SPC Binomial Q-Charts for Short or Long Runs

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Approximately normalized control charts, called Q-Charts, are proposed for charting a binomial random variable. Transformations are given for the two cases when the parameter p is known before charting is begun and when it is not known before charting is begun. These charts permit real-time charting essentially from the beginning of sampling and are especially useful for the case when p is not known in advance. These charts are all plotted in a standard normal scale and permit a flexible charts management program. For the case when p is assumed known, we present tables to compare the closeness of the control probabilities for these Q-Charts with those of the approximating normal distribution, and with those achieved by either classical 3-sigma p-Charts or with charts based on a standardized arcsin transformation chart. These results show that Q-charts give better approximations than the classical p charts to the nominal normal probabilities, and are comparable with, but differ from, the arcsin charts.

Introduction

In a recent paper Quesenberry (1989), Q89, proposed using standardized SPC charts, called Q-charts, to control a process mean or variance for a normally distributed quality variable. Formulas were given for computing the Q-statistics for the classical case when the parameters are known and for the case when the process parameters, i.e., mean and variance, are not known, but it is desired to plot $\overline{X}$ and S-charts in real time from the start-up point of the operating process, especially for short runs. In the present paper we propose using a similar approach to charting for attributes, and give the required formulas for charts for a binomial parameter p.

The cases when p is known and when it is not known are both treated. In all cases we can chart the observed values in real time, even for short runs. These attributes Q-charts share the advantages and disadvantages of the charts for the normal distribution in Q89. They are particularly helpful in applications of charts for the purpose of studying the stability of a process and bringing it into control by identifying and eliminating special causes at the earliest possible point in time, since these charts can be made without prior knowledge of parameter values. We will use the notation of Table 1 for probability and distribution functions. The symbol $[\cdot]$ denotes the greatest integer function.

For the case when the binomial parameter p is known, the chart proposed here would replace the standardized p-chart, which was recently discussed in JQT by Nelson (1989) and has been discussed in many books including Duncan (1974) and Montgomery (1985). The standardization transformation is a linear transformation and the accuracy of the standard normal distribution approximation to assess the probabilities associated with a chart based on it depends only upon the well-known normal approximation for a binomial distribution. However, control charts are usually made for situations where p is small (perhaps $p = 0.01, 0.05, 0.10, \cdots$) and for these cases the binomial distribution is skewed and a normal distribution does not give accurate approximations, especially in the tails of the distribution, which are important for control chart applications. In order to achieve better approximations by the normal distribution we must consider nonlinear transformations. In this work we introduce such a nonlinear transformation that gives improved normal approximations for this case when the parameter p is known, and has in addition the advantage that it permits a natural way to generalize to the case when p is unknown by the use of an estimating function with efficient statistical inference properties. We give tables to compare the chart based on this new transformation with the standardized p chart, and with another chart based on another nonlinear transformation that appears in the statistical literature.

TABLE 1: Notation for Probability and Distribution Functions

Normal:

$\Phi(\cdot)$ — The standard normal distribution function.

$\Phi^{-1}(\cdot)$ — The inverse of the standard normal distribution function.

Binomial:

$$\begin{aligned} b(x;\, n,\, p) &= \binom{n}{x} p^x (1-p)^{n-x},\ x = 0,\ 1,\ 2,\ \cdots,\ n \\ &= 0, \text{ elsewhere} \end{aligned}$$

$$\begin{aligned} B(x;\, n,\, p) &= 0,\ x < 0 \\ &= 1,\ x \geq n \\ &= \sum_{x'=0}^{[x]} b(x';\, n,\, p),\ 0 \leq x < n \end{aligned}$$

Hypergeometric:

$$h(x;\, n,\, N_1,\, N_2) = \frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N_1+N_2}{n}},\ h_L \leq x \leq h_U$$

for $h_L = \max\{0,\ n - N_2\}$, $h_U = \min\{n,\ N_1\}$

$$\begin{aligned} H(x;\, n,\, N_1,\, N_2) &= 0,\ x < h_L \\ &= 1,\ x \geq h_U \\ &= \sum_{x'=h_L}^{[x]} h(x';\, n,\, N_1,\, N_2),\ h_L \leq x \leq h_U \end{aligned}$$

Exact Charts to Control a Binomial Parameter p

To prepare the way for the introduction of attributes Q-charts, we consider first the basic nature of Shewhart control charts for attributes. We illustrate the ideas with binomial distributions.

Suppose an in-control manufacturing process has nominal fallout of $p = p_0$ proportion of production units, and we wish to design a control chart for p. For definiteness, suppose $p_0 = 0.1$ (10%

fallout) and we consider samples of size $n = 63$. Then Figure 1a shows a bar-chart of the binomial distribution of nonconformities in one sample. To make a Shewhart control chart for the number of defects in each sample, we must select a lower control limit, LCL, a centerline, CL, and an upper control limit, UCL. For X the number of nonconformities in a sample, if $P(X < LCL) = \alpha_L$ and $P(X > UCL) = \alpha_U$, we call these limits (α_L, α_U) probability limits. The usual 3-sigma formulas give here:

$$UCL = np_o + 3\sqrt{np_o(1-p_o)} = 6.3 + 3\sqrt{(63)(.1)(.9)} = 13.44$$

$$CL = 6.3$$

$$LCL = np_o - 3\sqrt{np_o(1-p_o)} = 6.3 - 3\sqrt{(63)(.1)(.9)} = -0.84$$

So, for this example the 3-sigma formulas give no lower control limit and an upper control limit of UCL = 13.44. By using an algorithm for the binomial distribution function, see column 3 of Table 2, we find

$$P(X > UCL = 13.44) = 1 - B(13;\, 63,\, 0.1) = 1 - 0.99671 = 0.00329 \simeq 1/304$$

However, $P(X > 14) = 1 - 0.99885 = 0.00115$.

Therefore, if we wish to make a chart with the probability of exceeding UCL as close as possible to, but less than, the value for a 3-sigma normal chart, we should take UCL = 14. Note also that, in general, for a binomial chart the smallest possible integer-valued lower control limit is LCL = 1, and the probability of a point below it is

$$(1 - p)^n \tag{1}$$

Thus, if we wish to determine the smallest value of n so that the probability below LCL is less than or equal to a value α_L, then we must take

$$n > \frac{\ln(\alpha_L)}{\ln(1-p)} \tag{2}$$

For a process with fallout of $p = 0.1$, we must take $n > \ln(0.00135)/\ln(0.9) = 62.7$, in order to have the normal 3-sigma value of 0.00135 below LCL = 1. A value of $n = 63$ will suffice.

Before we continue to discuss other charts for the case when p is known, we point out, and emphasize, that when a computer algorithm to evaluate the binomial distribution function B(x; n, p) is available, it can be used in the manner illustrated above to design a chart with exactly known probability limits (α_L, α_U). This is, we think, the best possible way to design a chart for the case when p is known and n is constant.

The Binomial Q-Chart for p Known

We consider next transforming observations from binomial distributions to values that can be plotted on standardized normal Q-charts. Let x_i denote an observation on a binomial random variable from a sample of size n_i. Then transform the observed values to Q-statistics as in equations (3).

$$\begin{aligned} u_i &= B(x_i; n_i, p) \\ Q_i &= \Phi^{-1}(u_i) \\ &\text{for } i = 1, 2, \cdots \end{aligned} \tag{3}$$

These values Q_1, Q_2, $\cdots$ can be plotted on a Q-chart with control limits at UCL = 3, CL = 0, and LCL = −3. More generally, a chart with approximate (α_L, α_U) probability limits can be made by putting LCL $= -z_{\alpha_L}$, CL = 0 and UCL $= z_{\alpha_U}$. We will give some numerical results below that reflect the accuracy of the normal approximation for the case with 3-sigma limits, and permit the comparison of the performance of these charts with other competing charts.

To study the nature of this transformation, we consider again the binomial b(x; 63, 0.1) distribution. The b(x; 63, 0.1) probability function is shown in Figure 1a, and the transformed Q-binomial distribution is shown in Figure 1b. This Q-distribution is, of course, a discrete distribution. It has the same probabilities of points as the binomial distribution and the points with positive probability appear in the same order on the horizontal axis, however, they are now in a different scale, which is related to a standard normal distribution. For example, the probability at zero, namely b(0; 63, 0.1), is now at the point $-z_{B(0;63,0.1)}$. In general, for x's with B(x; 63, 0.1) $\leq$ 1/2, the probability at $-z_{B(x;63,0.1)}$ is b(x; 63, 0.1); and for x's with B(x; 63, 0.1) > 1/2 the probability at $z_{B(x;63,0.1)}$ is b(x; 63, 0.1).

Figure 1a: The Binomial b(x; 63, 0.1) Probability Function

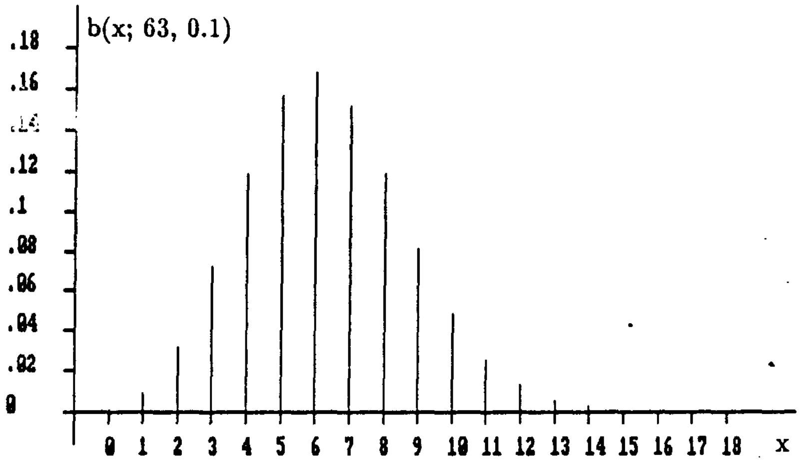


Figure 1b: The Q-Binomial Qb(q; n, p) Probability Function

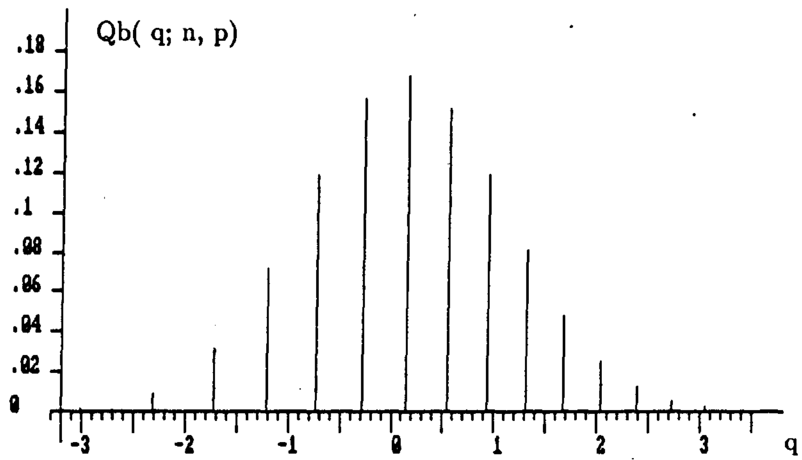


TABLE 2

x	b(x;63,0.1)	B(x;63,0.1)	Q
0	.00131	.00131	-3.01
1	.00917	.01048	-2.31
2	.03159	.04207	-1.73
3	.07136	.11343	-1.21
4	.11894	.23236	-.73
5	.15594	.38830	-.28
6	.16749	.55579	.14
7	.15154	.70732	.55
8	.11786	.82519	.94
9	.08003	.90522	1.31
10	.04802	.95323	1.68
11	.02571	.97894	2.03
12	.01238	.99132	2.38
13	.00540	.99671	2.72
14	.00214	.99885	3.05
15	.00078	.99963	3.38
16	.00026	.99989	3.72
17	.00008	.99997	4.01
18	.00002	.99999	4.32
19	.00001	1.00000	4.63

When these Q-statistics are plotted on an (α_L, α_U) control chart, it will be possible for a point to fall below the LCL only if equation (2) is satisfied. Table 2 shows the distribution of the Q-binomial distribution for n = 63 and p = 0.1. We shall denote this distribution in general by Qb(q; n, p). Note that for each value of x in the first column of Table 2, the value in the second column of the table is the binomial probability of this x, and this probability is assigned to the value of Q given in column 4, by the Q-binomial distribution Qb(q; n, p).

Example 1: To illustrate the use of the binomial Q-chart for the case when the parameter p is known, we have drawn 30 samples from a b(x; 63, 0.1) distribution and then 30 more samples from a b(x; 63, 0.15) distribution. The data are given in Table 3. The first row gives the observation number i, the second row the actual observed value x of the binomial random variable, the third the value of the statistic Q_i computed from formulas (3), and the fourth row gives the value of the Q-statistic for the case when the parameter p is not known, and will be explained below. The Q-chart for these data is shown in Figure 2a. We have drawn lines at $\pm$ 1, $\pm$ 2, $\pm$ 3. For LCL = −3 and UCL = 3, we see from Table 2 above that these are actually (0.00131, 0.00329) probability limits.

Figure 2a: Binomial Q-Chart for Example 1

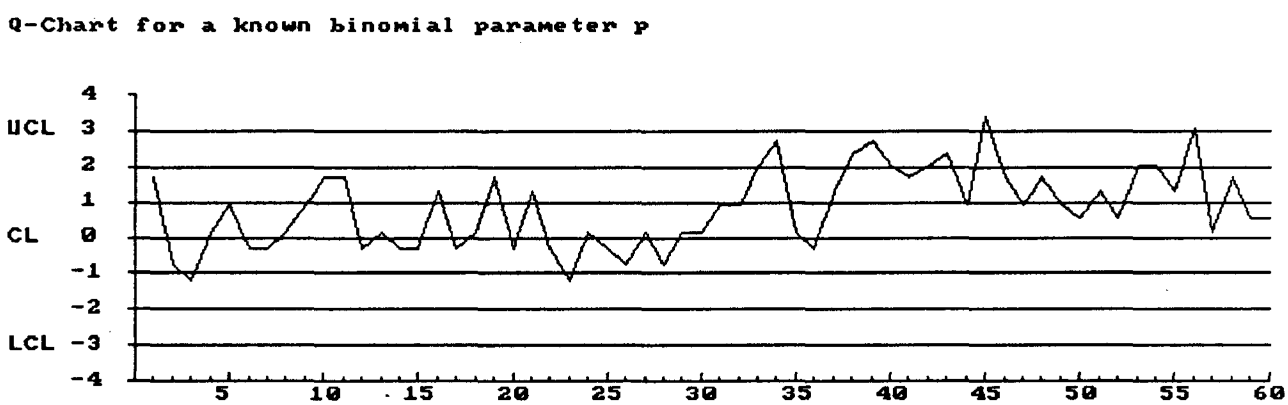


TABLE 3
Data for Examples 1 and 2

i:	1	2	3	4	5	6	7	8	9	10	11
x:	10	4	3	6	8	5	5	6	8	10	10
Q_i:	1.68	-.73	-1.21	.14	.94	-.28	-.28	.14	.94	1.68	1.68
Q'_i:	—	-1.42	-1.18	.41	1.08	-.19	-.12	.33	1.08	1.68	1.52
i:	12	13	14	15	16	17	18	19	20	21	22
x:	5	6	5	5	9	5	6	10	5	9	5
Q_i:	-.28	.14	-.28	-.28	1.31	-.28	.14	1.68	-.28	1.31	-.28
Q'_i:	-.46	0	-.39	-.34	1.23	-.37	.08	1.57	-.40	1.18	-.42
i:	23	24	25	26	27	28	29	30	31	32	33
x:	3	6	5	4	6	4	6	6	8	8	11
Q_i:	-1.21	.14	-.28	-.73	.14	-.73	.14	.14	.94	.94	2.03
Q'_i:	-1.29	.09	-.32	-.74	.16	-.70	.19	.20	.98	.96	2.01
i:	34	35	36	37	38	39	40	41	42	43	44
x:	13	6	5	9	12	13	11	10	11	12	8
Q_i:	2.72	.14	-.28	1.31	2.38	2.72	2.03	1.68	2.03	2.38	.94
Q'_i:	2.61	.01	-.40	1.18	2.2	2.46	1.73	1.34	1.65	1.95	.51
i:	45	46	47	48	49	50	51	52	53	54	55
x:	15	10	8	10	8	7	9	7	11	11	9
Q_i:	3.38	1.68	.94	1.68	.94	.55	1.31	.55	2.03	2.03	1.31
Q'_i:	2.86	1.15	.41	1.12	.39	.01	.75	0	1.44	1.42	.70
i:	56	57	58	59	60						
x:	14	6	10	7	7						
Q_i:	3.05	.14	1.68	.55	.55						
Q'_i:	2.36	-.48	1.01	-.09	-.09						

The probabilities associated with the zones defined by the ± 1, ± 2, ± 3 standard deviation lines can be approximated by the standard normal probabilities. The goodness of the approximation depends upon both n and p and generally improves as np increases. Here np = (63) (0.1) = 6.3, and the approximation may be considered marginally adequate. When the approximation is good enough, special tests for point patterns, such as some of those of Nelson (1984) can be applied to these Q-charts. We will give some results below that are helpful in judging the adequacy of the approximation. □

This chart, of course, makes use of the known value of p = 0.1. In the next section, we present a charting technique that does not assume that p is known.

The Binomial Q-Chart for p Unknown

Some further notation is needed to treat the present case. Again, we consider a sequence of values (n_i, x_i) for $i = 1, 2, \cdots$, and when the ith value is obtained we plot a point on the Q-chart. Let

$$N_i = \sum_{j=1}^{i} n_j \text{ and } t_i = \sum_{j=1}^{i} x_j \tag{4}$$

Then we compute the Q-statistics by the equations (5).

$$\begin{aligned} u_i &= H(x_i; t_i, n_i, N_{i-1}) \\ Q_i &= \Phi^{-1}(u_i) \\ &\text{for } i = 2, 3, \cdots \end{aligned} \tag{5}$$

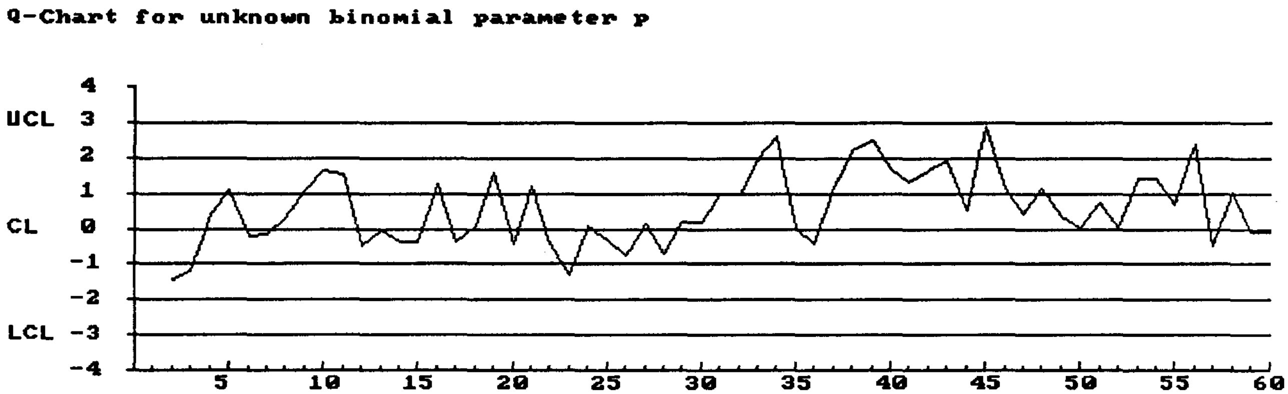
First, we explain the reason for using the transformation given in (5). Note that the transformation in (5) is the same as that in (3), except that we now use in (5) a hypergeometric distribution function where we used the binomial distribution function $B(x; n, p)$ in (3). The hypergeometric distribution function used in (5) is the ***uniform minimum variance unbiased***, UMVU, estimating distribution function of $B(x; n, p)$, and we denote it by $\tilde{B}(x)$. This function $\tilde{B}(x)$ has excellent both small and large sample properties as an estimator of $B(x; n, p)$. Briefly stated, the estimator $\tilde{B}(x)$ is an unbiased estimator of $B(x; n, p)$ with minimum variance at every point x, and it converges to the function $B(x; n, p)$ uniformly in x with probability one as sample size n increases. Those interested in the background theory on which this statement is based should see Lehmann (1983, Chapter 2) and O'Reilly and Quesenberry (1972). Plainly stated, the function $\tilde{B}(x)$ is the best possible estimator of the probability given by $B(x; n, p)$ by very reasonable criteria, and we will see below in examples that the chart obtained from this transformation agrees very closely with that for known constant p after the first few points.

From these properties it follows that when the process is in control, that is, when x_1, x_2, $\cdots$ are independent binomial random variables with constant parameter p, the Q_i's are approximately independent with discrete Q distributions. Therefore the values Q_2, Q_3, $\cdots$ can be plotted on a Q-chart. The interpretation of this chart is similar to the interpretation of the chart for p known, but certain special considerations must be observed. We will see in examples below that for the same binomial data from an in-control process the pattern of points on a chart is very similar when the Q's are computed from the formulas of (5) and when they are computed from (3), using the correct value of the parameter p. *Note that the equations (5) essentially solve the problem of charting short runs of binomial observations, since they do not require knowledge of the binomial parameter.*

Note that this chart permits plotting from the second binomial sample on, but no point is plotted for the first sample. This is because the parameter p must be "estimated" from the present data sequence. Of course, from the above remarks it is seen that we do not actually make and use an estimate of p itself. The situation is similar to that in Quesenberry (1989) which gives Q-charts for normal distribution processes with unknown parameters.

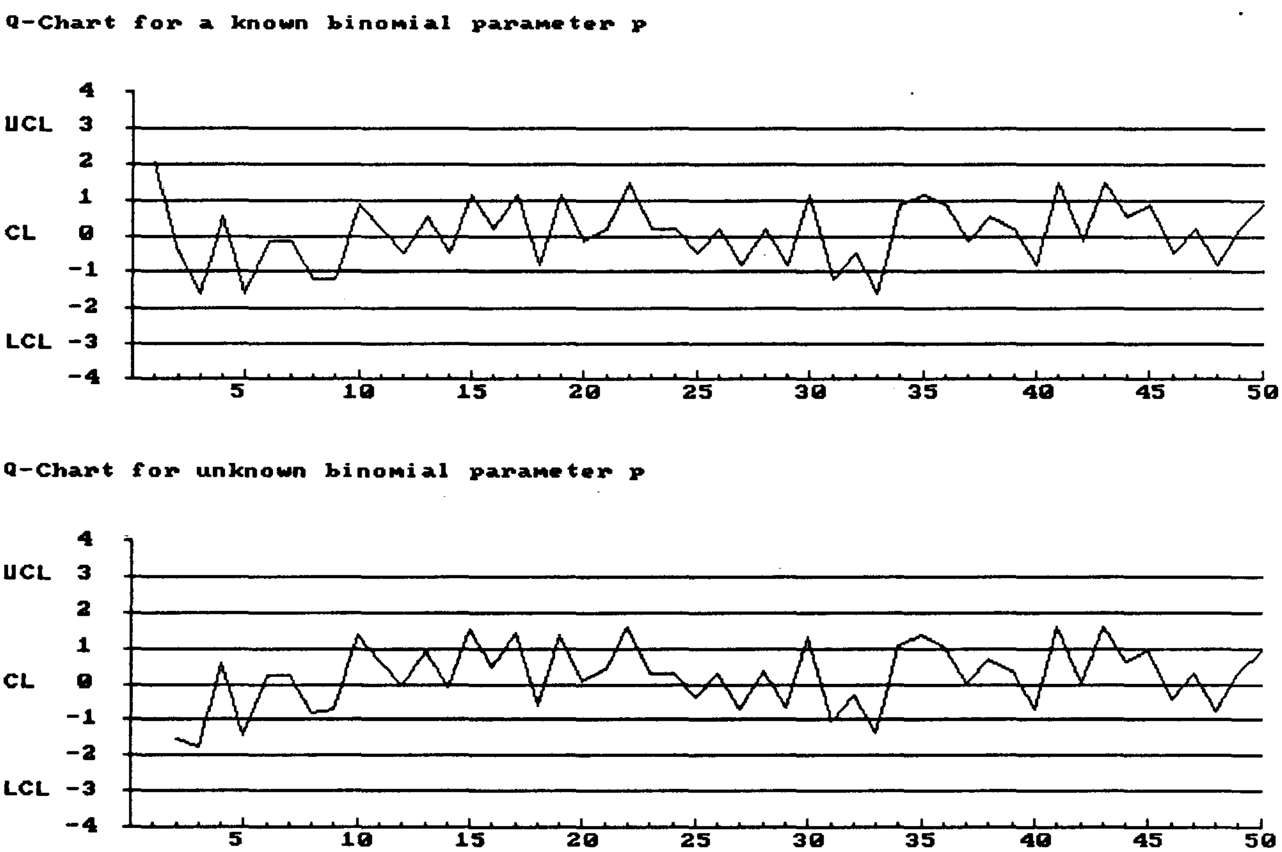
Example 2: To illustrate the use of the binomial Q-chart for unknown p we consider again the data in Table 3 above. The Q-statistics have been computed using equations (5) and are given in the last row of Table 3 as Q_i' and are plotted on a chart in Figure 2b. Recall that the first thirty binomial observations were drawn from a b(x; 63, 0.1) distribution and the last thirty from a b(x; 63, 0.15) distribution. The chart in Figure 2b should be compared with the chart in Figure 2a. Note that the point patterns for the points before p shifts from 0.1 to 0.15 on observation 31 are very similar for the charts for p known and for p unknown. Also, the point patterns after the shift are also very similar but that eventually the pattern for Figure 2b is less pronounced than that in Figure 2a. This is because, as was discussed in Q89 for continuous distributions, due to the increasing number of observations after the shift, the process is becoming stable at the new value of p. □

Figure 2b: Binomial Q-Chart for Example 2



It is important to note that the Q-chart using the statistics computed in equations (3) and (5) can be made in real time, that is, when each binomial observation x_i is observed a point for it can be plotted immediately. For the equations of (5), a point is plotted for each sample, beginning with the second. This strategy is ideal for detecting changes of p at the earliest possible time.

Figure 3: Q-Charts for 50 Binomial Observations



Example 3: To further demonstrate the similarity of the Q-charts when the parameter is not known to when it is known, we have generated 50 samples from a b(x;100,0.1). Charts for the two cases are shown in Figure 3. Note the similarity of the patterns of the two charts! □

The Normal Approximation and Comparisons with Other Charts

We have presented formulas that can be used to transform binomial observations to values that are approximately standard normal random variables and can therefore be plotted on standard normal Shewhart type control charts. For the case when p is known, the Q-chart would in most applications today be used in place of a classical 3-sigma p, np or standardized p chart. Therefore, for this case, it would be useful to study the accuracy of the normal approximation for these Q-charts, and compare this approximation with the normal approximation for these classical charts. The classical chart that is comparable with the Q-chart is the standardized np or p chart. For x the binomial random variable with probability function b(x; n, p), this standardized statistic is given by

$$z = \frac{x - np}{\sqrt{np(1 - p)}} \tag{6}$$

This standardization transformation, that is, subtracting the mean of x and dividing by its standard deviation, do not give a random variable z that is more normal than x. This, or any, linear transformation of a random variable change only the location and spread of the distribution, and do not affect its shape. This standardization transformation has been discussed in the statistics and quality literature by many writers. See, for example Duncan (1986) and Nelson (1989) for recent discussions in the quality literature. In order to transform the binomial random variable x to a new random variable that has a more normal distribution it is necessary to make a nonlinear transformation of x. The Q-statistic of (3) is such a nonlinear transformation.

A number of other nonlinear transformations to improve the normality of a binomial random variable have been considered in the statistical literature. In order to compare the Q-charts proposed here with what we think is about the best alternative potential method, we consider here a transformation given in Johnson and Kotz (1970), and recently presented in the quality literature by Ryan (1989). This is called an *arcsin* transformation and is defined by

$$y = 2\sqrt{n}\left[\sin^{-1}\left(\sqrt{\frac{x + 3/8}{n + 3/4}}\right) - \sin^{-1}(\sqrt{p})\right] \tag{7}$$

where $\sin^{-1}$ is the inverse sign function. When x is a b(x; n, p) random variable y is approximately a N(0, 1) random variable.

We now give some results for comparison of charts based on the statistics z, Q and y. Suppose that we plot these statistics on control charts with lines drawn at 0, ± 1, ± 2, and ± 3. Then we partition the vertical axis into cells in terms of Q as follows:

Cell 1 is all values for $Q < -3$
Cell 2 is all values for $-3 \leq Q < -2$
Cell 3 is all values for $-2 \leq Q < -1$
Cell 4 is all values for $-1 \leq Q \leq 0$
Cell 5 is all values for $0 < Q \leq 1$
Cell 6 is all values for $1 < Q \leq 2$
Cell 7 is all values for $2 < Q \leq 3$
Cell 8 is all values for $3 < Q$

We have computed the actual probability that each of the statistics z, Q and y will fall in each of these eight cells for a range of values of n and p, in order to study the accuracy of the normal approximation for each case. These values are given in Table 4 for p=0.01, in Table 5 for p=0.05 and in Table 6 for p=0.10 for selected values of n. At the top of each table, beside the "n", the probabilities for a standard normal distribution are given for these cells, for easy comparisons. Three rows of values are given for each sample size. The first row is for z, the second is for Q and the third is for y. The values under cell numbers are the probabilities for these cells when the binomial distribution with this n and p is correct.

The values in the last two columns labeled "Lower" and "Upper" are probabilities of detecting certain shifts in the parameter values. The value in the "Lower" column is the probability for these values of n and p that a shift from p to p/2 will be detected by having the statistic fall below the −3 control limit, that is, fall in cell 1. The value in the "Upper" column is the probability that for these values of n and p that a shift from p to 2p will be detected by having the statistic plot above the +3 control limit, that is, fall in cell 8.

To illustrate reading these tables, consider the value in Table 4 for n = 700. Then np = (700)(0.01) = 7. The first row shows that the probability in cell 1 of a standard p-chart is zero, and that, of course, the probability of detecting a shift of p = 0.01 to p/2 = 0.01/2 = 0.005 is also zero. The probability of a point on a standard p-chart plotting above the upper control limit when p has not changed, that is, the probability of a false alarm, is 0.00547, which is about four times the nominal normal rate of 0.00135. For the Q-chart for this case the nominal in-control cell probabilities are 0.00088 and 0.00228 for cell 1 and cell 8, respectively, and the probabilities of detecting shifts are 0.02993 and 0.32963 from the last two columns. For the arcsin y-chart the nominal in-control cell probabilities are 0.00088 and 0.00089, respectively, and the probabilities of detecting shifts are 0.02993 and 0.24232.

A number of interesting points can be noted by studying these tables. Due to the discreteness of the distributions, small changes in sample size n can result in rather large changes in the cell probabilities for any of these statistics, even for rather large values of the binomial mean np. Although we have mostly tabulated the tables for n changing by values of 10 or more, to illustrate this phenomenon we have given values in Table 6 (p = 0.10) for n = 500(1)520 (read this as "for n equal 500 to 520 in steps of 1"). For example, the value for cell 1 of the y statistic shifts from a value at n = 519 of 0.00132 to a value at n = 520 of 0.00216. This is a shift for the ARL from 758 to 463, which is a surprisungly large shift given that the binomial mean np = 51.9.

We have given these charts only for the three small values of p = 0.01, 0.05 and 0.10 because the applications of these charts typically are for small values of p. Although we have computed these probabilities for eight cells, we are especially interested in the probabilities in cells 1 and 8, which are the probabilities beyond the lower and upper 3-sigma control limits. Comparison of the probabilities in some of the other cells with the normal probabilities at the top of the table allows some assessment of the accuracy of special runs tests, such as those of Nelson (1984). We make a few general comparative conclusions from these results. We think that these results show that the classical p-chart is clearly inferior to both of the other charts. It requires much larger sample sizes to achieve any power to detect decreases in p, and cell probabilities always differ by as much, and usually by more, from the nominal normal values than for the other charts. In particular, for all of the results in these tables the Q-chart probabilities for cells 1 and 8 are either the same as for the p-chart or - most often - they are between 0.00135 and the value for the p-chart. For cell 1 the p-chart values are too small and for cell 8 they are too large.

It is more difficult to summarize the comparison of the Q-charts and arcsin charts (based on y). Neither can be said to be a better overall approximation, but we venture a few summary remarks. The Q-chart tends to have values in cell 8 closer to, but somewhat larger than, the nominal value of 0.00135 than the y-chart, which tends to give values that are often considerably less than 0.00135, and therefore the y-chart gives less protection to detect increases in p. On the other hand, the Q-chart tends to give values in cell 1 that are smaller than 0.00135 while the y-chart tends to give values for this cell closer to but frequently larger than 0.00135. Thus the y-chart will be more sensitive to detect decreases in p, while making more false alarms. Due to this trend the y-chart gives positive probability in cell 1, and therefore some power to detect decreases of p at smaller sample sizes, for the values of p considered here.

These tables are only for charts for p known. The Q-chart for unknown p converges rapidly to the chart for known p and its limiting behavior will be the same as that for the Q-chart with known p. Due to the excellent theoretical properties of the UMVU estimator $\widetilde{B}(x)$ of $B(x; n, p)$ discussed above, we think it is unlikely that one can make other charts for the unknown p case that would perform as well as the Q-chart proposed here. We close by summarizing some of the properties of the Q-charts proposed here for the consideration of potential users.

Some Properties of Binomial Q-charts:

1. These charts can be made in real time beginning with either the first binomial observation (when p is known) or the second (when p is unknown). This is especially useful for short runs or start-up processes when p is <u>not</u> known, since there are no established methods available today for this situation.
2. The samples do not have to be of constant size.
3. Since these charts are plotted on standard normal scale, the training of personnel to use them is simplified. This may permit savings in a charts management program.
4. It is permissible to plot several different binomial variables on the same chart. Some people may not consider this a desirable thing to do because of the potential for mix-ups and confusion that are possible. However, it does give greater flexibility in planning chart management programs, and users have the choice at their disposal. For example, plotting the data for different types of nonconformities counted on the same units of production on the same chart may give a quicker way to detect production system troubles.

4. Tests for point patterns to detect special causes can be implemented on these charts. When the normal approximation is adequate, tests such as those of Nelson (1984) can be used.

5. The implementation of these methods will generally require at least a small computer and algorithms for the computations. Algorithms for the binomial and hypergeometric distributions are easily written in any computer language and are available in most statistical packages, as well as

inverse normal distribution functions.

Acknowledgement

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TABLE 4

Classical p-Chart, Q-Chart and Arcsin Transformation Cell Probabilities and Power Comparisons for p=0.01

Cell:	1	2	3	4	5	6	7	8		
n	.00135	.02140	.13591	.34134	.34134	.13591	.02140	.00135	Lower	Upper
20	.00000	.00000	.00000	.81791	.00000	.16523	.00000	.01686	.00000	.05990
	.00000	.00000	.00000	.00000	.81791	.00000	.16523	.01686	.00000	.05990
	.00000	.00000	.00000	.00000	.81791	.16523	.01682	.00004	.00000	.00060
40	.00000	.00000	.00000	.66897	.27029	.00000	.05324	.00750	.00000	.04567
	.00000	.00000	.00000	.00000	.66897	.27029	.05324	.00750	.00000	.04567
	.00000	.00000	.00000	.66897	.00000	.32353	.00745	.00005	.00000	.00118
60	.00000	.00000	.00000	.54716	.33161	.09881	.00000	.02242	.00000	.11874
	.00000	.00000	.00000	.00000	.54716	.33161	.11811	.00312	.00000	.03219
	.00000	.00000	.00000	.54716	.33161	.09881	.02207	.00035	.00000	.00702
80	.00000	.00000	.00000	.44752	.36163	.14429	.03789	.00866	.00000	.07685
	.00000	.00000	.00000	.44752	.36163	.14429	.03789	.00866	.00000	.07685
	.00000	.00000	.00000	.44752	.36163	.18218	.00850	.00016	.00000	.00545
100	.00000	.00000	.36603	.36973	.00000	.18486	.06100	.01837	.00000	.14104
	.00000	.00000	.00000	.36603	.36973	.18486	.07594	.00343	.00000	.05083
	.00000	.00000	.00000	.36603	.36973	.24586	.01784	.00053	.00000	.01548
120	.00000	.00000	.29938	.36289	.21810	.08665	.02560	.00738	.00000	.09383
	.00000	.00000	.00000	.29938	.36289	.30475	.03160	.00138	.00000	.03410
	.00000	.00000	.00000	.29938	.58098	.08665	.03276	.00022	.00000	.01073
140	.00000	.00000	.24487	.34627	.24309	.11295	.03908	.01374	.00000	.15042
	.00000	.00000	.00000	.24487	.58937	.11295	.04981	.00300	.00000	.06317
	.00000	.00000	.24487	.34627	.24309	.15203	.01318	.00056	.00000	.02311
160	.00000	.00000	.20028	.32368	.25992	.19310	.01728	.00574	.00000	.10333
	.00000	.00000	.00000	.20028	.58360	.19310	.01728	.00574	.00000	.10333
	.00000	.00000	.20028	.32368	.25992	.19310	.02279	.00023	.00000	.01578
180	.00000	.00000	.16381	.29783	.43062	.07213	.02565	.00996	.00000	.15392
	.00000	.00000	.00000	.46164	.26925	.23350	.03320	.00241	.00000	.07127
	.00000	.00000	.16381	.29783	.43062	.09777	.00945	.00051	.00000	.02934
200	.00000	.00000	.13398	.54270	.18136	.09022	.04745	.00430	.00000	.10856
	.00000	.00000	.13398	.27067	.27203	.27158	.04745	.00430	.00000	.10856
	.00000	.00000	.13398	.27067	.45339	.12594	.01581	.00021	.00000	.02017
250	.00000	.00000	.08106	.46211	.34902	.06663	.03716	.00403	.00000	.13125
	.00000	.00000	.08106	.20469	.47236	.20070	.03716	.00403	.00000	.13125
	.00000	.00000	.08106	.46211	.21495	.22818	.01345	.00025	.00000	.03037

TABLE 4 (Continued)

Classical p-Chart, Q-Chart and Arcsin Transformation Cell Probabilities and Power Comparisons for p=0.01

Cell:	1	2	3	4	5	6	7	8		
n	.00135	.02140	.13591	.34134	.34134	.13591	.02140	.00135	Lower	Upper
300	.00000	.00000	.19765	.44958	.16888	.15114	.02915	.00360	.00000	.15067
	.00000	.00000	.04904	.37302	.39405	.15114	.02915	.00360	.00000	.15067
	.00000	.04904	.14861	.22441	.39405	.17241	.01045	.00102	.00000	.08184
350	.00000	.00000	.13456	.40153	.32252	.11532	.02293	.00314	.00000	.16745
	.00000	.00000	.13456	.18489	.40647	.24801	.02293	.00314	.00000	.16745
	.00000	.02967	.10489	.40153	.32252	.11532	.02512	.00095	.00000	.09637
400	.00000	.01795	.21868	.39221	.15708	.16384	.04243	.00780	.00000	.28211
	.00000	.01795	.07253	.34201	.35343	.16384	.04755	.00268	.00000	.18210
	.00000	.01795	.07253	.34201	.35343	.19331	.01992	.00085	.00000	.10973
450	.00000	.01086	.16131	.35946	.30039	.12854	.03306	.00638	.00000	.29280
	.00000	.01086	.04936	.28080	.49100	.12854	.03718	.00227	.00000	.19502
	.01086	.00000	.16131	.35946	.30039	.15141	.01583	.00075	.10481	.12202
500	.00000	.00657	.11682	.49258	.25172	.10122	.02589	.00521	.00000	.30207
	.00000	.00657	.11682	.31623	.32331	.20598	.02920	.00190	.00000	.20652
	.00657	.03318	.08363	.31623	.42807	.11908	.01260	.00065	.08157	.13332
550	.00000	.00398	.19630	.32821	.28193	.16497	.02037	.00424	.00000	.31019
	.00000	.00398	.08332	.26905	.45408	.16497	.02303	.00158	.00000	.21683
	.00398	.02208	.17422	.32821	.28193	.16497	.02406	.00055	.06349	.14375
600	.00000	.01698	.13288	.45644	.24197	.10993	.03834	.00345	.00000	.31739
	.00000	.01698	.13288	.29501	.29980	.21353	.03834	.00345	.00000	.31739
	.00241	.01458	.13288	.29501	.40341	.13219	.01906	.00047	.04941	.15338
650	.00000	.01101	.09963	.41549	.35234	.08840	.03033	.00280	.00000	.32383
	.00000	.01101	.09963	.25730	.42453	.17439	.03033	.00280	.00000	.32383
	.00146	.04087	.06832	.41549	.26634	.19199	.01445	.00108	.03846	.23460
700	.00000	.00710	.16451	.42710	.23280	.14214	.02087	.00547	.00000	.42956
	.00088	.00622	.07362	.36824	.38256	.14214	.02407	.00228	.02993	.32963
	.00088	.02820	.14254	.27734	.38256	.14214	.02545	.00089	.02993	.24232
750	.00000	.01983	.11095	.39349	.33905	.09484	.03744	.00440	.00000	.43191
	.00053	.01930	.11095	.24634	.40015	.18089	.03999	.00185	.02330	.33490
	.00457	.01526	.11095	.39349	.33905	.11570	.02025	.00074	.11110	.24942
800	.00000	.01343	.17642	.40269	.22434	.14967	.02990	.00354	.00000	.43403
	.00032	.01311	.08505	.35377	.36463	.14967	.03194	.00150	.01813	.33971
	.00293	.01051	.17642	.26240	.36463	.16636	.01615	.00060	.09103	.25596

TABLE 4 (Continued)

Classical p-Chart, Q-Chart and Arcsin Transformation Cell Probabilities and Power Comparisons for p=0.01

Cell:	1	2	3	4	5	6	7	8		
n	.00135	.02140	.13591	.34134	.34134	.13591	.02140	.00135	Lower	Upper
850	.00000	.00905	.13923	.37448	.32698	.12350	.02392	.00285	.00000	.43597
	.00019	.00885	.13923	.23634	.37958	.20904	.02392	.00285	.01411	.43597
	.00187	.02767	.11874	.37448	.32698	.12350	.02627	.00050	.07439	.26202
900	.00012	.02066	.18463	.38200	.21658	.15536	.03557	.00509	.01098	.53230
	.00119	.01959	.09369	.34052	.34900	.15536	.03836	.00230	.06066	.43775
	.00119	.01959	.09369	.34052	.34900	.17456	.02046	.00099	.06066	.34818
950	.00007	.01444	.14909	.35789	.31599	.12976	.02865	.00409	.00855	.53144
	.00076	.01376	.07293	.30345	.44660	.12976	.03089	.00185	.04936	.43939
	.00404	.01048	.14909	.35789	.31599	.14529	.01641	.00080	.14668	.35194
1000	.00004	.01003	.11880	.45416	.28252	.10804	.02310	.00329	.00665	.53064
	.00048	.00959	.11880	.32842	.33521	.18110	.02489	.00150	.04009	.44090
	.00268	.02601	.10019	.32842	.40826	.10804	.02574	.00065	.12402	.35544
1100	.00019	.01456	.12715	.43737	.27587	.11338	.02701	.00447	.02631	.61473
	.00117	.01358	.12715	.31739	.32294	.18630	.02935	.00213	.08784	.52922
	.00117	.03568	.10505	.31739	.39585	.12768	.01620	.00098	.08784	.44362
1200	.00008	.01982	.13382	.42225	.26954	.11784	.03375	.00290	.01717	.60988
	.00050	.01940	.13382	.30731	.31191	.19041	.03526	.00139	.06152	.52797
	.00221	.01768	.13382	.30731	.38449	.13378	.02008	.00064	.15053	.44600
1300	.00021	.01024	.15403	.40856	.26353	.13903	.02060	.00379	.04268	.68116
	.00101	.00945	.08816	.36394	.37403	.13903	.02251	.00189	.11125	.60559
	.00362	.02174	.13913	.29807	.37403	.13903	.02349	.00091	.22297	.52687
1400	.00009	.01379	.16047	.39609	.25784	.14355	.02570	.00248	.02936	.67483
	.00045	.01343	.09433	.35570	.36436	.14355	.02570	.00248	.08124	.67483
	.00174	.02927	.14334	.28956	.36436	.15549	.01564	.00059	.17231	.52590
1500	.00020	.01739	.16583	.38467	.25245	.14743	.02888	.00316	.05871	.73505
	.00082	.01677	.09966	.34789	.35540	.14743	.03041	.00162	.13142	.66912
	.00270	.01489	.16583	.28172	.35540	.16053	.01813	.00081	.24075	.59834
1600	.00038	.02113	.17028	.37417	.24734	.15076	.03201	.00392	.09906	.78633
	.00133	.02018	.10424	.34050	.34706	.15076	.03386	.00208	.19055	.72822
	.00133	.02018	.10424	.34050	.34706	.16495	.02068	.00106	.19055	.66393
1700	.00018	.01210	.12149	.43026	.29857	.11278	.02204	.00259	.07387	.77903
	.00065	.01163	.12149	.33349	.33927	.16885	.02326	.00137	.14894	.72195
	.00199	.02360	.10818	.33349	.33927	.16885	.02393	.00070	.25551	.65920

TABLE 4 (Continued)

Classical p-Chart, Q-Chart and Arcsin Transformation Cell Probabilities and Power Comparisons for p=0.01

Cell:	1	2	3	4	5	6	7	8		
n	.00135	.02140	.13591	.34134	.34134	.13591	.02140	.00135	Lower	Upper
1800	.00031	.01470	.12631	.42093	.29397	.11620	.02440	.00318	.11508	.82196
	.00100	.01401	.12631	.32686	.33198	.17226	.02586	.00172	.20610	.77225
	.00279	.02698	.11155	.32686	.38804	.12696	.01593	.00090	.32331	.71617
1900	.00015	.01775	.13056	.41215	.28953	.11928	.02846	.00211	.08799	.81513
	.00050	.01740	.13056	.32057	.32513	.17526	.02846	.00211	.16428	.81513
	.00145	.01644	.13056	.32057	.38112	.13085	.01787	.00114	.26801	.76594
2000	.00024	.02067	.13430	.40388	.28526	.12204	.03105	.00256	.12951	.85114
	.00075	.02017	.13430	.31459	.31869	.17789	.03220	.00141	.21954	.80870
	.00201	.01890	.13430	.31459	.37454	.13439	.02050	.00075	.33226	.76004

TABLE 5

Classical p-Chart, Q-Chart and Arcsin Transformation Cell Probabilities and Power Comparisons for p=0.05

Cell:	1	2	3	4	5	6	7	8		
n	.00135	.02140	.13591	.34134	.34134	.13591	.02140	.00135	Lower	Upper
20	.00000	.00000	.35849	.37735	.00000	.18868	.05958	.01590	.00000	.13295
	.00000	.00000	.00000	.35849	.37735	.18868	.07291	.00257	.00000	.04317
	.00000	.00000	.00000	.35849	.37735	.24826	.01557	.00033	.00000	.01125
40	.00000	.00000	.12851	.54822	.18511	.09012	.04464	.00339	.00000	.09952
	.00000	.00000	.12851	.27055	.27767	.27524	.04464	.00339	.00000	.09952
	.00000	.00000	.12851	.27055	.46279	.12427	.01317	.00071	.00000	.04190
60	.00000	.00000	.19155	.45573	.17238	.15064	.02685	.00285	.00000	.14164
	.00000	.00000	.04607	.37137	.40223	.15064	.02685	.00285	.00000	.14164
	.00000	.04607	.14548	.22588	.40223	.15064	.02896	.00074	.00000	.07307
80	.00000	.01652	.21411	.39826	.16034	.16418	.04006	.00653	.00000	.27655
	.00000	.01652	.06954	.34239	.36078	.16418	.04449	.00210	.00000	.17338
	.00000	.01652	.06954	.34239	.36078	.19237	.01779	.00061	.00000	.10044
100	.00000	.00592	.11234	.49774	.25604	.09977	.02391	.00427	.00000	.29697
	.00000	.00592	.11234	.31772	.33003	.20580	.02672	.00146	.00000	.19818
	.00592	.03116	.08118	.31772	.43606	.09977	.02773	.00046	.07952	.12388
120	.00000	.01553	.12888	.46193	.24626	.10896	.03568	.00277	.00000	.31266
	.00000	.01553	.12888	.29714	.30604	.21396	.03568	.00277	.00000	.31266
	.00212	.01340	.12888	.29714	.41105	.13010	.01631	.00099	.04792	.21816
140	.00000	.00637	.15966	.43272	.23701	.14053	.01918	.00453	.00000	.42947
	.00076	.00561	.07016	.36935	.38988	.14053	.02192	.00179	.02888	.32520
	.00076	.02611	.13915	.27985	.38988	.14053	.02305	.00066	.02888	.23469
160	.00000	.01218	.17204	.40835	.22846	.14849	.02760	.00287	.00000	.43395
	.00027	.01191	.08167	.35552	.37167	.14849	.02760	.00287	.01741	.43395
	.00257	.03625	.14540	.26514	.37167	.16420	.01360	.00116	.08882	.33552
180	.00010	.01892	.18078	.38764	.22061	.15456	.03317	.00422	.01049	.53632
	.00102	.01799	.09053	.34272	.35579	.15456	.03557	.00182	.05891	.43767
	.00102	.01799	.09053	.34272	.35579	.17280	.01840	.00075	.05891	.34421
200	.00004	.00901	.11469	.45932	.28704	.10609	.02113	.00266	.00632	.53446
	.00040	.00864	.11469	.33097	.34177	.17972	.02113	.00266	.03875	.53446
	.00234	.02411	.09730	.33097	.34177	.17972	.02264	.00116	.12148	.44083
250	.00027	.01281	.18150	.32295	.29375	.16157	.02325	.00389	.04970	.69368
	.00128	.01180	.10554	.28293	.40972	.16157	.02528	.00187	.12704	.61428
	.00128	.03010	.08724	.39890	.29375	.16157	.02629	.00086	.12704	.53083

TABLE 5 (Continued)

Classical p-Chart, Q-Chart and Arcsin Transformation Cell Probabilities and Power Comparisons for p=0.05

Cell:	1	2	3	4	5	6	7	8		
n	.00135	.02140	.13591	.34134	.34134	.13591	.02140	.00135	Lower	Upper
300	.00016	.01580	.16201	.39014	.25684	.14586	.02661	.00257	.05696	.74518
	.00069	.01528	.09633	.35072	.36194	.14586	.02661	.00257	.12885	.74518
	.00233	.03173	.14391	.28504	.36194	.15820	.01557	.00127	.23798	.67761
350	.00037	.01751	.14530	.35160	.32272	.13222	.02707	.00321	.12854	.83640
	.00120	.01668	.08774	.31119	.42068	.13222	.02859	.00169	.22707	.78624
	.00120	.01668	.14530	.35160	.32272	.14431	.01733	.00086	.22707	.72870
400	.00020	.01885	.13088	.40918	.28987	.12032	.02864	.00207	.12697	.86177
	.00062	.01843	.13088	.31804	.32472	.17660	.02864	.00207	.21680	.86177
	.00172	.01733	.13088	.31804	.38101	.13199	.01793	.00110	.32996	.81952
450	.00032	.01933	.11835	.37501	.34649	.10985	.02824	.00241	.20714	.91221
	.00089	.01877	.11835	.28863	.38234	.16037	.02824	.00241	.31106	.91221
	.00219	.01746	.11835	.37501	.34649	.12102	.01877	.00072	.42856	.84685
500	.00046	.01940	.15899	.37409	.27059	.14621	.02756	.00270	.29404	.94498
	.00114	.01871	.10738	.34412	.35218	.14621	.02871	.00155	.40369	.92491
	.00261	.01725	.15899	.29250	.35218	.15683	.01878	.00086	.51826	.89989
550	.00060	.01916	.14424	.34775	.32486	.13375	.02669	.00294	.38123	.96587
	.00060	.01916	.09771	.31617	.40297	.13375	.02790	.00173	.38123	.95258
	.00138	.01838	.14424	.34775	.32486	.14381	.01858	.00099	.49014	.93558
600	.00031	.01914	.13127	.39765	.30009	.12271	.02695	.00189	.36079	.97036
	.00073	.01872	.13127	.32313	.32893	.16839	.02695	.00189	.46435	.97036
	.00158	.01787	.13127	.32313	.37461	.13221	.01822	.00111	.56810	.95906
650	.00039	.01860	.11977	.37205	.34840	.11286	.02591	.00202	.44060	.98161
	.00085	.01814	.11977	.30020	.37848	.15464	.02591	.00202	.54086	.98161
	.00176	.01723	.11977	.37205	.30663	.16359	.01775	.00121	.63674	.97423
700	.00046	.01797	.15206	.37433	.28590	.14235	.02482	.00212	.51539	.98866
	.00096	.01747	.10954	.34783	.35492	.14235	.02482	.00212	.60953	.98866
	.00190	.02731	.14127	.30531	.35492	.15078	.01721	.00130	.69617	.98391
750	.00052	.01726	.13931	.35297	.33272	.13131	.02371	.00220	.58367	.99305
	.00104	.01674	.13931	.28609	.35794	.17297	.02454	.00137	.67002	.99002
	.00201	.02579	.12929	.35297	.33272	.13924	.01714	.00084	.74699	.98592
800	.00057	.01652	.12786	.39698	.31185	.12134	.02261	.00226	.64478	.99575
	.00112	.01598	.12786	.33240	.33798	.15980	.02344	.00143	.72257	.99384
	.00209	.02431	.11856	.33240	.37643	.12134	.02398	.00089	.79005	.99122

TABLE 5 (Continued)

Classical p-Chart, Q-Chart and Arcsin Transformation Cell Probabilities and Power Comparisons for p=0.05

Cell:	1	2	3	4	5	6	7	8		
n	.00135	.02140	.13591	.34134	.34134	.13591	.02140	.00135	Lower	Upper
850	.00062	.01576	.15663	.33643	.31885	.14788	.02152	.00230	.69865	.99741
	.00118	.01520	.11754	.31270	.38167	.14788	.02235	.00148	.76773	.99622
	.00215	.02288	.14799	.33643	.31885	.14788	.02289	.00093	.82626	.99455
900	.00066	.01500	.14430	.37959	.30062	.13705	.02045	.00233	.74556	.99843
	.00122	.01444	.10822	.35477	.36153	.13705	.02127	.00151	.80621	.99768
	.00218	.02150	.13627	.31869	.36153	.13705	.02182	.00096	.85655	.99663
950	.00069	.02170	.12564	.36089	.34213	.11898	.02762	.00234	.78603	.99905
	.00126	.02114	.12564	.30148	.36593	.15459	.02843	.00153	.83877	.99859
	.00220	.02020	.12564	.36089	.34213	.12718	.02077	.00099	.88177	.99793
1000	.00072	.02044	.15214	.36424	.29041	.14363	.02610	.00233	.82067	.99943
	.00128	.01988	.11598	.34261	.34820	.14363	.02689	.00154	.86619	.99914
	.00220	.01896	.11598	.34261	.34820	.15131	.01974	.00101	.90271	.99873
1100	.00075	.01808	.13035	.38661	.31428	.12437	.02327	.00229	.87506	.99979
	.00129	.01754	.13035	.33150	.33621	.15755	.02402	.00154	.90835	.99968
	.00215	.02484	.12219	.33150	.36938	.13110	.01781	.00102	.93435	.99953
1200	.00076	.01597	.14345	.37409	.30567	.13711	.02073	.00222	.91369	.99992
	.00127	.01546	.11204	.35273	.35844	.13711	.02144	.00151	.93759	.99988
	.00207	.02169	.13642	.32133	.35844	.13711	.02193	.00102	.95584	.99982
1300	.00075	.02016	.14934	.36268	.29767	.14216	.02511	.00212	.94076	.99997
	.00123	.01968	.11789	.34343	.34838	.14216	.02576	.00147	.95768	.99996
	.00197	.01894	.14934	.31198	.34838	.14881	.01958	.00100	.97037	.99994
1400	.00073	.01767	.12972	.38363	.31940	.12458	.02227	.00201	.95953	.99999
	.00118	.01722	.12972	.33476	.33909	.15375	.02288	.00141	.97139	.99998
	.00185	.02344	.12282	.33476	.36826	.12458	.02331	.00097	.98016	.99998
1500	.00070	.02145	.13464	.37388	.31242	.12882	.02620	.00189	.97245	1.00000
	.00111	.02104	.13464	.32667	.33049	.15796	.02620	.00189	.98071	1.00000
	.00172	.02043	.13464	.32667	.35963	.13525	.02032	.00134	.98672	.99999
1600	.00067	.01874	.14548	.36481	.30583	.13949	.02320	.00177	.98131	1.00000
	.00104	.01837	.11765	.34692	.35155	.13949	.02320	.00177	.98701	1.00000
	.00159	.02445	.13885	.31909	.35155	.13949	.02371	.00126	.99112	1.00000
1700	.00097	.02180	.14969	.35635	.29962	.14319	.02609	.00228	.99127	1.00000
	.00097	.01606	.12761	.33982	.34397	.14319	.02672	.00165	.99127	1.00000
	.00147	.02131	.12187	.33982	.34397	.14935	.02104	.00118	.99407	1.00000

TABLE 5 (Continued)

Classical p-Chart, Q-Chart and Arcsin Transformation Cell Probabilities
and Power Comparisons for p=0.05

Cell:	1	2	3	4	5	6	7	8		
n	.00135	.02140	.13591	.34134	.34134	.13591	.02140	.00135	Lower	Upper
1800	.00090	.01904	.13186	.37621	.31977	.12702	.02310	.00211	.99413	1.00000
	.00134	.01859	.13186	.33310	.33685	.15305	.02367	.00153	.99604	1.00000
	.00134	.02490	.12556	.33310	.36288	.13244	.01868	.00111	.99604	1.00000
1900	.00082	.01664	.14110	.36870	.31417	.13617	.02046	.00194	.99606	1.00000
	.00122	.01624	.14110	.32674	.33013	.15632	.02683	.00142	.99735	1.00000
	.00179	.02116	.13562	.32674	.35613	.13617	.02137	.00103	.99825	1.00000
2000	.00076	.01933	.14488	.36160	.30884	.13951	.02331	.00178	.99736	1.00000
	.00111	.01897	.11993	.34566	.34973	.13951	.02331	.00178	.99823	1.00000
	.00162	.02443	.13893	.32070	.34973	.14467	.01862	.00131	.99883	1.00000

TABLE 6

Classical p-Chart, Q-Chart and Arcsin Transformation Cell Probabilities and Power Comparisons for p=0.10

Cell:	1	2	3	4	5	6	7	8		
n	.00135	.02140	.13591	.34134	.34134	.13591	.02140	.00135	Lower	Upper
5	.00000	.00000	.00000	.59049	.32805	.00000	.07290	.00856	.00000	.05792
	.00000	.00000	.00000	.00000	.59049	.32805	.07290	.00856	.00000	.05792
	.00000	.00000	.00000	.59049	.32805	.07290	.00810	.00046	.77378	.00672
10	.00000	.00000	.34868	.38742	.00000	.19371	.05740	.01280	.00000	.12087
	.00000	.00000	.00000	.34868	.38742	.19371	.06856	.00163	.00000	.03279
	.00000	.00000	.00000	.34868	.38742	.25111	.01265	.00015	.00000	.00637
15	.00000	.00000	.20589	.34315	.26690	.12851	.04284	.01272	.00000	.16423
	.00000	.00000	.00000	.20589	.61005	.12851	.05331	.00225	.00000	.06105
	.00000	.00000	.20589	.34315	.26690	.17134	.01241	.00031	.00000	.01806
20	.00000	.00000	.12158	.55535	.19012	.08978	.04079	.00239	.00000	.08669
	.00000	.00000	.12158	.27017	.28518	.27990	.04079	.00239	.00000	.08669
	.00000	.00000	.12158	.27017	.47530	.12170	.01084	.00042	.00000	.03214
25	.00000	.00000	.07179	.46530	.36491	.06459	.03114	.00226	.00000	.10912
	.00000	.00000	.07179	.19942	.49239	.20301	.03114	.00226	.00000	.10912
	.00000	.07179	.00000	.46530	.22650	.22693	.00902	.00046	.00000	.04677
30	.00000	.00000	.18370	.46374	.17707	.14967	.01804	.00778	.00000	.23921
	.00000	.00000	.04239	.36896	.41315	.14967	.02381	.00202	.00000	.12865
	.00000	.04239	.14130	.22766	.41315	.14967	.02537	.00045	.00000	.06109
35	.00000	.00000	.12238	.40861	.33738	.11165	.01369	.00630	.00000	.25499
	.00000	.00000	.12238	.18387	.42450	.21407	.05344	.00174	.00000	.14573
	.00000	.02503	.09734	.40861	.33738	.11165	.01957	.00042	.00000	.07471
40	.00000	.01478	.20803	.40621	.16471	.16437	.03684	.00506	.00000	.26822
	.00000	.01478	.06569	.34266	.37060	.16437	.04043	.00147	.00000	.16077
	.00000	.01478	.06569	.34266	.37060	.19078	.01511	.00038	.00000	.08751
45	.00000	.00873	.15031	.36809	.31435	.12651	.02795	.00404	.00000	.27953
	.00000	.00873	.04364	.27657	.37879	.26028	.02795	.00404	.00000	.27953
	.00873	.00000	.15031	.36809	.31435	.14648	.01081	.00122	.09944	.17412
50	.00000	.00515	.10657	.50439	.26173	.09761	.02132	.00322	.00000	.28933
	.00000	.00515	.10657	.31947	.33903	.20524	.02132	.00322	.00000	.28933
	.00515	.02863	.07794	.31947	.44666	.09761	.02353	.00100	.07694	.18606
60	.00000	.01378	.12362	.46905	.25191	.10743	.02853	.00568	.00000	.42356
	.00000	.01378	.12362	.29977	.31439	.21424	.03218	.00203	.00000	.30557
	.00180	.01198	.12362	.29977	.42120	.12705	.01392	.00067	.04607	.20654

TABLE 6 (Continued)

Classical p-Chart, Q-Chart and Arcsin Transformation Cell Probabilities and Power Comparisons for p=0.10

Cell:	1	2	3	4	5	6	7	8		
n	.00135	.02140	.13591	.34134	.34134	.13591	.02140	.00135	Lower	Upper
70	.00000	.00550	.15329	.44006	.24259	.13810	.01699	.00347	.00000	.42912
	.00063	.00487	.06573	.37058	.29445	.21967	.04060	.00347	.02758	.42912
	.00550	.01868	.13461	.28302	.39963	.13810	.01919	.00127	.12921	.31856
80	.00000	.01068	.16623	.41576	.23394	.14664	.02462	.00213	.00000	.43363
	.00022	.01047	.07729	.35759	.38106	.14664	.02462	.00213	.01652	.43363
	.00216	.03315	.14161	.26864	.38106	.14664	.02595	.00080	.08605	.32925
90	.00008	.01680	.17561	.39504	.22598	.15322	.03004	.00323	.00989	.54199
	.00084	.01604	.08637	.34542	.36485	.15322	.03004	.00323	.05673	.54199
	.00460	.01228	.17561	.25617	.36485	.17017	.01502	.00131	.16643	.43737
100	.00003	.00781	.10932	.46600	.29297	.10328	.01862	.00198	.00592	.53984
	.00032	.00751	.10932	.33413	.35053	.15829	.03791	.00198	.03708	.53984
	.00194	.02177	.09344	.33413	.35053	.17758	.01979	.00081	.11826	.44054
110	.00012	.01158	.11814	.44954	.28632	.10922	.02227	.00281	.02407	.63164
	.00081	.01089	.11814	.32371	.33777	.18359	.02227	.00281	.08294	.63164
	.00355	.02735	.09894	.32371	.41215	.10922	.02387	.00122	.19447	.53799
120	.00033	.01571	.12539	.43464	.27996	.11426	.02591	.00380	.05751	.71044
	.00033	.01571	.12539	.31409	.32631	.18845	.02798	.00173	.05751	.62616
	.00157	.01447	.12539	.31409	.40051	.12820	.01502	.00075	.14441	.53638
130	.00013	.02059	.13134	.42108	.27390	.11853	.03208	.00235	.03948	.70276
	.00069	.02004	.13134	.30520	.31594	.19236	.03208	.00235	.10576	.70276
	.00264	.01809	.13134	.30520	.38978	.13403	.01786	.00107	.21653	.62130
140	.00029	.01062	.15094	.40868	.26813	.13910	.01915	.00308	.07652	.76723
	.00121	.00970	.08635	.36156	.37985	.12214	.03773	.00146	.16602	.69587
	.00121	.02444	.13620	.29696	.37985	.13910	.02157	.00066	.16602	.61697
150	.00012	.01389	.15688	.39729	.26266	.14352	.02372	.00192	.05477	.75935
	.00055	.01347	.09194	.35425	.37063	.14352	.02372	.00192	.12559	.75935
	.00192	.02882	.14016	.28932	.37063	.14352	.02473	.00091	.23444	.68963
160	.00024	.01711	.16192	.38678	.25746	.14736	.02666	.00246	.09385	.81204
	.00091	.01644	.09685	.34728	.36204	.14736	.02666	.00246	.18422	.81204
	.00281	.01454	.16192	.28220	.36204	.15979	.01550	.00120	.30710	.75213
170	.00042	.02044	.16620	.37705	.25252	.15070	.02959	.00308	.14291	.85485
	.00042	.02044	.10114	.34062	.35401	.15070	.03111	.00155	.14291	.80456
	.00138	.01948	.10114	.34062	.35401	.16418	.01843	.00075	.24935	.74547

TABLE 6 (Continued)

Classical p-Chart, Q-Chart and Arcsin Transformation Cell Probabilities and Power Comparisons for p=0.10

Cell:	1	2	3	4	5	6	7	8		
n	.00135	.02140	.13591	.34134	.34134	.13591	.02140	.00135	Lower	Upper
180	.00019	.01161	.11759	.43293	.30466	.11125	.01981	.00195	.10954	.84765
	.00067	.01114	.11759	.33428	.34649	.15360	.03429	.00195	.19980	.84765
	.00198	.02252	.10490	.33428	.34649	.16808	.02078	.00098	.31787	.79759
190	.00032	.01391	.12218	.42428	.30022	.11469	.02199	.00241	.15814	.88242
	.00099	.01324	.12218	.32823	.33942	.17154	.02199	.00241	.26201	.88242
	.00269	.02555	.10817	.32823	.39627	.11469	.02316	.00124	.38706	.84084
200	.00048	.01630	.12629	.41610	.29593	.11780	.02418	.00292	.21330	.91007
	.00048	.01630	.12629	.32246	.33276	.17461	.02556	.00154	.21330	.87610
	.00139	.03066	.11103	.32246	.38957	.11780	.02631	.00078	.32702	.83439
250	.00035	.01715	.15438	.38111	.27656	.14374	.02465	.00205	.29093	.95466
	.00091	.01660	.10316	.34851	.36038	.14374	.02465	.00205	.40156	.95466
	.00213	.02875	.14101	.29728	.36038	.14374	.02557	.00113	.51753	.93632
300	.00057	.01655	.12680	.40450	.30624	.11995	.02297	.00242	.46302	.98418
	.00127	.01585	.12680	.32794	.33679	.16597	.02398	.00141	.56811	.97696
	.00127	.02741	.11524	.32794	.38280	.11995	.02459	.00080	.56811	.96719
350	.00075	.01542	.14744	.38125	.29201	.13949	.02101	.00264	.61016	.99464
	.00075	.01542	.10525	.35254	.36292	.13949	.02205	.00160	.61016	.99196
	.00152	.02458	.13749	.31034	.36292	.13949	.02270	.00095	.69789	.98821
400	.00044	.01450	.12329	.40375	.31806	.11822	.02003	.00171	.64591	.99726
	.00088	.01406	.12329	.33740	.34577	.14750	.02938	.00171	.72462	.99726
	.00168	.02180	.11475	.33740	.34577	.15686	.02069	.00105	.79269	.99589
450	.00051	.02046	.13218	.38644	.30683	.12563	.02619	.00177	.74786	.99908
	.00096	.02001	.13218	.32387	.33079	.16423	.02619	.00177	.80898	.99908
	.00175	.01922	.13218	.32387	.36940	.13376	.01871	.00112	.85950	.99860
500	.00055	.01808	.14783	.37111	.29658	.14080	.02328	.00177	.82353	.99970
	.00100	.01763	.11187	.34770	.35595	.14080	.02328	.00177	.86915	.99970
	.00176	.02567	.13903	.31174	.35595	.14080	.02391	.00114	.90555	.99953
501	.00053	.01748	.14486	.36877	.29868	.14370	.02412	.00187	.82077	.99972
	.00096	.01704	.10946	.34481	.35804	.14370	.02412	.00187	.86687	.99972
	.00169	.02486	.13631	.30941	.35804	.15101	.01747	.00121	.90373	.99956
502	.00091	.01648	.14194	.36637	.30073	.14663	.02498	.00197	.86456	.99974
	.00091	.01648	.10709	.34189	.36006	.14663	.02498	.00197	.86456	.99974
	.00161	.02408	.13363	.30704	.36006	.15417	.01814	.00127	.90189	.99959

TABLE 6 (Continued)

Classical p-Chart, Q-Chart and Arcsin Transformation Cell Probabilities and Power Comparisons for p=0.10

Cell:	1	2	3	4	5	6	7	8		
n	.00135	.02140	.13591	.34134	.34134	.13591	.02140	.00135	Lower	Upper
503	.00087	.01592	.13904	.36392	.33748	.11482	.02587	.00207	.86223	.99976
	.00087	.01592	.13904	.30463	.36201	.14958	.02587	.00207	.86223	.99976
	.00154	.02332	.13098	.36392	.30272	.15735	.01882	.00134	.90002	.99962
504	.00083	.01539	.13619	.36142	.33993	.11728	.02678	.00218	.85988	.99978
	.00083	.01539	.13619	.30218	.36390	.15255	.02754	.00141	.85988	.99965
	.00148	.02258	.12835	.36142	.33993	.12529	.02003	.00091	.89813	.99946
505	.00080	.02248	.12576	.35887	.34233	.11976	.02771	.00229	.85750	.99979
	.00080	.01487	.13337	.29970	.36572	.15555	.02851	.00149	.85750	.99967
	.00141	.02186	.12576	.35887	.34233	.12802	.02079	.00096	.89622	.99950
506	.00076	.02175	.12320	.35628	.34467	.13077	.02016	.00240	.85510	.99981
	.00076	.02175	.12320	.29718	.36747	.15856	.02951	.00157	.85510	.99970
	.00135	.02116	.12320	.35628	.34467	.13077	.02156	.00101	.89428	.99953
507	.00072	.02105	.12068	.35363	.34695	.13355	.02089	.00253	.85268	.99982
	.00129	.02048	.12068	.35363	.31014	.17035	.02177	.00165	.89232	.99972
	.00129	.02048	.12068	.35363	.34695	.13355	.02235	.00107	.89232	.99956
508	.00069	.02036	.15526	.31387	.34917	.13635	.02255	.00174	.85023	.99974
	.00123	.01982	.11818	.35094	.34917	.13635	.02255	.00174	.89034	.99974
	.00213	.01892	.11818	.35094	.34917	.13635	.02317	.00113	.92181	.99960
509	.00066	.01970	.15225	.31169	.35133	.13918	.02337	.00183	.84776	.99976
	.00118	.01918	.11572	.34821	.35133	.13918	.02337	.00183	.88834	.99976
	.00204	.01832	.15225	.31169	.35133	.13918	.02401	.00119	.92024	.99962
510	.00063	.01905	.14927	.36825	.29464	.14204	.02420	.00193	.84527	.99978
	.00113	.01856	.11330	.34543	.35343	.14204	.02420	.00193	.88631	.99978
	.00196	.01773	.14927	.30946	.35343	.14932	.01759	.00125	.91864	.99965
511	.00060	.01843	.14633	.36597	.29669	.14491	.02505	.00203	.84275	.99979
	.00108	.01795	.11090	.34262	.35546	.14491	.02505	.00203	.88426	.99979
	.00187	.02597	.13751	.30719	.35546	.15243	.01825	.00132	.91702	.99968
512	.00057	.01782	.14342	.36363	.29868	.14781	.02593	.00213	.84021	.99981
	.00103	.01736	.10854	.33976	.35743	.14781	.02668	.00139	.88218	.99970
	.00179	.02517	.13485	.30488	.35743	.15556	.01943	.00089	.91539	.99954
513	.00055	.01723	.14055	.36124	.33539	.11596	.02684	.00224	.83764	.99982
	.00098	.01679	.14055	.30253	.35934	.15073	.02761	.00146	.88008	.99972
	.00172	.02439	.13222	.36124	.33539	.12395	.02015	.00094	.91373	.99957

TABLE 6 (Continued)

Classical p-Chart, Q-Chart and Arcsin Transformation Cell Probabilities and Power Comparisons for p=0.10

Cell:	1	2	3	4	5	6	7	8		
n	.00135	.02140	.13591	.34134	.34134	.13591	.02140	.00135	Lower	Upper
514	.00052	.01665	.13772	.35881	.33779	.12662	.01954	.00236	.83505	.99984
	.00094	.01623	.13772	.30015	.36118	.15367	.02858	.00154	.87796	.99974
	.00165	.02363	.12962	.35881	.33779	.12662	.02090	.00099	.91204	.99960
515	.00090	.01570	.13492	.35632	.34013	.12933	.02024	.00247	.87581	.99985
	.00090	.01570	.13492	.29772	.36295	.15663	.02956	.00162	.87581	.99976
	.00157	.02289	.12705	.35632	.34013	.12933	.02166	.00105	.91034	.99963
516	.00086	.01517	.13215	.35379	.34241	.13206	.02185	.00171	.87365	.99978
	.00086	.01517	.13215	.29527	.36466	.16833	.02185	.00171	.87365	.99978
	.00151	.02217	.12451	.35379	.34241	.13206	.02245	.00111	.90861	.99965
517	.00082	.02209	.12200	.35121	.34463	.13481	.02264	.00180	.87145	.99980
	.00082	.01466	.12943	.35121	.34463	.13481	.02264	.00180	.87145	.99980
	.00144	.02147	.12200	.35121	.34463	.13481	.02327	.00117	.90686	.99968
518	.00078	.02139	.11952	.34858	.34680	.13759	.02344	.00189	.86924	.99981
	.00078	.02139	.11952	.34858	.34680	.13759	.02344	.00189	.86924	.99981
	.00138	.02079	.11952	.34858	.34680	.13759	.02410	.00123	.90509	.99970
519	.00075	.02070	.15359	.30940	.34891	.14039	.02427	.00199	.86700	.99982
	.00132	.02013	.11708	.34592	.34891	.14039	.02427	.00199	.90330	.99982
	.00132	.02013	.11708	.34592	.34891	.14766	.01769	.00130	.90330	.99972
520	.00071	.02003	.15064	.36545	.29273	.14322	.02512	.00209	.86474	.99984
	.00126	.01949	.11466	.34321	.35095	.14322	.02584	.00137	.90149	.99974
	.00216	.01858	.15064	.30723	.35095	.15071	.01835	.00137	.93009	.99974
600	.00058	.02043	.13239	.38090	.31192	.12674	.02534	.00169	.91645	.99997
	.00100	.02002	.13239	.32669	.33283	.16004	.02534	.00169	.94005	.99997
	.00166	.01936	.13239	.32669	.36613	.13377	.01888	.00112	.95795	.99995
700	.00092	.02150	.15032	.35904	.29641	.14377	.02584	.00220	.97306	1.00000
	.00092	.02150	.11900	.34015	.34662	.14377	.02652	.00152	.97306	1.00000
	.00147	.02095	.11900	.34015	.34662	.15057	.02019	.00104	.98149	.99999
800	.00081	.02230	.13503	.37160	.31211	.12977	.02649	.00190	.98802	1.00000
	.00125	.01575	.14113	.32463	.32984	.15901	.02649	.00190	.99190	1.00000
	.00192	.02119	.13503	.32463	.35908	.13627	.02055	.00134	.99461	1.00000
900	.00069	.01679	.12762	.38293	.32609	.12378	.02049	.00160	.99471	1.00000
	.00105	.01643	.12762	.33865	.34425	.14374	.02666	.00160	.99646	1.00000
	.00157	.02171	.12182	.33865	.34425	.14990	.02095	.00115	.99767	1.00000

TABLE 6 (Continued)

Classical p-Chart, Q-Chart and Arcsin Transformation Cell Probabilities and Power Comparisons for p=0.10

Cell:	1	2	3	4	5	6	7	8		
n	.00135	.02140	.13591	.34134	.34134	.13591	.02140	.00135	Lower	Upper
1000	.00086	.02224	.13514	.36836	.31514	.13045	.02596	.00184	.99846	1.00000
	.00127	.01634	.14063	.32634	.33107	.15654	.02596	.00184	.99899	1.00000
	.00185	.02125	.13514	.32634	.35716	.13626	.02065	.00134	.99935	1.00000
2000	.00094	.02148	.13466	.36173	.32403	.13145	.02408	.00162	1.00000	1.00000
	.00123	.02119	.13466	.33201	.33552	.14968	.02408	.00162	1.00000	1.00000
	.00160	.02083	.13466	.33201	.35376	.13545	.02041	.00128	1.00000	1.00000
3000	.00102	.02157	.13502	.35776	.32699	.13238	.02368	.00158	1.00000	1.00000
	.00127	.02133	.13502	.33349	.33639	.14725	.02368	.00158	1.00000	1.00000
	.00156	.02103	.13502	.33349	.35126	.13566	.02067	.00131	1.00000	1.00000
4000	.00119	.02167	.14198	.34847	.32202	.13949	.02346	.00172	1.00000	1.00000
	.00119	.01888	.13185	.34037	.34304	.13949	.02372	.00146	1.00000	1.00000
	.00143	.02143	.12906	.34037	.34304	.14236	.02108	.00123	1.00000	1.00000
5000	.00116	.02037	.13383	.35656	.33266	.13188	.02195	.00161	1.00000	1.00000
	.00116	.02037	.13383	.33775	.34008	.14326	.02217	.00139	1.00000	1.00000
	.00136	.02278	.13121	.33775	.35146	.13188	.02236	.00119	1.00000	1.00000

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