Calculation of crack width and rigidity of partially prestressed and reinforced concrete structural members

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1 INTRODUCTION

In order to develop a simple method for calculating the crack width and the rigidity of partially prestressed and reinforced members, the test was carried out to measure the distribution of steel strain within the cracked region by milling out slots in the tensile bar and sticking the electrical gauges to the slots in series of chain type. Based on the test results, the trigonometric function is assumed to express the tensile strain distributions of steel bar and concrete between the adjacent cracked sections. According to the assumption, a series of simple formulas have been derived to predict the crack width and rigidity of partially prestressed and reinforced concrete members and verified in good agreement with test data obtained at the structural laboratory of Dalian Institute of Technology and elsewhere.

2 CALCULATION OF CRACK WIDTH

The calculation of crack width is a complex problem in the field of the theory of the reinforced and partially prestressed concrete members. There are much factors causing cracks in concrete. The crack caused by load is the main factor considered in design. The other factors are usually eliminated or reduced by selecting suitable material and improving the quality of construction. Because of the complexity of this problem, a number of formulas are used for calculating crack width. In the research of calculation method on crack width, the empirical formula based on statistical anslysis was proposed by authors and adopted by the Design Code of Port Construction in China (Guofan Zhao 1980, Design Code of Prot Construction JTJ220-82,1982) as follows:

(1) $$W_{max} = C_1 C_2 C_3 \frac{\sigma_s}{E_s} \left(\frac{30 + d}{0.28 + 10\mu} \right)$$

Meanwhile, the semi-theoretical formula has also been investigated and introduced in this paper. The calculation formulas for crack width proposed previously by some authors were derived as following form :

(2) $$W_{cr} = (\bar{\varepsilon}_s - \bar{\varepsilon}_c) l_{cr} = (\psi \sigma_s / E_s - \bar{\varepsilon}_c) l_{cr}$$

where $\psi = \bar{\varepsilon}_s / \varepsilon_s$

W_{cr} --- average crack width
l_{cr} --- average crack spacing
$\bar{\varepsilon}_s$ --- average tensile strain of tensile steel between two adjacent cracked sections,
$\bar{\varepsilon}_c$ --- average tensile strain of concrete at the level of the tensile bar between two adjacent cracked sections,
σ_s --- stress in the tensile steel in the cracked sections,
E_s --- modulus of elasticity of steel.
If the tensile strain of concrete is neglected, the formula (2) can be simplified to the following formula :

$$W_{cr} = \psi \varepsilon_s l_{cr} = \psi \frac{\sigma_s}{E_s} l_{cr} \tag{3}$$

According to formulas (2) and (3) the studies have been emphasising on the work of predicting the parameter ψ for a long time. It should be noticed that the rather strict relationship is expressed as follows :

$$W_{cr} = \int_0^{l_{cr}} \varepsilon_{sx}\, dx - \int_0^{l_{cr}} \varepsilon_{cx}\, dx = \int_0^{l_{cr}} (\varepsilon_{sx} - \varepsilon_{cx})\, dx \tag{4}$$

where ε_{sx} , ε_{cx}---tensile strain of steel and concrete at distance x away from crack section respectively

The key problem to calculate the crack width from formula (4) is to find out a function to express ε_{sx} in good agreement with the experimental results on the strain of tensile steel, and this function should be easy to integrate. The function of ε_{cx} is not important since it is smaller in comparison with ε_{sx} and can be neglected. On the study of crack width and deformation by some authors thirty years ago, the mechanical instrument was only used for measuring the average steel strains, it was impossible to get ε_{sx} along the tensile steel. Using the ratio of the average strain along the steel between the adjacent cracked section $\bar{\varepsilon}_s$ to the steel strain in the cracked section ε_s is reasonable in past time. As the electrical measuring technology develops, the electrical gauges arranged in series of chain type can be used to measure ε_{sx} along the tensile steel. Hence, it gives a possibility to derive the calculation formulas of crack width by using formula (4) instead of formulas (2) and (3). The test of four beams reinforced with plain bars (LA, d=20mm) and deformed bars (LB, d=20mm) was carried out by authors to measure the distribution of steel strain within the cracked region by milling out the slots in the tensile steel bar and sticking the electrical gauges to the slots in series of chain type as shown in Fig 1.

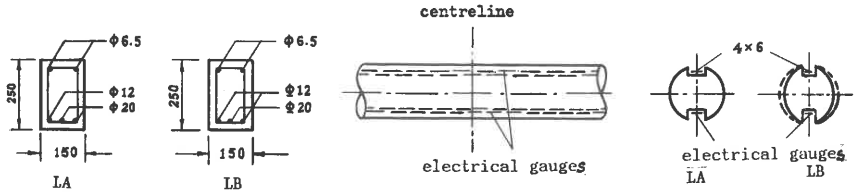

Fig 1. Details of specimens

The typical strain distribution diagram of steel in the specimen LA-1 is shown in Fig 2.

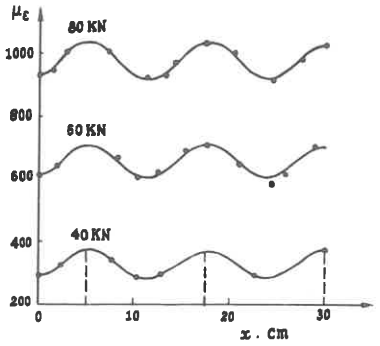

Fig 2. The tensile strain diagram of steel of specimen LA-1

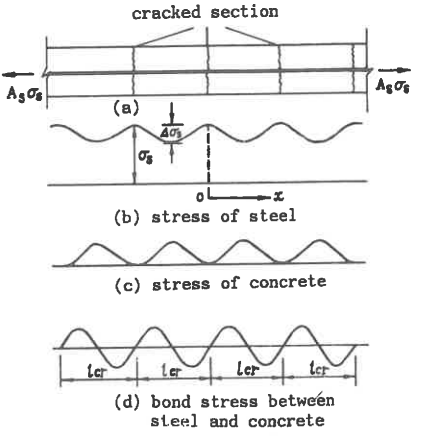

Fig 3.Distribution diagrams of steel stress, concrete stress, and bond stress between two adjacent cracked section

From the analysis of the curve of the steel strain shown in Fig 2. and reference (Guofan Zhao 1984) it can be seen that the strain diagram of steel is a gradually varied curve and the reasonable stress diagram can be simply expressed by the trigonometric function as shown in Fig 3. The distribution of steel stress σ_{sx} can be derived as follows :

$$\sigma_{sx} = \sigma_s - \Delta\sigma_{sx} = \sigma_s - B\left(1 - \cos\frac{2\pi x}{l_{cx}}\right) \tag{5}$$

where $\sigma_{sx} = \varepsilon_{sx} \cdot E_s$, $\sigma_s = \varepsilon_s \cdot E_s$

ε_s --- strain of the tensile steel in the cracked section

$\Delta\sigma_{sx}$ --- the decrease of the steel stress at a distance x away from the cracked section caused by influence of the uncracked concrete

B --- parameter determined by formula (7)

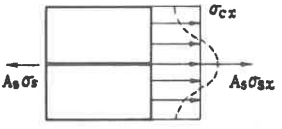

Fig 4.Distribution of concrete stress in the cross section

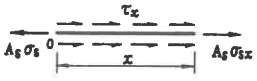

Fig 5. Force equilibrium diagram of steel bar

When the distribution of σ_{sx} is known, the stress distribution of concrete can be obtained as follows (Fig 4):

$$A_s\sigma_s = A_s\sigma_{sx} + A_c\sigma_{cx}$$

$$\sigma_{cx} = \frac{A_s}{A_c}(\sigma_s - \sigma_{sx}) = \mu_e B\left(1 - \cos\frac{2\pi x}{l_{cr}}\right) \tag{6}$$

where $\mu_e = A_s / A_c$

A_s --- area of tensile steel bar

A_c --- area of cross of an axial tension member

At $x=l_{cr}/2$, the peak value of σ_{cx} is equal to $2\mu_e B \leqslant f_t$, $\therefore B \leqslant 0.5 f_t/\mu_e$

(7) or $B = \alpha f_t / \mu_e \quad (\alpha \leqslant 0.5)$

where f_t --- tensile strength of concrete
The bond stress τ_x can be obtained as follows (Fig 5.):

$$\sigma_s A_s = A_s \sigma_{sx} + \int_0^x \tau_x u \, dx$$

(8) $$\tau_x = \frac{2\pi A_s B}{l_{cr} u} \operatorname{Sin} \frac{2\pi x}{l_{cr}} = \tau_{max} \operatorname{Sin} \frac{2\pi x}{l_{cr}}$$

(9) $$\tau_{max} = \frac{2\pi A_s B}{l_{cr} u} = \frac{2\pi d B}{4 l_{cr}} = \frac{\pi d \alpha f_t}{2 l_{cr} \mu_e} \qquad (\alpha \leqslant 0.5)$$

where u --- the perimeter of the steel bar
The theoretical formula of the crack spacing l_{cr} can be obtained from formulas (9) and modified by test data as follows:

(10) $$l_{cr} = (2c + 0.08\, d/\mu_e)\, k_2$$

Where c --- thickness of concrete cover in cm
d --- diameter of the steel bar in cm
k_2 --- coefficient denoting the bond effect of the bars, k_2 =1.0 for deformed bars; k_2=1.3 for plain bars
The formulas (5) , (6) and (8) are respectively the expressions of the steel stress, concrete stress and bond stress of the axial tension member with the crack spacing l_{cr}, and have been verified in good agreement with the experimental results given in Fig 2. and Ref. (Guofan Zhao et al. 1984).

2.1 Practical formula for evaluating the crack width of R. C. member

When the tensile stress diagrams of steel and concrete are known as shown in Fig 3. the formula for evaluating the crack width of a reinforced concrete member can be directly obtained from equation (4) as follows :

(11) $$W_{cr} = \int_0^{l_{cr}} (\varepsilon_{sx} - \varepsilon_{cx})\, dx = \int_0^{l_{cr}} \left(\frac{\sigma_{sx}}{E_s} - \frac{\sigma_{cx}}{E_c} \right) dx$$

$$= \frac{\sigma_s}{E_s} \left[1 - \frac{B(1+\mu_e n)}{\sigma_s} \right] l_{cr} = \frac{\sigma_s}{E_s} \left[1 - \frac{\alpha f_t (1+\mu_e n)}{\mu_e \sigma_s} \right] l_{cr}$$

where $\mu_e = A_s / A_{ce}$
E_c --- modulus of elasticity of concrete
A_{ce} --- effective area of tensile concrete
$A_{ce} = 0.4bh + (b_f - b)h_f$ for bending, eccentric compression and tension members
$A_{ce} = bh$ for pure tension members
b --- web width of section
h --- overall depth of section
b_f --- width of tensile flange of inverted T or I section
h_f --- thickness of tensile flange of inverted T or I section

If the strain of tensile concrete may be neglected, W_{cr} can be obtained as follows :

(12) $$W_{cr} = \frac{\sigma_s}{E_s}\left(1 - \frac{\alpha f_t}{\mu_e \sigma_s}\right) l_{cr}$$

For the purpose to compare formulas (11) or (12) with some references, the value of ψ in formulas (2) or (3) can be directly obtained from formula (12)

(13) $$\psi = 1 - \frac{B}{\sigma_s} = 1 - \frac{\alpha f_t}{\mu_e \sigma_s}$$

The coefficient α can be approximately chosen as 0.5 with the experimental results. The value of μ_e is determined by the ratio of the area of tensile steel to the effective area of tensile concrete. Finally, the effect of variation of crack widths and the effect of long-term loads on the average crack width should be taken into account , thus, the unified formula for evaluating the maximum crack width of reinforced concrete members subjected to pure tension, bending, eccentric tension or compression may be expressed as follows :

(14) $$W_{max} = 1.5 K_1 K_2 K_3 \frac{\sigma_s}{E_s}\left(1 - \frac{0.5 f_t}{\mu_e \sigma_s}\right)\left(2C + 0.08 \frac{d}{\mu_e}\right)$$

$$= K_1 K_2 K_3 \frac{\sigma_s}{E_s}\left(1 - \frac{0.5 f_t}{\mu_e \sigma_s}\right)\left(3C + 0.12 \frac{d}{\mu_e}\right)$$

where K_1 ---- coefficient denoting the load effect
- K_1 = 1.0 for flexural members
- K_1 = 0.9 for eccentric compression members
- K_1 = 1.1 for eccentric tension members
- K_1 = 1.2 for axial tension members

K_3 ---- coefficient denoting the effect of the duration of the loading
- K_3 = 1.0 for the short-term load
- K_3 = 1.5 for the long-term load

2.2 Practical method for evaluating the crack width of partially prestressed concrete member

For evaluating the crack width of a partially prestressed concrete members , it is necessary to give the change in net stress $\Delta\sigma_p$ in the prestressing steel and the magnitude of the tensile stress σ_s in the non-prestressed steel at any crack width load level in which the decompression load is taken as the reference point.

It is easy to calculate $\Delta\sigma_p$ or σ_s for axial tension members, but for flexural members, the prestressing compressive force acting on the section and the bending moment result in an equivalent eccentric load. The equation for solving the depth X of the neutral axis of a cracked section or $\xi = X/h_p$ is a cubic equation of X or ξ. This cubic equation of ξ (for rectangular, inverted T, I and T sections) may be expressed as follows (Fig 6.):

(15) $$\xi^3 + A\xi^2 + B\xi + C = 0$$

where $\xi = X/h_p$, $e_N = M/N_{po}$, $A = 3(e_N - h_p)/h_p$

$B = 6[(b'_f - b)h'_f(e_N - h_p + h'_f/2) + n_p A_p e_N + n_s A_s (e_N - h_p + h_s)]/(bh_p^2)$

$C = -6[(b'_f - b)h'^2_f(e_N - h_p + 2h'_f/3)/2 + n_p A_p h_p e_N + n_s A_s h_s (e_N - h_p + h_s)]/(bh_p^3)$

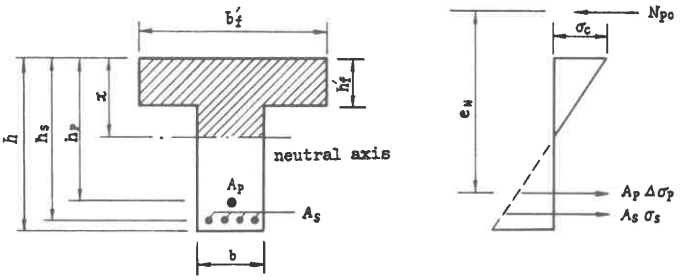

Fig 6. Stress diagram in cracked section under the service load

If $\xi = X/h_p$ is solved from equation (15) the stress increment $\Delta\sigma_p$ in the prestressing steel can be calculated from the following formula :

$$\Delta\sigma_p = \frac{2N_{po}(h_p - X)n_p}{bX^2 + (2X - h_f')(b_f' - b)h_f' - 2n_s A_s(h_s - X) - 2n_p A_p(h_p - X)} \tag{16}$$

The stress of the non-prestressed steel can be calculated from the following formula :

$$\sigma_s = \Delta\sigma_p \frac{n_s}{n_p}\left(\frac{h_s - X}{h_p - X}\right) \tag{17}$$

When $X < h_f'$, the calculation can be made as a rectangular section with $b = b_f'$.

where M --- bending moment under characteristic load

$N_{po} = A_p\sigma_{po}$, $\sigma_{po} = \sigma_{con} - \sum\sigma_s$ for pre-tension members;

$\sigma_{po} = \sigma_{con} - \sum\sigma_s + n_p\sigma_c$ for post-tension members

σ_{con} --- control stress for prestressed steel

$\sum\sigma_s$ --- the total losses of prestress

σ_c --- the effective prestress of concrete at the level of the prestressed steel

$n_s = E_s/E_c$, $n_p = E_p/E_c$

h_s , h_p --- the distance from the centroid of the prestressed and non-prestressed steel area to the extreme compression fiber, respectively

For practical purposes, there is a need of a simpler formula for evaluating the crack width, so the cubic equation (15) may be simply solved by reducing the power in the separated regions, then the value of ξ can be simply determined by the linear or square equation formula (see Ref.Guofan Zhao 1986a):

In partially prestressed concrete beams, where the prestressing steel is usually combined with non-prestressed bars, the value of μ_e, d and σ_s introduced in Eq.(14) for ordinary reinforced concrete members should be replaced by $\mu_e = (A_p + A_s)/[0.4bh + (b_f - b)h_f]$, $d_e = (n_1 d_1 + n_2 d_2)/(n_1 + n_2)$ and $\Delta\sigma_p$ respectively, where A_p = area of prestressing steel, n_1 and n_2 =number of prestressing tendons or bars and non-prestressed bars or wires, respectively, d_1 and d_2 = diameter of prestressing tendons or bars and non-prestressed bars or wires, respectively, $\Delta\sigma_p$ =change in net stress in the prestressing steel or the magnitude of the tensile stress in the non-prestressed steel at any crack width load level in which the decompression load (decompression here means compressive stress equal to zero at the level of the reinforcing steel) is taken as the reference point.

For pre-tensioned beams with prestressing strands combined with

non-prestressed bars or wires, the bond characteristics of these steel components may be different from each other, so the factor k_2 introduced in Eq.(14) should be replaced by $k_2=(1.3n_1d_1+n_2d_2)/(n_1+n_2)d_e$ for strands combined with deformed bars or $k_2=(1.3n_1d_1+1.3n_2d_2)/(n_1+n_2)d_e$ for strands combined with plain wires .

3 THE RIGIDITY OF REINFORCED CONCRETE MEMBER WITH FLEXURE

For evaluating the deflection of a cracked reinforced concrete beam, the rigidity EI for beam made of elastic materials should be replaced with B_l. Taking the same distribution diagrams of the strain of steel and concrete proposed for evaluating the crack width of R. C. members, a simply derived method based on the model $1/\rho=\bar{\varepsilon}_s/(h_0-\bar{X}_c)$ for evaluating the rigidity of R. C. beam is proposed. The rigidity B_s under short-term loads was derived as follows (see Ref.Guofan Zhao 1986b).

$$B_s = A_s E_s h_0^2 \Big/ (1.6 + 6n\mu - 0.4 f_t b h_0^2/M) \tag{18}$$

The formula for evaluating the rigidity of a flexural member with rectangular section has been verified by the test data obtained in Dalian Institute of Technology and elsewhere . As for T ,inverted T and I section, the procedures for deriving the rigidity formulas are the same as for the rectangular beam. But for the purpose of design practice, the rigidity B_s for these sections may be calculated by multiplying the rigidity B_s for rectangular section with an increment factor β_f(Yongchun Chen et al 1983).

Under the action of long-term loads, the rigidity B_l may be evaluated by the following empirical formula:

$$B_l = \frac{M}{M_l(Q-1)+M} B_s \tag{19}$$

where M_l --- bending moment under long-term loads
$Q = 2.0 - 0.4\mu'/\mu$, $\mu = A_s/(bh_0)$, $\mu' = A_s'/(bh_0)$
A_s' --- area of compressive steel
M --- bending moment under total characteristic loads

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