

Thermal Stresses in Case of Buckling: Primary or Secondary?

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ABSTRACT

Section III of the ASME B & PV Code makes a distinction between "primary" and "secondary" stresses. In Code Case N47, another distinction exists between load-controlled and strain controlled buckling, leading to two sets of rules, but the situation where strain-controlled loads influence load-controlled buckling is not clear. This paper aims at shedding further light on the question of how to consider thermal stresses when elastic-plastic buckling is the collapse mode of interest. Our approach is based on the study of two simple models : a beam compressed either by uniform heating or by an imposed force and an arch subjected to an external pressure and a through-thickness thermal gradient.

It is known that imperfect beams have a force-deflection curve featuring a maximum if plasticity is taken into account. When our beam model is first compressed thermally and then by an imposed force, two different behaviours appear, depending on the deflection attained under the thermal load F_{th} alone : if this deflection is smaller than that corresponding to the maximum force F_{max} , then the beam can sustain a supplementary force equal to $F_{max} - F_{th}$; if the deflection is greater, then the beam cannot sustain the smallest supplementary force without undergoing immediate collapse : this deflection can also be obtained by cycling a smaller thermal loading.

The case of the arch is perhaps more realistic since it is possible, in contrast to the beam, to control separately the two loading parameters : the pressure and the thermal gradient. The computations lead to the following result : collapse occurs when the plastic zones developed by the total stresses nearly constitute a plastic hinge, no matter whether the stresses are due only to the pressure or to the combination of pressure and thermal gradient. In this case the thermal stresses are not to be considered as "secondary".

In both cases, thermally induced stresses act concurrently with load-controlled ones to cause instability. The distinction between primary and secondary stresses no longer applies here because the theory on which it is based i.e. classical Limit Analysis precludes geometric non-linearities which are of prime importance in buckling phenomena.

1. Introduction

Section III of the ASME B & PV Code [1] makes a distinction between "primary" and "secondary" stresses. Only the former are limited when excessive distortion or plastic instability is under consideration.

No reference is made to this distinction in the case of elastic-plastic buckling. In Code Case N 47 [1] another distinction is made between load-controlled and strain-controlled buckling, leading to two sets of rules but the situation where strain-controlled loads influence load-controlled buckling is not clear. This paper aims at shedding further light on the question of how to consider thermal stresses when elastic-plastic buckling is the collapse mode of interest. Our approach is based on the study of two simple models : a beam compressed either by uniform heating or by an imposed force and an arch subjected to an external pressure and a through-thickness thermal gradient. The main interest of having studied these two problems is that they illustrate two extreme cases of thermal loading : pure membrane and pure bending. Since buckling experiments with thermal stresses are very difficult to devise, we started to study these two problems through finite element computations only. We used the code CASTEM [2] which is adapted to deal with cases involving material and geometric nonlinearities. Both elastic-plastic bifurcation and incremental large displacement analysis were performed.

2. The beam model

The beam we studied has a rectangular cross-section and is clamped at both ends. The behaviour is assumed to be an elastic-perfectly plastic behaviour (see fig. 1), with a yield strength so adjusted that the ratio between the EULER load and the limit load is near 1. When this condition is fulfilled, the interaction between elastic buckling and plastic collapse is maximum and the influence of thermal stresses is strong (see 3). The beam is submitted to compression either by uniformly heating it or by applying an axial force at one end. A special feature of the computation program allowed us to change from one loading condition to the other at a given stage of the computation.

Since we were more interested in the ultimate behaviour of the beam than in simply determining its bifurcation load, we assumed that it had an imperfect shape. We chose a first mode, sinusoidal imperfection with an amplitude equal to half the beam thickness (see fig. 2).

2.1 Monotonic thermal loading

We first studied the case where the beam is subjected to a uniform increase in temperature T . The $T - \delta$ curve (δ : additional deflection) rises continually, even above the EULER critical temperature T_E of the perfect beam (see fig. 3). This behaviour is linked to a rising bifurcated solution as can be shown by hand computation, assuming an elastic behaviour. This apparently unusual feature is not in contradiction with the well-known hyperbolic shape of the $F - \delta$ curve (F : compressive force, see fig. 4) ; the difference is due to the strain-limited nature of the thermal load T .

If the beam is first heated up to temperature T and then submitted to a compressive force, two different behaviours are observed :

- for $T = 50^\circ\text{C}$, the beam can sustain a supplementary force ΔF ,

- for $T = 100^{\circ}\text{C}$, the beam cannot sustain the smallest increase in force without undergoing immediate collapse. Although the point representing (T, δ) on fig. 3 is on a rising, stable curve, the beam reacts in an unstable manner to a force perturbation.

The explanation resides in the shape of the elastic-plastic $F - \delta$ curve, which is very different from the elastic $F - \delta$ curve : this curve features a maximum, so it has a descending, unstable branch. The point representing $T = 100^{\circ}\text{C}$ is on this branch, whereas the point representing $T = 50^{\circ}\text{C}$ is on the stable branch (fig. 5). Therefore, the beam, once heated up to 100°C , cannot sustain an imposed, "load-controlled" force equal to F_{100} .

A key role is played by the temperature T_{max} corresponding to the maximum axial force F_{max} :

- if $T < T_{\text{max}}$, the beam is stable under a force disturbance,
- if $T > T_{\text{max}}$, the beam is unstable.

This situation is very different from that of an elastic beam, which even for $T > T_E$ never reaches a limit-point like F_{max} and so always reacts in a stable way.

The same case can be discussed by considering the value of δ associated with each value of F on the descending part of the $F - \delta$ curve. If the thermal loading can create a deflection δ greater than that corresponding to F , the equilibrium under force F is unstable. The $F - \delta$ descending branch corresponds to the simultaneous plastic yielding of both ends. It can be represented approximately by the following equation, which expresses that both ends are perfectly plastic hinges :

$$\frac{2F(\delta + \delta_0)}{S_y} + \frac{F^2}{S_y^2} = 1 \quad (\text{for further detail see [4]})$$

One might tend to consider rather credulous the designer considering the beam to be stable only on the basis of a rising $T - \delta$ curve. But it should be remembered that : in most cases the thermal loading does not act alone, but is combined with load-controlled loadings ; and even if it acts alone, it is difficult to preclude any load-controlled perturbation. In the event of multiple loading, the design rules often require that all the loads be increased proportionally by the same loading parameter λ . It is then very tempting to look only at the $\lambda - \delta$ curves and to check whether λ can go beyond the required safety coefficient. This is clearly not enough : even if no limit point is reached on the $\lambda - \delta$ curves, the structure may be unstable. The only means to make sure is to check the stability of the equilibrium along the loading path, for instance by performing bifurcation computations.

2.2 Cyclic thermal loading

We also looked at what could happen if we imposed cyclic heating on the beam. We chose three different cyclic temperature values :

- $T = 50^{\circ}\text{C}$
- $T = 100^{\circ}\text{C}$
- $T = 200^{\circ}\text{C}$

In all these cases, the elastic-plastic large displacement computations revealed that progressive distortion occurred although no load-controlled force was acting. In fact, the presence of a load-controlled force is not a necessary condition for progressive distortion (MILLER's three bars are a good illustration of this [5]). Small displacement computations led to the following results :

- elastic shakedown for $T = 50 \text{ }^\circ\text{C}$
- plastic shakedown for the other two cases.

As in the previous cases, when the deflection reached the critical value given by (1), the equilibrium was no longer stable and the beam could not withstand the smallest supplementary force.

In conclusion to this section two facts are to be retained :

- Membrane thermal stresses may bring a structure into an unstable configuration without the load parameter (here T) reaching a maximum. Only a test can determine whether the equilibrium is stable or not.
- Progressive distortion may occur even in the absence of load-controlled stresses, and cause a shape imperfection to develop to a critical size.

3. The arch model

The second model of our study is an arch, with a rectangular cross-section and clamped at its ends. It is made of an elastic-perfectly plastic material (see fig. 6). It has in fact, two buckling possibilities : by snap-through or by bifurcation. We managed to have the two critical loads (computed assuming an elastic behaviour) be close to each other and close to the limit pressure (i.e. pressure causing plastic collapse assuming small displacement), because we expected after a previous study [3] the interaction between buckling and ratchetting to be strong.

This arch is subjected to two different loadings :

- an external "dead" pressure P ,
- a through-thickness thermal gradient, uniform over the length of the arch. This gradient corresponds to the difference t between the inner and outer skins of the arch.

Since these two parameters can be controlled separately, this model is perhaps more realistic than the first. Actually the loading was applied following different loading paths which are depicted on fig. 7 :

- 1 : P increasing alone
- 2 : P increasing first, then kept constant while Δt increases
- 3 : same as 2 but with $\Delta t < 0$
- 4 : Δt increasing first, then kept constant while P increases.

Our first study [3] of the ratchetting-buckling interaction concerned compressed beams. The main result was as follows : in all the cases but one, collapse occurred only when the beam had enough plastic hinges to behave as a complete mechanism (in the Limit Analysis sense). Although the thermal stresses influenced strongly the value of the axial force necessary to

create the hinges, the relationship between force and deflection at collapse was independent of the thermal stresses. This point was discussed in § 2.

The exception mentioned above corresponded to the case where the beam was first compressed by an axial force F exceeding F'_E , the EULER load when one hinge is present, and then submitted to the through-thickness gradient. In that case, collapse took place as soon as the first hinge was formed, because the beam had become unstable.

The behaviour of the arch is fairly similar to that of the unstable beam : in the four cases described above it collapses when a plastic hinge is nearly completed in some section of the arch, because the remainder of the arch has become unstable. Fig. 7 shows the (P, δ) points corresponding to collapse for the four cases. The arch becomes unstable before a complete mechanism has been built and so the (P, δ) relationship at collapse is not independent of the thermal stresses. As an illustration of this, fig. 8 shows (P, δ) curves in cases 1 (P alone) and 2 (P constant, Δt increasing) : the collapse point in the latter case is not on the first case's curve. Actually the plastic hinge is not yet perfectly complete at collapse : the quantity $\frac{\sigma_b}{1,5 S_y} + \frac{\sigma_m}{S_y}^2$ does not equal 1, but lies in the range 0,8 - 0,95. However the important point is that this value is obtained taking into account both thermal stresses and P -related ones. Both play exactly the same role in building the hinge that triggers collapse.

4. Conclusion

For both models, thermally induced stresses act concurrently with the load-controlled stresses in producing unstable configurations, essentially by creating plastic hinges. In the circumstances no distinction is to be made between them.

The two cases we discuss are fairly extreme, but they show that under certain conditions thermal stresses have limited or no possibility of relief before collapse ; the term "secondary" is consequently meaningless in this context. In other, less "extreme" cases thermal stresses are partly relieved before collapse ; this part of the stresses can be viewed as "secondary" but it often gives rise to additional deflections which are equivalent to shape imperfections. The remaining part has the features of a primary stress.

Concerning computations, two important practical consequences arise :

- Small displacement bifurcation analysis are unable to quantify how much "secondary" thermal stresses are. So they must not be used alone in buckling cases involving thermal stresses.
- Load parameter-deflection curves computed assuming large displacements can result in apparently stable configurations, but which would not withstand some perturbations. These computations are usefull but must not be employed without specific structure stability checking.

The reason for those unusual thermal stresses feature resides in the geometrical non-linearity which makes the well established Limit Analysis theorems false. Since these theorems are the basis of the distinction between primary and secondary stresses, this distinction becomes irrelevant. This remark has consequences regarding not only instability but also other failure modes like fatigue and progressive distortion.

REFERENCES

- 1 : ASME Boiler and Pressure Vessel Code, Section III, Division 1
Code Case N 47, 1980 Edition

- 2 : HOFFMANN, A.
"Castem (CEASEMT) A system of Finite Element Computer Programs for
Non-linear Analysis of Technical Structures"
Porto Alegre (BRASIL), April 1978, Paper 40

- 3 : CHAVANT, C., DEVOS, J., HOFFMANN, A.
"On the Influence of Geometric Imperfections and Thermal Gradients
on Elastic-Plastic Buckling of Shells"
6th SMIRT, Paris (FRANCE) 17-21 August 1981, Paper E 5/6

- 4 : COMBESURE, A., BASTIEN, R.
"Etude simple d'une poutre avec défaut, chauffée uniformément"
CEA Internal Report n° DDMT/SMTS/BAMS/81-40

- 5 : MILLER, D.R.
"Thermal-Stress Ratchet Mechanism in Pressure Vessels"
Transactions of the ASME, Vol. 8 (June 1959) pp 190-196

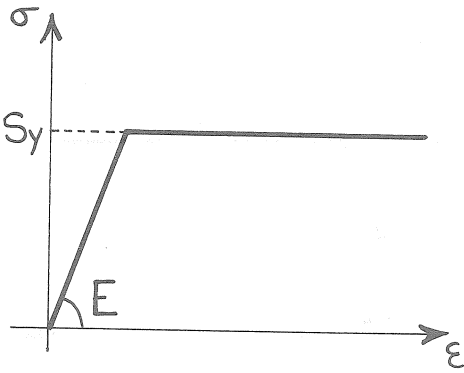


FIGURE 1 - BEAM MODEL : STRESS-STRAIN CURVE
 ($E = 20\ 000$, $S_y = 2$, $\alpha = 2 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$)

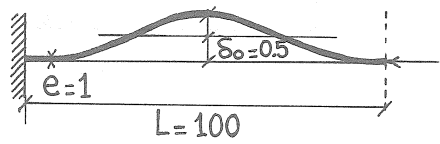


FIGURE 2 - BEAM MODEL : GEOMETRY

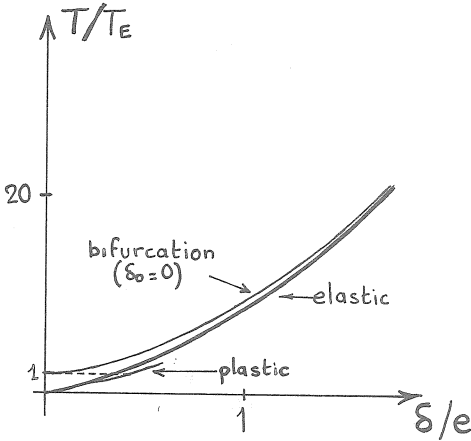


FIGURE 3 - T - δ CURVES

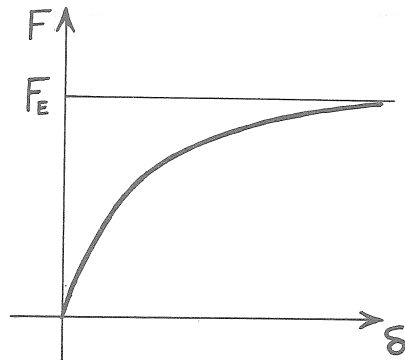


FIGURE 4 - F - δ CURVE (ELASTIC CASE)

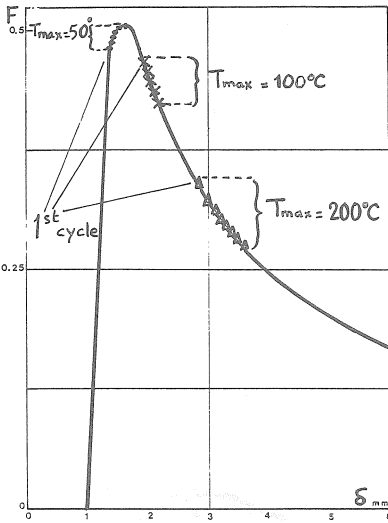


FIGURE 5 - F - δ CURVE (ELASTIC-PLASTIC CASE)
THE POINTS CORRESPOND TO SUCCESSIVE CYCLES

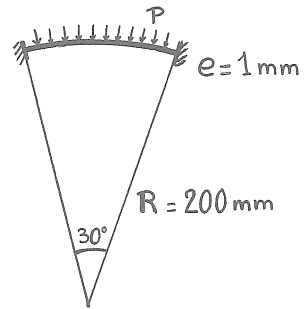
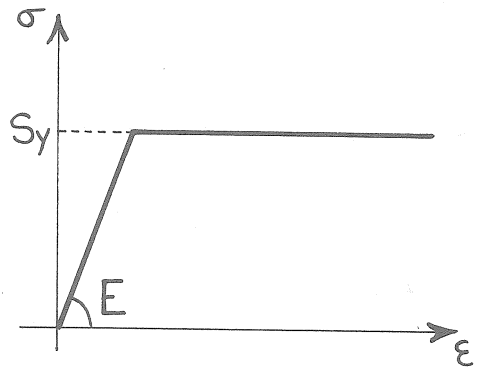


FIGURE 6 - ARCH MODEL : GEOMETRY AND STRESS-STRAIN CURVE $(E = 20\ 000, S_y = 20, \alpha = 18,6 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})$

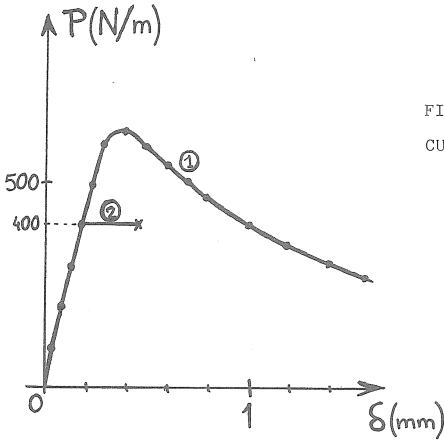


FIGURE 7 - ARCH MODEL : LOADING PATHS (THE CIRCLED NUMBERS REFER TO THE PATHS, THE POINTS TO COLLAPSE)

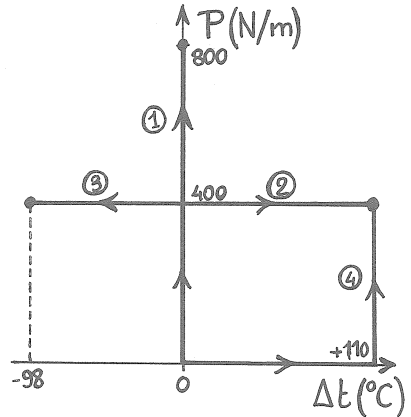


FIGURE 8 - ARCH MODEL : P - δ CURVES (1 : P alone ; 2 P first, then Δt)