Transactions, **SMiRT-22**
San Francisco, California, USA - August 18-23, 2013
Division IV

A Predictive Model for the Relative Differences between Nonlinear and Equivalent-Linear Site Response Analyses

Byungmin Kim[1], Youssef M. A. Hashash[2], Albert R. Kottke[3], Dominic Assimaki[4], Wei Li[5], Ellen M. Rathje[6], Kenneth W. Campbell[7], Walter J. Silva[8], and Jonathan P. Stewart[9]

[1]Postdoctoral researcher, Dept. of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, IL (bkim54@illinois.edu)
[2]Professor, Dept. of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, IL (hashash@illinois.edu)
[3]Engineer, Bechtel Corporation, San Francisco, CA
[4]Professor, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA
[5]Project Engineering Associate, Paul C. Rizzo Associates
[6]Professor, Dept. of Civil, Architectural and Environmental Engineering, University of Texas, Austin, TX
[7]Vice President, EQECAT, Inc., Oakland, CA
[8]Pacific Engineering and Analysis, Inc., El Cerrito, CA
[9]Professor and Chair, Dept. of Civil and Environmental Engineering, University of California at Los Angeles, CA

ABSTRACT

This study investigates the nonlinear amplification of ground motion in site response analysis, with the particular goal of providing guidance on the conditions for which nonlinear (NL) approaches provide results distinct from otherwise similar equivalent-linear (EL) approaches. Relative differences in spectral accelerations computed by NL and EL site response analyses are assessed for different periods using NL and EL site response simulation results by Assimaki and Li (2012) for 23 strong motion accelerograph sites in California and one downhole array site in Japan subject to 510 synthetic ground motions. Site and ground motion parameters investigated for their effect on nonlinear response include the time-averaged shear-wave velocity in the top 30 m of the soil profile (V_{S30}), site amplification at the fundamental frequency (Amp), peak ground acceleration and peak ground velocity of incident motions (PGA_{in} and PGV_{in}, respectively), and estimated strain ($\gamma_{est}=PGV_{in}/V_{S30}$). We find that the soil nonlinearity is most clearly dependent on γ_{est}, with an upper-bound threshold of approximately 0.1 % identifying conditions where EL and NL results are similar. For $\gamma_{est} > 0.4$ %, it is recommended that a greater weight be given to the NL approach because the spectral accelerations computed by NL and EL analyses differ by more than 30 %, particularly for short periods (T ≤ 0.55 sec). For longer periods (T ≥ 1.0 sec), the differences between spectral accelerations by nonlinear and equivalent-linear site responses are relatively small.

INTRODUCTION

An equivalent-linear (EL) site response analysis approximates the nonlinear response of soil using strain-compatible time-invariant soil properties, and provides reasonable estimation of ground response for many situations. Time domain nonlinear (NL) site response computes the dynamic response of the soil at each time step. This more rigorous approach can better capture soil behavior under large strains such as for soft soil sites subject to strong ground motions, but it is computationally expensive and requires the use of input parameters (such as Rayleigh damping) that are relatively unfamiliar for most engineers. Due to the advantages and disadvantages of these two approaches, the conditions under which the two methods produce consistent and divergent estimates of site amplification are of practical interest.

Matasovic and Hashash (2012) showed that that there was a consensus amongst practitioners who responded to a survey that a NL analysis is to be used when computed shear strain exceed 1 %, although they noted that this threshold is likely too high. Matasovic and Hashash (2012) also reported that ground motion intensity measures alone cannot be sufficient to indicate the soil nonlinearity because strain levels in soft soils can be quite high even for low levels of shaking.

Recently, Kaklamanos et al. (2013) tested the accuracy and precision of linear and equivalent-linear site response analyses using 100 KiK-net downhole arrays in Japan to provide thresholds for the selection of linear, equivalent-linear, and nonlinear methods with respect to the maximum shear strain computed from site response analysis. They proposed that the EL approach is sufficient below the EL-NL transition zone, defined when the maximum shear strains range from 0.1 % to 0.4 %. A disadvantage of this approach is that a full site response analysis is required to estimate the strains from which the appropriate type of analysis is judged, which will be inefficient in many cases.

Assimaki and Li (2012) proposed an empirical relationship between soil nonlinearity and site and ground motion parameters that can be identified a priori (i.e., prior to running a response analysis). They compared the amplification of spectral accelerations computed by (1) commonly used site response analysis methods (i.e., the linear visco-elastic (LIE) and the equivalent-linear); (2) empirical equations given by Boore and Atkinson (2008); and (3) nonlinear site response analyses using the modified Kondnor and Zelasko (MKZ) hyperbolic model (Matasovic and Vucetic 1993). Assimaki and Lee (2012) performed their calculations in such a way that the small-strain modulus reduction and damping behavior in the NL and EL models is practically identical. They found that the divergence of predicted amplification levels is least pronounced between EL and NL models, whereas the divergence between LIE and NL is more pronounced and shows clear dependency on various parameters such as the peak ground acceleration of rock outcrop and the V_{S30}.

This study extends the work by Assimaki and Li (2012) to develop a predictive model for the relative differences between amplification levels estimated by EL and NL site response analyses. Our intent is to provide guidelines for a priori identification of conditions for which NL methods should be considered in lieu of EL for seismic site response analysis.

SITE RESPONSE SIMULATIONS

Assimaki and Li (2012) performed site response analyses for 24 strong motion accelerograph sites (23 sites in California and one in Kobe, Japan) used in previous site response calibration work by Baturay and Stewart (2003) (http://www.cee.ucla.edu/faculty/stewart/research). Eight sites are in National Earthquake Hazards Reduction Program (NEHRP) site class C (Building Seismic Safety Council (BSSC) 2004), 11 are class D, and five are class E. The V_{S30} values for these sites vary from 142 m/sec to 692 m/sec. Broadband ground motion synthetics were computed on the rock outcrop for multiple rupture scenarios of a strike-slip fault using a crustal model extracted from the Southern California Earthquake Center Community Velocity Model 4 (SCEC CVM-4) (http://www.data.scec.org/research-tools/3d-velocity.html), and were evaluated using a dynamic rupture source model by Liu et al. (2006). More details on the ground motion synthetics and the dynamic soil properties at these accelerograph sites can be found in Assimaki and Li (2012), Assimaki et al. (2008), and Anderson (2003). The response spectra of the 510 incident ground motions used for the site response simulations by Assimaki and Li (2012) are shown in Figure 1. The PGA values of incident ground motions range from approximately 0.03 g to 0.87 g with a median of approximately 0.1 g. The examples of response spectra computed on the ground surface by the EL and the aforementioned NL approaches are shown in Figure 2. Two sites with different V_{S30} values (691.7 m/sec for Lick Observatory in Santa Cruz (LOB) and 258.3 m/sec for La Ciengega in Los Angeles (LAC)) and two incident motions with different levels of shaking (motion #1 with PGA = 0.07 g and motion #2 with PGA = 0.3 g) were selected for comparison. When the incident motion #1 (weaker motion) is propagated through the station LOB (stiffer site), the differences between spectral accelerations computed by EL and NL methods (Sa^{EL} and Sa^{NL}, respectively) are negligible. However when the incident motion #2 (stronger motion) and the station LAC (softer site) are considered, the differences between Sa^{EL} and Sa^{NL} become significant at short periods (under approximately 0.4 sec).

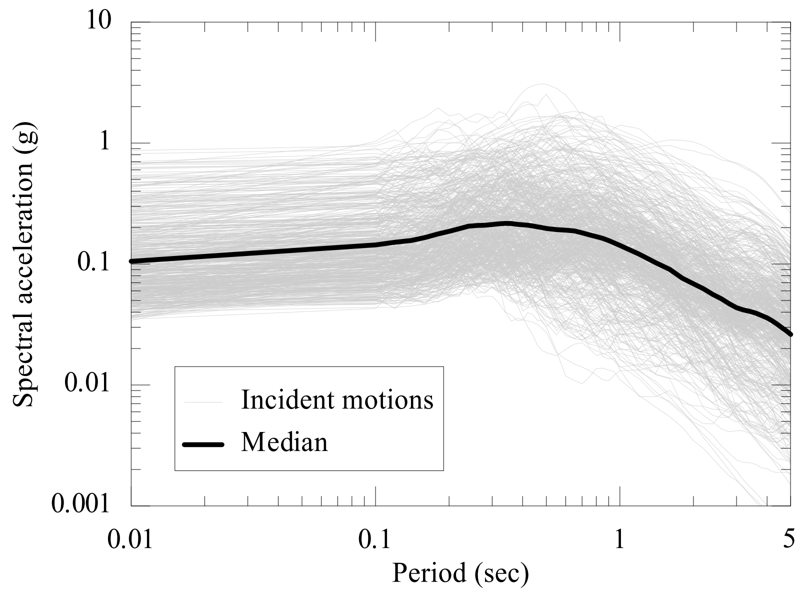

Figure 1. Response spectra for 510 incident synthetic motions used by Assimaki and Li (2012).

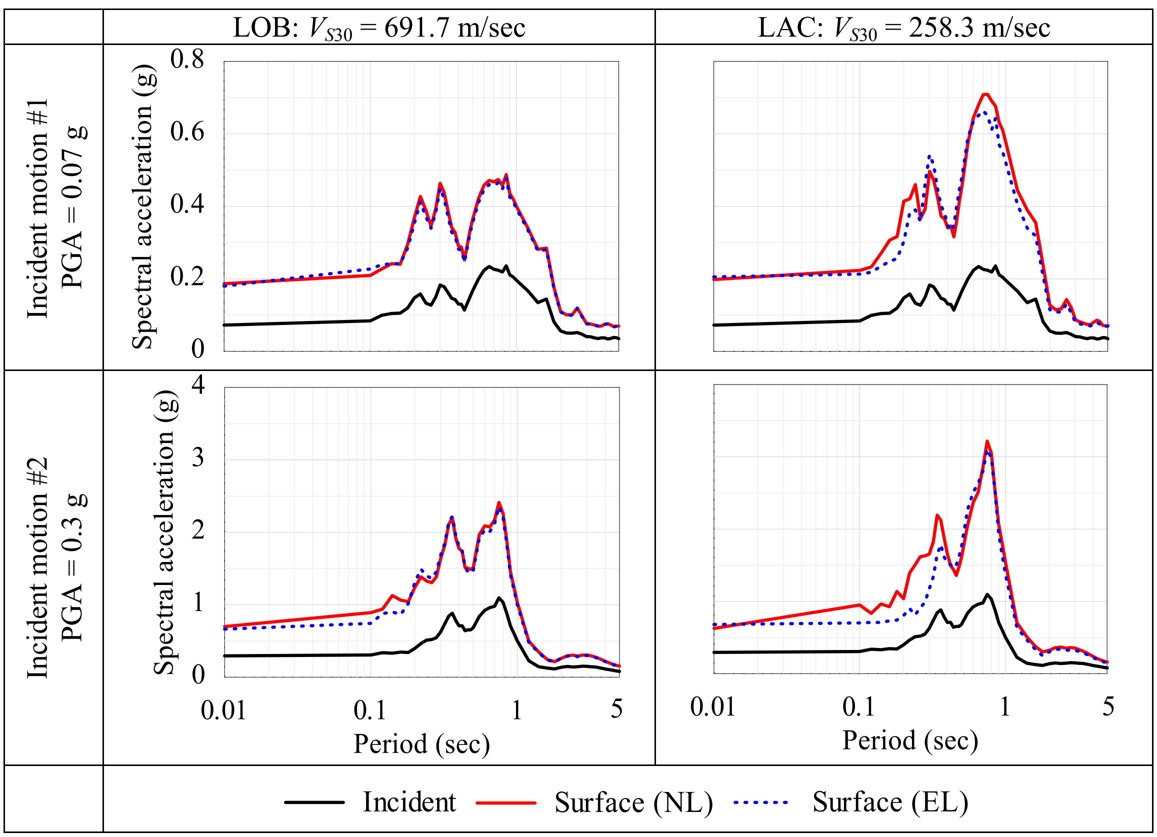

Figure 2. Response spectra for incident ground motions and the response spectra computed on the ground surface using equivalent-linear (EL) and nonlinear (NL) approaches, for two selected stations (LOB: UCSC Lick Observatory in Santa Cruz and LAC: La Cienega in Los Angeles) subject to two ground motions.

PROPOSED MODEL FOR DIFFENCES BETWEEN EQUIVALENT-LINEAR AND NONLINEAR SITE RESPONSES

We computed Sa^{EL}/Sa^{NL} (also equivalent to the ratio of amplification factors) for all of the 12,240 site response simulation results (510 incident motions × 24 sites) to differentiate between the computed EL and NL responses. This comparison focused only on relative differences between the EL and NL methods and does not consider performance relative to observed ground motions.

We regressed Sa^{EL}/Sa^{NL} against numerous ground motion and site parameters to test their predictive power. Parameters that were considered include: (1) ground motion intensity measures such as PGA and Peak Ground Velocity (PGV) of the incident ground motion (PGA_{in} and PGV_{in}, respectively); (2) site parameters such as V_{S30} and amplification at the fundamental mode site frequency; and (3) composite parameters representing intensity measures and site parameters such as estimated shear strain $\gamma_{est} = PGV_{in}/V_{S30}$ (e.g., Idriss 2011) and frequency index (FI). Assimaki and Li (2012) introduced the frequency index (FI) which is defined as the normalized cross-correlation between the amplitude of the linear transfer function of the site and Fourier amplitude spectrum of the incident motion.

Among the considered parameters, we found that estimated strain (γ_{est}) correlates most strongly with relative differences between Sa^{EL} and Sa^{NL}. The Assimaki and Li (2012) data set produces the distribution of γ_{est} shown in Figure 3, which appears to be log-normal with median = 0.06 % and a range of 0.003 to 1.0 %. The relationship between Sa^{EL}/Sa^{NL} and γ_{est} can be expressed as:

$$\frac{Sa^{EL}}{Sa^{NL}} = \frac{1}{1+(c_1 \cdot \gamma_{est})^{c_2}} + \varepsilon \tag{1}$$

where γ_{est} is the estimated strain, c_1 and c_2 are regression coefficients, and ε is the residual.

Figure 4 shows Sa^{EL}/Sa^{NL} from simulations against γ_{est} along with the proposed regression models from Eq. (1) for nine selected periods. The regression coefficients c_1 and c_2 are summarized in Table 1. The proposed models are generally in good agreement with binned mean values of Sa^{EL}/Sa^{NL} (equally spaced on log scale of γ_{est}). The difference between Sa^{EL} and Sa^{NL} for PGA (T = 0.00 sec Sa) is negligible for the entire estimated strain range. For other short periods (T = 0.10, 0.18, 0.32, and 0.55 sec), Sa^{EL}/Sa^{NL}

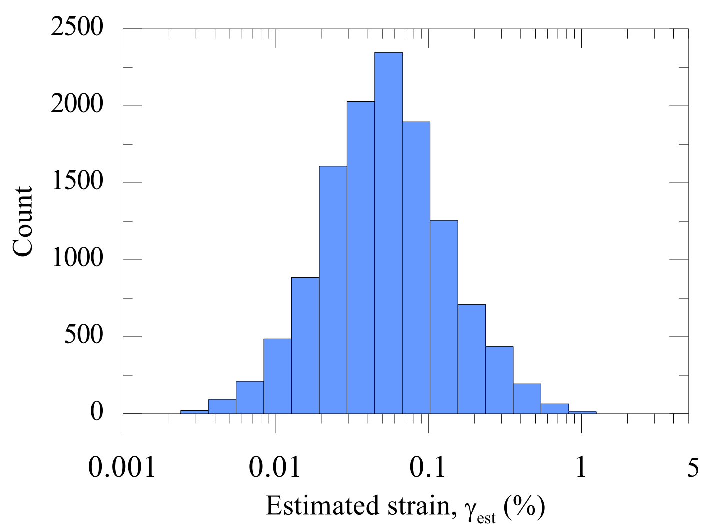

Figure 3. Distribution of the estimated strain, γ_{est}, for 12,240 cases (510 incident motions × 24 sites).

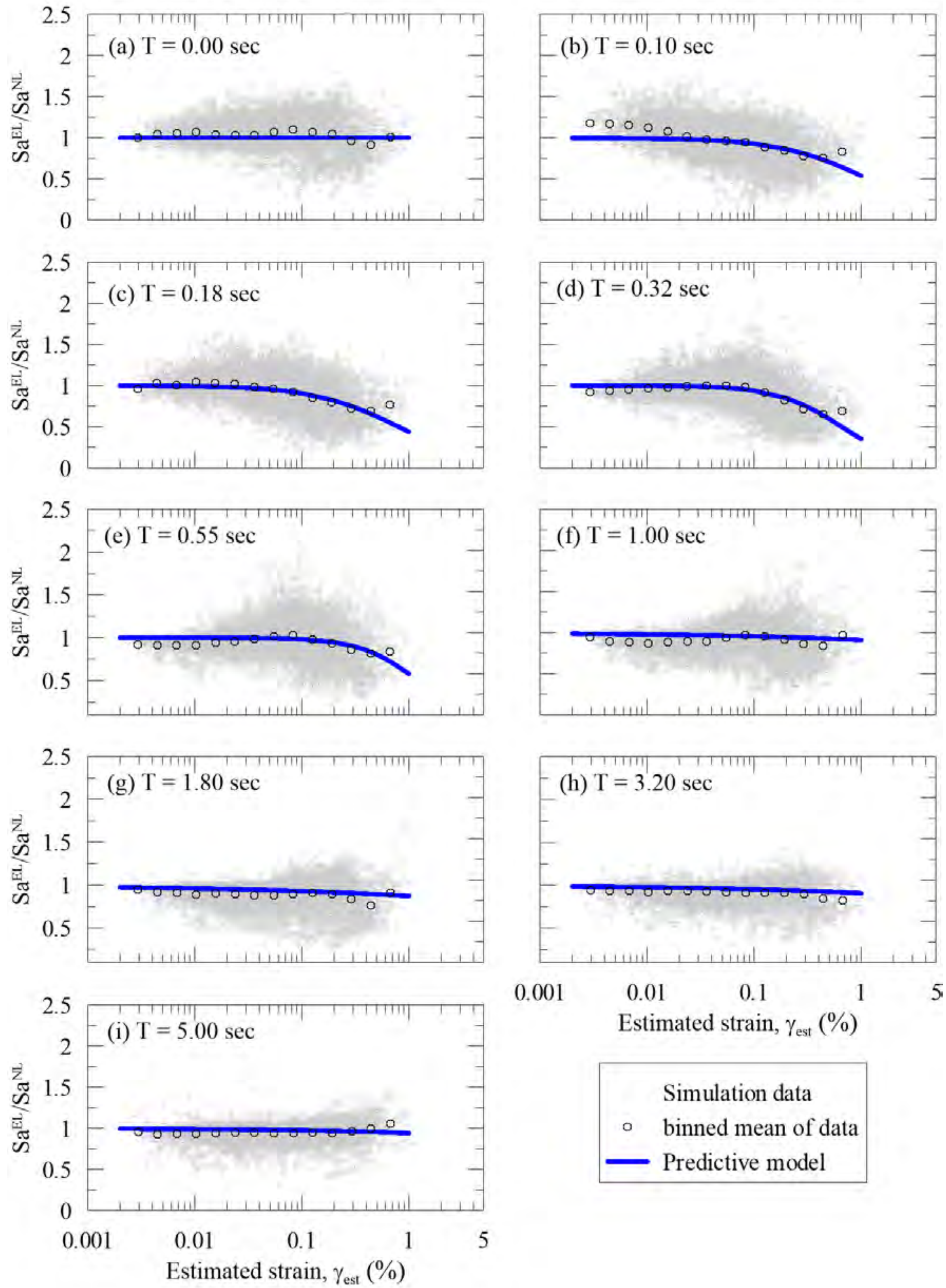

Figure 4. Ratio of Sa^{EL} to Sa^{NL} from site response simulation results and the regression models in terms of estimated strain, γ_{est}, for nine selected periods (T).

Table 1. Regression coefficients for the relationship between Sa^{EL}/Sa^{NL} and estimated strain, γ_{est} (Equation 1) for different periods.

Period (sec)	Regression coefficients	
	c_1	c_2
0.01	0.0010	2.0000
0.10	0.8587	1.0541
0.18	1.2510	1.0920
0.32	1.5237	1.4465
0.55	0.8214	1.6012
1.00	0.0010	0.3325
1.80	0.0010	0.2795
3.20	0.0010	0.3316
5.00	0.0010	0.4064

decreases abruptly at about γ_{est} = 0.1 %. For longer periods (T = 1.00, 1.80, 3.20, and 5.00 sec), the Sa^{EL}/Sa^{NL} decreases slightly with γ_{est}, and does not become less than 0.8 at large γ_{est} ranges (~1 %). Figure 5(a) presents the regression models for all nine selected periods. For relatively short periods (approximately 0.18 to 0.32 sec), Sa^{EL}/Sa^{NL} decreases continuously as γ_{est} increases, and becomes 0.9 (Sa^{EL} is smaller than Sa^{NL} by 10%) at γ_{est} = 0.1 % and 0.7 at γ_{est} = 0.36 %. Standard deviations of residuals ε, denoted σ_{res}, are shown in Figure 5(b) for different periods. The standard deviations of residual remain small ($\sigma_{res} < 0.2$ for T ≤ 0.55 sec and $\sigma_{res} < 0.15$ for T ≥ 1.00 sec) for γ_{est} < 0.1 % in spite of large amount of data. At γ_{est} > 0.1 %, σ_{res} increases to approximately 0.3.

The regression models for other parameters, using the same functional form as employed for γ_{est} (Equation 1) are shown in Figure 6. The significant differences between Sa^{EL} and Sa^{NL} (Sa^{EL}/Sa^{NL} ~ 0.7) occur at large PGA_{in} (at about 0.7 g) and PGV_{in} (at about 1 m/s). The dependency of Sa^{EL}/Sa^{NL} on PGA_{in} and PGV_{in} is not as clear as that on estimated strain. Sa^{EL}/Sa^{NL} does not show clear dependency on other parameters such as V_{S30}, Amp, and FI.

DISCUSSION

To more clearly identify the conditions for which NL analysis results were found to be distinct from EL, we synthesize in Figure 7 the γ_{est} values at which Sa^{EL}/Sa^{NL} from predictive models are 0.7, 0.8, and 0.9, as a function of spectral period. EL and NL results are similar for all periods when γ_{est} is smaller than 0.1 % (marked as 'EL sufficient'). An EL method is also sufficient for long periods (T > 1.0 sec) regardless of level of γ_{est}. The transition zone between EL and NL methods is proposed in terms of γ_{est} for short periods (T < 1.0 sec), where it is recommended equal weights be given to both methods. For T ~ 0.2 sec, the transition zone between EL and NL methods is defined at γ_{est} = 0.1 – 0.4 %. When γ_{est} exceeds 0.4 %, the differences between Sa^{EL} and Sa^{NL} become significant. Therefore, a greater weight for site amplifications derived from NL methods should be considered.

This recommendation is comparable with that by Kaklamanos et al. (2013) who proposed a threshold for EL and NL methods based on the comparison between observed recordings and results of site response analyses at KiK-net stations in Japan. They proposed the transition zone for EL and NL methods at maximum shear strain computed from site response analysis = 0.1 – 0.4 % for short periods (T ≤ 0.6 sec) below which an EL method is sufficient and above which an NL method is necessary. They also proposed that an EL method is sufficient for T > 0.6 sec at any level of maximum shear strain. The threshold recommended by this study is also consistent with the results of site response analyses conducted by Kim and Hashash (2013) for KiK-net stations subject to the 2011, M_W9.0, Tohoku-oki earthquake and other smaller earthquakes in Japan. They reported that the differences between Sa^{EL} and Sa^{NL} are significant for stations with the maximum shear strains computed from site response analysis greater than 0.3 %.

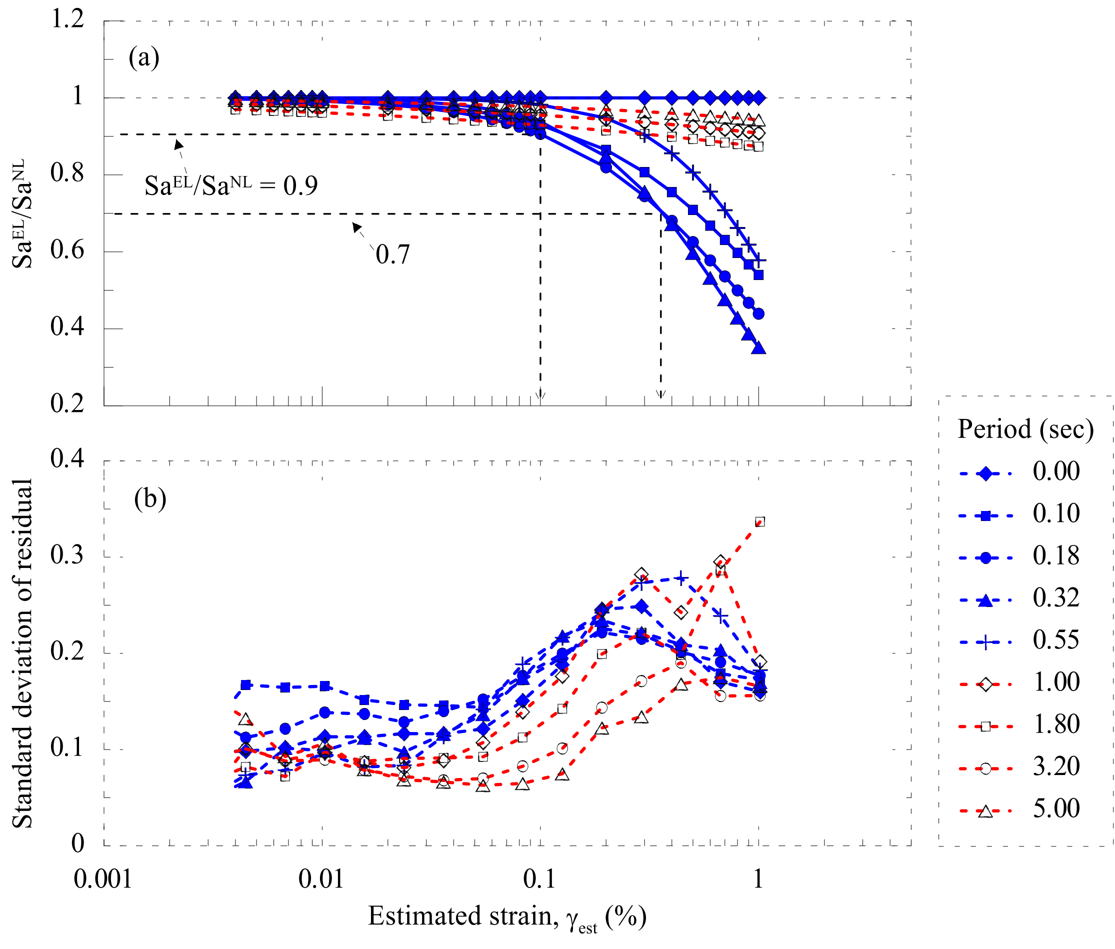

Figure 5. (a) Predictive models for the ratio of spectral accelerations computed by EL (Sa^{EL}) to those by NL (Sa^{NL}) and (b) standard deviations of residual of predictive models for Sa^{EL}/ Sa^{NL} with respect to the estimated strain, γ_{est}, at different periods.

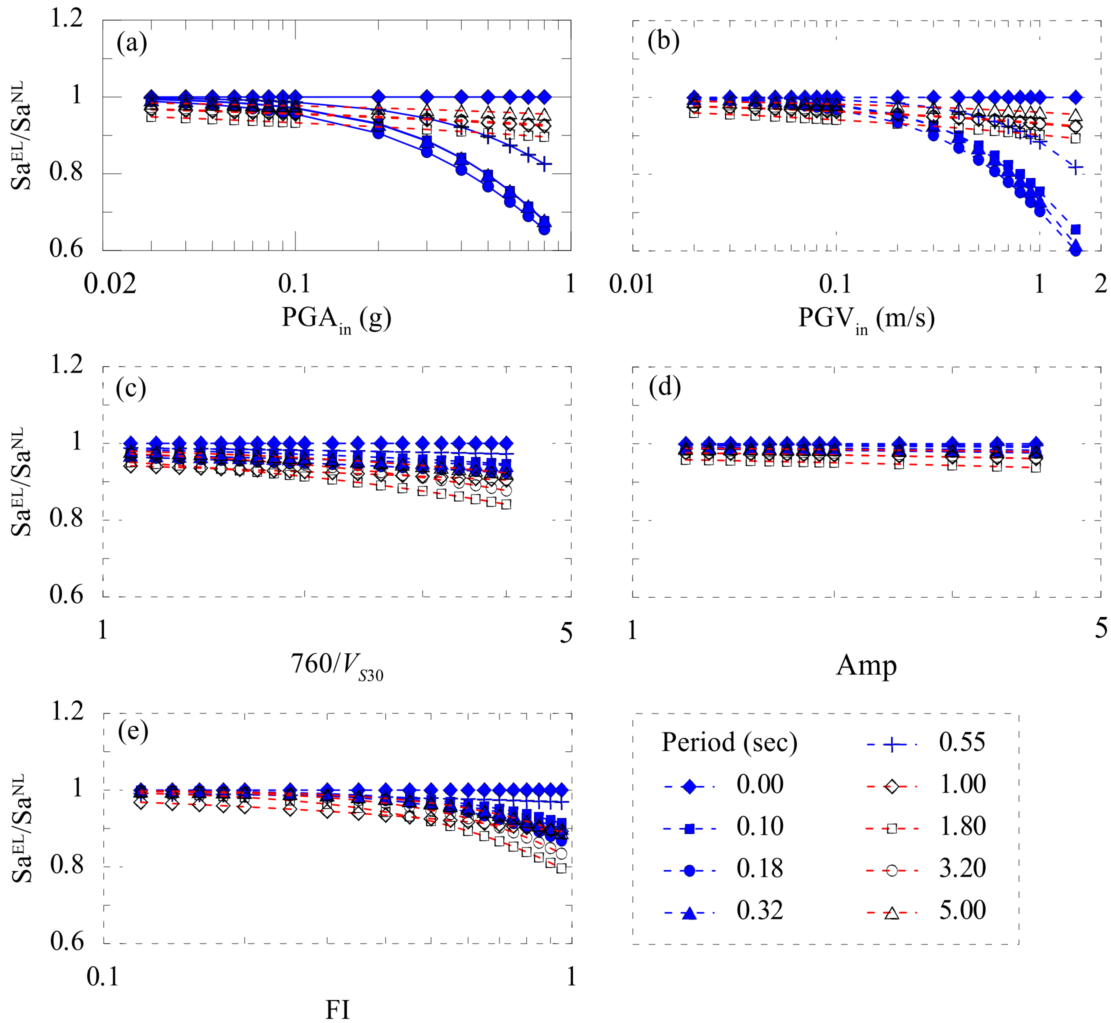

Figure 6. Predictive models of Sa^{EL}/ Sa^{NL} for : (a) PGA$_{in}$; (b) PGV$_{in}$; (c)V_{S30}; (d) Amp; and (e) FI.

CONCLUSIONS

This paper extended the study by Assimaki and Li (2012) to investigate the relative differences between equivalent-linear and nonlinear site response analysis approaches. Among various ground motion and site parameters, we found that the γ_{est}, which can be estimated by PGV$_{in}$/V_{S30}, is the best indicator for divergent ground motion estimates from the two methods of analysis, as measured by Sa^{EL}/Sa^{NL}.

There is no significant difference between Sa^{EL} and Sa^{NL} at T = 0.00 sec. For longer periods (T = 1.00 sec to 5.00 sec), Sa^{EL}/Sa^{NL} slightly decreases as γ_{est} becomes larger. For shorter periods (T = 0.10 sec to 0.55 sec), Sa^{EL}/Sa^{NL} abruptly decreases at large strains, and becomes 0.9 at $\gamma_{est} \sim 0.1$ % and 0.7 at $\gamma_{est} \sim 0.36$ %. Therefore, we propose $\gamma_{est} = 0.1$ % as a threshold below which soil nonlinearity is equally captured by both EL and NL methods. At $\gamma_{est} \geq 0.1$ %, the Sa^{EL} is smaller than the Sa^{NL} by 10 – 30 % at periods between 0.10 and 1.00 sec. Thus, the EL methods cannot be solely relied on at $\gamma_{est} \geq 0.1$ %, and it is recommended that NL method be integrated into site response analysis. At $\gamma_{est} \geq 0.4$ %, we recommend that a greater weight be given to the NL method than for the EL method to better capture the effects of soil nonlinearity.

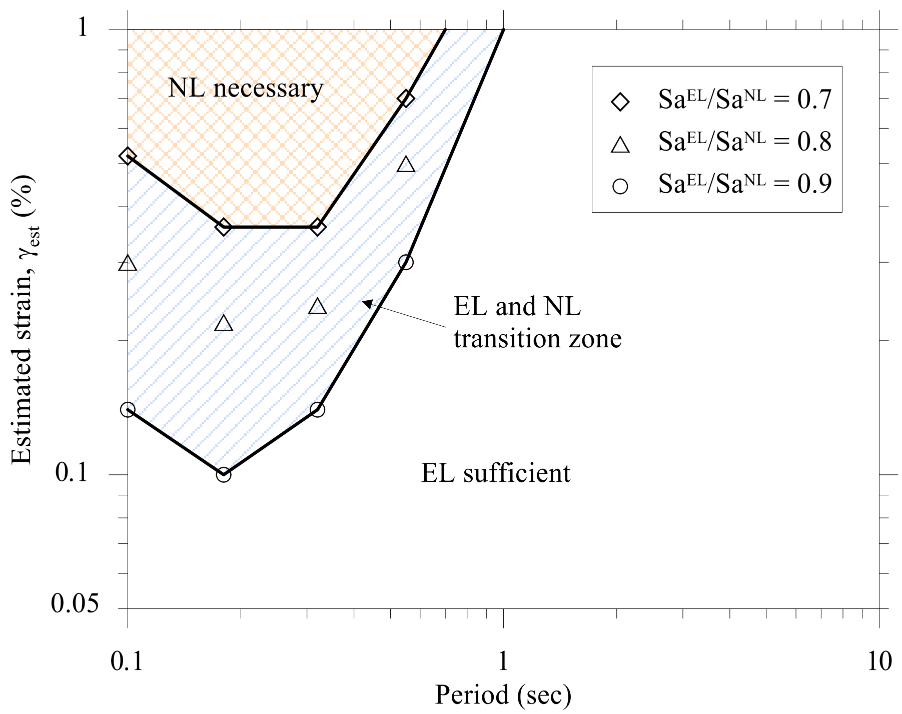

Figure 7. Guideline for a threshold between equivalent-linear (EL) and nonlinear (NL) site response analysis in terms of estimated strain, γ_{est}, and period. The γ_{est} values at which Sa^{EL}/Sa^{NL} of predictive models = 0.7, 0.8, and 0.9 are presented. (Framework of the figure after Kaklamanos (2013))

ACKNOWLEDGEMENT

This study was sponsored by the Pacific Earthquake Engineering Research Center (PEER) as part of NGA-East, a project funded by the U.S. Nuclear Regulatory Commission (NRC), the U.S. Department of Energy (DOE) and the Electric Power Research Institute (EPRI), with the participation of the U.S. Geological Survey (USGS). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect those of the organizations listed above.

REFERENCES

Anderson, D. G. (2003). Laboratory testing of nonlinear soil properties: I & II, Technical report, CH2M HILL, Lifelines Research Program, Pacific Earthquake Engineering Research Center, University of California at Berkeley,**:** 34 pages.

Assimaki, D. and W. Li (2012). "Site- and ground motion-dependent nonlinear effects in seismological model predictions." *Soil Dynamics and Earthquake Engineering* 32(1): 143-151.

Assimaki, D., W. Li, J. Steidl and J. Schmedes (2008). "Quantifying nonlinearity susceptibility via site-response modeling uncertainty at three sites in the Los Angeles basin." *Bulletin of the Seismological Society of America* 98(5): 2364-2390.

Baturay, M. B. and J. P. Stewart (2003). "Uncertainty and bias in ground-motion estimates from ground response analyses." *Bulletin of the Seismological Society of America* 93(5): 2025-2042.

Boore, D. M. and G. M. Atkinson (2008). "Ground-motion prediction equations for the average horizontal component of PGA, PGV, and 5%-damped PSA at spectral periods between 0.01 s and 10.0 s." *Earthquake Spectra* 24(1): 99-138.

Building Seismic Safety Council (BSSC) (2004). NEHRP recommended provisions and commentary for seismic regulations for new buildings and other structures (FEMA 450), 2003 edition. Washington, D.C., National Institute of Building Sciences.

Idriss, I. M. (2011). Use of Vs30 to represent local site conditions. 4th LASPEI/IAEE International Symposium Effects of Surface Geology on Strong Ground Motions. Santa Barbara, California.

Kaklamanos, J., B. A. Bradley, E. M. Thompson and L. G. Baise (2013). "Critical parameters affecting bias and variability in site response analyses using KiK-net downhole array data." *Bulletin of the Seismological Society of America* 103(3).

Kim, B. and Y. M. A. Hashash (2013). "Site response analysis using downhole array recordings during the March 2011 Tohoku-oki earthquake and the effect of long-duration ground motions." *Earthquake Spectra* 29(S1): page S1-S18.

Liu, P., R. J. Archuleta and S. H. Hartzell (2006). "Prediction of broadband ground-motion time histories: Hybrid low/high-frequency method with correlated random source parameters." *Bulletin of the Seismological Society of America* 96(6): 2118-2130.

Matasovic, N. and Y. Hashash (2012). NCHRP428: Practices and procedures for site-specific evaluations of earthquake ground motions (A synthesis of highway practice). Washington, D.C., National Cooperative Highway Research Program, Transportation Research Board**:** 78 pages.

Matasovic, N. and M. Vucetic (1993). "Cyclic characterization of liquefiable sands." *Journal of Geotechnical Engineering - ASCE* 119(11): 1805-1822.