Channel Estimation and Multiuser Detection for Frequency-Nonselective Fading Synchronous CDMA Channels

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Abstract

Since the Code Division Multiple Access (CDMA) system was proposed for the next generation of mobile communications, several multiuser detection methods have been developed to cope with the Multi-Access Interference inherent in the CDMA system. Fading and multipath characteristics of the mobile radio channel present significant challenges to the designer of multiuser receivers. The fading phenomenon prevents the traditional Phase-Lock Loop (PLL) carrier recovery, so for the coherent detection, more accurate channel estimation is required. In this paper, we consider only frequency non-selective fading channels. Removing the usual assumption that the channel coefficients are perfectly known to the receiver, we model the fading channel as a second order Auto-Regressive (AR) process, and use the decision-directed Kalman filter to estimate the channel coefficients. The effect of estimation errors on the performance of several multiuser detectors is investigated by analysis and simulations. Issues concerning the implementation of the Kalman filter are also addressed.

1. Introduction

CDMA is a promising technique for the future generation of mobile communication systems. It increases the overall capacity, removes the hard upper limit on the numbers of users (soft capacity), and eliminates the tedious frequency management used in the present mobile systems [1]. However, the essential problem in the CDMA system is the Multi-Access Interference (MAI), which stems from the accumulation of the non-zero cross-correlation noise. The interference is usually severe in the near-far environment in which the received signal powers are dissimilar. In the presence of fading, the interference is enhanced and can severely degrade the receiver's detection ability. The interference due to the presence of other users can be successfully canceled if the conventional single-user correlation receiver is replaced by a more sophisticated receiver structure.

A number of interference cancellation methods were proposed recently. These approaches include the optimal [2], linear [3], multistage [4] and decision-feedback detectors [5]. In [3-5], the performance of these detectors was analyzed for additive white Gaussian noise synchronous CDMA channel. In [6], the performance of the optimal detector and the linear decorrelator was analyzed for the flat fading channel. In [7], it was shown that the decision-feedback detector solves the near-far problem and improves upon the decorrelator for the flat Rayleigh fading synchronous CDMA channel.

Most of the performance evaluation of the multiuser detection was based on the assumption that the fading channel coefficients are perfectly known to the base-station receiver. However, in the mobile environment, especially in the presence of fast fading, the tracking of the carrier phase through the PLL is not as good as for the non-fading channel because of the hang-up effect of the PLL [8]. This tracking error can harm the performance of the PSK systems [8-9]. In [8-9], Kam, Haeb and Meyr proposed using the Kalman filter to replace the PLL. Instead of estimating the amplitude and phase, the Kalman filter is used to estimate the In-phase and quadrature components of the channel coefficients respectively. The Kalman filter is known to be the optimum MMSE estimator if the channel coefficients obey Gauss-Markov model [10].

In [11-12], the Kalman filter was used to estimate the fading channel coefficients for the frequency-nonselective fading channel. The performance of the linear multiuser decorrelator was evaluated assuming that the correct decisions are available for the decision-directed estimator. The BER was shown to be heavily dependent on the estimation error. In this paper, we also concentrate on the frequency-nonselective fading channel. We examine the effect of the estimation error and the actual decisions on the performance of the decorrelator, the two-stage detector and the decision-feedback detector.

The paper is arranged as follows. In section 2, we review the synchronous CDMA model and several multiuser detector structures. In section 3, we formulate the signal model of the flat fading channel, and show how the Kalman filter can be used to estimate the channel coefficients. Some implementation issues are discussed. The theoretical BER for multiuser detectors and the actual simulation results are presented in section 4.

2. Synchronous CDMA System Model and Multiuser Detectors

Consider a synchronous CDMA system with K users for a Rayleigh fading channel. The received signal is given by

$$r(t) = \sum_{i=0}^{\infty} \sum_{n=1}^{K} C_n(i)\, b_n(i)\, s_n(t - iT) + n(t),$$

where $C_n(i)$ is the complex fading coefficient, $b_n(i)$ is the information bit of the n-th user drawn from {+1,-1}, $s_n(t)$ is the normalized signature waveform of the n-th user (the energy of $s_n(t)$ is one), and n(t) is the additive white Gaussian noise with power N_0.

The front-end of a multiuser detector consists of a bank of matched filters. Each matched filter is designed to correlate each individual user's signature waveform. The output of the matched filter bank at the k-th sampling instance can be represented by the column vector $\mathbf{y}$ of length K:

$$\mathbf{y}(k) = \mathbf{R}\mathbf{W}(k)\mathbf{b}(k) + \mathbf{n}(k), \tag{1}$$

where the input vector $\mathbf{b}(k)$ consists of the information bits of K users. The matrix $\mathbf{R}$ has components $\mathbf{R}_{l,m} = \int_{-\infty}^{\infty} s_l(t)\, s_m(t)\, dt$ (l,m = 1, ... ,K), the cross-correlations between the signature waveforms of users l and m. The diagonal matrix $\mathbf{W}(k)$ has components $\mathbf{W}_{n,n} = \sqrt{Eb_n}\, |C_n(k)| \exp(-j\theta_n(k))$ (n=1,...,K), where Eb_n is the bit energy of user n. The vector $\mathbf{n}(k)$ is a K-dimensional complex Gaussian noise vector. We assume that the channel coefficients $|C_n(k)| \exp(-j\theta_n(k))$ are constant during the transmission interval, and are generated by independent complex Gaussian random processes. The amplitude $|C_n(k)|$ is Rayleigh distributed, and the phase $\theta_n(k)$ is uniform on $[0,2\pi)$.

We consider three multiuser detectors: the decorrelator, the two-stage detector and the decision-feedback detector. The linear decorrelator [3] can be obtained by multiplying the signal (1) by the inverse of the correlation matrix $\mathbf{R}$:

$$\mathbf{z}(k) = \mathbf{R}^{-1}\, \mathbf{y}(k) = \mathbf{W}(k)\mathbf{b}(k) + \mathbf{v}(k), \tag{2}$$

where the noise vector $\mathbf{v}(k) = \mathbf{R}^{-1}\mathbf{n}(k)$. The decision for the n-th user is made by $\hat{b}_n(k) = sgn(\mathrm{Re}[z_n(k)\, \hat{C}_n^*(k)])$, where $\hat{C}_n(k)$ is the estimate of the channel coefficient $C_n(k)$. The output of the decorrelator for the n-th user, $z_n(k)$, contains no interference from other users; however, the colored Gaussian noise $\mathbf{v}(k)$ is enhanced.

The two-stage detector considered here uses the decorrelator as the first stage [4]. The tentative decisions obtained from the output of the decorrelator are then used to reconstruct the MAI in the second stage. The reconstructed MAI is subtracted from the output of the matched filter bank. The final decisions are made after canceling the interference.

The decision-feedback detector first arranges the users in the descending energy order: $|\mathbf{W}_{1,1}(k)| \geq |\mathbf{W}_{2,2}(k)| \geq \ldots \geq |\mathbf{W}_{K,K}(k)|$ (the estimates of the energies are used in the actual implementation), and then performs the Cholesky factorization of the matrix $\mathbf{R}(k)$ to find the noise-whitening filter $(\mathbf{F}(k)^T)^{-1}$ (i.e., $\mathbf{R}(k)=\mathbf{F}^T(k)\mathbf{F}(k)$) [5,7]. Note that the matrix $\mathbf{R}(k)$ varies with the sampling time k depending on the energy order. This was not the case for the decorrelator and the two-stage detector. If the noise-whitening filter is applied to the matched filter output y(k), the resulting output vector is

$$\tilde{\mathbf{y}}(k) = \mathbf{F}(k)\mathbf{W}(k)\mathbf{b}(k) + \tilde{\mathbf{n}}(k), \tag{3}$$

where F(k) is a left lower triangular matrix with its entries denoted by $f_{ij}(k)$, and $\tilde{\mathbf{n}}(k)$ is white complex Gaussian noise vector. The lower triangular structure of F(k) leads to the successive interference cancellation. The signal of each user in (3) is free from interference due to weaker users. The first user (the strongest user) can make the decision first since it does not contain any interference from the other users ($\tilde{y}_1(k) = f_{11}(k)\sqrt{E_{b_1}}\, C_1(k) b_1(k) + \tilde{n}_1(k)$). Using the decision of the first user, the second strongest user can cancel the interference due to the first user. Similarly, the multiuser interference in the signal of the n-th user can be completely deleted if decisions of the users 1 to n-1 are correct.

3. Channel Estimation

Channel estimation is an essential problem for the coherent detection. For the three detectors described in section 2, the estimates $\hat{C}_n(k)$ are required to make reliable decisions. In particular, for mobile communications where channel varies rapidly, the performance of a receiver depends heavily upon the accuracy of the channel estimation. We use the Kalman filter, which, unlike the LMS and the RLS algorithms, incorporates the channel dynamics to compute the least mean square estimation error and the channel estimates.

Signal model of a fading channel and the Kalman filter

The power spectrum of a Rayleigh flat fading channel contains narrow band spectral peaks upon the maximum Doppler frequencies [13]. A simple signal model which describes the spectral peak behavior is a lightly damped second order AR process [14]. Once the fading channel model is determined, the Kalman filter is ready to implement.

For user n (n is not shown for simplicity), the discrete-time signal model of the flat fading channel coefficient is given by a second order Auto-Regressive (AR) process:

$$C(k) = -a_1\, C(k-1) - a_2\, C(k-2) + w(k). \tag{4}$$

The 2nd order AR process can also be expressed as the Gauss-Markov model,

$$\mathbf{X}(k+1) = \mathbf{F}\,\mathbf{X}(k) + \mathbf{G}\, w(k), \tag{5}$$

where $\mathbf{X}(k) = [C(k)\ C(k-1)]^T$ is the state variable , $\mathbf{F} = \begin{bmatrix} -a_1 & -a_2 \\ 1 & 0 \end{bmatrix}$ and $\mathbf{G} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ are model parameter matrices. The measurement used to estimate the channel coefficients is the output of the decorrelator for user n. This signal can be expressed as

$$z(k) = \sqrt{E_b}\, b(k) C(k) + v(k) = \mathbf{H}'(k)\,\mathbf{X}(k) + v(k), \tag{6}$$

where $\mathbf{H}(k) = \sqrt{E_b}\, b(k)\, [1 \quad 0]^T$, and v(k) is the Gaussian noise at the output of the decorrelator with variance $N_0[\mathbf{R}^{-1}]_{n,n}$.

Once the fading model (5) and the measurements (6) are acquired, the Kalman filter can be used as the channel estimator. The estimates are generated recursively by the following equations [10]:

$$\mathbf{M}(k) = \Sigma(k)\ \mathbf{H}(k)\ [\mathbf{H}'(k)\ \Sigma(k)\ \mathbf{H}(k) + R]^{-1}, \tag{7}$$

Kalman filter gain: $\mathbf{K}(k) = \mathbf{F}\,\mathbf{M}(k),$ (8)

Prediction: $\hat{\mathbf{X}}(k+1) = [\mathbf{F} - \mathbf{K}(k)\ \mathbf{H}'(k)]\ \hat{\mathbf{X}}(k) + \mathbf{K}(k)\ z(k),$ (9)

Error Covariance: $\Sigma(k+1) = \mathbf{F}\ [\Sigma(k) - \mathbf{M}(k)\ \mathbf{H}'(k)\ \Sigma(k)]\ \mathbf{F}' + \mathbf{G}\ Q\ \mathbf{G}'$ (Ricatti equation), (10)

where R and Q are the variances of the excitation noise and channel noise, i.e., $R = E\left(|v(k)|^2\right),\ Q = E\left(|w(k)|^2\right)$.

The structure of the decorrelator with the Kalman channel estimator is shown in Figure 1. It is convenient to use the output of the decorrelator z(k) as the input of the Kalman filter since it does not contain MAI shown in (2). The measurement z(k) is the same as for the single user case except for the enhanced noise. Therefore, the channel estimate of each user can be obtained separately, i.e., joint estimation is not necessary. In [12], the outputs of the matched filter bank were used to estimate the channel coefficients; however, in this case, the joint estimation is required. The joint estimation involves the irreducible data dependency in updating the Kalman filter gain (8), the channel estimate (9), and the error covariance (10) [12]. The matched filter bank is prone to make decision errors when the interference is high, so the unreliable decisions will lead to the inaccurate estimation. Compared with the non-joint estimation, it was shown in [15] that the joint estimation has only limited improvement in the estimation error in the high SNR region.

Implementation Issues

Equations (7-10) show that the channel estimates depend on the matrices **F**, **G** and **H**(k). The matrix **H**(k) depends on the unknown data bit b(k). When the estimator is in the training mode, the correct data bits are available. In the decision-directed mode, the decision $\hat{b}(k)$ is used in place of b(k). In this section, we will discuss the details of the Kalman filter implementation.

(i) The Training Mode:

The main purpose of the training mode is to let the initial channel estimates converge to the channel coefficients (9). The error covariance matrix can also be computed at this time, and can later be used in the decision-directed mode. From (10), the error covariance matrix Σ(k) appears to be data-dependent since the matrix Σ(k) is dependent on the matrix **H**(k). However, by examining (7) and (10) carefully, we observe that in our case, the product **M**(k)**H**(k) is given by the quadratic form which eliminates the data-dependency. Using this fact, we can show by a simple induction argument that the error covariance matrix Σ(K) does not depend of the transmitted data stream. This also implies that in the decision-directed mode, wrong decisions will not affect the error covariance matrix. Therefore, in principle, the steady state error covariance matrix can be computed off-line prior to the reception, and used in the decision-directed mode to save the computational load.

(ii) Decision-directed mode:

In the decision-directed mode, the data-dependent matrix **H**(k) is unknown, so it is replaced by $\hat{\mathbf{H}}(k)=\sqrt{E_b}\,\hat{b}(k)\,[1 \quad 0]^T$, where $\hat{b}(k)$ is the decision. The substitution modifies (7-9). The prediction equations are now given by (11-13),

$$\hat{\mathbf{M}}(k)=\Sigma(k)\,\hat{\mathbf{H}}(k)\,[\hat{\mathbf{H}}'(k)\,\Sigma(k)\,\hat{\mathbf{H}}(k)+R]^{-1} \tag{11}$$

Kalman filter gain:

$$\hat{\mathbf{K}}(k)=\mathbf{F}\,\hat{\mathbf{M}}(k) \tag{12}$$

Prediction:

$$\hat{\mathbf{X}}(k+1)=[\mathbf{F}-\hat{\mathbf{K}}(k)\,\hat{\mathbf{H}}'(k)]\,\hat{\mathbf{X}}(k)+\hat{\mathbf{K}}(k)\,z(k) \tag{13}$$

Note that this does not affect the steady-state covariance matrix as discussed in the previous section. One should note that the stationarity of the error covariance does not imply that the "measured" mean square error (MSE) ($\frac{1}{N}\sum_{k=1}^{N}\left|C(k)-\hat{C}(k)\right|^2$) approaches the theoretical MSE $\Sigma(k)_{1,1}$. Actually the measured MSE sometimes may be much larger than the theoretical MSE. When the measured MSE is much larger than the theoretical one, an interesting "reversal" phenomenon occurs. The term "reversal phenomenon" refers to the fact that the estimates are close to the opposites of the actual channel coefficients (phase shift of π). It is illustrated in Figure 2. The channel estimates and channel coefficients are reversed between iteration 300 and 600. During this interval, almost all the decisions are incorrect. Observations show that the reversal phenomenon starts during the deep fades, i.e., when the magnitudes of the channel coefficients are quite small. In the deep fades, the decision errors are likely to occur, and the Kalman estimator will produce the inaccurate estimates, which in turn will cause the subsequent decision errors.

The following arguments explain the reversal phenomenon. Substituting the output of the decorrelator (6) into (13) (ignoring the noise term), we have

$$\hat{\mathbf{X}}(k+1) = \mathbf{F}\hat{\mathbf{X}}(k) + \hat{\mathbf{K}}(k) \{ \mathbf{H}(k)\ \mathbf{X}(k) - \hat{\mathbf{H}}(k)\ \hat{\mathbf{X}}(k) \}. \tag{14}$$

Suppose the decision is wrong, i.e., $\hat{b}(k) = -b(k)$. Then $\hat{\mathbf{H}}(k) = -\mathbf{H}(k)$, and (14) becomes

$$\hat{\mathbf{X}}(k+1) = \mathbf{F}\ \hat{\mathbf{X}}(k) - \hat{\mathbf{K}}(k)\ \hat{\mathbf{H}}(k)\{\ \mathbf{X}(k) + \ \hat{\mathbf{X}}(k)\ \}. \tag{15}$$

The sum of the actual channel coefficient state vector X(k) (5) (without the noise term w(k)) and the estimate (15) gives

$$\mathbf{X}(k+1) + \hat{\mathbf{X}}(k+1) = \{\ \mathbf{F}\ - \hat{\mathbf{K}}(k)\ \hat{\mathbf{H}}(k)\ \}\{\ \mathbf{X}(k) + \hat{\mathbf{X}}(k)\ \} \tag{16}$$

If the Kalman filter is stable, i.e., the magnitudes of all the eigenvalues of the matrix $\mathbf{F} - \hat{\mathbf{K}}(k)\ \hat{\mathbf{H}}(k)$ are less than one, $\mathbf{X}(k) \to -\hat{\mathbf{X}}(k)$ as k increases. Thus, the sequence of the incorrect decisions may trigger the reversal phenomenon.

On the other hand, if the decision is correct, then $\hat{\mathbf{H}}(k) = \mathbf{H}(k)$, (14) becomes

$$\hat{\mathbf{X}}(k+1) = \mathbf{F}\ \hat{\mathbf{X}}(k) + \hat{\mathbf{K}}(k)\ \hat{\mathbf{H}}(k)\{\ \mathbf{X}(k) - \ \hat{\mathbf{X}}(k)\ \} \tag{17}$$

Subtracting (17) from (5), we obtain

$$\mathbf{X}(k+1) - \hat{\mathbf{X}}(k+1) = \{\ \mathbf{F}\ - \hat{\mathbf{K}}(k)\ \hat{\mathbf{H}}(k)\ \}\{\ \mathbf{X}(k) - \hat{\mathbf{X}}(k)\ \} \tag{18}$$

Equation (18) implies that a sequence of correct decisions will result in the estimate $\hat{\mathbf{X}}(k)$ approaching the actual channel coefficient X(k) provided that the ignored noise terms are small. These arguments and the experimental results show that a few consecutive errors in the deep fades are likely to trigger the reversal phenomenon. Once the reversal of the channel coefficients is established, the incorrect decisions continue to occur, even when the channel is "outside" the deep fades. This is due to the fact that the decision is given by $\hat{b}\ (k) = sgn(\mathrm{Re}[z\ (k)\ \hat{C}^*(k)])$. When the estimate $\hat{C}(k) \approx -C(k)$, the decision is likely to be $\hat{b}(k) = -b(k)$. The reversal typically continues until another deep fade during which several "decision errors" (which are actually the correct decisions) may disrupt the convergence in (16) and allow the estimator to recover back to (18).

To avoid the reversal, we set up a threshold level P. When the amplitude of the current channel estimate becomes smaller than the threshold level ($|\hat{C}(k)| < P$), i.e., the channel is in a deep fade, the transmitter is informed to send a training bit next time. The transmission throughput is related to the threshold level. The higher the threshold level, the lower the throughput.

4. Performance Analysis

The fading channel model is given by a second order AR process with a1=-1.9935, a2=0.996. The model corresponds to a second order IIR filter with poles given by $r_d e^{\pm j\omega_d T}$, where rd=0.998 and normalized fading rate $\omega_d T = 2\pi f_d T = 0.0503$ [14]. The data rate is 10 kb/s and maximum Doppler shift (f_d) is 80 Hz.

The synchronous CDMA system has 2 users. We consider two cases. In the first case, the MAI is moderate, and the cross-correlation between the signature waveforms r=0.3. In the second case, the MAI is strong, r=0.9. We examine the performance of the multiuser detectors with Kalman channel estimators by computing the theoretical limits and by simulations.

Theoretical Results

When the channel coefficients are estimated using the MMSE criterion, the BER of the decorrelator for user n was given by [11]:

$$PE_{dec,BPSK} = \frac{1}{2}[1 - \sqrt{\frac{1-\Gamma_n}{1+[\mathbf{R}^{-1}]_{n,n}/\bar{\gamma}_n}}], \tag{19}$$

where $[\mathbf{R}^{-1}]_{n,n}$ is the n-th diagonal element of the inverse of the cross correlation matrix, $\bar{\gamma}_n$ is the average SNR defined by $E_{b_n}\, E(|C_n(k)|^2)/N_0$, and Γ_n is the normalized estimation error variance for the n-th user, defined by

$$\Gamma_n = \frac{E\left(\left|C_n(k) - \hat{C}_n(k)\right|^2\right)}{E\left(\left|C_n(k)\right|^2\right)}$$

The steady-state estimation error variance can be calculated by the Ricatti equation (10). From the orthogonality principle of the MMSE estimator, i.e., $E\left(\hat{C}(k)e^*(k)\right) = 0$ where $e(k) = C(k) - \hat{C}(k)$, we obtain

$$E\left(C(k)\hat{C}^*(k)\right) = E\left(\hat{C}(k)\hat{C}^*(k)\right) = (1-\Gamma)E\left(|C(k)|^2\right) \tag{20}$$

Equation (20) implies that the normalized covariance matrix of $[C(k)\ \hat{C}(k)]$ is $\begin{bmatrix} 1 & 1-\Gamma \\ 1-\Gamma & 1-\Gamma \end{bmatrix}$.

Assuming that C(k) and $\hat{C}(k)$ are jointly Gaussian, we obtain the vector

$$\begin{bmatrix} C(k) \\ \hat{C}(k) \end{bmatrix} = \mathrm{A} \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix} \tag{21}$$

where $w_1(k)$ and $w_2(k)$ are i.i.d. while Gaussian random variables, and $AA^T = \begin{bmatrix} 1 & 1-\Gamma \\ 1-\Gamma & 1-\Gamma \end{bmatrix}$.

Using (21) and the Monte Carlo simulation technique, we generate the channel coefficients C(k) and the corresponding channel estimates $\hat{C}(k)$, and calculate the BER of the decorrelator, the 2-stage and the decision-feedback detectors. The simulation results for the decorrelator agree with (19).

To compare with the differentially coherent modulation scheme for the fading channel, the decorrelator with DPSK is considered [12, 15]. The BER of this decorrelator is [12]

$$PE_{dec,DPSK} = \frac{1}{2}[1 - \frac{\rho}{1+[\mathbf{R}^{-1}]_{n,n}/\bar{\gamma}_n}] \quad \text{where} \quad \rho = \frac{E\left(C(k)C^*(k-1)\right)}{E\left(|C(k)|^2\right)}.$$

Figures 3 and 4 show the theoretical BER of these multiuser detectors with Kalman channel estimators for the two user channels with r=0.3 and r=0.9 receptively. With moderate MAI (r=0.3), the decorrelator, the 2-stage and the decision-feedback detector have similar performance which is close to the single user system with the same imperfect channel estimation. For high MAI (r=0.9), the 2-stage and the decision-feedback detectors outperform the decorrelator even when compared with the decorrelator with perfect channel estimation for SNR below 25dB. In high SNR region, the BER of all three detectors are similar. This probability error floor is mainly due to the irreducible mean square estimation error (Γ_n in (19)) rather than to the MAI [12, 15]. We also observe that multiuser detectors outperform the decorrelator with DPSK in both cases. Note that the BER of the three multiuser detectors are higher than the BER of the single user system. This is due to the fact that the noise component in the input to the channel estimator for the single user system is not enhanced as for the multiuser detectors. Therefore, the single user system has smaller estimation error and lower BER. In summary, the performance gains of the 2-stage and the decision-feedback detectors are still preserved. However, the improvement with respect to the decorrelator is not as significant as for the perfect-estimation case.

Actual Simulation Results

The error covariance matrix used in the decision-directed mode is computed during training (about 100 iterations) and could be computed off-line. The threshold level is set in the following way. Suppose that the amplitude of the channel coefficient and the channel estimate are approximately the same. Using (6), we can compute the probability of error when the amplitude of the channel coefficient |C|=P. We choose the threshold level P so that this probability is 10^{-2}. Thus, we avoid the situations when the channel undergoes deep fades. For example, for the two-user case with cross-correlation r, this criterion leads to $Q(\sqrt{\frac{P^2 E_b}{N_0/2}(1-r^2)}) = 10^{-2}$. If the amplitude of the current estimate $|\hat{C}(k)|$ <P, the next bit b(k+1) sent by the transmitter is used for retraining. Although this is not a realistic ARQ protocol, it gives an upper bound on the throughput (the ratio of the number of data bits to the number of all transmitted bits (data+training)). Table 1 shows the throughputs given this threshold criterion. Note that the throughput of the decorrelator with r=0.9 is much lower than the throughput of the single user system in the low SNR region. This is due to the noise enhancement in the output of the decorrelator.

Average SNR (dB)		10	15	20	25	30	35
Throughput(%)	Single user	86.9	95.8	98.6	99.5	99.8	99.9
	2 user (r=0.3)	86.1	95.3	98.5	99.5	99.8	99.9
	2 user (r=0.9)	47.3	79.6	92.9	97.7	99.3	99.7

Table 1: Throughputs of the single user system and the decorrelator (2 user) for the simulation shown in Figure 5.

Figure 5 shows the actual simulation results. Compared with the theoretical results, the BER are lower because the receivers do not make decisions during deep fades. The BER of the decorrelator (r=0.3) is very close to that of the single user system. We did not encounter any reversals during the simulation with 10^6 bits. However, the probability of reversal is still under investigation.

5. Work in Progress

In this paper, we use the Kalman filter as the channel estimator for the Rayleigh flat fading channel. The error propagation problem (termed the "reversal phenomenon") present in the decision-directed mode was found to be a major obstacle to the reliable performance of this estimator. To prevent this problem, we set up a threshold to avoid the deep fades. Another method, the DPSK with coherent detection, is under the investigation. This is due to the fact that the reversal (phase shift of π) will not affect the difference between the phases of two consecutive bits. Our future study will concentrate on practical considerations, such as applying Kalman filter and other estimation methods to the Jakes' fading model, and investigating the performance of multiuser detectors for asynchronous multipath fading channels.

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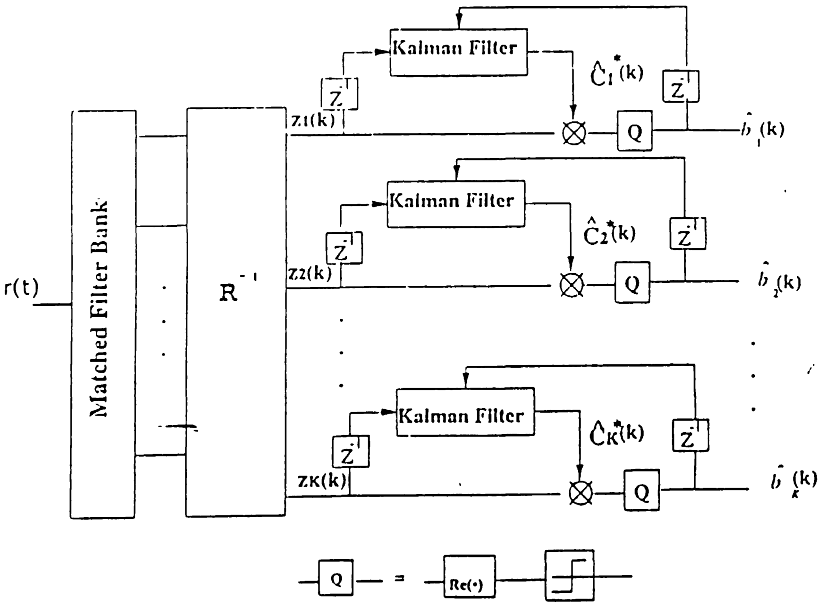

Figure 1: The decorrelator with the Kalman estimators

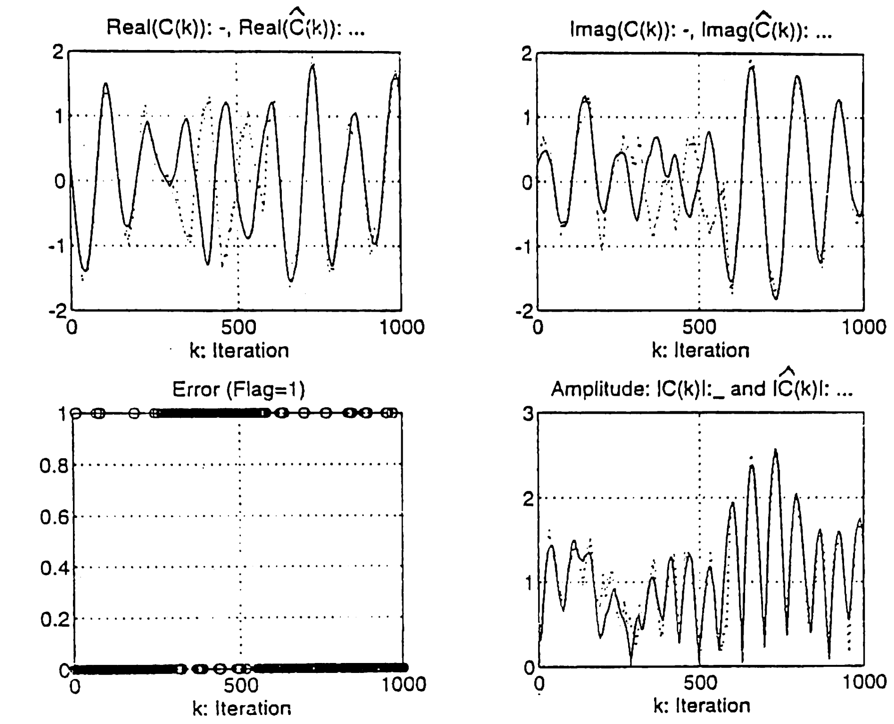

Figure 2: The reversal phenomenon of the Kalman estimator in the decision-directed mode

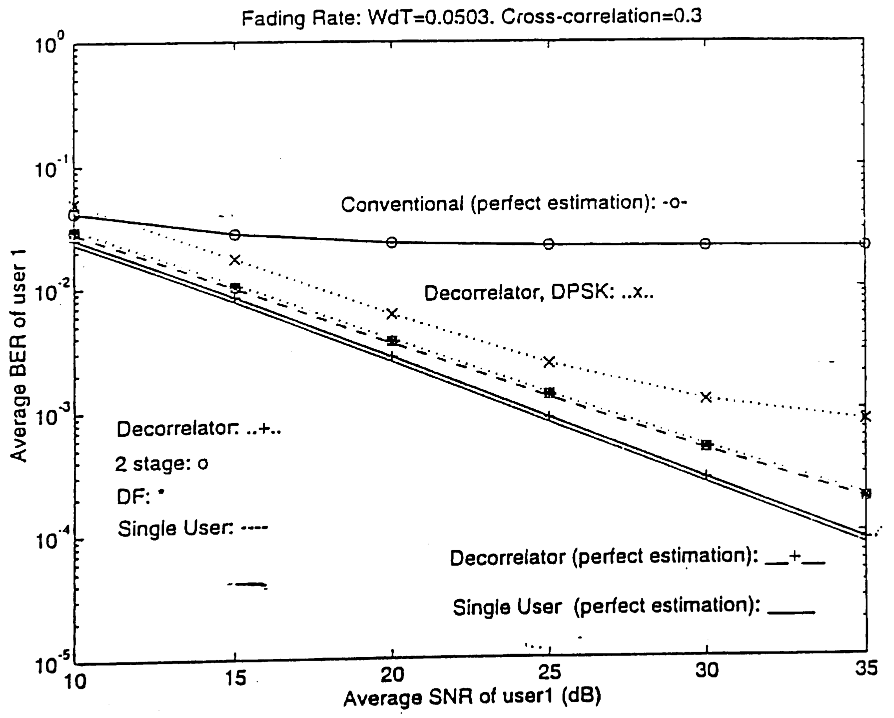

Figure 3: BER of multiuser detectors for a two-user channel (r=0.3)

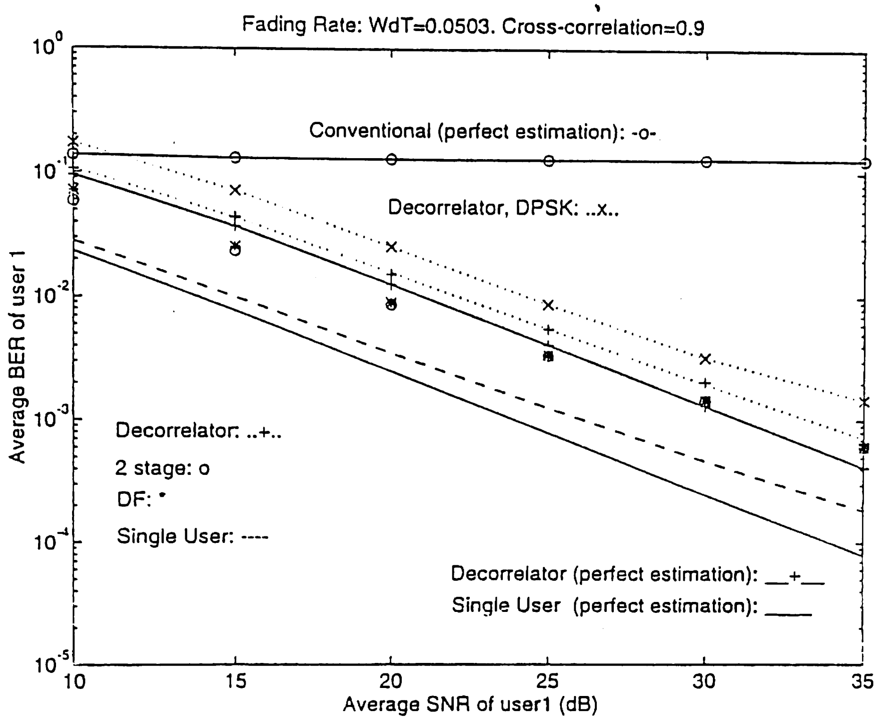

Figure 4: BER of multiuser detectors for a two-user channel (r=0.9)

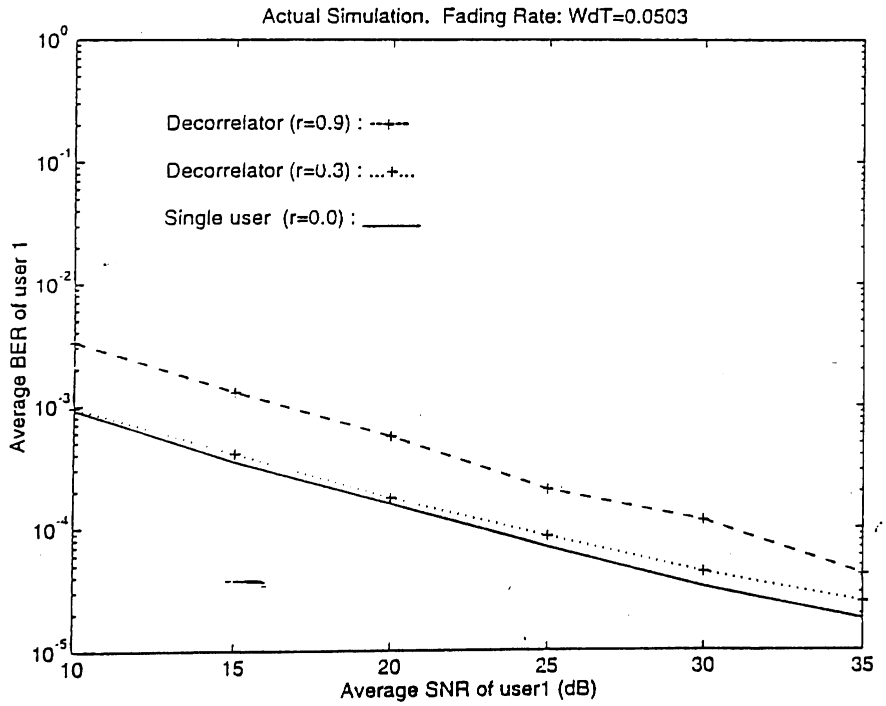

Figure 5: Actual BER of the decorrelator with Kalman estimator using the threshold criterion