SPICE Simulation of Analog Circuits for MFA and for Optimal Interpolation

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TR-95/5
March 1995
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Abstract

We report SPICE simulation results of two similar circuits that implement Mean Field Annealing to maximize a Bayesian posterior probability density for to determine the MAP estimate of a one dimensional image from a degraded image. The first circuit treats the case of blurring by convolution with an exponential kernel followed by the addition of noise. The second case treats the additional degradation of missing data by estimating the entire original image using only half the deleting every other blurred, noisy measurement.

1 Image Formation Model

Images degraded by imperfect sensors or transmission can be modeled usefully[2] as the sum of random noise image and a convolution of the true image with a point spread function or blurring kernel. We will write

\[ g = h * f + n, \tag{1} \]

for the degraded image and restrict ourselves to one dimensional discrete images so that \( g_i \) for \( i = 1, 2, \ldots, L \) is the \( i^{th} \) pixel of \( L \) pixels in image \( g \). Blurring is represented by the convolution of kernel \( h \) with the true signal \( f \) by the \( * \) operator, defined by the relation

\[ (h * f)_i = \sum_j h_{i-j} f_j \tag{2} \]

so that the \( i^{th} \) pixel of the convolved image \( h * f \) is a weighted sum of pixels of \( f \). Here we restrict ourselves to the case of a symmetric blur \( h_{-i} = h_i \). Additive noise is effectively an image of random variables \( n_i \) where \( i = 1, 2, \ldots, L \) are collected in \( n \).

2 Image Restoration

Piecewise smooth maximum a posteriori or MAP restorations produce high quality results when implemented as numerical algorithms on general purpose
digital computers. But even efficient serial implementations require several seconds to process a frame of video and therefore cannot be implemented in real time. Parallel implementations require perhaps two orders of magnitude less time[1] but this is still less than frame rate. Analog VSLI may allow real time implementation of mean field annealing of video images.

The simplest deconvolution method is the spatial domain analog of inverse filter which attempts to solve \( g = h \ast f \) for \( f \) at each pixel. The simplest circuit to implement this method is to put a resistive network model of the convolution in the negative feedback path of a differential amplifier whose noninverting input is the blurred image.

Deconvolution by inversion is not stable, whether in the frequency domain or the space domain. Stability can be improved by regularization, which is equivalent to a Bayesian estimate. Regularization terms that result in linear problems cannot completely distinguish between high frequency edge features and noise. When the true signal is known to be piecewise smooth, a better regularization term is the sum of Gaussians of differences of adjacent pixels. The excellent quality results of such formulations are compromised only by the high numerical complexity of solving the resulting global optimization problem which we propose to implement in silicon neural networks.

In this approach the optimal estimate is the image that minimizes

\[
H[f] = \sum_i (a \| g - h \ast f \|^2 + b R_T[f,i])
\]

where \( a \) and \( b \) are problem dependent constants and the sum is over all pixels. Here \( R_T \) is the piecewise constant regularization term that will be annealed on temperature \( T \). At pixel \( i \)

\[
R_T[f,i] = \exp \frac{(D_H \ast f)^2}{T} - \exp \frac{(D_V \ast f)^2}{T}
\]

where \( D_H \) is a horizontal difference kernel and \( D_V \) is a vertical difference kernel.

Figure 1 shows the result for a 10 pixel image of a scene with a foreground feature of unit intensity comprising pixels 1, 2, and 3 and a larger background region of zero intensity comprising pixels 4...10 where pixel 10 is labeled with the letter a. To avoid frame boundary effects, the last pixel is taken to be adjacent to the first pixel for purposes of difference operators and convolutions. The blur is implemented as a convolution with an exponential kernel with characteristic length of two pixels. The noise is approximately zero mean and amplitude of .1 or 10 % of the step edges. The combination of blur and noise produces errors as high as 40 percent for a simple inverse filter. At high enough temperature the prior or regularization term of this approach vanishes and the estimate coincides with an inverse filter for deconvolution. This can be seen at near \( T=5 \) in Figure 1. Similar MFA circuits that do not treat blur but only remove additive noise without deconvolution can compute estimates of similar quality for measured signals with several times as much noise.
Figure 1: Plot of 10 pixel estimate of a blurred noisy image annealed from all 10 degraded measurements. Here the measured inputs were blurred by convolving with a kernel of radius 2 pixels and then randomly displaced with noise of approximate amplitude .1. Temperature decreases in steps of -.1 from T=5 on the right to T=.1 on left. The output is represented by an array of ten voltages \( v(e_1) \) to \( v(e_{10}) \) corresponding to estimates of the ten signals \( f_1 \) to \( f_{10} \). The final estimates at low temperatures near the left of the plot show excellent agreement with the true values of unity for \( i = 1, 2, 3 \) and zero for the remaining \( i = 4 \ldots 10 \).
3 Optimal Interpolation

For this circuit the same regularization term will be used but half the measured \( g \) values will be eliminated

\[
H[f] = \sum_{i=1,3,5,...} |g - h * f|_i^2 + \sum_i R_T[f]_i
\]  

(5)

where the first sum is over only the odd pixels, but the second sum is over all pixels.

Figure 2 shows the result of the same input signal, blur, and noise for the case of the additional degradation of missing pixels. In this case the figure shows excellent restoration from only half the pixels even after blurring and added noise.

References


Figure 2: Plot of 10 pixel estimate from a blurred noisy image with every other measured pixel missing. As in Figure 1, the measured inputs were blurred by convolving with a kernel of radius 2 pixels and then randomly displaced with noise of approximate amplitude .1. Temperature decreases in steps of -.1 from T=5 on the right to T=.1 on left. The output is represented by an array of ten voltages $e_1 \ldots e_a$ corresponding to estimates of the ten signals $f_1 \ldots f_{10}$ computed from five degraded values $g_1, g_3, g_5, g_7, g_9$; the five even-indexed values were missing from the input available to this circuit. In spite of the missing data, the restoration at low temperatures is very close to the correct values of unity for the three foreground pixels (1,2,3) and zero for the remaining 7 pixels.