

Model Formulation of Drinking Behavior Using Longitudinal Data

H. T. Banks, Keri L. Rehm, Karyn L. Sutton

Center for Research in Scientific Computation
Center for Quantitative Science in Biomedicine
North Carolina State University
Raleigh, NC 27695-8212

and

Christine Davis, Lisa Hail, Alexis Kuerbis, Jon Morgenstern

Substance Abuse Services
Department of Psychiatry
Columbia University Medical Center
180 Fort Washington Ave. HP-240
NYC, NY 10032

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Abstract

We present a novel dynamical systems modeling approach to an intensive longitudinal data set of inter- and intrapersonal factors in problem drinkers. To our knowledge, there have been no such models developed in this area on the individual level. As a result, considerable effort was required to determine the variables of interest in such a model, possible, if any, relationships between them, and even the timescale on which to observe them. We discuss the construction of an initial mathematical model with the potential to help in understanding the underlying mechanisms responsible for behavior change in problem drinkers.

1 Introduction

In the field of drug and alcohol abuse, many researchers have collected vast amounts of information on substance use, participant's willingness to change behavior, and participant's success in a particular treatment. Using this information, field experts have formulated ideas about difficult-to-measure factors that control a patient's motivations and behavior, such as salience – the value of drinking in relation to other activities, commitment to a favorable outcome and reduced alcohol consumption, and recognition of reasons to not use a substance. However, the relative contributions of these driving factors as they form mechanisms for behavior change are unclear. The interplay of these factors as they change over time is a natural question to address via a mathematical model, which enables us to describe these processes quantitatively as a dynamical system. We seek to model these ideas of a drinking behavior control system as informed by a dataset, Project MOTION, to gain insight into the underlying mechanisms.

The MOTION data set is comprised of information collected via an interactive voice recording (IVR) from drinkers who wanted to reduce their drinking, although not necessarily cease drinking altogether. Ideally, each individual answered this 41-question survey once per day for eight weeks,

or 56 days, and gave information about inter- and intrapersonal factors that could potentially influence their drinking as well as their drinking behavior in the past 24 hrs and their commitment to avoid heavy drinking or drinking at all in the next day.

There is no straightforward way of organizing data on such a large variety of aspects of daily life into a reasonable number of variables to study within a mathematical modeling framework. Further complicating this issue is the lack of previous modeling efforts in this area, resulting in a lack of pertinent quantities commonly agreed upon. To begin formulating conceptual variables, we applied linear methods such as calculating coefficients of determination, linear interpolation via least-squares approximation and principal component analysis. However, these methods do not provide any useful information on relationships between an individual's responses to questions. It is sensible to construct our model variables on questions based on similar ideas used in the IVR. We shall examine how these categories relate to each other, and how to best use this data in formulating a model. Our initial interpretation of this data is based on knowledge and predictions from the field of substance abuse and recovery, visually assessing qualitative trends between similar questions within topical categories, and grouping them into variables. Once variables were decided upon, qualitative trends between them were examined to determine the existence and nature of relationships between them within a single patient.

Generally, for mathematical modeling to be effective, it should be done as an iterative process in which an initial model is formulated and then compared with observed or experimental data. This comparison gives insight into any discrepancies between the observed processes and model predictions, suggesting refinements to the model. This process is repeated until the model provides sufficient information to capture key aspects of behavior of the observed system and to answer questions of interest. Our approach, as described in this document, can be briefly outlined as follows: we initiate this iterative process and formulate a mathematical model of differential equations using the newly formed variables from the IVR data set.

We construct this first model based on prior knowledge of hypothesized relationships between the model variables based on knowledge of substance abuse therapy and relationships observed in the data from Project MOTION. We seek to construct a model such that solutions agree with our prior knowledge and are similar to the data. The processes involve nonlinear and delayed components contributing to decision-making and behavior modification mechanisms. This initial model is then compared to data of selected patients, and further refinements and next steps are discussed.

2 Background literature

Many groups have explored the use of various mathematical models and concepts to explain some aspects of drinking behavior and of behavior change. Many mathematical models found in literature are relatively simplistic and focus only on a small number of forces that may influence a person's or cohort's behavior patterns, and a number of these models lack comparisons to data. The most popular modeling techniques are based on diffusion through social structures and SIR (Susceptible - Infected - Recovered) models for infectious diseases. Unlike these models, we intend for our proposed dynamical system model to describe individual-level dynamics.

Substance abuse researchers commonly rely on statistical techniques for analyzing data sets; many of those rely on the existence of static linear relationships between variables. They often set up experimental trials that test a particular hypothesis in the hopes of isolating one effect, and compare a treatment group with a control group. Their studies identify such factors as demographics, interactions with friends and family, and treatment methods that appear to have an effect on a

participant’s substance use. These studies provide useful information to be drawn upon in our modeling efforts.

2.1 Mathematical Models in Alcohol Studies

A number of compartmental models have been developed to roughly explain substance use in a cohort. These models generate information regarding long term outcomes in substance use; namely, they attempt to determine the chances a person will become a regular user or even heavy user if he/she starts using a substance with a particular frequency. These models typically do not provide information regarding the changes a person experiences while transitioning from different levels of use and therefore cannot be used to inform reasons for behavior change. Additionally, these models are cohort-centric and may not be applicable to other groups in a population, of different drinking behavior.

In Scribner et al. [72] and Ackleh et al., [2] an ODE compartmental model which classifies students according to drinking habits (abstainers, moderate, etc.) is developed. The model is intended to develop predictive capabilities based on campus-wide information on individual factors, social interactions, and social norms in addition to ‘campus wetness’. The model was shown to have predictive capabilities and is used to explore other campus alcohol policies, including reducing ‘campus wetness’ and imposing harsher punishments for students who are found with alcohol, to effectively reduce drinking [2], [72]. The model was fit to campus-wide individual data by minimizing a least squares cost function. Additionally, Rasul et al. [68] utilize this model and the previous work in parameter estimation for this model to determine situations in which lowering the legal drinking age may be beneficial. Using this model, Rasul concludes that only very wet campuses with a great amount of heavy drinking in underage drinking groups and mainly social drinking in legal drinking groups would benefit from a reduction in drinking age. As our research is focused on helping individuals who are typically legal drinkers find self-motivated ways to control their drinking, such a compartmental model that focuses on interpersonal factors alone is not likely to explain the behavior of subjects in Project MOTION.

A Markov chain model focusing on cocaine use was developed by Caulkins et al., in [21]. The model is simple and considers initiation to light using, the conversion to heavy using and, of course, quitting out of both of these stages. Parameters were estimated from data via a least squares approach but the details of the estimator used are suppressed. This paper reconciles the most prominent lower- and higher-intensity estimates for annual prevalence of cocaine users and considers different mechanisms for conversion from lower- to higher-intensity substance intake using linear and nonlinear relationships. This paper is closely related to the model in [22], which uses a similarly structured model to describe general drug use in the Australian population. In this paper, the authors use the model to correctly predict known prevalences, thus establishing the model’s predictive capability, and to explore possible intervention policies. Data is based on above-14-year olds in selected households throughout Australia. Parameters are estimated first based on simple probabilities (e.g. probability of first regular injection given time since initial injection, etc.), and some parameters estimated based on some minimization of least squares, while other parameters are held constant.

In 2006, Gorman et al., [38] an agent-based modeling approach is taken to simulate the interaction of individuals with their environment (neighborhoods, bars, etc.) and with other people. The individuals were classified according to their drinking type: susceptible nondrinkers, drinkers, and former drinkers. Interactions converted individuals among drinking types (similar to compartments). Mentioned is a critical mixing speed among their agents, beyond which the conversion rate to drinkers is saturated. Similar ideas, such as a social network exhibiting an epidemic of substance

dependence are mentioned in [17].

Some mathematical modeling attempts have focused heavily on the theoretical formulations of models and rarely refer to data collected by samples. These attempts stretch the mathematical boundaries of modeling in the field of substance abuse therapy but have not established their relevance to any observed phenomena. Many of these attempts would benefit from comparisons against data. These models would be inadequate in their abilities to explain the dynamics seen in the Project MOTION data set and are likely not appropriate initial models to use in our modeling efforts.

In 2004, Gorman et al., [37] outlined the ways in which studies on alcohol could be improved via dynamic systems modeling and control theory. This note came out of a meeting on Ecological Modeling of Alcohol-Related Behavior sponsored by the NIAAA and possibly began the collaboration that gave rise to a series of papers. Sanchez’s initial paper [69], classifying individuals as susceptible (nondrinkers, S), drinkers (D), and recovered alcohol users (R) has the same structure as a typical SIR model. The difference is that the relapse term, (recruitment of R to D) was an interaction term, giving rise to a backward bifurcation (or hysteresis). This implies that treatment for recovering alcohol users can be ineffective unless their social interactions with drinkers is nearly diminished. This work was extended in [27], to explore the role of nonlinear relapse among networks of drinking communities with varying connectivities. Mubayi (in [66, 65]) extends this work to examine the contagion of drinking (using the same basic ‘SDR’ principles), amongst individuals in low- and high-risk communities, and explored the role of *residence times* in each community. Also, it is shown that social activity or extent of mixing within communities drastically affects the outcomes. In [66], the deterministic model was extended to consider variability in social interactions of drinkers, and increasing/decreasing levels of drinking. Then the distribution of drinking levels under prevention, intervention, and a combination of both was presented and discussed.

In 2007, Witkiewitz and Marlatt [89] propose a model that includes instability, hysteresis, and multiple critical points, reflecting the seemingly random bifurcation of substance users into relatively stable groups who either practice abstinence, moderately use, or heavily use a substance. The form of this model is the cusp catastrophe model $V(z) = \frac{1}{4}z^4 - \frac{b}{2}z^2 - az$, where z is a metric of the participant’s behavior and a and b are parameters. In the case of [89], this metric was a composite of self-efficacy, depression, psychological status, medical history, and family life scores as determined by standardized tests; however, the combination and weighting of these scores in the model is unclear. While it is an improvement over linear regression and statistical tests of the standard articles in the field of substance dependence research, its ability to describe patterns shown in substance use data still is lacking.

2.2 Mathematical Modeling in Learning and Social Behavior Change

There is a well-developed collection of literature regarding the use of mathematics to model learning behavior (language acquisition, cognitive development, etc.) and also behavior change, decision making, and other human behavior processes. Some of this work attempts to reconcile the form of the mathematical model with observed data; however, the attempts are performed heuristically. These efforts in trying to mathematically represent the observed system are not the typical statistical methods seen in most psychology and social sciences literature. But the models are quite simple, and while appropriate for the questions at hand, they are probably not sufficient for the level of intricacy of the Project MOTION data set.

In 1951, Bush and Mosteller [18] make one of the first attempts at using probability models to quantify the chance an individual changes their behavior based on a change in stimulus. The stimulus considered is typically food, or some sort of punishment (to represent ‘reward’ and ‘work’),

and the subjects are rats. The experimental setup is representative of the focus of psychology at that time. The experiments and models are developed in the effort of modeling work performed using the ‘Skinner box’ and the ‘Graham-Gagné runway’. While this model accounts for the non-identical behavior of different individuals, the use of alcohol and other substances produces effects that cannot be categorized as strictly positive or negative.

Grossberg, in his 1980 papers [40] and [41], discusses subject response to environmental stimuli. In [40], Grossberg examines theories on error detection in the brain. He explains that as a subject learns particular stimulus, it is more able to detect that signal in high levels of noise; however, if the signal is too small (underaroused), then the subject is unable to detect the stimulus, and if the signal is too large (overaroused), then the subject’s ability to detect other stimuli may be damaged. Over time, the brain develops a set of signals from which it may determine responses to new stimuli. Errors from this set are removed via competitive feedback networks that offer differing interpretations of stimuli, thereby placing conditions on the interpretation of these signals and determining what *behaviorally useful* cues will be stored. Through these self-learning and self-tuning mechanisms, psychologists may start formulating mathematical learning models.

In [41], Grossberg quantitatively explains how chemical transmitters, innate expectation, and learned conditioning affect a subject’s response to the introduction and removal of stimuli. He highlights the special cases of when an individual is either underaroused or overaroused and during conditioning. Grossberg describes an experiment in which mice are trained to press a button in order to receive food on a particular schedule and then theorizes – in his quantitative framework – possible responses if the feeding schedule is changed. In his work, Grossberg emphasizes the necessity of studying the interactions of many underlying behavioral mechanisms as subjects react to stimuli in real time.

Van Geert has made efforts in modeling the cognitive development of children. His focus is on language acquisition and trying to model the data sets he has on the number of words learned by a certain age. In [82], he determines that for his observed data a logistic model with a delay best explains the observed acquisition of words. One of the strong points of this paper is that he carefully defines these common models in terms of the this application by giving careful attention to interpreting the intrinsic growth rate, carrying capacity, etc. However, to work with a delayed model, he switches to a discrete-time formulation, the reason for which is unclear - and in fact may be accessibility.

In his 1998 paper [83], van Geert attempts to develop a model based on concepts and mechanisms in development that would encompass ideas in Piaget’s and Vygotsky’s theories. These theories are two main schools of thought in cognitive development and are similar to the nature versus nurture debate. The framework here, however, was a cellular automaton model in a one-dimensional array. The array is ordered along ‘developmental distance’ and exchanges information with another array, representing the external world. While such a model includes both internal and external factors in an individual’s behavior change, the approach is primarily computational and still would not involve formulating and testing hypotheses based on mechanisms driving the processes, a key goal of the current effort.

Frey and Lau propose a model of integral equations describing the ways in which governments make decisions in [36]. Their underlying guiding principles are that governments are motivated by their desire to be re-elected, by maximizing their utility function. The re-electibility and factors that affect it are concepts like a voting function, a good-will function, etc. A ‘government’ appears to be treated as one decision-making entity with internal factors taken into account by boundary conditions and so may better represent an individual under the influence of external forces optimizing her behavior.

2.3 Existing literature on predictors of changes in drinking behavior

Research on substance use over the past several decades has attempted to identify predictors of successful achievement of abstinence and moderation. There are four sets of categories or constructs that have been broadly identified as being related to sustaining the status quo or prompting changes in drinking behavior. These categories are: 1) stable characteristics of the patient, 2) mood and affect, 3) environmental factors, such as social networks and stressful events, and 4) internal process factors, such as motivation to change, commitment to changing, self-efficacy, etc.

Stable and/or personal characteristics of individuals have been known to affect drinking outcomes. For example, age, gender, and history of a drinking problem are all known and established predictors of both natural recovery and treatment outcomes [26, 52, 49, 78]. Overall, as individuals age, their drinking tends to lessen over time, and younger age is associated with a greater likelihood for achieving successful moderation. In terms of gender, men and women have different problem and treatment trajectories, with women tending to initiate treatment later than men and having better outcomes the longer they remain in treatment [1]. Severity of alcohol problems (e.g., severity of alcohol dependence, having a high number of negative consequences to ones drinking), as well as simply having a history of drinking problems, is also thought to influence successful achievement of abstinence or moderation [49, 50].

Both positive and negative mood and/or affect and how individuals tolerate and react to them, have been associated with maintenance of drinking behaviors. Mood disorders and negative affect have often been associated with poor prognosis for changes in drinking behavior. This relates in part to Khantzian's [48] theory of self-medication in which he posits substances are used to ameliorate unpleasant or intolerable feelings. Such feelings can include boredom, social anxiety and depressive symptoms. Negative affect, in addition to mood and anxiety disorders, has been associated with poor treatment retention, shorter time to relapse post-treatment, and increased substance use [24, 25]. Positive mood or affect may also increase substance use. An individual may choose to drink to enhance the feeling of celebration.

Environmental factors, such as one's social networks and external stressors, are also known to have quite an influence on substance use outcomes. Social networks, which can be supportive of continued substance use or supportive of reduced or cessation of substance use, are known to be highly influential in individuals' decision making processes, access to drugs and alcohol, and recovery itself, such as in the case of Alcoholics Anonymous and other community, peer-led recovery groups [39, 54, 55]. Drinking buddies and partners may discourage an individual from changing his or her drinking habits. Partners may pressure an individual to reduce his or her drinking. In addition, highly stressful environmental experiences, such as divorce, poverty, unemployment, trauma, or even day to day stressors, may also influence substance use [23, 70]. Situations such as being on a date or highly stressful employment are important contextual factors that may increase an individuals drinking.

Finally, there are a number of internal process factors that help to facilitate or impede changes in ones drinking. These factors are usually the primary targets for behavioral treatments. Examples of such factors are motivation to change, readiness to change, commitment to changing ones drinking, self-efficacy, and urges/craving for alcohol. Motivation or readiness to change refers to the incentives one has both internally and externally to change one's behavior. Motivational Interviewing (MI) [59] is an example of a treatment that is designed to enhance ones motivation for change by addressing internal ambivalence about such a change. While some of the literature is mixed, changes in motivation/readiness have generally demonstrated an association with better substance use outcomes [43]. Commitment to change usually increases once one is less ambivalent about change, and motivation is high. It is thought that the greater the commitment to change the

greater the likelihood an individual will change his or her drinking. Indeed, commitment language uttered (statements such as “I must change” or “I will change”) within sessions of MI have been associated with better substance use outcomes [3, 4]. Self-efficacy, as defined by Bandura, refers to a belief in ones ability to change [9, 8]. Bandura argued that self-efficacy was a highly context dependent and changed as one’s situation changed. For example, an individual may have high self-efficacy to avoid drinking in his or her home but low self-efficacy to avoid drinking in a bar. Studies have demonstrated that high self-efficacy is associated with positive substance abuse outcomes [85]. Finally, frequency, intensity and ability to cope with urges and cravings for alcohol are also thought to influence ones ability to reduce alcohol use or avoid a relapse to old behavior patterns [61]. Those who are able to cope with cravings effectively are thought to be in a better position for avoiding a return to heavy drinking.

Within Project MOTION, both the IVR-based daily survey and the fixed assessments attempted to collect data across these broad categories in order to inform the process of change for individuals aiming to moderate their drinking. These data were collected from participants both within the context of a brief treatment and through independent self-monitoring. Data yielded from each of these measures and individual items were utilized to help create an informed picture or guide to how individuals attempt to change their drinking and how the trajectories of change may differ across groups.

3 Project MOTION: Rationale, Procedures, and Data

3.1 Rationale for Initial Study

Motivational interviewing (MI) has been demonstrated to be an effective stand-alone intervention for alcohol use disorders (AUD). The consistency, magnitude, and durability of its effects, especially given its brevity, suggest powerful mechanisms of behavior change (MOBC) are operating to reduce drinking. Thus, gaining a better understanding of the underlying MOBC in MI is important. However, existing MOBC studies of MI have yielded limited and contradictory findings. Project Motion aimed to rigorously examine MOBC in MI by improving on prior methods and using an enhanced conceptual framework that considered non-specific therapy factors and self-change mechanisms. To accomplish this, Project Motion was a small pilot study that disaggregated MI into its component parts and tested full MI compared to MI without its directive strategies. This comparison aimed to determine whether the directive elements of MI are critical or whether MI effects may be attributable solely to its Rogerian, non-directive components (therapist empathy, genuineness). Rogerian elements are commonly referred to as MI spirit [63]. Thus, the treatment conditions were labeled Full MI (FMI) and Spirit-Only MI (SOMI). In addition, it was tested whether hypothesized main effects of FMI were mediated via increases in commitment to reduce drinking early in treatment using state-of-the-art assessment methods, such as daily data collection using the Interactive Voice Recording (IVR).

3.2 Participants

Recruitment. General advertising online and in local media was used to recruit 89 participants seeking to reduce but not stop drinking. Advertisements emphasized a moderation approach and avoided labeling drinking as a problem. In response to advertisements, individuals called into the main study telephone number and were provided initial information about the project. Once the caller provided consent over the phone, he or she was then assessed for initial study eligibility. Eligible participants were then scheduled for a full in-person assessment with study staff.

Study eligibility. Participants were considered eligible for the study if they were: (1) between the ages of 18 and 65; (2) had an estimated average weekly consumption of greater than 15 or 24 standard drinks per week for women and men, respectively; and (3) had a primary alcohol use disorder (AUD). Participants were excluded from the study if they: (1) presented with significant substance use or a current substance use disorder (for any substance other than alcohol, marijuana, nicotine or caffeine), which was defined for our assessment purposes as greater than once weekly use in the past month; (2) presented with a serious psychiatric illness or substantial suicide or violence risk; (3) demonstrated clinically severe alcoholism, as evidenced by physical withdrawal symptoms or a history of serious withdrawal symptoms; (4) were legally mandated to complete a substance abuse treatment program; (5) reported social instability (e.g., homeless); (6) expressed a desire to achieve abstinence at baseline; or (7) expressed a desire or intent to obtain additional substance abuse treatment while in the study. See Table 1 for baseline characteristics of the final Project Motion sample.

3.3 Procedures

Fixed Assessments and Randomization to Condition. During their initial in-person assessment, participants provided informed consent and participated in a full evaluation with a research assistant (RA) and a mental health clinician who assessed for any high risk mental health disorders, such as major depression. At the end of this evaluation (week 0), participants were trained on the interactive daily questionnaire system (called interactive voice recording (IVR), described further below) and asked to come in one week later to attend the full baseline assessment (week 1). At week 1, participants were: (1) provided general feedback about their drinking and given normative comparisons according to NIAAA guidelines [67] by the RA, and (2) randomly assigned to one of three conditions: Motivational Interviewing (MI), Spirit only Motivational Interviewing (SOMI), or Self Monitoring only. All participants were followed for a total of 9 weeks and participated in 90 minute assessments at weeks 0, 1, 4 and 8. Participants in MI and SOMI were called an additional 4 weeks post-treatment (week 12) to collect drinking data via the Timeline Follow Back (TLFB, described further below). Follow up rates for assessments at weeks 1, 4, 8 and 12 were 100%, 96%, 92.1%, and 68% respectively.

Daily Questionnaire/Interactive Voice Recording (IVR) Procedures. All participants were asked to complete a daily telephone survey at the end of each day for the duration of the eight weeks (one week of baseline information and seven weeks of treatment) of the study, for a total of 56 possible days. At the end of their initial screening visit (a week prior to randomization), a research assistant (RA) provided each participant a 15 minute training session on how to use the Interactive Voice Recording (IVR) system, a system developed using TELESAGE SmartQ 5.2, a software package specifically designed for the administration of automated surveys [79]. Each day, participants responded to a series of questions about potential mediators of drinking behavior such as mood, commitment to not drinking heavily or not at all, and confidence to do so, in addition to the number and type of drinks they consumed in the last 24 hours (described in greater detail in section 2.6). Once familiar with the system, the daily IVR session required approximately 5-minutes to complete. Each participant was provided a toll-free phone number and an anonymous participant identification number to ensure confidentiality. The IVR system could be accessed between 4:00 p.m. and 10:00 p.m. This time period was judged to be when participants most likely would be able to reflect on their alcohol use that occurred post the prior days assessment and before most individuals would be likely to consume large amounts of alcohol. This time window had the advantage of providing consistent report timing and facilitated compliance by creating routines for participants. If participants failed to call into the system by 8:00 p.m., an automated reminder call

Table 1: Baseline characteristics of study sample ($N = 89$)

	MI ($N = 29$)	SOMI ($N = 30$)	SC ($N = 30$)	Overall Sample ($N = 89$)
Variable	M or %	M or %	M or %	M or %
Demographics				
Age (years) \pm SD	40.8 \pm 11.9	39.8 \pm 11.8	37.4 \pm 11.4	39.2 \pm 11.7
Male	41.4	50.0	60.0	50.6
Race/Ethnicity				
Non-Hispanic, White/Caucasian	79.3	86.7	80.0	82.0
Hispanic/Latino, any race	6.8	10.0	10.0	9.0
Other	13.7	3.3	10.0	9.0
Education				
High school diploma, GED and under	3.4	6.7	6.6	4.5
Some college/Associate's	27.6	23.4	16.7	22.5
Bachelor's degree	31.0	36.7	43.3	37.1
Some graduate school or higher	37.9	33.3	33.3	34.8
Employment				
Employed	89.6	90.0	76.7	65.2
Unemployed/ Looking for work	6.9	3.3	13.3	7.9
Not in labor force/ not looking for work	3.4	6.7	10.0	6.7
Drinking severity				
Mean sum of standard drinks per week (\pm SD)	33.9 \pm 20.4	31.2 \pm 19.0	29.2 \pm 12.9	31.4 \pm 17.6
Mean drinks per drinking day (\pm SD)	6.9 \pm 3.0	5.7 \pm 2.6	5.8 \pm 2.8	5.9 \pm 2.8
Short Inventory of Problems (\pm SD)	16.5 \pm 8.9	15.0 \pm 7.1	13.2 \pm 5.2	14.9 \pm 7.2
Alcohol Dependence Scale (\pm SD)	13.6 \pm 4.6	11.8 \pm 5.3	12.1 \pm 5.3	12.5 \pm 5.1
Number of alcohol dependence criteria met (\pm SD) $F(2, 86) = 3.67, p < 0.05$	4.6 \pm 1.5	3.6 \pm 1.6	3.7 \pm 1.4	4.0 \pm 1.6
Any drug use	34.5	43.3	43.3	40.4
Beck's Depression Inventory-II Score (\pm SD)	15.2 \pm 9.8	12.2 \pm 8.2	11.0 \pm 7.2	12.8 \pm 8.5
Ever received formal treatment for substance abuse	31.0	16.7	13.3	20.2

was made. Participants data were coded as missing if they were not able to complete a call. This daily questionnaire was considered a form of self monitoring ones drinking.

3.4 Study Interventions

Treatment was delivered in 4 sessions that lasted between 45 minutes to an hour long at weeks 1, 2, 4, and 8. Participants were blind to condition assignment, meaning they were not told which therapy they would be receiving.

- **Motivational interviewing (MI).** This condition consisted of all the standard elements of MI, both the non-directive and directive strategies [58]. Rogerian elements, such as warmth, egalitarianism, genuineness, and a client centered approach to the therapeutic relationship, are commonly referred to as MI spirit [63]. MI is comprised of a foundation of MI spirit and includes specific directive strategies geared to focus the client toward targeted behavior change, such as confidence and importance rulers, visualization of behavior change, a decisional balance, personalized feedback, a change plan, or asking for commitment. The directive elements of MI are those that selectively reinforce positive change talk or enhance discrepancy between a clients wish to change and stay with the status quo. Included in the directive strategies were other tools used by MI practitioners to explore ambivalence such as double sided and amplified reflections, as well as what is described in Motivational Interviewing Treatment Integrity (MITI) coding as evocation – a prioritizing of a clients personal reasons for change and the means to go about it [64].
- **Spirit only MI (SOMI).** While this condition retained the Rogerian elements to MI, directive elements were excluded. For example, SOMI consisted of the non-directive elements including therapist stance (warmth, genuineness, egalitarianism), emphasis on client responsibility to change, extensive use of reflective listening skills (e.g., open ended questions, simple reflections), and avoidance of MI-inconsistent behaviors (advise, confront, take expert role, interpretation). Reflective listening was focused on the whole experience of the client and the clients affect, rather than targeting a particular behavior or eliciting change talk about the target behavior. Furthermore, tools utilized frequently in MI to develop discrepancy, such as amplified or double sided reflections, were avoided.
- **Self monitoring only (SM).** Participants in this condition were not assigned to treatment and asked to participate in the daily data collection using IVR during the eight weeks of treatment to provide a comparison group to the other two interventions. This condition was pitched to participants as an active treatment on its own and encouraged to try to cut down drinking. After completing their week eight assessment, at the end of the study period, they were provided the option of receiving four sessions of MI. Only about half of this sample elected to receive treatment at the end of the study.

Therapists and training. Six masters and doctoral level therapists provided both MI and SOMI. All therapists with the exception of one had five or more years experience providing MI. All therapists participated in an initial 3 hour training on the protocol, which was then followed up by once weekly individual and group supervision. All therapists were provided with volunteer practice cases to assure fidelity to the protocol prior to the beginning of the study.

Videotapes of Therapy Sessions. All therapy sessions were videotaped for two reasons related to the original study: (1) fidelity and discriminability of the treatment conditions, and (2) coding of language uttered by clients during the sessions for *commitment language*. Client

increase in commitment language was one of the predicted mediators of MI. Phrases that indicate strong commitment include “I promise,” “I guarantee,” and “I am prepared to,” while phrases that indicate a large lack of commitment include “I don’t intend to change,” or “I plan to keep drinking.” These phrases tend to reflect the patient’s plan for a long period of time. Phrases that indicate a moderate amount of commitment, such as “I mean to,” or “I favor,” often address a shorter period of time. Tapes were also useful in gaining an additional qualitative information about participants process of change.

3.5 Fixed assessment measures and demographic information

Sociodemographics. A self-report, demographic questionnaire used in a series of completed studies by Morgenstern et al., was used during the initial phone and in-person encounter with the participant. This included data on marital status, educational and occupational information, medical history, family psychiatric and substance abuse history, and the participants substance abuse treatment history.

Substance use diagnosis. Two instruments were used to screen and later identify alcohol and other substance use disorders. The Alcohol Use Disorders Identification Test (AUDIT) was developed by the World Health Organization to assess for heavy drinking, and it has demonstrated strong reliability and validity [7]. The Composite International Diagnostic Instrument, Substance Abuse Module (CIDI-SAM) [28] was used to evaluate substance dependence exclusion criteria. It is a well established diagnostic interview that has demonstrated excellent reliability and validity [91].

Alcohol and drug use patterns and problems. The Time-Line Follow-Back Interview (TLFB) [76] assessed quantity and frequency of alcohol use during the previous eight weeks. The TLFB has demonstrated good test-retest reliability [20], agreement with collateral reports of alcohol [30], convergent validity, and reliability across mode of administration (i.e., in person or over the phone, [86]). The TLFB is an interviewer assisted calendar based method that utilizes specific recall techniques (e.g., memory cues) for participants to recall daily drinking (in standard drink equivalents) and types and frequency of other drug use. The TLFB has demonstrated reliability and validity for the collection of alcohol and other drug use data for recall periods of up to one year [32]. Daily data is then aggregated into summary variables, reflecting weekly drinking frequency and intensity.

The widely implemented Form 90 [56] was used to evaluate lifetime and recent (past 8 weeks) severity of other drug use. The Form 90 has demonstrated strong reliability and validity [81].

The Short Inventory of Problems (SIP) [57] is a 15-item self-report measure of lifetime or past three months negative consequences of drinking. The SIP has demonstrated strong psychometric properties [47]. Higher SIP scores indicate a greater number of experienced negative consequences.

Severity of alcohol dependence was measured using the Alcohol Dependence Scale (ADS) [73]. The ADS is a 25-item self report measure of various symptoms and intensity of alcohol dependence. It has demonstrated strong reliability and validity across studies and populations [45, 74]. Higher scores indicate progressively greater risk for alcohol dependence.

Psychiatric and cognitive impairment exclusion criteria. Two screening tools, the Structured Clinical Interview for DSM-IV, Psychotic Screening and Mood Disorders sections (SCID) [33], and the Mini-Mental Status Examination (MMSE) [34] were used to screen for serious psychiatric symptoms and cognitive impairments, respectively. Both of these instruments are well established as having strong psychometric properties [35, 80, 84].

Other Psychiatric and Personality Factors. Depressive symptoms were measured using the Becks Depression Inventory, second edition (BDI-II) [15]. Items relate directly to symptoms of

depression, including hopelessness, irritability, feelings of being punished, and physical symptoms, such as fatigue, weight loss, and lack of interest in sex. The BDI-II is a self-report, 21-item questionnaire, which yields a continuous score, ranging from 0 to 63. Scores that range from 14 to 19 indicate mild depression, 20 to 28 indicate moderate depression, and 29 and above indicate severe depression. The State Trait Anxiety Inventory (STAI) [75] measures the extent to which an individual experiences or does not experience anxiety using two subscales: absent and present. Anxiety as a state is a more transitory, emotional state, which fluctuates over time. Higher scores in the 'absent' subscale indicate an absence of anxiety, and high scores in the 'present' subscale indicate presence of anxiety.

Process Variables. Several scales were used to capture participants readiness, perceived ability to, and process of change. The Processes of Change Scale (POC-27) is an adapted version [62] of the 40 item self-report measure assessing frequency of coping strategies for avoiding heavy drinking. This 27 item version includes two subscales, one behavioral and one cognitive coping score. Items use a 5-point Likert scale, with strongly agree and strongly disagree as anchors. Scores from all the items of each subscale are summed together. Higher scores on either subscale indicate a greater perceived ability to cope using cognitive or behavioral techniques, respectively.

The Readiness to Change Questionnaire (RCQ) [42] is a 12-item instrument for measuring stage of change reached by an excessive drinker, and it was utilized as the primary measure of motivation to change. The RCQ has demonstrated good psychometric properties including predictive validity, and it consists of three subscales: precontemplation, contemplation, and action. For the purposes of this study we utilized the action subscale. A higher score on the action subscale indicated a progressively stronger level of motivation to change.

The Situational Confidence Questionnaire (SCQ-39) [5] was used to assess participants confidence to resist the urge to drink heavily. It is a 39 item scale, and for each item, participants rate their level of confidence on a six point scale, from zero (not at all confident) to one hundred percent (very confident). Higher scores indicate a general higher sense of self efficacy in relation to reducing drinking.

The Obsessive-Compulsive Drinking Scale (OCDS) [6] measures incidence and frequency of obsessive thoughts and compulsive behaviors concerning drinking and ability to control behavior. It is a measure of alcohol craving. This 14 item scale yields a sum score, where higher scores indicate more craving, urges, and a perceived inability to control drinking.

3.6 Measures Used within the IVR Questionnaire

In order to capture and assess constructs that are known predictors or mediators in sustaining or reducing problem drinking, the IVR daily questionnaire was created. Measuring these variables on a daily basis was an attempt to enhance our current understanding of how these may change over shorter periods of time. A daily focus on these variables collected in a longitudinal fashion and was presumed to allow for a better understanding of processes (and mechanisms) of change.

The IVR questionnaire, transcribed in the Appendix, was comprised of 41 questions, which are divided into groups by topic and response set. These groups are: activities since yesterday, current mood, perceived stress, desire to drink right now, commitment and confidence, and actual drinks consumed.

Activities since yesterday include 11 questions about whether stressful or pleasant events occurred since yesterday. Responses to these items include (0) no; (1) yes, but last night only; (2) yes, but today only; and (3) yes, both last night and today. Stressful events include arguments, and general stressful events that could occur at home, work or school. Pleasant events include performing a task that inspired a sense of accomplishment; pleasant interaction with family; and pleasant

interaction with friends. A third category, drinking situations, inquired about the participant's exposure to situations in which there was pressure to drink, situations in which the participant commonly drinks, and contexts in which participants might be likely to drink such as celebrations, nightclubs or bars, and on dates.

Next were 11 questions on the participant's current mood. The participant was asked how active, sad, nervous, tense, lonely, happy, angry, enthusiastic, bored, tranquil, and relaxed he or she feels, and the participant rates each emotion on a scale from (0) not at all to (4) extremely.

Using the same response scale, four questions on a participant's perceived stress follow the questions on mood. These questions asked about a participant's feeling of being unable to control important events in her life, confidence in being able to handle personal problems, perceived luck or that things are "going their way", and ability to overcome difficulties.

Next, in an attempt to assess craving, were three questions on the participant's desire to drink at the moment. These questions were measured on the scale of (0) definitely false to (4) definitely true with (2) neither true nor false in between. The statements are, "I really don't feel like drinking," "I feel like I could really use a drink," and "the idea of drinking is appealing."

Another set of questions relate to how confident and committed the participant felt in regards to changing his or her drinking habits. These three questions were measured on the same scale as the questions pertaining to the participant's emotions and ask specifically about the participant's confidence in being able to resist drinking heavily in the next 24 hours, commitment to not drinking heavily for the next 24 hours, and commitment to not drinking at all for the next 24 hours.

Another set of questions focused on the number of alcoholic beverages consumed in the past 24 hours, divided into the categories of beer, wine, and liquor and the time periods of last night and today. Valid responses were any number of whole drinks greater than or equal to zero.

The final question asked the participant if he or she feels that their drinking in the past 24 hours has been excessive, with the responses (0) definitely not, (1) possibly, (2) probably, and (3) definitely.

4 Interpretation of Data to Inform Modeling

While the wealth of information in this rich data set presents a unique opportunity to examine aspects of drinking behavior change longitudinally, it also presents challenges. As outlined in Section 1, and in particular Section 2.1, previous modeling efforts in this area is limited and modeling efforts of the scope of these data are nonexistent. In contrast to other areas, such as the physical sciences, in which the state variables representing key players in the processes are clear, no such information is known. In addition, the relevant timescales on which the variables are changing and therefore should be observed are unclear. Thus, it is possible that some of the measures, if not deemed to be important on the timescale of the current data set, may be driving other aspects of drinking behavior either on a shorter or longer timescale.

As we initially approached the task of developing dynamical systems models of these processes, we were prompted to think critically about the information content in the Project MOTION data. Individuals' drinking behavior, and their response to treatment, is highly variable, and it is likely that one model describing all individuals is not possible. At any rate, to be so ambitious with first attempts at discerning key mechanisms is unreasonable and focusing on groups of individuals who respond in similar ways is a more feasible approach. With that in mind, we proceeded to organize the data in ways that would best inform modeling efforts, described in the rest of this section. Specifically, we examined the more quantitative measures - those with numerical scores - to see if they would be best used in an individual-level model or if they could be used to inform

a meaningful grouping of individuals into cohorts. Additionally, we considered how, if at all, the measures complemented the information in the daily IVR data

4.1 Timeline Followback

There were some reasonably identifiable groups using the TLFB data. Two such groups were responders and non-responders (Figure 1). We defined responders as those individuals who demonstrated at least a 40% decrease in their drinking by their last treatment session or the week 8 assessment. These patients are: 6006, 6026, 6029, 6083, 6119, 6124, 6128, and 6136. Non-responders were identified as demonstrating no change in their drinking or increasing their drinking by the week 8 assessment. Thus, there are several individuals who would be classified neither as a responder nor as a non-responder.

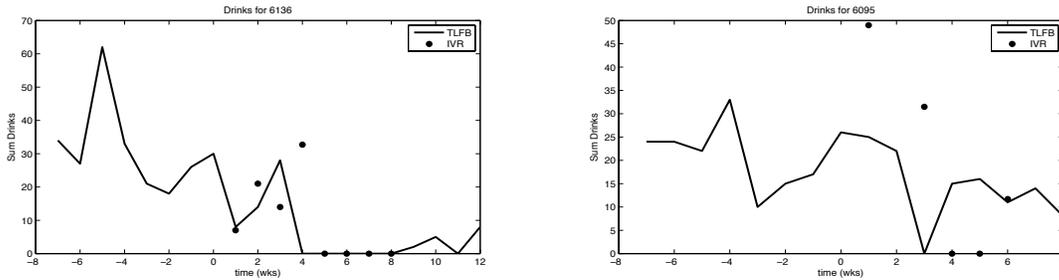


Figure 1: TLFB and IVR drinking for an individual termed a **responder** (Left) compared with an individual who reduced their drinking, but by less than that of a responder (Right).

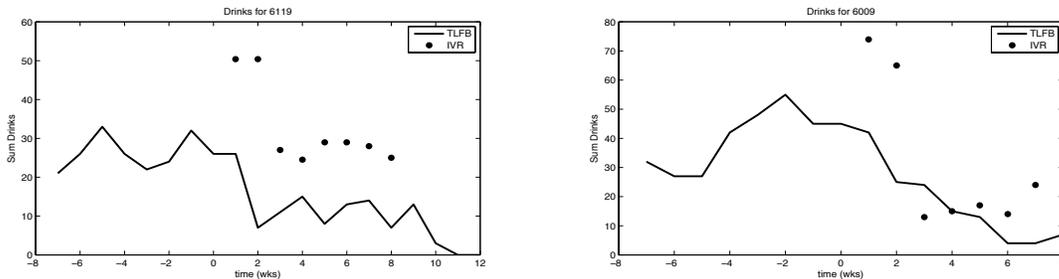


Figure 2: Left: Consistent underreporting of TLFB as compared to IVR data; Right: Underreporting of TLFB drinking data during periods of heavier drinking only;

Also, the timeline fallback (TLFB) data is compared to the IVR drinking data here to determine how reliable the timeline fallback data is compared with IVR drinking data. There were thought to be a couple advantages possible from the timeline fallback data; namely, that it is actually more complete during the study/treatment time period than the IVR data, in which subjects did not necessarily call in every day.

Also, with some patients, the use of the drinking data from before or after the IVR data collection period may be informative. We expect that the IVR data is more accurate and that individuals are more likely to accurately recall their drinking when it has occurred in the last 24 hours versus the timeline fallback where they are recalling their drinking weeks and months in the past. Indeed, underreporting was definitely seen in some patients consistently or only when drinking heavily (Figure 2). But some patients appeared to accurately recall their drinking during

TLFB (e.g., Patient 6136, Figure 3), and therefore their data prior to and after Project MOTION may be used as a reliable measure of their drinking.

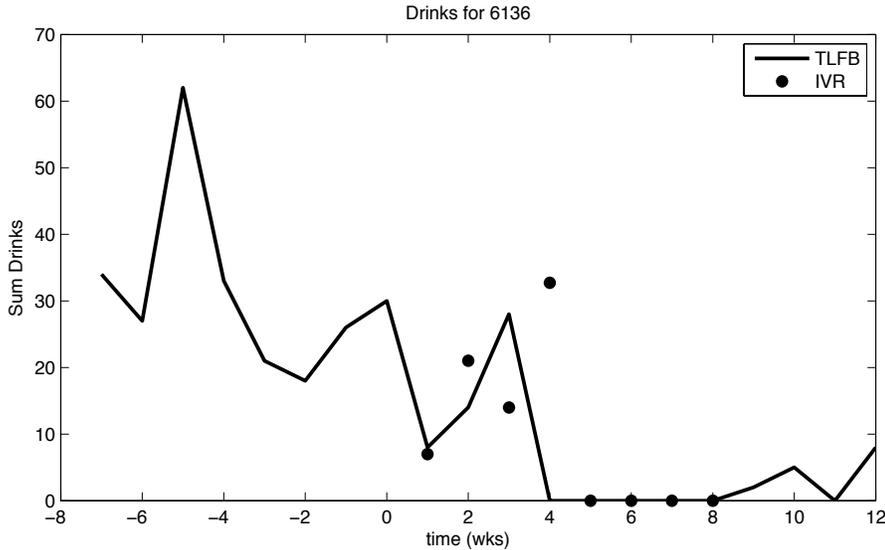


Figure 3: Patient 6136 sum of drinks by week as reported via TLFB and IVR.

4.2 Fixed Assessment Measures

While the IVR daily data initially appears a more desirable data set to use for dynamical systems modeling, data gathered via the fixed assessment measures could be used to determine characteristics that might inform a cohort-level model, or may provide distinct information not in the IVR. We looked for characteristics, traits, and states that would differentiate participants from one another, and these differences could be static or dynamic. We wanted to distinguish between a static difference, where for example certain patients could have higher overall scores but show similar changes in time (if at all), and a dynamic difference which would be the case if patient’s scores changed in a distinctly different way with time, even though the scores may be roughly of the same magnitude as all other patients. Specific measures were selected for the purpose of cohort formulation due to their potential to differentiate groups of participants who may share common trajectories of change in drinking or the measures’ close relation (i.e., measuring a similar or the same construct) to characteristics also measured by the IVR. Results of this exploration are described by measure selected below.

Due to the small number of time points of these measures for each patient, plots of these data are not particularly informative and are not shown here. We note that a general observation was that where a trend was observed in the responders, there was usually a lack of trend with the non-responders as opposed to a different characteristic trend. That suggests that responders are likely to behave in a more predictable way, more likely to be motivated by identifiable mechanisms and therefore, will be the focus of initial model building efforts. A short description and interpretation of the corresponding patient data is contained below.

- The Short Inventory of Problems [associated with drinking] (SIP) measure could be a supplemental construct to measure the severity of an individual’s drinking (in addition to drinking).

Responders, or those who show improvement in drinking habits, were found to have significantly higher SIP scores at baseline, and overall these decrease over time (along with their drinking). Non-responders typically reported very low SIP scores at baseline and these scores remained relatively static over time. Using the SIP score as a variable to split individuals up into cohorts could be useful, as it seems to be predictive of the changes made, and the individual's perception of their drinking. That is, the non-responders seem to realize that they are drinking excessively, but if they do not perceive it as a problem, then that may contribute to a lack of behavior change. There is no clear relationship of the SIP to any construct measure in the IVR.

- The State Trait Anxiety Inventory (STAI) measures the extent to which an individual experiences anxiety. No trends were clear with the non-responders. The responders in both treatment conditions report higher scores for presence of anxiety than for absence initially. Over time, they report decreased presence of anxiety and increased absence of anxiety - perhaps reflective of the success of their ability to control their drinking. Information collected by the STAI is also reflected in the IVR questions relating to stress and anxiety.
- The Situational Confidence Questionnaire (SCQ) questionnaire inquires about *self-efficacy*, by asking individuals how confident they are they could resist the urge to drink heavily in certain situations. No trends to differentiate groups were demonstrated. Information about confidence and commitment to reduce or abstain from drinking measured by the IVR may also get to self efficacy.
- The Beck Depression Inventory-II (BDI-II) test measures the severity of depression. The subjects included in Project MOTION are mostly not depressed as measured by the BDI-II, with an average around 15. While negative mood is measured on the IVR and could be related to information alluded to in the BDI-II, there is no direct relationship of depressive symptoms to a construct on the IVR.
- The Obsessive-Compulsive Drinking Scale (OCDS) test measures the obsessiveness and compulsivity of individuals concerning their drinking. In the initial perusal of the data, the OCDS scores of participants who responded to treatment all decreased with time. OCDS scores of participants who did not decrease their drinking did not show consistent trends. There was no corresponding item in the IVR to the information collected by the OCDS.
- The Readiness to Change Questionnaire (RCQ) quantifies the *stages of change* through which the subject is transitioning. In general, RCQ scores across a small sample of participants did not show any particular trends. The IVR questions addressing commitment to limit or abstain from drinking may be a stronger indicator of readiness to change.
- The Processes of Change (POC) questionnaire contains two subscales, cognitive and behavioral. No clear trends emerged differentiating participants in any significant way among responders or non-responders. There was no corresponding item in the IVR to the information collected by the POC.

Overall, the fixed assessments provided no further way to divide responders into similarly behaving cohorts. Also, there were no clear trends among non-responders from these data to group them in a meaningful way. From these initial efforts, we resolved to focus on a few select responders to identify key characteristic patterns, perhaps indicative of underlying mechanisms, contributing to long-term changes in drinking behavior.

4.3 Grouping of IVR data categories

Focusing on the IVR data to develop an initial model, we note that the selection of possible state variables is a nontrivial task due to the lack of previous efforts in modeling at this level. It would be easy to lose information by grouping too many questions together as indicative of one trait, when in reality there are distinct traits or processes measured within that group. Conversely, the treatment of every piece of information as distinct would likely prove prohibitive to building a computationally tractable model that would produce results that can be meaningfully interpreted. In an effort to balance these two pitfalls, we proceeded by combining all - or most - questions in one topical category into one “observed data category” to be used as a state variable in the model.

The connotation of the possible answers to each question help us determine a “neutral value.” For one’s perception of drinking as excessive, 0 is seen as neutral. In the case of mood, perception of stress, desire to drink, items were treated such that the middle value (typically 2) is treated as neutral and the tail values are the extremes. In implementation, questions that reflect positive states of mind were unchanged, so that the lowest value indicated a negative mindset and the highest value a positive mindset; questions that reflect negative states were reversed about the middle value so that the lowest value also indicated a negative mindset and the highest value a positive mindset.

An average, as opposed to a sum, was used to decrease sensitivity with respect to omitted entries in a single IVR response. For example, if items in an observed data category could have a maximum sum of 15, then omitting an entry would change this maximum value, thus biasing the entry for that day. Instead, the average would divide the sum by the number of non-omitted questions, thus making a 9 out of 15 and a 6 out of 10 mean approximately the same state in that category.

Such categorical groupings are commonly used in current research in the field of substance abuse. When additional breakdown is needed, the questions in one category are split into smaller groups and summed as before. The following composite scores could be used to inform us of important patterns that must be reflected in the model:

- Stressful and pleasant events: While the occurrence of the events was measured on an ordinal scale, it might be acceptable to treat the items as an interval scale so that they may be averaged. It was most useful to look at events today and last night, taking a weighted sum of stressful events (negative weight of -1) and pleasant events (positive weight of $+1$) occurring during the time period of interest. Initial investigations showed that averaging positive and negative events during these time periods obscured much of the dynamics of this data, so this idea will be extended such that stressful events last night, stressful events today, positive events last night, and positive events today are all considered. The category of stressful events contains responses from questions 1–3 in the section pertaining to activities since yesterday of the telephone interview; the pleasant events category contains responses from questions 4–6 in the section pertaining to activities since yesterday of the telephone interview.
- Pressure to drink: Like stressful and pleasant events, the occurrence of drinking situations were measured on an ordinal scale but may also be treated as an interval scale so that the model may include an item describing the drinking pressure from external forces felt by the participant. This category includes responses from questions 7–11 in the section pertaining to activities since yesterday of the telephone interview. This category is also divided into last night and today.
- Current mood: Each emotion was measured on a discrete interval scale, so we may also add or otherwise combine the scores for each emotion to create a composite score that describes

mood. If the information from all mood-related questions were added, then active, happy, enthusiastic, tranquil, and relaxed will carry positive weight, and sad, nervous, tense, lonely, angry, and bored will be scored inversely (4 minus reported score). Like in the case of stressful and pleasant events, current mood may be divided into more categories so that more information is maintained. Ideas of such groups are

- Negative mood: This category includes information from the sad, nervous, tense, lonely, and angry columns.
- Active mood: This category includes information from the active, happy, and enthusiastic columns.
- Inactive mood: This category includes information from the bored, tranquil, and relaxed columns. All questions have the same weight.
- Depression: This category includes information from sad and lonely columns.
- Anxiety: This category includes information from the nervous and tense columns

It is expected that only one or two of these subgroupings at most would be used when writing the model.

- Perceived stress: While perceived stress may be related to stressful and pleasant events, it is an internal interpretation of environmental circumstances. As the perceived stress questions are answered on an ordinal scale, the two statements with positive connotations will possess positive weights and statements with negative connotations will possess negative weights. The four questions of the perceived stress section of the telephone interview, with questions 1 and 4 scored inversely (4 minus reported score), and 2 and 3 having positive weight, provide information for this category.
- Desire to drink (Desire, *D*): In the field of substance abuse treatment, counselors often try to help patients improve their control system or ability to self-regulate in regards to reducing or resisting the urge to drink. One part of the control system is the desire to drink, which is measured by the three questions in the section on desire to drink right now. These questions are measured on the scale of (0) definitely false to (4) definitely true with (2) neither true nor false in between. The statements are, “I really don’t feel like drinking,” “I feel like I could really use a drink,” and “the idea of drinking is appealing.” The questions that express a desire to drink (numbers 2 and 3) will have positive weight and the question that expresses a lack of desire to drink (number 1) will have a negative weight.
- Commitment to not drink (Quit, *Q*): Commitment to not drink is another part of a person’s control system. The assessment of a participant’s commitment over her drinking is expected to have a longer-term relationship with a person’s actual number of drinks than desire and thus should be considered separately. The question regarding the participant’s commitment to not drinking at all for the next 24 hours is the only question in this category.
- Commitment and confidence to limit drinking (Limit, *L*): It was shown that often a person’s commitment to quit drinking and her commitment to limit drinking differed. Like commitment to not drink, Limit is expected to have a longer-term influence on drinking habits. The two questions ask about the participant’s confidence in being able to resist drinking heavily in the next 24 hours and her commitment to not drinking heavily for the next 24 hours.

- Guilt about drinking (Guilt, G): A person’s feeling that her drinking was excessive shows a slightly different dynamic than either desire or control. The question asks the participant if he or she feels that her drinking in the past 24 hours has been excessive, with the responses having values (0) definitely not, (1) possibly, (2) probably, and (3) definitely. While the construct we captured relates to a personal limit placed on drinking violation, we refer to this construct as ”guilt” as an easy shorthand to also differentiate this from ”Limit.” We recognize that guilt in and of itself is a different construct, and do not infer that guilt is what is measured here.
- Alcoholic beverages consumed (Alcohol, A): Alcohol intake is recorded in the IVR data by drinks during the time interval beginning at the time of the IVR call on the previous day to when the individual’s night ended, and the interval beginning at the start of the individual’s current day until the time of the current IVR call. The participant may input any number of whole standard drinks greater than or equal to zero. Participants were trained on the definitions of standard drinks. They entered the amount of beers, malt liquor, wine or cocktails they consumed. For these analyses, standard drinks were totaled.

The names (and shorthand symbols of composite scores) of these categories are Stressful Events (E_-), with time divisions into Stressful Events last night (E_{l-}) and today (E_{t-}), Pleasant Events (E_+) with time divisions E_{l+} and E_{t+} , Pressure (P) with time divisions P_l and P_t , Mood (M), Negative Mood (M_n), Active Mood (M_c), Inactive mood (M_i), Depressed mood (M_d), Anxious mood (M_a), Stress (S), Desire (D), Quit (Q) and Limit (L) which represent different aspects of Commitment, Guilt (G), and Alcohol (A) respectively. These categories and hypothesized relations are depicted in Figure 5.

While we consider the control system as an internal trait in theory, initially we exclude the control system from the internal category, thus yielding the broad categories of environment, internal, and control system. The existence and direction of the depicted relationships were heavily based on theory and conceptualization of common human behavior. These categories and relationships were examined in our initial analyses of the data using a variety of techniques including inspection, linear interpolation, and linear methods of data analysis such as principal component analysis.

4.4 Initial model ideas

The structure of an appropriate model requires careful consideration, even if the determination of state variables does not warrant further discussion. However, more appropriately, the construction of a model and its state variables should go hand-in-hand as a variable may not be relevant in some formulations. On the other hand, the same variable could be imperative in a different framework, depending on the timescale, or the scope of the model. That is, it may be that some variables/processes are more relevant on a large scale (say, at the population level) versus a small scale (perhaps at the individual level), or vice versa. It is important to keep this in mind as one is in the initial stages of developing a modeling framework.

As the field experts, the Columbia team has knowledge of common themes discussed in common language among colleagues and in literature. Thus, based on this prior knowledge, we began with those suggestions of how to organize the wealth of information into interacting components, potentially into a hierarchical structure of processes important in driving an individual’s decision making concerning their drinking. A logical way to think of this is an individual is controlled by 3 broad units: their environment, internal factors, and a ‘control’ system. Here we wish to distinguish that the word ‘control’ is meant to quite literally control one’s drinking as opposed

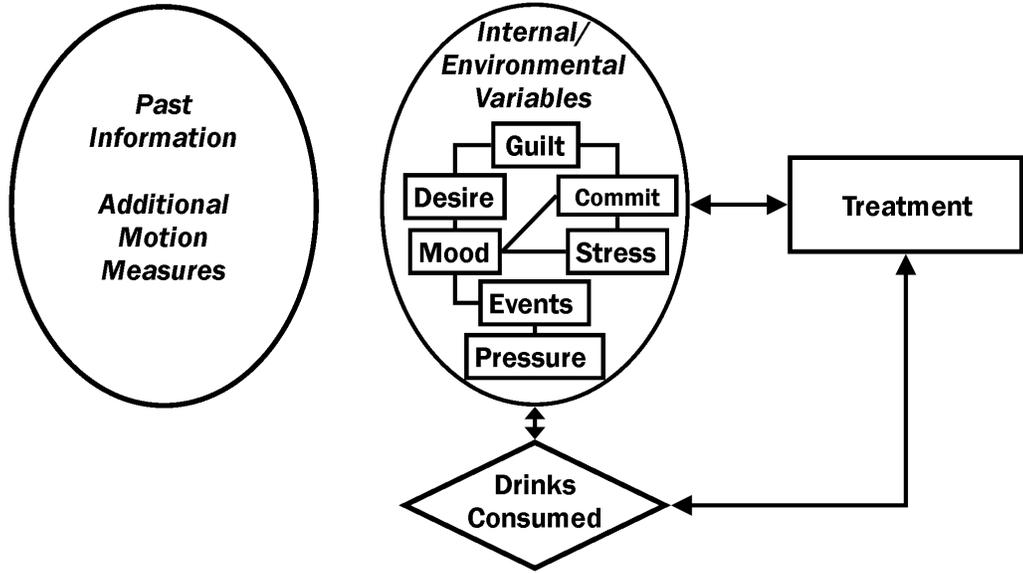


Figure 4: An illustration of how a categorical model based on the IVR data may be used in a model with a broader scope and stronger emphasis on treatment.

to referring to the field of control theory, an area of applied mathematics. An initial attempt to resolve the information in the IVR data as it might be part of these units is found in Figure 4.

Throughout discussions and initial analyses of the relationships between the data categories (which are the precursors of model state variables), these *conceptual models* underwent multiple revisions. As is natural in interdisciplinary work, some formulations are more natural from one perspective than another. Another preliminary attempt is shown in Figure 5, which arose from the intuition of the NC State group of the information content in Project MOTION. Some key differences are that the novel (meaning distinct from the IVR data) information from the additional measures, or fixed point assessments, all are relevant to the time before the study began. The drinking rate is considered more of a traditional ‘output’ variable or the key variable we are interested in overall. The other variables, including those in the previous conceptual model (Figure 4) were considered as internal variables, since in practice, they are not anything we can directly influence. Treatment, on the other hand, is thought of as an aspect of the process we can adapt and change in practice, and an external control. The goal of our modeling efforts as interpreted in this framework is to use the internal/environmental factors to construct an individual-level model of drinking behavior, and how drinking is effected by these interacting factors. The overall goal is then to understand how the traditional ‘input’ or treatment influences these factors, and in turn, drinking.

5 Methods to examine relationships between observed data categories

We next explore the feasibility of these category-based groups as model state variables (Stressful Events (E_-) with E_{l-} and E_{t-} , Pleasant Events (E_+) with E_{l+} and E_{t+} , Pressure (P) with P_l and P_t , Mood M with M_n , M_c , M_i , M_d , and M_a , Stress (S), Desire (D), Quit (Q), Limit (L),

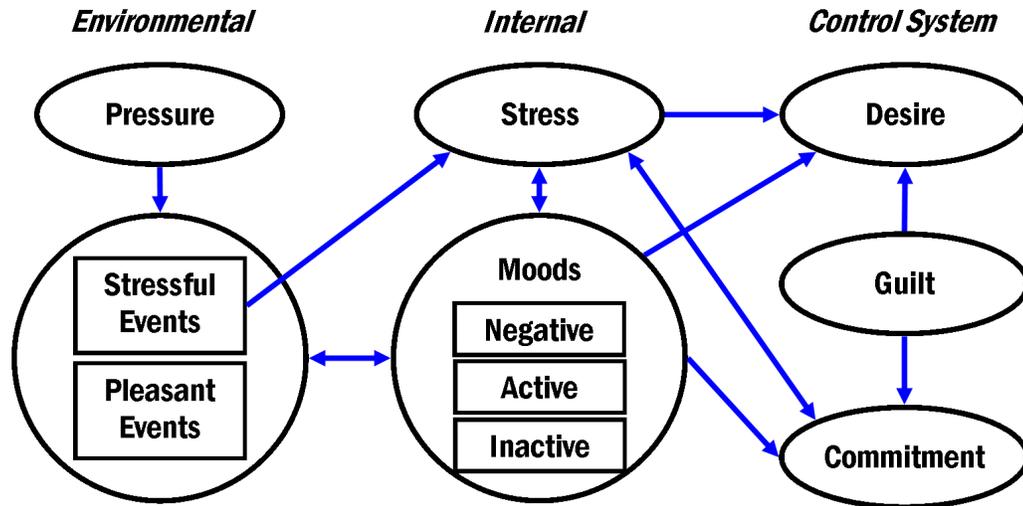


Figure 5: Observed data categories and hypothesized relationships between them.

and Guilt (G) in relating to the number of drinks consumed A . We looked to see if any of these should be combined into one variable or if there are clear relationships to drinking apparent. We assessed the new composite scores' relationship with each other and with the number of drinks consumed in several time periods and compare to the strength of relationships seen between data from individual questions using inspection, linear interpolation, and principal component analysis.

It was previously determined based on existing literature and discussion that time of day is related to the number of drinks a person consumes and thus including time in the study of any other composite scores' relationships with number of drinks may be important. Additionally, it allows us to see any possible changes in the data - drink relationships as a function of time.

5.1 Initial inspection of the data

Visual interpretation of the data is useful when searching for trends that may not be easily detected or expressed by statistical means and is a logical starting point when investigating a large data set. In our initial search, we detected several trends that should be considered when building our conceptual and mathematical models. Sometimes, relationships between data categories were observed on a short time scale of typically less than two days. Some individuals exhibited similar trends in behavior, indicating that they may be grouped into cohorts at a later stage that may affect the compartments of our conceptual model or how those compartments are related. Finally, different temporal trends in the data became clear at different scales.

We examined the time scale on which categories are related to search for initial evidence that delays may be needed in our model. For example, we would not expect the pressure to drink that a person experienced last night to have much relation to the number of drinks a person had the current night - the pressure last night is more likely related to the number of drinks consumed last night. The relationship between pressure and alcohol consumption is more or less instantaneous. On the other hand, if one has a lot to drink on a given night, one may then feel like their drinking was excessive the next day and thus reduce their drinking in the near future. There is a delay in this behavioral change. Several of these time-specific trends were evident in the data: affine relationships between a composite score versus drinks per different time periods would sometimes

appear stronger or weaker (Figures 6, 7, 8). Figure 6 shows that guilt has a clearer relationship with number of drinks over the past 24 hours than drinks consumed during the evening after the survey for two participants, while Figure 8 indicates that a person’s stress level is related to drinks consumed over the whole day, including the day leading up to and evening following the survey, more strongly than it is related to drinks consumed the night before. The pairing of a category and time period that seems to have the strongest relationship with drinking is located in Table 2.

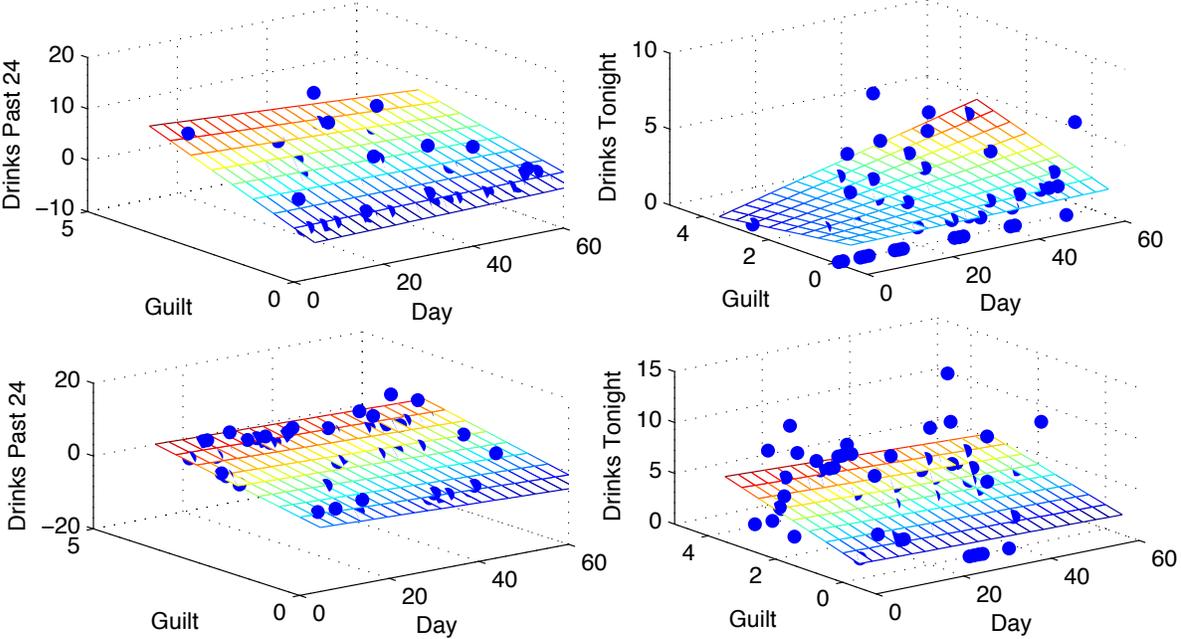


Figure 6: Guilt versus drinks over past 24 hours (left) and drinks tonight (right) for participants 6029 (top) and 6090 (bottom).

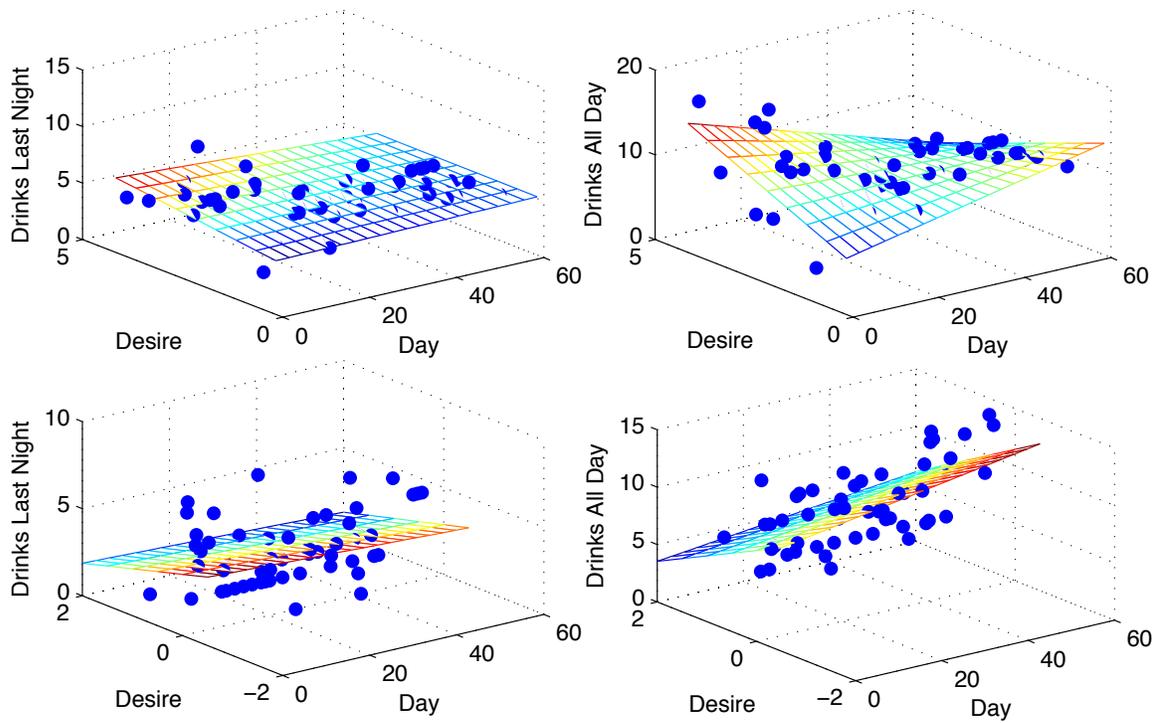


Figure 7: Desire versus drinks last night (left) and drinks over all of today (right) for participants 6021 (top) and 6072 (bottom).

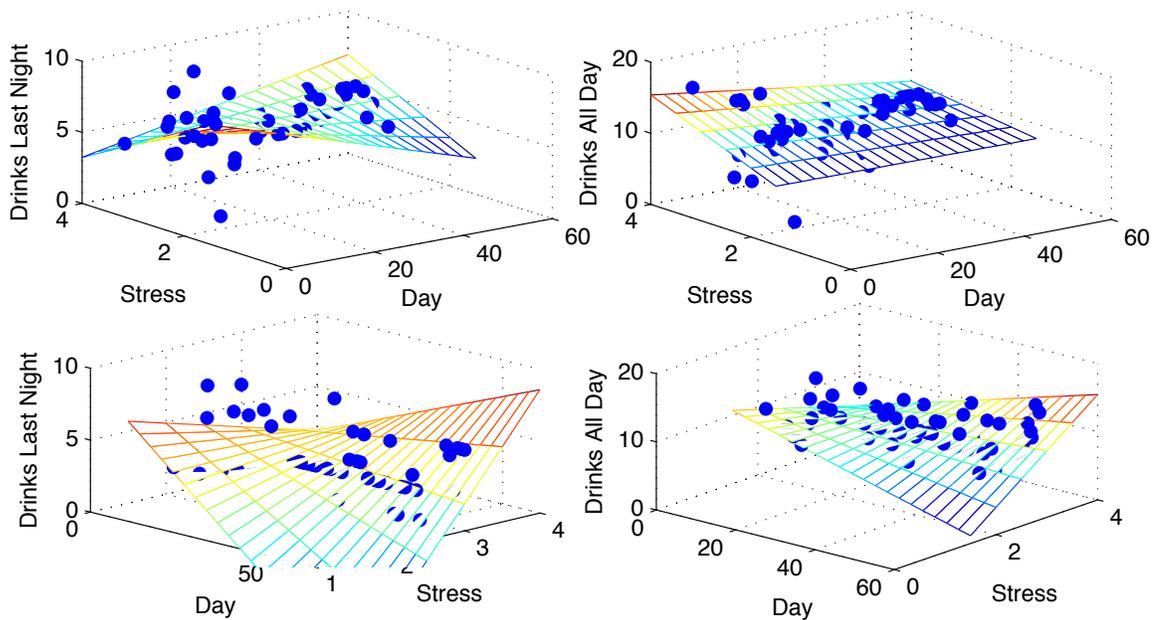


Figure 8: Stress versus drinks last night (left) and drinks over all of today (right) for participants 6021 (top) and 6072 (bottom).

Table 2: Best time period for each observed data category

Data Category	Time Period
Events (Stressful and Pleasant)	Past 24 hours
Pressure	Past 24 hours
Events last night	Last night
Events today	All of today
Pressure last night	Last night
Pressure today	Today
Mood	Past 24 hours
Stress	All of today
Desire	All of today
Guilt	Past 24 hours
Commitment	All of today

While looking at drinking patterns on this daily or even twice-daily level, we noticed that people show different drinking trends depending on the time of day. Some people tend to be night drinkers and other people appear to be day drinkers. Namely, a participant may exhibit a pattern of being in many more drinking situations or consuming more drinks during one part of the day than the other (Figures 9, 10). In Figure 7, we see that the desire of patient 6021 is related to the drinks 6021 consumed the night before, but the desire of patient 6072 is more closely related to the drinks consumed over the whole day, both before and after the IVR survey. These signs helped us determine that we should first build a number of individual-level models before attempting cohort or population models.

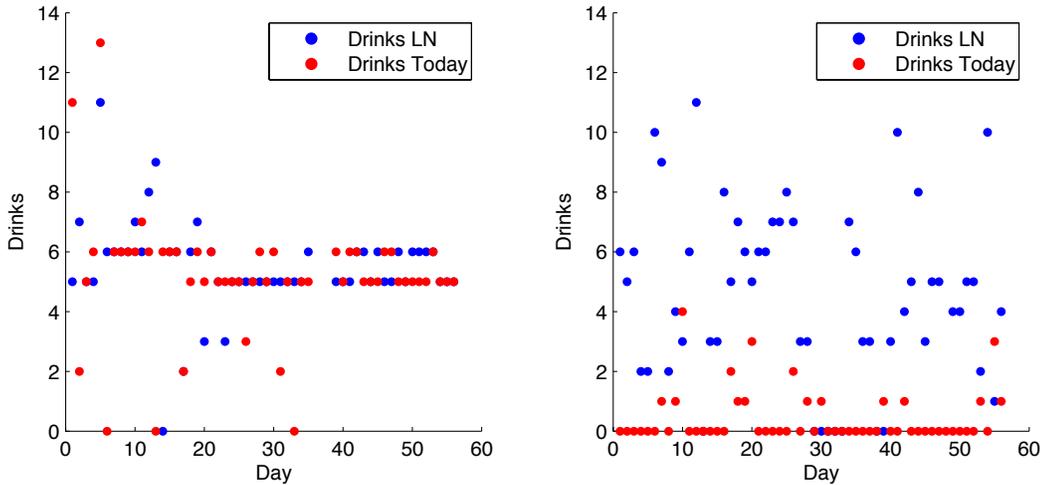


Figure 9: Drinks over time for participants 6021 (left) and 6090 (right).

Expanding the idea of observed data categories affecting number of drinks for different time periods, we also considered the idea that some categories may explain more behavior if averaged over a several-day period. From a theoretical perspective, perhaps day-to-day events influence the

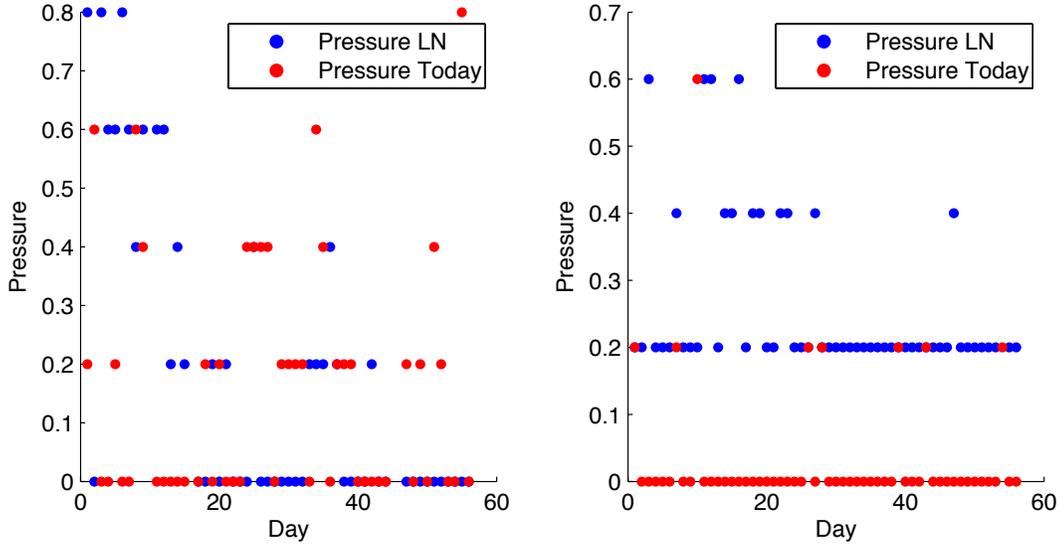


Figure 10: Pressure last night and pressure today over time for participants 6002 (left) and 6090 (right).

drinks consumed on that day or night. It is expected that people behave differently on weekends, which consists of a couple of days. In particular, a person typically does not work on the weekends and thus lacks a particular stress that may induce or suppress the desire to drink; however, that person may spend much more of the day around friends and family, and without the obligation of work, a person may feel more inclined to drink. In our initial explorations of the data, we found that that some people may change their drinking habits over the weekend, but other people do not change (Figure 11). This is yet another indication that the behavior of different individuals would be better reflected using different models.

To help search for broader-scale relationships between data columns or data categories, we considered several different time scales. The daily measurements are useful when examining fine-scale relationships or very strong affine relationships, but vary greatly and posed difficulties when attempting to fit preliminary mathematical models (Figure 12). A weekly average, however, provided only a maximum of eight data points for over the fifty-plus days of the IVR study. So few data points does not adequately reflect the complexity of human behavior, and we did not find much useful information from such general trends other than if a person decreased their overall drinking (Figure 13). With the knowledge that some people change drinking habits over the weekend, we formulated the tri-weekly drinking scheme that averaged small groups of the seven data points from a week into three data points: one point representing Friday night through Sunday afternoon, a second including Sunday night through Wednesday afternoon, and a third point containing Wednesday night through Friday afternoon. With this grouping we continued to have a large number of data points (approximately 24) and did not obscure any changes in drinking behavior during the weekend but reduced daily noise (Figure 14). This grouping also provided a realistic standard to which we may start fitting initial mathematical models.

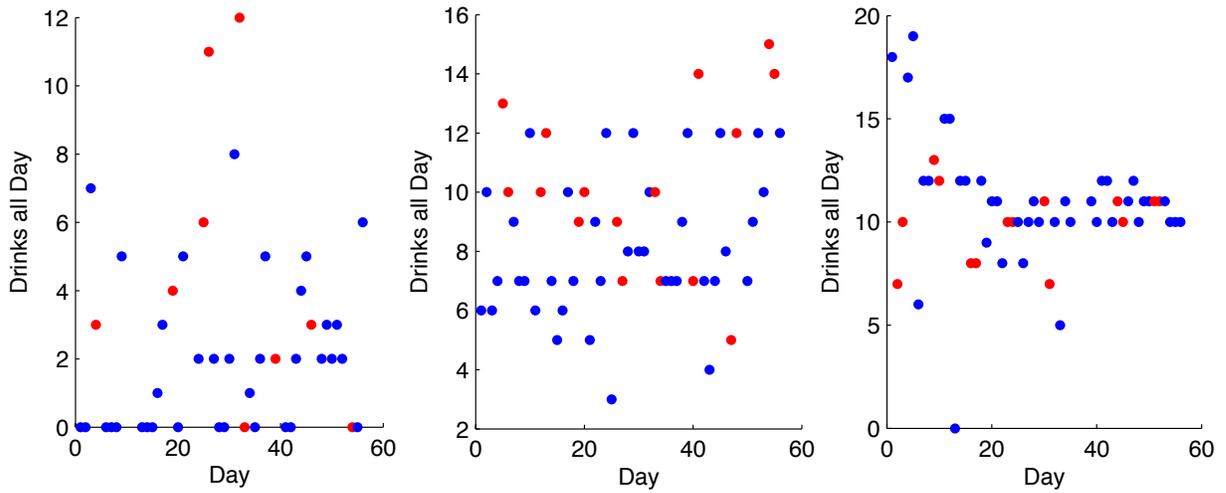


Figure 11: Drinks per day over time for participants 6029 (left), 6072 (center) and 6021 (right). Red dots indicate weekend days, and blue dots indicate weekdays.

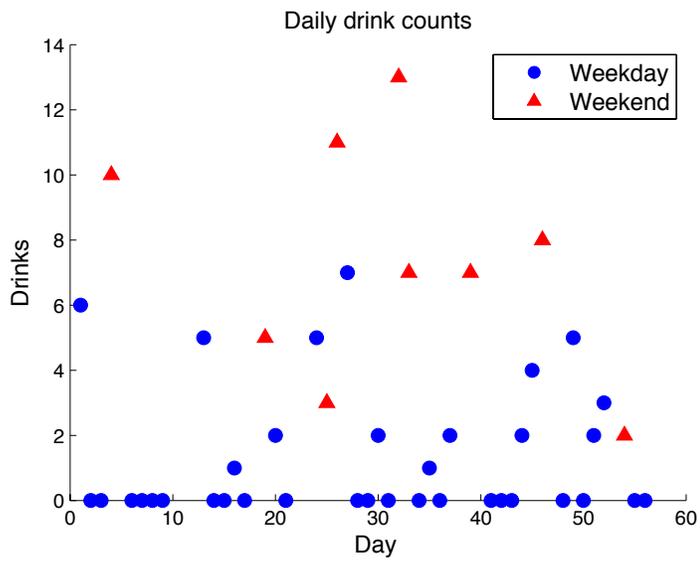


Figure 12: 24 hour drink counts for patient 6029. Red triangles indicate weekend days, blue dots indicate weekdays.

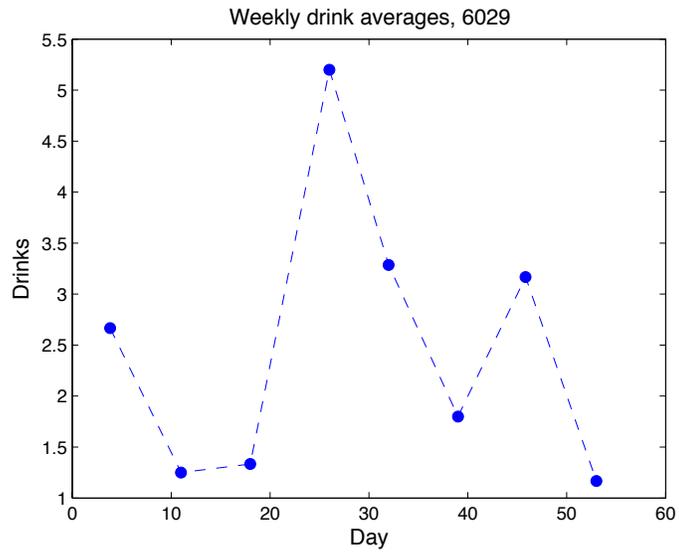


Figure 13: Average drinks per weekly time period for patient 6029.

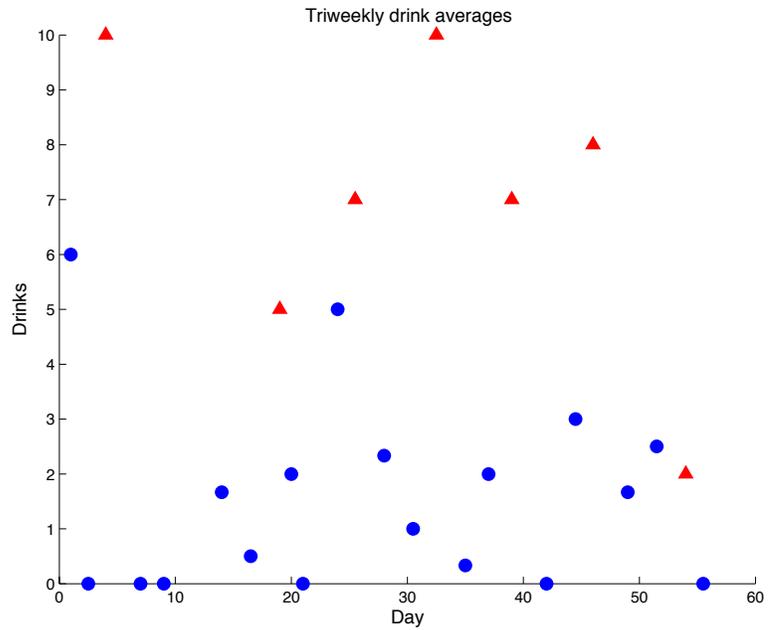


Figure 14: Average drinks per tri-weekly time period for patient 6029. Red triangles indicate weekend days, and blue dots indicate weekday days.

5.2 Linear interpolation methods

Scatter plots, multivariate linear regression, and coefficients of determination are the first tools we used to inform us of relationships between data and also between data categories. We examined scatter plots of time versus an observed data category versus drinks and the two dimensional projections in each direction. Multivariate regression methods help to reveal this same dynamic. Regression models are in the form of

$$f(x, t) = c_0 + c_1t + c_2x + c_3xt, \quad (1)$$

which is an affine transformation if the interaction term xt is considered as its own dimension.

First we examined affine relationships between two data columns – either from the original data set or from the averages across a category – by setting $c_1 = c_3 = 0$ and between a data column and time by setting $c_2 = c_3 = 0$. Since these simple models did not adequately describe trends seen in the data, we turned to the full regression model (1). To judge the strengths of these relationships we utilized the coefficient of determination.

The coefficient of determination r^2 is used to describe the strength of an affine relationship between two observed data categories by formulating an affine model $f(x) = ax + b$ where $a, b \in \mathbb{R}$ and then measuring the difference between the actual values y_i observed at x_i and $f(x_i)$. These differences are squared and sum, and this sum is then divided by the sum of the squared differences between the mean \bar{y} and each y_i , thus normalizing the value of variance explained by $f(x)$ by the total variance seen in the data set of all y_i . The coefficient of determination is calculated via

$$r^2 = 1 - \frac{\sum_{i=1}^n (y_i - f(x_i))^2}{\sum_{i=1}^n (\bar{y} - y_i)^2}$$

where n is the number of data points, x_i is the explanatory variable value, y_i is the corresponding response variable value, and $f(x_i)$ is the predicted response variable value given x_i and the affine transformation f calculated in MATLAB. The determination of explanatory variables and response variable in this procedure is arbitrary and thus r^2 provides no information on the direction of the relationship.

Since we expected that different participants in Project MOTION possess different characteristics, we focused on finding coefficients of determination for individual data sets and calculate r^2 for the categories. We sought strongly ($r^2 > 0.50$) affine relationships and weaker ($0.20 < r^2 < 0.50$) affine relationships. Note we could not account for any relationships that may vary with time or non-affine relationships such as polynomial of order two or greater or exponential relationships.

5.2.1 Interpolation of original IVR data

We focused our search for linear relationships on the IVR data from the previously mentioned 11 participants who decreased their drinking over the course of Project MOTION. These 12 participants had relatively complete data: they each started the survey at least 35 out of 56 times and failed to complete the survey no more than twice. The data of these participants show different trends and the correlation matrices for one dimensional linear relationships for each individual differ greatly, indicating that each of these individuals operate by different underlying mechanics. The average r^2 across individuals, however, indicated that only 3 relationships are strongly affine and only 21 are weakly linear. All of these relationships occurred between questions that are in the same category, therefore affirming that across a number of individuals, category-based variables may account for a sufficient amount of information provided by the full IVR data set. In order to account for direct interactions with time, we also considered the full function (1). In this case we

Table 3: r^2 for each composite score of one individual in relation to number of drinks

Variable	r^2
Events	0.0954
Pressure	0.1506
Mood	0.0215
Stress	0.00002
Desire	0.0673
Guilt	0.1879
Day Number	0.1845

accounted for gradual linear changes in time that may be due to external factors such as treatment. By visual inspection of data we found that patients’ characteristics show different trends over time; however, the operations performed to determine the associated affine transformations are often rank deficient and therefore provide unclear results.

5.2.2 Interpolation of IVR data categories

Examining one composite score versus drinks did not reveal any clear relationships – except for possible relationships between time and drinks and between guilt and drinks. The coefficients of determination calculated using data from one participant is featured in Table 3. Thus we believe that one category’s interactions with a second category (or with time) and number of drinks may have a stronger relationship.

Creating linear transformations using MATLAB’s regress function showed that when the drinks versus composite score data is spread along time, some relationships are stronger and can be represented relatively well by an affine function, as in Figure 15. Some relationships still are weak, indicating that these relationships are not represented well by affine transformations, like in Figure 16. When excluding weekend data from calculating affine relationship models, it was found that some of these static models explained noticeably more variance in the data. In particular, stressful events today, pressure at all times of day, and guilt had stronger relationships as the average r^2 value across participants increased by over 0.05 for each category.

5.3 Principal component analysis

Principal component analysis (PCA) is a multivariate data analysis tool that may be used to transform a set of potentially correlated data objects – or columns in the data set – into a smaller set of uncorrelated data objects via rotations and scaling of the original data. This smaller set of objects maintains a proportion of the information of the original data as determined by the person performing the PCA. We used PCA to determine if the topical data categories appropriately relate to the dynamics of drinking behavior, or if not, what categories may better explain the behavior. Moreover, we hope to begin to realize what unobservable variables might be useful in explaining drinking behavior.

We performed PCA on the IVR response data from the six participants 6002, 6009, 6021, 6029, 6072, and 6090. After the principal components were determined for each data set, the linear combinations that produce the components per data set were compared to detect similarities between the components formed.

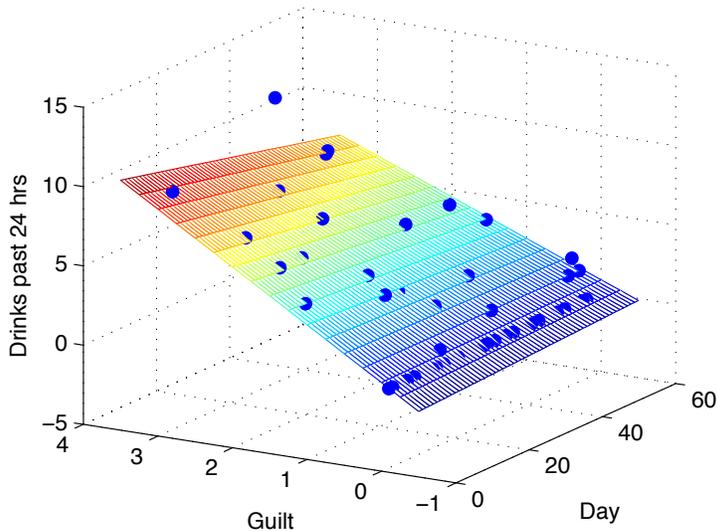


Figure 15: Guilt versus drinks versus time with affine interpolation $A(t) = c_1G(t) + c_2t + c_3tG(t) + c_4$ for one patient.

5.3.1 Mathematical background

Principal component analysis may be interpreted as a rotation of the axes on which the data reside. Namely, before PCA is performed, each column of data is represented by its own axis, and after, each axis attempts to explain the maximal amount of variance in the data. Thus the primary axis will explain the most variance, the secondary will explain the most variance of all the variance not accounted for by the primary axis, and so on. While each component is computed to be a linear combination of all data objects, and a number of components equal to the number of objects are computed, in practice only a select number of principal components are used in this new expression of the data.

The use of PCA requires a few assumptions about the data set $\mathbf{X} \in \mathbb{R}^{m \times n}$ which contains n items (variables) measured in m samples. The primary assumption is that linear relationships exist between the data objects. Before PCA is used, transformations on each variable such as the logarithm may be used to improve linearity between items of \mathbf{X} . There are no distributional assumptions that must be made when performing PCA; however, the calculation of error estimates and confidence intervals of the components and the amount of variance explained by the component typically require that the original data are normally distributed. Additionally, the presence of outliers and missing entries in the data will influence the outcome of PCA. While missing entries may be omitted when performing PCA, it is recommended to either estimate the value using the expectation maximization (EM) algorithm or delete the row containing the missing value.

In order to perform PCA, either the covariance or correlation matrix of \mathbf{X} must first be computed. The use of the covariance matrix is appropriate when data objects are measured in comparable units and the variance between data objects has practical significance. With the use of the covariance matrix, it is assumed that the distribution of each column is mean-centered; namely, that the mean of each column need not be equal to the means of the other columns. It is strongly recommended that each column is centered around mean zero.

The use of the correlation matrix is recommended when data objects are measured in different

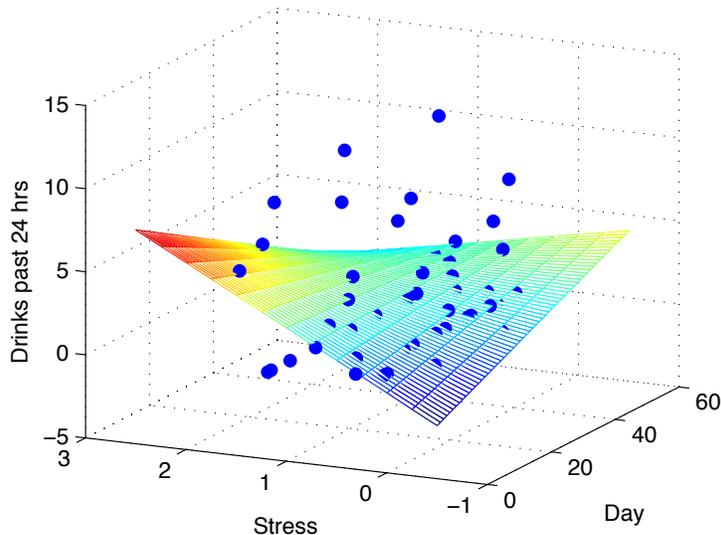


Figure 16: Stress versus drinks versus time with affine interpolation $A(t) = c_1G(t) + c_2t + c_3tG(t) + c_4$ for one patient.

units or when the variances between columns have no practical significance and requires that all columns have been standardized to a mean of zero and variance of one. For example, the covariance matrix may be used when comparing heights of 200 men from each country, however; the correlation matrix must be used when comparing physical, psychological, and economic traits of each of those men, since columns in each of these categories will have different units of measurement. The use of each matrix in PCA will yield different components, as the covariance matrix will lead to components that more strongly reflect the variance between data objects. In practice, the correlation matrix is typically used.

To construct the covariance matrix $\mathbf{C}^{(1)}$, one must compute

$$\mathbf{C}^{(1)} = \mathbb{E}[(\mathbf{X} - \mathbb{E}(\mathbf{X}))^T(\mathbf{X} - \mathbb{E}(\mathbf{X}))],$$

or equivalently,

$$\mathbf{C}^{(1)} = \frac{1}{n-1}[(\mathbf{X} - \mathbb{E}(\mathbf{X}))^T(\mathbf{X} - \mathbb{E}(\mathbf{X}))],$$

where $\mathbb{E}(\mathbf{X})$ is the expected value of each element in \mathbf{X} . Thus computing the covariance matrix automatically centers each data object about its mean. The construction of the correlation matrix $\mathbf{C}^{(2)}$ is similar to that of the covariance matrix $\mathbf{C}^{(1)}$ in that $c_{i,j}^{(2)} = c_{i,j}^{(1)} / (\sigma_i \sigma_j)$, where σ_i and σ_j are the standard deviations of columns \mathbf{X}_i and \mathbf{X}_j , respectively.

The components and the amount of variance they explain is found by performing spectral decomposition on the correlation or covariance matrix \mathbf{C} . This matrix \mathbf{C} is symmetric and positive semi-definite by its construction as a correlation or covariance matrix. Thus we computed the eigenvalue and eigenvector pairs λ_i, \vec{v}_i that satisfy $\mathbf{C}\vec{v}_i = \lambda_i\vec{v}_i$ using singular value decomposition (SVD).

After the eigenvalues λ_i and eigenvectors $\vec{v}_i, i = 1, 2, \dots, n$ of the correlation or covariance matrix \mathbf{C} are found, they are interpreted in the context of the original data set. Each \vec{v}_i contains the weights of the data columns that will be used for the calculation of that component, and the ratio $\lambda_i / \sum_{j=1}^n \lambda_j$ is the fraction of the total variance for which the component accounts. Additionally, if

the correlation matrix were used in the PCA, the ratio $v_{i,j}/\|\vec{v}_i\|_1$ is the correlation of data object j to component i ; if the covariance were used, then the ratio is the covariance between data object j and component i . In both cases, the greater $|v_{i,j}|$ is from zero, the more data object j contributes to the value of component i . Ideally, $v_{i,j}/\|\vec{v}_i\|_1$ would be near ± 1 for a few data objects and would be near 0 for most of the data objects per component, and $\text{sign}(v_{i,j})$ would be the same for these few important data objects. In practice, many components are composed of a number of moderately weighted data objects, in which case a second orthogonal rotation may be helpful.

As noted earlier, not all of the components are included in the final set of components: failure to exclude some of the components is equivalent to no reduction in dimensionality of the data set. Several rules have been developed to determine which components to include, the easiest of which to implement are based on the eigenvalues. Commonly, enough components to account for a set amount of variance as determined by $\lambda_i/\sum_{j=1}^n \lambda_j$ are selected, where common values for this ratio are between 0.80 and 0.90. If the selection is done by hand, then the cutoff may be made near a large jump in the eigenvalues. The common practice is to include all components up to the jump plus the first component following the jump. This is most useful when there are groups of eigenvalues that are close to each other. An additional rule when performing PCA on a correlation matrix is to include all components that have an eigenvalue $\lambda_i \geq 1$. While more formal methods exist to determine the appropriate number of components, they were unnecessary in this exploratory analysis of the MOTION IVR data set.

5.3.2 Numerical implementation of principal component analysis

Principal component analysis is most commonly implemented by performing singular value decomposition (SVD) on the matrix \mathbf{A} , where

$$\mathbf{A} = \mathbf{X} - \mathbf{E}(\mathbf{X}),$$

if we are using the covariance matrix and

$$\mathbf{A} = \left(\frac{\mathbf{X}_1 - \mathbf{E}(\mathbf{X}_1)}{\sigma_1}, \frac{\mathbf{X}_2 - \mathbf{E}(\mathbf{X}_2)}{\sigma_2}, \dots, \frac{\mathbf{X}_n - \mathbf{E}(\mathbf{X}_n)}{\sigma_n} \right)$$

if we are using the correlation matrix, where \mathbf{X}_i is the i^{th} column of the matrix of data \mathbf{X} and $\mathbf{E}(\mathbf{X}_i)$ and σ_i are the expected value of and standard deviation of that column, respectively. Letting \mathbf{C} refer to our covariance or correlation matrix, we then have

$$\begin{aligned} \mathbf{C} &= \frac{1}{n-1} \mathbf{A}^T \mathbf{A} \\ (n-1)\mathbf{C} &= \mathbf{A}^T \mathbf{A}. \end{aligned}$$

Performing SVD on \mathbf{A} yields Σ , \mathbf{U} , and \mathbf{V} such that $\mathbf{A} = \mathbf{U}\Sigma\mathbf{V}^T$, where \mathbf{V} contains the eigenvectors of $\mathbf{A}^T \mathbf{A}$ and $\Sigma = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_n)$ contains the singular values of \mathbf{A} or, alternatively, contains the square roots of the eigenvalues of $\mathbf{A}^T \mathbf{A}$ (note that these σ_i are not the standard deviations of \mathbf{X}). Additionally, \mathbf{U} and \mathbf{V} are orthogonal matrices. Therefore

$$\begin{aligned} \mathbf{A}^T \mathbf{A} &= \mathbf{V}\Sigma^T \mathbf{U}^T \mathbf{U} \Sigma \mathbf{V}^T \\ (n-1)\mathbf{C} &= \mathbf{V}\Sigma^T \Sigma \mathbf{V}^T \\ (n-1)\mathbf{C} &= \mathbf{V}\Sigma^2 \mathbf{V}^T \end{aligned}$$

Therefore the eigenvalues of \mathbf{C} , λ_i , $i = 1, \dots, n$ may be found by $\lambda_i = \sigma_i^2/(n-1)$, and the eigenvectors of \mathbf{C} are contained in the matrix \mathbf{V} .

5.3.3 Results of performing PCA on IVR data

The most effective way to examine trends in the IVR data would be to perform PCA on all or most of a subject's IVR response data. This would potentially enable us to identify any underlying themes in the data that are not captured by the constructed data categories. There are, however, 42 questions on the IVR survey, and so using all of this data simultaneously may lead to obscuring the questions. A second approach of performing PCA on a set of questions under the same topic is illustrated to better understand the relationships between these states. We also performed PCA on the composite scores calculated per each observed data category to better understand any linear relationships between these categories and to identify any underlying forces that may relate these topics.

To better understand how these characteristics also relate to time, we grouped or averaged data with respect to time and performed PCA on the data. In particular, we grouped the data according to week and performed PCA on each week per subject in order to examine how components change each week, and we averaged the data of each week and performed PCA in order to reduce the noise in the system and look at any overall trends that the components may show.

5.3.4 PCA on whole IVR data set

In order to judge the efficacy of categorical groupings to the groupings formed by PCA, we performed PCA on the whole data set from an individual. If PCA provided groupings that explain a large percentage of variability in the data and are strongly influenced by a small number of columns of data, then the PCA groupings could be better than categorical groupings.

In observing the results of PCA, we used three types of graphs. The first, like Figure 17, displays a bar for each component. Each color in a bar corresponds to a particular data column, so wider blocks of color correspond to a heavier weighting of a column of data in the linear combination used to create a component; thus large bars of a single color mean that corresponding column of data is important in the composition of that component. The similar progression through colors in Figure 17 indicates that there are no groupings of particular columns that create useful components: every component is composed of a weighted sum of nearly all columns of data with weights of nearly equal magnitude. The second type of graph, like Figure 18, shows information similar to that of the first type; each bar again represents a component. The blocks of color, however, are sums of the magnitudes of weights for data columns in each topical data category. In this case, PCA was performed on the data set and then the sums of the coefficients were calculated. These categories are time, events last night, events today, mood, perceived stress, desire to drink, commitment to not drink, and evaluation of drinking as excessive (drinking guilt). The third graph, like Figure 20, displays bars representing the proportion of variance explained by each component and a curve representing the sum of the height of the bars, indicating the variance by all components from the first up to the current component. In this case, the first ten components explained less than 70% of the variance in the data set.

Recall that PCA yields information about the composition of the new components and how much variance each component explains. For example, consider subject 6002 (Figure 18). In the first bar, which corresponds to the first principal component, we see that time, the bottom-most section of the bar has a small influence on it as the magnitude of its coefficient is about 0.1. Immediately above time is the portion of the bar representing events last night, for which the sum of the coefficient magnitudes is about 1.5, and above events last night are events today, for which the sum is about 1.1. This indicates that while events last night contribute to the first principal component more than events today, each explains no more than 25% of the behavior of the first

component, indicating that the first component is not closely related to either of these categories. The first principal component itself is only marginally useful in explaining the variance, as it only explained about 15% of the variance of the data set (Figure 20).

Performing PCA on a whole IVR data set yielded no strong information about what underlying components may be influencing a patient’s behavior. For each subject, approximately 13 components were required to account for 80% of the variance in the data, and approximately 20 components were required to account for 90% of the variance. For all participants, no set of questions stood out as strong influences in the first 20 components’ behavior (Figure 17). When computing the correlation $v_{i,j}/\|\vec{v}_i\|_1$, where i is the principal component and j is the original data object, the magnitude of these values rarely exceeded 0.07 and never exceeded 0.11. When the magnitude of the coefficients were summed by topic, it was apparent that no topic contributed excessively to any of the first 20 components, as well (Figures 18,19). Additionally, no component contributed greatly, or explained more than > 0.20 of the total variance, to the variance of the data set (Figure 20).

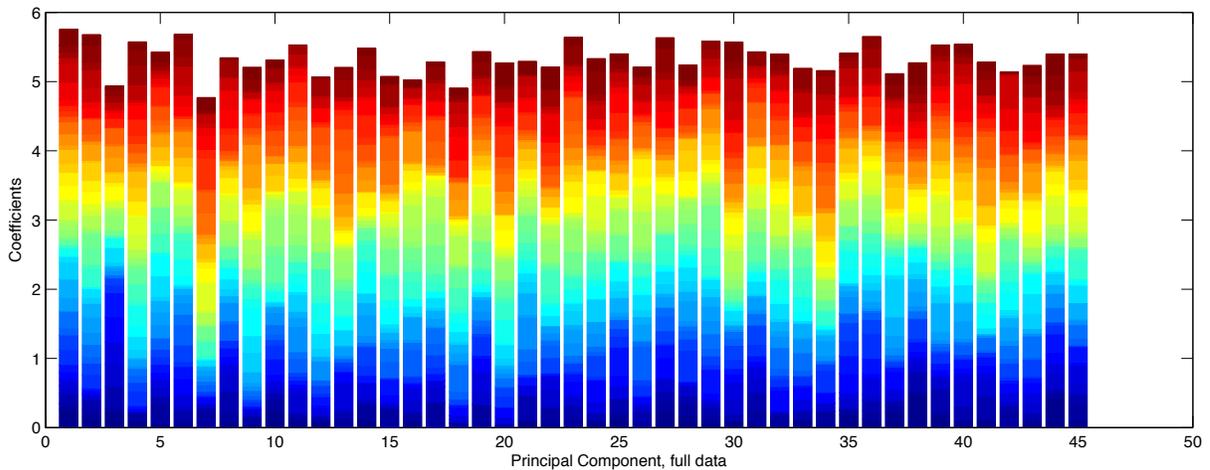


Figure 17: Stacked bar chart of the absolute value of coefficients per component for subject 6002.

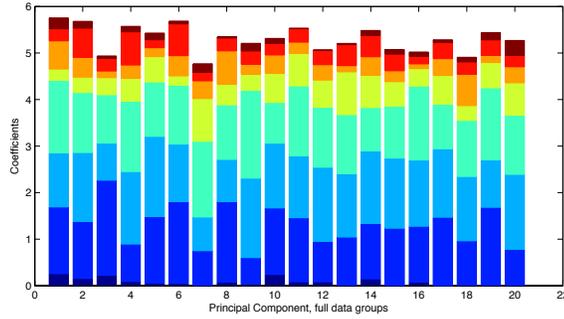


Figure 18: Topical sum of the magnitudes of coefficients of PCA performed on data columns for the first 20 components of subject 6002. From bottom to top are time, E_l , E_t, M , S , C_d , C_c , and C_g .

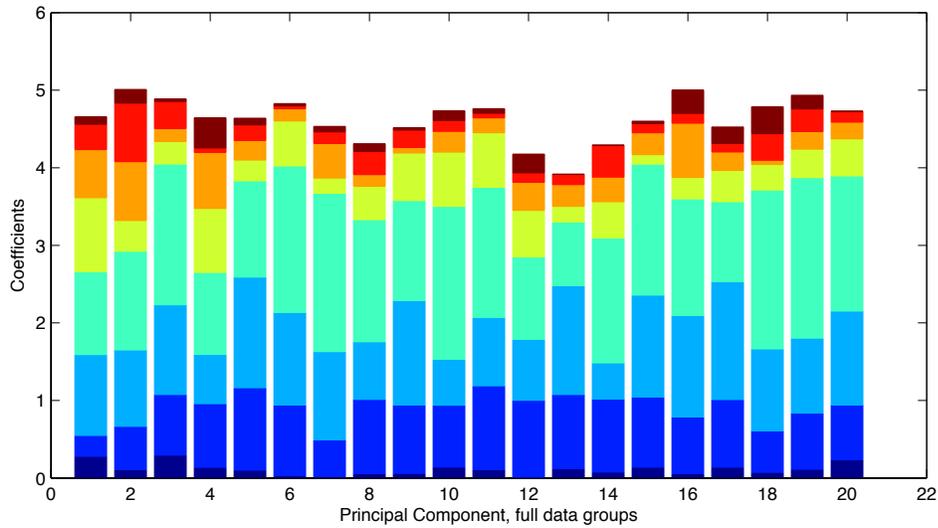


Figure 19: Topical sum of the magnitudes of coefficients of PCA performed on data columns per the first 20 components for subject 6072. From bottom to top are time, E_l , E_t, M , S , C_d, C_c , and C_g .

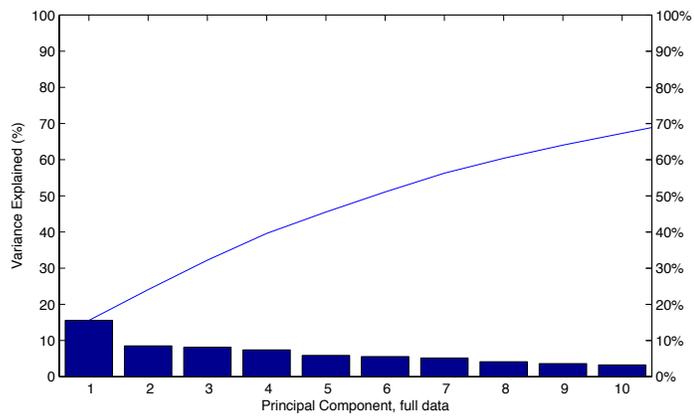


Figure 20: The individual (bar) and cumulative (line) variance explained by the first 10 components of PCA performed on the data for subject 6002.

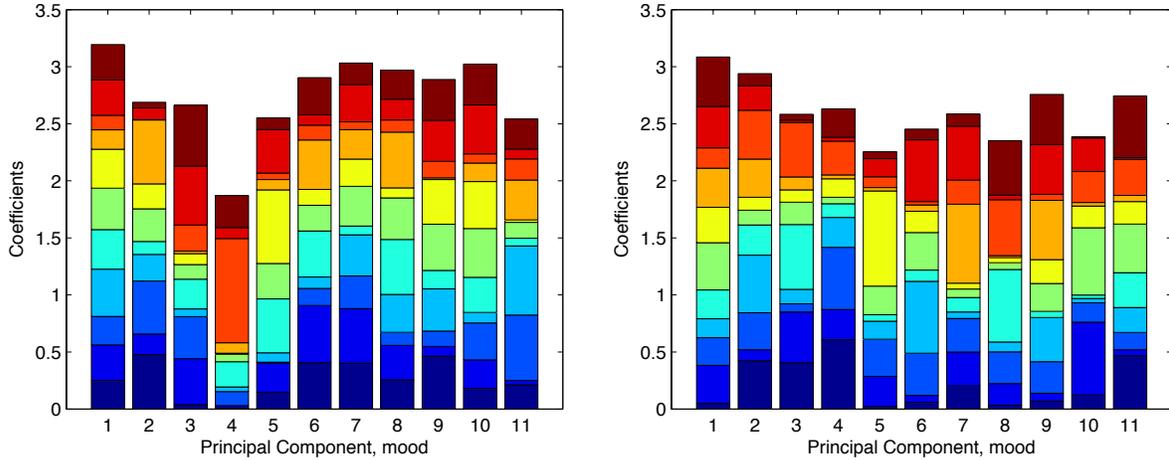


Figure 21: Coefficient magnitude of mood principal components of subjects 6009 (left) and 6029 (right).

5.3.5 PCA on mood information

One of the areas in which we were having the greatest difficulty finding observable data categories is mood. While some mood-based composite scores were created using the IVR data set, we were unsure if they captured enough of the dynamics of mood. Performing PCA on the mood data of a small sample of subjects revealed that about 30% of the variation in mood was explained by an evenly weighted sum of the columns of mood data per subject (Figure 21). No other notable trends in the PCA of the mood data were discovered, and about five or six (of a maximum of 11) components were required to account for 80% of the variation of mood, indicating that while the first principal component performs relatively well, the following components are not as remarkable. This indicates that the moods do not fall into cleanly defined components when PCA is performed on only the mood data and so PCA did not create any mood groupings that were more informative than or supplemental to an average of the mood data.

5.3.6 PCA on data categories

With the ungrouped data, it is likely that principal component analysis did not perform well because there are such a great number of columns in the data set, almost all of which vary on a very small discrete scale. After consolidating the data into the observable data categories formulated earlier, we utilized PCA in order to detect any underlying mechanics that relate two groups together.

Principal component analysis, however, also performed poorly on the observed data categories. The first principal component accounted for approximately 20% to 30% of the variance in the data; however, the makeup of this component did not point to any trends other than a rough average of all topics (Figure 22). Additionally, the second and following components did not show any remarkable patterns that strongly related only a few categories together. Each of these components also only accounted for 10% or less of the remaining variance. In all, approximately seven of the twelve components were required to explain 80% of the variance in the data. Performing PCA on the categorical groupings did not reveal any relationships that could be useful when formulating a dynamical system model using variables based on those categorical groupings.

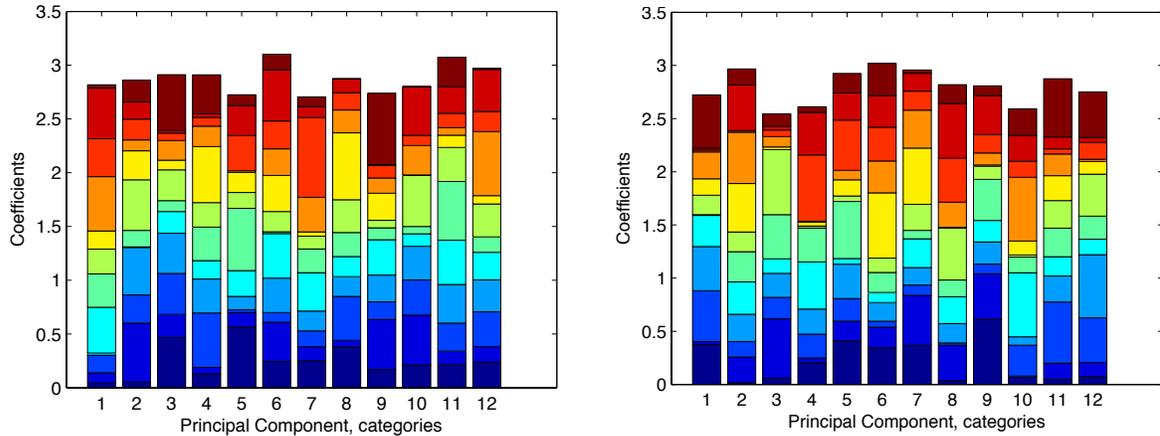


Figure 22: Coefficient magnitude of category principal components of subjects 6029 (left) and 6090 (right).

5.4 PCA by week per subject

We also examined the results of principal component analysis of each week of data per subject in order to see some effects of time on the principal components in a less direct way. It is very important to note that since we reduced our $m \times n$ data matrix to no more than a $7 \times n$ matrix, PCA yields no more than 7 nonzero eigenvalues, and hence we found a maximum of 7 components. Therefore, while only a small number of components are found, they describe all the variance in the small data set. Even though seven eigenvectors (components) were computed, some of these had a corresponding eigenvalue of 0, indicating that these components do not explain any of the variance in the data set. In these data sets the number of drinks consumed in the past 24 hours are included as an additional column so that the components' relationship to drinks may be explored. Therefore the categories are time, events last night, events today, mood, perceived stress, desire to drink, commitment to not drink, evaluation of drinking as excessive, and number of drinks in 24 hours.

When PCA was performed on these smaller data sets of different weeks, the composition of the components changed from week to week (Figures 23, 24, 25). These changes, however, had few patterns between subjects. Additionally, each component was only weakly influenced by the number of drinks (top rectangle in all bar charts), indicating that there was only a weak linear relationship between each component and number of drinks.

An additional concern was that there are a high number of columns of zeros when the data was divided into weeks, so these columns had no dynamics that could contribute to the creation of the component. In performing PCA, the coefficients assigned to these columns were zero. This led to components that did not accurately reflect the true dynamics of the subject's behavior for the week, as that column may truly have had a large impact on the true behavior of the component, except that column happened to be equal to zero for all seven days.

5.5 PCA on weekly averages per subject

In order to reduce the amount of daily variability in subject information and to locate any longer-term linear relationships that may be exhibited by the data, principal component analysis was tried on the weekly averages of patient data. As the IVR survey occurred over 8 weeks, this method produced a maximum of 8 principal components, some of which may not explain any variance in

the data set. When performing PCA on the data sets of the six subjects of interest, we found that the first component explained approximately 30% to 40% of the variance in the reduced data sets, and that the second component explained about 15% to 20% (Figures 26, 27). Since these data sets lacked columns that are all zeros, the dynamics reflected by the principal components more closely reflected the results of the PCA performed on a full data set, and like the results of the full data set, these components also did not show any unique trends regarding columns of data and components.

When the weekly summaries of each subject in the responder and non-responder groups were concatenated for PCA, the results again reflect those of when PCA was performed on a subject's full data set; namely, that no particular data columns contributed exceptionally to any of the first ten principal components and that no components explained more than 30% of the variance of the data.

While principal component analysis may be useful in some situations to reduce the dimensionality of a data set, in the case of the MOTION IVR data set there are no clear linear relationships between any data items. It is expected that if we were to perform a nonlinear transform on the data, such as taking the log or raising the data to a power, we would also not see any strong trends due to the interrelatedness of different columns and the importance of time in these relationships. When we performed PCA on a full IVR data set, none of the first twenty components showed any clear relationship with any columns of data, and a large number of components were required to explain even 80% of the variance in the data set. Similarly, PCA did not show any strong patterns between the observed data categories. These results indicated that linear relationships did not explain the dynamics between columns of the data well.

In using PCA to help find important components in the mood category of questions, we found that an average of all questions preserved the most information, also indicating that there are not any particular subgroupings of these questions that better maintained information of these questions. However, this average still only preserved about 30% of the mood information, indicating that while it may be the strongest component when considering linear relationships, subgroups may still be more useful when constructing a nonlinear model. PCA may also be used on other topical groups of data to affirm or perhaps find improvements to our current groupings, but has not been explored.

In dividing the data per week, we found that components vary over time; however, we could only calculate a small number of components due to the size of the weekly data sets. Additionally, there were a high number of strictly zero columns in these data sets, which artificially impacted the components that were found. If the data from each week were averaged, the result was similar to that of performing PCA on a full data set, thus yielding no new information. This supported the idea that a static linear model does not adequately describe the dynamics in the MOTION IVR data set, and in fact PCA did not help us interpret the data set in a meaningful way. Since static linear models did not fit the data well, and since graphing had shown the great amount of variability in the data set, we decided that a nonlinear static model also would not sufficiently explain the data. Therefore we continued with investigating a dynamical model.

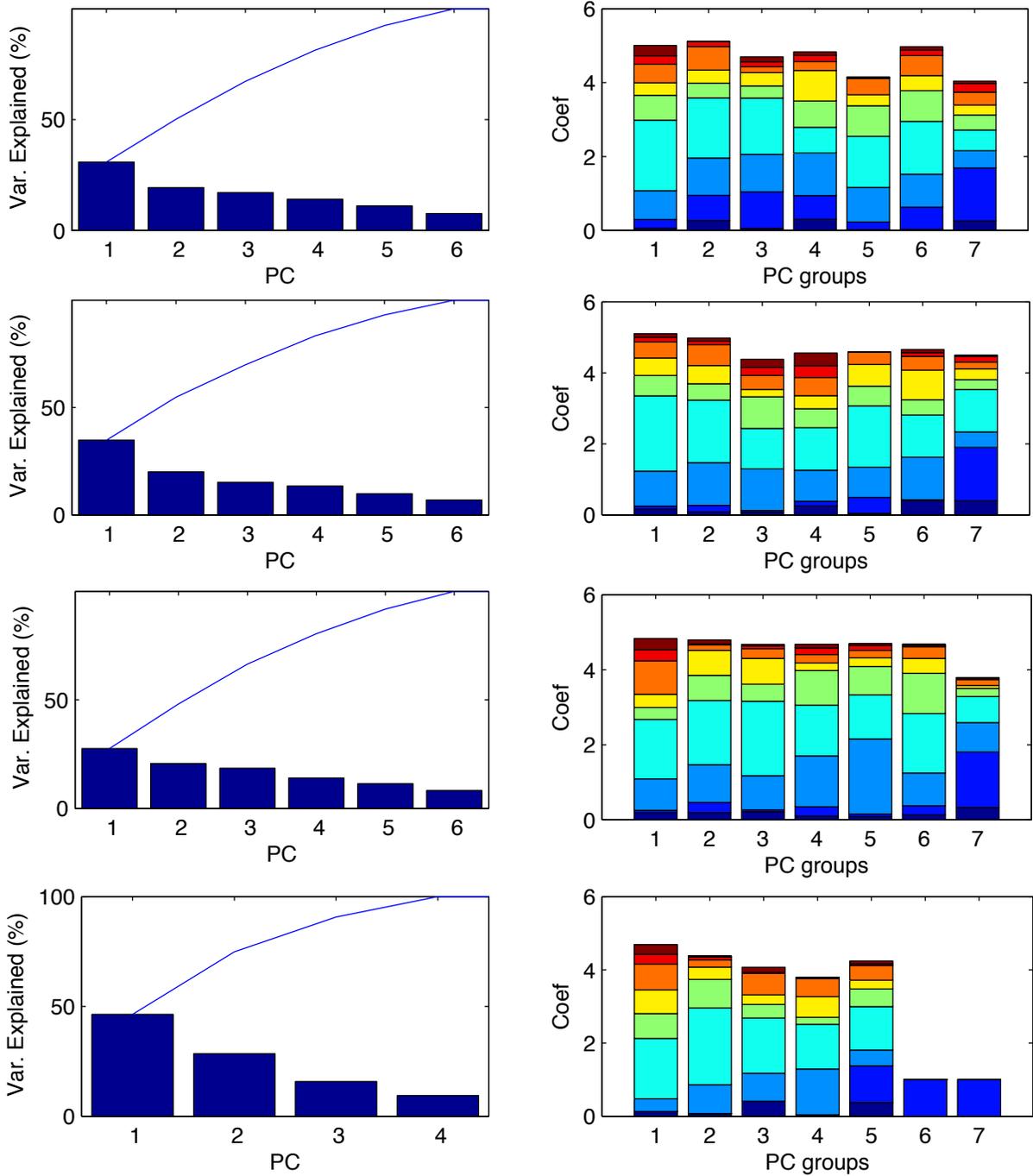


Figure 23: The variance explained by the components (left) and the coefficient group magnitude of the first seven components (right) for subject 6009 for weeks 1, 3, 6, and 8, (top to bottom).

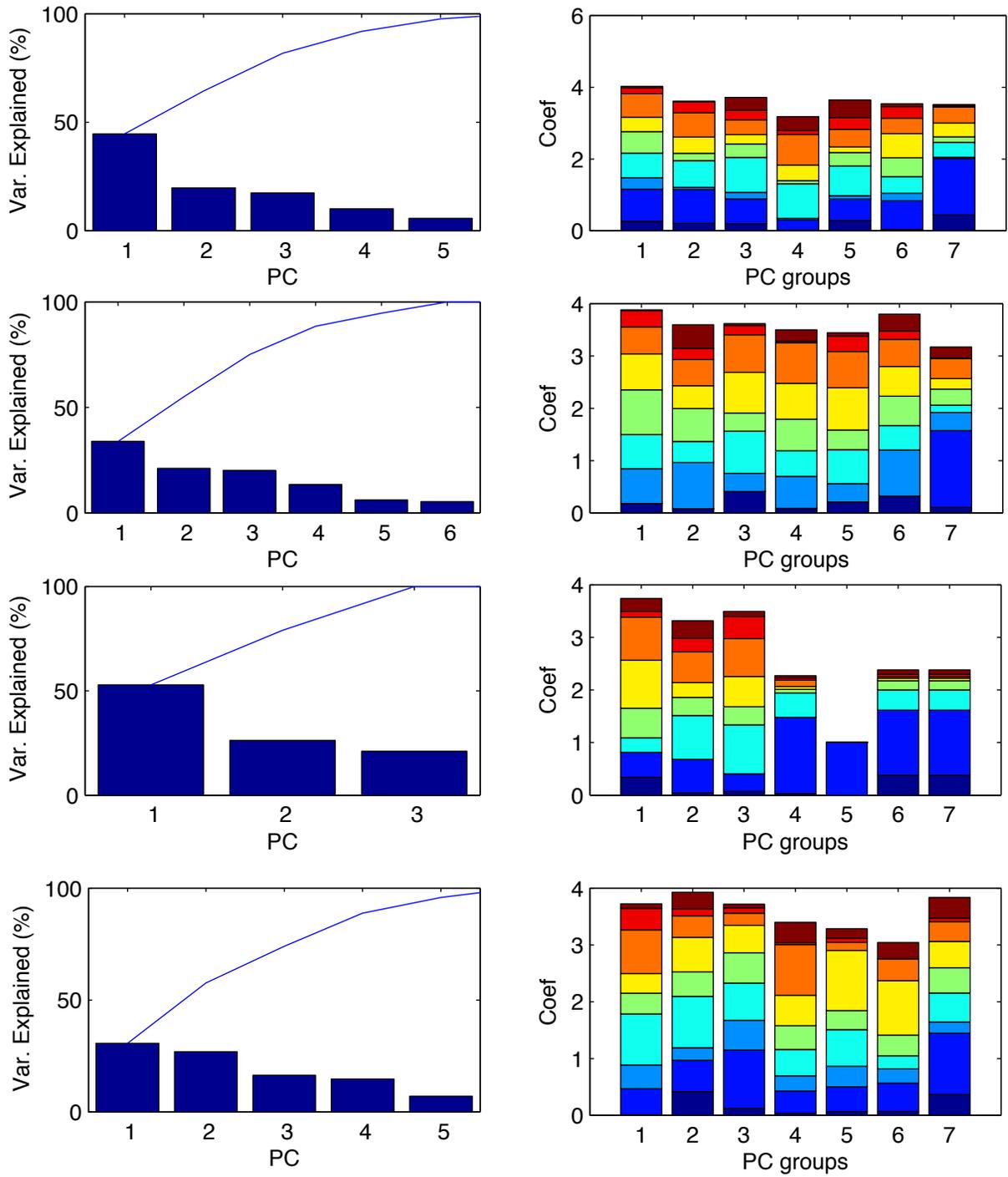


Figure 24: The variance explained by the components (left) and the coefficient group magnitude of the first seven components (right) for subject 6021 for weeks 1, 3, 6, and 8, (top to bottom).

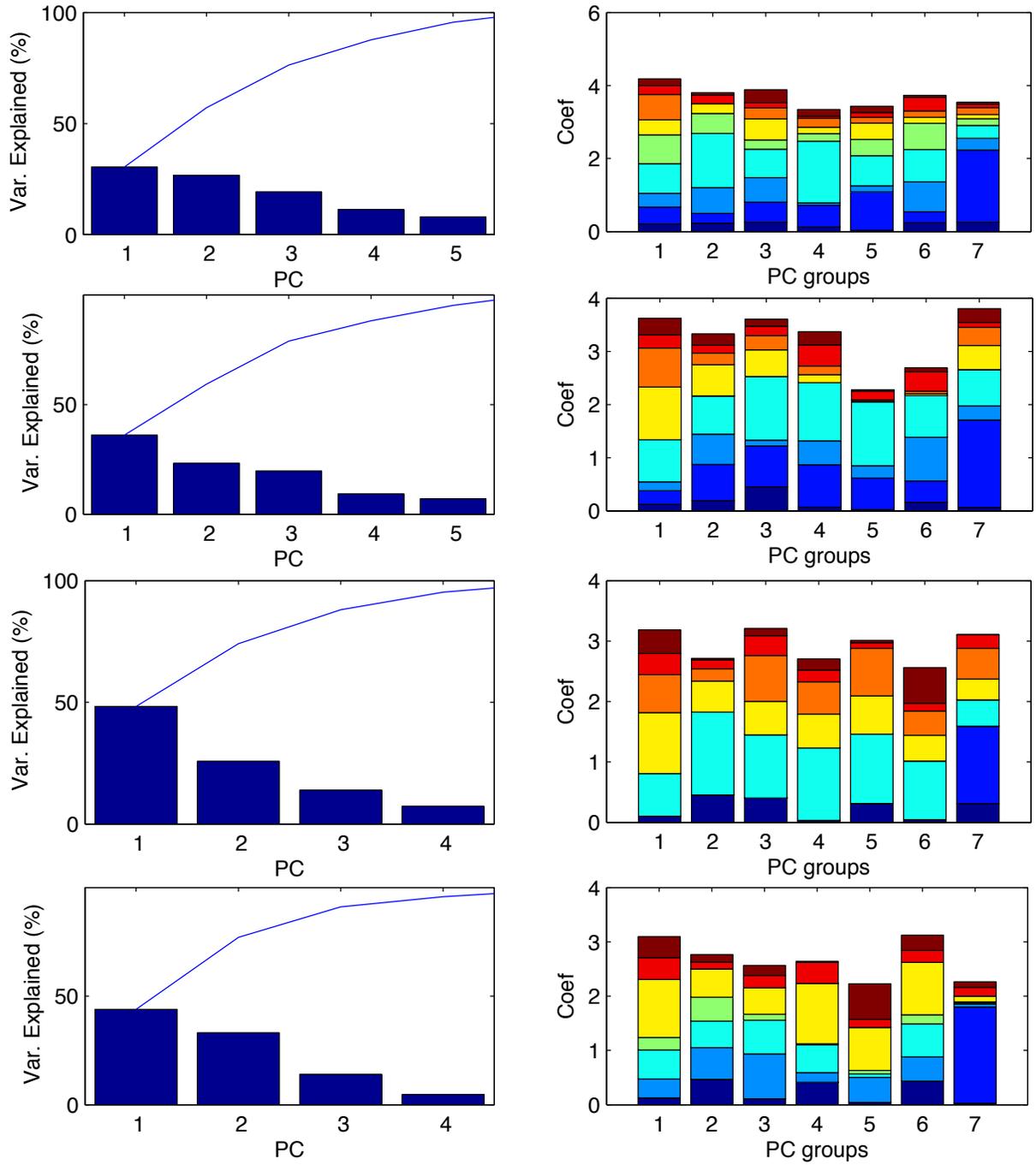


Figure 25: The variance explained by the components (left) and the coefficient group magnitude of the first seven components (right) for subject 6072 for weeks 1, 3, 6, and 8, (top to bottom).

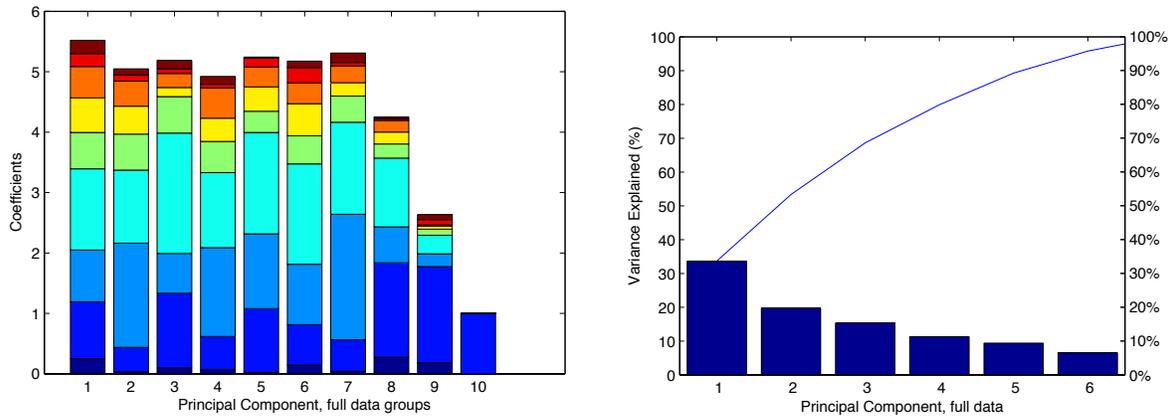


Figure 26: Topical grouping magnitude of principal components of subject 6009 (left) and corresponding chart of percent variance explained per first six components (right).

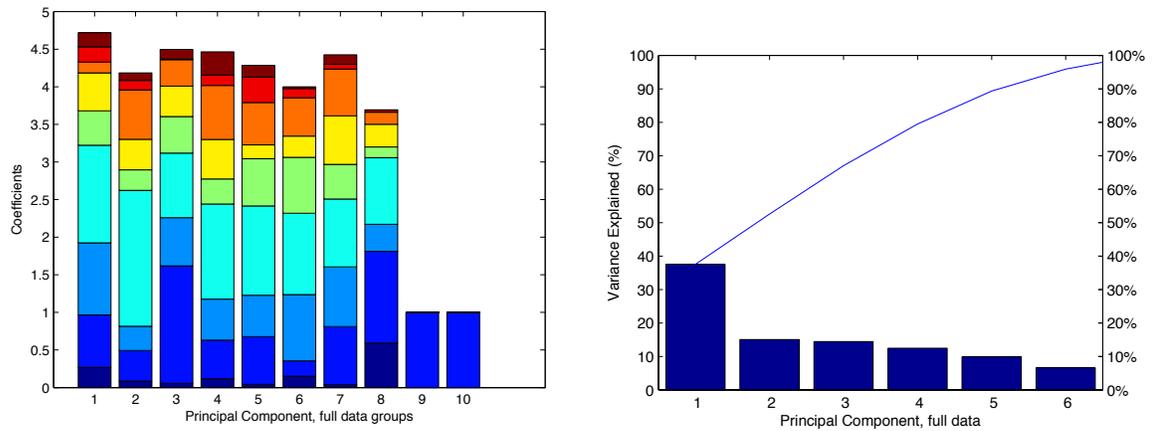


Figure 27: Topical grouping magnitude of principal components of subject 6021 (left) and corresponding chart of percent variance explained per first six components (right).

5.6 Factor analysis

After principal component analysis was performed on the data, it was determined that the relationships between data columns and topical categories cannot be explained by a static linear model, and it is unlikely that a non-dynamic model will be able to appropriately explain these relationships. An additional method by which we could examine the data is factor analysis. The Columbia group has used this in some exploratory analysis of the MOTION IVR data; and therefore it was decided between the NCSU and Columbia groups that it might be of use to perform some rudimentary analysis.

In principal component analysis, the objective is to rotate the axes that represent our data columns such that these new axes, or components, better explain the overall variance in the data set. The components are formed by linear combinations of the data columns. The expected result is that the directions of highly correlated columns tend to be grouped together to form one of the new axes. Then a smaller number of axes are chosen so that a relatively high proportion of the variance is maintained while reducing the dimensionality of the data set.

In factor analysis, however, we aim to identify underlying common factors that explain the behavior of our observed processes (our data columns). Namely, we construct static linear combinations of these factors that, with the addition of some observation noise, will yield the original data sets with which we were working. Thus we want to reduce the dimensionality and redundancy of the data.

Given the observation vectors \vec{y}_i of length n , where i corresponds to the day of the IVR survey and $1 \leq i \leq 56 = m$, and assume that these vectors have mean $\vec{\mu}$ and covariance matrix Σ . Then, suppressing the index of the observation vector, each of these vectors may be expressed using the p factors f_1, f_2, \dots, f_p and the observation-level noise ϵ_j , $1 \leq j \leq n$, where j indicates the position in the observation vector and moreover the data column of a participant's data set, by

$$\begin{aligned} y_1 - \mu_1 &= \lambda_{1,1}f_1 + \lambda_{1,2}f_2 + \dots + \lambda_{1,p}f_p + \epsilon_1 \\ y_2 - \mu_2 &= \lambda_{2,1}f_1 + \lambda_{2,2}f_2 + \dots + \lambda_{2,p}f_p + \epsilon_2 \\ &\vdots \\ y_n - \mu_n &= \lambda_{n,1}f_1 + \lambda_{n,2}f_2 + \dots + \lambda_{n,p}f_p + \epsilon_n \end{aligned}$$

where each $\lambda_{j,k}$ is called a loading (loadings are typically represented by λ in literature) and for each observation vector, $\lambda_{1,k}, \lambda_{2,k}, \dots, \lambda_{n,k}$ describe the amount of influence factor f_k has on each of the n elements of the observation vectors. Naturally, this system of equations may also be expressed as the matrix equation

$$\vec{y} - \vec{\mu} = \Lambda \vec{f} + \vec{\epsilon}.$$

Since we want the common factors to each describe unique information (or be orthogonal), we assume that the covariance matrix of all of the f_k to be the identity matrix. Also, since $E(y_j - \mu_j) = 0$ for all j , $E(f_k) = 0$ for all k . Additionally, as each ϵ_j represents observation noise, $\text{cov}(\epsilon_{j_1}, \epsilon_{j_2}) = 0$, $j_1 \neq j_2$ and we assume $E(\epsilon_j) = 0$ and $\text{cov}(\epsilon_j, f_k) = 0$. We shall denote $\text{var}(\epsilon_j) = \psi_j$ and $\text{cov}(\epsilon_j) = \Psi$. These ψ_j are known as the specific variances. With these assumptions, we may estimate the variance of each of the elements in the observation vector by

$$\text{var}(y_j) = \lambda_{j,1}^2 + \lambda_{j,2}^2 + \dots + \lambda_{j,p}^2 + \psi_j.$$

In matrix form, this may be written

$$\begin{aligned}
\Sigma &= \text{cov}(\Lambda \vec{f} + \vec{\epsilon}) \\
&= \text{cov}(\Lambda \vec{f}) + \text{cov}(\vec{\epsilon}) \\
&= \Lambda \text{cov}(\vec{f}) \Lambda^T + \text{cov}(\vec{\epsilon}) \\
&= \Lambda \Lambda^T + \Psi.
\end{aligned} \tag{2}$$

By this relation (2), it may be shown that $\text{cov}(y_j, f_k) = \lambda_{j,k}$, or $\text{cov}(\vec{y}, \vec{f}) = \Lambda$. It is important to note that since we only know sample covariances, these relations are typically not true; however, the use of factor analysis requires that we assume that these relations are true. The calculations of Λ and Ψ are unique up to a rotation.

Factor analysis has several methods of calculation. The two that will be examined are the principal component method, as it is one of the simplest methods, and the maximum likelihood method, as it is utilized in the factor analysis code, factoran, in MATLAB.

In the principal component method, the matrix Ψ is ignored, simplifying (2) to $\Sigma = \Lambda \Lambda'$. We perform spectral decomposition on Σ and find the eigenvalue and eigenvector pairs θ_l, \vec{v}_l such that $\Sigma \vec{v}_l = \theta_l \vec{v}_l$ using singular value decomposition, since this simplified formulation of Σ implies that Σ is symmetric and positive semi-definite. We then determine our desired p , which is less than n , and chose the first p pairs θ_l, \vec{v}_l to form Λ .

Assuming that our observations are normally distributed, we may use the maximum likelihood method to determine Λ and Ψ . Using (2), we may determine that $\Sigma \Lambda \Psi = \Lambda (\mathbf{I} + \Lambda^T \Psi^{-1} \Lambda)$, and $\Lambda^T \Psi^{-1} \Lambda$ is a diagonal matrix. This equation is then solved by finding the maximum likelihood estimate of Λ .

Choosing an appropriate p is much like choosing an appropriate number of principal components in principal component analysis. Two of the simplest methods involve the maintenance of an appropriate amount of dynamics in the data. When using the principal component method, we may choose enough factors to account for a proportion of the total variance $\text{trace}(\Sigma)$, where common values for this ratio are between 0.80 and 0.90. An additional rule when performing PCA on a correlation matrix is to include all components that have an eigenvalue θ_l that is greater than or equal to the average eigenvalue. If the selection is done by hand, then the cutoff may be made near a large decrease in the rate of change of the eigenvalues. The common practice is to include all factors up to but excluding the point of this change.

After calculating Λ and Ψ , it is common practice to rotate these axes representing the factors such that the new axes that potentially are more strongly related to some elements of an observation vector and less related to other elements than our originally determined factors. Therefore these new factors are more easily interpreted in the context of our data. Both orthogonal and oblique rotations may be used, and in both cases the amount of variance in the data set that all the factors describe in total remains unchanged.

To describe the relationship between \vec{f} and \vec{y} , we examine the matrix of loadings Λ . Each row of Λ corresponds to the coefficients in the linear combination of factors used to formulate a y_j . Find the loading $\lambda_{j,k}$ of the highest magnitude. If the magnitude is significantly higher than those of the other loadings in the row, and if the magnitude is sufficiently near one, then that factor is considered important in the formulation of that y_j . This process is then repeated for each entry in the observation vector \vec{y} , thus providing us with information on which factors most strongly influence which observation elements. Interpretations are less clear when several or no factors are weighted heavily in the formulation of a y_j .

While factor analysis is potentially a useful tool, it is looked down upon by some statistical communities. Due to the multiple methods by which Λ may be found, there is no standard algorithm

by which factor analysis is carried out, and so the factor analysis performed on two related samples may yield alarmingly different results. Additionally, there is no standard of interpretation. Many loose rules guide the determination of p and the interpretation of results. Finally, factor analysis does not yield unique results. Therefore, when performing factor analysis on the Motion IVR data - or on any data set - it is important to remember that noise in the data will affect results of factor analysis and that these factors are merely constructs until we have evidence to suggest that they may truly exist.

Factor analysis on the IVR data may be implemented via MATLAB's `factoran` command. This implementation uses the maximum likelihood method. In the case of the IVR data, we were unable to perform factor analysis using this method. Namely, the matrix Ψ , the covariance of $\vec{\epsilon}$, is estimated in an iterative process. There is a possibility that a value on the diagonal of Ψ becomes less than or equal to 0, which indicates that the maximum likelihood method is converging to an inappropriate result. This is one of the signs of a Heywood case, which means that the results of `factoran` would not be valid. Two typical scenarios that result in Heywood cases are when the determined p is too large or too small compared to the unknown true number of factors. In the case of the IVR data, it is likely that the non-normal distribution of data was negatively affecting the estimations of Ψ and Λ . Therefore the MATLAB implementation of factor analysis was ineffective for use on the IVR data.

6 Modeling the drinking behavior control system

As static linear models do not appear to adequately explain the patterns shown in the IVR data, we propose the use of a possibly nonlinear dynamical system model which as we shall see below requires delays and cumulative or hysteretic functions. Additionally, our static linear analysis did not show any clear patterns between IVR questions. As noted when explaining the data collected during project MOTION, some of the data provides redundant or closely related information, and thus it is reasonable to base our model variables on the categorical groupings of IVR questions. By using this scheme we may also utilize information from the field of substance abuse and recovery to further inform our model: if our model includes dynamics that are not reflected in literature of the field, then it is likely that our model does not reflect the recovery process. After the model is formulated, we may use techniques to determine the importance of each variable in the model.

It is reasonable to allow the composite scores of the categorical groupings (and hence the variables) to fluctuate on a continuum as opposed to a discrete set of numbers, as a discrete set may not completely reflect how an individual is feeling. For example, a person is not simply happy or sad, there are many strengths at which happiness or sadness may be felt that do not just correspond to five digits. Discrete scales simplify the data that may be gathered from questionnaires and make this data more understandable to a person performing simple analyses; however, when developing a dynamic model, the use of a continuum of values is preferred, and, as can be argued, more reasonable. For convenience, and also because the variables of interest often change in a continuous manner, we will consider the model as a function of continuous time as opposed to discrete time.

Drinking or *rate* of alcohol consumption is the main dependent variable of interest and is denoted by $A(t)$ representing 'alcoholic drinks per unit time at time t '. The other state variables are taken initially as simply the data categories from the IVR data: Desire $D(t)$, Guilt $G(t)$, Limit $L(t)$, Commitment $Q(t)$. The commitment category is represented by the variable Q to remind us that this category is 'commitment not to drink at all for the next 24 hours' or, 'commitment to quit'. This is not, of course, commitment to quit drinking entirely, it is a more extreme statement than

to reduce drinking in the next 24 hours, which is represented by Limit, $L(t)$.

We wish to combine (due to the similarity in questions) and scale data for use in estimation procedures with dynamical models. Let χ_d be the characteristic function indicating if the participant provided data d_j on a particular day j such that $\chi_{d_j} = 1$ if that data was provided and $\chi_{d_j} = 0$ if that data was not provided.

- Desire to drink (Desire, D): The questions that express a desire to drink (numbers 2 and 3) will have positive weight and the question that expresses a lack of desire to drink (number 1) will have a negative weight. Let $d_j^{(2,k)}$, $k = 1, 2, 3$ represent the participant's responses to these three questions, respectively, for each day $j = 1, 2, \dots, 56$. Then the observations and the model have the relationship

$$D(t_j) \approx D_j = \frac{(4 - d_j^{(2,1)}) + d_j^{(2,2)} + d_j^{(2,3)}}{\chi_{d_j^{(2,1)}} + \chi_{d_j^{(2,2)}} + \chi_{d_j^{(2,3)}}}$$

$$D_j = 2 \quad \text{if } \chi_{d_j^{(2,1)}} + \chi_{d_j^{(2,2)}} + \chi_{d_j^{(2,3)}} = 0$$

- Commitment to not drink (Quit, Q): As noted previously, the question regarding the participant's commitment to not drinking at all for the next 24 hours is the only question in this category and the daily responses are denoted by d_j^3 , $j = 1, \dots, 56$. The response to this question is scaled like those in the desire category, and so this data will have the relationship to the model

$$Q(t_j) \approx Q_j = d_j^3$$

$$Q(t_j) = 2 \quad \text{if } \chi_{d_j^3} = 0$$

- Commitment and confidence to limit drinking (Limit, L): The two questions ask about the participant's confidence in being able to resist drinking heavily in the next 24 hours ($d_j^{(4,1)}$) and her commitment to not drinking heavily for the next 24 hours ($d_j^{(4,2)}$). These two questions are averaged to create a composite score that is related to the model variable $L(t)$ by

$$L(t_j) \approx L_j = \frac{d_j^{(4,1)} + d_j^{(4,2)}}{\chi_{d_j^{(4,1)}} + \chi_{d_j^{(4,2)}}}$$

$$L(t_j) = 2 \quad \text{if } \chi_{d_j^{(4,1)}} + \chi_{d_j^{(4,2)}} = 0.$$

- Guilt about drinking (Guilt, G): The responses d_j^5 having values (0) definitely not, (1) possibly, (2) probably, and (3) definitely. This corresponds to the model via

$$G(t_j) \approx G_j = d_j^5$$

$$G(t_j) = 2 \quad \text{if } \chi_{d_j^5} = 0$$

- Alcoholic beverages consumed (Alcohol, A): The alcohol consumption function $A(t)$ describes the rate at which the participant is consuming alcoholic beverages and has units of drinks/time. Note that the 'units' of the information collected are drinks. It is useful to think of the alcoholic beverage data $d_{2i}^{(1)}$ as the number of alcoholic beverages consumed over

the daytime $t \in (t_{2i-1}, t_{2i})$ and $d_{2i-1}^{(1)}$ as the number of beverages consumed the preceding night $t \in (t_{2(i-1)}, t_{2i-1})$, where $i = 1, 2, \dots, 56$ are used to indicate half-day periods. Thus

$$d_{2i-1}^{(1)} \approx \int_{t_{2(i-1)}}^{t_{2i-1}} A(s) ds,$$

$$d_{2i}^{(1)} \approx \int_{t_{2i-1}}^{t_{2i}} A(s) ds,$$

or, in more general terms, integrating [drinks/time] with respect to time yields a number of drinks. Note that $d_{2i-1}^{(1)}$ and $d_{2i}^{(1)}$ are replaced by the participant’s average number of drinks in a night and day for the week, respectively, if $\chi_{d_{2i-1}^{(1)}} = 0$ and $\chi_{d_{2i}^{(1)}} = 0$.

6.1 Conceptual models

Concentrating on the patients selected to be ‘responders’ as determined by the Columbia team from TLFB data, we attempted to determine relationships among data categories for individual level models. The following is a discussion of relationships hypothesized from visual qualitative assessments of longitudinal plots for each patient. We selected 4 patients (6029, 6009, 6124, 6083) from this initial group as they reported with relatively high frequency to the IVR system.

In general, whether categories were included depended entirely on whether the category appeared to be directly related to drinking (represented as daily averages over the 2 or 3 day time period). Relationships were considered among the variables that were initially selected as being pertinent to the preliminary categorical model. We did not discuss possible relationships among variables that did not appear to be drinking related, and therefore were not considered in these categorical models. So, it could be that some of the variables are related to each other but if each of them is not related to drinking, that relationship is not represented in the current categorical model. In all of the depictions of these categorical models, ‘strong’ relationships are represented with solid lines and ‘weaker’ relationships are represented with dotted lines. Arrows indicated causality or direction. For example, in Figure 29, all relationships appeared bidirectional – drinks consumed appeared to influence one’s desire, and desire also influenced the drinks consumed. The timescale used here is termed ‘triweekly’ since there are 3 time points per week: weekend (Friday night through Sunday prior to IVR call), Sunday and Monday, and Tues through Thurs.

Patient 6029

We initially chose to focus on this patient since, upon first assessment, it appeared that their drinking decreased during the time period in a seemingly systematic way. That is, there appeared to be some heavy drinking initially that the individual gradually reduced so that there was consistently and noticeably less drinking at the end of the observation period. Also, we observed this individual controlled their drinking better during the week and less during the weekend, which we think is typical of most problem drinkers before they develop dependence.

The representation of the categorical model can be seen in Figure 29. The graphs from which we decided there are noticeable relationships are shown in Figure 30. All relationships for this individual seemed to be relatively strong when present. The plots of relationships among the categories are shown in Figure 31. There may be a relationship between pressure and drinks (Figure 33), for roughly the first 30 days, before the patient is really showing signs of recovery. This could be interesting to explore in the future, but for now will likely be left out of a model.

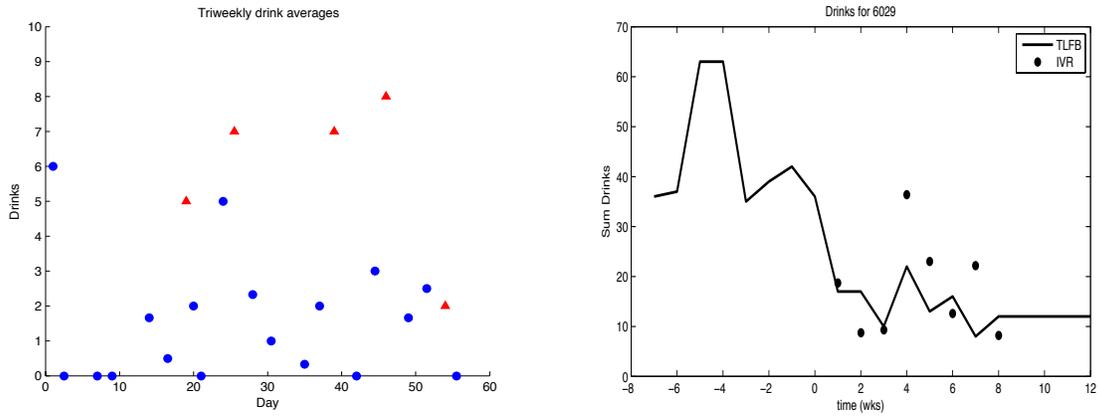


Figure 28: Left: Patient 6029's drinking as reported by IVR triweekly. Right: Drinking as reported by TLFB and IVR. Number of IVR days recorded for weeks 1 through 8 are: 6,4,6,5,7,5,6,6

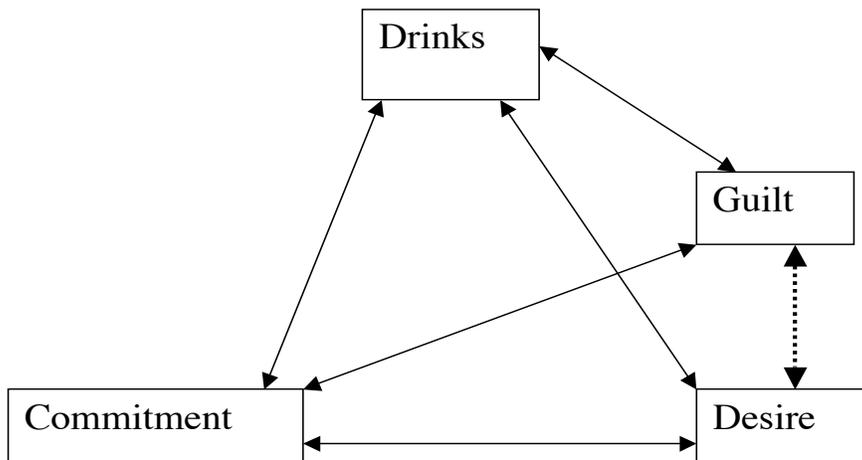


Figure 29: Categorical model for patient 6029 based on hypothesized relationships in the IVR data. 'Strong' relationships are represented by solid lines, and 'weak' relationships are represented by dashed lines. All relationships are taken to be bidirectional.

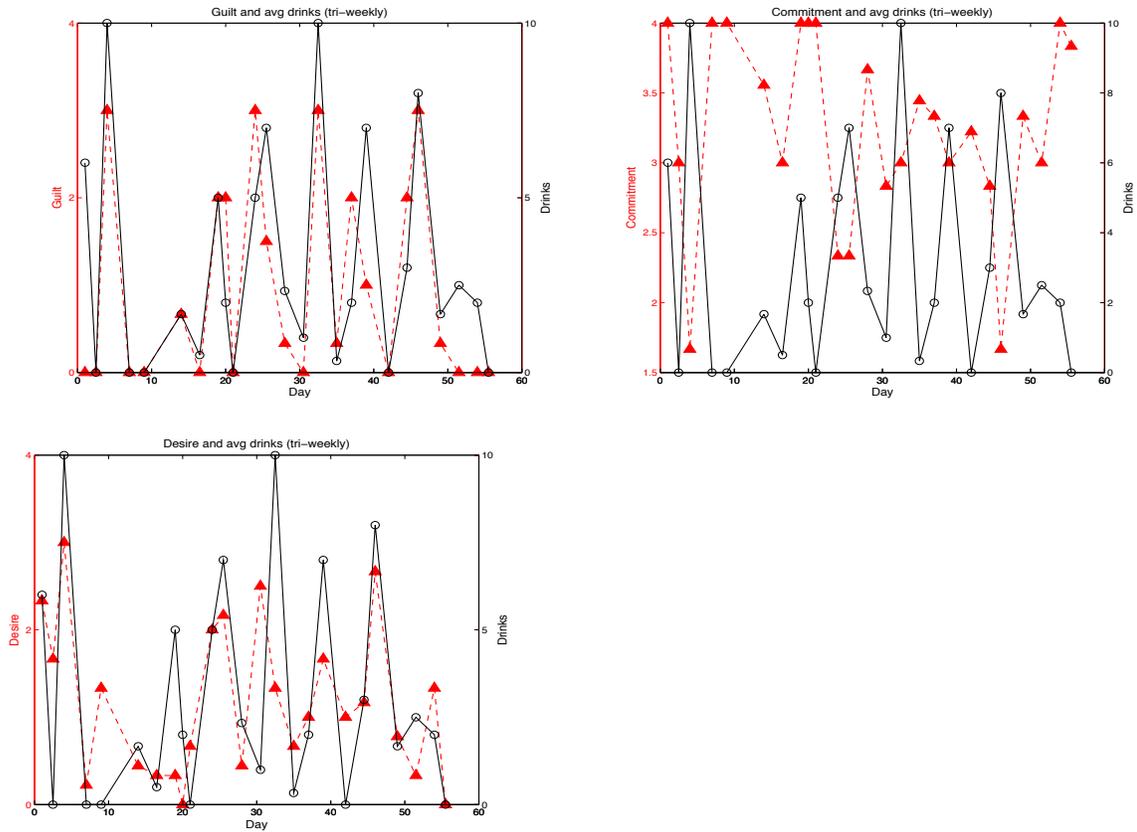


Figure 30: Categories thought to be (strongly) related to drinking for patient 6029. Triangles denote the IVR data categories, and circles represent the drinks (per day averaged over the 2 or 3 day time period).

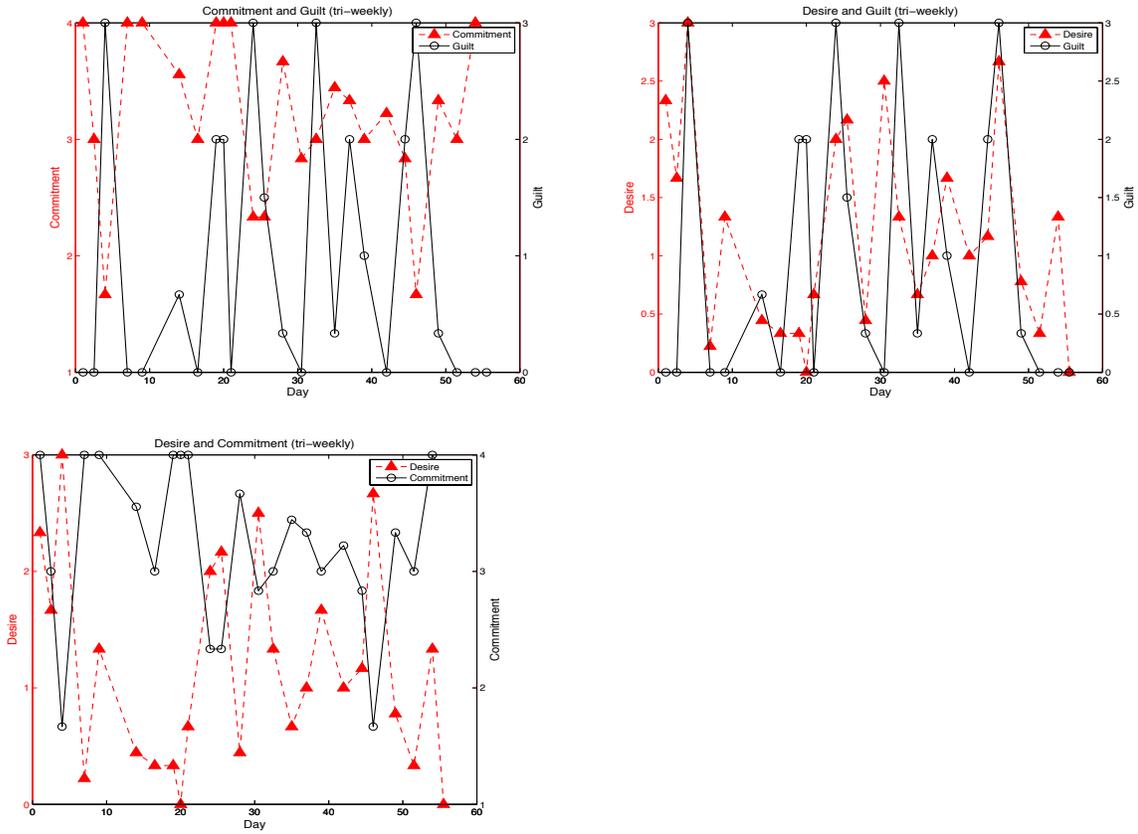


Figure 31: Hypothesized category-category relationships for patient 6029. All relationships are thought to be strong with the exception of a weak relationship between guilt and desire.

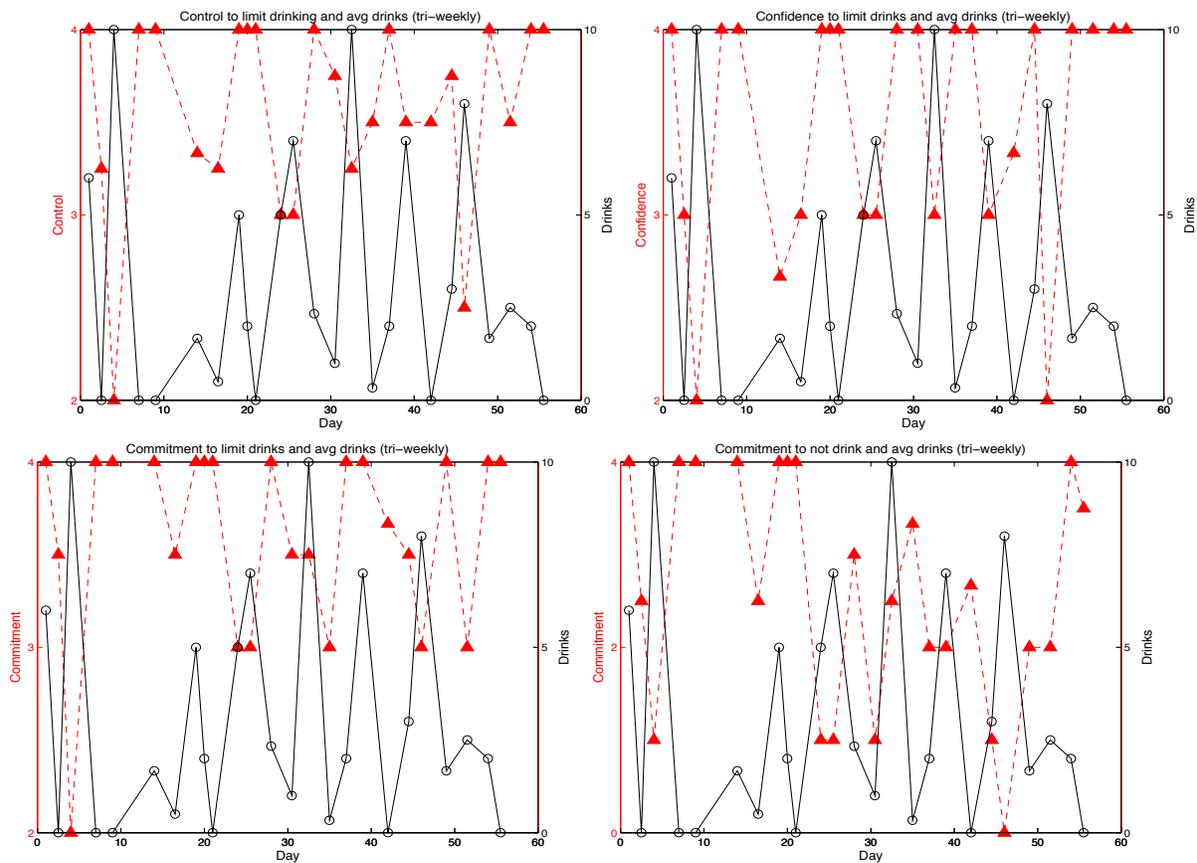


Figure 32: Metrics of confidence and commitment to limit drinking for patient 6029. Control (upper-left) is the average of commitment to not drink heavily (lower-left) and confidence to not drink heavily (upper-right). The lower-right graph shows commitment to not drink at all.

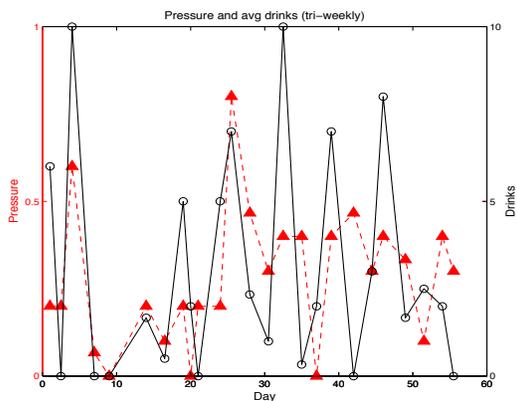


Figure 33: Potential weak time-dependent (for the first 30 days, roughly) relationship with drinking.

Patient 6009

This patient and the other 2 patients were selected primarily because of their frequency in IVR reporting. However, a clear and methodical reduction in drinking seems to have occurred in the first 20 days of treatment. The drinking (shown in Figure 34) appears to decrease by a few drinks every couple days, not just a quick drop-off so hopefully there will be some dynamics indicative of behavior change that can be observed during that time period. There were quite a few categories that showed similarities to the patterns in drinking behavior (Figure 35) although not all appeared to be strongly related as with 6029. The plots of each category and drinks can be seen in Figures 36 and 37. There were 2 possible weak time-dependent relationships, that between stress-drinks and stress-mood (Figure 40) after the first 30 days, that could potentially be considered in a more sophisticated model. The data for categories between which relationships are thought to exist are shown in Figure 38.

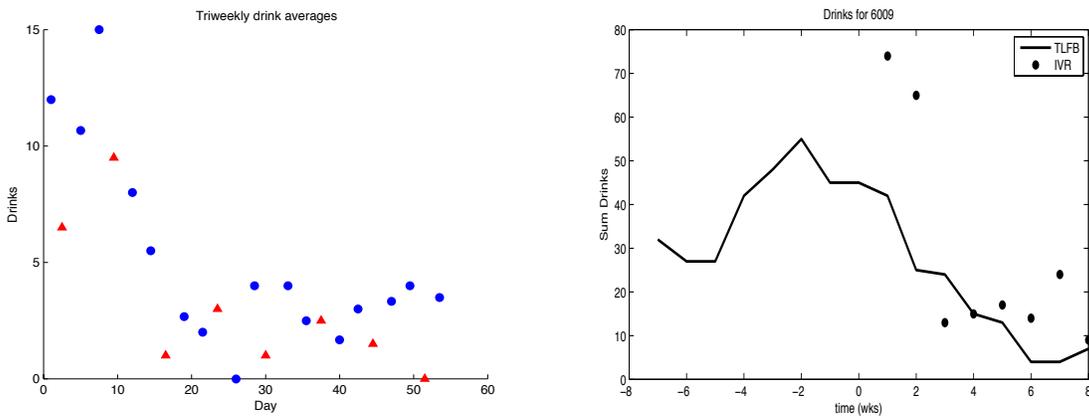


Figure 34: Left: Patient 6009's drinking as reported by IVR triweekly. Right: Drinking as reported by TLFB and IVR. Number of IVR days recorded for weeks 1 through 8 are: 7,7,7,7,6,7,7,5

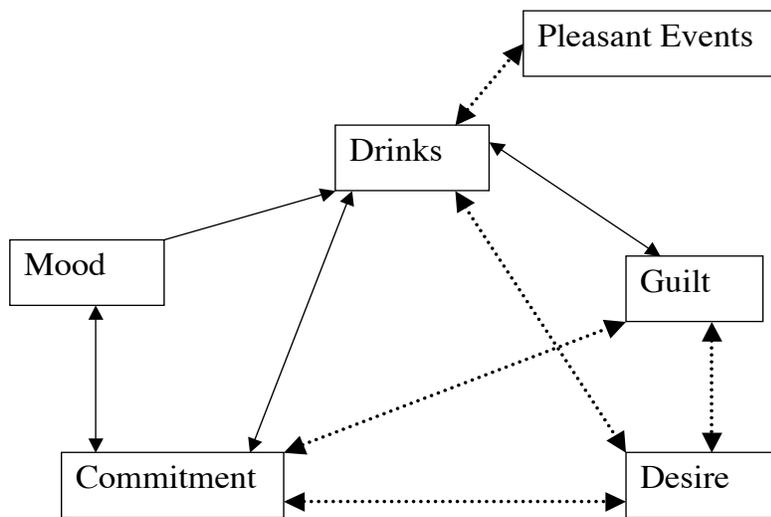


Figure 35: Categorical model for patient 6009 based on hypothesized relationships in the IVR data.

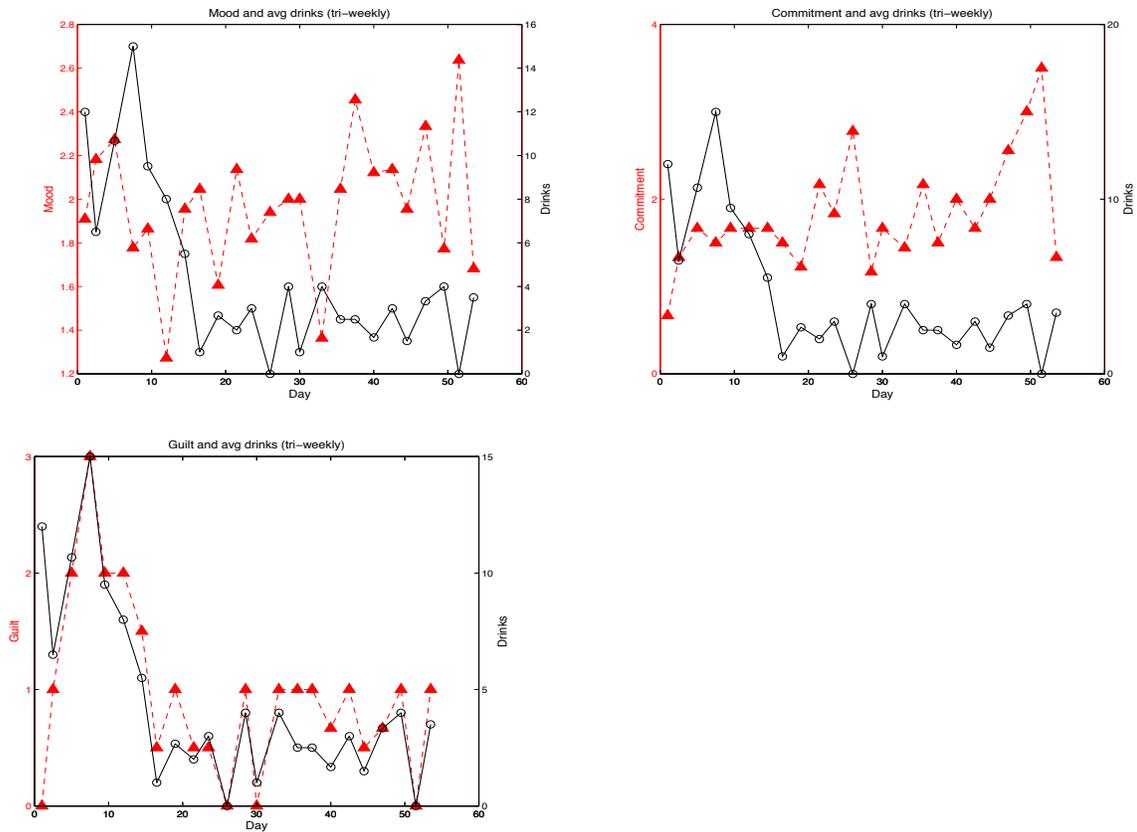


Figure 36: Categories thought to (strongly) be related to drinking for patient 6009.

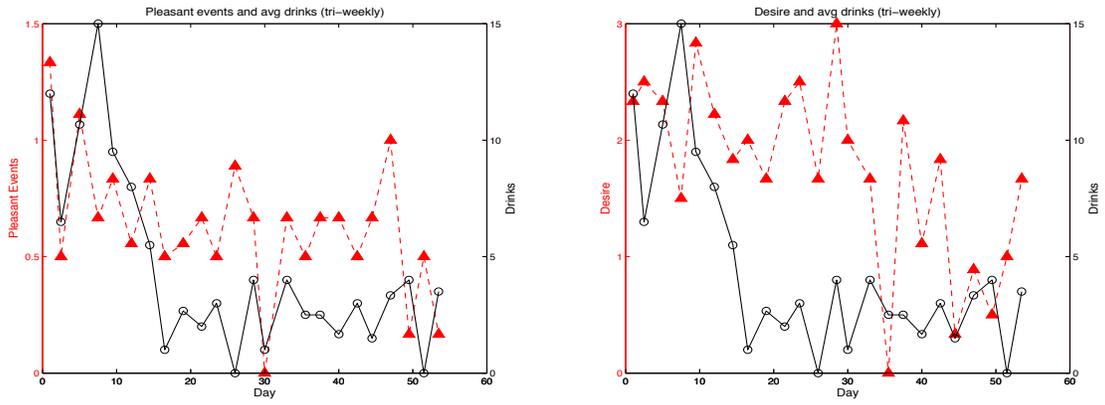


Figure 37: Categories thought to (weakly) be related to drinking for patient 6009.

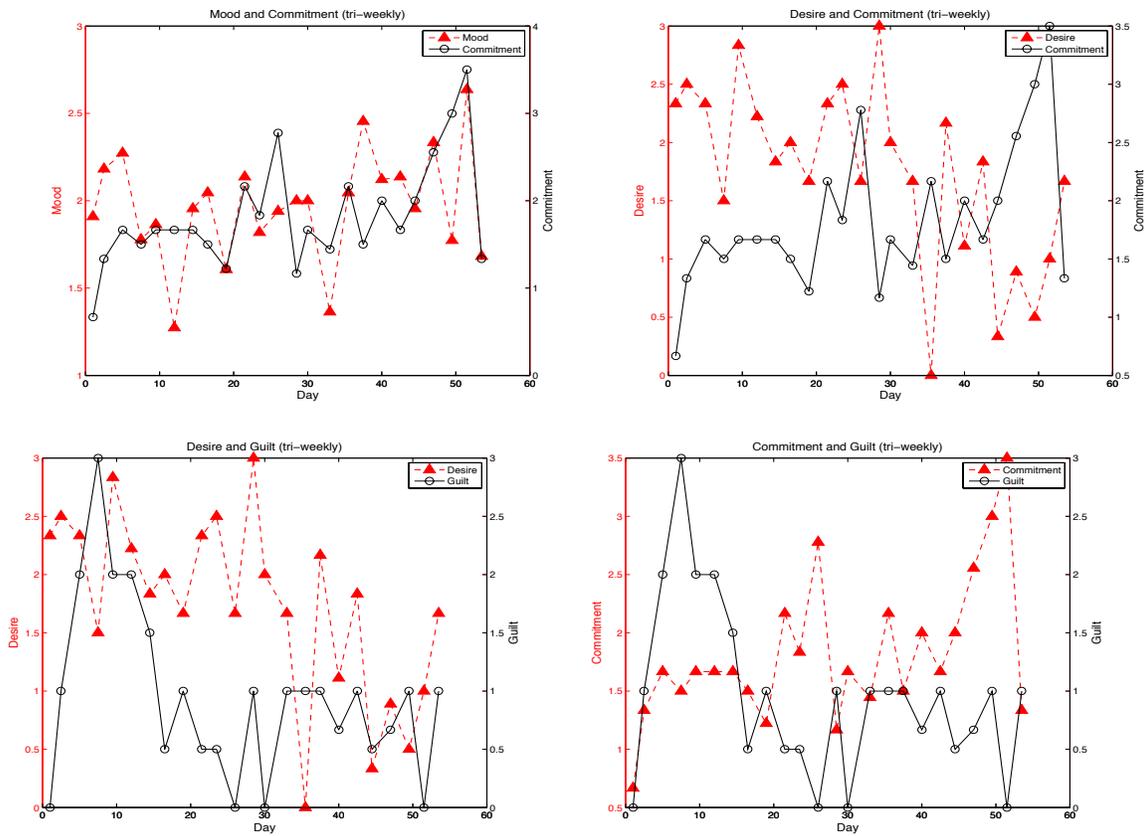


Figure 38: Hypothesized category-category relationships for patient 6009. All relationships are thought to be weak with the exception of a strong relationship between mood and commitment.

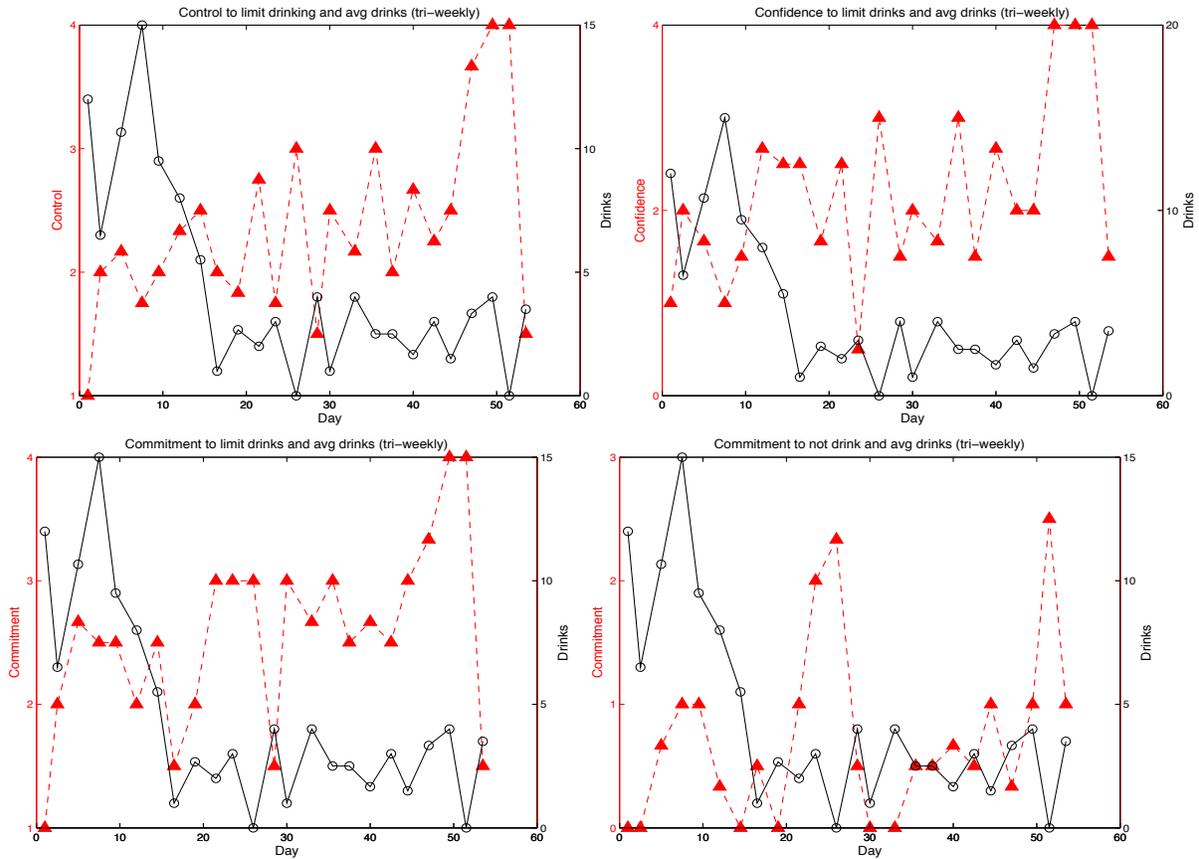


Figure 39: Metrics of confidence and commitment to limit drinking for patient 6009. Control (upper-left) is the average of commitment to not drink heavily (lower-left) and confidence to not drink heavily (upper-right). The lower-right graph shows commitment to not drink at all.

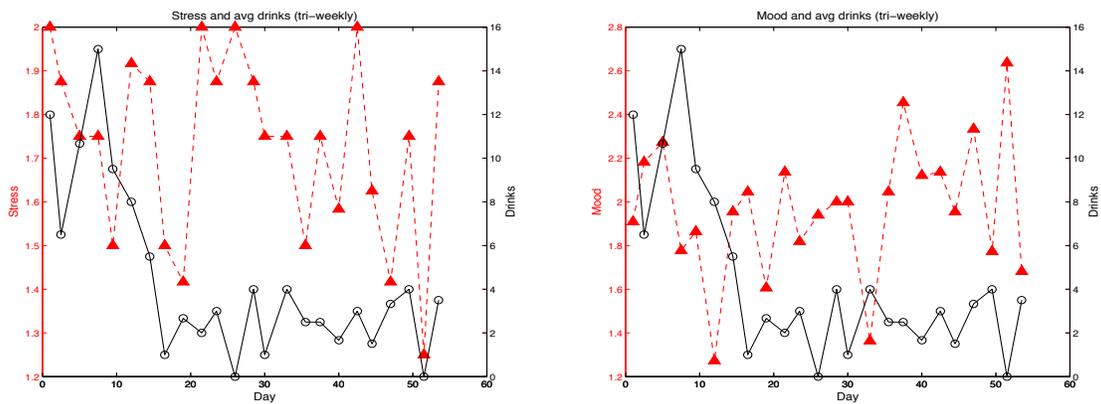


Figure 40: Left: Stress may be weakly related to drinking after the first 30 days or so. It also may be related to mood. Right: Mood and drinks displayed for comparison to stress for consideration of a weak relationship after 30 days.

Patient 6124

This patient was selected because of the regularity with which they reported to the IVR system. The drinking pattern overall shows a decrease (Figure 41), but quite sharply within the last 2 weeks of treatment and there is no follow-up. So, it is not clear if this person did in fact reduce their “long-term” drinking behavior. Also, during the weeks where the individual called into the IVR system, they reported considerably more drinking than they recalled during the TLFB.

There were relatively few categories seen to correlate with drinking, partly because the drinking behavior during most of the treatment period appeared to be highly variable. Only guilt appeared to be strongly related to drinking, but even commitment, which usually seems to agree well with drinking behavior in other patients, appears to be only loosely connected (Figures 42 and 43).

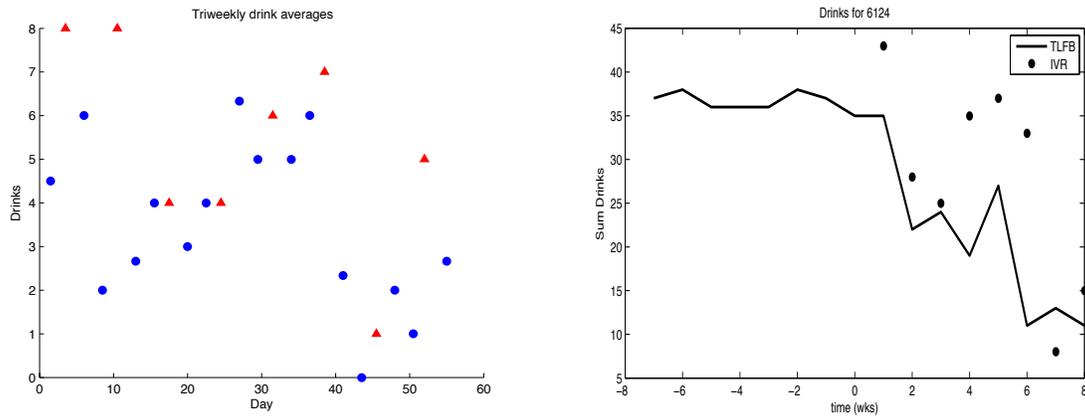


Figure 41: Left: Patient 6124’s drinking as reported by IVR triweekly. Right: Drinking as reported by TLFB and IVR. Number of IVR days recorded for weeks 1 through 8 are: 7,7,7,7,7,7,7,6

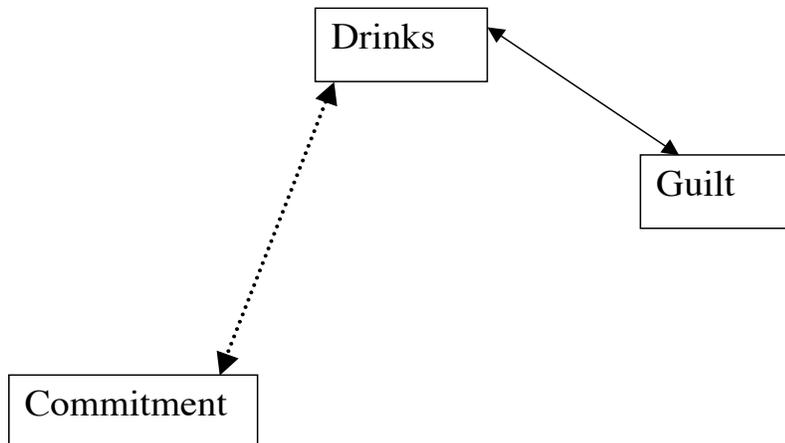


Figure 42: Categorical model for patient 6124 based on hypothesized relationships in the IVR data.

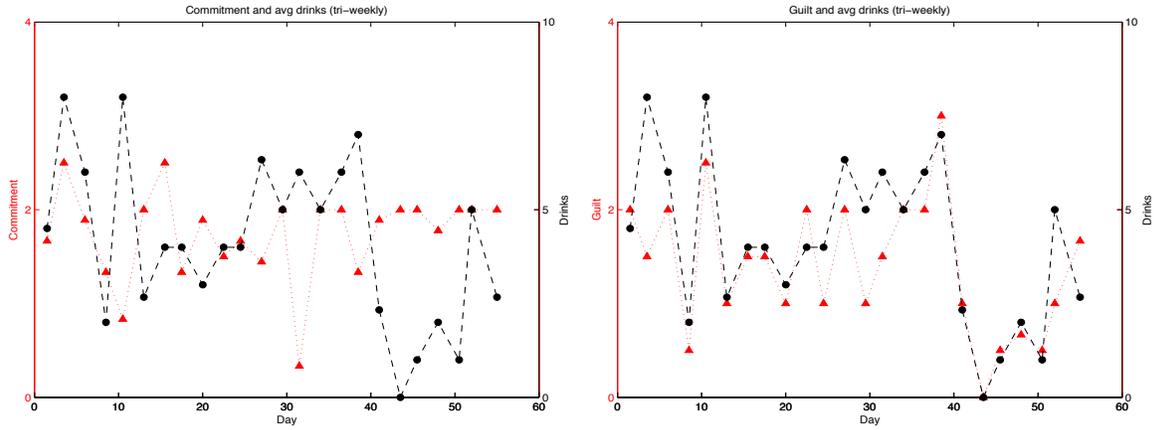


Figure 43: Categories thought to be related to drinking for patient 6124. Commitment looks to be weakly correlated with drinks whereas guilt appears to be strongly related to drinks.

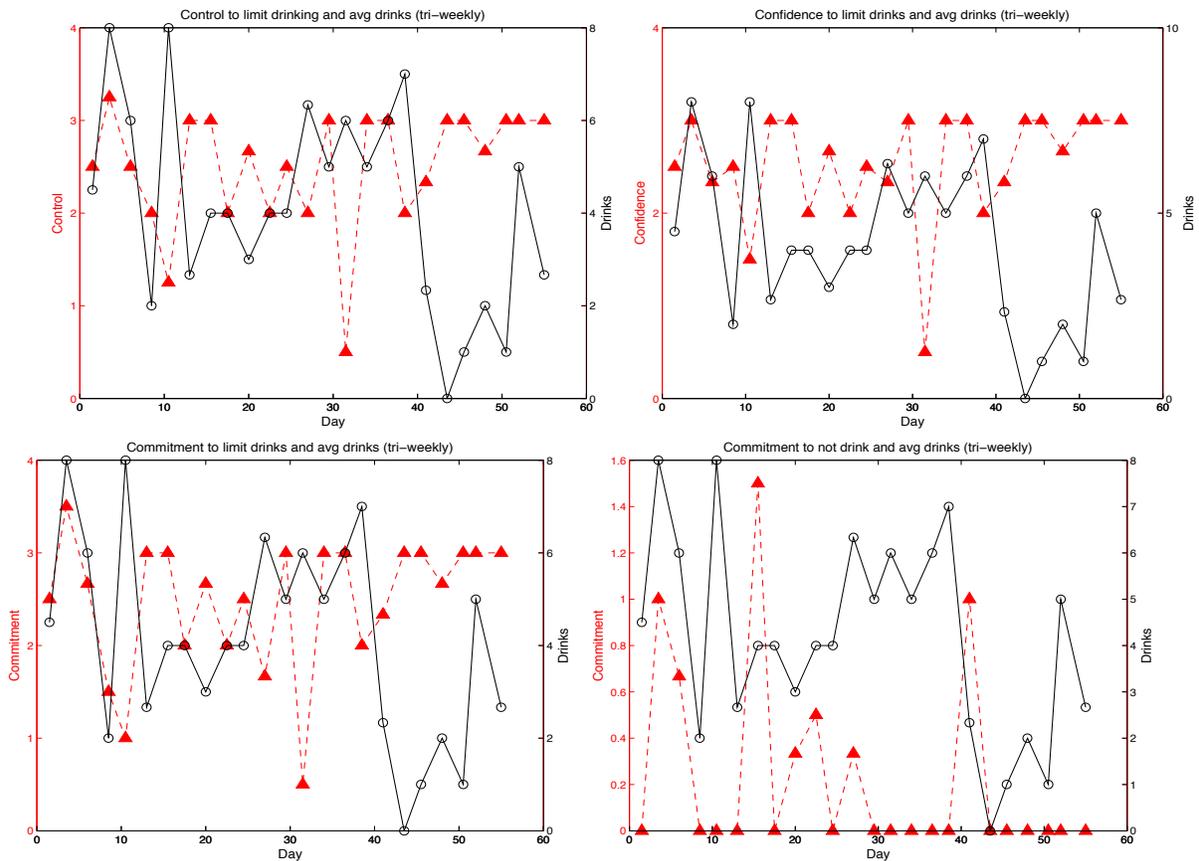


Figure 44: Metrics of confidence and commitment to limit drinking for patient 6124. Control (upper-left) is the average of commitment to not drink heavily (lower-left) and confidence to not drink heavily (upper-right). The lower-right graph shows commitment to not drink at all.

Patient 6083

The extent to which this patient reduces their drinking may need some discussion. It is clear why the patient is termed a responder from their TLFB data (right panel of Figure 45), but this reduction is not clear from the triweekly IVR data (left panel of Figure 45). On the other hand, a comparison of weekly drinks as reported by IVR seem relatively accurate (albeit with a bit more variation), and in fact, the TLFB seems to underestimate heavier drinking. So it is likely that the individual was actually drinking heavier than reported via TLFB in the weeks preceding treatment, and that they did in fact, significantly reduce their drinking, although the reduction occurred almost instantaneously in the first week.

There were strong relationships apparent between drinking and 3 categories: commitment, guilt, and pressure, with a weak relationship possibly between pressure to drink and guilt (depicted in Figure 46). The IVR data displaying these relationships is seen in Figures 47 and 48

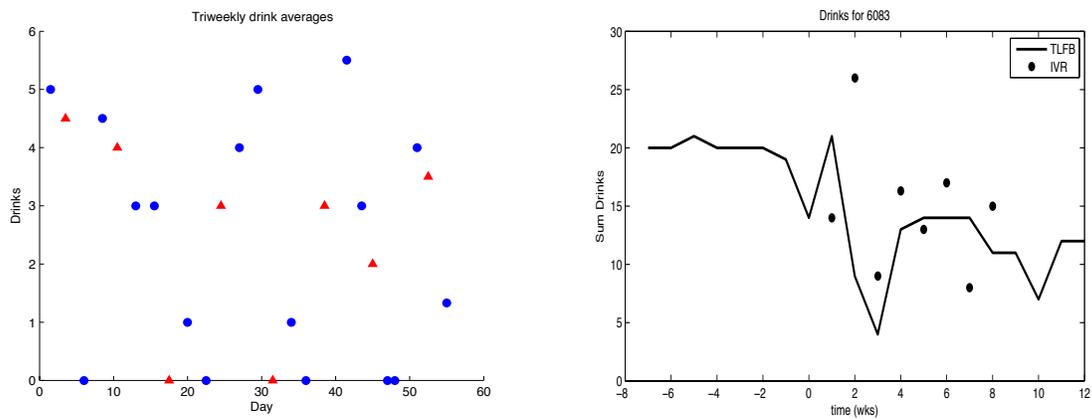


Figure 45: Left: Patient 6083's drinking as reported by IVR triweekly. Right: Drinking as reported by TLFB and IVR. Number of IVR days recorded for weeks 1 through 8 are: 7,7,7,6,7,5,5,6

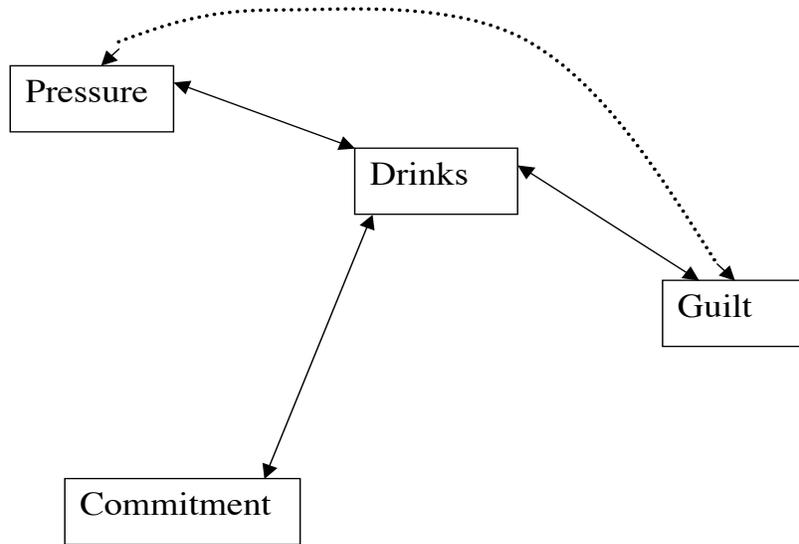


Figure 46: Categorical model for patient 6083 based on hypothesized relationships in the IVR data.

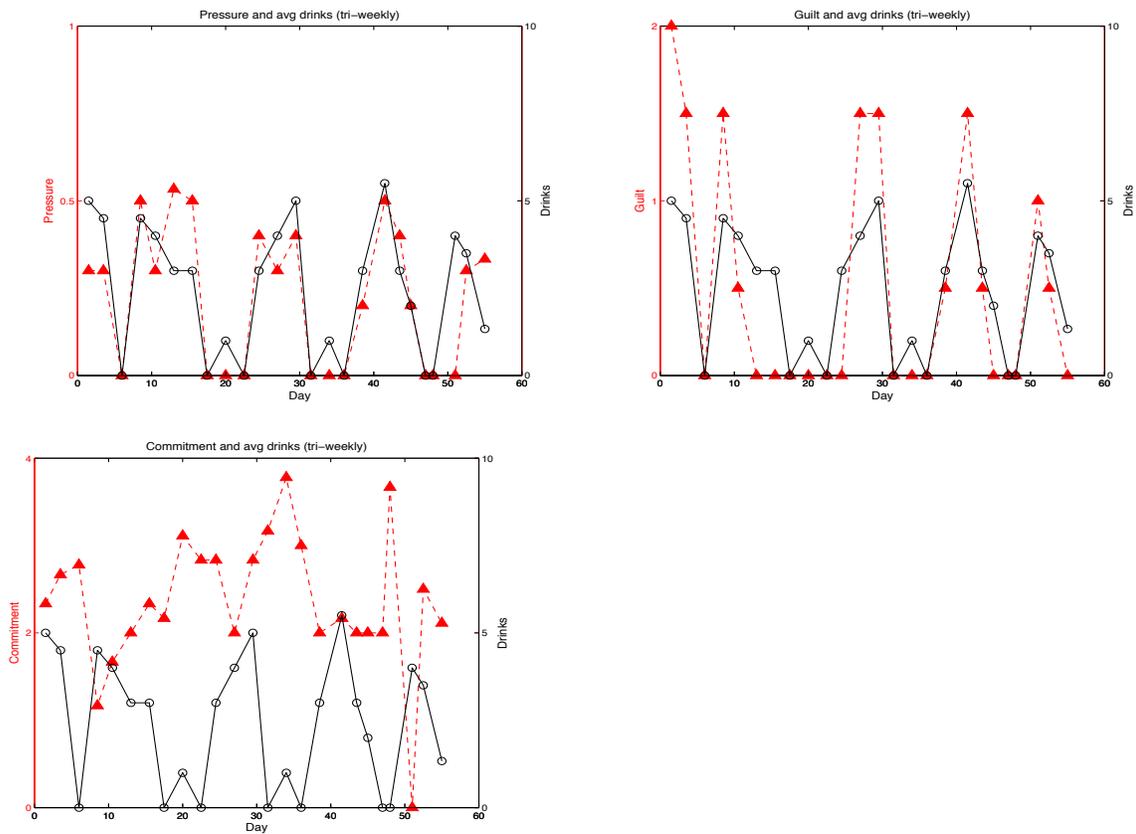


Figure 47: Categories thought to be (strongly) related to drinking for patient 6083.

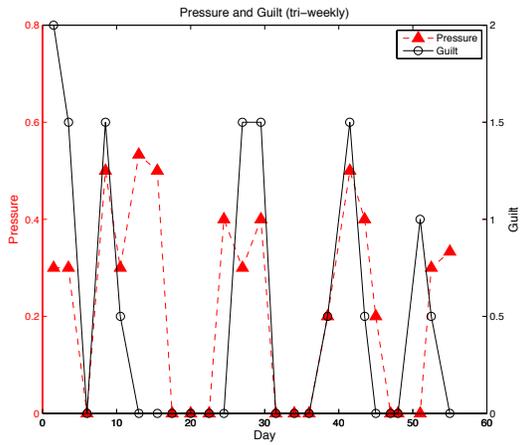


Figure 48: Potential weak relationship between pressure and guilt for patient 6083.

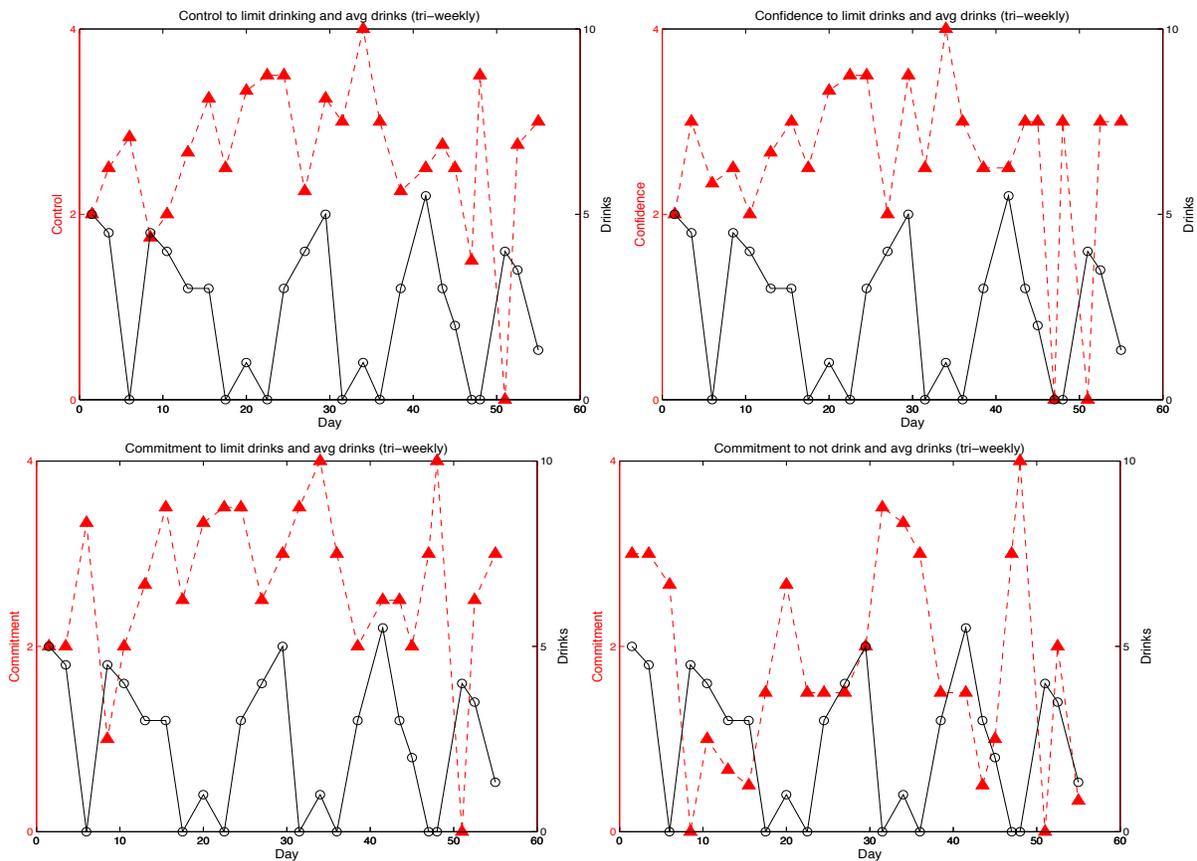


Figure 49: Metrics of confidence and commitment to limit drinking for patient 6083. Control (upper-left) is the average of commitment to not drink heavily (lower-left) and confidence to not drink heavily (upper-right). The lower-right graph shows commitment to not drink at all.

6.2 Suspected sources of error

There are many sources of error to consider when using the IVR data. Most patients neglect to call in to take the IVR survey at least one day during the IVR. During later phases of modeling, however, we may use an expectation-maximization algorithm to determine likely values for the missing data. We may also make inferences about the patient's missing drink data based on the patient's TLFB data. We expect that the error in the reported IVR data is not normally distributed or even constantly distributed as time passes and comes from many sources, including limitations of the scale, their feelings or memories about their recent actions, and even improved self-awareness and drink awareness as they take the IVR numerous times. As participants' responses are limited to a small number of discrete answers, perhaps a patient truly felt that his confidence to limit his drinking was only a 1.5; however, due to possible responses via the phone, they report a 2. On the other hand, he may feel extremely committed and would enter 10 as a response if he could, but he is limited to a maximum entry of 3. We expect more patients to under-report the number of drinks consumed than to over-report, as patients may feel like they are expected to show improvement. If a participant has 20 drinks but feels like he should hide his behavior or if he can't remember all the drinks he had, he is likely to under-report the number of drinks consumed.

During the initial examination of IVR data, we found other anomalies that may be difficult to address. Several data sets include excessively high drink counts that end with a digit of 0, indicating that a participant may have accidentally pressed an extra digit when attempting to proceed to the next question of the survey, and one data set included several weeks of the participant entering 0 for every question. Data sets with such anomalies may be difficult to use during modeling and inverse problems.

6.3 Mathematical model

We focus our initial modeling efforts on patient 6029, as he frequently responded to the IVR survey and showed a decrease in his frequency of drinking from the weeks preceeding the IVR to the weeks following the IVR, as measured by the TLFB. The goal of patient 6029 was to reduce drinking during the work week, and the patient felt that he was sufficiently limiting his drinking if he mostly abstained from alcohol during the week and drank mainly during the weekends. This particular drinking pattern, although reduced from a previously constant heavy drinking pattern, is certainly different from our general perception of limiting drinking as a daily reduction in alcoholic beverage consumption. This indicates that limit should have a time-dependent effect on alcohol consumption in the mathematical model for this individual. For example, the patient's high limit score during the week indicates that he plans to have at most 2 drinks in a 24 hour period, but on the weekend he may plan to have as many as 5 or 7 in 24 hours. By the end of the study, he manages to limit his drinking to mainly weekends, but the number of beverages consumed over the weekend still remained high and fluctuated greatly (Figure 50). This indicates that the weekend is an important event for this patient and is associated with heavier drinking, and that other quantities like threshold parameters should likely be time-dependent as well. The IVR data supports the patient's reflections on his drinking, as many instances of heavy drinking occur Friday night, Saturday during the day, Saturday night, and sometime Sunday night; however, the patient's confidence and commitment to control his drinking remains high during these time periods. Additionally, the patient shows that it requires more drinks during the weekend to make him feel like his drinking was excessive than it does during the week, and that the patient's commitment to abstain from drinking for the next 24 hours is very low over at the time of the Friday and Saturday IVR sessions.

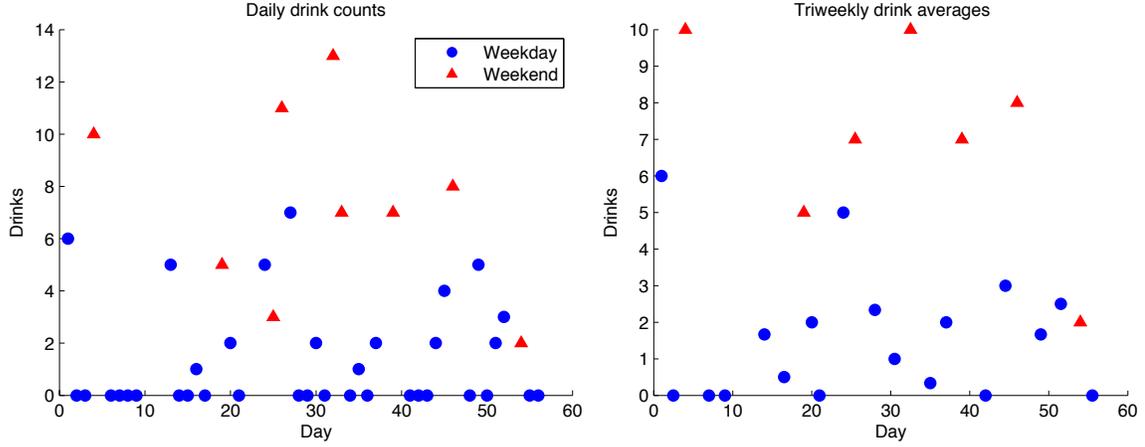


Figure 50: Drinks consumed by patient 6029 daily (left) and tri-weekly (right), with weekend measurements (Saturday call, Sunday call) highlighted.

The form of the current mathematical model for patient 6029 is given by

$$\begin{aligned} \frac{d}{dt}A(t) &= -a_{1,2}\chi_{(G>0)} \left(\int_{-\tau_1}^0 G(t+s)\kappa_1(s)ds \right)^2 + a_{1,3}\chi_{(D>0)}D(t) \\ &\quad - a_{1,4}(Q(t) + \chi_{(Q>0)}Q^2(t)) - a_{1,5}L(t) \end{aligned} \quad (3)$$

$$\frac{d}{dt}G(t) = a_{2,1} \left(\int_{-\tau_2}^0 A(t+s)\kappa_2(s)ds - (1 + \chi_W(t))A_G^* \right) \quad (4)$$

$$\frac{d}{dt}D(t) = -a_{3,4}Q(t) - a_{3,5}L(t) - a_{3,2} \left[\exp \left(\frac{1}{G_{D1}^*} \int_{-\tau_3}^0 G(t+s)\kappa_3(s)ds \right) - G_{D2}^* \right] \quad (5)$$

$$\begin{aligned} \frac{d}{dt}Q(t) &= -a_{4,3} (1 + \chi_W(t)\chi_{(D>0)}) D(t) \\ &\quad - a_{4,1} \left[\exp(A_{Q1}^* \min(0, (A(t - \tau_4) - A_{Q1}^*))) - (1 - \chi_W(t))A_{Q2}^* \right] \end{aligned} \quad (6)$$

$$\frac{d}{dt}L(t) = a_{5,2}G(t) - a_{5,3}D(t) - a_{5,1} \left[\exp \left(\frac{1}{A_{L1}^*} \min(A_{L1}^*, (A(t - \tau_5) - A_{L1}^*)) \right) - A_{L2}^* \right]. \quad (7)$$

where $a_{i,j}$ are parameters relating the magnitude of the effect of variable j on variable i , where $A(t)$ is variable 1, $G(t)$ is variable 2, and so on, and G_{D1}^* , G_{D2}^* , A_{Q1}^* , A_{Q2}^* , A_{L1}^* , and A_{L2}^* are also parameters. Currently the $a_{i,j}$ and other parameters are *unknown* constants (that may be changed to time-dependent functions in the future). The equations were determined from the categorical model and from either reasonable assumptions and/or information during our conversations between the NCSU team and the Columbia team. So for example, the term $a_{3,5}L(t)$ in the $\frac{d}{dt}D(t)$ equation means that the Limit variable affects the rate of change of the Desire variable in a directly proportional way, with a proportionality constant $a_{3,5}$. The interpretation of exponential function terms is, for example, that if the guilt (over some previous interval) is relatively low then desire increases, but if guilt is high then there is a corresponding notable decrease in desire.

The threshold functions $\chi_{(x>x^*)}$ are meant to model the case where the state variable does not affect the other one unless it gets above a certain level. For example, in the $\frac{d}{dt}A(t)$ equation, the term $a_{1,3}D(t)$ means that for $D(t) < 0$, desire does not have any effect on (it neither increases or decreases) drinks $A(t)$. If $D(t) \geq 0$ then it increases the number of drinks the individual takes at

a rate proportional (with proportionality constant $a_{1,3}$) to the magnitude of $D(t)$.

The τ_k , $k = 1, \dots, 5$, represent time delays, meaning that the effect on the rate of a certain variable depends on the value of another variable at a previous time or during a previous time interval (if there is an integral over the delay term), and κ_k , $k = 1, \dots, 3$ are density functions. For example, in (6), the model can be interpreted as saying that it is not the drinks consumed at the current time, but rather drinks had at a particular time in the past, that would affect the one's commitment to reduce drinking entirely. In (3), however, the model indicates that a patient's accumulated guilt from some time in the past to the present time affects their alcoholic beverage consumption. It is important to note that while we have primarily taken this tri-weekly timescale to organize the observations, we are not required to fix a delay as a scalar multiple of this timescale. The delays are not equal in the above model, allowing for a longer history of drinking to have an effect on guilt or commitment than the effect on the 'limit' variable, or vice versa, and similarly for the other relationships. Importantly, we note that in a single phone call, a patient reports on their drinking over the previous 24 hour period and their guilt concerning those drinks. Thus, even though on the surface it *appears* that guilt and drinks are then at the same point in time, the intent of the question is actually how drinks over a past time interval (leading up to the present time), influence guilt at the present time. Thus, the $\frac{d}{dt}G(t)$ equation is determined by a function of $\int_{-\tau_2}^0 A(t+s)\kappa_2(s)ds$.

When comparing data to the model, the data was shifted so that 0 is the neutral value. In order to enforce the restriction that $A(t) \geq 0$ and $G(t) \geq 0$, we impose the additional model specifications

$$\begin{aligned} \frac{d}{dt}A(t) &= 0 \text{ if } \frac{dA}{dt} \leq 0 \text{ and } A(t) \leq 0, \\ \frac{d}{dt}G(t) &= 0 \text{ if } \frac{dG}{dt} \leq 0 \text{ and } G(t) \leq 0, \end{aligned}$$

The mechanisms following the coefficients a_{41} and a_{51} are shown in Figure 51 left and right, respectively.

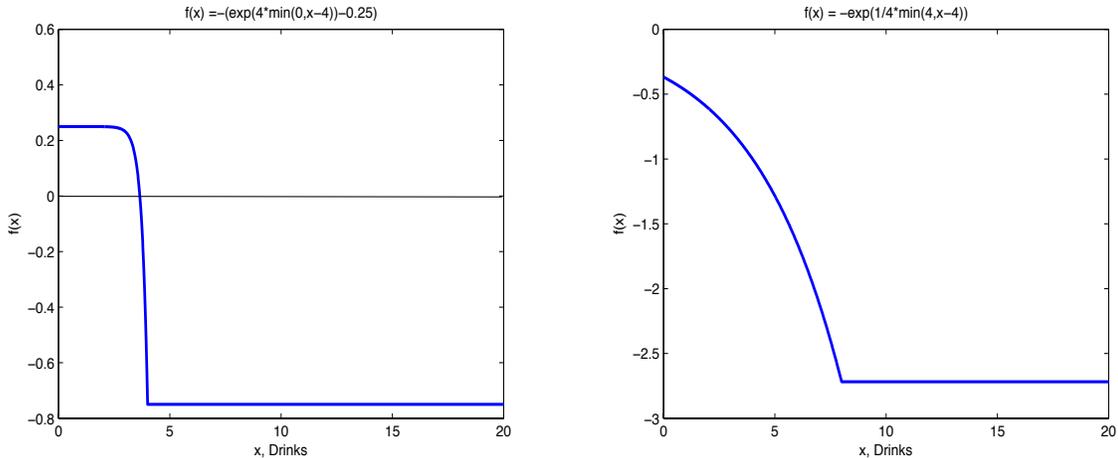


Figure 51: Graphics showing behavior of the mechanisms following the coefficients a_{41} (left) and a_{51} (right) in the model (3) - (7).

We include time-dependent terms to the model to reflect the time-dependent changes in patient 6029’s drinking habits. Patient 6029’s IVR session started on a Wednesday, and so this patient’s first weekend starts Friday evening, $t = 3$, and ends Sunday evening, $t = 5$. Weekends then start every seven days after that first Friday evening. Define the characteristic function $\chi_W(t)$ so that $\chi_W(t) = 1$ during a weekend ($3 < t \leq 5$, $10 < t \leq 12$, and so on) and $\chi_W(t) = 0$ otherwise. The model solutions should mirror the changes in a patient’s mindset during the weekend, which in turn results in a change in drinking rate, and so we would leave $\frac{dA(t)}{dt}$ unchanged. The model describing this particular patient, however, may have a very small $a_{(1,5)}$ since he feels that he is limiting his drinking if he drinks primarily on weekends. For $\frac{dG(t)}{dt}$ and $\frac{dQ(t)}{dt}$, we would multiply A_G^* , a_{43} and A_{Q2}^* by a factor of $\chi_W(t)$, reflecting that the patient’s threshold level of acceptable drinking rises and attachment to abstaining from alcohol decreases over the weekend.

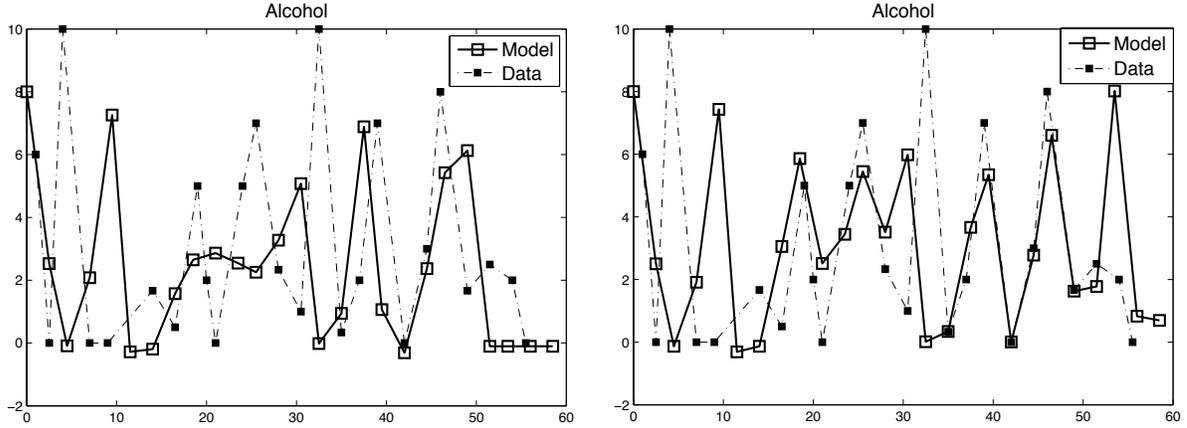


Figure 52: A comparison of the alcohol consumption ($\int_{t_{2(i-1)}}^{t_{2i-1}} A(s)ds$ and $\int_{t_{2i-1}}^{t_{2i}} A(s)ds$) as calculated by the AGDQ model (for two different sets of parameter values) and the IVR alcohol consumption data ($d_{2i-1}^{(1)}$ and $d_{2i}^{(1)}$) versus time in days.

In the clinical perspectives notes on patient 6029, it is noted that the participant is motivated to reduce his drinking and that before the IVR period started, he had identified internal motivating factors to promote controlling his drinking. He had started reducing his weekly alcohol consumption from between 35 and 65 drinks per week before the IVR period to no more than 35 per week during the IVR period. The therapists’ impression of this patient was that his salience - a term used by experts in the study of alcohol and substance abuse to represent the relative importance of reducing the substance or alcohol use compared with other priorities in one’s life - is particularly high. This normally suggests that the individual is well on their way to We are not sure how salience should factor into the model at this stage and thus omit it for now. However, we expect that it will be incorporated in our modeling efforts in the future.

7 Numerical implementation

We computed solutions using the standard fourth order Runge-Kutta method for ODE’s after approximating our equations using a linear spline basis. This method of approximating delay differential equations used here was established in [10] for linear systems, extended to nonlinear

systems in [11, 46] and has been used for computations in [12] and [13]. Technical details may be found in [12], [10], [13], and the recent extension [14].

Consider the general system (which includes as a special case our system above)

$$\dot{x}(t) = f(x(t), x_t, x(t - \tau_1), \dots, x(t - \tau_m)) + g(t) \quad (8)$$

for $0 \leq t \leq T$, $x_0 = \phi$, $x(0) = \eta$ where f is a nonlinear function, $f = f(\eta, \phi, y_1, \dots, y_\nu)$, which maps $Z \times \mathbb{R}^{\nu} \rightarrow \mathbb{R}^n$. The domain Z here is $Z = \mathbb{R}^n \times L_2(-r, 0; \mathbb{R}^n)$ if $0 < \tau_1 < \dots < \tau_m = r$. Here x_t denotes the usual function $x_t = x(t + \theta)$, $-r \leq \theta \leq 0$ and $\phi \in H^1(-r, 0)$.

Let us define the function $F : \mathbb{R}^n \times C(-r, 0; \mathbb{R}^n) \subset Z \rightarrow \mathbb{R}^n$ as

$$F(z) = F(\eta, \psi) = f(\eta, \psi, \psi(-\tau_1), \dots, \psi(-\tau_m))$$

where f is the RHS of (8).

The nonlinear operator $\mathcal{A} : D(\mathcal{A}) \subset Z \rightarrow Z$ where $D(\mathcal{A})$ is given by

$$D(\mathcal{A}) = \{(\psi(0), \psi) | \psi \in H^1(-r, 0)\}$$

is defined by

$$\mathcal{A}(\psi(0), \psi) = (F(\psi(0), \psi), D\psi)$$

Let $y(t; \phi, g) = (x(t; \phi, g), x_t(\phi, g))$ where x is the solution of (8) with $\phi \in H^1$ and $g \in L_2$. Then for initial data $\zeta = (\phi(0), \phi)$, $y(\phi, g)$ is the unique solution on $[0, T]$ of

$$z(t) = \zeta + \int_0^t \{\mathcal{A}(z(\sigma)) + (g(\sigma), 0)\} d\sigma. \quad (9)$$

Denote our delay of greatest magnitude in the system (3)–(7) as $0 < r < \infty$. We approximate our system at the nodes $\theta_i = -r \frac{i}{N}$, $i = 0, 1, \dots, N$ so that $t_i \in [t - r, t]$, so there are intervals of length $\delta = \frac{r}{N}$ between each node, and we have $n = 5$ equations in our system. Let Z_1^N denote the linear spline subspace of Z , with $\dim Z_1^N = k_n = n(N + 1)$. We interpolate our system of equations using a basis of linear splines

$$e_j^N(\theta_i) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}.$$

The matrix function β^N are then defined by $\beta^N = (e_0^N, e_1^N, \dots, e_N^N) \otimes I$ where I is the $n \times n$ identity matrix and \otimes denotes the Kronecker product. Define our basis functions to be $\hat{\beta}^N = (\beta^N(0), \beta^N)$. Then the dimension of the basis functions is $n \times k_n$. Pieces of the solution of length $[t - r, t]$, z^N are elements of Z_1^N with coordinate vector α_n , or

$$z^N = \hat{\beta}^N \alpha^N = \sum_{j=0}^N (e_j^N(0), e_j^N) a_j$$

where $a_j \in \mathbb{R}^n$ such that $\alpha^N = \text{col}(a_0^N, \dots, a_N^N)$.

Let $P^N = P_\omega^N$ be the orthogonal projection of Z onto Z^N so that $P^N z \rightarrow z$ for all $z \in Z$. The weighted inner product is defined by

$$\langle (\eta_1, \phi_1), (\eta_2, \phi_2) \rangle_\omega = \eta_1^T \eta_2 + \int_{-r}^0 \phi_1(\theta)^T \phi_2(\theta) \omega(\theta) d\theta.$$

where in order to create a dissipative operator in Z we apply the nondecreasing weight function $\omega(\theta)$ to the space Z as defined in [14]. (For further details of the theoretical advantages of using the weight function, see [14].) Since we have five delays (two discrete and three distributed) in our system, we rewrite our equations as

$$\begin{aligned}\frac{d}{dt}A(t) &= -g_1 \left(\int_{-r}^0 G(t+s)\kappa_1(s)ds \right) - g_2(L(t)) + f_1(D(t)) - g_3(Q(t)) \\ \frac{d}{dt}G(t) &= g_4 \left(\int_{-r}^0 A(t+s)\kappa_2(s)ds \right) \\ \frac{d}{dt}D(t) &= -f_2(Q(t)) - f_3(L(t)) - h_1 \left(\int_{-r}^0 G(t+s)\kappa_3(s)ds \right) \\ \frac{d}{dt}Q(t) &= -f_4(D(t)) - f_5(A(t-\tau_4)) \\ \frac{d}{dt}L(t) &= -g_5(D(t)) + g_6(G(t)) - f_6(A(t-\tau_5))\end{aligned}$$

where density functions κ_i , $i = 1, 2, 3$, are extended to be zero between $-\tau_i$ and $-r$. The discrete delays τ_4 and τ_5 we relabel such that the delay that is smaller in magnitude is $-r_1$ and the other delay is $-r_2$ so that we may, WLOG, write our delays using the inequality $0 < r_1 < r_2 < r$. Thus in [10], we have $m = 3$ and

$$\omega(\theta) = \begin{cases} 3, & -r_1 < \theta \leq 0 \\ 2, & -r_2 < \theta \leq -r_1 \\ 1, & -r \leq \theta \leq -r_2 \end{cases} .$$

Then we approximate the operator \mathcal{A} by $A^N = P^N \mathcal{A} P^N$ and the equation (9) by the approximating equations

$$z^N(t) = P^N \zeta + \int_0^t \{A^N(z^N(\sigma)) + P^N(g(\sigma), 0)\} d\sigma.$$

Since Z^N is finite dimensional this is equivalent to the system

$$\dot{z}^N(t) = A^N(z^N(t)) + G^N(t), \quad z^N(0) = P^N \zeta, \quad (10)$$

where $\hat{\beta}^N G^N(t) = P^N(g(t), 0)$. Note that (10) has a unique solution. Then we may use a Runge-Kutta or other ODE solver on this system. For any $\hat{\phi}^N = (\phi^N(0), \phi^N) \in Z^N$, the following orthogonality relationship holds,

$$\{P^N(\phi(0), \phi) - (\phi(0), \phi)\} \perp Z^N. \quad (11)$$

The orthogonality relationship in (11) is equivalent to writing

$$\langle \hat{\beta}^N \alpha^N - (\phi(0), \phi), \hat{\beta}^N \rangle_{\omega} = 0.$$

By the properties of inner products,

$$\langle \hat{\beta}^N, \hat{\beta}^N \rangle_{\omega} \alpha^N - \langle (\phi(0), \phi), \hat{\beta}^N \rangle_{\omega} = 0.$$

We let $Q^N = \langle \hat{\beta}^N, \hat{\beta}^N \rangle_{\omega}$ and $h^N(\zeta) = \langle (\phi(0), \phi), \hat{\beta}^N \rangle_{\omega}$. By applying the definition of the inner product, we have the following expressions for Q and h^N :

$$Q^N = \beta^N(0)^T \beta^N(0) + \int_{-r}^0 \beta^N(\theta)^T \beta^N(\theta) \omega(\theta) d\theta, \quad (12)$$

and

$$h^N(\phi(0), \phi) = \beta^N(0)^T \phi(0) + \int_{-r}^0 \beta^N(\theta)^T \phi(\theta) \omega(\theta) d\theta. \quad (13)$$

where $(\phi(0), \phi)$ are initial data for the system, $x(0) = \phi(0)$ and $x(\theta) = \phi(\theta)$, for $\theta \in [-r, 0]$. Note that the dimension of Q^N is $\dim Q^N = (k_n \times n) \times (n \times k_n) = k_n \times k_n$. In the case of linear spline approximations, Q_1^N is given by

$$Q_1^N = \frac{r}{N} \left\{ \begin{array}{c} \left[\begin{array}{cccc} \frac{N}{r} & 0 & \dots & 0 \\ 0 & 0 & & 0 \\ \vdots & & \ddots & \\ 0 & 0 & \dots & 0 \end{array} \right] + \left[\begin{array}{cccc} \frac{1}{3} & \frac{1}{6} & 0 & \dots & 0 \\ \frac{1}{6} & \frac{2}{3} & \frac{1}{6} & & \vdots \\ 0 & \frac{1}{6} & \frac{2}{3} & & \\ \vdots & & & \ddots & \\ 0 & \dots & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \\ 0 & & 0 & \frac{1}{6} & \frac{1}{3} \end{array} \right] \cdot * W(\omega(\theta)) \end{array} \right\} \otimes I_{n \times n} \quad (14)$$

where W is a weighting matrix containing zeros off the diagonal, sub-diagonal, and super-diagonal and

$$\begin{aligned} W_{i,i}(\omega(\theta)) &= \frac{1}{2\delta} \int_{t_{i+1}}^{t_{i-1}} \omega(t) dt \\ W_{i,i+1}(\omega(\theta)) &= W_{i+1,i}(\omega(\theta)) = \frac{1}{\delta} \int_{t_{i+1}}^{t_i} \omega(t) dt. \end{aligned}$$

Then one can proceed by calculating the coordinate vector α^N for the initial piece of the solution by solving the system

$$Q_1^N \alpha^N = h^N(\phi(0), \phi).$$

One can solve this via Gaussian elimination, but since the size of Q_1^N for our problem is $5(N+1) \times 5(N+1)$, we used the fact that Q_1^N is a tri-banded matrix to reduce the number of operations performed.

After finding α^N , we computed $Q_1^N \gamma^N = h^N(F(\phi), D\phi)$. In solving this system, we need to compute $A^N(z^N(t))$ at each time t due to the nonlinearity of A^N . For each time t , the element $z^N(t) \in Z^N$ can be written as $z^N(t) = \hat{\beta}^N \alpha^N(t)$ for some coordinate vector $\alpha^N(t)$. The operator \mathcal{A} applied to $z^N(t)$ is

$$\mathcal{A}(z^N(t)) = \mathcal{A}(\beta^N(0)\alpha^N(t), \beta^N\alpha^N(t)) = (F(\beta^N(0)\alpha^N(t), \beta^N\alpha^N(t)), D\beta^N\alpha^N(t)).$$

Then

$$A^N = P^N \mathcal{A} P^N z = P^N (F(\beta^N(0)\alpha^N(t), \beta^N\alpha^N(t)), D\beta^N\alpha^N(t)).$$

Since $A^N(z^N(t)) \in Z^N$ it also has a representation of the form $A^N(z^N(t)) = \hat{\beta}^N \gamma^N(t)$. Using the orthogonality relation as before, we have

$$\begin{aligned} Z^N &\perp \{A^N(\hat{\beta}^N \alpha^N(t)) - \hat{\beta}^N \gamma^N(t)\} \\ 0 &= \langle \hat{\beta}^N \gamma^N(t) - (F(\beta^N(0)\alpha^N(t), \beta^N\alpha^N(t)), D\beta^N\alpha^N(t)) \beta^N \rangle_\omega \\ \langle \hat{\beta}^N, \hat{\beta}^N \rangle \gamma^N(t) &= \langle \beta^N, (F(\beta^N(0)\alpha^N(t), \beta^N\alpha^N(t)), D\beta^N\alpha^N(t)) \rangle_\omega \\ Q^N \gamma^N(t) &= h^N(F(\beta^N(0)\alpha^N(t), \beta^N\alpha^N(t)), D\beta^N\alpha^N(t)). \end{aligned} \quad (15)$$

The matrix Q_1^N is unchanged from before. The definition of the inner product is again used to obtain h^N , which is given by

$$h^N(\mathcal{A}z^N(t)) = \beta^N(0)^T F(\beta^N(0)\alpha^N(t), \beta^N\alpha^N(t)) + \int_{-r}^0 \beta^N(\theta)^T D\beta^N(\theta)\omega(\theta)d\theta\alpha^N(t).$$

In the above expression the integral $\int_{-r}^0 \beta^N(\theta)^T D\beta^N(\theta)d\theta$ does not depend on time and therefore is only actually computed once.

Similarly, we have $h^N(g(t), 0) = \beta^N(0)^T f(t)$ and therefore

$$Q^N G^N(t) = \beta^N(0)^T g(t).$$

With the coefficient vectors α^N for both $A^N(z^N(t))$ and $P^N(g(t), 0)$, we can then implement the Runge Kutta scheme to solve for $\gamma^N(t)$ in the equation (10). In short, $\beta^N(0)^T F(\beta^N(0)\alpha^N(t), \beta^N\alpha^N(t))$ is found by evaluating our system of equations at our current time, and the remainder is calculated using a weighted sum of α_j^N , α_{n+j}^N , and α_{2n+j}^N , with the weights based on the function $\omega(\theta)$. Then we may use Gaussian Elimination, again optimized for the tridiagonal matrix Q_1^N , to find γ^N . These γ^N will then be used in the Runge-Kutta method.

7.1 Runge-Kutta Method

Here we describe the Runge-Kutta method for one time step of the ODE system (10). This scheme is given by

$$z^N(t_{i+1}) = z^N(t_i) + \frac{1}{6}\Delta t(k_1 + 2k_2 + 2k_3 + k_4). \quad (16)$$

where $z^N(t_i)$ is the solution to the previous time step, $z^N(t_{i+1})$ is the solution we wish to find to the current time step, and Δt is the time step size (for convenience, set $\Delta t = \delta$). The quantities k_1, k_2, k_3, k_4 are found using our spline interpolation

$$\begin{aligned} k_1 &= A^N(z^N(t)) = \hat{\beta}^N \gamma_1^N, \\ k_2 &= A^N(z^N(t) + \frac{1}{2}\Delta t k_1) = A^N(\hat{\beta}^N \alpha^N(t) + \frac{1}{2}\Delta t k_1) = \hat{\beta}^N \gamma_2^N, \\ k_3 &= A^N(z^N(t) + \frac{1}{2}\Delta t k_2) = A^N(\hat{\beta}^N \alpha^N(t) + \frac{1}{2}\Delta t k_2) = \hat{\beta}^N \gamma_3^N, \\ k_4 &= A^N(z^N(t) + \Delta t k_3) = A^N(\hat{\beta}^N \alpha^N(t) + \Delta t k_3) = \hat{\beta}^N \gamma_4^N. \end{aligned}$$

Recall that $\hat{\beta}^N$ is our basis of dimension $n(N+1)$ and remains the same throughout all time steps of these calculations. Additionally, $\hat{\beta}^N \alpha^N(t)$ remains the same per time step and so is calculated once per $z^N(t_{i+1})$, and then the additional term based on k_i is added at each stage of the step. We solve equation (15) for $\gamma_i(t)$ to use for k_i with $i = 1, \dots, 4$. For example γ_1^N is obtained by solving $Q^N \gamma_1^N = h^N$ by Gaussian elimination (again, using the tri-bandedness of Q^N to reduce the computations), where h^N is

$$h^N = \beta^N(0)^T F(\beta^N(0)\alpha^N(t), \beta^N\alpha^N(t)) + \int_{-r}^0 \beta^N(\theta)^T D\beta^N(\theta)\omega(\theta)d\theta\alpha^N(t).$$

We will solve $F(\beta^N(0)\alpha^N(t), \beta^N\alpha^N(t))$ by using our data $\alpha^N(t)$ in our system of equations. For evaluating integrals of distributed delays in (3)–(7), using Composite Simpson's Rule will produce

adequate results without being too computationally intensive. For k_2, k_3, k_4 , we solve $Q^N \gamma_i^N = h^N$ with h^N given by

$$h^N = \beta^N(0)^T F(\beta^N(0) \alpha_i^N(t), \beta^N \alpha_i^N(t)) + \int_{-r}^0 \beta^N(\theta)^T D \beta^N(\theta) \omega(\theta) d\theta \alpha_i^N(t).$$

Here we have used the notation $\hat{\beta}^N \alpha_i^N(t) = \hat{\beta}^N \alpha^N(t) + \frac{1}{2} \Delta t k_{i-1}$, $i = 2, 3$, and $\hat{\beta}^N \alpha_i^N(t) = \hat{\beta}^N \alpha^N(t) + \Delta t k_{i-1}$, $i = 4$.

8 Performance of the mathematical model in comparison with patient data

8.1 Finding parameter ranges for model solutions that mimic patient behavior

As we seek to simplify our parameter searches that will be performed when we attempt to solve the inverse problem, we first compared the model to data to find ranges of acceptable parameter values. We identified a set of parameters for the simplified model

$$\begin{aligned} \frac{d}{dt} A(t) &= -a_{12} \chi_{(G>0)} \left(\int_{-\tau_1}^0 G(t+s) \kappa_1(s) ds \right)^2 + a_{13} \chi_{(D>0)} D(t) \\ \frac{d}{dt} G(t) &= a_{21} \left(\int_{-\tau_2}^0 A(t+s) \kappa_2(s) ds - A_G^* \right) \\ \frac{d}{dt} D(t) &= a_{32} \left[\exp \left((G_{D1}^*)^{-1} * \int_{-\tau_3}^0 G(t+s) \kappa_3(s) ds \right) - G_{D2}^* \right] \end{aligned}$$

so that $G(t) \in [0, 5]$, $D(t) \in [-3, 3]$, and $A(t) \in [0, 20]$. The results of using one such parameter set is shown in Figure 53. The parameters used for this example are $(a_{12}, a_{13}, a_{21}, a_{32}, A_G^*, G_{D2}^*, G_{D1}^*) = (20, 2.5, 0.625, 1.5, 4, 1.25, 5)$ and delays $\tau_i = 2$ for $i = 1, 2, 3$ with the initial conditions $(A_0, G_0, D_0) = (7, 0.5, 0.15)$. As a person's alcoholic consumption varies over a larger range than the data for G , D , Q , or L , we expect to see larger parameters in the equation $\frac{dA(t)}{dt}$. The primary goal of the study that produced the MOTION data is to help patients reduce their drinking to a safer level, and so the thresholds A_G^* and G_{D2}^* reflect these thresholds in both drinking behavior and the resulting guilt. The values of G_0 , D_0 , Q_0 , and L_0 are the average of the first week of the patient's data, and A_0 is based on the average number of drinks the patients imbibed on days when he did drink, though for a closer fit to the initial week of data A_0 was reduced from 8 to 7.

While the model produces a solution that stays within ranges that are similar to those used in the IVR data, the cycles in the model appear to have a longer period than that of the data. Additionally, it appears that the simplified model consisting of only A , G , and D is not sufficient to model a patient's drinking behavior. Changing each parameter in the simplified model typically changes the period of the model and the amplitude of at least one of $A(t)$, $G(t)$, and $D(t)$; though drastic changes from the example (Figure 53) may result in the model behaving unrealistically (Figure 54). Increasing a_{12} , a_{21} , A_G^* and G_{D1}^* , and decreasing a_{32} shorten the period of the solution, and increasing any of a_{13} , a_{21} , A_G^* , and G_{D1}^* increase the amplitude of $A(t)$. Additionally, increasing a_{13} , a_{32} , G_{D2}^* , or G_{D1}^* increase the amplitude of D , though changing a_{13} or G_{D2}^* too much can lead to the model traveling well outside the desired range. Increasing a_{32} also increases the amplitude of G .

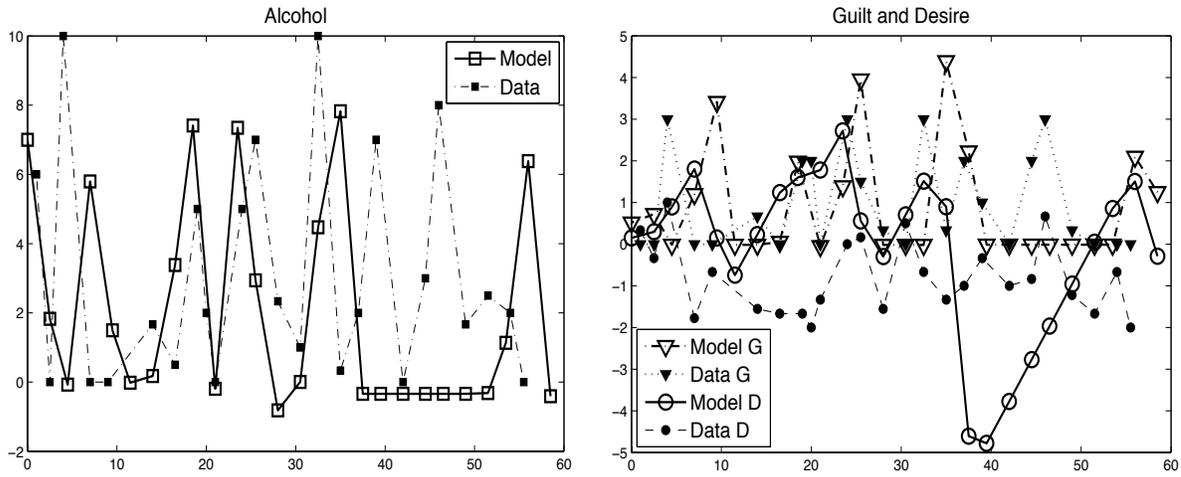


Figure 53: Solution to the simplified model with patient 6029 tri-weekly data. Notice that all three functions map to ranges similar to those of the data.

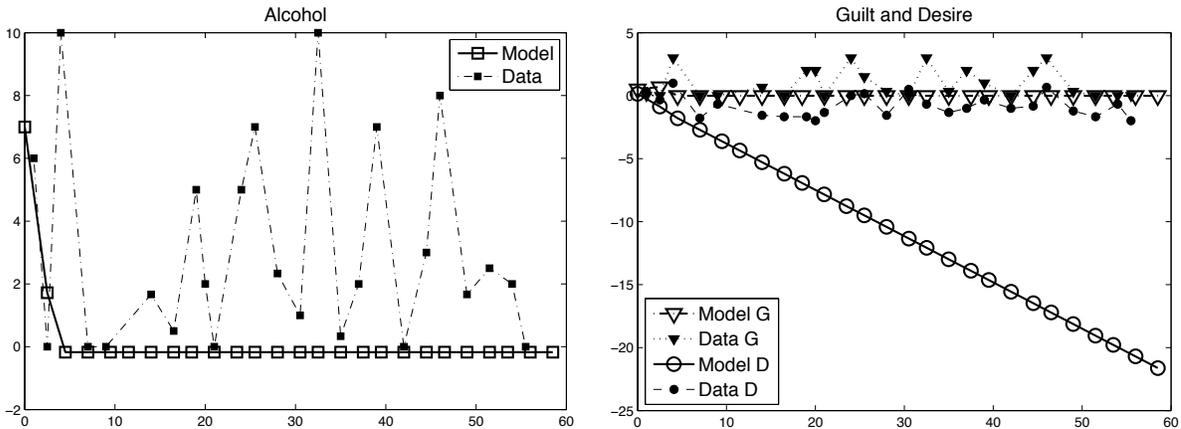


Figure 54: Undesirable solution to the simplified model with patient 6029 tri-weekly data.

Parameters may be selected to control the simplified model so that this version of the model reflects general trends seen in a patient’s data. Our immediate next task is to perform this same qualitative analysis on the full model with an emphasis on finding parameter value ranges to use as initial values in the inverse problem.

9 Conclusion

The field of alcohol and drug abuse research or substance use research possesses a great amount of qualitative information on human behavior and statistical information regarding patient performance in treatment programs; however, there does not exist a previously formulated dynamic model that explains patient behavior in relation to other factors such as events, mood, past behavior, desire for a particular substance, and confidence and commitment to change one’s behavior. Additionally, linear methods of interpreting data, such as linear interpolation and principal com-

ponent analysis, fail to capture the trends in the longitudinal data of the IVR data set. Hence a nonlinear dynamical system model is a potential tool to describe human behavior. Our model focuses on the dynamics of an individual, and we intend to later use the individual model to build cohort- and population-level models. Currently our model includes alcohol consumption, desire to drink, recognition of excessive drinking (referred to as guilt in the model), inclination to limit drinking, and commitment to abstain from drinking in the near future. Initial simulations to identify possible parameter value ranges indicate that this model is capable of showing periodic oscillating behavior.

We are focusing our current modeling efforts on the data of patient 6029. The patient sometimes acts in a way that cannot be explained by our current model. It may be wise to also incorporate some sort of stochasticity into the model to help account for the small fluctuations in daily behavior. Such random behavior could be incorporated into $\frac{dD(t)}{dt}$, $\frac{dQ(t)}{dt}$, and $\frac{dL(t)}{dt}$ to a lesser extent, as all three data constructs show consistent fluctuation that is not tied to the other constructs. Guilt data seems to be strongly related to alcohol consumption and does not appear to randomly fluctuate as much as other data. We expect these fluctuations to be relatively small in each variable but still have a noticeable cumulative effect on $A(t)$. Notice that during one weekend around day 32, patient 6029 drinks on average 10 drinks per day, which is higher than most weekends during the IVR period. If this patient's drinking behavior control system were completely deterministic, there would be identifiable factors that would help explain the heavy drinking; however, the complete IVR data set shows no large irregularities in responses in comparison to other weekends. While the current model reflects general behavior we see in the data, it still requires some tweaking to sufficiently match the data.

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A IVR Questionnaire

Here we have included the transcript of the IVR questionnaire to which the subjects of Project MOTION called in each day.

Thank you for participating in this study. During this interview, please press the pound or number key once you have entered your answer. If you would like to have the **current question repeated** at any time, press *7. If you would like to have the **previous question repeated**, please press *1 at any time. You may repeat a question as many times as you need to. If you would like to **pause the survey**, press *9. If you accidentally terminate the questionnaire, please call back immediately to continue.

Q1-11: ACTIVITIES SINCE YESTERDAY

For the next 11 questions, I'm going to ask you about things that you've done since this time yesterday. For each question, answer using a scale of 0 to 3 where:

- 0 = no,
- 1 = yes,
- 2 = yes - but last night only, or
- 3 = yes, both last night and today.

(Stressful events)

- 1) Did you have or nearly have an argument or disagreement with anyone?
- 2) Did anything else happen at home, work or school that you felt was stressful?
- 3) Did anything else happen to you that most people would consider stressful?

(Pleasant events)

- 4) Did you meet a goal or complete a task that left you with a sense of accomplishment?
- 5) Did you have a pleasant interaction with a family member?
- 6) Did you have a pleasant interaction with a friend or colleague?

(Encountered Drinking Situation)

- 7) Did anyone pressure you to drink?
- 8) Were you in a situation where you commonly drink?
- 9) Were you at a celebration or a party?
- 10) Were you at a nightclub or bar?
- 11) Were you on a date?

CURRENT MOOD

Now I'm going to ask you about your mood right now. On a scale from 0 to 4 where:

0 = **not at all**

1 = **slightly**

2 = **moderately**

3 = **very much**

4 = **extremely**

Right now, do you feel:

- 1) Active?
- 2) Sad?
- 3) Nervous?
- 4) Tense?
- 5) Lonely?
- 6) Happy?
- 7) Angry?
- 8) Enthusiastic?
- 9) Bored?
- 10) Tranquil?
- 11) Relaxed?

PERCEIVED STRESS

- 1) That you are unable to control the important things in your life?
- 2) Confident about your ability to handle your personal problems?
- 3) That things are going your way?
- 4) That difficulties are piling up so high that you can not overcome them?

YOUR DESIRE TO DRINK RIGHT NOW

Next I'm going to ask you about your desire to drink right now. Please answer the following questions rating your current desire to drink from 0 to 4, where:

0 = **definitely false**

1 = **slightly false**

2 = **neither true nor false**

3 = **slightly true**

4 = definitely true

Right now..

- 1) I really don't feel like drinking.
- 2) I feel like I could really use a drink.
- 3) The idea of drinking is appealing.

COMMITMENT AND CONFIDENCE

On a scale of 0 to 4, where:

0 = not at all

1 = somewhat

2 = moderately

3 = very

4 = totally

- 1) How confident are you that you can resist drinking heavily (that is, resist drinking more than 5 drinks) over the next 24 hours?
- 2) How committed are you not to drink heavily (that is, not to drink more than 5 drinks) over the next 24 hours?
- 3) How committed are you not to drink AT ALL over the next 24 hours?

DRINKS

Now I would like to ask you some questions about your drinking over the past day. Let's start with last night's drinking. Please indicate how many standard drinks you had in each beverage category **last night** - from the time you took the survey yesterday until you went to sleep.

How many beers did you drink last night? Remember, a standard drink is 12 ounces of beer: ____

0 = no beer

What kind of beer was it mainly?

1 = light beer

2 = regular beer

3 = ale or malt liquor

How many standard drinks of wine did you have last night? Remember, a standard drink is 5 ounces of wine: ____

0 = no wine

How many standard drinks of liquor did you have last night? Remember, a standard drink is 1 1/2 ounces of liquor: ____

0 = no liquor

Now let me ask you about your drinking **today**. Please indicate how many standard drinks you had in each beverage category today.

How many beers did you drink today? A standard drink is 12 ounces of beer: ____

0 = no beer

What kind of beer was it mainly?

1 = **light beer**

2 = **regular beer**

3 = **ale or malt liquor**

How many standard drinks of wine did you have today? A standard drink is 5 ounces of wine: _____

0 = **no wine**

How many standard drinks of liquor did you have today? A standard drink is 1 1/2 ounces of liquor: _____

0 = **no liquor**

ASSESSMENT OF DRINKING AS EXCESSIVE

Do you consider the total amount you have had to drink since this time yesterday to be excessive? That is, was it more than you think you should have had?

0 = **definitely not**

1 = **possibly**

2 = **probably**

3 = **definitely**

Please press **one** to exit this survey.

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