ANALYSIS OF VARIANCE OF A NON-ORTHOGONAL THREE-FACTOR EXPERIMENT USING A COMPUTER

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INTRODUCTION

An article of W. L. Stevens [7] originated this work. Stevens has shown by a suitable worked example that the solution of fitting constants to a set of non-orthogonal data can be reduced to an iterative procedure. But this procedure, as he admitted is always lengthy.

We have considered the problem from a much more general point of view; this was possible because we have now at our disposal high-speed and high-capacity computers.

1. Linear models and three-way cross classification with interactions and unequal number of replications.

The three-way lay-out is a particular case of a linear model.

If we use the vectorial notation, the general form of such a model is:

\[ y = X\beta + e \]

where \( y \) is a random observed vector, \( e \) is a random vector, \( X \) is a

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\((n \times p)\) matrix of known fixed quantities and \(\beta\) is a \((p \times 1)\) vector of unknown parameters.

In the case of a three-way layout with main effects and two-factor interaction, the model is often written in the more explicit form:

\[(2\, y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + \delta_{ij} + \epsilon_{ik} + \eta_{jk} + \epsilon_{ijkl} \]

where

\[i = 1, \ldots, r\]
\[j = 1, \ldots, s\]
\[k = 1, \ldots, t\]
\[\ell = 1, \ldots, n_{ijk}\]

\(n_{ijk}\) being the number of observations in the \(ijk^{th}\) cell.

In this model the number of unknown parameters is equal to:

\[(3\, p = 1 + r + s + t + rs + rt + st = (l+r)(l+s)(l+t) - rst.\]

But all these parameters are not independent; we have the following restrictions:

\[\sum_{i=1}^{r} \alpha_i = \sum_{j=1}^{s} \beta_j = \sum_{k=1}^{t} \gamma_k = 0\]
\[\sum_{i=1}^{r} \delta_{ij} = \sum_{j=1}^{s} \delta_{ij} = 0\]
\[\sum_{i=1}^{r} \epsilon_{ik} = \sum_{k=1}^{t} \epsilon_{ik} = 0\]
\[\sum_{j=1}^{s} \eta_{ij} = \sum_{k=1}^{t} \eta_{jk} = 0.\]

If we return to the vectorial notation these restrictions can be expressed in the following way:
where \( L' \) is a \((m \times p)\) matrix.

It can be noticed that the matrices \( X \) and \( L' \) have a particular pattern: they contain only 0's and 1's as elements.

For calculating the least-squares estimates of \( \beta \), say \( \hat{\beta} \), we must find the vector \( \hat{\beta} \) that minimizes the quantity:

\[
(6) \quad S(\beta) = (y - X\beta)'(y - X\beta)
\]

and at the same time satisfies the equations:

\[
(7) \quad L'\beta = 0.
\]

This may be done by using the Lagrange multipliers. We form the function:

\[
(8) \quad S*(\beta) = (y - X\beta)'(y - X\beta) + \lambda' L'\beta
\]

where \( \lambda \) is a \((m \times 1)\) vector of unknown quantities.

The set of equations:

\[
\frac{\partial S*}{\partial \beta} = 0
\]

give us a set of \( p \) equations:

\[
(9) \quad X'X\beta + L\lambda = X'y.
\]

With the set of \( m \) equations (7), we have a system of \((m + p)\) equations with \((m + p)\) unknowns.

The relations (7) and (9) can be brought together in the following matricial form:

\[
(10) \quad \begin{pmatrix} X'X & L' \\ L & 0 \end{pmatrix} \begin{pmatrix} \beta \\ \lambda \end{pmatrix} = \begin{pmatrix} X'y \\ 0 \end{pmatrix}.
\]
While the matrix $X'X$ is singular, the matrix:

$$
\begin{bmatrix}
X'X & L' \\
L & 0
\end{bmatrix}
$$

is as a rule regular; consequently, the solution of the system (10) can easily be found theoretically; in fact we can write:

$$
\begin{bmatrix}
b \\
\lambda
\end{bmatrix} = \begin{bmatrix}
X'X & L' \\
L & 0
\end{bmatrix}^{-1} \begin{bmatrix}
X'y \\
0
\end{bmatrix}.
$$

(11)

Practically, the problem is much more difficult because of the dimensions of the matrix:

$$
\begin{bmatrix}
X'X & L' \\
L & 0
\end{bmatrix} = G.
$$

(12)

For they become quickly very large when the number of levels of any of the three factors increases. For instance, in the example given by Stevens the numbers of levels are:

$$
r = 2, \ s = 4, \ t = 4.
$$

Then the dimensions of the matrix $X'X$ are given by the relation (3):

$$
p = 3 \cdot 5 \cdot 5 - 2 \cdot 4 \cdot 4 = 75 - 32 = 43
$$

and the dimension $m$ of the matrix $L'$ can be computed from this formula:

$$
m = 2(r+s+t).
$$

In our case, we obtain $m = 20$.

Consequently, the dimensions of the matrix $G$ are $(63 \times 63)$.

For this reason, we thought it would be interesting to find a method to compute the inverse matrix $G^{-1}$ from the matrices $X'X, L'$ and $L$. In the
next section we give such a method allowing us to calculate the inverse matrix \( G^{-1} \) from the generalized inverses of some singular matrices but with smaller dimensions than \( G \).

2. Generalized inverses and solution of normal equations.

First we will give some definitions because all the authors do not use the same terminology.

*Generalized inverse.* Let \( A \) be a \((m \times n)\) matrix. A matrix \( A^- \) is defined to be a generalized inverse of \( A \), if and only if it satisfies:

\[
AA^-A = A.
\]

Note: Some authors call such a matrix a conditional matrix or c-inverse.

*Moore-Penrose inverse.* Let \( A \) be a \((m \times n)\) matrix. If a matrix \( A^+ \) exists that satisfies the four conditions below, we shall call \( A^+ \) a Moore-Penrose inverse of \( A \).

1. \( AA^+ \) is symmetric
2. \( A^+A \) is symmetric
3. \( AA^+A = A \)
4. \( A^+AA^+ = A^+ \).

Note: Some authors call such a matrix a generalized inverse or g-inverse.

It is clear that every Moore-Penrose inverse is a generalized inverse; but a generalized inverse may not be a Moore-Penrose inverse.

It can be stated that the Moore-Penrose inverse of a matrix \( A \) is unique while there may exist more than one generalized inverse.

The results we shall use are valid for generalized inverses and, of course, for Moore-Penrose inverses. In our computations, we shall calculate always the Moore-Penrose inverse which is equal to the inverse when the matrix is regular.

Chakravarti [1] gives the following results.
Let \( A \) be a matrix having the following particular structure:

\[
A = \begin{bmatrix}
X'X & L \\
L' & 0
\end{bmatrix}
\]

where the column-vectors of \( L \) do not belong to the vector space generated by the column-vectors of \( X'X \) and \( A \) is regular. Then the generalized inverse \( A^- \) of \( A \) is given by:

\[
A^- = \begin{bmatrix}
C^- - C^-LQ^-L'C^- & C^-L - C^-LQ^-Q + C^-LQ^-L'C^- \\
Q^-L'C^- & Q^-Q\sigma^-Q^-L'C^- \\
Q^-L'C^- & Q^-Q\sigma^-Q^-L'C^- \\
\end{bmatrix}
\]

where:

\[
Q = L'C^-L \\
C = X'X + LL'.
\]

For our problem, we only need the first term of this partitioned matrix, that is to say:

\[
A_{11} = C^- - C^-LQ^-L'C^-.
\]

Indeed, the values of the vector solution \( \mathbf{b} \) in (11) will be given by:

\[
\mathbf{b} = A_{11}X'y.
\]

The calculation of \( A_{11} \) only needs matrices with dimensions \((p \times p)\), \((m \times p)\), \((p \times m)\) and \((m \times m)\). To compute the matrices \( C^- \) and \( Q^- \) we used a method given by Greville [4] permitting to calculate the Moore-Penrose inverse; then we have calculated the matrix:

\[
A_{11} = C^+ - \mathbf{C}^+LQ^+L'C^+.
\]
3. Analysis of variance and tests for interactions and main effects.

The model we chose is given by the expression (2):

\[ y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + \delta_{ij} + \epsilon_{ik} + \eta_{jk} + \varepsilon_{ijkl} \]

where the \( \varepsilon_{ijkl} \) are independent and normally distributed \( \sim N(0,1) \).

First, let us consider the following test:

**Test for three-factor interaction**

In our model we have not included parameters for the three-factor interaction because it is not necessary to know explicitly the estimations of these parameters to test the hypothesis of no three-factor interaction.

Indeed, if we use the vectorial notation it is known that the sums of squares due to fitting \( \beta \) is:

\[ b'X'y \]

the total sum of squares is:

\[ y'y \]

and consequently the sum of squares of error is:

\[ y'y - b'X'y. \]

On the other hand, as we supposed that there is at least one cell with more than one observation, we have for the sum of squares within the cells the expression:

\[ \sum_{i,j,k} (y_{ijkl} - \bar{y}_{ijk})^2 = \sum \]

where:

\[ \bar{y}_{ijk} = \frac{1}{n_{ijk}} \sum_{\ell=1}^{n_{ijk}} y_{ijkl} \text{ for } n_{ijk} \neq 0. \]
Then we have the following analysis of variance to test the three-factor interaction:

<table>
<thead>
<tr>
<th>Due to</th>
<th>S.S.</th>
<th>d.f.</th>
<th>M.S.</th>
<th>F (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>b'X'y</td>
<td>p - m</td>
<td>I(3)</td>
<td></td>
</tr>
<tr>
<td>3-fact. interaction</td>
<td>y'y - b'X'y - ∑ n - p + m - u</td>
<td>I(3)</td>
<td>I(3)</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>∑ u</td>
<td>u</td>
<td>F_MS</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>y'y</td>
<td>n</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where:

- n is the total number of observations
- p is the number of parameters in (2)
- m is the number of restrictions
- u is the number of degrees of freedom of the sum of squares for error and is given by the formula:

\[ u = \sum_{i_j_k} (n_{ijk} - 1). \]

\[ n_{ijk} \neq 0 \]

Now let us consider:

**Test for two-factor interaction**

To test hypothesis of no two-factor interaction we use the method generally employed in the regression analysis to test the nullity of a set of parameters.

Let ω be the hypothesis that any two-factor interaction is null and denote the corresponding model by:

\[ y = X\beta^* + e^* \]

and by \( \beta^* \) the least-squares estimates of the vector \( \beta^* \); then, we have the following analysis of variance to test the hypothesis ω:
Due to

<table>
<thead>
<tr>
<th></th>
<th>S.S.</th>
<th>d.f.</th>
<th>M.S.</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>$b'X'y$</td>
<td>p - m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b^*$</td>
<td>$b^*X'^y$</td>
<td>p - m' - m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-fact. inter.</td>
<td>$b'X'y - b^*X'^y$</td>
<td>m'</td>
<td>$I_{MS}^{(2)}$</td>
<td>$E_{MS}$</td>
</tr>
<tr>
<td>3-fact. inter.</td>
<td>$y'y - b'X'y - \sum$</td>
<td>n - p - m - u</td>
<td>$E_{MS}$</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>$\sum$</td>
<td>u</td>
<td>$E_{MS}$</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$y'y$</td>
<td>n</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where $m'$ is the number of degrees of freedom of the considered interaction and can be calculated from this formula:

$$m' = (r'-1)(r''-1)$$

$r'$ and $r''$ being the numbers of levels of the two considered factors.

At last, let us consider:

**Test for main effects**

We used exactly the same method for the test for main effect as for the two-factor interaction, but we suppose that the two-factor interactions and the three-factor interaction are null.

Let $\omega^o$ be the null hypothesis of any of the main effects: if we denote by:

$$y = X^o\beta^o + e^o$$

the model corresponding to the hypothesis $\omega^o$, we shall have the following analysis of variance:
Due to S.S. d.f. M.S. F

\[ b \quad b'X'y \quad p' - m'' \]

\[ b' \quad b'X'o'y \quad p' - m'' - m''' \]

Main effect \[ b'X'y - b'o'X'o'y \quad m''' \]

\[ \frac{(ME)_{MS}}{E_{MS}} \]

Résidual \[ y'y - b'X'y \quad n - p' - m'' \]

\[ E_{MS} \]

Total \[ y'y \quad n \]

where \( p' \) is the number of parameters in the general model with no interaction and is given by the formula:

\[ p' = r + s + t. \]

\( m'' \) is the number of restrictions in this model and is equal to 3 and \( m''' \) is the number of degrees of freedom of the considered factor, that is to say \((r''' - 1)\) if \( r''' \) is the number of levels of this factor.

We must also notice that \( b \) denotes the estimation of \( \beta \) in the model where all the interactions were supposed null and not the model (2) of the paragraph 1. We have kept the same notation as for the general model (2) in order to simplify.

4. Description of the program

First we will give some considerations about the conception of this program.

To obtain the various tables of analysis of variance it is necessary to get the various matrices we called \( X, X^* \) and \( X^o \) and also the corresponding restrictions matrices \( L', L^* \) and \( L^o' \).

It can be noted that the matrices \( X^* \) and \( X^o \) can be obtained from the matrix \( X \) by suppressing some columns of that matrix.
The matrices $L^s$ and $L^a$ can also be obtained from the matrix $L'$, but it is a little more difficult because we must suppress not only some columns but also some rows.

To avoid the long and tedious work of punching the elements of the matrices $X, X^s, X^a, L', L^s$ and $L^a$ we have written a program permitting

(i) to obtain the matrices $X$ and $L'$ knowing only the numbers $r, s$ and $t$ of levels of the three-factors and the numbers $n_{ijk}$ of observations in each cell.

(ii) to obtain the matrices $X^s, X^a, L^s$ and $L^a$ from the matrices $X$ and $L'$ respectively for the various tests mentioned above.

As we saw in the paragraph 1, to solve the system of equations given by (10), we must find the inverse of the matrix $G$ given by (12). If the capacity of the computer is sufficient we can use a classical method, but if the dimensions of $G$ are too large we can use the method given in the paragraph 2 employing the generalized inverses.

The main programs corresponding to these two methods are almost the same only the subprograms are different. Although it would have been possible to write a main program valid for the two methods, we have thought that it would be more practical to have two distinct programs.

We tested these two programs with the example given by Stevens. The agreement between the results was very good. The results of the tables of analysis of variance were exactly the same and the values of the components of the vector $b$ disagreed sometimes in the fifth, but mostly in the sixth decimal.

All the quantities are computed with double precision because the simple precision was not sufficient.

The programs were written in FORTRAN IV G language and the computer used is an IBM System/360 Model 75 computer.
Card preparation

Dimension cards: The dimensions were foreseen for the computation of the Steven's example, that is to say for the levels:

\[ r = 2 \quad s = 4 \quad t = 4 \]

and for a number of observations \( n \) equal to 36.

Consequently if the problem has values greater than our problem, the dimension cards must be changed as far as the capacity of the used computer allows it.

Input cards: The first card contains in the format (1615) the following quantities:

IM = \( r \) level of the first factor

JM = \( s \) level of the second factor

KM = \( t \) level of the third factor

N = \( n \) total number of observations

ID1M = number greater or equal to the dimensions of \( G \); it must be equal to the dimensions of the matrix \( A \) of the program.

The second card on to the \((th-1)th\) card where \( h \) is so defined:

\[ h = \left[ \frac{rs}{16} \right] + 1 \]

the symbol \([\ ]\) denoting the integer part, contain in the format (1615) the numbers of observations per cell in the following way:

2\(^{th}\) to \((h-1)th\): \( n_{11}, n_{21}, \ldots, n_{r1}, n_{l1}, \ldots, n_{rs}, n_{ls} \)

\( h^{th}\) to \((2h-1)th\): \( n_{12}, \ldots, n_{r2}, n_{l2}, \ldots, n_{rs2}, n_{ls2} \)

\( \vdots \)

\((th-h)th\) to \((th-1)th\): \( n_{1t}, \ldots, n_{rt}, n_{l1t}, \ldots, n_{ls1}, n_{rst} \)

The \(th\) card and the following cards contain the values of the observations in the format \((20F4.0)\) in the way given below:
\[ y_{1111}, \ldots, y_{11n_111}, y_{2111}, \ldots, y_{21n_{211}}, \ldots, y_{r111}, \ldots \\
\ldots, y_{r1n_{n11}}, y_{1211}, \ldots, y_{12n_{121}}, \ldots, y_{r211}, \ldots \\
\ldots, y_{r2n_{r21}}, \ldots, y_{1s11}, \ldots, y_{1s1n_{s11}}, \ldots, y_{rs11}, \\
\ldots, y_{rsln_{rs1}}, \ldots, y_{rst1}, \ldots, y_{rstn_{rst}} \]

The number of cards in this last set is equal to:

\[ \lceil n/20 \rceil + 1 = f. \]

The \((\text{th}+f)\)th card and the following cards allow us to obtain the six analyses of variance mentioned in the paragraph 3.

They are punched according to the following pattern in the format (16I5):

\[ N_1 \quad N_2 \quad N_3 \quad N_4 \quad N_5 \quad N_6 \quad N_7 \]

where the \(N_i\) \((i = 1, \ldots, 7)\) are so defined:

- \(N_1 = 1\) always
- \(N_2 = r\) or 0
- \(N_3 = s\) or 0
- \(N_4 = t\) or 0
- \(N_5 = rs\) or 0
- \(N_6 = rt\) or 0
- \(N_7 = st\) or 0

that is to say:

- \(N_1 = 0\) if the parameters in the considered model, resulting from the considered null hypothesis, are 0.

More explicitly, the \((\text{th}+f)\)th card corresponding to the complete model given by (2) in the paragraph 1 is:

\[ 1 \quad r \quad s \quad t \quad rs \quad rt \quad st \]

the three following cards corresponding to the three analyses of variance for the two-factor interactions are:
the \((th+f+4)\text{th}\) card corresponding to the general model with no interactions is:

\[
1 r s t 0 0 0
\]

and the three following cards corresponding to the three analysis of variance for the main factors are:

\[
1 0 s t 0 0 0 \\
1 r 0 t 0 0 0 \\
1 r s 0 0 0 0.
\]

**Note:** By a slight modification or adjunction to the program (because the output print ought to be modified or enlarged), we can easily obtain the values of the parameters for any model supposing the nullity of one or several main effects or interactions.

The last card must always be a blank card.

The output print

The program gives:

i) the values of the parameters for the complete model given by (2).

ii) The three tables of analysis of variance for the test of the three two-factor interactions followed by the values of the parameters in the corresponding null hypothesis.

In each of these tables appears the test of the three-factor interaction.

iii) The three tables of analysis of variance for the test of the three main effects followed by the values of the parameters in the corresponding null hypothesis.
As an example we give the results obtained from the data of the problem
given by Stevens in [7].

Time estimates

As we already said, we have used two methods. The method using the gener-
alized inverses is much longer than the method using the classical inversion.
Then, if the capacity of the computer allows it, the second method is less ex-
ensive. With the Steven's example we got the following times:

First method: 22'30"
Second method: 3'30".

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ACKNOWLEDGMENTS

I am grateful to Professor I. M. Chakravarti for his help and valuable suggestions in the preparation of this work and for a critical reading of this manuscript.

APPENDIX

1. Listing of the program employing generalized inverses and data for the Steven's example.

2. Listing of the program employing the classical inverse and data for the Steven's example.

3. Results obtained with the data of the Steven's example.
DATA SET LISTING

C ANALYSIS OF VARIANCE OF A NON-ORTHOGONAL THREE-FACTOR EXPERIMENT

DOUBLE PRECISION F(43),H(43),Y(43),P(43)
DOUBLE PRECISION A(43,43),B(43,43),BT(43,43),EL(43,43),ELT(43,43)
DOUBLE PRECISION YT(4,4,4)
DOUBLE PRECISION CCA,SCD,SCDC,CMDC,CMD,SCF,CMF
DOUBLE PRECISION SY,SCY,SYT,TCOR,SCC,SCCEL,SCI,CMI,SCA,SC
DIMENSION NOBS(4,4,4),NX(7),NXX(7)
READ 1,IM,JM,KM,N,IDIM

1 FORMAT(1615)
DO 21 K=1,KM
21 READ 1,((NOBS(I,J,K),I=1,IM),J=1,JM)
READ 55,(Y(I),I=1,N)
55 FORMAT(20F4.0)
SY=0.
SCY=0.
SCYT=0.
KOMPT=0
SCA=0
IJK=0
NCCEL=0
DO 70 K=1,KM
DO 70 J=1,JM
DO 70 I=1,IM
IF(NOBS(I,J,K).EQ.0)GO TO 70
NM=NOBS(I,J,K)
NCCEL=NCCEL+1
YT(I,J,K)=0.
DO 71 L=1,NM
IJK=IJK+1
YT(I,J,K)=YT(I,J,K)+Y(IJK)
SCY=SCY+Y(IJK)**2
71 SY=SY+Y(IJK)
SCYT=SCYT+YT(I,J,K)**2/FLOAT(NM)
70 CONTINUE
TCOR=SY**2/FLOAT(IJK)
SCC=SCY-TCOR
NDLT=IJK-1
SCCEL=SCYT-TCOR
NDLCEL=NCCEL-1
SCI=SCC-SCCEL
NDLI=NDLT-NDLCEL
CMI=SCI/FLOAT(NDLI)

99 READ 1,(NX(I),I=1,7)
IF(NX(1).EQ.0)GO TO 60
NC=0
NCC=0
MI=1
KOL=1
NXT=0
NXX(1)=0
DATA SET LISTING

    DO 41 I=2,7
    NXX(I) = 0
    KOL = KOL + NX(I)
    IF(NX(I).EQ.0) GO TO 411
    IF(I.GT.4) GO TO 42
    NCC = NCC+1
    NEP = NCC
    GO TO 41

42   IF (I-6)43,44,45
43   NCC = NCC+IM+JM-1
    INT IJ = NCC
    GO TO 41
44   NCC=NCC+IM+KM-1
    INTIK=NCC
    GO TO 41
45   NCC=NCC+JM+KM-1
    INTJK=NCC
    GO TO 41

411  NXX(I)=1

41   CONTINUE
    IMIN=KOL+1
    IMAX=KOL+NCC
    DO 22 IND=1,7
    NXT=NXT+NXX(IND)
    NC=NC+NX(IND)
    IF(NX(IND).EQ.0) GO TO 22
    IJK=0
    MM=NC
    DO 23 K=1,KM
    DO 23 J=1,JM
    DO 23 I=1,IM
    IF(NOBS(I,J,K).EQ.0) GO TO 23
    NM=NOBS(I,J,K)
    DO 24 L=1,NM
    IJK=IIJK+1
    DO 25 M=MI,MM
    MA=M1,MM
    GO TO (26,27,28,29,30,31,32),IND
26    BT(IJK,M)=1.
    GO TO 25
27    IF(MA-I)33,26,33
33    BT(IJK,M)=0.
    GO TO 25
28    IF(MA-J)33,26,33
29    IF(MA-K)33,26,33
30    IF(MA-((I-1)*JM+J))33,26,33
31    IF(MA-((I-1)*KM+K))33,26,33
32    IF(MA-((J-1)*KM+K))33,26,33
25    CONTINUE
24    CONTINUE
DATA SET LISTING

23 CONTINUE
   DO 46 I=IMIN,IMAX
   K=I-KOL
   IMM=I-IMIN+1
   DO 47 M=M1,MM
   MA=M-MI
   GO TO (34,35,35,35,36,37,38),IND
34 ELT(K,M)=0.
   GO TO 47
35 IF(IMM-IND+1+NXT)34,48,34
48 ELT(K,M)=1.
   GO TO 47
36 IK=INTIJ-IM-JM+1
   IL=INTIJ
   IS=JM
   IP=INTIJ-JM
   GO TO 39
37 IK=INTIK-IM-KM+1
   IL=INTIK
   IS=KM
   IP=INTIK-KM
   GO TO 39
38 IK=INTJK-JM-KM+1
   IL=INTJK
   IS=KM
   IP=INTJK-KM
   IN=IMM-IK
   IQ=IMM-IP
   IT=MA/IS+1
   IU=MA+1-IS*(MA/IS)
   IF(IMM.LE.IK.OR.IMM.GT.IL) GO TO 34
   IF(IMM.LE.IP) GO TO 40
   IF(IQ.EQ.IU) GO TO 48
   GO TO 34
40 IF(IN-IT)34,48,34
47 CONTINUE
46 CONTINUE
22 CONTINUE
   DO 50 I=1,N
   DO 50 J=1,KOL
50 B(J,I)=BT(I,J)
   CALL MATVMU(B,Y,P,KOL,N,IDIM)
   CALL MATMUL(B,BT,A,KOL,N,KOL,IDIM)
   M=NCC
   DO 7 I=1,M
   DO 7 J=1,KOL
7 EL(J,I)=ELT(I,J)
   CALL MATMUL(EL,ELT,BT,KOL,M,KOL,IDIM)
   DO 8 I=1,KOL
DATA SET LISTING

DO 8 J=1,KOL
8 A(I,J)=A(I,J)+BT(I,J)
   CALL GINV(A,B,KOL,KOL,IDIM)
   CALL MATMUL(B,EL,A,KOL,KOL,M,IDIM)
   CALL MATMUL(ELT,A,BT,M,KOL,M,IDIM)
   CALL GINV(BT,A,M,M,IDIM)
   CALL MATMUL(EL,A,BT,KOL,M,M,IDIM)
   CALL MATMUL(B,BT,A,KOL,KOL,M,IDIM)
   CALL MATMUL(A,ELT,BT,KOL,M,KOL,IDIM)
   CALL MATMUL(BT,B,A,KOL,KOL,KOL,IDIM)
DO 9 I=1,KOL
9 B(I,J)=B(I,J)-A(I,J)
   CALL MATVMU(B,P,H,KOL,KOL,IDIM)
   SC=0.
   DO 2 I=1,KOL
2 SC=SC+P(I)*H(I)
   SC=SC-TCOR
   IF(KOMP,T.GT.0)GO TO 72
   SCA=SC
   NDLA=KOL-1-NCC
   CCA=SCA/FLOAT(NDLA)
   SCD=SCC-SCA
   NDLD=NDLT-NDLA
   SCDC=SCD-SCI
   NDLDC=NDLD-NDLI
   CMDC=SCDC/FLOAT(NDLDC)
   FI=CMDC/CFI
   CMD=SCD/FLOAT(NDLD)
   PRINT 93
93 FORMAT(1X,'VALUES OF THE PARAMETERS FOR THE COMPLETE MODEL')
   PRINT 6, (H(I),I=1,IMAX)
   PRINT 67
   KOMP=T.KOMP+1
   GO TO 99
72 SCF=SCA-SC
   GO TO (85,86,87,173,73,74,75),KOMP
173 SCA=SC
   KOMP=T.KOMP+1
   NDLA=IM+JM+KM-3
   CCA=SCA/FLOAT(NDLA)
   SCD=SCC-SCA
   NDLD=NDLT-NDLA
   CMD=SCD/NDLD
   GO TO 99
73 NDLF=IM-1
   GO TO 76
74 NDLF=JM-1
   GO TO 76
75 NDLF=KM-1
DATA SET LISTING

76 SCF=SCA-SC
    CMF=SCF/FLOAT(NDLF)
    NDLAF=NDLA-NDLF
    FF=CMF/CMD
    KK=KOMP-4
    PRINT 77,KK
77 FORMAT(20X,'ANALYSIS OF VARIANCE FOR THE FACTOR',I5/)
    PRINT 90
    PRINT 78,SCF,NDLF,CMF,FF
78 FORMAT(1X,'CONSIDERED FACTOR',8X,F20.6,2X,I4,F20.6,F12.3)
    PRINT 79,SC,NDLAF
    PRINT 80,SCA,NDLA
    GO TO 92
85 NDLINT=(IM-1)*(JM-1)
    INT=12
    GO TO 88
86 NDLINT=(IM-1)*(KM-1)
    INT=13
    GO TO 88
87 NDLINT=(JM-1)*(KM-1)
    INT=23
88 CMF=SCF/FLOAT(NDLINT)
    NDLAF=NDLA-NDLINT
    FF=CMF/CMI
    PRINT 89,INT
89 FORMAT(20X,'ANALYSIS OF VARIANCE FOR INTERACTION',I4/)
    PRINT 90
90 FORMAT(39X,'SS',10X,'DF',10X,'MS',15X,'F'/)
    PRINT 91,INT,SCF,NDLINT,CMF,FF
91 FORMAT(1X,'INTERACTION',I3,11X,F20.6,2X,I4,F20.6,F12.3)
    PRINT 79,SC,NDLAF
79 FORMAT(1X,'OTHER FACTORS',12X,F20.6,2X,4/)
    PRINT 80,SCA,NDLA
80 FORMAT(1X,'PARTIAL TOTAL (A)',8X,F20.6,2X,I4/)
    PRINT 82,SCDC,NDLDC,CMDC,FI
82 FORMAT(1X,'INTERACTION 123 (D)-(C)',2X,F20.6,2X,I4,F20.6,F12.3)
    PRINT 81,SCI,NDLI,CMI
81 FORMAT(1X,'WITHIN (C)',15X,F20.6,2X,I4,F20.6/)
92 FORMAT(39X,'SS',10X,'DF',10X,'MS',15X,'F'/)
    PRINT 93,SCD,NDLDC CMD,FI
83 FORMAT(1X,'REST (D)=(B)-(A)',9X,F20.6,2X,I4,F20.6//)
    PRINT 84,SCC,NDLT
84 FORMAT(1X,'TOTAL (B)',16X,F20.6,2X,I4///)
    PRINT 94
94 FORMAT(1X,'VALUES OF THE PARAMETERS'//)
    PRINT 6,(H(I),I=1,IMAX)
6 FORMAT(1X,8F16.6)
    KOMP=KOMP+1
    PRINT 67
67 FORMAT(1H1)
    GO TO 99
DATA SET LISTING

60 CALL EXIT
END

SUBROUTINE GINVC(A,B,IMAX,IMAX,IMAX,IMAX)
DOUBLE PRECISION A(IMAX,IMAX),B(IMAX,IMAX)
DOUBLE PRECISION BT(43,43),C(43),D(43),G(43),H(43)
DOUBLE PRECISION E,F,DEN,BB
E=0.
DO 4 I=1,IMAX
4 E=E+A(I,1)**2
DO 5 J=1,IMAX
5 B(I,J)=A(J,1)/E
JC=2
IL=1
DO 6 I=1,IMAX
6 H(I)=A(I,JC)
CALL MATVMU(B,H,D,IL,IMAX,IMAX)
CALL MATVMU(A,D,C,IMAX,IL,IMAX)
DO 9 I=1,IMAX
IF(H(I)-C(I))8,9,8
9 CONTINUE
1 FORMAT(1X,10I5)
DO 10 I=1,IL
DO 10 J=1,IMAX
10 BT(J,I)=B(I,J)
CALL MATVMU(BT,D,C,IMAX,IL,IMAX)
CALL MATVMU(B,C,G,IL,IMAX,IMAX)
CALL MATVMU(BT,G,H,IMAX,IL,IMAX)
F=0.
DEN=0.
DO 11 I=1,IMAX
F=F+A(I,JC)*C(I)
11 DEN=DEN+A(I,JC)*H(I)
DO 12 I=1,IMAX
12 G(I)=(1.+F)*C(I)/DEN
GO TO 13
8 CALL MATMUL(A,B,BT,IMAX,IL,IMAX,IMAX)
DO 14 I=1,IMAX
DO 14 J=1,IMAX
IF(I.EQ.J)GO TO 15
BT(I,J)=-BT(I,J)
GO TO 14
15 BT(I,J)=1.-BT(I,J)
14 CONTINUE
CALL MATVMU(BT,H,G,IMAX,IMAX,IMAX)
BB=0.
DO 17 I=1,IMAX
17 BB=BB+G(I)**2
DO 18 I=1,IMAX
18
DATA SET LISTING

G(I)=G(I)/BB
18 CONTINUE
DO 19 J=1,IMAX
DO 20 I=1,IL
20 B(I,J)=B(I,J)-D(I)*G(J)
19 B(IL+1,J)=G(J)
IF(JC.EQ.JMAX)GO TO 21
JC=JC+1
IL=IL+1
GO TO 26
21 RETURN
END

SUBROUTINE MATMU(A,B,C,IMAX,JMAX,IDIM)
DOUBLE PRECISION A(IDIM,IDIM),B(IDIM),C(IDIM)
DO 1 I=1,IMAX
C(I)=0.
CO 1 J=1,JMAX
1 C(I)=C(I)+A(I,J)*B(J)
RETURN
END

SUBROUTINE MATMUL(A,B,C,IM,JM,KM,ID)
DOUBLE PRECISION A(ID,ID),B(ID,ID),C(ID,ID)
DO 1 I=1,IM
DO 1 K=1,KM
C(I,K)=0.
DO 1 J=1,JM
1 C(I,K)=C(I,K)+A(I,J)*B(J,K)
RETURN
END

2 4 4 36 43
2 1 2 0 1 1 1 1 1
2 1 2 1 1 1 1 0
1 1 1 2 2 0 1 1
2 0 1 2 2 0 1 1
43 58 58 73 59 81 62 67 71 93 83 60 75 89 71 101 76 100 91 70
85 50 58 92 88 106 73 89 89 98 69 72 105 108 109 76
1 2 4 4 8 8 8 16
1 2 4 4 0 8 8 16
1 2 4 4 8 8 16
1 2 4 4 8 8 0
1 2 4 4 0 0 0
1 0 4 4 0 0 0
1 2 0 4 0 0 0
1 2 4 0 0 0 0
DATA SET LISTING

C ANALYSIS OF VARIANCE OF A NON-ORTHOGONAL THREE-FACTOR EXPERIMENT
DOUBLE PRECISION A(65,65),B(65,65),BT(65,65),P(65),Y(65),X(65)
DOUBLE PRECISION YT(4,4,4),H(65)
DOUBLE PRECISION CCA,SCD,SCDC,CMDC,CMD,SCF,CMF
DOUBLE PRECISION SY,SYT,TCOR,SCC,SCCEL,SCI,CMI,SCA,SC
DIMENSION NOBS(4,4,4),NX(7),NXX(7)
READ 1,IM,JM,KM,N,IDIM
1 FORMAT(16I5)
   DO 21 K=1,KM
21 READ 1,(NOBS(I,J,K),I=1,IM),J=1,JM)
   READ 55,(Y(I),I=1,N)
55 FORMAT(20F4.0)
   SY=0.
   SCY=0.
   SCYT=0.
   KOMPT=0
   SCA=0
   IJK=0
   NCEL=0
   DO 70 K=1,KM
   DO 70 J=1,JM
   DO 70 I=1,IM
   IF(NOBS(I,J,K).EQ.0)GO TO 70
   NM=NOBS(I,J,K)
   NCEL=NCEL+1
   YT(I,J,K)=0.
   DO 71 L=1,NM
   IJK=IJK+1
   YT(I,J,K)=YT(I,J,K)+Y(IJK)
   SCY=SCY+Y(IJK)**2
71 SY=SY+Y(IJK)
   SCYT=SCYT+YT(I,J,K)**2/FLOAT(NM)
70 CONTINUE
   TCOR=SY**2/FLOAT(IJK)
   SCC=SCY-TCOR
   NDLT=IJK-1
   SCCEL=SCYT-TCOR
   NDLCEL=NCEL-1
   SCI=SCC-SCCEL
   NDL=NDLT-NDLCEL
   CMI=SCI/FLOAT(NDL)
99 READ 1,(NX(I),I=1,7)
   IF(NX(1).EQ.0)GO TO 60
   NC=0
   NCC=0
   MI=1
   KOL=1
   NXT=0
   NXX(1)=0
   Do 41 I=2,7
DATA SET LISTING

NXX(I)=0
KOL=KOL+NX(I)
IF(NX(I).EQ.0)GO TO 411
IF(I.GT.4)GO TO 42
NCC=NCC+1
NEP=NCC
GO TO 41
42 IF(I-6)43,44,45
43 NCC=NCC+IM+JM-1
INTIJ=NCC
GO TO 41
44 NCC=NCC+IM+KM-1
INTIK=NCC
GO TO 41
45 NCC=NCC+JM+KM-1
INTJK=NCC
GO TO 41
411 NXX(I)=1
41 CONTINUE
IMIN=KOL+1
IMAX=KOL+NCC
DO 22 IND=1,7
NXT=NXT+NXX(IND)
NC=NC+NX(IND)
IF(NX(IND).EQ.0)GO TO 22
IJK=0
MM=NC
DO 23 K=1,KM
DO 23 J=1,JM
DO 23 I=1,IM
IF(NOBS(I,J,K).EQ.0)GO TO 23
NM=NOBS(I,J,K)
DO 24 L=1,NM
IJK=IJK+1
DO 25 M=MI,MM
MA=M-MI+1
GO TO (26,27,28,29,30,31,32),IND
26 BT(IJK,M)=1.
GO TO 25
27 IF(MA-I)33,26,33
33 BT(IJK,M)=0.
GO TO 25
28 IF(MA-J)33,26,33
29 IF(MA-K)33,26,33
30 IF(MA-((I-1)*JM+J))33,26,33
31 IF(MA-((I-1)*KM+K))33,26,33
32 IF(MA-((J-1)*KJ+K))33,26,33
25 CONTINUE
24 CONTINUE
23 CONTINUE
DATA SET LISTING

    DO 46 I=IMIN,IMAX
    IMM=I-IMIN+1
    DO 47 M=MI,MM
    MA=M-MI
    GO TO (34,35,35,35,36,37,38),IND
34    A(I,M)=0.
    GO TO 47
35    IF(IMM-IND+1+NXT)34,48,34
48    A(I,M)=1.
    GO TO 47
36    IK=INTJ-IM-JM+1
    IL=INTJ
    IS=JM
    IP=INTJ-JM
    GO TO 39
37    IK=INTK-IM-KM+1
    IL=INTK
    IS=KM
    IP=INTK-KM
    GO TO 39
38    IK=INTJK-JM-KM+1
    IL=INTJK
    IS=KM
    IP=INTJK-KM
39    IN=IMM-IK
    IQ=IMM-IP
    IT=MA/IS+1
    IU=MA+1-IS*(MA/IS)
    IF(IMM.LE.IK.OR.IMM.GT.IL)GO TO 34
    IF(IMM.LE.IP)GO TO 40
    IF(IQ.EQ.IU)GO TO 48
    GO TO 34
40    IF(IN-IT)34,48,34
47    CONTINUE
46    CONTINUE
    MI=MM+1
22    CONTINUE
    DO 50 I=1,N
    DO 50 J=1,KOL
50    B(J,I)=BT(I,J)
    CALL MATVMU(B,Y,P,KOL,N,IDIM)
    CALL MATMUL(B,BT,A,KOL,N,KOL,IDIM)
    DO 52 J=IMIN,INAX
    DO 53 I=1,KOL
53    A(I,J)=A(J,I)
    DO 54 I=IMIN,INAX
54    A(I,J)=0.
    P(J)=0.
52    CONTINUE
    JMAX=IMAX
DATA SET LISTING

DO 4 I=1,KOL
4   A(I,IMAX+1)=-P(I)
DO 5 I=IMIN,IMAX
5   A(I,IMAX+1)=0.
    L=IMAX-1
    CALL SOLSL(A,X,L,IDIM)
    SC=0.
    DO 7 I=1,KOL
7       H(I)=P(I)*X(I)
       SC=SC+H(I)
    CONTINUE
    SC=SC-TCOR
    IF(KOMPT.GT.0)GO TO 72
    SCA=SC
    NDLA=KOL-1-NCC
    CCA=SCA/FLOAT(NDLA)
    SCD=SCC-SCA
    NDLD=NDLT-NDLA
    SCDC=SCD-SCI
    NDLDC=NDLD-NDLI
    CMDC=SCDC/FLOAT(NDLDC)
    FI=CMDC/CMI
    CMD=SCD/FLOAT(NDLD)
    PRINT 93
93   FORMAT(1X,'VALUES OF THE PARAMETERS FOR THE COMPLETE MODEL'/)
    PRINT 6,(X(I),I=1,IMAX)
    PRINT 67
    KOMPT=KOMPT+1
    GO TO 99
72   SCF=SCA-SC
    GO TO (85,86,87,173,73,74,75),KOMPT
173  SCA=SC
    KOMPT=KOMPT+1
    NDLA=IM+JM+KM-3
    CCA=SCA/FLOAT(NDLA)
    SCD=SCC-SCA
    NDLD=NDLT-NDLA
    CMD=SCD/NDLD
    GO TO 99
73   NDLF=IM-1
    GO TO 76
74   NDLF=JM-1
    GO TO 76
75   NDLF=KM-1
76   SCF=SCA-SC
    CMF=SCF/FLOAT(NDLF)
    NDLAF=NDLA-NDLF
    FF=CMF/CMD
    KK=KOMPT-4
    PRINT 77, KK
DATA SET LISTING

77 FORMAT(20X,'ANALYSIS OF VARIANCE FOR THE FACTOR',I5/)    
PRINT 90
PRINT 78,SCF,NDLF,CMF,FF
78 FORMAT(1X,'CONSIDERED FACTOR',8X,F20.6,2X,I4,F20.6,F12.3)  
PRINT 79,SCF,NDLAF
PRINT 80,SCA,NDLA
GO TO 92
85 NDLINT=(IM-1)*(JM-1)  
INT=12
GO TO 88
86 NDLINT=(IM-1)*(KM-1)  
INT=13
GO TO 88
87 NDLINT=(JM-1)*(KM-1)  
INT=23
88 CMF=SCF/FLOAT(NDLINT)  
NDLAF=NDLA-NDLINT
FF=CMF/CMI
PRINT 89,INT

89 FORMAT(20X,'ANALYSIS OF VARIANCE FOR INTERACTION',I4/)  
PRINT 90
90 FORMAT(39X,'SS',10X,'DF',10X,'MS',15X,'F')/   
PRINT 91,INT,SCF,NDLINT,CMF,FF
91 FORMAT(1X,'INTERACTION',13,11X,F20.6,2X,I4,F20.6,F12.3)  
PRINT 79,SC,NDLAF
79 FORMAT(1X,'OTHER FACTORS',12X,F20.6,2X,I4/)  
PRINT 80,SCA,NDLA
80 FORMAT(1X,'PARTIAL TOTAL (A)',8X,F20.6,2X,I4/)  
PRINT 82,SCDC,NDLDC,CMDC,FI
82 FORMAT(1X,'INTERACTION 123 (D)-(C)',2X,F20.6,2X,I4,F20.6,F12.3)  
PRINT 81,SCI,NDLI,CMI
81 FORMAT(1X,'WITHIN (C)',15X,F20.6,2X,I4,F20.6/)  
PRINT 83,SCD,NDLD,CMD
82 FORMAT(1X,'REST (D)=(B)-(A)',9X,F20.6,2X,I4,F20.6/)  
PRINT 84,SCC,NDLT
83 FORMAT(1X,'TOTAL (B)',16X,F20.6,2X,I4///)  
PRINT 94
84 FORMAT(1X,'VALUES OF THE PARAMETERS///')  
PRINT 6,(X(I),I=1,IMAX)
6 FORMAT(1X,8F16.6)
KOMPT=KOMPT+1
PRINT 67
67 FORMAT(1H1)
GO TO 99
60 CALL EXIT  
END

SUBROUTINE SOLSL(A,X,L,ID)
DOUBLE PRECISION A(ID,ID),X(ID)
DATA SET LISTING

M=L+1
J=H+1
DO 1 K=1,L
KP=K+1
DO 4 I=KP,M
P=A(K,K)
C=A(I,K)
IF(C)3,4,5
5 IF(C+P)4,4,6
3 IF(C+P)7,4,4
6 IF(C-P)4,4,8
7 IF(C-P)8,4,4
8 DO 9 JJ=K,J
E=A(I,JJ)
A(I,JJ)=A(K,JJ)
9 A(K,JJ)=E
4 CONTINUE
DO 1 I=KP,M
DO 1 JJ=KP,J
1 A(I,JJ)=A(I,JJ)-A(I,K)*A(K,JJ)/A(K,K)
X(J)=1.
DO 10 I=1,M
NI=J-I
X(NI)=0.
DO 11 JJ=1,I
NJ=J+1-JJ
11 X(NI)=X(NI)-A(NI,NJ)*X(NJ)
10 X(NI)=X(NI)/A(NI,NI)
RETURN
END

SUBROUTINE MATMVU(A,B,C,IMAX,JMAX,IDIM)
DOUBLE PRECISION A(IDIM,IDIM),B(IDIM),C(IDIM)
DO 1 I=1,IMAX
C(I)=0.
DO 1 J=1,JMAX
1 C(I)=C(I)+A(I,J)*B(J)
RETURN
END

SUBROUTINE MATMUL(A,B,C,IM,KM,ID)
DOUBLE PRECISION A(ID,ID),B(ID,ID),C(ID,ID)
DO 1 I=1,IM
DO 1 K=1,KM
C(I,K)=0.
DO 1 J=1,JM
1 C(I,K)=C(I,K)+A(I,J)*B(J,K)
RETURN
END
## DATA SET LISTING

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VALUES OF THE PARAMETERS FOR THE COMPLETE MODEL

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4.770985 0.990214 5.562733 -1.174429 -4.395984 5.746488 -0.176073 1.174430
4.395982 -5.746489 0.176072 -11.072997 0.336730 3.841893 6.894376 11.073005
-0.336731 -3.841894 -6.894377 -5.660567 0.107945 8.983373 -3.430749 5.823996
-4.972384 -2.981629 2.130016 4.567067 5.472151 -9.457112 -0.582111 -4.730496
-0.607714 3.455364 1.882845 0.000000 0.000000 -0.000000 0.000000 -0.000000
-0.000000 -0.000000 -0.000000 -0.000000 0.000000 -0.000000 0.000000 -0.000000
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ANALYSIS OF VARIANCE FOR INTERACTION 12

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<td>TOTAL (B)</td>
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VALUES OF THE PARAMETERS

3.744644 2.444644 7.441073 -8.869641 0.880355 2.680355 5.308925 8.869649
-0.880355 -2.680355 -5.308928 -4.241667 0.800000 7.433333 -3.991666 3.923809
-3.867858 -2.301191 2.245238 2.812499 2.437498 -7.062502 1.812499 -2.494641
0.630359 1.930359 -0.066069 0.000000 0.000000 -0.000000 0.000000 -0.000000
-0.000000 -0.000000 -0.000000 -0.000000 -0.000000 0.000000 0.000000 0.000000
### Analysis of Variance for Interaction 13

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### Values of the Parameters

4.057450 2.467803 9.986742 -2.869317 0.456439 1.539774 0.873105 2.869320
-0.456438 -1.539774 -0.873106 -5.230432 -0.133208 5.486746 -1.023105 0.498106
-1.126895 -0.259470 0.888258 4.386998 0.817552 -7.092808 1.888256 0.345329
0.442550 1.865532 -2.653408 0.000000 0.000000 -0.000000 -0.000000 0.000000
0.000001 0.000000 0.000000 -0.000002 -0.000001 -0.000001 -0.000000 0.000000

### Analysis of Variance for Interaction 23

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### Values of the Parameters

3.533980 2.052747 8.210268 -2.020564 -1.067710 1.700805 1.387469 2.020565
1.067710 -1.700805 -1.387470 -7.847813 1.487878 1.791514 4.568422 7.847820
-1.487879 -1.791515 -4.568423 0.000000 0.000000 -0.000000 -0.000000 0.000000
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### Analysis of Variance for the Factor 1

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<td>Rest (D) = (B)−(A)</td>
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Values of the Parameters
80.830720 -6.221669 -7.883800 8.294279 5.811187 -16.583868 4.493353 1.489702
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### Analysis of Variance for the Factor 2

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<td>Rest (D) = (B)−(A)</td>
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Values of the Parameters
77.108486 9.209456 -9.209463 -16.622747 2.932809 3.312689 10.377253 0.000000
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### Analysis of Variance for the Factor 3

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Values of the Parameters
78.229611 8.874068 -8.874075 -8.379236 -4.581796 6.458357 6.502668 0.000000
0.000000