The dynamics of structures – Necessity and methodology for amendment by comparing the calculated model with experimental model

B.Canéparo
SOPEMEA, France
S.Zirilli
ENEA, Italy

1 INTRODUCTION

When one wishes to find out the dynamic behaviour of a structure in specific zones, for example in the case of bearing structures for sismic tests in which the points in question are the anchorage zones of the structures to be tested, adjustment of the finite elements model (MEF) solely on the basis of natural frequencies and dampings obtained by the experimental modal analysis and the consecutive refining thereof according to the response (to impulse stress), enables taking into account the effects introduced by :

- assembly of parts
- limit conditions
- coupling with structures not included in the model (test means)

The model thus carried out is able to supply a correct evaluation of the forced response to Time History.

The aim of studying the dynamic behaviour of structures is to anticipate their response to stresses which may be applied to them. To attain a good qualitative level for this result, two complementary techniques are necessary : computer simulation, plus experimental measurements. The extend to which the respective techniques are used depends on the following factors :

- availability of the necessary means
- ability of the laboratory to carry out the tests
- utilization of the result (requirements of standards).

Experimental measurements supply more precise information, yet these are not always realisable and are more costly in terms of means and staff ; non-expanded numerical methods of experimental verification can be the source of totally erroneous evaluations.

In as much as one proposes operating according to the hypothesis of a linear behaviour both as regards the theoretical and the experimental approach, it is possible to proceed in two stages :

- determination of the intrinsic behaviour of the structure
- application of the ambient stresses.

In this work relating to support structures for sismic tests, we present a mixed procedure necessitating the experimental measurement of natural frequencies, dampings, and the response to impulse stresses (in the case of a sismic stress, the subject of this study, a single impulse is sufficient) in the zone in question.

Experimental measurements are used to adjust the finite elements model;

it may then be used for later studies.

As regards determination of the intrinsic behaviour of the structure, some authors have already taken up the problem of adjusting the model, either with algorithms to directly modify the elements of the matrices defining the MEF (L. Bugeat et al.... 1978) (H. Bergers et al.... 1984), or with a modification of the physical parameters upon which the MEF is dependent (L. Bugeat et al... 1982), in any case using measurements of modal distortions.

In the presence of interaction with structures not included in the model, such as, for example, the means used for the actual test, it is impossible to adjust it according to the methods proposed and it is up to the experienced author to introduce the modifications judged opportune to take into account everything which is not a part of the model.

We have, however, carried out a programme based on the local modification of Young's module, which uses only natural frequencies, useful in the adjustment process.

Once the zone of poor modelling has been found, this programme enables optimizing the value of E as a function of the experimental data, whilst also furnishing an estimate of residual differences.

Dynamic tests have shown that the model thus obtained can be refined by the forced impulse to an impulse stress.

In addition to setting out the theories and formulae used, we then give account of verification of the methodology using a plate, and of its application to a support structure in the form of a frame for sismic tests.

The appendices include both experimental measurements and tests.

We carried out the modal analysis with even greater care than necessary in view of the methodology verification phase.

2 STUDY OF THE DYNAMIC BEHAVIOUR OF STRUCTURES

Technological progress, the adoption of new materials and manufacturing methods, increasing demands from users (for example the nuclear and aerospace sectors and the army) have highlighted the dynamic problems of structures, to the point that in many cases it has proved necessary to solve these problems at the design stage, or, if the structure had already been built, to check its dynamic behaviour.

Complex measurement and calculation systems capable of carrying out the qualification tests and the dynamic study of structures (EDS) were created and developed to cater for the task.

The EDS consists of parallel development of computer simulation with experimental measurements (see diagram 1). The basic design of the structure is generally used as the starting point.

This design enables building a basic finite elements model with a first verification of its dynamic characteristics, in particular enabling the evaluation of mode parameters at 10 % (Zirilli-Caraglio, 1984).

The experimental modal analysis carried out after building the prototype enables adjusting the MEF with correction methods making it converge on the experimental modal model. One can thus obtain a finite elements model which furnishes modal parameters close on the experimental parameters and an estimate of the differences between the experimental modal parameters and those of the MEF.

Forced response tests will enable a later refinement of the model.

In addition, we still have a modal experimental model enabling the study of modifications to be made to the structure in terms of mass, rigidity, damping ; for example, it enables evaluating by how much the rigidity of a zone of the structure needs to be increased in order to obtain a desired variation at the frequency of the nth mode.

The adjusted finite elements model enables studying the modifications to be made to the structure in real terms ; for example : determination of a geometric modification leading to the variation of rigidity evaluated at the previous point. The two modified models may predict the forced response via a calculation and they enable building a structure having the desired dynamic behaviour or, perhaps, to make opportune modifications to one which exists already.

As a further safeguard, this definitive structure can be submitted to dynamic control tests.

The fields of application of the dynamic study of structures are very wide-ranging, for example, to quote just a few :

- aeroelasticity
- fatigue
- precision
- comfort
- coupling phenomena
- etc.

We summarize, below, the main objectives achieved with the :

1. Design of structures having a given dynamic behaviour ;
2. Realization of a model of a structure ;
3. Forecasting the behaviour of the structure for everv type of stress to which it could be subjected in its environment ;

3 THEORY AND FORMULAE

3.1 Adjustment of the model

The structure calcualtion algorithm may be considered as a function which makes all the natural frequencies correspond to each value of the Young module in the modification zones.

The explicit shape of this function is generally not known. A representation may, however, be elaborated by estimation, by carrying out the Taylor series expansion.

The formula is thus :

$$MEF = MEF_{\circ} + \frac{\delta MEF}{\delta E}_{(E+E_{\circ})} . \Delta E + \ldots$$

The derivative of the MEF compared with E may be calcualted by interpolation after calculation of three MEFs with variations of E opportunely chosen around the derivation value.

The size of X2 is defined as follows :

$$X^2 = \sum_i \left| \frac{\left(MEF_{\circ} + \frac{\delta MEF}{\delta E}\Big|_{E=E_{\circ}} . \Delta E\right)_i - S_i}{f\ (S_i)} \right|^2$$

According to the smallest square number method, the value of ΔE which makes X2 smallest furnishes the best adjusted model. For $F(S_i)$ one takes the proportionality to S_i (constant error in percent).

The fact of the following condition being imposed :

$$\frac{\delta X^2}{\delta \Delta E} = 0$$

furnishes the following expression for E :

$$\Delta E = \frac{\Sigma_i \; (\; (S_i \; \frac{\delta MEF}{\delta E} \; {}_{E=E_\circ} - MEF_\circ \; \frac{\delta MEF}{\delta E} \; {}_{E=E_\circ}) \; / \; f(S_i)^2)}{\Sigma_i (\; \frac{\delta MEF}{\delta E} \; \Big|_{E=E_\circ} \; / \; f(S_i) \;)^2}$$

The MEF function being known approximately, the value found for ΔE is generally not that which makes X2 minimum, but it enables setting up a convergent iterative process.

The size order of the remaining deviations between experimental values and theoretical values of MEF, whatever their origin in effect (residual imperfection in the model or experimental errors) may be evaluated as long as X2 has the expected N value (number of points used in the fit) in its minimum.

A calculation programme has been carried out operating on the principles laid out for a Honey Bull Mini 6 computer.

3.2 Theory of calculation of the forced response

Forecasting the forced response using the finite elements method (on this point, see Zirilli-Caraglio, TIB/AFF (84) 4) consists of preparing a mathematical model whose behaviour is regulated by the following linear differential equations system :

$$|M| + \; |C|\{ X\} + |K| \; \{ \dot{X}\} = \{ F \; (t)\} \qquad (1)$$

$|M|$ - matrix of mass

$|C|$ - matrix of dampings

$|K|$ - matrix of rigidity

$\{X\}$ is the vector of movement in relation to time, with a component for each degree of liberty (D.O.F.) of the model.

The intrinsic behaviour is determined using the same equation, with $\{F \; (t) \} = \{0\}$

In the case of an inertia force linked to a movement of the support (sismic stress), here $\{ F \; (t) \} = |M| \; \{U\} \; \gamma \; (t)$

where γ (t) is the acceleration at the base and the vector $\{U\}$ has components 1 or 0.

Once the finite elements model has been carried out, i.e. once matrices $|M|$, $|C|$, and $|K|$ have been determined, there are two methods for determining $\{ X \; (t) \}$:

a) the step-by-step numerical intergration method

b) the spectral method.

a) Numerical intergration method

Numerical integration methods are the more usual, but they require the use of much more data processing resources. One can make the difference between :

- the direct integration method where the N differential equations with N being unknown are resolved at the instants in question thanks to complex calculation algorithms (which can also be applied to non-linear cases).
- the modal superposition method where calculation takes place in three stages :

First Stage : determination of the free vibration modes of the structure
Second Stage : calculation of the contribution of each mode to the forced response (resolution of uncoupled equations at the instants in question)
Third Stage : modal superposition.

This latter is the method most usually chosen, as much for the relatively short calculation time involved as for the fact that determination of the natural modes is an important step forward in the dynamic study of structures.

b) Spectral method

The spectral method furnishes synthetic information giving the maximum response of the structure. In general, results are conservative compared with those that can be obtained using other methods, in the sense that the maximum responses obtained are generally a higher limit than those obtained using the other methods.

The stages of the spectral method are :
First Stage : determination of the natural modes of vibration
Second Stage : resolution of N uncoupled equations to read the maximum response at the frequency in question, on a response spectrum characteristic of the stress
Third Stage : the maximum response is obtained using rules for combining the maximum responses obtained for each mode (sums of absolute values, quadratic combination and combination in percent in the case of neighbouring modes).

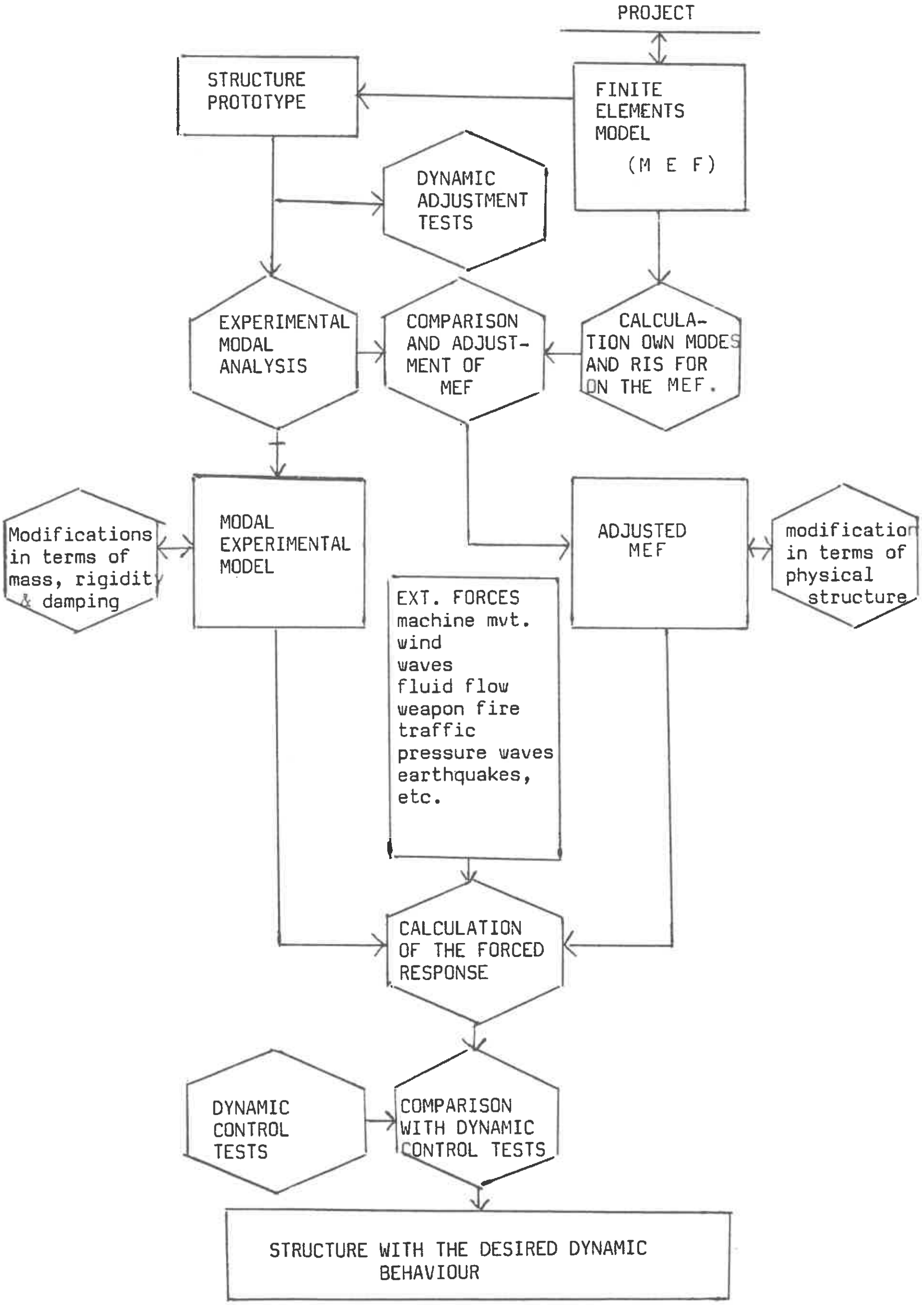
4 VERIFICATION OF THE METHODOLOGY AT THE SUMMIT OF A RECESSED PLATE

A plate with one end recessed into a concrete block was subjected to modal analysis. The results of this analysis, reported in the RTI (TIB/AFF (84) 4) indicate a difference between the natural frequencies calculated for the finite elements model and those measured (Table I and column II of Table I).

In order to adjust the model, we introduced a band of elements of the Young module to be optimized into the recessed zone. Column II of Table I shows the results of applying the programme (see here above, paragraph 3.1).

The estimate of the residual difference between the experimental data and the data supplied by the model is given in column IV, Table I.

The basic model and the model adjusted on the basis of the modal analysis, were used to calculate the forced response to the impulse stress lasting 120 seconds.


PROJECT
STRUCTURE PROTOTYPE
FINITE ELEMENTS MODEL (M E F)
DYNAMIC ADJUSTMENT TESTS
EXPERIMENTAL MODAL ANALYSIS
COMPARISON AND ADJUST-MENT OF MEF
CALCULA-TION OWN MODES AND RIS FOR ON THE MEF.
Modifications in terms of mass, rigidity & damping
MODAL EXPERIMENTAL MODEL
ADJUSTED MEF
modification in terms of physical structure
EXT. FORCES machine mvt. wind waves fluid flow weapon fire traffic pressure waves earthquakes, etc.
CALCULATION OF THE FORCED RESPONSE
DYNAMIC CONTROL TESTS
COMPARISON WITH DYNAMIC CONTROL TESTS
STRUCTURE WITH THE DESIRED DYNAMIC BEHAVIOUR

Diagram

The comparison of results obtained via calculation with the actual response of the plate measured experimentally, shows the inadequacy of the basic model in calculating the forced response : errors in range of approximately a factor 2 (Fig. 1a).

However, the model adjusted (on the basis of what we set out in §3.1) supplies results which conform better to the experimental results (Fig. 1b).

It is noted that because of the phenomenon of the compounding of errors responses to an impulse stress are very sensitive to frequency variations, and most specifically, the calculated response is in this case progressively behind the measured response.

This suggested the possibility of refining the model afterwards, by integrating the "fitting" of the response to impulse stress to the model adjustment procedure, thus obtaining for the frequencies, the values recorded in the final column of Table 1.

Figure 1f shows what is obtained thanks to the suitable variation of the parameter used in the adjustment with variation of the frequency of the first mode, of around one tenth Hz. In the case of a monoaxial sismis stress, the response of the structures is determined essentially by the first vibration mode.

The validity of what is obtained is verified by dynamic control tests, in particular forced responses to a stress impulse lasting 60 milliseconds, fig. 2a, and forced response to Time History with a response spectrum between 1 and 100 Hz, fig. 2b.

5 APPLICATION OF THE ADUSTMENT METHODOLOGY TO A SUPPORT STRUCTURE FOR SISMIC TESTS - STUDY OF MOVEMENT AT THE SUMMIT

5.1 Adjustment of the model on the basis of data from the modal analysis

A support structure for sismic tests, in the shape of a trellis, was subjected to modal analysis. The results of this analysis highlight the presence of modes totally different from those forecast by the finite elements model (Fig. 3).

Certain considerations enable making a distinction between the experimental modes determined essentially by the structure under study, and those determined by its union with the sismic table.

The relation between the movement in the zone and the direction seen in conjunction with the sum of absolute values for the maximum forces applied for the modal analysis, is a useful size in evaluating the possible contribution of each mode to the movement of the structure. This parameter is called C.

It can be observed on Table III that modes n. 101, 103 and 104 present :
- a high generalized mass
- a different value for C for an order of size in relation to the other modes
- modal distortions characterized by sideways movements and rotations or the whole of the sample.

Overall, this data demonstrates that these modes are born of the union of the sample with the sismic table to which it is fixed and of the possibilities of movement due to play stemming from the fixings of this latter mechanism.

These modes will not be chosen when adjusting the model; if only because the low C value indicates a small contribution to the movement of the structure at its summit, thus enabling us to ignore their eventual contribution in calculating the forced response.

The modes higher than n. 107 are not taken into consideration for they are above the "range" of the study. Modes n. 105, 102 and 106 correspond to the modes calculated using the basic MEF, even if the frequency values can be seen to be influenced by the link with the table.

We therefore proceed with an operation to adjust the model, carried out partly on the basis of the sensitivity of the experimeter, and partly according to the procedure proposed in § 2.

Table II records the frequencies furnished by the basic model, the experimental frequencies, the frequencies furnished by the adjusted model and the evaluation of the residual difference, and the frequencies furnished by the model refined according to the impulse response.

5.2 Refining the model according to the impulse response

On the basis of the results obtained in the previous paragraph, we proceded to refine the model according to a forced response to impulse stress lasting 40 milliseconds.

Fig. 4 shows the effects of the adjustment procedure on the forced response : basic model, Fig. 4a ; model adjusted according to the modal analysis, Fig. 4d ; refinement according to the response itself, Fig. 4f.

Fig. 5 shows a dynamic control test performed on the trellis : the start of an earthquake with response spectrum between 1 and 100 Hz.

6 CONCLUSION

All that is required in the way of equipment to carry out this methodology, is a simple Fourier's analyzer plus a personal computer equipped with software to enable modal analysis and calculation of the structure using the finite elements method. This, as has been verified by the dynamic tests, enables evaluating the forced response of the structures.

Fig. 6 shows the behaviour of the trellis to the NORTH/SOUTH component of the EL CENTRO earthquake with the distortion of the structure after 2.2 seconds, evaluated using the adjusted model.

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Table I . Confrontation between the experimental values and the values calculated from the basic model and the adjusted models of the plate.

F_{EB}	F_{AA}	F_{EFA}	ΔF	F_{RI}
13.25	12.13	12.3	.4	12.4
52.9	54.53	52.0	1.6	52.2
78.6	75.2	76.9	2.3	77.3
170.5	175.3	170.3	5.2	170.8

Table II. Confrontation between the experimental values and the values calculated from the basic model and the adjusted models of the treillis.

F_{EFB}	F_{AA}	F_{EFA}	ΔF	F_{RI}
23.7	20.65	20.73	0.10	21.2
65.3	52.08	51.96	0.25	53.2
65.5	58.82	58.41	0.28	59.7

Table III. Results of modal analysis.

N	Frequencies Hz	Normalization point	Amplitude mm	Amplitude top point mm	Strenght x N	C mm/N x 100	α %	μ kg. m²	Direction
101	26.60	45	0.35	0.09	380	0.02	1.81 0.61	2664	y
102	52.08	8	0.72	0.32	200	0.16	0.45	109	x, y
103	55.31	3	0.13	0.03	400	0.008	2.10	447	x, y
104	70.42	13	0.22	0.18	400	0.05	0.90	459	x, y
105	20.65	6	1.12	1.12	160	0.70	2.10	186	x
106	58.82	33	1.43	1.36	80	1.7	2.55	77	x
107	100.12	33	0.51	0.45	100	0.45	0.19	97	x, y
108	114.93	52	0.43	0.29	300	0.1	0.20	223	x, y
109	126.68	53	0.30	0.11	360	0.03	0.29	124	x, y

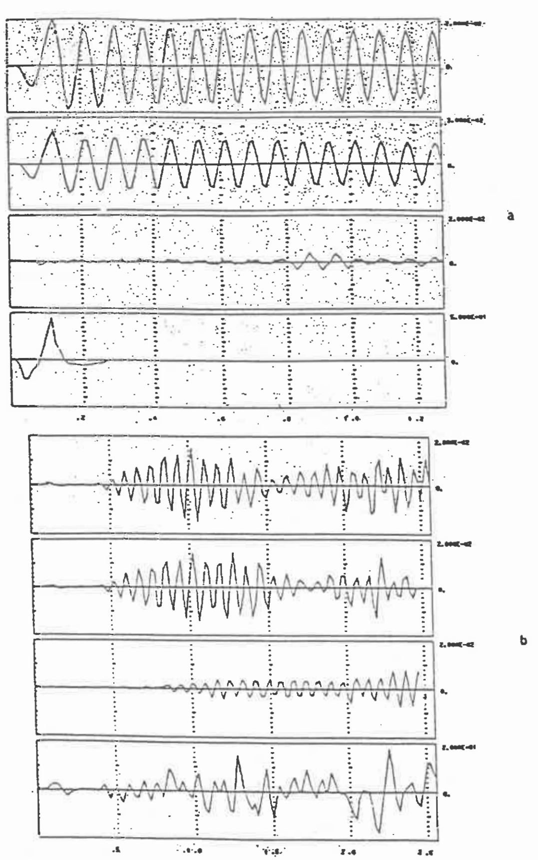

Fig. 2 - Dynamic tests to control the definitive model.
In both figures (a, b), Time History I represents the calculated response ; Time History II represents the measured response ; Time History II is the difference between the two ; IV is the ground acceleration.

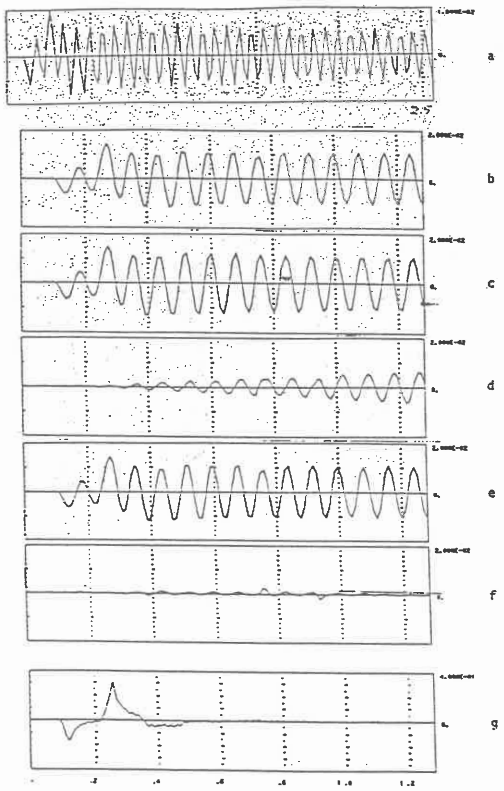

Fig. 1 - Confrontation between the calculated response and the measured response of the recessed plate and impulse lasting 120 milliseconds.
a) basic model ; b) experimental model ; c) model adjusted on the modal analysis ; d) difference between b and c ; e) model adjusted on the measured response ; f) difference between b and e ; g) ground acceleration.

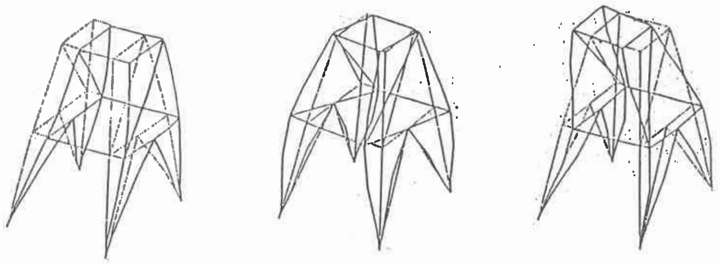

Fig. 3 - Modal distortions furnished by the finite elements model for the first three modes.

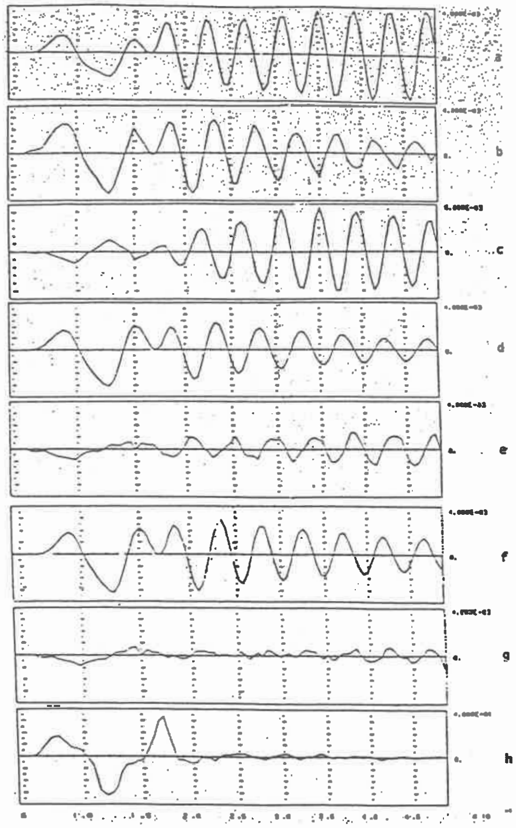

Fig. 4 - Confrontation between calculated and measured responses of the trellis. a) basic model ; b) experimental measurements ; c) difference a-b ; d) model adjusted in the modal analysis ; e) difference d-b ; f) model adjusted on the measured response ; g) difference f-b ; h) ground acceleration.

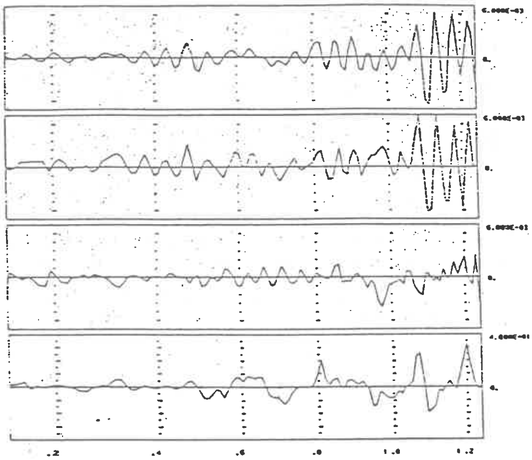

Fig. 5 - Dynamic test of the trellis with beginning of earthquake simulated with response spectrum between 1 and 100 Hz. a) calculated response ; b) measured response ; c) difference ; d) ground acceleration.

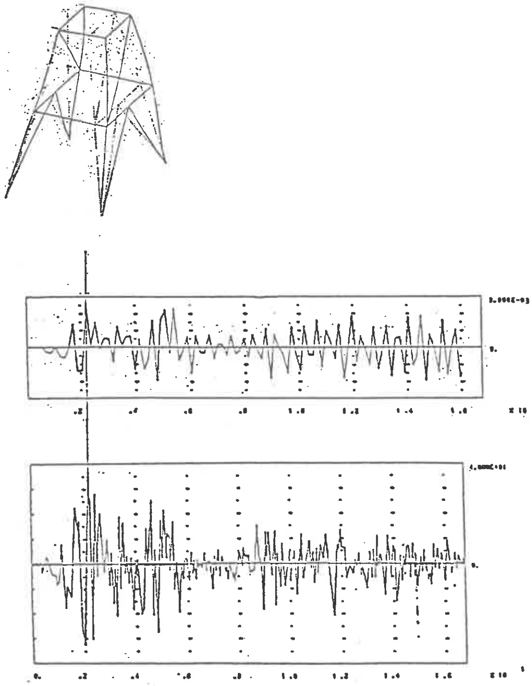

Fig. 6 - Simulated response of the trellis summit to the North/South component of the "El centro" earthquake with distortion after 2.2 secs.