

## NON-LINEAR DEFORMATION AND PLASTIC COLLAPSE OF NON-AXISYMMETRICALLY LOADED CONICAL SHELLS WITH STIFFENERS AND CUTOUTS

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### SUMMARY

Based on a two-dimensional discretization scheme, a finite-difference energy method is utilized for the non-linear, elastic deformation and elastic-plastic collapse analysis of non-axisymmetrically loaded conical shells with discrete stiffeners and cutouts. The corresponding system of non-linear algebraic equations is solved by means of a modified Newton-Raphson method of iteration for increasing values of the load parameter. Collapse is determined as a limit point in the calculated non-linear load-deflection diagram. It is assumed that the material of the shell consists of several perfectly plastic constituents with identical elastic properties but different yield limits. This model can thus represent strain-hardening effects in the original material by means of a piecewise linear stress-strain relation.

Specific numerical examples considered include a ring-stiffened shell subjected to external pressures which vary arbitrarily in the circumferential direction, and a shell with a rectangular cutout subjected to a combination of uniform axial load and arbitrary distributions of external pressure including a concentrated inward radial load in the vicinity of the cutout.

Conical shells under unevenly distributed and localized external overpressures are currently of interest in the structural safety analysis of certain nuclear reactor containment (bottom of the pressure suppression chamber) and pressure vessel (supporting skirt) components. Such loads can be caused by pressure relief or suppression processes, or by the rupture of a pipe.

A similar approach has also been used in recent investigations of the non-linear behavior of non-axisymmetrically loaded spherical shells for both static (L.M. Habip, Non-linear instability and plastic collapse of partially loaded spherical shells, *European Mechanics Colloquium on Finite Deformations in Plasticity*, Polish Academy of Sciences, Warsaw, 1974) and dynamic (L.M. Habip, Non-linear dynamic response of partially loaded spherical shells, *European Mechanics Colloquium on Dynamic Problems in Non-Linear Elasticity*, Polish Academy of Sciences, Warsaw, 1975) loading conditions.

## 1. Introduction

The stability of conical shells under unevenly distributed and localized external overpressures is currently of interest in the structural safety analysis of certain nuclear reactor containment and pressure vessel components. Such loads can be caused by pressure relief or suppression processes, or by the rupture of a pipe.

In the present paper, a method which has recently become available [1,2] is utilized in order to obtain numerical solutions for non-axisymmetrically loaded conical shells in both the elastic and elastic-plastic cases.

## 2. Summary of Method

The method of analysis is based on the non-linear shell theory of Marlowe and Flügge [3]. It is assumed that the rotation about the normal is of the same order of magnitude as the square root of a typical middle surface strain and that the out-of-plane rotation can be moderately large. The kinematical relations of this theory are summarized by Almroth, Brogan, and Marlowe [4].

The plasticity theory utilized in the analysis follows the approach of Besseling [5]. It is assumed that the material of the shell consists of several perfectly plastic constituents with identical elastic properties but different yield limits. This model can thus represent strain-hardening effects in the original material by means of a piecewise linear stress-strain relation.

The numerical solution is based on a two-dimensional finite-difference approximation of the energy expression. A system of non-linear algebraic equations is obtained when the energy is rendered stationary. These equations are then solved by means of a modified Newton-Raphson method of iteration for increasing values of the load parameter. The collapse load of the shell is determined as a maximum in the calculated non-linear load-deflection diagram. The procedure is further described by Bushnell, Almroth, and Brogan [6].

## 3. Applications

A stiffened conical shell and a conical shell with a cutout which are part of a reactor containment and pressure vessel, respectively, are considered here.

### 3.1 Stiffened Shell

The primary containment structure of interest consists of a spherical steel shell enclosing an integral annular tank partially filled with water. As partly shown in fig. 1, this annular tank has a ring-stiffened, truncated conical shell bottom, and serves as a condensation chamber for pressure relief or suppression. The circumferential distribution of external lateral pressure acting on the conical shell during such blow-down processes is also shown in fig. 1. For points along generators at which the loading is a maximum and a minimum, the deflection calculated by means of a non-linear elastic analysis is given by curves a and b in fig. 1, respectively. The influence of discrete stiffeners is clearly noticeable.

### 3.2 Shell with Cutout

The pressure vessel component of interest is the truncated conical steel shell, the skirt, on which the vessel rests. As shown in fig. 2, it has a rectangular cutout serving as a manhole. Following the rupture of a pipe, the shell is subjected to unevenly distributed external overpressures, in addition to the weight of the pressure

vessel, and a concentrated inward radial load along the dark zone in the vicinity of the cutout (see fig. 2). Assuming an elastic and perfectly plastic material behavior (see fig. 2), the calculated non-linear load-deflection diagram for a point in the neighborhood of the concentrated load is given in fig. 2. It can be seen that, as the collapse load is approached, this curve flattens out to reach a limiting point. The results exhibit the highly non-linear nature of the collapse behavior of shells.

#### 4. Conclusion

The present method is thus a useful tool for the study of non-linear deformation and collapse phenomena in shells subjected to non-axisymmetric loads, including the effects of plasticity. Considerable progress in understanding such phenomena appears possible, since many problems which otherwise would be theoretically untractable have now become solvable by means of a numerical technique.

A similar approach has been used by the author in recent investigations of the non-linear behavior of non-axisymmetrically loaded spherical shells for both static[7] and dynamic [8] loading conditions. In the latter case, the need for generating structural damage curves relating the dynamic pressure amplitude and the impulse necessary to cause a prescribed level of response has also been discussed [8-11]. Such considerations become necessary when the transient nature of the events causing the loading conditions of interest here must be accounted for.

#### References

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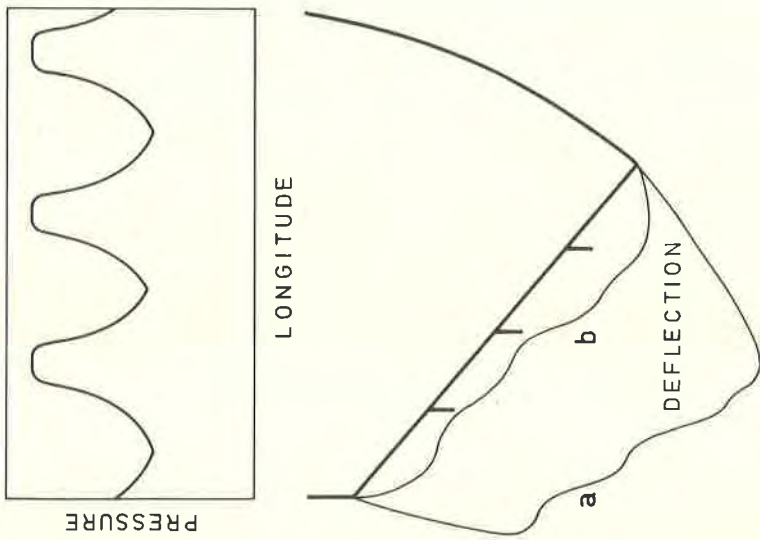


Fig. 1 Stiffened Shell: Non-Linear Elastic Analysis

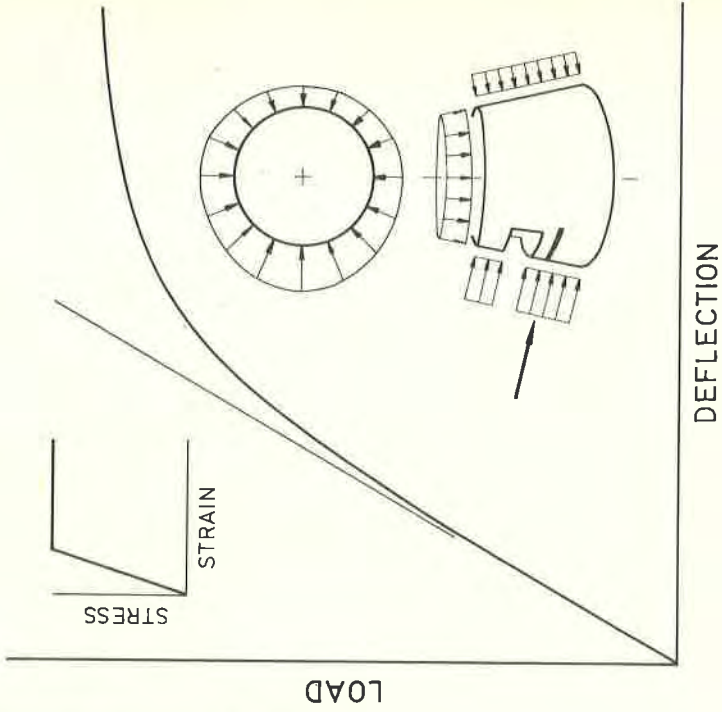


Fig. 2 Shell with Cutout: Non-Linear, Elastic-Plastic Collapse Analysis



