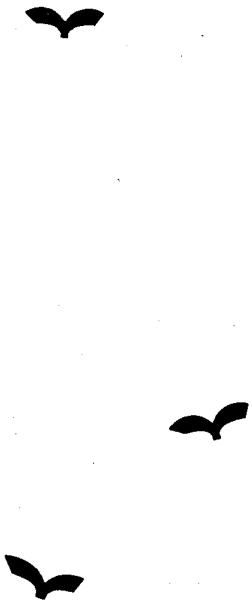
REPORT NO. 75



COMPUTER SIMULATION OF GROUND WATER AQUIFERS OF THE COASTAL PLAIN OF NORTH CAROLINA

BY

JABBAR K. SHERWANI

DEPARTMENT OF ENVIRONMENTAL SCIENCES AND ENGINEERING
UNIVERSITY OF NORTH CAROLINA AT CHAPEL HILL

APRIL 1973

Water Resources Research Institute
OF THE UNIVERSITY OF NORTH CAROLINA

THE UNIVERSITY OF NORTH CAROLINA
WATER RESOURCES RESEARCH INSTITUTE

Office of the Director
124 Riddick Building
North Carolina State University
Raleigh, North Carolina, 27607
Telephone: 919:737-2815

June 1973

TO: WHOM IT MAY CONCERN

JMS

FROM: James M. Stewart
Assistant Director for Research Application

SUBJECT: Institute Report No. 75 - "Computer Simulation of Ground Water Aquifers of the Coastal Plain of North Carolina," by Dr. Jabbar K. Sherwani, Department of Environmental Sciences and Engineering, School of Public Health, University of North Carolina at Chapel Hill

This report summarizes the results of an intensive research effort involving computer simulation of the Castle Hayne groundwater aquifer of North Carolina.

Highlights of the report include:

(1) a description of the hydrogeologic units of the area under study in the Coastal Plain of North Carolina,

(2) water level data and water level changes of the hydrogeologic units within and surrounding the Castle Hayne aquifer,

(3) a description of the recharge patterns and characteristics for the Castle Hayne aquifer,

(4) application of steady state mathematical models,

(5) application and use of analog simulation and digital computer simulation of groundwater in the region,

(6) water quality and changes in the Castle Hayne aquifer system,

(7) suggestions for water management for optimal use of groundwater, and

(8) conclusions and recommendations.

The report is expected to be of special value to state and federal agencies responsible for groundwater management. The digital computer model is fully described including an operational computer program. Card decks are available for the utilization of this valuable tool in studies of probable effects of alternative plans for development and water use in the Coastal Region.

JMS:p
Attachment

THE UNIVERSITY OF NORTH CAROLINA *is comprised of the sixteen public senior institutions in North Carolina*

UNC-WRRI-73-75

COMPUTER SIMULATION OF GROUND WATER AQUIFERS OF THE COASTAL PLAIN OF NORTH CAROLINA

by

Jabbar K. Sherwani
Department of Environmental Sciences and Engineering
University of North Carolina at Chapel Hill

April 1973

Also Published as

E.S.E. Publication No. 332

The work upon which this publication is based was supported in part by funds provided by the Office of Water Resources Research, Department of the Interior, through the Water Resources Research Institute of The University of North Carolina as authorized under the Water Resources Research Act of 1964.

Project No. B-005-NC
Agreement No. 14-01-0001-1931

ACKNOWLEDGMENTS

The author is indebted to several individuals for assistance in carrying out this investigation. Jacques H. Faigenblum constructed the analog model, and the results of analog studies are largely based on his work. Graduate students Roland Brandt, E. W. Arp, and R. W. Wentworth rendered valuable assistance, and their contributions are reflected in the areas of mathematical models, strategies of development and groundwater quality, respectively.

The discussions in the meetings of the Ground Water Technical Advisory Committee provided valuable guidance in the earlier phases of the project. However, Mr. Ralph Heath of the U. S. Geological Survey and Mr. Harry Peek of the Office of Water and Air Resources continued to serve as valuable technical resources.

My debt to the late Professor C. E. Jacob is much greater than is shown by a few references to his work in the body of the report. My several stimulating discussions with him were instrumental in my approaching the problem from many different angles.

Professor David H. Howells provided constant encouragement, and without his help the study would have been almost impossible.

ABSTRACT

The determination of the hydrologic characteristics of a leaky confined aquifer is approached from several viewpoints: (a) analysis of pumping tests, (b) fitting of steady-state mathematical models, (c) analog simulation, and (d) digital computer simulation. The Castle Hayne aquifer in eastern North Carolina is used as a case study. The aquifer is analyzed as a heterogeneous isotropic system. The inputs and outputs of the system are considered; the hydraulic connection between the aquifer and the underlying and overlying aquifers is examined. The verified digital model is used for prediction purposes under hypothetical strategies of development.

The groundwater quality is considered to be of major importance in the optimal management of the aquifer on a regional basis. The vertical movement of the estuarine water and the lateral flow from lenses of poor quality water within the aquifer are examined using simple mathematical relationships. It is found that single-valued estimates based on specific values of controlling parameters can be subject to large errors because of the accuracy of the data. The possible range of variation is investigated by carrying out sensitivity analysis.

The conceptual problems in the measurement of social benefits and costs are outlined, and possible regulation techniques are explored. A possible framework for the regional management of the aquifer taking quality of water and economic consequences into account is presented.

TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

LIST OF FIGURES

LIST OF TABLES

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

1. The Castle Hayne aquifer system is a leaking system. The development of the cone of depression in the overlying Yorktown and Pungo River formations and the underlying Lower Castle Hayne and Beaufort formations proves conclusively the existence of hydraulic connection between all these units. The hydraulic potentials in the overlying and underlying formations show the existence of both the leakage from above and from below into the Upper Castle Hayne aquifer. The downward leakage is the stronger of the two in the western part of the area.
2. An adequate representation of the aquifer is possible by a hydrologic recharge boundary superimposed on a moderate leakage effect. The model for the 1970 piezometric surface giving the best fit is that of semi-infinite leaky aquifer. A better characterization of the earlier phases of heavier pumping is provided by an infinite leaky quadrant with two recharge boundaries.
3. The Upper Castle Hayne aquifer is in hydraulic contact with the Pamlico estuary. Before the start of dewatering operations at Lee Creek mine, at least 20 million gallons per day were discharged from the aquifers to the upper reaches of the Pamlico River. Under the present conditions, the hydraulic gradients have reversed in large areas, including the major portion of the estuary. The base flow entering the estuary has reduced by approximately 30 mgd; there is still some fresh water entering the estuary in the uppermost reaches.
4. The leakage in the updip portions of the aquifer is an order of magnitude higher in the vicinity of the estuary than in the adjoining aquifer. There is, however, no evidence that the aquifer is more leaky near the Lee Creek mine.
5. The values of the vertical hydraulic conductivity of the confining beds contained in the 1971 Technical Report are greatly underestimated, particularly in an area in the vicinity of the upper reaches of the Pamlico estuary.
6. There is very little hard data available. Where observed data do exist, the control is only fair to poor. The numerical values of the parameters, except for the discharge from the Lee Creek mine, are largely interpretive

derived from hydrologic variables that are either unknown or only estimated. The precision of the data is largely illusory, and the estimates are subject to possible large errors.

7. Little confidence can be placed in the predictions of future water quality based on a single specification of the parameters. Sensitivity analyses to discover the possible ranges of concentrations and travel times are in order.
8. Groundwater is only a partially renewable resource. Significant water quality changes caused by long-lasting, concentrated and large withdrawals would only be partially offset in a very long period of recovery after the cessation of the hydrologic stresses.
9. The goal of efficient, comprehensive development of the Castle Hayne aquifer system cannot be realized in the absence of projections of future economic development and water needs, both with respect to quantity and quality. An evaluation of the social costs imposed on other users, present and potential, by mining operations and a plan for the equitable distribution of these costs cannot be developed without the availability of the demand and its spatial and temporal distribution.
10. Quantitatively speaking, the water supply in the Castle Hayne aquifer is more than adequate for all existing uses.
11. Although the groundwater levels may stabilize, the water quality can continue to deteriorate. In groundwater development, benefits and costs occur at markedly different times and are widely separated in space. For a proper evaluation of benefits and costs, the adoption of an extended time scale for analysis is essential.

Recommendations

1. As the data to delineate the occurrence of high chloride zones in the aquitards are very meager, a monitoring program to detect these areas should be devised.
2. Because no observed data on the vertical permeabilities exist, it is recommended that additional testing of the aquitards be undertaken to determine the distribution of vertical hydraulic conductivities in the critical areas; for example, in the vicinity of the upper reaches of the Pamlico estuary.
3. A more complete delineation of fresh water--saline water contact in the major water-bearing formations should be undertaken and the actual monitoring of the migration of the contact zones be launched.
4. As there is an increasing need to maintain close control on the deterioration of water quality in response to the existing and proposed large withdrawals, it is suggested that more sophisticated methods of predicting the transport of chlorides be used.
5. As the more complete utilization of groundwater resource is approached, a public policy for the optimal management of this resource should be established. The objectives of such a policy should be formulated in operational terms. It is recommended that guidelines stipulating constraints on water levels and water quality be prepared for different sections of the aquifer.
6. Because projection of water needs and their quality requirements are essential for any strategy for the regional management of the groundwater resource, it is recommended that alternative projections for the agricultural, municipal, and industrial uses of water be developed for the area.
7. As preventive measures to retard the rate of water quality deterioration form a part of any coordinated plan for water management and because measures may be required to keep open the maximum number of options for future development, a detailed examination of large water uses should be conducted to see if a part of the pumped water can be made available for recharge.
8. The feasibility of a policy for instituting charges for the licensing of large withdrawals and provision of economic incentives for reducing withdrawals or for returning surplus water back to the aquifer system should be given serious consideration.

Chapter 1

GENERAL SETTING

The study area lies within the Atlantic Coastal Plain. The terrain is generally flat to gently rolling. The altitude of the land surface ranges from mean sea level to 100 feet above it. The terrain generally slopes to the southeast.

The drainage pattern in the area is essentially dendritic. Most of the tributaries originate in the large swampy areas, generally known as pocosins. These inter-stream areas, although topographically high relative to the adjacent drainages, show poor circulation of water. The water table is near the surface, and during periods of heavy rainfall, these areas are largely flooded.

As the water table reservoir is full, or nearly full, the rainfall is mostly rejected as overland runoff to the slow-running, shallow and swampy streams. The streamflows are substantial. The streams are affected by tides during periods of low flow or strong winds. The quality of surface water is poor and most of the streams are classified as tidal salt water or as swampy waters.

The effect of groundwater withdrawals from the aquifer depends generally on the geologic and hydrologic environment in the area of withdrawal. The factors to be considered in the analysis of the hydrogeologic system are:

1. the position and fluctuation of the piezometric surface,
2. regional geology, and
3. natural inflows and outflows.

There are interrelationships between these factors. The only constant factor in the whole fabric is the rock skeleton.

The internal state of an aquifer, determined by its stratigraphy, is expressed by the spatial distribution of the coefficients of transmissivity and storage. The geologic properties of confining layers are summarized by the spatial distribution of the coefficients of vertical hydraulic conductivity and specific storage. The driving force is provided by gravity and the head differences in various beds. The location and the geometry of external hydrogeologic boundaries and the specification of force field around them are essential for a complete solution of the flow problem. The man-made changes in the aquifer system such as those caused by pumping, drainage, and artificial recharge provide new hydrologic boundaries and alter the aquifer flow system.

1.1 Aquifer as a System

The aquifer as a system is represented in Figure 0. The solution of a groundwater problem involves the determination of relationship between three classes of physical entities:

(1) excitation or stimulus,
(2) system, and
(3) response.

The excitation comprises (a) initial conditions, (b) point, line, and area sources and sinks, and (c) applied forcing functions. The input variables generally vary both in time and space and can be controlled or uncontrolled deterministic, or stochastic in nature.

The groundwater problems can be categorized into three types. In the system identification problem, the excitation and response are given and it is required to determine the system parameters and characteristics. The solution for this problem is not unique. One has to select a probable solution from a multitude of possible solutions. For a large part, our problem is of this type. In the input-discovery problem, system response and system characteristics are known and it is desired to determine the excitation which was responsible. The solution to this problem is not unique either. In the prediction of response problem, the system characteristics and the excitation are assumed to be known and it is desired to forecast the system behavior. The management of groundwater aquifers belongs to this category. The prediction of the response of the aquifer system procedures is required to manage the system in an optimal manner.

EXCITATION

(A) SOURCES AND SINKS

Stochastic:

- Recharge from precipitation +
- Rejected recharge -
- Evapotranspiration -

Uncontrolled, Deterministic:

- Flow into the aquifer through boundaries ±

Controlled, Deterministic:

- Pumping -
- Artificial recharge +

(B) INITIAL CONDITIONS

Initial piezomatric head

Head distribution in source bed

Potential or flux on or across boundaries

SYSTEM

AQUIFER GEOMETRY

(a) Flow Area

- Aquifer Width
- Saturated Thickness

(b) Aquifer Boundaries (Shape and Location)

- Lateral boundaries
- Upper boundary
- Lower boundary

HYDRAULIC CHARACTERIZATION

Hydraulic Conductivity

Storativity

Aquitard Characteristics

Potential and Flux Functions for Boundaries and Interfaces

RESPONSE

Potential Distribution

- Transient
- Steady state

Changes in

- Recharge conditions
- Discharge conditions

FIGURE 0: AQUIFER AS A SYSTEM

Chapter 2

GEOLOGIC FRAMEWORK

No attempt is made in the following to give a comprehensive picture of the geology of the area. An excellent summary and an extended bibliography is contained in reference (1). Only the salient features are given below derived largely from (1) to (4).

The Coastal Plain of North Carolina is underlain by a wedge-shaped sequence of stratified sedimentary rocks deposited on the crystalline bedrock surface during numerous transgressions and regressions of the sea. The sedimentary rocks have been subdivided into stratigraphic units ranging in age from Recent to Early Cretacious. As shown in Figure 1, reproduced from the 1971 Technical Report (1), the geologic units have:

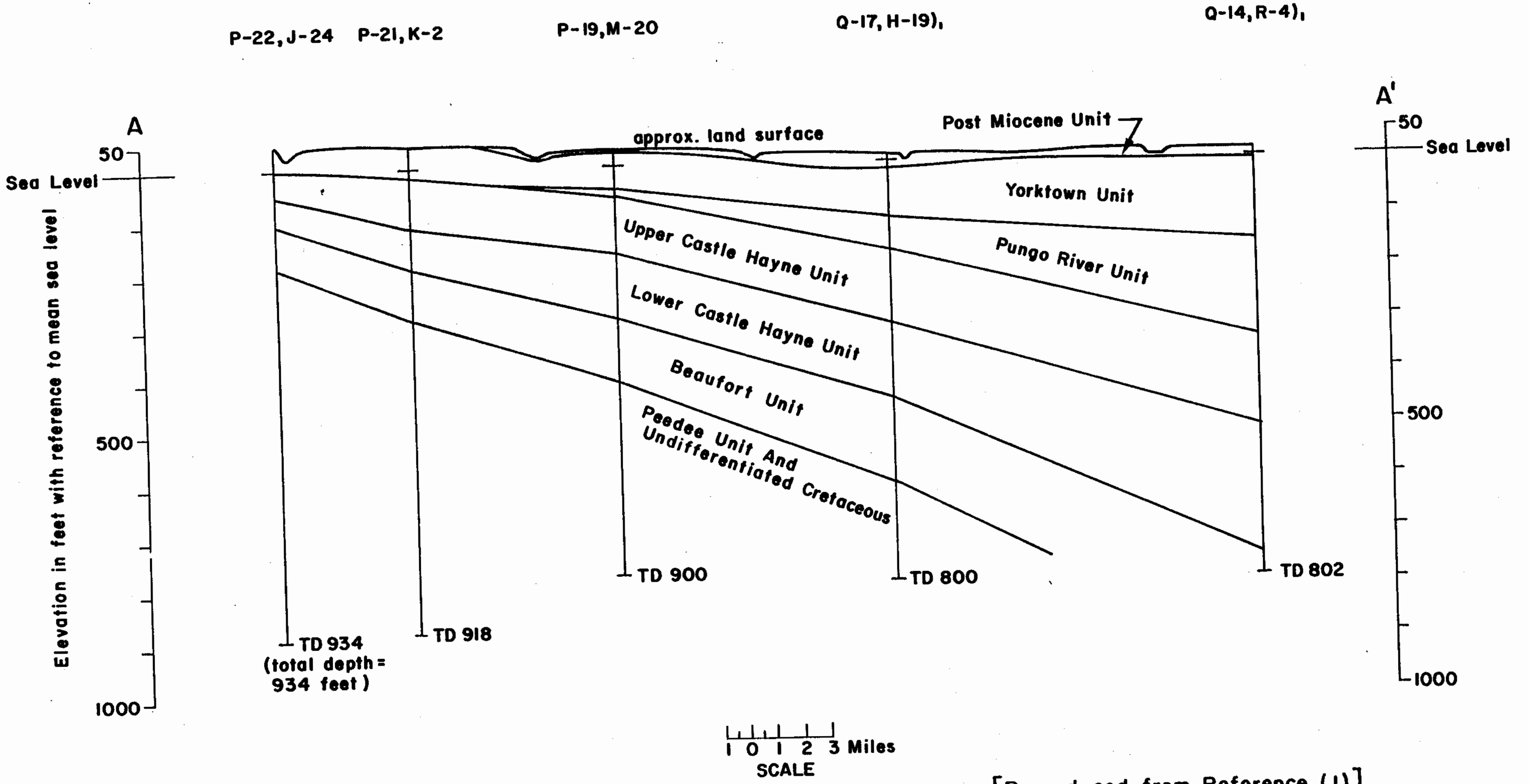
1. a northeastern strike and dip southeastward at a gradient of about 20 feet per mile,
2. are generally wedge-shaped, beginning as a feather-edge in the west and gradually thickening and becoming more deeply buried toward the east, and
3. gradually change in lithologic character, the hydraulic conductivity decreasing from west to east.

Based mainly on the difference in the hydrologic characteristics and hydraulic connectivity, the sedimentary rocks have been subdivided into hydrogeologic units. The hydrologic units generally coincide with the stratigraphic units as shown in Table 1 reproduced for reference (1).

The sediments between the basement rocks and the water table are saturated with water, are interconnected, and form one continuous groundwater reservoir. In a study of the behavior of Castle Hayne aquifer, the hydrologic characteristics of the following formations are important:

2.1 Water-Table Aquifer: Post-Miocene Deposits

This aquifer covers the entire study area. In it, the ground water occurs under unconfined conditions. Its upper surface is the water table which fluctuates with changes in ground water storage. The water level ranges from land surface in the swamps to a depth of more than 15 feet along the sandy ridges bordering the Pamlico and Neuse rivers. It varies in thickness in the Study Area from less than 5 feet to more than 50 feet. Further east, on the southeast shore of Lake Mattamuskeet was encountered



[Reproduced from Reference (1)]

Figure 1. Cross-Section A-A'

Table 1. Stratigraphic and Hydrogeologic Subdivision

System	Series	Stratigraphic Units	Hydrogeologic Units*		Character
QUATERNARY	RECENT PLEISTOCENE	Undifferentiated Unnamed Unit	POST-MIOCENE UNIT (pM)		Sand, silt, shells & some clay. This unit comprises the unconfined or "water-table" aquifer & includes sands of the Yorktown formation in some localities
TERTIARY	MIOCENE	YORKTOWN FORMATION	YORKTOWN UNIT (My)		Imbedded sand & clay with some shell beds. Clays are generally sandy & comprise confining beds over most of the area. The beds of shell & sand are confined aquifers except in the southwestern part of the area.
		PUNGO RIVER FORMATION	PUNGO RIVER UNIT (Mpr)		Phosphate & quartz sand, silt, cla & limestone. Permeability of unit is low because of silt & clay content & layers of dolomitic limestone. Unit serves as confining bed over most of area. May serve as aquifer in some localities.
	EOCENE	CASTLE HAYNE LIMESTONE	CASTLE HAYNE AQUIFER SYSTEM	UPPER CASTLE HAYNE UNIT (U Ech)	Permeable & porous shell limestone Excellent aquifer & the most productive unit of Castle Hayne aquifer system.
				LOWER CASTLE HAYNE UNIT (L Ech)	Shell limestone interbedded with calcareous sands. A moderately productive aquifer but less permeable than Upper Castle Hayne unit.
	PALEOCENE	BEAUFORT FORMATION		BEAUFORT UNIT (Pb)	Fine glauconitic sand, silty & clayey in part. Includes sands of upper part of Peedee formation in some localities. Permeability of unit is relatively low & unit is not a highly productive aquifer.
CRETACEOUS	UPPER CRETACEOUS	PEEDEE FORMATION	PEEDEE UNIT (Kpd) AND UNDIFFERENTIATED CRETACEOUS UNITS		Interbedded clay, fine sand & silt that form a confining bed beneath the Castle Hayne aquifer system.
		BLACK CREEK FORMATION			
		TUSCALOOSA FORMATION			
	LOWER CRETACEOUS	UNNAMED FORMATION			
	BASEMENT				

*Symbols used on water-level, chloride content, & water use maps

(Reproduced from (1))

180 feet of this section (4). The average thickness is about 20 feet. The base of this aquifer in most cases is a clay layer which in the western part is discontinuous. In the western part, the aquifer is in direct contact with the Beaufort formation or the Castle Hayne.

The yields of the wells tapping the aquifer are low, generally less than 10 gallons per minute (gpm), and the specific capacities generally range from 0.1 to 0.5 gpm per foot of drawdown after the day of continuous pumping.

A significant amount of water is stored in these sediments. The aquifer is replenished by infiltrated water from rainfall.

2.2 Yorktown Formation

It underlies the water-table aquifer and is separated from it by relatively impermeable layers of silt and clay. The water occurs under confined or semi-confined conditions depending on the thickness of the overlying silt and clay. The thickness of the formation of Lee Creek site is about 70 feet. All strata have strong lenticular character.

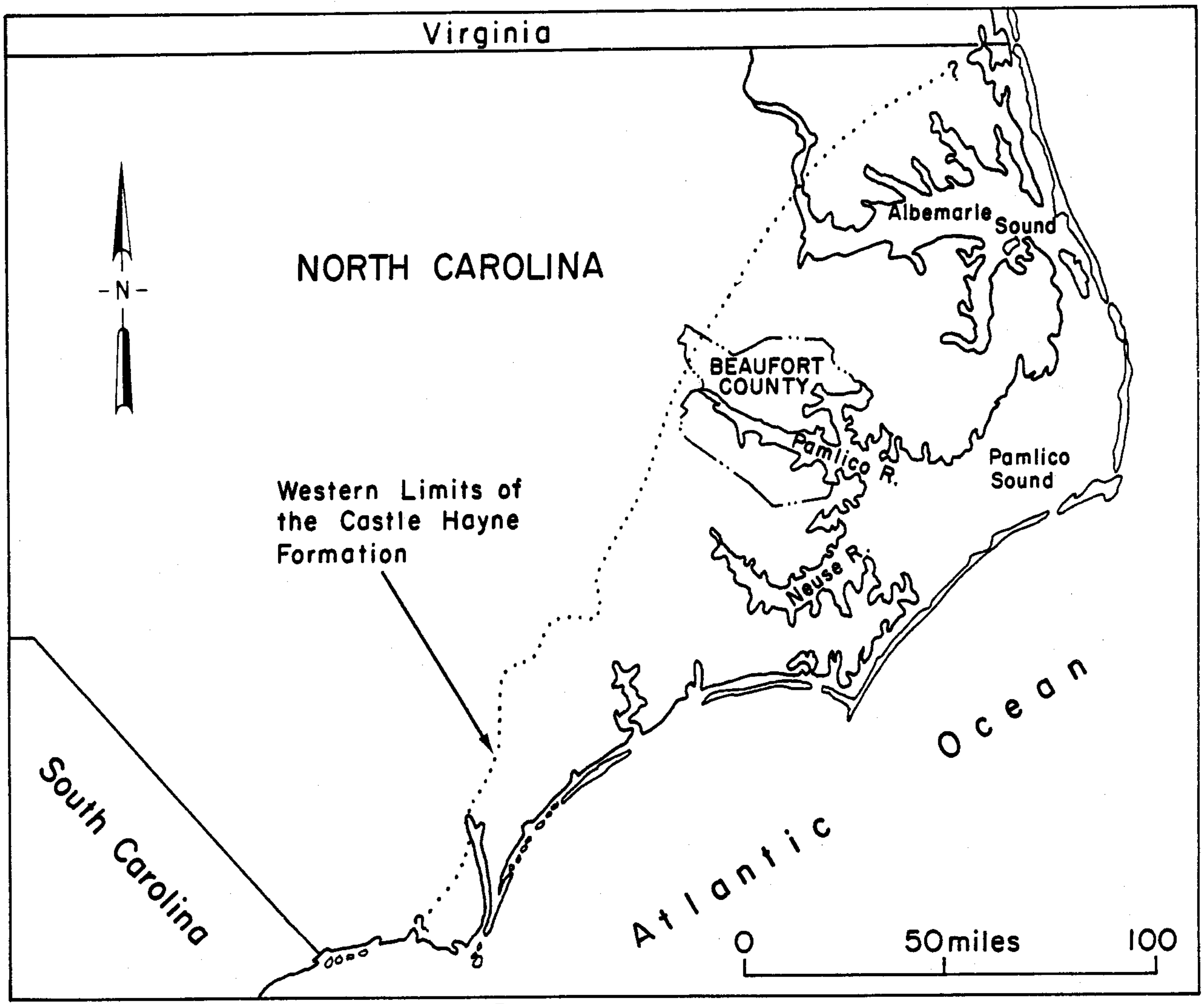
Large quantities of water are stored in the formation. The transmissivities are about one-tenth of the Castle Hayne formation. In some locations, wells tapping the aquifer may yield up to 250 gpm.

2.3 Pungo River Formation

This unit has been assigned to the Middle Miocene. It contains interbedded phosphatic and monmorillonite clays. The formation does not crop out at the surface and has poor water-bearing properties. The formation contains phosphorites in a large area of about 1000 square miles. The phosphate ore is composed of unconsolidated mixture of clay, quartz sand and francolite sand $(Ca\ CO_3)\ Ca_{10}\ (PO_4)_6\ F_2$. There exist 50,000 acres of potentially recoverable ore, of which Texas Gulf has mineral rights for 30,000 acres (5). The phosphate formation thickens from a feather edge at the upper reaches of the estuary to 160 feet at the outfall into the Pamlico Sound.

2.4 Castle Hayne Formation

The Castle Hayne limtstone is of middle and late Ecocene age extending along nearly the entire coast of North Carolina (Figure 2). The formation is feather-edged in its western extremities, thickening and dipping eastward at the rate of about 10 feet per mile and extends beneath the Atlantic



[Reproduced from Hird (5)]

Figure 2. Map of the Coastal Plain Region of North Carolina Showing the Location of the Castle Hayne Formation and Beaufort County.

Ocean. In the western part, the confining beds are discontinuous. The formation crops out in a belt, with its northern upper part immediately to the southwest of Beaufort County. At the Lee Creek mine, it is about 230 feet thick. The upper contact of the formation forms the lower part of the semi-confining layer between the Eocene and Miocene leaky aquifer systems. Generally, the Castle Hayne formation can be divided into two zones, the upper zone being the more permeable. The water at depths greater than 300 feet below mean sea level is generally brackish or saline. The Upper Castle Hayne Unit is the most important aquifer in the area. Yields from 8-inch wells tapping the aquifer range up to 300 gpm; 1000 gpm or more can be developed in favorable locations. The lower horizon grades into a confining glaconite silty clay.

2.5 Beaufort Formation

The Beaufort formation is of Paleocene age. The formation has no known outcrop area. While the formation has generally good porosity and permeability, the least permeable part is generally the upper part. In the western and central part of the area the formation is a fresh water aquifer. At the Lee Creek mine site, a sample of water in this formation at a depth of about 470 feet below mean sea level had a chloride content of 3000 ppm (3). It has a thickness of about 160 feet at the mine site.

2.6 Peedee, Black Creek, and Tuscaloosa Formations

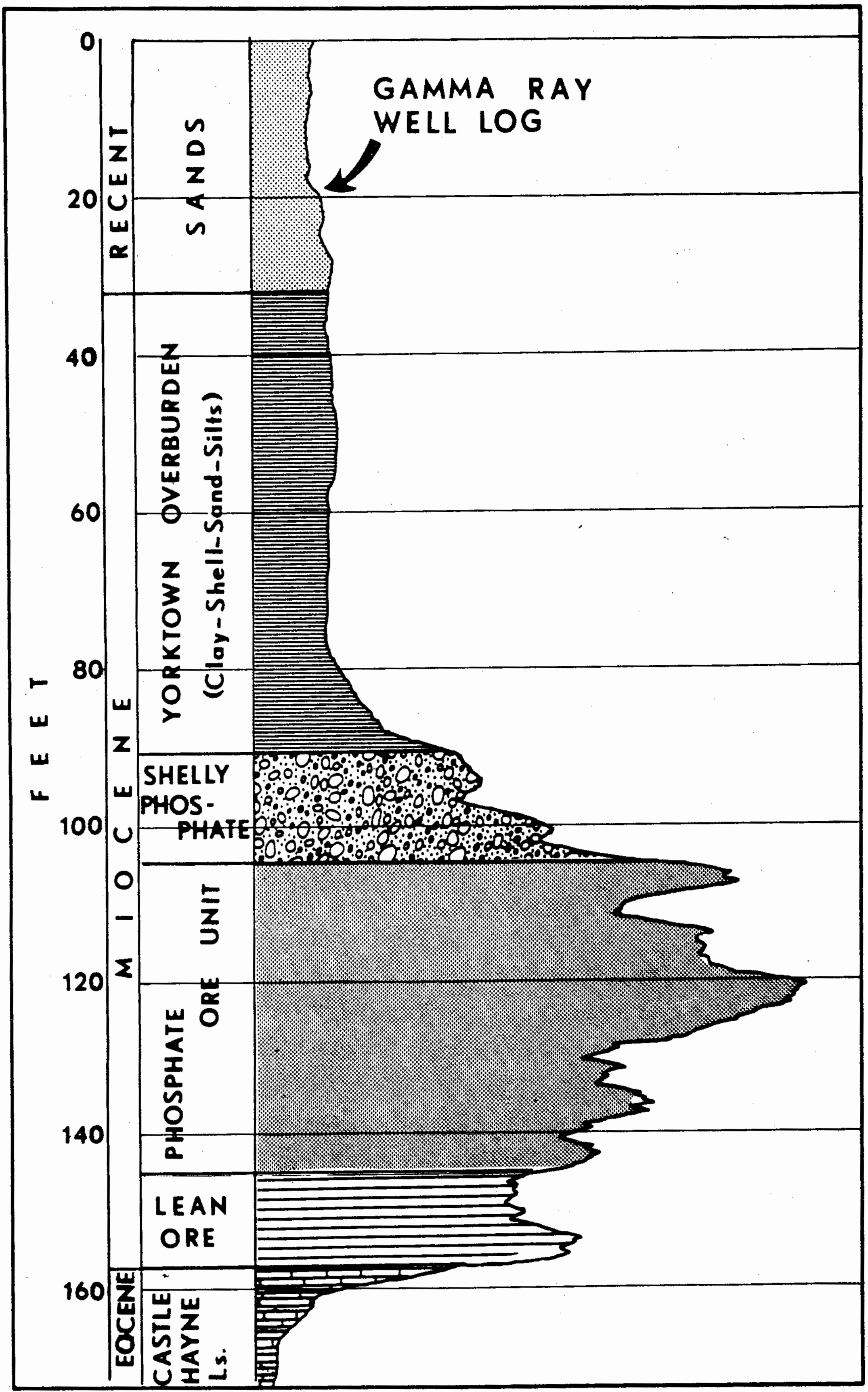
The oldest formation penetrated by wells in the Beaufort County is the Peedee formation which has been assigned to the late Cretaceous age. This unit is underlain by Black Creek formation which is, in turn, underlain by the Tuscaloosa formation, both of late Cretaceous age. These formations are tapped by wells west of Beaufort County. Although these formations may not be important for the development of fresh groundwater supplies, their hydrologic characteristics can be important in the study of upward leakage into the overlying strata.

Summary

Table 2 gives characteristics of the various hydrogeologic units and Figure 3 contains a geologic section and gamma ray well log through the Lee Creek ore body.

Table 2. Characteristics of Aquifers

	Top (MSL)	Thickness Range (West-East)	Avg. Thickness	T
Peedee	-125 ft to -800 ft			
Beaufort	- 35 to > -800 ft	< 80 - 220 ft	130	2 - 7000 gpd/ft
Lower C-H	- 40 to < -540 ft	0 - > 240 ft	190 ft	10 - 35,000
Upper C-H	0 450 ft Irregular 0 - < 350 ft	0 - > 180		90 - 310,000
Pungo	20 - > 100	0 - 180 ft		A few thousand (at Lee Creek 1000 gpd/ft)
Yorktown	+ 40 - < 120	20 - 150 ft	70	Few thousand (20,000 at Lee Creek)
Aquitard	Clay member of My & underlying Pungo	0 - 220 ft		
Post Miocene	Land Surface	1 - 50 ft	35	Few thousand gpd/ft



[Reproduced from Hird (5)]

Figure 3. Geologic Section and Gamma-Ray Well Log Thru the Lee Creek Ore Body.

2.7 Geologic Parameters

For a proper evaluation of the hydrogeologic characteristics of the sedimentary units and the behavior of the aquifer system, the geological parameters needed are:

1. the updip limits of the formation,
2. The lithology including porosity,
3. the hydraulic conductivity, including secondary permeability potential, and
4. hydraulic boundaries of the unit, both lateral and vertical.

2.8 Precision of Geologic Data

Sampair (4) observes that:

1. the geologic data is not well distributed, and the vertical and horizontal control must be judged no better than 85 percent accurate. The potential discrepancies introduced by this inaccuracy should be acknowledged;
2. the stratigraphic information is indirectly derived from the geophysical logs. The areas where no data were available are interpretive;
3. the data in the study area becomes sparse with depth;
4. the density and distribution of the geologic data is poor in the western part of the study area where the Castle Hayne and Pungo River formations pinchout. The geology of these areas is largely interpretive; and
5. the subcrop pattern on the top of Castle Hayne and on top of Pungo River formations is significant from a hydrologic point of view, but the important changes occur in the areas where the data are inadequate.

Chapter 3

WATER LEVEL CHANGES

The water level data for hydrogeologic units other than the Upper Castle Hayne are extremely limited and were obtained after 1965. Therefore, the precise effect of pumping at Lee Creek cannot be directly measured in any of the units. However, a few generalizations can be made.

Before the start of heavy pumping at Lee Creek, the piezometric surfaces of all hydrologic units were probably everywhere above sea level. The withdrawals caused a widespread lowering of the piezometric surface of the Upper Castle Hayne unit. This has affected the piezometric surfaces of the overlying and underlying hydrogeologic units.

3.1 Post-Miocene Water Table Unit

Water level measurements have been made only in few water table wells. The elevation and configuration of water levels in most of the area is essentially unknown. Water levels in water table wells respond relatively quickly to varying infiltration rates and varying discharge rates to the tributaries. Where records are available, they are not of adequate length to establish trends. There is no conclusive evidence to show that there has been no measurable effect on the water table in the recharge areas of the Castle Hayne aquifer or at the upstream end of the estuary. A reduction in the overland flow and the salvage of any appreciable amount of rejected recharge is contingent on the creation of storage in the water table aquifer.

The only map of water table configuration that exists (Figure 26 of 1971 Report) has been chiefly derived from the U. S. Geological Survey topographic maps. This is the best that can be done under the circumstances. One has to concede that this can give rise to elevations significantly different from those actually obtained over large areas.

The water table map plays a crucial role in the analysis of a leaky aquifer system like Castle Hayne. It determines the area extent of the downward and upward leakage and the head difference driving the leakage. The effect of these on water quality determination is obvious. Because of the nonuniform characteristics of the aquitards and the variation of drawdown over the cone of depression, an error of ± 1 ft in the estimation of the elevation of water table has significantly different consequences on the downward leakage in the different parts of the area. This effect would be the least in the vicinity of the mine and maximum in the head reaches of the estuary and near the contour of zero piezometric surface elevation.

3.2 Yorktown Unit

The water levels in the Yorktown formation have been affected by the decline in the Castle Hayne piezometric surface. Water levels have also responded after the pumpage at Lee Creek was reduced (1). Superimposed on this regional effect is the local effect of the drainage of Yorktown water by the mine. The differential response to pumping at Lee Creek also lends credence to the variability in aquitard characteristics. Because of the development of a cone of depression in the piezometric surface, the reduction of storage in the aquitard should be taken into account.

3.3 Pungo River Unit

It is believed that the decline of water levels in the Pungo River unit has been greater than in the Yorktown and that water levels in this formation are now between those of Yorktown and the Upper Castle Hayne (1).

3.4 Upper Castle Hayne Unit

The concentrated high volume pumping at Lee Creek mine has developed a widespread cone of depression in the piezometric surface of the Upper Castle Hayne aquifer. It has lowered the water level to some degree for up to 40 miles from the mine. The cone of depression is assymmetrical with the short axis westward and the long axis eastward. The water levels are stabilized, and a pseudo-steady state was reached after about 15 months of pumping. The virtual radius of cone of depression, i.e., extrapalated where it intersects the original piezometric surface, was (2)

November	1965	19.0 miles
February	1966	23.4
May	1966	28.0

The piezometric surface is below sea level in an area of about 100 square miles; it has been lowered 5 feet or more in an area of 1400 square miles (1). The piezometric surface at the periphery of the mine is about 120 feet below sea level; it was about 7 feet above mean sea level at the start of pumping.

Except for a small area in the upper end of the estuary, groundwater is moving toward Lee Creek from all directions. The hydraulic gradient between the recharge areas in the west and the mine has steepened; the gradient east of the pit has reversed. This has increased the rate of flow from the west and reversed the direction of movement to the east. In 1965, the upward leakage out of the Castle Hayne to the overlying aquifers and the estuaries was greater than the downward leadage into the aquifer. The areas of upward leakage have largely been converted to the areas of downward leakage as due to the reversal of the vertical hydraulic gradient.

The lowering of the water levels has had adverse effect on other water users. Formerly, many of the farmers enjoyed the convenience of flowing wells in pastures or could pump their wells with suction pumps. The impact of pumping has made the conversion of 960 domestic wells and 45 irrigation wells to deep well units necessary (5).

3.5 Lower Castle Hayne Unit

The pumping has formed a cone of depression similar to the upper unit in the piezometric surface of the lower unit. In the recharge areas of the west, the piezometric surface for the Lower Castle Hayne is lower and to the east of Lee Creek mine higher than that of the Upper Castle Hayne. This shows recharge through the Upper Castle Hayne in the subcrop areas and upward leakage into it in the eastern portions. The direction of groundwater movement is about the same in both units.

3.6 Beaufort Unit

Due to the pumping at Lee Creek, a cone of depression has developed in the piezometric surface of the Beaufort formation. In the recharge areas to the west, the elevation of the piezometric surface is lower than the piezometric surface of the Lower Castle Hayne, indicating recharge through the overlying formations. To the east of the mine, the piezometric surface is higher than either that of Lower or of the Upper Castle Hayne. This indicates upward leakage to the overlying formations.

3.7 Conclusion

The development of the cones of depression in the overlying Yorktown and Pungo River formations and the underlying Lower Castle Hayne and Beaufort formations proves conclusively the existence of hydraulic connection between all these units. The measurements of the hydraulic potentials in the overlying and underlying formations show the existence of both the leakage from above and from below into the Upper Castle Hayne aquifer. The downward leakage is the stronger of the two in the western part of the area.

Chapter 4

THE HYDROLOGIC SETTING

An aquifer system consists of recharge areas, water-bearing materials which transmit groundwater and the discharge areas and sinks.

4.1 Precipitation and Evapotranspiration

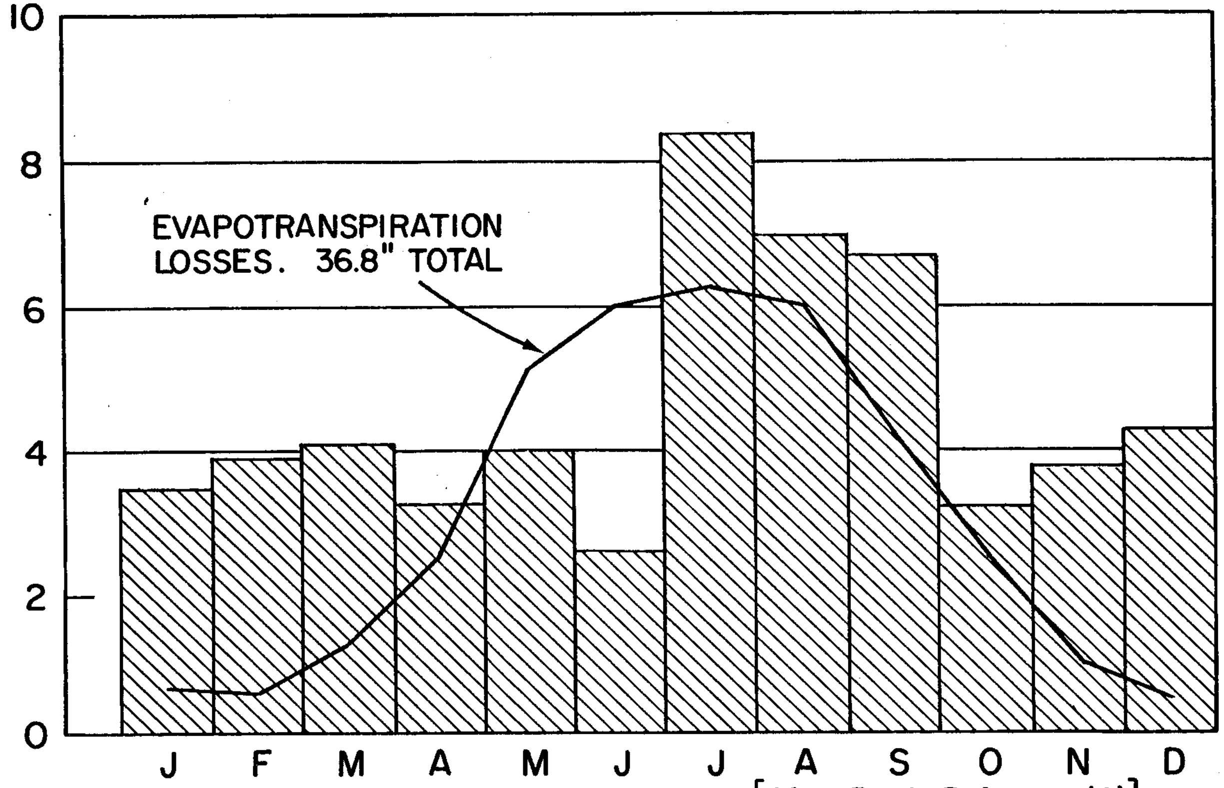
The ultimate source of all water in the ground is the precipitation on the recharge areas. The average annual precipitation in the study area is about 53 inches. The loss of moisture through evapotranspiration is largely a function of air temperature. Using the method developed by Thornthwaite and Mather, the 1967 Consultant's Report computed the evapotranspiration from the Aurora, North Carolina, area to be about 36.8 inches. The monthly distribution of precipitation and evapotranspiration for New Bern, North Carolina is shown on Figure 4 (6).

The difference between the precipitation and the potential evapotranspiration is the water which runs off as overland flow to the streams or is available for addition to the groundwater reservoirs. Most of the recharge to the groundwater reservoir occurs in the fall and winter months when the evapotranspiration is small. Knowing the runoff from the area (9 inches per year), contribution to groundwater was calculated to be 8 inches by the 1967 Consultant's Report.

4.2 Recharge to the Water-Table Aquifer

The recharge to the water-table aquifer--post-miocene unit or the areas of unconfinement of Castle Hayne unit--takes place over the entire land area except that occupied by water and swamps and the areas adjacent to the streams.

The annual natural recharge to the groundwater basin is a stochastic variable with a high degree of variation from year to year. This is in response to the variability of the meteorological factors which govern the replenishment. To indicate the amounts of groundwater available, DeWiest (3) separated the monthly hydrographs of the Trent River, near Trenton, North Carolina, and of Swift Creek, near Vanceboro, North Carolina, for the water years 1952-66 into their base flow and surface runoff components. The entire period of record was broken down into tow segments, an initial period of five years 1952-56 and a second period of ten years



[After Floyd - Reference (6)]

Figure 4. Climatic Conditions at New Bern, N.C., 1885 to 1964. Average Monthly Precipitation (Bar). Average Monthly Evapotranspiration Loss (Curve).

1957-1966. His results are given in Table 3. The precipitation figures used are those for Trenton, North Carolina (Trend River), and New Bern, North Carolina (Swift Creek). The yearly variation of groundwater recharge is obvious from his data. His conclusions are:

1. the total distribution of total runoff in overland flow and base flow is the same for both periods; namely, 56 percent of the total runoff for the base flow and 44 percent for the surface flow;
2. the average increase in precipitation of the order of 7 percent in the second period caused a 52 percent increase of base flow in the Swift Creek; and
3. this constitutes an adequate proof that the replenishment of groundwater basins by recharge through rainfall infiltration in the outcrop areas of the formations is very efficient.

The recharge calculations based on the estimates of average annual quantities can only be considered of a preliminary character but provide valuable insight as to the order of magnitudes involved. Various investigators have estimated the potential infiltration from precipitation when the water table reservoir is not already full to be 15 to 20 percent of the annual precipitation. The estimates range from 8 inches per year to 11 inches per year. If the water table reservoir is full or nearly full, the rainfall is mostly rejected as runoff to the shallow, slow-running tributaries to the Pamlico, Neuse, and other rivers. The replenishment from the water table aquifer is disposed of to (a) sustain the base flow of tributaries and streams, (b) provide recharge to the connecting confined aquifers, and (c) to provide downward leakage to underlying aquifers in areas where downward vertical hydraulic gradients exist. This is the case in the areas near the western boundary. The downward rate of percolation from the post-miocene unit is governed by the magnitude of the vertical hydraulic gradient and the vertical permeability of materials. Upward leakage into the post-miocene from the underlying aquifers occurs where upward hydraulic gradients exist. This was true of the regions to the east in pre-1965 conditions.

4.3 Recharge to the Castle Hayne Aquifer

The recharge to the Castle Hayne aquifer takes place in the outcrop area and in the subcrop areas beneath saturated sands. The aquitard is

Table 3. Hydrograph Separation - Annual Series Wateryears 1952-1966

Trent River near Trenton, North Carolina

Wateryear	P	R 0	0 L	B	B/P in %
1952	48.20	9.51	2.80	6.71	14.0
1953	39.10	7.89	3.61	4.28	11.0
1954	33.10	10.84	4.62	6.22	18.8
1955	84.50	23.23	13.27	9.96	11.8
1956	42.27	10.70	3.57	7.13	16.9
1957	49.19	6.82	2.68	4.14	8.4
1958	57.21	20.73	6.87	13.86	24.2
1959	48.34	20.47	8.26	12.21	25.3
1960	59.52	25.59	11.01	14.58	24.4
1961	55.59	20.39	8.80	11.59	20.9
1962	59.25	21.64	11.08	10.56	17.8
1963	43.92	14.12	4.48	9.64	22.0
1964	59.50	20.78	11.29	9.49	15.9
1965	49.84	21.92	11.12	10.80	21.7
1966	51.01	13.41	5.80	7.61	14.9
TOTAL	780.54	248.04	109.26	138.78	17.8
Percent of total R 0			44.2	55.8	
Percent of total P			14.0	17.8	

Swift Creek, North Carolina

Wateryear	P	R 0	0 L	B	B/P in %
1952	52.61	9.90	4.13	5.77	11.0
1953	47.55	7.58	2.68	4.90	10.3
1954	44.29	8.11	3.75	4.36	9.9
1955	73.26	16.08	9.90	6.18	8.4
1956	46.15	8.47	2.83	5.64	12.2
1957	46.95	9.66	4.26	5.40	11.5
1958	58.39	20.23	6.63	13.60	23.3
1959	58.74	20.30	8.97	11.33	19.3
1960	62.08	25.77	11.65	14.12	22.8
1961	59.47	22.90	9.27	13.63	23.0
1962	54.99	16.66	7.40	9.26	16.4
1963	56.22	12.90	4.35	8.55	15.3
1964	59.10	21.70	13.06	8.64	14.6
1965	49.46	20.93	10.01	10.92	22.2
1966	53.66	13.99	5.75	8.24	15.4
TOTAL	822.94	235.18	104.64	130.54	15.9
Percent of total R 0			44.5	55.5	
Percent of total P			12.7	15.9	

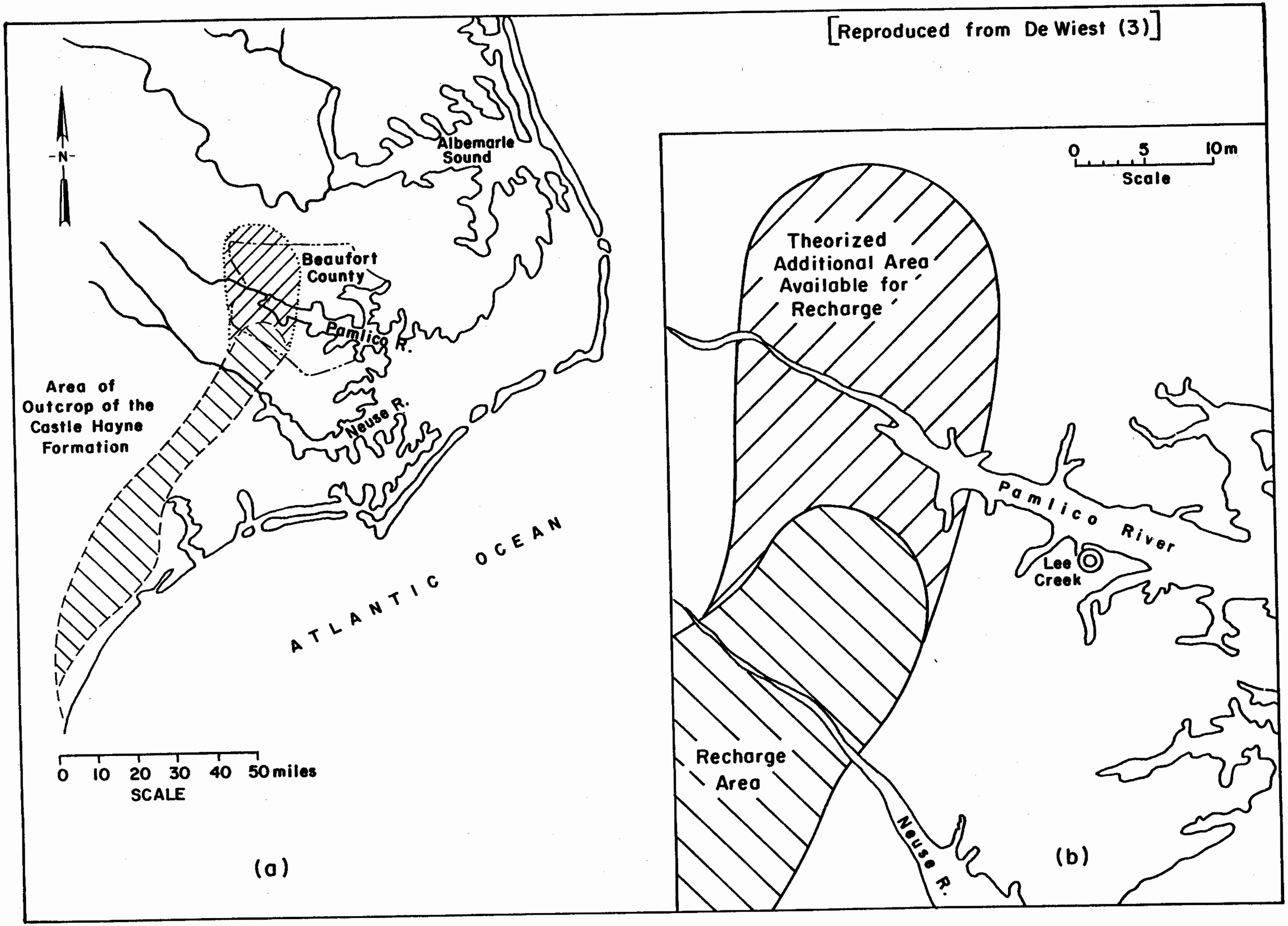
Legend: P = precipitation, R 0 = runoff, 0 L = overland flow, B = base flow

All numbers expressed in inches

[Reproduced from De Wiest (3)]

not present in a large area south of the Pamlico River and the Upper Castle Hayne unit is essentially unconfined. Concerning the magnitude of recharge, there is wide disagreement. The first estimate was provided by the 1967 Consultant's Report. Taking an infiltration efficiency of 15 percent and a recharge area of 170 square miles, they arrived at a value of 65 mgd. Later, DeWiest revised this estimate upwards and Jacob downward. For the Craven County, from a flow-net analysis Floyd (6) estimated that in the northern part of the county where the confining beds are thin or absent, the recharge may be as much as 300,000 gpd per square mile which corresponds to 6.35 inches per year. He remarked that in the southern part of the county the rate of recharge was probably less than 250,000 gpd per square mile; i.e., about 4.23 inches per year. The average recharge over an area of 380 square miles was estimated as 100 mgd, or an average rate of recharge of 5.6 inches per year. Jacob from an analysis of the curvature of the piezometric surface between the Neuse and Pamlico Rivers arrived at a value of 3.5 inches per year.

DeWiest (3) maintains that the Yorktown formation where it mantles the Castle Hayne formation near the outcrop area of Castle Hayne, contributes directly to the replenishment of the Castle Hayne. The hydrologically "effective" outcrop area of the Castle Hayne limestone is much larger than 170 square miles as may be taken from the geologic map of the State of North Carolina . . . The part of the hydrologically effective outcrop area that may affect the flow in the Castle Hayne aquifer at the TGS mine is of the order of 550 square miles. This area is given in Figure 5. Based on an infiltration efficiency of 0.2 or a recharge rate of 10.7 inches per annum and a recharge area equal to 500 square miles, the potential annual recharge rate is estimated to be 280 mgd. From this the base flow of the intersecting streams must be satisfied. If the runoff of Tar River is estimated as 1.05 cubic feet per second per square mile and if it is assumed that 55 percent of this stream flow is derived from groundwater flow, then the contribution to be made by the recharge to the base flow is computed as 206 mgd. This leaves 74 mgd for the satisfaction of the phosphate mining operations and other uses in Castle Hayne. This represents a potential contribution of 2.8 inches per year. On the other hand, if the stream flow representative of the recharge area is only 0.72 cubic feet per second per square mile, as is the case



[Reproduced from De Wiest (3)]

Fig. 5. (a) Castle Hayne Outcrop Area and (b) Recharge Area in the Vicinity of Lee Creek.

for Herring River near Washington, North Carolina, the contribution to the base flow of the Tar River by the direct recharge from the Castle Hayne would only be 141 mgd. The residual recharge potentially available for development would thus be 139 mgd. This is equivalent to a recharge rate of 5.3 inches per year. As 55 mgd is required to meet the existing uses, the excess varying between 19 mgd and 84 mgd represents the rejected recharge which is potentially available for development.

The geometric shape of the 1965 water level contours reveals that the areas of recharge for the Castle Hayne aquifer lie both to the north and south of the Pamlico River. An estimate of the recharge into the Pamlico groundwater basin using flow net analysis of the 1965 piezometric contour map indicated the total recharge to be 44 mgd, 15 mgd to the north of Pamlico and 29 mgd to the south. The electric analog model for the 1965 steady state conditions gave a value of 38 mgd.

On the assumption of no leakage, Hird (5) estimated that approximately 60 percent of the recharge to the Castle Hayne aquifer in the area affected by Texas Gulf pumping is from the southern area. From a flow net analysis of the 1970 piezometric surface map, the 1971 Report estimates that the total recharge in the intake area to the west is about 35.5 mgd, about two-thirds of the withdrawals from the Lee Creek Mine.

4.4 Leakage

The Castle Hayne aquifer is hydraulically connected to the overlying and underlying beds. In areas where the elevation of piezometric surface is lower than that of the water table, water moves from the overlying beds into the aquifer. Wherever the elevation of the piezometric surface is higher than that of the water table, water will move out of the aquifer into the overlying beds. Similarly the movement to and from the underlying beds depends on the direction of the vertical hydraulic gradient. Water moves into the Upper Castle Hayne aquifer from the underlying Lower Castle Hayne and Beaufort formations in areas where the piezometric surface in these formations is higher. The magnitude of leakage is governed by the vertical hydraulic conductivity and thickness of the materials through which the flow takes place and by the head difference which provides the driving force. With the heavy draft on the Upper Castle Hayne aquifer at Lee Creek, a reversal of hydraulic gradient has taken place and, in a very large area including the estuary, the upward leakage to overlying aquifers has been changed to downward leakage. Leakage is usually helpful in improving the performance of the wells by reducing drawdowns for the same discharge, but it may result in the contamination of the pumped aquifer by intercepting surface and groundwater having undesirable qualities.

The contribution from leakage to the Texas Gulf withdrawals have been estimated to range from 13.5 mgd; i.e., 25 percent (1) to 64 percent (2). Analog and digital studies place this figure in the neighborhood of 50 percent.

4.5 Rejected Recharge

The salvage of rejected recharge is a critical factor in predicting the capacity of the aquifer system to sustain large withdrawals indefinitely. Rejected recharge has to be examined in two separate contexts. One situation is in the outcrop areas where the water table is in the Castle Hayne aquifer. It means the capture of a part of the overland flow that now goes directly to the tributary streams. This can only be accomplished if storage space is created, i.e., the water levels in the area of unconfinement are lowered, as a result of heavy draft within the area of confinement. There is no conclusive empirical evidence to show whether this has or has not happened after the commencement of Texas Gulf pumping. The other situation for the rejected recharge is in the subcrop areas of the Castle Hayne aquifer where (a) there is a thin cover (say, 25 feet or less of Yorktown), (b) there are perennial streams intersecting this cover, (c) the distance between perennial streams is not too large, and (d) Castle Hayne has limited capacity to absorb all the possible recharge. Here, the rejected recharge appears as the base flow of these tributaries. A heavy draft within the area of confinement of Castle Hayne as at Lee Creek has the effect of diverting some of this water.

The only available estimate of the total rejected recharge is that of DeWiest. He estimates that additional rejected recharge available for future development adjusted for the current rate of pumping at Lee Creek ranges from 19 mgd to 84 mgd as given earlier. The Consultant's Report estimated that one-third of the 1966 Lee Creek withdrawal (65 mgd) was coming from the salvage of rejected recharge. An analysis of the results of analog simulation places this figure at 6 mgd. As the pertinent water level data in the area of interest is not available, it is difficult to confirm these estimates.

A distinction has to be made between the water coming from recharge and from leakage. If most of the water were coming from rejected recharge and very little from leakage, then as DeWiest (3) asserts, the reversals in the hydraulic gradient of the aquifer flow will only be very localized and limited to the immediate vicinity of the mine in spite of the heavy withdrawals. The reversal of the hydraulic gradient over an extensive area indicates that leakage does play a substantial part.

Recovery of all the potential rejected recharge will be possible only with an assumption of drawdowns in the aquifer which from its effect on water quality will be intolerable.

4.6 Base Flow of Tributaries

In the recharge areas, tributaries act as virtual drains and take out most of the water. The area is cut up by tributaries which pull down the water table. A preliminary analysis of the Durham Creek, Swift Creek, and Trent River from the exponential decay of dry weather flow and time constants of the tributaries shows that the groundwater runoff in tributaries amounts to an upward of 9 inches per year. DeWiest's analysis of Swift Creek and Trent River given in Table 3 give values of 9.3 inches and 8.7 inches, respectively. Most of the water does not enter the aquifer which accounts for only 3 to 5.5 inches of the recharge.

4.7 Discharge to the Estuaries

The indentations in the generalized piezometric contours of the prepumping period as they cross the streams show upward leakage into those streams. Upward leakage is significant in the Pamlico River estuary, especially in its upper reaches. The Consultant's Report estimated that the upward leakage in the 16-mile stretch of the estuary between its inlet and the site of the mine was 30 mgd from both sides of the estuary (2,3). The analog model studies place this figure at 21 mgd from both sides, a large proportion coming from the south of the river. Further leakage was found in areas adjacent to the Pamlico River with 8 mgd discharged over an area of 530 square miles to the overlying Yorktown formation represented as a hydraulic head at mean sea level.

The vertical permeability of the aquitard that will be required to transport this amount of water from the top of the Upper Castle Hayne to the estuary under the hydraulic gradients prevailing in 1965 can be approximated. This indicates that the aquitard is more leaky in the vicinity of the upstream portions of the Pamlico estuary than elsewhere. This can be due to (a) thinness of the cover on Castle Hayne, (b) because of the relative perviousness of that cover "in places," or (c) greater solution activity in response to relatively higher velocities. Whichever hypothesis is correct has great significance for water quality. Flow downward is as easy as flow upward. With the reversal of hydraulic gradients, brackish water in the estuary would be pulled downward into the Castle Hayne aquifer.

4.8 Seaward Flow

The sounds and the sea serve as a sink for the flow not directed to the Neuse and Pamlico estuaries. The Consultant's Report estimated the average intensity of seaward flow to be 0.45 mgd/per mile in sections remote from but between the Pamlico and Neuse estuaries. The analog studies indicate that this is an extremely high estimate.

4.9 Pumping

The ore is mined by dry strip mining method requiring the depressurizing of the Castle Hayne aquifer. Initially, natural water levels were lowered requiring a reduction of about 150 feet of hydraulic head to prevent seepage from the bottom of the open mine cut. To accomplish this 20 deep wells at 3000 gpm each (total of 60,000 gpm) spaced 400 feet apart around the periphery of a 2,600-foot mining block were used (5).

The pumpage averaged about 60 mgd till the end of 1968. With experience it was found that up to 12 feet of artesian head could be adequately held without upward seepage by 8 ft section dense clayey lean ore left in place. This reduced the pumping demand to approximately 42,000 gpm (14 wells)(5). This corresponds to a reduction of 140 feet of hydraulic head to prevent seepage from the bottom of the open mining cut at 133 feet below mean sea level. The pumpage was reduced to about 53 mgd in 1969 and 52 mgd in 1970. The pumping rates are given in Figure 6 (1). The values are correct only to ± 10 percent for the first year and to about ± 5 percent for later years. The Texas Gulf pumping shows seasonal fluctuations. A better idea of actual pumping can only be had if we monitor fluctuations in an observation well at some distance from the area of pumping.

The arrangement of pumping wells and the drawdown pattern around the mine are given in Figure 7 (5). It will be seen that the drawdown in the

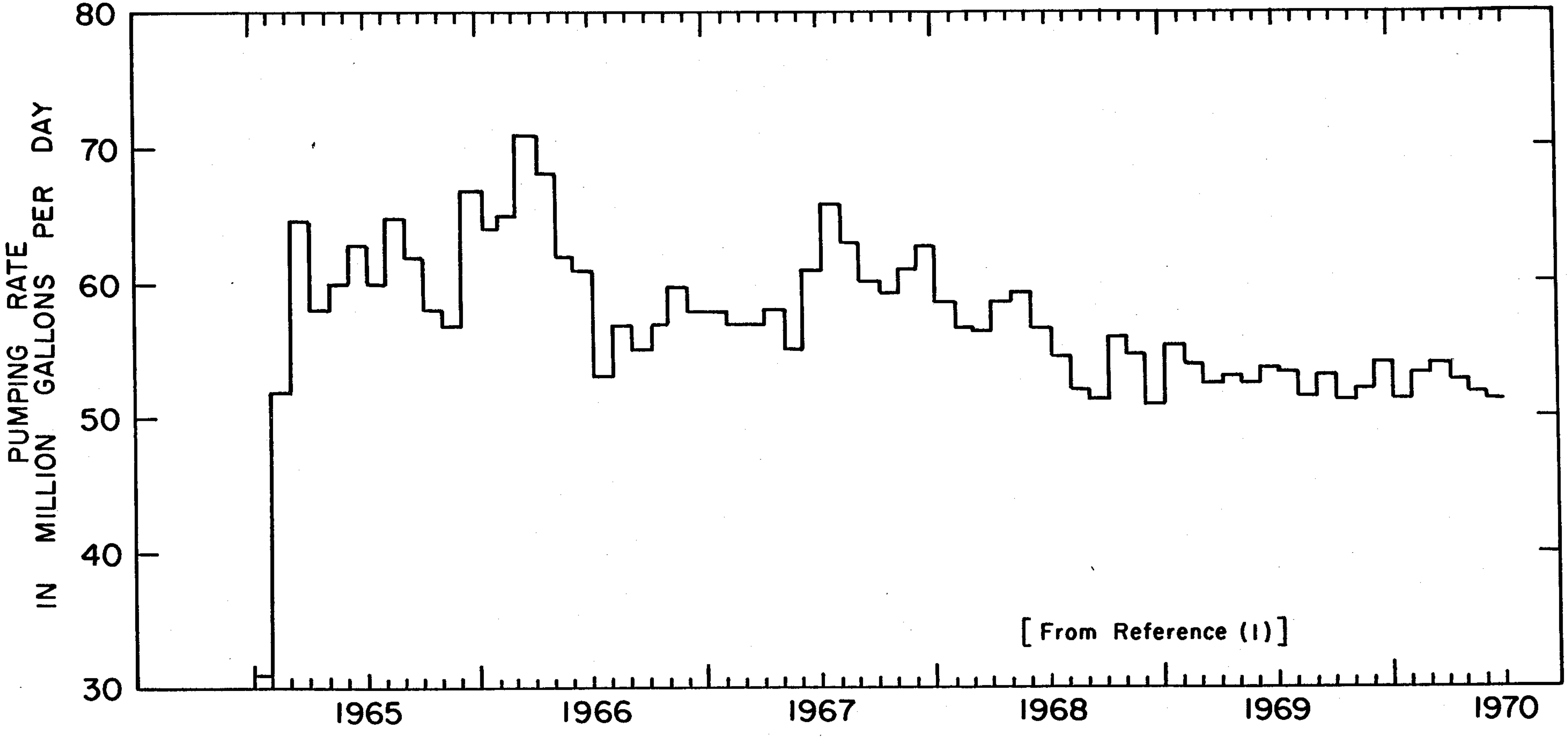
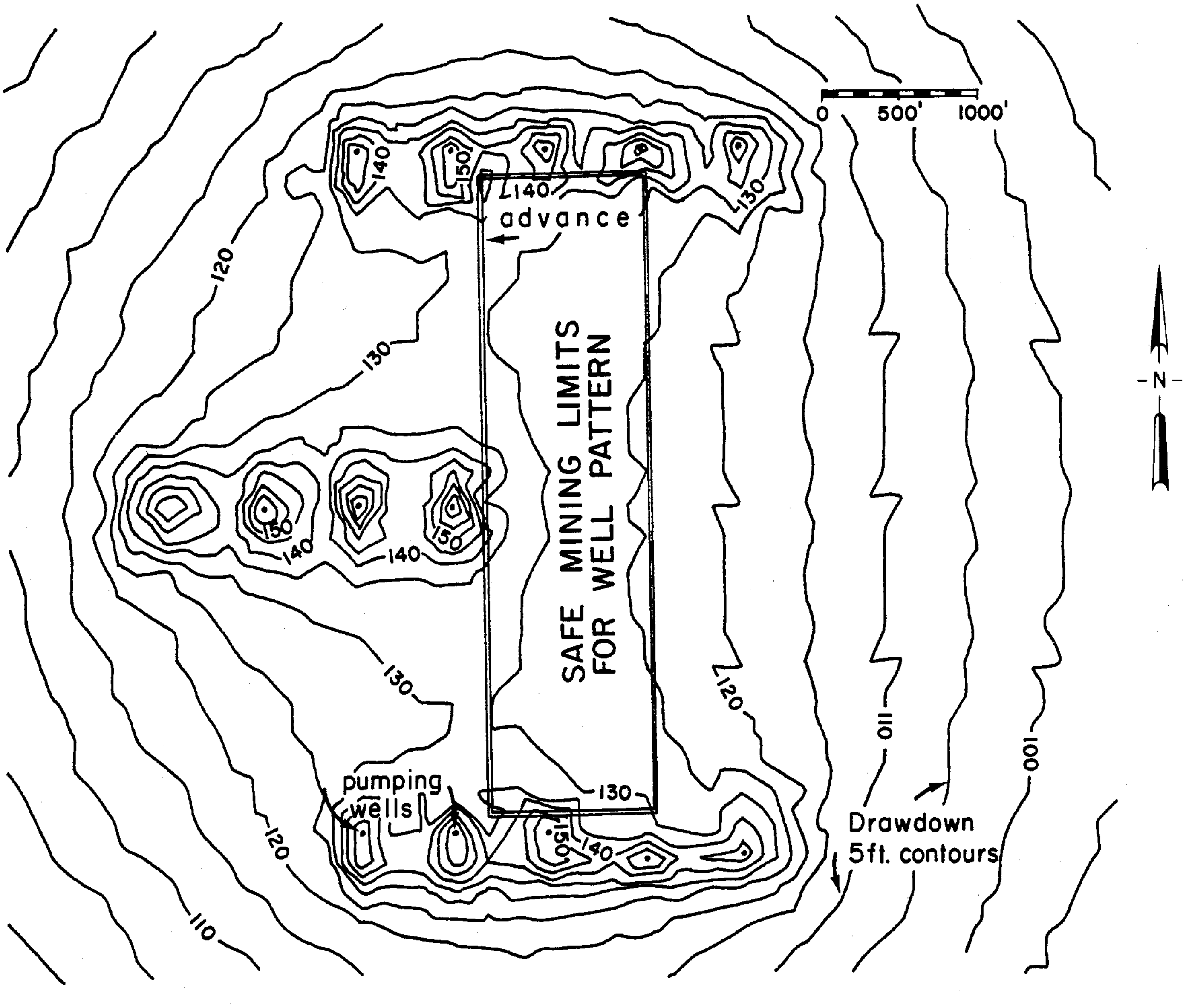


Figure 6. Monthly Pumping Rate at Lee Creek Mine, North Carolina.



[After Hird (5)]

Figure 7. Drawdown Effects Created by 14 Depressurizing Wells.

wells ranges from 145 feet to 165 feet. The actual pumping levels in the wells are not measured.

4.10 Conclusion

The areas of recharge for the Castle Hayne aquifer lie both to the north and the south of the Pamlico River, the principal contribution coming from south of the Pamlico River. In the areas of direct recharge where the Castle Hayne formation outcrops or subcrops, the contribution to the confined portion of the aquifer is between 3 inches and 5.5 inches annually.

Chapter 5

AQUIFER CHARACTERISTICS

A study of the piezometric surface contours for 1965, 1968 and 1970 reveal the heterogeneity of the aquifer. There are definite directional differences in the transmissivity of the aquifer. There is a wide range of permeabilities and porosities. Sampair (4) observes that Castle Hayne formation can be expected to have a considerable amount of fracturing developed within its indurate members. This fracturing will give the formation some vertical permeability in the vicinity of the fractures. The aquifer is highly anisotropic near the head of the estuaries. The transmissivity is higher in the transverse direction towards the Pamlico River than downgradient. The solution activity is probably more here than where the aquifer is thick. There are higher hydraulic gradients and velocities and the transmissivity is roughly proportional to velocity. As one goes downdip, the thickness is greater, materials finer and hydraulic gradients smaller. In these areas the aquifer is reasonably isotropic.

5.1 Analyses of Pumping Tests

Various investigations have arrived at different values of transmissivity (T), storativity (S), and leakage thickness (B) from essentially the same data. The disagreement on the magnitude of transmissivity is less than that with respect to the values of storage coefficient and leakage. This points out the fact that graphical solutions, including the flow net analysis, of the governing differential equations are subject to possible large errors.

Pumping tests in the Castle Hayne aquifer have been carried out at a few locations in and around the Lee Creek mine. Four of these will be discussed here. The earliest test was conducted by Harshbarger (7) in 1962 near Aurora, North Carolina, on a 10-inch well producing 1,070 gpm and lasting for 28 hours. The maximum drawdown observed was 13.7 feet. The estimated values of transmissivity and storage coefficient were 245,000 gallons per day per foot (gpd/ft) and 8 x 10^{-4}, respectively. The value of hydraulic conductivity was estimated to be 2,500 gallons per day per square foot (gpd/ft^2). The aquifer was considered to be completely confined with no leakage; hence, the value of B was infinite. The radius of strong influence was 2 miles (2).

Another aquifer test was conducted by Tippetts-Abbelt-McCarthy-Stratton for Texas Gulf and reviewed by Leggett, Brashears and Graham. The test was run on a 20-inch well at 3,500 gpm for 35 days. Drawdowns were measured in a number of observation wells from which aquifer characteristics were evaluated. Several analyses of the test have been made. The results are summarized below:

Miller's Analysis (8)

$T = 370{,}000 - 410{,}000$ gpd/ft

$S = 0.004 - 0.009$

$B - \infty$

1967 Consultant's Report (2)

$T = 390{,}000$ gpd/ft

$S = 0.00019$

$B = 14{,}000$ ft

$K = 1{,}300$ gpd/ft^2

Area of Strong Influence = 2.7 miles

Jacob's Second Analysis (9)

$T = 350{,}000$ gpd/ft

$S = 5 \times 10^{-5}$

$B = 30{,}000$

DeWiest's Analysis (3)

$T = 390{,}000$ gpd/ft

$S = 1.24 \times 10^{-4}$

$B = 182{,}400$ ft

A pilot well pump test was conducted by North Carolina Phosphate Corporation for a duration of 80 days in July-October 1968 at a distance of 2.4 miles from Texas Gulf mine site. A 19-inch well partially penetrating the Castle Hayne formation was pumped at 3,500 gpm and drawdowns were observed in several wells on both sides of the river. In his analysis, Harshbarger assumed the aquifer to be nonleaky; i.e., with $B = \infty$. His results are given in Table 4. The directional variation of transmissivity is obvious from his calculations. The average value of transmissivity was found to be 347,000 gallons per day per foot and the value of storage coefficient of 5.1×10^{-4}.

Table 4. Summary of Castle Hayne Aquifer Characteristics as Determined NCPC Pilot Well Pump Test, July-September, 1968

NCPC Well Number	NCPC Well Location	Theis Model Analysis Pump Test, July 5-13, 1968		Jacob Model Analysis Pump Test, July 17-Sept. 27, 1968	
		Transmissivity gpd/ft	Storage Coefficient	Transmissivity gpd/ft	Storage Coefficient
191 (Pump Well)	P-17,S-16	-	-	-	-
B-9 (Observation Well)	P-17,r-24	287,000	$5.4X10^{-4}$	290,000	$4.0X10^{-4}$
170 "	Q-17,e-1	335,000	$5.5X10^{-4}$	355,000	$4.6X10^{-4}$
172 "	Q-17,c-15	400,000	$4.1X10^{-4}$	320,000	$5.0X10^{-4}$
173 "	Q-18,a-10	410,000	$6.8X10^{-4}$	-	-
174 "	Q-17,e-11	250,000	$4.5X10^{-4}$	330,000	$4.2X10^{-4}$
182 "	P-16,p-18	400,000	$4.5X10^{-4}$	370,000	$4.2X10^{-4}$

(John W. Harshbarger)
(February, 1969)

The sustained dewatering operation by Texas Gulf provides an ideal aquifer test extending over several years. The 1967 Consultant's Report (2) analyzed the situation after one year of continuous pumping. Their results are:

$$T = 300{,}000 \text{ gpd/ft}$$
$$S = 9 \times 10^{-4}$$
$$B = 63{,}000 - 76{,}000$$

Radius of Strong Influence = 24 miles

It was remarked that the analysis for first year of operation to a distance of 24 miles gives a much higher value of B than the analysis of 35-day test, where strong influence reached only 2.7 miles. The aquifer appears to be more leaky on a small radial scale than a larger one. This strongly suggests an area of more concentrated leakage fairly near the pit (2).

5.2 Jacob's Analysis of North Carolina Phosphate Corporation's Test

Jacob (9) carried out an extensive analysis of the North Carolina Phosphate Corporation's 1968 pumping test. He derived the characteristics of the aquifer on the basis of the following mathematical models:

1. infinite leaky aquifer system,
2. circular drive with no leakage and the test well at the center, and
3. line drive with no leakage.

He divided the observation wells into three groups to investigate the directional variation of the hydrologic parameters. The results for the leaky aquifer system analysis are given in Table 5. He points out that the results of transmissivity for the southeast and southwest wells were high for the limestone and the value of apparent storativity included both the recharge and leakage. Furthermore, leakage was not effectively seen by drawdown and recovery curves at such a great distance.

Jacob also analyzed the response of the observation wells to two instantaneous impulses on July 15 and August 19, 1968. From the amplitude reduction and time lag of response he obtained the following results:

Storativity	7.8×10^{-4}
Leakage Thickness	45,000 ft
Transmissivity	45,000 ft^2/day
Leakance	2.2×10^{-5} 1/day

He found that the river wells showed a greater time lag and less amplitude than the land wells.

Table 5. Characteristics of Castle Hayne Aquifer

	Quadrant		
	SW	NE	SE AND SW
Leaky Aquifer			
Transmissivity, ft^2/day	45,000	56 - 60,000	71 - 98,000
Storativity	6.3×10^{-4}	$4.9 - 9.0 \times 10^{-4}$	$8.9 - 19.6 \times 10^{-4}$
Diffusivity, ft^2/day	7.1×10^{7}	$11 - 6.6 \times 10^{7}$	$8.0 - 5.0 \times 10^{7}$
Leakage Thickness, ft	29,000	27,000	
Leakance, day^{-1}	5.5×10^{-5}	7.7×10^{-5}	

5.3 1971 Technical Report

The 1971 Report relies almost exclusively on the flow-net analysis of the 1970 piezometric surface. The following is summarized from Ganus (10). The area of the flow net was divided into eleven flow tubes. Selected piezometric contours were used to provide five subdivisions within each flow tube. It was assumed that within the -50 ft piezometric contour the leakage was negligible and the transmissivity values were constant. The total discharge of 52 mgd was apportioned among the tubes on the basis of flow tube width and the average hydraulic gradient across that width at the -50 line. The values of vertical hydraulic conductivity were assumed for each of the subdivisions of the flow net, and the resulting transmissivity values were obtained from the continuity relationships. A digital computer program was developed to solve the continuity equations.

Six assignments of vertical hydraulic conductivity of the aquitard were tested. Four computer runs were made assuming constant values of 0, 0.0001, 0.001 and 0.01 gallons per day per square foot for the vertical permeability, giving 0, 0.54, 5.4 and 54 mgd, respectively, for the total leakage. The resulting transmissivity patterns were found to be unsatisfactory. It was concluded that the assumption of a constant value of the vertical permeability was untenable. The fifth run assumed a linear relationship between log K' (vertical hydraulic conductivity) and the aquitard thickness, assigning a value of 0.01 gpd/ft^2 to zero thickness and 0.0001 to a thickness of 250 feet. The total leakage within the area of the flow net was found to be 14.3 mgd. The sixth and the final run assigned the following control values:

0.018 gpd/ft^2 at Pungo River Unit subcrop pinchout
0.001 gpd/ft^2 at Pungo River thickness of 30 feet
0.0005 gpd/ft^2 at the aquitard thickness of 250 feet

The resulting pattern of transmissivity was deemed to be reasonable. These assumed values of vertical hydraulic conductivity and the resulting transmissivity distribution were adopted for all analyses. The effective aquitard thickness responsible for all the vertical head loss was assumed to be 70 to 80 percent of the total aquitard thickness. The transmissivity values varied from 100,000 in the west to 400,000 gpd/ft in the extreme east. The values of leakage thickness ranged between 40,000 feet in the western area of maximum leakage, 170,000 ft near the pumping center and 300,000 feet in the extreme eastern area. Of the total discharge of 52.1 mgd at Lee Creek mine, 38.4 mgd entered as underflow at the boundary of the flow net and 13.7 mgd is attributable to leakage within the area of the flow net.

5.4 Formation Constants Near the Pumping Center

Using the empirical data, Hird (5) developed the following equation to give the drawdown caused by a single well of 3,000 gpm capacity:

$$s = 31.4 - 2.92 \ln r$$

where s is the drawdown in feet at a distance r from the well. This relationship was used to calculate the parameters in the leaky aquifer equation

$$s = \frac{Q}{2\pi T} \ln \frac{1.12\ 3\ B}{r}$$

where Q is the discharge of the well
T, the transmissivity, and
B, the leakage thickness.

From

$$Q/(2\pi T) = 2.92,$$

the value of T was calculated to be 235,000 gpd/ft and from

$$2.92 \ln 1.123\, B = 31.4,$$

B was found to be 42,000 ft (cf. Technical Report value of 170,000). The value of leakance

$$L = T/B^2 = K'/b' = 1.81 \times 10^{-5} \text{ day}^{-1}$$

For an aquitard thickness of 125 ft, the vertical hydraulic conductivity

$$K' = 0.017 \text{ gpd/ft}^2$$

to allow a comparison with the Technical Report, this value should be multiplied by 0.75 to give K' = 0.0128 (cf .0009).

5 Formation Constants from 1970 Cone of Depression

Approximate values for the formation constants can be obtained by using the following equation for the drawdown, s, in an observation well at a distance r from the pit.

$$s = \frac{Q}{2\pi T} \frac{K_o\left(\frac{r}{B}\right)}{\frac{r_w}{B} K_1 \left(\frac{r_w}{B}\right)}$$

where r_w is the effective radius of the pit. If one has two pairs of values (r,s), then one can compute the quantities B and T. For two points 1 and 2, the ratio of drawdowns is given by

$$s_1/s_2 = K_o\left(\frac{r_1}{B}\right)/K_o\left(\frac{r_2}{B}\right)$$

from which the value of B can be computed. A trial value of B can be obtained by using the approximation

$$K_o\left(\frac{r}{B}\right) \simeq 2.3 \log \frac{1.123B}{r}$$

which is valid only for small values of (r/B). After finding B, T can be calculated from the drawdown equation for either point.

Proceeding in four different directions from the mine several pairs of points were selected on equal drawdown contours and 'spot' values of the formation constants were calculated. For a vast majority of the points, the leakage thicknesses obtained were much smaller than those implied by the Technical Report. The general conclusion that can be derived from this analysis is that the aquifer appears to be much more leaky than assumed in the Technical Report, but the leakage is less than that postulated by Jacob.

Chapter 6

NONSTEADY STATE ANALYSIS

The fundamental flow equation governing the movement of groundwater in a homogeneous, isotropic, leaky aquifer has been assumed to be

$$\nabla^2 s - \frac{s}{B^2} = \frac{S}{T}\frac{\partial s}{\partial t}$$

The solution of this equation is

$$s = [Q/4\pi T]\ W(u,r/B)$$

where

$$W(u,r/B) = \int_u^\infty \frac{1}{y}\, e^{-y - \frac{r^2}{4B^2 y}}\, dy$$

$u = r^2\ S/(4\ Tt)$

s, the drawdown in ft

Q, the discharge from the mine, ft^3/day

T, the transmissivity, ft^2/day

S, the storativity

B, the leakage thickness, ft

and t, the time.

The derivation of the equation is based on the following assumptions:

1. A constant value of S/T. Actually, S/T is an unknown function of time.
2. Aquifer is infinite in extent. Aquifers are not infinite in extent and boundary effects cannot be excluded even in the case of large aquifers as the Castle Hayne.
3. Storage in the confining bed is negligible. This implies that at any instant of time the drawdown varies linearly across the confining beds. This certainly is not satisfied for small values of time as evidenced by the development of the cone of

depression in the overlying and underlying beds. The assumption is realized at large values of time if the system approaches a quasi-steady state, as the Castle Hayne aquifer has.

4. No drawdown in the source bed aquifer. This implies that the transmissivity of the source bed is effectively infinite. This assumption is valid for small values of time. This can lead to significant errors at large values of time. A preliminary analysis undertaken for this study shows that the transmissivity of the water table aquifer as seen from the Castle Hayne formation is about the same as that of the aquifer itself despite the presence of the rejected recharge.
5. The direction of flow is vertical in the aquitard and horizontal in the aquifer. We know that both the Yorktown and the Beaufort formations have developed cones of depression. When the conductivities are two or more orders of magnitudes greater than those of the aquitard, the error is less than 5 percent (11). This is not true for the Castle Hayne aquifer system.

Three types of analyses were conducted for the aquifer system. These were:

1. the conventional type curve method for determining the characteristics of the system,
2. Hantush's inflection point method, and
3. analysis of the time rate of change of drawdown.

The data used for these studies were for the first 15 months of pumping. As the rate of pumping and the number of wells operating has varied, particularly in the earliest period, the drawdown values were normalized by dividing them by an equivalent number. The equivalent number was obtained by dividing the total average discharge of a period by 4.3 mgd, this being the daily discharge of a 3,000 gpm well. The aquifer was assumed to be effectively infinite for these calculations.

6.1 Type Curve Method

The mechanics of the method are given in the Consultant's Report. Several wells on both sides of the Pamlico were included in the analysis. The values of transmissivity ranged from 240,000 gpd/ft to 290,000 gpd/ft. The values of storativity were from 4×10^{-4} to 9×10^{-4} and the leakage thickness varied from 40,000 ft to 90,000 ft.

6.2 Inflection Point Method

The procedure for applying the inflection point method is given by Hantush (12). The values of transmissivity averaged about 280,000 gpd/ft. The leakage thickness was 80,000 ft and the average values of the storage coefficient was 4×10^{-4}.

6.3 Rate of Change of Drawdown with Time

The time rate of change of drawdown at any point depends on:

(i) the values of S/T

(ii) the value of leakance, L = K'/b'

where K' is the hydraulic conductivity of the aquitard and b' is the thickness of the aquitard, and

(iii) distance from the boundary. The smaller the distance of the pumping center from a recharge boundary, the smaller will be the time required to reach stabilized drawdowns. The boundary effects were not studied in this analysis. By the differentiation of the expression of the drawdown with respect to time

$$\frac{\partial s}{\partial t} = \frac{Q}{4\pi Tt}\, e^{-\frac{Sr^2}{4Tt}}\, e^{-\frac{K't}{Sb'}}$$

in which s is the drawdown, t is the time, Q is the well discharge, T is the transmissivity, S is the storage coefficient, K' is the vertical hydraulic conductivity of the confining layer, and b' is the thickness of the confining layer. The second exponential expresses the influence of leakage. It is independent of the transmissivity and of the distance of the observation well from the center of pumping. It depends only on the ratio L/S where L is the leakance (K'/b') and the storage coefficient. Its numerical value becomes small with time. The first exponential expresses the decay of the rate of drawdown in a confined aquifer without leakage. Its value increases with time and approaches asymptotically a value of unity.

The alternative expression which is more convenient to use is

$$\partial s/\partial t = Q/(4\pi Tt)\, e^{-\frac{r^2}{4Dt}}\, e^{-\frac{Dt}{B^2}}$$

where D is the diffusivity T/S.

For an observation well the values of the reduced drawdown were computed at the beginning of each month from the observed values. Values of D and B were assumed and the theoretical change in reduced drawdown were calculated. These were compared with the actual change. New values of D and B were adopted to get a better fit. No conclusive results could be obtained from this analysis. Different values of D and B were obtained for the same well for different periods. This may have been due to the variability of the discharge or due to the boundary effects. However, the values of B obtained were consistently lower than those implied by the Technical Report.

Chapter 7

STEADY STATE MATHEMATICAL MODELS

In this chapter, it is assumed that the water-bearing formation and the confining layers have uniform hydrogeologic characteristics. The relationships developed and the results are based on this premise. In actual fact, the aquifers and confining layers have non-uniform characteristics. The analysis will produce values for the formation constants: transmissivity, storage coefficient, and leakage thickness which might be called "effective values" for non-homogeneous system. The hydrologic significance of these values is that if the actual non-uniform aquifers and semi-confining layers were replaced in the field by uniform aquifers and aquitards having transmissivity, storage coefficient and leakage thickness equal to the effective values, the hydrologic quantities, heads, discharges, etc. at the point of observation used for analysis would remain practically unchanged.

This type of analysis is justified on several grounds:

1. This analysis allows us to concentrate on the main features of a problem without having to pay inordinate attention to complicated details. It can provide valuable insights into the possible solution of the more realistic situation.
2. We are primarily interested in the regional effects and should not allow ourselves, at least in initial stages, to be too greatly influenced by the individual data points.
3. Even in the more detailed studies, analog and digital simulation included, considerable aggregation of data is necessary. For the most part we lack independent controls against which this postulated detail can be checked.
4. The variations in groundwater flow are generally small and gradual. Generally, small slopes of the piezometric surface, small velocities that change slowly with time and averaging out of local variations in the flow system are typical characteristics of basins that may be exploited in developing a suitable mathematical model.
5. The detailed specification of the hydrologic data of the Castle Hayne aquifer is largely interpretive. The uncertainties and the inaccuracies in the values of the parameters are quite substantial.
6. The cone of depression centered at Lee Creek mine samples a large area and represents the integrative response of the aquifers and

the aquitards. The non-homogeneity gets substantially averaged out. The particular configuration of the cone of depression gives the characteristics of the system as seen at the pit. This enables a fairly wide areal distribution of both the transmissivity and vertical permeability to be replaced by the average values.

As the Castle Hayne aquifer system attained a quasi-steady state after about one year of pumping, several hypotheses were formulated to see if the conditions in the aquifer could be adequately represented by a simple mathematical model. The boundary conditions postulated and their mathematical expressions for the drawdown are listed below:

1. Infinite non-leaky aquifer with no recharge

$$s = Q/(2\pi T) \quad \ln\,(r_e/r)$$

2. Non-leaky semi-infinite confined aquifer with a single recharge boundary (Figure 8a)

$$s = Q/(2\pi T) \quad \ln \frac{r_1}{r}$$

3. Infinite non-leaky quadrant with:
   a. two recharge boundaries (Figure 8b)

$$s = Q/(2\pi T) \quad \ln\,(r_1 r_2/r r_3)$$

   b. one recharge and one impermeable boundary (Figure 8c)

$$s = Q/(2\pi T) \quad \ln\,(r_1 r_3/r r_2)$$

4. Semi-confined infinite aquifer with uniform leakage

$$s = Q/(2\pi T) \quad K_o(\frac{r}{B})$$

5. Semi-confined, semi-infinite aquifer with a single recharge boundary (Figure 8a)

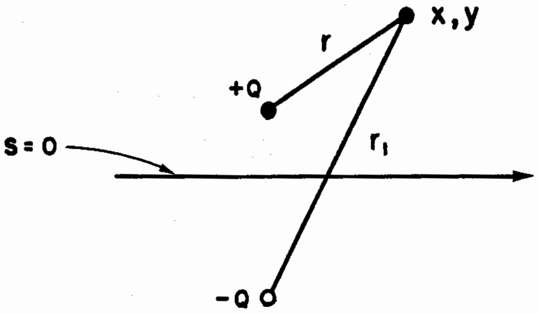
$$s = \frac{Q}{2\pi T} \quad [K_o(\frac{r}{B}) - K_o(\frac{r_1}{B})]$$

6. Semi-confined, infinite quadrant with:
   a. two recharge boundaries (Figure 8b)

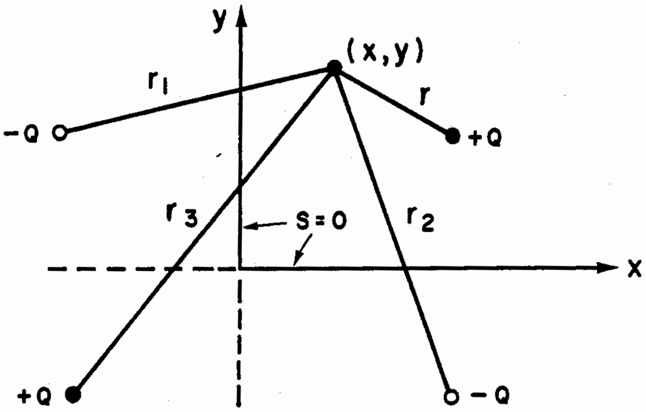
$$s = \frac{Q}{2\pi T} \quad [K_o(r/B) - K_o(r_1/B) - K_o(r_2/B) + K_o(\frac{r_3}{B})]$$

   b. one recharge and one impermeable boundary (Figure 8c)

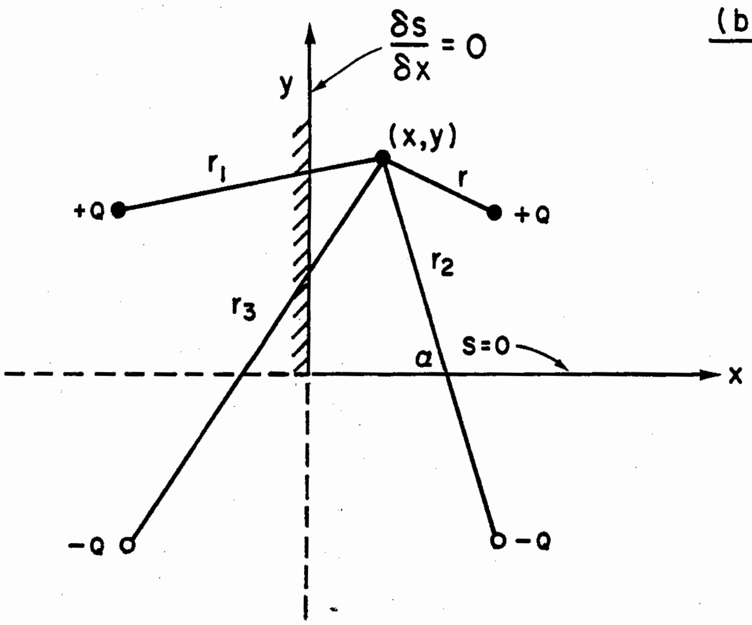
$$s = \frac{Q}{2\pi T} [K_o(\frac{r}{B}) + K_o(\frac{r_1}{B}) - K_o(\frac{r_2}{B}) - K_o(\frac{r_3}{B})]$$



(a) Cases 2,5



(b) Cases 3(a) and 6(a)



(c) Cases 3(b) and 6(b)

Figure 8. Semi-infinite and Infinite Quadrant Models.

7. Circular leaky aquifer with a fixed potential along its boundary and an eccentric well.
8. Infinite two aquifer (Yorktown and Castle Hayne) and two semi-previous layer systems with water withdrawn from the lower aquifer (Castle Hayne). The symbols used in the mathematical expressions are:

s,	drawdown in ft
r,	radial distance from the observation well to the pit
r_1, r_2, r_3	radial distances from the observation wells to the image wells
K_o,	the modified Bessel function of the second kind and zero order
Q,	the constant discharge at the pit
T,	the aquifer transmissivity
and B,	the leakage thickness in ft defined as $[T/(K'/b')]^{1/2}$ where k' and b' are the vertical hydraulic conductivity and thickness of the semi-previous confining bed.

The goodness of fit of the various models was tested by determining the correlation coefficients for 15 observation wells for which the drawdown values were available.

Figure 30 of the 1971 Technical Report giving the net change in the piezometric surface from June 1965 to July 1970 was taken as the basis of the observed drawdowns. Only the results of alternatives 1 to 6 are reported here.

Models (1), (2), and (3) gave very poor fit for the drawdown values and required transmissivity values which were excessive. It was concluded that the behavior of the aquifer cannot be reproduced as a non-leaky aquifer. Model 4, i.e., infinite aquifer with uniform leakage, provided the best fit for the observation wells north of the Pamlico estuary. The goodness of fit elsewhere was fair. Several positions and orientations of the recharge boundary were tried for Model 5. The best fit was obtained with the recharge boundary parallel to the Pungo River unit subcrop pinchout and centered on the area of zero thickness of the aquitard at a perpendicular distance of 13.3 miles from the Lee Creek pumping center. The position of this boundary is shown as 'F-F' on Figure 9. This model provided the best overall fit to the Figure 30 of the Technical Report. The drawdown profile given by the model was compared

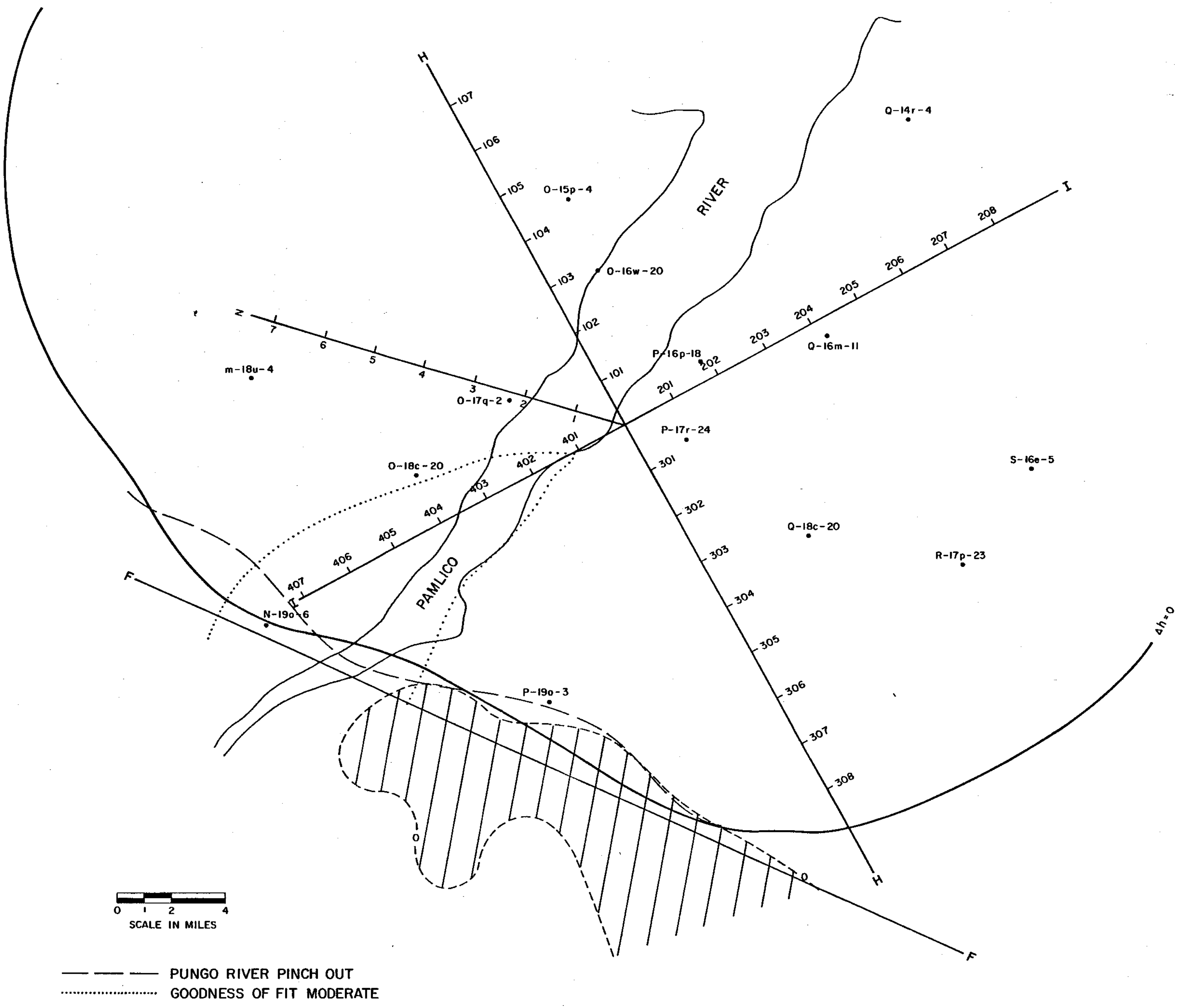


Figure 9. Semi-infinite Leaky Aquifier with a Recharge Boundry.

for several sections. The goodness of fit was excellent except for a small area along the estuary northwest of Lee Creek. The drawdown profiles are shown in Figure 10. The characteristics of the aquifer system obtained from the model are as follows:

Transmissivity = 275,000 gpd/ft
Leakage thickness = 90,000 ft

It can be shown (13) that the quantity of water flowing from the recharge boundary, Q_B, is equal to

$$Q_B = Q_w e^{-\frac{a}{B}}$$

where Q_w is the discharge of the pumping center
a is the distance of the recharge boundary from the pumping well

and B is the leakage thickness. With Q = 52.1 mgd, a = 13.3 miles and B = 17 miles (90,000 ft)

$$Q_B = 23.6 \text{ mgd.}$$

Thus, about 45 percent of the discharge of Texas Gulf wells comes from the recharge boundary and 55 percent from leakage through the aquitard system. Two points about these results need mention. The area where the fit is poor is at and near the estuary and is in the same area where the analog model studies require greater leakance. The model also shows that the leakage thickness in this area has to be reduced to provide a better fit. The vertical permeabilities in the Technical Report for this area are low. These values are not sufficient to carry all the upward leakage to the estuary that was obtained before the start of pumping as shown by the 1965 piezometric surface map. Secondly, the leakage thickness given by the model is lower than the average leakage thickness in the Technical Report, 17 miles vs. about 22.7 miles.

The quadrant model with two recharge boundaries also provides a good fit, but the goodness of fit is not as high as with a single recharge boundary.

There is reason to doubt that the position of the recharge boundary is actually where the model gives it. This position is dictated by the zero drawdown line of the Figure 30 of the Technical Report. The position of that line itself is debatable. There are several observation wells beyond that line that show drawdown now or were affected in the past when pumpage was higher. Furthermore, the drawdown map does not extend far enough to the north to encompass all the area that shows the effect of Texas Gulf pumping. These outlying wells both to the west and to the north play a crucial role in understanding the true

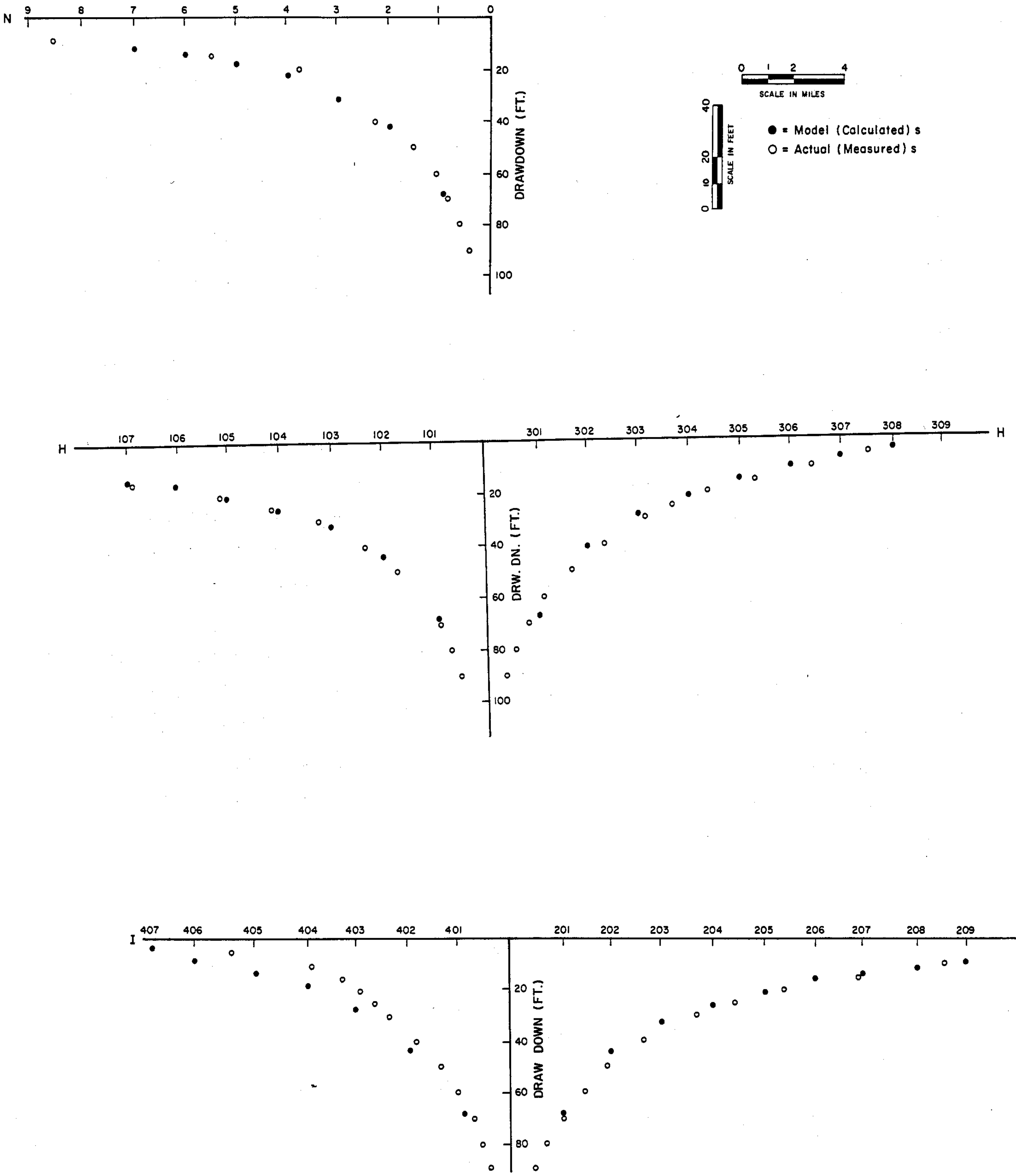


Figure 10. Drawdown Profile of Semi-infinite Model

behavior of the aquifer system. During 1965-1968 the aquifer was subjected to much greater stress than it is at present. There is greater information content in the water levels of that period than there is in 1970 conditions. Jacob (9) fitted several models to the September 1966 water level data. He obtained the best fit with a leaky infinite quadrant with two recharge boundaries. He placed the southwest-northeast recharge boundary at a distance of 24 miles from the mine and the northwest-southeast boundary at a distance of 15-20 miles. The characteristics of the aquifer obtained were

Transmissivity = 281,000 gpd/ft

and Leakage thickness = 70,000 ft

A small area in the south was assigned a B of 35,000 ft. For this study several models were fit to the 1968 data. The quadrant model gave the best overall fit and the results were intermediate between the two models detailed. The closer the boundary is placed to the pumping center, the smaller will be the amount of leakage to obtain the same drawdowns, assuming that transmissivity remains unchanged.

The conclusion is that hydrologically the aquifer can be characterized as a recharge boundary effect superimposed on a moderate leakage effect. The model giving the best fit to the 1970 piezometric surface as drawn in the Technical Report is that of a semi-infinite leaky aquifer. A better characterization of the earlier phases of much heavier pumping is provided by an infinite leaky quadrant with two recharge boundaries.

Chapter 8

ANALOG SIMULATION

The future planning and management of the groundwater resource relies heavily on the ability to predict the behavior of the aquifer system under alternative strategies of groundwater use and development. This assumes a complete knowledge on the part of the planner of aquifer parameters which govern the occurrence and movement of groundwater in a basin. In the Castle Hayne aquifer system the parameters which must be specified are transmissivity, storage coefficient, leakage thickness and the boundary configuration. These characteristics can be determined through the solution of the partial differential equation governing the flow of water through porous media.

The actual aquifer system is extremely complex. It is a spatially distributed, leaky system which is (a) non-homogeneous and anisotropic, (b) nonlinear in its response, and (c) time variant. Even after making several simplifying assumptions, the simulation of the system appeared to be the only satisfactory way of achieving any realistic solution. This was done in two phases:

1. the analog phase was used to test the assumptions made about the system and to arrive at an approximate specification of the system, and
2. the digital computer simulation phase was used to refine the results obtained from the analog model and tested several other alternate hypotheses.

Both methods provide the solution of a system of partial differential equations using finite difference techniques. The analog method simulates time continuously and discretizes the space coordinates. The digital computer techniques discretizes time as well. The analog solution gives a visual picture and feel of the physical system, is more rapid but less precise. Considerable preparatory work is required in both approaches.

It was assumed, not very correctly, that the problem was essentially that of system identification in which the system's excitation stimuli and responses were known but not the characteristics of the regulating mechanisms. Problems of this type do not yield unique solutions since theoretically there is an infinite number of models capable of causing any given excitation-response pattern. However, only a few of these combinations are physically feasible and make geological sense.

The problem offered special difficulties because not even the input or response functions or hydrogeologic boundaries were adequately known. In the case of the response function, that is, the piezometric surface, which was constructed from well level readings, several equally valid representations of the surface for a point in time were prepared by competent hydrologists from the same data. The location of the control points is not evenly distributed and is sparse. The "true" response surface in large parts of the area remains unknown.

For the magnitude of input, several widely varied values had been proposed which offered, at the best, order-of-magnitude limits.

The output of the system was partially known through daily records of the mining operation withdrawals, but the natural discharge of the system was a matter of educated guesswork.

Of the three characteristics of the real potential field, quantity of flow, transmissivity and hydraulic gradient, only the latter characteristic was fixed by the piezometric surface maps and the model had to accurately reproduce the gradients by manipulation of the values of the other two characteristics: leakage and transmissivity.

Under these circumstances, the objective for the model was the production of the most likely characterization of the real system. This necessitated having some means by which to judge the goodness-of-fit of the model. What amounted to "before" and "after" piezometric surface maps had been constructed; the "before" conditions were assumed to be those extant in the system in 1965, prior to the commencement of the mining operation withdrawals. The "after" conditions were the system conditions in 1968, at which time the well logs showed that the system was essentially in a steady state with the drawdowns remaining at stable values. The maps of the piezometric contours are presented as Figures 11 and 12.

Testing of the model consisted of simulating the 1965 conditions and, without further adjustment to the network, being able to satisfactorily simulate 1968 conditions upon application of the appropriate pumping-discharge stress to the model. Construction of such a model constituted the first analog phase

The second analog phase concerned the investigation of the correctness of the model by observing the transient behavior of the system, and this means taking the storage properties of the system into account. If the model could not only simulate the two steady-state conditions but could also simulate the

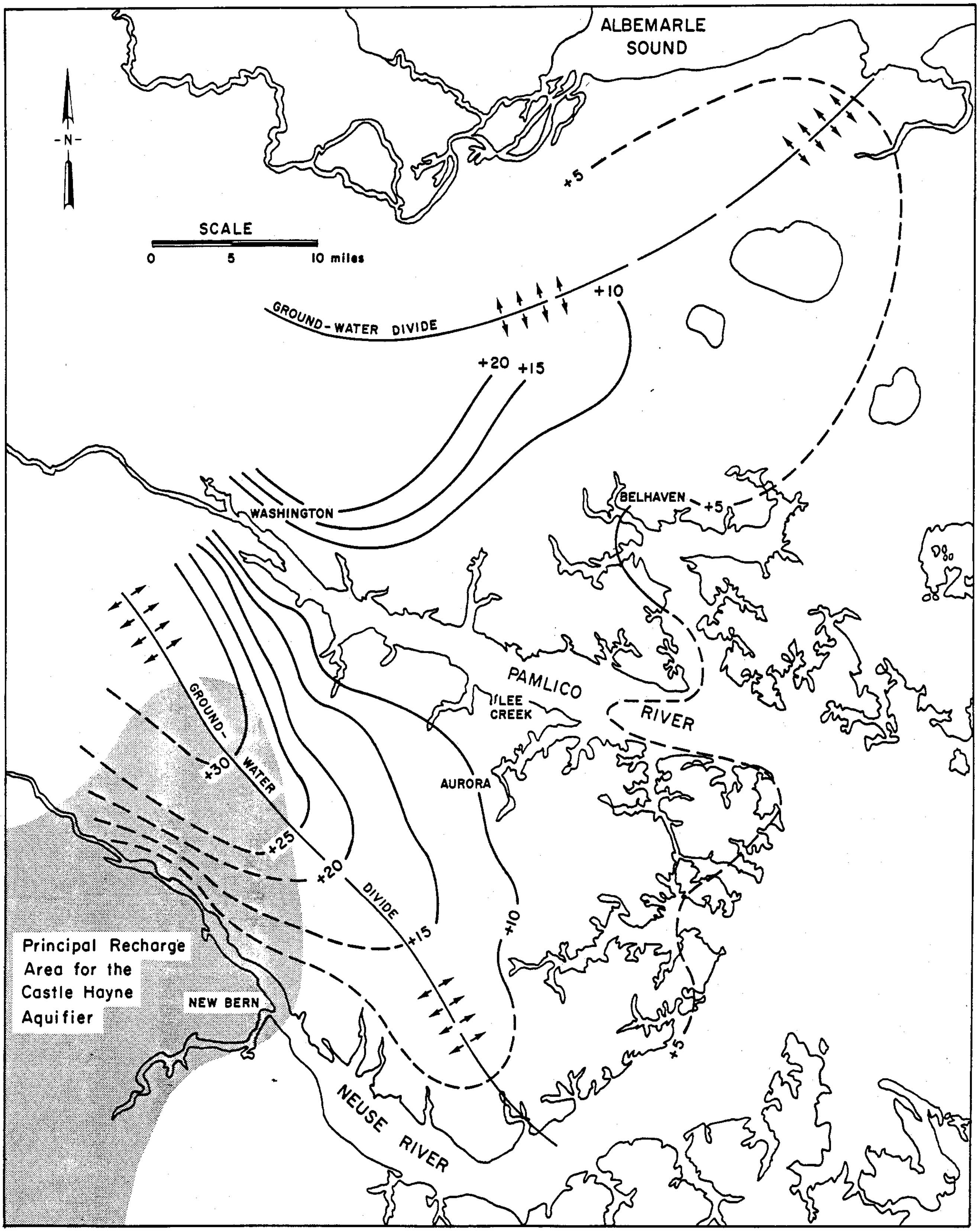


Fig. 11. Map Showing the Piezometric Surface of the Castle Hayne Aquifer in 1965. (N.C. Department of Water and Air Resources)

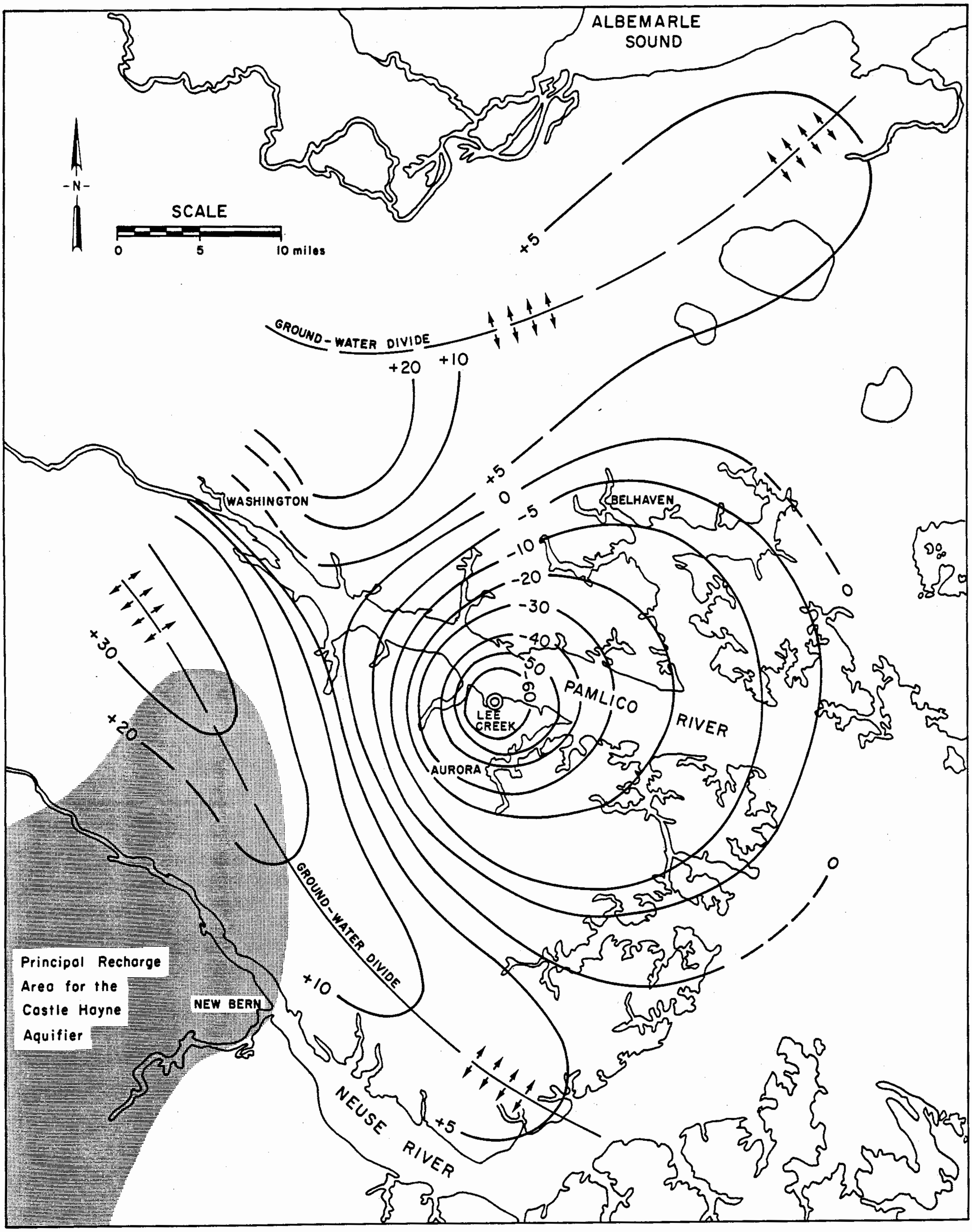


Fig. 12. Map Showing the Piezometric Surface of the Castle Hayne Aquifer in 1968. (N.C. Dept of Water and Air Resources.)

path taken by the real system during the intervening time period, then the model would have achieved the project's objective. Once this stage was reached, the attempt would be made to predict the dynamic behavior of the system under differing withdrawal regimens. Very limited work was done on the nonsteady state on the analog model.

8.1 Boundaries

Every system needs defined boundaries to differentiate it from its environment. In this case, the areal extent of the Castle Hayne aquifer (Figure 2) made simulation of the entire aquifer impracticable.

The effect of the large pumped withdrawals at Lee Creek was known to be felt up to forty miles away from the pump site; thus, it was reasonable to concentrate attention on some three thousand square miles of the aquifer with the site situated at the center.

For conceptual ease, Albemarle Sound was taken as the northern boundary; the eastern boundary was eighty miles east of the pumping site at Lee Creek; the southern boundary was ninety miles south of Lee Creek; the western boundary was the pinchout line of the Castle Hayne formation (Figure 13).

This does not mean, however, that all the area contained within these bounds was of equal interest and importance; as stated before, the principal region of interest lay close to the pumping site.

The groundwater divides are shown in Figures 11 and 12. This study was concerned with flow of water in the Pamlico River Basin and the piezometric maps implied that change of position of these divides, in the two steady-state conditions, was not large--about five miles--suggesting that the pumping did not cause much diversion of flow of water going to the Neuse Basin or the Albemarle South under non-pumping conditions. Thus, the area of maximum interest was the portion of the system lying between the groundwater divides: the Pamlico groundwater basin.

8.2 Choice of Grid-Size

Descriptions of other research work carried out with the electric analog were vague about appropriate sizes for the lattice but repeated the warning that the total error of the results were functions of this dimension and that it should be small compared to the overall dimensions of the potential surface to be simulated. It so happened that most of the crucial maps available to the investigators in this study, containing information about well locations, piezometric surfaces, etc. were either

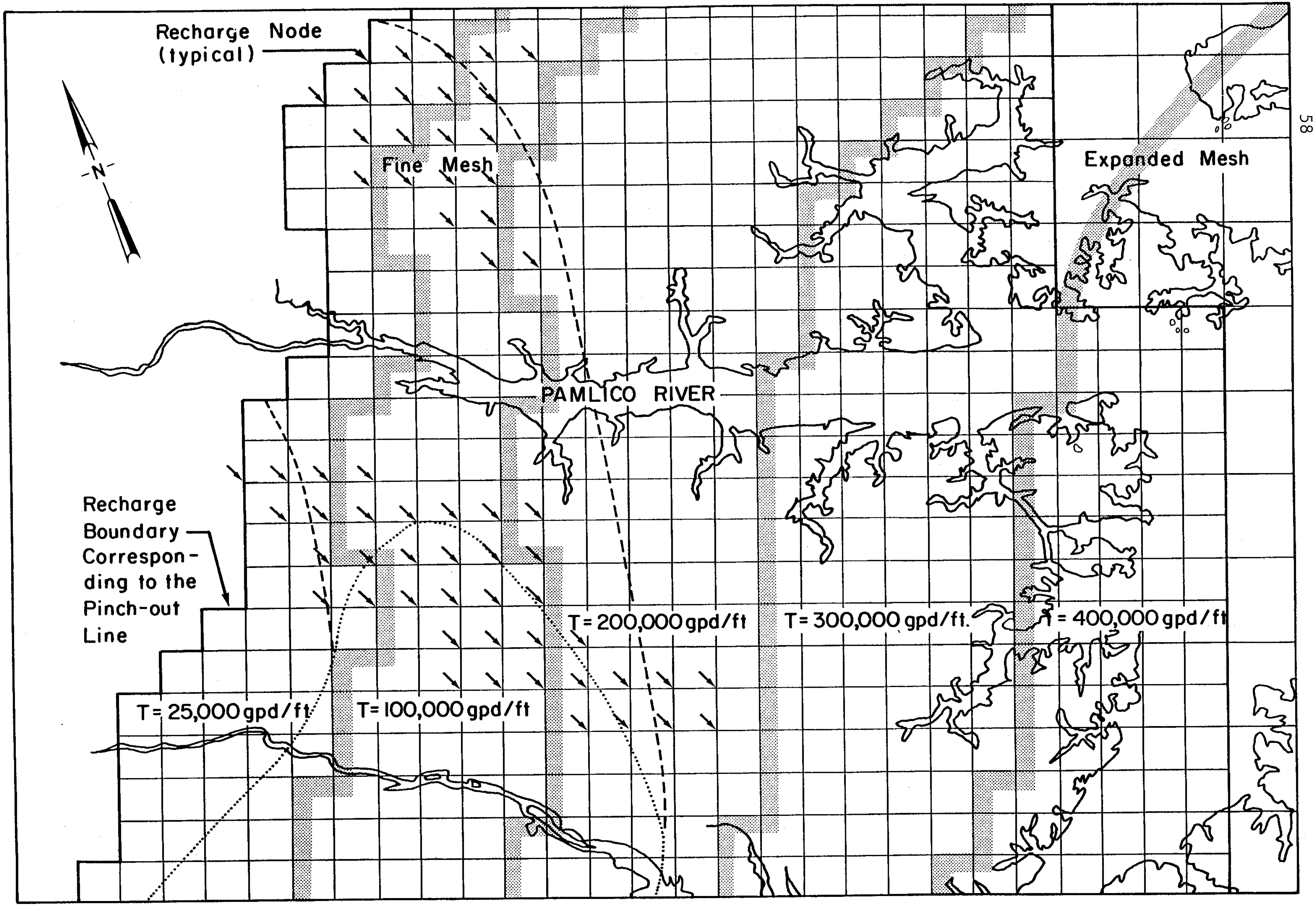


Fig. 13. Map Showing the Boundaries, Meshes, Transmissivities, and Recharge Nodes.

at scales of one inch equal to one mile or one inch equal to two miles. The former scale would have resulted in a network of massive and unwieldy proportions, so the latter scale was chosen. If found necessary, the scale could be changed for some area of the field if finer resolution of the potential field was required. At the scale chosen square mesh unit represented four square miles of aquifer and would be less than 0.1 percent of the total area.

8.3 Grid Orientation

Figure 13 shows a section of the system overlain by the finite-difference grid. One axis was made to align with the center line of the Pamlico River, this being the principal direction of groundwater movement and transmissivity change. The grid was superimposed in such a way that the principal data-well clusters would lie under or close to nodal points, since these were the control points in both analog and prime systems. The Lee Creek mine was represented by a node since the discharge at that location has to be simulated.

8.4 Pumped Withdrawal

Records kept by the mining company at Lee Creek gave the average daily withdrawal rate, in 1968, as 60 million gallons per day. Other quantities of water being removed from the system for agricultural and potable uses were considered to be negligible compared to that amount and were not, therefore, specifically included.

8.5 Transmissivity

Pumping tests that had been carried out at a few locations in and around Lee Creek designated values for the average transmissivity of the Castle Hayne aquifer in the range 250,000 to 400,000 gallons per day per foot with a corresponding coefficient of permeability range of 1,300 to 2,500 gallons per day per square foot. The tests also yielded results of 30,000 gallons per day per foot and 250 gallons per day per square foot for the overlying Yorktown formation. This justified the assumption that the Yorktown formation is an aquitard relative to the underlying aquifer (2).

An isopach map and a contour map of the upper surface of the Castle Hayne formation shows it to slope from 20 feet below mean sea level, with a thickness of 25 feet, in the vicinity of the pinch-out line, to 360 feet below mean sea level, with a thickness of 400 feet, at the western shoreline of the Pamlico Sound; the axis along which increase takes place is parallel to the center line of the Pamlico River. The depth of the formation at Lee Creek was determined to be 300 feet; this included the whole depth of the formation. As stated before the lower portion of the formation is less permeable than the top portion. Assuming that 50 percent of the formation acted as the aquifer, but keeping the full depth of the formation for convenience, the coefficient of permeability for the whole aquifer was taken to be equal to 1000 gallons per day per square foot; the transmissivities based on this value ranged from 25,000 gallons per day per foot at the pinch-out line to 400,000 gallons per day per foot at the Pamlico Sound. It simply involved taking the values off the isopach map and multiplying them by 1000 gallons per day per square foot.

The method chosen to arrive at the areal distribution of transmissivity values was to let a transmissivity value extend its influence from midway between the distances to the next transmissivity contours. For example, the value of 200,000 gpd/ft covered the area between 150,000 and 250,000 contours. The lowest and the highest values became border regions. This resulted in five regions illustrated in Figure 13: 25,000, 100,000, 200,000, 300,000, and 400,000 gallons per day per foot.

8.6 Recharge Areas

It was assumed that the most likely place for direct recharge to the Castle Hayne aquifer to take place was in the areas where the cover on the aquifer was less than 25 feet. These hypothesized recharge areas were located on the map showing the top of the Castle Hayne formation and transferred to the model to identify the recharge nodes. Two recharge areas were postulated, one to the north of the Pamlico River and the other to the south. Several values of the recharge per unit area were tried. The initial areas had to be somewhat enlarged to give a good fit.

8.7 Leakage

Leakance values tested on the model varied from 1×10^{-5} gallons per day per square foot per foot to 2×10^{-3}.

8.8 Errors

Stallman (14) considers the several sources of error that must be examined in evaluating the accuracy of analog or finite-difference solutions:

a. truncation or round-off errors,

b. observational errors made in measurements taken off the model,

c. errors due to the instability of the electrical characteristics of the model or control equipment,

d. errors due to representing the continuous groundwater system as a group of finite elements, and

e. inaccurate proportioning between the aquifer properties and the components in the model.

Walton and Prickett (15) note the accuracy as being:

> ". . . a matter of the quality of resistors and capacitors, the effects of finite-difference approximations and the signal-noise ratio. Inaccuracies due to the quality of resistors and capacitors is small, owing to the error-equalizing property of the resistor-capacitor network. Errors due to the finite-difference approximations are minimum when the network spacing is small in comparison to the size of the aquifer; errors depend in part on the nature of the potential field being analyzed."

The accuracy of the data derived from an electric analog model is highly dependent on the accuracy with which the model reproduces the aquifer. Tolerance for error in groundwater studies is generally high, since the parameters and structure of the underground system are seldom known closer than plus or minus 10 percent. Under such circumstances, it does not make a great deal of sense to strive for analog accuracies much higher than this: add to this the error inherent in the finite-difference equations and errors due to highly irregular potential fields, which do not submit well to these types of solution methods.

> "No general criteria exist for exactly predicting errors of these types . . . Error analysis has generally been made from a viewpoint that looks towards the unknown solution. For most groundwater problems this position is untenable. A solution should be obtained through model design guided by experienced intuition, and error evaluations should be made from the completed solution." (14)

It is generally accepted that of the first five sources of error mentioned by Stallman the last two offer most serious concern for error production. Given the quality of present electronic equipment, the errors in measurement and stability are lower than one percent, and the probability is small that lack of consideration about their performances will lead to serious oversight of an error source.

Errors are also inherent in the mathematical interpretation of the real system, such as the lumping of the system area at nodes, and in the manner in which the boundaries and hydraulic connections to the other systems are simulated.

The next important source of error is the method by which the Laplace equation is solved; namely, the finite-difference approach, since this method involves an approximation of the equation.

It is very difficult to put a number on the magnitude of error involved in estimating the physical parameters of the system because even if the results obtained from the field tests were known to be absolutely correct they only represent isolated points in the system. These isolated values have been generalized to represent large areas of the system. Under such circumstances the actual value of the parameter at some point in the model may be much different.

8.9 Results

1. Simulation was conducted under the assumptions that (a) the aquifer was completely confined (i.e., no leakage) and (b) the aquifer was in hydraulic contact with the overlying and underlying beds (i.e., the aquifer was leaky). It was not possible to reproduce the historical response of the aquifer under the no-leakage hypothesis. It was concluded that the aquifer was leaky.
2. The lateral flow across the pinch-out line of the Castle Hayne aquifer was extremely small; i.e., about .2 mgd. This was further tested by assuming that the pinch-out boundary was (a) an aquipotential, and (b) an impermeable boundary. The effect on the potential distribution within the aquifer was not significant.
3. The potential field was found to be surprisingly insensitive to reasonable changes in the transmissivity values even when the area covered was quite extensive. It is concluded that moderate changes in the transmissivity values were not critical to the response of the system.
4. The aquifer was in hydraulic connection with the Roanoke River and Albemarle Sound.
5. Under 1965 steady-state conditions, about 20 mgd were entering over an area of approximately 90 square miles north of the Pamlico River and about 45 mgd over an area of approximately 200 square miles south of the river as direct recharge. This is equivalent to an average recharge rate of 4 inches per square mile above the Pamlico and about 4.5 inches per year south of Pamlico. These results are at variance with either the magnitude of recharge or the area over which it takes place contained in the 1967 Consultant's Report or in DeWiest's paper.

6. Approximately 20 million gallons per day were discharged from the aquifer to the Pamlico estuary between Washington, North Carolina, and Lee Creek as compared to 30 mgd contained in the 1967 Consultant's Report.
7. The pumping has had little effect in the main basin recharge areas by the pinchout line. The actual increase in recharge from the capture of rejected recharge is approximately 6 million gallons per day.
8. Under the 1968 conditions, the flow to the Yorktown formation was reversed with approximately 25 mgd entering the system.
9. The base flow entering the Pamlico estuary has been reduced by about 30 mgd under 1968 conditions while there is still fresh water entering the estuary in the uppermost reach.
10. The leakance in the updip portions of the aquifer is an order of magnitude higher than in the rest of the aquifer. There is, however, no evidence of the aquifer being more leaky near the Lee Creek mine as was concluded by the 1967 Consultant's Report.

Chapter 9

DIGITAL COMPUTER SIMULATION

The purpose of this phase of the study was to simulate the response of a leaky confined aquifer to pumping from one or more wells by solving the governing two-dimensional groundwater flow equations. The Castle Hayne aquifer system is irregular in shape, non-homogeneous and anisotropic in character. The technique of solving the differential equations should be able to take into account the transient leakage from confining beds.

The differential equation for non-steady flow of a compressible fluid in an elastic non-homogeneous porous medium can be written (16).

$$\frac{\partial}{\partial x_i}\left(T_{ij}\frac{\partial h}{\partial x_j}\right) = S\frac{\partial h}{\partial t} + W(x,y,t)$$

where T_{ij} is the transmissivity tensor
h is the hydraulic head
S is the storage coefficient
t is the time
W is the volume flux per unit area

W is the source function which may consist of pumping, leakage from a confining layer, or recharge.

If the coordinate axes are aligned with the principal directions of the transmissivity tensor, the finite difference approximation to the governing equation may be written by discretizing the space and time variables. A rectangular net or grid is superposed on a plan view of the groundwater reservoir. If N is the number of nodes, N simultaneous equations have to be solved.

The basic computer program which computes the hydraulic head in the aquifer as a function of space and time has been devised by Pinder of the U. S. Geological Survey (17). The program is written in FORTRAN IV for the IBM 360 system. The equations are solved implicitly at each node of the grid for the specified boundary conditions, stepping through time in increments of Δt seconds until the period of analysis is complete. To determine the precision of the calculations, a mass balance is also computed. The procedure known as iterative alternating implicit directions is used for solving the equations.

The basic computer program was modified in several respects for use in this study. At each node the following information is recorded as data sets:

(i) Transmissivity of the aquifer

(ii) Storage coefficient of the aquifer

(iii) Thickness of the leaky confining bed

(iv) Hydraulic head in the source bed; i.e., water table elevations

(v) Initial hydraulic head values in the aquifer

(vi) The vertical hydraulic conductivity of the confining bed

(vii) Specific storage in the confining layer

(viii) Discharge from wells

(ix) Recharge

(x) Dimensions of the rectangular grid elements of the superposed finite-difference mesh.

(xi) Temporary hydraulic head values for use in extending earlier run to a longer period.

A 27 x 27 grid was used. The size of the mesh was finer near the pit. Starting from the pit the following spacing in feet was used in each of the four directions:

1300, 2000, 2650, 5300, 5300, 5300, 5300, 10600, 10600, 10600, 10600, 10600, 10600, 10600.

The area covered was 36 miles x 36 miles; i.e., about 1300 square miles.

Three sets of transmissivity values were used:

(i) The values in the Technical Report

(ii) Values in the Technical Report modified to partially smooth the extreme bends in the transmissivity contours

(iii) Transmissivity distribution pattern based on the Upper Castle Hayne aquifer thickness with a decrease in the hydraulic conductivity values downdip.

Two sets of water-table elevations were used. For earlier runs these were read from the water table contour map. It was discovered that these values did not give the same head difference as contained in Figure 35 of the Technical Report. To assure compatibility with the leakage studies, water-table elevations were derived by adding the head difference to the piezometric surface elevations. These were the values which were used in all subsequent runs. In a few runs, the water table elevations were assumed to be equal to the initial piezometric surface elevations. This is what is required by the theory;

otherwise strictly speaking, the solution of the differential equation is not valid.

Numerous runs were made assuming diverse distributions of vertical permeability. The recharge assumptions in the zero aquitard thickness areas made in the Technical Report were not stated. An equipotential boundary through the center of gravity of these areas was postulated. Keeping the ratios of vertical conductivity inviolate, the permeability values were multiplied by a factor of 1.0, 2.0, 5.0, 6.5, 7.0, and 10.0 to investigate the resulting stability characteristics of the drawdowns. In an alternate set of runs, different recharge intensities were tried to avoid unreasonable build-up of head in the zero aquitard thickness areas. With these values of recharge, the vertical permeabilities were multiplied by different factors.

In another set of runs, uniform values of vertical permeabilities were assumed to get an idea of the magnitude of permeabilities that will be required to get the observed values of drawdowns.

To see the effect that the inclusion of transient leakage from the aquitards will make, runs were made with and without transient leakage.

It was not possible to reproduce the drawdowns using the data in the Technical Report. The vertical permeabilities have to be increased 5 to 7 times to attain the stability of the drawdowns. Even then, the drawdowns in most of the areas in the east are much more than shown by the 1970 piezometric surface. The only conclusion that can be reached is that the vertical permeabilities of the aquitards are greatly underestimated in the Technical Report and the relative distribution of the magnitudes is also at fault. One possible explanation may be that the upward leakage from the underlying Beaufort may be more significant in the eastern areas than the downward leakage from Yorktown and Pungo River formations. Alternatively, the leakage from Yorktown may be higher than presumed.

Inclusion of the storage in the aquitards and the resulting transient leakage reproduces the time rate better than the assumption of negligible storage in the aquitard. In this might be still another explanation for the disparity discussed earlier.

The leakage in the upper reaches of the estuary and the adjoining areas has to be fixed at a higher level to reproduce the drawdowns of the observation wells in those areas. As no flow net was constructed for the 1965 piezometric surface, this can easily escape detection by the flow net of 1970 piezometric surface. As has been pointed out earlier, the extent of upward leakage implied by the 1965 contours cannot be transported to the Pamlico estuary with the vertical permeability magnitudes assumed.

The disparity between the results of the flow net analysis and analog and digital simulation should be investigated further. Flow-net analysis is really a graphical method of solving the differential equation governing the flow of groundwater. The trouble most probably lies in the acceptance criterion and the fact that it deals only with steady-state conditions. After one assumes a particular distribution of vertical permeabilities and arrives at a distribution of transmissivities, how does one decide that this is a reasonable distribution of transmissivities? The criteria for rejection or for acceptance are not sharp enough. Not only is it enough to reproduce the steady state in 1970, it is essential to be able to trace the path that the system has taken to reach that steady state. The testing ground is the non-steady state data.

Both the alternative distributions of transmissivities give slightly better fit than the distribution in the Technical Report.

9.1 Alternative Strategies of Development

No future projections of water needs for industrial, municipal, and agricultural development in the area were available for use in this study. The only requirements known with any degree of assurance are for the dewatering operations of existing and proposed open pit mines. The model was applied to three configurations of development within the Beaufort County. The strategies considered were (1) 60 mgd at the Lee Creek mine site, (2) a quasi-steady state drawdown of 120 feet at the Lee Creek mine and a drawdown of 150 feet at the proposed North Carolina Phosphate Corporation mine at South Creek, and (3) an additional pumping center away from the natural recharge and discharge areas with the stipulation that the drawdown at the new location not exceed 130 feet.

The first alternative was selected to test the ability of the model to reproduce the drawdowns historically observed in the earlier phases of pumping at Lee Creek. The comparison with the observed drawdowns was considered to be satisfactory.

In the second strategy, an additional pumping center was placed at a location 4 miles south of the existing Texas Gulf pit at Lee Creek.

This location at South Creek is the potential site of the mining operations of the North Carolina Phosphate Corporation. For the stipulated drawdowns, the pumping rates required were:

35 mgd at Texas Gulf site
and 56 mgd at the South Creek

The piezometric surface map and the values of drawdowns for the various cross-sections are given in Figure 14. The model values show greater variations from the values in the Technical Report in the vicinity of the pumping centers and near the recharge areas.

The third pumping center was selected away from the areas of natural recharge and discharge where the sensitivity of the drawdowns to the withdrawal rates will be greater. So as not to concentrate all the development south of the Pamlico River, the pumping center was placed north of the river, 20 miles north-northeast of Lee Creek mine near the map grid-point M-15. The stipulation drawdowns were 120 feet at Lee Creek, 150 feet at South Creek and 130 feet at the new site. The pumping rates for these assumptions corresponded to

28.3 mgd at Lee Creek
52.5 mgd at South Creek
and 49.0 mgd at the third pumping center

As this pumping center location is different than in the Technical Report, it is not possible to compare these values.

9.2 Summary and Conclusions

This study would have been much more meaningful and useful had the projections of the economic development and water needs of the area been available. The goal of efficient, comprehensive development of the Castle Hayne aquifer cannot be realized without this information. The social costs imposed on other users by mining operations and an equitable distribution of these costs depends on the forecast of pumping lifts and water quality. This, in turn, is dependent on the prediction of hydrologic variables which cannot be made realistic in the absence of demand information.

It is not possible to reproduce the 1970 piezometric surface with the data of the Technical Report. The values of vertical hydraulic

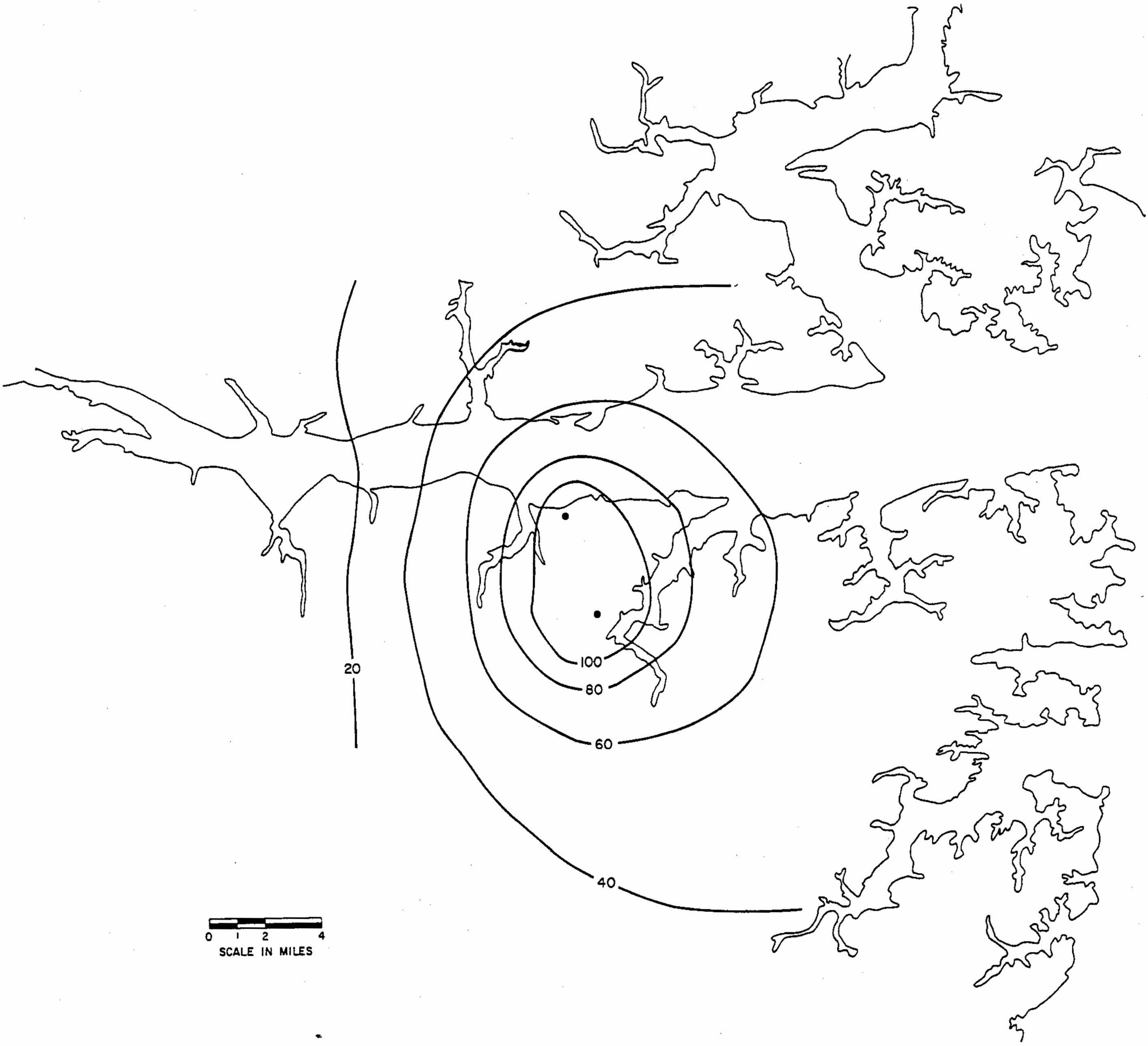


Figure 14. Drawdown for Two Pumping Centers - Texas Gulf-35 MGD, NCP-55 MGD.

conductivity of the confining layers, particularly in the vicinity of the upper reaches of the Pamlico estuary, are greatly underestimated. Analyses from several different approaches confirms this as well. The distribution of transmissivities is also different from that given in the Technical Report.

The consideration of storage in the aquitards gives a better fit to the historical data than does its exclusion.

The assumption of constant water-table elevations will lead to eventual quasi-steady state conditions, no matter what the amount of withdrawals. This assumption cannot be sustained outside reasonable limits. The two alternatives available to circumvent this are (a) use of a more sophisticated analysis taking the transmissivities of source bed into account, or (b) specification of maximum available drawdowns through the area.

Chapter 10

WATER QUALITY

The major limiting factor in the continued development of the groundwater resource in the Castle Hayne aquifer system is the future quality of water. Any realistic evaluation of groundwater management alternatives requires the ability to predict the long-term local and regional effects on water quality. In a complex system like the Castle Hayne, development of acceptable methods of analysis to trace the movement of contaminants is not straightforward.

Three phases of analysis are involved in predicting contaminant transport through porous media:

1. Macroscopic fluid flow analysis which gives the so-called convective flux,
2. Microscopic flow analysis for diffusion and hydrodynamic dispersion (these are the only mechanisms which can lead to any mixing in the two-dimensional flow), and
3. Interaction of contaminants with the physical, chemical, and biological environment.

The quality constituents in groundwater can be classified as to their relative movement with respect to that of the transporting water: the pollutants which travel with the water (fluid coincident) and the constituents which move at a rate that is significantly different from that of the transporting fluid (non-coincident). Many artificial recharge studies have shown that not all quality constituents are fluid coincident. Even the contaminants that are fluid coincident have significantly different dispersion characteristics.

As a first approximation, fluid-coincident constituents, like chlorides, can be analyzed for convective flux only. To determine where chlorides may go and how long will be required, it is necessary to determine:

1. pattern of flow,
2. velocity distribution and travel times, and
3. flux or material distribution.

The principal factors that control the flow system are:

a. the physical properties of the rock skeleton of the system; for example, porosity, thickness, and stratification of the aquitards and the aquifers,
b. degree of homogeneity and anistotropy of the porous media,
c. spatial distribution of hydraulic conductivity and storativity of the aquifers and the aquitards,

d. the distribution of hydraulic heads and gradients for the present, or future, conditions of interest, and

e. quantities of water flowing into and out of the water-bearing materials and the places where the inflow and outflow occur.

As a result of the concentrated heavy withdrawals, the piezometric level has been drawn down to about 140 feet below sea level near the pumping center, and the general flow of groundwater from all directions is towards the Texas Gulf mine. The aquifer has geohydrologic boundaries with the overlying Yorktown and the underlying Beaufort formations. As fresh water is pumped out, it is replaced by the water moving and contributing to the discharge as follows:

1. fresh water from the recharge areas,
2. water with relatively low chloride content from the east followed by the westerly migration of the fresh water-salt water interface,
3. leakage water of varying quality and composition from the aquitards above and below, and
4. brackish water from the Pamlico estuary and other surface water bodies in areas where the hydraulic gradient has been reversed.

Water with high chloride content (or having other objectionable qualities) whether in the aquifer, the aquitards, or on the surface, when intercepted by the cone of depression, will be induced by the prevailing hydraulic gradients to flow toward the point of withdrawal. This leads to the deterioration of the water quality in the aquifer system between the points of interception and pumping. The potential sources of deterioration of the water quality in the Upper Castle Hayne aquifer can be enumerated under the following headings:

1. lateral movement of the lenses of brackish waters occurring within the fresh water zone in the aquifers,
2. lateral movement of the salt water-fresh water interface in the Castle Hayne aquifer,
3. upconing of the salt water that underlies the fresh water in the Castle Hayne in the vicinity of the mine,
4. regional migration of the salt water downward from the Yorktown formation and upward from the Beaufort formation because of the relative head differences between these aquifers and the Castle Hayne,

5. the "squeezing" of salt water from the confining beds into the aquifer due to compaction, and
6. vertical movement of the estuarine water from the Pamlico.

The first three sources relate to the saline water that occurs naturally in the aquifer; the last three involve the behavior of the confining layers.

10.1 Lateral Movement

To solve the problem of the transport of a dissolved solid, convective and dispersive transport should be considered simultaneously. The governing differential equation for two-dimensional flow is (18):

$$\frac{v_x}{\Theta}\frac{\partial c}{\partial x} + \frac{v_y}{\Theta}\frac{\partial c}{\partial y} + \frac{\partial c}{\partial t} = D_x \frac{\partial^2 c}{\partial x^2} + D_y \frac{\partial^2 c}{\partial y^2}$$

where v_x and v_y are average darcian velocities in the x and y direction, respectively, Θ is the porosity, D_x and D_y are dispersion coefficients and c is the concentration of salt in mass per unit volume of solution. Pinder (18) has devised a digital computer solution of this equation. The convective term is larger than the dispersive term. However, if dispersion is ignored, the solution can differ considerably than when the two are considered simultaneously.

The one-dimensional version of the differential equation becomes

$$\frac{\partial c}{\partial t} = -\frac{v_e}{\Theta}\frac{\partial c}{\partial l} + D\frac{\partial^2 c}{\partial l^2}$$

where c represents the concentration of a constituent. The first term on the right-hand side represents convective transport, the second term represents transport of material by dispersion, and the term on the left-hand side represents accumulation. The differential equation governs one-dimensional displacement of a fluid in a porous medium by means of a second, miscible fluid. The effect on concentration variation is given by convective transport with respect to travel, and by dispersive transport with respect to decay. Of special interest in groundwater quality are the cases in which the dispersion term is small relative to the convective term. If we ignore dispersion, the equation reduces to

$$\partial c/\partial t = -(v_e/\Theta)\,\partial c/\partial x$$

This is a partial differential equation of the hyperbolic type. It is very difficult to find a suitable finite difference approximation which combines high stability with a small error. To make valid predictions of future water quality, the transient has to be followed rather accurately. The finite difference technique adopted by Ganus (10), which forms the basis of predictions of groundwater quality in the 1971 Technical Report does not have the requisite properties. His formulation implicitly includes the dispersion term with

$$D = (v/2\Theta)\ \Delta l$$

where Δl is the length of the segment of the streamline. This implicit inclusion of dispersion proportional to Δl should either be justified on physical grounds or should be avoided. Nothing is known about the convergence, cumulative errors or stability of his solution. An estimate of possible range of variation of quality is, therefore, not available. Several numerical techniques (19, 20, 21) have been developed for solving the one-dimensional second order equation (i.e., including dispersion) given above.

The choice of a correct boundary condition for the saline areas is also important; different boundary conditions correspond to solving different physical problems. Adequate attention has not so far been given to this matter. Furthermore, the assumptions with respect to mixing adopted by Ganus (10) are debatable and are inconsistent with the flow pattern adopted.

To get an insight into the possible reliability of estimates of groundwater quality due to the known errors in the estimation of parameters, consider the flow along a streamline. The travel time from point 1 (x_1,y_1) to point 2 (x_2,y_2) can be found by evaluating the integral of the reciprocal of velocity along the length of the flow path involved. That is

$$T_m = \int_{\text{point 1}}^{\text{point 2}} \frac{dl}{|v|} = \int_{x_1}^{x_2} \frac{dx}{v_x} = \int_{y_1}^{y_2} \frac{dy}{v_y}$$

where T_m is the time of travel from point 1 to point 2 and dl is the differential length along the path. The integral is valid only along the path of flow.

If the flow system results are available only as a piezometric surface map, the travel time along the path m between two equipotentials

$$t_m = (\Theta b/T)(l^2/\Delta h)$$

where Θ is the porosity, T is the transmissivity, t_m is the travel time for a distance l along the streamline between the equipotentials with a head difference of Δh and b is the aquifer thickness. Beginning at the pumping center and proceeding along a streamline to selected equipotentials, the cumulative lateral travel times are computed from the relationship

$$T_m = \Sigma \frac{\Theta b}{T} \frac{l^2}{\Delta h}$$

From the points generated in this manner "isochrones" or lines of equal time may be plotted. These are given in Figure 15 for 1970 pumping conditions and for a porosity of 30 percent.

From the location of the areas of high chloride concentration which presently exist in the aquifer, approximate travel times to any point may be estimated. The location of high chloride lobe near the west shore of the Pungo River and a well containing 350 ppm of chloride are shown on the Figure. Under 1970 pumping conditions these points have lateral travel times of approximately 73 and 33 years, respectively. With the two pumping centers the travel times are projected as 54 and 33 years.

The actual concentrations of chlorides will depend on the assumption as to leakage. The consideration of convective flux alone automatically assumes that the two miscible fluids moving through a porous medium will be separated by a sharp interface. No mixing is permissible.

Longitudinal Dispersion

The foregoing calculations and the calculations in the 1971 Technical Report are based on the assumption of no longitudinal dispersion of velocities; i.e., all particles of water in a section will travel at the same speed. By virtue of the property of dispersivity exhibited by all

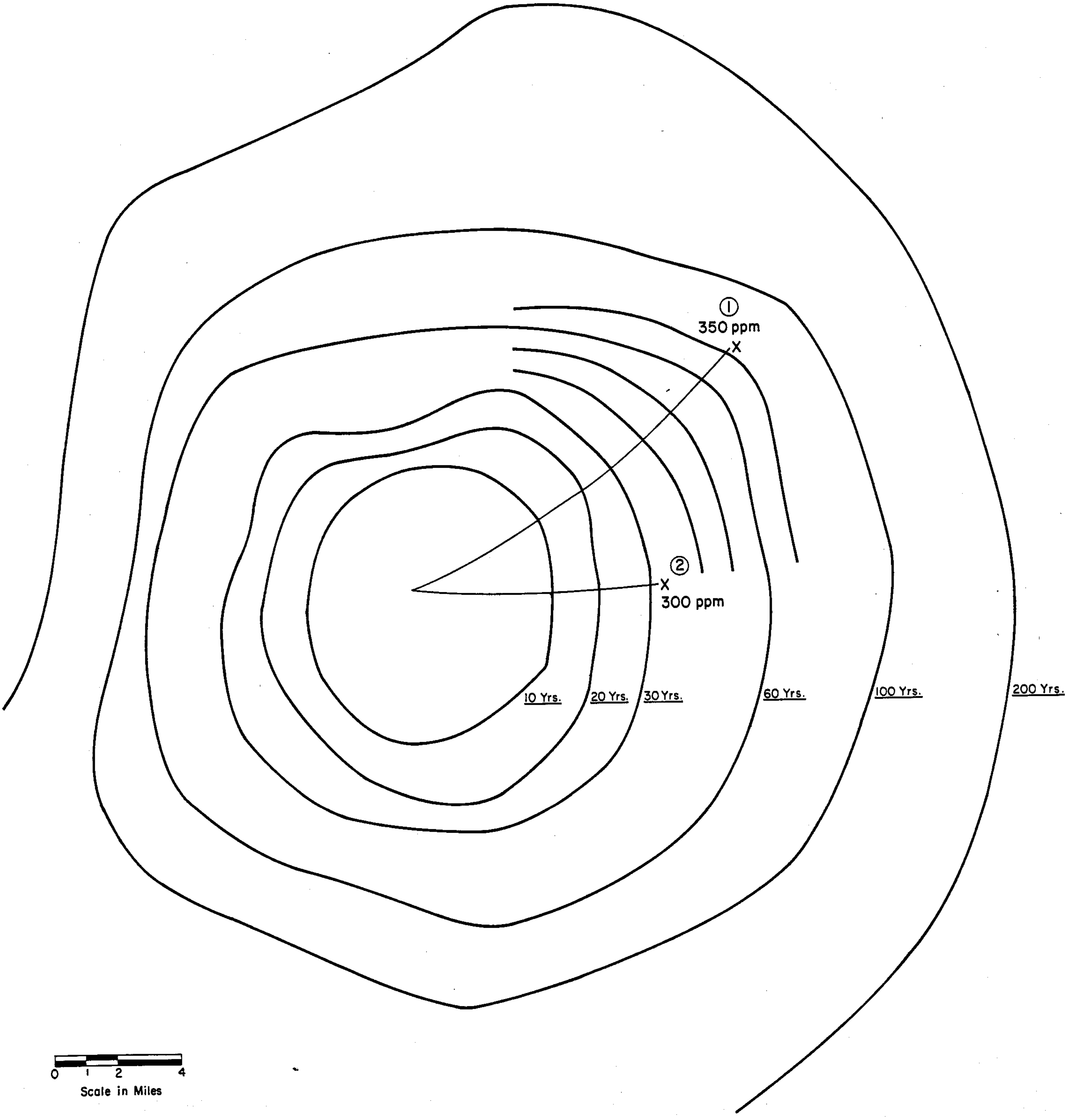


Figure 15. Isochrones for 1970 Pumping Conditions.

sediments, a fraction of the water in the limestone will travel at speeds many times the average velocity. The size of this fraction is significant in limestones. If this is sufficient to cause deterioration of fresh water into which it advances, the times at which deterioration will take place will be much shorter.

Sensivity of Lateral Travel Times

The magnitude of error in the estimated travel times can be investigated in terms of the possible errors in the parameter values. Mathematical expressions can be developed to express these relationships. These are given in Table 6.

Table 6. Sensitivity of Lateral Travel Times to the Errors in Parameters

Parameter	Sensitivity Relationship $\partial t_m/t_m$
l	$2\ \partial l/l$
Δh	$-\ \partial \Delta h/\Delta h$
T	$-\ \partial T/T$
b	$\partial b/b$
Θ	$\partial \Theta/\Theta$

The possible error in distance between the equipotential lines l and Δh depends on the accuracy of the Piezometric map. Because of the inadequate distribution or the absence of the observation wells in large areas of the Castle Hayne aquifer, the position of the contour line is largely interpretive. Moreover, even where actually observed values are available, they have sometimes to be disregarded in preserving the regional character of the piezometric surface. Several such values exist in the 1970 piezometric surface map given in the 1971 report. Thus, the position of the piezometric contour lines is at best approximate. The table above tells us that a 10 percent error in the spacing of contours can lead to 20 percent error in the calculation of the travel times. No actual measurements of the porosity of the limestone exist. This parameter is fundamental in governing the actual velocities. A 15 percent variation from the assumed values can lead to 15 percent variation in the estimation of travel times. Similarly, the actual thickness of the Castle Hayne formation can be significantly different from the assumed thicknesses which are largely interpretive from a few geophysical logs. A 10 percent error will lead to 10 percent error in the estimation of travel times and, hence, concentrations. Errors in the estimation of transmissivities will lead to similar errors in the estimation. Similar comments apply to the possible errors in the estimation of Δh. The obvious conclusion is that the values derived from a single set of values of the parameters could be highly unreliable. There is no escape from carrying out a sensitivity analysis to give the possible range that the values of travel times may assume.

10.2 Fresh Water - Saline Water Interface

At present inadequate data are available to delineate the fresh water-saline water contact in the major water-bearing formations. The zero piezometric surface contour (i.e., mean sea level) in 1970 has shifted significantly to the west from its position in 1965. The seaward flow of fresh water has largely reversed due to the drawdowns caused by pumping at Lee Creek. With increased withdrawals as the zero drawdown line moves eastward, it will intercept increasingly poor quality water. Maintaining the hydraulic head at or above sea level in all areas where Castle Hayne aquifer is directly connected with the brackish water may not be feasible, the position of the interface and its westerly migration should be adequately monitored.

10.3 Semi-Confining Layers

The Castle Hayne aquifer is in direct hydraulic contact with the overlying and underlying formations. As a result of pumping, cones of depression have been developed in the Yorktown and Pungo River formations above and in the Beaufort formation below. As a result of this, there is substantial lateral flow in these formations, and the assumption that there is only one-dimensional vertical flow in these formations is incorrect. Because of the low hydraulic conductivities in the vertical direction, the vertical flow is much weaker than the lateral flow. Be as it may, all computations carried out by all investigators assume only vertical flow. The rate of flow in this direction is given by Darcy's law as

$$v' = \frac{K'}{b'\Theta} \Delta H$$

where v' is the vertical velocity through the confining layer and Θ, K' and b' are the porosity, the vertical hydraulic conductivity and thickness of the aquitard, respectively, and ΔH is the head difference between the water level in the source bed (post-Miocene water table) and the piezometric head in the Castle Hayne. The travel time, t', for a vertical distance, z, above the top of Castle Hayne is

$$t' = \frac{z}{v'} = \frac{zb'\Theta}{K'\Delta H}$$

If one knew the position of the zones of high chloride waters in the aquitard, the time of travel to the Castle Hayne aquifer could be calculated. These will be much smaller than the time required for the water from the Pamlico estuary to reach the aquifer which has to travel through the total thickness of the aquitard. There is lack of conclusive data on the occurrence of zones of anomalous water quality characteristics in the overlying aquitards. But this does not mean that such zones do not exist. The existence of such zones in the Castle Hayne with hydraulic conductivities which are at least an order of magnitude higher would lead one to believe that there are high chloride concentration zones in the aquitards where there is much less potential for flushing. There is stronger evidence for the existence of high chloride zones in the Lower Castle Hayne and Beaufort formations. The consideration of upward migration of such waters into the Upper Castle Hayne was precluded in the Technical Report because of their inclusion in the aquifer system, notwithstanding the fact that their transmissivities are an order of magnitude lower. Additional testing of aquitards and sampling of their water quality is needed. The potential of this source for the deterioration of water quality of the aquifer should be recognized even though quantitative estimates cannot be made because of lack of data.

The available estimates of vertical hydraulic conductivity, K', aquitard thickness, b', and the porosity of aquitards, Θ, are subject to large errors. No observed data for the vertical hydraulic conductivities of any of the formations are available. Analysis of pumping test data or of the quasi-steady state cone of depression give only the values of leakage thickness $B = [T/(K'/b')]^{1/2}$. In reality, none of the quantities b', K' and T are known with sufficient accuracy. There is always uncertainty in apportioning the value of B to several factors. If the values of transmissivities are known with sufficient accuracy, one can get good estimates of leakance, $L = (K'/b')^{1/2}$. In nature, the variations in b' and K' of a semi-pervious layer are often larger than the variations in L. Thus, the leakance of an aquitard is a more workable parameter in the solution of practical problems than the individual quantities b' and K'.

In the 1971 Technical Report the values of vertical hydraulic conductivities have been assumed to decrease uniformly from updip to downdip. This is a reasonable assumption. However, one has to recognize that natural materials seldom conform to such uniform variation. At places with a lower resistance, b'/K', the leakage through the confining layer will be stronger compared with the leakage elsewhere. (There is evidence of such areas near the estuary.) The leakage will be more or less concentrated at the weak spots. These spots are of much greater importance than their area might lead one to expect. Furthermore, if the flow pattern changes due to the introduction of other large pumping centers, the weak spots which previously contributed little to the total leakage may attract relatively much larger quantities of water thus altering the effective resistance (13). Thus, the behavior of the confining bed cannot be simulated accurately by assuming a uniform variation in its properties, and the hydraulic characteristics may change when the total stress and its distribution is changed.

Another phenomenon which may give rise to local deterioration may deserve mention. The water confined in an artesian system being under pressure tends to support the confining beds. When the pressure in the aquifer is reduced, if there are fine-grained materials in the aquitard, water is 'squeezed' from them into the aquifer. The presence of lenses of bad quality water may give rise to local deterioration. Furthermore, the lowering of the head in the aquitards, in response to the cone of depression in the Castle Hayne, will cause compression of the formations and will release water of compaction into the aquifer which may be saline. This is more of a possibility from the Beaufort formation.

10.4 Vertical Movement of Estuarine Water

The 1965 piezometric surface map shows that the Pamlico River has a hydraulic connection with the Castle Hayne aquifer. There was an upward leakage of about 22 mgd from the aquifer to the estuary. As a result of pumping at Lee Creek, the gradients have reversed over a major portion of the estuary. The direction of flow is now from the estuary to the aquifer. The question to be answered is how long will it take for the estuary water to appear in the aquifer. That it will eventually happen is not disputed.

The vertical travel time may be calculated from the relationship given previously which now becomes

$$t' = \frac{b'^2 \Theta}{K' \Delta H}$$

where t' is the total travel time, and K', b', and Θ are the vertical hydraulic conductivity, thickness and porosity of the aquitard, respectively, and ΔH is the vertical head difference between the water surface and the piezometric head at the top of Castle Hayne aquifer. A porosity value of 30 percent was assumed which is considered to be a better representation of the actual conditions than the 40 percent adopted in the 1971 Technical Report. Vertical travel times were calculated for several points along the estuary to locate the minimum. The results of the calculations are given in Figure 16 for 1970 conditions with Texas Gulf pumping of 52 mgd. The data used were taken from the Technical Report figures. The minimum travel time was found to be approximately 250 years at a point 3 miles downstream from Blount's Bay. This should be compared with the value of 500 years given in the Technical Report. The calculations were also carried out for an additional pumping center at South Creek, the site of proposed North Carolina Phosphate Company's mine. The minimum vertical travel time is projected to be 70 years at a point about one mile downstream from Blount's Bay, as compared to 135 years in the Technical Report. Here, the piezometric surface used was derived from computer runs.

The possible errors in these estimates can be derived from a sensitivity analysis of the relationship to the possible errors in the estimated parameters used. The sensitivity relationships are easily derived from partial differentiation and are given in Table 7.

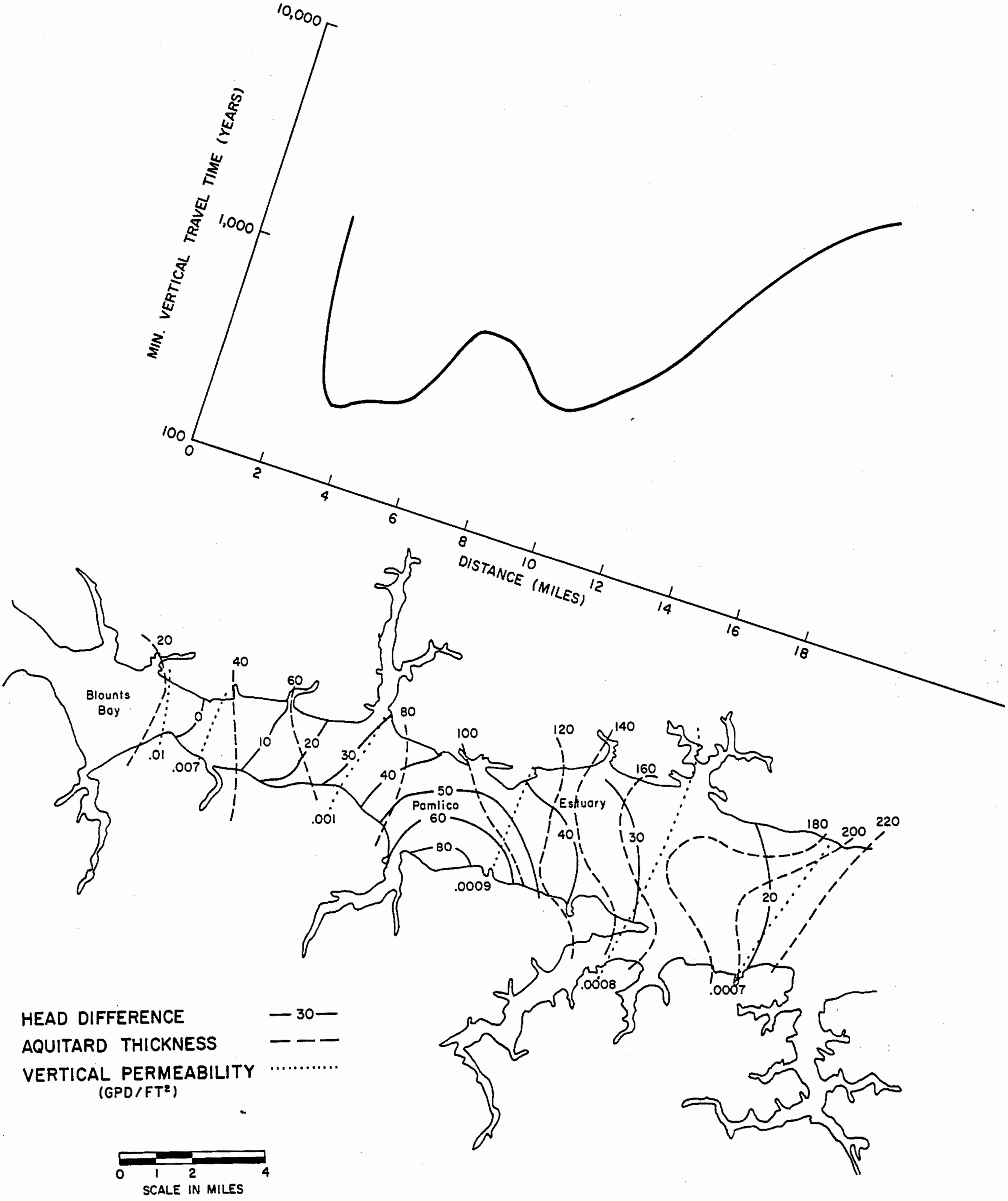


Figure 16. Minimum Vertical Travel Times Pamlico Estuary.

Table 7. Sensitivity of Vertical Travel Times

Parameter	Sensitivity $\partial t_v/t_v$
b	$2\ \frac{\partial b'}{b'}$
K'	$-\ \partial K'/K'$
Θ	$\partial\Theta/\Theta$
ΔH	$-\ \partial\Delta H/\Delta H$

A one percent error in the aquitard thickness leads to two percent error in the estimate of travel time. A one percent error in the adopted porosity, vertical permeability and head difference leads to one percent error in travel times, respectively. Similar observations apply as those made under lateral travel times. However, the errors in vertical permeability deserve special mention. The vertical permeabilities in the vicinity of the estuary adopted in the Technical Report are inadequate to transport large quantities of water flowing into the estuary implied by the 1965 piezometric surface map. Both analog and digital computer runs show that vertical permeabilities in and around upstream portions of the estuary are at least several times those given in the Technical Report. If this be the actual state of affairs, the implication is obvious. A detailed reexamination of the situation is warranted before additional large-scale withdrawals are authorized.

After the estuarine water arrives at the top the aquifer mixing occurs between the induced water and the water already in the aquifer. The prediction of the chloride concentrations in the aquifer is only possible if the type of mechanism causing mixing and the extent of mixing are specified. The mixing assumptions have not been made explicit in the Technical Report. If the underlying assumptions are the ones outlined by Ganus (10), they are unrealistic and in contradiction to the two-dimensional pattern adopted for flow through the aquifer. The vertical component of the hydraulic conductivity of the Upper Castle Hayne aquifer is a critical determinant in the movement of the chlorides in the zone of saturation. The stratification of the Castle Hayne aquifer is another.

10.5 Summary

The increasing need to maintain close control on the deterioration of water quality in response to the existing and proposed large withdrawals requires that more sophisticated methods of predicting the transport of chlorides be used.

The parameters used in predicting future water quality are either assumed or are derived from hydrologic variables that are either unknown or only estimated. They are subject to large errors. Single-valued estimates based on specific values of the parameters give unreliable results in which little confidence can be placed. The possible range of variation should also be specified. This can be accomplished through sensitivity analyses.

The data to delineate the occurrence of high chloride zones in the aquitards are very meager. Additional testing of aquitards to determine their vertical permeabilities and sampling for the determination of water quality are needed.

The delineation of the fresh water-saline water contact in the major water-bearing formations and the actual monitoring of the migration of the contact zones should be undertaken.

Chapter 11

WATER MANAGEMENT

Within the study area, the groundwater resource constitutes by far the greatest source for domestic, municipal, agricultural, and industrial use. Of the groundwater aquifers, the Upper Castle Hayne is the most important. The growth potential of the area is almost unlimited. The use of water from the Castle Hayne aquifer will increase with the industrial and urban growth of the region. Activities based on the use of groundwater could make substantial contributions to the economy of the area provided the continued availability of groundwater in adequate quantities and of suitable quality can be assured. The extent of the future economic growth of the region will depend on the efficiency with which optimal use of groundwater is managed.

The optimal management of groundwater resources is extremely difficult. The extractions from the aquifer system are controlled by a large number of individual decision makers and there is extreme interdependence between users. The pumping by one user affects the availability of the resource for others. The failure to protect the quality in one use adversely affects the value of the resource to other widely separated in space and time. The long-term best interests of all users can be served only under a system of regional groundwater management. This will require the cooperative effort of all large users and the informed guidance from state agencies.

It is hoped that in the Coastal Plain of North Carolina we are not condemned to repeat the history of many other areas in the nation going through the complete cycle of development of groundwater resource, overdevelopment, deterioration of water quality and attempted corrective measures.

The Water Use Act of 1967 provides an adequate regional water management tool for the state agencies. The act gives the Board of Water and Air Resources the authority to regulate the use of water by large water users (in excess of 100,000 gallons per day) in the capacity use areas where that agency finds that:

a. the aggregate uses of water have developed, or threaten to develop, to a degree where they exceed, or threaten to exceed, the renewal of the resource,

b. the chemical nature of the resource is impaired, or threatens to be impaired, adversely affecting its fitness for other water users in the area, including present as well as potential users.

The Board has been given wide regulatory powers to specify timing of withdrawals, maximum pumping rates and spacing of wells and to control the elevation below which water may not be drawn down. It can require:

a. the provision of remedial measures to protect against unreasonable adverse effects on other water users, and
b. compensation for the damages inflicted.

The current practice of groundwater management can be classified into two broad categories: the safe yield concept and the static storage utilization (or mining) concept. The Water Use Act of 1967 seems to embrace the safe yield concept.

The safe yield of a groundwater basin has been defined in a variety of ways, each definition laying emphasis on a particular aspect of groundwater resource development. In essence, the safe yield of a water-bearing formation is the maximum rate at which water may be withdrawn without incurring undesirable consequences. No hard and fast rules exist for the determination of safe yield--since the underlying principle is the avoidance of an undesired result. In the case of Castle Hayne aquifer system the considerations giving the maximum extent of development will be (a) the lowering of the piezometric surface to a degree that it imposes excessive costs on existing users, or (b) the likelihood of changes in the quality of groundwater which are unreasonable. Hydrologically safe yield has been regarded as the maximum rate at which water can be salvaged under long-term steady state conditions. At this point, the inputs to the aquifer balance the output, and there is no progressive lowering of the water levels. The aquifer is essentially in equilibrium and the storage is utilized only to provide regulation of fluctuating inflow to meet the demand. If the pumping is limited to safe yield, the utilization of water in static storage is eliminated.

The yield of an aquifer depends on (1) the hydrologic characteristics of the aquifer, (2) dimension of the aquifer and the hydraulic characteristics of its boundaries, (3) the vertical position of the aquifer and the hydrologic characteristics of the overlying and underlying beds, and (4) the effect of proposed withdrawals on the recharge and the discharge of the aquifer. It is evident that the safe yield of an aquifer is not a fixed quantity. It is a variable quantity dependent on the level of development. The response of recharge and discharge conditions varies with the rate of withdrawals and spatial arrangement of wells. The location of demand with respect to the boundaries plays a crucial role. The same amount of aggregate use in different spatial configurations will have widely varied consequences on the piezometric surface and water quality. What the safe yield of Castle Hayne aquifer is cannot be answered in the abstract. The location and magnitude of future water uses have to be known, or assumed, to give even an approximate answer.

Falling of the piezometric surface by itself does not indicate overdevelopment of the groundwater resource. Recharge and discharge are subject to adjustment as the aquifer is subjected to increasing withdrawals. The result may be the establishment of equilibrium conditions at a lower level with no systematic fall from year to year. The changes in recharge conditions are hard to forecast as all input mechanisms, for example, rejected recharge, have to be taken into account. Obviously, the safe yield of an aquifer remains a speculative quantity until there is actual overdevelopment. The time required to reach new equilibrium conditions can, however, provide a useful indication. In North Carolina, our experience of basins near the maximum potential of development is limited. Prolonged periods are needed to establish with certainty the safe yield of a complex basin which may be as long as 20 years.

The water supply in the Castle Hayne aquifer is more than adequate for all existing uses. The water levels stabilized in 1966, about a year after the commencement of pumping at Lee Creek. The total withdrawals from the aquifer at that time were about 70 mgd. The capture of rejected recharge, reduction of natural discharge and increased leakage from the overlying and underlying beds were more than adequate to support this off-take. The present withdrawals from the aquifer are somewhat lower, about 56 mgd, of which 52 mgd is at Texas Gulf mine. It must be accepted that, within limits, the increased withdrawals will cause the establishment of new equilibrium conditions with the piezometric surface at a lower level, and resulting increase in capture of rejected recharge, increased leakage and decreased natural outflow from points of discharge.

The safe yield concept generally results in an unsatisfactory level of utilization of the total groundwater resource potential. The quantity of water in storage is much greater than the annual replenishment. This high ratio of storage capacity to annual recharge has two consequences. It allows the discharge to exceed recharge for long periods of time but makes a condition of overdevelopment hard to recognize. If the recharge is limited by the intake capacity of an aquifer, as it is in Castle Hayne system, the creation of additional storage space through pumping has the desirable effect of increasing the salvage of normally rejected recharge. The rejected recharge unless used is eventually wasted as runoff to the sea. One way of managing the aquifer will be to make a decision on the maximum allowable decrease in groundwater storage in any one time period. This is equivalent to specifying that the drawdowns in the different sections of the aquifer should not exceed a certain value in a particular time period. By varying the maximum allowable annual change in storage, the behavior of the entire aquifer system can be analyzed and the optimal decision rule for the operation of the groundwater basin stipulated. There are two limitations to this method of development. One is the maximum economic pumping lifts. The other is the range of drawdowns below which groundwater reservoirs cannot be economically replenished. If the stability of drawdowns is the eventual objective, mining of storage has to be undertaken in conjunction with artificial recharge.

The likelihood of the deterioration of water quality limits the extent of development of groundwater resource. Although the groundwater levels may become stabilized, the water quality can continue to deteriorate. The aquifer might become damaged before its full fresh water potential has been realized. The fresh water within the Castle Hayne aquifer is in direct hydraulic contact with the lenses of brackish water in other parts of the formation. The aquifer is not isolated from the underlying and overlying formations which contain water of inferior quality. In response to the concentrated withdrawals, the vertical hydraulic gradients become reversed causing this water to move into the aquifer. Any significant development of the groundwater resource which changes the flow conditions will inevitably result in the redistribution of water quality within the aquifer. In any policy for long-term water development, a decision has to be made as to the acceptable variation in water quality, its areal extent, and timing. In the absence of such guidelines, it is impossible to determine whether a particular use is unreasonable or whether the impending changes in water quality are intolerable.

Preventive measures, if necessary, to retard the rate of quality deterioration should form a part of any coordinated water management plan. These can take the form of either (a) reduction in the rates of pumping at strategic points, or (b) artificial recharge of the Upper Castle Hayne aquifer with supplemental water of suitable quality in areas suitable for recharge selected so as to minimize quality deterioration. This can, in part, be done by returning a part of the water pumped from this aquifer.

Before an optimal preventive measure can be selected, the following questions have to be answered:

What returns can be obtained if the withdrawals at a point are reduced? For example, what benefits can accrue if the Texas Gulf use is reduced from 52 mgd to, say, 40 mgd?

At what cost can this reduction be obtained? For example, what would the economic consequences be if the size of the pit is reduced?

Would the direction of travel of the pit have significant impact on the hydrologic consequences?

A detailed examination of water use in the plant, and its possible modification, should be conducted to see if a part of the water pumped can be made available for recharge.

Are there possibilities of artificial recharge during times of low production and slack in demand of the fertilizers? Such measures may be considered to be unwarranted in terms of the existing uses, but would merit consideration if the objective is to keep open the maximum number of options available for future development of the area. They would play an exceedingly important role as the full utilization of the aquifer potential is approached.

As the more complete utilization of groundwater resource in the Coastal Plain becomes a reality, the establishment of a public policy for the optimum management of this resource becomes a necessity. A primary objective of such a policy should be to use the groundwater resource for the overall purposes of the society and to ensure the continuance of uses into the indefinite future. Operationally, the task will be to operate the system optimally in relation to the demand with the stipulated constraints on water levels and water quality. The possible combination of uses, both present and future, can be very large. However, only a few of these are physically realizable and economically efficient. For each alternative groundwater use plan the variables to be considered are:

a. spatial arrangement of wells,

b. pumping schedules, and

c. possible protective methods for retarding the deterioration of water quality.

The solution consists of determining the optimal location and size of withdrawals and recharge facilities. The solution can be arrived at only through a trial and error process.

Hopefully, the groundwater resource will be brought into the productive process in the direction of the greatest net benefits to society. Unfortunately, the estimation of benefits and costs of groundwater development presents formidable problems. In some activities, for example, open pit mining for phosphate ore, quarrying and swamp drainage, the groundwater is a nuisance commodity and has negative or zero value. In other uses, groundwater is an economic commodity and has a positive value as an essential ingredient of the production process. Both types of uses can coexist as is the case in the Texas Gulf operations. The desirability of groundwater for any use cannot be determined until we know what other uses are being sacrificed, or what additional costs are being imposed on other users, present or perspective.

The effect of groundwater development on water levels, water quality, recharge and streamflows must be accounted for in any benefit-cost calculations. As a result of pumping, the piezometric surface declines, pumping lifts are increased, and there is corresponding rise in total energy requirements. Additional capital expenditures may be required to deepen the existing wells or to modify the pumping equipment. In the case of artificial recharge, the effect is just the opposite. The pumping lifts and energy requirements decrease and the pumping costs are reduced. Hence, the cost of a recharge project is partially offset by reduced costs.

The social costs of water quality deterioration are difficult to evaluate. The forecasting of future groundwater quality is never precise inasmuch as it depends on several factors not adequately quantified. To complicate the evaluation, the time scale involved for different types of occurrences is not the same. For certain effects the time is measured in decades; for others, in centuries. For a proper evaluation of quality effects, an extended time scale has to be adopted. Projections of water needs and their quality requirements are not subject to precise formulation at this time scale. An alternative approach can, however, be adopted. The cost of remedial measures that the society may have to undertake to restore the water quality to stipulated levels at different points in time provides a lower bound of the social costs.

In planning for groundwater resource management the time scale is of critical importance. In groundwater development benefits and costs occur at markedly different times and are widely separated in space. There is usually a substantial time lag between the start of groundwater withdrawals and the time that deleterious effects on water levels, water quality and surface flows are noted. In such circumstances, a particular use may appear to be very attractive when considered on a short time scale; the long-term social costs directly attributable to the project may, however, far outweigh the short-term benefits.

The decision whether a particular use of water should be permitted or whether the water should be left in storage depends on the capitalized value assigned to the water left in the ground. The answer to this question cannot be given until we know what the alternative future needs are. The impact of any use on reducing the number of options available for future development has also to be taken into account. If the water left in the ground has a greater value than can be obtained from proposed use, the decision will be not to pump water (22).

Burt (23) has developed a decision rule for the storage regulation of water table aquifers which states that the extraction of groundwater from an aquifer should be expanded to a point where the marginal net output per unit of water is equal to the negative of the capitalized marginal pumping cost with respect to the water in storage. A corresponding simple rule for leaky confined aquifers need be developed.

In planning for artificial recharge, the relevant considerations are:

1. the cost of bringing water to the recharge site,
2. the cost of bringing it to a level of quality suitable for mixing with the naturally occurring groundwater, and
3. the changes in the cost of pumping at the point of abstraction.

A policy for instituting charges for the licensing of withdrawals for large users providing economic incentives to encourage the disposal of surplus water by recharge through a system of credits set against the charges needs serious consideration. At present no feasible mechanism exists by which all those benefiting from artificial recharge schemes can be made to contribute their share of the cost

Groundwater should be declared a public property. The permit holders should be assigned only a quota right in a common pool and should be entitled to its use so long as the exercise of the right does not impose unreasonable costs on the existing or future uses. The economic regulatory framework should include devices to protect other users from the adverse effects of legitimate actions by owners of water rights. The regulatory measures should include:

1. economic incentives to encourage:
    a. reductions in total withdrawals by large users, and

   b. the return, at a suitable point, of a part of the withdrawals to the aquifer system, and

2. power to tax property
   a. in proportion to the damage done to other users or the value of water in future uses, or
   b. to pay for the cost of remedial measures that the society may have to undertake to prevent, or undo, the damage directly attributable to a particular use. If this can be achieved only at prohibitive cost, the particular use should be condemned.

Framework for Computer Models of Aquifer Management

There is increasing need to apply a more sophisticated approach to develop optimum plans for aquifer development and management.

The objectives of a management policy are

1. to meet the future demands with respect to water quantity and quality;
2. to anticipate, minimize or correct the undesirable effects of overdraft conditions. (The overdraft condition is defined as violation of (a) water level evaluation constraints, or (b) groundwater quality constraints, or (c) both.)
3. to assign ranks with respect to some criterion to the desirability of alternative strategies of development of an aquifer.

A planning and management model of an aquifer system will consist of three segments:

1. simulation of the dynamic behavior of the aquifer system,
2. simulation of future groundwater quality, and
3. an optimization model.

In cases where the aquifer is extensive and where there are several interdependencies, the aquifer has to be managed on a regional basis. The rules governing the operation of the aquifer will be quite complex, and there is no escape from some form of computer simulation.

The first prerequisite for developing alternative plans to meet anticipated demands is creating the demand schedules for different time periods specifying the location of pumping, amount of withdrawals, timing of extractions and the quality of water. In the absence of a demand study, hypothetical programs will have to be formulated.

Simulation of Dynamic Behavior of the Aquifer

For complex, heterogeneous aquifer systems, the simulation of aquifer behavior under alternative operating policies should be accepted as a necessary part of any resource planning exercise. A detailed knowledge of the characteristics of the aquifer is assumed in predicting the behavior of aquifers with time and in developing and testing management plans. Good estimates of aquifer parameters are often not available in real life. The analysis should, therefore, include the development of a mathematical model of the groundwater basin, estimates of the parameters of the model and verification of the model with historical data.

The first decision one has to make is the number of components (layers) of the aquifer system to be modeled. In this study, the overlying and underlying aquitards were lumped together into a single component overlying the aquifer. From the mass balance and Darcy's equations, a generalized groundwater flow equation is derived relating the aquifer parameters; for example, transmissivity, storage coefficient and leakance.

Next, the technique for solving the governing differential equation is specified. Finite difference methods are almost universally adopted. Enough computational experience does not exist on the newer techniques of finite element based on variational calculus. In the finite difference approach, the heterogeneous aquifer is divided into a number of subareas. The number and shape of subareas depend on the detail and accuracy of the geohydrologic data and the details and accuracy of the desired results. The finite difference network chosen may be symmetric or asymmetric. For both the analog and digital computer models in this study the grid was symmetric with variable spacing. Then, a specific equation with proper value of coefficients is either written or implied for each of the subareas in the basin. The complex field situation is thus reduced to a simpler, idealized system. These equations are collectively referred to as the mathematical model of the groundwater basin and are solved simultaneously. For a problem of some size, the solution of the system of equations can only be accomplished either on

a. analog model,
b. general-purpose analog computer,
c. digital computer, or
d. hybrid computers.

Analog model and digital computer simulation was used in this study.

The model parameters are modified to give reasonable correspondence with the data observed in the field. After the model has been verified, it is used to predict the spatial and temporal changes in drawdown or water levels under alternative strategies of development. The output of this segment is the predicted elevation of future water levels. These water level elevations are needed for each of the assumed future combination of extraction and artificial replenishment in order to:

1. detect any violations of stipulated water levels or maximum permissible drawdowns;
2. provide input for the water quality model; and
3. provide input into the optimization segment to enable calculation of
    a. pumping lifts for new pumpage, and
    b. changes in pumping lifts of existing users. This information is used for the calculation of costs.

The approach for the development of aquifers as separate units has traditionally been based on purely hydrological considerations. Most decisions on pumping and artificial recharge are derived solely from a hydrogeologic point of view. Simulation is essentially a heuristic approach. It cannot determine the optimum of an objective function. The final objectives of operating the system are presumably socio-economic and have to be provided from outside the hydrogeologic consideration.

A comparison of the analog and digital simulation techniques for aquifer modeling and strategy evaluation shows that there are certain problems for which analog models are more suitable than digital models, and there are other problems for which the reverse is true. There is also a wide range of problems for which either the digital or the analog model could be used. The choice between the two techniques often depends on whether it is important to obtain further physical insight into the working of the system. In an analog model, the hydrologic parameters are converted directly into electrical equivalents by means of appropriate modeling and scaling. The hydrologist is able to modify the parameters in the system and monitor the resulting changes, thus defining the cause and effect relationships regulating the behavior of the system. On the other hand, the digital computer is merely a device for doing arithmetic and carrying out certain types of logical operations. The problem loses all physical content when it enters a digital computer. The output is simply a set of figures which need interpretation to become meaningful.

The accuracy of the solution obtained from a digital computer is much superior than can be obtained from the analog model. In groundwater problems, however, this is not crucial. Both the analog and digital techniques exceed the accuracy of the available field data.

The analog model technique of solving groundwater problems is the so-called discrete space continuous time method in which the space derivatives are replaced by finite difference expressions, but the time is continuous. The digital model simulation is the discrete time-discrete space method in which all the space and time variables are discretized. The analog method has an advantage in case of very rapidly changing heads.

The range of problems which digital simulation can solve is much wider. Analog models are restricted to only one independent variable. It is difficult to simulate non-linear conditions (input, boundary, parameters) and time-varying fields. Analog methods are essentially limited to linear, constant parameter fields (24). In this study dealing with a leaky, confined aquifer system, no non-linearities were assumed. Analog techniques were found to be more advantageous in the modeling phase when the inputs, boundaries and parameters had to be changed. Digital model simulation would have been too expensive and would not have provided the insight into the working of the system. In subsequent stages, digital simulation was used.

Prickett and Lonnquist (25) compare the results of comparison between analog and digital simulation. Their conclusions are:

1. digital method does not require as much time for model construction, data readout, and is more convenient than equipment manipulation phases needed in the analog technique, and
2. for very large problems that would require many time increments and large core storage for an accurate and practical solution, electric analog techniques have an advantage.

The hybrid computation techniques combine the accuracy of the digital computer and the speed of an analog computer. This will be a technique to be considered when many non-linearities are involved (24). The accessibility of hybrid computer equipment, however, is limited.

Simulation of Groundwater Quality

In the management of several aquifers, quality considerations weigh heavily in the derivation of operating rules. Along with quantitative effects on water levels and drawdowns, one should also predict changes in groundwater quality with time. No general water quality models are available. The problem in each aquifer system has to be examined as a specific case. Some salient features of groundwater quality models were given in Chapter 10.

The input to this segment consists of

a. spatial and temporal distribution of hydraulic heads from segment one, and

b. areal distribution of high chloride areas in the aquifer and the aquitard and initial concentrations.

The output from this segment will be chloride levels at different times and places. The output will be used to see

a. if any quality constraints are violated, and

b. if the protective or remedial measures are required. The cost of these shall be taken into account in the optimization segment.

Optimization Segment

Implied is the assumption that there are several planning alternatives, each having a different combination of pumping and artificial recharge and operating procedures that are capable of providing the desired outputs. Direct optimization against defined objectives is, therefore, necessary. There is need for criteria. The objectives of development are socio-economic in nature and are the final reason for operating the system. For a public agency, the objective may be the maximization of net social benefits. The conceptual problems encountered in the estimation of social benefits and costs were outlined earlier in this chapter.

For a certain specified level of physical outputs, minimization of total costs may be a relevant objective. The decision variable will be

a. location of pumping and pumping schedules, and

b. location and rates of artificial recharge (or other protective and remedial measures).

The relevant costs for new pumping facilities will be construction and installation costs, electrical connection service charges and supply costs. For existing users, the costs include capital expenditures to deepen the wells and to modify pumping equipment, and changes in pumping energy costs. For artificial recharge the relevant costs to be considered are the cost of bringing the water to the recharge site, the cost of changing quality of recharging water and savings in the pumping costs of the existing users.

The output of this segment will either be quantitative net benefits, economic costs, or simply assignment of ranks to the alternative strategies of development of an aquifer indicating the relative desirability of the outputs resulting from the system operation.

REFERENCES

1. Report on Hydrogeology and Effects of Pumping from Castle Hayne Aquifer System, Beaufort County, North Carolina, prepared for N. C. Board of Water and Air Resources, Texas Gulf Sulphur Company and N. C. Phosphate Corporation, 1971. (1971 Technical Report)

2. DeWiest, R. J. M., Sayre, A. N., and Jacob, C. E. Evaluation of Potential Impact of Phosphate Mining on Ground-water Resources of Eastern North Carolina, N. C. Department of Water Resources, 1967. (1967 Consultant's Report)

3. DeWiest, R. J. M., Hydrologic Relationship Between the Pamlico River and the Castle Hayne Aquifer in Eastern North Carolina, Preprint 830, American Society of Civil Engineers, Annual Meeting and National Meeting on Water Resources Engineering, February 1969.

4. Sampair, J. L., Subsurface Geology of Beaufort County and Environs, North Carolina, Consulting Geologist's Report to Texas Gulf Sulphur, 1968.

5. Hird, J. M., Control of Artesian Groundwater in Strip Mining Phosphate Ores--Eastern North Carolina, Paper presented to AIME Annual Conference, 1970.

6. Floyd, E. O., Ground-water Resources of Craven County, North Carolina, U. S. Geological Survey Hydrol. Inv. Atlas HA-343, 1969.

7. Harshbarger, J. W., Analysis of Groundwater Systems in the Beacham Savanna Area, Beaufort County, North Carolina, Consultant Report for Bear Creek Mining Company, 1963.

8. Miller, L. J., Report of Investigations on the Castle Hayne Aquifer as Affected by Phosphate Mining Operations, Texas Gulf Sulphur Company Report, 1966.

9. Jacob, C. E., Papers on file with N. C. Dept. of Water and Air Resources, 1969-70.

10. Ganus, W. J., Analysis of Factors Controlling Ground-water Flow for the Purpose of Predicting Rates of Ground-water Movement and Changes in Quality, Atlantic Coast Plain, Doctoral Dissertation, University of Arizona, 1972.

11. Neuman, S. P., and Witherspoon, P. A., Applicability of Current Theories of Flow in Leaky Aquifers, Water Resources Research, Vol. 5, No. 4, 1969, 817-829.

12. Hantush, M. S., Analysis of Data from Pumping Tests in Leaky Aquifers, Transactions American Geophysical Union, Vol. 37, 1956, p. 702-714.

13. Comilee for Hydrologic Research T.N.O., Steady Flow of Water Towards Wells, 1964.

14. Stallman, R. W., Calculation of Resistance and Error in an Electric Analog of Steady Flow Through Non-homogeneous Aquifers, Geol. Survey Water Supply Paper 1544-G, 1963.

15. Walton, W. C., and Prickett, T. A., Hydrogeologic Electric Analog Computers, Journal of the Hydraulic Division, American Society of Civil Engineers, 1963.

16. Pinder, G. F., and Bredehoeft, J. D., Application of Digital Computer for Aquifer Evaluation, Water Resources Research, 4(5) 1968, p. 1069-1093.

17. Pinder, G. F., An Iterative Digital Model for Aquifer Evaluation, U. S. Geological Survey, 1970.

18. Pinder, G. F., and Cooper, H. H., A Numerical Technique for Calculating the Transient Position of the Salt Water Front, Water Resources Research, Vol. 6, No. 3, 1970, pp. 875-882.

19. Stone, R. L., and Brian, P. L. T., Numerical Solutions of Conventive Transport Problems, Jour., A.I.Ch.E., 9(5), 1963.

20. Price, H. S., Cavendish, J. C., and Varga, R. S., Numerical Methods of Higher-Order Accuracy for Diffusion-Convection Equations, J. Soc. Petrol. Eng., 8(3), 1968.

21. Perrine, R. L., and Gay, G. M., Unstable Miscible Flow in Heterogeneous Systems, J. Soc. Petrol. Eng., 3(3), 1963.

22. Renshaw, E. F., The Management of Groundwater Reservoirs, Journal of Farm Economics, 45(2), 1963.

23. Burt, O. R., Optimal Resource Use Over Time With an Application to Groundwater, Management Science, 11(1), 1964, pp. 80-93.

24. Vemuri, V., and Dracup, J. A., Analysis of Nonlinearities in Groundwater Hydrology: A Hybrid Computer Approach, Water Resources Research, 3(4) 1967.

25. Prickett, T. A., and Lonnquist, C. G., Comparison Between Analog and Digital Simulation Techniques for Aquifer Evaluation, Intern'l Assoc. Sci. Hydrology, Publication No. 81, 1968.

APPENDIX A

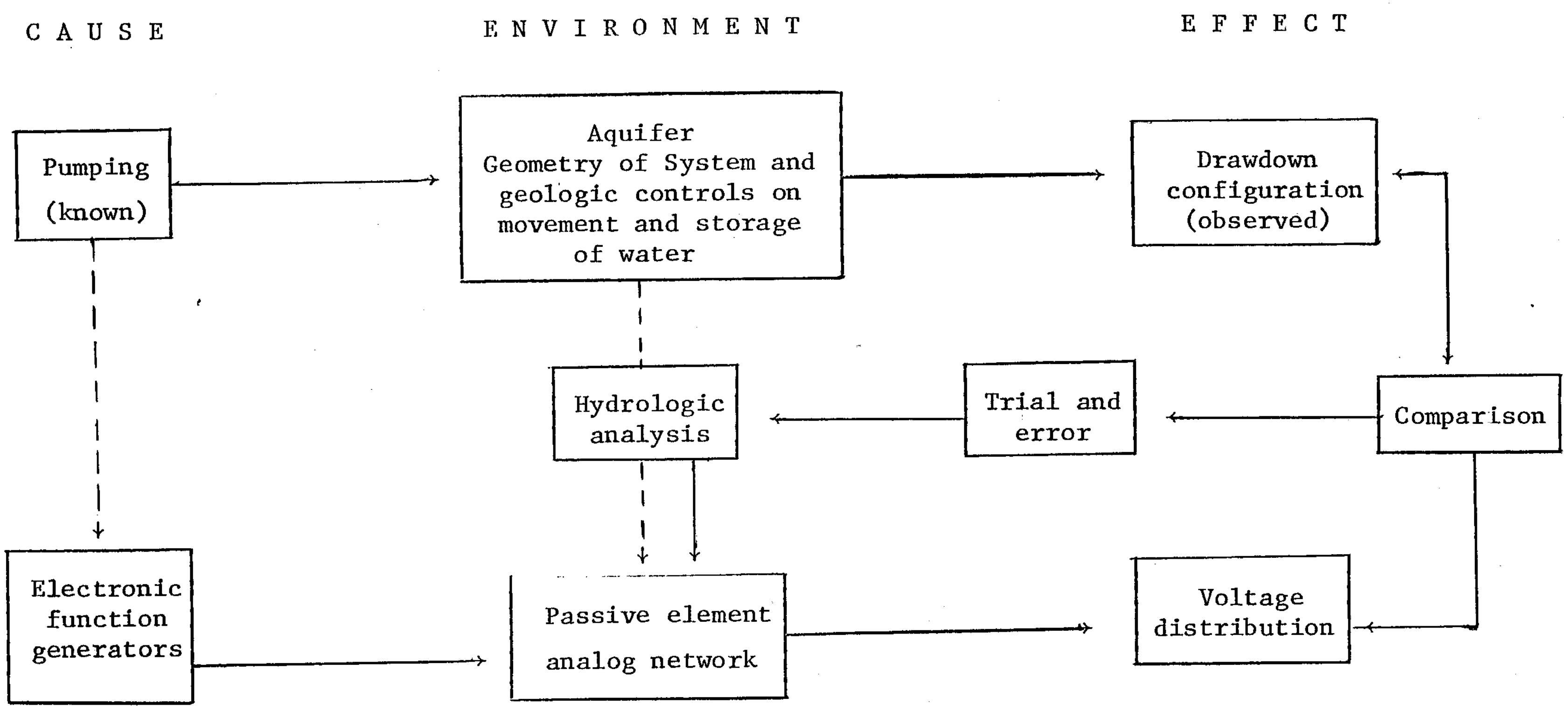
Analog Model Simulation

The Electric Analog Model

The aquifer as a passive unit in the hydrological cycle is the concept that is basic to the electric analog technique. The aquifer as a system deals with the study of system excitations, system responses to these excitations and the values of the system parameters. Recharge and discharge from an aquifer represent the stimuli; the observed changes in water levels within the aquifer constitute its response to these stimuli. The concept of passivity separates the non-changing geologic environment from the varying hydrologic parameters, (Patten, 1965).

In electrical theory, the resistance network is a passive unit in which recharge and discharge of current represent the stimuli to the system and the measured changes in potential-difference (voltage) levels within the network constitutes its response to the stimuli.

The electrical analog consists of a direct correspondence between elements of these two systems. The potential-difference, or voltage, corresponds to the hydraulic potential, or piezometric head, in the hydraulic system; the flow of current corresponds to the flow of water and the resistance to current flow corresponds to the resistance to groundwater flow. The validity of the analogy depends on the rigour with which the correspondence of elements in the two physical systems is defined. Figure A1 graphically summarizes the use of the electric analog technique to the solution of a groundwater problem, (Patten, 1965).



(After Skibitzke and Robinson, 1954)

Figure A1: Schematic Diagram of Trial and Error Method of Duplicating Aquifer Response on the Analog Model

Equations of Groundwater Flow

The partial differential equation governing the two-dimensional flow of an incompressible fluid through an isotropic non-homogeneous aquifer is given by

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T}\frac{\partial h}{\partial t}$$

where x, y are cartesian coordinates

H is the hydraulic head,

T is transmissivity of the aquifer,

S is the storage coefficient,

and t is time.

In steady state this reduces to

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

When an aquifer is confined by a semi-pervious layer, the governing flow equation is given by

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y} + \frac{h'-h}{B^2} = \frac{S}{T}\frac{\partial h}{\partial t}$$

where h' is the hydraulic head in the source bed and B is the leakage thickness. Other symbols have been defined above. The leakage thickness represents the combined response of the system and can be shown to be

$$B = (T/L)^{1/2} = (Tb'/K')^{1/2}$$

where b' is the thickness of the semi-pervious bed

K' is the vertical hydraulic conductivity of the

and L = K'/b' is defined as the leakance and represents the characteristics of confining formation.

Finite Difference Analogs

The finite difference forms of the governing equations used in analog simulation are (Figure A2a)

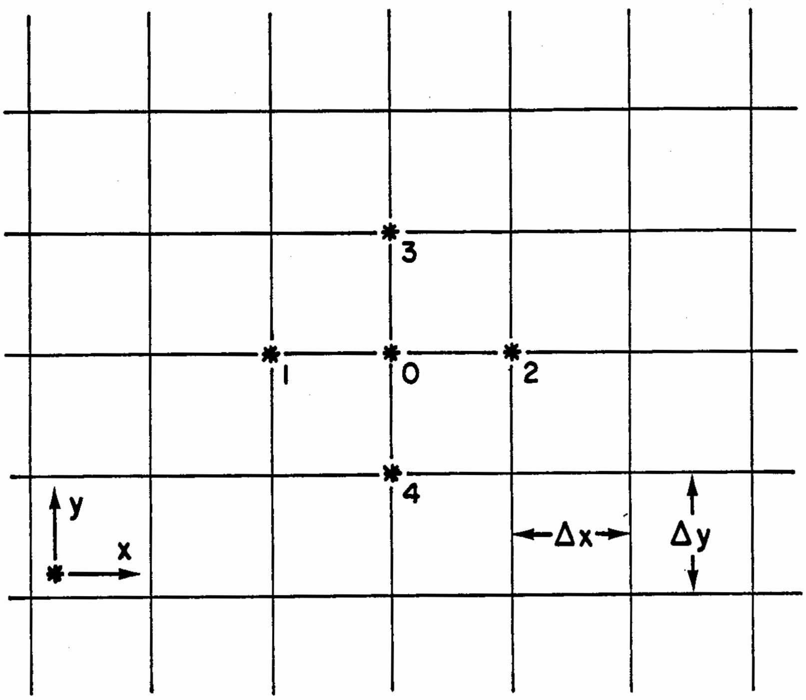


Figure A2a. Two-dimensional Finite-difference Nomenclature.

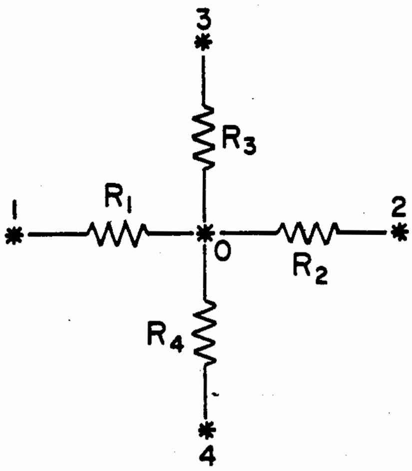


Figure A2b. Resistance Node

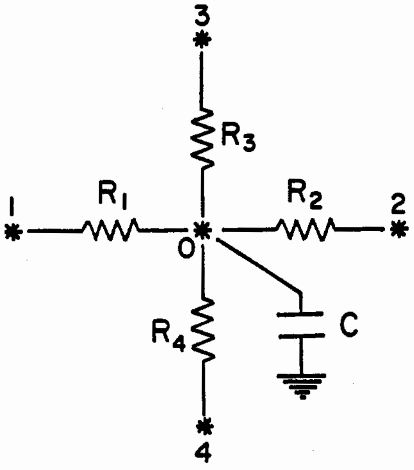


Figure A2c. Capacitance Node

$$\sum_{i=1}^{4} h_i - 4h_o = 0 \qquad \text{for the steady state}$$

and $$\sum_{i=1}^{4} h_i - 4h_o = \frac{a^2 S}{T} \frac{\partial h}{\partial t}$$

where a is the grid spacing.

The Electric System

Application of the Ohm's and Kirchoff's laws to the node shown in Figure A2c in an isotropic aquifer yields

$$\sum_{i=1}^{4} v_i - 4v_o = 0$$

and $$\sum_{i=1}^{4} v_i - 4v_o = RC\frac{\partial v}{\partial t}$$

A comparison of the two sets of equations gives the following correspondence between variables

Δh analogous to v

T analogous to 1/R

and a^2S analogous to C

Scaling

For the construction of the electrical model of the hydraulic system, it is necessary to make the analogous properties of the two systems proportional to each other by means of scale factors. The scale factors used in practice are derived from the modified form of the relationships given previously. These are:

1. $\Delta h = F_1 v$, the units of F_1 are feet per volt
2. $q = F_2 I$, the units of F_2 are gpd/ampere
3. $t = F_3 \tau$, the units of F_3 are days/second

It can be shown that

Resistance, $R = \frac{F_2}{F_1} \frac{1}{T}$ $\qquad C = \frac{F_1}{F_2 F_3}\, 7.48\, a^2 S$

These relationships are used to determine the values of the resistors and capacitors to go into the model. In practice, the values are limited by the operating range of the electrical equipment available to the investigator and the commercially available values of resistors and capacitors.

Resistor-areal Relationships

Each element in the electric analog model represents some characteristic of the hydraulic system in a particular region of that system. Therefore, it is necessary to consider precisely what extent of the aquifer the resistors simulate. Equation above states that the resistance in the 1-direction is given by

$$R_1 = \frac{F_2 \Delta 1}{F_1 K_1 A_1}$$

For the two-dimensional isotropic system,

$$R_x = \frac{F_2 \ \Delta x}{F_1 K \ A_x} \quad ; \qquad R_y = \frac{F_2 \ \Delta y}{F_1 K \ A_y}$$

When the finite-difference grid is square,

$\Delta x = \Delta y = a, \quad A_x = A_y = am$

Figure A3a shows the vector volume, $a^2 m$, associated with a resistor in the Y-direction.

In the case of a boundary resistor where the area it represents is half that represented by a non-boundary resistor, then, typically:

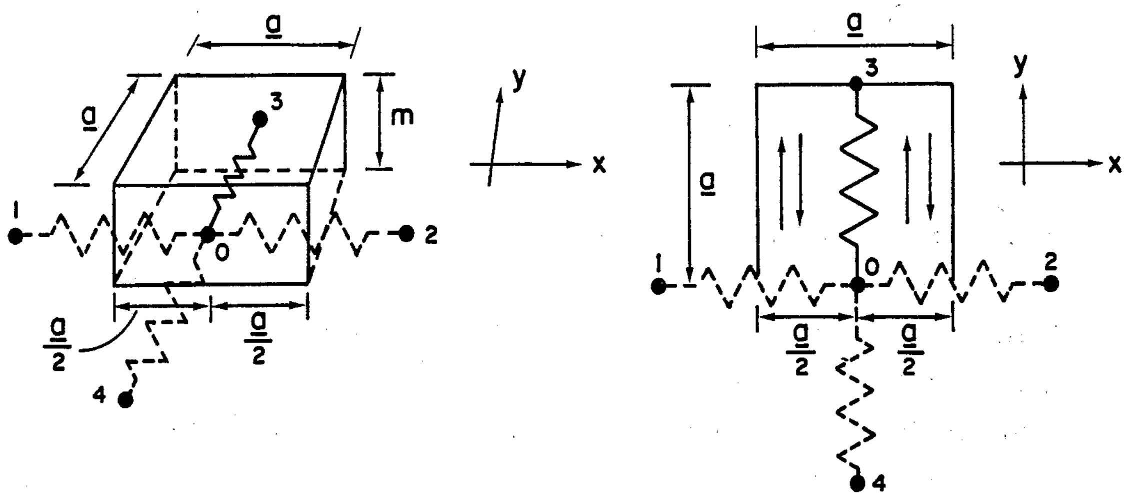
$$R_y^B = \frac{F_2 \quad a}{F_1 K(a/2)m} = \frac{2F_2}{F_1} \frac{1}{T_y}$$

That is, for a region of constant transmissivity value, the boundary resistor has to be twice the value of a neighboring internal resistor. Indeed, a resistor can be designed to simulate any area by similar methods.

Resistor-grid Relationships

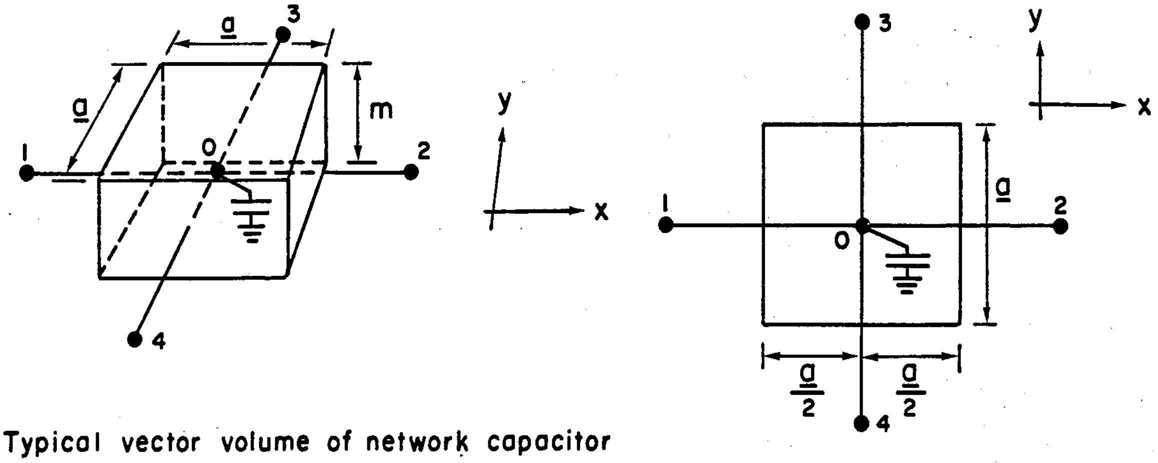
a. Change of grid-size:

Consider, for example, the case in which the grid-size is doubled but keeping the same scale proportionality between the model and the real system, e.g., 1 inch = 2 miles, then



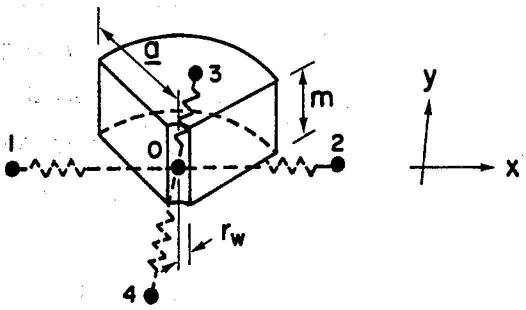
(A) Vector volume of resistor

(B) Top view of resistor

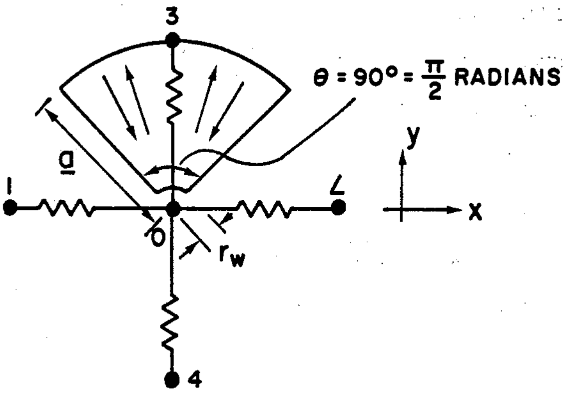


(C) Typical vector volume of network capacitor

(D) Top view of capacitor vector volume



(E) Modified pumped well junction vector volume



(F) Top view of vector volume

Figure A3. Vector Volumes

$$R_x = \frac{F_2}{F_1 K} \frac{2a}{2am} = \frac{F_2}{F_1} \frac{1}{T_x}$$

This is the same result as was obtained for the original grid-size.

b. Change of scale:

Consider the case where the grid-size remains unchanged but where the scale proportionality is varied, e.g., 1 inch now representing 32 miles in the X-direction only, the scale remaining as 1 inch = 2 miles in the Y-direction, then:

$$R_x = \frac{F_2}{F_1 K} \frac{16a}{am} = \frac{16F_2}{F_1} \frac{1}{T_x}$$

Capacitor-volume Relationship

The vector volume associated with a capacitor is shown in Figure A3c, the volume of aquifer it represents being equal to a^2m cubic-feet.

Problems sometimes arise because the capacitor concentrates storage at a point in the system, while the real storage property of the aquifer is distributed over an area of a^2 square-feet; the errors involved are functions of the grid-size and decrease with smaller grid-sizes.

Resistor error at well-sites

Prickett (1968) discusses the different types of errors involved in replication of a well, or system of wells, by a point-node. Difficulties occur because of the distortion of the potential field in the vicinity of the well, but more particularly because the flow to the well is obviously not in the direction of the principal coordinates only, but is radial flow towards it, and the vector volume requires suitable adjustment; Figure A3e shows the new vector volume.

Another difficulty is caused by depiction of the well as a point-sink; that is, acceptable as long as the effective well-radius is very small compared to the grid-scale. In this study, the effective well-radius was known to be 1500 ft with the grid-sides scaling 10,560 feet.

Prickett recommends replacing the resistor calculated using equation given earlier by:

$$R_w = \frac{2}{\pi} R \log_e \left(\frac{a}{r_w}\right)$$

where,

R = the existing grid-resistance,

a = the grid-dimension, in feet, and

r_w = the effective well-radius, in feet.

REFERENCES

A1 Patten, E. P., Design, Construction and Use of Electric Analog Models, in Analog Model Study of Ground Water in Houston District, by L. A. Wood, Texas Water Commission Bulletin 6508, 1965.

A2 Prickett, T. A., Designing Pumped Well Characteristics into Electric Analog Models, Ground Water, v.5, No. 4, 1967.

APPENDIX B

Listing of the Computer Program

This is a modified version of the computer program in "An Iterative Digital Model for Aquifer Evaluation" by George F. Pinder, U. S. Geological Washington, D. C.

```
      INTEGER DIML,DIMW
      REAL*8 KEEP,IMK,NUM,MINS,M,K
      DIMENSION S(27,27), SS(27,27),HYCOND(27,27), RATE(27,27), KEEP(27,
     127), G(27), TEMP(27), BE(27), RHOP(27), CHK(10), STRT(27,27), T(27
     2,27), PHI(27,27), PUMP(27,27), WTABLE(27,27), DELX(27), DELY(27),Q
     3COEF(27,27), QRE(27,27), HEADNG(33)
      DIMENSION ASTRIX(27)
      DOUBLE PRECISION PHI,RHOP,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,DELT
     1,PARAM,CHK,PNCH,CONTR,CHCK
      COMMON /INTEGR/ DIML,DIMW,NUMT,LENGTH,ITMAX,INO1,JNO1,KTH,NCYCLE,
     1JFINAL
      COMMON /SINGLE/ SUM,DELX,DELY,RATE,S,STRT,SPACNG,T,FLUXT,QRE,PUMP,
     1WTABLE,TESTP,FACTOR,ERR,FACS,FACSS,FACK,FACT,TMAX,SS,HYCOND,TEST,Q
     2COEF,TT,HEADNG,FACQRE,FACDLT
      COMMON /DOUBLE/ PHI,KEEP,DELT,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,
     1RHOP,PARAM,IMK,CHK,PNCH,CONTR,NUM,CHCK
      COMMON /CHKWRT/ CONET,PUMPT,DELQT,DIFF,DIFFT
      DATA ASTRIX/27*1H*/
      JFINAL = 0
       NCYCLE = -1
    5 NCYCLE = NCYCLE + 1
C
C
C        READ IN DATA
C     *************
      CALL DATAIN
C     *************
C             ENDOF FILE ON SYSIN
      IF (JFINAL.GT.0) GO TO 60
C
C        PRINT INPUT DATA
C     *************
      CALL INOUT
C     *************
C
C        COMPUTE ITERATION PARAMETERS
C     *************
      IF (NCYCLE.EQ.0) CALL IPARAM
C     *************
C
      KT=0
      TEST=0
      IF (NCYCLE.EQ.0) TMAX = TMAX * 3600.
      JNO1=DIMW-1
      INO1=DIML-1
   10 IF (TEST.EQ.0) GO TO 50
C
C        HAVE WE EXCEEDED PERMITTED NO. OF ITERATIONS
      IF(KOUNT.LT.ITMAX) GO TO 20
      WRITE (6,170)
      JFINAL = 1
      GO TO 60
   20 KOUNT=KOUNT+1
      IF (MOD(KOUNT,LENGTH)) 30,30,40
   30 NTH=0
   40 NTH=NTH+1
      PARAM=RHOP(NTH)
      TEST=0.
```

```
C         COMPUTE IMPLICITLY ALONG ROWS
C       *************
        CALL ROW
C       *************
C
C         COMPUTE IMPLICITLY ALONG COLUMNS
C       *************
        CALL COLUMN
C       *************
        GO TO 11
C
C         ADJUST FOR NEW TIME STEP
   50   IFINAL=0
        IF (CHCK.EQ.CHK(4).AND.KT.NE.0) CALL CHECK
        IF (KT.GT.NUMT.OR.SUM.GT.TMAX) IFINAL=1
        IF (SUM.GT.TMAX) JFINAL = 1
        IF ((MOD(KT,KTH).NE.0.OR.KT.EQ.0).AND.IFINAL.NE.1) GO TO 80
        WRITE (6,180) KT,DELT,SUM,MINS,HRS,DAYS,KOUNT,TT
        IF (CONTR.EQ.CHK(2)) CALL PRNTA
        IF (NUM.EQ.CHK(3)) CALL PRNT1
        IF (CHCK.EQ.CHK(4)) WRITE (6,120) DIFF,DIFFT,CONET,PUMPT,DELQT,FLU
       1XT
        WRITE (6,130) ASTRIX
        IF (IFINAL.NE.1) GO TO 80
        IF (PNCH.NE.CHK(1)) STOP
   60   IFINAL = 0
        IF (JFINAL.EQ.0) GO TO 5
        WRITE(6,150)
        STOP
   80   CONTINUE
        KT=KT+1
        KOUNT=0
        DO 90 I=1,DIML
        DO 90 J=1,DIMW
   90   KEEP(I,J)=PHI(I,J)
  110   DELT = FACDLT  * DELT
        SUM=SUM+DELT
C       *********
        CALL CLAY
C       *********
        HRS=SUM/3600.
        MINS=HRS*60.
        DAYS=HRS/24.
        GO TO 30
C
C       ...............................................................
C
C
  120   FORMAT (1H0,///,51X,29HINFORMATION FROM MASS BALANCE//50X,21HMASS
       1BALANCE RESIDUAL1PE11.3/50X,19HCUMULATIVE RESIDUAL1PE11.3/50X,28HV
       2OLUME OF CONE OF DEPRESSION1PE11.3/50X,18HCUMULATIVE PUMPING1PE11.
       33/50X,28HCUMULATIVE TRANSIENT LEAKAGE1PE11.3/50X,25HCUMULATIVE STE
       4ADY LEAKAGE1PE11.3)
  130   FORMAT (1H ,15X,50A2)
  140   FORMAT (4E20.10/4E20.10)
  150   FORMAT('1END OF JOB')
  170   FORMAT (1H0,39HEXCEEDED PERMITTED NUMBER OF ITERATIONS)
  180   FORMAT (1H1,55X,17HTIME STEP NUMBER ,I10/50X,29HSIZE OF TIME STEP
       1IN SECONDS ,E10.3/40X,48HDURATION OF PUMPING AT THIS PRINTOUT IN S
```

```
      2ECONDS ,E10.3/80X,8HMINUTES ,E10.3/80X,6HHOURS ,E10.3/80X,5HDAYS ,
      3E10.3/56X,17HITERATION NUMBER ,I10/45X,26HMAXIMUM DIMENSIONLESS TI
      4ME,1PE10.3)
       END
**WARNING**  FORMAT STATEMENT    140 IS UNREFERENCED

       SUBROUTINE DATAIN
C      SUBPROGRAM NO. 1
C      ------------------------------------------------------------------
       INTEGER DIML,DIMW
       REAL*8 KEEP,IMK,NUM,MINS,M,K
       DIMENSION S(27,27), SS(27,27),HYCOND(27,27), RATE(27,27), KEEP(27,
      127), G(27), TEMP(27), BE(27), RHOP(27), CHK(10), STRT(27,27), T(27
      2,27), PHI(27,27), PUMP(27,27), WTABLE(27,27), DELX(27), CELY(27),Q
      3COEF(27,27), QRE(27,27), HEADNG(33)
       DOUBLE PRECISION PHI,RHOP,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,DELT
      1,PARAM,CHK,PNCH,CONTR,CHCK
       COMMON /INTEGR/ DIML,DIMW,NUMT,LENGTH,ITMAX,INO1,JNO1,KTH,NCYCLE,
      1JFINAL
       COMMON /SINGLE/ SUM,CELX,DELY,RATE,S,STRT,SPACNG,T,FLUXT,QRE,PUMP,
      1WTABLE,TESTP,FACTOR,ERR,FACS,FACSS,FACK,FACT,TMAX,SS,HYCCND,TEST,Q
      2COEF,TT,HEADNG,FACQRE,FACDLT
       COMMON /DOUBLE/ PHI,KEEP,DELT,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,
      1RHOP,PARAM,IMK,CHK,PNCH,CONTR,NUM,CHCK
       COMMON /CHKWRT/ CONET,PUMPT,DELQT,DIFF,DIFFT
C      ------------------------------------------------------------------
C
C      PURPOSE--TO READ INPUT DATA
       IF (NCYCLE.GT.0) GO TO 125
       READ (5,130) HEADNG
       READ (5,140) TMAX,DIML,DIMW,NUMT,ITMAX,FACQRE,KTH,FACTCR,LENGTH,ER
      1R,FACS,FACSS,FACK,FACT,FACW,SPACNG
       READ(5,170) SUM,CONET,PUMPT,DELQT,FLUXT,DIFFT,DELT,FACDLT
       IF  (FACDLT.EQ.0.0) FACDLT  = 1.5
       READ (5,160) PNCH,CONTR,NUM,CHCK
       READ (5,190) (DELX(J),J=1,DIMW)
       READ (5,190) (DELY(I),I=1,DIML)
       DO 10 I=1,DIML
       READ (5,180) (STRT(I,J),J=1,DIMW)
       DO 10 J=1,DIMW
    10 PHI(I,J)=STRT(I,J)
       DO 20 I=1,DIML
       READ (5,150) (RATE(I,J),J=1,DIMW)
       DO 20 J=1,DIMW
       IF(RATE(I,J).LT.0.) QCOEF(I,J)=10.
    20 IF (RATE(I,J).GT.0) RATE(I,J)=RATE(I,J)*FACTOR
       DO 30 I=1,DIML
       READ (5,150) (SS(I,J),J=1,DIMW)
       DO 30 J=1,DIMW
    30 SS(I,J)=SS(I,J)*FACSS
       DO 40 I=1,DIML
       READ (5,150) (HYCOND(I,J),J=1,DIMW)
       DO 40 J=1,DIMW
    40 HYCOND(I,J)=HYCOND(I,J)*FACK
    50 DO 60 I=1,DIML
       READ (5,150) (T(I,J),J=1,DIMW)
       DO 60 J=1,DIMW
    60 T(I,J)=T(I,J)*FACT
       DO 70 I=1,DIML
       READ (5,150) (WTABLE(I,J),J=1,DIMW)
```

```
      DO 70 J=1,DIMW
   70 WTABLE(I,J)=WTABLE(I,J)*FACW
   80 DO 90 I=1,DIML
      READ (5,150) (S(I,J),J=1,DIMW)
      DO 90 J=1,DIMW
   90 S(I,J)=S(I,J)*FACS
      DO 95 I=1,DIML
      READ (5,150) (QRE(I,J),J=1,DIMW)
      DO 95 J=1,DIMW
   95 QRE(I,J)=QRE(I,J)*FACQRE
      DO 100 I=1,DIML
  100 READ (5,150) (PUMP(I,J),J=1,DIMW)
      IF (SUM.EQ.0.) GO TO 120
      DO 110 I=1,DIML
  110 READ (5,180) (PHI(I,J),J=1,DIMW)
  120 RETURN
C
  125 READ(5,200,END=129)TDELT,NUMTMP, FCDELT,IPUMP
      IF  (NUMTMP.NE.0) NUMT = NUMTMP
      IF (TDELT.NE.0.0) DELT = TDELT
      IF (FCDELT.NE.0.0) FACDLT  = FCDELT
      IF (IPUMP.GT.0) GO TO 120
      DO 128 I = 1,DIML
  128 READ(5,150,END=129)(PUMP(I,J),J=1,DIMW)
      RETURN
  129 JFINAL = 1
      RETURN
C     ............................................................
C
C
  130 FORMAT (20A4/20A4)
  140 FORMAT (F10.2,4I10,E10.3,I10,F10.2/I10,3F10.2,2E10.3,2F10.2)
  150 FORMAT (20F4.0)
  160 FORMAT (A5/A7/A7/A5)
  170 FORMAT (4E20.10/3E20.10,F10.0)
  180 FORMAT (8F10.4)
  190 FORMAT (8F10.0)
  200 FORMAT(E20.10,I10,F10.0,I10)
      END

      SUBROUTINE INOUT
C     SUBPROGRAM NO. 2
C     ------------------------------------------------------------
      INTEGER DIML,DIMW
      REAL*8 KEEP,IMK,NUM,MINS,M,K
      DIMENSION S(27,27), SS(27,27),HYCOND(27,27), RATE(27,27), KEEP(27,
     127), G(27), TEMP(27), BE(27), RHOP(27), CHK(10), STRT(27,27), T(27
     2,27), PHI(27,27), PUMP(27,27), WTABLE(27,27), DELX(27), DELY(27),Q
     3COEF(27,27), QRE(27,27), HEADNG(33)
      DOUBLE PRECISION PHI,RHOP,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,DELT
     1,PARAM,CHK,PNCH,CONTR,CHCK
      COMMON /INTEGR/ DIML,DIMW,NUMT,LENGTH,ITMAX,INO1,JNO1,KTH,NCYCLE,
     1JFINAL
      COMMON /SINGLE/ SUM,CELX,DELY,RATE,S,STRT,SPACNG,T,FLUXT,QRE,PUMP,
     1WTABLE,TESTP,FACTOR,ERR,FACS,FACSS,FACK,FACT,TMAX,SS,HYCOND,TEST,Q
     2COEF,TT,HEADNG,FACQRE,FACDLT
      COMMON /DOUBLE/ PHI,KEEP,DELT,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,
     1RHOP,PARAM,IMK,CHK,PNCH,CONTR,NUM,CHCK
C     ------------------------------------------------------------
C
```

```
      PURPOSE--TO PRINT OUT INPUT DATA
      WRITE (6,160) HEADNG
      WRITE (6,180) DELT,TMAX,NUMT,FACQRE,DIML,DIMW,LENGTH,ERR,FACTOR,FA
     1CS,FACSS,FACK,FACT,KTH,ITMAX
      IF (NCYCLE.GT.0) GO TO 85
      IF (CHK(1).EQ.PNCH) WRITE (6,110)
      IF (CHK(2).EQ.CONTR) WRITE (6,130)
      IF (CHK(3).EQ.NUM) WRITE (6,140)
      IF (CHK(4).EQ.CHCK) WRITE (6,150)
      WRITE (6,210) (DELX(J),J=1,DIMW)
      WRITE (6,220) (DELY(I),I=1,DIML)
      WRITE (6,190)
      DO 10 I=1,DIML
   10 WRITE (6,200) I,(T(I,J),J=1,DIMW)
      WRITE (6,290)
      DO 20 I=1,DIML
   20 WRITE (6,270) I,(WTABLE(I,J),J=1,DIMW)
   30 WRITE (6,230)
      DO 40 I=1,DIML
   40 WRITE (6,200) I,(S(I,J),J=1,DIMW)
      WRITE (6,260)
      DO 50 I=1,DIML
   50 WRITE (6,200) I,(RATE(I,J),J=1,DIMW)
      WRITE (6,250)
      DO 60 I=1,DIML
   60 WRITE (6,200) I,(SS(I,J),J=1,DIMW)
      WRITE (6,240)
      DO 70 I=1,DIML
   70 WRITE (6,200) I,(HYCOND(I,J),J=1,DIMW)
   85 CONTINUE
   80 WRITE (6,280)
      DO 90 I=1,DIML
   90 WRITE (6,270) I,(PUMP(I,J),J=1,DIMW)
      IF (NCYCLE.GT.0) RETURN
      WRITE (6,170)
      DO 100 I=1,DIML
  100 WRITE (6,200) I,(STRT(I,J),J=1,DIMW)
      WRITE (6,295)
      DO 95 I=1,DIML
   95 WRITE (6,200) I,(QRE(I,J),J=1,DIMW)
      RETURN

      ........................................................


  110 FORMAT (1H0,24HPUNCHED OUTPUT REQUESTED)
  130 FORMAT (1H0,26HCONTOURED OUTPUT REQUESTED)
  140 FORMAT (1H0,24HNUMERIC OUTPUT REQUESTED)
  150 FORMAT (1H0,28HMASS BALANCE CHECK REQUESTED)
  160 FORMAT (1H1,//1X,33A4,//)
  170 FORMAT (1H1,48X,37HINITIAL PIEZOMETRIC HEAD DISTRIBUTION)
  180 FORMAT (1H0,60X,16HINPUT PARAMETERS//40H LENGTH OF INITIAL TIME ST
     1EP IN SECONDS=,E10.3//46H MAXIMUM PERMITTED PERIOD OF PUMPING IN H
     2OURS=,E10.3//40H MAXIMUM PERMITTED NUMBER OF TIME STEPS=,I4//50H M
     3ULTIPLIER FOR LEAKAGE FLUX FROM CONFINING LAYER=,E10.3//37H NUMBER
     4 OF NODES IN COLUMN OF MATRIX=,I4//34H NUMBER OF NODES IN ROW OF M
     5ATRIX=,I4//32H NUMBER OF ITERATION PARAMETERS=,I4//28H ERROR CRITE
     6RIA FOR CLOSURE=,E10.3//45H MULTIPLIER FOR THICKNESS OF CONFINING
     7LAYER=,1PE10.3//36H MULTIPLIER FOR STORAGE COEFFICIENT=,E10.3//33H
     8 MULTIPLIER FOR SPECIFIC STORAGE=,E10.3//39H MULTIPLIER FOR HYDRAU
```

```
     9LIC CONDUCTIVITY=,E10.3//31H MULTIPLIER FOR TRANSMISSIVITY=,E10.3/
     $/40H NUMBER OF TIME STEPS BETWEEN PRINTOUTS=,I4//40H MAXIMUM PERMI
     $TTED NUMBER OF ITERATIONS=,I4)
  190 FORMAT (1H1,61X,21HTRANSMISSIVITY MATRIX)
  200 FORMAT (1H0,I4,11E11.3/(5X,11E11.3))
  210 FORMAT (1H1,40X,40HGRID SPACING IN PROTOTYPE IN X DIRECTION///(1H0
     1,12F10.3))
  220 FORMAT (1H0,40X,40HGRID SPACING IN PROTOTYPE IN Y DIRECTION///(1H0
     1,12F10.3))
  230 FORMAT (1H1,54X,26HSTORAGE COEFFICIENT MATRIX)
  240 FORMAT (1H1,52X,29HHYDRAULIC CONDUCTIVITY MATRIX)
  250 FORMAT (1H1,50X,33HSPECIFIC STORAGE OF CONFINING BED)
  260 FORMAT (1H1,61X,11HRATE MATRIX)
  270 FORMAT (1H0,I4,11E11.3/(5X,11E11.3))
  280 FORMAT (1H1,51X,27HDISCHARGE FROM AQUIFER, CFS)
  290 FORMAT (1H1,56X,21HWATERTABLE ELEVATIONS)
  295 FORMAT (1H1,57X,23HRATE OF RECHARGE MATRIX)
      END

      SUBROUTINE IPARAM
C     SUBPROGRAM NO. 3
C     -------------------------------------------------------------------
      INTEGER DIML,DIMW
      REAL*8 KEEP,IMK,NUM,MINS,M,K
      DIMENSION S(27,27), SS(27,27),HYCOND(27,27), RATE(27,27), KEEP(27,
     127), G(27), TEMP(27), BE(27), RHOP(27), CHK(10), STRT(27,27), T(27
     2,27), PHI(27,27), PUMP(27,27), WTABLE(27,27), DELX(27), DELY(27),Q
     3COEF(27,27), QRE(27,27), HEADNG(33)
      DOUBLE PRECISION PHI,RHOP,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,DELT
     1,PARAM,CHK,PNCH,CONTR,CHCK
      COMMON /INTEGR/ DIML,DIMW,NUMT,LENGTH,ITMAX,IND1,JND1,KTH,NCYCLE,
     1JFINAL
      COMMON /SINGLE/ SUM,DELX,DELY,RATE,S,STRT,SPACNG,T,FLUXT,QRE,PUMP,
     1WTABLE,TESTP,FACTOR,ERR,FACS,FACSS,FACK,FACT,TMAX,SS,HYCOND,TEST,Q
     2COEF,TT,HEADNG,FACQRE,FACDLT
      COMMON /DOUBLE/ PHI,KEEP,DELT,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,
     1RHOP,PARAM,IMK,CHK,PNCH,CONTR,NUM,CHCK
C     -------------------------------------------------------------------
C
C     PURPOSE--TO COMPUTE ITERATION PARAMETERS
C     COMPUTE HMIN
      HMIN=2.
      XVAL=3.1416**2/(2.*DIMW**2)
      YVAL=3.1416**2/(2.*DIML**2)
      DO 10 I=2,DIML
      DO 10 J=2,DIMW
      IF (T(I,J).EQ.0.) GO TO 10
      XPART=XVAL*(1/(1+DELX(J)**2/DELY(I)**2))
      YPART=YVAL*(1/(1+DELY(I)**2/DELX(J)**2))
      HMIN=AMIN1(HMIN,XPART,YPART)
   10 CONTINUE
      IF(HMIN.LT..0001) HMIN=.0001
      ALPHA=EXP(ALOG(1/HMIN)/(LENGTH-1))
      RHOP(1)=HMIN
      DO 20 NTIME=2,LENGTH
   20 RHOP(NTIME)=RHOP(NTIME-1)*ALPHA
      WRITE (6,30) (RHOP(J),J=1,LENGTH)
      RETURN
C
C     ooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooo
```

```
 30 FORMAT (1H1,56X,20HITERATION PARAMETERS///(1H ,10E12.3))
    END

    SUBROUTINE ROW
    SUBPROGRAM NO. 4
    ------------------------------------------------------------
    INTEGER DIML,DIMW
    REAL*8 KEEP,IMK,NUM,MINS,M,K
    DIMENSION S(27,27), SS(27,27),HYCOND(27,27), RATE(27,27), KEEP(27,
   127), G(27), TEMP(27), BE(27), RHOP(27), CHK(10), STRT(27,27), T(27
   2,27), PHI(27,27), PUMP(27,27), WTABLE(27,27), DELX(27), DELY(27),Q
   3COEF(27,27), QRE(27,27), HEADNG(33)
    DOUBLE PRECISION PHI,RHOP,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,DELT
   1,PARAM,CHK,PNCH,CONTR,CHCK
    COMMON /INTEGR/ DIML,DIMW,NUMT,LENGTH,ITMAX,INO1,JNO1,KTH,NCYCLE,
   1JFINAL
    COMMON /SINGLE/ SUM,DELX,DELY,RATE,S,STRT,SPACNG,T,FLUXT,QRE,PUMP,
   1WTABLE,TESTP,FACTOR,ERR,FACS,FACSS,FACK,FACT,TMAX,SS,HYCOND,TEST,Q
   2COEF,TT,HEADNG,FACQRE,FACDLT
    COMMON /DOUBLE/ PHI,KEEP,DELT,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,
   1RHOP,PARAM,IMK,CHK,PNCH,CONTR,NUM,CHCK
    ------------------------------------------------------------
    PURPOSE--TO CALCULATE IMPLICITLY ALONG ROWS, EXPLICITLY ALONG COLU
    MNS
    DO 10 J=1,DIMW
    BE(J)=0.
    G(J)=0.
 10 TEMP(J)=PHI(1,J)
    DO 70 I=2,DIML
    DO 30 J=2,JNO1

    oooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooo

    DETERMINE WHETHER NODE IS OUTSIDE AQUIFER BOUNDARY
    IF (T(I,J)) 20,30,20

    oooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooo

 20 RHO=S(I,J)/DELT

    oooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooo

    CALCULATE AVERAGE VALUES OF T BETWEEN ADJACENT NODES
    NODE CONFIGURATION.  T1=LEFT, T2=RIGHT, T3=UPPER, T4=LOWER
    T1=((2.*T(I,J-1)*T(I,J))/(T(I,J)*DELX(J-1)+T(I,J-1)*DELX(J)))/DELX
   1(J)
    T2=((2.*T(I,J+1)*T(I,J))/(T(I,J)*DELX(J+1)+T(I,J+1)*DELX(J)))/DELX
   1(J)
    T3=((2.*T(I-1,J)*T(I,J))/(T(I,J)*DELY(I-1)+T(I-1,J)*DELY(I)))/DELY
   1(I)
    T4=((2.*T(I+1,J)*T(I,J))/(T(I,J)*DELY(I+1)+T(I+1,J)*DELY(I)))/DELY
   1(I)
    IMK=PARAM*(T1+T2+T3+T4)

    oooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooo

    CHECK WHETHER NODE IS ALONG A STREAM OR ON A LAKE
```

```
    CALCULATE VALUES FOR PARAMETERS A,B,C, AND BE
    B=-T1-T2-RHO-IMK-QCOEF(I,J)
    A=T1
    C=T2
    W=B-A*BE(J-1)
    BE(J)=C/W
    DUMMY=0.
    IF (RATE(I,J).GT.0.) DUMMY=HYCOND(I,J)/RATE(I,J)

    ooooooooooooooooooooooooooooooooooooooooooooooooooooooooooo

    D=-T3*PHI(I-1,J)+(T4+T3-IMK)*PHI(I,J)-T4*PHI(I+1,J)-RHO*KEEP(I,J)-
   1DUMMY*(WTABLE(I,J)-STRT(I,J))+PUMP(I,J)/(DELX(J)*DELY(I))-QCOEF(I,
   2J)*STRT(I,J)-QRE(I,J)
    G(J)=(D-A*G(J-1))/W
 30 CONTINUE

    ooooooooooooooooooooooooooooooooooooooooooooooooooooooooooo

    CALCULATE HEAD VALUES FOR ROWS OF MATRIX AND PLACE THEM IN
    TEMPORARY LOCATION TEMP
    NO3=DIMW-2
    DO 60 KNO4=1,NO3
    NO4=DIMW-KNO4
    PHI(I-1,NO4)=TEMP(NO4)
    IF (T(I,NO4)) 50,40,50
 40 TEMP(NO4)=PHI(I,NO4)
    GO TO 60
 50 TEMP(NO4)=G(NO4)-BE(NO4)*TEMP(NO4+1)
 60 CONTINUE
 70 CONTINUE
    RETURN

    END

    SUBROUTINE COLUMN
    SUBPROGRAM NO. 5
    ===========================================================
    INTEGER DIML,DIMW
    REAL*8 KEEP,IMK,NUM,MINS,M,K
    DIMENSION S(27,27), SS(27,27),HYCOND(27,27), RATE(27,27), KEEP(27,
   127), G(27), TEMP(27), BE(27), RHOP(27), CHK(10), STRT(27,27), T(27
   2,27), PHI(27,27), PUMP(27,27), WTABLE(27,27), DELX(27), DELY(27),Q
   3COEF(27,27), QRE(27,27), HEADNG(33)
    DOUBLE PRECISION PHI,RHOP,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,DELT
   1,PARAM,CHK,PNCH,CONTR,CHCK
    COMMON /INTEGR/ DIML,DIMW,NUMT,LENGTH,ITMAX,INO1,JNO1,KTH,NCYCLE,
   1JFINAL
    COMMON /SINGLE/ SUM,DELX,DELY,RATE,S,STRT,SPACNG,T,FLUXT,QRE,PUMP,
   1WTABLE,TESTP,FACTOR,ERR,FACS,FACSS,FACK,FACI,TMAX,SS,HYCOND,TEST,Q
   2COEF,TT,HEADNG,FACQRE,FACDLT
    COMMON /DOUBLE/ PHI,KEEP,DELT,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,
   1RHOP,PARAM,IMK,CHK,PNCH,CONTR,NUM,CHCK
    ===========================================================
    PURPOSE--TO CALCULATE IMPLICITLY ALONG COLUMNS, EXPLICITLY ALONG
    ROWS
    DO 10 I=1,DIML
    BE(I)=0.
    G(I)=0.
```

```
10 TEMP(I)=PHI(I,1)
   DO 70 J=2,DIMW
   DO 30 I=2,INO1
   IF (T(I,J)) 20,30,20

   oooooooooooooooooooooooooooooooooooooooooooooooooooooooo

20 RHO=S(I,J)/DELT

   oooooooooooooooooooooooooooooooooooooooooooooooooooooooo

   CALCULATE AVERAGE VALUES OF T BETWEEN ADJACENT NODES
   T1=((2.*T(I,J-1)*T(I,J))/(T(I,J)*DELX(J-1)+T(I,J-1)*DELX(J)))/DELX
  1(J)
   T2=((2.*T(I,J+1)*T(I,J))/(T(I,J)*DELX(J+1)+T(I,J+1)*DELX(J)))/DELX
  1(J)
   T3=((2.*T(I-1,J)*T(I,J))/(T(I,J)*DELY(I-1)+T(I-1,J)*DELY(I)))/DELY
  1(I)
   T4=((2.*T(I+1,J)*T(I,J))/(T(I,J)*DELY(I+1)+T(I+1,J)*DELY(I)))/DELY
  1(I)
   IMK=PARAM*(T1+T2+T3+T4)

   oooooooooooooooooooooooooooooooooooooooooooooooooooooooo

   CHECK WHETHER NODE IS ALONG A STREAM OR ON A LAKE

   oooooooooooooooooooooooooooooooooooooooooooooooooooooooo

   CALCULATE VALUES FOR PARAMETERS A,B,C,AND BE
   A=T3
   C=T4
   B=-T3-T4-RHO-IMK-QCOEF(I,J)
   W=B-A*BE(I-1)
   BE(I)=C/W
   DUMMY=0.
   IF (RATE(I,J).GT.0.) DUMMY=HYCOND(I,J)/RATE(I,J)

   oooooooooooooooooooooooooooooooooooooooooooooooooooooooo

   D=-T1*PHI(I,J-1)+(T1+T2-IMK)*PHI(I,J)-T2*PHI(I,J+1)-RHO*KEEP(I,J)-
  1DUMMY*(WTABLE(I,J)-STRT(I,J))+PUMP(I,J)/(DELX(J)*DELY(I))-QCOEF(I,
  2J)*STRT(I,J)-QRE(I,J)
   G(I)=(D-A*G(I-1))/W
30 CONTINUE


   oooooooooooooooooooooooooooooooooooooooooooooooooooooooo
   CALCULATE HEAD VALUES FOR COLUMNS OF MATRIX AND PLACE IN TEMPORARY
   LOCATION TEMP
   NO3=DIML-2
   DO 60 KNO4=1,NO3
   NO4=DIML-KNO4
   PHI(NO4,J-1)=TEMP(NO4)
   IF (T(NO4,J)) 50,40,50
40 TEMP(NO4)=PHI(NO4,J)
   GO TO 60
50 TEMP(NO4)=G(NO4)-BE(NO4)*TEMP(NO4+1)
   SNGL=TEMP(NO4)-PHI(NO4,J)
   IF (ABS(SNGL).GT.ERR) TEST=1.
60 CONTINUE
```

```
   70 CONTINUE
      RETURN
      END
      SUBROUTINE PRNT1
      SUBPROGRAM NO. 6
      ------------------------------------------------------------------
      INTEGER DIML,DIMW
      REAL*8 KEEP,IMK,NUM,MINS,M,K
      DIMENSION S(27,27), SS(27,27),HYCOND(27,27), RATE(27,27), KEEP(27,
     127), G(27), TEMP(27), BE(27), RHOP(27), CHK(10), STRT(27,27), T(27
     2,27), PHI(27,27), PUMP(27,27), WTABLE(27,27), DELX(27), CELY(27),Q
     3COEF(27,27), QRE(27,27), HEADNG(33)
      DIMENSION DDN(27)
      DOUBLE PRECISION PHI,RHOP,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,DELT
     1,PARAM,CHK,PNCH,CONTR,CHCK
      COMMON /INTEGR/ DIML,DIMW,NUMT,LENGTH,ITMAX,INO1,JNO1,KTH,NCYCLE,
     1JFINAL
      COMMON /SINGLE/ SUM,DELX,DELY,RATE,S,STRT,SPACNG,T,FLUXT,QRE,PUMP,
     1WTABLE,TESTP,FACTOR,ERR,FACS,FACSS,FACK,FACT,TMAX,SS,HYCCND,TEST,Q
     2COEF,TT,HEADNG,FACQRE,FACDLT
      COMMON /DOUBLE/ PHI,KEEP,DELT,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,
     1RHOP,PARAM,IMK,CHK,PNCH,CONTR,NUM,CHCK
      ------------------------------------------------------------------
      THIS SUBROUTINE PRINTS OUT DRAWDOWN IN NUMERICAL FORM
      WRITE (6,30)
      DO 20 I=1,DIML
      DO 10 J=1,DIMW
   10 DDN(J)=STRT(I,J)-PHI(I,J)
   20 WRITE (6,40) I,(DDN(L),L=1,DIMW)
      RETURN
      oooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooo
   30 FORMAT (1H1,58X,16HDRAWDOWN IN FEET//)
   40 FORMAT (1H0,I5,11E11.3/(6X,11E11.3))
      END
      SUBROUTINE PRNTA
      SUBPROGRAM NO. 7
      ------------------------------------------------------------------
      INTEGER DIML,DIMW
      REAL*8 KEEP,IMK,NUM,MINS,M,K
      DIMENSION S(27,27), SS(27,27),HYCOND(27,27), RATE(27,27), KEEP(27,
     127), G(27), TEMP(27), BE(27), RHOP(27), CHK(10), STRT(27,27), T(27
     2,27), PHI(27,27), PUMP(27,27), WTABLE(27,27), DELX(27), CELY(27),Q
     3COEF(27,27), QRE(27,27), HEADNG(33)
      DOUBLE PRECISION PHI,RHOP,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,DELT
     1,PARAM,CHK,PNCH,CONTR,CHCK
      COMMON /INTEGR/ DIML,DIMW,NUMT,LENGTH,ITMAX,INO1,JNO1,KTH,NCYCLE,
     1JFINAL
      COMMON /SINGLE/ SUM,DELX,DELY,RATE,S,STRT,SPACNG,T,FLUXT,QRE,PUMP,
     1WTABLE,TESTP,FACTOR,ERR,FACS,FACSS,FACK,FACT,TMAX,SS,HYCCND,TEST,Q
     2COEF,TT,HEADNG,FACQRE,FACDLT
      COMMON /DOUBLE/ PHI,KEEP,DELT,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,
     1RHOP,PARAM,IMK,CHK,PNCH,CONTR,NUM,CHCK
      DIMENSION PRNT(27), SYM(39), BLANK(35)
```

```
      DATA SYM/1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,1HA,1HB,1HC,1HD,1HE,1
     1HF,1HG,1HH,1HI,1HJ,1HK,1HL,1HM,1HN,1HO,1HP,1HQ,1HR,1HS,1HT,1HU,1HV
     2,1HW,1HX,1HY,1HZ,1H0,1H ,1H*,1HG/,BLANK/35*1H /
      ------------------------------------------------------------------

      THIS SUBROUTINE PRINTS OUT DRAWDOWN AS ALPHAMERIC CONTOURS
      WRITE (6,50)
      IND=(65-DIMW)/2
      DO 40 IB=1,DIML
      DO 30 JB=1,DIMW
      K=STRT(IB,JB)-PHI(IB,JB)
      K=K/SPACNG
      IF (K.LT.0) GO TO 10
      REALK4=K
      K=AMOD(REALK4,36.)
      IF (K.LT.1.) PRNT(JB)=SYM(36)
   10 IF (K.LT.0) PRNT(JB)=SYM(39)
      IF (PHI(IB,JB).EQ.STRT(IB,JB)) PRNT(JB)=SYM(37)
      N=K
      IF (N.LT.1) GO TO 20
      PRNT(JB)=SYM(N)
   20 IF (PUMP(IB,JB).GT.0.) PRNT(JB)=SYM(32)
      IF(RATE(IB,JB).LT.0.)PRNT(JB)=SYM(27)
   30 CONTINUE
   40 WRITE (6,60) (BLANK(I),I=1,IND),(PRNT(JB),JB=1,DIMW)
      WRITE (6,70) SPACNG
      RETURN

      oooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooo

   50 FORMAT (1H0,50X,32HALPHABETIC CONTOURS FOR DRAWDOWN,////)
   60 FORMAT (1H ,65A2)
   70 FORMAT (10H0LEGEND***/18H0CONTOUR INTERVAL ,F10.3/32H0LOCATION OF
     1RECHARGE BOUNDARY R/16H0WELL LOCATION W/21H0CONE OF IMPRESSION G)
      END

      SUBROUTINE CHECK
      SUBPROGRAM NO. 8
      ------------------------------------------------------------------
      INTEGER DIML,DIMW
      REAL*8 KEEP,IMK,NUM,MINS,M,K
      DIMENSION S(27,27), SS(27,27),HYCOND(27,27), RATE(27,27), KEEP(27,
     127), G(27), TEMP(27), BE(27), RHOP(27), CHK(10), STRT(27,27), T(27
     2,27), PHI(27,27), PUMP(27,27), WTABLE(27,27), DELX(27), DELY(27),Q
     3COEF(27,27), QRE(27,27), HEADNG(33)
      DOUBLE PRECISION PHI,RHOP,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,DELT
     1,PARAM,CHK,PNCH,CONTR,CHCK
      DOUBLE PRECISION DELS
      COMMON /INTEGR/ DIML,DIMW,NUMT,LENGTH,ITMAX,IND1,JND1,KTH,NCYCLE,
     1JFINAL
      COMMON /SINGLE/ SUM,DELX,DELY,RATE,S,STRT,SPACNG,T,FLUXT,QRE,PUMP,
     1WTABLE,TESTP,FACTOR,ERR,FACS,FACSS,FACK,FACT,TMAX,SS,HYCOND,TEST,Q
     2COEF,TT,HEADNG,FACQRE,FACDLT
      COMMON /DOUBLE/ PHI,KEEP,DELT,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,
     1RHOP,PARAM,IMK,CHK,PNCH,CONTR,NUM,CHCK
      COMMON /CHKWRT/ CONET,PUMPT,DELQT,DIFF,DIFFT
      ------------------------------------------------------------------

      THIS SUBROUTINE COMPUTES THE ERROR IN THE SOLUTION ON A MASS
```

```
      BALANCE BASIS
      DIFF=0
      DO 10 I=2,DIML
      DO 10 J=2,DIMW
      IF (T(I,J).EQ.0.) GO TO 10
      AREA=DELX(J)*DELY(I)
      DUMDUM=0.
      IF(RATE(I,J).NE.0.) DUMDUM=HYCOND(I,J)/RATE(I,J)

      T1=((2.*T(I,J-1)*T(I,J))/(T(I,J)*DELX(J-1)+T(I,J-1)*DELX(J)))*DELY
     1(I)
      T2=((2.*T(I,J+1)*T(I,J))/(T(I,J)*DELX(J+1)+T(I,J+1)*DELX(J)))*DELY
     1(I)
      T3=((2.*T(I-1,J)*T(I,J))/(T(I,J)*DELY(I-1)+T(I-1,J)*DELY(I)))*DELX
     1(J)
      T4=((2.*T(I+1,J)*T(I,J))/(T(I,J)*DELY(I+1)+T(I+1,J)*DELY(I)))*DELX
     1(J)
      QIN=-T1*(PHI(I,J)-PHI(I,J-1))-T3*(PHI(I,J)-PHI(I-1,J))
      QOUT=-T2*(PHI(I,J+1)-PHI(I,J))-T4*(PHI(I+1,J)-PHI(I,J))
      DELS=-S(I,J)*AREA*(PHI(I,J)-KEEP(I,J))
      DELPMP=-PUMP(I,J)
      DELQ=QCOEF(I,J)*(STRT(I,J)-PHI(I,J))*AREA
      STDYQ=-DUMDUM*(STRT(I,J)-WTABLE(I,J))*AREA+QRE(I,J)*AREA

      CONET=CONET+DELS
      PUMPT=PUMPT+DELPMP*DELT
      DELQT=DELQT+DELQ*DELT
      FLUXT=FLUXT+STDYQ*DELT

      DIF=(QOUT-QIN-(DELPMP+DELQ+STDYQ))*DELT-DELS
      DUM=ABS(DIF)
      DIFF=AMAX1(DIFF,DUM)

      DIFFT=DIFFT+DIF
   10 CONTINUE
      RETURN

      END

      BLOCK DATA

      ------------------------------------------------------------------
      INTEGER DIML,DIMW
      REAL*8 KEEP,IMK,NUM,MINS,M,K
      DIMENSION S(27,27), SS(27,27),HYCOND(27,27), RATE(27,27), KEEP(27,
     127), G(27), TEMP(27), BE(27), RHOP(27), CHK(10), STRT(27,27), T(27
     2,27), PHI(27,27), PUMP(27,27), WTABLE(27,27), DELX(27), DELY(27),Q
     3COEF(27,27), QRE(27,27), HEADNG(33)
      DOUBLE PRECISION PHI,RHOP,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,DELT
     1,PARAM,CHK,PNCH,CONTR,CHCK
      COMMON /INTEGR/ DIML,DIMW,NUMT,LENGTH,ITMAX,INO1,JNO1,KTH,NCYCLE,
     1JFINAL
      COMMON /SINGLE/ SUM,DELX,DELY,RATE,S,STRT,SPACNG,T,FLUXT,QRE,PUMP,
     1WTABLE,TESTP,FACTOR,ERR,FACS,FACSS,FACK,FACT,TMAX,SS,HYCOND,TEST,Q
     2COEF,TT,HEADNG,FACQRE,FACDLT
      COMMON /DOUBLE/ PHI,KEEP,DELT,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,
     1RHOP,PARAM,IMK,CHK,PNCH,CONTR,NUM,CHCK
      ------------------------------------------------------------------

      DATA CHK(1)/5HPUNCH/,CHK(2)/7HCONTOUR/,CHK(3)/7HNUMERIC/,CHK(4)/5H
```

```
     1CHECK/,QCOEF/729*0.0/
      END
      SUBROUTINE CLAY
      SUBPROGRAM NO. 9
      ------------------------------------------------------------------
      INTEGER DIML,DIMW
      REAL*8 KEEP,IMK,NUM,MINS,M,K
      DIMENSION S(27,27), SS(27,27),HYCOND(27,27), RATE(27,27), KEEP(27,
     127), G(27), TEMP(27), BE(27), RHOP(27), CHK(10), STRT(27,27), T(27
     2,27), PHI(27,27), PUMP(27,27), WTABLE(27,27), DELX(27), DELY(27),Q
     3COEF(27,27), QRE(27,27), HEADNG(33)
      DOUBLE PRECISION PHI,RHOP,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,DELT
     1,PARAM,CHK,PNCH,CONTR,CHCK
      COMMON /INTEGR/ DIML,DIMW,NUMT,LENGTH,ITMAX,INO1,JNO1,KTH,NCYCLE,
     1JFINAL
      COMMON /SINGLE/ SUM,DELX,DELY,RATE,S,STRT,SPACNG,T,FLUXT,QRE,PUMP,
     1WTABLE,TESTP,FACTOR,ERR,FACS,FACSS,FACK,FACT,TMAX,SS,HYCOND,TEST,Q
     2COEF,TT,HEADNG,FACQRE,FACDLT
      COMMON /DOUBLE/ PHI,KEEP,DELT,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,
     1RHOP,PARAM,IMK,CHK,PNCH,CONTR,NUM,CHCK
      ------------------------------------------------------------------
      PIE=3.1415927
      TT=0.0
      PRATE=0.0
      PCOND=0.0
      PSS=0.0
      DO 50 I=1,DIML
      DO 50 J=1,DIMW
      IF (HYCOND(I,J).LE.0..OR.SS(I,J).EQ.0.) GO TO 50
      IF (RATE(I,J).LE.0..OR.T(I,J).EQ.0.) GO TO 50
      IF ((RATE(I,J).EQ.PRATE).AND.(HYCOND(I,J).EQ.PCOND).AND.(SS(I,J).E
     1Q.PSS)) GO TO 40
      DIMT=HYCOND(I,J)*SUM/(RATE(I,J)*RATE(I,J)*SS(I,J)*2.)
      IF (DIMT.GT.TT) TT=DIMT
      SUMN=0.0
      DO 20 L=1,200
      PPT=PIE*PIE*DIMT
      IF (DIMT.LT.1.0E-03) PPT=1.0/DIMT
      CK=(2.3-PPT)/(2.*PPT)
      POWER=L*L*PPT
      IF (POWER.LE.174.) GO TO 10
      POWER=150
   10 PEX=EXP(-POWER)
      SUMN=SUMN+PEX
      IF (PEX.GT.0.00009) GO TO 20
      IF (L.GT.CK) GO TO 30
   20 CONTINUE
   30 DENOM=1.0
      IF (DIMT.LT.1.0E-03) DENOM=SQRT(PIE*DIMT)
   40 Q1=HYCOND(I,J)/(RATE(I,J)*DENOM)
      QCOEF(I,J)=Q1+2.0*Q1*SUMN
      PRATE=RATE(I,J)
      PCOND=HYCOND(I,J)
      PSS=SS(I,J)
   50 CONTINUE
   60 RETURN
      END
```