The Jakes Fading Model for Antenna Arrays Incorporating Angular Spread

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Abstract

The effect of angular spread and nominal angle of arrival on the spatial correlation of fading waveforms is examined. A new model, based on the Jakes model, is introduced which generates a group of M fading waveforms for a linear array of M antennas, where the waveforms are appropriately correlated according to angle spread and nominal angle of arrival. Angular spread of the signal will manifest itself in the correlation of these fading waveforms: no angular spread will give totally correlated waveforms while angular spread will give some decorrelation among the antenna/receiver elements. Correlation of simulated waveforms is examined as well as examples of the effect of the model in simulated wireless radio links.

1 Introduction

With antenna array/multielement receivers in a free-space environment, a specific phase shift is introduced into the received signals at each antenna element, dependent on the arrival angle of the received signal. When simulating such a multielement receiver in a scattering, Rayleigh fading environment, the question arises: how do we model the fading at each element taking into consideration the direction of arrival and the potential for the scattered signal to be coming from different directions (a non-planar[1] wave)? With a multielement receiver in a fading environment, there are two extremes:

- Perfectly correlated fading, where the signal at each antenna element is faded in the same manner. Here the closely spaced antenna elements are considered a phased array since the high correlation of phase information in the multiple received signals allows the retrieval of direction of arrival information. In this case, all of the signal comes from a single direction and has a planar carrier wave.

- Totally uncorrelated fading, where the signal at each antenna element is independently faded, and the carrier wave is non-planar at the array. Here, the widely spaced antenna elements are considered a diversity array, since diversity is obtained by the lack of correlation in the envelope of the received signals; if one signal is in a deep fade, it is unlikely that the other signals are in a deep fade as well.

It is logical to assume that due to local scatterers around a mobile transmitter there should be some angular spread to the arrival angle, ψ, of the signal. That is, if local scattering were not present, the signal would be represented as a discrete impulse of energy on the arrival angle axis, shown in Figure 1(a). If local scatterers are present, the energy is no longer contained at one discrete angle, but is dispersed, or "smeared", as shown by the example in Figure 1(b).

A number of previous works use this notion in their modeling of the directional channel. In some cases, a ring of scatters of significant diameter is used to model the spreading of the the arrival angle [1, 2, 3, 4], while some

[1]We are considering that a discrete source of a signal is sufficiently far away that the carrier of a signal has a planar wave. The non-planar characteristic, here, comes from the superposition of planar-waves from scatter sources in different directions

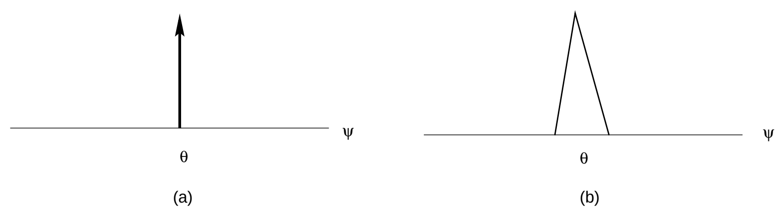

Figure 1: Spreading in Angle of Arrival, ψ: (a) zero angular spread (b) non-zero angular spread

use an area (typically circular) of uniformly distributed scatterers to model the scattering[1, 5, 6].

This paper examines the spreading of the received signal in arrival angle, translating to the correlation of the fading of the received signal at different elements in a phased array. It introduces a new model, based on the Jakes model, which generates the Rayleigh (flat) fading waveforms for the received signals at each of the elements of an array, for use in simulating wireless links[7]. The spread in arrival angle due to the scatterers around the mobile transmitter is incorporated directly to appropriately correlate the multiple fading waveforms for the array. In this work, the ring of scatterers model is focused upon, though the method of generating the waveforms need not be restricted to this distribution of scatterers.

Section 2 of this paper examines analytically the spatial correlation along a linear antenna array for the ring of scatterers model. Section 3 briefly reviews the Jakes fading model and discusses the modifications to that model to account for angular spreading. Section 4 examines the correlation of fading waveforms generated with the model and compares the results to the expected analytical correlation. Section 5 discusses additional modifications to the fading model in order that uncorrelated groups of fading waveforms may be obtained. The effects of the modified fading model are shown in the simulation of example wireless system in Section 6, and the paper is concluded with Section 7.

2 Analytical Correlation

To examine the effect of angle spread on the spatial correlation of fading waveforms at an antenna array, an approach similar to the development in

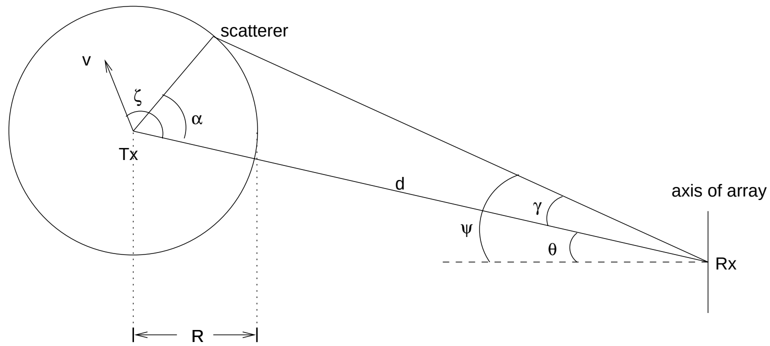

Figure 2: Geometry of Scatterers

[8] for the spatial correlation at the base station is used. First, we assign a specific radius to a ring of scatterers which produces the fading waveform, giving us the differing angles of arrival of signals scattered toward the base station. In Figure 2, γ is the angle of spreading around some nominal angle of arrival, θ, so that

$$\psi = \theta + \gamma \tag{1}$$

By simple trigonometry, γ can be described in terms of the radius of the circle of scatterers, R, the distance between the mobile transmitter, d, and the angle, α, of a scatterer with respect to the line between the receiver and transmitter,

$$\gamma = \arctan\left[\frac{R\sin(\alpha)}{d - R\cos(\alpha)}\right]. \tag{2}$$

Assuming that R is small with respect to d, then (2) simplifies with trigonometric approximations to

$$\gamma = \gamma_{max}\sin(\alpha), \tag{3}$$

where γ_{max} is the maximum angle spread defined to be equal to R/d. Equation (3) is a reasonable approximation of (2) if $R/d \leq 0.1$ (corresponding to a maximum spreading of ± 0.1 rad or $\pm 5.7°$ around a nominal arrival angle).

Of interest is the correlation of waveforms separated in distance and received along an axis, specifically, the line of antennas in a linear array. Consider, in Figure 3, a signal from a particular point on the ring of scatterers approaching the line with an angle of arrival ψ (which is a function of α, the location of the scatterer on the circle, and the nominal angle of arrival,

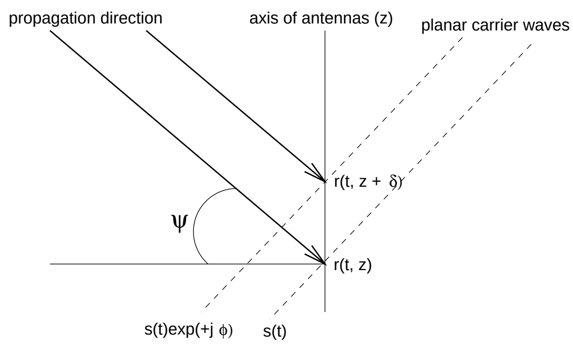

Figure 3: Correlation Geometry

θ). The signal received, $r(t, z + \delta)$, at some location $z + \delta$ (where z and δ are expressed in terms of multiples of the carrier wavelength) will simply be a phase shifted version of the signal at location z. This is often known as the "narrowband assumption" for arrays: δ is considered small enough where the time delay in signal arrival along the array is negligible with respect to the data signal modulating the carrier, but is significant with regard to the carrier phase[9]. If we let $r(t, z) = s(t)$, then the version received at $z+\delta$ will be a phase advanced version (since it arrives first), $r(t, z+\delta) = s(t)\exp(j\phi)$. The crosscorrelation of the signal is then

$$\begin{aligned}
\tilde{\rho}(z, z+\delta) &= E[r(t,z)r^*(t,z+\delta)] \\
&= E[s(t)s^*(t)\exp(-j\phi)] \\
&= E[s(t)s^*(t)\exp(-j2\pi\delta\sin(\psi))],
\end{aligned} \tag{4}$$

where we have defined the phase shift, $\phi = 2\pi\delta\sin(\psi)$, by the geometry and delay for propagation. Assuming that $E[|s(t)|^2] = 1$, we are left with

$$\tilde{\rho}(\delta) = E[\exp(-j2\pi\delta\sin(\psi))]. \tag{5}$$

Since these scatterers are assumed uniformly distributed around the circle of scatterers, the average is taken with respect to a uniform alpha (see Appendix A), resulting in:

$$\tilde{\rho}(\delta) = J_0[2\pi\delta\gamma_{max}\cos(\theta)]\exp(-j2\pi\delta\sin(\theta)), \tag{6}$$

where $J_0[\cdot]$ is the Bessel function of order zero.

To further understand the nature of the spreading of the received signal in arrival angle, and subsequently its spatial correlation along the antenna array, consider the power angle density (PAD). The PAD is analogous to the power spectral density (PSD) and simply gives the power received in a differential element of arrival angle ($d\psi$) as the PSD gives us the power of a signal in a differential element of frequency (df). We consider all the power scattered towards the receiver (P_0) to be evenly distributed among all the scatterers in the circle about the mobile. With a continuum of scatterers in this ring, the power is uniformly distributed in the angle α about the mobile, with the power in a differential element of angle $d\alpha$ being:

$$\frac{P_o}{2\pi} d\alpha. \tag{7}$$

Transforming this distribution in alpha (see Appendix B) to a distribution in ψ through the relations in (1) and (3) results in a PAD of:

$$S(\psi) = \frac{P_0}{\pi\sqrt{\gamma_{max}^2 - (\psi - \theta)^2}}, \tag{8}$$

when $\theta - \gamma_{max} < \psi < \theta + \gamma_{max}$ and 0, otherwise. This PAD is shown in Figure 4. Note that this is the same form as the power spectral density of the E-field component of a fading waveform as given in [8](Eq. 1.2-11). In fact, this PAD has an inverse Fourier transform relationship with the spatial correlation derived at (6), provided that the angle spread (γ_{max}) is small. For simplicity of illustration, consider the case where $\theta = 0$, so that $\psi = \gamma$, then:

$$\tilde{\rho}(\delta) = \int_{-\gamma_{max}}^{\gamma_{max}} S(\gamma) \exp(j2\pi\delta\sin(\gamma)) d\gamma \approx \int_{-\gamma_{max}}^{\gamma_{max}} S(\gamma) \exp(j2\pi\delta\gamma) d\gamma. \tag{9}$$

Here, δ is analogous to τ in the time correlation, and γ is analogous to f in the PSD. For the more general case of the nominal angle of arrival $\theta \neq 0$, the Fourier relationship still holds at small angle spreads, with some modifications. By following the same line of reasoning as in (23)-(25), Appendix A, $\gamma\cos(\theta)$ now maps to f, and the result of the inverse transform must be multiplied by a phase term of $\exp(-j2\pi\sin(\theta))$.

It is noted that spatial correlation for a ring of scatterers has the same form as the complex correlation of the E-field of the fading component, as given in [8], with a re-mapping of the variables. Given this relationship, the

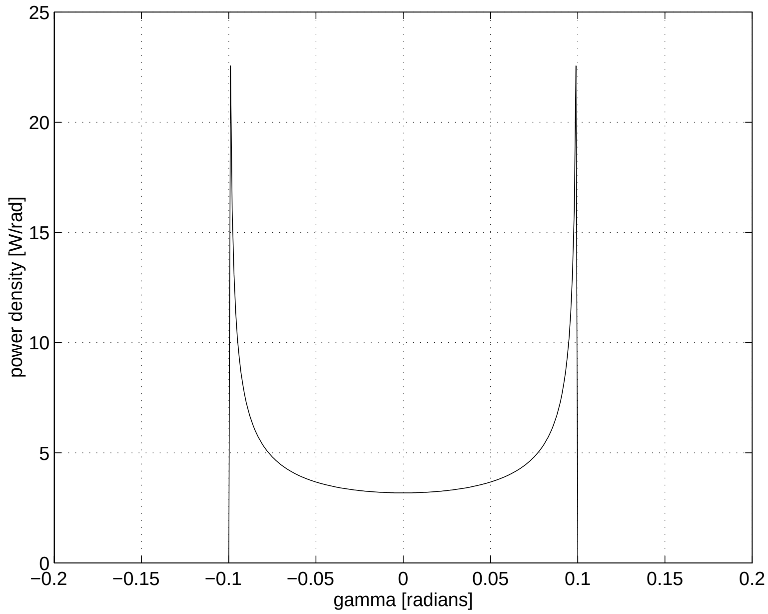

Figure 4: Power Angle Density for Ring of Scatterers $\theta = 0$, $R = 50m$, $d = 500m$ ($\gamma_{max} = 0.1$)

complex correlation along the axis of a linear array can be found for any PAD (*i.e.*, any distribution of scatterers) provided the angle spread is small enough for the approximation $\sin(\gamma) \approx \gamma$ to hold.

For example, a uniformly distributed disk of scatterers is another model which has been used to describe scattering [5, 6]. Given this model, the power from this disk of scatterers can be mapped into a PAD, to obtain $\tilde{\rho}(\delta)$ from the modified inverse Fourier relationship. Using the same conventions for R, d, and γ_{max} as before, a uniformly distributed disk of scatterers gives a PAD of

$$S(\gamma) = \frac{2P_0}{\pi\gamma_{max}^2}\sqrt{\gamma_{max}^2 - (\psi - \theta)^2}, \tag{10}$$

when $\theta - \gamma_{max} < \psi < \theta + \gamma_{max}$ and 0, otherwise. The PAD for this model of scatterers has the same form as the PSD for the x-direction H-field in [8] (Eq. 1.2-12). Using the modified inverse Fourier transform, a complex correlation (setting $P_0 = 1$)

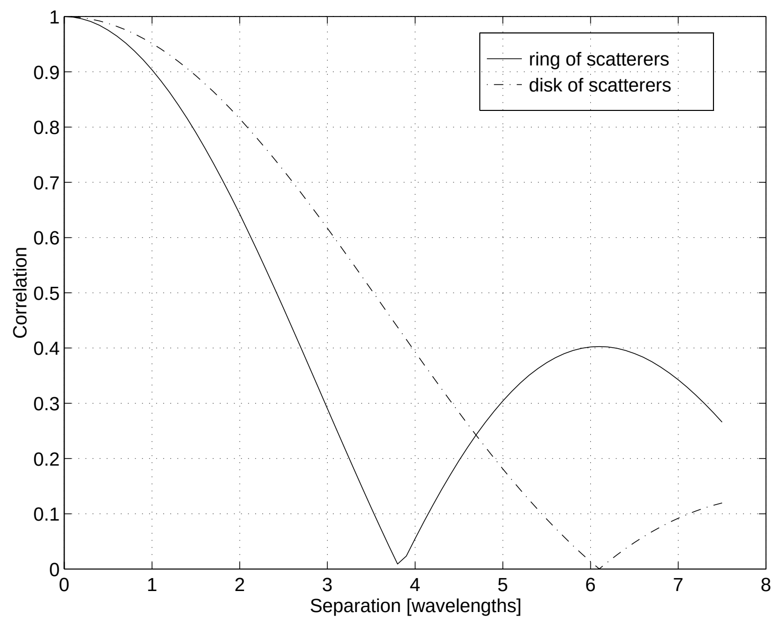

Figure 5: Correlation vs. Separation, Ring and Disk of Scatterers

$$
\begin{aligned}
\tilde{\rho}(\delta) &= [J_0(2\pi\delta\gamma_{max}\cos(\theta)) + J_2(2\pi\delta\gamma_{max}\cos(\theta))] \\
&\quad \cdot \exp(-j2\pi\delta\sin(\theta))
\end{aligned} \tag{11}
$$

is obtained, which is the same form as the complex correlation of the H-field in [8], with the appropriate substitution of variables.

Figure 5 shows the correlation expressed as $\rho(\delta) = |\tilde{\rho}(\delta)|$ for both the ring of scatterers and disk of scatterers model. In this case, the nominal angle of arrival is $\theta = 0$ and the angle spread $\gamma_{max} = 5.7°$ ($R/d = 50/500 = 0.1$ rad). The difference between these curves suggests that the ring of scatterers may be a sufficient model for the simulation of wireless links, however for purposes of directional estimation, it suggests that a more accurate model of scattering may be needed.

3 Modifications to the Jakes Model

To generate fading waveforms that take into account the angle of arrival and angle spread, consider the ring of scatterers model previously described, with N discrete scatterers, where the n^{th} scatterer is at the angle α_n in the circle:

$$\alpha_n = \frac{2\pi(n-0.5)}{N} \tag{12}$$

A fading waveform model is given by

$$T(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} exp(j(\omega_n t + \phi_n)), \tag{13}$$

where

$$\omega_n = \omega_{max} \cos(\alpha_n - \zeta) \tag{14}$$

is the Doppler shift associated with the signal coming from the n^{th} scatterer, ω_{max} is the maximum Doppler frequency, ϕ_n is a random phase, and ζ is the angle of motion of the transmitter with respect to the line between the mobile transmitter and the base receiver. This model is the basis for Jakes model [8] and the modified Jakes model in [10].

To incorporate the angle of arrival of each scatterer, ψ, an array response vector is created for each scattered signal. The array response vector simply contains the phase shift appropriate for each antenna element due to the angle of arrival of the signal. For an uniformly spaced linear array (ULA) of M-antenna elements, this is given by

$$\mathbf{a}_n = \frac{1}{\sqrt{M}} \begin{bmatrix} 1 \\ \exp(-j\delta_0 2\pi \sin(\psi_n)) \\ \exp(-j2\delta_0 2\pi \sin(\psi_n)) \\ \vdots \\ \exp(-j(M-1)\delta_0 2\pi \sin(\psi_n)) \end{bmatrix}, \tag{15}$$

where δ_0 is the interelement spacing of the antennas in terms of wavelengths[9]. While ψ was based on an approximation of γ to obtain the analytic results in Section 2, the exact expression (2) for γ is used to calculate ψ here and for the simulations cited in Section 4.

Each scattered signal is multiplied by its response vector and summed with the other scattered signals,

$$\mathbf{T}(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \mathbf{a}_n \exp(j(\omega_n t + \phi_n)). \tag{16}$$

In this manner, for an array of M antenna elements, a set of M fading waveforms is created,

$$\mathbf{T}(t) = \begin{bmatrix} T_1(t) \\ T_2(t) \\ \vdots \\ T_M(t) \end{bmatrix}. \tag{17}$$

Note that in general, the only way to get a perfectly correlated set of waveforms is if the array response vector for each scattering element is the same. This will only happen if the signal from each scatterer comes from exactly the same direction, *i.e.* there is no angular spreading. This case arises only if the circle of scatterers is considered to be essentially a "point source", or $R = 0$ and $\gamma_{max} = 0$. The modeling of a group of fading waveforms reduces, then, to a single fading waveform multiplied by an array response vector. So the result is the same fading waveform, with only a phase difference between the fading waveforms in the group. If there *is* angular spreading ($R \neq 0$ and $\gamma_{max} \neq 0$), then, in general, there must be decorrelation of the set of fading waveforms since the scattering signals arrive from subtly different directions.

In the case of M antennas in an array, it is necessary to calculate the correlation of the fading waveforms from one antenna to another, or the correlation expression (6) at fixed, interelement distances. We can examine the correlation matrix for a set of M fading waveforms generated by our model. This can be formed from

$$\begin{aligned} \mathbf{R}(t) &= E[\mathbf{T}(t)\mathbf{T}^H(t)] \\ &= \frac{1}{N} E\left[\left(\sum_{n=1}^{N} \mathbf{a}_n \exp(j(\omega_n t + \phi_n))\right)\left(\sum_{m=1}^{N} \mathbf{a}_m^H \exp(j(\omega_m t + \phi_m))\right)\right] \\ &= \frac{1}{N} \sum_{n=1}^{N} \sum_{m=1}^{N} \mathbf{a}_n \mathbf{a}_m^H E\left[\exp(j((\omega_n - \omega_m)t + \phi_n - \phi_m))\right] \end{aligned} \tag{18}$$

where $(\cdot)^H$ indicates the conjugate transpose of a matrix. Averaging with respect to the random phase offset of each scatterer gives

$$E\left[\exp(j((\omega_n - \omega_m)t + \phi_n - \phi_m))\right] = \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases}, \tag{19}$$

and

$$\mathbf{R} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{a}_n \mathbf{a}_n^H. \tag{20}$$

The correlation matrix of the array is the summation of the outer products of the array response vectors for the signal from each scattered element. This correlation matrix expression is not restricted to the ULA, but holds for any planar configuration of antennas, with redefinition of $\mathbf{a}_n$ according to the geometry of the array.

For a ULA, the elements in the rows of this matrix should correspond with the expression (6) taken at the specific spacings of the antennas,

$$\mathbf{R}(t) = \begin{bmatrix} \tilde{\rho}(0) & \tilde{\rho}(\delta_0) & \tilde{\rho}(2\delta_0) & \cdots & \tilde{\rho}((M-1)\delta_0) \\ \tilde{\rho}(-\delta_0) & \tilde{\rho}(0) & \tilde{\rho}(\delta_0) & \cdots & \tilde{\rho}((M-2)\delta_0) \\ \tilde{\rho}(-2\delta_0) & \tilde{\rho}(-\delta_0) & \tilde{\rho}(0) & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{\rho}((M-1)\delta_0) & \cdots & \cdots & \cdots & \tilde{\rho}(0) \end{bmatrix}. \tag{21}$$

As the number of scatterers in the model, N, approaches infinity, the elements of the matrix in (20) converge to the values shown in (21).

4 Correlation of Simulated Waveforms

Sets of fading waveforms were generated using this modified fading model for an array of $M = 16$ elements at a $\delta_0 = 0.5$ (wavelength) spacing, using $N = 32$ complex oscillators in the model. With a scattering circle radius of $R = 50m$, waveforms were generated for transmitters at various distances (thus, various angular spreads) and various nominal angles of arrival.

The correlation between the waveform for the first antenna and the waveforms generated for each antenna was calculated as the time average of the the product of the first antenna waveform with the complex conjugate of the waveform for each of the respective antennas. Waveforms of 2 second durations were generated for a maximum Doppler shift of 100 Hz. The magnitude of the averaged product is plotted versus the separation between the antennas to which the waveforms corresponded. Plotted for comparison is the magnitude of the analytic expression, (6), taken at the appropriate separation (δ), angle spread (γ_{max}), and nominal arrival angle (θ).

Figure 6 shows the case where the distance of the transmitter to receiver was varied over d = 4000m, 2000m, 1000m, and 500, with nominal angle of arrival $\theta = 0°$. This corresponds to increasing angle spreads of $\gamma_{max} = 0.7°, 1.4°, 2.9°$, and $5.7°$, respectively. In Figure 7, with $\theta = 0°$, the simulated

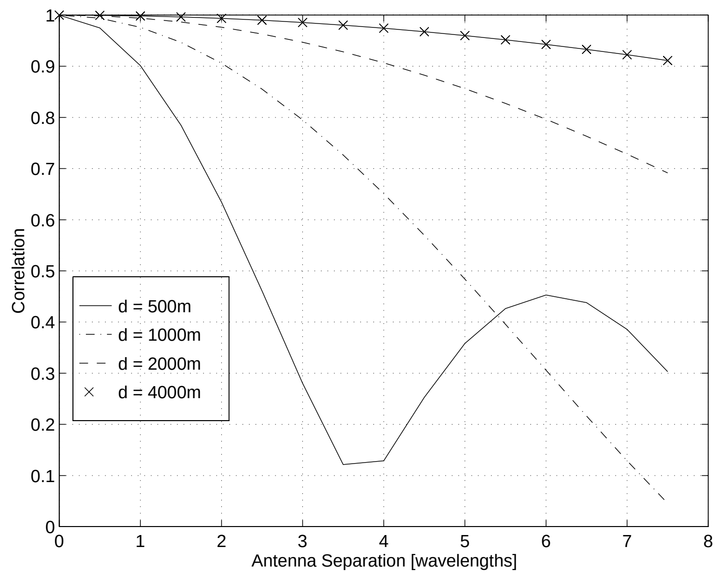

Figure 6: Correlation vs. Antenna Separation, $\theta = 0$, $R = 50m$

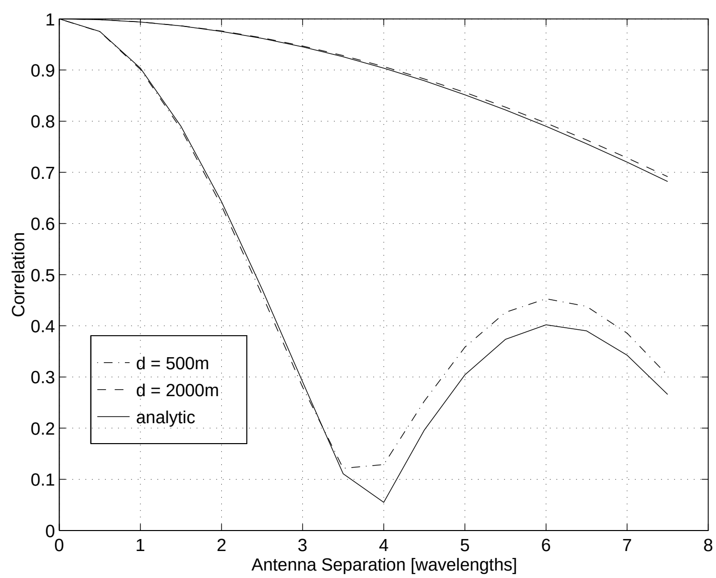

Figure 7: Simulation and Analytic Correlation in Distance, $\theta = 0$, $R = 50m$

cases of $d = 500m$ and $2000m$ are compared with the analytic expression given by (6), where close agreement with theory is observed.

Note that the decorrelation at a given antenna separation becomes more pronounced as the angular spread becomes greater. This makes sense considering the analogy between the relationship of frequency to time separation and the relationship of angle of arrival to spatial separation. If an impulse exists as the power spectral density in the frequency domain, this will transform to a constant amplitude correlation in time separation. If that PSD impulse is smeared, or spread, the resulting correlation waveform will no longer be constant. This is true with the angle of arrival. If the discrete angle of arrival (an impulse in the PAD or "angle domain") is smeared, the resulting spatial correlation waveform will not be constant (in the "spatial domain").

Figure 8 shows the effect of varying nominal angle of arrival, θ. In each case, $R = 50m$ and $d = 500m$, so the angular spread about that nominal angle of arrival is constant. As θ increases to $90°$ from boresite, the correlation becomes increasingly flat, with the fading becoming totally correlated at the endfire angle of arrival. This agrees with the notion that no diversity is obtained when a signal arrives along the line of antennas in a diversity array. Again, comparing a couple of simulation cases against the theoretical analytic expression in Figure 9, with nominal angles of arrival $\theta = 60°$ and $30°$, close agreement is shown to theory.

All simulated results presented above use a model with $N = 32$ oscillators to generate the fading waveforms. As expected, with fewer oscillators in the model, the correlation shows increasingly less agreement with the theoretical curves. A model with $N = 32$ oscillators was deemed sufficient for use in subsequent wireless link simulations (see Section 6).

Simulation with the model shows independence between the spatial correlation and the direction of motion of the mobile transmitter. The direction of the mobile will only have an effect on the Doppler frequency of each scattered component. Evaluating (19), it can be seen that the dependence on Doppler frequency drops out when $\tau = 0$ as in this case.

5 Further Model Modifications

This model generates a group of appropriately correlated fading waveforms for a group of antennas with the intent of multiplying a given multipath

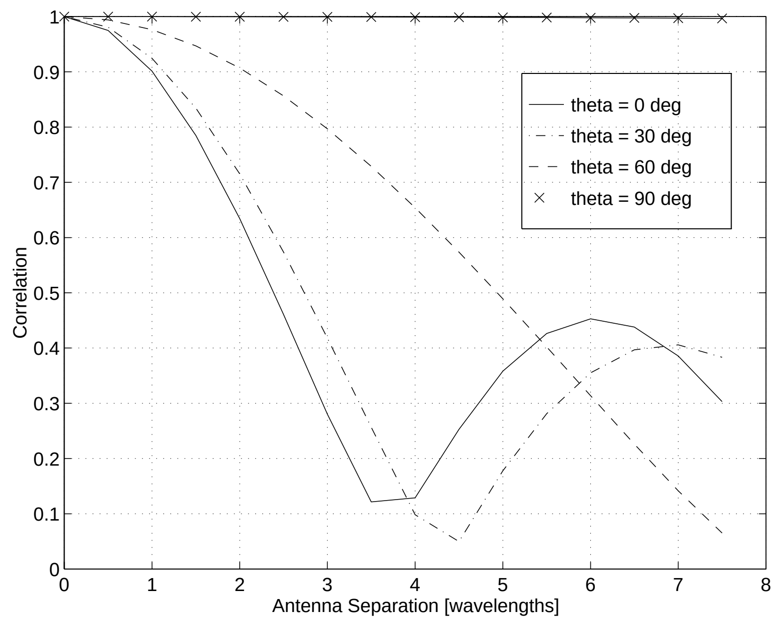

Figure 8: Correlation and Nominal Angle of Arrival, $R = 50m$, $d = 500m$

component coming from some nominal angle of arrival in order to obtain a faded received signal for each antenna element. For other signals arriving from other nominal arrival angles (other paths of the desired signal, or interference signals), it is desired that we have another group of fading waveforms that is uncorrelated from the first. That is, the group of fading waveforms should be appropriately correlated within that group according to angle spread and angle of arrival, but the waveforms should be uncorrelated from group to group. In this case, the innovations in [10] can be applied. The "modified Jakes model" created there incorporates an N-length Walsh sequence to multiply the N complex oscillators in the model (one bit of the sequence multiplying each oscillator) to generate the fading waveform. To obtain another fading waveform, another Walsh sequence, orthogonal to the first, is multiplied onto the oscillators.

In our model here, the method of using Walsh sequences can be used to generate groups of waveforms that are uncorrelated from group to group. Basically, one Walsh sequence is used to generate a group of M correlated waveforms, then for the next group of M correlated waveforms, another

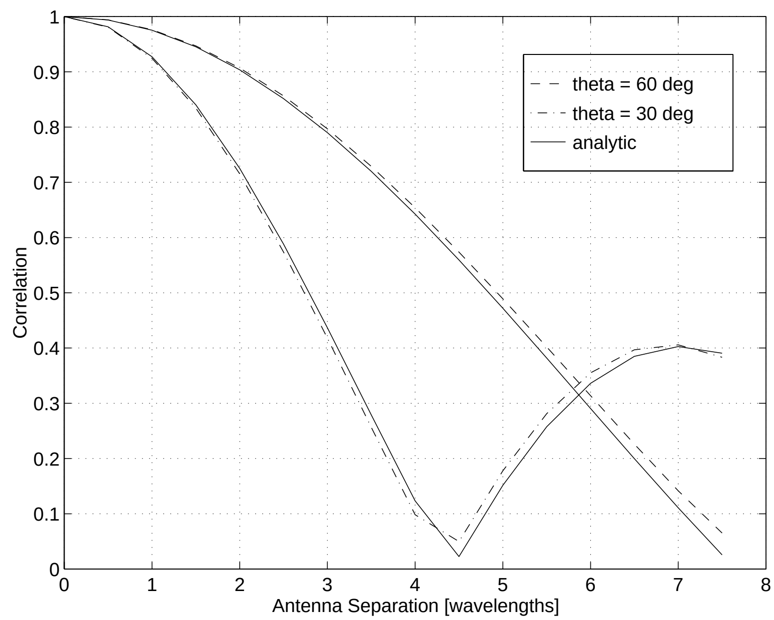

Figure 9: Simulation and Analytic Correlation in Nominal Angle of Arrival, $R = 50m$, $d = 500m$

Walsh sequence is chosen. Modifying the model given in (16), we can generate the j^{th} set of fading waveforms,

$$\mathbf{T}_j(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \mathbf{a}_n u_j(n) \exp(j(\omega_n t + \phi_n), \tag{22}$$

where $u_j(n)$ is the n^{th} bipolar bit of the j^{th} Walsh sequence of length N. In this way, up to N sets of M fading waveforms may be generated, where the waveforms within each group are correlated appropriately, while the waveforms across groups will be uncorrelated.

In [10], the fact that equation (16) contains four terms of the same Doppler frequency (two identically positive, and two identically negative) is used to make an approximation that reduces the number of required oscillator waveforms (and consequently their computation time) by a factor of four. Essentially four complex exponential terms are combined into a single real cosine waveform. It is not clear, however, whether the use of "quadrantal symmetry" is applicable in this case. While terms of (16) share the same Doppler frequency, they will not generally share the same direction of arrival, which is an essential aspect of the model here. It may be possible to use a "half-plane" symmetry approximation to reduce this model complexity, and this is a matter of further study.

6 Simulation of Example Systems

To illustrate the effect of the angular spreading in this fading model on the performance of wireless communication links, a phased array base station receiver was simulated.

The modulation and slot structure was similar to U.S. digital cellular[11]: a slot of 162 $\frac{\pi}{4}$-QPSK symbols at 25 ksym/sec with root raised cosine (RRC) pulse shape. The leading 14 symbols of the slot are used for training purposes. The receiver utilizes a 4 element, half wavelength spaced phased array receiver with a RRC receive filter on each element. The outputs of each element are weighted and combined according to a minimum mean square error (MMSE) criterion, and the weights are adapted in a recursive least squares (RLS) fashion. This receiver structure is similar to that used in [12], but in that case, direct matrix inversion and least mean squares methods where used to solve for combining weights.

Since there are four receiver elements, we consider four channels from the mobile to the base station, and the channel coefficients for each are generated as Rayleigh fading waveforms with the new angle spread model, with a maximum Doppler rate of 5 Hz. With the nominal angle of arrival set to $\theta = 0°$, angle spreads of $0°$, $2.8°$, and $5.7°$ (corresponding to the angle spreads used for Figure 6) were introduced into the fading model to give appropriately correlated fading waveforms. The results are summarized in Figure 10,where $\frac{E_s}{N_0}$ is the symbol energy to noise power density ratio. In this single user scenario, no interference is present.

It can be seen that improved performance in terms of BER is realized as the angular spreading of the signal, and therefore the decorrelation of the fading along the antennas, increases. This is due to the increased diversity effects as a result of the decorrelation of the fading among the antenna elements. In essence, as the angular spread of the signals increases, the antenna array becomes less like a phased array and more like a diversity array.

Another example system of interest is the multiple user case, where simultaneous signals may be separated and detected by multiple antennas. In

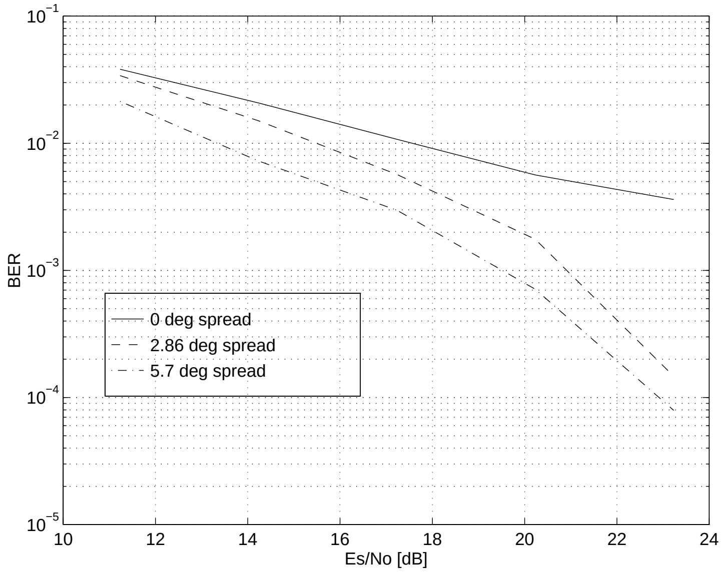

Figure 10: Angle Spread and P_e Performance in Fading

this case, we consider two users with the same transmission format as above, only transmitting simultaneously in the same time slot. In the example simulated, the user of interest and the interfering signal are in the same direction from the base station, i.e., they both have the same nominal arrival angle. The user of interest is farther away, however, so that the average C/I is -6 dB. Since the users are separated in distance (though not in angle) it is fair to assume that the fading is uncorrelated between the users, since they are surrounded by different scatterers.

If the fading at each antenna of the array is totally correlated for a user (corresponding to the 0° spread case) and the signals come from the same direction, then the two user signals will not be separable. Even though there is uncorrelated fading between the users at an antenna, the same fading exists at all the other antennas times a complex (phase) constant. Since the user signals come from the same direction, this complex constant is the same for both user signals, so we have phase shifted versions of the same received signal at each antenna. With essentially the same signal at each antenna, all the extra degrees of freedom required to solve for multiple signals are gone, and the signals are inseparable. However, if the fading for each user

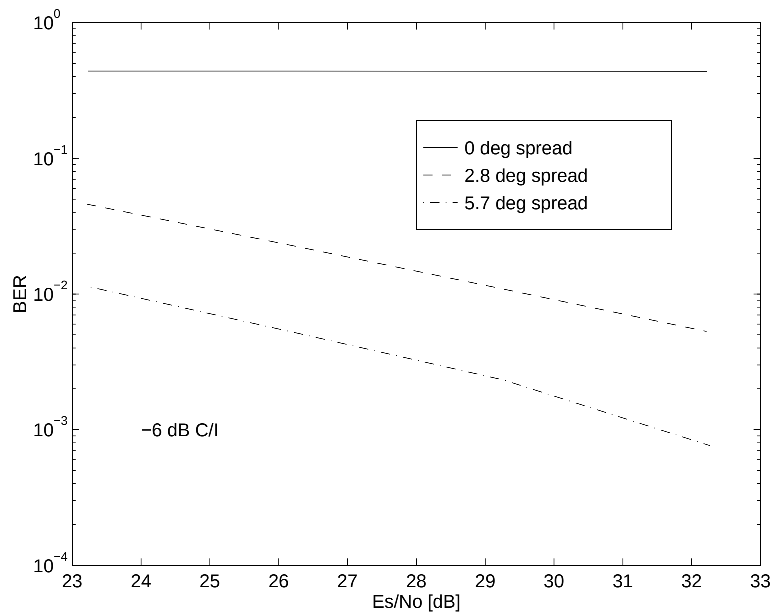

Figure 11: Angle Spread and BER Performance in Fading for Two Users at the same Arrival Angle

is decorrelated along the array, even by a small amount, the signals will be separable even if they come from the same direction. Figure 11 shows the effect of the decorrelation from the model on the detectability of the user of interest. At $0°$ spread case, as expected, the signal is undetectable with a BER of ~ 0.5. However, as angular spread is introduced, the signal becomes detectable, and BER performance increases as the angle spread is increased from $2.8°$ to $5.7°$.

7 Conclusion

In this paper, the relationship between angular spread, power angle density, and spatial correlation in Rayleigh fading waveforms is shown. A method is developed of generating a group of appropriately correlated fading waveforms for use in the context of phased antenna array reception which takes into account this notion of angular spreading of a signal. This method is a modification of the Jakes model which introduces the angle of arrival of each scatterer and uses this angle to generate an array response vector. The array response vector of a scattered signal is used to multiply that signal and add it to the group of fading waveforms. The correlation of generated waveforms was examined in the case of a uniform linear array by both simulation and analytical means, with close agreement between the two results. While the correlation was examined for the ULA case, the model itself is general and will work with any planar arrangement of antennas. As long as the array response vector for an arrival angle can be calculated, fading waveforms for that array can be generated with appropriate correlation across the antenna elements. Groups of fading waveforms that are uncorrelated from group to group are obtainable by incorporating the use of Walsh sequences as in [10]. The model is of principal use in the simulation and analysis of wireless links involving phased antenna arrays. The model is use here in an example wireless link simulation, showing the importance of considering angular spread (and the consequent decorrelation of fading) in simulation of phased array receivers.

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A Appendix

To arrive at the expression for the correlation of a received signal along the axis of a linear array given by (6), the expected value in (5) must be evaluated. For this evaluation, the average over a uniform distribution in angle α is taken:

$$\tilde{\rho}(\delta) = \frac{1}{2\pi} \int_0^{2\pi} \exp(-j2\pi\delta \sin(\psi)) d\alpha. \tag{23}$$

Observing that $\psi = \theta + \gamma$ and assuming that γ is small,

$$\sin(\psi) \approx \sin(\theta) + \gamma \cos(\theta). \tag{24}$$

(23) then becomes:

$$\tilde{\rho}(\delta) = \frac{1}{2\pi} \exp(-j2\pi\delta \sin(\theta)) \int_0^{2\pi} \exp(-j2\pi\delta\gamma \cos(\theta)) d\alpha \tag{25}$$

Since γ is a function of α by (3),

$$\tilde{\rho}(\delta) = \frac{1}{2\pi} \exp(-j2\pi\delta \sin(\theta)) \int_0^{2\pi} \exp(-j2\pi\delta\gamma_{max} \sin(\alpha) \cos(\theta)) d\alpha. \quad (26)$$

The integrand in this equation is of the form where the relation

$$\exp(ju \sin(v)) = \sum_{n=\infty}^{\infty} J_n(u) \exp(jnv) \quad (27)$$

can be used. Letting $u = -2\pi\delta\gamma_{max} \cos(\theta)$, (26) becomes:

$$\tilde{\rho}(\delta) = \frac{1}{2\pi} \exp(-j2\pi\delta \sin(\theta)) \sum_{n=\infty}^{\infty} J_n(u) \int_0^{2\pi} \exp(-jn\alpha) d\alpha. \quad (28)$$

The integral evaluates to 0 for all n except $n = 0$, and we are left with a general expression for the correlation as a function of separation, δ, angular spread, γ_{max}, and nominal angle of arrival, θ:

$$\tilde{\rho}(\delta) = J_0[2\pi\delta\gamma_{max} \cos(\theta)] \exp(-j2\pi\delta \sin(\theta)). \quad (29)$$

B Appendix

To arrive at the distribution of power in the arrival angle (ψ) or power angle density, the distribution of power among the scatterers around the transmitter must be transformed. With a continuous ring of scatterers, there is uniform distribution of power among the scatterers or through α, the angle around the transmitter. The power scattered by a differential element of α, $d\alpha$, is then

$$\frac{P_o}{2\pi} d\alpha, \quad (30)$$

where P_o is the total signal power scattered toward the receiver. Defining the power angle density as $S(\psi)$, the energy arriving in a differential element of arrival angle, $d\psi$, is $S(\psi)d\psi$. Consider Figure 12 which shows the differential elements. For any given angle of arrival for scattered energy, the energy arriving in that differential element $d\psi$ comes from two separate differential portions of the ring, one at α_1 and α_2. The power in a differential portion of arrival angle is then two times that given in (30),

$$S(\psi)d\psi = \frac{P_o}{\pi} d\alpha, \quad (31)$$

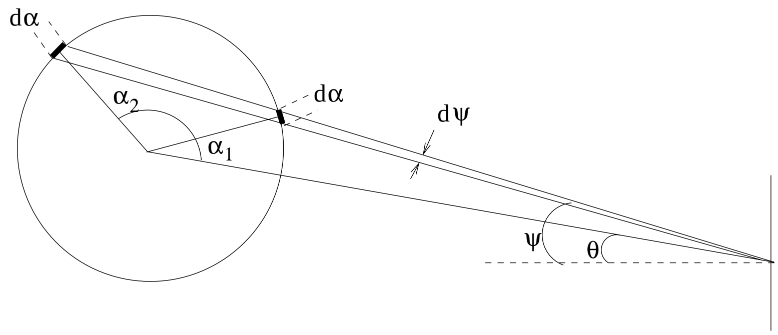

Figure 12: Differential elements of the ring of scatterers and arrival angle

so

$$S(\psi) = \frac{P_o}{\pi}\frac{d\alpha}{d\psi}. \tag{32}$$

From (1) and (3) we know that

$$\psi = \theta + \gamma = \theta + \gamma_{max}\sin(\alpha), \tag{33}$$

and

$$\alpha = \sin^{-1}\left(\frac{\psi - \theta}{\gamma_{max}}\right). \tag{34}$$

Evaluating $\frac{d\alpha}{d\psi}$, we obtain

$$S(\psi) = \frac{P_0}{\pi\sqrt{\gamma_{max}^2 - (\psi - \theta)^2}}, \tag{35}$$

when $\theta - \gamma_{max} < \psi < \theta + \gamma_{max}$, and 0 otherwise.

References

[1] D. Asztely, *On Antenna Arrays in Mobile Communicaion Systems Fast Fading and GSM Base Station Receiver Algorithms*, PhD thesis, Royal Institute of Technology, Stockholm, February, 1995.

[2] S. Anderson, M. Millnert, M. Viberg, and B. Wahlberg, "An adaptive array for mobile communication systems", *IEEE Transactions on Vehicular Technology*, vol. 40, pp. 230–236, 1991.

[3] A. Klouche-Djedid and M. Fujita, "Adaptive array sensor processing applications for mobile telephone communications", *IEEE Transactions on Vehicular Technology*, vol. 45, pp. 405–416, 1996.

[4] Chris Ward, Martin Smith, John Hudson, "Propagation characteristics influencing smart antenna design and performance", Presentation Slides – Third Smart Antennas Workshop, Stanford University, 1996.

[5] J. Fuhl and A.F. Molisch, "Capacity enhancement and BER in a combined SDMA/TDMA system", in *Proceedings of the IEEE 46th Vehicular Technology Conference*, 1996, pp. 1481–1485.

[6] P. Petrus, J.H. Reed, and T.S. Rappaport, "Effects of directional antennas at the base station on the Doppler spectrum", *IEEE Communications Letters*, vol. 1, pp. 40–42, 1997.

[7] T.L. Fulghum, K. Molnar, and A. Duel-Hallen, "The Jakes fading model incorporating angular spread", in *Proceedings of CISS*, 1997.

[8] W. Jakes, *Microwave Mobile Communications*, IEEE Press, 1974.

[9] R.A. Monzingo and T.W. Miller, *Introduction to Adaptive Arrays*, John Wiley and Sons, 1980.

[10] P. Dent, G.E. Bottomley, and T. Croft, "Jakes model revisited", *Electronics Letters*, vol. 29, pp. 1162–1163, 1993.

[11] *EIA/TIA Interim Standard: Cellular System Dual-Mode Station-Base Station Compatibility Standard*, Telecommunications Indus. Assoc., 1992.

[12] J.H. Winters, "Signal acquisition and tracking with adaptive arrays in the digital mobile radio system IS-54 with flat fading", *IEEE Transactions on Vehicular Technology*, vol. 42, pp. 377–384, 1993.