

EFFECTS OF PERMANENT TRAP RESPONSE IN CAPTURE PROBABILITY ON JOLLY-SEBER
CAPTURE-RECAPTURE MODEL ESTIMATES

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SUMMARY

(1) Large-sample approximations and computer simulations were used to investigate the effects of permanent trap response in capture probability on estimates from the Jolly-Seber capture-recapture model.

(2) A "trap-happy" response produces negatively biased estimates of population size and a "trap-shy" response results in positive bias of population size estimates.

(3) For a specified difference between capture probabilities of marked and unmarked animals, relative biases of population size estimates are most serious when capture probabilities are low.

(4) Relative biases of population size estimates are more severe following periods when the population exhibits substantial turnover (high birth/immigration and high mortality/emigration) than when the population exhibits low turnover.

(5) Trap response does not bias survival rate estimates but does affect their precision. A trap-happy response results in precise survival rate estimates and a trap-shy response produces estimates with low precision.

INTRODUCTION

The Jolly-Seber model (Jolly 1965, Seber 1965) is probably the most important open population model for use in capture-recapture experiments. Because of its importance and its wide usage, it is desirable to have some knowledge of the effects of deviations from underlying model assumptions on resulting parameter estimates. Effects of heterogeneity of capture probability (all individuals do not have the same capture probability) have been studied by Carothers (1973) and Gilbert (1973). Effects of tag loss and temporary emigration have been investigated by Arnason and Mills (1981) and Balser (1981), respectively. The effects of age-specific differences in mortality rates have been addressed by Manly (1970).

Trap response refers to the situation in which marking an animal causes a change in its capture or survival probability for some time period after its initial capture. Robson (1969), Pollock (1975), and Brownie and Robson (1980) have generalized the Jolly-Seber capture-recapture model to permit temporary (e.g. 1 or more time periods after initial capture) trap response in survival and capture probability. Although a temporary trap response seems reasonable for survival rate and in some instances for capture probability, a permanent trap response in capture probability seems likely in many situations. Here we examine bias in Jolly-Seber estimates of survival rate and population size resulting from permanent trap response in capture probability.

NOTATION AND DEFINITIONS

We are interested in a K -sample capture-recapture experiment on an open animal population. Animals are captured at each sampling period,

i ($i=1, \dots, K$). Unmarked animals are tagged, the tag numbers of marked animals are recorded, and animals are then released back into the population.

We obtain the following statistics directly from the sampling effort:

- r_i = total number of animals caught in sample i ,
- m_i = total marked animals caught in sample i ,
- u_i = total unmarked animals caught in sample i ,
- R_i = number of marked animals released after sample i ,
- r_i = number of marked animals released at i and subsequently recaptured,
- z_i = number of animals caught before and after, but not in, sample i ,
- a_{ij} = number of marked animals caught in sample j that were last caught in sample i .

The following unobservable random variables must be estimated:

- N_i = total population size just before sample i is taken,
- M_i = total number of marked animals in the population just before sample i ,
- B_i = total number of new animals entering the population during the period separating samples i and $i+1$, and alive at the time of sample $i+1$.

The following are model parameters:

- ϕ_i = the probability that an animal alive and in the population at the time of sample i , survives and is in the population at the time of sample $i+1$,
- p_i = the probability that an unmarked animal alive and in the population at the time of sample i , is caught in sample i ,
- p'_i = the probability that a marked (previously caught) animal alive and in the population at the time of sample i , is caught in

sample i ,

v_i = the probability that an animal caught in sample i will be returned to the population after the sampling (i.e. that the animal does not die from handling).

Only our parameterization of capture probability differs from that of the standard Jolly-Seber model. The Jolly-Seber model assumes equal capture probabilities for all animals in the population at any specified sampling period, but we have designated different capture probabilities for marked and unmarked animals as a means of modeling permanent trap response.

METHODS

We use both analytic approximations and computer simulation to investigate bias in the Jolly-Seber estimates \hat{N}_i and $\hat{\phi}_i$. These methods are quite general and can be used to approximate bias in virtually any situation of interest. However, in order to make the problem tractable and the results interpretable, we have assumed the following about the parameters:

$$\phi_i = \phi \text{ for all } i,$$

$$p_i = p \text{ for all } i,$$

$$p'_i = p' \text{ for all } i,$$

$$v_i = 1 \text{ for all } i$$

We are thus assuming capture and survival probabilities that are constant over time, as well as no losses on capture. In addition we assumed that $E(N_i) = N_i$ for all i and thus that $B_i = (1-\phi) N_i$, for all i , where E denotes expectation. Thus, population size and number of deaths vary stochastically, while the number of births is a constant which balances expected deaths.

The large-sample approximations were computed using methods similar to those of Carothers (1973) and Arnason and Mills (1981). We first express the expectations of the summary statistics in terms of model parameters (Table 1). We then evaluate these expected values for a given set of parameters (and N_1 and K) and use these values in conjunction with the estimators of interest (e.g. the equations of Jolly 1965 for \hat{N}_1). We used these approximate expectations of the estimators to approximate relative bias (e.g. for \hat{N}_1) as:

$$[E(\hat{N}_1) - N_1] / N_1 \quad (1)$$

Average relative bias is defined as the arithmetic mean of the $K-2$ relative bias expressions in the entire experiment.

The large-sample approximations are expected to perform well for large values of $E(m_1)$, $E(n_1)$, $E(R_1)$, $E(r_1)$ (see Carothers 1973). We used computer simulation as a means of checking these approximations and providing better bias approximations for situations of small sample size. In the simulation program, capture and survival of each individual for each sampling period were treated as independent Bernoulli trials using pseudorandom numbers. Each population was initialized with N_1 individuals and a constant number of new individuals ($B = (1-\phi)N_1$) was added to the population each time period. In the simulations, it was possible to have no recaptures occurring in a particular sampling period ($m_1 = 0$) or resulting from a particular release ($r_1 = 0$), and these situations occurred in some of the simulations with low population size and capture probability. In these cases, the iteration in which the problem occurred was simply thrown out, and no information from the iteration was included in any of the summary statistics dealing with bias. The above procedure was used to generate a table of capture histories for each iteration. These histories were then used,

in conjunction with the "bias-adjusted" estimators of Seber (1973:204), to compute estimates of N_1 , ϕ_1 , and their asymptotic variances. We used the full variance estimation equations of Jolly (1965:237) and thus included variance components associated both with errors of estimation and with variation of the conditional parameters. Relative bias was computed as in (1), except that $E(\hat{N}_1)$ was approximated by $\bar{\hat{N}}_1$, the arithmetic mean of the \hat{N}_1 over all simulations. Average relative bias was computed over the K-2 sampling periods, as with the large-sample approximations. All simulation results reported here are based on 1000 iterations.

RESULTS

Population Size

The Jolly-Seber estimator for population size (e.g. Jolly 1965) is:

$$\hat{N}_1 = \hat{M}_1 n_1/m_1 \quad , \quad (2)$$

where \hat{M}_1 is estimated as:

$$\hat{M}_1 = R_1 z_1/r_1 + m_1 \quad . \quad (3)$$

The estimator (3) for \hat{M}_1 is based only on statistics for marked animals, and permanent trap response, $p \neq p'$, should thus not result in bias in this estimator. However, the estimator for \hat{N}_1 (2) also includes the ratio n_1/m_1 , and this ratio will be affected by trap response. If $\bar{p} < p'$ (i.e. a "trap-happy" response), then n_1/m_1 will be smaller than it should be (i.e. m_1/n_1 will overestimate the true proportion of marked animals in the population) and \hat{N}_1 will be negatively biased. If $p > p'$ (a "trap-shy" response), then n_1/m_1 will be too large (i.e. m_1/n_1 will underestimate the true proportion of marked animals in the population) and \hat{N}_1 will be positively biased.

Average relative bias of \hat{N}_1 for different combinations of p and p' ,

with $\phi = 0.80$ and $K = 10$ are presented in Table 2. There is good agreement between the results based on computer simulation and large-sample approximation, and they were virtually identical for $p = 0.80$. As expected, the trap-happy response ($\Delta > 0.0$ in Table 2) produced negatively-biased \hat{N}_1 , and the trap-shy response ($\Delta < 0.0$ in Table 2) resulted in positive bias in \hat{N}_1 . For fixed p , the absolute value of the average relative bias increases as the absolute value of Δ ($\Delta = p' - p$) increases. For fixed Δ , the absolute value of the average relative bias decreases as p increases. Thus, as with heterogeneity of capture probability, high p offers some protection against trap-response bias in the Jolly-Seber \hat{N}_1 . When trap response occurs in conjunction with low capture probability, the resulting bias can be quite serious (Table 2, $p = 0.20$).

In Table 3 we present average relative biases of \hat{N}_1 for specific combinations of p and p' with 5 different ϕ and 3 different N_1 . One purpose of this Table is to illustrate the effect on average relative bias of variation in ϕ and in B , the number of new (and hence unmarked) individuals added to the population each period. Since we defined B as $B = N_1(1-\phi)$, high values of ϕ correspond to low B and vice versa. As pointed out earlier, \hat{M}_1 is estimated without bias when trap-response occurs, and the bias in \hat{N}_1 (see equation 2) is produced by the failure of m_1/n_1 to estimate the true proportion of marked animals in the population. When the true proportion M_1/N_1 is small, m_1/n_1 will be small and have a larger influence on \hat{N}_1 than when M_1/N_1 is large. Large B and low ϕ should result in low M_1 and hence in a large influence of the sample ratio m_1/n_1 in (2). As expected, for fixed N_1 and Δ , large B -low ϕ populations are characterized by greater (in absolute value) average relative biases than low B -high ϕ populations (Table 3).

Table 3 also illustrates the general nature of the relationship between population size, N_1 , and average relative bias. At large N_1 , population size is unimportant, and our large-sample approximations of bias are invariant to differences in N_1 . Under a trap-happy response ($p' - p = \Delta > 0$) the absolute value of the average relative bias is greatest for small N_1 (Table 3). However, under a trap-shy response ($\Delta < 0$) the absolute value of the average relative bias is smallest for small N_1 (Table 3). These relationships result from the negative "small-sample" bias of the bias-adjusted estimator for population size (see Gilbert 1973).

Relative biases of the time-specific \hat{N}_i are greatest (in absolute value) for \hat{N}_2 and decrease to the end of the experiment (see Table 4). Thus, the greatest bias occurs when the true proportion of marked animals in the population, M_1/N_1 , is smallest (and thus when the factor, n_i/m_i , by which \hat{M}_i must be multiplied in order to estimate \hat{N}_i is largest; see equation 2). Under the assumptions of our model, M_1/N_1 should be smallest at the beginning of the experiment and should then approach an asymptotic value as the experiment proceeds. This asymptote should be approached more rapidly with low ϕ - high B than with the converse situation. As a result, relative bias of \hat{N}_i approaches a constant value more quickly with low ϕ - high B (see contrast of $\phi = 0.70$ and $\phi = 0.30$ in Table 4, especially large-sample approximations). Similarly, there is a larger relative difference between the relative biases of \hat{N}_2 and \hat{N}_{K-1} in the high ϕ - low B situation (see example in Table 4) because of the greater change which occurs in the M_1/N_1 ratio in this case between the beginning and end of the experiment. Naturally, the tendency of relative bias of \hat{N}_i to decrease with increasing i is a consequence of our restrictive assumptions, $\phi_i = \phi$, $p_i = p$, $p'_i = p'$, $B_i = B = N_1(1-\phi)$. When ϕ_i and B_i fluctuate, if we can predict their effect on the ratio M_1/N_1

then we should be able to predict the general effect on the relative bias of \hat{N}_1 . The relative bias should be low after periods of high ϕ_1 and low B_1 and high after periods of low ϕ_1 and high B_1 .

We note that variation in K does not affect the relative bias of specific \hat{N}_i ($i < K$). For example, the relative bias of \hat{N}_3 is the same for $K = 5$ as for $K = 20$. However, the average relative bias of the \hat{N}_1 will decrease as K increases, simply because the tendency for relative bias to decrease with i results in a larger number of low-bias estimates being included in the computation of the average. For example, assuming $\phi = 0.80$, $p = 0.50$, $p' = 0.40$, the large-sample approximations of average relative bias for $K = 5, 10, 15, 20$ are 0.118, 0.097, 0.092 and 0.090, respectively.

The precision of \hat{N}_1 is also affected by permanent trap response, as shown in Table 5. Here we have computed the sample variances of the point estimates of N_5 over all iterations for the simulations reported in Table 2. We have also computed the arithmetic mean of the asymptotic variance estimates, $\hat{\text{var}} \hat{N}_5$, for the same set of simulations (Table 5). Both estimates of $\text{var} \hat{N}_5$ are seen to be larger in the case of a trap-shy response and smaller under a trap-happy response (Table 5). In addition both estimates increase with increasing $|\Delta|$ under a trap-shy response and decrease with increasing $|\Delta|$ under a trap-happy response. The sample variance should estimate the true $\text{var} \hat{N}_5$, and should provide a means of ascertaining the bias of the asymptotic variance estimates. As seen in Table 5, the asymptotic variance estimates, $\hat{\text{var}} \hat{N}_5$, exhibit positive bias under a trap-shy response and negative bias under a trap-happy response. However, the absolute value of the bias is not large except for a trap-shy response with low p (Table 5).

Survival Rate

The Jolly-Seber estimator for survival rate (e.g. Jolly 1965) is:

$$\hat{\phi}_1 = \hat{M}_{i+1} / (\hat{M}_i - m_i + R_i) \quad (4)$$

The estimate, $\hat{\phi}_1$, is thus based only on marked individuals in the population at time $i+1$, and marked individuals in the population just after the sample at time i (4). This estimator is thus unbiased even when permanent trap response occurs. However, because of the importance of p'_i to sample sizes of released and recaptured animals, the nature of trap response can have an important influence on the precision of $\hat{\phi}_1$. This statement is illustrated in Table 6, where we present the arithmetic mean of the asymptotic variance estimates of $\hat{\phi}_1$ ($\text{var } \hat{\phi}_1$) over all simulations, for the simulations reported in Table 2. As expected, $\text{var } \hat{\phi}_1$ increases with increasing $|\Delta|$ when a trap-shy response is exhibited and decreases with increasing $|\Delta|$ under a trap-happy response. Therefore, although trap response does not bias $\hat{\phi}_1$, it can still be an important consideration with respect to use of $\hat{\phi}_1$, because of the effect on $\text{var } \hat{\phi}_1$. In fact, if $\hat{\phi}_1$ is the only parameter of interest, then a trap-happy response is desirable.

DISCUSSION

Trap response is thought to occur commonly in capture-recapture studies of vertebrate populations. Evidence of both "trap-happiness" and "trap-shyness" is available for a number of bird species (see review of Nichols et al. 1981: 123, and references therein). Trap-response appears to be widespread among small mammals, with the majority of species exhibiting a trap-happy response (see review by Tanaka 1980:42-52, and discussion and references in Seber 1973:83-85, and Otis et al. 1978:29). Moreover, in small mammals the difference between capture probabilities of marked and unmarked individuals,

$|\Delta|$, can be quite large (Tanaka 1980:47). Evidence suggests that trap-response behavior in small mammals is persistent for long periods of time (Tanaka 1980:48-51) and that it can be considered permanent, or effectively so, in the species in which response duration has been examined.

It is difficult to speak generally of field-experimental methods to reduce trap response, because such methods will clearly be specific to the species and field situation encountered. In general, trap-shyness can be minimized by using capture and handling techniques that reduce stress upon the animal as much as possible. Prebaiting, the practice of placing open baited traps in the experimental area prior to the beginning of the experiment, is thought to reduce trap-happiness in some situations, but can also cause other kinds of problems (Tanaka 1980:52-55). Shifting the location of traps or mist-nets has been suggested as a means of reducing both positive and negative trap responses.

We can think of no obvious way to test for permanent trap response using standard open model capture-recapture data. Jolly-Seber model goodness-of-fit tests (Seber 1973:223-224 and Jolly 1982) and the indirect test of Leslie et al. (1953) deal only with marked animals. However, trap response can be tested using closed population models. Thus, it might be reasonable to follow a sampling design similar to that of Pollock (1982) and take several samples within some of the major sampling periods (e.g. 5 consecutive nights of trapping each month). It would then be possible to test for trap response in capture probability using closed population models for all individuals within the first major sampling period and for unmarked individuals within subsequent major sampling periods. Several of the tests provided by Otis et al. (1978:55; e.g. tests 2,5,7) would be useful in identifying trap-response. In addition, models M_b (Pollock 1974,

Otis et al. 1978) and M_{bh} (Otis et al. 1978) could be useful in providing estimates of p and p' (more specifically estimates of daily capture probability would be provided and these could be used to estimate p and p' for the entire major sampling period).

Our results indicate that a trap-happy response can produce serious underestimates of N_1 in the Jolly-Seber model, while a trap-shy response can produce substantial positive bias. For a given difference between capture probabilities of marked and unmarked animals, the greatest relative biases are produced when capture probabilities are low. Moreover, relative biases are most severe when the studied population exhibits substantial turnover (low ϕ - high B) between sampling periods. Within an experiment, the worst bias occurs in N_2 and, given the assumptions of our model, bias tends to be less severe for subsequent \hat{N}_1 . In general, bias will be worst when the ratio M_1/N_1 is low. Precision of the \hat{N}_1 is also affected, with a trap-happy response producing greater precision and a trap-shy response resulting in loss of precision. Although trap response can seriously affect \hat{N}_1 , it does not result in biased $\hat{\phi}_1$. Trap response does affect the magnitude of $\text{var } \hat{\phi}_1$, however, and positive trap response is actually desirable in the sense that it results in precise estimates of $\hat{\phi}_1$.

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Table 1. Expectations of capture-recapture summary statistics under a model with permanent trap response

Statistic	Expectation
a_{ij}	$R_i \rho^{j-1} p' (1-p')^{j-i-1}$
r_i	$\sum_{j=i+1}^K E(a_{ij})$
z_i	$\sum_{h=1}^{i-1} \sum_{j=i+1}^K E(a_{hj})$
m_i	$\sum_{h=1}^{i-1} E(a_{hi})$
u_i	$[N_1 - \sum_{j=1}^{i-1} E(u_j) \rho^{i-j}] p$ for $i > 1$
	$N_1 p$ for $i = 1$
n_i	$E(m_i) + E(u_i)$
R_i	$E(m_i) + E(u_i)$

Table 2. Average relative bias of \hat{N}_1 assuming $N_1 = 500$, $\phi = 0.80$, $k = 10$, and values of p and p' as specified.^a

p	Δ								
	-0.15	-0.10	-0.05	-0.02	0.00	0.02	0.05	0.10	0.15
0.20	1.824 (1.955)	0.648 (0.651)	0.217 (0.217)	0.070 (0.072)	-0.003 (0.000)	-0.063 (-0.059)	-0.133 (-0.130)	-0.219 (-0.217)	-0.281 (-0.279)
0.50	0.164 (0.167)	0.095 (0.097)	0.041 (0.043)	0.014 (0.016)	-0.002 (0.000)	-0.016 (-0.015)	-0.036 (-0.035)	-0.066 (-0.065)	-0.091 (-0.090)
0.80	0.060 (0.059)	0.038 (0.037)	0.017 (0.017)	0.007 (0.007)	0.000 (0.000)	-0.006 (-0.006)	-0.015 (-0.015)	-0.028 (-0.029)	-0.040 (-0.041)

^aAverage relative bias obtained by computer simulation and approximation (in parentheses) methods.

^b Δ is used to define p' , where $p' = p + \Delta$.

Table 3. Average relative bias of \hat{N}_1 with $K=10$ and specified values of N_1 , ϕ , p , and p' .

p	Δ^a	Method ^b	N_1	ϕ^c						
				0.90	0.70	0.50	0.30	0.10		
0.20	-0.10	LSA	-	0.549	0.732	0.847	0.924	0.979		
		SIM	500	0.550	0.709	0.791	_d	_d		
			100	_d	_d	_d	_d	_d		
			50	_d	_d	_d	_d	_d		
	0.10	LSA	-	-0.183	-0.244	-0.282	-0.308	-0.326		
		SIM	500	-0.185	-0.248	-0.291	-0.324	_d		
			100	-0.193	-0.269	_d	_d	_d		
			50	_d	_d	_d	_d	_d		
			LSA	-	-0.183	-0.244	-0.282	-0.308	-0.326	
		0.50	-0.10	LSA	-	0.066	0.124	0.170	0.207	0.237
				SIM	500	0.065	0.123	0.166	0.201	0.221
					100	0.062	0.112	0.145	0.150	_d
50	0.052				0.096	0.110	_d	_d		
0.10	LSA		-	-0.044	-0.083	-0.113	-0.138	-0.158		
	SIM		500	-0.046	-0.085	-0.116	-0.144	-0.169		
			100	-0.048	-0.092	-0.131	-0.171	_d		
			50	-0.056	-0.101	-0.147	_d	_d		

Table 3. (Cont.)

0.80	-0.10	LSA	-	0.021	0.052	0.080	0.107	0.131
		SIM	500	0.021	0.053	0.081	0.101	0.115
			100	0.019	0.045	0.064	0.079	0.091
			50	0.014	0.038	0.054	0.066	- ^d
0.10	0.10	LSA	-	-0.016	-0.040	-0.062	-0.083	-0.102
		SIM	500	-0.017	-0.040	-0.062	-0.081	-0.100
			100	-0.017	-0.045	-0.073	-0.096	-0.110
			50	-0.022	-0.049	-0.076	-0.099	- ^d

^aAverage relative bias obtained by computer simulation (SIM) and large-sample approximation (LSA) methods.

^b Δ is used to define p' , where $p' = p + \Delta$.

^cNote that specification of ϕ also implies specification of B , as $B = N_1(1-\phi)$.

^dResults are not presented because large numbers of iterations (>50) had to be discarded (when $m_1 = 0$ or $r_1 =$ for any i).

Table 4. Relative bias of \hat{N}_1 assuming $N_1 = 500$, $K = 10$, $p = 0.50$ and specified values of ϕ and Δ^{ab} .

Sample period	$\phi = 0.70$		$\phi = 0.30$	
	$\Delta = 0.10$	$\Delta = -0.10$	$\Delta = 0.10$	$\Delta = -0.10$
1	-	-	-	-
2	-0.109(-0.108)	0.163(0.132)	-0.154(-0.142)	0.177(0.213)
3	-0.089(-0.088)	0.133(0.132)	-0.143(-0.138)	0.215(0.207)
4	-0.082(-0.081)	0.121(0.121)	-0.144(-0.137)	0.192(0.206)
5	-0.083(-0.078)	0.110(0.117)	-0.136(-0.137)	0.220(0.206)
6	-0.077(-0.077)	0.118(0.116)	-0.149(-0.137)	0.198(0.206)
7	-0.077(-0.077)	0.114(0.116)	-0.140(-0.137)	0.198(0.206)
8	-0.081(-0.077)	0.114(0.115)	-0.148(-0.137)	0.195(0.206)
9	-0.079(-0.077)	0.114(0.115)	-0.148(-0.137)	0.195(0.206)
10	-	-	-	-

^aRelative bias obtained by computer simulation and large-sample approximation (parentheses) methods.

^b Δ is used to define p' , where $p' = p + \Delta$.

Table 5. Estimates of $\sqrt{\widehat{\text{var}} N_5}$ from simulations assuming $N_1 = 500$, $\phi = 0.80$, $K = 10$, and values of p and p' as specified.^a

p	Δ^b								
	-0.15	-0.10	-0.05	-0.02	0.00	0.02	0.05	0.10	0.15
0.20	949.4 (752.3)	277.7 (252.6)	144.8 (138.7)	109.7 (108.9)	93.3 (93.6)	80.5 (81.1)	66.6 (66.7)	50.4 (51.5)	39.9 (41.5)
0.50	46.5 (45.9)	38.5 (38.2)	32.6 (32.2)	29.8 (29.7)	28.2 (28.4)	26.7 (27.2)	24.8 (25.4)	22.0 (22.6)	19.8 (20.9)
0.80	21.2 (20.4)	19.3 (18.7)	17.8 (17.5)	17.0 (17.0)	16.6 (16.6)	16.1 (16.3)	15.5 (16.0)	14.6 (15.2)	13.8 (14.7)

^aFirst value is $\sqrt{\widehat{\text{var}} N_5}$, where $\widehat{\text{var}} N_5$ is the arithmetic mean of the asymptotic variance estimates for N_5 over all simulations. Value in parentheses is the square root of the sample variance computed using the point estimates, \hat{N}_5 , for all simulations (i.e. $\sqrt{\frac{1}{n-1} \sum_{i=1}^n (\hat{N}_{5,i} - \bar{\hat{N}}_5)^2}$; where n is the number of iterations, $\hat{N}_{5,i}$ is the estimate of N_5 for iteration i , and $\bar{\hat{N}}_5 = \frac{1}{n} \sum_{i=1}^n \hat{N}_{5,i}$).

^b Δ is used to define p' , where $p' = p + \Delta$.

Table 6. $\sqrt{\widehat{\text{var}}_{\Delta} \hat{\rho}_4}$ from simulations assuming $N_1 = 500$, $\phi = 0.80$, $K = 10$, and values of p and p' as specified.^a

p	Δ^b								
	-0.15	-0.10	-0.05	-0.02	0.00	0.02	0.05	0.10	0.15
0.20	0.4188	0.2371	0.1687	0.1447	0.1316	0.1209	0.1077	0.0905	0.0779
0.50	0.0607	0.0529	0.0468	0.0437	0.0418	0.0401	0.0378	0.0344	0.0315
0.80	0.0293	0.0270	0.0251	0.0239	0.0232	0.0226	0.0219	0.0205	0.0192

^a $\widehat{\text{var}}_{\Delta} \hat{\rho}_4$ is computed as the arithmetic mean of the asymptotic variance estimates for $\hat{\rho}_4$ over all simulations.

^b Δ is used to define p' , where $p' = p + \Delta$.