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UNIFIED MODEL FOR THE HEAT TRANSFER  
PROCESSES THAT OCCUR DURING CANNING, SSE AND ELECTRICAL RESISTANCE ASEPTIC  
PROCESSING OF FOOD PRODUCTS THAT CONTAIN LARGE PARTICLES

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MANUSCRIPT I

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CANNING, SSHE AND ELECTRICAL RESISTANCE ASEPTIC PROCESSING OF FOOD  
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## ABSTRACT

A unified general model for the heat transfer processes that occur within a food product subjected to canning or aseptic thermal treatment, is presented. Two principles are extensively used in the model building process: system segregation and energy balancing. The model is summarized in an algorithm, whose specification is showed for different combinations of processing system type (PST) and product formulation (PF) with a single particle type. A discussion on the practical relevance of proper product identification in the case of aseptic processing, is included. Finally, an illustration is given on the results that can be obtained from the model algorithm application, in a comparative study of different PST-PF combinations.

## INTRODUCTION

A food product may be thermally treated by pasteurization, conventional canning or aseptic processing. The purpose of aseptic processing is to endow the food product with commercial sterility, a condition in which the product is free of viable microorganisms with either public health significance, as well as those of non-health significance, capable of reproducing under normal non-refrigerated conditions of storage and distribution (FPI, 1989). When a food product is subjected to a thermal treatment, there are heat transfer processes that take place. The driving force of such processes is the temperature gradients within the product. This paper describes a model of the heat transfer processes in thermal treatment of a food product consisting of a fluid medium with large particles. An example would be beef stew. The model is general enough to describe aseptic processing as well as conventional canning. It is intended to be used for simulating aseptic processing, as a guide for making decisions

relating to the design of aseptic processing equipment and as the basis for estimating the sensitivity of the degree of sterilization and of the quality degradation to errors in process control, to variation in product formulation and to variability of the physical characteristics of the food material. An important feature of the model is its ability to estimate temperature at the slowest heating locations, which is difficult or impossible to measure with current techniques.

In developing the model, the system structure is represented by defining relevant components, and by defining the input-output relations (exchanges of energy) between the components, as well as the inputs and outputs for the overall system. Processes modeled within the components include fluid flow, heat diffusion within fluid and particulate phases, heat transfer between the phases and, for electrical resistance (ER) heating, conversion of electrical to heat energy (for discussions on the modeling approach, see Gold, 1985, Zeigler, 1976). In a subsequent paper, we will report on the structure of a computer program based on the model discussed here (Pastrana et. al, 1992b).

The modeling of the heat transfer processes that take place when a thermal treatment is applied to a particulate-laden food product was pioneered by de Ruyter and Brunet (1973) and by Mason and Cullen (1974). Sastry (1986) made a substantial contribution in this area, and Sastry (1988) presented an overview of modeling approaches and problems encountered. Sastry (1986, 1988) introduced the idea of using energy balances over incremental volumes in a heater (H), which consisted in a scraped surface heat exchanger (SSHE), and a holding tube or thermoequilibrator (THEQ), to obtain fluid medium temperatures. The same idea of energy balances to obtain fluid temperature was later applied, and extended to the cooler (C), by Chandarana and Gavin (1989a), Chandarana et. al (1989b), and Larkin (1990). Instead of using incremental energy balances to obtain carrier medium temperatures, some authors have applied average temperature profiles for the fluid, computed according to different equations:

Larkin et. al (1989), Armenante et. al (1990) and Lee et. al (1990). In constructing the model presented in this paper, extensive use was made of the idea of local energy balances:

A solid particle as a subsystem, is first considered as the union of mutually exclusive and exhaustive regions, then energy balances are established for each region to arrive at a system of ordinary differential equations (ODE), which describes the heat transfer by conduction taking place within the solid particle.

Since some of the equations in the ODE system (those that correspond to the solid particle surface) depend on the surrounding fluid temperature, there is a need to know or estimate that temperature. This may be done in three alternative ways: by direct measurement (although the most accurate, it requires that the system be physically constructed), by fluid energy balances on incremental volumes in the system equipment (Sastry, 1986; Chandarana and Gavin, 1989a; Chandarana et. al, 1989b; Larkin, 1990), and by assuming an average fluid temperature profile (Larkin, 1989; Armenante et. al, 1990; Lee et. al, 1990). In our model, we use the local energy balance principle following Sastry (1986), to obtain an estimate of the fluid temperature profile as the product flows through the system. We make the simplifying assumptions that the fluid is well mixed (in the radial direction for aseptic processing and in all directions for canning), and that there is piston (plug) flow throughout.

The model proposed in this paper differs from Sastry's (1986), in several respects:

In addition to the SSHE system, it includes electrical resistance aseptic and canning processing, and also adds the cooling stage.

It applies mean interstitial fluid velocity, as a normalizing constant in the thermoequilibrators velocity ratio (THEQVR) (Barry, 1991). A mean bulk product velocity is implied when mean bulk residence time is used as normalizing constant in the residence time ratio (RTR).

A target for the fastest heating zone (FHZ) temperature at heater exit ( $y_{e,f}^h$ ), is established as in Chandarana and Gavin (1989a), Chandarana et. al (1989b), Larkin (1989,1990) and Lee et. al (1990).

A target for the fluid temperature at system exit ( $y_{e,f}^c$ ), is established. A similar target for the warmest zone proved to be too strong a requirement for a particular product formulation (PF), under ER aseptic processing (Pastrana et. all, 1992b)

Irregular shapes are not considered for the solid particles, since any irregular shape can be included in an appropriate imaginary regular shape, such as a sphere or parallelepiped.

A subsequent paper (Pastrana et. all, 1992b) will report on the applications of the model to specific PF (beef in gravy without starch, and beef in gravy with starch having equal or greater electrical conductivity than the beef).

## THEORY

### SYSTEM DEFINITION AND STRUCTURE

We are concerned with modeling changes in a food product, which consists of particles in a fluid medium. The relevant changes which are induced by the thermal treatment (canning, ER or SSHE aseptic thermal treatment) include microbial and spore load, enzyme concentration, nutrient retention and other measures of food quality. The thermal treatment consists of the following stages (shown with abbreviations which will be used): heating (H), thermoequilibrium (THEQ), and cooling (C). The H stage consists in the application of a heat source by means of pressurized steam (canning and SSHE aseptic processing) or an electrical current (ER aseptic processing). The product temperature at any point is expected to increase during the H stage. The THEQ stage follows immediately the H stage and consists of a holding stage during which the product is expected to reach thermal equilibrium, in which thermal gradients would disappear. The C stage follows immediately the THEQ stage. It consists of applying a heat sink by means of cooling water, so that the product temperature at any point is expected to decrease.

The thermal state of the system at any given time is specified by the product temperature distribution. The devices associated with each stage for the different types of thermal treatment are as follows:

STAGE	DEVICE	
	<u>Canning</u>	<u>Aseptic</u>
H	Rotating retort at temperature below retort temperature (RT) and under pressurized steam.	SSHE or ER heater.
THEQ	Rotating retort at temperature equal to RT, and under pressurized steam.	Stainless steel insulated tube.
C	Rotating retort under cooling water.	SSHE.

FIG.1 shows a flowchart of the general system:

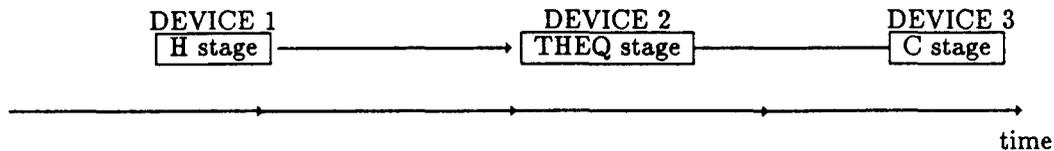


FIG. 1: Flowchart of the general system

## SYSTEM INPUTS, OUTPUTS AND ENVIRONMENT

The input to the system is energy. For canning or SSHE aseptic processing, the main input energy is in the form of heat transfer from a heating medium. For ER aseptic processing, the main input energy is in the form of electrical energy delivered by subjecting the product to an alternating electrical current. The output from the system is energy. The main output energy is in the form of heat transfer to the cooling medium. The environment is considered to be everything apart from the system that may transfer heat to, or receive heat from, the system. In particular, the environment includes the supporting systems needed to preheat the product, raise the heating medium temperature and lower the cooling medium temperature.

## SYSTEM COMPONENTS, THEIR INPUTS AND OUTPUTS

### Component 1

It includes DEVICE 1 (FIG. 1) plus the particulate-laden product being heated. The input is the same as the system energy input and the output is heat transferred to component 2 through a heated product.

### Component 2

It includes DEVICE 2 plus the heated particulate-laden product being already in thermoequilibrium. The input is the same as the output from component 1 and the output is heat transferred to component 3 in the form of a food product in thermoequilibrium.

### Component 3

It includes DEVICE 3 plus the particulate-laden product in thermoequilibrium being cooled. The input is equal to the output from component 2 and the output coincides with the system energy output.

The heating stage for canning and SSHE aseptic processing involves convective and conductive heat transfer processes as follows:

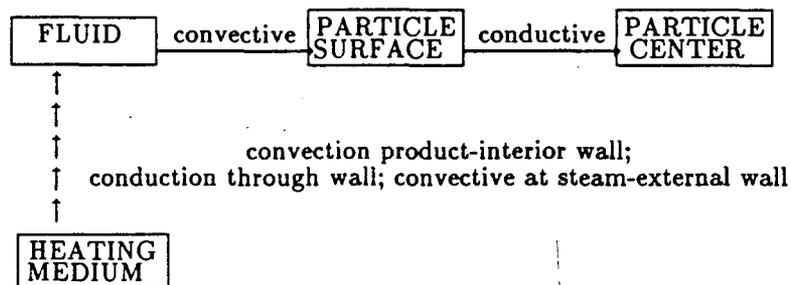


FIG. 2: Heat transfer during canning and SSHE H stage

The product regions that appear in FIG. 2, can be classified as either donors or receptors of heat: the fluid is a heat donor to the particle surface, and the particle surface is a heat donor to the particle center during the heating stage. During cooling, the heat donor condition in FIG.2, is replaced by a heat receptor condition (FIG. 3):

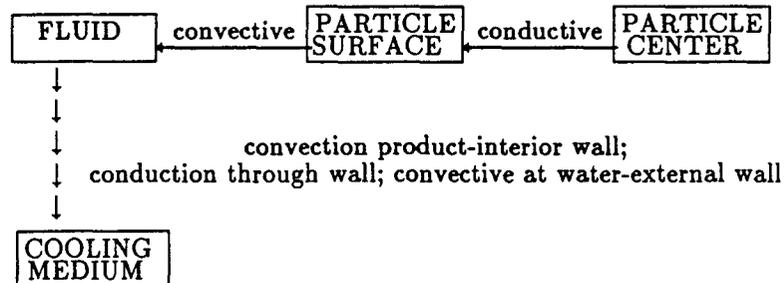


FIG. 3: Heat transfer during canning SSHE C stage

When the fluid is the FHZ during ER heating, the fluid and particle surface are also heat donors as for SSHE heating (FIG. 1), except that in that case the heating medium is the product itself.

FIG. 4 shows the convective and conductive heat transfer processes for the THEQ stage; the arrows go from fluid to ambient for aseptic processing, and from constant steam temperature to fluid for canning:

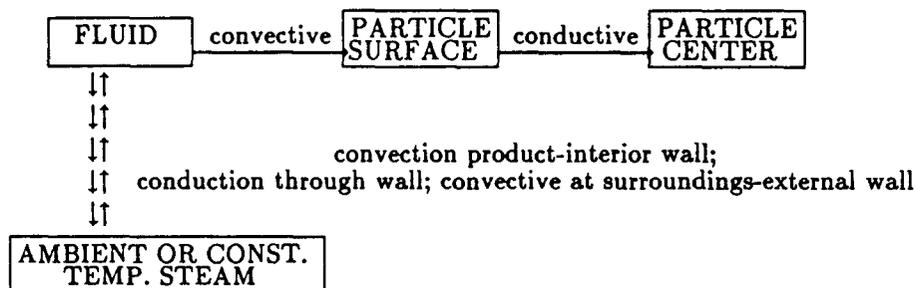


FIG. 4: Heat transfer during THEQ stage

## MODEL BUILDING PROCESS

### Introduction

The model is designed to yield estimates , at any point in the processing system, of variables which depend upon temperature history of the food product. Some important variables which we consider are:

#### Nutritional quality variables:

Nutrient percent concentration.

Point or integrated nutrient equivalent thermal destruction times.

#### Sterility variables:

Spores percent concentration.

Point or integrated spore equivalent thermal destruction times.

#### Chemical variables:

Enzyme percent concentration

Point or integrated enzyme equivalent thermal destruction time

As already described, the system consists of a particulate-laden food product subject to a thermal treatment that comprises three stages, each stage carried out in a particular DEVICE. In canning, a specific food product volume is well identified because it is contained in a hermetically closed container ( a 211 × 214 tin plate can). However, for SSHE aseptic and ER aseptic processing, the volume of food product, whose state and quality are of concern, is not well identified, since volume elements mix with each other as the food travels through the equipment. However, when the main interest centers on determining the fastest solid particle thermal state ( that is, its temperature), then the food product volume identification is not a serious problem, as long as two conditions hold: first, the solid particle considered for modeling

purposes is the one that contains the product's slowest heating zone (SHZ) , and second, the surrounding fluid temperature is known. As indicated in the INTRODUCTION of this paper, three ways to generate a carrier fluid temperature profile are: by direct measurement; by computation of fluid energy balances, and by assuming an average temperature profile. The second, which is the one followed in this paper, idealizes, for the purpose of generating a carrier fluid temperature profile, the existence of a thermodynamic control volume (Van Wylen and Sonntag, 1985). The coordinate system that allows the volume localization is a translating system along the horizontal axis (imaginary incremental volume<sup>1</sup>, assumed to move horizontally in a horizontal processing system).

Although the control volume contains, at a given instant, a thermally treated product, it is true that the product is still not well identified, since mass gets in and out the control volume. Should plug (piston) flow hold throughout for aseptic processing, then the problem of control-volume product identification would disappear.

#### Modeling assumptions

The following assumptions are made in order to simplify the model:

- a) All solid particles are identical with respect to size , shape, and other relevant characteristics.
- b) The product fluid in the reference volume is well mixed, so that fluid temperature is uniform.
- c) For canning, the resistance to heat transfer offered by the metal can wall, is ignored.
- d) For aseptic processing, H stage exit FHZ temperature and C stage (system) exit fluid temperature equal to their targets, as indicated in the INTRODUCTION, and are set by

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<sup>1</sup>The volume of a 211 × 214 tin plate can, and of the hypothetical incremental volume are here on referred to as "reference volume".

the operator. For canning, constant pressurized steam and cooling water temperatures, were fixed; although the same fluid exit temperature target was required as for aseptic processing, no corresponding target was set, at the H stage exit.

- e) Initial temperature distribution within the reference volume is uniform and equal to a constant  $y_I$  at every point in the product.
- f) For aseptic processing, a particular configuration is assumed where devices are straight, lined up horizontally, directly connected one after the other, and with no bends.
- g) For ER heating, the following specific assumptions are made: all electrical energy is converted into thermal energy; the effect on temperature of the particle orientation relative to the electrical field lines is negligible (reasonable for cubic shapes); the ratio between the solid and fluid electrical conductivities is invariant with temperature.
- h) Applicable  $h_{fp}$  is the same for canning as for aseptic processing.

#### Basic modeling principles

Two basic principles have been applied in the modeling building process. These are appropriate segregation of the system, and thermodynamic modeling of local energy balances.

Appropriate segregation of the system:

First, the system is divided into three components, as indicated in the section SYSTEM COMPONENTS, THEIR INPUTS AND OUTPUTS.

Second, within each component, one of the identical (see assumption a)) solid particles is considered. In aseptic processing, the particle considered is the fastest moving particle.

Third, a volume of product containing this particle is considered.

The segregation is carried further by segregating the food particle, into mutually exclusive and exhaustive regions, where each region within the solid particle is

considered to be a thermodynamic subsystem (a partition of the solid particle that is suitable for numerical integration, is convenient here). The volume of product that contains the fastest moving solid particle, is also segregated into two thermodynamic subsystems: the fluid phase and the solid phase; the latter consists of the solid particles (beef cubes), each being partitioned identically.

Local energy balances:

An energy balance is an equality between the sum of the rates of energy inputs (sources), and the sum of the rates of energy uses. It is a generalization of the work-energy theorem of mechanics, which sometimes is referred to as the general form of the first law of Thermodynamics (Sears and Salinger, 1986).

Possible energy sources are:

Energy input ( $E_{in}$ ) such as heat transfer input and energy generated ( $E_g$ ) by the system resistance to an electrical current.

Possible energy uses are:

Energy output ( $E_{out}$ ) such as heat transfer output and thermal energy ( $E_s$ ) stored in the form of internal energy.

An energy balance takes the form(Myers, 1976):

$$\dot{E}_{in} + \dot{E}_g = \dot{E}_{out} + \dot{E}_s$$

For a thermodynamic subsystem during canning or SSHE aseptic processing:

$\dot{E}_{in} = \dot{E}_{out} + \dot{E}_s$ , since there is no heat generation in such cases (no electric current is applied, as during ER heating).

During ER heating:

$\dot{E}_g = \dot{E}_{out} + \dot{E}_s$ , since there is no heat transfer applied in such case.

Following Myers (1976), the different rates in the energy balance equation are given

by:

$$\dot{E}_{in} \text{ or } \dot{E}_{out} = \begin{cases} -kA \frac{\partial y}{\partial \theta}, \text{ where } \theta = x_1, x_2, x_3 \\ -h A (\bar{y} - y_f) \end{cases}$$

$$\dot{E}_g = g'''V$$

$$\dot{E}_s = \rho V \gamma \frac{\partial y}{\partial t}$$

Model algorithms to obtain reference volume temperature spatial distribution

The following algorithm allows the estimation of the temperature distribution in the reference volume for canning and, under the assumption that THEQVR=1, for aseptic processing. For the first algorithm iteration,  $t_i$  is set equal to 0:

- a) Consider the product in the reference volume to be subjected to a thermodynamic process that consists of heating (if product is in DEVICE 1), thermoequilibrium (if product is in DEVICE 2), or cooling (if product is in DEVICE 3) from time  $t_i$  to time  $t_e$ ,  $t_e = t_i + \delta t$ , where  $\delta t$  is a time increment (also called variable time step). Assume plug flow for aseptic processing.
- b) Partition each food particle in the reference volume identically into disjoint and exhaustive regions, and consider each region within the food particle to be a thermodynamic subsystem.
- c) Establish energy balances for each region included in the food particle, and express them as a system of ordinary differential equations (ODE's). Since the energy balances for the regions of the food particle that contain a portion of the particle surface, depend on  $y_f$ , there is one more unknown than equations, so that an additional ODE is needed, or one of the variables (fluid or local regions temperature) must be known, in order to solve the system of ODE's.

- d) Obtain the needed extra ODE (see c) above), by performing an energy balance on the reference volume fluid.
- e) Solve the system of ODE's, storing  $y_f$  in a vector of fluid temperatures  $y_f$  for the aseptic processing case, so that the fastest particle surrounding fluid temperature can be computed later. These stored fluid temperatures will be referred to as plug flow fluid temperatures. In the numerical integration, a variable time step method that uses 4<sup>th</sup> and 5<sup>th</sup> order Runge Kutta schemes may be applied (Pastrana et. al, 1992c). At time  $t_{out}$  this gives an estimate of the spatial temperature distribution.
- f) Set  $t_i = t_e$ , and take last temperature estimates as initial estimates.
- g) Repeat the whole process until  $t_e$  becomes equal to the processing system exit time.

For aseptic processing, the temperature spatial distribution of the fastest food particle is computed by applying the following algorithm, first setting  $t_i = 0$ :

- a) Consider the product enclosed in a reference volume  $Q \times \delta t$  that contains the fastest particle, where  $Q$  is the volumetric flow rate, and a thermodynamic process (heating, thermoequilibrium or cooling) from  $t_i$  to  $t_e$ ,  $t_e = t_i + \delta t$ , on that product.
- b) Partition the fastest food particle into disjoint and exhaustive regions, and consider each region included in the food particle to be a thermodynamic subsystem.
- c) Establish energy balances for each of the regions of the fastest particle, expressing them as a system of ODE's.
- d) Find the temperature of the fluid surrounding the fastest particle by applying the stored plug flow fluid temperature profile:
- fastest particle position = particle velocity  $\times$   $t_i$
- plug flow fluid corresponding time =  $\frac{\text{particle position}}{\text{fluid velocity}}$

The surrounding fluid for the fastest particle, has a temperature approximately equal to the stored plug flow fluid temperature that corresponds to the above plug flow fluid time (entry in  $y_f$  associated to time less or equal to such plug flow time).

- e) Solve the system of ODE's applying the surrounding fluid temperature obtained in the previous step. At time  $t_e$  this gives an estimate of the spatial temperature distribution of the product enclosed in the reference volume containing the fastest particle.
- f) Set  $t_i = t_e$  and take the last temperature estimates as initial estimates.
- g) Repeat the whole process until  $t_e$  is equal to the processing system exit time.

#### Residence time considerations in aseptic processing

The stored plug flow fluid temperatures allow the computation of the temperature distribution for the fastest particle, under various assumptions concerning residence time at high product flow rates:

- a) Plug flow in each device. This could be a reasonable scenario when the product flow is turbulent throughout.
- b) Plug flow in the H and the C stages, but bimodal normally distributed residence time ratio (RTR) in the THEQ. This scenario may be appropriate when the flow is turbulent in the H and C. There is experimental evidence, such as with the PF's considered by us, that the RTR distribution is likely to be bimodal normal in the THEQ (Berry, 1989; Dutta and Sastry, 1990; Palmieri, 1991). This is the residence time scenario chosen for the model applications (Pastrana et. al, 1992b).
- c) Exponentially distributed RTR (perfectly mixed model) in the SSHE (H and C stages), and bimodal normally distributed RTR in the THEQ. The exponential distribution may be an appropriate approximation to the RTR distribution in the SSHE, when there is mixing in both the radial and axial direction (Defrise and Taeymans, 1988;

Taeymans et. al, 1985). As pointed out in b) immediately above, there is experimental evidence that suggests the possibility of a bimodal normal RTR in the THEQ, for the PF's considered by us. This scenario may be modified by assuming plug flow in the THEQ, when the flow there, is turbulent.

- d) Plug flow in the electrical resistance (ER) heater, and THEQ, and exponentially distributed RTR in the C. This residence time scenario may be appropriate when there is turbulent flow in the ER heater and THEQ, and there is mixing in the radial and axial direction in the C.

#### Model equations

The model equations consist of a system of ODE's derived from the energy balances performed according to the algorithms described previously. The specific form of the equations depend on several factors:

Particle shape. This may be regular (spherical, cubic, etc.) or irregular.

Manner in which the particle is segregated into regions. The type of segregation depends on the particle shape, and presupposes a strategy to solve the system of ODE's (finite difference, finite element, etc.).

Region location within the particle. The region location is described by the corresponding node location: in the interior, at the boundary and/or surface. In the case of the cubic shape, considered in the model applications (Pastrana et. al, 1992b, 1992c, 1992d), surface regions correspond to: non-edge, edge but not at the corner, and corner nodes.

Thermal treatment stage. The heat transfer processes change, and there are different pertinent parameters in each stage.

The system of ODE's shown below is for the H stage of a product that consists of cubic particles (beef cubes) in a fluid (gravy with or without starch). Under the assumption of

uniform fluid temperature, there is symmetry in the convective heat transfer from (to) the fluid to (from) the particle faces, which allows consideration of just one octant of the cube in setting the system of ODE's. The cube octant was segregated into small volume units or regions, by applying the finite difference method (Myers, 1971).

To illustrate, when the octant edge is divided into 3 congruent segments, the number of discrete points on the octant edge ( $n$ ) is 4: two endpoints and two interior points. Letting  $(i,j,k)$  represent an arbitrary point in the resulting octant grid,  $i,j,k=1,2,3$  or 4, there are 64 points  $(i,j,k)$ , called nodes, each of which can be monitored as far as spore, enzyme, and nutrient thermal destruction. The idea is to assign an octant volume unit, that is, an octant region, to each node, and assume uniform thermal conditions for the octant region. Following Myers's nodal point arrangement (Myers, 1971), octant regions are assigned to the nodes so that any octant cross section, yields regions in two dimensions, as described in FIG. 5A. The regions assigned to nodes  $(1,1,1)$ , and  $(n,n,n)$  (lower left and upper right regions in FIG. 5B) are of particular interest in the applications, because one of them corresponds to the fastest beef cube SHZ, while the other to the SHZ (See ILLUSTRATION Section in this paper, and Pastrana et. al, 1992b and d).

The volume of a region assigned to a node included in any face of the cube octant, is just a fraction of the common volume of an interior (non-face) region:  $\frac{1}{8}$  if node is at the face corner,  $\frac{1}{4}$  if node is at the face edge but is not in the corner, and  $\frac{1}{2}$  if node is in the face but not at the edge. This fact was carefully considered when establishing the energy balance of any region assigned to a node in the face of the cube octant.

There are six faces in a beef cube octant; three of them have direct contact with the fluid, and the other three, have direct contact with neighboring octant cubes. Regions in any of the former three faces, correspond to nodes  $(i,j,k)$  with at least one of  $i, j$  and  $k$  equal to 1; the energy balances for these regions must include a convective boundary condition. Regions in any of the latter three faces, correspond to nodes  $(i,j,k)$  with at least one of  $i, j$  and  $k$  equal to  $n$ ; the energy balances for these regions include a symmetric boundary condition.

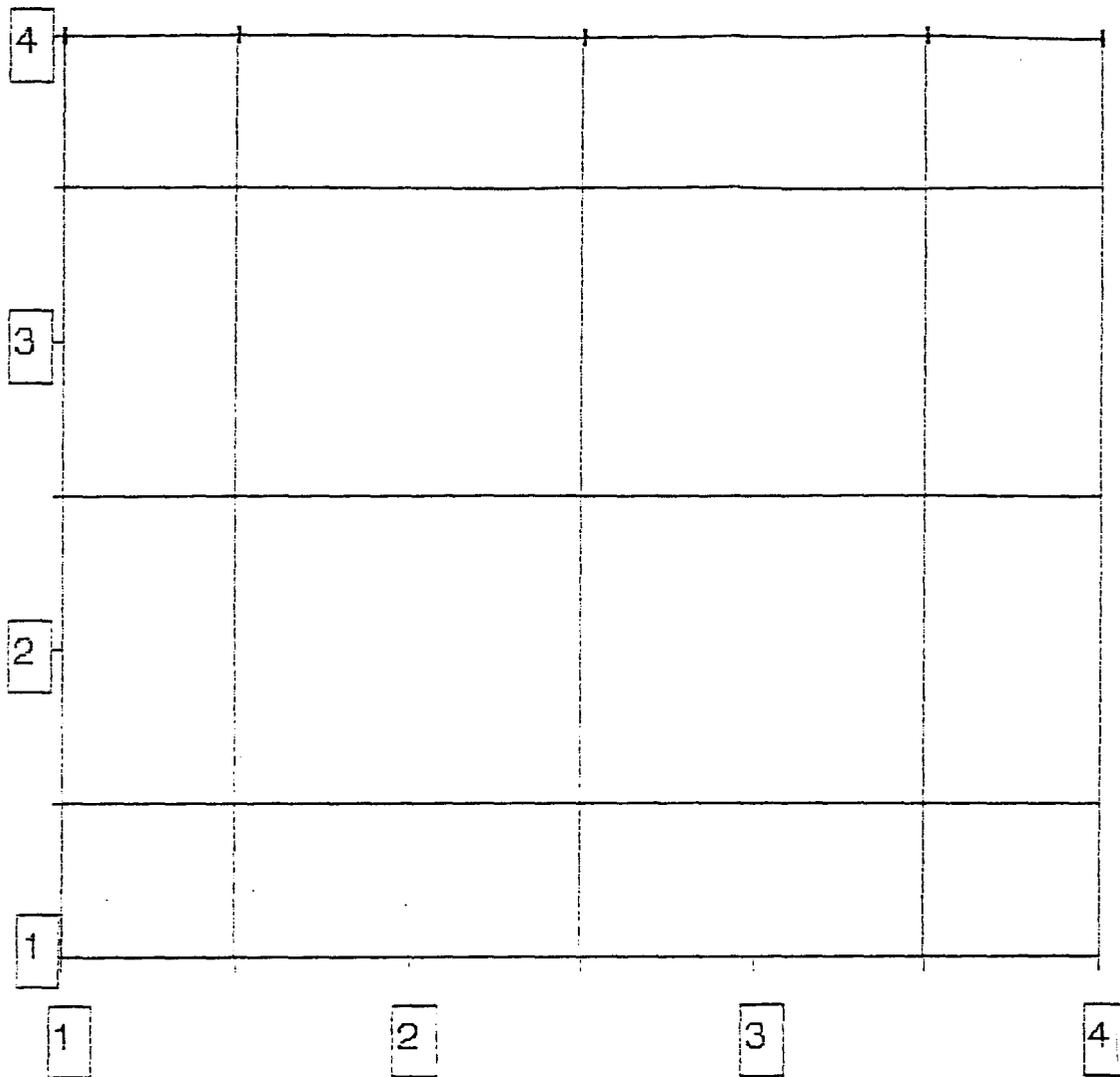


FIG. 5A: Two dimensional regions obtained by any cross section of the cube octant.

Axis identification is as follows, assuming that the cube is centered at the origin, and its sides lie on the coordinate system axes:

For a horizontal cross section ( $k$  fixed),  $I$  is the horizontal axis, and  $J$  is the vertical axis.

For a vertical cross section ( $I$  fixed),  $J$  is the horizontal axis, and  $k$  is the vertical axis.

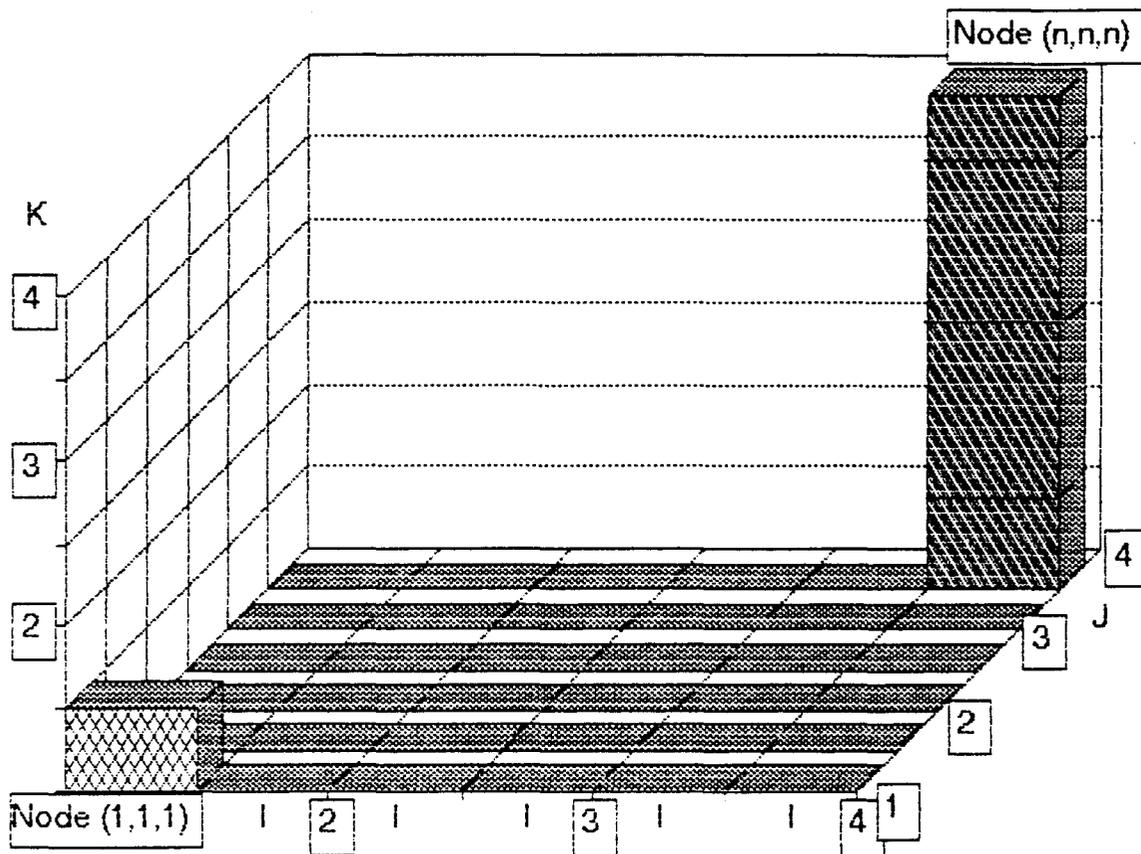


FIG. 5B: Regions within the cube octant that correspond to nodes  $(1,1,1)$  and  $(n,n,n)$ . Each octant side is partitioned into 3 congruent segments, so that there are  $n=4$  nodes on each octant edge.

The energy balances for the fluid and regions within a particle, established according to the above algorithms steps, include when applicable, the rate of thermal energy generated, the rate of heat transferred by convection to or from the fluid, the rate of heat transferred by conduction to or from neighboring particle regions, and the rate of energy stored in the form of internal energy. As an example, the fluid energy balance includes the following energy sources and uses:

**Sources:**

Thermal energy generated (as in the ER).

Heat transferred by convection from a heating medium (as in canning, SSHE, and depending on PF, in ER processing).

**Uses :**

Energy stored in the form of internal energy.

Energy transferred by convection to the particles (as in canning and SSHE and, depending on PF, in ER processing).

The equations that appear below were derived by Jose Pastrana. In them,  $us=0$  for the canning H stage and SSHE aseptic processing (since no electrical current is applied),  $U_h = 0$  for ER heating (there is no heat transfer through the ER wall), and the parameters  $m_{\delta t}$ ,  $A_{\delta t}$ ,  $N_{\delta t}$ ,  $V_{\delta t}$ , and  $y_{st}$  depend on the processing system considered.

**Derivative of fluid temperature**

$$\frac{\partial y_f}{\partial t} = 1.0 / (m_{\delta t} \times \gamma_f) \times (U_h \times A_{\delta t} \times (y_{st} - y_f) - h_{fp} \times 24.0 \times l^2 \times N_{\delta t} \times (y_f - \bar{y})) + \\ ((V_{\delta t} / (A_{er} \times L_{er}))^2) \times us \times \sigma_f \times (1.0 + b_f(y_f - 25.0)) \times \varepsilon \times (A_{er}^2) / V_{\delta t}$$

In the case of SSHE and ER aseptic processing, this derivative depends on the time step  $\delta t$ , since the volume of fluid (included in the reference volume product), depends on  $\delta t$ . The necessary steps to obtain this equation are presented in an APPENDIX at the end of this Manuscript.

### Derivatives of cube octant temperatures

a) At octant interior nodes:

$$\begin{aligned} \frac{\partial y}{\partial t}(i,j,k) = & (\alpha/\delta^2) \times (y(i-1,j,k) - 6.0 \times y(i,j,k) + y(i+1,j,k) + \\ & y(i,j-1,k) + y(i,j+1,k) + y(i,j,k-1) + y(i,j,k+1) + \\ & ((\delta/L_{er})^2) \times vs \times \sigma_p \times (1.0 + m_p \times (y(i,j,k) - 25.0)) / k_p, \quad i,j,k=2, n-1 \end{aligned}$$

b) At nodes in the three cube octant faces that have direct contact with the fluid. A convective boundary condition was considered when establishing the corresponding energy balances.

b1) Nodes not at the edges:

$$\begin{aligned} \frac{\partial y}{\partial t}(1,j,k) = & (\alpha/\delta^2) \times (2.0 \times Bi \times (y_f y(1,j,k)) + y(1,j-1,k) - \\ & 6.0 \times y(1,j,k) + y(1,j+1,k) + y(1,j,k-1) + y(1,j,k+1) + 2.0 \times y(2,j,k) + \\ & 4.0 \times ((\delta/(2.0 \times L_{er}))^2) \times vs \times \sigma_p \times (1.0 + m_p \times (y(1,j,k) - 25.0)) / k_p, \quad j,k=2, n-1 \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(i,j,1) = & (\alpha/\delta^2) \times (2.0 \times Bi \times (y_f y(i,j,1)) + y(i-1,j,1) - \\ & 6.0 \times y(i,j,1) + y(i+1,j,1) + y(i,j-1,1) + y(i,j+1,1) + 2.0 \times y(i,j,2) + \\ & ((\delta/L_{er})^2) \times vs \times \sigma_p \times (1.0 + m_p \times (y(i,j,1) - 25.0)) / k_p, \quad i,j=2, n-1 \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(i,1,k) = & (\alpha/\delta^2) \times (2.0 \times Bi \times (y_f y(i,1,k)) + y(i-1,1,k) - \\ & 6.0 \times y(i,1,k) + y(i+1,1,k) + y(i,1,k-1) + y(i,1,k+1) + 2.0 \times y(i,2,k) + \\ & ((\delta/L_{er})^2) \times vs \times \sigma_p \times (1.0 + m_p \times (y(i,1,k) - 25.0)) / k_p, \quad i,k=2, n-1 \end{aligned}$$

b2) Nodes at edges, but not in the corners. A symmetric boundary condition was considered, in addition to the convective one, when establishing the energy balances for the regions assigned to those nodes included also in any of the faces that have direct contact with neighboring octant cubes. The symmetry boundary condition was applied by imposing equalities such as:

### Derivatives of cube octant temperatures

a) At octant interior nodes:

$$\begin{aligned} \frac{\partial y}{\partial t}(i,j,k) = & (\alpha/\delta^2) \times (y(i-1,j,k) - 6.0 \times y(i,j,k) + y(i+1,j,k) + \\ & y(i,j-1,k) + y(i,j+1,k) + y(i,j,k-1) + y(i,j,k+1) + \\ & ((\delta/L_{er})^2) \times vs \times \sigma_p \times (1.0 + m_p \times (y(i,j,k) - 25.0)) / k_p, \quad i,j,k=2, n-1 \end{aligned}$$

b) At nodes in the three cube octant faces that have direct contact with the fluid. A convective boundary condition was considered when establishing the corresponding energy balances.

b1) Nodes not at the edges:

$$\begin{aligned} \frac{\partial y}{\partial t}(1,j,k) = & (\alpha/\delta^2) \times (2.0 \times Bi \times (y_f y(1,j,k)) + y(1,j-1,k) - \\ & 6.0 \times y(1,j,k) + y(1,j+1,k) + y(1,j,k-1) + y(1,j,k+1) + 2.0 \times y(2,j,k) + \\ & 4.0 \times ((\delta/(2.0 \times L_{er}))^2) \times vs \times \sigma_p \times (1.0 + m_p \times (y(1,j,k) - 25.0)) / k_p, \quad j,k=2, n-1 \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(i,j,1) = & (\alpha/\delta^2) \times (2.0 \times Bi \times (y_f y(i,j,1)) + y(i-1,j,1) - \\ & 6.0 \times y(i,j,1) + y(i+1,j,1) + y(i,j-1,1) + y(i,j+1,1) + 2.0 \times y(i,j,2) + \\ & ((\delta/L_{er})^2) \times vs \times \sigma_p \times (1.0 + m_p \times (y(i,j,1) - 25.0)) / k_p, \quad i,j=2, n-1 \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(i,1,k) = & (\alpha/\delta^2) \times (2.0 \times Bi \times (y_f y(i,1,k)) + y(i-1,1,k) - \\ & 6.0 \times y(i,1,k) + y(i+1,1,k) + y(i,1,k-1) + y(i,1,k+1) + 2.0 \times y(i,2,k) + \\ & ((\delta/L_{er})^2) \times vs \times \sigma_p \times (1.0 + m_p \times (y(i,1,k) - 25.0)) / k_p, \quad i,k=2, n-1 \end{aligned}$$

b2) Nodes at edges, but not in the corners. A symmetric boundary condition was considered, in addition to the convective one, when establishing the energy balances for the regions assigned to those nodes included also in any of the faces that have direct contact with neighboring octant cubes. The symmetry boundary condition was applied by imposing equalities such as:

$$y(1,n,k)=y(1,n+1,k) \text{ and}$$

$$y(i,1,n)=y(i,1,n+1).$$

$$\begin{aligned} \frac{\partial y}{\partial t}(1,1,k) &= (\alpha/\delta^2) \times (4.0 \times \text{Bi} \times (y_f - y(1,1,k)) + 2.0 \times y(1,2,k) - \\ & 6.0 \times y(1,1,k) + y(1,1,k-1) + y(1,1,k+1) + 2.0 \times y(2,1,k) + \\ & 4.0 \times ((\delta/(2.0 \times L_{er}))^2) \times \text{vs} \times \sigma_p \times (1.0 + m_p \times (y(1,1,k) - 25.0))/k_p), \quad k=2, n-1 \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(1,n,k) &= (\alpha/\delta^2) \times (2.0 \times \text{Bi} \times (y_f - y(1,n,k)) + \\ & 4.0 \times y(1,n-1,k) - 8.0 \times y(1,n,k) + y(1,n,k-1) + y(1,n,k+1) + 2.0 \times y(2,n,k) + \\ & 4.0 \times ((\delta/(2.0 \times L_{er}))^2) \times \text{vs} \times \sigma_p \times (1.0 + m_p \times (y(1,n,k) - 25.0))/k_p), \quad k=2, n-1 \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(1,j,1) &= (\alpha/\delta^2) \times (4.0 \times \text{Bi} \times (y_f - y(1,j,1)) + 2.0 \times y(1,j,2) - \\ & 6.0 \times y(1,j,1) + y(1,j-1,1) + y(1,j+1,1) + 2.0 \times y(2,j,1) + \\ & 4.0 \times ((\delta/(2.0 \times L_{er}))^2) \times \text{vs} \times \sigma_p \times (1.0 + m_p \times (y(1,j,1) - 25.0))/k_p), \quad j=2, n-1 \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(1,j,n) &= (\alpha/\delta^2) \times (2.0 \times \text{Bi} \times (y_f - y(1,j,n)) + 4.0 \times y(1,j,n-1) - \\ & 8.0 \times y(1,j,n) + y(1,j-1,n) + y(1,j+1,n) + 2.0 \times y(2,j,n) + \\ & 4.0 \times ((\delta/(2.0 \times L_{er}))^2) \times \text{vs} \times \sigma_p \times (1.0 + m_p \times (y(1,j,n) - 25.0))/k_p), \quad j=2, n-1 \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(i,1,1) &= (\alpha/\delta^2) \times (4.0 \times \text{Bi} \times (y_f - y(i,1,1)) + 2.0 \times y(i,2,1) - \\ & 6.0 \times y(i,1,1) + y(i-1,1,1) + y(i+1,1,1) + 2.0 \times y(i,1,2) + \\ & ((\delta/L_{er})^2) \times \text{vs} \times \sigma_p \times (1.0 + m_p \times (y(k,1,1) - 25.0))/k_p), \quad i=2, n-1 \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(i,n,1) &= (\alpha/\delta^2) \times (2.0 \times \text{Bi} \times (y_f - y(i,n,1)) + 2.0 \times y(i,n,2) - \\ & 8.0 \times y(i,n,1) + 4.0 \times y(i,n-1,1) + y(i-1,n,1) + y(i+1,n,1) + \\ & ((\delta/L_{er})^2) \times \text{vs} \times \sigma_p \times (1.0 + m_p \times (y(k,nn,1) - 25.0))/k_p), \quad i=2, n-1 \end{aligned}$$

c) At nodes in the three octant faces that have direct contact with neighboring cube octants. A symmetric boundary condition was considered when establishing the corresponding energy balances. The symmetric boundary condition was applied by imposing equalities such as:

$$y(n,1,k)=y(n+1,1,k) \text{ and}$$

$$y(n,n,k)=y(n,n+1,k).$$

c1) Nodes not at the edges.

$$\begin{aligned} \frac{\partial y}{\partial t}(n,j,k) &= (\alpha/\delta^2) \times (y(n,j-1,k) - 8.0 \times y(n,j,k) + \\ & y(n,j+1,k) + y(n,j,k-1) + y(n,j,k+1) + 4.0 \times y(n-1,j,k) + \\ & 4.0 \times ((\delta/(2.0 \times L_{er}))^2) \times vs \times \sigma_p \times (1.0 + m_p \times (y(n,j,k) - 25.0)) / k_p), \quad j,k=2, n-1 \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(i,j,n) &= (\alpha/\delta^2) \times (4.0 \times y(i,j,n-1) - 8.0 \times y(i,j,n) + \\ & y(i-1,j,n) + y(i+1,j,n) + y(i,j-1,n) + y(i,j+1,n) + \\ & ((\delta/L_{er})^2) \times vs \times \sigma_p \times (1.0 + m_p \times (y(i,j,n) - 25.0)) / k_p), \quad i,j=2, n-1 \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(i,n,k) &= (\alpha/\delta^2) \times (y(i,n,k-1) - 8.0 \times y(i,n,k) + \\ & y(i,n,k+1) + y(i-1,n,k) + y(i+1,n,k) + 4.0 \times y(i,n-1,k) + \\ & ((\delta/L_{er})^2) \times vs \times \sigma_p \times (1.0 + m_p \times (y(i,n,k) - 25.0)) / k_p), \quad i,k=2, n-1 \end{aligned}$$

c2) Nodes at the edges, but not in the corners. A convective boundary condition was considered, in addition to the symmetric one, when establishing the energy balances corresponding to regions assigned to those nodes included also in the faces that have direct contact with the fluid.

$$\begin{aligned} \frac{\partial y}{\partial t}(n,1,k) &= (\alpha/\delta^2) \times (2.0 \times \text{Bi} \times (y_f - y(n,1,k))) + 2.0 \times y(n,2,k) - \\ & 8.0 \times y(n,1,k) + y(n,1,k-1) + y(n,1,k+1) + 4.0 \times y(n-1,1,k) + \\ & 4.0 \times ((\delta/(2.0 \times L_{er}))^2) \times v_s \times \sigma_p \times (1.0 + m_p \times (y(n,1,k) - 25.0)) / k_p, \quad k=2, n-1 \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(n,n,k) &= (\alpha/\delta^2) \times (4.0 \times y(n,n-1,k) - 10.0 \times y(n,n,k) + \\ & y(n,n,k-1) + y(n,n,k+1) + 4.0 \times y(n-1,n,k) + \\ & 4.0 \times ((\delta/(2.0 \times L_{er}))^2) \times v_s \times \sigma_p \times (1.0 + m_p \times (y(n,n,k) - 25.0)) / k_p, \quad k=2, n-1 \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(n,j,1) &= (\alpha/\delta^2) \times (2.0 \times \text{Bi} \times (y_f - y(n,j,1))) + 2.0 \times y(n,j,2) - \\ & 8.0 \times y(n,j,1) + y(n,j-1,1) + y(n,j+1,1) + 4.0 \times y(n-1,j,1) + \\ & 4.0 \times ((\delta/(2.0 \times L_{er}))^2) \times v_s \times \sigma_p \times (1.0 + m_p \times (y(n,j,1) - 25.0)) / k_p, \quad j=2, n-1 \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(n,j,n) &= (\alpha/\delta^2) \times (4.0 \times y(n,j,n-1) - 10.0 \times y(n,j,n) + \\ & y(n,j-1,n) + y(n,j+1,n) + 4.0 \times y(n-1,j,n) + \\ & 4.0 \times ((\delta/(2.0 \times L_{er}))^2) \times v_s \times \sigma_p \times (1.0 + m_p \times (y(n,j,n) - 25.0)) / k_p, \quad j=2, n-1 \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(i,1,n) &= (\alpha/\delta^2) \times (2.0 \times \text{Bi} \times (y_f - y(i,1,n))) + 4.0 \times y(i,1,n-1) - \\ & 8.0 \times y(i,1,n) + 2.0 \times y(i,2,n) + y(i-1,1,n) + y(i+1,1,n) + \\ & ((\delta/L_{er})^2) \times v_s \times \sigma_p \times (1.0 + m_p \times (y(i,1,n) - 25.0)) / k_p, \quad i=2, n-1 \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(i,n,n) &= (\alpha/\delta^2) \times (4.0 \times y(i,n,n-1) - 10.0 \times y(i,n,n) + \\ & y(i-1,n,n) + y(i+1,n,n) + 4.0 \times y(i,n-1,n) + \\ & ((\delta/L_{er})^2) \times v_s \times \sigma_p \times (1.0 + m_p \times (y(i,n,n) - 25.0)) / k_p, \quad i=2, n-1 \end{aligned}$$

d) At corner nodes.

$$\begin{aligned} \frac{\partial y}{\partial t}(1,1,1) &= (2.0 \times \alpha / \delta^2) \times (3.0 \times \text{Bi} \times (y_f - y(1,1,1)) + y(1,2,1) - \\ & 3.0 \times y(1,1,1) + y(1,1,2) + y(2,1,1) + \\ & 2.0 \times ((\delta / (2.0 \times L_{er}))^2) \times \text{vs} \times \sigma_p \times (1.0 + m_p \times (y(1,1,1) - 25.0)) / k_p) \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(1,1,n) &= (2.0 \times \alpha / \delta^2) \times (2.0 \times \text{Bi} \times (y_f - y(1,1,n)) + y(1,2,n) - \\ & 4.0 \times y(1,1,n) + 2.0 \times y(1,1,n-1) + y(2,1,n) + \\ & 2.0 \times ((\delta / (2.0 \times L_{er}))^2) \times \text{vs} \times \sigma_p \times (1.0 + m_p \times (y(1,1,n) - 25.0)) / k_p) \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(1,n,1) &= (2.0 \times \alpha / \delta^2) \times (2.0 \times \text{Bi} \times (y_f - y(1,n,1)) + 2.0 \times \\ & y(1,n-1,1) - 4.0 \times y(1,n,1) + y(1,n,2) + y(2,n,1) + \\ & 2.0 \times ((\delta / (2.0 \times L_{er}))^2) \times \text{vs} \times \sigma_p \times (1.0 + m_p \times (y(1,n,1) - 25.0)) / k_p) \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(1,n,n) &= (2.0 \times \alpha / \delta^2) \times (\text{Bi} \times (y_f - y(1,n,n)) + \\ & 2.0 \times y(1,n,n-1) - 5.0 \times y(1,n,n) + 2.0 \times y(n,n-1,n) + y(2,n,n) + \\ & 2.0 \times ((\delta / (2.0 \times L_{er}))^2) \times \text{vs} \times \sigma_p \times (1.0 + m_p \times (y(1,n,n) - 25.0)) / k_p) \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(n,1,1) &= (2.0 \times \alpha / \delta^2) \times (2.0 \times \text{Bi} \times (y_f - y(n,1,1)) + y(n,2,1) - \\ & 4.0 \times y(n,1,1) + y(n,1,2) + 2.0 \times y(n-1,n,1) + \\ & 2.0 \times ((\delta / (2.0 \times L_{er}))^2) \times \text{vs} \times \sigma_p \times (1.0 + m_p \times (y(n,1,1) - 25.0)) / k_p) \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(n,1,n) &= (2.0 \times \alpha / \delta^2) \times (\text{Bi} \times (y_f - y(n,1,n)) + y(n,2,n) - \\ & 5.0 \times y(n,1,n) + 2.0 \times y(n,1,n-1) + 2.0 \times y(n-1,1,n) + \\ & 2.0 \times ((\delta / (2.0 \times L_{er}))^2) \times \text{vs} \times \sigma_p \times (1.0 + m_p \times (y(n,1,n) - 25.0)) / k_p) \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(n,n,1) = & (2.0 \times \alpha / \delta^2) \times (\text{Bi} \times (y_f - y(n,n,1)) + y(n,n,2) - \\ & 5.0 \times y(n,n,1) + 2.0 \times y(n,n-1,nn) + 2.0 \times y(n-1,n,1) + \\ & 2.0 \times ((\delta / (2.0 \times L_{er}))^2) \times v_s \times \sigma_p \times (1.0 + m_p \times (y(n,n,1) - 25.0)) / k_p) \end{aligned}$$

$$\begin{aligned} \frac{\partial y}{\partial t}(n,n,n) = & (2.0 \times \alpha / \delta^2) \times (2.0 \times y(n,n-1,n) - \\ & 6.0 \times y(n,n,n) + 2.0 \times y(n,n,n-1) + 2.0 \times y(n-1,n,n) + \\ & 2.0 \times ((\delta / (2.0 \times L_{er}))^2) \times v_s \times \sigma_p \times (1.0 + m_p \times (y(n,n,n) - 25.0)) / k_p) \end{aligned}$$

### Kinetics

As indicated in the Introduction of this section, the model was designed with the goal of obtaining estimates of variables which depend upon the temperature history of the food product. Once an estimate of the temperature spatial distribution is available for the product enclosed in the reference volume (containing the fastest particle in the case of aseptic processing), the evaluation of any temperature dependent variable, is straightforward:

- a) Point equivalent thermal destruction time for spores, enzymes and nutrients. The computation is done by applying the General Method of accumulated lethality computation (Pflug, 1990). For spores (of *Clostridium Botulinum*, say), this method was applied in the time interval from  $t_i$  to  $t_e$  for any of the product elements (fluid and beef cube regions), by obtaining the point equivalent destruction time or kill time ( $F_o$ ) (Pflug, 1990), at the reference temperature ( $y_o$ ,  $y_o = 121.1$  °C). The kill time is equal to the product of the time interval length ( $t_e - t_i$ ), by the lethality rate ( $L$ ) (ratio of exposure time  $F_o$  at the reference temperature  $y_o$  to exposure time  $F$  at a temperature  $y$  during  $t_i$  to  $t_e$ ):

$F_o = (t_e - t_i) \times L$ , at the fluid or cube octant region with center node (i,j,k).

When the Bigelow model holds for relative times of thermal destruction (Pflug, 1990):

$$L = 10^{\frac{(y-y_o)}{Z}}$$

The accumulated kill time (sterility), for a relatively long time interval is computed by adding the kill times of a partition of subintervals. For constituents such as enzymes (peroxidase for example) or nutrients (thiamine for example), the same procedure is applied to obtain the corresponding equivalent destruction times.

- b) Concentration of spores, enzymes and nutrients. The following formula is applied to compute the concentration  $C_e$  in any product element at time  $t_e$ , given an initial concentration  $\varphi$  at time  $t_i = 0$ :

$$C_e = \varphi \times 10^{\frac{F_o}{D_o}}$$

An alternative way of obtaining  $C_e$  is by applying actual time  $F$ , instead of kill time  $F_o$ , but adjusting the  $D$  values (Teixeira et al., 1964; Teixeira and Shoemaker, 1989). The total concentrations are obtained by adding the products  $C_e \times$  product element volume.

- c) Integrated equivalent destruction times  $F_I$  at time  $t_e$  for the product included in the reference volume. The initial concentration  $\varphi$  at time  $t_i = 0$ , and the concentration  $C_e$  at time  $t_e$  must be known in order to apply the following formula (Stumbo, 1965):

$$F_I = D_o \times [\log(\varphi) - \log(C_e)]$$

## ILLUSTRATION

Two alternative PF's of beef in gravy were considered, one without starch, denoted as BBROTH, and other containing 3% crosslinked starch, denoted as STARCH<sup>2</sup>. The thermal processing of each PF was simulated for each processing system type (PST): SSHE aseptic (PST=1), ER aseptic (PST=2), and canning (PST=3). A lethality target of 360 seconds (s) at a reference temperature equal to 121.1 °C, was required for the THEQ stage. The results that appear in FIGURES 6 to 10 were obtained under worst case conditions: minimum convective heat transfer coefficient ( $h_{fp}$ ), and for aseptic processing, maximum fastest particle velocity ratio in the thermoequilibrators (THEQVR=2). The equivalent destruction times shown in the figures, are given at a reference temperature ( $y_r$ ), equal to 121.1 °C (250 °F).

Although all PST's were required to have the same system exit fluid temperature (32.2°C), a pressurized steam constant temperature was assumed for PST=3 (115.6 °C), and a target fluid temperature (140 °C) at H exit, was imposed for PST=1 and PST=2. Constant cooling water temperature was also assumed for PST=3 (18.3 °C). Adjustable pressurized steam and cooling water temperatures were considered for PST=1 and PST=2.

For any of the PST's and PF's considered in this illustration, the fluid is the fastest heating zone (FHZ) of the product included in the reference volume, the octant region assigned to node (1,1,1) is the octant (and whole beef cube) FHZ, and the region assigned to node

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<sup>2</sup>For comparative purposes, a modification of STARCH is considered in FIGS. 11E and 12E, that consists in assuming equality between the gravy and the beef electrical conductivities.

(n.n,n), is the octant (particle) slowest heating zone (SHZ) (FIG.6). For a PST and PF combination where the fluid is the SHZ, see Pastrana et. al (1992 b).

SSHE aseptic processing (PST=1) for BBROTH, takes around half the time as canning (PST=3) (FIGS. 6A, 6B, 6C), and for STARCH, it takes approximately 80% of the canning time (FIGS. 6D, 6E, 6F). ER aseptic processing (PST=2) for STARCH, takes the least time where compared to PST=1 and PST=3 (FIGS. 6D, 6E, 6F). The time PST=2 takes is about 33% that for PST=1, and about 25% that for PST=3 (FIGS. 6D, 6E, 6F).

As far as spores equivalent destruction times for the fluid, cube octant (particle) FHZ and cube octant (particle) SHZ, PST=3 consistently shows lower values than PST=1 for BBROTH (FIGS. 7A, 7B, 7C) and STARCH (FIGS. 7D, 7E, 7F), and than PST=2 for STARCH (FIGS. 7D, 7E, 7F). PST=1 appears to be a more effective sterilizing system than PST=3, for BBROTH (FIGS. 7A, 7B, 7C): at system exit, PST=3 has an accumulated lethality, at the particle SHZ, approximately equal to 82% of that for PST=1. When PF is changed to STARCH, the lethality accumulated for PST=1 and PST=3 in the particle SHZ at system exit, is approximately the same (FIGS. 7D, 7E, 7F). PST=2 is the most effective of the three PST's for STARCH (FIGS. 7D, 7E, 7F): at system exit, PST=2 delivers, at the particle SHZ, approximately twice the lethality delivered by PST=1 or PST=3 (FIGS. 7D, 7E, 7F).<sup>3</sup>

With respect to enzyme equivalent destruction times at processing system exit, for fluid, particle FHZ, and particle SHZ, PST=3 shows lower values than PST=1 (FIGS. 8A, 8B, 8D, 8E), except for the particle SHZ (FIGS. 8C, 8F). PST=2 corresponding values for STARCH do not show a definite pattern, since for fluid the value is between those of PST=3 and PST=1 (FIG. 8D), for the particle FHZ the value is the greatest (FIG. 8E), and for the

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<sup>3</sup>PST=2 would appear with less advantage if a particle SHZ consisting of only fat were considered.

particle SHZ it is the lowest (FIG. 8F).

With respect to nutrient equivalent destruction times for particle FHZ, and particle SHZ, PST=3 shows higher values than PST=1 at system exit for BBROTH (FIGS. 9B, 9C); for STARCH, however, the corresponding difference is greatly reduced (FIGS. 9E, 9F). PST=3 shows lower fluid nutrient equivalent destruction times than PST=1 (FIGS. 9A, 9D). PST=2 shows consistently lower fluid, particle FHZ, and particle SHZ nutrient equivalent destruction times, at system exit, than PST=1, and PST=3 (FIGS. 9D, 9E, 9F); in particular, PST=2 has a system exit nutrient equivalent destruction time, at particle SHZ, of about 60% of that for PST=1 and about 53% of that for PST=3 (FIG. 9F).

By following criteria for individual responses optimization, choices may be established for the PST's:

<u>CRITERIA</u>	<u>PF</u>	<u>PST CHOSEN</u>
High sterility at SHZ	1	1
	2	2
High enzyme destr. at SHZ	1	3
	2	3
Low product nu- trient destruct.	1	1
	2	2

A simplistic decision rule based on the above results would be: for BBROTH choose PST=1, and for STARCH choose PST=2. With any of these choices there is a potential

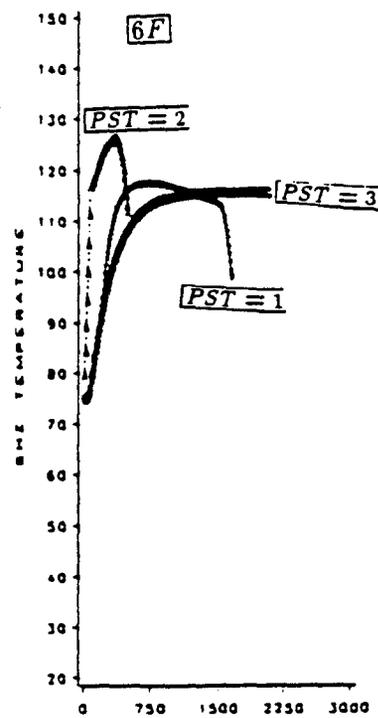
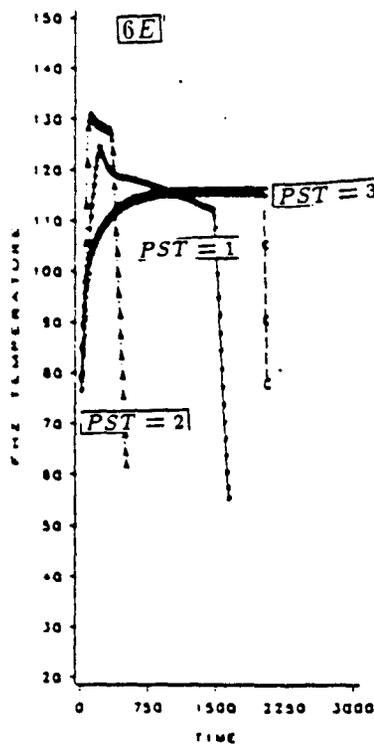
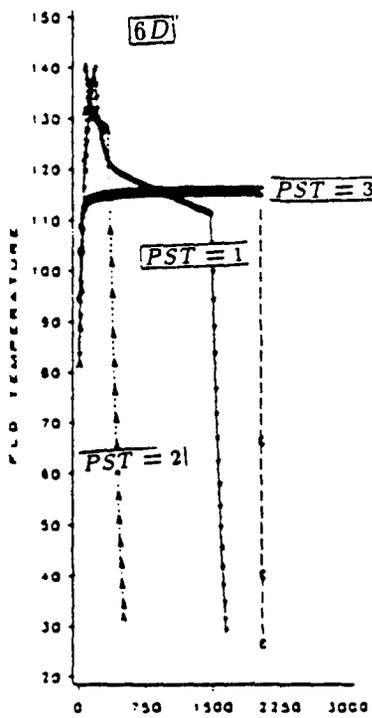
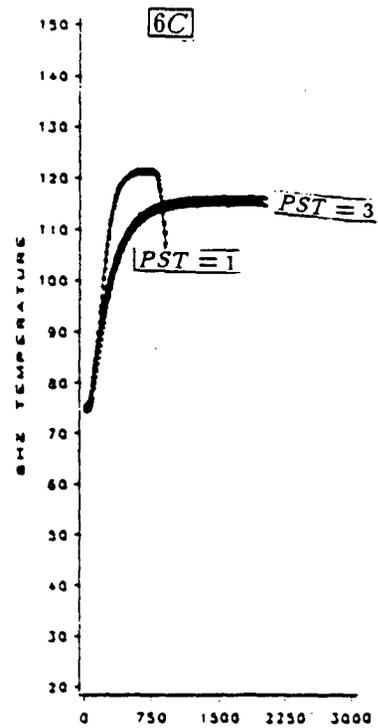
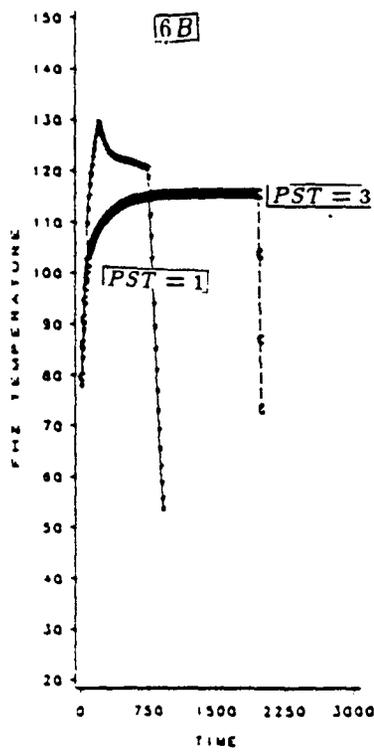
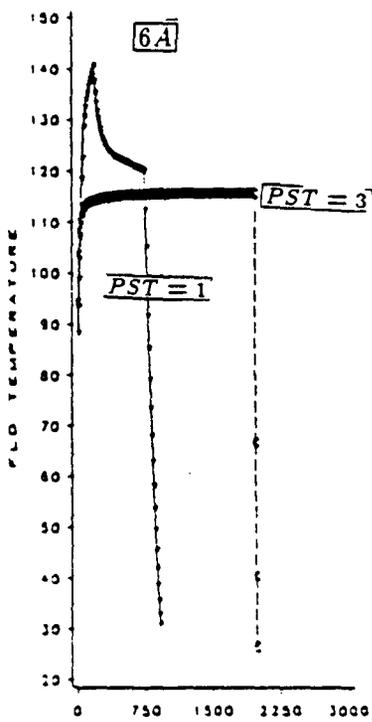
enzyme reactivation problem, although at system exit, the enzyme equivalent destruction time for particle SHZ ( FIG. 8), are well above a target enzyme equivalent destruction time of 371 s to ensure at least 99.9% enzyme (peroxidase) destruction, assuming a first order reaction and a decimal reduction time, when  $y_r=121$  °C, equal to 185.4 s (Yamamoto et. al, 1962; Chandarana and Gavin, 1989a).

A choice for PST=3 can be justified by the following arguments: spore equivalent destruction time for the particle SHZ, at THEQ exit, is above the target  $F_o$ ,  $F_o=360$  s (by design), so that there is no lethality problem; the nutrient equivalent destruction time for the fluid is below a maximum target nutrient equivalent destruction time of 2348 s (FIG. 9A and FIG. 9B), to ensure no more than a 50% nutrient destruction at the product FHZ, assuming a first order reaction for nutrient destruction and a decimal reduction time, when  $y_r=121.1$  °C, equal to 7800 s (Felicciotti and Esselen, 1957; Chandarana and Gavin, 1989a) . When cost is considered, the choice is in favor of PST=3, which is the cheapest of the three PST's considered.

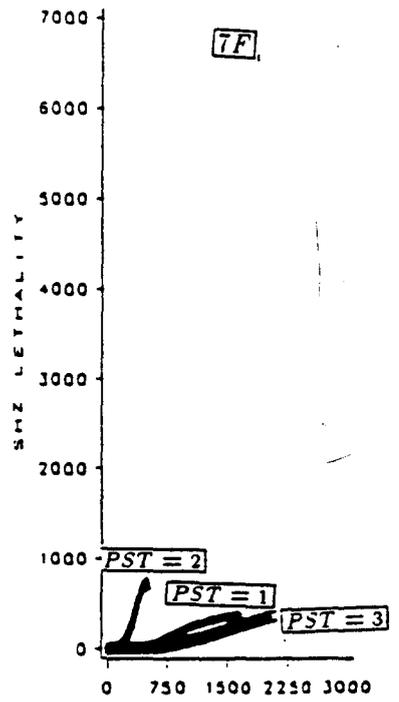
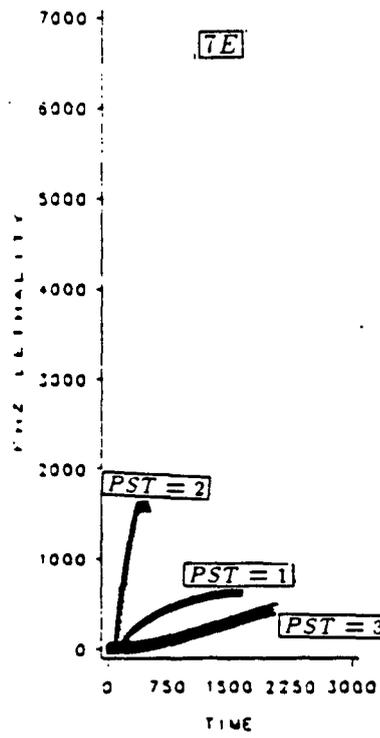
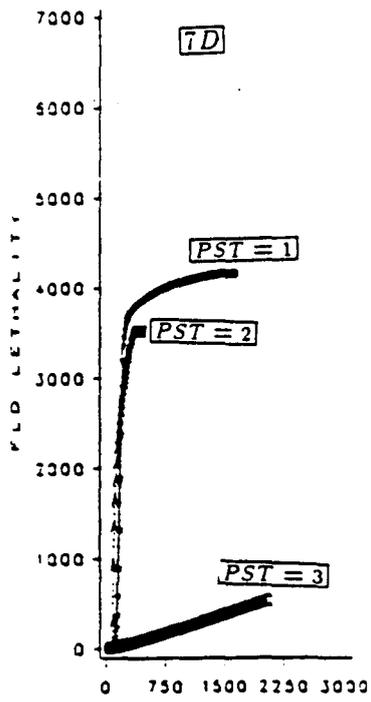
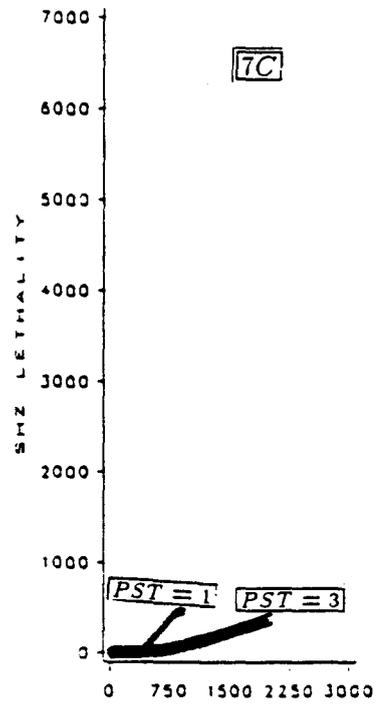
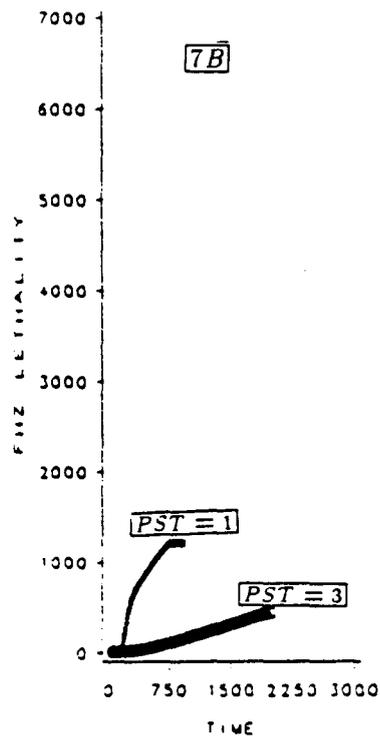
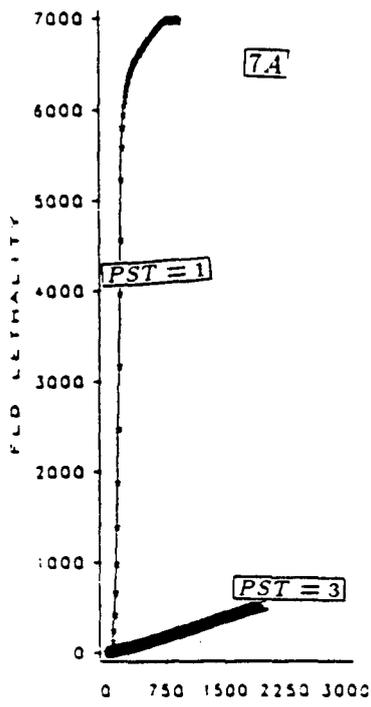
FIGS. 11A to 11E show an increasing relationship between the fluid and octant (1,1,1) region lethality, and the lethality of the octant (n,n,n) region (beef cube center). This relationship is characterized by a decreasing growth rate; in fact, the growth rate is high at the H stage beginning, decreases abruptly, continues decreasing or becomes constant, and finally becomes zero during the C stage, after another abrupt decrease. When a modification of STARCH is considered to allow for equality between the gravy (with starch) and the beef electrical conductivities (FIG. 11E), a remarkable contrast occurs in the corresponding relationship: octant (n,n,n) region shows greater lethality than the fluid and octant (1,1,1) region, and the fluid curve is below the octant (1,1,1) curve (compare FIG. 11E to FIGS. 11A, 11B, 11C and 11D). If nutrient equivalent destruction time is considered instead of spores

equivalent destruction time or lethality, as has been done in FIG. 12, a similar relationship to that between lethalities, is obtained between nutrient equivalent destruction times (compare FIG. 12 to FIG. 11).

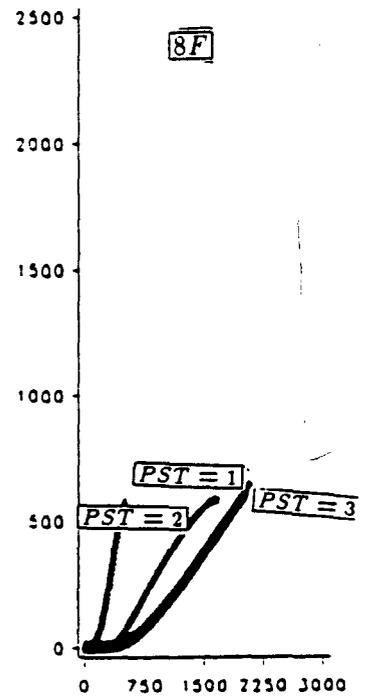
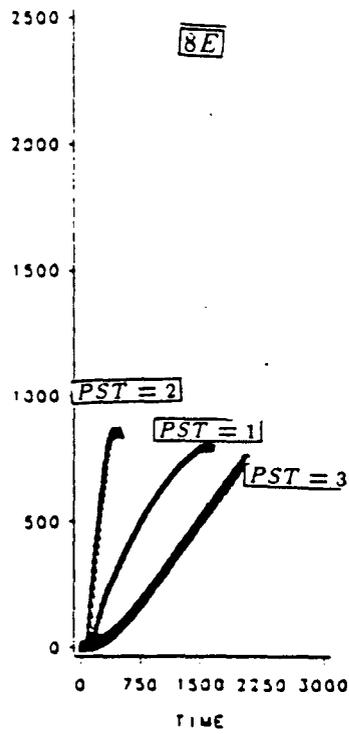
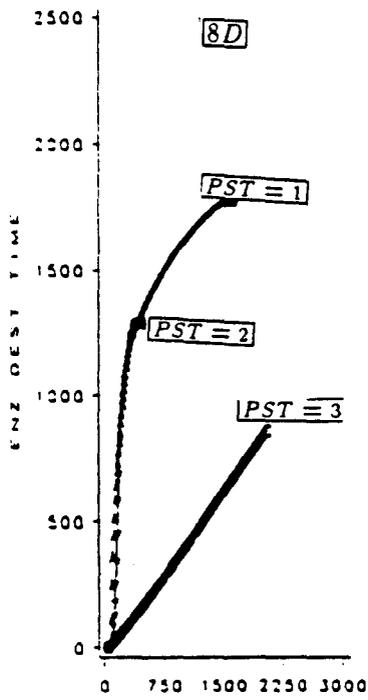
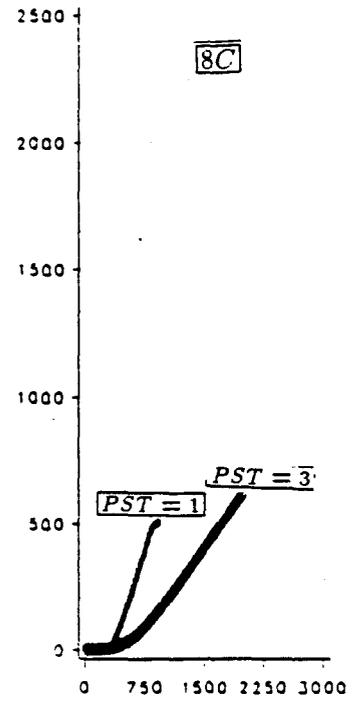
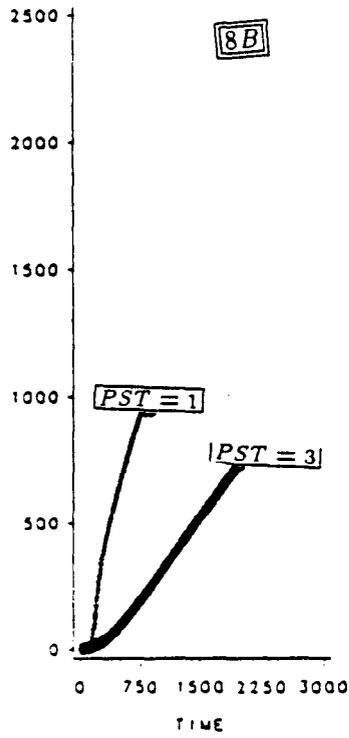
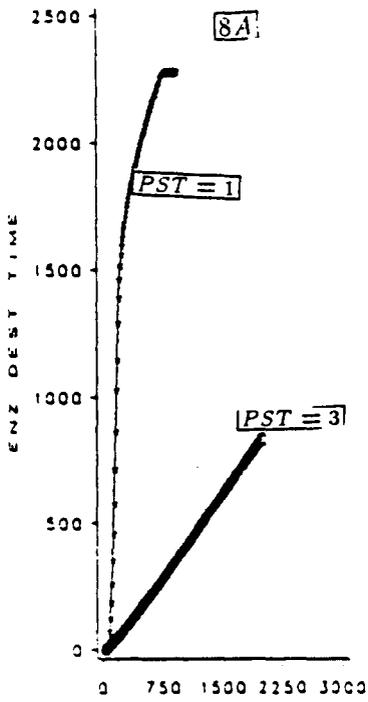
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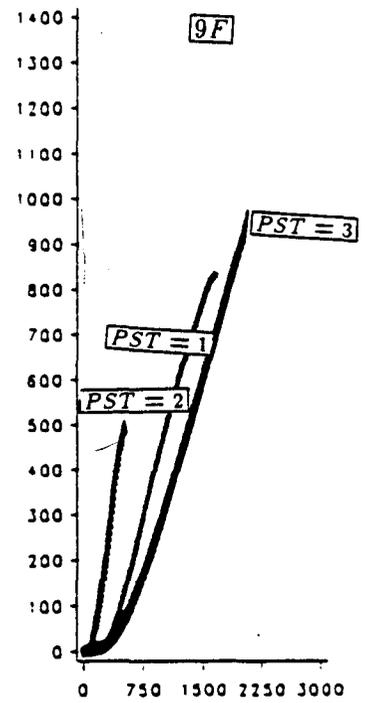
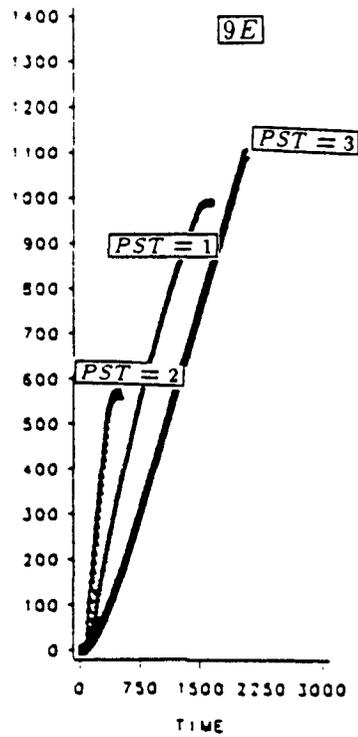
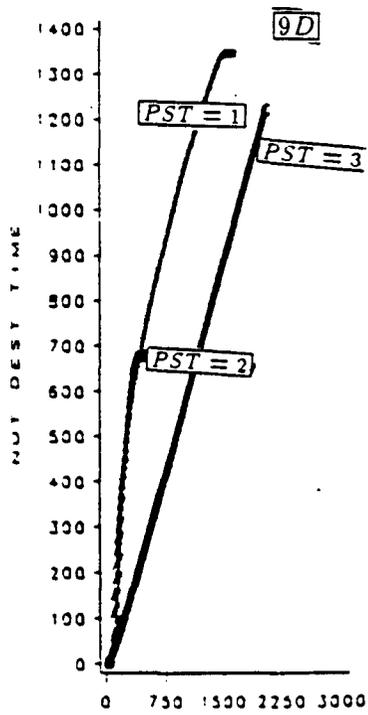
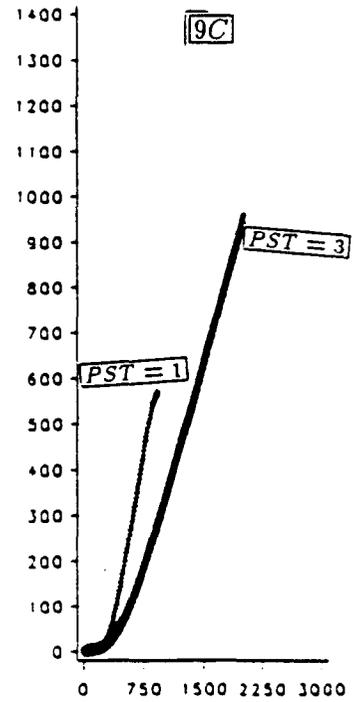
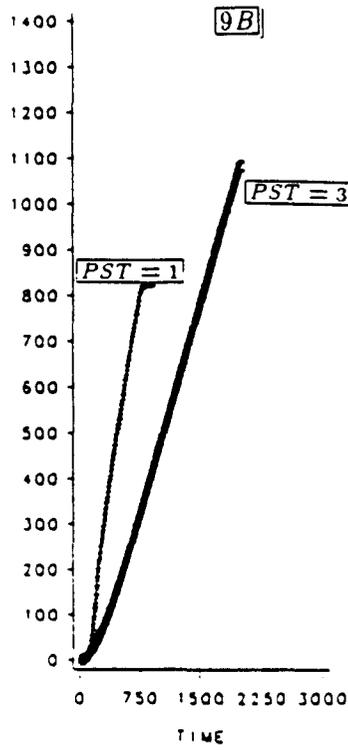
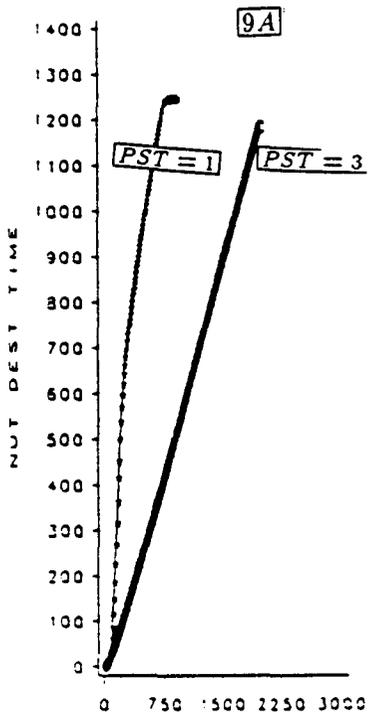


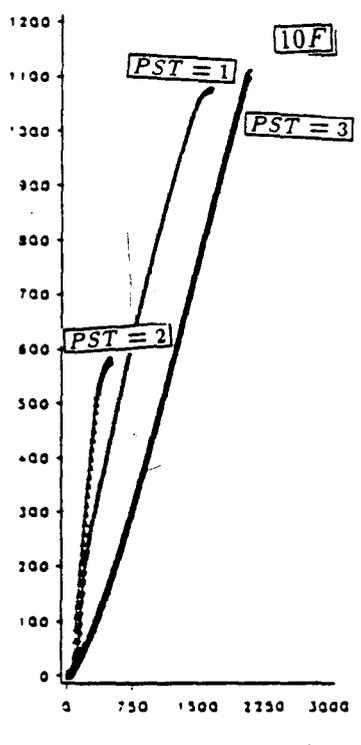
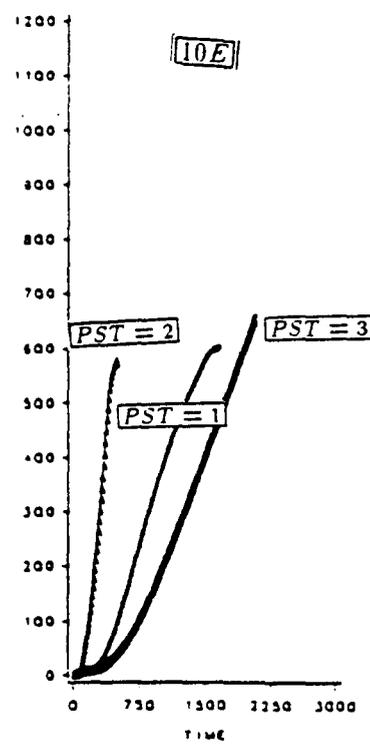
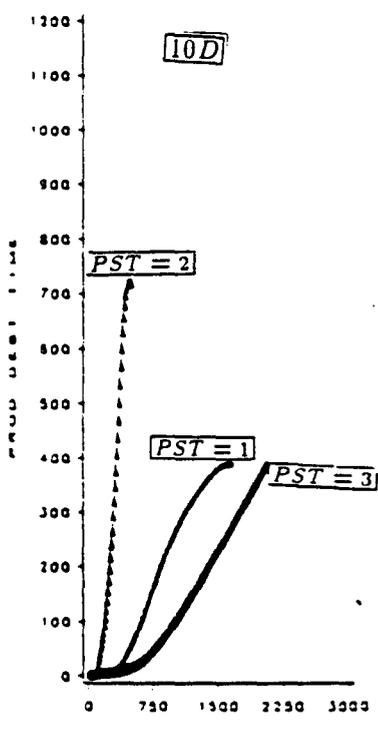
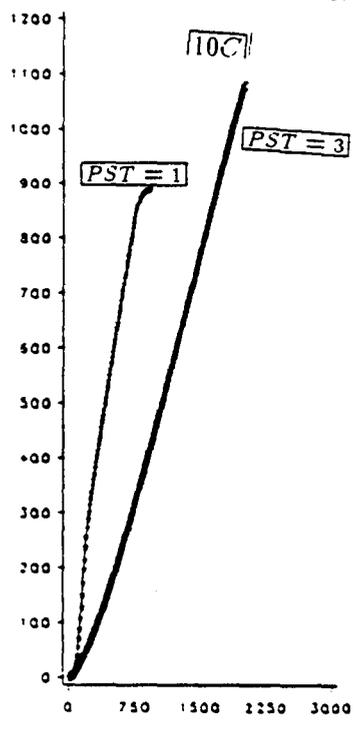
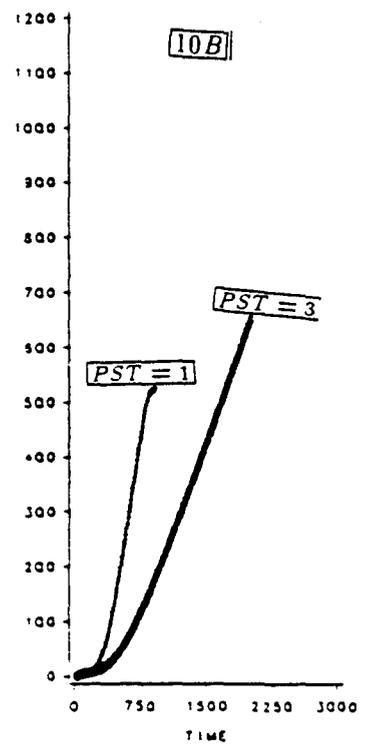
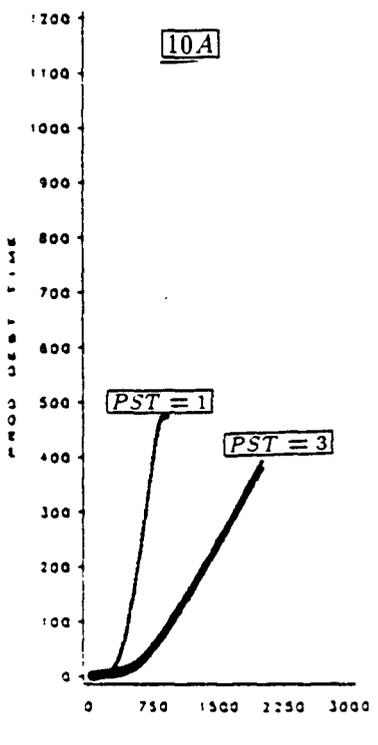




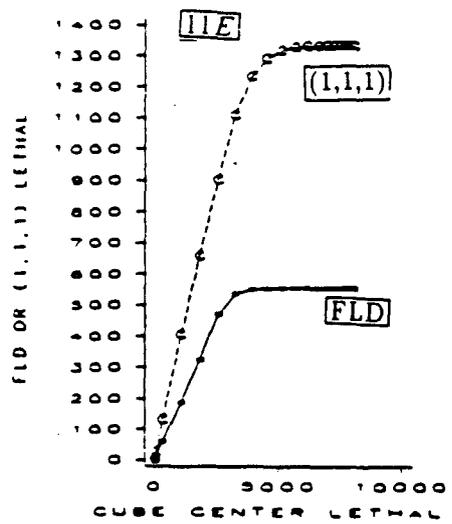
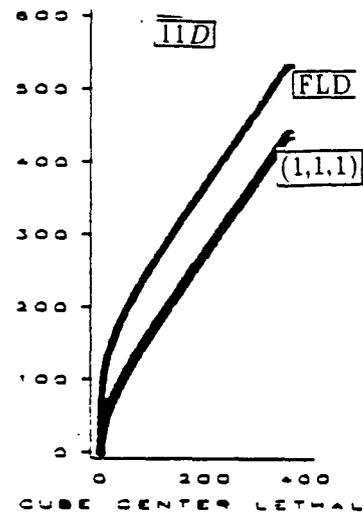
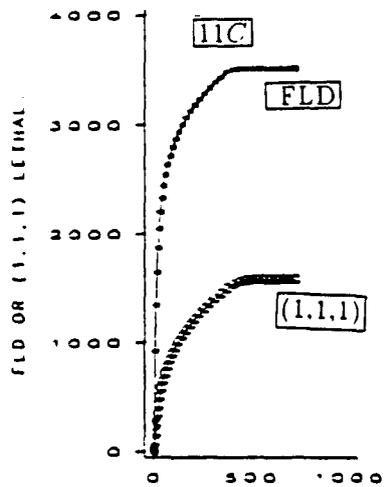
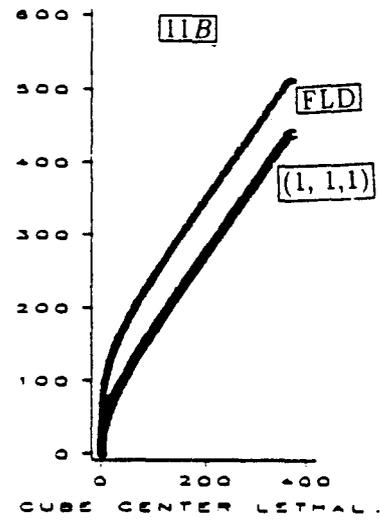
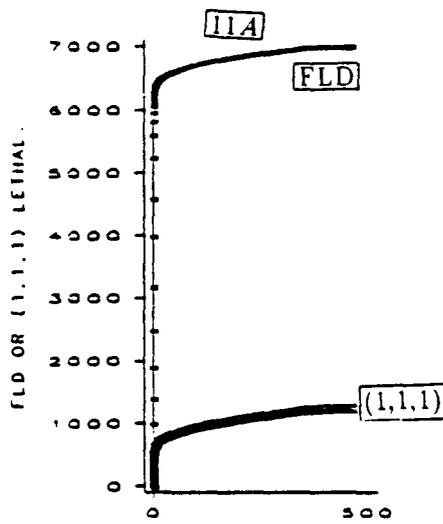


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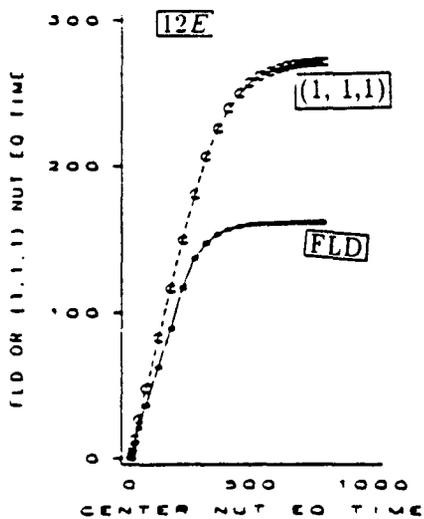
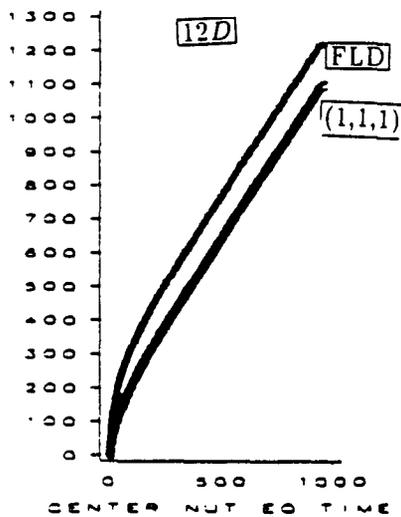
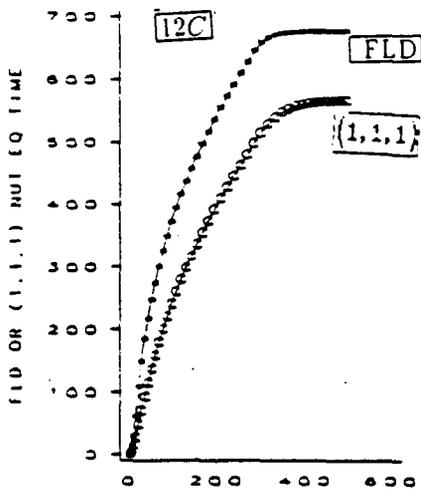
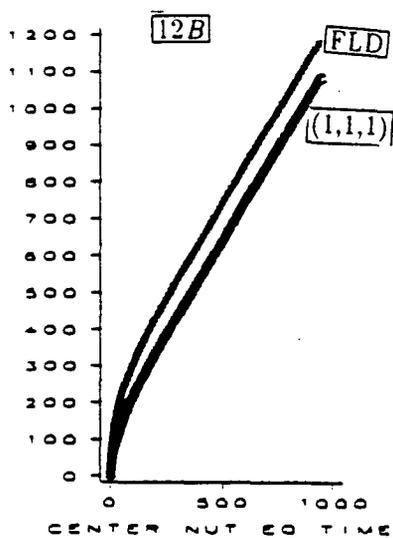
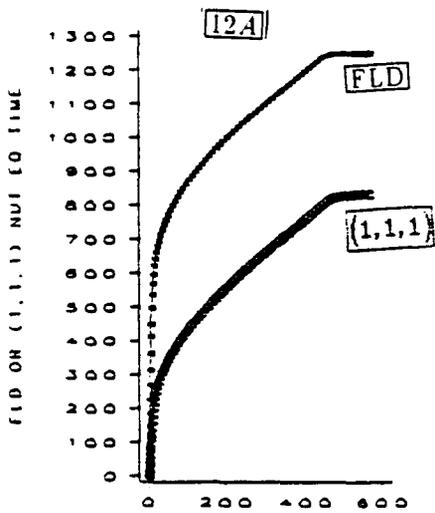












## NOMENCLATURE

$A$	: Cross sectional area ( $m^2$ )
$A_{er}$	: ER cross sectional area ( $m^2$ )
$A_{\delta t}$	: Incremental volume or can area ( $m^2$ )
$b_{u, u = f, p}$	: slope factor in the electrical conductivity versus temperature line $(\frac{1}{^\circ C})$
$Bi$	: Biot number for any cube unit inside the beef cube octant. $Bi = \frac{h_{fp}\delta}{k_p}$ (dimensionless)
$b_f$	: Slope of the following fluid electrical conductivity equation: $\frac{\sigma_f(y)}{\sigma_f(25^\circ C)} = 1 + b_f(y - 25)$ . Units: $1/^\circ C$
$C_{u, u = i, e}$	: Constituent concentration (No. of spores or g per cc)
$D$	: Time(s), at $y_o = 121.1^\circ C$ , necessary to obtain a 90% (1 decimal log cycle) reduction a constituent concentration
$D_{u, u = M, ENZ, N}$	: Time (s), at $y_o = 121.1^\circ C$ , necessary to obtain a 90% (1 decimal log cycle) reduction in constituent $u$ concentration
$E_{u, u = in, g, out, s}$	: Energy ( $J$ )
$\dot{E}_{u, u = in, g, out, s}$	: Rate of Energy ( $w, w = \frac{J}{s}$ )
$F$	: Exposure time (s) of a product region (fluid or octant region), to a thermal treatment consisting in the application of a constant temperature $y$
$F_{u, u = o, I}$	: Sterility ( $F_o$ is a target at $y_o = 121.1^\circ C$ , and $Z = 10^\circ C$ ). Units: s

$g'''$	: Energy generation rate per volume unit ( $\frac{w}{m^3}$ )
$h$	: Convective heat transfer coefficient ( $\frac{w}{m^2 \cdot C}$ )
$h_{fp}$	: convective heat transfer coefficient at the fluid particle interface  ( $\frac{w}{m^2 \cdot C}$ )
$k$	: Thermal conductivity ( $\frac{w}{m \cdot C}$ )
$k_p$	: Particle thermal conductivity ( $\frac{w}{m \cdot C}$ )
$l$	: octant edge length ( $m$ )
$L_{er}$	: ER heater length ( $m$ )
$L$	: Lethality rate. It is defined as the ratio of exposure time at $y_o$ to exposure time at a temperature $y$ during the time interval from $t_i$ to $t_e$ . Units: s at temperature $y_o$ per s at temperature $y$
$m_{\delta t}$	: Mass of the fluid contained in the reference volume product  (product in can or incremental volume) ( $Kg$ )
$n$	: number of nodes or points equally spaced on the octant edge. It is equal to the number of congruent segments into which the edge is divided plus 1
$N_{\delta t}$	: Number of beef cubes contained in the reference volume product  (product in can or incremental volume)
$Q$	: Volumetric flow rate ( $\frac{m^3}{s}$ ) in aseptic processing
$t$	: General time (s)
$\delta t$	: Time increment or step (s)
$t_u, u = i, e, E$	: time (s)
$\mathfrak{R}_{uv}, u = M, N, v = f, 1, n$	: Reduction exponents (dimensionless)
$y$	: Temperature ( $^{\circ}C$ )
$y(i,j,k)$	: Temperature ( $^{\circ}C$ ) at (i,j,k) octant node, and applicable to the

	corresponding octant region. $i, j, k=1, 2, \dots, n$
$\bar{y}$	: Convective octant face mean surface temperature ( $^{\circ}C$ )
$y_I$	: Initial temperature ( $^{\circ}C$ ). At time $t_i = 0$
$y_u, u = st, cw, f, o, E$	: Temperature ( $^{\circ}C$ ).
$y_f$	: vector containing the plug flow fluid temperatures ( $^{\circ}C$ ), in aseptic processing
$y_{e, f, u}^u, u = h, c$	: Fluid temperature ( $^{\circ}C$ ) target at device $u$ exit
$U_h$	: Overall heat transfer coefficient corresponding to H stage ( $\frac{w}{m^{\circ}C}$ )
$V$	: Volume ( $m^3$ )
$V_{\delta t}$	: Incremental or can volume ( $m^3$ )
$vs$	: Square voltage required for a given power in the ER heater. Units: Square Ohms
$Z$	: Temperature increase ( $^{\circ}C$ ) required to reduce $D$ by 90% (by 1 decimal log cycle)
$Z_u, u = M, ENZ, N$	: Temperature increase ( $^{\circ}C$ ) required to reduce $D_u$ by 90% (by 1 decimal log cycle)

#### SUBSCRIPTS

$\delta t$	: Time increment. In aseptic processing, corresponding characteristic (mass, volume, area, number of beef cubes) refer to time interval from $t_i$ to $t_i + \delta t$
$e$	: at upper end of time interval or exit
$er$	: electrical resistance
$E$	: Equivalent point
$ENZ$	: Enzyme

<i>f</i>	: fluid
<i>g</i>	: generated
<i>h</i>	: heater
<i>i</i>	: at lower end of time interval
<i>I</i>	: Integrated measurement over the entire unit (octant, beef cube or can product)
<i>in</i>	: input
<i>M</i>	: Spores
<i>n</i>	: at (n,n,n) octant region
<i>N</i>	: Nutrient
<i>o</i>	: Reference value. $y_o=121.1$ °C.
<i>out</i>	: Output
<i>p</i>	: Particle
<i>s</i>	: stored
<i>st</i>	: Pressurized steam

#### SUPERSCRIPTS

<i>C</i>	: Cooling stage
<i>H</i>	: Heating stage

#### GREEK LETTERS

$\gamma$	: specific heat ( $\frac{J}{kg \cdot C}$ )
$\gamma_{u, u = f, p}$	: Phase <i>u</i> specific heat ( $\frac{J}{kg \cdot C}$ )

$\delta$	: congruent subintervals length on octant edge (m). $\delta = \frac{l}{ND}$
$\varepsilon$	: Fluid volume fraction of product in can or in incremental volume (dimensionless)
$\theta$	: Any of the three coordinate axis ( $x_j, j = 1, 2, 3$ )
$\rho$	: density ( $\frac{kg}{m^3}$ )
$\rho_u, u = f, p$	: Phase u density ( $\frac{kg}{m^3}$ )
$\sigma_u, u = f, p$	: Phase u electrical conductivity at $y_f = 25^\circ C$ ( $\frac{S}{m}$ ), S: Siemens, S= 1/ohm
$\varphi$	: Constituent (spore, nutrient or enzyme) concentration at the time origin ( $t_i = 0$ )
$\alpha_p$	: Particle diffusivity, $\alpha_p = \frac{k_p}{\rho_p \gamma_p}$ , units: 1/s

#### ABBREVIATIONS

BBROTH	: Product formulation (PF) consisting of beef cubes in a fluid that does not contain starch
C	: Cooling device or stage
ENZ DEST TIME	: Enzyme equivalent destruction time (s at $y_o = 121.1^\circ C$ )
ER	: Electrical resistance
FHZ	: Product or beef cube fastest heating zone
FLD	: Fluid
H	: Heating device or stage
ND	: Number of congruent subintervals into which the octant edge is divided
NUT DEST TIME	: Nutrient equivalent destruction time (s at $y_o = 121.1^\circ C$ )

ODE	: Ordinary differential equation
PF	: Product formulation
PROD DEST TIME	: Product integrated equivalent destruction time (s at $y_o = 121.1$ °C)
PST	: Processing system type
RTR	: Fastest particle residence time ratio, defined as the ratio of the fastest particle residence time to either the mean particle residence time (H or C stage, assuming plug flow) or the mean fluid interstitial residence time (THEQ stage, under other than plug flow) (dimensionless)
SHZ	: Product or beef cube slowest heating zone
SSHE	: Scraped surface heat exchanger
STARCH	: Product Formulation (PF) consisting of beef cubes in gravy with 3% starch (different electrical conductivities between the gravy and the beef)
THEQ	: Thermoequilibrium device or stage
THEQVR	: Ratio of the fastest beef cube speed in the THEQ to the interstitial mean fluid velocity in the THEQ. It is equal to 1/RTR (dimensionless)
RT	: Retort Temperature (°C)

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## APPENDIX

The purpose of this appendix is to show the construction of the equation for the fluid temperature derivative during the H stage:

$$\frac{\partial y_f}{\partial t} = 1.0 / (m_{\delta t} \times \gamma_f) \times (U_h \times A_{\delta t} \times (y_{st} - y_f) - h_{fp} \times 24.0 \times l^2 \times N_{\delta t} \times (y_f - \bar{y})) + ((V_{\delta t} / (A_{er} \times L_{er}))^2) \times v_s \times \sigma_f \times (1.0 + b_f(y_f - 25.0)) \times \epsilon \times (A_{er}^2) / V_{\delta t} \quad (1)$$

This equation is obtained from an energy balance of the form:

$$\dot{E}_{in} + \dot{E}_g = \dot{E}_{out} + \dot{E}_s \quad (*)$$

For SSHE aseptic processing (PST=1) and canning (PST=3),  $\dot{E}_g = 0$  since there is no heat generation (no alternate electric current is applied); in this case, equation (\*) becomes:

$$\dot{E}_{in} = \dot{E}_{out} + \dot{E}_s \quad (**)$$

where:

$$\begin{aligned} \dot{E}_{in} &= U_h \times A_{\delta t} \times (y_{st} - y_f) \\ \dot{E}_{out} &= h_{fp} \times 24.0 \times l^2 \times N_{\delta t} \times (y_f - \bar{y}) \\ \dot{E}_s &= m_{\delta t} \times \gamma_{\delta t} \times \frac{\partial y_f}{\partial t} \end{aligned}$$

After substitution of the corresponding expressions for  $\dot{E}_{in}$ ,  $\dot{E}_{out}$  and  $\dot{E}_s$ , and proper algebraic manipulation, the following expression is obtained from (\*\*):

$$\frac{\partial y_f}{\partial t} = 1.0 / (m_{\delta t} \times \gamma_f) \times (U_h \times A_{\delta t} \times (y_{st} - y_f) - h_{fp} \times 24.0 \times l^2 \times N_{\delta t} \times (y_f - \bar{y})) \quad (1a)$$

For ER aseptic processing (PST=2),  $\dot{E}_{in} = 0$  since there is no heat transfer applied from an external source. The general energy balance equation (\*) becomes:

$$\dot{E}_g = \dot{E}_{out} + \dot{E}_s \quad (***)$$

where  $\dot{E}_{out}$  and  $\dot{E}_s$  are given by the same expressions as above for PST=1 and PST=2, and  $\dot{E}_g$  is given by the following expression (Skudder, 1987; Palaniappan and Sastry, 1991):

$$\dot{E}_g = g'''_{\delta t} V_{\delta t, f} = \frac{(vs)_{\delta t} \times \sigma_{\delta t, f} \times A_{\delta t, f}}{l_{\delta t}} \quad (****)$$

where:

$g'''_{\delta t}$  is the fluid energy generation rate per volume unit ( $\frac{w}{m^3}$ );

$V_{\delta t, f}$  is the volume of the fluid in the product contained in the heater

incremental volume (volume corresponding to time interval from  $t_i$  to  $t_i + \delta t$ )

(cubic m);

$(vs)_{\delta t}$  is the square of the voltage potential corresponding to the heater incremental

volume (square ohms);

$\sigma_{\delta t, f}$  is the fluid electrical conductivity at time  $t_i$ , when the fluid temperature is

$y_f$  ( $\frac{S}{m}$ );

$A_{\delta t, f}$  is the cross sectional area for the fluid in the product contained in the

heater incremental volume (square m);

$l_{\delta t, f}$  is length corresponding to the fluid volume  $V_{\delta t, f}$ .

The following equalities hold:

$$(vs)_{\delta t} = \left( \frac{l_{\delta t, f}}{L_{er}} \right)^2 \times vs = \left( \frac{V_{\delta t}}{L_{er}} \right)^2 \times vs$$

$$\sigma_{\delta t, f} = \sigma_f \times (1.0 + b_f \times (y_f - 25.0)) \quad (\text{Skudder, 1987; Palaniappan, 1991})$$

$$l_{\delta t, f} = \frac{V_{\delta t}}{A_{er}}$$

$$A_{\delta t, f} = \frac{\varepsilon \times V_{\delta t}}{l_{\delta t, f}} = \varepsilon \times A_{er}$$

After substitution of these last expressions in (\*\*\*\*), the expression for  $\dot{E}_g$  when PST=2 becomes:

$$\dot{E}_g = (V_{\delta t} / (A_{er} \times L_{er}))^2 \times vs \times \sigma_f \times (1.0 + b_f \times (y_f - 25.0)) \times \varepsilon \times (A_{er}^2) / V_{\delta t}$$

Once the corresponding expressions for  $\dot{E}_g$ ,  $\dot{E}_{out}$  and  $\dot{E}_s$  are included in (\*\*\*), and after proper algebraic manipulation, the following expression is obtained:

$$\begin{aligned} \frac{\partial y_f}{\partial t} = & 1.0 / (m_{\delta t} \times \gamma_f) \times [-h_{fp} \times 24.0 \times l^2 \times N_{\delta t} \times (y_f \bar{y}) + \\ & (V_{\delta t} / (A_{er} \times L_{er}))^2 \times vs \times \sigma_f \times (1.0 + b_f \times (y_f - 25.0)) \times \varepsilon \times (A_{er}^2) / V_{\delta t}] \quad (1b) \end{aligned}$$

Equations (1a) and (1b) can be written as a single equation equal to (1), so that  $vs = 0$  if PST=1 or PST=3, and  $U_h = 0$  if PST=2, and parameters  $m_{\delta t}$ ,  $A_{\delta t}$ ,  $N_{\delta t}$ ,  $V_{\delta t}$ , and  $y_{\delta t}$  are assigned values in accordance with the PST considered.

	corresponding octant region. $i, j, k=1, 2, \dots, n$
$\bar{y}$	: Convective octant face mean surface temperature ( $^{\circ}C$ )
$y_I$	: Initial temperature ( $^{\circ}C$ ). At time $t_i = 0$
$y_u, u = st, cw, f, o, E$	: Temperature ( $^{\circ}C$ ).
$y_f$	: vector containing the plug flow fluid temperatures ( $^{\circ}C$ ), in aseptic processing
$y_{e, f, u}^u, u = h, c$	: Fluid temperature ( $^{\circ}C$ ) target at device $u$ exit
$U_h$	: Overall heat transfer coefficient corresponding to H stage ( $\frac{w}{m^{\circ}C}$ )
$V$ : Volume ( $m^3$ )	
$V_{\delta t}$	: Incremental or can volume ( $m^3$ )
$vs$	: Square voltage required for a given power in the ER heater. Units: Square Ohms
$Z$	: Temperature increase ( $^{\circ}C$ ) required to reduce $D$ by 90% (by 1 decimal log cycle)
$Z_u, u = M, ENZ, N$	: Temperature increase ( $^{\circ}C$ ) required to reduce $D_u$ by 90% (by 1 decimal log cycle)

#### SUBSCRIPTS

$\delta t$	: Time increment. In aseptic processing, corresponding characteristic (mass, volume, area, number of beef cubes) refer to time interval from $t_i$ to $t_i + \delta t$
$e$	: at upper end of time interval or exit
$er$	: electrical resistance
$E$	: Equivalent point
$ENZ$	: Enzyme

<i>f</i>	: fluid
<i>g</i>	: generated
<i>h</i>	: heater
<i>i</i>	: at lower end of time interval
<i>I</i>	: Integrated measurement over the entire unit (octant, beef cube or can product)
<i>in</i>	: input
<i>M</i>	: Spores
<i>n</i>	: at (n,n,n) octant region
<i>N</i>	: Nutrient
<i>o</i>	: Reference value. $y_o = 121.1$ °C.
<i>out</i>	: Output
<i>p</i>	: Particle
<i>s</i>	: stored
<i>st</i>	: Pressurized steam

#### SUPERSCRIPTS

<i>C</i>	: Cooling stage
<i>H</i>	: Heating stage

#### GREEK LETTERS

$\gamma$	: specific heat ( $\frac{J}{kg \cdot ^\circ C}$ )
$\gamma_u, u = f, p$	: Phase <i>u</i> specific heat ( $\frac{J}{kg \cdot ^\circ C}$ )

$\delta$	: congruent subintervals length on octant edge (m). $\delta = \frac{l}{ND}$
$\epsilon$	: Fluid volume fraction of product in can or in incremental volume (dimensionless)
$\theta$	: Any of the three coordinate axis ( $x_j, j = 1, 2, 3$ )
$\rho$	: density ( $\frac{kg}{m^3}$ )
$\rho_u, u = f, p$	: Phase u density ( $\frac{kg}{m^3}$ )
$\sigma_u, u = f, p$	: Phase u electrical conductivity at $y_f = 25^\circ C$ ( $\frac{S}{m}$ ), S: Siemens, S= 1/ohm
$\varphi$	: Constituent (spore, nutrient or enzyme) concentration at the time origin ( $t_i = 0$ )
$\alpha_p$	: Particle diffusivity, $\alpha_p = \frac{k_p}{\rho_p \gamma_p}$ , units: 1/s

#### ABBREVIATIONS

BBROTH	: Product formulation (PF) consisting of beef cubes in a fluid that does not contain starch
C	: Cooling device or stage
ENZ DEST TIME	: Enzyme equivalent destruction time (s at $y_o = 121.1^\circ C$ )
ER	: Electrical resistance
FHZ	: Product or beef cube fastest heating zone
FLD	: Fluid
H	: Heating device or stage
ND	: Number of congruent subintervals into which the octant edge is divided
NUT DEST TIME	: Nutrient equivalent destruction time (s at $y_o = 121.1^\circ C$ )

ODE	: Ordinary differential equation
PF	: Product formulation
PROD DEST TIME	: Product integrated equivalent destruction time (s at $y_o = 121.1$ °C)
PST	: Processing system type
RTR	: Fastest particle residence time ratio, defined as the ratio of the fastest particle residence time to either the mean particle residence time (H or C stage, assuming plug flow) or the mean fluid interstitial residence time (THEQ stage, under other than plug flow) (dimensionless)
SHZ	: Product or beef cube slowest heating zone
SSHE	: Scraped surface heat exchanger
STARCH	: Product Formulation (PF) consisting of beef cubes in gravy with 3% starch (different electrical conductivities between the gravy and the beef)
THEQ	: Thermoequilibrium device or stage
THEQVR	: Ratio of the fastest beef cube speed in the THEQ to the interstitial mean fluid velocity in the THEQ. It is equal to $1/RTR$ (dimensionless)
RT	: Retort Temperature (°C)

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## APPENDIX

The purpose of this appendix is to show the construction of the equation for the fluid temperature derivative during the H stage:

$$\frac{\partial y_f}{\partial t} = 1.0 / (m_{\delta t} \times \gamma_f) \times (U_h \times A_{\delta t} \times (y_{st} - y_f) - h_{fp} \times 24.0 \times l^2 \times N_{\delta t} \times (y_f - \bar{y}) + ((V_{\delta t} / (A_{er} \times L_{er}))^2) \times v_s \times \sigma_f \times (1.0 + b_f(y_f - 25.0)) \times \varepsilon \times (A_{er}^2) / V_{\delta t}) \quad (1)$$

This equation is obtained from an energy balance of the form:

$$\dot{E}_{in} + \dot{E}_g = \dot{E}_{out} + \dot{E}_s \quad (*)$$

For SSHE aseptic processing (PST=1) and canning (PST=3),  $\dot{E}_g = 0$  since there is no heat generation (no alternate electric current is applied); in this case, equation (\*) becomes:

$$\dot{E}_{in} = \dot{E}_{out} + \dot{E}_s \quad (**)$$

where:

$$\begin{aligned} \dot{E}_{in} &= U_h \times A_{\delta t} \times (y_{st} - y_f) \\ \dot{E}_{out} &= h_{fp} \times 24.0 \times l^2 \times N_{\delta t} \times (y_f - \bar{y}) \\ \dot{E}_s &= m_{\delta t} \times \gamma_{\delta t} \times \frac{\partial y_f}{\partial t} \end{aligned}$$

After substitution of the corresponding expressions for  $\dot{E}_{in}$ ,  $\dot{E}_{out}$  and  $\dot{E}_s$ , and proper algebraic manipulation, the following expression is obtained from (\*\*):

$$\frac{\partial y_f}{\partial t} = 1.0 / (m_{\delta t} \times \gamma_f) \times (U_h \times A_{\delta t} \times (y_{st} - y_f) - h_{fp} \times 24.0 \times l^2 \times N_{\delta t} \times (y_f - \bar{y})) \quad (1a)$$

For ER aseptic processing (PST=2),  $\dot{E}_{in} = 0$  since there is no heat transfer applied from an external source. The general energy balance equation (\*) becomes:

$$\dot{E}_g = \dot{E}_{out} + \dot{E}_s \quad (***)$$

where  $\dot{E}_{out}$  and  $\dot{E}_s$  are given by the same expressions as above for PST=1 and PST=2, and  $\dot{E}_g$  is given by the following expression (Skudder, 1987; Palaniappan and Sastry, 1991):

$$\dot{E}_g = g'''_{\delta t} V_{\delta t, f} = \frac{(vs)_{\delta t} \times \sigma_{\delta t, f} \times A_{\delta t, f}}{l_{\delta t}} \quad (****)$$

where:

$g'''_{\delta t}$  is the fluid energy generation rate per volume unit ( $\frac{w}{m^3}$ );

$V_{\delta t, f}$  is the volume of the fluid in the product contained in the heater

incremental volume (volume corresponding to time interval from  $t_i$  to  $t_i + \delta t$ )

(cubic m);

$(vs)_{\delta t}$  is the square of the voltage potential corresponding to the heater incremental volume (square ohms);

$\sigma_{\delta t, f}$  is the fluid electrical conductivity at time  $t_i$ , when the fluid temperature is

$y_f$  ( $\frac{S}{m}$ );

$A_{\delta t, f}$  is the cross sectional area for the fluid in the product contained in the

heater incremental volume (square m);

$l_{\delta t, f}$  is length corresponding to the fluid volume  $V_{\delta t, f}$ .

The following equalities hold:

$$(vs)_{\delta t} = \left( \frac{l_{\delta t, f}}{L_{er}} \right)^2 \times vs = \left( \frac{V_{\delta t}}{L_{er}} \right)^2 \times vs$$

$$\sigma_{\delta t, f} = \sigma_f \times (1.0 + b_f \times (y_f - 25.0)) \quad (\text{Skudder, 1987; Palaniappan, 1991})$$

$$l_{\delta t, f} = \frac{V_{\delta t}}{A_{er}}$$

$$A_{\delta t, f} = \frac{\varepsilon \times V_{\delta t}}{l_{\delta t, f}} = \varepsilon \times A_{er}$$

After substitution of these last expressions in (\*\*\*\*), the expression for  $\dot{E}_g$  when PST=2 becomes:

$$\dot{E}_g = (V_{\delta t} / (A_{er} \times L_{er}))^2 \times vs \times \sigma_f \times (1.0 + b_f \times (y_f - 25.0)) \times \varepsilon \times (A_{er}^2) / V_{\delta t}$$

Once the corresponding expressions for  $\dot{E}_g$ ,  $\dot{E}_{out}$  and  $\dot{E}_s$  are included in (\*\*\*), and after proper algebraic manipulation, the following expression is obtained:

$$\frac{\partial y_f}{\partial t} = 1.0 / (m_{\delta t} \times \gamma_f) \times [-h_{fp} \times 24.0 \times l^2 \times N_{\delta t} \times (y_f \bar{y}) + (V_{\delta t} / (A_{er} \times L_{er}))^2 \times vs \times \sigma_f \times (1.0 + b_f \times (y_f - 25.0)) \times \varepsilon \times (A_{er}^2) / V_{\delta t}] \quad (1b)$$

Equations (1a) and (1b) can be written as a single equation equal to (1), so that  $vs = 0$  if PST=1 or PST=3, and  $U_h = 0$  if PST=2, and parameters  $m_{\delta t}$ ,  $A_{\delta t}$ ,  $N_{\delta t}$ ,  $V_{\delta t}$ , and  $y_{st}$  are assigned values in accordance with the PST considered.