UK Research Programme on Fracture Mechanics – Review of Progress

John Sharples[1], Chris Watson[2] and Keith Wright[2]

1) Serco Assurance, Birchwood Park, Warrington, Cheshire, WA3 6GA, UK.
2) Rolls-Royce plc, P. O. Box 2000, Raynesway, Derby, DE21 7XX, UK

ABSTRACT

A long-term research programme was launched in the UK in late 2004, a significant part of which is aimed at developing and improving fracture mechanics based methods for assessing the structural integrity of metallic engineering structures and components. The main thrust of the programme initially, is to develop a better understanding of the levels of conservatisms (and possibly non-conservatisms) inherent in current methodologies, primarily focusing on the R6 method. Following on from this, the intention is then to develop improved guidance on relevant aspects so as to reduce conservatisms and eliminate any non-conservatisms appropriately.

The programme involves Rolls-Royce plc. and Serco Assurance, supported by a number of UK universities and the fracture mechanics studies are strongly linked to the R6 Development Programme.

The current areas of investigation within the fracture mechanics studies include the following:

- Deriving alternative methods for the treatment of combined primary and secondary stresses in fracture mechanics evaluations.
- Assessing the effects that cyclic loading and crack growth have on crack driving force parameters so that the significance of ignoring these aspects (as in conventional fracture mechanics methodologies) can be evaluated.
- Assessing the validity of the characterisation rules for multiple defects, particularly in view of the fact that recent work by Bezensek and Hancock (2004) has indicated that there may be non-conservatisms in the current rules when applying them to cleavage fracture situations.
- The development and use of micro-mechanistic local approach models for future application to plant structural integrity problems but also to be used as a tool for underpinning the understanding of relevant issues being addressed within the research programme.
- Develop guidance on the most relevant way of dealing with weld residual stress profiles in fracture mechanics calculations (e.g. by stress partitioning or by using stress intensity factors that can solve for high order polynomial equations).

The work is being undertaken by a combination of computational and experimental studies. The benefits of this work are intended to be the provision of more realistic assessment methodologies. In many instances this will be achieved by first validating the assumptions in current methodologies to provide a sound basis for the development of future methodologies.

The paper presents an overview of the research work being undertaken and provides an outline of progress made to-date in terms of some of the key technical results obtained.

INTRODUCTION

Rolls-Royce and Serco Assurance are working in partnership to develop underlying research programmes that focus on environmentally assisted cracking (EAC), residual stresses and fracture mechanics. A more comprehensive and in-depth understanding of these areas should improve the ability to predict the reliability of working components. An initial five-year programme on fracture mechanics has been defined and work started in November 2004 [1]. This paper provides an overview of some of the current work packages and a selection of results obtained so far.

THE TREATMENT OF COMBINED PRIMARY AND SECONDARY STRESS

Structural integrity assessments of components containing flaws need to consider the effects of both primary loading (e.g. pressure) and secondary loading (e.g. residual or thermal stresses). The aim of the work is to build upon previous work to provide an improved technique for the treatment of combined primary and secondary stresses.

The R6 defect assessment procedure [2] defines failure[1], of a cracked component, by the co-incidence of the assessment point (L_r K_r) and the Failure Assessment Diagram (FAD), $K_r = f(L_r)$, where the assessment point parameters, L_r and K_r, are measures of proximity to failure by plastic collapse and linear elastic fracture mechanics (LEFM), respectively. The K_r parameter is simply the ratio of the value of the LEFM stress intensity factor to material toughness, and in the case of loading by primary stresses acting alone, a plasticity correction is applied to stress intensity factor through the FAD. For the treatment of combined primary and secondary loading, since secondary stresses are not included in the FAD function $f(L_r)$, a plasticity correction factor ρ (or V) is applied to the secondary stress component of the K_r parameter. Two procedures for the evaluation ρ (or V) are presented in R6: a simplified conservative method for moderate levels of secondary stress; and a general more detailed method which requires the use of 'Look-up' tables. Recent studies have indicated that the current R6 methodology for the treatment of combined primary and secondary stresses may be excessively conservative in some situations, particularly where the secondary stresses are relatively high compared to the primary stresses.

An alternative, and potentially less conservative definition of the R6 assessment point parameter K_r for combined primary and secondary loading has been proposed as [3]:

$$K_r = \frac{f(L_r)}{K_{mat}} \left\{ \left(\frac{K_I^p}{f(L_r)} \right)^2 + \left(K_J^s\right)^2 + 2K_I^p K_J^s \right\}^{1/2} \tag{1}$$

where the stress intensity factors and material toughness symbols K have the same definitions as those of R6, and the function $f(L_r)$ is any of the FAD options of R6. The definition of the L_r parameter remains unchanged. i.e. $L_r = \sigma_{ref}^p / \sigma_y$, where again stresses σ retain their R6 definitions.

This definition of K_r in the alternative method has the distinct advantage over the conventional R6 approach in that it does not require the determination of a ρ (or V) factor, and hence does not require the use of 'Look-up' tables. In further studies, Eq. (1) has been further simplified to the expression:

$$K_J = \frac{K_I^P}{f(L_r)} + K_J^S \tag{2}$$

For the evaluation of K_J in this simplified form, the inputs are elastic stress intensity factor for primary stresses -K_I^p, elastic-plastic stress intensity factor for secondary stresses -K_J^s and the L_r parameter, with $f(L_r)$ determined for the R6 FAD [2].

Detailed finite element (FE) analysis has been carried out on various simple geometries and loading to test the validity of K_J evaluated from the simplified method of Eq. (2) compared to that evaluated from the current R6 methodology. Comparisons between the FE and Simplified Expression (Eq. (2)) demonstrate good agreement of CDF predictions (see Figures 1a and 1b as an example which show, for various values of L_r,, K_J calculated from Eq. (2) or from the current R6 methodology normalised to K_J evaluated from the FE analysis). The Simplified Expression is shown to reduce the conservatism in the current R6 procedure. This is a result of the Simplified Expression giving a more rapid reduction in the relative contribution of the secondary stress with increasing primary stress.

Some relative non-conservatisms are indicated by the results mainly at high values of the primary load parameter L_r, although it has been shown that these do not necessarily represent absolute non-conservatism of the method. In general, given the good agreement with the Simplified Method and FE results and a reduction in the conservatism compared to the R6 procedure, it is considered that this work presents further positive evidence towards the acceptability of the Simplified Method.

Further details of this work will be provided in future papers as the studies are progressed further.

LOAD HISTORY EFFECTS

Assessments of the integrity of structures containing defects or cracks require estimates to be made of the elastic-plastic

[1] Here the term failure is used to define a limiting or unsafe condition, which is a conservative estimate of the actual predicted component failure.

crack driving force (CDF) parameter J. This is the characterising parameter that controls the intensity of the fields of stress and strain close to the tip of a crack. Such estimates of J are inherently made in assessment procedures such as R6 [2]. These values of J can readily be converted to the K_J parameter referred to in the previous section.

Many components are subjected to a large number of load cycles in their lifetime. It is thus important to understand the effect that this has on the fracture mechanics assessment (covering defect growth and fracture initiation), and avoid any excessive conservatism (or indeed non-conservatism) in the approaches used. This requires an understanding of how the CDF can be quantified under such conditions. The current basis of the CDF J parameter in fracture mechanics is the J-Contour Integral and this generally assumes proportional loading on a stationary defect.

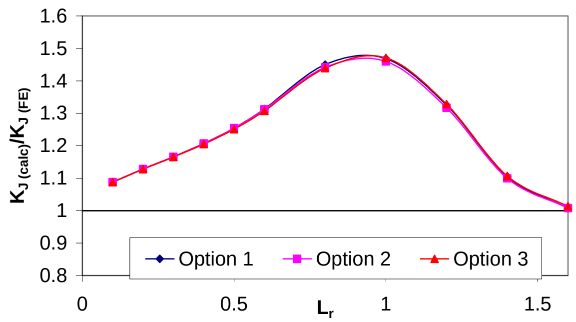


Figure 1a – Comparison of K_J estimates with FE values (R6 method)

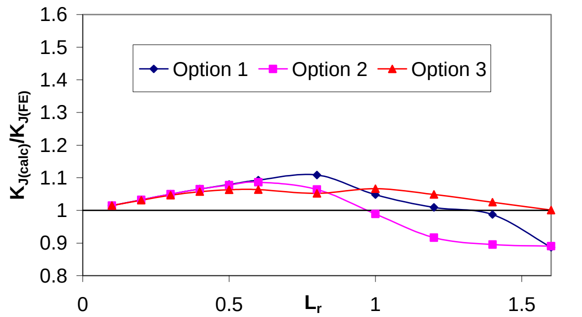


Figure 1b – Comparison of K_J estimates with FE values (Simplified method)

The objective of the study is to be able to provide engineering methodologies that can in the future be used to provide more realistic assessments of plant integrity. Two aspects that have recently been considered have addressed an overload followed by cyclic primary loading of a centre-cracked plate containing a simplified residual stress field, and the evaluation of CDF for growing cracks.

Finite Element (FE) analyses and R6 [2] calculations were undertaken and compared to examine the effects of inserting a crack at different times during the life of an engineering structure (simulated by the centre-cracked plate). The range of load cases allowed comparisons of the effects of (i) the material hardening model, (ii) the point in service at which the crack appears and (iii) how the crack is introduced. Through comparing the R6 and FE results, inaccuracies within existing assessment procedures in accounting for load history effects could be highlighted. Within the FE analyses, various local crack tip parameters, such as Crack Tip Opening Displacement (CTOD) and the simple Rice and Tracey ductile damage parameter [5], were calculated in addition to the J crack driving force.

The stresses acting across the plane of the potential crack are shown in Figure 2a at various times during the load history of the un-cracked plate where "PT" denotes Pressure Test (i.e. a single overload corresponding to 2/3 yield stress prior to the cyclic loads that corresponded to a value of 1/3 yield stress). "RS" denotes the residual stress. Figure 2b shows how inserting the crack at different times in the loading history affects the crack characterising parameters.

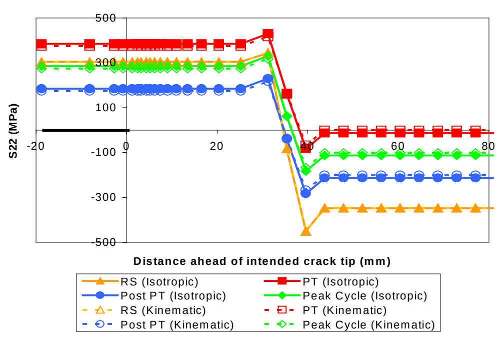


Figure 2a – Stress Distributions in the Uncracked Plate.

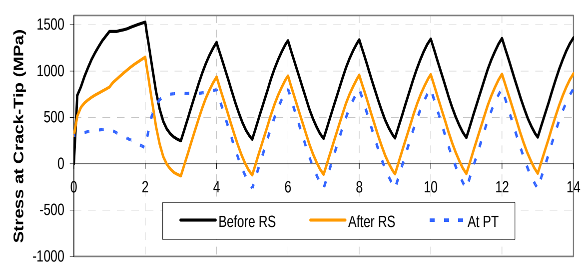


Figure 2b – Crack Driving Force Comparisons for Cracks Inserted at Different Times during the Load History with the Kinematic model.

The study concerned with a growing crack was performed using the finite element programme ABAQUS [6] and the in-house post-processing program JEDI [7]. The analyses examine the following aspects (i) a crack is introduced progressively, element by element. (ii) J-integrals are evaluated using both crack tip contours and rectangular contours that envelope the whole crack trajectory. (iii) The effect of residual stress is investigated by applying a compressive pre-load in some load cases.

Analyses have been carried out for both J-integrals results and crack-tip opening angle (CTOA) using two levels of crack tip mesh refinement. Firstly, the results show that crack growth should be analysed with the non-proportional option of JEDI rather than the proportional option, because at each successive crack tip position, a residual stress field is created for subsequent crack growth. These form a plastic wake which invalidates the proportional loading approach. A comparison for J-integrals between proportional and non-proportional loading is given in Figure 3a.

Secondly, the analysis points out that the J-integrals result obtained by ABAQUS with crack growth are invalid due to the incorrect assumption of proportional loading that ABAQUS makes. In addition, the mesh size has a significant effect on J-integrals for the growing crack; but the effect of mesh size on CTOA is small (see Figure 3b).

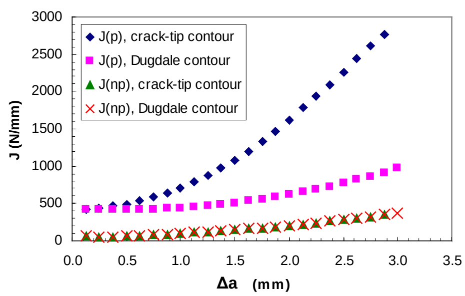


Figure 3a. Comparison of J-integrals from proportional and non-proportional loading.

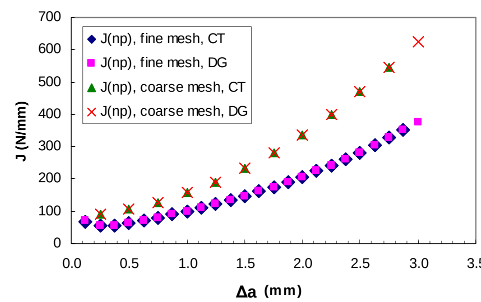


Figure 3b. Comparison of J-integral results between the fine and coarse mesh analyses.

The results of the studies referred to in this section show that (a) different material hardening models give similar uncracked distributions of residual stress at various times in the loading history, at least for the simply applied residual stress field and primary loads used in the study. (b) The tensile overload reduces the pre-existing uniform, uncracked, residual stress acting over the crack site by about 38%; (c) The later a crack is instantaneously introduced into the structure, the lower the value of peak crack driving force obtained during subsequent cyclic loading. (d) Cracks introduced progressively into a residual stress field give significantly lower crack driving forces for subsequent combined secondary and primary loading. (e) These reductions in crack driving force for a progressively introduced crack, compared with instantaneously introduced, point to the potential benefit of reduced conservatism in assessments of defects that develop slowly during services by creep, stress corrosion and fatigue.

More detailed information on some of the studies being performed on load history effects are provided in another paper in this conference [4] and within [8].

MULTIPLE FLAWS

In procedures such as R6[2] and BS 7910[9] for assessing the structural integrity of structures, complex multiple flaws located in close proximity to each other may be required to be characterized as one, larger, single flaw. However, there are concerns that current characterization procedures may be non-conservative under certain situations, in particular where the possibility of cleavage fracture may have to be considered [10, 11].

A combined experimental and analytical programme of work is underway in order to further investigate this potential non-conservatism for situations where the possibility of cleavage failure may have to be taken into account when assessing structures or components containing multiple flaws.

Details of early stages of the analytical programme were reported in [12] and comprised a number of elastic-plastic finite element analyses to evaluate cleavage failure probability, via a master-curve based approach, for interacting twin flaws and the corresponding characterised single flaw under an applied tensile load. These analyses considered surface-breaking semi-elliptical flaws of the same depth but with three values of crack aspect ratio (crack depth, a / crack semi-length, c) and a range of separations (0 to 10mm). It was found that non-conservatism was indicated for flaws of high aspect ratio in contact.

The failure probability P_F can be evaluated as an integral of the stress intensity factor along the flaw, via the following relation derived based upon weakest link methodology [12]:

$$P_F = 1 - \exp\left\{ -\frac{1}{B_0} \int_0^L \left(\frac{K_J(l) - K_{min}}{K_0 - K_{min}} \right)^4 dl \right\} \tag{3}$$

Recasting Eq. (3) into a form more appropriate to the FE model results in:

$$P_F = 1 - \exp\left\{ -\frac{2}{B_0} \sum_{i=1}^{n} \left(\frac{K_J(i) - K_{min}}{K_0 - K_{min}} \right)^4 \Delta l_i \right\} \tag{4}$$

where $K_J(i)$ is the value of K_J for a one-element 'slice' number i of the crack front, evaluated as the mean of the nodal values at each end of the slice and Δl_i is the slice length. The factor K_{min} is taken as 20MPa√m and B_0 as 25mm. K_0 is given by [11]:

$$K_0 = 31 + 77 * \exp(0.019 * (T - T_0)) \tag{5}$$

where T is the temperature (-196°C) and T_0 is -130°C [5, 6]. The factor of 2 is introduced to account for symmetry.

Further work has been centred on (i) Extending the three aspect ratios (a/c = 0.2, 0.44 and 0.8) for the twin surface flaws previously considered to a/c = 1. (ii) Extending the study to include an applied bending load. (iii) Extending the study on surface breaking flaws to include embedded flaws (for a/c = 0.8 and tensile loading only). Shown in Figure 4a-4c are the bend loading case for the a/c=1.0. Figure 4a shows the values of P_F for the actual a/c=1.0 twin flaw geometries under bending loading. A small increase in P_F is shown as the separation reduces from 10mm to 6mm. There are then progressively larger increases as separation reduces to 2mm. This is followed by a very large increase as the flaws come into contact. Figure 4b shows values of P_F for the corresponding characterised flaws. For the characterised flaws, the predicted failure probability is shown to increase with decreasing flaw aspect ratio (for a fixed crack depth, a) and, less strongly, with increasing separation. The overall trends for all the bending cases are broadly in line with those previously seen for the tensile loads, though probabilities are higher (for the bend loading).

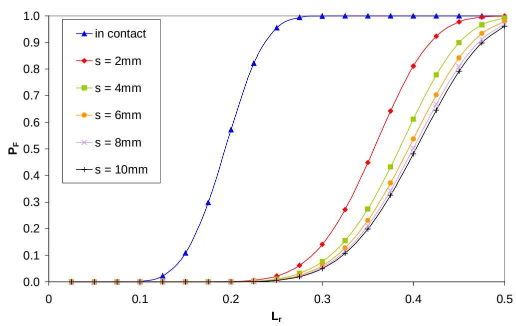


Figure 4a: Failure probabilities for Twin Flaws with Various Separations, a/c=1.0.

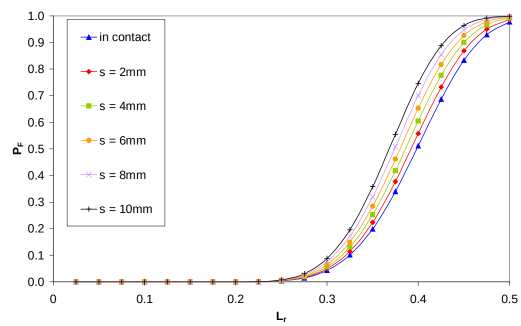


Figure 4b: Failure probabilities for Characterised Flaws. from Flaws in Figure 4a.

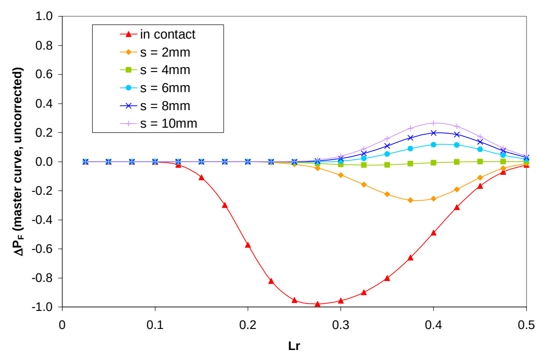


Figure 4c: ΔP_F for a/c = 1.0, Bending

Plotting the difference in P_F between the original and characterised flaw (Figure 4c) reveals that the conservatism (defined as when ΔP_F has a positive value) reduces strongly with the reduction of separation from 10mm to 4mm until at 4mm separation, no significant conservatism remains. There is then a large switch to non-conservatism as the separation reduces to 2mm and as separation goes to zero, the non-conservatism becomes very large, reaching almost -1.0 between L_r=0.25 and 0.3.

More details of the further work referred to above are given in [14] and another paper in this Conference [15] provides this information together with details of additional analyses that have been undertaken based on the local approach Beremin modelling approach [16].

Results from the cases studied so far indicate that (a) The use of the Beremin method to evaluate cleavage failure probability is more robust than the master curve method. (b) The characterisation rules for surface flaws are potentially more non-conservative for bend loading than for tensile loading. (c) The characterisation rules are potentially less non-conservative for embedded flaws than for surface flaws.

LOCAL APPROACH

Local approach methods provide a complementary approach to traditional methods based on crack tip driving force and constraint. The Beremin cleavage model [16] is a typical local approach model. In the Beremin model, the probability of cleavage fracture depends on the stress field around the crack tip and material parameters m and σ_u which can be expressed as a two-parameter Weibull distribution of the form:

$$P_F = 1 - \exp\left\{-\left(\frac{\sigma_W}{\sigma_u}\right)^m\right\} \tag{6}$$

Where σ_u and m are material constants or model parameters. The quantity σ_W is the scalar Weibull stress, which can be calculated from the spatial distribution of principal stresses in the component. The material parameters must be chosen so that the model can predict the fracture behaviour of laboratory fracture toughness specimens. Determination of the parameters provides a calibrated or tuned Beremin model.

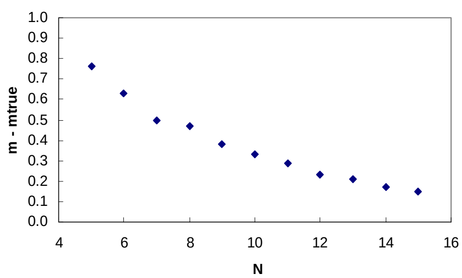
However, it has been found that the robustness of the calibration process is sensitive to a number of factors, mainly including (i) the quality and sample size of the fracture toughness data. (ii) the quality of the stress vs. strain (tensile) data for the material. The sensitivity of calibration processes to the size of the fracture toughness data sets has thus been investigated using a series of Monte-Carlo simulations for different sample sizes, and estimates of the Beremin parameters $\hat{m}$ and $\hat{\sigma}_u$ have been obtained. The estimates are compared with true values for the parameters and errors in the estimates are obtained. The numerical simulations have been repeated a large number of times to determine standard errors in the estimates. Two methods of calibration are described and analysed. The first method, the Gao and Dodds J-scaling method [17], converts high and low constraint fracture toughness or critical J values to equivalent small scale yielding values and ensures that the scale factors of the two equivalent J distributions are equal for each constraint geometry. A second method is also studied in which equal values of m and σ_u are obtained for high and low constraint geometries without having to introduce J-scaling to small scale yielding values.

Monte Carlo simulations of each calibration method have been carried out using samples of between 5 and 15 for each constraint geometry. The estimates $\hat{m}$ and $\hat{\sigma}_u$ have been calculated and standard errors in the estimates have been determined by carrying out 10000 simulations.

Results, as presented in Figure 5a and 5b show, respectively that when the sample size is increased, the standard errors decrease and the values of $\overline{m}$ and $\overline{\beta}$ (corresponding to J in the MBL small scale yielding condition) become closer to the theoretical true values of $m = 5$ and $\beta = 50$. However, the true values of the parameters are never attained and the estimates always over-estimate the true values.

The main conclusions that can be drawn from the work are (a) There is a link between the Beremin model parameter values and the stress vs. strain behaviour of the material. Consequently, it is possible that no calibration will be possible unless confidence can be demonstrated in the quality of measured fracture toughness data and tensile properties of a material. (b) Numerical simulations of calibration procedures using the Gao and Dodds method [17] have demonstrated that the accuracy of the results is sensitive to the size of the fracture toughness samples. (c) With small samples, it was not always possible to carry out a successful calibration. This implies that it may not always be possible to calibrate the model to a given set of experimental fracture toughness data. This possibility becomes less likely as the sample size is increased, so calibration should always be possible if more fracture toughness tests can be carried out. (d) The Gao and Dodds calibration procedure [17] depends on critical J samples from specimen geometries with different constraint levels. The results show that better results are obtained when the levels of constraint are more different from the SSY condition.

Detailed information on the work described in this section is provided in [18].



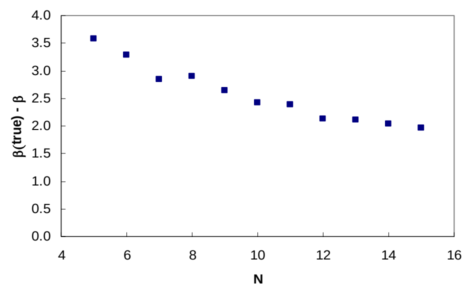


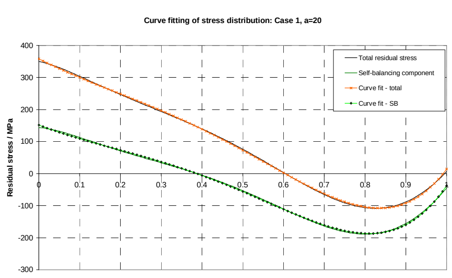
Figure 5a. Error on $\hat{m}$ as a function of sample size N. **Figure 5b. Error on $\hat{\beta}$ as a function of sample size N.**

RESIDUAL STRESS PROFILES

There is a significant amount of work within the U.K. and elsewhere aimed at improving methods for predicting the magnitude and distribution of welding residual stresses, such as that being undertaken in the Rolls Royce/Serco Assurance Research Programme as described in [19]. The level of assessment effort to provide a detailed residual stress profile can be considerable. It is more straight-forward to provide a simpler profile describing the distribution in terms of membrane and bending stress. Obtaining a profile of the self balancing component often necessitates time consuming bead by bead modelling of the welding procedure.

A study has therefore been undertaken to investigate the importance of accurately representing residual stress distributions in structural integrity assessments. This has been carried out by comparing regular polynomial representations of residual stress distributions, combined with available weight function stress intensity factor solutions (as provided in the R6 procedures [2]) with alternative polynomial representations of residual stress distributions, which provide a more accurate fit in the region of the crack.

Such improvements in representation of residual stress profiles provide an indication as to how stress intensity factor solutions could, in future, be modified in order to result in improved accuracy of calculated stress intensity factors. Representation by partitioning residual stress profiles into membrane, bending and self-balancing components, in terms of providing a more straight-forward route for curve-fitting of residual stress profiles is considered. The investigation considers several transverse, through-thickness residual stress distributions. Stress intensity factors are calculated for a variety of crack sizes.



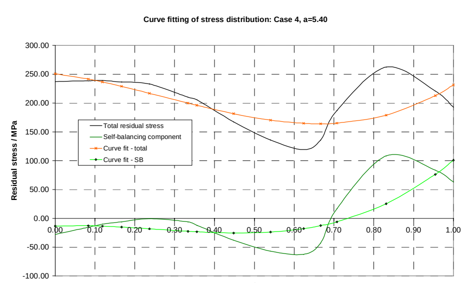


Figure 6. Curve fitting Ex. 1 of residual stress profile **Figure 7. Curve fitting Ex. 2 of residual stress profile**

An example of the curve fitting of the residual stress profile for an edge crack in an infinitely wide plate with crack depth a=20mm and thickness t=25 mm is illustrated in Figure 6. The fifth order polynomial expressions used to describe the stress distribution over the crack depth for both the total residual stress profile and the self-balancing component are provided in this figure. The curve fitting illustrated in Figure 6 exhibits a close approximation of the actual residual stress distribution. However in some cases studied, the curve fit was a poor representation of the underlying data.

An example of such an instance is illustrated in Figure 7 , where it can be observed that the third order polynomial fit to the total residual stress and self-balancing components do not represent the true residual stress distributions compared, for example, with that illustrated in Figure 6.

In light of these observations it was considered worthwhile to examine if an alternative curve fitting method could be identified which might provide improved accuracy in some cases. An alternative method has been considered. The new method was developed in-house to approximate the residual stress distribution as a rational polynomial (a ratio of two polynomials):

$$\sigma = \sigma_{(u)} = \left[\frac{\sigma_{a,0} + \sigma_{a,1}\left(\frac{u}{a}\right) + \sigma_{a,2}\left(\frac{u}{a}\right)^2 + \sigma_{a,3}\left(\frac{u}{a}\right)^3}{\sigma_{b,0} + \sigma_{b,1}\left(\frac{u}{a}\right) + \sigma_{b,2}\left(\frac{u}{a}\right)^2 + \sigma_{b,3}\left(\frac{u}{a}\right)^3 + \sigma_{b,4}\left(\frac{u}{a}\right)^4} \right] \qquad \text{for } 0 \le u \le a, \quad (7)$$

Where $\sigma_{a,0}$, $\sigma_{a,1}$ etc… are the fitting coefficients of polynomial for residual stress, u is the through thickness t position ($0 \le u \le t$) and a is the crack length. Least-squares values for the coefficients in this equation were obtained by minimising the sum of the squares of the residual values between actual and calculated values of stress, using the Microsoft Excel Solver Add-In.

From Figures 8 and 9 it can be observed that this method offers an improved approximation of the stress distributions, as compared with the third order polynomial descriptions which had been calculated previously, particularly in the two examples calculated. The additional computational effort required to achieve this improvement is not unduly onerous.

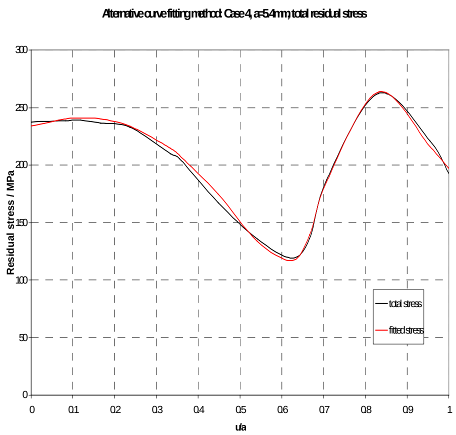


Figure 8: Alternative curve fitting method for total residual stress of Ex. 2.

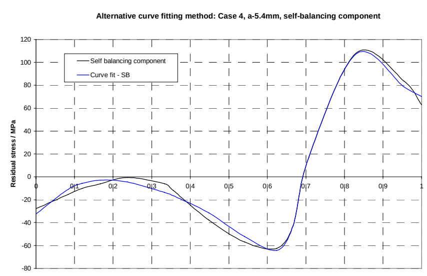


Figure 10: Alternative curve fitting method for self-balancing residual stress component of Ex. 2.

The alternative curve fitting method described above provides a useful illustration of how a curve fit may be improved, and could be used in future finite element analyses to investigate the significance of differences between stress intensity factors calculated using both good and poor curve fits. However, this study shows that it is not possible to provide general guidance on the relative significance of membrane, bending and self-balancing components of a residual stress profile when evaluating values of stress intensity factor. Therefore, when planning and undertaking detailed analysis, or indeed experimental work, to evaluate residual stress profiles to feed into a structural integrity assessment it is prudent to consider what crack depths are likely to be relevant. This may be particularly important when relatively shallow cracks are required to be considered.

Detailed information on the work described in this section is provided in [20].

CONCLUSIONS

The studies undertaken to date have led to a better understanding being gained of the levels of conservatisms in some of the current methodologies and have identified suitable ways forward of improving guidance in order to reduce such conservatisms. For example, a proposed alternative way of treating combined primary and secondary stresses looks to have good potential based on validation involving elastic-plastic finite element evaluations. In the same vein, finite element analyses have highlighted the potential benefit of developing methodology that allows for the fact that crack driving force may be significantly lower for growing cracks than for stationary cracks and that it can vary depending on what stage through component life the crack is introduced. On the other side of the coin, both analytical and experimental studies have indicated that the current characterization rules for multiple defects may be non-conservative for assessing plant components where the possibility of cleavage fracture has to be taken into consideration.

REFERENCES

1. Yellowlees S F, Watson C T, "New UK Research Programme on Fracture Mechanics", ASME PVP2006-ICPVTII-93559, 2006.
2. R6: Assessment of the integrity of structures containing defects, British Energy Generation Limited, Revision 4 (2006).
3. Hooton D G, Sharples J K, Yellowlees S F and Watson C T, "An Alternative Method to that of the R6 Procedure for the Treatment of Combined Primary and Secondary Loading", ASME PVP2006-ICPVTH-93569, 2006.
4. Wood P M, Beardsmore D W and Watson C T, "Load History Effects on Crack Tip Characterising Parameters", SMIRT Conference, Toronto, August 2007.
5. Rice J R and Tracey D M, "On the ductile enlargement of voids in triaxial stress fields", J Mech. Phys. Solids, Vol. 17, 1969, p.201-217.
6. ABAQUS version 6.6, ABAQUS inc, 2006.
7. Beardsmore D W and Sherry A H, "Allowance for Residual Stresses and Material Interfaces when Calculating J in and Close to Welded Joints", ASME Pressure Vessels and Piping, Vol. 464, pp. 11-21, 2003.
8. Sharples J K, James P M, Higham L A, Wood P M, Teng H, Beardsmore D W, Goldthorpe M R, Sherry A H, Watson C T, "Load and Crack History Effects on Fracture", ASME PVP2007-26069, 2007.
9. BS 7910:2005, "Guide to Methods for Assessing the Acceptability of Flaws in Metallic Structures", British Standards Institution.
10. Bezensek B and Hancock J W, "The Re-characterisation of Complex Flaws. Part 1: Fatigue and Ductile Tearing", Eng. Fract. Mech., Vol. 71, 2004, p.981-1000.
11. Bezensek B and Hancock J W, "The Re-characterisation of Complex Flaws. Part 2: Cleavage", Eng. Fract. Mech., Vol. 71, 2004, p.1001-1019.
12. Sharples J K, Wilkes M A, Yellowlees S F, Beardsmore, D. W., Melvin, G. T., Hurlston, R., and Watson, C. T., "Analysis of Multiple Co-Planar Surface Breaking Flaws Loaded in Tension Under Cleavage Failure Conditions", ASME PVP2006-ICPVTII-93562, 2006.
13. ASTM Standards, E1921-98, "Standard Test Method for Determination of Reference Temperature, T_0, for Ferritic Steels in the Transition Range", Section 3, Vol. 03.01, 1998, p.1068-1084.
14. Sharples J K, Wilkes M A, Beardsmore D W, Melvin G T, Sherry A H, Goldthorpe M R and Watson C T, "Further Analysis of Multiple Co-Planar Flaws under Cleavage Fracture Conditions", ASME PVP2007-26068.
15. Wilkes M A, Beardsmore D W, Watson C T, Sherry A H and Goldthorpe M R, "Characterisation of Co-Planar Surface-Breaking Flaws under Cleavage Fracture Conditions", SMIRT Conference, Toronto, August 2007.
16. Beremin F M, "A Local Criterion for Cleavage Fracture of a Nuclear Pressure Vessel Steel", Met. Trans. A., Vol. 14A, 1983, p.2277-2287.
17. Gao X, Ruggieri C, and Dodds Jr. R H, "Calibration of Weibull stress scale parameter parameters using fracture toughness data", Int J Fract; Vol. 92, 1998 p.175–200.
18. Beardsmore D W, Teng H and Martin M, "On the Expected Errors in Callibration of the Beremin Cleavage Model Parameters", ASME PVP Conference, San Antonio, July 2007.
19. Bate S K, and Watson C T, "New UK Research Programme On Residual Stresses," PVP Conference 2006-ICPVT11-93567.
20. Toft A, Beardsmore D W, James P M, Sharples J K and Martin M, "Presentation and Partitioning of Welding Residual Stress Distributions for use in Structural Integrity Assessments", PVP2007-26562.,