DORFMAN-STERRETT SCREENING (GROUP TESTING) SCHEMES AND THE EFFECTS OF FAULTY INSPECTION

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Key Words and Phrases: Acceptance sampling; Faulty inspection.

ABSTRACT

The Dorfman screening procedure is based on first testing a group of items as a whole, proceeding to individual testing only if the group-test indicates existence of at least one nonconforming item. A modification suggested by Sterrett allows for reintroduction of group testing of all items, not yet tested individually, when an item is classified as nonconforming by an individual test. Effects of faulty test inspection on the properties of the modified procedures are studied.

INTRODUCTION

Dorfman (1943) proposed a procedure for reducing the expected amount of testing needed to identify the nonconforming (NC) items in a group of n items. (In the case he considered, the items were blood samples and NC signified a positive reaction to a test for syphilis.)

Dorfman's procedure requires mixing part of each of the n items together and testing the mixture for existence of at least one NC item among them. If no such item is indicated, no further testing is needed - all items are classed as conforming (C). If existence of at least one NC item is indicated, then each item is tested individually. When the population proportion of NC items is small, this will result in a reduction of the expected number of tests below n (the number needed if each item is tested individually, *ab initio*). In any case, no more than (n+1) tests will be needed.

Sterrett (1957) suggested a modification of the Dorfman procedure aimed at further reduction in the expected number of tests, when the population proportion of NC items is small. He proposed that whenever an item is identified as NC on individual testing, a Dorfman (group-testing) procedure be applied to the remaining groups of items, as yet not tested individually. In the original proposal this rule was to be followed until only one item remained untested.

For practical purposes it may be desirable to restrict the number of reversions to a Dorfman procedure. (There cannot be more than (n-2) such reversions.) If only one reversion is allowed, we will call the procedure a one-stage Dorfman-Sterrett procedure; if up to k reversions are allowed, we have a k-stage Dorfman-Sterrett procedure. The original proposal might be described as (n-2)-stage Dorfman-Sterrett procedure.

We shall investigate the effects of faulty testing on the properties of Dorfman-Sterrett procedures, using techniques developed in parallel investigations for standard acceptance sampling procedures (e.g. Johnson et al. (1985, 1986)).

Other variants of the standard Dorfman procedure have been suggested and studied by Sobel and Groll (1959), Sobel (1960,1968), Lee and Sobel (1972), Graff and Roeloffs (1972), Pfeifer and Enis (1978) and Mehravari (1986). Hwang (1974,1984) gives useful surveys of the literature.

NOTATION

The formulas we obtain are essentially simple, although their derivation involves some rather elaborate probabilistic arguments. It will be necessary to use a special system of notation, which we now explain. For group inspection, let

p_0 denote probability of identifying a NC group as NC
p_0' denote probability of identifying a C group as NC.

For individual inspection let the corresponding quantities be p,p' respectively. (It would be straightforward to develop formulas allowing for group test probabilities to depend on the size of the group, but we will not do this here.)

We use Y to denote the actual number of NC items in the group of n items. If the items have been selected by random sampling (without replacement) from a lot of N items containing D items which are NC, then

$$\Pr[Y=y|n] = \binom{D}{y}\binom{N-D}{n-y}/\binom{N}{n} \quad (\max(0,n-N+D) \le y \le \min(D,n)) \tag{1}$$

If $N \to \infty$ and $D \to \infty$ with $D/N \to \omega$ (or if sampling is with replacement and $D/N = \omega$) we have

$$\Pr[Y=y|n] = \binom{n}{y}\omega^y(1-\omega)^{n-y} \quad (0 \le y \le n) \tag{2}$$

In the formulas to be obtained below, we will leave Pr[Y=y] unspecified. Either (1) or (2), or indeed, some other distribution may be inserted, as appropriate.

To describe the properties of procedures, we will use the following indices:

E: the expected number of tests (and also 100(1-E/n), the expected percent reduction as compared with individual testing *ab initio*)

PC(NC): probability of correct identification of a NC item

PC(C): probability of correct identification of a C item

To indicate that indices are relevant to a k-stage Dorfman-Sterrett procedure we will use a prefix - thus:

$$_kE \quad , \quad _kPC(NC) \quad \text{and} \quad _kPC(C)$$

(k = 0 corresponds to the original Dorfman procedure.)

If it is necessary to indicate the number of items in the group this will be done by a suffix - $_kE_n$, etc . If, also, we wish to indicate that the value is conditional on there being a fixed number, y, of truly NC items in the group we will use compound symbols $_kE_{n|y}$ etc .

The quantities n,y will be called the parameters of the procedure.

Our formulas will be conveniently expressed in terms of the quantities

$$P_{NC}(m,t|n,y) = \begin{cases} \binom{m-1}{t-1}\binom{n-m}{y-t}(1-p)^{t-1}p(1-p')^{m-t}\Big/\binom{n}{y} & (t > 1) \\ 0 & (t = 0) \end{cases} \tag{3.1}$$

$$P_C(m,t|n,y) = \begin{cases} \binom{m-1}{t}\binom{n-m}{y-t}(1-p)^{t}(1-p')^{m-t-1}p'\Big/\binom{n}{y} & (t < m - 1) \\ 0 & (t = m) \end{cases} \tag{3.2}$$

$P_{NC}(\cdot)$ (resp. $P_C(\cdot)$) is the probability that, in individual testing, the first item to be identified as NC is the m-th item tested, with t truly NC items among the m items tested and the m-th item tested being, in fact, NC (resp. C).

Clearly,

$$P(m,t|n,y) = P_{NC}(m,t|n,y) + P_C(m,t|n,y) \tag{4}$$

is just the probability that the first item declared NC is the m-th item tested, with t NC items among the m tested. There are natural limitations on possible combinations of values of m,t, n and y. For example if $y > t$ or $m<t$, then $P(m,t|n,y) = 0$.

ONE-STAGE DORFMAN-STERRETT PROCEDURES

We first note some formulas for simple Dorfman (k=0) procedures. We have, for $n > 1$

$$_0E_{n|y} = \begin{cases} 1 + np_0 & \text{for } y > 0 \\ 1 + np_0' & \text{for } y = 0 \end{cases} \tag{5}$$

$$ {}_0PC(NC)_{n|y} = p_0 p \quad \text{for} \quad y > 0 \tag{6} $$

$$ {}_0PC(C)_{n|y} = \begin{cases} 1 - p_0 p' & \text{for } y > 0 \\ 1 - p_0' p' & \text{for } y = 0 \end{cases} \tag{7} $$

whence

$$ {}_0E_n = \sum_{y=0}^{n} \Pr[Y=y|n]\,{}_0E_{n|y} = 1 + n[p_0\{1-\Pr[Y=0|n]\} + p_0' \Pr[Y=0|n]] \tag{8} $$

$$ {}_0PC(NC)_n = p_0 p \tag{9} $$

$$ {}_0PC(C)_n = 1 - p'[p_0\{1 - \Pr[Y=0|n-1]\} + p_0' \Pr[Y=0|n-1]] \tag{10} $$

(Note that $\Pr[Y=0|n-1]$ is the probability that there are no NC items in the group of n, given that a specified item is C.)

Proceeding to one-stage Dorfman-Sterrett procedures, we have

$$ {}_1E_n = \sum_{y=0}^{n} \Pr[Y=y|n]\ {}_1E_{n|y} \tag{11} $$

In evaluating ${}_1E_{n|y}$, it is important to note that if the first item identified as NC on individual testing is the m-th then (i) if m=n-1, the last item also will be tested individually (no further group-testing stage) and so (ii) when m=n-1 or m=n the total number of tests must be (1+n). And (iii) for $m \le n-2$, we have to follow a Dorfman procedure with parameters n-m, y-t (t being the number of NC items among the first m tested).

Hence, for $y > 0$

$$ {}_1E_{n|y} = 1 + p_0\left[\sum_{m=1}^{n-2} \sum_t^{(y)} P(m,t|n,y)\{m + {}_0E_{n-m|y-t}\} + n\{1 - \sum_{m=1}^{n-2} \sum_t^{(y)} P(m,t|n,y)\}\right] $$

$$ = 1 + p_0\left[n - \sum_{m=1}^{n-2} \sum_t^{(y)} P(m,t|n,y)(n-m-{}_0E_{n-m|y-t})\right] $$

$$= 1 + p_0[n - \sum_{m=1}^{n-2} [\sum_t^{(y-1)} P(m,t|n,y)\{(n-m)(1-p_0)-1\}$$

$$+ P(m,y|n,y)\{(n-m)(1-p_0') - 1\}]] \quad (12)$$

where $\sum_t^{(z)}$ denotes summation over $\max(0,y-n+m) \leq t \leq \min(m,z)$ and $P(m,z|n,y) = 0$ for $m < z$ or $z > y$.

For $y = 0$

$${}_1E_{n|0} = 1 + p_0'[\sum_{m=1}^{n-2} P(m,0|n,0)(m+{}_0E_{n-m|0}) + n(1-p')^{n-2}]$$

$$= 1+p_0'[\sum_{m=1}^{n-2} (1-p')^{m-1}p'\{m + 1+(n-m)p_0'\} + n(1-p')^{n-2}]$$

$$= 1 + p_0'p'^{-1}\{1 + p'-p_0' + np'p_0-(1-p')^{n-2}(1-p'-p_0'+2p'p_0)\} \quad (13)$$

(Remember that for $m > n-2$, the number of individual tests is n.)

Turning to the evaluation of ${}_1PC(NC)_n$ we have

$${}_1PC(NC)_n = \sum_{y=1}^{n} P_y^* \; {}_1PC(NC)_{n|y} \quad (14)$$

where $P_y^* = \binom{D-1}{y-1}\binom{N-D}{n-y}/\binom{N-1}{n-1}$ for lot size N; $P_y^* = \binom{n-1}{y-1}\omega^{y-1}(1-\omega)^{n-y}$ for infinite lot size. (These probabilities relate to the distribution of Y, given that a specified item is NC.) We evaluate ${}_1PC(NC)_{n|y}$ as

$y^{-1} \times$ (expected number of NC items correctly classified, when there are y NC items in the group)

$= y^{-1} \times p_0$ (expected number of NC items correctly classified given that Y=y <u>and</u> individual testing starts). (15)

If the first item classified as NC is the m-th tested, and there are t NC items among the first m tested, then for $m \leq n-2$ the expected number of NC items correctly classified in the <u>subsequent</u> Dorfman procedure is

(number of remaining NC items) $\times$ (probability each is correctly identified) $= (y-t)p_0p$.

Also, if the m-th item was correctly identified as NC (probability $P_{NC}(m,t|n,y)$) there is one more correct classification of a NC item - so the total contribution to expected number is

$$P_{NC}(m,t|n,y)+P(m,t|n,y)(y-t)p_0p$$

For $m=n-1$ we have the contribution $P_{NC}(n-1,y-1|n,y) + P(n-1,y-1|n,y)p + P_{NC}(n-1,y|n,y)$. For $m=n$, the contribution is $P_{NC}(n,y|n,y)$.

Hence

$$ {}_1PC(NC)_{n|y} = y^{-1}p_0[\sum_{m=1}^{n-2} \sum_t^{(y)} \{P_{NC}(m,t|n,y) + P(m,t|n,y)(y-t)p_0p\} + P_{NC}(n-1,y-1|n,y) + P(n-1,y-1|n,y)p + P_{NC}(n-1,y|n,y) + P_{NC}(n,y|n,y)] \quad (y \geq 1) \qquad (16)$$

and ${}_1PC(NC)_n$ is evaluated from (14) and (16).

Similarly

$$ {}_1PC(C)_n = \sum_{y=0}^{n-1} P_y^{**} \; {}_1PC(C)_{n|y} \qquad (17)$$

where $P_y^{**} = \binom{D}{y}\binom{N-D-1}{n-1-y}\Big/\binom{N-1}{n-1}$ for lot size N; $P_y^{**} = \binom{n-1}{y}\omega^y(1-\omega)^{n-1-y}$ for infinite lot size. (These probabilities relate to the distribution of Y given that a specified item is C.)

For $y > 0$

$$ {}_1PC(C)_{n|y} = 1-(n-y)^{-1} p_0\Big[\sum_{m=1}^{n-2} [\sum_t^{(y-1)} \{P_C(m,t|n,y) + P(m,t|n,y)(n-m-y+t)p_0p'\} + P_C(m,y|n,y) + P(m,y|n,y)(n-m)p_0'p'] + P_C(n-1,y|n,y) + P_C(n-1,y-1|n,y) + P(n-1,y|n,y)p' + P_C(n,y|n,y)\Big] \qquad (18)$$

and, for $y = 0$

$$_1PC(C)_{n|0} = 1 - n^{-1} \; p_0'[\sum_{m=1}^{n-2} (1-p')^{m-1}p'\{1 + (n-m)p_0'p'\} + $$

$$+ 2p'(1-p')^{n-2}]$$

$\left(\text{note that } (1-p')^{n-2}\{p'+p'^2+(1-p')p'\} = 2p'(1-p')^{n-2}\right)$

$$= 1 - n^{-1}p_0'\{1 - p_0'+np'p_0' - (1-p')^{n-2}(1-2p')\} \qquad (19)$$

For the case of lots of infinite size, $_1PC(NC)_n$ can be found using simpler arguments. Consider a NC item, $\mathcal{J}$, say, in the group of size n. Since there is a NC item in the group, the probability that the group test gives a positive result, so that individual testing starts, is p_0. Given that individual testing starts, the probability that $\mathcal{J}$ is the first item declared NC is

$$n^{-1}(1-\tilde{\omega})^{m-1}p$$

where $\tilde{\omega} = \omega p + (1-\omega)p'$ is the probability that a randomly chosen item is declared NC when tested individually. So Pr[$\mathcal{J}$ is the first item declared NC] =

$$= n^{-1}p \sum_{m=1}^{n} (1-\tilde{\omega})^{m-1}$$

$$= \frac{p\{1 - (1-\tilde{\omega})^n\}}{n\,\tilde{\omega}} \qquad (20)$$

The probability that the first item declared NC is the m-th tested, and $\mathcal{J}$ is among the remaining n-m items is

$$(1 - \frac{m}{n})(1-\tilde{\omega})^{m-1}\tilde{\omega}$$

In this case, the conditional probability that $\mathscr{I}$ is correctly identified as NC is just $p_0 p$, except when $m = n-1$, in which case it is p, since in this case an individual item (rather than a group) is tested. Summing over m gives

$$n^{-1}\tilde{\omega}\left[\sum_{m=1}^{n-2}(1-\tilde{\omega})^{m-1}(n-m)p_0 p + (1-\tilde{\omega})^{n-1}p\right] \qquad (21)$$

Combining (20) and (21)

$$PC(NC) = \frac{p_0 p}{n\,\tilde{\omega}}\left[(1-p_0)\{1+(2\tilde{\omega}-1)(1-\tilde{\omega})^{n-2}\} + p_0 n\,\tilde{\omega}\right] \qquad (22)$$

Unfortunately, this method cannot be used to calculate PC(C) for lots of infinite size, because the constitution of the remaining items in the group, given that the individual testing stage is reached, is not that of a random sample with constant probability $\tilde{\omega}$ of obtaining a NC item. (It depends on whether or not there is at least one NC item among the remainder.)

MULTI-STAGE DORFMAN-STERRETT PROCEDURES

Evaluation of ${}_kE_{n|y}$, ${}_kPC(NC)_{n|y}$ and ${}_kPC(C)_{n|y}$ can be effected in an iterative manner, using the fact that after the first reversion in a k-stage Dorfman-Sterrett procedure, the subsequent procedure is a (k-1)-stage procedure with appropriate change of the parameters n,y to n-m,y-t (where m and t have the same meaning as in the preceding section).

Thus for $y > 0$

$${}_kE_{n|y} = 1 + p_0 \sum_{m=1}^{n-2} \sum_t^{(y)} P(m,t|n,y)\{m + {}_{k-1}E_{n-m|y-t}\}$$

$$+ n\{1 - \sum_{m=1}^{n-2} \sum_t^{(y)} P(m,t|n,y)\}.$$

$$= 1 + p_0[n - \sum_{m=1}^{n-2} \sum_t^{(y)} P(m,t|n,y)\{n-m-{}_{k-1}E_{n-m|y-t}]\} \qquad (23.1)$$

and

$$ {}_kE_{n|0} = 1 + p_0'[n- \sum_{m=1}^{n-2} P(m,0|n,0)\{n-m-{}_{k-1}E_{n-m|0}\}] \qquad (23.2)$$

Formula (23.2) may be written explicitly using formula (13) and $P(m,0|n,0) = (1-p')^{m-1}p'$.

The stated form is easily suited to computer programming.

Also,

$$ {}_kPC(NC)_{n|y} = y^{-1}p_0[\sum_{m=1}^{n-2} \sum_t^{(y)} \{P_{NC}(m,t|n,y) + $$

$$ + P(m,t|n,y)(y-t)\,{}_{k-1}PC(NC)_{n-m|y-t}\} $$

$$ + P_{NC}(n-1,y|n,y) $$

$$ + P_{NC}(n-1,y-1|n,y) + P(n-1,y-1|n,y)p + P_{NC}(n,y|n,y)]; \qquad (24)$$

and

$$ {}_kPC(C)_{n|y} = 1-(n-y)^{-1}p_0[\sum_{m=1}^{n-2} \sum_t^{(y)} \{P_C(m,t|n,y) + $$

$$ + P(m,t|n,y)(n-m-y+t)\{1-{}_{k-1}PC(C)_{n-m|y-t}\} $$

$$ + P_C(n-1,y|n,y) + P(n-1,y|n,y)p' + P_C(n-1,y-1|n,y) + $$

$$ + P_C(n,y|n,y)] \qquad (25)$$

For $y=0$

$$ {}_kPC(C)_{n|0} = 1 - n^{-1}p_0'[\sum_{m=1}^{n-2} (1-p')^{m-1}p'\{1 + (n-m)\{1-{}_{k-1}PC(C)_{n-m|0}\} $$

$$ + 2p'(1-p')^{n-2}]. \qquad (26)$$

These formulas (with k=2) were used in computing Tables 4,5 and 6. Much more detailed tables are scheduled to appear in a survey paper by the authors in a forthcoming issue of J. of Qual. Techn. - the third in the series Johnson et. al. (1985,1986).

TABLES

Tables 1-6 contain the following values:

	k=1	k=2
${}_kE_{n\|y}$	Table 1	Table 4
${}_kPC(NC)_{n\|y}$	Table 2	Table 5
${}_kPC(C)_{n\|y}$	Table 3	Table 6

for n=6; $p_0=p$ = 0.75, 0.90, 0.95; $p_0'=p'$ = 0.05, 0.10, 0.25 and all relevant values of y. Values of ${}_kE_n$, ${}_kPC(NC)_n$ and ${}_kPC(C)_n$ can readily be obtained by averaging the tabulated values over the appropriate distribution of Y. For ${}_kE_n$ this is given by (1) and (2); for ${}_kPC(NC)_n$ it is P_y^* from (14); for ${}_kPC(C)_n$ it is P_y^{**} from (17). In most cases, practical accuracy can be attained by averaging over just y=0, y=1 and $y \geq 2$ (using a rough average value for the latter set of values). For example, in Table 1, with $p_0=p=0.9$, $p_0'=0.25$ and $p'=0.05$, we find

$${}_1E_{6|0} = 2.422;\ {}_1E_{6|1} = 5.944 \text{ and } 6.86 \leq {}_1E_{6|y} \leq 6.91 \text{ for } y \geq 2.$$

Hence

$${}_1E_6 = 2.422 \times \Pr[Y=0] + 5.944 \times \Pr[Y=1] + \theta \times \Pr[Y \geq 2]$$

with $6.86 < \theta < 6.91$. Taking the case of large lot size and sample size (n) equal to 6, we get the limits on ${}_1E_0$ set out below, for a few values of ω. Exact values of ${}_1E_6$ (to 3 decimal places) and ${}_0E_6$ (corresponding to the "standard" Dorfman procedure) are also shown, for comparison.

Values of ${}_1E_6$

ω	Lower limit	Upper limit	Exact	${}_0E_6$
0.05	3.385	3.387	3.385	3.460
0.10	4.177	4.183	4.178	4.274
0.15	4.820	4.832	4.823	4.929

Similar situations hold for values of ${}_kPC(NC)_{n|y}$ and ${}_kPC(C)_{n|y}$; it is important to remember that the appropriate distributions of Y (from (14) and (17) respectively) must be used in each case.

DISCUSSION

When y=0, increase in the value of k makes little difference to the expected number of tests. With n=6, the greatest difference (k=2 vs k=1) is 2.159 vs. 2.177, occurring when $p_0'=p'=0.25$. (The expected number of tests does not depend on p_0 or p when y=0). For larger values of y, differences are somewhat greater, especially for higher values of p_0 and p. As y increases, it becomes more and more likely that the last stage of the procedure will be reached, with corresponding increase in the number of tests needed. For example, if y=6 with n=6, $p_0=p=0.95$ we have ${}_1E_{6|6} = 7.415$, but ${}_2E_{6|6}=8.141$ (whatever the values of p_0' and p'). However, in the cases where the Dorfman (and Dorfman-Sterrett) procedures are likely to be used because of expected advantages, ω is small, so high values of y have little weight, because they have small probabilities of being attained. The modified procedure outlined in the last section of this paper may perhaps be advantageous from this aspect for somewhat larger values of ω or D/N.

Increasing k increases the probability of correct classification of C items, but decreases that for NC items. This is because each time a group test is used there is the possibility of items being classified C without being tested individually. It should however be noted that this decrease in PC(NC) is rather gradual and small even for large values of y.

Tables 7 and 8 illustrate this effect, for n=6; $p_0=p=0.9$; $p_0'=p'=0.5$ (with an extra decimal place).

Table 7 VALUES OF ${}_kPC(NC)_{6|y}$

y\k	(standard) 0	1	2
1	0.810	0.801	0.801
2		0.772	0.769
3		0.758	0.740
4		0.751	0.719
5		0.747	0.706
6		0.744	0.698

TABLE 8 VALUES OF ${}_kPC(C)_{6|y}$

y\k	(standard) 0	1	2
0	0.997(5)	0.998	0.998
1	0.955	0.972	0.973
2		0.959	0.967
3		0.958	0.961
4		0.958	0.961
5		0.959	0.961

Table 9 gives the corresponding values of ${}_kE_{6|y}$

TABLE 9 VALUES OF ${}_kE_{6|y}$

y\k	(standard) 0	1	2
0	1.300	1.278	1.278
1	6.400	5.584	5.231
2		6.846	6.672
3		6.909	7.291
4		6.894	7.402
5		6.875	7.393
6		6.860	7.361

When ω is small the low values of y have greater weight, so increasing k can decrease the overall expected number of tests. We thus recommend Dorfman-Sterrett procedure for those applications when PC(NC) and/or the expected number of tests is of primary concern.

For applications in which control of PC(k) (i.e. reduction of false positives) is of crucial importance, modified hierarchical Dorfman procedures (described in Kotz et al. (1987)) may be appropriate.

FURTHER MODIFICATIONS

When the proportion (ω or D/N) of NC items in the population is not small (<1%) but not very large (say 5-10%) it is possible that extra savings in expected number of inspections may be attained by waiting until the second (generally, the s-th) NC decision is reached in testing individual items before reverting to group testing of the remaining items. The rationale is that it is more likely that all NC items have already been tested. On the other hand if ω is really small, the possibility of saving by group testing when more individual items are tested remain is lost.

In place of the probability $P(m,t|n,y)$ defined in (4) we need to compute:

$_{(2)}P(m,t|n,y)$: - the probability that the <u>second</u> NC decision occurs at the m-th inspection, and that there are just t truly NC items among the first m tested.

This quantity can be calculated from the formula

$$_{(2)}P(m,t|n,y) = \sum_{m'=1}^{m-1} \sum_{t'=0}^{t} P(m',t'|n,y)P(m-m',t-t'|n-m',y-t')$$

(with $t \leq y$ and $t \leq m$)

This probability is then used in place of $P(m,t|n,y)$ in (12) and (13) to evaluate the corresponding $_{1(2)}E_{n|y}$. For evaluation of $_{(2)}PC(NC)$ and $_{(2)}PC(C)$, the probability needs to be split into four parts according as the first and second NC decisions apply to truly C,C or C,NC or NC,C or NC,NC items (in that order). Thus

$$_{(2)}P_{C,C}(m,t|n,y) = \Sigma\Sigma\ P_C(m',t'|n,y)P_C(m-m',t-t'|n-m',y-t')$$

$$_{(2)}P_{NC,C}(m,t|n,y) = \Sigma\Sigma\ P_{NC}(m',t'|n,y)P_C(m-m',t-t'|n-m',y-t')$$

and similarly for the remaining two cases.

Analysis and numerical results for this modification are planned for study in a later paper.

<u>TABLE 1</u> VALUES OF $_1E_{6|Y}$

Y	$p_0=p=$	0.75			0.90			0.95		
	$p_0'=p'=$	0.25	0.10	0.05	0.25	0.10	0.15	0.25	0.10	0.05
0		0.42	1.52	1.28	2.42	1.52	1.28	2.42	1.52	1.28
1		4.95	4.78	4.70	5.94	5.75	5.58	6.31	6.11	5.91
2		4.47	5.44	5.44	6.86	6.85	6.95	7.39	7.39	7.38
3		5.49	5.47	5.48	6.91	6.91	6.91	7.44	7.44	7.44
4		5.45	5.44	5.45	6.89	6.89	6.89	7.43	7.43	7.43
5		5.41	5.40	5.41	6.87	6.87	6.87	7.42	7.42	7.42
6		5.37	5.37	5.37	6.86	6.86	6.86	7.42	7.42	7.42

TABLE 2 VALUES OF $_1PC(NC)_{6|y}$

y	$p_0=p=$ 0.75			0.90			0.95		
	$p_0'=p'=$ 0.25	0.10	0.05	0.25	0.10	0.05	0.25	0.10	0.05
1	.501	.533	.547	.775	.793	.801	.883	.893	.897
2	.483	.500	.506	.761	.769	.772	.875	.879	.881
3	.471	.479	.482	.754	.757	.758	.871	.872	.873
4	.463	.467	.468	.749	.751	.751	.868	.865	.865
5	.457	.459	.459	.746	.747	.747	.866	.867	.867
6	.453	.453	.453	.744	.744	.744	.865	.865	.865

TABLE 3 VALUES OF $_1PC(C)_{6|y}$

y	$p_0=p=$ 0.75			0.90			0.95		
	$p_0'=p'=$ 0.25	0.10	0.05	0.25	0.10	0.05	0.25	0.10	0.05
0	.958	.992	.998	.958	.992	.998	.958	.992	.998
1	.866	.949	.975	.836	.942	.972	.826	.940	.972
2	.847	.938	.969	.793	.917	.959	.773	.909	.955
3	.846	.938	.969	.792	.917	.958	.772	.909	.954
4	.848	.939	.969	.793	.917	.958	.772	.909	.954
5	.849	.940	.970	.793	.917	.954	.772	.909	.944

TABLE 4 VALUES OF $_2E_{6|y}$

y	$p_0=p=$ 0.75			0.90			0.95		
	$p_0'=p'=$ 0.25	0.10	0.05	0.25	0.10	0.05	0.25	0.10	0.05
0	2.16	1.52	1.28	2.16	1.52	1.28	2.42	1.52	1.28
1	4.77	4.56	4.63	5.86	5.41	5.23	5.84	5.64	5.42
2	5.38	5.31	5.29	7.04	6.77	6.67	7.40	7.35	7.23
3	5.52	5.54	5.53	7.38	7.31	7.29	8.03	8.04	8.02
4	5.47	5.51	5.55	7.41	7.40	7.40	8.16	8.16	8.16
5	5.40	5.43	5.50	7.38	7.39	7.39	8.16	8.16	8.16
6	5.34	5.36	5.44	7.35	7.36	7.36	8.14	8.14	8.14

TABLE 5 VALUES OF $_2PC(NC)_{6|y}$

y	$p_0=p=$ 0.75			0.90			0.95		
	$p_0'=p'=$ 0.25	0.10	0.05	0.25	0.10	0.05	0.25	0.10	0.05
1	.488	.531	.546	.765	.791	.801	.877	.892	.897
2	.461	.491	.502	.744	.762	.769	.864	.875	.879
3	.438	.456	.463	.726	.736	.740	.853	.854	.861
4	.420	.429	.432	.713	.717	.719	.846	.484	.849
5	.406	.409	.410	.704	.706	.706	.841	.842	.842
6	.395	.395	.395	.698	.698	.698	.838	.838	.838

TABLE 6 VALUES OF $_2PC(C)_{6|y}$

y	$p_0=p=$	0.75			0.90			0.95		
	$p'_0=p'=$	0.25	0.10	0.05	0.25	0.10	0.05	0.25	0.10	0.05
0		.966	.994	.998	.966	.994	.998	.966	.994	.998
1		.879	.951	.976	.858	.945	.973	.847	.944	.973
2		.867	.947	.973	.825	.932	.967	.806	.927	.965
3		.863	.945	.972	.804	.922	.961	.779	.912	.956
4		.865	.946	.973	.805	.922	.961	.779	.911	.956
5		.868	.947	.974	.807	.922	.961	.780	.912	.956

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