System Response to Multiple Support Response Spectra Input

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SUMMARY

A composite formula to treat and combine the support participation factors has been developed and reported in a previous paper to compute system response when multiple support response spectra have been applied. This approach is a substantial improvement from methods proposed by others. For instance, in one method, the support participation factor for each support excitation is treated absolutely which results in the maximum upper bound solution when all support point inputs are added on an absolute basis. This is clearly a contradiction to the case when all supports have the identical input where the support participation factors should be combined on an algebraic basis. On the other hand, the use of algebraic sum for all support participation factors under different support inputs results in the least conservative and frequently unconservative solutions. Finally, the widely used square root sum of the squares approach of combining support participation factors assumes, at all times, that the support input motions are statistically independent. For inputs arrived at from a same controlling cantilever mode, for instance, this assumption may not be entirely justifiable. Also, the method is not reducible to the solution when all supports have the same input in a given input direction.

In this paper, the composite formula approach proposed in a previous paper is reviewed. A numerical example is also presented, using a typical piping system, to illustrate the advantages of using the multiple support response spectra input.

1. Introduction

A composite formula to treat and combine the support participation factors has been developed and reported previously in Ref. 1 to compute system response when multiple support response spectra have been applied. This approach is a substantial improvement from methods proposed by others.

For instance, system response subjected to multiple support response spectra input was investigated in 1973 by Lin (Ref. 2). In that publication, a closed form solution was derived for beams with various support conditions. It was shown that the use of different response spectra at different supports results in less restrictive structural response. Further studies based on the formulation of Clough and Penzien (Ref. 3) have since been published. These include Vashi (Ref. 4), Thailer (Ref. 5), Wu, et. al. (Ref. 6) and Leimbach and Sterkel (Ref. 7). All except Ref. 5 have arrived at the same response equation, but the final solution differs in the treatment of the support participation factors. In Ref. 4, for instance, the participation factor for each support excitation is treated absolutely which results in the maximum upper bound solution when all support point inputs are added on an absolute basis. On the other hand, Ref. 6 retains the sign of each support participation factor. This results in the least conservative and often times unconservative solutions. Finally, Ref. 7 proposed to sum the response from each support input by the square root sum of the squares approach. This assumption can be valid if and when the input motions are statistically independent. For inputs arrived at from a same controlling cantilever mode, for instance, this assumption may not be entirely justifiable. Also, the method is not reducible to the solution when all supports have the same input in a given input direction.

To include the various possible combination of phase relationships between support motions, the approach presented previously in Ref. 1 utilized a composite formula which is also directly reducible to the solution when all supports are subjected to the same response spectrum input in a given input direction. This approach has since been incorporated in a proprietary multidiscipline computer program.

In this paper, the composite formula approach is reviewed. A numerical example is also presented, using a typical piping system, to illustrate the advantages of using the multiple support response spectra input.

2. Theory

The equation of motion in its general form can be written as the following:

$$\left[\begin{array}{c|c} M & M_g \\ \hline M_g^T & M_{gg} \end{array}\right] \left\{\begin{array}{c} \ddot{V} \\ \hline \ddot{V}_g \end{array}\right\} + \left[\begin{array}{c|c} C & C_g \\ \hline C_g^T & C_{gg} \end{array}\right] \left\{\begin{array}{c} \dot{V} \\ \hline \dot{V}_g \end{array}\right\} + \left[\begin{array}{c|c} K & K_g \\ \hline K_g^T & K_{gg} \end{array}\right] \left\{\begin{array}{c} V \\ \hline V_g \end{array}\right\} = \left\{\begin{array}{c} P \\ \hline P_g \end{array}\right\} \tag{1}$$

where M, C and K are the mass, damping, and stiffness matrices, respectively, V and P are the displacement and the time history modal forces. Subscripts g and gg correspond to the items pertaining to the support points.

Equation 1 can be simplified to the following form by manipulations and by dropping the velocity terms in view of the small damping effect.

$$[M]\ \{\ddot{V}_r\} + [C]\ \{\dot{V}_r\} + [K]\ \{V_r\} = \{P\} - [M]\ [\eta]\ \{\ddot{V}_g\} - [M_g]\ \{\ddot{V}_g\} \tag{2}$$

where

$$\{V_r\} = \{V\} - [\eta]\ \{V_g\} \tag{3}$$

which represents the relative displacement, and where

$$[\eta] = -[K]^{-1}\ [K_g] \tag{4}$$

is the matrix for the influence coefficients.

Equation (2) can now be solved using the normal mode approach which results in the following solution for the generalized coordinates when the externally applied load (P) is taken to be zero.

$$Y_j^{ki} = L_j^{ki} \left[\frac{-1}{\omega_j} \int_0^t \ddot{V}_g^{ki}(\tau)\ e^{\zeta_j \omega_j (t-\tau)} \sin \omega_j (t-\tau)\ d\tau \right] \tag{5}$$

where ω_j is the jth mode natural frequency, ζ_j is the damping ratio, subscripts k and i represent the kth support and ith direction, respectively. Also

$$[L] = [\phi]^T\ [M]\ [\eta] + [\phi]^T [M_g] \tag{6}$$

Hence, L_j^{ki} is the support participation factor for the kth support, along ith direction and the jth mode, and ϕ is the orthonormalized mode shape. In this formulation, it is obvious that the total degrees of freedom of the system have been partitioned based on the ith directional motion.

It should be noted that equation (6) can be shown as identical to the participation factor of a system with same support inputs. Also, equation (6) can be simplified further to avoid inverting the stiffness matrix. This can be done by use of an identity. Consider the eigenvalue problem,

$$[K]\ \phi_j = \lambda_j\ [M]\ \phi_j \tag{7}$$

which can be written as

$$\lambda_j^{-1}\ \phi_j = [K]^{-1}\ [M]\ \phi_j \tag{8}$$

Taking the transpose,

$$\lambda_j^{-1}\ \phi_j^{\ T} = \phi_j^{\ T}\ [M]\ [K]^{-1} \tag{9}$$

so that

$$[\lambda]^{-1}\,[\phi]^{T} \;=\; [\phi]^{T}[M][K]^{-1} \tag{10}$$

Post-multiplying by $[K_g]$,

$$[\lambda]^{-1}[\phi]^{T}[K_g] \;=\; [\phi]^{T}[M][K^{-1}]\,[K_g] \tag{11}$$

Therefore,

$$[L] = -[\lambda]^{-1}[\phi]^{T}[K_g] \;+\; [\phi]^{T}[M_g] \tag{12}$$

The determination of $[L]$ will be done with expanded mode shapes to permit stiffness elements from massless degrees-of-freedom to support degrees-of-freedom. This situation occurs frequently with snubbers.

In applying the response spectrum technique, the integral on the right-hand side of Equation (5) has to be evaluated in the absolute maximum sense. However, in view that the displacement response is a sum of all significant modes and all supports and directions in inputs, Equation (5) should not be treated alone in the response spectrum approach. Instead, it is more appropriate to evaluate the final solution of the relative displacement vector in the following form:

$$\{V_r\} \;=\; \Sigma_i\;\Sigma_j\;\Sigma_k\;\;\{\phi_j\}\;\;Y_j^{ki} \tag{13}$$

In a response spectrum approach, Equation (13) has to be evaluated on an absolute maximum basis. However, it is an acceptable practice (Ref. 8) to combine the modes and the directional input by square root sum of the squares (SkSS) except where closely spaced mode criteria apply. Therefore, Equation (13) can be evaluated as

$$\{V_r\}_{\max} \;=\; \left[\Sigma_i\Sigma_j\;\;\{\phi_j^{2}\}\;\;\left(\Sigma_k\,Y_j^{ki}{}_{\max}\right)^{2}\right]^{1/2} \tag{14}$$

To properly evaluate Equation (14) using a response spectrum approach the quantity $|\cdot|_{\max}$ needs to be determined first. A composite formula has been proposed in Ref. (1) as the following:

$$\{V_r\}_{\max} \;=\; \left\{\Sigma_i\Sigma_j\;\;\{\phi_j^{2}\}\;\left[\Sigma_k\;(Y_j^{ki})^{2} \;+\; 2\sum_{n=m+1}^{K}\;\sum_{m=1}^{K-1}\;\varepsilon_{mn}\,Y_j^{mi}\,Y_j^{ni}\right]\right\}^{1/2} \tag{15}$$

where

$$Y_j^{ki} \;=\; L_j^{ki}\;\;S_d^{ki}\;\;(\omega_j)\;, \tag{16}$$

where $S_d^{ki}(\omega_j)$ is the displacement response spectrum value, representing the absolute maximum value of the bracketed quantity in Equation (5) for a given ω_j,

and ε_{mn} = 1: all supports m and n which both have the same response spectrum input; or if the response spectra for the supports m and n are proportional.

= 0: all other support response spectra for supports m and n.

Finally, define a_j as

$$a_j = \sum_{i=1}^{3} \left[\sum_{k=1}^{K} (Y_j^{ki})^2 + 2 \sum_{m=1}^{k-1} \sum_{n=m+1}^{K} \varepsilon_{mn} Y_j^{mi} Y_j^{ni} \right] \quad (17)$$

Then equation (9) may be written as

$$\{V_r\}_{max} = \{\Sigma_j \{\phi_j^2\} a_j\}^{1/2} \quad (18)$$

so that the modal coefficient for the j-th mode can be taken to be a_j.
Note that expanded mode shapes are used in Equation (6) to permit stiffness connections from massless degrees-of-freedom to support degrees-of-freedom. Also, Guyan reduction can affect $[M_g]$ in Equation (6).

3. Numerical Example and Conclusions

Reference 1 has shown the superiority of the multiple response spectra method in terms of reduction in the response loads. However, the models used were extremely simple, two-span beams. To provide for a better comparison, a safety injection line is analyzed using the computer program written based on the approach discussed in this paper. The line has four supports (each subjected to a set of response spectral input), 47 mass points, and a total of 141 dynamic degrees of freedom. Due to the complexity of the line, it will not be shown in this paper. However, displacement response for significant node points is presented in Table 1 and compared with results from other methods. As can be expected, the approach of absolute sum(Ref.4)is far too conservative. Likewise, the envelope response spectrum approach is also far more conservative than the multiple response spectra approach. The envelope response spectra approach is based on using the envelope of all four support response spectra at every support. Therefore, it is by definition a rather conservative approach.

Both multiple response spectra approaches (SRSS, LIN (Ref.1) and NUPIPE) show comparable results. If it is not because of the inseparable effect of dead weight which has been included in the NUPIPE analysis, it's results should be the same as the SRSS method. Also, with the assumption of dependence only for the supports 3 and 4, the composite formula presented herein does not show significant difference from combination of those supports by the SRSS method. However, this only reflects that the response spectra used are primarily independent in nature. The composite formula will show significant difference if either the spectra are dependent or if the spectra can be grouped into widely different dependent and independent sets.

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TABLE 1

COMPARISON OF RESULTS

Method	Max UX		Max UY		Max UZ	
	Node	Value	Node	Value	Node	Value
Lin's Method (Ref. 1) with sets 3 and 4 spectrum values added by SRSS Method	780	.748	790	.215	1860	.706
Lin's Method (Ref. 1) with sets 3 and 4 spectrum values added algebraically	780	.744	790	.226	1860	.807
NUPIPE	780	.745	1630	.234	2150	.739
Envelope	780	.931	1980	.323	1860	1.127
Absolute Sum	780	1.322	800	.375	1860	1.357