Ductile Fracture Assessment of Piping Systems Including the Compliance Effects

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ABSTRACT

This paper is concerned with the study of the effect of compliance on the ductile tearing instability load calculations. In conventional tearing analysis the piping system is assumed to have infinite compliance or zero stiffness and the unstable tearing moment is compared with the applied moment based on the analysis of un-cracked piping system, which in general is very conservative for evaluation of safety margin. The presence of the crack in the piping causes reduction in the moment at its location. The amount of the reduction, for any given piping, depends upon the effective length, crack size and material of piping. Further this reduction directly affects on the fracture parameters like J integral, CMOD and tearing modulus. In order to understand these effects, finite element analysis of a typical nuclear pipe was done with different effective lengths. The pipe is assumed to have a through wall circumferential crack. Finally a simple equation has been derived for J-tearing modulus.

INTRODUCTION

The Leak-Before-Break (LBB) methodology is now accepted as a technical approach for eliminating postulation of double-ended guillotine break in the high energy piping systems. The fundamental premise of LBB is that the materials used in high-energy piping system are sufficiently tough to tolerate a large through-wall flaw/crack, which will give easily detectable leak rate without causing unstable crack growth during maximum accident loading. Nuclear Power Plant piping is generally made of ductile material. Its fracture assessment is essential for demonstration of LBB capability and safety margins against unstable fracture failure, under design basis safe shut down earthquake loading. In order to determine the unstable ductile tearing load, detailed tearing stability analysis is performed with postulated crack at the location of maximum stresses under normal operating condition load plus design basis safe shut down earthquake load.

In conventional tearing analysis, the through wall circumferential (TWC) cracked pipe is assumed to have free rotation at the ends (infinite compliance of the connected piping) and subjected to the bending load as shown in Fig.1. The unstable ductile tearing moment is determined by applying the J – Tearing modulus approach. This can be done either using finite-element analysis or by engineering estimation schemes such as GE/EPRI, R-6 and A-16. However, in reality the ends of the pipe are connected to the rigid pressure vessels etc. Hence the piping on the either side of the cracked section has finite compliance or non-zero stiffness. In piping system, the presence of crack causes moment redistribution and results in the reduction of moment at its location while at the same time the moment at supports and anchors will increase. It is found that the stiffness of the attached piping system (or compliance of the system) increases the safety margins available against ductile fracture. The moment at the cracked section will further reduce as a result of small increment of crack growth (Δa). The remaining piping basically acts as repository for the moment which sheds from the cracked section. This leads to change the applied J-integral, crack mouth opening displacement (CMOD) and applied tearing modulus (T) curve. Nestell and Coward [1] had included the effect of compliance in the stability criteria and derived an equation for applied tearing modulus.

In order to understand the behavior of the J-integral, CMOD and tearing modulus T under finite compliance conditions, finite element analysis of typical nuclear pipe has been carried out for different effective lengths (basically representing the compliance). The pipe size and material selected corresponds to the Steam Generator Inlet (SGI) pipeline of 500 MWe PHWR at TAPP, Tarapur India.

THEORITICAL CONSIDERATIONS

As discussed earlier, in case of the cracked piping, subjected to an arbitrary load, the actual moment at the cracked section (M_C) is less as compared to the moment (M_U) at the same section under un-cracked condition under the same load. The evaluation of the M_C is generally not easy and calls for detailed elastic plastic piping analysis using 3-D elements.

However, designer generally performs linear piping system analysis using 1-D pipe elements. In the following section a brief discussion will be made to evaluate M_C in a simplified manner.

Evaluation of Moment at Cracked Section

Consider a general case of piping where a pipe of length L is subjected to the distributed loading that is basically earthquake type loading as shown in Fig.2. The pipe has built-in ends that simulate the steam generator. For healthy pipe the moment at the mid span is M_U. Postulation of a TWC crack at mid-span location has two effects, first there is loss in load carrying area of the cross section and secondly there is formation of plasticity ahead of the crack tip. Both of these factors contribute to the reduction of the moment at this location from M_U to M_C value. Small incremental crack growth (Δa) further reduces the moment at crack section. This further reduction in the moment is accompanied by an increase in the crack rotation angle ϕ_{crack} shown in Fig. 2. The crack rotation ϕ_{crack} is absorbed by the remaining piping system when it has built-in ends. If the pipe ends is free (infinite compliance, Fig. 1), this rotation will not lead to change in the moments But in case of the built-in ends pipe this additional rotation will cause redistribution of the moments, at the ends the moment increases while it decreases at the crack location. So the postulation of crack sheds the (M_U – M_C) moment from cracked section to the connected piping system. The moment will further shed as a result of crack growth.

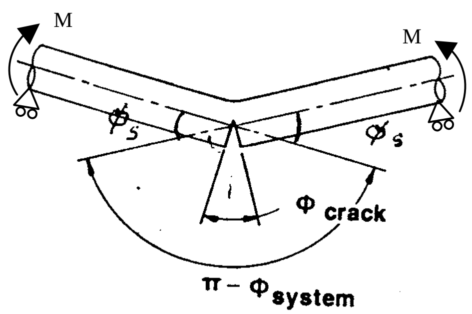

Fig. 1: Free end TWC cracked Pipe subjected to Pure Bending Moment load.

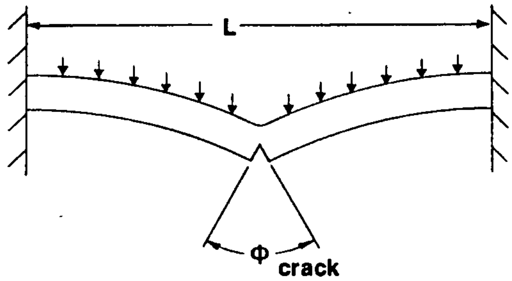

Fig. 2: Built-in end TWC cracked Pipe subjected to Distributed loading.

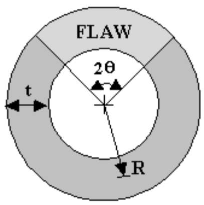

Fig. 3: Through Wall Circumferential (TWC) Crack

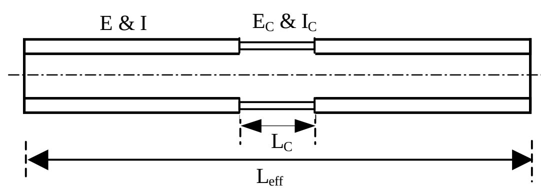

Fig. 4: 1-D simulation of the TWC cracked Pipe

The loss in the load carrying area due to crack can be modeled by decreasing the thickness while keeping the mean radius constant and the effect of the plasticity can be modeled by changing the elastic modulus. Fig.4 shows the 1-D modeling of the crack in the piping systems. In 1-D simulation of crack, the increase in rotation will be given as:

$$\phi_{crack} = \frac{M_c L_c}{E_c I_c} - \frac{M_c L_c}{EI} \qquad (1)$$

Here the E_C and I_C parameter accounts for both the plasticity and cracked ligament. The E_C and I_C effects can be combined into one parameter as I_{eff} which is given as $I_{eff} = E_C I_C / I$. The choice of L_C is arbitrary and generally taken equal to mean radius of pipe R. Then the Eq.1 becomes:

$$\phi_{crack} = \frac{M_c L_c}{EI}\left(\frac{I}{I_{eff}} - 1\right) \qquad (2)$$

Let the compliance of the piping system supporting the crack, equal to Cs at its location. Some times it is also referred as residual elastic compliance. The equation for the reduction in the moment at crack location can be given as:

$$M_U - M_C = C_S \phi_{crack} \tag{3}$$

The compliance of the piping system can be determined by the method described by Paris and Tada [2]. The elastic rotational stiffness of the piping at crack location is determined by applying equal and opposite moment on each side of the hinge joint representing the crack. The stiffness is determined about two axis and a Mohr's circle approach [2] is used to find out the maximum compliance of the piping that leads to conservative side. Paris and Tada [2] had expressed the system compliance in term of dimension less parameter L_{eff}/R. The L_{eff}/R is the ratio of length to mean radius of a hypothetical pipe with built-in ends and having the same mean radius and wall thickness as that at cracked location being analyzed. L_{eff} is chosen such that the compliance at the mid-span is equivalent to the compliance at crack location of the piping. The two different layout of piping, having similar L/R ratio for some crack locations, will demonstrate similar tearing instability behavior. The L_{eff} is calculated using the following equation:

$$L_{eff} = EIC_S \tag{4}$$

From Eq. 2, Eq. 3 and Eq. 4 and choosing L_C equal to R, one can find out the actual moment at the crack location as:

$$M_C = \frac{M_U\left(1 + \frac{R}{L_{eff}}\right)}{\left(1 + \frac{R}{L_{eff}}\frac{I}{I_{eff}}\right)} \tag{5}$$

The above equation clearly illustrates that for low L_{eff}/R and high I/I_{eff} the loss in moment is higher. The I/I_{eff} is function of crack size (θ /π) and plastic zone size, which in turn is function of applied load. Without going into the detail derivations, it is mentioned that I/I_{eff} can be calculated by Eq.2 in which ϕ_{crack} is evaluated by GE/EPRI estimation scheme at moment load equal to M_C, which is unknown. Therefore the actual value of M_C is determined by solving Eq.2 and Eq.5 iteratively.

Stability Criteria

The earlier discussion clearly demonstrates that the actual load / moment carried by cracked section is function of crack length, the compliance Cs and the applied loading. Then the actual moment (M_C or M) at the crack is expressed as:

$$M = M(a, \phi) \tag{6}$$

And the crack rotation (hinge angle) ϕ (same as ϕ_{crack}) is expressed as:

$$\phi = \phi(a, M) \tag{7}$$

The J integral can be given as

$$J = \frac{1}{t}\int_0^M \left.\frac{\partial \phi}{\partial a}\right|_M dM = -\frac{1}{t}\int_0^\phi \left.\frac{\partial M}{\partial a}\right|_\phi d\phi \tag{8}$$

$$J = J(a, M) \tag{9}$$

Or J can be expressed as a function of crack length 'a' and crack rotation 'ϕ'

$$J = J(a, \phi) \tag{10}$$

There are two independent variables 'a' and 'M' in the above Eq.6, Eq.7, eq.9 and Eq.10. Nestell and Coward [1] have been derived an equation for tearing stability in term of the applied T and the material tearing modulus without making any assumptions. They have not used any constraint equation, like constant displacement or constant load condition during the crack growth. In the derivation they have used a stability condition based on the postulate that the moment shed by the cracked section during the crack growth, causes an increase in the crack rotation angle (hinge angle $d\phi_{crack}$) that must be picked up by the system.

$$-dM_{crack} < dM_{system} \tag{11}$$

The stiffness of the piping system Ks is equal to reciprocal of compliance C_S. During the crack extension the increase in the crack rotation $d\phi_{crack}$ must be matched by an equal increase in the system rotation $d\phi_{syatem}$ at its location. Hence dM_{system} becomes equal to $K_S d\phi_{crack}$. Then the above stability criteria can be rewritten as:

$$-\left.\frac{dM}{d\phi}\right|_{crack} \leq K_S \qquad \text{(stable)} \tag{12}$$

Using Eq. 6 through 12 Nestell and Coward [1] have been derived the tearing modulus equation as following.

$$T = \frac{E}{\sigma_f^2}\frac{dJ}{da} = \frac{E}{\sigma_f^2}\left.\frac{\partial J}{\partial a}\right|_M - \frac{t\dfrac{E}{\sigma_f^2}\left.\dfrac{\partial J}{\partial M}\right|_a^2}{C_S + \left.\dfrac{\partial \phi}{\partial M}\right|_a} = T^{\infty} - T^{Cs} \tag{13}$$

In the above expression the first term is the conventional tearing modulus T^{∞} as for infinite compliance and the second term T^{CS} is the reduction in the tearing modulus due to the compliance.

FINITE ELEMENT ANALYSIS

Finite element analysis using ADINA code [3] was performed for the steam generator inlet (SGI) pipeline. This pipe has mean radius equal to 234 mm and thickness equal to 40 mm. The pipe is made of carbon steel SA333 Gr.6 and its stress strain behavior is assumed to obey a Ramberg-Osgood power law as $\varepsilon/\varepsilon_o = \sigma/\sigma_o + (\sigma/\sigma_o)^n$ .The power law coefficient and other material properties is given below:

$n = 3.273$
$\alpha = 8.1505$
Yield stress σ_o = 240 MPa
Flow stress σ_f = 349 MPa
Young modulus E = 179000 MPa

A TWC crack, of total subtended angle (2θ) at center equal to 90^o, was postulated at the mid-span. Symmetric one-quarter domain of the Pipe was modeled using 20 noded 3D elements shown in Fig.5 and virtual crack extension method was used for J integral calculations. The analysis was done for two types of loadings and boundary conditions. In first type all degree of freedom at the ends were fixed and the pipe was subjected to distributed loading as shown in Fig.2. The analysis was done for different L/R ratios equal to 20, 40, 60, 100, 150, 200 and 400. The J integral and the CMOD was obtained. Another case was analyzed in which the pipe ends are free and is subjected to a constant bending moment as shown in Fig.1. This corresponds to infinite compliance of piping i.e. L/R equal to ∞. Here the J integral CMOD and end rotations were obtained.

One case of 3D layout of the piping as shown in Fig. 6a was also analyzed to demonstrate the L_{eff}/R equivalence. One half domain was modeled by 20 noded 3D elements and the 90^o crack was modeled at the symmetric section as shown in Fig. 6b.

For the case of fixed-fixed pipe, with different L/R, the results of J integral versus Mu and CMOD versus M_U are shown in Fig. 7 and Fig. 8. Here M_U/M_O, the normalized moment at the concerned location in un-cracked condition, is the common reference in all the case. The M_O is limit moment $M_o = 4\sigma_o^2 R^2 t[\text{Cos}(0.5\theta) + 0.5\text{Sin}(\theta)]$ for 90^o crack. The case of free free, that is L/R $\rightarrow \infty$, is also shown in these figures. It is clearly seen that at low L/R ratios there is considerable reduction in the J and CMOD. This implies that the actual moment at the cracked section reduces depending on the value of L/R. The J versus M_U and CMOD versus M_U graphs for 3-D pipe layout are also shown by dotted line in the Fig.7 and Fig.8.

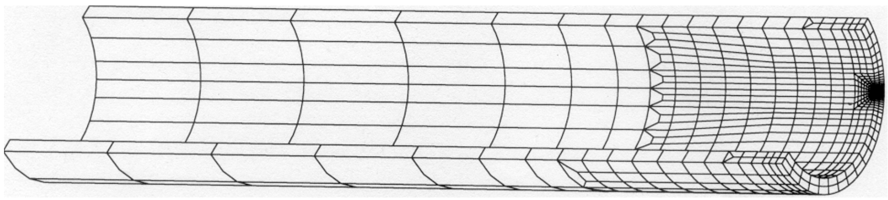

Fig. 5a: Finite Element mesh for quarter domain of the Straight Pipe

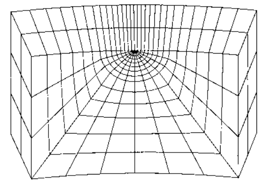

Fig. 5b: Mesh near The Crack tip

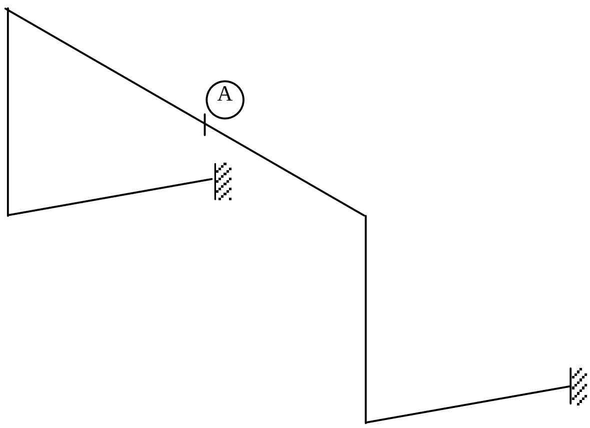

Fig. 6a: 3-D Piping Layout having TWC crack at 'A'

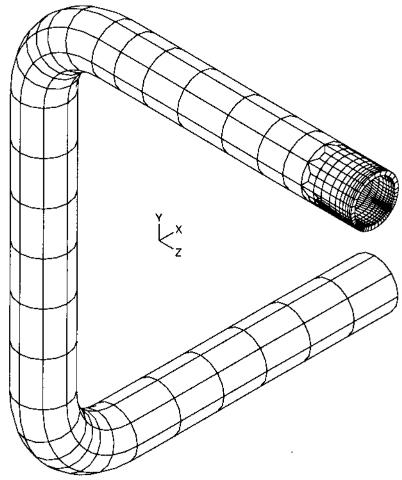

Fig. 6b: FE Mesh of Half domain of 3-D Layout

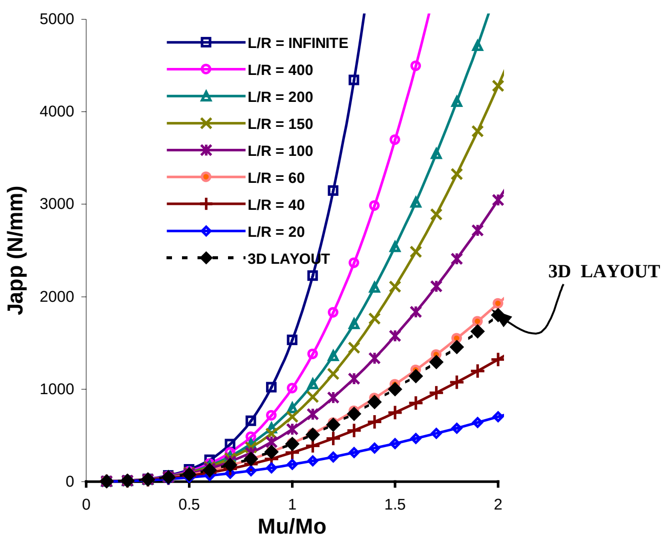

Fig. 7: FE J value versus normalized un-cracked Moment for different L/R

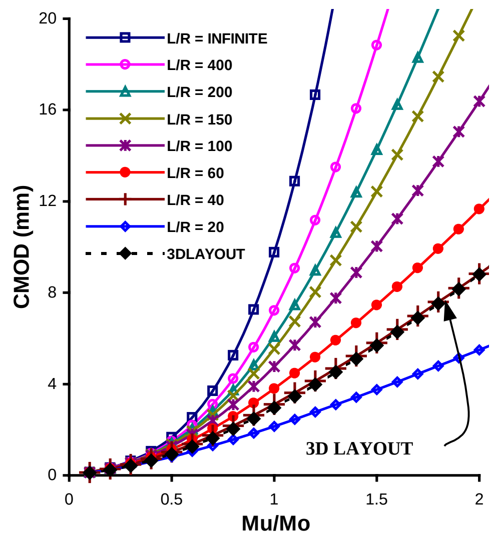

Fig. 8: FE CMOD value versus normalized un-cracked Moment for different L/R

For the 3-D piping layout, shown in Fig. 6a, first of all L_{eff}/R was calculated. At the cracked section the hinge was assumed at 'A' location, then equal and opposite bending moment were applied and followed by linear elastic analysis on 1-D pipe model. The L_{eff} was evaluated using the Eq. 4. It was found that L_{eff}/R is around 50. Referring to the Fig.7 and Fig.8, it can be seen that J and CMOD versus M_U behavior of this 3-D pipe layout indeed narrow bound to L/R ~ 50.

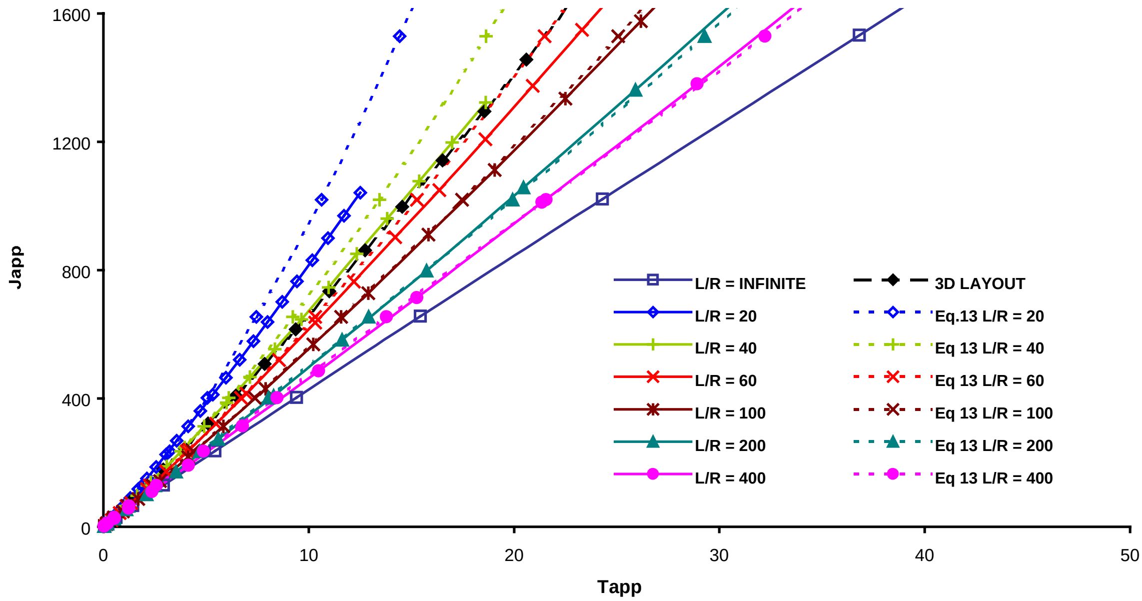

Fig. 9: Comparison of J versus T as calculated directly using FE J values for fixed-fixed conditions and calculated by using Eq.13

APPLIED TEARING MODULUS

The applied tearing modulus is generally compared with the material tearing modulus value to ascertain margin against unstable ductile tearing load. The Eq.13 implies that the applied tearing modulus depends upon the compliance of the piping. The finite element analysis, discussed in the previous section, was repeated for slightly different crack sizes. The T_{app} was evaluated directly from finite element results, using center difference scheme. The direct finite element results are shown in Fig.9, for different L/R ratios. In addition, the free-free pipe results of the ϕ versus M and J versus M were also used in Eq.13 to calculate the T_{app} and shown in Fig.9 by dotted lines. It is observed that T_{app}, as evaluated using Eq.13 and directly using FEM are in fair agreement with each other. Both Eq.13 and the Fig.9 illustrate that the applied tearing modulus is lower for same value of J integral and the reduction depends upon the value of L/R. Conversely, stating that for same value of T the J integral would be higher. So critical value of J, that is when applied T exceeds the material T, would be higher. Hence one ends up with higher safety margins.

After ascertaining the Eq.13 with FE results, it was further simplified. In this equation, all the quantities, except Cs, depend on J and ϕ values of pipe with free-free end conditions. The expressions for T^{∞} and T^{Cs} were simplified using GE/EPRI estimation scheme equations. In an earlier paper by present authors, Gupta et. al. [4], the simplified equation for T^{∞} was presented as following:.

$$T^{\infty} = \frac{E}{\sigma_f^2 R\theta}\left[\left(1+5\frac{\theta}{\pi}\right)\left(0.85+0.01\frac{R}{t}\right)+(n-1)\frac{\theta}{2}\left(1+4\frac{\theta}{\pi}\right)-0.1n\right]J^{\infty} \tag{14}$$

The term T^{Cs} was simplified and presented below.

$$T^{Cs} = \frac{E}{\sigma_f^2 R\,\theta}\left[\frac{\dfrac{f_b\theta^2}{B_3}\,\dfrac{(n+1)^2}{n}\,\dfrac{\Psi 1}{\Psi 2}}{1+\dfrac{Leff}{2\,R\,B_3}\,\dfrac{1}{[(n-1)\;\alpha\,\Psi_2+1]}}\right]J^{\infty} \tag{15}$$

$$J^{\infty} = \left[1+\alpha\Psi_1\left(\frac{M}{M_O}\right)^{n-1}\right]J^{e} \tag{16}$$

$$\phi^{\infty} = \left[1+\alpha\Psi_2\left(\frac{M}{M_O}\right)^{n-1}\right]\phi^{e} \qquad (17)$$

The ψ_1 and ψ_2 can be determined by equivalence of the Eq.16 and Eq.17 with the corresponding elastic plastic J and ϕ equations for TWC cracked straight pipe under pure bending, given by Zahoor [5]. These are basically dimensionless terms and function of n, θ/π and R/t. The f_b and B_3 expression are also given in Zahoor [5].

Extensive validation of this simplified equation was carried out with respect to Eq.13, in which J and ϕ values were taken from Zahoor [5]. The comparison is shown in Fig.10 and Fig 11, in which n values was varied from 2 to 7 and θ/π varied from 0.1 to 0.5 and R/t varied from 5 to 20. In some cases the negative value of the applied tearing modulus was observed as shown in Fig. 11. In these cases the moment reduction at the crack section, because of small incremental in crack growth (Δa), is so high that the J (a +Δa, M_U) becomes lesser than J (a, M_U). It means, the chances of unstable crack growth are less since the crack driving force (J) is decreasing as a result of crack growth.

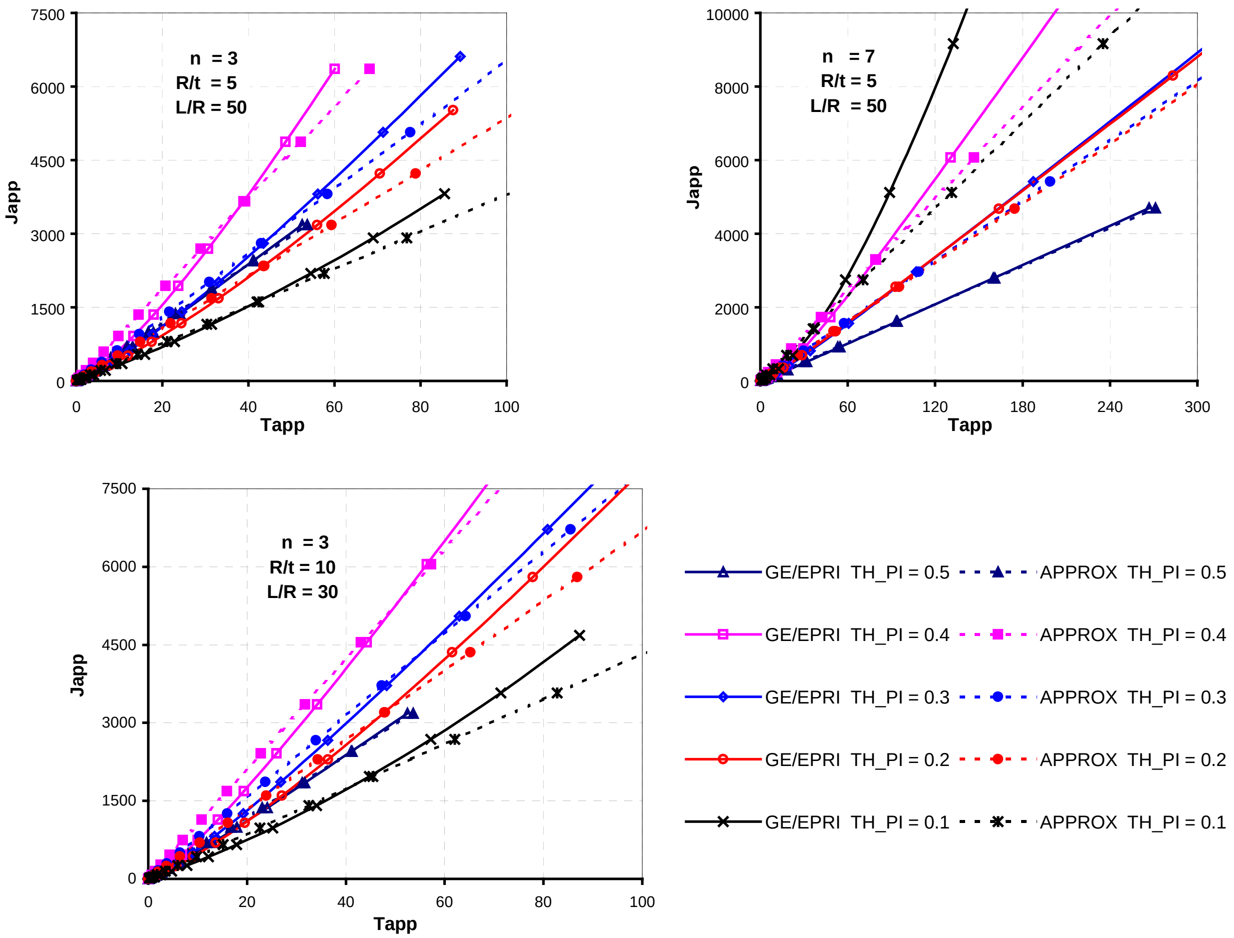

Fig. 10: Comparison of J versus T as calculated using Eq.13, T^{Cs} calculated by GE/EPRI and by approx proposed Eq.15

CONCLUSIONS

The aim of the present work was to study the effects of compliance on the fracture parameters such as J, CMOD and on ductile tearing instability load calculations. For this, finite element analysis for a typical nuclear power plant pipe with different L/R ratios was performed. Fracture parameters such as J and CMOD for a 3D layout of the piping were also obtained. The concept of representing 3D piping systems by a single straight pipe with built-in ends and crack at center span, works reasonably well. It is observed that the compliance of the piping cause significant reduction in the J and CMOD particularity when the L/R is small. The critical value of J integral, that is the J value when applied T becomes equal to material T, increases when the compliance term T^{Cs} is included in the applied tearing modulus calculations. The increase in the J-critical and reduction of the J-applied, together leads to substantial increase in the unstable tearing load. Finally, a simple formula is proposed for estimation of applied tearing modulus T, which includes the compliance term. Another equation is also proposed for calculating the reduction in moment at the cracked section. The two equations jointly can be used for calculation of the realistic safety margins. It is concluded that the associated benefit in terms of an increase in safety margin due to finite compliance or non zero stiffness of the connected piping system should be utilized.

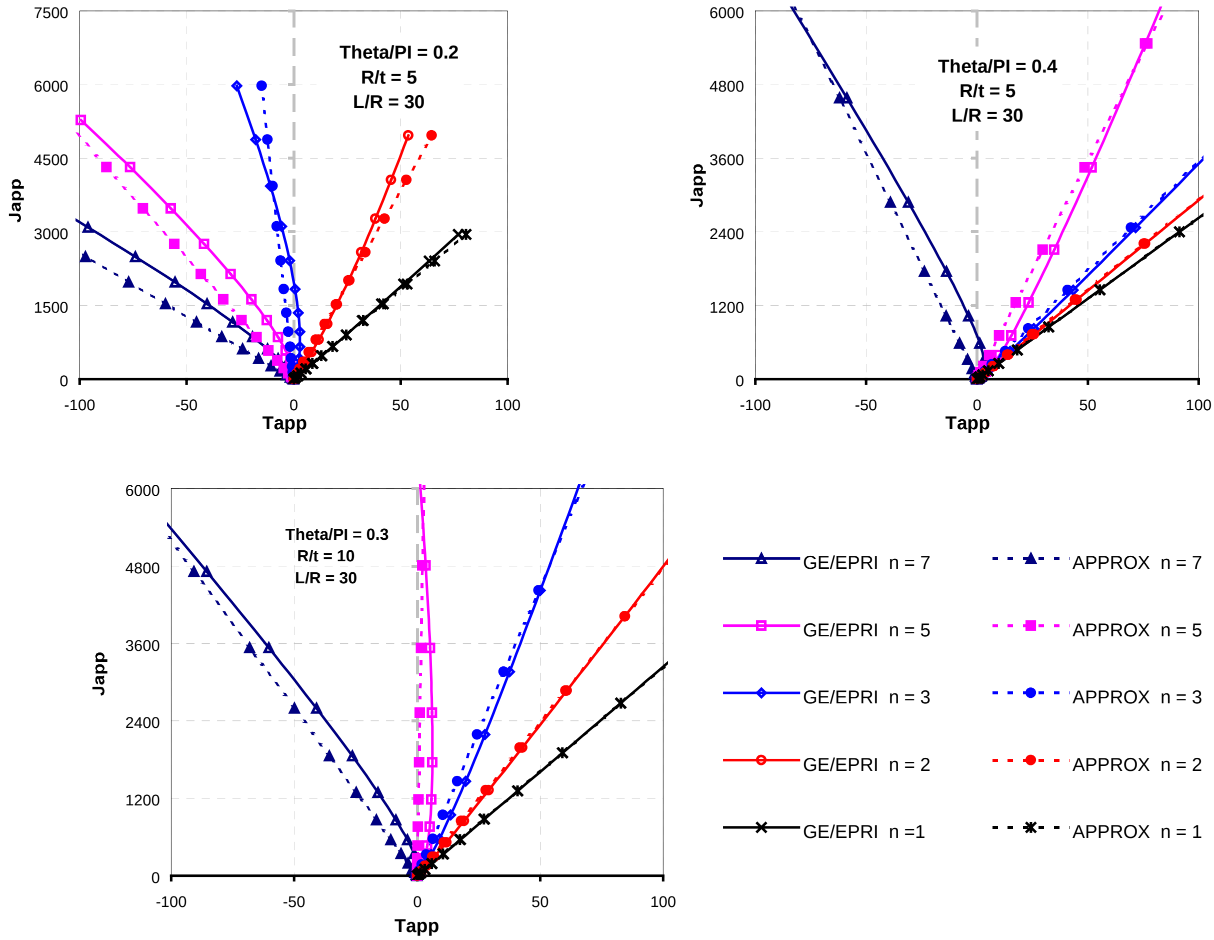

Fig. 11: Comparison of J versus T as calculated using Eq.13, T^{Cs} calculated by GE/EPRI and by approx Proposed Eq.15

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