Multiuser Detection and Channel Estimation for Flat Rayleigh Fading Mobile Radio CDMA Channels

by

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Abstract

We compare the performance of suboptimal multiuser detectors for rapidly time-varying fading channels. Disjoint Kalman filters are incorporated in the detector structure to obtain phase and amplitude estimates for coherent detection and multi-access interference cancellation. We also evaluate performance of several multiuser detectors which do not require channel estimation. Our results indicate that relative gains of various detectors depend on the maximum Doppler frequency since the estimation error grows as the maximum Doppler frequency increases. In contrast to the case when the Doppler shift is small and the channel can be accurately estimated, the more complex detectors (two-stage and decision-feedback) do not provide significant performance gain over the simpler decorrelator when estimation errors are large. For both classes of detectors (with and without channel estimation), the decorrelator outperforms other considered schemes for high signal-to-noise ratios in fast fading environment and asymptotically approaches the performance of the single user system.

Several fading channel models are considered. We prove that a realistic fading channel can be approximated by the second order autoregressive process (AR) process and discuss coefficient adjustment for the AR model. It is also shown that error bursts often occur due to the "reversal phenomenon" when the Kalman filter is used in the decision-directed mode. We demonstrate that differential encoding remedies this problem.

Keywords: Code division multiaccess, Fading channels, Detectors, Estimation, Kalman filtering
I. Introduction

Multiuser detection has the potential to reduce the Multi-Access Interference (MAI) and solve the near-far problem in the reverse link of a Code Division Multiple Access (CDMA) channel [1]. Since the optimal multiuser detector requires much higher complexity than the conventional detector, recent studies focus on the suboptimal multiuser detection schemes with reasonable complexity. Among these schemes are the linear decorrelator [2], the multi-stage detector [3], the decision-feedback detector (DF) [4], the successive interference cancellation scheme (SIC) [5], and the parallel interference cancellation scheme (PIC) [6-7].

The performance of these suboptimal detectors has been extensively studied for the Additive White Gaussian Noise (AWGN) channels, and recent studies address on their performance for fading channels. The analysis of multiuser detectors for fading channels is often conducted under the ideal assumption of perfect channel estimation [2-10]. The assumption is not valid in practice due to the difficulties of estimating rapidly time-varying channels in mobile radio systems. Therefore, the performance of multiuser detection algorithms should be assessed under more realistic conditions. In [11-12] the performance of the decorrelator was evaluated in the presence of the channel mismatch, and in [13-15] the two-stage (2S) and the DF were considered. Imperfect channel estimation degrades performance of multiuser detectors since many multiuser detectors require channel estimates to cancel the MAI and/or to perform coherent reception. The main purpose of this paper is to compare robustness of multiuser detectors with channel estimation for realistic rapidly time-varying flat Rayleigh fading channels.

The performance comparison can be summarized as follows. We found that when the fading channel is perfectly estimated (e.g., for very slow fading), the more complex detectors, such as the 2S and the DF, can achieve the single user bound and are near-far resistant [14]. However, the simpler schemes, such as the SIC and the PIC, are not suitable for the high bandwidth efficient channels, and should be applied to systems with relatively low cross-correlations. When the channel mismatch is present due to fast fading, the performance of the coherent multiuser detectors--the decorrelator, the 2S, and the DF, becomes comparable. As the Doppler frequency increases, the performance gain of the 2S and the DF over the decorrelator decreases, and is limited to lower SNR region. The performance degradation of the 2S and the DF is due to the channel estimation errors of all the users. The decorrelator, with less complexity than the 2S and the DF, has a lower error floor, which is identical to the error floor of the single user system. The near-far resistance is still preserved in the decorrelator, but not in the 2S and the DF. For noncoherent multiuser detectors (the decorrelator, the SIC, and the PIC), the decorrelator outperforms the other methods, and retains the near-far resistant property. These results indicate that the decorrelator is the most robust multiuser detector among the considered detectors for the realistic fading channels.
To implement multiuser detectors for rapidly varying fading channels, we studied reliable channel estimation algorithms. It was first mentioned in [16] that the estimation of channel coefficients is equivalent to carrier phase tracking, and more work [17-18] was devoted to applying the Kalman filter to channel estimation. In most of these studies the fading channel was modeled as an auto-regressive (AR) process in order to apply the Kalman filter. In this paper, in addition to the AR model, we also consider a more realistic channel model--Jakes model [19] with modification of [20]. We verify that a simple second order AR process (AR2) can approximate the Jakes model and can be used as a hypermodel embedded into the Kalman filter. It was observed in [21] that the spectral peak frequency of the AR2 process should be adjusted by a factor of $\sqrt{2}$ from the maximum Doppler frequency [21]. This adjustment is justified in this paper. This simple AR hypermodel for the realistic fading channel reduces the complexity of the Kalman filter implementation and makes computationally tractable analysis feasible.

Regarding the operation of the Kalman filter, we showed in [22] that the Kalman filter operating in the decision-directed mode would produce the channel estimates that often have about 180-degree phase shift relative to the channel coefficients. We term this behavior "the reversal phenomenon." The phase shift causes long burst of errors and results in a very high BER (on the order of $10^{-1}$). In [22] and this paper, we derive the cause of the reversal phenomenon. We also show that differential encoding eliminates long burst of errors, and consider both coherent or noncoherent detection of the differentially encoded data for several multiuser detectors.

Combinations of Kalman filters and multiuser detectors were also studied previously. In [23-24], it was shown that a multiuser detector and a channel estimator can be decoupled, and a joint Kalman filter was shown to be the optimal channel estimator. In [25], it was shown that the Kalman filter can be configured to estimate all the users' channel coefficients jointly or disjointedly. We adopt simpler disjoint estimation in our work. With the AR channel model, the Bit Error Rate (BER) analysis of the decorrelator, the DF, and the two-stage detector (2S) with the disjoint Kalman channel estimators can be found in [12-14, 22]. These results were for Binary Phase Shift Keying (BPSK) modulation, and are extended to the differentially encoded data in this paper.

The paper is organized as follows: Section II reviews the multiuser detectors to be compared. Section III is devoted to the issues of channel estimation using the Kalman filter. It includes the construction of the hypermodel for the AR process and the modified Jakes fading model, the reversal phenomenon, and other implementation issues. In Section IV, we analyze the BERs of multiuser detectors for perfect channel estimation and with the channel mismatch using the unified analysis. In Section V, several numerical examples are presented, and the analytical results are verified using simulations. Finally, we conclude our work and discuss future directions in Section VI.
II. Synchronous CDMA System Model and Multiuser Detectors

In this section, we review the mathematical model of a synchronous CDMA system and the structures of the considered multiuser detectors.

A. Synchronous CDMA system

Consider N users which transmit simultaneously using different signature waveforms over independent flat Rayleigh fading channels. Assuming that the signals of all users arrive at the receiver at the same time, i.e., they are synchronized, the received signal, in complex baseband notation, is

\[ r(t) = \sum_{i=0}^{\infty} \sum_{n=1}^{N} \sqrt{2E_{b_n}/T} \ C_n(i) \ b_n(i) \ s_n(t-iT) + n(t), \]

where

- \( i \): the sampling time index;
- \( n \): the user index; \( n=1, ..., N \);
- \( E_{b_n} \): the bit energy of the \( n \)-th user;
- \( T \): symbol interval;
- \( C_n(i) \): the fading coefficient for the \( n \)-th user. For Rayleigh fading channel, \( C_n(i) \) has zero mean and unity variance, i.e., \( E(C_n)\) = 0 and \( E(|C_n|^2) = 1 \) or \( E(|C_n|^4) = E(|C_n|^2) = 1/2 \) (Superscripts I and Q denote the in-phase and quadrature components);
- \( b_n(i) \): the information bit of the \( n \)-th user at the \( i \)-th sampling instance; \( b_n(i) \in \{+1,-1\} \);
- \( s_n(t) \): the signature waveform of the \( n \)-th user; \( \int_0^T s_n(t)s_n^*(t)dt = T \);
- \( n(t) \): the complex additive white Gaussian noise with zero mean and variance \( N_0 \).

The received signal is fed to a bank of matched filters, where each matched filter correlates each individual user's signature waveform. After normalization, the output of the matched filter bank (MFB) at the \( k \)-th sampling instance can be represented by the column vector \( y \) of length \( N \):

\[ y(k) = RW(k)b(k) + n(k), \]  

(1)

- \( R \): the signature cross-correlation matrix (NxN); \( R_{l,m} = (1/T)\int_0^T s_l(t)s_m^*(t)dt \) \( (l, m = 1, ..., N) \);
- \( W(k) \): the channel gain matrix (diagonal, NxN); \( W_{n,n}(k) = \sqrt{E_{b_n}C_n(k)} = \sqrt{E_{b_n}|C_n(k)|}\exp(-j\theta_{n}(k)) \) \( (n = 1, ..., N) \);
- \( b(k) \): the vector of information bits of \( N \) users (Nx1); \( b_n(k) = b_n(k) \) \( (n = 1, ..., N) \);
- \( n(k) \): the complex Gaussian noise vector (Nx1); \( n(l,k) = (1/\sqrt{2T})\int_{kT}^{(k+1)T} n(t)s_l^*(t-kT)dt \) \( (l = 1, ..., N) \). The noise vector has the covariance matrix \( E(n^*n) = R N_0 \) (* denotes the conjugate transpose).
B. Multiuser detectors

(1) The conventional detector

In the conventional detector the decision for user n is made by using the output of the matched filter directly, i.e., \( \hat{b}_n(k) = \text{sgn}[\Re(y_n(k)\hat{C}_n^*(k))] \), where \( \hat{C}_n(k) \) is the channel estimate of user n.

(2) The decorrelator

In the decorrelator, the matrix filter \( R_s \) is applied first as in (2) [2]. The output of the decorrelator is
\[
z(k) = R_t^{-1} y(k) = W(k)b(k) + v(k),
\]
where the noise vector \( v(k) = R_t n(k) \), and the decision for the n-th user is
\[
\hat{b}_n(k) = \text{sgn}(\Re[z_n(k)\hat{C}_n^*(k)]).
\]

(3) The two-stage detector

The two-stage detector considered here uses the decorrelator as the first stage [3]. In the first stage, the tentative hard decisions are formed, and in the second stage they are used to reconstruct the MAI for cancellation. The final decision is made after canceling the MAI from the MFB (1).

The final decision for the n-th strongest user is
\[
\hat{b}_n(k) = \text{sgn}[y_n(k) - MAIf_n(k)\hat{C}_n^*(k)],
\]
where \( MAIf_n \) is the reconstructed MAI for user n, defined by \( MAIf_n = \sum_{i \neq n} R_{n,i} \hat{W}_{i,i}(k)\hat{b}_i(k)(\hat{W}_{i,i}(k) = \sqrt{E_{b_i}} \hat{C}_i(k)) \).

(4) The decision-feedback detector

The operation of the DF was described in detail in [4]. In this paper, for efficient implementation of the DF and the Kalman filter, we consider the noise-canceling version of the DF [4]. Since the Kalman filters use the outputs of the decorrelator (see section III), the matrix filter is applied first as in (2), and the enhanced noise of each user is estimated and canceled successively. The performance of the noise-canceling DF was shown to be identical to the original DF [4].

The DF first sorts the MFB outputs in the descending signal strength order. The sorted MFB output vector is \( y^s(k) = R^s(k)W^s(k)b^s(k) + n^s(k) \), where the superscript s denotes the sorted result, and \( |W^s_{1,1}(k)| \geq |W^s_{2,2}(k)| \geq \ldots \geq |W^s_{N,N}(k)| \). By Cholesky factorization, the inverse of the autocorrelation matrix \( R^s(k) \) is given by \( [R^s(k)]^{-1} = [F^s(k)]^{-1} [F^s(k)^T]^T \), and the sorted decorrelator output (2) can be rewritten as
\[
z^s(k) = W^s(k) b^s(k) + [F^s(k)]^{-1} \tilde{n}^s(k),
\]
where \( \tilde{n}^s(k) = [F^s(k)^T]^T n^s(k) \). For the n-th strongest user, the noise term depends on n-1 noise components that can be estimated using the decisions of n-1 stronger users. The decision of n-th user is given by
\[
\hat{b}_n^s(k) = \text{sgn}(\Re[z_n^s(k)\hat{C}_n^*(k)]),
\]
where \( z_n^s(k) = z_n^s(k) - \sum_{i \neq n} [F^s(k)]^{-1} \tilde{n}^s_i(k) \) and the noise estimate is \( \hat{n}^s_i(k) = [F^s(k)]^{-1}_{i,i} \{ z_i^s(k) - \sqrt{E_{b_i} F^s_i(k) \hat{C}_i^s(k) \hat{b}_i^s(k)} \} \) (n=1, ..., N).
(5) The successive interference cancellation (SIC) schemes

The basic concept of the SIC [5-6] is similar to that of the DF where the users are demodulated successively according to their strengths. Unlike the DF, it does not require any matrix inversion or factorization. In reconstruction of the MAI in the SIC detector, several different signals can be used, namely, a soft decision, a hard decision, or a matched filter output. We denote these three types by SIC-SD, SIC-HD and SIC-MF, respectively. In [5], the SIC-SD with BPSK modulation was studied. In addition, the performance of the SIC was evaluated for the IS-95 standard. It was shown that this detector can significantly outperform the conventional detector for this channel.

These SIC schemes can be implemented in discrete time as follows:

**SIC-SD** Initially, set M=N (the number of users). At each iteration, the remaining M users are sorted so that\( d_1^s(k) \geq d_2^s(k) \geq ... \geq d_M^s(k) \), where \( d_i^s(k) \) is the decision variable of the i-th strongest user defined by \( d_i^s(k) = \text{Re}[e^{-j\theta_i^s} y_i^s(k)] \). The phase term \( e^{-j\theta_i^s} \) is determined as \( e^{-j\theta_i^s} = \frac{\mathbf{W}_{i,i}^s}{|\mathbf{W}_{i,i}^s(k)|} \). Make a decision for the strongest user among M remaining sorted users using \( d_1^s(k) : \hat{b}_1^i(k) = \text{sgn}[d_1^i(k)] \). Reconstruct the MAI for other remaining users due to the strongest user, i.e., \( \hat{\text{MAI}}_i = \mathbf{R}_{i,1}^s d_i^s(k) \) (i=2, ..., M) and subtract it from the signals of other remaining users: \( y_i^s(k) \leftarrow y_i^s(k) - \hat{\text{MAI}}_i(k) \) (i=2, ..., M). Then set \( M \leftarrow M - 1 \) and repeat the loop until every user is demodulated.

Note that the decision variable of the strongest user is actually equal to \( d_1^i(k) = \sqrt{E_{b_i^1}} |C_i^1(k)| \mathbf{R}_{i,1}^s \hat{b}_1^i(k) + \text{MAI} + \text{background noise} \). The decision variable only contains the amplitude information of the strongest user, and no phase information is given. Therefore, the reconstruction of the MAI requires the phase information in addition to the cross-correlation values.

**SIC-HD** The SIC-HD is identical to the SIC-SD except that the hard decision \{+1, -1\} is used for MAI construction. The hard decision contains no information of the phase and the amplitude, therefore, the reconstruction of the MAI requires the estimates of both. The algorithm described above for the SIC-SD is modified as follows: reconstruct the MAI using \( \hat{\text{MAI}}_i = \mathbf{R}_{i,1}^s \mathbf{W}_{i,2}^s (k) \hat{b}_1^i(k) \) (i=2, ..., M).

**SIC-MF** The SIC-MF uses the matched filter outputs in sorting the users and in reconstructing the MAI. The following two modifications of the algorithm for SIC-SD are implemented. First, in sorting the users, use \( |y_1^s(k)| \geq |y_2^s(k)| \geq ... \geq |y_M^s(k)| \). (In the first iteration, M=N, and the actual matched filter outputs are used, \( y_i^s(k) = y_i(k), \) i=1, ..., N). Second, to reconstruct the MAI, use \( \hat{\text{MAI}}_i = \mathbf{R}_{i,1}^s y_1^i(k) \) (i=2, ..., M).
Note that in this algorithm channel estimation can be completely avoided if noncoherent (e.g., DPSK) detection is used. For the SIC-SD and the SIC-HD, even with noncoherent demodulation, channel estimation is still required for MAI reconstruction.

(6) The parallel interference cancellation (PIC) scheme

The PIC considered here is identical to the approximate decorrelator discussed in [7]. The PIC directly uses the outputs of the matched filter bank to reconstruct the MAI for all users in a parallel fashion and therefore does not require ordering of users as in the DF and the SIC. With the definition \( \mathbf{R} = \mathbf{I} + \mathbf{L} \) (\( \mathbf{I} \) is \( N \times N \) identity matrix), the signal after MAI cancellation in the PIC is given by [26]

\[
\tilde{y}(k) = \mathbf{\hat{R}} \mathbf{y}(k) = \mathbf{\hat{R}} \mathbf{W}(k) \mathbf{b}(k) + \mathbf{\hat{n}}(k),
\]

where \( \mathbf{\hat{R}} = \mathbf{I} - \mathbf{L}, \) \( \mathbf{\hat{R}} = \mathbf{\hat{R}} \mathbf{R}, \) and \( \mathbf{\hat{n}}(k) = \mathbf{\hat{R}} \mathbf{n}(k). \) The matrix filter \( \mathbf{\hat{R}} \) is the first order approximation of \( \mathbf{R}^{-1} = \mathbf{I} - \mathbf{L} + \mathbf{L}^2 - \mathbf{L}^3 + \ldots [7]. \)

### III. Channel Estimation

Several of the multiuser detectors considered in Section II require fading channel estimation for reconstruction of MAI. In general, channel estimation is an important issue in coherent detection. For the mobile fading channel, the tracking of the phase is difficult to achieve with the traditional Phase Locked Loop (PLL) [18]. It was shown in [18] that the slow convergence problem of the PLL ("hang-up effect") can be overcome by using a Kalman filter, which estimates the in-phase and the quadrature components of the channel coefficients. When a fading channel obeys the Gauss-Markov model, the Kalman filter is the MMSE estimator [27]. The use of the Kalman filter requires the statistical knowledge of the fading channel and since the statistical information of the fading channel is available, the Kalman filter has recently been adopted for estimating a rapid time-varying channel [21]. The statistical knowledge leads to the construction of the hypermodel, which is embedded in the Kalman filter. In this section, we first briefly introduce the Kalman filter, and show how to construct the hypermodel for the realistic fading channel using a AR2 process. Finally, we discuss the practical considerations regarding the use of the Kalman filter.

#### A. The Kalman filter

The hypermodel used in the Kalman filter consists of two parts. The first part describes the process to be estimated, and is defined by the Gauss-Markov model. The second part is the model of the measurement process. They can be expressed by the following state space equations [27],

\[
\mathbf{X}(k+1) = \mathbf{A}(k) \mathbf{X}(k) + \mathbf{G}(k) \mathbf{W}(k)
\]

\[
\mathbf{z}(k) = \mathbf{H}^T(k) \mathbf{X}(k) + \mathbf{V}(k)
\]

where the column vector \( \mathbf{X}(k) \) is a state variable, the column vectors \( \mathbf{W}(k) \) and \( \mathbf{z}(k) \) are the input and output signal of the hypermodel, respectively. The output \( \mathbf{z}(k) \) is called the observation
process or the measurement process, which is corrupted by the observation noise \( V(k) \). The matrices \( A(k), G(k), \) and \( H(k) \) contain the parameters of the system dynamics.

Once the hypermodel ((5) and (6)) is acquired, the MMSE estimate of the state variable can be generated recursively by the following equations [27]:

\[
M(k) = \Sigma(k) H(k) \left[ H^T(k) \Sigma(k) H(k) + P(k) \right]^{-1},
\]

Kalman filter gain:

\[
K(k) = A(k) M(k),
\]

Prediction (Kalman Filter):

\[
\hat{X}(k+1) = \left[ A(k) - K(k) H^T(k) \right] \hat{X}(k) + K(k) z(k),
\]

Error Covariance:

\[
\Sigma(k+1) = A(k) \left[ \Sigma(k) - M(k) H^T(k) \Sigma(k) \right] A^T(k) + G(k) Q(k) G^T(k),
\]

(Riccati equation) (10)

where the matrices \( P(k), Q(k), \) and \( \Sigma(k) \) are covariance matrices of the observation noise, the input signal, and the estimation error, respectively. They are defined by

\[
P(k) = E\{V(k)V^T(k)\},
\]

\[
Q(k) = E\{W(k)W^T(k)\},
\]

\[
\Sigma(k) = E\{[X(k) - \hat{X}(k)][X(k) - \hat{X}(k)]^T[\mathbf{Z}(k-1)]\}, \mathbf{Z}(k-1)=\{z(0), z(1),..., z(k-1)\}
\]

B. Fading channel models and selection of the hypermodel for the Kalman filter

It is well known that a Rayleigh fading process \( C(t) \) can be characterized by its autocorrelation function or its power spectrum density [19]:

The autocorrelation function:

\[
\Phi(\tau) = J_0(2\pi f_d \tau),
\]

The power spectrum density:

\[
\Psi(f) = \frac{1}{\pi f_d} \frac{1}{\sqrt{1-(f/f_d)^2}}, \quad |f| \leq f_d, \text{ (Baseband)}
\]

where \( J_0(\bullet) \) is the Bessel function of the zero-th kind, and \( f_d \) is the maximum Doppler frequency in hertz. When using the Kalman filter to estimate a fading process, equations (14) and (15) can be utilized in constructing the Gauss-Markov model (5) in the hypermodel.

(1) The modified Jakes model

In [19], Jakes proposed to model a Rayleigh fading process \( C(t) \) by a number of oscillators with different phases and frequencies which reflect the Doppler spread. Jakes model was further modified in [20] to satisfy the desired properties of a fading channel, i.e., the in-phase and the quadrature components are uncorrelated and their variances are identical. The modified model of the Rayleigh fading process is [20]

\[
C(t) = \sqrt{\frac{2}{N_d}} \sum_{n=1}^{N_d} e^{j\phi_n} \cos(2\pi f_d t \cos \alpha_n),
\]

where \( N_d \) is the number of distinct oscillators (the total number of oscillators are \( 4N_d \)), \( \phi \) is the phase of each oscillator, and \( \alpha_n = 2\pi(n-0.5)/4N_d \). The autocorrelation function of the fading process is
\[ \Phi(\tau) = \langle C(t)C^*(t + \tau) \rangle = \frac{1}{N_d} \sum_{n=1}^{N_d} \cos(2\pi f_d \tau \alpha_n) \to J_0(2\pi f_d \tau) \text{ as } N_d \text{ increases.} \] (17)

Since the autocorrelation function of the modified Jakes model approaches the autocorrelation function of a realistic Rayleigh fading process (14) if the number of oscillator is not too small, it is a realistic model for mobile radio channels.

For a single user system when a binary symbol is transmitted through a slow fading channel, the fading process is assumed to be constant during a symbol period, and the received signal at the sampling time \( kT \) can be expressed by
\[ z(k) = \sqrt{E_b} \, C(k) \, b(k) + v(k), \] (18)
where \( E_b \), \( b(k) \) and \( v(k) \) are the symbol energy, the information bit, and the additive noise. Equation (18) also represents the output of the decorrelator for multiuser systems (2). The fading coefficient \( C(k) \) is the sample of the fading process, i.e., \( C(k) = C(t)|_{t=kT} \) of (16). For coherent reception or MAI reconstruction, it is necessary to estimate \( C(k) \). When the Kalman filter is used for estimation, the received signal is considered as the observation process, and can be expressed in the form of (6) by choosing an appropriate system matrix \( H(k) \). The hypermodel of the fading process also has to be chosen to specify parameters in equation (5). The hypermodel is a model that is suitable for embedding into the Kalman filter. In contrast to the Jake model, an autoregressive (AR) model can serve as a hypermodel since it can be represented in the form of (5). A realistic AR model has to approximate the features of the fading process described by (14-17). One way to approximate the spectral-peak shape power spectrum is to use a high order low pass filter. For example, an eighth-order elliptic filter was used in [29]. This high order low-pass filter approximation increases the dimension of the Kalman filter and is not suitable in practical implementation. It is possible to approximate the modified Jakes model by a lower order AR process (greater than one), such as the second order (AR2) process. In this paper we adopt the AR2 as the approximate hypermodel for the Rayleigh fading process. The construction of the hypermodel for the fading process created using the modified Jakes fading model will be further discussed in the following sections.

(2) The second order auto-regressive model (AR2)

The fading coefficient modeled by a AR2 process can be written as
\[ C(k) = -a_1 \, C(k-1) - a_2 \, C(k-2) + w(k), \] (19)
where \( w(k) \), the driving noise of the fading process, is a complex zero-mean white Gaussian process. The AR process parameters \( a_1 \) and \( a_2 \) are determined by the locations of the poles of the transfer function on the unit circle. The parameters \( a_1 \) and \( a_2 \) are closely related to the physical parameters of the underlying fading process:
\[ a_1 = -2 \, r_d \cos(2\pi f_d' T) \quad a_2 = r_d^2, \] (20)
where \( f_d' \) is the spectral peak frequency, \( T \) is the symbol period, and \( r_d \) is the pole radius which corresponds to the steepness of the peaks of the power spectrum. We will show below that the spectral peak frequency should be chosen as \( f_d'/\sqrt{2} \) where \( f_d \) is the maximum Doppler frequency.

The AR2 process (19) and the observation process (18) can be converted into the hypermodel (5) and (6) by defining the state variable \( X(k) = [C(k) C(k-1)]^T \) and choosing the system matrices 
\[
A = \begin{bmatrix} -a_1 & -a_2 \\ 1 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad H(k) = \sqrt{E_b b(k)} \begin{bmatrix} 1 \\ 0 \end{bmatrix}.
\]
With these arrangements, the Kalman filter can be implemented by (7-10).

(3) Coefficient adjustment for the AR2-based hypermodel

In [21], Lindbom observed that in choosing the parameters of the AR2 hypermodel model for a realistic fading channel, the spectral peak frequency used in the AR2 signal model should be compensated by a factor of \( \sqrt{2} \) relative to the Doppler frequency. We justify the factor of \( \sqrt{2} \) compensation by comparing the autocorrelation functions of a Rayleigh fading channel and a second order AR process [15, 26].

Let \( C'(k) \) be a discrete-time sequence generated by the AR2 process (19). The autocorrelation function of the AR2 process is defined by 
\[
\tilde{\Phi}(m) = \mathbb{E}\{C(k)C^*(k-m)\}.
\]
Since the variance of the actual channel coefficient is one, we define \( \tilde{\Phi}(0) = 1 \). From (19), 
\[
\tilde{\Phi}(1) = -a_1 \tilde{\Phi}(1) - a_2 \tilde{\Phi}(2), \quad m \geq 2.
\]
The solution to (21) can be approximated by
\[
\tilde{\Phi}(m) \approx r_d^{m-2} \cos(2\pi f_d' T (m-2)), \quad m \geq 2.
\]
The autocorrelation function \( \Phi(m) \) of a fading coefficient \( C(k) \) is [19]
\[
\Phi(m) = \mathbb{E}\{C(k)C^*(k-m)\} = J_0(m 2\pi f_d T).
\]
Using Taylor series expansion for (22) and (23), we have
\[
\Phi(m) = 1 - \frac{(2\pi f_d' T m)^2}{2^2} + \frac{(2\pi f_d' T m)^4}{2^2 4^2} - \ldots
\]
\[
\tilde{\Phi}(m) = 1 - \frac{(2\pi f_d' T r_d (m-2))^2}{2!} + \frac{(2\pi f_d' T r_d (m-2))^4}{4!} - \ldots
\]
To make \( \tilde{\Phi}(m) = \Phi(m) \) for moderate values of \( m \), we compare only the first two expansion terms since the terms \( 2\pi f_d T m \) and \( 2\pi f_d' T r_d (m-2) \) are small. We have
\[
f_d' = \sqrt{\frac{m}{m-2}} \sqrt{\frac{1}{r_d \sqrt{2}}} f_d = \frac{1}{\sqrt{2}} f_d.
\]
The approximation in the last term in (26) is achieved when \( r_d \) approaches 1. This result indicates that a second order AR process with the spectral peak frequency \( f_d' \) is approximately equivalent to a fading process with maximum Doppler shift \( f_d = \sqrt{2} f_d' \). Figure 1 shows the
autocorrelation functions of a fading process and the second order AR process with and without the factor of $\sqrt{2}$ compensation.

C. Implementation issues

(1) Steady-state error covariance matrix

In the decision-directed operation of the Kalman filter, the data bit $b(k)$ is unknown, and it is replaced by the decision $\hat{b}(k)$. Therefore, the data-dependent matrix $H(k)$ is replaced by $\hat{H}(k) = \sqrt{E_b} \hat{b}(k) [1 \ 0]^T$ in the decision-directed mode. This replacement will not affect the steady state value of the error covariance matrix. The fact that the error covariance matrix is independent of the transmitted binary data stream can be observed from (7) and (10). The error covariance matrix is a function of the square of the binary bit, which is the constant +1. Therefore, the steady state error covariance matrix can be computed off-line prior to the reception, and used in the decision-directed mode to save the computation.

In the decision-directed mode, the prediction equations (7-10) are now modified by

$$\hat{M}(k) = \Sigma(k) \hat{H}(k) [\hat{H}^T(k) \Sigma(k) \hat{H}(k) + \alpha]^{-1}$$  \hspace{1cm} (27)

Kalman filter gain:

$$\hat{K}(k) = A \hat{M}(k)$$  \hspace{1cm} (28)

Prediction:

$$\hat{X}(k+1) = [A - \hat{K}(k) \hat{H}^T(k)] \hat{X}(k) + \hat{K}(k) z(k)$$  \hspace{1cm} (29)

Error Covariance:

$$\Sigma(k+1) = A(k) [\Sigma(k) - \hat{M}(k) \hat{H}^T(k) \Sigma(k)] A^T(k) + G(k) \beta G^T(k)$$  \hspace{1cm} (30)

where $\alpha$ and $\beta$ are the variances of the observation noise and the driving noise of the fading process, i.e., $\alpha = \mathbb{E}\{|v(k)|^2\}$ and $\beta = \mathbb{E}\{|w(k)|^2\}$. Note that although this modification does not affect the steady-state covariance matrix, this does not imply that the actual "measured" mean square error (MSE), $\frac{1}{N} \sum_{k=1}^{N} |C(k) - \hat{C}(k)|^2$, always approaches the theoretical estimation error variance, which is the steady-state value of $\Sigma(k)_{1,1} = \mathbb{E}\{|C(k) - \hat{C}(k)|^2\}$. Actually, the measured MSE sometimes may be much larger than the theoretical one, in which case, we encounter a long burst of decision errors, as will be discussed below.

(2) The reversal phenomenon

In the decision-directed mode, the channel estimate (the prediction) and the data estimate (the decision) are coupled (27-29). The coupling causes an interesting "reversal phenomenon," in which the measured MSE is much larger than the theoretical one [22]. The term "reversal phenomenon" reflects the fact that the estimates are close to the opposites of the actual channel coefficients; they have phase difference close to $\pi$. As illustrated in Figure 2, the channel estimates and channel coefficients are "reversed" between iterations 300 and 600. During this interval, almost all the decisions are incorrect. Observations show that the reversal phenomenon starts during deep fades — when the magnitudes of the channel coefficients are quite small. In deep fades, the decision errors are likely to occur, and the Kalman estimator will produce inaccurate estimates, which in turn will cause subsequent decision errors.
The following arguments help to explain the reversal phenomenon. Substituting the received signal \( z(k) \) (18) into (29) (ignoring the noise term \( v(k) \)), we have
\[
\hat{X}(k+1) = A \hat{X}(k) + \hat{K}(k) \{ H(k) X(k) - \hat{H}(k) \hat{X}(k) \}. 
\] (31)
Suppose the decision is wrong, i.e., \( \hat{b}(k) = -b(k) \). Then \( \hat{H}(k) = -H(k) \), and (31) becomes
\[
\hat{X}(k+1) = A \hat{X}(k) - \hat{K}(k) \hat{H}(k) \{ X(k) + \hat{X}(k) \}. 
\] (32)
The sum of the actual channel coefficient state vector \( X(k) \) (5) (ignoring the noise term \( w(k) \)) and the estimate (32) gives
\[
X(k+1) + \hat{X}(k+1) = \{ A - \hat{K}(k) \hat{H}(k) \} \{ X(k) + \hat{X}(k) \}. 
\] (33)
If the Kalman filter is stable, i.e., the magnitudes of all the eigenvalues of the matrix \( A - \hat{K}(k) \hat{H}(k) \) are less than one, the iteration of (33) goes to zero. This implies that \( X(k) \) will have 180 degree phase shift relative to \( \hat{X}(k) \) as \( k \) increases. Thus, a sequence of incorrect decisions may trigger the reversal phenomenon.

On the other hand, if the decision is correct, then \( \hat{H}(k) = H(k) \), and (31) becomes
\[
\hat{X}(k+1) = A \hat{X}(k) + \hat{K}(k) \hat{H}(k) \{ X(k) - \hat{X}(k) \}. 
\] (34)
Subtracting (34) from (5), we obtain
\[
X(k+1) - \hat{X}(k+1) = \{ A - \hat{K}(k) \hat{H}(k) \} \{ X(k) - \hat{X}(k) \}. 
\] (35)
Equation (35) implies that a sequence of correct decisions will result in the tracking of \( \hat{X}(k) \) to \( X(k) \) provided that the ignored noise terms are small.

These arguments and experimental results show that a few consecutive errors caused by a deep fade are likely to trigger the reversal phenomenon. Once the reversal of the channel estimates is established, the incorrect decisions continue to occur, even when the channel exits the deep fade. This is because the decision is given by \( \hat{b}(k) = \text{sgn}(\text{Re}[z(k) \hat{C}^*(k)]) \), and when the estimate \( \hat{C}(k) \approx -C(k) \), the decision is likely to be \( \hat{b}(k) = -b(k) \). The reversal typically continues until another deep fade during which several "decision errors" (which are actually the correct decisions now) may disrupt the convergence in (33) and bring the estimator back to (35). The long burst of errors due to the reversal will raise the BER to the order of \( 10^{-1} \) even for high SNR values. Therefore, the detector-estimator system must be designed to avoid the reversal phenomenon.

(3) Differential encoding

To avoid the catastrophic error events associated with the reversal phenomenon, we originally proposed an ARQ scheme with the threshold detection since the amplitude of the channel estimate can be used as the indicator of the fading channel condition [22]. If the amplitude is smaller than the pre-determined threshold level, the channel is in a deep fade and the transmitter is informed by the receiver to send a training bit next time. The method was not pursued further due to the problems arising in the threshold level selection, the low throughputs, and the increased delays. A more realistic and efficient solution is to encode the data bits differentially [30]. The data bits \( b(k) \) are encoded as the phase differences between the transmitted bits \( a(k) \), i.e., \( a(k) = \text{sgn}(\text{Re}[z(k) \hat{C}^*(k)]) \), and when the estimate \( \hat{C}(k) \approx -C(k) \), the decision is likely to be \( \hat{b}(k) = -b(k) \). The reversal typically continues until another deep fade during which several "decision errors" (which are actually the correct decisions now) may disrupt the convergence in (33) and bring the estimator back to (35). The long burst of errors due to the reversal will raise the BER to the order of \( 10^{-1} \) even for high SNR values. Therefore, the detector-estimator system must be designed to avoid the reversal phenomenon.
b(k) ⊕ a(k-1), where ⊕ is the addition mod-2 (exclusive OR). During the reversal, although the phases of most decisions are shifted by π relative to the transmitted bits, most of the differences between the consecutive transmitted bits are not changed. Therefore, the data bits encoded as the differences between the consecutive transmitted bits should be preserved during the reversal interval. The errors occur only at the beginning and the end of the reversal or are sparsely scattered within the reversal interval.

Both coherent and noncoherent detection can be applied to demodulate the differentially encoded data [30]. We denote coherent detection by Differentially Coherent Phase Shift Keying, or DC-PSK, and noncoherent detection by Differential Phase Shift Keying, or DPSK. Throughout the paper, we consider binary symbols only. In coherent detection, the estimate of the phase reference is needed to demodulate the transmitted bits. The received bits then go through the differential decoder to generate the data bits. In noncoherent reception, detection is performed directly using the received signal. Since it requires no channel estimation, it is much simpler and is widely used in communications for fading channels; however, its performance is usually poorer than that of coherent reception.

In the context of multiuser detection, the use of the DPSK is limited to the detectors which do not require channel estimation for the MAI reconstruction. For example, in our study, possible choices are the decorrelator [31], the SIC-MF, and the PIC. The output of the decorrelator is identical to that of a single user system with enhanced noise. Therefore, the decision process does not involve interference reconstruction. The SIC-MF and the PIC reconstruct the MAI, but they use the matched filter outputs and do not require channel estimates. Thus, noncoherent detection can be applied. On the other hand, for the 2S and the DF detectors, or any other decision-based MAI canceling type multiuser detector, channel estimation must be performed in order to reconstruct and cancel the MAI. Since the channel estimates are available, it makes sense to use coherent reception (DC-PSK) for these multiuser detectors.

(4) Disjoint estimation

A single user or a multiuser receiver for fading channels can be decomposed into a detector and a channel estimator [32, 23]. In the context of multiuser receivers, it was shown in [23] that the optimal joint channel estimator is also the Kalman filter provided the fading process obeys the Gauss-Markov model. In joint estimation, a single Kalman channel estimator generates channel estimates using the MF outputs and data bits of all users. Since joint estimation requires high complexity, it is more desirable to use disjoint estimation. Instead of using a single Kalman filter with a large dimension, we use N 'smaller' Kalman filters to estimate the channel coefficients for each of the N users [11, 25]. Different from the joint Kalman filter, the disjoint Kalman filter uses the output of the decorrelator as the observation process since it is independent of other users’ data bits. In applying the disjoint estimation in the 2S and the DF, we still use the output of the
decorrelator as the observation process because they both use the decorrelator in the front end (see Figure 3). The outputs of succeeding stage of the DF and the 2F could be used as the observation process for the Kalman filters in these two multiuser receivers; however we did not consider this implementation in our work due to the dependency on the data bits of other users.

IV. BER Analysis

In this section, we briefly summarize our analysis tool--the unified analysis [33-34], and apply it to obtain the BERs of several multiuser detectors. The BERs of the decorrelator, the 2S, and the DF with MMSE channel estimators for BPSK modulation assuming perfect decisions of other users were derived in [14]. These theoretical results are extended to the differentially encoded PSK, and verified by simulations in which the Kalman filters are operated in the decision-directed mode. We also analyze the BERs of the conventional detector and the PIC with DPSK, and verify them using simulations.

A. The unified analysis

In [33], Stein presented a generalized error rate expression of a binary communication system which can be applied to both coherent and noncoherent reception under all the possible conditions of correlations in signals and in the additive noise process. Recent applications of the unified analysis include performance evaluation for communication systems in the presence of the carrier noise [10, 14, 18, 35]. We summarize the result for the Rayleigh fading channel [25]. More detailed derivation can be found in [26, 34].

Suppose the decision variable in the receiver is obtained by $d = \text{Re}\{X Y^*\}$, where $X$ and $Y$ are zero-mean correlated complex Gaussian random variables. The error probability is

$$PE = \Pr(d < 0) = \frac{1}{2} \left[ 1 - \frac{M_{xy} + M_{yx}}{\sqrt{(M_{xy} - M_{yx})^2 + 4M_{xx}M_{yy}}} \right]$$

(36)

where $M_{xx}, M_{yy}, M_{xy},$ and $M_{yx}$ are the moments of functions of $X$ and $Y$ which are defined as

$$M_{xx} = \mathbb{E}\{X^2\}, \quad M_{yy} = \mathbb{E}\{Y^2\}, \quad M_{xy} = \mathbb{E}\{XY^*\}, \quad M_{yx} = \mathbb{E}\{YX^*\}$$

(37)

B. BERs of multiuser detectors with perfect channel estimation

In this section we use the unified analysis to calculate the BERs of several multiuser detectors with perfect channel estimation and BPSK modulation for flat Rayleigh fading channels. Let us first consider the conventional detector with perfect channel estimation. The output of the matched filter for user 1 is

$$y_1(k) = \sqrt{E_b} C_1(k) b_1(k) + \sum_{j=2}^{N} R_{1,j} \sqrt{E_b} C_j(k) b_j(k) + n_1(k)$$

Let $X = y_1(k)$ and $Y = C_1(k)$. To calculate the BER using the unified analysis, we assume $b_1(k)=1$ and condition on $b_i(k)$ ($b_i(k)=\pm 1, i=2, ..., N$), so that $y_1(k)$ satisfies the Gaussian assumption. Since the conditional error probabilities $\Pr(d<0|b_1(k)=1,B)$ ($B=(b_2(k), ..., b_N(k))$) are the same for all values of $B$, the error probability $\Pr(d<0|b_1(k)=1) = \Pr(d<0|b_1(k)=1,B)$. For any value for $B$,
the moments \( M_{xx} = E_{b_1} \sum_{j=2}^{N} [\mathbf{R}_{1,j}]^2 E_{b_j} + N_0 \), \( M_{xy} = \sqrt{E_{b_1}} = M_{yx}, \) and \( M_{yy} = 1. \) Note that the value of \( B \) does not affect the above moments. Substituting these expressions into (36), we obtain:

\[
PE_{\text{conv}} = \frac{1}{2} \left[ 1 - \frac{1}{\sqrt{1 + \sum_{j=2}^{N} [\mathbf{R}_{1,j}]^2 (E_{b_j}/E_{b_1}) + 1/\gamma_1}} \right].
\]  

(38)

From (38), the performance of the conventional detector is limited by the cross-correlation values and the strengths of other users.

The BER of the PIC can be calculated similarly. From (4), the signal of user 1 after canceling the MAI is

\[
\hat{y}_1(k) = \sqrt{E_{b_1}} \mathbf{C}_1(k) b_1(k) + \sum_{j=2}^{N} \hat{\mathbf{R}}_{1,j} \sqrt{E_{b_j}} C_j(k) b_j(k) + \hat{n}_1(k).
\]

Let \( X=\hat{y}_1(k) \) and \( Y=\mathbf{C}_1(k). \) Then \( M_{xx} = \sum_{j=1}^{N} [\hat{\mathbf{R}}_{1,j}]^2 E_{b_j} + \rho N_0, \) where \( \rho = [\hat{\mathbf{R}} \hat{\mathbf{R}}^T]_{1,1}, \)

\( M_{xy} = \hat{\mathbf{R}}_{1,1} \sqrt{E_{b_1}} = M_{yx}, \) and \( M_{yy} = 1. \) From (36), the BER of the PIC is:

\[
PE_{\text{PIC}} = \frac{1}{2} \left[ 1 - \frac{1}{\sqrt{1 + \sum_{j=2}^{N} (\hat{\mathbf{R}}_{1,j}/\hat{\mathbf{R}}_{1,1})^2 (E_{b_j}/E_{b_1}) + \left( \rho/[\hat{\mathbf{R}}_{1,1}]^2 \right)(1/\gamma_1)} \right].
\]  

(39)

Similarly to the conventional detector, the performance of the PIC is affected by the powers of the interferers.

The BER of the decorrelator for flat Rayleigh fading channel was derived in [9]:

\[
PE_{\text{dec}} = \frac{1}{2} \left[ 1 - \frac{1}{\sqrt{1 + [\mathbf{R}^{-1}]_{1,1}/\gamma_1}} \right].
\]  

(40)

The BERs of the ideal DF and the 2S were discussed in [10, 14]. Their BERs are asymptotically equal to the BER of the single user system.

C. BERs of multiuser detectors for differentially encoded data

As discussed in section III, differential encoding is required to avoid the error bursts due to the reversal phenomenon. For multiuser detectors with coherent reception (DC-PSK), the BERs can be approximated using the error rates of the BPSK system [30]:

\[
PE_{\text{DC–PSK}} = 2 \left( 1 - PE_{\text{BPSK}} \right) PE_{\text{BPSK}}.
\]  

(41)

This expression is based on the assumption that the decisions are independent. Although in the decision-directed mode this assumption is not valid, the approximation (41) is still accurate for high SNR values [26].

Consider the decorrelator, the 2S and the DF with the disjoint Kalman filters. The Kalman filter is the MMSE estimator if the fading channel obeys the Gauss-Markov model. Assuming BPSK modulation and correct past decisions of all users in the estimator (i.e., no reversal
phenomenon), we can calculate the BERs of these detectors using the unified analysis. For user $i$, define the estimation error as $e_i(k) = C_i(k) - \hat{C}_i(k)$, and the normalized estimation error variance as $\Gamma_i = E[|e_i(k)|^2] = E[|C_i(k)|^2]$. The steady-state estimation error variance $E[|e_i(k)|^2]$ ($=\Sigma(k)_{i,i}$) is calculated by the Riccati equation (10). To compute the BER of the decorrelator [11], let $X = z_i(k) = \sqrt{E_b} \tilde{C}_i(k) b_i(k) + v_i(k)$ (see (2)) and $Y = \hat{C}_i(k)$. Then,

$$PE_{\text{dec. Mismatch}} = \frac{1}{2} \left[ 1 - \frac{1 - \Gamma_1}{1 + \left[ \frac{R^{-1}}{\Sigma(h_i)} \right]_{1,1} / \bar{\gamma}_1} \right]$$

(42)

For the 2S, we let $X$ be the output signal of the second stage after the MAI cancellation and $Y$ be the channel estimate generated from the Kalman filter, i.e.,

$$X = \sqrt{E_b} \tilde{C}_i(k) b_i(k) + \sum_{i=2}^{N} \sqrt{E_b} (C_i(k) - \hat{C}_i(k)) b_i(k) + n_i(k), \quad Y = \hat{C}_i(k).$$

It is assumed that the current decisions of other users are correct. The lower bound on the BER of the 2S for BPSK modulation is [14]

$$PE_{2S, \text{Mismatch}} \geq \frac{1}{2} \left[ 1 - \frac{1 - \Gamma_1}{1 + \sum_{i=2}^{N} (E_b / E_b_i) [R_{i,i}]^2 \Gamma_i} \right] + 1 / \bar{\gamma}_1$$

(43)

The lower bound of the DF is the same as that of the 2S [14]. Note that in addition to the channel estimation error of the desired user (user 1), the performance of the 2S and the DF is also determined by the estimation errors of other users. Once the BERs of these detectors with BPSK are calculated ((42) and (43)), the BERs of the coherent reception for the differential encoding (DC-PSK) can be approximated by (41).

Note that the BERs of all three detectors saturate for high SNR values. The error floors are determined by the limiting value of the estimation error variance, which increases as the Doppler shift grows [14]. Although performance of all detectors degrades as the fading rate increases, the degradation is less pronounced for the decorrelator than for the 2S and the DF detectors, since its performance does not depend on the estimation errors of the interferers. In fact, the error floors of the decorrelator and the single user system are the same [25], and are lower than the error floors of the 2S and the DF [14].

For noncoherent reception with differential encoding (DPSK), the BERs of the decorrelator, the PIC, and the conventional detector are obtained as follows. The decision variable of user 1 for the decorrelator with DPSK is $d = -\text{Re} \{z_i(k)z_i^*(k-1)\}$, and the error probability is $PE_{\text{DPSK}} = \Pr \{d \geq 0 \mid \text{data} = -1\}$ where the event that data=-1 is equivalent to $b_i(k-1)=b_i(k)$. Let $X = z_i(k)$ and $Y = z_i(k-1)$ and apply (36). Then the BER of decorrelator with DPSK is [25]

$$PE_{\text{dec. DPSK}} = \frac{1}{2} \left[ 1 - \frac{\Phi(1)}{1 + \frac{[R^{-1}]_{1,1}}{\bar{\gamma}_1}} \right]$$

(44)
where $\Phi(1)$ is the autocorrelation function of $C(k)$ evaluated at one sampling period (see equation 23). Note that the BER of noncoherent reception (DPSK) depends on the fading channel parameters, quantified by $\Phi(1)$.

To calculate the BER of the PIC with DPSK, we substitute the random variable $X$ by the signal of user 1 after MAI cancellation (4):

$$X = \tilde{y}_i(k) = \hat{R}_{1,1} \sqrt{E_{b_1}} C_i(k) b_i(k) + \sum_{j=2}^{N} \hat{R}_{1,j} \sqrt{E_{b_j}} C_j(k) b_j(k) + \hat{n}_i(k).$$

The random variable $Y$ is the delay version of the signal, i.e., $Y = \tilde{y}_i(k-1)$. Since the conditional error probabilities (conditioned on $b_i(k)$ and $b_i(k-1)$ ($i=2, ..., N$)) are identical, the BER of the PIC with the DPSK is [15]

$$PE_{\text{PIC, DPSK}} = \frac{1}{2} \left[ 1 - \frac{\Phi(1)}{1 + \sum_{j=2}^{N} \frac{\hat{R}_{1,j}}{\hat{R}_{1,1}} \left( \frac{E_{b_j}}{E_{b_1}} \right) + \left( \frac{\rho}{\left| \hat{R}_{1,1} \right|^2} \right) \left( \frac{1}{\tilde{\gamma}_1} \right)} \right]. \quad (45)$$

Similarly, let $X$ and $Y$ be the consecutive MF output of the desired user, i.e., $X = y_i(k)$ and $Y = y_i(k-1)$, we obtain the BER of the conventional detector with DPSK [15]

$$PE_{\text{conv, DPSK}} = \left[ 1 - \frac{\Phi(1)}{1 + \sum_{j=2}^{N} \hat{R}_{1,j} \left( \frac{E_{b_j}}{E_{b_1}} \right) + \frac{1}{\tilde{\gamma}_1}} \right]. \quad (46)$$

From (44–46), the BERs of all these detectors with DPSK depend on the fading rate, which is quantified by $\Phi(1)$. If the fading channel is modeled by the AR2, $\Phi(1) = -a_1/(1 + a_2)$. For actual fading channel (modified Jakes model), $\Phi(1) = J_0(2\pi f_d T)$. The performance of the conventional detector and the PIC with DPSK also depends on the signal strengths of the interferers. The BER of the decorrelator with DPSK is not affected by the powers of the interferers and therefore, it is near-far resistant.

### V. Numerical Examples

In this section, we compare the performance of multiuser detectors for several CDMA channels. The analytical results are plotted using lines and the simulations are presented by symbols (circles and plus signs), except for the SIC curves which are evaluated entirely by simulations. The BERs of the 2S and the DF are almost the same and only the results for the 2S are shown here.

**Example 1: The effect of the cross-correlation (Figure 4)**

First consider the simplest CDMA channel - the 2-user channel. Figures 4(a) and 4(b) show the results with the cross-correlation values of 0.5 and 0.9, respectively. Both users have equal average path strengths or SNRs ($\tilde{\gamma}_1 = \tilde{\gamma}_2$). The performance of all multiuser detectors degrades as the cross-correlation ($r$) increases, with the exception of the 2S, the DF, and the SIC-HD.
The PIC and the SIC-MF perform almost the same as the decorrelator in this case. Among three successive interference scheme, the SIC-HD has the best performance and the SIC-MF has the worst. This is expected since we assume the amplitude and phase of the interferer are known in the SIC-HD. Therefore, the MAI estimate generated in the SIC-HD gives the best approximation of the actual MAI and the cancellation is most complete. Note that the SIC-HD, as well as the 2S and the DF, are very close to the single user bound. Although this two-user example describes dependence of multiuser detectors on the cross-correlation, it does not completely illustrate several other relationships, especially for the SIC and PIC detectors. Therefore, we consider the cases of more users in the following examples.

**Example 2: The effect of the interferers’ powers (Figure 5)**

Figure 5 shows the simulation results for the four-user case. Figure 5(a) shows the case of equal strengths and Figure 5(b) shows the near-far scenario where interferes have stronger power (6 dB more) than the desired user. The set of signature waveforms is derived from the Gold sequences of length seven [10]. The performance of the decorrelator, the DF, and the 2S are not affected by the strength of other users whereas the performance of the PIC and the SIC is.

The SIC and the PIC degrade significantly for the four user channel in comparison with the 2-user channel. The two-user channel is a special case for the SIC and the PIC schemes since there is only one MAI component in the received signal, and it can be mostly removed. For a greater number of users, the MAI is canceled successively in the SIC. Therefore, a portion of the MAI due to stronger interferers remains uncanceled and deteriorates the performance of weaker users. For the PIC, since the MAI estimate is very noisy, the parallel MAI cancellation removes the major part of the MAI, but at the same time, adds in a significant amount of interference. The additional interference is due to the rough MAI estimates obtained from the outputs of the matched filter bank, which contain not only the desired signal, but also signals of the other users.

Although our results for the SIC-SD show that its performance degrades significantly when the number of users increases, it has been demonstrated that the SIC can provide acceptable performance in [5]. The difference between these results is due to the fact that a much larger processing gain is utilized in [5]. Since the scenario considered in [5] is close to the IS-95 standard, the SIC schemes should still be considered as strong multiuser detection candidates for current CDMA systems because of their reasonably good performance and low complexity.

**Example 3: The effect of the estimation errors and the near-far problem (Figure 6)**

In this example, we model the channel by the AR2 process. The fading channel model parameters in (20) are chosen as follows: the spectral peak frequency \(f'_d\) is 80 Hz; the Doppler shift \(f_d\) is approximately 113 Hz, which corresponds to the carrier frequency of 900 MHz and the vehicle speed of 85 miles per hour. The data rate \(1/T=10\) Kbps, and \(r_d=0.998\) reflecting the
peaks in the power spectrum. By (20), the coefficients of the second order AR process are \(a_1=-1.99348\), \(a_2=0.996\).

The cross-correlation matrices \(R\) for 4-user and 24-user cases are derived from Gold sequence of length 7 and 31 respectively. In particular, for the case of 24 users, we choose 24 Gold sequences of length 31, and cyclically shift each sequence by +1 chip or -1 chip. The purpose is to account for the effect of imperfect symbol-level synchronization, and to mimic the asynchronous CDMA channel. With this arrangement, the off diagonal elements of \(R\) will have one of the three values: \(-1/31, -9/31, 7/31\).

Figure 6(a) and Figure 6(b) show the results for 4 and 24 users with equal strength. Let us first focus on the coherent multiuser detectors (DC-PSK): the decorrelator, the 2S, and the DF. It is noted that for a single user system, the limiting value of the normalized error variance, \(\Gamma\), is the same as that of the decorrelator [25]. Therefore, the decorrelator has the same error floor as the single user system, i.e. the decorrelator asymptotically approaches the single user bound in this case. On the other hand, the limiting BERs of the 2S and the DF are higher than that of the single user system because of the contribution of other users' estimation errors \(\left(\sum_{i=2}^{N} (E_{b_i}/E_{b_1}) [R_{1,i}]^2 \Gamma \right)\) [14]. This result is quite different from the case of perfect channel estimation in which the performance of the 2S and the DF asymptotically approaches that of a single user system [10]. Further comparison with the case of perfect channel estimation shows that the performance gain of the 2S and the DF over the decorrelator is not as significant when channel is mismatched.

For multiuser detectors with noncoherent reception (DPSK), such as the conventional detector, the PIC, the SIC, and the decorrelator, the tradeoff between the complexity and performance is obvious. The conventional detector has BER of the order of \(10^{-1}\), which is unacceptable for most applications. For more than two users and in the highly interference-limited environment, the PIC and the SIC schemes also reach high error floors due to the MAI. The superior performance of the decorrelator indicates that the enhancement of the background noise is a small price to pay for the MAI elimination. The error floor of the decorrelator is due to the nature of the fading channel.

We also investigate the near-far scenario, illustrated by a 4-user case, shown in Figure 6(c). When the interferers are stronger than the desired user (user 1) by 6 dB, we find that the performance of the 2S and the DF degrades compared with the case of equal strength (see Figures 6(a) and 6(c)). This is again a different result when compared with the perfect channel estimation case [10]. For the perfect channel estimation case, in the near-far scenario, decisions of interferers in the 2S and the DF are more reliable and the MAI cancellation is more complete than in the case of equal strength. Therefore, the 2S and the DF are closer to the single user bound when interferers are stronger than in the case of equal strength. In the channel mismatched situation, the
residual interference is the dominant factor of the performance of the 2S and the DF for high SNR. In the near-far scenario, the residual interference is greater than that of the equal strength case, and therefore the 2S and the DF perform worse in the near-far scenario. This is due to the following reason. Although in the near-far scenario the estimation errors $\Gamma_i$ of other users are reduced compared with those of the equal strength case, the total power of the resultant residual interference $\sum_{i=2}^{N} (E_b / E_b) [R_{i,j}]^2 \Gamma$ is greater than that of the equal strength case due to higher power ratios $(E_b / E_b)$. The performance degradation of the 2S and the DF in the near-far scenario is more obvious in high SNR since the estimation error converges to the limiting value $\Gamma_i$ and cannot be further reduced. The performance of the PIC and the SIC shows that they are less suitable for high bandwidth efficiency systems than more complex detectors.

**Example 4: The effect of the Doppler shift (Figure 7)**

In Figures 7(a) and 7(b), the performance of the multiuser detectors with different Doppler frequencies for a two-user channel (cross-correlation $r=0.9$) is compared. In Figures 7(a) and 7(b), the spectral peak frequencies for the AR2 process are 80 Hz and 40 Hz, respectively. They are approximately equal to the maximum Doppler frequency 113 Hz and 56 Hz. Comparing these two figures, we find that for the lower Doppler shift, the error floors are lower and the cross-over point of the decorrelator and the 2S occurs at higher SNR value (45 dB instead of 40 dB). This means that for the lower Doppler frequency, the channel estimation errors are reduced, and the 2S (or the DF) performs better than the decorrelator for a larger operation region.

The dependency of the error floor values on the Doppler shift is explained by considering the limiting value of the normalized estimation error variance, $\Gamma_1$, in (42-43), as the additive Gaussian noise becomes negligible. For the AR2 process, this value is given by the variance of the excitation noise in (19) [14]. From the expression derived in [14], it is evident that the limiting value of $\Gamma_1$ monotonically increases with the fading rate. Thus, the higher the fading rate, the higher the error floors are. In addition, as the Doppler shift increases, the terms due to the channel estimation errors of the interferers increase (see (43)), and limit the performance of the 2S and the DF detectors.

**Example 5: The Jakes model (Figure 8)**

In this example, we simulate several cases using a more realistic channel model—the modified Jakes model. We use 24 distinct oscillators ($N_o=24$), or 96 oscillators in total. By using the Walsh-Hadamard codewords, we can generate at most 24 uncorrelated fading waveforms. The parameters in the Kalman filter are chosen based on the approximate AR2 hypermodel of the modified Jakes model, as discussed in Section III. When the Kalman filter estimates the channel coefficients generated by the modified Jakes model in the decision-direction mode, the reversal phenomenon still occurs. Therefore, the data bits need to be encoded differentially.
Figures 8(a) and (b) show the results for 4 and 24 users. The Doppler frequency is 80 Hz and the sampling rate is 10 Kbps. The Doppler frequency corresponds to the carrier frequency 900 MHz and vehicle speed of 60 mph. The signature cross-correlation matrices used here are the same as in examples 1-4. The results are similar to those in Examples 3 and 4. Although the error floors of the 2S and the decorrelator are not obvious when the modified Jakes model is used, they do exist. In Figure 8(c), we simulate a two-user scenario (cross-correlation r=0.9) and demonstrate that these error floors exist by increasing the Doppler frequency to 500 Hz. It is interesting to see that in this extreme case, the 2S or the DF performs worse than the decorrelator for all the considered operation SNR values due to the effect of channel estimation errors.

In section III we showed that the fading process created using the modified Jakes model with the Doppler frequency $f_d$ can be approximated well by the second order AR process with the spectral peak frequency $f'_d = f_d / \sqrt{2}$. In Figure 8(d), we verify this fact by plotting the BERs of the two-stage detector with different channel models for a two-user channel (cross-correlation r=0.9). We observe that the performance is similar for different models, and the factor of $\sqrt{2}$ adjustment provides a better approximation.

VI. Conclusions and Future Work

In this paper, we evaluated the performance of several multiuser detectors under the ideal perfect channel estimation assumption and under channel mismatch. In the case of perfect channel estimation, the more complex detectors (the 2S and the DF) can effectively eliminate the MAI and achieve the performance of the single user system, even for high bandwidth efficient channels. The simpler schemes, such as the SIC and PIC, are not as suitable for the use in the high bandwidth efficient channels. In the presence of the channel mismatch, the performance of the coherent multiuser detectors--the decorrelator, the 2S, and the DF, is comparable. The performance degradation of the 2S and the DF is due to the channel estimation errors of all users whereas the decorrelator is only affected by the desired user. The decorrelator outperforms the 2S and the DF in the high SNR region and preserves the near-far resistance. For noncoherent multiuser detectors, the decorrelator has a large performance gain over the SIC and the PIC. These results indicate that the decorrelator is the most robust multiuser detector for the realistic fading channels.

We also verify that the Kalman filter can be used to estimate a realistic fading channel even if the channel does not exactly obey the Gauss-Markov model. A second order AR process can be used as the hypermodel in the Kalman filter. To avoid the burst errors due to the reversal phenomenon when using the decision-directed Kalman filter, differentially encoded data is required.
Future work will concentrate on the performance evaluation of multiuser detectors for the frequency-selective fading CDMA channels, and other channel estimation methods.

References


The autocorrelation functions of fading models

Rayleigh fading channel (or Jakes model): _____
Second order AR model with $\sqrt{2}$ modification: ...........
Second order AR model without $\sqrt{2}$ modification: -------

Figure 1 Comparison of the AR2 model and the modified Jakes model
Figure 2 The reversal phenomenon (fading channel model: the second order AR process)
Figure 3 The two-stage detector with the disjoint Kalman filters
Performance comparison of multiuser detectors for a 2 user channel with equal signal strength and perfect channel estimation

(a) Cross-correlation r=0.5
(b) Cross-correlation r=0.9

Performance comparison of multiuser detectors for a 4-user channel with perfect channel estimation

(a) Equal Signal Strength
\( \bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma}_3 = \bar{\gamma}_4 \)

(b) Stronger interferers: Near-Far
\( \bar{\gamma}_i = 4 \bar{\gamma}_1, \ i = 2, 3, 4 \)
Performance comparison for multiuser detectors with Kalman channel estimator (AR2 model)

- Modulation: differentially encoded PSK
- Channel model: the second order AR process
(a) Spectral peak frequency $f_d = 80$ Hz  
(b) Spectral peak frequency $f_d = 40$ Hz

Figure 7
Performance comparison for multiuser detectors with different Doppler frequencies
- 2-user with equal strength ($\gamma_1 = \gamma_2$)
- Modulation: differentially encoded PSK
- Channel model: the second order AR process
Figure 8
Performance comparison for multiuser detectors with Kalman channel estimator (modified Jakes model)

- Modulation: differentially encoded PSK
- Channel model: the modified Jakes model
- Equal strength