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Modal analysis of cyclically symmetric structures

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ABSTRACT: Finite group theory applied to cyclically symmetric structures provides an exact method to calculate the complete structure's eigen modes by considering just one of its sectors. Indeed, for each eigen mode, all sectors have the same mode shapes but their amplitude depend on their angular position. To take this property into account, a wave propagation equation relates the sector displacement fields to the adjoining sectors. Craig-Bampton and Mac Neal sub-structure synthesis approaches have been used to implement this method in the Aster Code. Two examples are presented below to demonstrate the efficiency of these methods. One of them in particular, deals with the industrial study of a low pressure rotor of a turbine generator set.

1 INTRODUCTION

The E.D.F. Research and Development department is in charge of performing finite element calculations, to ensure, by the design stage, the correct sizing of nuclear components submitted to vibratory loads. Due to the size and the geometric complexity of some of these components, it may be essential to reduce their model's number of degrees of freedom. In order to limit the computer time and storage, the geometrical and mechanical properties of these components have to be taken into account. In the same way, E.D.F. has developed sub-structure synthesis methods for modal analysis of cyclically periodic structures. Such methods, implemented in the Aster Code [EDF 1993], enable the optimal use of the geometrical properties of such structures as cooling tower, pressure vessel, turbine generator set ...

1.1 *Notations*

u : displacement vector
η : generalized displacement vector
f : force at the interface of two sub-structures
K : stiffness matrix
M : mass matrix
ω : pulsation

the superscript k denotes the sector number
the subscript i denotes the sector inner degrees of freedom
the subscript R denotes the right interface degrees of freedom of the sector
the subscript L denotes the left interface degrees of freedom of the sector

2 MODAL ANALYSIS OF CYCLICALLY SYMMETRIC STRUCTURES

2.1 *Definition*

A structure is said to be cyclically periodic around an Oz axis, when a $0 < \alpha < \pi$ angle exists such as the structure is geometrically and mechanically invariant as regards the rotation of this angle around Oz. When α is the smallest angle verifying this property, thus all angular parts of an α angle of the structure are called "base sectors".

The global structure is then composed of N sectors :

(1) $$N = \frac{2\pi}{\alpha}$$

2.2 *Wave propagation equation*

We call θ the rotation around Oz axis and of angle α defined in R^3. Let us consider a base sector of a cyclical repetitivity structure of axis Oz (fig. 1), and two similar points of two adjacent sectors L (left) et R (right) :

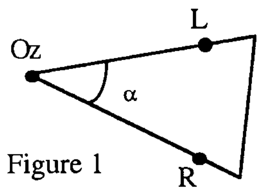

Figure 1

The L and R points confirms : $R = \theta(L)$.

We can say that the rotations leaving the structure invariant (geometrically and mechanically) are in definite numbers :

$$\theta^m \quad m \in \{0,1,\ldots,N-1\}$$

From then, we can demonstrate, with the finite group theory ([Elhami 1993], [Henry 1980], [Ohayon 1983], [Thomas 1979]), the following relations for L and R :

(2) $$\forall Z, \text{ complex variable of the mechanical system}$$
$$\exists\, m \in \{0,1,\ldots,\tfrac{N}{2}\}, \text{ integer} \quad \text{such as} \quad Z(R) = e^{jm\alpha}\theta Z(L)$$

This equation shows that the complex phase difference between two adjacent sectors can only take a finite number of known values between 0 and $N\alpha/2$ (in fact, the eigen modes associated to the phase differences $(N-m)\alpha$ and $m\alpha$ are identical).

Then, for each eigen mode, all sectors have the same mode shapes but their amplitude depend on their angular position. The wave propagation equation (2) which takes this property into account, relates the sector displacement field to the adjoining sectors. Consequently, the calculation of the eigen modes of the complete structure can be reduced to the modal analysis of only one sector to which the appropriate boundary conditions will be applied.

Let us consider a cyclical repetitivity structure, and two successive base sectors :

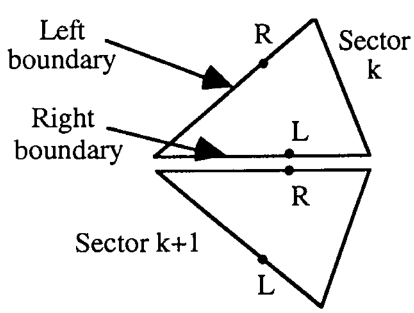

Figure 2

The coupling between sectors being considered as perfect, we have the following conditions :

$$u^k(L) = u^{k+1}(R)$$
$$f^k(L) = -f^{k+1}(R)$$

The superscript indicates the number of the considered sector (k or k+1).

The use of the formula (2) concerning the wave propagation in the structure enables to re-write the coupling conditions between sectors k and k+1 under the form of boundary conditions of the sector k :

$$\text{(3)}\qquad \begin{aligned} u^k(L) &= e^{jm\alpha}\theta u^k(R) \\ f^k(L) &= -e^{jm\alpha}\theta f^k(R) \end{aligned}$$

The boundary conditions (3) enable to calculate the eigen modes of the whole structure from only one base sector.

3 SUB-STRUCTURE SYNTHESIS METHODS

The sub-structure synthesis methods consist in using simultaneously dynamic sub-structuring and modal superposition. The displacement of a sub-structure in the whole movement, thus results from interface forces which relate it to the other components. Moreover, each sub-structure is represented by a finite set of suitable admissible functions (Ritz 's transformation) which defines its projection base.

Two eigen modes calculation methods by component mode synthesis are implemented in the general mechanics code of the E.D.F. Research and Development department (the *Aster Code*) : Craig-Bampton and Mac Neal's methods. They can be distinguished by the use of different bases for the sub-structures.

On the one hand, Craig-Bampton uses, as projection base for the sub-structures, constrained modes and eigen modes with fixed interfaces [Craig 1968]. On the other hand, Mac Neal uses, as projection base of the sub-structures, attachment modes and eigen modes with free interfaces [Neal 1971].

Whatever method is chosen, the sector projection base is composed of eigen modes noted φ, and of static modes relative to the right and left interfaces degrees of freedom, noted ψ_R and ψ_L. In this base the displacements will be noted :

$$\text{(4)}\qquad u = \begin{Bmatrix} u_i \\ u_R \\ u_L \end{Bmatrix} = \begin{bmatrix} \varphi & 0 & 0 \\ 0 & \psi_R & 0 \\ 0 & 0 & \psi_L \end{bmatrix} \begin{Bmatrix} \eta_i \\ \eta_R \\ \eta_L \end{Bmatrix} = \Phi\eta$$

The eigenvalue problem of the global structure is expressed on the base sector. The latter is subjected to the interface forces that are applied by adjacent sectors. On the other hand, the coupling equations (3) are verified. Consequently we have :

$$\text{(5)}\qquad \begin{aligned} (K-\omega^2 M)u &= f \\ u_L &= e^{jm\alpha}\theta u_R \\ f_L &= -e^{jm\alpha}\theta f_R \end{aligned} \quad \Rightarrow \quad \begin{aligned} (\overline{K}-\omega^2\overline{M})\begin{Bmatrix} \eta_i \\ \eta_R \\ \eta_L \end{Bmatrix} = \Phi^T f &= \begin{bmatrix} \varphi & 0 & 0 \\ 0 & \psi_R & 0 \\ 0 & 0 & \psi_L \end{bmatrix}^T \begin{Bmatrix} 0 \\ f_R \\ f_L \end{Bmatrix} \\ u_L &= e^{jm\alpha}\theta u_R \\ f_L &= -e^{jm\alpha}\theta f_R \end{aligned}$$

The generalized mass and stiffness matrices which appear here, have the following values :

$$\text{(6)}\qquad \overline{K} = \Phi^T K \Phi \quad \text{and} \quad \overline{M} = \Phi^T M \Phi$$

Let B_R and B_L be the extraction matrices of the right and left interfaces degrees of freedom :

$$\text{(7)}\qquad u_R = B_R u \quad \text{and} \quad u_L = B_L u$$

The displacements boundary conditions become, with these notations and (4) :

$$\text{(8)}\qquad B_L u = e^{jm\alpha}\theta B_R u \;\Rightarrow\; B_{RL}\Phi\eta = 0 \quad \text{where} \quad B_{RL} = e^{jm\alpha}\theta B_R - B_L$$

Concerning interface forces, the boundary conditions become :

(9) $$f = B_L^T f_L + B_R^T f_R \Rightarrow f = \left(B_L^T - e^{-jm\alpha} B_R^T \theta^T\right) f_L = -B_{RL}^T f_L$$

By applying formulas (8) and (9), the equation (5) is exactly equivalent to the following reduced size hermitian system :

(10) $$\left(\begin{bmatrix} \overline{K} & \Phi^T B_{RL}^T \\ B_{RL}\Phi & 0 \end{bmatrix} - \omega^2 \begin{bmatrix} \overline{M} & 0 \\ 0 & 0 \end{bmatrix}\right)\begin{Bmatrix} \eta \\ f_L \end{Bmatrix} = 0$$

The size of the system (10) is determined by the number of eigen modes and static modes of the projection base of the considered sector. It is thus the result of a compromise between performance (cpu time) and accuracy (truncation errors).

4 VALIDATION TEST

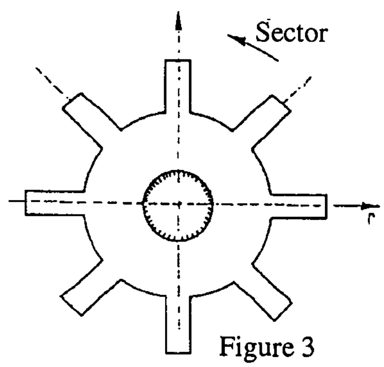

Figure 3

The example we have chosen to validate our developments deals with a cyclically symmetric structure which consists of 8 sectors (fig. 3). This structure has been the subject of an experimental modal analysis [Henry 1981]. Calculations have been lead thanks to both Craig-Bampton and Mac Neal sub-structure synthesis methods for modal analysis of cyclically symmetric structures implemented in the Aster Code. A direct calculation has also been achieved on the complete structure with a mesh as accurate as the sub-structure 's one.

The obtained results are sum up in the following table :

Mode	experimental results	direct calculation	Craig-Bampton	Mac Neal
1	264.7 Hz	281.0 Hz	281.0 Hz	281.0 Hz
2	267.2 Hz	281.8 Hz	281.8 Hz	281.8 Hz
3	295.1 Hz	306.1 Hz	306.1 Hz	306.1 Hz
Nb nodes		1,376	183	183
Tcpu		271 s	32 s	32 s

The results issued from the component mode synthesis methods and direct calculations agree perfectly. Moreover, the run times (Cray YMP) prove the efficiency of the sub-structure synthesis methods for modal analysis of cyclically symmetric structures. The comparison with the experimental results is less than satisfactory. However, we can attribute the differences to the measurement uncertainties. On the contrary, the comparison of the mode shapes is perfectly satisfactory (fig. 4) :

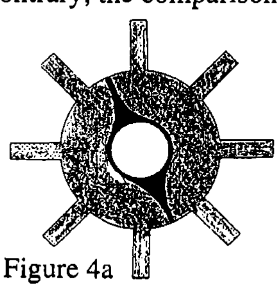

Figure 4a

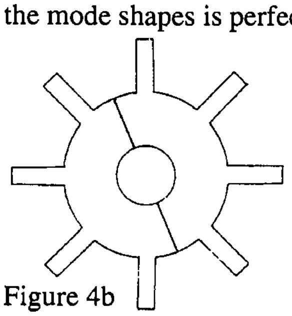

Figure 4b

The figure 4a represents the first calculated eigen mode and the figure 4b, the first measured eigen mode. The eigen mode nodal diameter is represented in black. We can notice that the mode shapes issued from the calculations and from the experiment are very close .

5 INDUSTRIAL ANALYSIS

The second example deals with an industrial study. The goal is to calculate the eigen modes of the low pressure rotor (L. P. rotor) of a turbine generator set, around its equilibrium position while rotating at 1500 RPM. The results are compared to measurements which have shown a coupling between the bending of the last turbine-stages blades and the shaft torsion eigen modes (being characterised by a "frequency split" of the first no nodal diameter eigen mode).

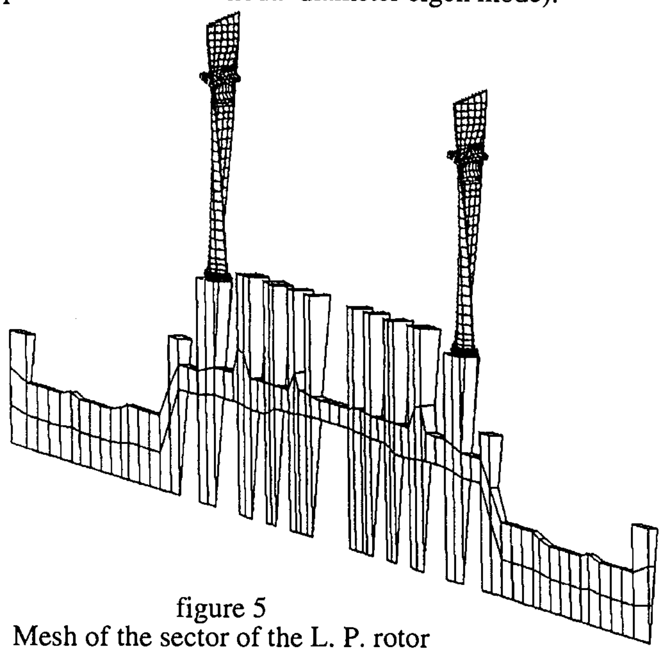

figure 5
Mesh of the sector of the L. P. rotor

Considering the size of the L. P. rotor, the direct calculation of its eigen modes is not feasible. Thus it is essential to resort to component mode synthesis. The last stages of the rotor being composed of 65 blades, the mesh consists of a blade from each last turbine-stage and a 1/65 angular section of the shaft. The middle stages are represented by punctual elements to which are assigned the appropriate mass and inertia. The mesh is presented figure 5.

The model of the dynamical behaviour of the rotor at 1500 RPM requires to represent the untwisting of the blades under centrifugal force. The calculation is made in two stages :

- calculation of the static position of the rotor at 1500 RPM,
- calculation of the eigen modes around this static position.

The eigenvalue problem of the global structure, solved by component mode synthesis, is expressed on a Craig-Bampton base of the sub-structure. It is composed of 6 eigen modes and of 762 static modes which ensure the coupling between sub-structures at the two shrouded blades and on both sides of the shaft section. Concerning the costs, such calculations represent 1000 seconds cpu on the Cray C98 and no memory problem whatsoever.

The obtained results allow to find again the coupling modes between the bending of the last stages blades and the torsion eigen modes of the shaft of the L. P. rotor, which had been showed by the achieved experimental measurements. The coupling has the effect of dividing the zero nodal diameter bending modes of the final stages of the L. P. rotor into two.

Let us consider one of these modes, with a frequency f, the result is :

- a coupling mode between the bending of the last stages blades and the first torsion eigen mode of the rotor, and of frequency : $0.95f$,
- a coupling mode between the bending of the last stages blades and the zero torsion eigen mode of the rotor, and of frequency : $1.05f$.

The results of the modal calculation, which hitherto was not feasible, agree with the experimental measurements (error < 2,5%).

6 CONCLUSIONS

We have demonstrated, in this abstract, how to calculate the eigen modes of a cyclically periodic structure by limiting the modal analysis to an irreducible unique sector. It implies to apply to the left and right interfaces of the sector mathematically complex boundary conditions. The size of the system to be solved can even be more reduced, while keeping its hermitian characteristics, using the component mode synthesis methods of Craig-Bampton or Mac Neal.

The achieved developments have been validated on a simple basic case, then have been used within an industrial study. The calculations time savings are very important and the accuracy of the calculations is very satisfactory.

REFERENCES

Craig, R.R. & M.C.C. Bampton. July 1968. Coupling of substructures for dynamic analysis. AIAA Journal. Vol. 6. N°7. pp 1313-1319.

EDF. 1993. "*Code_Aster* (A general finite element code for thermo-mechanics developed by EDF Research and Development Division)" - *Internal Report*, HI-75/93/034

Elhami, A. & G. Lallement & P. Minotti & S. Cogan. 1993. Methods that combine finite group theory with component mode synthesis in the analysis of repetitive structures. Computers and Structures. Vol. 48. N°6. pp 975-982.

Hale, A.L. & L. Meirovitch. 1980. A general substructure synthesis method for the dynamic simulation of complex structures. Journal of sound and vibration. 69 (2). pp 309-326.

Henry, R. 1981. Contribution à l'étude dynamique des machines tournantes. Thèse de Docteur d'Etat. Université Claude Bernard. Lyon I.

Henry, R. 1980. Frequencies and mode shapes calculation for rotationally periodic structures. Journal de mécanique appliquée. Vol. 4. N°1. pp 61-82.

Neal, R.H. 1971. A hybrid method of component mode synthesis. Computers and Structures. Vol. 1. pp 581-601.

Ohayon, R. & R. Valid. 1983. Static and dynamic computational methods for cyclically symmetric structures. La recherche Aérospatiale. pp 479-486.

Ohayon, R. & R. Valid; 1983. Static and dynamic analysis of cyclically symmetric structures. La recherche Aérospatiale. N° 4. pp 251-263.

Thomas, D.L. 1979. Dynamics of rotationally periodic structures. Journal for numerical methods in engineering. Vol. 14. pp 81-102.