uniformity, and the temporal rate of evolution. However, not all the factors are of equal importance, thereby not included in the different deterministic models researchers developed. Table 3-1 presents the summary of the aforementioned deterministic models in terms of the consideration of pier-scour influencing factors. The differences in the choice of consideration of pier scour influencing factors in the deterministic models result in a different scour magnitude prediction for a similar set of hydraulic, structural, and geotechnical parameters.

Observation-based approach such as the one introduced by Govindasamy (2009) and Govindasamy et al. (2013) is based on the idea of capitalizing on the available information from observations at existing bridges. Thereafter, based on the knowledge of maximum flow velocity the bridge has been subjected to in its history, future scour depths are predicted, and then compared with the allowable scour depth of the foundation to assess the criticality of the foundation. However, the approach requires performing hydrologic analyses (if flow data are available) or developing a recurrence interval flood map (if flow data are not available). The process, although does not require site-specific erosion testing, an assumption of the bed material soil classification is required. Texas department of Transportation adopted this method for the first-order bridge
scour assessment. Public agencies are leaning towards using this method to prioritize bridge repairs, and countermeasure decisions.

Deterministic scour prediction models do not account for inherent model uncertainties and errors. The use of deterministic models does not yield the inherent model bias (conservativeness/unconservativeness). For example, Kwak and Briaud (2002) reported that for pier number 3W of the old Woodrow Wilson Bridge, the measured scour depth ranged from 4.3ft-8.9ft (1.31m-2.72m). Using a deterministic model to estimate the scour depth, termed as SRICOS (Scour Rate in Cohesive Soils) method (Briaud et al. 1999), the maximum scour depth was reported to be 9.3ft (2.85m) (Kwak et al. 2001). While from the scour depth estimation, it is apparent that the deterministic SRICOS model is conservative, the designer when applying the SRICOS model does not know the degree of biasness inherent in the system. Briaud (2008) reported that using HEC-18 model, and Briaud model (2001), the estimated scour at the bascule pier of the new Woodrow Wilson Bridge are 64ft (19.5m), and 38ft (11.6m) respectively. Without the prior knowledge of the degree of biasness of these models (each model predicts different scour depths for similar set of parameters), one cannot discern whether or not these predictions are conservative and the extent of conservatism in view a given model’s application to a wider set of site conditions. This raises the importance of developing of probabilistic models for scour assessment that account for the aleatory and epistemic uncertainties (statistical error, measurement error, human error, model error), albeit a sufficient level of data is needed for such development.

Johnson (1992) developed a relationship between probability of bridge failure and safety factors. In the study, bridge failure is defined as the point at which the scour depth reaches the bottom of the pier footing, simultaneously admitting that the probability of failure to be different depending on the definition of failure. Briaud et al. (2007) developed a probabilistic bridge scour model that considered the uncertainty of hydrologic conditions; however, no analyses were presented on the model estimates’ bias and error distribution. Bolduc et al. (2008) introduced a probabilistic model based on the bias in the deterministic models (HEC-18 sand, and Briaud et al. 1999 which is also referred to as “HEC 18 clay”) and scatter around the mean. Johnson et al. (2015) examined the hydraulic and hydrologic uncertainties associated with HEC-18 and FDOT (2011) scour prediction models. Johnson et al. suggested “scour design factors” based on target reliability index, defined as the ratio of difference between design and simulated scour depth to standard
Briaud et al. (2014) developed a reliability-based pier scour model focusing on the risk (product of the probability of occurrence times the value of consequences) associated with failure. In summary, probabilistic aspects to predictive models make a designer aware of the uncertainties associated the model’s estimate and the extent to which estimation of scour depth is conservative.

Work herein is focused on the applicability of five models in practice and the bias inherited in their estimate of expected scour magnitude. The scope includes quantifying scatter in the predicted scour depth from Wilson (1995) model, Melville (1997) model, HEC-18 (2012), Briaud (2014) model, and Sheppard et al. (2014) model versus measured data. Relationship between probability of predicted scour depth deceedance (i.e., predicted scour magnitude being less than measured value) and modification factors applied to such prediction is proposed. The modification factors are used to adjust the predicted scour magnitude from the five models such that the probability of such prediction deceeding or exceeding a measured value is known. Finally, a step-by-step application of the proposed model and approach is demonstrated and discussed.

Data for analyses
Benedict and Caldwell (2014) compiled laboratory scour measurements from 17 previous investigations consisting of data from 23 different states within the United States. Data reported in Benedict and Caldwell (2014) were classified into two categories: clear water and live bed scour. The analysis in this paper is focused on the data corresponding to clear water scour condition (no. of data points = 340) (Benedict and Caldwell, 2014). In general, laboratory investigations allow a controlled environment, relatively accurate measurements of downscaled parameters like pier width, flow depth, flow velocity, and scour depth compared to the field investigations. Studies reported in Mueller and Wagner (2005), Sheppard et al. (2011) suggest that deterministic models developed based on laboratory data can well capture the responses in the field, thereby providing a means to avoid the uncertainties associated with field data. The uncertainties associated with field data might include (i) the accuracy of measured scour (depending on the tool used like Ground Penetrating Radar, soundings with fathometers, survey levels etc.), (ii) maturity of scour depth (the measured scour might not reflect the equilibrium scour depth), (iii) accuracy of hydraulic properties (depending on the method used to approximate, like based on historical information, based on one-dimensional flow models). Despite the limitations of using field measurements, their importance in comprehending unprecedented and complex field setups cannot be overlooked. However, owing to measurement accuracy, and representative scour depth reported in relation to
measured hydraulic, structural, and geotechnical properties, clear water laboratory measurements are considered in the study. The ranges of the selected variables for the clear water laboratory data, including pier width, velocities and median grain size are presented in Table 3-2. The values in Table 3-2 reflect the minimum, median, and maximum values for a selected variable within the database. It is important to note that the data across a single row do not correspond to each other; i.e., the minimum, median, and maximum values in Table 3-2 across one row are not for a specific site. These values rather indicate the minimum, median, and maximum values of the specified variable within the whole database.

**Scour prediction models for analyses**

Five pier scour prediction models, namely Wilson (1995) model, Melville (1997) model, HEC-18 (2012) model, Briau (2014) model, and Sheppard et al. (2014) model are utilized herein. The model equations are provided in Table 3-3. Wilson (1995) model is based on field data collected from 22 bridges in Mississippi during the period 1938-1994. The drainage area of the selected sites ranged from 60.8 to 5720 square miles. The measured pier scour depth in the sites ranged from 0.6 to 20.4 ft. Melville (1997) model is based on extensive laboratory and field data collected over a period of 25 years. The tests were mostly performed in the University of Auckland, New Zealand (Melville and Sutherland 1988; Melville 1992, Dongol 1994). The method uses a number of factors (refer to Tables 3-1 and 3-3) that influences scour. The value of these factors are determined from the envelope curve fitted to the data (Melville 1997; Sheppard et al. 2011). The latest update of HEC-18 model by Arneson et al. (2012) is the most widely used pier-scour prediction model and it is recommended by the US Federal Highway Administration (FHWA) for estimating equilibrium scour depth. The initial HEC-18 model, known as Richardson et al. (1995) model used a subset of the data collected by Chabert and Engeldinger (1956) and Shen et al. (1966). The subset database consisted of data from laboratory testing on sediments with median grain size, $d_{50}$ ranging from 0.24-0.52 mm. Thereafter, based on the studies of Melville and Sutherland (1988), and Jain and Fisher (1979), limits on the maximum scour depth based on Froude number was imposed. Briau (2014) model is based on data from 94 large scale laboratory flume testing, as well as dimensional analysis. The $d_{50}$ ranged from 0.10-0.60 mm. The critical shear stress of the bed material varied from 0.1 to 0.8 Pa. Sheppard et al. (2014) evaluated the performance of 23 scour prediction models using 441 laboratory and 791 field test results and melded two best performing models (Sheppard and Miller 2006 and Melville 1997) to develop the
Sheppard et al. (2014) model. The melding was done in such a way that the resulting equation yields the least total error.

**Statistical model**

**Approach formulation**

In scour prediction models, uncertainties arise from multiple sources including hydraulic, geotechnical, structural, or predictive model errors (Yao 2013). Hydraulic uncertainty is associated with errors in estimating flow velocity, or flow depth. Geotechnical uncertainty is related to the soil characteristics and erodibility parameters. Briaud et al. (2019) documented the parameters which affect the erodibility of soil, and noted that erodibility is dependent on water content, undrained shear strength, unit weight, plasticity index, activity and other parameters which might not be frequently available. Structural uncertainty arises due to error in estimating the skew angle of the flow with respect to bridge pier, or in accurately representing the geometry of the bridge pier in the analysis. Finally, the model uncertainty involves the error generated when a given model is used to predict complex phenomena of bridge scouring. The hydraulic, geotechnical, and structural uncertainties can be addressed through improved field characterization and measurement accuracy, by accumulating more data, or by performing more targeted tests on the soil bed of interest. In the work presented herein, we will focus on model uncertainty. The model uncertainties will be quantified through analyzing the probability of scour magnitude exceedance associated with the predicted scour depth while using one of the five aforementioned deterministic models.

Figure 3-1 shows the data scatter associated with scour prediction when applying Wilson (1995) model, Melville (1997) model, HEC-18 (2012) model, Briaud (2014) model, and Sheppard et al. (2014) model to the measurement database. The scatter in the scour prediction is not uniform across the range of measured scour depth. The non-uniform scatter gives rise to a non-constant variance, which statistically is known as heteroscedasticity (Stone 1996). Figure 3-2 shows a plot of the residuals (difference between measured and predicted values obtained from best-fit curve) for HEC-18 model; in this case, a linear relationship is assumed between the predicted and the measured scour depth. It seems that the residual is significantly dispersed in the positive residuals’ region varying between 0 to 5; whereas the data lie mostly within 0 to -1 in the negative residuals region. In addition, if a linear relationship between predicted and the measured scour depth is adequate, a quantile-quantile plot should reveal a normal distribution. However, Figure 3-2 shows that the relationship between standardized residual and theoretical quantile is not linear, rather a
significant right skewness in the data is noted. In light of these observations, a robust model is needed which can reduce the heteroscedasticity and non-normal errors. Accordingly, the method suggested by Box and Cox (1964), commonly referred to as Box-Cox method, is adopted for this purpose. The main objective of applying the Box-Cox transformation to the database is after the transformation, the following is obtained:

(i) The error variance is constant (homoscedasticity)
(ii) The observations are normally distributed
(iii) A model, which is linear in the independent variable, can represent the expected value of the transformed response, i.e., no interaction terms are required.

The probabilistic model, considering the scatter in scour prediction is formulated as:

\[ y^{(\lambda)} = \kappa (y_{sm}/b) + \zeta \]  \hspace{1cm} (6)

And

\[ y^{(\lambda)} = \frac{(y_{sp}/b)^\lambda - 1}{\lambda} \]  \hspace{1cm} (7)

Where, \( b \) denotes the pier width, \( y_{sp} \), and \( y_{sm} \) indicate the predicted and the measured scour depth respectively, \( \kappa \), and \( \lambda \) are constant terms depending on the deterministic model being considered, and \( \zeta \) is the error term describing the uncertainty of model prediction. To ascertain the most likely value of \( \lambda \), large-sample maximum likelihood theory is considered. The likelihood in terms of the original observations \( y \) can be expressed as in Eq. (8).

\[
\frac{1}{n} \exp \left\{ \frac{-(y^{(\lambda)} - E\{y^{(\lambda)}\})'(y^{(\lambda)} - E\{y^{(\lambda)}\})}{2\sigma^2} \right\} \prod_{i=1}^{n} \left| \frac{dy_i^{(\lambda)}}{dy} \right| \]  \hspace{1cm} (8)

Where, \( n \) is the total number of observations, \( y^{(\lambda)} \) is the column vector of transformed observations, \( E\{y^{(\lambda)}\} \) is the expected value of \( y^{(\lambda)} \), \( (y^{(\lambda)} - E\{y^{(\lambda)}\})' \) is the transpose of the vector \( y^{(\lambda)} - E\{y^{(\lambda)}\} \), and \( \sigma^2 \) is the variance of data. More details of the associated steps can be found in Box and Cox (1964). The estimated parameter \( \lambda \) differ for each of the five models considered herein as presented in Figure 3-3.
**Interpretation of calculated parameters**

The calculated parameters for Wilson (1995) model, Melville (1997) model, HEC-18 (2012) model, Briaud (2014) model, and Sheppard et al. (2014) model are presented in Table 3-4. The $R^2$ of the fitted models vary from 0.16 for Wilson (1995) model to 0.56 for Sheppard et al. (2014) model. Magnitude of standard error suggest that Melville (1997) model has the maximum variance, whereas, Briaud (2014) model has the minimum variance. The 95% confidence limit of the parameter $\lambda$ can be estimated from Figure 3-3. Gardoni et al. (2002), Bolduc et al. (2008) considered logarithmic transformation of the data while fitting models to the Gudavalli database (Gudavalli 1997), Landers-Mueller database (Landers and Mueller 1996), and Kwak database (Kwak 2000) to remove heteroscedasticity. Gudavalli database consisted of 43 laboratory flume experiments, Landers-Mueller database is populated with 305 bridge pier scour measurements from 56 bridges in the United States, and Kwak database entails bridge pier scour measurements from 10 bridge piers from 8 bridges in the state of Texas. If the parameter $\lambda$ is calculated to be 0, then the Box-Cox transformation converts to logarithmic transformation. Referring to Figure 3-3 and Table 3-4, the parameter $\lambda$ is not 0 in any of the five models, indicating that the logarithmic transformation is not the best fit for the five models based on using the clear water laboratory database reported in Benedict and Caldwell (2014). Figure 3-4 shows the residual diagnostic plots for the five models. The fitted models seem to satisfy the homoscedasticity assumption as indicated by equal dispersion of residuals on either side of the axis. Although Wilson (1995) model, Melville (1997) model, HEC-18 (2012) model, and Sheppard et al. (2014) model comprehensively satisfies the normality assumption (linear relationship between standardized residuals and theoretical quantiles), Briaud (2014) model seems to have a right skewness to the fitted data. To check the possibility of other transformation functions, logarithmic transformation, inverse transformation, square root transformation, and inverse square root transformation were performed. However, in all these cases, the right skewness seems to increase, resulting in a clear non-normal error (diagnostic plots are checked). Nevertheless, considering the $R^2$, F statistic, and standard residual error, Box-Cox transformation provided the best-fit model for Briaud (2014) model. Figure 3-5 shows the fitted model to the clear water laboratory datasets. It is important to note that the model equations were not forced to satisfy the condition that the predicted scour depth should be zero, if the measured scour depth is zero. Doing so, the heteroscedasticity problem and non-normal error arises.
Quantification of model error

From the clear water laboratory data, it is observed that the discrepancy between the predicted and the measured scour depth increases as the magnitude of the scour depth increases. This can be owing to the fact that the deterministic models are devised to achieve a conservative estimate of the scour depth. For example, from extensive laboratory investigation, Melville and Sutherland (1988) reported that the ratio of scour depth to pier width ($y_s/b$) rarely exceeds 2.3; however, based on these test results, the upper bound of $y_s/b$ for HEC-18 model was set to be 2.4 (for $F < 0.8$), which is higher than 2.3. Therefore, in general, when the predicted scour depth from a deterministic model is high, “actually” the measured scour depth in the field is low, resulting in a higher discrepancy. Statistically, the described hypothesis is not concrete because the scatter in the predicted versus the measured normalized scour depth plot (Figures 3-1 and 3-5) is significant. However, some statistical measures need to be taken to quantify the dependence of the discrepancy between the predicted and the measured scour depth on the magnitude of the predicted scour depth. Diagnostic plots of the prediction discrepancy-predicted scour depth present non-normal errors. Several transformation functions (logarithmic transformation, exponential transformation, inverse transformation, square root transformation, inverse square root transformation, cubic transformation, inverse cubic transformation) were applied and based on associated residual statistics, Eq. (9) can be suggested.

$$\zeta = \left[ \frac{y_{sp}}{b} - \frac{y_{sm}}{b} \right]^{1/3} = m \frac{y_{sp}}{b} + \vartheta + \epsilon$$  \hspace{1cm} (9)

Where, $m$ and $\vartheta$ are constants, $\epsilon$ is the random error of the model. Figure 3-6 shows the residual diagnostics for the model of Eq. (9). It seems that the model in Eq. (9) satisfies the homoscedasticity assumption and normality assumption. The residuals versus leverage plot suggests that there are no outliers or influential data points. Figure 3-7 shows the relationship between $\zeta$ and predicted scour depths for the five models considered herein. In addition to the model best-fit regression line, 95% confidence limits and 95% prediction limits are also presented. It seems that the discrepancy function, $\zeta$ increases as the magnitude of the predicted normalized scour depth increases. The calculated model parameters for Wilson (1995) model, Melville (1997) model, HEC-18 (2012) model, Briaud (2014) model, and Sheppard et al. (2014) model are presented in Table 3-5. It is apparent that the discrepancy associated with HEC-18 model is most sensitive to the predicted scour depth, indicated by maximum slope ($m = 0.262$), while Sheppard
et al. (2014) model is the least sensitive ($m = 0.055$). For the five models, the standard error ranges from 0.184-0.210, indicating the models have similar variance. $R^2$ value indicates that the Melville (1997), and Sheppard et al. (2014) models are poorly correlated ($R^2 = 0.05$, and 0.01 respectively), whereas the maximum correlation was noticed for Wilson (1995) model ($R^2 = 0.57$). It is apparent from Figures 3-6 (a-c) and 7 (a-c) that a significant number of data points congregate to form a vertical line. Referring to Table 3-3, the predicted scour depth in Wilson (1995) model is a function of upstream flow depth, and projected pier width. Therefore, irrespective of the other parameters (Aspect ratio, $d_{50}$, pier face shape etc.), the predicted scour depth will be the same if the upstream flow depth, and projected pier width are the same. Eventually, site conditions with similar upstream flow depth, and projected pier width presented some discrete congregation of data points when Wilson (1995) model is used (Figures 3-6(a), and 2-7(a)). Referring to Table 3-3, Melville (1997) model depends on multiple factors (refer to the Remarks section of Table 3-3); however, Melville (1997) suggested a limiting value for each of these factors. For example, if $b/d_{50} > 25$, then sediment size factor, $K_d = 1$. Similarly, if all the limiting values for each of the factors are reached, the corresponding normalized predicted scour depth is yielded to be 2.4. So, a single vertical congregation of data points is noticed in Figures 3-6(b) and 2-7(b), representing the site conditions when the limiting values are reached. Arneson et al. (2012) model considered that if the Froude number, $F \leq 0.8$, the maximum normalized predicted scour depth, $\frac{y_{sp}}{b}$ is 2.4; while for $F > 0.8$, the value is 3.0. Therefore, in situations when the Arneson et al. (2012) model expression predicted $\frac{y_{sp}}{b} > 2.4$ although $F \leq 0.8$, the predicted normalized scour depth was restricted to be 2.4. A similar explanation is valid for $F > 0.8$, while the expression suggests $\frac{y_{sp}}{b} > 3$. That is why two vertical congregation of data points are perceived in Figures 3-6(c) and 3-7(c). For Briaud (2014) model, although there are limiting values for different factors being considered in the model, the final expression of $\frac{y_{sp}}{b}$ is dependent upon the difference between $F_{pier}$ and $F_{c(pier)}$, which was different for different data points. In case of Sheppard et al. (2014) model, a limiting value for each of the factors being considered in the model was not imposed. Subsequently, for Briaud (2014) and Sheppard et al. (2014) models, a vertical congregation of the data points was not apparent.

**Bias in the selected deterministic models**

National Research Council (2000) suggested that exceedance and deceedance probabilities provide a useful way to assess the engineering performance of a given model. Figure 3-8 shows
the relationship between probability of deceedance and $\mu_p$, which is the ratio of the reported scour depth in Benedict and Caldwell (2014) to the predicted scour depth from the respective deterministic model (e.g., Wilson model, Melville model, HEC 18 model, Briaud model, and Sheppard et al. model). A value of $\mu_p < 1$ indicates that the predicted scour depth from the deterministic model is greater than the reported scour depth in the database, implying that the deterministic model is providing a conservative estimate of the scour depth. In other words, the probability of obtaining $\mu_p < 1$ is the probability of exceedance when a specific deterministic model is used. In contrast, a value of $\mu_p > 1$ implies an unconservative estimate from the deterministic model. The probability of deceedance magnitude can be estimated by deducting the probability of exceedance (refer to Figure 3-8, probability corresponding to $\mu_p = 1$) from one. Figure 3-8(a) indicates that for Wilson model, the probability of deceedance is 0.38, which in turn indicates that if the deterministic Wilson model is used in estimating scour depth, in 38% occasions the predicted scour depth will be less than the measured scour depth. Similarly, the probability of deceedance for Melville model, HEC-18 model, Briaud model, and Sheppard et al. model are 7%, 17%, 29%, and 7% respectively. It indicates that the Melville, and Sheppard et al. models are the most conservative among the five models considered in the study. In addition, although it is established that HEC-18 model provides conservative estimates (Briaud et al. 2014); the associated probability of deceedance is still 17%. The analyses suggest the necessity of devising a relationship between probability of deceedance with a modification factor which can be incorporated into the deterministic models to achieve a certain probability of exceedance/ deceedance.

**Proposed modification factor**

Kernel density estimation is a non-parametric process to identify the density function from the frequency distribution. Firstly, the distribution of $\mu_{POD}$, which is the ratio of predicted to measured scour depth (modification factor) is quantified. Thereafter, the probability density function is ascertained. Integration of probability density function gives the cumulative density function. Deduction of cumulative density function from one will provide the magnitude of the probability of deceedance (POD) in this case. POD is defined as the probability that the predicted scour depth will be less than the measured ones. The expression presented in Eq. (10) relates expected scour depth corresponding to a POD to the measured scour depth.
\[
\frac{y_{se}}{b} |_{POD} = \mu_{POD} \frac{y_{smed}}{b}
\]  

(10)

Where, \(y_{se}\) is the expected scour depth corresponding to a certain POD, and \(y_{smed}\) is the median scour depth obtained considering \(\zeta = 0\) in Eq. (6). Figure 3-9 shows the relationship between the POD of the measured scour depth and \(\mu_{POD}\). It seems that as the expected POD decreases, the corresponding modification factor increases. The rate of increase of the modification factor increases when the POD is less than 0.2. Figure 3-9 is useful in a sense that the designer can select a suitable modification factor to satisfy a desired POD, with higher POD indicating a larger probability that the predicted scour will be less than the one occurring in the field.

**Database dependence**

It is important to note that the POD chart is developed considering the laboratory clear water scour data reported in Benedict and Caldwell (2014), which may lead to the conclusion that the proposed factors are database dependent. However, the database utilized is the largest collection of pier scour data available in the United States, with a complete set of parameters to allow for comparative analysis of models that require different input parameters. In addition to the database-specific statistics, the approach for the probabilistic framework leading to the development of the easy-to-use factors that render the probabilistic approach readily applicable to practice is described. Therefore, it is possible to extend a deterministic model to assess probability of deceedance, and to develop associated modification factors for a different set of database, using the approach discussed in the study. To this end, and to illustrate the effect of the developed approach on database dependence, a similar analysis using the field clear-water database reported in Benedict and Caldwell (2014) was performed. A statistical outlier identification technique was used to disregard any obvious unreasonable data. Error in the models’ estimation was considered as a parameter to identify the outliers. A second criterion that was applied to identify unreasonable data points was the measured scour depth being equal to the accuracy of the measurement instrument. Through these screening processes, the number of data count was reduced to 405 from 469; these 405 data points were considered for further analyses. The resulting modification factor-POD charts are shown in Figure 3-10. In general, the modification factors in Figure 3-10 (based on field data) are comparatively higher than the modification factors presented in Figure 3-9, where the clear-water laboratory datasets were utilized. The higher modification factor, to some extent, can be attributed to limitations associated with field scour measurements. While the laboratory
measurements provide relatively accurate measurements of downscaled equilibrium scour magnitude, field measurements may not represent an equilibrium scour depth. Even though extreme outliers were eliminated from the dataset, there is a possibility that non-equilibrium scour field measurements remaining in the dataset led to increase the inherent uncertainty of the model and thereby the increase in the proposed modification factor. Therefore, it is recommended that the modification factors developed using the laboratory clear water data (Figure 3-9) be used in design.

**Application of the proposed model**

To utilize the model suggested herein, the following steps need to be considered.

**Step 1:** Estimate the mean approach velocity at the threshold (critical) condition using Eq. (11-13) (Henderson 1966; Melville and Sutherland 1988) and compare with upstream mean flow velocity to identify clear water or live bed condition. The models presented herein are applicable to clear water case only.

\[
V_c = 5.75u_c^* \log \left( \frac{y}{d_{50}} \right)
\]  
(11)

And

\[
u_c^* = K_u (0.0377 + 0.041d_{50}^{1.4})
\]

For 0.1 mm < \(d_{50}\) < 1 mm  
(12)

\[
u_c^* = K_u (0.01d_{50}^{0.5} - 0.0213/d_{50})
\]

For 1 mm < \(d_{50}\) < 100 mm  
(13)

Where, \(u_c^*\) is the critical shear velocity in m/s, \(K_u = 0.3048\), \(d_{50}\) is the median particle size in mm, \(y\) is the flow depth directly upstream of the pier, and \(V_c\) is the mean approach velocity at the threshold condition (parameters need to be put in consistent units in Eq. 11).


**Step 3:** Insert the predicted scour depth in Eq. (6-7) to get the median measured scour depth.

**Step 4:** Use Eq. (9) to assess the median error associated with model scatter, also note the range of measured scour depth.

**Step 5:** Select a POD that needs to be satisfied.
**Step 6:** Use the appropriate POD chart of Figure 3-9 to estimate the corresponding modification factor ($\mu_{POD}$).

**Step 7:** Multiply $\mu_{POD}$ with the median measured scour depth to get the expected scour depth.

For example, a pier of width = 2.5 ft, skew angle = 30 degree, is subjected to a mean flow velocity = 0.97 ft/sec. The upstream flow depth is 10.4 ft, and $d_{50}$ is reported to be 0.18 mm. The pier nose shape is round and has an aspect ratio of 4. From step 1, it is identified that the mean flow velocity at the threshold condition is 1.19 ft/sec, which is greater than the mean flow velocity; thus, clear water condition is assumed. Using HEC-18 (2012) model (Eq. 3), the normalized scour depth is estimated to be 2.05 (corresponding $y_{sp} = 5.1$ ft). Eq. (6-7) reveal the median normalized measured scour depth to be 1.62 (corresponding $y_{sm} = 4.1$ ft). Using Eq. (9) and parameters from Table 3-4, median error is calculated to be ±0.48. Therefore, the measured normalized scour depth may vary in the range of (1.57, 2.53). Two POD were selected to be 0.1 and 0.5, respectively for illustration. From Figure 3-9, the corresponding $\mu_{POD}$ was noted to be 2.6 and 1.3 respectively. The expected normalized scour depths (Eq. 9) corresponding to a POD of 0.1 and 0.5 are 4.2 and 2.1 respectively (corresponding to scour depths of 10.5 ft and 5.3 ft respectively). Therefore, if the predicted scour depth is 10.5 ft then there is only a 10% probability that this value will be exceeded, while if the predicted scour depth is 5.3 ft then there is a 50% probability that this value will be exceeded.

**Summary and Conclusion**

Current practice of bridge pier scour prediction is mostly based on the use of deterministic models. Work herein presents statistical model that extends five deterministic approaches reported in literature to predict the expected scour depth while quantifying inherent model uncertainty in view of measurements reported in database. Statistical analyses extended Wilson (1995) model, Melville (1997) model, HEC-18 (2012) model, Briaud (2014) model, and Sheppard et al. (2014) model by assessing probability of decedance associated with the predicted scour depth. A modification factor (that is applied into the deterministic prediction) is proposed to allow for the use of the deterministic models while quantifying the probability of a computed scour depth being less than or more than a most likely value (per measurements reported in database.) Based on the results obtained herein, the following conclusions are advanced:
1. The scatter associated with the scour prediction using the five deterministic scour prediction models is non-uniform, leading to heteroscedasticity and non-normal errors. A probabilistic model, considering the scatter in the five deterministic models is formulated to address such heteroscedasticity and non-normal errors.

2. Analyses using the clear water measured laboratory data suggest that the discrepancy between the predicted and measured scour depth is a function of the magnitude of the scour depth. Several statistical models are applied. Based on the residual statistics, a prediction discrepancy-predicted scour depth function, referred to as discrepancy function, is proposed.

3. The use of Wilson model, Melville model, HEC 18 model, Briaud model, and Sheppard et al. model in scour depth estimation involves a probability of deceedance of 38%, 7%, 17%, 29%, and 7% respectively. Therefore, Melville and Sheppard et al. models are the most conservative among the five models. The widely used HEC 18 model show a 17% probability of deceedance, which is less than half the value obtained by using the Melville model.

4. A relationship between probability of deceedance of a given measured scour depth and a modification factor (that is applied into the deterministic prediction) is proposed. The modification factor allows for the use of the deterministic models while quantifying the probability of the computed scour depth being less than, or more than a most likely value per measurements reported in the utilized database.

The developed probabilistic model of this study is applicable for estimating the expected scour depth for given hydraulic, geotechnical, and structural parameters. The application of the model is simple (refer to the demonstrated example problem) and the proposed approach can be used by garnering information from publicly available databases.

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Declaration of interests
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
Notations
The following symbols are used in this paper:

\[ b = \text{Pier width} \]
\[ b' = \text{Pier width normal to the flow} \]
\[ d_{50} = \text{Median grain size} \]
\[ E\{y^{(\lambda)}\} = \text{Expected value of } y^{(\lambda)} \]
\[ F = \text{Froude number} \]
\[ F_{c(pier)} = \text{Critical pier Froude number} \]
\[ F_{pier} = \text{Pier Froude number} \]
\[ k = \text{Statistical parameter to be used in Eq. (5-6)} \]
\[ K_d = \text{Sediment size factor} \]
\[ K_G = \text{Channel geometry factor (= 1 for pier)} \]
\[ K_f = \text{Flow intensity factor} \]
\[ K_s = \text{Pier nose shape factor in Melville (1997) model} \]
\[ K_{yb} = \text{Pier depth-size factor} \]
\[ K_1 = \text{Pier nose shape factor in Arneson et al. (2012) model} \]
\[ K_2 = \text{Pier alignment factor in Arneson et al. (2012) model} \]
\[ K_3 = \text{Bed condition factor} \]
\[ K_\theta = \text{Pier alignment factor in Melville (1997) model} \]
\[ m = \text{Statistical parameter to be used in Eq. (8)} \]
\[ n = \text{Total number of observations} \]
\[ u_c^* = \text{Critical shear velocity} \]
\[ v = \text{Approach mean flow velocity} \]
\[ V_c = \text{Mean approach velocity at the threshold condition} \]
\[ V_{1p} = \text{Live-bed peak velocity (Sheppard et al. 2014)} \]
\[ y = \text{Approach flow depth} \]
\[ y_{se} = \text{Expected scour depth corresponding to a certain POD} \]
\[ y_{sm} = \text{Measured scour depth} \]
\[ y_{smed} = \text{Median scour depth} \]
\[ y_{sp} = \text{Predicted scour depth using the deterministic models} \]
\[ y^{(\lambda)} = \text{Column vector of the transformed scour depth observations} \]
\[ \lambda = \text{Statistical parameter to be used in Eq. (5-6)} \]
\[ \varepsilon = \text{Random error of the model in Eq. (8)} \]
\[ \vartheta = \text{Statistical parameter to be used in Eq. (8)} \]
\[ \mu_p = \text{Ratio of the measured scour depth to the predicted scour depth} \]
\[ \mu_{POD} = \text{Proposed modification factor} \]
\[ \sigma^2 = \text{Variance} \]
\[ \zeta = \text{Model error describing the uncertainty of the prediction} \]
Table 3-1. Consideration of pier scour influencing parameters in selected deterministic models.

<table>
<thead>
<tr>
<th>Reference</th>
<th>(y/b)</th>
<th>(b/d_{50})</th>
<th>Pier face shape</th>
<th>Aspect ratio</th>
<th>Skew angle</th>
<th>(v/V_c)</th>
<th>Sediment non-uniformity</th>
<th>Euler/Reynolds number</th>
<th>Temporal rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wilson (1995)</td>
<td>C</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>Melville (1997)</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>Arneson et al. (2012)</td>
<td>C</td>
<td>NC</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>Briaud (2014)</td>
<td>C</td>
<td>NC</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>NC</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>Sheppard et al. (2014)</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
</tr>
</tbody>
</table>

Note: C means considered in the respective model; NC means not considered in the respective model

Table 3-2. Range of some selected variables associated with clear-water laboratory data in Benedict and Caldwell (2014).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pier width normal to the flow, (b') (ft)</th>
<th>Approach flow velocity, (v) (ft/s)</th>
<th>Approach depth, (y) (ft)</th>
<th>Median grain size, (d_{50}) (mm)</th>
<th>Measured pier-scour depth, (y_{sm}) (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>0.052</td>
<td>0.49</td>
<td>0.066</td>
<td>0.22</td>
<td>0.013</td>
</tr>
<tr>
<td>Median</td>
<td>0.246</td>
<td>0.98</td>
<td>0.656</td>
<td>0.96</td>
<td>0.327</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.002</td>
<td>3.96</td>
<td>6.234</td>
<td>7.80</td>
<td>4.626</td>
</tr>
</tbody>
</table>
Table 3-3. Deterministic pier scour model equations considered in the present study.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Model</th>
<th>Equation No.</th>
<th>Remarks</th>
</tr>
</thead>
</table>
| Wilson (1995)   | \( \frac{y_{sp}}{b'} = 0.9 \left( \frac{y}{b'} \right)^{0.4} \)     | (1)          | \( b' \) = projected pier width  
\( y_{sp} \) = predicted scour depth  
\( y \) = flow depth upstream of pier |
| Melville (1997) | \( y_{sp} = K_{yb} K_I K_d K_s K_\theta K_G \)                       | (2)          | \( K_{yb} \) = pier depth-size factor  
\( K_I \) = flow intensity factor  
\( K_d \) = sediment size factor  
\( K_s \) = pier nose shape factor  
\( K_\theta \) = pier alignment factor  
\( K_G \) = channel geometry factor (= 1 for pier) |
| Arneson et al.  | \( \frac{y_{sp}}{b} = 2K_1 K_2 K_3 \left( \frac{y}{b} \right)^{0.35} F^{0.43} \) | (3)          | \( K_1 \) = pier nose shape factor  
\( K_2 \) = pier alignment factor  
\( K_3 \) = bed condition factor  
\( b \) = pier width  
\( F \) = Froude number |
| Briaud (2014)   | \( \frac{y_{sp}}{b'} = 2.2K_{pw} K_{psh} K_{pa} K_{psp} (2.6F_{pier} - F_{c(pier)})^{0.7} \) | (4)          | \( K_{pw} \) = water depth influence factor  
\( K_{psh} \) = pier shape influence factor  
\( K_{pa} \) = aspect ratio influence factor  
\( K_{psp} \) = pier spacing influence factor  
\( F_{pier} \) = pier Froude number  
\( F_{c(pier)} \) = critical pier Froude number |
Table 3-3 (continued).

<table>
<thead>
<tr>
<th>Sheppard et al.</th>
<th>$\frac{y_{sp}}{b'} = 2.5f_{1}f_{2}f_{3}$ for $0.4 \leq \frac{v}{V_c} &lt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$y_{sp} = f_{1}\left[2.2\left(\frac{v}{V_c} - 1\right) + 2.5f_{3}\left(\frac{V_{1p}}{V_c} - \frac{v}{V_c}\right)\right]$</td>
</tr>
<tr>
<td></td>
<td>for $1.0 \leq \frac{v}{V_c} \leq \frac{V_{1p}}{V_c}$</td>
</tr>
<tr>
<td></td>
<td>$f_{1} = tanh\left[\frac{y}{b'}^{0.4}\right]$</td>
</tr>
<tr>
<td></td>
<td>$f_{2} = \left{1 - 1.2\left[ln\left(\frac{v}{V_c}\right)\right]^2\right}$</td>
</tr>
<tr>
<td></td>
<td>$f_{3} = \left[\frac{b'}{d_{50}}\right]^{1.2} + 10.6\left(\frac{b'}{d_{50}}\right)^{-0.13}$</td>
</tr>
</tbody>
</table>

$V_{1p}$ = live-bed peak velocity (Sheppard et al. 2014)
Table 3-4. Calculated parameters in Equations (6-7) for the five models considered in the study.

| Reference           | Parameter | Statistics | | | | |
|---------------------|-----------|------------|------|----------------------------------|--------|-----------------|----------------------|
|                     | $\lambda$ | $k$        | Significance level | Standard error | Median residual error | $F$ statistic | $R^2$           |
| Wilson (1995)       | 0.101     | 0.510      | 0.01 | 0.590                           | -0.061 | 63.48           | 0.16                 |
| Melville (1997)     | 2.909     | 1.538      | 0.01 | 1.07                            | -0.009 | 176.1            | 0.34                 |
| Arneson et al. (2012)| 0.909     | 0.626      | 0.01 | 0.507                           | -0.067 | 130.1            | 0.28                 |
| Briaud (2014)       | 0.061     | 0.524      | 0.01 | 0.429                           | -0.064 | 127.4            | 0.27                 |
| Sheppard et al. (2014)| 2.121     | 1.004      | 0.01 | 0.448                           | -0.021 | 426.5            | 0.56                 |

Table 3-5. Calculated parameters in Equation (9) for the five models considered in the study.

| Reference           | Parameter | Statistics | | | | |
|---------------------|-----------|------------|------|----------------------------------|--------|-----------------|----------------------|
|                     | $m$       | $\theta$   | Significance level | Standard error | Median residual error | $F$ statistic | $R^2$           |
| Wilson (1995)       | 0.202     | 0.428      | 0.01 | 0.210                           | 0.037  | 439.5            | 0.57                 |
| Melville (1997)     | 0.114     | 0.591      | 0.01 | 0.204                           | 0.041  | 16.37            | 0.05                 |
| Arneson et al. (2012)| 0.262     | 0.244      | 0.01 | 0.204                           | 0.014  | 220.4            | 0.39                 |
| Briaud (2014)       | 0.239     | 0.264      | 0.01 | 0.207                           | 0.002  | 388.0            | 0.53                 |
| Sheppard et al. (2014)| 0.055     | 0.635      | 0.01 | 0.184                           | 0.019  | 4.334            | 0.01                 |
Figure 3-1. Predicted-measured normalized scour depth relation for: (a) Wilson (1995) model, (b) Melville (1997) model, (c) HEC-18 (2012) model, (d) Briaud (2014) model, and (e) Sheppard et al. (2014) model using clear water dataset. \( b \) denotes the pier width, and \( y_{sp}, y_{sm} \) indicate the predicted and the measured scour depth respectively.
Figure 3-2. Demonstration of heteroscedasticity and non-normal errors associated with predicted-measured normalized scour depth relation using HEC-18 model.
Figure 3-5. Variation of $f(y_{sp}/b, \lambda)$ with the measured normalized scour depth for: (a) Wilson (1995) model, (b) Melville (1997) model, (c) HEC-18 (2012) model, (d) Briaud (2014) model, and (e) Sheppard et al. (2014) model using clear water dataset.
(a) 

(b) 

(c) 

(d)
Figure 3-9. Probability of deceedance-modification factor relation for different models using clear water laboratory dataset.
Figure 3-10. Probability of deceedance-modification factor relation for different models using clear water field dataset.
Chapter 4: Estimating live bed local scour considering model bias and uncertainty

This chapter was previously published as:

Abstract

To design the foundation system of waterway bridges, Load and Resistance Factor Design guidelines suggest use of deterministic scour depth prediction models. Understanding the inherent bias of deterministic scour depth prediction models will advance development of reliability index-based foundation design regime. Four bridge scour depth prediction models were assessed in terms of two statistical parameters, termed herein Mean Absolute Percentage Error (MAPE), and conservatism, percentage of cases the predicted scour depth exceeded the measured scour depth. Live bed laboratory and field scour depth databases were used in analyses to quantify model scatter by comparatively assessing the computed scour depth versus measured data. For live bed laboratory data, values of MAPE ranged from 23.5% to 59.8%, whereas conservatism ranged from 28.4% to 97.8%. For live bed field data, conservatism varied from 93.3% to 95.1%, while MAPE ranged from 205.6% to 319%. Statistical models were applied to ascertain the biasness of the four deterministic models. Accuracy and conservatism of a given model were consequently adjusted through proposed modification factors. The proposed approach allows for the selection of a suitable modification factor to satisfy a target probability of deceedance or a target conservatism.

Keywords: Bridge scour; LRFD; Probability of exceedance; Live-bed; Conservatism; Statistical models
Introduction

Bridge pier scour is a complex process of interaction among flow, sediments, and structure leading to the mobilization and transport of sediments surrounding the structure. When the flow is obstructed by a bridge pier, the downflow at the pier face causes the bed material to erode, and then transported past the pier by the horseshoe vortex and wake vortices. The scouring process may cause instability of the foundation systems supporting bridges, and other coastal structures, leading to operational disruption and economic losses (Shirole and Holt, 1991; Briaud et al. 1999; Najafzadeh et al. 2013; Azamatullah et al. 2014). Murillo (1987) investigated the bridge failures that occurred during the period 1961-1974 and concluded that 53% of the bridge failures are attributed to the waterway bed scouring. Wardhana and Hadipriono (2003) noted that in 243 out of 503 bridge failures in the United States during 1989-2000, the principal cause was flood-induced scour. Prendergast and Gavin (2014) reported that the 1993 flood that occurred within the Missouri river basin caused the distress of 22 out of the 28 bridges on the waterway. With extreme weather events becoming frequent, scouring of soils supporting bridge piers is poised to be one of the leading causes of bridge damage in near future (Liang et al. 2015).

The American Association of State Highway and Transportation Officials (AASHTO) 2007 guidelines suggested that foundation strength and serviceability limit states need to be assessed while taking into account scour depth corresponding to the 100-year flood event, and the factored foundation resistance after scour should be greater than the factored load determined with the scoured soil removed. Nowak (1999) introduced the concept of Load and Resistance Factor Design (LRFD) approach in which the concept of a reliability estimates the factored loads and resistance offered from the foundation. Scour estimation is crucial since the soil volume removed from the scour prism controls the subsurface stress condition (Lin et al. 2014; Lin and Wu 2019), which eventually impacts the axial and lateral foundation response. The Federal Highway Administration (FHWA) provided guidelines (Arneson et al. 2012) on estimation of design scour depth magnitude from parameters collected from a 100-year flood event. The suggested model is developed deterministically using data from laboratory flume testing on sediments with median grain size ranging from 0.24-0.52 mm. Subsequently, the measured data from full-scale case studies suggested that the FHWA-recommended scour prediction approach is overly conservative (Briaud 2008, Briaud 2014). Briaud (2014) attributed such aspect to the lack of consideration of soil properties and soil resistance to erosion in the deterministic models. Briaud et al. (2014)
demonstrated using Landers and Mueller database (Landers and Mueller 1996, 380 field scour depth data on cohesionless sediments) that the predicted scour depth using FHWA recommended model was 3.26 times the reported measured scour depth. Yao (2013) used 73 large scale laboratory data on cohesive sediments and demonstrated that the predicted scour depth using the FHWA recommended model is 1.55 times the reported measured scour depth. For both new and existing hydraulic structures, estimating the scour depth magnitude with known degree of biasness and uncertainty is therefore critical in developing robust and reliable foundation system design.

It is well established that pier scour depth is influenced by multiple factors; some factors define the structure and geometric scale of the pier flow field, thereby controlling the maximum scour depth (Ettema et al. 2011). Factors, including flow depth to pier width ratio, pier width to median grain size ratio, pier face shape, pier aspect ratio, skew angle of pier are termed as primary factors. On the other hand, parameters including flow intensity (approach mean flow velocity to critical velocity ratio), Euler number or Reynolds number, sediment uniformity, and the temporal rate of evolution characterize the scour depth sensitivities within the geometric scale limit and are thereby known as secondary factors (Ettema et al. 2011; Sheppard et al. 2011). The choice of scour-influencing factors that are included in a given deterministic model leads to estimating different scour depth magnitudes under similar set of hydraulic, structural, and geotechnical attributes. With no prior knowledge of the extent of bias yielded by a given deterministic model, one cannot discern whether these predictions are conservative or unconservative, let alone the degree of conservatism/unconservatism in view of the application realm. Even with the general knowledge that a given deterministic model is conservative, it is important to quantify the extent of such conservatism in terms of percent probability the expected measured scour depth magnitude will be exceeded.

Multiple researchers investigated the probabilistic aspects of bridge pier scour albeit the considered sources of uncertainty varied. In scour depth prediction models, the hydraulic, geotechnical, and structural uncertainties can be decreased through better-quality field characterization and measurement accuracy, or by accumulating more data; it is often the case that scour depth prediction is performed with a data set with a limited coverage of conditions (e.g., Wilson 1995, Arneson et al. 2012). Johnson and Dock (1998) proposed a probabilistic model to estimate the bridge scour depth considering hydraulic and geometrical parameter uncertainty. The
probabilistic model proposed by Briaud et al. (2007) considered hydrologic uncertainty; however, uncertainties associated with input parameters (e.g., geometry of obstacle, soil erodibility, model error) were not considered. The reliability-based pier scour depth model suggested by Briaud et al. (2014) focused on the risk associated with stability of shallow and deep foundations subjected to scour with specific interest on the foundation system. Johnson et al. (2015) suggested scour design factors based on attaining a target reliability index while using HEC-18 (Arneson et al. 2012) and FDOT (2011) scour depth prediction models. However, to the authors’ knowledge, model uncertainty arising from model error, which depends on how well a given scour depth prediction model estimates the scour depth, has not received appropriate attention. A comprehensive database with a wide range of hydraulic, structural, and geotechnical parameters is needed to properly assess model uncertainty. With the availability of Benedict Caldwell (2014) database, model uncertainty may now be quantified in order to comprehend the bias associated with a given deterministic model.

The study herein is focused on assessment of four bridge scour depth prediction models, viz Wilson (1995) model, Melville (1997) model, HEC-18 (2012) model, and Briaud (2014) model. The assessment includes investigation of Conservatism and Accuracy of a given model within the context of comparing the models’ predicted results with measured live-bed laboratory and field scour-depth data. The database reported in Benedict and Caldwell (2014) is used with the exception of few data points for which insufficient site information was reported to be able to comparatively apply the predictive models. Statistical analyses are used to evaluate and quantify the scatter and uncertainty associated with scour depth prediction. A statistical model is proposed to quantify model prediction bias and required parameters to assess the probability a given prediction will exceed a measured value. Modification factors are proposed for application to a given model deterministic prediction to quantify the probability that a computed scour depth will be less than the most likely estimate of scour depth.

**Scour prediction models**

Arneson et al. (2012) model, commonly referred to as HEC-18 model is widely used for estimating equilibrium scour depth. The initial HEC-18 model, known as Richardson et al. (1995) model used a subset of the data collected by Chabert and Engeldinger (1956) and Shen et al. (1966). The subset database consisted of data from laboratory testing on cohesionless sediments. Briaud (2014) model is based on data from 94 large scale laboratory flume testing, as well as dimensional analysis. In summary, these four models were chosen due to the differences in the parameters and group of data upon which each was developed.

**Database considered for analyses**

Benedict and Caldwell (2014) collected scour data from 23 states within the United States, and 6 other countries, e.g., Canada, China, New Zealand, Nigeria, Russia, and Yugoslavia. The database includes a total of 569 laboratory, and 1,858 field scour depth measurements. Data reported in Benedict and Caldwell (2014) can be classified into two categories: dataset from clear water scour and another from live bed scour. If the mean velocity of the approaching flow is such that there is no transport of bed material from upstream of the bridge crossing, the condition is considered *clear water*, while if the transport of bed material occurs from the upstream area, the condition is *live bed*.

The four deterministic models considered herein emphasize different factors that influence pier scour. As such, any missing information related to one or more of the variables that are used to generate input parameters for a given model will impair the scour depth calculation. Therefore, an initial screening was performed on the database, and those sites that lacked any of the parameters necessary to compute scour depth were not further considered. The initial screening yielded 1,319 usable field scour depth measurements, while the laboratory database did not have any missing information. Among the 1,319 field scour depth measurements, data from 469 sites have clear water condition, whereas data from 850 sites have live bed condition. For the 569 laboratory measurements, data from 340 sites have clear water condition, and data from 229 sites have live bed condition. The study herein is focused on the live bed laboratory and field data, while analyses on the clear water datasets have been presented elsewhere in Shahriar et al. (2021b). Although the laboratory tests were focused on cohesionless sediments, 67 field sites contained cohesive sediments. Among the 67 field sites with cohesive sediments, complete site characteristics were reported for 54 field sites (which are not sufficient for a meaningful statistical analysis). The analysis herein was, as such, focused on the laboratory and field tests with
cohesionless sediments. The ranges of the selected variables (pier width, approach flow depth, approach flow velocity, median grain size, and measured scour depth) for the live bed laboratory and field data are presented in Tables 4-2 and 4-3 respectively.

**Data screening**

Under clear water or live bed conditions, the scour depth evolves at a different rate due to equilibrium balances arising from multiple sources (e.g., erosion of sediments from scour hole, sediment inflow and outflow rates etc.). In comparison to the case of clear water condition in which the scour depth develops asymptotically towards the equilibrium magnitudes, the equilibrium depth under live bed conditions is attained at a faster rate, followed by an oscillation of scour depth owing to the passage of bed features past the pier (Ettema et al. 2011). Furthermore, the measured scour depth under live bed condition is dependent on the size and steepness of the bed features per given flow velocities (Chee 1982, Melville and Sutherland 1988). Most scour depth prediction models are developed using test results under steady flow conditions (e.g., Melville 1997, Arneson et al. 2012). The flow condition in a river during a flood event is however unsteady and the discharge changes rapidly. Since the maximum flood discharge during an unsteady flow event does not last long enough for the scour hole to develop to equilibrium, the scour depth obtained using the steady state models are perceived as conservative. Kothyari et al. (1992) developed a model for temporal variation of scour depth in unsteady flow condition by discretizing the hydrograph unsteadiness into different segments so that each discretized segment of flow can be considered as “steady” within a given time period.

In general, laboratory investigations on pier scour evolution processes are performed under well-controlled environment, thereby, relatively accurate representation of downscaled parameters (e.g., flow velocity, scour depth) is possible. However, field observations do come with some limitations (Shahriar et al. 2021b). Depending on the type of tool used to measure scour depth (e.g., survey tool, soundings, Acoustic Doppler Current Profiler, ADCP or fathometer), the accuracy of the measured scour depth will vary (Yu and Yu 2011). In addition, the measured scour depth might not represent the equilibrium scour depth, as in most cases, the maturity of scour depth is unknown. Finally, depending on the method used to obtain the hydraulic properties (e.g., historical information, type of flow models), the accuracy of the scour depth measurements will vary. The collected database parameters might not contain all the descriptions regarding the associated uncertainties. Sheppard et al. (2014) concluded that identification of outliers in the field
data sets might not always be possible owing to the unknown nature of whether the measured scour depth represents equilibrium scour depth values for the specified flow, sediment, and structure conditions. Therefore, statistical measures, and when applicable, site-specific observations are necessary to identify outliers.

Error in models’ estimation, as expressed in Equation (1) is a parameter that was considered to quantify the adequacy of a given deterministic model.

\[
\text{Error, } \% = \frac{y_{sp} - y_{sm}}{y_{sm}} \times 100\%
\]  

(1)

Where, \(y_{sp}\) and \(y_{sm}\) are the predicted and measured scour depths respectively. Figure 4-1 shows the distribution of error for the four deterministic models (corresponding to the field data) considered herein, on the basis of prediction associated with database parameters. It is apparent that the error varied within a wide range (-100% to 4771%) for the four models considered herein. Furthermore, the pattern of over-prediction is very similar in some regions, with the peak errors, corresponding to each model, occurring usually at the same test sites. Although for the four deterministic models, different parameters (e.g., \(y/b'\), \(b'/d_{50}\), pier face shape, skew angle, \(V/V_{c}\), Euler number, sediment uniformity) were utilized in their development, the over-prediction trend appeared similar. While the scour depth prediction models are developed for steady state equilibrium conditions, Kothyari et al. (1992), and Arneson et al. (2012) reported that application of such models to estimate the scour depth for an unsteady flow condition will result in even deeper scour depth than the actual scour. Application of unsteady flow model (e.g., Kothyari et al. 1992) is restrictive in the study herein owing to the fact that the utilized database (Benedict and Caldwell 2014) does not include the flow history at a given site in association with the measured scour depth data in the database. The high peaks of the error magnitude increase the average error for the deterministic model, which might not be a representative performance of a given model. Subsequently, a statistical outlier identification technique was adopted.

The data screening process adopted herein was used to eliminate values most likely not representing equilibrium condition (albeit the screening process does not assure that all field scour depth data will represent equilibrium condition). Figure 4-2 presents the error boxplot for the four models considered herein. A boxplot is a standardized process to display the dataset based on five statistical parameters, e.g., minimum, maximum, median, first and third quartile. The distance
between the first and third quartile is referred to as interquartile range (IQR). Any error observation farther than 1.5 IQR from the closest fourth (first or third quartile) is an outlier. As evident from Figure 4-2, the four models have positive outliers. Wilson (1995), Melville (1997), HEC 18 (2012), and Briand (2014) models show 48, 54, 41, and 54 outliers respectively. These outliers were eliminated from further analyses.

In the utilized database, there were some data points for which the magnitude of the reported scour depth is equal to the accuracy tolerance of the measurement instrument (e.g., survey tool, soundings, ADCP or fathometer). The accuracy tolerance of the survey tools, soundings, ADCP and fathometer varied from 0.03-0.08 m, 0.03-0.91 m, 0.61-0.91 m, and 0.061-0.91 m respectively. Data points with scour depth estimate equaling the measurement accuracy tolerance were identified and screened out. By adopting this 2-step screening process, the number of field live-bed scour depth data count was reduced to 704, which was considered for further analyses.

To ascertain if data points with some specific trend had been excluded, the relationship between error magnitude and $b'$, $y/b'$, $V/V_c$, $b'/d_{50}$ was explored. Figure 4-3 shows the typical relationship when HEC 18 model is used. In this case, 41 data points were screened out through the outlier identification technique of Figure 4-2. Data in Figure 4-3(a) suggest that most of the excluded data are associated with an $b'<5$ m. In the database being considered, 824 data points out of the total 850 data points (97%) had an $b'<5$ m, thereby suggesting that the excluded data points were dispersed within the range of $b'$investigated. Melville and Coleman (2000) suggested that pier scour flow field can be classified into three categories, i) narrow pier ($y/b'>1.4$), (ii) intermediate pier ($0.2 \leq y/b' \leq 1.4$), and (iii) wide pier ($y/b' <0.2$). For a narrow pier, an interacting and unsteady set of flow features transports the sediments away from the pier foundation. The flow approaching the pier decelerates, impinges against the pier and deflects both down and up of the pier’s face. Flow contraction occurs as it passes around the pier’s sides, resulting in increase of local flow velocity and bed shear stress around the pier’s sides. The increase of local flow velocity and bed shear stress causes the inception of scour at the sides of the pier, followed by a development of hole fully around the pier owing to the strengthening of the down flow and horseshoe vortices. For an intermediate pier, the main flow field features are similar to the flow features of narrow pier, however, the features alter due to the reductions of flow depth and or increase in pier width. The lower $y/b'$ (compared to the narrow pier) disrupt the formation
of the flow features which causes the reduction in bed material erosion surrounding the foundation. For a wide pier, at a given flow depth, greater pier width increases flow blockage resulting in a weak development of the down flow at the pier face. The blockage due to greater pier width modifies the lateral distribution of approach flow, causing the deepest scour depth to occur at the pier flanks.

It is established that the laboratory and field data points corresponding to wide piers is scarce (Ettema et al. 2011). In the present study as well, among the 850 field cases, only 10 sites had a wide pier condition. Figure 4-3(b) suggests that among the 41 data points excluded, only one had a wide pier condition, while the rest 40 sites had \( y/b' > 0.2 \). Figure 4-3(c) shows that the excluded data points had a \( V/V_c < 6 \), while among the 850 live bed data points, 814 data points (96%) had a \( V/V_c < 6 \). Therefore, any dependence of the excluded data points on \( V/V_c \) was not discerned. Lee and Sturm (2009) experimentally observed that \( b'/d_{50} \leq 25 \) can impede the scour progression. Ettema et al. (2011) observed that if the \( b'/d_{50} < 8 \), individual particles are so large relative to the pier that scour occurs due to erosion at pier sides and yields a reduced scour depth magnitude. However, the considered database entailed only one site with \( b'/d_{50} < 8 \), and the excluded data points had \( b/d_{50} \) ranging from 32 to 39200. Therefore, it can be concluded that the excessive over-prediction (based on the screening process) was not related to any site-specific attribute, rather can be related to the fact that the reported scour depths might not be the equilibrium scour depth magnitudes.

**Accuracy and conservatism analyses**

A perfectly conservative scour depth prediction model would be the one that never yields a predicted scour depth that is less than the measured scour depth in the field, while a perfectly accurate scour depth prediction model would be one that yields a predicted scour depth that is equal to the measured scour depth in the field. Tan and Dunca (1991) adopted similar logic while assessing the performance of the deterministic models to estimate the settlement of footings on sand.

In this study, *accuracy* was measured in terms of Mean Absolute Percentage Error (MAPE), which can be expressed as in Equation (2).
\[
        MAPE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{y_{sp_i} - y_{sm_i}}{y_{sm_i}} \right|
        \tag{2}
    \]

Where, \( n \) is the total number of data points, \( y_{sp_i} \) and \( y_{sm_i} \) indicate the predicted and the measured scour depth for the \( i \)th data point, respectively. Level of conservatism was defined as the ratio of the number of cases the calculated scour depth is more than the measured scour depth, expressed as percentage of the total number of data sets. It is important to note that MAPE, rather than the true error magnitude (obtained from Equation 1) is considered to represent the accuracy of the model, while the “sign” of the error magnitude (which identifies whether a model estimate is conservative or unconservative) is taken into consideration in conservatism calculation.

**Analyses with laboratory data**

The results of the MAPE, and level of conservatism of the four deterministic models are presented in Figure 4-4(a). Values of MAPE ranged from 23.5% to 59.8%, whereas level of conservatism ranged from 28.4% to 97.8%. Wilson (1995) model provided the least error; however, the model was also least conservative (28.4%). Briaud (2014) model under-predicted the measured scour depth in 18.3% of the number of data points. Melville (1997) model provided the most conservative scour depth, under-predicting the measured scour depth in only 2.2% occasions. The associated MAPE for Melville (1997) model is 59.8%. HEC 18 (2012) model provided an error of 53.1%, however presented a level of conservatism of 95.2%, which is more than the conservatism levels of Wilson (1995), and Briaud (2014) models.

The accuracy and level of conservatism of a given model can be adjusted by multiplying the scour depth computed from the deterministic model by a factor. Figure 4-5(a) demonstrates the relationship between MAPE and conservatism when a multiplication factor is applied for the four models considered herein. When the factor is applied to the predicted scour depth, \( y_{sp} \) from any of the four deterministic models, refer to Equation (1), the difference between \( y_{sp} \) and \( y_{sm} \) increases, while the denominator \( y_{sm} \) remains the same, leading to an increase in the MAPE magnitude. However, at the same time, a deterministic scour prediction, which was unconservative initially (without application of a modification factor), may become conservative, leading to an increase in the level of conservatism of the estimation (in the sense of the scour depth magnitude estimated during the design phase is not exceeded).
For example, Briaud (2014) model has a MAPE and level of conservatism of 47.6% and 81.7% respectively (refer to Figure 4-4a). If the values of calculated scour depth for all the 229 cases are multiplied by a factor of 1.4, the MAPE increases to 101.1%, while the level of conservatism increases to 99.1%. Multipliers ranging from 1 to 2 were applied and the corresponding MAPE and level of conservatism are calculated and presented. It is apparent that there is a tradeoff between MAPE and level of conservatism. Improvement of level of conservatism through application of the modification factors involves an increase in the overall MAPE of a model. For Melville (1997), HEC 18 (2012) and Briaud (2014) models, using a multiplying factor of 1.4 applied to the calculated scour depth leads to a level of conservatism of >99%. For Wilson (1995) model, application of a multiplying factor of 2.0 leads to a level of conservatism of 98.3% from 28.4% (with no multiplication factor applied). The MAPE increases from 23.5% to 85.2% in the process. Selection of the multiplier is dependent on the desired level of accuracy and level of conservatism. For example, if it desired to select a model that provides 98% conservative estimate, then any of the four deterministic models in this study can be selected; albeit each prediction should be multiplied with different factors. If Wilson (1995), Melville (1997), HEC 18 (2012), or Briaud (2014) model is selected, the prediction from the respective deterministic model should be multiplied by a factor 1.70, 1.0, 1.18, and 1.3 respectively to achieve 98% level of conservatism (refer to Figure 4-5(a), the abscissa corresponding to 98% level of conservatism ordinate). The corresponding MAPE from these models are 59%, 59%, 75% and 98% respectively (refer to Figure 4-5(a), the secondary axis ordinates corresponding to the multiplying factors). Now, if it is also desired to select a model that provides the minimum MAPE (with 98% level of conservatism), then Wilson (1995), and Melville (1997) models are the potential candidate options to be selected.

**Analyses with field data**

When the field scour depth measurement data were considered, the level of conservatism-MAPE relation changed. The variation of level of conservatism and MAPE is depicted in Figure 4-4(b). In contrast to the obtained relationship based on the laboratory measurements, the use of field data yielded a higher MAPE, which is understandable in view of the limitations of the field measurements not necessarily being at equilibrium. Level of conservatism varied from 93.6% to 95.3%, while MAPE ranged from 202.9% to 313.2%. Wilson (1995) model presented the most conservative estimate (95.3%), at the same time provided the least MAPE of 202.9%. Melville (1997) model provided the highest MAPE (313.2%); however, the associated level of conservatism
was 94.6%. Although HEC 18 (2012) model showed the least conservative estimate among the four models being considered, its level of conservatism is still 93.6%. It is important to note that when the laboratory data were considered, Wilson (1995) model presented a level of conservatism of only 28.4%, while for field measurement, associated level of conservatism increased to 95.3%. Wilson (1995) model considers \( y/b' \) as the only parameter influencing the scour depth. It is congruent with the approach Wilson (1995) followed, who used scour depths from 22 bridge sites to develop the model. Therefore, while the use of the \( y/b' \) captures the field conditions well, it is possible it cannot represent the laboratory hydraulic and geometric conditions properly.

A similar technique, as described in the previous section (Analyses with laboratory data) was adopted to generate the relationship among the level of conservatism, MAPE, and the multiplying factor. Figure 4-5(b) presents the variation of level of conservatism and MAPE when a multiplying factor is applied. For all the four models considered in the study, application of a multiplying factor of 1.6 ensured a level of conservatism > 99%. The associated MAPE for Wilson (1995), Melville (1997), HEC 18 (2012) and Briaud (2014) models are 382%, 558%, 382%, and 438% respectively.

Statistical model

Figure 4-6 shows the relationship between the predicted and the measured scour depths when laboratory and field scour depth measurements are considered using the four deterministic models investigated herein. In general, it is apparent that the deterministic models have the tendency to over-predict the measured scour depth. The use of Wilson (1995) model yielded the maximum number of under-predicted scour depth cases when laboratory data are considered (Figure 4-6a), which agrees with the low level of conservatism shown in Figure 4-4(a). Observation of Figure 4-6 reveals that the scatter of the data is not uniform, which leads to heteroscedasticity (Stone 1996). Figure 4-7(a) presents the residuals versus fitted value plot for Briaud (2014) model. Residual in this case is defined as the difference between measured and predicted normalized scour depth obtained from the best-fit curve, which is a linear expression between the measured and the predicted values. The disproportionate scatter in positive and negative sides of residuals suggest that the residuals have heteroscedasticity, with such observation being prominent for field measurements\(^1\). Figure 4-7(b) shows the standardized residuals versus theoretical quantile plot for

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\(^1\) See Figure A1 (e-h)
Briaud (2014) model. It appears that the dataset has long tailed distribution when Briaud (2014) model is used. When all four models are considered, the dataset presented left-skewness and long-tailed distribution\(^2\); indicating that a linear relationship between the predicted and the measured normalized scour depth is not adequate. Therefore, a statistically robust model needs to be selected that can address the heteroscedasticity and non-normal error. Subsequently, the method suggested by Box and Cox (1964) was adopted for this purpose. The probabilistic model was formulated as in Equation 3(a-c).

\[
\frac{(y_{sp}/b)^\lambda - 1}{\lambda} = k \frac{y_{sm}}{b} + \vartheta + \xi
\]  
\[
\frac{(y_{sp}/b)^\lambda - 1}{\lambda} = y^{(\lambda)}
\]

Where, \(y^{(\lambda)}\) represents the column vector of transformed observations and \(k\), \(\vartheta\), and \(\lambda\) are constant terms depending on the deterministic model being considered, \(\xi\) is the residual error term.

The likelihood in relation to the original observation \((y)\) is obtained by multiplying the normal density with the Jacobian of the transformation, \(J(\lambda; y)\), and can be expressed as in Equation (3c).

\[
\frac{1}{(2\pi)^{n/2} \sigma^n} \exp \left\{ -\frac{(y^{(\lambda)} - E\{y^{(\lambda)}\})^T (y^{(\lambda)} - E\{y^{(\lambda)}\})}{2\sigma^2} \right\} J(\lambda; y)
\]

Where, \(n\) is the total number of observations, \(E\{y^{(\lambda)}\}\) is the expected value of \(y^{(\lambda)}\), and \(\sigma^2\) is the variance of the transformed observations. The large sample maximum likelihood theory is applied to Equation (3c) to obtain the point estimate of the parameter \(\lambda\). Readers are referred to Box and Cox (1964) for the full derivation of the steps considered to estimate \(\lambda\).

After obtaining the most likelihood estimate of \(\lambda\), Equation (3a) was fitted to the predicted \((y_{sp})\) and the measured \((y_{sm})\) scour depth observations. The computed parameters \((k\), \(\vartheta\), and \(\lambda\)) for the four deterministic models are presented in Tables 4-4 and 4-5 for laboratory data and field data, respectively. The \(R^2\) of the fitted models vary from 0.20 for Briaud (2014) model to 0.46 for HEC 18 (2012) model when laboratory data are considered; while for the field data-based statistics, \(R^2\) vary from 0.12 for Melville (1997) model to 0.33 for Wilson (1995) model. Interpretation of

\(^2\) For left-skewness see Figure A1 (e-g), for long-tailed distribution see Figure A1 (a-d, h)
standard error suggested that Briaud (2014) model had the least variance, while Melville (1997) model presented the highest variance when laboratory data were considered. For field scour depth measurements, Briaud (2014) model had the least variance, while Melville (1997) model presented the highest variance. Shahriar et al. (2021b) observed that Box-Cox transformation presented the best response while fitting models to the clear-water laboratory scour depth measurements. When the live bed laboratory and field measurements are utilized, the trends based on data from the four models considered herein seem to agree with the observation reported in Shahriar et al. (2021b).

The distribution of parameter \( \lambda \) is obtained from Figure 4-8. The 95% confidence bound of the \( \lambda \) parameter is also shown on the same plot. Application of logarithmic transformation for Briaud (2014) model agrees with the observation of Gardoni et al. (2002), and Bolduc et al. (2008), who observed that a logarithmic transformation best depict the scour depth measurements reported in Landers-Mueller, Gudavalli, and Kwak databases (Landers and Mueller 1996; Gudavalli 1997; Kwak 2000). However, for the live bed laboratory and field measurements considered herein, application of logarithmic transformation was not effective in reducing heteroscedasticity and non-normal errors. Referring to Figure 4-8, if the parameter \( \lambda \) is 0, then the Box-Cox transformation will be converted to the logarithmic transformation form; however, in neither of the cases (laboratory and field), a \( \lambda = 0 \) was noticed, suggesting logarithmic transformation is not suitable for the large database considered herein.

The residual diagnostics of the fitted statistical model suggest that the non-normal error has been addressed for most cases except for results from Melville (1997) and HEC 18 (2012) models when applied to the field scour depth measurement data\(^3\). Although HEC 18 (2012) model presented a prevalence of short-tailed distribution, the effect of which is minimal in statistical analyses (Devore 2012), the left skewness of Melville (1997) model data could not be reduced significantly.

Figure 4-9 shows the dispersion of the measured normalized scour depth around the fitted model using the parameters listed in Tables 4-4 and 4-5. The data points are well dispersed on both sides of the fitted model. For Melville (1997) and HEC 18 (2012) models, for a particular transformed response, a congregation of data points is observed. This is indicative of sites with

\(^3\) See Figure A2
conditions that limit the applicability of the respective models. Melville (1997) model yielded the highest residual error (for both laboratory and field cases) among the four models considered herein. Figure 4-7(c-d) suggests that application of the proposed model is effective in reducing heteroscedasticity and non-normal error.

**Inherent probability of deceedance**

A Probability of Deceedance (POD) is the probability that the predicted scour depth will be less than the measured scour depth and can be expressed as in Equation (4). Figures 4-10 shows the variation of probability of occurrence with the ratio of the reported scour depth in the considered database to the predicted scour depth from the deterministic models for both the laboratory and field cases ($\chi_p$). Therefore, $\chi_p > 1$ represents an unconservative scour depth estimate, whereas $\chi_p < 1$ represents a conservative scour depth estimate.

$$POD = P(y_{sp} \leq y_{sm})$$ (4)

Where, $P(\cdot)$ symbolizes probability. Referring to Figure 4-10, POD is the probability magnitude corresponding to $\chi_p = 1$. As shown in Figure 4-10(a), Wilson (1995) model provides the maximum POD (71%), while Melville (1997) model provides the minimum POD (3%). However, for field database-based probability estimates (Figure 4-10b), the difference among the PODs for different models studied are not clearly discerned. The PODs corresponding to Wilson, Melville, HEC 18, and Briaud models are 5%, 5%, 7% and 6% respectively. The POD values for field database are also reflective of the observation made previously based on *level of conservatism*-MAPE response (Figure 4-4b). Nevertheless, from the probability estimates obtained using laboratory and field data, it is apparent that although in literature the deterministic models are considered overly conservative, at a minimum a 5% POD is associated with all of the deterministic models investigated in the study.

**Proposed modification factor**

The frequency distribution of the proposed modification factor, $\chi_{POD}$ for the four models considering the laboratory database is presented in Figure 4-11. Observation of Figure 4-11 suggests that the distribution of $\chi_{POD}$ is right-skewed. The skewness of Wilson (1995), Melville (1997), HEC 18 (2012), and Briaud (2014) models are 2.25, 1.15, 1.61, and 1.94 respectively. To develop a relationship between POD and $\chi_{POD}$, a non-parametric process, termed as Kernel density estimation was adopted. The process allows development of the population probability density
function using the sample frequency distribution demonstrated in Figure 4-11. Thereafter, cumulative density function is obtained through integration of probability density function. POD is calculated by deducting the cumulative density function from one. For both the laboratory and field dataset, similar process was considered and the developed relationship between the POD and \( \chi_{POD} \) is presented in Figure 4-12. The design scour depth corresponding to a POD, \( y_{sd} \) can be obtained by multiplying \( y_{smed} \) by the \( \chi_{POD} \).

Figure 4-12 suggests that the modification factor increases significantly at POD< 0.1 based on applying the four deterministic models to both laboratory and field data. For comparison, the modification factors proposed in Shahriar et al. (2021b) are also presented in Figure 4-12. Shahriar et al. (2021b) used clear water scour depth measurements reported in Benedict and Caldwell (2014). Figure 4-12(a) suggests that the factors proposed for clear water laboratory scour depth are higher than the live bed laboratory scour depth throughout the POD range. The reason can be attributed to the fact that the live bed estimates of the scour depth using the models proposed herein are inherently more conservative than the clear water scour depth estimates obtained from Shahriar et al. (2021b) model. As a result, to attain a similar POD, the scour depth estimates from the clear water models need to be multiplied with a higher factor than the one corresponding to the live-bed case. In addition, for the live bed analyses using laboratory dataset, a \( \chi_{POD} \) of 1.85 ensure a POD\( \leq 0.1 \)for all the four models, while to ensure POD\( \leq 0.1 \), a factor ranging from 2.2-2.6 is required for the clear water analyses-based statistics suggested in Shahriar et al. (2021b).

Interpretation of Figure 4-12(b) does not suggest a discernible difference between the clear water and live bed \( \chi_{POD} \) except for HEC 18 (2012) and Briaud (2014) model in the POD range of 0.15-0.65. The modification factors reported herein for field measurement-based statistics are comparatively higher than the modification factors proposed for laboratory measurement-based statistics. This could be owing to the non-equilibrium scour depth reported for the field measurements, whereas laboratory measurements represent relatively accurate estimation of the downscaled scour depth. In the study herein, field scour depth measurements were used in analyses after two-step screening process, followed by assessing the outliers based on site specific attributes. However, in the database considered herein, all the site-specific attributes (e.g., particle size distribution, prevailing steady or unsteady condition etc.) were not reported, as such, sources of uncertainties remain to be quantified. Thus, it can be concluded that it was not possible to
remove all the sites with non-equilibrium scour depth measurements. Therefore, it will be prudent to adopt the proposed modification factors developed using laboratory data-based statistics.

**Example application**

Two approaches have been explored to estimate the design scour depth, either based on the target *level of conservatism* or based on the target probability of exceedance. An example illustrating the POD approach is presented, followed by applying the *level of conservatism* approach to the same example.

A circular pier of diameter 0.61 m is subjected to an approach mean flow velocity and flow depth of 0.61 m/sec and 1.83 m respectively. The median grain size, particle size distribution of the bed material, hydraulic characteristics, and fluid viscosity are such that live bed condition with a dune bed configuration is prevalent. The use of HEC 18 (2012) model leads to a normalized scour depth of 1.40 (corresponding to a scour depth of 0.85 m). Using Equation (3a), by adopting appropriate parameters from Table 4-4, reveals a median normalized scour depth of 0.48. If the target POD is considered equal to 10%, then the corresponding $\chi_{POD}$ from Figure 4-12(a) is obtained as 1.42. The design normalized scour depth can then be obtained as 0.57 (corresponding to scour depths of 0.35 m). Therefore, if the design scour depth is 0.35 m, there is a 10% possibility that this scour depth magnitude will be exceeded.

If the objective, while estimating the scour depth, is to obtain a *level of conservatism* of 98%, then the accuracy-conservatism approach described herein can be considered. Referring to Figure 4-4a, the predicted scour depth from HEC 18 (2012) model provides a *level of conservatism* of 95.2%, with an associated MAPE of 53.1%. Referring to Figure 4-5a next, the multiplying factor corresponding to 98% *level of conservatism* is 1.13, and the associated MAPE is 70%. Therefore, the normalized scour depth of 1.40 (corresponding to a scour depth of 0.85 m) in the example should be multiplied by a factor of 1.13 to obtain a normalized scour depth that will have a *level of conservatism* of 98%. The design normalized scour depth, thus, would be 1.58 (corresponding to a scour depth of 0.96 m).

**Summary and Conclusions**

Four bridge scour depth prediction models were assessed in terms of two statistical parameters, termed Mean Absolute Percentage Error (MAPE), and *level of conservatism*. Live bed laboratory scour depth measurements (Data points: 229), and live bed field scour depth measurements (Data
points: 704) were utilized herein. Level of conservatism is defined as the extent to which prediction from a given model is unlikely to yield a scour depth that is less than the expected scour depth in the field. Furthermore, statistical models were used to propose factors to adjust deterministic scour depth prediction. The statistical parameters are presented, and applicability of the proposed factors is described. Based on the results presented herein, the following conclusions are drawn:

1. Deterministic models used for bridge pier scour depth do not account for the inherent model errors. The scatter associated with the scour depth prediction produces heteroscedasticity, and non-normal errors. As shown, statistical measures were developed to reduce the error associated with data distribution.

2. For live bed laboratory data, values of MAPE ranged from 23.5% to 59.8%, whereas level of conservatism ranged from 28.4% to 97.8%. For live bed field data, level of conservatism varied from 93.6% to 95.3%, while MAPE ranged from 202.9% to 313.2%.

3. Considering the live bed laboratory data, Melville (1997) model’s estimates provided the most conservative scour depth estimate, whereas Wilson’s (1995) model yielded the least conservative scour depth estimate. When the live bed field data are considered, Wilson (1995) model provided the most conservative scour depth estimate, while HEC 18 (2012) model provided the least conservative scour depth estimate.

4. With regards to MAPE, and considering the live bed laboratory data, the Wilson (1995) model yielded the least percent error magnitude, and the Melville (1997) model yielded the highest percent error magnitude. The same trend of models’ performance was observed when the live bed field data are used in the analyses.

5. The accuracy and level of conservatism of a specific model are adjusted by multiplying the scour depth computed using a given deterministic model by proposed modification factors. These modification factors will allow for attaining certain level of conservatism for scour depth prediction from the four models considered herein.

6. Utilizing both laboratory and field databases, to quantify the probability that a computed scour depth will be less than the most likely estimate of scour depth, modification factors were proposed. The proposed modification factors corresponding to field data measurements were higher than that of the laboratory data measurements owing to the non-equilibrium data points remaining in the field data that led to increased uncertainty.
The proposed approach allows for the practice of selecting a suitable modification factor to satisfy a target probability of deceedance/level of conservatism depending on the criticality of the scour estimates with respect to the stability of the foundation systems.

**Data availability statement**
Data generated or analyzed during this study are provided in full within the published article.

**Competing interests statement**
The authors declare there are no competing interests.

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Table 4-1. Deterministic pier scour models considered in this study (Shahriar et al. 2021a).

<table>
<thead>
<tr>
<th>Reference</th>
<th>Model</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wilson (1995)</td>
<td>( \frac{y_{sp}}{b'} = 0.9 \left( \frac{y}{b} \right)^{0.4} )</td>
<td>( b' = ) pier width normal to the flow ( y_{sp} = ) predicted scour depth ( y = ) flow depth upstream of pier</td>
</tr>
<tr>
<td>Melville (1997)</td>
<td>( y_{sp} = K_{yb}K_{I}K_{a}K_{s}K_{\theta}K_{G} )</td>
<td>( K_{yb} = ) pier depth-size factor ( K_{I} = ) flow intensity factor ( K_{a} = ) sediment size factor ( K_{s} = ) pier nose shape factor ( K_{\theta} = ) pier alignment factor ( K_{G} = ) channel geometry factor (= 1 for pier)</td>
</tr>
<tr>
<td>HEC 18 (Arneson et al. 2012)</td>
<td>( \frac{y_{sp}}{b} = 2K_{1}K_{2}K_{3}\left( \frac{y}{b} \right)^{0.35}F^{0.43} )</td>
<td>( K_{1} = ) pier nose shape factor ( K_{2} = ) pier alignment factor ( K_{3} = ) bed condition factor ( b = ) pier width ( F = ) Froude number</td>
</tr>
<tr>
<td>Briaud (2014)</td>
<td>( \frac{y_{sp}}{b'} = 2.2K_{pw}K_{psh}K_{pa}K_{p} (2.6F_{pier} - F_{c(pier)})^{0.7} )</td>
<td>( K_{pw} = ) water depth influence factor ( K_{psh} = ) pier shape influence factor ( K_{pa} = ) aspect ratio influence factor ( K_{p} = ) pier spacing influence factor ( F_{pier} = ) pier Froude number ( F_{c(pier)} = ) critical pier Froude number</td>
</tr>
</tbody>
</table>
Table 4-2. Range of some selected variables associated with live bed laboratory data in the selected database.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( b ) (m)</th>
<th>( b' ) (m)</th>
<th>( V ) (m/s)</th>
<th>( y ) (m)</th>
<th>( d_{50} ) (mm)</th>
<th>( y_{sm} ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>0.029</td>
<td>0.029</td>
<td>0.23</td>
<td>0.04</td>
<td>0.22</td>
<td>0.015</td>
</tr>
<tr>
<td>Median</td>
<td>0.051</td>
<td>0.051</td>
<td>0.62</td>
<td>0.17</td>
<td>0.6</td>
<td>0.089</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.914</td>
<td>0.914</td>
<td>2.16</td>
<td>1.22</td>
<td>3.2</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Table 4-3. Range of some selected variables associated with live bed field data in the selected database.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( b ) (m)</th>
<th>( b' ) (m)</th>
<th>( V ) (m/s)</th>
<th>( y ) (m)</th>
<th>( d_{50} ) (mm)</th>
<th>( y_{sm} ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>0.244</td>
<td>0.244</td>
<td>0.37</td>
<td>0.15</td>
<td>0.088</td>
<td>0.03</td>
</tr>
<tr>
<td>Median</td>
<td>0.914</td>
<td>1.304</td>
<td>1.28</td>
<td>4.14</td>
<td>0.49</td>
<td>0.82</td>
</tr>
<tr>
<td>Maximum</td>
<td>11.57</td>
<td>11.57</td>
<td>4.57</td>
<td>22.51</td>
<td>73.00</td>
<td>7.80</td>
</tr>
</tbody>
</table>

Table 4-4. Calculated parameters in Equations (3a-b) for the four models considered in the study (Laboratory data).

<table>
<thead>
<tr>
<th>Reference</th>
<th>( \lambda )</th>
<th>( k )</th>
<th>( \vartheta )</th>
<th>Significance level</th>
<th>Standard error</th>
<th>Median residual error</th>
<th>( F ) statistic</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wilson (1995)</td>
<td>0.67</td>
<td>0.432</td>
<td>-0.36</td>
<td>0.01</td>
<td>0.31</td>
<td>-0.02</td>
<td>78.42</td>
<td>0.26</td>
</tr>
<tr>
<td>Melville (1997)</td>
<td>4.0</td>
<td>2.78</td>
<td>2.52</td>
<td>0.01</td>
<td>1.86</td>
<td>0.57</td>
<td>90.7</td>
<td>0.28</td>
</tr>
<tr>
<td>HEC 18 (2012)</td>
<td>2.12</td>
<td>1.69</td>
<td>-0.32</td>
<td>0.01</td>
<td>0.77</td>
<td>-0.038</td>
<td>194.3</td>
<td>0.46</td>
</tr>
<tr>
<td>Briaud (2014)</td>
<td>0.61</td>
<td>0.58</td>
<td>0</td>
<td>0.01</td>
<td>0.48</td>
<td>-0.047</td>
<td>58.25</td>
<td>0.20</td>
</tr>
</tbody>
</table>
Table 4-5. Calculated parameters in Equations (3a-b) for the four models considered in the study (Field data).

<table>
<thead>
<tr>
<th>Reference</th>
<th>Parameter</th>
<th>Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda$</td>
<td>$k$</td>
</tr>
<tr>
<td>Wilson (1995)</td>
<td>0.48</td>
<td>0.564</td>
</tr>
<tr>
<td>Melville (1997)</td>
<td>4.0</td>
<td>1.17</td>
</tr>
<tr>
<td>HEC 18 (2012)</td>
<td>1.69</td>
<td>0.55</td>
</tr>
<tr>
<td>Briaud (2014)</td>
<td>0.30</td>
<td>0.33</td>
</tr>
</tbody>
</table>
Figure 4-1: Error distribution considering live bed field datasets (Data count: 782) for (a) Wilson (1995), (b) Melville (1997), (c) HEC 18 (2012), and (d) Briaud (2014) models.

Figure 4-2: Error boxplot for the four deterministic models considered (Dataset: Live bed field, Data count: 782).
Figure 4-3: For the excluded data points, relationship between error magnitude and (a) Effective pier width, (b) Upstream flow depth to effective pier width ratio, (c) Upstream mean flow velocity to the critical velocity, and (d) Effective pier width to median grain size ratio when HEC 18 (2012) model is used (Data count: 41).
Figure 4-4: Relationship between level of conservatism and mean absolute percentage error for the four models considered (a) Dataset: live bed laboratory data (Data count: 229), (b) Dataset: live bed field data (Data count: 704).
Figure 4-5: Effect of the multiplication factor on the level of conservatism and mean absolute percentage error for the four deterministic models considered (a) Dataset: live bed laboratory data (Data count: 229), (b) Dataset: live bed field data (Data count: 704).
Figure 4-6: Relationship between predicted and measured normalized scour depth for: a) Wilson (1995) model, (b) Melville (1997) model, (c) HEC-18 (2012) model, and (d) Briaud (2014) model using laboratory live bed dataset (Data count: 229), (e) Wilson (1995) model, (f) Melville model, (g) HEC-18 (2012) model, and (h) Briaud (2014) model using field live bed dataset (Data count: 704). Note: $b$ denotes the pier width, $y_{sp}$, and $y_{sm}$ are the predicted and the measured scour depth respectively.
Figure 4-7: For Briaud (2014) model, (a) demonstration of heteroscedasticity and (b) non-normal errors associated with predicted-measured normalized scour depth relation. For the same model, (c) state of heteroscedasticity, and (d) non-normal error when the proposed model is considered (Data count: 704).
Figure 4-9: Proposed model performance with respect to the measured normalized scour depth for: (a) Wilson (1995) model, (b) Melville (1997) model, (c) HEC-18 (2012) model, and (d) Briaud (2014) model using laboratory live bed dataset (Data count: 229) (e) Wilson (1995) model, (f) Melville (1997) model, (g) HEC-18 (2012) model, and (h) Briaud (2014) model using field live bed dataset (Data count: 758). Note: $f\left(\frac{y_{sp}}{b}, \lambda\right)$ represents the transformed response, $y^\lambda$ as expressed in Eq. 3a, $b$ denotes pier width, $\lambda$ is a deterministic model-based parameter, $y_{sp}$, and $y_{sm}$ are the predicted and the measured scour depth respectively.
Figure 4-10: Probability-$\chi_P$ relation for different models using (a) laboratory live bed dataset (Data count: 229), (b) field live bed dataset (Data count: 704).
Figure 4-11: Frequency distribution of the proposed modification factor, $\chi_{POD}$ for (a) Wilson (1995) model, (b) Melville (1997) model, (c) HEC-18 (2012) model, and (d) Briaud (2014) model using laboratory live bed dataset (Data count: 229).
Figure 4-12: Probability of deceedance-proposed modification factor relation for different models using (a) laboratory live bed dataset (Data count: 229), (b) field live bed dataset (Data count: 704). Clear water data points are from Shahriar et al. (2021b).
Chapter 5: Reliability index-based scour approach and corresponding impact on pile foundation response

This chapter is submitted as:

Abstract

Work herein is focused on developing and demonstrating the implications of the reliability index ($\beta$)-based scour assessment approach on the response of a pile group supporting a bridge pier. To facilitate the analyses, parameters from the Woodrow Wilson bridge pier were considered. A relationship between the probability of exceedance over the design life and the scour depth is presented to demonstrate the risk associated with the use of scour depth assessed using Hydraulic Engineering Circular (HEC) 18 approach. The estimated scour depth using both HEC-18 and the $\beta$-based approaches was applied to the modeled bridge pile foundation system and the response of the pile foundation under AASHTO loading scenarios were studied. For the studied case, consideration of $\beta$-based scour depth causes the longitudinal and axial displacement to exceed the allowable displacement limit at pile head. Furthermore, the point of contraflexure for $\beta$-based scour depth case is at a lower elevation than the HEC 18 predicted scour depth case. A parametric study was conducted by changing the pile dimensions to explore the effect of scour assessment corresponding to target $\beta$ on the lateral response of the pile foundation system. Risk analysis was also presented to demonstrate the advantages of $\beta$-based approach.

Keywords: Scour; Bridge pier; Reliability index; Probability of exceedance; Risk
Introduction

Bridge failures cause human life loss, disruption of commerce and enormous repair cost (Estes and Frangopol 2001; LeBeau and Wadia-Fascetti 2007; Deng et al. 2015). Bridge scour is one of the major reasons for bridge failure in the United States and worldwide (Melville and Coleman, 2000; Briaud 2014). Wardhana and Hadipriono (2003) focused on 503 failures of bridge structures that occurred during the period 1989-2000 in the United States and estimated that 85% of the failure is associated with externally triggered events, of which flood-induced scour is the primary contributor. Arneson et al. (2012) reported that 493,473 highway bridges are built over waterways in the United States (which is more than 80% of the total number of bridges within the country). Scour, thereby, is a primary design consideration for the waterway bridges and similar transportation infrastructures. The evaluation of scour magnitude at bridge sites involves technical expertise from multiples disciplines, e.g., hydraulics, structural, and geotechnical engineering (Parkes et al. 2018). The Federal Highway Administration (FHWA) provides guidelines for assessing the potential maximum scour depth around bridge foundations using the Hydraulic Engineering Circular (HEC) No. 18 model. The HEC 18 model is deterministic (Arneson et al. 2012), and often provides a conservative estimate of scour depth (e.g., Briaud et al. 2014; Yao 2013). However, the degree of inherent conservatism embedded in deterministic models such as HEC 18 and others is unknown. The level of conservatism depends on the inherent model bias as well as the uncertainty of hydraulic, hydrologic, and geotechnical parameters (Johnson 1992, 1996; Lagasse et al. 2013; Johnson et al. 2015; Shahriar et al. 2021a-b).

Bolduc et al. (2008) investigated the model bias in scour estimation using HEC 18 model and suggested a probabilistic scour estimation process using Gudavalli (1997) database, Landers and Mueller (1996) database, and Kwak (2000) database. Bolduc et al. (2008) suggested that a logarithmic transformation of the measured scour depth estimate is sufficient to remove associated heteroscedasticity. While Shahriar et al. (2021b), using Benedict and Caldwell (2014) database, demonstrated that Box-Cox transformation (Box and Cox 1964), rather than logarithmic transformation is necessary to remove heteroscedasticity and non-normal error associated with the measured scour depths. Contreras-Jara et al. (2021) utilized the first order reliability method to quantify the probability of exceedance of the estimated scour considering hydraulic and hydrologic parameter uncertainty. Contreras-Jara et al. (2021) reported that the probability of exceedance was dependent on uncertainties in annual maximum flow, Froude number, top width, and riverbed
longitudinal slope. Johnson et al. (2015) examined the hydraulic and hydrologic uncertainties associated with HEC-18 and Florida Department of Transportation (FDOT, 2011) scour prediction models. A large number of simulations (10,000) in HEC RAS were considered to assess the hydraulic conditions at a bridge considering the hydrologic input variable uncertainties. Three levels of hydrologic uncertainties were investigated, viz. low, medium, and high uncertainty. Johnson et al. (2015) showed that for different hydrologic uncertainty level, the estimated scour depth varies. Johnson (2005) demonstrated a methodology of assessing channel instability, which if not conducted appropriately, can lead to excessive scouring around bridge foundation. The developed methodology was intended for a quick assessment of channel instability status at bridges and for judging whether more extensive geomorphic studies or complete hydraulic and sediment transport analyses are needed to assess the potential for adverse conditions. Akay and Koçyiğit (2020) developed a hydrologic assessment approach using hydrologic indicators to assess hydrologic behavior and flash-flood potential of the bridge watershed. The developed methodology is useful in assessing the vulnerability of bridge watershed to flash flood. Briaud et al. (2014) developed a risk-based pier scour model through quantification of the “risk associated with failure”, where risk was defined as the product of the probability of occurrence times the value of consequence. FHWA developed HYRISK (Pearson et al. 2002), a methodology to estimate the annual scour failure risk by using pertinent items from the National Bridge Inventory (NBI) database. Yanmaz and Apaydin (2012) evaluated the scour risk for a bridge crossing Fol Creek in the Black Sea region of Turkey using HYRISK, while Akay (2021) applied HYRISK to evaluate the risk condition and proposed countermeasure design for the Çatalzeytin Bridge in Turkey. In summary, probabilistic aspects of scour estimation have been advanced to some extent to make a designer aware of model bias, hydraulic, and hydrologic uncertainties related to the model’s scour estimate and the associated annual risk.

On the other hand, the American Association of State Highway and Transportation Officials (AASHTO 2007) suggests that bridge foundation strength and service limit states need to be assessed for the scour corresponding to the design flood (100-year flood) event, and the factored foundation resistance with scour-assessed soil removed should be greater than the factored load. Lin et al. (2014, 2015) showed that changes to the subsurface stresses occur owing to the removal of the soils in scour prism, with such change leading to reduced axial and lateral foundation resistance. The Load and Resistance Factor Design (LRFD) guidelines suggest
achieving a certain level of reliability index (Nowak 1999) for the superstructure components, while for the design of foundation systems and considering scour, a deterministic HEC 18 approach is used. Reliability index, $\beta$ is a measure of the number of standard deviations that the mean margin of safety falls on the “safe” side (Paikowsky 2004), where the margin of safety is dependent on the limit state function considered as “critical” for a system. Reliability index has been in use across structural and geotechnical engineering literature (AASHTO LRFD 2007; AISC 2005; ACI 2005). Current AASHTO LRFD (2007) bridge design specifications suggest different target reliability index depending on soil types, pile types, the various design methods, and the consequence of failure. AASHTO LRFD (2007) suggests the use of $\beta=2.33$ for redundant systems, and $\beta=3.0$ for non-redundant systems. In general, a $\beta$ value of 2 to 4 is specified for the various structural applications. The calibration procedure of LRFD code is well documented in Nowak (1999) and Kulicki et al. (2007); and an example application procedure for driven pile axial capacity can be found in Kim et al. (2005). An inconsistency is posed by adopting a $\beta$-based approach for the design of superstructure components as opposed to a deterministic approach for assessing the scour magnitude at the foundation system. Subsequently, there is a need to develop a $\beta$-based model that considers the uncertainties of the parameters affecting the scour and that relates $\beta$ with a multiplicative factor necessary to achieve a target $\beta$ value.

Work herein is focused on demonstrating the implications of the $\beta$-based scour assessment approach on the response of pile groups supporting bridge piers. To facilitate illustrating the applicability of $\beta$-based approach, parameters from the Woodrow Wilson bridge pier in Prince George County, Alexandria, Virginia carrying traffic of I-95 and I-495 was considered in the analyses. A relationship between the probability of exceedance over the design life and the scour depth is presented to demonstrate the risk associated with the use of scour depth assessed using Hydraulic Engineering Circular (HEC) 18 approach. Scour depth was estimated using both traditional HEC 18 procedure and the $\beta$-based procedure. The estimated scour depth using both approaches was then applied to the Woodrow Wilson bridge foundation system and the corresponding displacement (transverse, longitudinal, axial) components and moments of the pile foundation under AASHTO loading scenarios were studied. The analyses were extended by conducting a parametric study by varying the pile length and pile diameter to explore the effect on the transverse, longitudinal, and axial displacement components of the pile foundation system. Finally, risk (the product of probability of failure and the cost of the consequences) analysis was
presented to show how the consideration of \( \beta \)-based approach is advantageous in assessing the potential of economic loss.

**The Woodrow Wilson Bridge Site**

The Woodrow Wilson bridge on the Potomac River is located in Prince George County, Alexandria, Virginia, and crosses over to Washington D.C. The bridge carries traffic through I-95 and I-495 and the relatively recent construction was built due to the fact that the previous Woodrow Wilson bridge reached its capacity (75,000 vehicles per day) within 8 years of its construction in 1969. The overall cost of the project including the approach embankment, and interchanges was estimated at US$ 2.2 billion (Kwak 2000; Kwak and Briaud 2002). The drawbridge is supported by two bascule piers (Jones 2000), which are located on the main channel of the Potomac River. The foundation system of the bascule piers consists of exposed pile foundation that is capped near the water surface. The soil stratigraphy in the main channel is made of soft clay overlying on thin layer of loose sand. Below the loose sand layer, a clay layer persists. The profile view, and pile cap section along with the soil stratigraphy for the pier M1, which supports the drawbridge, is presented in Figure 5-1. The soil properties, pile and pile cap properties are taken from Kwak and Briaud (2002) and Briaud (2008).

**Scour Estimation**

The Woodrow Wilson bridge pier M1 consists of three complex geometrical shapes of columns resting on a pile cap, supported by the pile groups. HEC 18 (Arneson et al. 2012) classifies such pier as complex pier and provides guidance on the calculation of scour depth. Jones and Sheppard (2000) suggested that a superposition of the scour depth component from the pier stem \( y_{s-pier} \), pile cap \( y_{s-pile cap} \) and the pile group \( y_{s-pile group} \) is necessary to estimate the total local scour depth, \( y_s \) (Equation 1).

\[
y_s = y_{s-pier} + y_{s-pile cap} + y_{s-pile group} \tag{1}
\]

The scour depth estimated from the HEC 18-suggested approach was 62.3 ft (19 m). Jones and Davis (2007) computed the contraction scour in the main channel of the Potomac River using the Laursen (1960) model, and concluded that the contraction scour magnitude is insignificant, thereby suggesting that total scour entails only local scour around the bridge pier for the Woodrow Wilson bridge case. Subsequently, analyses in this study were conducted using a total scour
magnitude of 62.3 ft (19 m), obtained from HEC 18 model (as currently FHWA suggests using HEC 18 model for scour assessment while designing bridge foundation).

Reliability Index-Based Scour Estimation

The reliability index incorporated in the LRFD methodology accounts for the uncertainties of loads and resistances while evaluating the safety of a system. In LRFD, safety is expressed by the level of desired reliability (e.g., Lagasse et al. 2013; Nowak and Collins 2000). The methodology to develop a reliability index based on scour estimation entails consideration of statistical attributes (mean, coefficient of variation, probabilistic distribution) of the input parameters in scour analyses (e.g., upstream flow velocity, upstream flow depth, median grain size of the bed material, Manning’s roughness coefficient, channel bed slope) to analyze the limit state function that defines the margin of safety. The limit state function, \( L \), can be expressed through Equation (2).

\[
L = D_p - D_s
\]  
(2)

Where, \( D_p \) is the depth from current bed level to the scour depth estimated using the mean value estimate of the input variables, and \( D_s \) is the scour depth predicted from HEC 18 (Arneson et al. 2012) scour prediction model. The probability of exceedance of \( L \), \( P_e \) when HEC 18 model is considered can be expressed as in Equation (3).

\[
P_e = P \left( D_p < \lambda 2K_1K_2K_3 \left( \frac{y}{b} \right)^{0.35} F^{0.43} \right)
\]  
(3)

Where, \( K_1 \), \( K_2 \), and \( K_3 \) are pier nose shape factor, pier alignment factor, and bed condition factor respectively; \( y \), \( b \), and \( F \) are flow depth upstream of the pier, pier width and Froude number respectively; \( \lambda \) is the bias factor which accounts for the model uncertainty. Model uncertainty depends on how well a given scour equation predicts scour and can be represented by the ratio of the measured to the predicted scour depth using HEC 18 model (Lagasse et al. 2013). In Equation (3), pier width, skew angle, pier face shape, and aspect ratio are deterministic, as these parameters are not subjected to any inherent temporal or spatial variability, while flow depth, flow velocity, and bias factor are associated with a mean magnitude, coefficient of variation, and probabilistic distribution. The Coefficient of Variation (COV) of \( y \) is dependent on the statistical distribution of Manning’s roughness coefficient, \( n \) and channel bed slope, \( S \) and can be estimated based on the uncertainty analyses presented by the U.S. Army Corps of Engineers (HEC 1986). The COV of flow velocity is also dependent on the distribution properties of \( n \) and \( S \), and can be estimated
based on the hydrologic uncertainty analyses presented in Johnson (1992), and Lagasse et al. (2013). The estimation procedure of $\lambda$ is dependent on the considered database. Data reported in Benedict and Caldwell (2014), who compiled laboratory scour measurements conducted since 1956, are utilized herein. Although the database entails a wide range of data, laboratory live bed data under narrow pier condition was considered for the estimation of $\lambda$ since the site condition of the Woodrow Wilson bridge falls in such category. Based on the definition of Melville and Coleman (2000), if $y/b > 1.4$, then a narrow pier condition exists. Figure 5-2(a) shows the observed distribution of $\lambda$ for the laboratory live bed data under narrow pier condition when the HEC 18 model is used. Figure 5-2(b) presents the quantile-quantile plot, which suggests a normal distribution of the parameter $\lambda$. Table 5-1 shows the list of deterministic and random variables and the associated statistical characteristics considered herein.

Owing to the non-linear nature of Equation (3), a Monte Carlo simulation approach was considered to compute the probability of exceedance of $L$. If the limit state function follows normal probability distribution, $\beta$ can be related to $P_e$ using Equation (4).

$$P_e = \Phi(-\beta)$$  \hspace{1cm} (4)

Where, $\Phi$ is a probability function representing the probability that the normalized random variable is below a given value. Similar to the concept of resistance factors in the LRFD approach, the method herein develops scour factors to be applied to the deterministically evaluated scour magnitudes A scour factor is defined as the ratio of $D_p'$ to $D_p$, where $D_p'$ is the depth from current bed level to the scour depth necessary to reach the scour factor. The Monte Carlo simulation was run for a scour factor ranging from 1 to 3, subsequently, a relationship between the reliability index and scour factor was devised. To ascertain the minimum number of simulations needed to evaluate the probability of exceedance of $L$ with sufficient accuracy (a discrepancy of <0.01% between the probability of exceedance of two successive simulation count increments), various simulation counts are considered, and the corresponding probability of exceedance was computed. The procedure was repeated for two scour factors, e.g., 1.0 and 1.1, and the results are presented in Figure 5-3. Although Figure 5-3 suggests that 5000 simulation cycles are enough to obtain a stable $P_e$, a total of 10,000 simulation cycles were used in order to ensure a low $P_e$ and a high $\beta$ (e.g., $P_e = 0.0001$, corresponding to $\beta = 3.72$).
Based on the statistical distribution properties of the input parameters (Table 5-1), Monte Carlo simulation will generate a scour estimate. If this scour estimate is more than the scour estimated from the mean value estimate of the input parameters times the scour factor, the condition is considered as "exceeding the defined limit state". For all the Monte Carlo simulation cycles, the $L$ was quantified to estimate $P_e$ (Equations 2-3). Using Equation (4), $P_e$ was then related to $\beta$ given the limit state function follows normal probability distribution and the result is presented in Figure 5-4. The scour factor associated with a certain $\beta$ can further be correlated to the Probability of Deceeedance (POD). POD is defined as the probability that the predicted scour depth will be less than the measured scour depth, estimated from the Benedict and Caldwell (2014) database. To quantify POD, initially, the frequency distribution of the ratio of the measured to the predicted scour depth was developed. Using Kernel density estimation, the population probability density function was then generated, followed by integration of the density function to obtain the cumulative density function. Finally, the POD is obtained by subtracting the cumulative density value from one. The relationship between the reliability index or the associated probability of deceeedance and scour factor is presented in Figure 5-4.

For example, in Figure 5-4, if the target reliability index, $\beta_T$ is 2.0, the corresponding scour factor is 1.70. Therefore, for the pier M1 of the Woodrow Wilson bridge, the scour depth corresponding to $\beta_T=2.0$ is 105.9 ft (32.3 m), which can be obtained by multiplying the deterministic HEC 18 predicted scour depth (62.3 ft or 19 m) by the scour factor (1.70). Figure 5-4 suggests that the use of a scour factor $>1$ causes the probability of deceeedance (defined as the probability of having a predicted scour magnitude being less than the value occurring in the field) to decrease. Figure 5-4 further reveals that the reliability index corresponding to a scour factor of 1 is zero.

**Probability of Exceedance over the Design Life of Structure**

A flood event with a recurrence interval of $T$ years has a $1/T$ probability of being exceeded in any given year. The 100-year flood is generally recommended in HEC 18 for hydraulic analyses. It has a probability of exceedance of $1/100$ in any year and this probability increases as the design life increases. If the design life of a bridge is $N$ years, the probability of exceedance in $N$ years, $P_{cumulative}$, can be represented as a function of annual probability of exceedance, $P_N$, as expressed in Equation (5).
\[ P_{\text{cumulative}} = 1 - (1 - P_N)^N \quad (5) \]

For a specific bridge, a chart relating the scour depth and the probability of exceedance in \( N \) years can be developed, allowing the engineer to select a scour depth corresponding to a target probability of exceedance in \( N \) years. Referring to Figure 5-4, the scour factors are corresponding to a certain annual probability of exceedance, which depending on the design life of the bridge, will provide the probability of exceedance in \( N \) years (Equation 5). The scour depth corresponding to a scour factor can be obtained by multiplying the HEC 18-predicted scour depth with the scour factor. Figure 5-5 shows the relationship between the probability of exceedance in \( N \) years and scour depth for different design life. For the pier M1 of Woodrow Wilson bridge, the HEC 18 (2012) suggested scour depth is 62.3 ft (19 m), which in turn has a 40%, 50%, and 76% probability of exceedance over a design life of 75, 100 and 200 years respectively. However, if the reliability index-based scour depth is considered with \( \beta = 2 \), the scour depth is 105.9 ft (32.3 m) and it has a 17%, 22%, and 38% probability of exceedance over a design life of 75, 100 and 200 years respectively. Therefore, it appears that the \( \beta \) based approach reduces the probability of exceedance by more than 23% over the design life investigated (75-200 years). For systematically estimating “risk,” the probability is multiplied by the consequence. A reduction in the probability of exceeding design scour depth leads to lower probability of the foundation system exceeding design criterion.

**FB-MultiPier Modeling**

Robinson et al. (2012) reported that FB-MultiPier can be chosen as a numerical tool to model bridge pier owing to multiple reasons, some of which are:- (i) has a built-in interactive bridge bent software wizard, (ii) automatically models the soil resistance (lateral and axial, single and group) using methods that represent the current state of practice, and (iii) allows for typical linear or nonlinear bent cap models to be utilized. Furthermore, FB-MultiPier produces similar results as commonly used finite difference program for pile analyses, LPILE with the additional benefit of modeling the superstructure (FB-MultiPier 2021).

The piles of the Woodrow Wilson bridge consist of 70 inch (1778 mm) diameter concrete filled pipe piles with a length of 210 ft (64 m) (Briaud 2008). The pile cap is 87 ft (26.5 m) wide and 129 ft (39.3 m) long. The flow depth was considered to be the water depth corresponding to the 100-year flood, which is 44.7 ft (13.6 m) (Kwak and Briaud 2002). The pile cap midplane is at
an elevation of 19.3 ft (5.9 m) from the water level. For the top and bottom clay layer, lateral soil resistance was modeled using Matlock (1970) model, and axial resistance was modeled using API (2003) model. For the thin sand layer in between the clay layers, lateral soil resistance was modeled using Reese et al. (1974) model, and axial resistance was modeled using API (2000) model. The exact reinforcement detailing of the pier cap, and the pier were not provided in literature; as such, a reinforcement ratio of 2% was assumed for the analysis.

FB-MultiPier facilitates generation of wind loads using the AASHTO LRFD design specification (2007) for pier. The generated wind pressure on structure, and wind pressure on live load were applied at the bearing location, which is at the center of the pier cap. Three wind angles (0, 30, 60 degrees) were considered to generate wind pressure in the transverse and longitudinal directions. In analysis settings, for both the pile and pier, the behavior was considered non-linear.

**AASHTO Load Cases**

FB-MultiPier, the modeling tool considered herein, allows for application of dead load, live load, impact load, braking load, vehicle collision load, and wind load among many other options (e.g., earth surcharge, locked in construction stresses, down drag, post tensioning, creep, shrinkage, temperature gradient etc.). In the present study, AASHTO strength (I-V) and service (I-III) load cases were considered. Analyses were conducted using the aforementioned load cases and the results were synthesized considering the load case that governed in terms of yielding the most demand. Axial force, moment, transverse displacement, longitudinal displacement, and axial displacement generated on the pile foundation system were analyzed for “no scour,” and scoured cases. In FB-MultiPier, axial forces and moments, along with non-linear pile section properties are used to calculate Demand Capacity Ratio (DCR). The DCR is an estimate of the percentage of the cross-section capacity that has been reached for a particular loading state (Robinson et al. 2012). A DCR of less than 1 implies that the section can sustain the range of AASHTO load cases considered for the analyses, while a DCR of more than 1 implies failure.

**Modeling Results**

The studied pier M1 of the Woodrow Wilson bridge has three pier columns with a complex cross-section placed on the pile cap. The “Pier” model type in FB-MultiPier does not allow modeling complex pier cross-section. Thereby, the analysis was divided into two phases. In the first phase, (Phase I), to represent the actual load transfer scenario, two cases with different pier count (on the same pile cap) were investigated. The first case consists of three piers transferring
the load from the bridge deck to the pile cap. The base area of the pier was kept the same as the base area of the pier M1. It is important to note that the cross section of the pier M1 changes along the depth, whereas a uniform cross-section was considered herein. The second case consists of equivalent single pier with the base cross-sectional area equal to the sum of the three piers (located centrally) with loads from the end piers transferred (as load, and moment) on the middle pier. The objective of the first phase analysis was to obtain similar results from both the 3-pier and equivalent single pier cases. Figure 5-6 shows a comparison of transverse, longitudinal, and axial displacement of the pile foundation along the depth from both model cases (Phase I). Considerable agreement in the displacement response can be obtained between the 3-pier case and the equivalent single pier case scenario. The maximum discrepancy in the displacement at any particular elevation was 4%. Figure 5-7 shows a comparison of the moments generated on the pile foundation around transverse and longitudinal axes. The maximum discrepancy of moments between 3-pier case and equivalent single pier case was 4.3%, which suggest that the equivalent single pier case is capable of replicating the load transfer mechanism of 3-pier case.

In the second phase (Phase II), scour analyses were conducted considering the equivalent single pier case as the base case model. Two further model cases are generated in FB-MultiPier to observe the difference in pile response when HEC 18-determined scour depth versus $\beta$ based scour depth is used. For the HEC 18-determined scour depth case, the riverbed surface elevation near the vicinity of the pile group was lowered to the depth of total scour depth obtained from HEC 18 (62.3 ft). For the $\beta$-based scour depth case, the riverbed surface at the vicinity of the pile group was lowered to a depth of 105.9 ft (32.3 m). Per Richardson and Davis (2001), considering practical applications, a scour prism with a top width of 2 times the scour depth was considered on either side of the pile group.

Figure 5-8 shows the comparison among base case, HEC 18-predicted scour case, and $\beta$-based scour case in terms of transverse displacement, longitudinal displacement, and axial displacement. Results show the pile head deflections for $\beta$-based scour depth case is considerably higher than the HEC 18-computed scour depth case. The second edition of AASHTO LRFD placed a limit of 1.5 inches allowable deflection for bridge piles at the pile head. However, the limit was removed in the subsequent versions of AASHTO LRFD specifications. Nevertheless, researchers
(e.g., Robinson et al. 2006; Robinson et al. 2012) consider 1 inch (25 mm) as an acceptable allowable deflection for bridge piles at the pile head.

Figure 5-9 shows the comparison among base case, HEC 18-computed scour case, and $\beta$-based scour case in terms of transverse and longitudinal moments generated on the piles. The point of contraflexure for the $\beta$ based scour depth case is at a lower elevation than the HEC 18-computed scour depth case. The maximum positive and negative moments generated also increases for $\beta$ based scour depth case than the HEC 18-predicted scour depth case. The maximum Demand to Capacity Ratio (DCR) for the pile was 0.148, 0.153, and 0.165 for the base case, the HEC 18-computed scour depth case, and the $\beta$ – based scour depth case respectively. Figure 5-10 shows the interaction diagram for pile 3, which governed for the moment capacity. Referring to Figure 5-1-pile cap section, pile 3 is the center pile in the first row that is parallel to the “87 ft (26.5 m)” side (black shaded). With the increase of the considered scour depth, the moment generated on the pile tends to reach the envelope, although the capacity of the section is significantly higher than the generated moment on the pile.

**Parametric Analysis**

FB-MultiPier analyses were performed to explore the mitigation measures of the increased scour magnitude with the use of the scour factor approach. Subsequently, the effect of variation of pile length and pile diameter on the transverse, longitudinal, and axial displacement of the pile was investigated. While performing the parametric analyses, the center-to-center distance between the piles was kept same as the base case.

**Effect of pile length variation**

The variation of transverse displacement, longitudinal displacement, axial displacement, and DCR with the changes in pile length is presented in Figure 5-11. The pile length of the base case model is 210 ft (64 m). The pile length was increased from 210 ft (64 m) to 250 ft (76.2 m) (19%). It appears that an increase of pile length by 40 ft (12.2 m) decreased the transverse and longitudinal displacement by 4.4% and 6.2% respectively, while the axial displacement was reduced by 41.4%. Nevertheless, if the pile length is increased to 220 ft (67.1 m) (increasing length by 4.8%), the axial displacement can be lowered to an acceptable displacement limit of 1 inch (25 mm). It is also apparent that increase of pile length is effective in reducing the axial displacement of the pile.
Effect of pile diameter variation

Figure 5-11 further shows the variation of pile displacement components with the changes in pile diameter. The pile diameter of the base case model is 70 inches (1778 mm). The pile diameter was increased from 70 inch (1778 mm) to 75 inch (1905 mm). As shown in Figure 5-11, and for an increase in pile diameter from 70 inches (1778 mm) to 74 inches (1880 mm), the transverse and longitudinal displacement was reduced by 13.2% and 14.4% respectively, while for axial displacement, the reduction was 7.8%. However, as the pile diameter was increased further to 75 inches (1880 mm), the displacement tends to increase. This could be due to the center to center spacing between the piles that was kept same as the base case, which resulted in a reduction of the effective volume surrounding the pile to mobilize the complete foundation resistance. However, it is evident that increase in pile diameter is effective in reducing transverse and longitudinal displacement of the pile.

Scour Risk

Annual risk, $R$, is defined as the product of the annual probability of failure, $P_F$, times the cost of the consequences, $C$, as expressed in Equation (6) (Baecher and Christian 2003).

$$ R = P_F \times C $$

The FHWA developed HYRISK methodology (Pearson et al. 2002) that quantifies scour failure risk. HYRISK methodology estimates the risk of scour failure by using pertinent items from the National Bridge Inventory (NBI) database. The cost of consequence, $C$, based on HYRISK can be estimated using the model proposed by Stein and Sedmera (2006).

$$ C = C_1WLM + C_2DAd + \left[ C_3O \left( 1 - \frac{T}{100} \right) + C_4 \frac{T}{100} \right] \frac{DAd}{S} $$

The parameters in Equation (7) are defined in Table 5-2, where typical value considered are also presented. The cost of consequence, $C$, was estimated at US$ 7,551,045. Pearson et al. (2002) defined that the first approximation of the probability of failure (trial probability of failure, $P_{TR}$) is dependent on overtopping frequency and scour criticality which should be assessed based on NBI items 26, 71, and 113. Yanmaz and Apaydin (2012) reported that if a bridge is classified as in “very good condition,” which corresponds to a code of “8” per NBI item 113, and the bridge deck and roadway approaches are above flood water elevations (Chance of overtopping is remote), the trial probability of scour induced failure is 0.00312. The current age of the bridge is a check
for the trial failure probability, $P_{TR}$. Pearson et al. (2002) suggested that the trial probability of failure can be related to the expected life of the bridge, $L_{bridge}$ using Equation (8).

$$L_{bridge} = \frac{\log (1 - 0.90)}{\log (1 - P_{TR})} \tag{8}$$

When the current age of the bridge is greater than $L_{bridge}$, $P_{TR}$ needs to be modified by taking the bridge age as $L_{bridge}$. The $P_{TR}$ then becomes $P_F$, which will then be used in Equation (6) to estimate the scour risk. For the $P_{TR}$ considered herein (0.00312), the expected age of the bridge is 737 years, which is assumed to be more than the age of a typical bridge in the United States. As such, $P_{TR}$ can be considered equal to $P_F$ and was used in Equation (6) to estimate the scour risk. The scour risk for a typical bridge was estimated at US$ 23,559 per year. Briau et al. (2014) reported that the acceptable target risk for a civil engineering structure in the United States is US$ 1,000 per year, indicating that the scour risk associated with a typical bridge in the United States needs proper consideration to avoid undesired consequences.

We present herein an example to illustrate the advantage of the proposed approach. Statistics presented in Arneson et al. (2012) suggest that in the United States, 493,473 bridges are built over waterways. In a given year, if HEC 18 approach is used to estimate scour depth, there is a possibility that the scour depth will be exceeded in 3,454 bridges. Whereas, if the scour factor corresponding to the $\beta_T$ of 2.0 is used, annually, the scour depth would be exceeded in 1,184 bridges, reducing the affected number of bridges by 2,270. As presented in Equation (7), the economic loss encompasses the bridge repair cost, detour cost, and time cost. Based on the per square foot repair cost for a bridge in North Carolina, as reported in FHWA (2017), the cost of a bridge having a span of 82 ft and a width of 40 ft (25 m x 12.2 m) is US$ 406,720. For the 2,270 bridges, the economic loss considering only the bridge repair would be US$ 0.92 billion. Therefore, adopting the scour depth corresponding to the $\beta_T$ of 2.0 that is consistent with the reliability index used in the super and sub-structure allows for assessing the risk of US$ 0.92 billion annual loss. It should be noted that the inherent assumptions in the demonstrated example are that the HEC 18 model has been used for scour assessment of all the waterway bridges in the United States, and the per square foot cost remains same across the country. Nevertheless, as presented in the parametric analyses section, consideration of $\beta$-based scour depth might cause the transverse, longitudinal, or axial displacements of the pile to exceed the allowable limit,
thereby, requiring a need to optimize pile diameter, pile length, or pile spacing, which might increase the construction cost.

**Summary and Conclusion**

Based on the results presented, following conclusions are advanced:

1. FHWA suggests use of HEC 18 model to estimate the scour depth around bridge foundations. For the studied case of pier M1 of Woodrow Wilson bridge, HEC 18 procedure predicts the scour depth to be 62.3 ft (19 m). During the design life of 100 years, there is a 50% chance that this scour depth will be exceeded.

2. Reliability index-based procedure considers the hydraulic, hydrologic, geotechnical parameter uncertainty in addition to inherent model bias. For the studied case of pier M1 of Woodrow Wilson bridge, and using a target reliability index of 2.0, the scour depth will be 105.9 ft (32.3 m). During the design life of 100 years, there is a 22% chance that this scour depth will be exceeded, reducing the exceedance probability by 28% compared to the HEC 18 suggested scour depth case.

3. FB-MultiPier modeling results suggest that for the studied site, consideration of reliability index-based scour depth causes the longitudinal and axial displacement to exceed the allowable displacement limit of 1 inch (at pile head), while for the HEC 18 suggested scour depth case, the transverse, longitudinal, and axial displacements remained within the 1-inch (25 mm) allowable limit.

4. FB-MultiPier modeling results further reveal that the point of contraflexure for reliability index-based scour depth case is at a lower elevation than the HEC 18 predicted scour depth case. The maximum positive and negative moments generated on the pile also increases for reliability index-based scour depth case compared to the HEC 18 predicted scour depth case. Subsequently, the moments generated on the piles approach the envelope of interaction, although the section moment capacity is significantly higher than the generated moments on the pile.

5. Parametric analyses were performed to explore the mitigation measures of the increased scour magnitude with the use of the scour factor approach. It was apparent that increase of pile length is effective in reducing axial displacement of the pile, while increase of pile diameter is effective in reducing transverse and longitudinal displacement of the pile.
6. The scour risk for a typical bridge in the United States was estimated at US$ 23,559 per year. In the United States, 493,473 highway bridges are built over waterways. Based on the repair cost data reported for a bridge in North Carolina, as reported in FHWA (2017), the cost of a bridge, it was showed that the use of reliability index-based scour depth in foundation design allows for assessing the risk of US$ 0.92 billion annual loss.

Data availability statement
All data, models, and code generated or used during the study appear in the submitted article.

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Declaration of interests
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
Table 5.1. Distribution properties of utilized deterministic and random variables in Monte Carlo simulation.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Live bed Distribution</th>
<th>Reference for COV and distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>COV</td>
</tr>
<tr>
<td>Pier width</td>
<td>0.6 ft</td>
<td>Deterministic</td>
</tr>
<tr>
<td>Pier face shape</td>
<td>Circular</td>
<td>Deterministic</td>
</tr>
<tr>
<td>Skew angle</td>
<td>0 degree</td>
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<tr>
<td>Aspect ratio (Length to width)</td>
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<td>Deterministic</td>
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<tr>
<td>Flow velocity</td>
<td>1.55 ft/s</td>
<td>0.186</td>
</tr>
<tr>
<td>Flow depth</td>
<td>1.24 ft</td>
<td>0.2</td>
</tr>
<tr>
<td>Median grain size</td>
<td>0.3 mm</td>
<td>0.081</td>
</tr>
<tr>
<td>Bias factor – HEC 18 (2012)</td>
<td>0.67</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 5.2. Parameters considered to calculate risk for a typical bridge.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit rebuilding cost ($/ft^2)</td>
<td>$C_1$</td>
<td>124</td>
<td>FHWA (2017)</td>
</tr>
<tr>
<td>Cost multiplier</td>
<td>$m$</td>
<td>2</td>
<td>Briaud et al. (2014)</td>
</tr>
<tr>
<td>Bridge width (ft)</td>
<td>$W$</td>
<td>40</td>
<td>Briaud et al. (2014)</td>
</tr>
<tr>
<td>Bridge length (ft)</td>
<td>$L$</td>
<td>82</td>
<td>Briaud et al. (2014)</td>
</tr>
<tr>
<td>Cost of running vehicle ($/mi^2$)</td>
<td>$C_2$</td>
<td>0.25</td>
<td>Stein and Sedmera (2006)</td>
</tr>
<tr>
<td>Detour length (mi)</td>
<td>$D$</td>
<td>9.3</td>
<td>Briaud et al. (2014)</td>
</tr>
<tr>
<td>Average daily traffic (vehicles/day)</td>
<td>$A$</td>
<td>6635</td>
<td>Briaud et al. (2014)</td>
</tr>
<tr>
<td>Duration of detour (days)</td>
<td>$d$</td>
<td>183</td>
<td>Briaud et al. (2014)</td>
</tr>
<tr>
<td>Value of time per adult in passenger car ($/hr$)</td>
<td>$C_3$</td>
<td>7.05</td>
<td>Stein and Sedmera (2006)</td>
</tr>
<tr>
<td>Average occupancy rate (adults count)</td>
<td>$O$</td>
<td>1.56</td>
<td>Stein and Sedmera (2006)</td>
</tr>
<tr>
<td>Average daily truck traffic (vehicles/day)</td>
<td>$T$</td>
<td>30</td>
<td>Briaud et al. (2014)</td>
</tr>
<tr>
<td>Value of time for truck ($/hr$)</td>
<td>$C_4$</td>
<td>20.56</td>
<td>Stein and Sedmera (2006)</td>
</tr>
<tr>
<td>Average detour speed (mi/hr)</td>
<td>$S$</td>
<td>40</td>
<td>Briaud et al. (2014)</td>
</tr>
</tbody>
</table>
Figure 5-1: Profile view and pile cap section along with the soil stratigraphy for the bascule pier M1 of Woodrow Wilson bridge (After Kwak and Briaud 2002; Briaud 2008).
Figure 5-2: For narrow pier under live bed condition when HEC 18 (2012) model is used, (a) statistical distribution of bias factor, $\lambda$ and (b) quantile-quantile plot.
Figure 5-3: Relationship between probability of exceedance and number of simulation cycles when HEC 18 (2012) model is used (SF stands for scour factor, which is defined as the ratio of the depth from average bed level to the bottom of the footing to the scour depth predicted from different scour prediction models, in this plot which is HEC 18 model).

Figure 5-4: Relationship among probability of deceedance, reliability index and scour factor. (Note: The POD chart has been developed based on HEC 18 model applied to live bed laboratory data).
Figure 5-5: Relationship between probability of exceedance and scour depth for different design life of the bridge when HEC 18 model is used. (Scour depths are for pier M1 of the Woodrow Wilson bridge).
Figure 5-6: Comparison between the 3-pier case and equivalent 1 pier case in terms of (a) transverse displacement, (b) longitudinal displacement, and (c) axial displacement of the pile. (Maximum among the investigated strength and service limit states is plotted; Datum is at an elevation of 0 ft).
Figure 5-7: Comparison between the 3-pier case and equivalent 1 pier case in terms of (a) transverse moment, and (b) longitudinal moment generated on the pile. (Maximum among the investigated strength and service limit states is plotted; Datum is at an elevation of 0 ft).
Figure 5-8: Comparison among base case, HEC 18 predicted scour case, and reliability index-based scour case in terms of (a) transverse displacement, (b) longitudinal displacement, and (c) axial displacement. (Maximum among the investigated strength and service limit states is plotted; Datum is at an elevation of 0 ft).
Figure 5-9: Comparison among base case, HEC 18 predicted scour case, and reliability index-based scour case in terms of (a) transverse moment, and (b) longitudinal moment generated on the pile. (Maximum among the investigated strength and service limit states is plotted; Datum is at an elevation of 0 ft).
Figure 5-10: Interaction diagram for pile 3 of pier M1, which governed for moment capacity. (Referring to Figure 5-1-pile cap section, pile 3 is the center pile in the first row parallel to the 87 ft side—black shaded).
Figure 5-11: Variation of (a) transverse displacement, (b) longitudinal displacement, (c) axial displacement, (d) demand to capacity ratio with pile length, and (e) transverse displacement, (f) longitudinal displacement, (g) axial displacement, (h) demand to capacity ratio with pile diameter.
Chapter 6: Incorporation of a reliability-based analysis of local scour in AASHTO LRFD framework

This chapter was previously published as:
Abstract

The analyses of axial and lateral capacity of a pile are significantly dependent on the appropriate estimation of scour depth, while the scour estimation procedure is uncertain due to the hydraulic, hydrologic, and geotechnical parameter uncertainty. Work herein is focused on developing a framework for reliability-based pier scour assessment methodology and demonstrate its integration with the concept of Load and Resistance Factor Design (LRFD) approach. Scour factors are proposed based on reliability level ($\beta$) and the associated probability of deceedance (POD). Three example applications of axially and laterally loaded pile design approach considering scour factor in the LRFD framework has been demonstrated. Based on axial pile capacity analysis, the increase of pile length when the $\beta$-based scour with the soil resistance factors is used, compared to using the deterministic scour with soil resistance factor was estimated to be 26.5-29.6%. Based on the lateral pile response analysis, it was discerned that as $\beta$ increases from 2.0 to 3.0, the lateral pile head deflection increased by 46%-132% compared to the deterministically estimated scour depth case. To obtain $\beta$ =3.0, the pile diameter needs to be increased by 35.7% compared to the base case pile’s diameter.

Keywords: Scour; Bridge pier; Reliability index; LRFD; Laterally loaded pile; Clay.
Introduction

Traditional practice of bridge scour estimation relies upon the use of deterministic models (Briaud et al. 2014; Shahriar et al. 2021a). Although many pier scour prediction models are available in literature, the Hydraulic Engineering Circular No. 18 (Arneson et al. 2012) is one of the most commonly used in the United States (Johnson et al. 2015; Karim et al. 2019). Investigation into the scour prediction models suggests that pier scour estimation is predominantly influenced by flow depth to pier width ratio, pier width to median grain size ratio, pier face shape, pier aspect ratio, skew angle of pier, and approach mean flow velocity to critical velocity ratio (Melville and Coleman 2000; Ettema et al. 2011; Ettema et al. 2017). The majority of the input parameters related to flow and streambed are not deterministic in nature, rather can be random variables with a particular statistical distribution.

Multiple advancements have been made in probabilistic scour assessment front. Briaud et al. (2007) considered the uncertainty of the hydrologic loading conditions to develop a probabilistic bridge scour depth prediction model. Bolduc et al. (2008) introduced a probabilistic model based on the bias in the HEC 18 (Arneson et al. 2012) model using three databases (Gudavalli 1997, Landers and Mueller 1996, and Kwak 2000); whereby it was discerned that a logarithmic transformation of the measured scour depth estimate is sufficient to remove heteroscedasticity. However, Shahriar et al. (2021b), using the Benedict and Caldwell (2014) database, demonstrated that Box-Cox transformation (Box and Cox 1964), rather than logarithmic transformation is necessary to remove heteroscedasticity and non-normal error associated with the measured scour depths, thereby, proposed a refined method to estimate model bias. Johnson et al. (2015) incorporated hydraulic and hydrologic uncertainties in the HEC-18 and Florida Department of Transportation (FDOT, 2011) scour depth prediction models. Johnson et al. (2015) showed that for different hydrologic uncertainty level, the estimated scour depth varies and reported scour design factors based on target scour non-exceedance. Contreras-Jara et al. (2021), utilizing first order reliability method demonstrated that the probability of exceedance was dependent on uncertainties in annual maximum flow, Froude number, top width, and riverbed longitudinal slope, which agrees with the philosophy of Johnson et al. (2015). In summary, probabilistic aspects of scour estimation have been advanced to make a designer aware of model bias, hydraulic, and hydrologic uncertainties associated the model’s estimate.
In the philosophy of bridge design considering Load and Resistance Factor Design (LRFD) approach, an inconsistency is posed by adopting a reliability index-based approach for the design of superstructure components as opposed to a deterministic approach for assessing the scour magnitude at the foundation system. AASHTO (2007) suggests that all material in the scour prism above the total scour line is assumed to be removed and unavailable for any axial or lateral support. AASHTO (2007) further proposes that scour design for the design flood must satisfy the requirement that the factored foundation resistance after scour is greater than the factored load determined with the scoured soil removed. The resistance factors will be those used in the Strength Limit State, without scour (AASHTO, 2007). Lin et al. (2010), Lin et al. (2014a), and Lin and Lin (2019) examined the effect of scour on the behavior of laterally loaded pile in sand and concluded that scour depth is the most influential factor impacting the lateral capacity of piles compared to scour width and scour hole slope angle, and consideration of the stress history results in a reduced lateral capacity of the pile. Lin and Jiang (2019) demonstrated that scour hole dimension and stress history affects the tension capacity of piles in sand. Lin et al. (2014b) and Zhang et al. (2017) observed a reduction of lateral pile capacity for piles embedded in soft clays owing to the consideration of the stress history, whereas an omission of scour hole dimension led to an over-conservative prediction of lateral pile capacity. Briaud et al. (2014) and Yao (2013) examined the axial capacity of the piles embedded in sand and clay and reported that the axial capacity of pile decreases as the scour depth increases and suggested a reduction of resistance factors depending on the magnitude of the scour depth. Nevertheless, it is apparent that the analyses of axial and lateral capacity of a pile are significantly dependent on the appropriate estimation of scour depth, while the scour depth estimation procedure is uncertain owing to the hydraulic, hydrologic, and geotechnical parameter uncertainty. Subsequently, there is a need to develop a reliability-based scour estimation model that considers the uncertainties of the parameters affecting the scour depth and that relates reliability index with a multiplicative factor necessary to achieve a target reliability index, which is currently non-existent in the current state-of-the-art. The reliability index-based scour depth can then be used in the lateral and axial capacity of the pile foundation design, whereby a safe, reliable, and consistent design of super- and sub-structure can be attained.

Work herein is focused on developing a framework for reliability-based pier scour assessment methodology by extending estimates from deterministic models. Four commonly used pier scour prediction models are considered. The laboratory clear-water and live-bed databases
documented by Benedict and Caldwell (2014) are utilized to quantify model bias and uncertainty. The relationship between probability of exceedance of the measured scour depth and scour factors for varying reliability levels is explored considering the effect of input parameter uncertainty. To advance the concept of LRFD application to the various components of hydraulic structures, scour factors were proposed relating to the reliability index. Responses for narrow pier, intermediate pier, and wide pier subjected to both clear water and live bed conditions are examined. Scour factors corresponding to various reliability levels are discussed in view of the deterministic scour evaluations. Three example applications on the axially and laterally loaded pile design approach considering probabilistic scour assessment factor and using the LRFD method has been demonstrated. For axial pile capacity problem, the method proposed by American Petroleum Institute (API) was considered, while for the lateral pile capacity problem, the method proposed by Matlock (1970), commonly known as the p-y method was adopted.

**Scour prediction models for analyses**

Four pier scour prediction models, namely Wilson (1995) model, Melville (1997) model, HEC-18 (Arneson et al. 2012) model, and Briaud (2014) model are utilized. The model equations are provided in Table 6-1. Further details of the hydraulic and geotechnical data considered for developing the models are presented elsewhere (Shahriar et al. 2021b). In summary, these four models are chosen due to the differences in the groups of data on which each of these models was developed.

**Database for analyses**

One of the objectives of the paper is to present a framework for development of a reliability-based scour assessment methodology, where comprehension of model bias is needed that requires use of a database. As such, an appropriate database needs to be selected followed by a discussion on the methodological framework. Johnson (1992) investigated the relationship between probability of bridge failure and safety factors using data from Chee (1982) and Chiew (1984). The Chee-Chiew database consists of 130 laboratory pier scour data. Bolduc et al. (2008) used a probabilistic model based on the bias in the HEC 18 model using three databases collected from Gudavalli (1997), Landers and Mueller (1996), and Kwak (2000). Gudavalli database consists of 73 large scale laboratory pier scour data, Landers-Mueller database consists of 380 field pier scour data, and Kwak database entails 18 field pier scour data.
In the study herein, database collected by Benedict and Caldwell (2014) is utilized. The database is the largest collection of pier scour database in the United States entailing 569 laboratory scour measurements and 1858 field scour measurements. The database is comprised of both clear water (approach mean flow velocity to sediment critical velocity ratio is less than one with mainly a sandy bed) and live bed (approach mean flow velocity to sediment critical velocity ratio is more than one with the stream bed having appreciable percent fines) scour condition. The laboratory dataset rather than field dataset was considered for analyses, considering the fact that significant uncertainties are associated with the field scour measurements. Uncertainties of field scour measurements include the accuracy of measured scour (Ground Penetrating Radar versus Fathometer approach), the maturity of the scour depth (equilibrium versus non-equilibrium scour), and accuracy of the hydraulic attributes (historic flow-based approach versus one dimensional flow model-based approach).

Melville and Coleman (2000) identified that the approach flow depth ($y$) to pier width ($b$) ratio is one of the most important parameters driving the maximum scour magnitude and suggested three classes of pier scour flow field, i.e., $y/b > 1.4$, $0.2 \leq y/b \leq 1.4$, and $y/b < 0.2$. Ettema et al. (2017) presented information on the differences in the turbulence structures formed for the three pier classes. Subsequently, the database considered herein was classified into three pier categories. The ranges of the selected variables for both clear water and live bed laboratory data are presented in Tables 6-2 and 6-3 respectively.

**Data screening**

Two approaches were considered to identify the outliers within the database. The first approach to identify the outlier data points was to adopt a statistical outlier identification technique, commonly known as box plotting, considering error in models’ estimation as the parameter that quantifies the adequacy of a given deterministic model. Error is defined as the difference between the predicted, $y_{sp}$, and the measured scour depth, $y_{sm}$, expressed as a percentage of $y_{sm}$. Figure 6-1 shows the error-boxplot corresponding to narrow pier clear water data for the four deterministic models considered in the study. The outlier data points were identified and listed in Table 6-4. Observation of Table 6-4 suggest that 67% of the outliers has $b/D_{50} \leq 52$. Ettema et al. (2011) reported that for $b/D_{50} \leq 50$ the individual soil particles are so large compared to the groove carved out by the down flow, thereby erosion is impeded as the rough bed dissipates some of the
energy. Data in Table 6-4 suggest that $v/v_c$ ranged from 0.41 to 0.96, and 45% of the data points had a $v/v_c = 0.41 - 0.55$. Melville and Coleman (2000), based on the investigation on piers placed on uniform deposits in clear water condition, reported that scouring process initiates when $v/v_c \approx 0.4$, and the scour magnitude increases linearly with the increase of approach mean flow velocity. As substantial data points presented $v/v_c < 0.55$ (refer to Table 6-4), it is expected that the scour rate was within the initial steep rising slope and has not reached equilibrium yet. Although the duration of flow was reported, it is not known whether such duration was sufficient for reaching the equilibrium state.

The second method used to identify the outliers is estimating the standard deviation of the error from the data set and identifying the data points that presented an error beyond two standard deviations from the mean error. The outliers, along with the key parameter attributes are listed in Table 6-5. Data in Table 6-5 suggest that 56% of the data has a $v/v_c < 0.55$, while 50% of the outliers has $b/D_{50} \leq 52$. Therefore, the explanation presented before regarding the outliers and $v/v_c < 0.55$ is possibly applicable to justify the outlier nature of the data points.

The objective of data screening process was to identify data points that “most likely” are not representative of equilibrium scour depth. A similar data screening methodology was adopted to identify the outliers for intermediate pier and wide pier corresponding to clear water and live bed conditions, and a summary of screening result is presented in Table 6-6.

**Methodology for reliability-based analyses**

The reliability theory incorporated in the LRFD methodology describes safety where the capacity exceeds demand (e.g., Lagasse et al. 2013; Nowak and Collins 2000). A limit state function, $L$, expressing safety can be expressed as shown in Equation (5).

$$L = D_p - D_s \quad (5)$$

Where, $D_p$ is the depth from bed level to the scour depth estimated using the mean value estimate of the input variables, and $D_s$ is the scour depth predicted from different deterministic scour depth prediction models. The probability of exceedance, $P_e$ for the limit state function, considering Arneson et al. (2012) can be expressed as in Equation (6).

$$P_e = P \left( D_p < \lambda 2 K_1 K_2 K_3 \left( \frac{Y}{B} \right)^{0.35} F_r^{0.43} \right) \quad (6)$$
Where, \( \lambda \) is a bias factor equal to the ratio of the measured to the predicted scour depth from a given deterministic model, and the rest of the parameters in Equation (6) are defined in Table 6-1. Due to the nonlinear nature of Equation (6), Monte Carlo simulation approach was used to compute \( P_e \).

Reliability index, \( \beta \), (which is a measure of the number of standard deviations that the mean margin of safety falls on the safe side), therefore, can be expressed as in Equation (7).

\[
P_e = \Phi(-\beta)
\]

(7)

Similar to “resistance factors” in the LRFD approach, the concept introduced herein seeks to develop “scour factors”. A scour factor, \( SF \) is defined as the ratio of \( D_p' \) to \( D_p \), where \( D_p' \) is the depth from current bed level to the scour depth necessary to reach the scour factor.

The pier width, skew angle, pier face shape, and aspect ratio were considered as deterministic, as these are not subjected to inherent temporal or spatial variability. The random variables considered herein are \( y, v, D_{50}, \) and \( \lambda \). A dimensionless measure of uncertainty is the coefficient of variation (COV), which is the ratio of standard deviation divided by the mean magnitude. The COV associated with \( y \), \( COV_y \) can be estimated based on the uncertainty analyses performed by U.S. Army Corps of Engineers (HEC 1986), as expressed in Equation (8).

\[
COV_y = 0.76y^{0.6}S^{0.11}(5N)^{0.65}
\]

(8)

Where, \( N \) is the reliable estimate of Manning’s coefficient, \( n \). The distribution of \( n \) was considered lognormal as presented in HEC (1986) and Johnson (1992). The distribution of the slope of the channel bed, \( S \) was considered lognormal based on the analyses of Johnson (1996). Discharge in a stream is a function of Manning’s \( n \) and \( S \). Johnson (1992), and Lagasse et al. (2013), based on hydrologic uncertainty analyses, considering the aforementioned distribution of Manning’s \( n \) and \( S \), showed that the distribution of \( v \) is lognormal, and suggested that the COV is 0.12 times the mean approach flow velocity.

Lagasse et al. (2013) indicated that sediments’ gradation is normally distributed, as such, the \( D_{50} \) was considered to be normally distributed. The bias factor, \( \lambda \) was observed to be normally distributed as shown in Figure 6-2 for live bed condition (statistical Q-Q plot warrants the observation). It appears that the distribution of \( \lambda \) is not purely symmetric, rather a mild positive
skewness of 0.14-0.65 was noticed. Table 6-7 shows the list of deterministic and random variables and the associated statistical characteristics considered.

Based on the statistical attributes (mean, COV, distribution) of the input variables (Table 6-7), Monte Carlo simulation will randomly generate a scour depth. If the estimated scour depth is more than the scour depth using the mean value estimate of the input variables times the scour factor, the condition is considered as "exceeding the defined limit state, L". For all the Monte Carlo simulation cycles, the L was quantified to estimate $P_e$ (Equations 6-7). Using Equation (8), $P_e$ was then related to $\beta$ given the limit state function follows normal probability distribution. The Monte Carlo simulation was run for $SF$ ranging from 1 to 3, and a relationship between $\beta$ and $SF$ was devised.

Analyses

Simulation cycles

To ascertain the minimum number of simulations needed to evaluate the probability of exceedance equilibrium pier scour magnitude with sufficient accuracy, various simulation counts were considered, and the corresponding probability of exceedance were computed. The procedure was repeated for two scour factors, e.g., 1.0 and 1.1, and the results are presented in Figure 6-3. Beyond a simulation cycle of 5,000, stability of the reported probability of exceedance was observed. However, to obtain a small probability of exceedance, e.g., in the range of 0.0001, the number of simulation cycles should be greater than 5,000. Therefore, 10,000 simulation cycles were considered in the study.

Probability of exceedance-scour factor relation

Figure 6-4 shows the relationship between probability of exceeding the measured scour depth versus the scour factor using four scour prediction models considered herein. The probability of exceedance decreases as the scour factor increases. The following is commentary on the results related to each deterministic model considered herein:

For Wilson (1995) model (Figure 6-4a-b), and above a probability of exceedance of 0.001, no discernible difference in the scour factor was noticed for narrow pier, intermediate pier, or wide pier cases. For narrow pier cases, a scour factor of 1.4 needs to be applied to the predicted scour depth to obtain a probability of exceedance in the range of 0.0001 (or 1 in 10,000.) Irrespective of the type of pier (narrow, intermediate, wide), a scour factor of 1.35 ensures a probability of
exceedance of 0.001 for both clear water and live bed conditions when Wilson (1995) model is considered.

For Melville (1997) model, and in clear water condition (Figure 6-4c), the use of a scour factor of 1.0 corresponds to a 50% probability of exceedance for narrow, intermediate, and wide piers. For wide pier cases in clear water condition, the scour factor required to attain a given level of probability of exceedance is less than that required for narrow and intermediate piers. This is due to the fact that the Melville (1997) model, without the application of any factor, is providing a conservative estimate of the scour depth for wide pier compared to narrow and intermediate piers (under clear water conditions). Therefore, to achieve a certain probability of exceedance, the scour factor is less for wide pier compared to the narrow and intermediate piers. For Melville (1997) model, in live bed condition (Figure 6-4d), the use of a scour factor of 1.0 corresponds to 30% probability of exceedance for wide pier and 50% probability of exceedance for narrow, and intermediate piers.

Figure 6-4(e-f), for both clear water and live bed conditions, shows results for HEC 18 (Arneson et al. 2012) model. It appears that among the three categories of piers, to achieve the same level of probability of exceedance, the highest scour factor is needed for the scour depth predicted for narrow piers. This suggest that results from HEC 18 model provide the most conservative estimate for wide pier condition and the least conservative for narrow pier condition. It is important to note that HEC 18 model was developed mostly based on narrow pier \((y/b > 1.4)\) data (Benedict and Knight 2017). Melville and Coleman (2000) reported that for a similar hydraulic condition, the use of HEC 18 for narrow piers will yield a higher scour depth compared to its use for intermediate and wide piers. Accordingly, the scour depth predicted from HEC 18 model is inherently conservative.

For Briaud (2014) model in clear water condition (Figure 6-4g), discernible differences in the scour factor for narrow and intermediate piers are not apparent. In this case, the scour factor corresponding to the wide pier condition is the lowest among the scour factors for narrow, intermediate and wide piers corresponding to a certain probability of exceedance. For Briaud (2014) model, and in live bed condition, the use of a scour factor of 1.0 corresponds to a 22% probability of exceedance for narrow pier and \(~50\%\) probability of exceedance for wide, and intermediate piers under both clear water and live bed conditions.
Reliability index-scour factor relation

Figure 6-5 shows the relationship between $\beta$ and scour factor for the four models considered in the study. As the target $\beta$ increases, the corresponding scour factor increases. Referring to Figure 6-5(a-b), for the Wilson (1995) model, the scour factors corresponding to different reliability indices are almost identical. This is owing to the fact that Wilson model is empirical in nature and considered the $y/b$ as the primary variable controlling the magnitude of scour, as such, only the uncertainty of $y$ had an impact on the $\beta$. This resulted in a maximum scour factor for the Wilson model to be the lowest (1.5) compared to the other three models, where a scour factor as high as 2.95 was computed to attain a $\beta$ of 3.7 as for the Melville model (clear water, intermediate pier—Figure 6-5c).

For Wilson (1995) model (Figure 6a-b), the scour factor corresponding to a $\beta_T = 3.0$ is 1.34 for clear water case and 1.35 for live bed case. For Melville (1997) model in clear water condition (Figure 6-5c), the scour factor corresponding to a $\beta_T = 3.0$ is 2.24, 2.40, and 2.65 for wide pier, narrow pier, and intermediate pier respectively. However, for live bed condition (Figure 6-5d), the scour factor corresponding to a $\beta_T = 3.0$ varies within a narrow range (2.10-2.22). For HEC 18 model, irrespective of the clear water or live bed condition (Figure 6-5e-f), the scour factor corresponding to a $\beta_T = 3.0$ is lowest for wide pier (1.61 and 1.75 for clear water and live bed respectively), and highest for narrow pier (2.05 and 2.1 for clear water and live bed respectively). For Briaud (2014) model (Figure 6-5g-h), the scour factor corresponding to a $\beta_T = 3.0$ is identical (2.4) for narrow and intermediate piers under clear water condition, while for wide pier the scour factor is 1.90.

Sensitivity analyses

Figure 6-6 shows the relationship between $\beta$ and scour factor for different $y/b$ and $F_r$ considering HEC 18 model under clear water narrow pier condition. Figure 6-6(a) shows the 5% error bars from the suggested $\beta$ and scour factor magnitudes (Figure 6-5e). It appears that differences in $y/b$ magnitudes lead to insignificant impact on the computed $\beta$ and scour factor magnitudes, as the responses corresponding to different $y/b$ fall within the bandwidth of ±5% from the base case value of $y/b = 2$. The effect of using a range of Froude number, $F_r$, is minimal as well. As shown in Figure 6-6(b), irrespective of the $F_r$ (0.10-0.80), the error associated with the proposed $\beta$-scour factor magnitude is less than 2% from the base case value of $F_r = 0.25$. 
**Probability of deceedance-reliability index relation**

The scour factor associated with a $\beta$ is correlated to the probability of deceedance obtained from the comparison of measured and the predicted scour depth reported in the database considered. A probability of deceedance (POD) is defined as the probability that the predicted scour depth will be less than the measured scour depth, mathematically which can be expressed in the form of Equation (9).

$$POD = P(y_{sp} \leq y_{sm})$$

Where, $P()$ symbolizes probability. First, the frequency distribution of $\chi_{POD}$, which is the ratio of predicted to the measured scour depth was developed. Using Kernel density estimation, the population probability density function for $\chi_{POD}$ was generated, followed by integration of the density function to obtain the cumulative density function. The POD is obtained by subtracting the cumulative density value from one. Wilson (1995) model, Melville (1997) model, HEC 18 (2012) model, and Briaud (2014) model were applied to live bed local scour measurements reported in the database. POD estimates using the four models are presented in Figure 6-7. The $\beta$ and scour factor relation for narrow pier under live bed condition (refer to Figure 6-5) is presented on the same plot. For illustration, if the HEC 18 (2012) model is considered, (refer to Figure 6-7c), the POD corresponding to a scour factor of one is 0.007, and the associated $\beta$ is zero. To obtain a target $\beta$ of 3.0, the corresponding scour factor would be 2.09, and the extrapolated POD chart suggests the corresponding POD would be 0.0012. It implies that use of a scour factor of 2.09 can reduce the POD by 6 times (to 0.0012) compared to the POD corresponding to the unfactored scour depth estimation (0.007); whereby, a $\beta$ of 3 can be achieved.

**Application examples: LRFD approach for pile foundation considering scour**

**Example 1: Axial loading case**

**Site description**

The example site and relevant parameters are from Briaud et al. (2014) and Yao (2013). The analyses will be focused on a pier with a square cross section with dimensions of $2 \times 2$ m. The pier is placed within a stream in which the flow toward the pier is at skew angle = 0 degree. The stream bed has a median grain size of 0.1 mm. The site soil has an undrained shear strength, $s_u = 40$ kPa. The live load model is an HS-20–44 truck (Paikowsky 2004), which yields $LL_{predicted} = 445$ kN. To keep a consistency with the NCHRP Report 507, Briaud et al. (2014), and Yao (2013), the dead load and live load ratio is chosen = 2; therefore, $DL_{predicted} = 890$ kN.
**Scour estimation**

Step 1: Based on hydraulic and hydrologic studies, determine approach flow depth and approach mean flow velocity using a design return period, as suggested by Arneson et al. (2012). HEC 18 (2012) suggests the use of U.S. Army Corps of Engineers (USACE) Hydrologic Engineering Center’s River Analysis System (HEC-RAS) for this task. For the sake of this example, hydraulic and hydrologic modeling using a return period of 100 years is used which indicated an approach mean flow velocity, \( v \) and flow depth, \( y \) of 2.1 m/sec and 6.10 m respectively.

Step 2: Estimate the mean approach flow velocity at the critical condition, referred to as critical velocity (Equation 10), using the expression suggested by Henderson (1966) and Melville and Sutherland (1988). Compare the critical velocity to the approach mean flow velocity estimated in step 1 to assess whether the prevailing flow condition is clear water or live bed.

\[
v_c = 5.75 u_c^* \log \left( \frac{y}{D_{50}} \right)
\]

and

\[
u_c^* = K_u (0.0377 + 0.041 D_{50}^{1.4}) \quad \text{For } 0.1 \text{ mm} < D_{50} < 1 \text{ mm }
\]

\[
u_c^* = K_u (0.01 D_{50}^{0.5} - 0.0213/D_{50}) \quad \text{For } 1 \text{ mm} < D_{50} < 100 \text{ mm }
\]

Where, \( u_c^* \) is the critical shear velocity in m/s, \( K_u = 0.3048 \), \( D_{50} \) is in mm, and \( v_c \) is the mean approach velocity at the threshold condition (parameters need to be in consistent units in Equation 10). The \( v_c \) was estimated to be < 2.1 m/sec; therefore, live bed upstream condition exists.

Step 3: Compute the equilibrium scour depth using one of the four deterministic models (e.g., Wilson 1995, Melville 1997, HEC 18 2012, Briaud 2014). For example, consider HEC 18 (2012) model presented in Equation (3), Table 6-1:

\[
\frac{y_{sp}}{b} = 2K_1K_2K_3\left(\frac{y}{b}\right)^{0.35}F_r^{0.43}
\]

Where, \( K_1 = 1.1, K_2 = 1.0, K_3 = 1.1, y = 6.10 \text{ m}, F_r = \frac{v}{\sqrt{gy}} = 0.271, v = 2.1 \text{ m/sec}, b = 2 \text{ m.} \) Inputting the numerical values in Equation (3) “HEC-18 model” yields \( y_{sp} = 4.10 \text{ m.} \)

Step 4: Specify the \( \beta \) level to be achieved. Use the appropriate chart from Figure 6-5 to estimate the scour factor based on the following: i. approach flow depth to pier width ratio, ii. the specific analytical model used to compute scour (HEC-18 in this case), and iii. the approach flow condition.
Multiply the scour factor by the deterministic scour depth computed in step 3 to obtain the design scour depth corresponding to the target reliability index ($\beta$).

The approach flow depth to pier width ratio is 3.05, indicating narrow pier condition. Selecting the target reliability indices of 2.0, 2.5, and 3.0, the corresponding scour factors from Figure 6-5f are 1.70, 1.82, and 2.09, respectively. Therefore, the design scour depth corresponding to reliability indices of 2.0, 2.5, and 3.0 are 6.97 m, 7.46 m, and 8.57 m respectively.

**Pile length LRFD (Clay $\alpha$ — API method)**

There are several design methods in geotechnical engineering for assessing undrained axial pile capacity. In this study, the square concrete pile with a width $b$ in clay is designed using the $\alpha$-API method. Based on the $\alpha$-API method, the predicted pile resistance, $R_{predicted}$ is obtained using Equation (13).

$$R_{predicted} = f_u A_f + p_u A_p = \alpha s_u A_f + 9s_u A_p$$ \hspace{1cm} (13)

Where, $A_f$ is side friction area surrounding pile, $A_p$ is tip resistance area, $f_u$ is the unit side friction, $p_u$ is the ultimate bearing pressure, and $\alpha$ is a coefficient considered to be 0.8 in this case study. Paikowsky (2004) reported that for target reliability indices of 2.0, 2.5 and 3.0, the resistance factors, $\varphi$, are 0.60, 0.52, and 0.44, respectively for concrete piles. Based on First Order Reliability Method of analysis, the nominal resistance factor can be estimated as in Equation (14).

$$\varphi = \frac{\gamma_{DL} \times DL_{predicted} + \gamma_{LL} \times LL_{predicted}}{R_{predicted}} = \frac{\gamma_{DL} \times DL_{predicted} + \gamma_{LL} \times LL_{predicted}}{R_{predicted}}$$ \hspace{1cm} (14)

Where, $\gamma_{DL}$= 1.25 is dead load factor, $\gamma_{LL}$ = 1.75 is live load factor (AASHTO 2007).

For a $\varphi$ of 0.60, 0.52, and 0.44, $\frac{R_{predicted}}{LL_{predicted}}$ was estimated from Equation (14) as 7.10, 8.17, and 9.66 respectively. If $L_{no-scour}$ is the pile length required by design to sustain the design load, then for a $\varphi$ of 0.60, 0.52, and 0.44 at no scour condition, the $L_{no-scour}$ can be estimated as 6.70 m, 8.58 m, and 11.20 m respectively (given the example’s parameters). AASHTO (2007) suggests, all material in the scour prism above the total scour line is assumed to be removed and unavailable for axial support. Accordingly, incorporation of scour in the design will update the $R_{predicted}$ per Equation (15).
\[ R_{predicted} = f_u A_f + p_u A_p - 4y_{sp} b f_u \quad (15) \]

In Equation (15), based on AASHTO (2007) recommendation, \( y_{sp} \) is the HEC 18 predicted scour depth. However, as \( y_{sp} \) is not estimated based on a target reliability index, the study herein proposes that \( y_{sp} \) in Equation (15) is replaced by the computed reliability index-based scour depth. Equation (16) is suggested by Briaud et al. (2014) and Yao (2013) to estimate the updated resistance factor for scoured condition. Figure 6-8 shows the pile length normalized with respect to the scour depth obtained from various approaches including the use of three different target reliability indices. The normalized pile length with respect to deterministic scour depth obtained from the method proposed by Briaud et al. (2014) and Yao (2013) was also presented in Figure 6-8 for comparison. If \( \frac{y_{sp}}{L_{no-scour}} \) is greater than 0.2, the resistance factors decrease as \( \frac{y_{sp}}{L_{no-scour}} \) values increase.

\[ \frac{\varphi_{scour}}{\varphi_{no-scour}} = 0.4 \left( \frac{y_{sp}}{L_{no-scour}} \right)^{-0.7} \quad (16) \]

For reliability indices of 2.0, 2.5, and 3.0 the \( \frac{y_{sp}}{L_{no-scour}} \) was estimated as 0.61, 0.48, and 0.37 which leads to resistance factors of 0.340, 0.347, and 0.355 respectively compared to a \( \varphi \) of 0.60, 0.52, and 0.44 suggested by Paikowsky (2004). Figure 6-8 reveals that as the reliability index increases, the required pile length to sustain the design loads increases for both the \( \beta \)-based approach, and AASHTO (2007) recommended approach. The percent increase of pile length to \( y_{sp} \) ratio when the reliability-based scour, with the resistance factors is used compared to using the deterministic scour magnitude with the same resistance factors was estimated to be 26.5-29.6%. When reliability index-based scour estimates considering Melville and Briaud models are used, the pile length to \( y_{sp} \) ratios increase by 37.6-39.8% and 51.3-59.4% respectively, compared to the case when deterministic scour with resistance factors (Paikowsky, 2004) is used. However, for Briaud et al. (2014) and Yao (2013) model, the normalized pile length seems to slightly decrease from 4.93 (87.2% increase compared to the deterministic scour with resistance factor case) to 4.70 (26.5% increase compared to the deterministic scour with resistance factor case) as \( \beta \) increases from 2.0 to 3.0. In this case, it is prudent to apply consistent target reliability index (\( \beta \)) for the resistance factors and the scour factors for consistent analysis and design.
The example problem demonstrated how pile length need to be adjusted while designing for axial capacity of piles for different target reliability indices. Effect of scour hole dimensions, and stress history on the axial response of pile were not considered. Zhang et al. (2017) observed that scouring process caused a reduction of undrained shear strength of the post-scour deposit. Referring to Equations (13-15), reduction in $s_u$ will lead to a reduction in $R_{predicted}$, which will further result in an increase of pile length to $\gamma_{sp}$ ratio for a particular reliability index. It is further expected that the effect of scour width and scour hole slope angle will have insignificant effects on the axial response of a pile.

**Example 2: Lateral loading case**

**Site description**

The example site and analysis parameters are from Matlock (1970), Zhang et al. (2017), and Lin et al. (2014). The laterally loaded pile test was conducted in Lake Austin, Texas, and was considered as the base case to demonstrate the effect of scoured soil condition. The case is used herein to demonstrate the application of the scour factors in concert with the LRFD approach. Zhang et al. (2017) and Lin et al. (2014) considered the Lake Austin test results as the base case and imposed scoured conditions to assess the effect of stress history on the ultimate soil resistance and lateral pile head deflection. The site soils are classified as CH per Unified Soil Classification System (USCS) with the properties listed in Table 6-8. The water table is located 0.06 m above the ground level. The properties of the test pile are given in Table 6-9 and depicted in Figure 6-9.

**Scour estimation**

The scour depth is estimated by using the steps demonstrated in Example 1. For illustration purpose, it is assumed that the hydraulic, hydrologic, geotechnical, and structural parameters are such that the scour depth using HEC 18 (2012) model is 0.76 m, which is 2.4 times the pile outer diameter, D (i.e., 2.40D). It is further assumed that a narrow pier and live bed conditions are prevalent. For target reliability indices $= 2.0$, 2.5, and 3.0, the corresponding scour factors, from Figure 6-5f, are 1.7, 1.82, and 2.09, respectively. Therefore, the design scour depth corresponding to reliability indices of 2.0, 2.5, and 3.0 are 1.30 m (4.08D), 1.39 m (4.37D), and 1.60 m (5.00D) respectively.
Lateral load analysis (Matlock method)

The p-y method proposed by Matlock (1970) is used to analyze the behavior of laterally loaded piles in soft clay. FB-Multipier, a commercial software for bridge analyses, was used to perform the lateral load analysis using Matlock (1970) method. Zhang et al. (2017) performed numerical simulation using commercial software LPILE, where their model was verified with the field test case considered herein (Lake Austin, Texas). Zhang et al. (2017) further reported the p-y response at a depth of 0.5D below the post scour ground surface. The p-y data from Zhang et al. (2017) are presented in Figure 6-10. The p-y curve for the initial case (pre-scour) is presented on the same plot. The p-y curves obtained from FB-Multipier modeling and LPILE modeling by Zhang et al. (2017) show satisfactory agreement. Figure 6-10 further shows that a scour magnitude of 5.00D (the scour depth used by Zhang et al. 2017) causes a 47.5% decrease of the ultimate soil resistance compared to the ultimate lateral resistance (p) obtained from pre-scour ground level.

Pile head deflection

The lateral pile head deflection under the applied lateral load was computed for the pre-scour case and compared to field measurements reported in Matlock (1970) to check whether the modeling approach is capable of producing the field data as means for validating the analytical model. Figure 6-11 shows the relationship between the lateral load at the pile head and the lateral pile head deflection. As shown, a satisfactory agreement is obtained between the field measurements and the FB-Multipier modeling results. The lateral pile head deflection for the scoured cases are also presented in Figure 6-11. The LPILE modeling response reported by Zhang et al. (2017) shows good agreement with the FB-Multipier modeling results for scour magnitude of 5.00D; with the maximum discrepancy being 7% at a load magnitude of 100 kN. Data in Figure 6-11 further suggest that for a given lateral load at the pile head, as the scour factor increases, the lateral pile head deflection increases in a non-linear fashion. For a β of 2.0, 2.5, and 3.0 (Scour depth of 4.08D, 4.37D, and 5.00D respectively), the increase in lateral pile head deflection compared to the deterministically estimated scour depth case (Scour = 2.40D) is computed to be 46%, 68%, and 132% respectively at a load magnitude of 100 kN. Referring to Figure 6-7c, the deterministic scour depth (Scour factor = 1.00), is associated with a POD of 0.007, and a reliability index of zero.
Past literature (e.g., Robinson et al. 2006; Robinson et al. 2012) consider 25 mm as an allowable lateral deflection for bridge piles at the pile head. Figure 6-11 shows that for an allowable deflection of 25 mm, the allowable lateral load at the pile head for the deterministically estimated scour case is 47 kN. Haldar and Basu (2014) developed resistance, dead load and live load factors for laterally loaded pile in clay using a probabilistic analysis. They reported that for an allowable deflection of 15 mm, a dead load to live load ratio of 2.0, and a $\beta$ of 3.0, resistance factor, $\varphi$ is 0.22, while dead and live load factors ($\gamma_{DL}$ and $\gamma_{LL}$) are 1.12 and 1.07 respectively. Haldar and Basu (2014) further reported that the resistance and load factors are not impacted by the allowable deflection, rather depend on the target reliability index. As such, for an allowable deflection of 25 mm, and a target $\beta$ of 3.0, the resistance and load factors are related per Equation (14).

From Equation (14), $\frac{R_{predicted}}{L_{L predicted}}$ can be estimated to be 15.045 corresponding to the lateral load at the pile head for the deterministically estimated scour case (47 kN). Therefore, the maximum live load exerted on the pile is 3.12 kN. Based on the results reported herein, deterministic scour depth (Scour factor = 1.00), is associated with a reliability index of zero. If the reliability index-based scour depth is used, and for $\frac{R_{predicted}}{L_{L predicted}} = 15.045$, the scour depth corresponding to $\beta =$3.0 is 5.00D (1.60 m). Subsequently, pile diameter and length need to be adjusted. For the study herein, the pile length is kept constant, while the pile diameter was increased as shown in Figure 6-12. In this case, the relationship between the lateral load at the pile head with pile diameter for $\beta =$3.0 is shown and the data suggest that to obtain $\beta =$3.0, the pile diameter needs to be increased to 0.433 m, which is 35.7% more than for the base case pile diameter (0.319 m).

The example problem on the laterally loaded pile demonstrated how the pile diameter needs to be adjusted while designing the lateral capacity of piles for a target reliability index. While the effect of scour hole dimensions, and stress history on the lateral behavior of the pile are outside the scope coverage herein, Zhang et al. (2017) observed that increase of scour width and scour hole slope angle resulted in a reduced surrounding soil resistance, and increased pile head deflection for piles installed in soft clay. Lin et al. (2014b) and Zhang et al. (2017) further indicated that consideration of stress history, in terms of increase in over-consolidation ratio at a given depth is due to the lower effective stresses in the remaining soil profile with the occurrence of soil
scouring resulted in a reduction in surrounding soil resistance, and increased pile head deflection for piles installed in soft clay. Zhang et al. (2017) found that scour depth is the most important factor among the various scour hole dimensions (scour depth, scour width, and scour hole slope angle) affecting the lateral response of piles. In parallel, consideration of stress history will result in an increased pile diameter (compared to the case of non-consideration of stress history) for the reliability-index based design framework presented herein.

**Example 3: Combined lateral and axial loading case**

**Site description:**

The example site are parameters are similar to Example 2, however, in addition to the lateral loading, axial DL and LL of 60 kN and 30 kN respectively are assumed to be acting on the top of the pile head. The initial pile diameter was 0.319 m. Scour analyses suggested that the design scour depth corresponding to $\beta = 3.0$ is 1.60 m (5.00D). Based on the laterally loaded pile analyses, it was concluded that to obtain $\beta = 3.0$, the pile diameter $D$ needs to be increased to 0.433 m from a base case $D = 0.319$ m.

**Axial capacity analysis**

Considering the resistance factors, $\varphi$, of 0.60 for a target reliability index of 3.0 (Paikowsky 2004), $R_{\text{predicted}}^{\text{LL}}$ was estimated from Equation (14) as 9.66. Using Equation (13) for the estimation of $R_{\text{predicted}}^{\text{LL}}$, the length of pile needed at no scour condition, $L_{\text{no-scour}}$ is estimated = 7.38 m. For a circular pile, Equation (15) can be updated as in Equation (17).

$$R_{\text{predicted}} = f_u A_f + p_u A_p - \pi y_{sp} D f_u$$  \hspace{1cm} (17)

Using Equation (17), to obtain $\beta = 3.0$, the pile length needs to be 9.55 m. However, based on lateral capacity analyses (presented in Example 2), to obtain $\beta = 3.0$, the pile length and diameter needed to be 12.80 m, and 0.433 m respectively. Therefore, no further refinement of the pile length and diameter is needed as the dimensions satisfy both the axial and lateral capacity corresponding to $\beta = 3.0$.

**Design check**

The axial and lateral loads were applied on the pile head and the lateral loading analyses were conducted to observe the changes in the pile response. Figure 6-13 shows the relationship between the lateral load at the pile head with lateral pile head deflection at different scour depth
based on the reliability index. Observation of Figure 6-13 reveals that for an allowable deflection of 25 mm, the allowable lateral load at the pile head for the deterministically estimated scour case is 47 kN. Using Equation (14) and considering the resistance factor ($\varphi = 0.22$) and load factors ($\gamma_{DL} = 1.12$ and $\gamma_{LL} = 1.07$) proposed by Haldar and Basu (2014), $R_{predicted_{LL, predicted}}$ can be estimated as 15.045. The pile length was kept constant ($L = 12.80$ m), while the pile diameter was increased until an allowable deflection of 25 mm for $\frac{R_{predicted}}{LL_{predicted}} = 15.045$ was achieved. Figure 6-13 suggests that the pile diameter needs to be increased to 0.43 m to achieve $\beta = 3.0$. Therefore, the pile length of 12.80 m, and pile diameter of 0.433 m satisfy both the axial and lateral capacity corresponding to $\beta = 3.0$. Another step of checking the design is to apply the lateral load and axial dead load only, which in this present problem did not affect the lateral load-pile head deflection chart.

Example 3 demonstrated for a target reliability index, how pile dimensions need to be adjusted when a pile is subjected to both axial and lateral loads. The example shows only one solution approach; however, it is possible to further optimize the pile dimensions. If it happened that the pile length needed to satisfy $\beta = 3.0$ is more than 12.80 m (corresponding to $D = 0.433$ m), the pile length needed to be increased, followed by conducting lateral load analyses per illustrated in Example 2. The pile length will then be checked for the axial capacity. The iteration will be continued until both the axial and lateral capacity are assessed and satisfied (corresponding to a certain reliability index) for the recommended pile dimension.

**Summary and Conclusion**

A reliability-based pier scour estimation methodology was presented accounting for input parameter uncertainty and inherent model bias. The methodology is applicable for local scour calculation in riverine flow under clear water and live bed condition using four deterministic scour prediction models: Wilson (1995) model, Melville (1997) model, HEC-18 (2012) model, and Briaud (2014) model. The integration of the computed scour magnitudes corresponding to a given reliability level with the LRFD approach is demonstrated through two examples of an axially loaded and laterally loaded pile, respectively, in clay. Based on the results obtained herein, the following conclusions are advanced:

1. A relationship between reliability index and scour factor is devised. The proposed relationship is dependent on the pier type (e.g., narrow pier, intermediate pier, and wide
pier), upstream sediment transport condition (e.g., clear water and live bed), and the deterministic model being used (e.g., Wilson (1995) model, Melville (1997) model, HEC-18 (2012) model, and Briaud (2014) model).

2. The probability of exceedance (the probability that a given scour depth will be exceeded in a future time) decreases as the scour factor increases. The relationship between the probability of exceedance and scour factor is dependent on upstream sediment transport condition, pier type and the deterministic model considered.

3. The scour factor associated with a given $\beta$ is correlated to the probability of deceedance, POD (the probability that the predicted scour depth will be less than the measured scour depth) obtained from the comparison of the predicted versus the measured scour depth per the utilized database herein. It was demonstrated for HEC 18 model application in live bed condition, and to obtain a target $\beta$ of 3.0, the scour factor would be 2.09, which is corresponding to a POD of 0.0012 (12 in 10,000). This indicates a reduction of POD by 6 times compared to the POD corresponding to the unfactored scour depth estimation.

4. Based on the axial pile design example and considering $\alpha$-API method, as the reliability index increases, the required pile length to sustain the design loads increases for both the $\beta$ – based scour estimates and the AASHTO (2007) recommended LRFD approach where deterministic HEC 18-computed scour depth is used. The increase of pile length/$y_{sp}$ ratio when the reliability-based scour depth is used along with the resistance factors and compared to using the deterministic scour with resistance factors, was estimated to be 26.5-29.6%. The $\beta$-based approach for scour magnitude assessment is recommended as it offers consistency in terms of utilizing the AASHTO soil resistance factor while estimating the scour magnitude based on the same reliability level.

5. In the case of the lateral pile analysis design example, and as the reliability index increases, the lateral pile head deflection increases by 108%-348% compared to the deterministically estimated scour depth case. Furthermore, it is demonstrated that to obtain a $\beta = 3.0$ while the pile length is maintained unchanged, the pile diameter needs to be increased by 35.7% as compared to the base case pile diameter in which deterministic scour magnitude is used.

6. Approach has been presented on how pile dimensions need to be adjusted when a pile is subjected to both axial and lateral loads. Using the reliability-based approach proposed herein, it was shown that the pile dimension can be adjusted while satisfying both the axial
and lateral capacity of the pile (corresponding to a certain reliability index). The design was further checked to withstand the lateral load and axial dead load.

The proposed method will help mitigate the inconsistency posed by adopting a β-based approach for the design of various superstructure bridge components as opposed to a deterministic approach for assessing the scour magnitude at the foundation system. Having a uniform level in the reliability index of the bridge components will facilitate the development of integral risk-based design approach for bridge structures.

**Data availability statement**

All data, models, and code generated or used during the study appear in the submitted article.

**Acknowledgements**

Funding from North Carolina Department of Transportation (NCDOT) is gratefully acknowledged. Any conclusions, findings, opinions, and recommendations expressed in this article are those of the authors and do not necessarily reflect the views of NCDOT.
Figure 6-1. Error box plot for the four models considered in the study (Data: Narrow pier under clear water condition).
Figure 6-2. Statistical distribution of bias factor, $\lambda$ for (a) Wilson, (b) Melville, (c) HEC 18, and (d) Briaud models for narrow pier under live bed condition.
Figure 6-3. Relationship between probability of exceedance and number of simulation cycles when HEC 18 model is used (SF stands for scour factor).
Clear water

Live bed

Melville (1997)
Clear water

Melville (1997)
Live bed
(e) HEC 18 (2012) Clear water

(f) HEC 18 (2012) Live bed

(g) Briaud (2014) Clear water

(h) Briaud (2014) Live bed
Narrow pier (y/b > 1.4)
Intermediate pier (0.2 < y/b < 1.4)
Wide pier (y/b < 0.2)

(b) Wilson (1995) Live bed
Narrow pier (y/b > 1.4)
Intermediate pier (0.2 < y/b < 1.4)
Wide pier (y/b < 0.2)

(c) Melville (1997) Clear water
Narrow pier (y/b > 1.4)
Intermediate pier (0.2 < y/b < 1.4)
Wide pier (y/b < 0.2)

(d) Melville (1997) Live bed
Narrow pier (y/b > 1.4)
Intermediate pier (0.2 < y/b < 1.4)
Wide pier (y/b < 0.2)
Reliability index

Scour factor

Narrow pier (y/b>1.4)
Intermediate pier (0.2<y/b<1.4)
Wide pier (y/b<0.2)

HEC 18 (2012)
Clear water

Briaud (2014)
Clear water

Briaud (2014)
Live bed

Live bed
Figure 6-6. Sensitivity of reliability index when the Monte Carlo simulation is run under different (a) \( \frac{y}{b} \), and (b) \( F_r \) for HEC 18 (2012) model in clear water narrow pier condition. Note: \( y \), \( b \) and \( F_r \) stand for approach flow depth, pier width, and Froude number respectively.
Figure 6-7. Relationship among probability of deceedance, reliability index, and scour factor when (a) Wilson (1995) model, (b) Melville (1997) model, (c) HEC 18 (2012) model, and (d) Briaud (2014) model is applied to estimate scour at narrow pier ($y/b > 1.4$) under live bed condition. Note: $y$, and $b$ stand for approach flow depth, and pier width respectively.
Figure 6-8. Relationship between pile length to deterministic scour depth ratio and the reliability index based on HEC 18 model, reliability index-based model proposed herein and Briaud et al. (2014) and Yao (2013) model ($y_{sp}$ is the predicted scour depth from deterministic HEC 18 model).
Figure 6-9. Soil and pile profiles considered for lateral load analyses (Data from Matlock 1970). Note: $L$, $D$, $F_{top}$, and $y_{sp}$ are pile length, pile diameter, lateral load at pile head, and predicted scour depth respectively.
Figure 6-10. The p-y curve (per Matlock 1970) at a depth of half pile diameter (0.5D) below the post scour ground surface. LPILE modeling results are from Zhang et al. (2017).

Figure 6-11. Relationship between the lateral load at the pile head with lateral pile head deflection at different scour depth based on the reliability index. Measured data are from Matlock (1970), and LPILE data are from Zhang et al. (2017). Note: Scour = 2.40D, 4.08D, 4.37D, and 5.00D are corresponding to reliability indices of 0.0, 2.0, 2.5, and 3.0 respectively.
Figure 6-12. Relationship between the lateral load at the pile head with pile diameter based on a target reliability index of 3.0. The lateral pile head deflection is 25 mm.

Figure 6-13. Relationship between the lateral load at the pile head with lateral pile head deflection at different scour depth based on the reliability index. Note: Scour = 2.40D, and 5.00D are corresponding to reliability indices of 0.0, and 3.0 respectively.
Table 6-1. Deterministic pier scour model equations considered in the present study (Shahriar et al. 2021b).

<table>
<thead>
<tr>
<th>Reference</th>
<th>Model</th>
<th>Equation No.</th>
<th>Remarks</th>
</tr>
</thead>
</table>
| Wilson (1995)   | \( \frac{y_{sp}}{b'} = 0.9\left(\frac{y}{b'}\right)^{0.4} \) | (1)          | \( b' \) = projected pier width  
\( y_{sp} \) = predicted scour depth  
\( y \) = flow depth upstream of pier |
| Melville (1997) | \( y_{sp} = K_{yb}K_{I}K_{d}K_{s}K_{g}K_{G} \) | (2)          | \( K_{yb} \) = pier depth-size factor  
\( K_{I} \) = flow intensity factor  
\( K_{d} \) = sediment size factor  
\( K_{s} \) = pier nose shape factor  
\( K_{g} \) = pier alignment factor  
\( K_{G} \) = channel geometry factor (= 1 for pier) |
| Arneson et al.  | \( \frac{y_{sp}}{b} = 2K_{1}K_{2}K_{3}\left(\frac{y}{b}\right)^{0.35}F_{r}^{0.43} \) | (3)          | \( K_{1} \) = pier nose shape factor  
\( K_{2} \) = pier alignment factor  
\( K_{3} \) = bed condition factor  
\( b \) = pier width  
\( F_{r} \) = Froude number |
| Briaud (2014)   | \( \frac{y_{sp}}{b'} = 2.2K_{pw}K_{psh}K_{pa}K_{psp}(2.6F_{pier} - F_{c(pier)})^{0.7} \) | (4)          | \( K_{pw} \) = water depth influence factor  
\( K_{psh} \) = pier shape influence factor  
\( K_{pa} \) = aspect ratio influence factor  
\( K_{psp} \) = pier spacing influence factor  
\( F_{pier} \) = pier Froude number  
\( F_{c(pier)} \) = critical pier Froude number |
Table 6-2. Mean, Standard deviation (SD) and Coefficient of variation (COV) of the parameters for clear water laboratory data considered in the study.

<table>
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<th>(b/D_{50}) SD</th>
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| Reference                        | \(v\) Mean (m/s) | \(v\) SD (m/s) | \(v\) COV | \(y\) Mean (m) | \(y\) SD (m) | \(y\) COV | \(b/D_{50}\) Mean | \(b/D_{50}\) SD | \(b/D_{50}\) COV | \(v/v_c\) Mean | \(v/v_c\) SD | \(v/v_c\) COV | \(y_s/b\) | \(y_s/b\) Mean | \(y_s/b\) SD | \(y_s/b\) COV |
| Coleman (unpublished)            | 0.31             | 0.0            | 0.0     | 0.26           | 0.0         | 0.0     | 911               | 0.0            | 0.0            | 0.9           | 0.0          | 0.0         | 0.4     | 0.0           | 0.0         |
| Ettema (1980)                    | 0.31             | --             | --      | 0.02           | --          | --      | 53                | --             | --             | 0.8           | --          | --          | 0.7     | --            | --          |
| Melville (1997)                  | 0.33             | --             | --      | 0.10           | --          | --      | 901               | --             | --             | 0.9           | --          | --          | 0.6     | --            | --          |
| Sheppard et al. (2004)           | 0.50             | --             | --      | 0.17           | --          | --      | 316               | --             | --             | 0.7           | --          | --          | 0.8     | --            | --          |

Note: \(v\), \(y\), \(D_{50}\), \(v_c\), and \(y_s\) mean approach mean flow velocity, flow depth, median grain size, sediment critical velocity, and measured scour depth respectively.
Table 6-3. Mean, Standard deviation (SD) and Coefficient of variation (COV) of the parameters for live bed laboratory data considered in the study.

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<td>SD (m/s)</td>
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<td>Mean (m)</td>
<td>SD (m)</td>
<td>COV</td>
<td>Mean</td>
<td>SD</td>
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<th>𝑣/𝑣ₖ</th>
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Note: 𝑣, 𝑦, 𝐷₅₀, 𝑣ₖ, and 𝑦ₛ mean approach mean flow velocity, flow depth, median grain size, sediment critical velocity, and measured scour depth respectively.
Table 6-4. List of outliers that are identified using the concept of error boxplot.

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<th>(b) (m)</th>
<th>(v) (m/s)</th>
<th>(v_c) (m/s)</th>
<th>(y) (m)</th>
<th>(D_{50}) (mm)</th>
<th>(\sigma_g)</th>
<th>Duration of flow (minutes)</th>
<th>(y/b)</th>
<th>(F_r)</th>
<th>(v/v_c)</th>
<th>(b/D_{50})</th>
<th>(y_s/b)</th>
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<td>--</td>
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<td>0.96</td>
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<tr>
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Note: \(b\), \(v\), \(v_c\), \(y\), \(D_{50}\), \(\sigma_g\), \(F_r\) and \(y_s\) are pier width, approach mean flow velocity, sediment critical velocity, approach mean flow depth, median grain size, sediment uniformity, Froude number, and measured scour depth respectively.
Table 6-5. List of outliers that are identified using the concept of error standard deviation.

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<th>$y$ (m)</th>
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<th>$\sigma_g$</th>
<th>Duration of flow (minutes)</th>
<th>$y/b$</th>
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</tr>
</tbody>
</table>

Note: $b$, $v$, $v_c$, $y$, $D_{50}$, $\sigma_g$, $F_r$, and $y_s$ are pier width, approach mean flow velocity, sediment critical velocity, approach mean flow depth, median grain size, sediment uniformity, Froude number, and measured scour depth respectively.
Table 6-6. Percentage of data points eliminated for each category of pier.

<table>
<thead>
<tr>
<th>Pier type</th>
<th>Upstream flow condition</th>
<th>Percentage of data screened out</th>
</tr>
</thead>
<tbody>
<tr>
<td>Narrow ((y/b &gt; 1.4))</td>
<td>Clear water</td>
<td>7.0</td>
</tr>
<tr>
<td>Intermediate ((0.2 \leq y/b \leq 1.4))</td>
<td>Clear water</td>
<td>25.3</td>
</tr>
<tr>
<td>Wide ((y/b &lt; 0.2))</td>
<td>Clear water</td>
<td>0.0</td>
</tr>
<tr>
<td>Narrow ((y/b &gt; 1.4))</td>
<td>Live bed</td>
<td>9.3</td>
</tr>
<tr>
<td>Intermediate ((0.2 \leq y/b \leq 1.4))</td>
<td>Live bed</td>
<td>24.2</td>
</tr>
<tr>
<td>Wide ((y/b &lt; 0.2))</td>
<td>Live bed</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Note: \(y\) and \(b\) denote approach flow depth and pier width respectively.
Table 6-7. Distribution properties of utilized deterministic and random variables.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Clear water</th>
<th>Live bed</th>
<th>Statistical distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>COV</td>
<td>Mean</td>
</tr>
<tr>
<td>Pier width</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Narrow pier</td>
<td>0.18 m</td>
<td>0.18 m</td>
<td></td>
</tr>
<tr>
<td>Intermediate pier</td>
<td>0.46 m</td>
<td>0.46 m</td>
<td></td>
</tr>
<tr>
<td>Wide pier</td>
<td>2.13 m</td>
<td>2.13 m</td>
<td></td>
</tr>
<tr>
<td>Pier face shape</td>
<td>Circular</td>
<td>Circular</td>
<td></td>
</tr>
<tr>
<td>Skew angle</td>
<td>0 degree</td>
<td>0 degree</td>
<td></td>
</tr>
<tr>
<td>Aspect ratio (Length to width)</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Flow velocity</td>
<td>0.47 m/s</td>
<td>0.186</td>
<td>0.47 m/s</td>
</tr>
<tr>
<td>Flow depth</td>
<td>0.38 m</td>
<td>0.2</td>
<td>0.38 m</td>
</tr>
<tr>
<td>Median grain size</td>
<td>1.18 mm</td>
<td>0.081</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>Bias factor</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Narrow pier</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wilson (1995)</td>
<td>0.75</td>
<td>0.45</td>
<td>1.11</td>
</tr>
<tr>
<td>Melville (1997)</td>
<td>0.69</td>
<td>0.31</td>
<td>0.68</td>
</tr>
<tr>
<td>HEC 18 (2012)</td>
<td>0.71</td>
<td>0.3</td>
<td>0.67</td>
</tr>
<tr>
<td>Briaud (2014)</td>
<td>0.8</td>
<td>0.37</td>
<td>0.73</td>
</tr>
<tr>
<td>Intermediate pier</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wilson (1995)</td>
<td>1.42</td>
<td>0.35</td>
<td>1.36</td>
</tr>
<tr>
<td>Melville (1997)</td>
<td>0.66</td>
<td>0.27</td>
<td>0.61</td>
</tr>
<tr>
<td>HEC 18 (2012)</td>
<td>0.95</td>
<td>0.26</td>
<td>0.71</td>
</tr>
<tr>
<td>Briaud (2014)</td>
<td>1.1</td>
<td>0.3</td>
<td>0.89</td>
</tr>
<tr>
<td>Wide pier</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wilson (1995)</td>
<td>2.051</td>
<td>0.1</td>
<td>1.33</td>
</tr>
<tr>
<td>Melville (1997)</td>
<td>0.78</td>
<td>0.18</td>
<td>0.76</td>
</tr>
<tr>
<td>HEC 18 (2012)</td>
<td>1.16</td>
<td>0.17</td>
<td>1.13</td>
</tr>
<tr>
<td>Briaud (2014)</td>
<td>1.4</td>
<td>0.2</td>
<td>1.18</td>
</tr>
</tbody>
</table>
Table 6-8. Properties of soft clay considered in the study (Data from Matlock 1970, Lin et al. 2014, and Zhang et al. 2017).

<table>
<thead>
<tr>
<th>Item</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective unit weight, $\gamma_{eff}'$</td>
<td>kN/m$^3$</td>
<td>10</td>
</tr>
<tr>
<td>Water content, $\omega$</td>
<td>%</td>
<td>44.5</td>
</tr>
<tr>
<td>Compression index, $C_c$</td>
<td></td>
<td>0.40</td>
</tr>
<tr>
<td>Swell index, $C_{ur}$</td>
<td></td>
<td>0.08</td>
</tr>
<tr>
<td>Undrained shear strength, $s_u$</td>
<td>kN/m$^2$</td>
<td>32.3</td>
</tr>
<tr>
<td>Strain at 50% of the maximum stress, $\varepsilon_{50}$</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu$</td>
<td></td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 6-9. Properties of pile considered in the study (Data from Reese and van Impe 2001).

<table>
<thead>
<tr>
<th>Item</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, $L$</td>
<td>m</td>
<td>12.8</td>
</tr>
<tr>
<td>Outer diameter, $D$</td>
<td>m</td>
<td>0.319</td>
</tr>
<tr>
<td>Thickness, $t$</td>
<td>m</td>
<td>0.0127</td>
</tr>
<tr>
<td>Moment of inertia, $I_p$</td>
<td>m$^4$</td>
<td>1.44×10$^{-4}$</td>
</tr>
<tr>
<td>Elastic modulus, $E_p$</td>
<td>kN/m$^2$</td>
<td>2.18×10$^8$</td>
</tr>
<tr>
<td>Yielding moment, $M_y$</td>
<td>kN-m</td>
<td>231</td>
</tr>
</tbody>
</table>
Chapter 7: Local scour around bridge abutments

This chapter is proposed to be submitted for publication as:

Abstract

More than 80 percent of the bridges in the United States are built over waterways. Support system of the structures crossing waterways are subjected to scour during their design life owing to the flowing water-induced bed shear stresses, resulting in scour. Work herein is focused on assessing the performance of abutment scour assessment methodology by considering three abutment scour prediction models suggested by Hydraulic Engineering Circular No. 18. An abutment scour database is utilized to quantify the predicted and measured scour depth relationship. Abutment scour prediction models are assessed in terms of two statistical parameters, termed herein Mean Absolute Percentage Error (MAPE, as a measure of accuracy of the prediction), and Level of conservatism, percentage of cases the predicted scour exceeded the measured scour (as a measure of conservativeness of the model). For vertical wall and spill through abutments, responses for long abutment, and intermediate abutment are examined separately. For vertical wall abutments, conservatism ranged from 4.76% to 100%, and MAPE ranged from 44% to 201%. For spill through abutments, conservatism ranged from 0% to 100%, and MAPE ranged from 10.3% to 347%.

Keywords: Abutment scour, MAPE, Conservatism, HEC 18.
Introduction

Records from the National Bridge Inventory (NBI) suggest that more than 80% of the total 583,000 bridges in the United States are built over waterways. Support system of structures crossing the waterways are subjected to scour during their design life owing to the flowing water-induced bed shear stresses. Scour around existing bridge foundation has been a primary cause of bridge collapse in the United States and worldwide (Melville and Coleman, 2000; Liang et al. 2015; Qi et al. 2016). Two types of local scour processes are predominantly observed, viz. pier scour and abutment scour. Abutment scour occurs when the abutment and roadway embankment obstruct the flow. The flow obstructed by the abutment and roadway embankment accelerates and forms vortex at the upstream side of the embankment that runs through the toe of the abutment, followed by wake vortex formation at the downstream end of the abutment (e.g., Arneson et al. 2012,) with past research conducted to comprehend the vortex formation steps and its effect on the scour magnitude (Arneson et al. 2012, Ettema et al. 2017). The National Bridge Inspection Standards (NBIS) regulation suggests that bridge owners identify bridges that are scour susceptible and further suggests preparing the plan of action to address potential deficiencies. Despite such stringent regulations, in a period of 10 years (from 2001 to 2011), the percentage of scour critical bridges has been reduced by merely 0.5% (from 5.2 to 4.7%). Bridge scour evaluation program, in 2011 reported that there are 23,034 bridges that are scour critical (Arneson et al. 2011).

Ettema et al. (2011) explained that the state-of-the-art of abutment scour prediction is less advanced than pier scour prediction. Pier scour studies have covered deterministic (e.g., Melville 1997, Arneson et al. 2012), probabilistic (e.g., Lagasse et al. 2013, Shahriar et al. 2021a-b), and observational (e.g., Govindasamy et al. 2013) aspects, while abutment scour analyses are still deterministic in nature. Although deterministic scour prediction models seem easy to apply, Ettema et al. (2011) and Arneson et al. (2012) suggested that abutment scour prediction models do not take into account the physical processes involved. Furthermore, for nearly 15 abutment scour prediction models available in the literature, no specific information on their inherent degree of conservatism/un-conservatism is available. Therefore, there is a necessity to comprehend the inherent conservatism/un-conservatism of the deterministic models used to estimate abutment scour.

Work herein is focused on comparing the predicted and the measured abutment scour depth using a database collected from the literature. Three abutment scour prediction models suggested
Scour prediction models

For estimating scour at bridge abutments, HEC 18 (2012) suggested use of three models, including Froehlich (1989) model, HIRE (2001) model, and NCHRP (2010) model. Froehlich (1989) model, shown in Equation 1 (Table 7-1), was developed based on 170 abutment scour measurements in laboratory flumes. HIRE (2001) model was developed based on field data of scour at the end of spurs in the Mississippi river. HEC 18 (2012) suggested that the HIRE (2001) model, shown in Equation 2-Table 7-1, is applicable mostly for abutments where the abutment length (L) to the flow depth (y) ratio is greater than 25. The abutment scour assessment procedure developed by Ettema et al. (2010) as a part of NCHRP 24-20 (2010) argue the importance of considering how abutments are built. All the abutment scour prediction formulae, except for Ettema et al. (2010), consider the abutment as a pier-like structure, extending as solid forms deeply into the bed. Ettema et al. (2010) suggested that the maximum scour depth that is attainable at an abutment is limited by the geotechnical stability of the embankment at the abutment. Ettema et al. (2010) developed the abutment scour model for a range of abutment types, abutment locations, and sediment transport conditions. One of the basic differences between Ettema et al. (2010) model and the other models available for abutment scour calculation is Ettema et al. (2010) considered the potential maximum scour depth near an abutment can be expressed in terms of an amplified contraction scour. Ettema et al. (2010) also identified three scour conditions (e.g., scour conditions A, B, and C) depending on the developed flow field at an abutment. Scour condition A is used when the abutment is in or close to the main channel, scour condition B is considered when the abutment is set back from the main channel, and scour condition C is selected when the approach embankment is breached. Specific site conditions necessary to identify the scour condition can be
found in Ettema et al. (2010) and HEC 18 (2012). The Ettema et al. (2010) model can be expressed as shown in Equation 3 through Equation 6 (Table 7-1).

An investigation by Ettema et al. (2011) classified factors affecting abutment scour into five categories. Table 7-2 shows the parameter group and associated parameter names as suggested by Ettema et al. (2011). The various deterministic scour prediction models were focused on different datasets, thereby, the performance of a given model to a common set of data will be different. Table 7-3 presents the models suggested by HEC 18 (2012) and list of parameters considered in developing those models.

Database for analyses

Table 7-4 shows a summary of the laboratory data sources available in literature. Important to note that the list of laboratory data sources presented in Table 7-4 is not exhaustive; all the data sources presented in Table 7-4 are from testing conducted in a rectangular channel. The tests performed by Sturm (2004) focused however on compound channel hydraulics, a more representative scenario compared to the rectangular channel hydraulic. As such, test data by Sturm (2004) are utilized herein. Sturm (2004) conducted the experiments in a 4.2-m-wide by 24.4-m-long flume of a fixed slope. Figure 7-1 shows the compound channel configuration used in scour experiments by Sturm (2004). Scour depths were measured as a function of discharge, sediment size, and abutment shape and length for two different compound-channel configurations constructed. Tests were conducted for vertical wall and spill through abutments.

Melville (1992) suggested that abutment scour can be classified into three categories based on the abutment length ($L$) to flow depth ($y$) ratio. These are (a) Long abutment ($L/y > 25$), where scour depth is proportional to flow depth, (b) Intermediate abutment ($1 \leq L/y \leq 25$), where scour depth is dependent on both flow depth and embankment length, and (c) Short abutment ($L/y < 1$), where scour depth is proportional to embankment length. The Sturm (2004) database was classified and analyzed separately for long, intermediate and short abutments.

Analyses

Predicted-measured scour depth relation

Vertical wall abutment

Figure 7-2 (a, b, c, d, e, f, g) shows the relationship between the predicted and the measured scour depth for the Vertical Wall (VW) abutments using the three models considered herein. Figure
suggests that in general for long abutments, Froehlich (1989) model provides a more conservative estimate of scour depth compared to HIRE (2001) model. Although Froehlich (1989) model did not underpredict the measured scour depth, the HIRE (2001) model underpredicted the measured scour depth in two occasions. The underpredicted two occasions were corresponding to long abutment in clear water condition (shown in Figure 7-2d). HEC 18 (2012) suggested that HIRE (2001) model be only used for estimating scour of long abutments. Accordingly, the scour estimations were compared only for long abutments. The application of NCHRP (2010) model is based on the location of embankment in relation to the floodplain. Ettema et al. (2010) suggested that if the projected embankment length \( L \) is 75 percent or greater than the floodplain width \( B \), live bed conditions prevail, while for the cases when \( L/B < 75\% \), clear water conditions exist. Data in Figure 7-2(f) suggest that for \( L/B \geq 75\% \) cases, NCHRP (2010) model under-predicted the scour depth for 2 out of 10 data points. Observation of Figure 7-2(g) suggest that for the \( L/B < 75\% \) cases, NCHRP (2010) model significantly under-predicts the scour depth. Referring to Figure 7-2(g), on an average, the predicted scour depth is 0.45 times the measured scour depth for \( L/B < 75\% \) case. Comparison of Figures 7-2(f) and 7-2(g) suggest that most data points (80%) are in \( L/B < 75\% \) category, while the rest 20% sites have \( L/B \geq 75\% \).

**Spill through abutment**

Figure 7-3 shows the relationship between the predicted and the measured scour depth for the Spill Through (ST) abutments using the three models considered herein. In the case of long abutment in clear water condition, Froehlich (1989) model over-predicted the measured scour depth on the average by a factor of 1.35. In comparison, for long abutment in live bed condition (Figure 7-3b), Froehlich (1989) model under-predicted the measured scour depth on the average by a factor of 0.89 times. On average, HIRE (2001) model under-predicted the measured scour depth by a factor of 0.97 and 0.59 for clear water and live bed conditions (Figure 7-3 c, d) respectively. Figure 7-3(e) shows that for \( L/B > 75\% \) cases, NCHRP (2010) model slightly over-predicts the measured scour depth, with the predicted scour depth being on the average 1.13 times the measured scour depth. Observation of Figure 7-3(f) suggest that for \( L/B < 75\% \) cases, NCHRP (2010) model significantly under-predicts the scour depth. Referring to Figure 7-3(f), on an average, the predicted scour depth is 0.53 times the measured scour depth for \( L/B < 75\% \) case.
Figure 7-4 shows the relationship between key variables, including affecting the scour prediction models and the quantitative discrepancy between the predicted and the measured scour depth. A positive quantitative discrepancy is indicative of overprediction of measured value, while a negative discrepancy means underprediction. It is apparent that in general, as the Froude number, $F_r$ increases, the prediction discrepancy increases. It is the case that the live bed tests were run at low $F_r$ (0.19-0.24) compared to the $F_r$ for the clear water tests (0.29-0.47). Data in Figure 7-4(c), and for HIRE (2001) model, indicate that the live bed tests were conducted at a low $F_r$ (1.19-0.25), which resulted in a underprediction of scour depth for all the test cases. The clear water tests were conducted at a higher $F_r$, which resulted in a prediction discrepancy ranging from -3.9 cm to 18.4 cm.

The NCHRP (2010) model uses the unit discharge ratio ($q_2/q_1$) to determine an amplification factor necessary to be able to apply to model to estimate the scour (which in this case the contraction scour.) Data in Figure 7-4(d) suggest that for clear water tests, $q_2/q_1$ was in the range of 1.38-2.29, while for the live bed tests, $q_2/q_1$ was in the range of 4.29-7.64. For the tests conducted at a $q_2/q_1>6$, overprediction of the measured scour depth was obtained using the NCHRP (2010) model. It means that for higher constriction, the deterministic NCHRP (2010) model predicts a higher scour depth (represented by higher overprediction, 0.2 to 22 cm) compared to the measured scour depth. In relation to $F_r$ and $L/y$, no specific trend of under- or overprediction was noticed from data in Figure 7-4(e-f). Figure 7-4(g) shows that $L/D_{50}$ ranged from 354 to 718 for clear water cases, while for the live bed cases, $L/D_{50}$ ranged from 798 to 1110. A higher $L/D_{50}$ is representative of smaller particle size around the abutment compared to the length of embankment. There are 7 cases with $L/D_{50}$ more than 1000, and overprediction of scour depth was noticed for all those cases. Benedict et al. (2014) observed that the magnitude of overprediction from the NCHRP (2010) model diminishes as the $D_{50}$ increases, which is in congruent with the relationship observed herein.

**Accuracy and conservatism analyses**

A perfectly accurate scour prediction model would be one that yields a predicted scour depth that is equal to the measured scour depth. As the scour processes depend on multiple factors, it is unlikely that a deterministic model would exactly predict the measured scour depth for varying hydraulic, structural, and geotechnical conditions. A perfectly conservative scour prediction model
would be the one that never yields a predicted scour depth that is less than the measured scour depth. Tan and Duncan (1991) adopted similar logic while assessing the performance of the deterministic models to estimate the settlement of footings on sand.

In this study, accuracy was measured in terms of Mean Absolute Percentage Error (MAPE). For each of the data sets, the absolute percentage error was calculated, and the mean value of the error was reported as the MAPE associated with a specific deterministic model. Conservatism was defined as the ratio of the number of cases the calculated scour depth is more than the measured scour depth, expressed as percentage of the total number of data points.

Figure 7-5 shows the relationship between the level of conservatism and mean absolute percentage error for Vertical Wall (VW) and Spill Through (ST) abutment using the three models considered herein. For VW abutment, conservatism ranges from 4.76% to 100%, and MAPE ranges from 44% to 201%. Froehlich (1989) model provided a 100% conservative estimate of the scour depth for both clear water and live bed conditions, while HIRE (2001) model presented a 100% conservative estimate of the scour depth for long abutments. The least conservative estimate was obtained using NCHRP (2011) model in live bed conditions. The least MAPE was presented by HIRE (2001) model under live bed condition (44% for long abutment case). The highest MAPE was demonstrated by Froehlich (1989) model under clear water condition (201% for intermediate abutment case).

Referring to Figure 7-5(b), for ST abutment, conservatism ranges from 0% to 100%, and MAPE ranges from 10.3% to 347%. Froehlich (1989) model provided a 100% conservative estimate of the scour depth for long abutments in clear water condition. HIRE (2001) model for clear water condition provided the least conservative estimate (0%; meaning all measured data were over predicted) of scour depth. Froehlich (1989) model for long abutments in live bed condition provided the least MAPE (10.3%), while Froehlich (1989) model for long abutments in clear water condition provided the highest MAPE (347%).

Table 7-5 summarizes the results of abutment scour analyses. It is apparent that except for the spill through abutment in live bed condition, Froehlich (1989) model provided the most conservative scour estimate with the highest MAPE associated. The NCHRP (2010) model provided the least conservative scour estimate for VW abutment and ST abutment in clear water.
condition. However, NCHRP (2010) model predicted most conservative scour estimate for ST abutment in live bed condition.

Conclusion

Work herein is focused on comparing the predicted and the measured abutment scour by comparing computed values from Froehlich (1989) model, Highway in the River Environment (HIRE) (FHWA 2001) model, and National Cooperative Highway Research Program (NCHRP 2010) model with measured scour depth by Sturm (2004). The comparative results were synthesized in terms of accuracy and conservatism. Accuracy was defined in terms of Mean Absolute Percentage Error, and Conservatism was defined as the ratio of the number of cases the calculated scour depth is more than the measured scour depth, expressed as percentage of the total number of data points. Based on the results presented herein, the following conclusions are advanced:

1. For Vertical Wall abutments, conservatism ranges from 4.76% to 100%. For clear water conditions, Froehlich (1989) model provides the most conservative (100%) scour estimate, while NCHRP (2010) model provides the least conservative (70%) scour estimate. For live bed conditions, Froehlich (1989) model and HIRE (2001) model provide the most conservative (100%) scour estimate, while NCHRP (2010) model provides the least conservative (4.76%) scour estimate.

2. For Vertical Wall abutments, MAPE ranges from 44% to 201%. For clear water conditions, Froehlich (1989) model provides the highest MAPE (201%), while HIRE (2001) model provides the least MAPE (61%). For live bed conditions, Froehlich (1989) model provides the highest MAPE (82%), while HIRE (2001) model provides the least MAPE (44%).

3. For Spill Through abutments, conservatism ranges from 0% to 100%. For clear water conditions, Froehlich (1989) model provides the most conservative (100%) scour estimate, while NCHRP (2010) model provides the least conservative (41.67%) scour estimate. For live bed conditions, NCHRP (2010) model provides the most conservative (70%) scour estimate, while HIRE (2001) model provides the least conservative (0%) scour estimate.

4. For Spill Through abutments, MAPE ranges from 10.3% to 347%. For clear water conditions, Froehlich (1989) model provides the highest MAPE (347%), while NCHRP
(2010) model provides the least MAPE (72%). For live bed conditions, HIRE (2001) model provides the highest MAPE (41%), while Froehlich (1989) model provides the least MAPE (10.3%).

5. For Vertical Wall abutment, under clear water long abutment condition, Froehlich (1989) model provides the most conservative scour depth estimate, and HIRE (2001) model provides the least MAPE. Under live bed long abutment condition, Froehlich (1989) model provides the most conservative scour depth estimate, and NCHRP (2010) model provides the least MAPE.

6. For Spill Through abutment, under clear water long abutment condition, Froehlich (1989) model provides the most conservative scour depth estimate, and NCHRP (2010) model provides the least MAPE. Under live bed long abutment condition, NCHRP (2010) model provides the most conservative scour depth estimate, and Froehlich (1989) model provides the least MAPE.

Data availability statement
All data, models, and code generated or used during the study appear in the submitted article.

Acknowledgements
The Authors appreciate the funding from North Carolina Department of Transportation (NCDOT). Any conclusions, findings, opinions, and recommendations expressed in this article are those of the authors and do not necessarily reflect the views of NCDOT.
Table 7-1. Deterministic abutment scour model equations considered in the present study.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Model</th>
<th>Equation No.</th>
<th>Remarks</th>
</tr>
</thead>
</table>
| Froehlich (1989)| $y_{sp} = 2.27K_1K_2(L'_y/a)^{0.43}F_r^{0.61} + 1$ | (1)         | $y_{sp} =$ predicted scour depth  
$y_a =$ average depth of flow on the floodplain  
$K_1 =$ coefficient for abutment shape  
$K_2 =$ coefficient for angle of embankment to flow  
$L'_y =$ length of active flow obstructed by the embankment  
$F_r =$ Froude number |
| HIRE (2001)     | $y_{sp} = 4F_r^{0.33}K_1/0.55K_2$ | (2)         |         |
| NCHRP (Ettema et al. 2010) | $y_{max} = \alpha y_c$ | (3)         | $y_{max} =$ Maximum flow depth resulting from abutment scour  
$\alpha =$ amplification factor depending on clear water/live bed condition  
$y_c =$ Flow depth including live-bed or clear-water contraction scour |
|                 | $y_{sp} = y_{max} - y_o$ | (4)         | $y_o =$ flow depth prior to scour |
|                 | $y_{cLB} = y_1(q_2/q_1)^{6/7}$ | (5)         | $q_2 =$ unit discharge in the constricted opening  
$q_1 =$ upstream unit discharge |
|                 | $y_{cCW} = (q_2/K_uD_{50})^{6/7}$ | (6)         | $y_{cLB} =$ flow depth including live bed contraction scour  
$y_{cCW} =$ flow depth including clear water contraction scour  
$D_{50} =$ median grain size |
Table 7-2. Classification of abutment scour parameters (summarized from Ettema et al. 2011).

<table>
<thead>
<tr>
<th>Parameter group</th>
<th>Parameter names</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow/ Sediment</td>
<td>Flow intensity, Froude number, Reynolds number</td>
</tr>
<tr>
<td>Abutment/ Sediment scale</td>
<td>Relative sediment size</td>
</tr>
<tr>
<td>Abutment/ Flow geometry</td>
<td>Floodplain aspect ratio, relative contraction length, abutment shape, skewness</td>
</tr>
<tr>
<td>Abutment flow distribution</td>
<td>Abutment, channel, and flow length scales</td>
</tr>
<tr>
<td>Geotechnical aspect</td>
<td>Abutment stability parameter</td>
</tr>
</tbody>
</table>

Table 7-3. Consideration of abutment scour influencing parameters in HEC 18 (2012) suggested deterministic models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Flow/ sediment</th>
<th>Abutment/ sediment scale</th>
<th>Abutment/ flow geometry</th>
<th>Abutment/ flow distribution</th>
<th>Geotechnical aspect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Froehlich (1989)</td>
<td>C</td>
<td>NC</td>
<td>C</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>HIRE (FHWA 2001)</td>
<td>C</td>
<td>NC</td>
<td>C</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>NCHRP (Ettema et al. 2010)</td>
<td>C</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>C</td>
</tr>
</tbody>
</table>

Note: C means *Considered* in the respective model, while NC means *Not Considered* in the respective model.
### Table 7-4. Summary of the laboratory tests on abutment scour.

<table>
<thead>
<tr>
<th>Type of abutment</th>
<th>Model</th>
<th>Researcher</th>
<th>Test count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical wooden board</td>
<td></td>
<td>Gill (1962)</td>
<td>85</td>
</tr>
<tr>
<td>Vertical metal plate</td>
<td></td>
<td>Liu et al. (1961)</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Garde et al. (1961)</td>
<td>88</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Tison (1962)</td>
<td>3</td>
</tr>
<tr>
<td>Vertical wall</td>
<td></td>
<td>Liu et al. (1961)</td>
<td>43</td>
</tr>
<tr>
<td>Vertical wall (rounded end)</td>
<td></td>
<td>Tey (1984)</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kwan (1984)</td>
<td>9</td>
</tr>
<tr>
<td>Triangular</td>
<td></td>
<td>Tey (1984)</td>
<td>3</td>
</tr>
<tr>
<td>Wing wall</td>
<td></td>
<td>Wong (1984)</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Tey (1984)</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kwan (1988)</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kandaswamy (1988)</td>
<td>5</td>
</tr>
<tr>
<td>Spill through</td>
<td></td>
<td>Liu et al. (1961)</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Wong (1982)</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Tey (1984)</td>
<td>5</td>
</tr>
</tbody>
</table>
Table 7-5. Summary of the abutment scour analyses.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intermediate</td>
<td>Froehlich (1989), NCHRP (2010)</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Short</td>
<td>Froehlich (1989), NCHRP (2010)</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Long</td>
<td>Froehlich (1989), NCHRP (2010)</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Intermediate</td>
<td>Froehlich (1989), NCHRP (2010)</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Short</td>
<td>Froehlich (1989), NCHRP (2010)</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Long</td>
<td>Froehlich (1989), NCHRP (2010)</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Intermediate</td>
<td>Froehlich (1989), NCHRP (2010)</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Short</td>
<td>Froehlich (1989), NCHRP (2010)</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Long</td>
<td>Froehlich (1989), NCHRP (2010)</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Intermediate</td>
<td>Froehlich (1989), NCHRP (2010)</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Short</td>
<td>Froehlich (1989), NCHRP (2010)</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 7-1. Compound channel configurations considered in Sturm (2004), (a) Channel A, slope = 0.005, and (b) Channel B, slope = 0.0022.
Figure 7-5. Relationship between the level of conservatism and mean absolute percentage error using Froehlich (1989), HIRE (2000) and NCHRP (2011) models for (a) Vertical wall abutment, and (b) Spill through abutment.
Chapter 8: Contributions and Future Work

Essential contributions

The key contributions of this research work are listed as follow:

1. Development of a statistical model that can quantify model bias and uncertainty for estimating clear water and live bed local scour around bridge piers.
2. Development of a relationship between reliability index and scour factors considering input parameter (e.g., approach flow velocity, approach flow depth, median grain size of the bed material, Manning’s roughness coefficient, channel bed slope) uncertainty and model bias while estimating clear water and live bed local scour around bridge piers.
3. Assessment of the abutment scour depth prediction models in terms of accuracy and level of conservatism for vertical wall and spill through abutment types under clear water and live bed upstream bed condition.
5. Demonstration of the risk, which is defined as the product of the annual probability of failure times the cost of the consequences for the highway bridges in the United States using Federal Highway Administration developed HYRISK methodology.
6. Establishment of a framework for the incorporation of the reliability index-based procedure in the AASHTO LRFD framework for analysis of axial capacity, lateral capacity and combined axial and lateral capacity of a pile embedded in clay.

Suggested Future Work

Several aspects of the research can be explored further:

1. In a coastal setting, the stability of the bridges is dependent upon an appropriate foundation design for which the knowledge of long-term scour and scour induced from different sequences of flow events (e.g., 10 years of tidal events followed by a 500-year flood event) is paramount. Analyses of scour related to tidal waterways entail consideration of 100- and 500-year storm surge effects, the geometry of tidal inlet, estuary, bay or tidal stream and the effect of flow constrictions (HEC 18, 2012). Such effects need to be investigated to facilitate development of a reliability index-based scour assessment procedure for coastal bridges.
2. For coastal bridges, temporal evolution of scour is of significant importance. There are a very few studies that focused on temporal evolution of scour (e.g., Kothyari et al. 2007). Further research is necessary on this aspect.

3. Collection of an appropriate database on abutment scour depth that might enable development of a reliability index-based scour depth assessment protocol for abutment scour.

4. The scour prediction models present in the current state-of-the-art assume that the first deck span is not short, i.e., there is no effect of the abutment being close to the first pier. However, there are scarce studies (e.g., Croad 1989) on the effect of pier proximity to the abutment scour. Such pier site complication needs further research.
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