APPROXIMATE METHOD FOR ELIMINATION OF A RISK OF DEATH IN CURRENT LIFE TABLE CALCULATIONS

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Institute of Statistics Mimeo Series No. 1324

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ABSTRACT

An approximate, short-cut method to produce the calculations for a current life table with the elimination of a cause of death is presented and illustrated. The method is to perform the usual calculations for a current life table using the schedule of rates obtained by subtracting the age-specific death rate for the cause to be eliminated from the corresponding death rate for all causes. When the risk of death for the cause of interest is not too large, the life table produced is comparable to that obtained from the theoretical development of Chiang [1968], involving the recalculation of the conditional probabilities of death in each age group to remove the effect of a particular risk of death. The mean value theorem for integrals provides the basis for the approximation; therefore, the approximation will be better with shorter age bands. This application of the theorem is equivalent to assuming exponential survivorship within each age band.

For causes of death that are not too common and utilizing five year age bands, the method provides a good approximation to Chiang's approach to the recalculating of the conditional probabilities of death. An example is provided that demonstrates very reasonable approximation of the revised, age-specific probabilities of death and subsequently calculated age-specific life expectancies.

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*This research was completed under an Intergovernmental Personnel Act appointment while MJS was on leave from the Department of Biostatistics, School of Public Health, University of North Carolina at Chapel Hill, N.C. 27514

1. Introduction

An evaluation of the benefit of a highway safety or cancer research program may in part consider the increased life expectancy due to the elimination of all automobile accidents or cancer deaths, respectively. Chiang [1968] presents the standard competing risks theory that underlies the revision of a current life table to measure the effect of the elimination of a cause of death. The procedure is similar to the usual life table calculations once the conditional probabilities of death are recomputed presuming a particular cause of death, say k, has been eliminated. The maximum likelihood estimate of these revised conditional probabilities of dying, $\hat{q}_{i\cdot k}$, is

$$\hat{q}_{i\cdot k} = 1 - \{1-\hat{q}_i\}^{(D_i-D_{ik})/D_i} , \qquad (1)$$

where $\hat{q}_i$ is the estimated probability of dying in the interval of age x_i to x_{i+1} with all risks of death present, and $D_i (D_{ik})$ is the number of deaths due to all risks (only the kth risk). An assumption inherent in the derivation of equation (1) is that of a constant relative intensity of the kth risk in each age band. The appropriateness of this assumption depends not only upon the magnitude of the proportion of deaths due to risk k but also on how rapidly this proportion changes with age. Generally the shorter the age bands and the smaller the proportion of deaths due to risk k, the more plausible this assumption becomes.

2. Approximate Method

Also from Chiang [1968],

$$p_i = \exp\{-\int_{x_i}^{x_{i+1}} \mu(t)dt\} , \qquad (2)$$

where $p_i=1-q_i$ is the conditional probability of surviving the interval of age x_i to x_{i+1} and $\mu(t)$ is the total force of mortality at time t. The proposed method approximates the integral in (2) by using the mean value theorem for integrals, specifically

$$\int_{x_i}^{x_{i+1}} \mu(t)dt \doteq (x_{i+1}-x_i)M_i \ , \qquad (3)$$

where M_i is an estimate of the age-specific force of mortality between the ages x_i and x_{i+1}. It follows that p_i in equation (2) can be estimated by

$$\tilde{p}_i \doteq \exp(-n_i M_i) \ , \qquad (4)$$

where $n_i = (x_{i+1}-x_i)$ is the length of the ith age interval, $M_i = D_i/P_i$ ($M_{ik} = D_{ik}/P_i$) is the estimated age-specific mortality rate for all causes (only the kth cause), and P_i is the mid-year population estimate for the same age band.

The conditional probabilities of dying from (2) can be approximated by substituting the estimate $\tilde{p}_i$ from (4) into the right hand side of equation (1) for $1-\hat{q}_i$ giving

$$\tilde{q}_{i \cdot k} \doteq 1 - \{\exp(-n_i M_i)\}^{(D_i-D_{ik})/D_i} \ . \qquad (5)$$

Simplifying this equation algebraically, the result is expressible as

$$\tilde{q}_{i \cdot k} \doteq 1 - \exp\{-n_i(M_i-M_{ik})\} \ . \qquad (6)$$

Thus, a current life table that eliminates a cause of death, denoted by k, can be approximated by calculating the reduced age-specific death rates

$$M_{i \cdot k} = M_i - M_{ik} = (D_i-D_{ik})/P_i \qquad (7)$$

and computing the reduced conditional probabilities of dying by (6).

The mathematical basis for approximation (4) is the mean value theorem for integrals. As the length of the age bands decrease, the accuracy of the approximation improves. Yule [1924, 1934] found this approximation quite useful, noting as an empirical observation that minus the natural logarithm of estimates of the p_i were approximately proportional to the corresponding age-specific death rates. This approximation is also a special case of the method of Reed and Merrell [1939] for constructing current life tables and has been advocated since then by Fergany [1971].

Two comments are appropriate.

a. The approximation is equivalent to assuming a constant hazard for each age band. This corresponds to presuming an exponential survivorship model within each age group.

b. The current life table methods developed by Greville [1943], Reed-Merrill [1939], or Chiang [1972] could be used to produce the desired life table by employing the vector of reduced age-specific death rates given in (7). Spiegleman [1955] describes these methods, except for the more recent one of Chiang [1972].

3. Numerical Illustrations

The mortality of U.S. white males in 1970 due to automobile accidents and ischemic heart disease (ICD 8th revision: E810-E823 and 410-413, respectively) will illustrate this approximate technique. Reduced conditional probabilities of dying obtained by employing Chiang's [1972] method appear in Table 1, denoted as $\hat{q}_{i \cdot k}$. The corresponding probabilities estimated by the proposed approximate method $(\tilde{q}_{i \cdot k})$ are also shown in Table 1. Perusal of the two vectors of revised conditional probabilities reveals very good agreement. The greatest differences between the two age-specific probabilities occurs in the older age groups where the mortality rates are largest.

The cumulative effects of the approximations in calculating the age-specific life expectancy are shown in Table 2. With ischemic heart disease the approximation over estimates each age-specific life expectancy. The difference is about 0.10 years. As expected, the agreement is even closer for the less frequently occurring cause of death, automobile accidents.

4. Discussion

The approximation serves to illustrate the practical usefulness of the mean value theorem for integrals. Equivalently the utility of the piece-wise exponential survival model with short age bands is re-emphasized. The accuracy of the proposed approximation is seen to be very reasonable when the age bands are short and risk of the cause of death being eliminated is not large. For rare causes of death, such as some site-specific cancers, the approximation using even 10 year age bands should be adequate for many purposes.

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Table 1

Comparison of Chiang's Formula and an Approximation to Calculate Revised, Age-Specific Probabilities of Dying of Ischemic Heart Disease and Automobile Accidents for U.S. White Males in 1970

			Ischemic Heart Disease (ICDA codes: 410-413,8th Revision)			Automobile Accidents (ICDA codes: E810-E823,8th Revision)		
				Revised Conditional Probabilities of Dying			Revised Conditional Probabilities of Dying	
Age Intervals	Population*,† (P_i)	All deaths** (D_i)	Deaths** (D_{ik})	($\hat{q}_{i \cdot k}$)††	($\tilde{q}_{i \cdot k}$)°	Deaths** (D_{ik})	($\hat{q}_{i \cdot k}$)††	($\tilde{q}_{i \cdot k}$)°
[0,1)	1,590,140	31,725	7	0.019591	0.019749	136	0.019512	0.019670
[1,5)	5,873,083	4,910	3	0.003335	0.003336	714	0.002853	0.002854
[5,10)	8,682,687	4,099	7	0.002353	0.002354	1,088	0.001732	0.001732
[10,15)	9,074,965	4,382	6	0.002408	0.002408	1,142	0.001784	0.001784
[15,20)	8,388,172	12,200	21	0.007237	0.007233	5,565	0.003949	0.003947
[20,25)	6,832,641	13,812	82	0.009996	0.009997	5,891	0.005779	0.005780
[25,30)	5,824,831	9,897	223	0.008270	0.008270	3,123	0.005798	0.005798
[30,35)	4,943,282	9,130	656	0.008536	0.008535	1,944	0.007243	0.007242
[35,40)	4,821,143	12,459	2,203	0.010586	0.010580	1,726	0.011075	0.011070
[40,45)	5,210,254	21,819	6,006	0.015073	0.015060	1,786	0.019057	0.019041
[45,50)	5,258,458	35,992	12,629	0.022002	0.021970	1,836	0.032002	0.031956
[50,55)	4,831,401	53,092	20,924	0.032804	0.032743	1,659	0.051932	0.051836
[55,60)	4,317,083	76,502	31,134	0.051309	0.051188	1,652	0.083233	0.083039
[60,65)	3,659,121	98,781	40,874	0.076387	0.076077	1,448	0.125029	0.124535
[65,70)	2,797,493	113,614	47,694	0.111937	0.111144	1,193	0.183276	0.182032
[70,75)	2,107,023	122,829	52,029	0.156096	0.154654	1,078	0.253121	0.250926
[75,80)	1,459,706	124,979	53,000	0.222395	0.218510	951	0.351716	0.346125
[80,85)	808,163	101,556	43,794	0.305774	0.300484	601	0.471580	0.464522
85+	491,124	90,339	39,756	1.0	1.0	319	1.0	1.0
(not stated)		320	67			19		

* U.S. Bureau of the Census, 1970 Census of the Population, Volume 1, Characteristics of the Population, Part 1, United States Summary: Section 2. Table 191, p. 1-596 and Section 1. Table 50, p. 1-265.

† U.S. National Center for Health Statistics, 1970 Vital Statistics of the United States, Volume I: Natality, Table 1-18, p. 1-19.

** U.S. National Center for Health Statistics, 1970 Vital Statistics of the United States, Volume II: Mortality, Part B: Table 7-5, p. 7-150 ff.

†† Formula given by equation (1): Chiang's formula.

° Formula given by equation (6): Approximation.

Table 2

Comparison of Chiang's Formula and an Approximation to Calculate[1] Revised Age-Specific Life Expectancies for Ischemic Heart Disease and Automobile Accidents for U.S. White Males in 1970

Age Intervals	Ischemic Heart Disease (ICDA codes: 410-413, 8th Revision) $\hat{e}_{i \cdot k}$	$\tilde{e}_{i \cdot k}$	Automobile Accidents (ICDA codes: E810-E823, 8th Revision) $\hat{e}_{i \cdot k}$	$\tilde{e}_{i \cdot k}$
[0,1)	74.27	74.36	69.03	69.09
[1,5)	74.76	74.85	69.40	68.47
[5,10)	71.00	71.10	65.59	65.67
[10,15)	66.16	66.26	60.70	60.78
[15,20)	61.32	61.41	55.81	55.88
[20,25)	56.74	56.84	51.02	51.09
[25,30)	52.29	52.39	46.30	46.38
[30,35)	47.71	47.81	41.55	41.63
[35,40)	43.09	43.20	36.84	36.92
[40,45)	38.53	38.63	32.22	32.30
[45,50)	34.07	34.18	27.79	27.87
[50,55)	29.78	29.88	23.62	23.70
[55,60)	25.70	25.81	19.77	19.86
[60,65)	21.95	22.06	16.33	16.42
[65,70)	18.55	18.66	13.29	13.38
[70,75)	15.56	15.67	10.69	10.78
[75,80)	12.97	13.07	8.45	8.54
[80,85)	10.95	11.01	6.66	6.71
85+	9.71	9.71	5.46	5.46

[1] Calculated using the method of Chiang [1972].