On the Superposition of Arrival Processes for Voice and Data

Harry G. Perros
and
Raif O. Onvural

Center for Communication and Signal Processing
Department of Electrical and Computer Engineering
North Carolina State University
Raleigh, North Carolina

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ON THE SUPERPOSITION OF ARRIVAL PROCESSES FOR VOICE AND DATA[1]

by

H.G. Perros[2]

Computer Science Department
and
Center for Communications and Signal Processing
North Carolina State University
Raleigh, NC, 27695-8206

and

Raif O. Onvural

BNR
Research Triangle Park, NC, 27709-3478

Abstract: The single inter-arrival distribution function of the superposition of N independent bursty arrival processes is obtained, assuming that each bursty arrival stream is an interrupted Poisson process. It is shown that this probability distribution function is hyperexponential with 2^N phases. Its parameters have a closed form solution, and they can be easily computed. Using this probability distribution function, we address the problem of how many bursty arrival processes are required so that the resulting superposition process may be approximated by a Poisson process.

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1. Introduction

In this paper, we consider the problem of superposing bursty arrival processes. This problem arises when modelling high-speed computer communication networks. Future networks will be capable of handling a large number of highly bursty sources, such as voice, bulk file transfer, and video, which will amount to throughputs of the order of several gigabits/sec. In such an environment, a transmission link may be fed by a large number of such sources. In order to model such a link, one has the option of modelling each bursty source separately. This, of course, leads to an intractable model as the number of sources increases. Alternatively, one may merge all the sources into a single or a few sources, thus reducing the dimensionality of the model. In this case, a common problem that arises is to determine the number of bursty sources one needs to merge so that the resulting superposition process can be approximated by a Poisson process. This is a rather hard question to answer, seeing that it is difficult to characterize the superposition process. This due to the fact successive inter-arrival times of the superposition process are correlated. If one freely makes the assumption that a superposition process is Poisson, then one runs the risk of introducing serious errors, in particular when one tries to estimate performance measures such as the percent of lost packets in a finite buffer and the mean queue-length.

The problem of superposing renewal processes also arises in the analysis of non-product form queueing networks. In particular, most of the approximation algorithms reported in the literature for non-product form queueing networks are based on the notion of decomposition, i.e. the network is decomposed into individual queues and each queue is analyzed in isolation (cf. Chandy and Sauer[5], Kuehn [8], Sevcik, Levy, Tripathi, and Zahorjan [11], Whitt [15]). In order to study each queue in isolation, one needs to calculate the superposition of all the arrival processes to this queue, which are basically the departure processes from the upstream queues in the network and the arrival process from outside the network.

In this paper, a bursty arrival process is assumed to be modelled by an inrerrupted Poisson process (hereafter referred to as an IPP). That is, arrivals occur during an exponentially distributed period (*active* period). This period is followed by an exponentially distributed *silence* period during which no arrivals occur. During the active period, arrivals occur in a Poisson fashion.

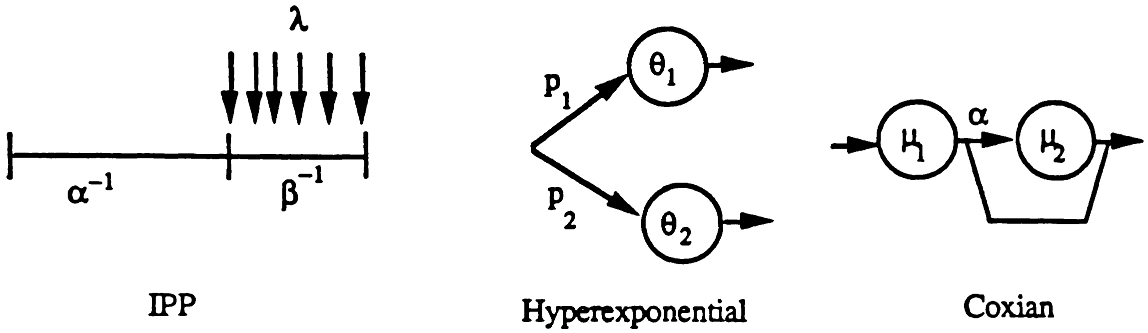

Figure 1: Three equivalent distributions

Let α^{-1}, β^{-1}, and λ be respectively the mean time of the silence period, the mean time of the active period, and the rate at which packets arrive during the active period. Then, the Laplace transform of the inter-arrival time is as follows:

$$f(s) = \frac{\lambda(\alpha+s)}{s^2+(\lambda+\alpha+\beta)s+\alpha\lambda} \tag{1}$$

Taking partial derivatives, the mean inter-arrival time, m, its variance, var, and squared coefficient of variation, c^2, are as follows:

$$m = (\alpha+\beta)/\alpha\lambda \tag{2}$$

$$var= (2\lambda\beta+(\alpha+\beta)^2) / (\lambda\alpha)^2 \tag{3}$$

$$c^2 = 1+ 2\lambda\beta/(\alpha+\beta)^2 \tag{4}$$

An IPP is equivalent to a hyperexponential distribution with two phases, which in turn is equivalent to a two-phase Coxian distribution (hereafter referred to as C_2). In particular, a hyperexponential distribution with parameters θ_1, θ_2, p_1 and p_2 (as shown in figure 1) has the same Laplace transform as the IPP with the following relations between the parameters of the two distributions:

$$\theta_1 = (1/2)\{(\lambda+\alpha+\beta) + [(\lambda+\alpha+\beta)^2 - 4\alpha\lambda]^{1/2}\} \tag{5}$$

$$\theta_2 = (1/2)\{(\lambda+\alpha+\beta) - [(\lambda+\alpha+\beta)^2 - 4\alpha\lambda]^{1/2}\} \tag{6}$$

$$p_1 = (\lambda-\theta_2)/(\theta_1-\theta_2), \text{ and } p_2=1-p_1. \tag{7}$$

Furthermore, this hyperexponential distribution has the same Laplace transform as a two phase Coxian distribution with parameters μ_1, μ_2, and α (as shown in figure 1) where:

$$\mu_1=\theta_1,\ \mu_2=\theta_2,\ \text{and } \alpha=p_2(\mu_1-\mu_2)/\mu_1 \tag{8}$$

In view of the above transformations, the problem of superposing bursty processes, modelled as interrupted Poisson processes, is equivalent to the problem of superposing C_2 distributions. This problem can be seen as part of a more general problem, that of superposing renewal processes. The superposition of N independent renewal processes is a renewal process if and only if all the component processes are Poisson processes. Furthermore, if the superposition process is composed of many independent and relatively sparse component processes then it converges to a Poisson process as the number of component processes tends to infinity (cf. Çinlar [4]). In general, if at least one of the component processes is not Poisson then the intervals between renewals are not independent, and the superposition process is not a renewal process. The dependence among the intervals tends to make the superposition process and the associated queueing model analytically intractable. There are number of approximations reported in the literature to obtain the superposition of N renewal arrival processes (cf. Kuehn [8,9], Whitt [14,15], and Albin [1,2,3]). In these approximations, the inter-arrival time of the superposition process is characterized by the exact mean and an estimate of the coefficient of variation of the inter-arrival time. More recently, Sriram and Whitt [12] studied the aggregate arrival process resulting from superposing separate voice streams. Each voice stream is characterized by a bursty process. Using the notion of the index of dispersion for intervals in conjunction with simulation, they investigated the correlation of successive inter-arrival times of the superposition process. Heffes and Lucantoni [6] studied the superposition of voice streams using correlated Markov Modulated Poisson Processes. A discussion of the properties of this process can be found in Rossiter [10].

In this paper, we study the superposition of N arrival streams, where each stream is characterized by a C_2. In particular, we obtain the exact probability distribution function (pdf) of a single inter-arrival time of the superposition by exploiting the Markovian structure of N Coxian arrival processes. It is shown that this pdf is a hyperexponential distribution with 2^N phases. The parameters of this hyperexponential distribution have a closed-form solution and they can be easily calculated. We note that the expression for the pdf of a single interval of a superposition of any number of renewal processes is known (see Whitt [14], eq. (4.4)). This expression is easy to use when two processes are being superposed. However, in order to use it in the case of more than two processes one needs to resort to approximations (see Whitt [14] for a discussion). The pdf of a single interval given here is obtained following a different approach, it is exact, and its parameters can be easily calculated.

In the following section, we obtain the pdf of a single inter-arrival of the superposition of N C_2 arrival processes. In section 3, we address the problem of how many bursty processes are required so that the resulting superposition process can be approximated by a Poisson process. Finally, the conclusions are given in section 4.

2. The Superposition of N C_2 Arrival Processes

Consider N arrival processes, each being a C_2 distribution. Let $1/\mu_{ij}$ and α_i be respectively the mean service time at phase j, j=1,2, and the branching probability of the ith arrival process, i=1,...,N, as illustrated in figure 2. Let $(n_1,n_2,\ldots,n_N)$ be the state of the N arrival processes, where n_i is the phase that the ith arrival process is currently in, n_i=1,2; i=1,2,...,N. Furthermore, let $X_i(n_i)$ be a random variable indicating the time elapsing until the next departure from the ith arrival process when the arrival process is in phase n_i. Also, let, $Y_N(n_1,n_2,...,n_N)$ be a random

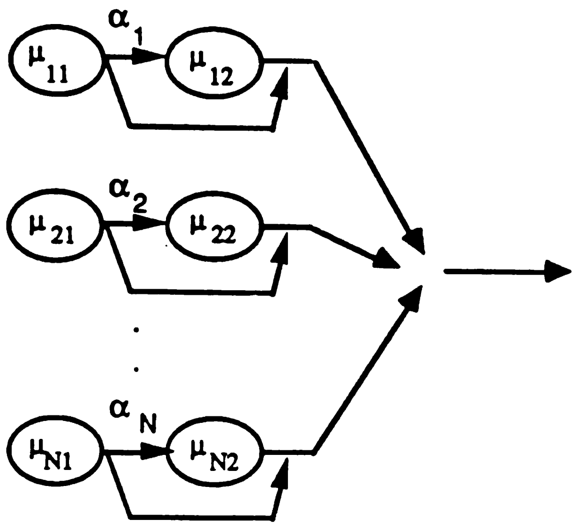

Figure 2: N C_2 arrival processes

variable indicating the time elapsing between two successive departures of the superposition process given that immediately after the first departure the state of the N arrival processes was $(n_1,n_2,...,n_N)$. Then:

$$Y_N(n_1,n_2,...,n_N) = \min \{ X_1(n_1), X_2(n_2),...,X_N(n_N) \} \tag{9}$$

By unconditioning on $(n_1,n_2,...,n_N)$, we have that the time elapsing between two successive departures of the superposition process is

$$Y_N = \sum_{(n_1,n_2,...,n_N)} \min\{X_1(n_1),X_2(n_2),...,X_N(n_N)\} p^D(n_1,n_2,...,n_N) \tag{10}$$

where $p^D(n_1,n_2,...,n_N)$ is the steady-state probability that the system is in state $(n_1,n_2,...,n_N)$ immediately after a departure. The probabilities $p^D(n_1,n_2,...,n_N)$ can be easily calculated by appropriately conditioning on the time average probabilities $p(n_1,n_2,...,n_N)$ as illustrated below.

We first observe that the system of the N arrival processes is Markovian. Let $S=\{(n_1,n_2,\ldots,n_N) \mid n_i = 1,2;\ i=1,\ldots,N\}$ be the set of all feasible states of the superposition process. For a given state $(n_1,n_2,\ldots,n_N)$, let S_1 and S_2 be the set of arrival processes which are in phase 1 and in phase 2 respectively . Then, the global balance equations for the time average probabilities $p(n_1,n_2,\ldots,n_N)$ are as follows:

$$\{\sum_{i\in S_1}\mu_{i1}\alpha_i + \sum_{i\in S_2}\mu_{i2}\}\ p(n_1,\ldots,n_N) =$$

$$\sum_{i\in S_1}\mu_{i2}\, p(n_1+e_1,\ldots,n_N+e_N) + \sum_{i\in S_2}\mu_{i1}\alpha_i\, p(n_1-e_1,\ldots,n_N-e_N), \qquad (n_1,n_2,\ldots,n_N)\in S$$

where $e_k=1$ if $k=i$, and $e_k=0$ otherwise. It can be easily shown by substitution that $p(n_1,n_2,\ldots,n_N) = p_1(n_1)\, p_2(n_2) \ldots p_N(n_N)$, where $p_i(1) = \mu_{i2}/(\alpha_i\mu_{i1}+\mu_{i2})$ and $p_i(2) = \alpha_i\mu_{i1}/(\alpha_i\mu_{i1}+\mu_{i2})$. That is,

$$p(n_1,n_2,\ldots,n_N) = \left(\prod_{i\in S_1}\frac{\mu_{i2}}{\alpha_i\mu_{i1}+\mu_{i2}}\right)\left(\prod_{i\in S_2}\frac{\alpha_i\mu_{i1}}{\alpha_i\mu_{i1}+\mu_{i2}}\right) \tag{11}$$

The departure point probabilities $p^D(n_1,n_2,\ldots,n_N)$ are calculated from the above time average probabilities $p(n_1,n_2,\ldots,n_N)$ as follows:

$$p^D(n_1,n_2,\ldots,n_N) = \{\sum_{i\in S_1}\mu_{i1}(1-\alpha_i)\, p(n_1,n_2,\ldots,n_N) + \sum_{i\in S_2}\mu_{i2}p(n_1+e_1,n_2+e_2,\ldots,n_N+e_N)\}\Big/$$

$$\sum_{(n_1,n_2,\ldots,n_N)\in S} p(n_1,n_2,\ldots,n_N)\ \left(\sum_{i\in S_1}\mu_{i1}(1-\alpha_i) + \sum_{i\in S_2}\mu_{i2}\right) \tag{12}$$

Substituting (11) to (12), we have

$$p^D(n_1,n_2,\ldots,n_N) = \frac{1}{G}\left(\sum_{i\in S_1}\prod_{\substack{j\in S_1\\ j\neq i}}\mu_{j1}^{-1}\right)\prod_{i\in S_2}\alpha_i\mu_{i2}^{-1} \tag{13}$$

where,

$$G=\sum_{i=1}^{N}\left(\prod_{\substack{j=1\\ j\neq i}}^{N}\mu_{j1}^{-1}+\alpha_j\,\mu_{j2}^{-1}\right).$$

We now proceed with the calculation of $Y_N(n_1,n_2,\ldots,n_N)$. Seeing that $Y_N(n_1,n_2,\ldots,n_N) =\min\{X_1(n_1),X_2(n_2),\ldots,X_N(n_N)\}$, we have:

$$P\{Y_N(n_1,n_2,\ldots,n_N)\geq t\}=\prod_{i\in S_1}P\{X_i(n_i)\geq t\}\prod_{i\in S_2}P\{X_i(n_i)\geq t\} \tag{14}$$

If $i\in S_2$, $X_i(n_i)$ is distributed exponentially with parameter μ_{i2}, whereas if $i\in S_1$, $X_i(n_i)$ is a C_2 distribution with parameters μ_{i1}, μ_{i2}, and α_i Hence, we have the following theorem:

Theorem:

i) $\prod_{i\in S_2}P\{X_i(n_i)\geq t\}=e^{-\lambda t}$, where $\lambda=\sum_{i=1}^{N}\mu_{i2}$, i.e. exponential with rate λ.

ii) $\prod_{i\in S_1}P\{X_i(n_i)\geq t\}=\sum_{i=1}^{2^{|S_1|}}p_i e^{-\mu_i t}$, i.e. hyperexponential with $2^{|S_1|}$ phases, where $|S_1|$ is the cardinality of set S_1.

iii) $Y_N(n_1,n_2,\ldots,n_N)$ is distributed hyperexponential with $2^{|S_1|}$ phases.

iv) Y_N is distributed hyperexponential with 2^N phases.

Proof:

i) If $X_i(n_i)$ is distributed exponentially with parameter μ_{i2}, $i=1,2,\ldots,N$, then $P\{X_i(n_i)\geq t\}=e^{-\mu_{i2}}$, and $\prod_{i\in S_2}P\{X_i(n_i)\geq t\}=\prod_{i\in S_2}e^{-\mu_{i2}}=e^{-\lambda t}$, where $\lambda=\sum_{i=1}^{N}\mu_{i2}$.

ii) If $X_i(n_i)$ has a Coxian distribution with parameters μ_{i1}, μ_{i2}, and α_i, then it has a hyperexponential distribution with parameters μ_{i1}, μ_{i2}, p_{i1}, and p_{i2}, where $p_{i2}=\alpha\mu_{i1}/(\mu_{i1}-\mu_{i2})$ and $p_{i1}=1-p_{i2}$. In view of this, $P\{X_i(n_i)\geq t\}=p_{i1}e^{-\mu_{i1}t}+p_{i2}e^{-\mu_{i2}t}$, and $\prod_{i\in S_1} P\{X(n_i)\geq t\} = \sum_{i=1}^{2^{|S_1|}} p_i e^{-\mu_i t}$, where p_i and μ_i are obtained as follows:

Algorithm 1:
i=0
for n_1=1 to 2 do
  for n_2=1 to 2 do
.
.
    for $n_{|S_1|}$=1 to 2 do
      begin
        i=i+1;
        $\mu_i=\sum_{j=1}^{|S_1|}\mu_{jn_j}$
        $p_i=\prod_{j=1}^{|S_1|}p_{jn_j}$
      end;

iii) Using (14), and the above two results, we have:

$$P\{Y_N(n_1,n_2,\ldots,n_N)\geq t\} = \sum_{i=1}^{2^{|S_1|}} p_i^{*} e^{-(\mu_i+\lambda)t} \tag{15}$$

i.e. hyperexponential with $2^{|S_1|}$ phases. The branching probabilities p_i^{*}, $i=1,2,\ldots,2^{|S_1|}$, are defined below in algorithm 2.

iv) It follows from (10) and (15) that Y_N is distributed hyperexponentially with 2^N phases. The parameters of the superposition process p_i^{*} and μ_i^{*}, $i=1,\ldots,2^N$, are given in terms of the parameters of each arrival process (expressed as hyperexponential, i.e. μ_{i1}, μ_{i2}, p_{i1} and $p_{i2}=1-p_{i1}$, $i=1,\ldots,N$) as follows. Let Q_j be the set of states where j arrival processes are in their second phase, $j=0,\ldots,N$. Then, in each set Q_j, $P\{Y_N(n_1,n_2,\ldots,n_N)\}$ is calculated as the product of j two-phase independent hyperexponential distributions and N-j independent exponential distributions. In particular, p_i^* is a function of the p_{ij}'s while μ_i^* is a function of the μ_{i1}'s and μ_{i2}'s. The service rates μ_i^* and the branching probabilities p_i^* associated with each phase i, i=1,2,...,N, are calculated using the following algorithm.

Algorithm 2:

i:=0;
for n_1=1 to 2 do
  for n_2=1 to 2 do
    .
    .
    for n_N=1 to 2 do
      begin
        i:=i+1; $\mu_i^* = \sum_{j=1}^{N} \mu_{jn_j}$ (* service rate at phase i *)
        p_i^* =0;
        for r_1=1 to n_1 do
          for r_2=1 to n_2 do
            .
            .
            for r_N=1 to n_N do
              $p_i^* = p_i^* + p^D\{ (r_1,r_2,\ldots,r_N) \} \prod_{\substack{j=1 \\ r_j \neq 2}}^{N} p_{jn_j}$ (*branching probability *)
      end;

3. Approximating a superposition process by a Poisson process

Quite often in high-speed networking, a single server queue may serve a large number of arrival processes (i.e. virtual circuits). In this case, one is tempted to assume that the superposition of these arrivals processes is Poisson. In this section, we address the problem of determining the number of bursty arrivals required so that the resulting superposition process can be approximated by a Poisson process. We shall do so using the pdf of a single inter-arrival interval obtained above. Each bursty process is assumed to be described by an interrupted Poisson process. All arrival processes are assumed identical. (This restriction can be easily removed to allow arrival processes with different parameters.)

Using the results given in the previous section, we can calculate the squared coefficient of variation, c^2, of a single inter-arrival time of a superposition, as a function of N, the number of superposed arrival processes. In figure 3 we plotted the logarithm of c^2 as a function of N for various values of cv^2, the squared coefficient of variation of the inter-arrival time of a bursty process. In particular, we give plots for $cv^2 = 19.36, 60.89, 120.89, 450.89, 3751$, and 6604. The parameters of the interrupted Poisson processes corresponding to these values of cv^2 are given in table 1. The case of $cv^2=19.36$ corresponds to the case where the bursty source is voice (see Sriram and Whitt [12]).

cv^2	α	β	λ
19.36	0.00154	0.0028	0.0625
60.89	0.016	0.024	2
120.89	0.016	0.024	4
450.89	0.016	0.024	15
3751	0.016	0.024	125
6604	0.05	0.05	40000

Table 1: Parameters of the interrupted Poisson processes

We note that in figure 3, the logarithm of c^2 tends to 0 (i.e. c^2 tends to 1) as the number of the arrival processes N increases. Also, the higher the value of cv^2, the longer it takes for c^2 to tend to 1. This behaviour is of course expected.

Now, we observe that for a specific value of cv^2, the value of c^2 becomes approximately equal to one when N is large. For instance, for $cv^2=19.36$, c^2 is close to one when N>30. The question, therefore, that arises is whether the superposition process can be approximated by a Poisson process when N is large, for instance N>30 when cv^2=19.36. In order to investigate this we considerd an infinte capacity queue with a single exponential server. The arrival process consisted of N independent identical interrupted Poisson processes. The parameters of these arrival processes were taken from table 1. The pdf of the inter-arrival time of this superposition process of the N arrivals is approximated by the pdf of a single inter-arrival time. In other words, we assume that the successive inter-arrival times of the superposition are not correlated. The resulting queue is the familiar G/M/1 queue, which is known to have a closed form solution (cf. Kleinrock [7]). In particular, let σ be the unique root between zero and one of the following functional equation, where A(s) is the Laplace transform of the probability density function of the interarrival process:

$$\sigma = A(\mu_0 - \mu_0 \sigma) \tag{16}$$

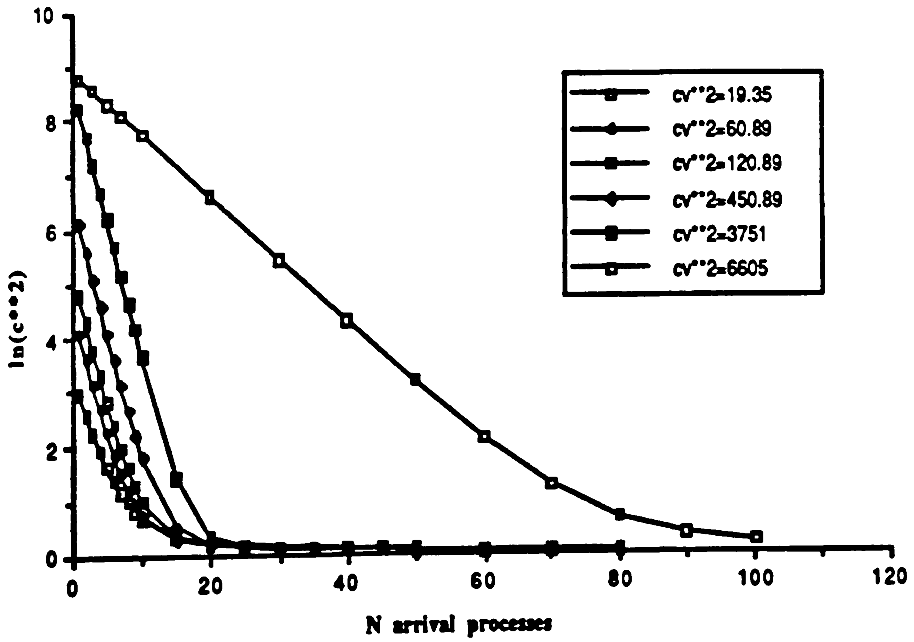

Figure 3: c^2 as a function of N for various values of cv^2

Seeing that the pdf of an inter-arrival time of the superposition process is hyperexponential with $N^*=2^N$ phases, as described in section 2, we have

$$\sigma = \frac{\dot{p}_1\dot{\mu}_1}{(\mu_0-\mu_0\sigma)+\dot{\mu}_1} + \frac{\dot{p}_2\dot{\mu}_2}{(\mu_0-\mu_0\sigma)+\dot{\mu}_2} + \ldots + \frac{\dot{p}_{N^*}\dot{\mu}_{N^*}}{(\mu_0-\mu_0\sigma)+\dot{\mu}_{N^*}} \tag{17}$$

Now, it is possible to find the roots of equation (17) and so the unique root of interest which is between zero and one numerically or as a fixed point problem. However, this is a rather time consuming task, particularly if N^* is large. To decrease the time complexity for obtaining σ, we obtain the first three moments of the superposition process and then we fit a hyperexponential distribution with two phases. In particular, let m_1, m_2, m_3 be the first three moments of the superposition process. Then:

$$m_1=\sum_{i=1}^{N^*}(p_i^*/\mu_i^*)\ , \quad m_2=2\sum_{i=1}^{N^*}(p_i^*/\mu_i^{*2}),\ m_3=6\sum_{i=1}^{N^*}(p_i^*/\mu_i^{*3}) \tag{18}$$

The parameters of the fitted hyperexponential distribution λ_1, λ_2, p_1, and p_2 are given in terms of the first three moments of the original distribution as follows (cf. Whitt [14]):

$$\lambda_i^{-1} = \left((x+1.5y^2+3m_1^2\,y)\pm\sqrt{(x+1.5y^2+3m_1^2\,y)^2 - 12\,m_1^2\,xy}\right)/(6m_1\,y) \geq 0 \tag{19}$$

$$p_1 = (m_1-\lambda_2^{-1})/(\lambda_1^{-1}-\lambda_2^{-1}) \geq 0, \tag{20}$$

and $p_2 = 1-p_1$, where $x= m_1m_3 - 1.5m_2^2$, and $y= m_2 - 2m_1^2$. In this case, utilizing the fact that $\sigma=1$ is one of the roots of equation (17) with $N^*=2$, the root of interest is the root between zero and one of a quadratic equation $a\sigma^2+b\sigma+c=0$, where

$$a=\mu_0^2;\quad b=\mu_0^2+\mu_0(\lambda_1+\lambda_2);\quad \text{and } c=\lambda_1\lambda_2+\mu_0(p_1\lambda_1+p_2\lambda_2). \tag{21}$$

Let p(j) be the probability that there are j customers in the queue (i.e. time average probabilities). Then:

$$p(0)=1-\rho \; ; \quad \text{and} \quad p(j)=\tau(1-\sigma)\,\sigma^{j-1} \; , \; j=0,1,2,.. \tag{23}$$

where ρ is the utilization of the server. For a given value of cv^2, p(j), j=0,1,..., was plotted as a function of N, for various values of ρ, with a view to identifying when this probablity distribution approaches the probability distribution of an M/M/1 queue with the same mean inter-arrival and service time. As an example, in figures 4, 5, and 6, we give plots for the case of cv^2=19.36 for ρ=0.2, 0.5, and 0.9, where ρ is the utilization of the server. Each figure gives a plot of p(1), p(2), p(3), and mql, the mean number of customers in the system, as a function of the number of arrival processes N. The arrows on the vertical axis give the corresponding values of the M/M/1 queue.

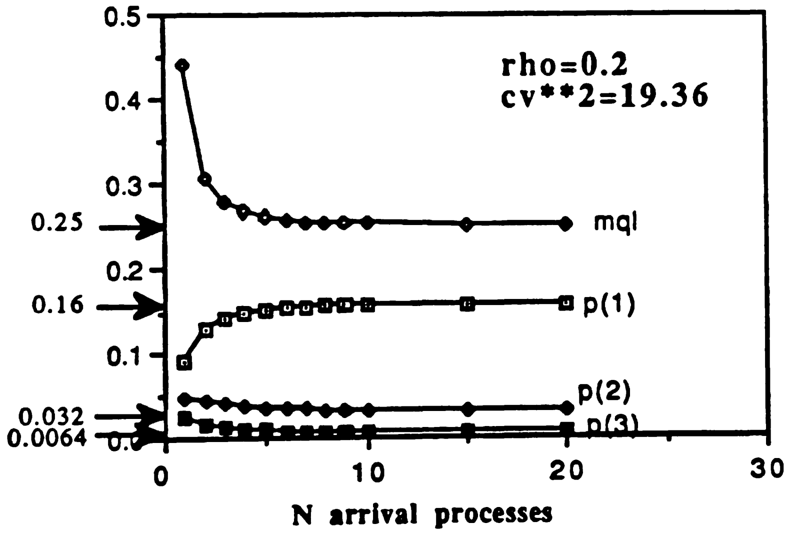

Figure 4: A plot of the mean queue-length, p(1), p(2), p(3) as function of N arrival processes for ρ=0.2.

We note that when ρ=0.2 the queue-length distribution can be approximated by that of the M/M/1 queue when N >10. However, for ρ=0.5 and 0.9, the value of N is much larger. These results are based on the assumption that the successive inter-arrival periods of the superposition process were not correlated. In order to examine the effect of this assumption on the value of N after which the superposition can be approximated by a Poisson process, we simulated the single exponential server with the N arrival processes. Each arrival process was simulated explicitly. The parameters of these processes were the same as those given in table 1. The obtained results

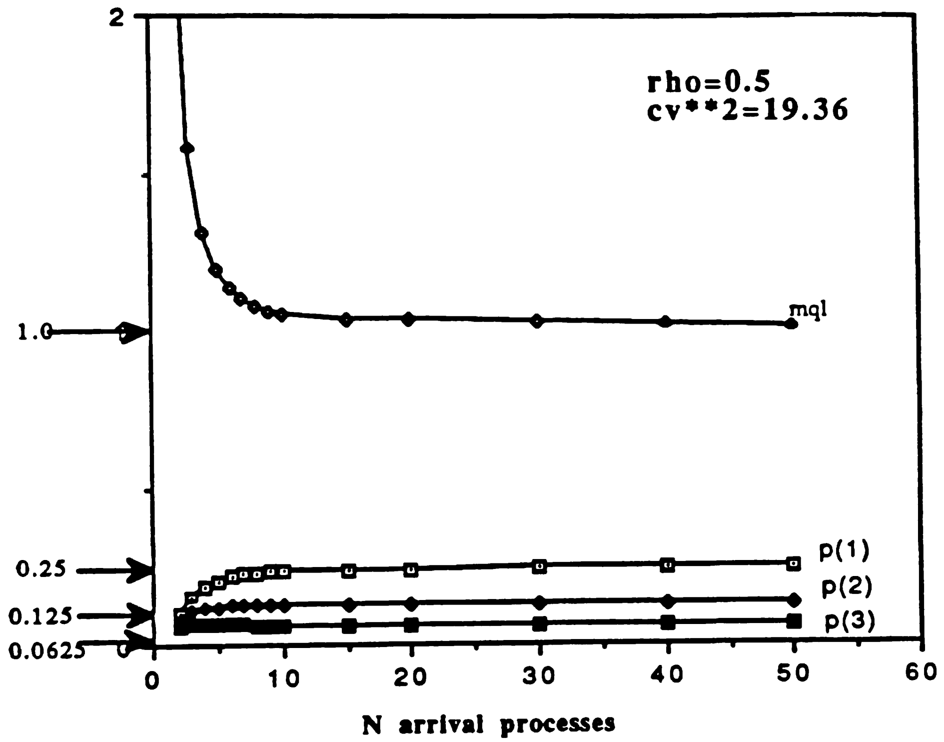

Figure 5: A plot of the mean queue-length, p(1), p(2), p(3) as function of N arrival processes for ρ=0.5.

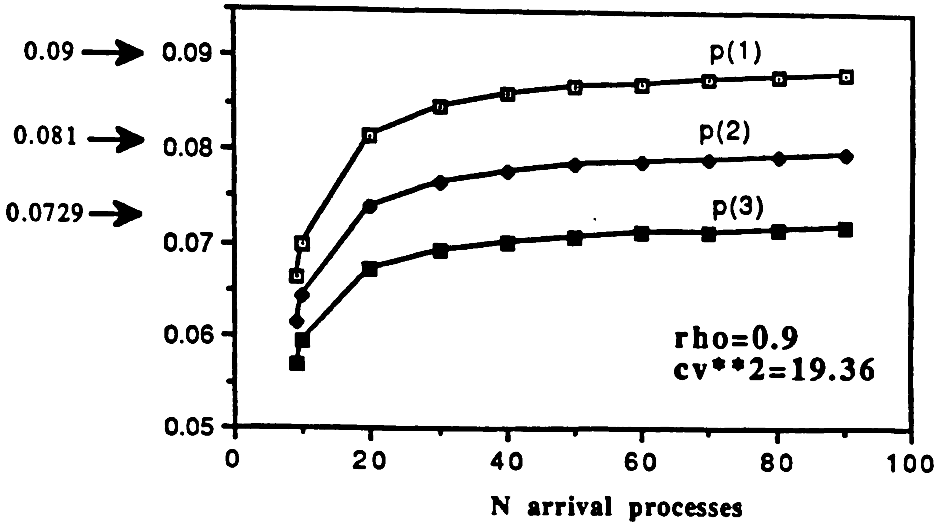

Figure 6: A plot of p(1), p(2), p(3) as function of N arrival processes for ρ=0.9.

suggest that the queue-length distribution of this single server queue can be approximated by the queue-length distribution of an M/M/1 queue with the same mean service and inter-arrival times when N is much larger than the value of N calculated above. For instance, N should be greater than 45 when ρ=0.2 for cv^2=19.36, and much larger for cv^2=60.89, or 120.89.

In general, the important question that needs to be addressed is what is the pdf of the superposition inter-arrival period when N is not too large. This requires the calculation of the correlation of successive intervals. This issue is beyond the scope of this paper.

4. Conclusions

A procedure was developed to obtain the pdf of a single inter-arrival time of the superposition of N independent bursty arrivals, each modelled by an interrupted Poisson process.It is shown that this pdf has a hyperexponential distribution with 2^N phases. Its parameters have closed form solution and they can be easily computed. Using this probability density function we addressed the problem of how many bursty arrivals are required so that the resulting superposition process can be approximated by a Poisson process.

We note that this work can be easily extended to the case where during the active period of a bursty process, the inter-arrival time of customers is constant rather than exponentially distributed as it was assumed in this paper. In this case, one can approximate the pdf of the inter-arrival time of any two successive customers by a C_2, thus enabling the use of the results given in this paper. This approach, however, needs to be validated.

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