

BIAXIAL FATIGUE OF MILD STEEL; DATA COMPILATION AND ANALYSIS

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SUMMARY

Design of pressure vessels, including components of nuclear reactors, where loading varies periodically during service, requires an estimate of fatigue life for conditions of biaxial stress. The ASME pressure vessel code, does this by the use of the maximum shear criterion, modified to include the strain dependence of fatigue behaviour. However, since the ASME code was developed a number of authors have reported biaxial fatigue data which are more accurately described by various other criteria. This study attempts to generate a relationship that describes at broad range of published biaxial fatigue data and to review the accuracy of the ASME code fatigue criterion.

This paper describes a study performed at the University of Waterloo which combined data from nine investigations of fatigue of mild steels at room temperature under fully-reversed, biaxial, stress or strain controlled, conditions. This assemblage of some 260 data points covers the fatigue life range from 10 to 10^7 cycles. Biaxial stress ratios, from equibiaxial (+1) to torsional (-1) are represented. By combining data a broad view of the effect of biaxiality on fatigue is obtained. Areas where data are scant or show inconsistencies become apparent and conclusions drawn have more general applicability than those based on a single investigation.

The data analyzed are published as amplitude or range of stress, total strain, plastic strain or equivalent plastic strain. A variety of testing techniques and interpretations of specimen failure were used. The chemical composition and heat treatment of the steels used also varied. In view of these factors the correspondence of the strain range versus life data is surprisingly good. All data were transformed to give both elastic and plastic strain values. This was achieved using a cyclic stress versus plastic strain curve determined earlier by the authors for a similar steel. All mild steels are assumed to have the same slope of curve but to be displaced in proportion to their yield stress. In addition Hooke's law for elastic strains and the constant volume assumption for plastic strains are applied.

The paper shows how some standard and other less usual strain based criteria describe the data. Two-part criteria were investigated in which the equivalent elastic and plastic strain components are based on either octahedral - or maximum - shear strain with the equivalent plastic strain modified to include a hydrostatic stress effect. It was found that this effect was small but detectable over the data scatter. A criterion that uses modified maximum shear for plastic strains and octahedral shear for elastic strains gave the best agreement. The octahedral shear strain criterion based on total strain, using 0.5 for Poisson's ratio, gave a slightly worse correlation.

The collected data were also compared to the ASME Pressure Vessel Code curves for design against fatigue. The safety factors included are shown to be less than the desired 2 on stress or 20 on life particularly at short fatigue lives. Some modifications to the design practice to restore these safety factors are suggested.

1. Introduction

In nuclear and fossil-fuel fired generating stations, as well as in other industrial applications, the major excursion of load results from the start-up and shutdown cycle of the plant. This load cycle, which occurs a relatively small number of times may nevertheless be responsible for much of the structure's fatigue life. These large load excursions can produce large inelastic strains at flaws or other stress raisers which initiate fatigue cracks. These cracks usually start at the surface, where the stress state is close to biaxial. The stresses initially follow the zero to maximum service loading cycle and later approach full reversals as the mean stress is relaxed by cyclic plasticity. Thus knowledge of low cycle biaxial, fully-reversed fatigue behaviour is required. The most common design for these conditions is that of the ASME Pressure Vessel Code [1] which employs the maximum shear criterion modified to include the strain dependence of fatigue behaviour. However, as shown in a recent survey by Krempl [2], since the Code was developed [3] a number of authors have reported low cycle biaxial fatigue data which are more accurately described by various other criteria.

Design for high cycle applications also requires knowledge of fatigue properties under biaxial conditions to permit rational treatment of discontinuities or other sources of biaxial stresses. The von Mises criterion is generally used to permit uniaxial fatigue properties to be applied in this life range.

2. Objectives

It is logical to expect that biaxial fatigue behaviour for all lives can be described by a single criterion which approaches the von Mises criterion at about 10^7 cycles but has some other form at shorter lives. This study attempts to combine data from nine series of fatigue tests [4-12] on mild steels at room temperature under fully-reversed biaxial conditions. At present the applicability of conclusions drawn from any single study in this area of research is restricted by the necessarily limited ranges of fatigue life and biaxiality investigated. By combining the data, a broad view of the effects of biaxiality on fatigue is obtained and conclusions drawn are likely to be more general. Also, areas of fatigue life and principal stress ratio where data are scant or show inconsistencies, will become apparent and so guide future studies. The objectives of this study are to analyze the correlation of the fatigue data achieved by several different biaxial criteria and to review the accuracy of the ASME Pressure Vessel Code design criterion [1] for cyclic biaxial stress states.

3. Sources of Data

Tests at long lives were reported by Gough and Pollard [4], Rotvel [9] and Sawert [10]. Intermediate life data were given by Havard and Topper [5], Ives et al [6], Pascoe and de Villiers [8], Taira et al [11] and Yokobori et al [12]. Kikukawa et al [7] produced data for very short lives. This

assemblage of 252 data points covers the fatigue life range from 10 to 10^7 cycles. Biaxial stress ratios from equibiaxial (+1) to torsional (-1) are well represented.

Table 1 summarizes the nine fatigue investigations used as sources of data. This Table includes the ranges of fatigue life and biaxiality reported, and the test technique, specimen geometry and failure criterion used in each investigation. All tests were conducted at room temperature and employed fully-reversed load or strain control. The chemical composition and mechanical properties of the mild steels tested are also included. The yield point of the mild steels investigated varies from 28.5 ksi to 45.6 ksi and carbon content from 0.10% to 0.35%. In most of the studies the steel was normalized or annealed but three investigators [7,8,9] do not indicate the heat treatment. The degree of anisotropy reported also varies, but is generally small.

The interpretation of failure in the nine studies showed considerable variation from microscopic crack detection to separation of the specimen. Where alternative end points are given, eg [8], the condition considered closer to the initial leakage of a pressure vessel was chosen.

Five investigators [6,7,8,11,12] presented only graphs of their data which were digitized for this study by scaling. To avoid this loss of accuracy in the further use of such results future stress and/or strain and fatigue life values, as well as the material's monotonic properties, should be reported digitally.

4. Ranges of Data Studied

The ranges of fatigue life and biaxiality covered by the nine studies are shown in Figure 1. Each point of the plot represents a single test except for those identified with an "S". These data from Sawert [10] are "endurance limit" values derived from unpublished fatigue curves. In total, there are 79 tests with positive principal stress ratios (the ratios usually found in pressure vessels) and 112 with negative principal stress ratios. The remaining 61 uniaxial tests are used for correlation with the biaxial fatigue data.

Figure 1 shows that this survey includes three domains where there are very few data points. Surprisingly, there are hardly any fatigue data reported for positive stress ratios between 10^5 and 10^7 cycles and consequently the correlations for long life biaxial fatigue have been based mainly on negative stress ratio data. The other two areas that lack values are positive stress ratios and negative stress ratios other than torsional, at lives less than 1000 cycles.

5. Conversion of Data to a Common Form

The currently most successful criterion for describing uniaxial fatigue behaviour of many classes of materials is that developed by Coffin [13] and Manson [14] which shows uniaxial plastic strain range, $\Delta \epsilon_{pu}$, as a power law function of fatigue life, N_f . This suggests that the fatigue data should be converted to strain values. The procedure used for deriving strains from the maximum principal stress and the stress ratio is as follows: In Figure 4 of

[5], the stable ranges of cyclic stress and plastic strain for a series of biaxial fatigue tests on mild steel were related by using the equivalent von Mises stress, $\Delta\bar{\sigma}$, in ksi and plastic strain, $\Delta\bar{\epsilon}_p$. The relationship providing the best fit to the data points is:

$$\Delta\bar{\epsilon}_p = 2.12 \times 10^{-8} (\Delta\bar{\sigma})^{2.9} \quad (1)$$

where $\Delta\bar{\sigma} = \Delta\sigma_1 \sqrt{1 - R + R^2}$ (2)

and $\Delta\bar{\epsilon}_p = \frac{2\Delta\epsilon_{1p}}{2 - R} \sqrt{1 - R + R^2}$ (3)

In eq (2) and eq (3), $\Delta\sigma_1$ and $\Delta\epsilon_{1p}$ are the ranges of maximum principal stress, in ksi, and plastic strain and R is the ratio of smaller to larger principal stress.

Conversion of stress based data to components of strain was accomplished by assuming the exponent of the cyclic stress-strain curve in eq (1) is applicable to all the mild steels tested. The stresses were adjusted in each case by the ratio of the yield strength (31.5 ksi) of the mild steel tested in [5] to the yield strength, σ_y , reported in each study. Eq (1) is then generalized to:

$$\Delta\bar{\epsilon}_p = 2.12 \times 10^{-8} \left(\Delta\bar{\sigma} \frac{31.5}{\sigma_y}\right)^{2.9} \quad (4)$$

As the strains in the long life region are mainly elastic, errors due to this approximation should be very small.

From Hooke's law, the elastic strain components, $\Delta\epsilon_{1e}$, $\Delta\epsilon_{2e}$, $\Delta\epsilon_{3e}$, are:

$$\Delta\epsilon_{1e} = \frac{\Delta\sigma_1}{E} (1 - \mu R) \quad (5)$$

$$\Delta\epsilon_{2e} = \frac{\Delta\sigma_1}{E} (R - \mu)$$

$$\Delta\epsilon_{3e} = -\frac{\Delta\sigma_1}{E} \mu(1 + R)$$

where E and μ are Young's modulus and Poisson's ratio (assumed = 0.3), respectively.

For the plastic strain components, $\Delta\epsilon_{1p}$, $\Delta\epsilon_{2p}$ and $\Delta\epsilon_{3p}$, $\Delta\bar{\sigma}$ was obtained from eq (2) which then allowed computation of $\Delta\bar{\epsilon}_p$ using eq (4). The maximum principal plastic strain range $\Delta\epsilon_{1p}$, was then calculated from eq (3).

Assuming the constant volume law of plasticity, the other two principal plastic strain ranges were determined from:

$$\Delta\epsilon_{2p} = \left(\frac{2R - 1}{2 - R}\right) \Delta\epsilon_{1p}$$

$$\Delta\epsilon_{3p} = \left(\frac{R + 1}{R - 2}\right) \Delta\epsilon_{1p}$$

All three principal strains could then be calculated and inserted in any selected strain based criterion.

Two investigators [6,8] provided their biaxial fatigue data in terms of maximum principal strain range, $\Delta\epsilon_1$, versus life. To obtain the components

of principal strain range it was first necessary to derive a value of the maximum principal stress range. Substituting eq (2) and eq (3) into eq (4) and rearranging yields:

$$\Delta \epsilon_{1p} = (1 - \frac{R}{2}) (1 - R + R^2)^{0.95} \times 2.12 \times 10^{-8} (\Delta \sigma_1 \frac{31.5}{\sigma_y})^{2.9} \quad (6)$$

Adding the right hand sides of eq (5) and eq (6) provides a relation for $\Delta \epsilon_1$ ($= \Delta \epsilon_{1e} + \Delta \epsilon_{1p}$) in terms of R and $\Delta \sigma_1$. $\Delta \sigma_1$ was determined using a numerical technique. The principal strain components were then computed as described above.

Several investigators [7,11,12] reported fatigue data in the form of von Mises equivalent plastic or total strain range. In these cases, the value of $\Delta \epsilon_1$ was first calculated using the related von Mises equivalence criterion. The procedure outlined above was then used to obtain the components of principal strain.

6. Biaxial Criteria Studied

All criteria studied assume isotropic behaviour and are based on strain data. The maximum principal strain values are used in Figure 2 to permit comparison of values of similar life and biaxiality from different sources. Examination reveals generally good correspondence with the exception of values in the 10^2 to 10^4 cycle life range with equibiaxial stressing from Ives et al [6] and Pascoe et al [8]. Almost comparable data from Havard and Topper [5] in the 10^3 to 10^4 cycle life range are closer to Ives' data.

The elastic strain values have been examined separately using the two common yield criteria; namely the maximum shear and the von Mises criteria (using $\mu = 0.3$). The second gave the better correlation. Using the same criteria (with $\mu = 0.5$ for von Mises) for the plastic strain values did not indicate clear superiority of one over the other. To exemplify the degree of scatter remaining, the sum of the von Mises elastic and von Mises plastic equivalent strains is plotted in Figure 3. The maximum scatter band width represents a factor of about 2.1 in stress each side of the mean curve.

Attempts were made to improve this by the use of a correlation which includes the effect of hydrostatic stress. This takes the form:

$$\Delta \bar{\epsilon}_t = \Delta \bar{\epsilon}_{eM} + \Delta \bar{\epsilon}_{pi} (1 + \alpha R) \quad (7)$$

where $\Delta \bar{\epsilon}_t$ is the total equivalent strain range, $\Delta \bar{\epsilon}_{eM}$ is the von Mises elastic equivalent strain range, $\Delta \bar{\epsilon}_{pi}$ is either the von Mises plastic equivalent strain range or the maximum plastic shear strain range and α is a constant. The optimum value of α was found for each of the plastic equivalent strain range criteria from the calculated correlation coefficients. The best fit was found with $\alpha = 0.1$ using the von Mises plastic equivalent strain range. The resulting correlation is shown in Figure 4. The maximum scatter band width is reduced, giving a factor of 2.0 on stress each side of the mean curve.

7. Design Codes

The modified maximum shear criterion used by the ASME Pressure Vessel Code [1] has been applied to the data giving the result shown in Figure 5. The maximum scatter band width here represents a factor of 2.3 on stress each side of the mean curve. Figure 5 also includes the failure curve that was plotted through the data on which it is based. It can be seen to lay close to the upper limit of the data surveyed in this study. The ASME design curve, which is below the failure curve by the greater of 2 on stress or 20 on life, is below all data points except one. This data point is one that does not correspond to comparable values from other studies as discussed above. An alternative design curve which restores the safety factors of 2 on stress or 20 on life based on the mean curve through the data points has been included in Figure 5. This would be conservative for all the data points. It represents a reduction of about 30% in permissible values of design "stress" in the 10^4 to 10^3 cycles life range and a lesser reduction from 10^3 to 2×10^5 cycles where the curve meets the present design curve.

An alternative improvement in the code would be to use an equivalence criterion such as (7), converted to fictitious "stress" units if desired, instead of the present stress amplitude. This would then be compared with a curve such as "A" shown in Figure 4 which employs the same safety factors on the centre of the data band as the ASME Code now uses. These factors now protect against both material variations and incomplete knowledge of biaxial fatigue behaviour. Perhaps, using this approach lower factors protecting only against material variations could be used. A second curve, using the factors of at least 10 on life and 1.5 on stress as originally suggested by Langer [3] is shown in Figure 4 as "B".

8. Conclusions

8.1 Biaxial fatigue data from nine independent studies of mild steels, when presented on a common basis in terms of strain by the use of an approximation to each cyclic stress-strain curve, are complementary and give broad coverage of the domain.

8.2 Three areas of sparse data and one region of conflicting data are identified.

8.3 A biaxiality criterion using von Mises equivalent elastic strain plus von Mises equivalent plastic strain modified by a hydrostatic stress effect best correlates the data. Design curves based on this criterion are presented.

8.4 The ASME pressure vessel code is shown to be less conservative than desired in predicting the set of data and a modification to the design curve is suggested.

9. Acknowledgements

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TABLE 1 - SOURCES OF FULLY REVERSED BIAXIAL FATIGUE DATA AND PROPERTIES OF THE MILD STEELS TESTED

INSTITUTIONS	SAUGH-POLLARD	HAYWARD, TOPPER	IVES	KIRIKAWA	FASSOZ	POTVEL	SAWERT	TAJIMA	YOKOCHI
REFERENCE	#	#	#	#	#	#	#	#	#
EXPERIMENTAL INVESTIGATION	NATIONAL PHYSICAL LABORATORY	DAIICHI DENSO DENKAI	WILCOX CO. STEEL UNIV. U.S.A.	OSAKA UNIV. JAPAN	CAMBRIDGE UNIV., ENGLAND	TECHNICAL UNIV. OF DENMARK	SAKI AIRCRAFT MOTOR FACTORY, OSAKA	KYOTO UNIV. JAPAN	TOYOKU UNIV. YAMAGUCHI, JAPAN
STEEL TESTED	NORMALIZED SAE 1020	NORMALIZED SAE 1020	NORMALIZED SAE 1020	NOT ROLLED SAE 1020	0.1% C	0.1% C	NORMALIZED SAE 1020	0.1% C	0.1% C
TESTING METHOD OF BIAXIAL TESTS	10 ⁷ - 10 ⁸	10 ⁷ - 10 ⁸	10 ⁷ - 10 ⁸	10 ⁷ - 10 ⁸	10 ⁷ - 10 ⁸	10 ⁷ - 10 ⁸	10 ⁷ - 10 ⁸	10 ⁷ - 10 ⁸	10 ⁷ - 10 ⁸
GEOMETRY	THIN-WALLED CYLINDER	THIN-WALLED CYLINDER	48 DIAMETER CIRCULAR PLATE, 1/8" THICK	THIN-WALLED CYLINDER	THIN-WALLED CYLINDER	THIN-WALLED CYLINDER	VARIOUS	THIN-WALLED CYLINDER	THIN-WALLED CYLINDER
FAILURE DEFINITION	SEPARATION	SEPARATION	SEPARATION	SEPARATION	SEPARATION	SEPARATION	SEPARATION	SEPARATION	SEPARATION
TEST TYPES	TORSION, BENDING AND COMBINATIONS	AXIAL, INTERNAL AND EXTERNAL PRESSURE, TORSION	OUT-OF-PLANE BENDING	INTERNAL AND TORSION	INTERNAL AND TORSION	INTERNAL AND TORSION	INTERNAL AND TORSION	INTERNAL AND TORSION	INTERNAL AND TORSION
BIAXIAL STRESS RATIOS	0.5, 0.2, -0.2, -0.7, -0.3, -0.7, -1	0.5, 0.5, 0.2, 0, -0.2, -1	1, 0.5	0, -1	0, 0.5, -1	0.5, 0.2, 0, -0.5, -0.7, -1	0.5, 0.2, 0, -0.5, -0.7, -1	0, -0.5, -1	0, -1
CONTROLLED VARIABLE	LOAD	AXIAL OR TORSIONAL	DEFLECTION	STRAIN	MAXIMUM STRAIN THEN LOAD	INTERNAL STRESS VALUES	LOAD	AXIAL OR TORSIONAL	STRAIN
FORM OF DATA REPORTED	MAX. SHEAR STRESS AT FAILURE OF LOADING OFFSET	PRINCIPAL STRESS AND STRAIN	MAX. PRINCIPAL STRESS AND STRAIN	SUBJECTIVE STRAIN RANGE	INTERNAL STRESS RANGE	INTERNAL STRESS VALUES	DISPLACEMENTS AT (1/3) CYCLES	INTERNAL STRESS RANGE (MISES), TORSIONAL	INTERNAL STRESS RANGE
CHEMICAL COMPOSITION (PERCENTAGE BY WEIGHT)									
CARBON	0.12	0.18	0.14	0.20	0.18	0.21	0.15	0.16	0.12
MANGANESE	0.30	0.30	0.38	0.30	0.34	0.45	0.45	0.35	0.35
SILICON	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
SULPHUR	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
PHOSPHORUS	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
COPPER	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
NICKEL	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
ALUMINIUM	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
PHYSICAL PROPERTIES (AVERAGE VALUES)									
TENSILE STRENGTH (T _{0.2})	37.2	31.5	37.24	43.8	27.5	35.24	34.8	35.8	28.58
YIELD STRENGTH (T _{0.2})	24.7	21.5	24.74	28.5	18.0	24.74	24.0	24.0	20.7
ELONGATION AT FRACTURE (%)	24.75 x 10 ³	30 x 10 ³	24.75	31.2	24.75 x 10 ³	24.75 x 10 ³	24.75 x 10 ³	24.75 x 10 ³	24.75
YOUNG'S MODULUS (10 ¹¹)	28.75 x 10 ³	28.75 x 10 ³	28.75	28.75	28.75 x 10 ³	28.75 x 10 ³	28.75 x 10 ³	28.75 x 10 ³	28.75

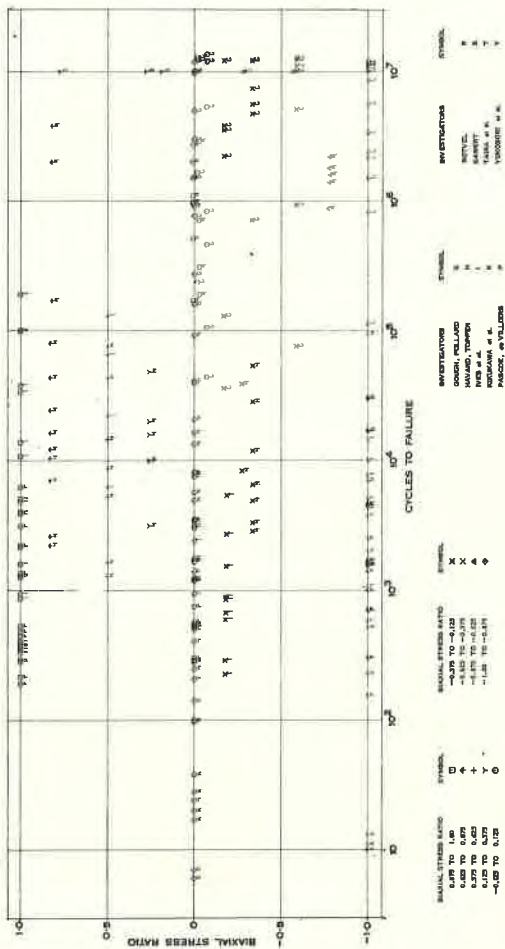


FIGURE 1 Ranges of Fatigue Life and Biaxiality of Data Studied. All tests on mild steels and with fully reversed loading.

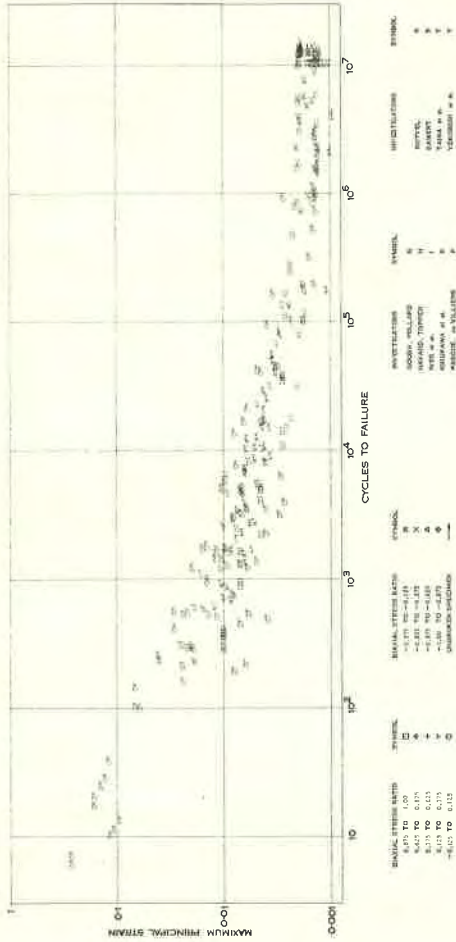


FIGURE 2 Data Comparison Based on Maximum Principal Strain Range

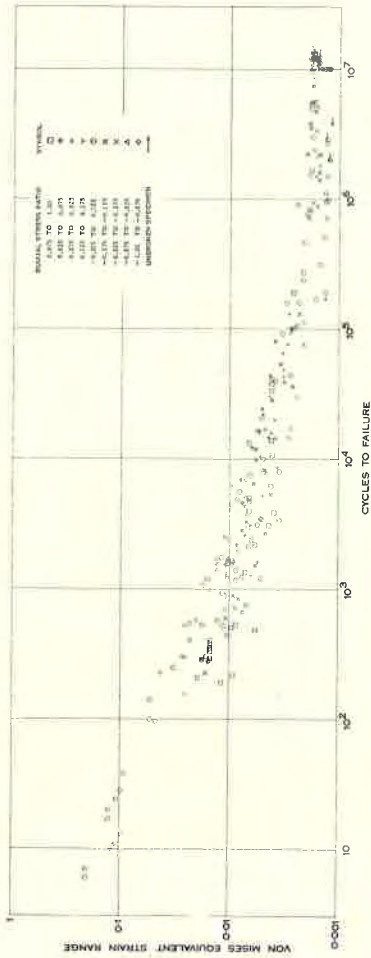


FIGURE 3 Data Correlation based on von Mises Equivalent Strain

