Power Spectral Density Functions Compatible with Design Response Spectra

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Abstract

Artificially generated ground acceleration time histories are often used for the analysis and design of structures subjected to earthquake ground motions. Among other possibilities, the Kanai-Tajimi power spectral density enhanced at the higher frequency range is found to be useful for generating ground acceleration time histories that satisfy NRC RG 1.60 requirements. The values of the parameters involved in the spectral density function are recommended for this purpose. Also, a suggestion is made as to how the power spectral density requirements can be placed in combination with those of NRC RG 1.60 to ensure both response and power spectrum requirements.

1. Introduction

It is often practical to use a simulated ground acceleration time history for the analysis and design of structures subjected to earthquake ground motions. From the viewpoint of frequency as well as time domain characterization, it is advisable to consider such a simulated time history as a sample function of a random process. With this in mind, quite sophisticated digital simulation techniques were developed over the years by Shinozuka et al (e.g., [1]). The techniques were used in a variety of applications as summarized in Shinozuka [2]. These techniques are primarily based on the spectral representation of Gaussian random processes.

In the present paper, which summarizes an NUREG report by Shinozuka et al [3], a nonstationary Gaussian random process characterized by the spectral representation is used as the parent random process from which sample functions are generated. The purpose of this paper is then to construct the power spectral density function of the parent stationary process in such a way that sample functions of the parent process satisfy NRC RG 1.60. The spectral density function must satisfy the additional requirement that it be a smooth function of the frequency so that no apparent lack of power exists over any frequency windows. This requirement is to obviate the construction and eventual use in design of power spectral density functions deficient in their power spectral distribution over certain frequency windows. Therefore, if a power spectral density function without such a deficiency can be developed and is used in generating an acceptable acceleration time history (acceptable in the sense that it satisfies NRC RG 1.60), it will help dispel the concern that the corresponding power spectral density of an acceptable time history may be deficient over certain critical frequency windows.

2. Use of Kanai-Tajimi Spectra

In the present study, a power spectral density function is to be determined which has an analytically prescribed form and can be used, together with a deterministic envelope function, to generate ground acceleration time histories compatible with NRC RG 1.60. In order to accomplish this task, the following steps are taken. (a) Select among other possible candidates the one-sided Kanai-Tajimi spectral density function (Kanai [4]).

$$S_0(\omega) = S_0 \frac{1 + 4\zeta_g^2(\omega/\omega_g)^2}{[1 - (\omega/\omega_g)^2]^2 + 4\zeta_g^2(\omega/\omega_g)^2} \quad (1)$$

where $S_0 = 1100\ \text{in}^2/\text{sec}^3$, $\omega_g = 10.66$ rad/sec and $\zeta_g = 0.9793$ are chosen. Generate time histories in the following form:

$$\ddot{\zeta}_0(t) = \sqrt{2} \sum_{k=1}^{N} \sqrt{S_0(\omega_k)\Delta\omega}\ \cos\ (\omega_k t + \phi_k) \quad (2)$$

The summation in eq. (2) is accomplished by means of the Fast Fourier Transform (FFT) technique [1] with $\omega_k = k\Delta\omega$, $\Delta\omega = \omega_u/N_u$, and ϕ_k representing a sequence of independent realizations of the random variable Φ uniformly distributed between 0 and 2π. Each sequence of ϕ_k will produce a sample function, $\ddot{\zeta}_0(t)$, of a stationary Gaussian random process with mean zero and power spectral density function $S_0(\omega)$. The quantity ω_u is the largest natural frequency value considered in this study; $\omega_u = N\Delta\omega = 51.2 \times 2\pi$ rad/sec ($N = 1{,}024$) where each simulated record of 20 second duration is generated at 2N (= 2,048) time points equally spaced at an interval of $\Delta t = 20/2{,}048 = 0.00988$ seconds, thus satisfying the Nyquist requirement. (b) The time histories generated by eq. (2) usually outcross the $\pm$1G levels from time to time. However, the acceleration time histories generated in this study should have peak ground accelerations equal to 1G (in the absolute value sense). The most straightforward way to accomplish this is to clip the values of $\ddot{\zeta}_0(t)$ outside the $\pm$1G range. The clipping, however, may give the artificial ground acceleration an overly unnatural appearance. Therefore, the procedure of "fractional folding" is introduced. This procedure will produce a stationary time history $\ddot{\zeta}_2(t)$ as shown in Fig. 1; that part of $\ddot{\zeta}_0(t)$ outside the $\pm$1G range will be folded into the range after being multiplied by a factor r ($0 < r < 1$). In this way, the resulting artificial acceleration $\ddot{z}_2(t)$ given by

$$\ddot{z}_2(t) = g(t)\ \ddot{\zeta}_2(t) \quad (3)$$

will have a less unnatural appearance, where g(t) is a deterministic non-dimensional envelope function as shown in Fig. 2. (c) The response spectra of the artificial ground acceleration $\ddot{z}_2(t)$ thus generated with $r = 0.1$, $t_r = 5$ sec and $t_m = 10$ sec are then evaluated. If a generated artificial earthquake does not satisfy NRC RG 1.60 requirements, another artificial earthquake with a different set of ϕ_k will be generated. This is repeated until the desired number of artificial earthquakes satisfying the requirements are generated. (d) Finally, the power spectral density function $S_2(\omega)$ of $\ddot{\zeta}_2(t)$ is computed to be compared with $S_0(\omega)$. The same values of ω_n, N and Δt are used in this computation.

The raw power spectral density of the underlying stationary process $\ddot{\zeta}_2(t)$ exhibits considerable fluctuation particularly in the high frequency range. Also, a comparison between $S_0(\omega)$ and $S_2(\omega)$ indicates that the fractional folding tends to downgrade the spectral density over the entire frequency range.

3. Use of Enhanced Kanai-Tajimi Spectra

The procedure described above for the generation of artificial earthquakes does not have direct control over the shape and intensity of the resulting power spectral densities. The procedure is now revised so as to achieve more direct control over the power spectral density. The revision consists mainly of "spectral density enhancement" and "smoothing."

The spectral density enhancement is achieved by multiplying the Kanai-Tajimi spectrum $S_0(\omega)$ by a factor $F(\omega)$;

$$F(\omega) = [1 + p(\frac{\omega}{\omega_c})^2]/[1 + q(\frac{\omega}{\omega_c})^2] \tag{4}$$

in which p, q and ω_c are parameters. This factor is a monotonically increasing function of ω such that $F(\omega) \to p/q$ as $\omega \to \infty$, $F(\omega) \to 1$ as $\omega \to 0$, and $F(\omega) = (1+p)/(1+q)$ for $\omega = \omega_c$. Since we assume here that $p = 8.0$, $q = 1.0$ and $\omega_c = 500$ rad/sec, $S_0(\omega)$ is enhanced as much as eight times at the higher frequency range.

Then, using the enhanced initial power spectral density $S_0^*(\omega)$ in eq. (2) (in place of $S_0(\omega)$) such that

$$S_0^*(\omega) = S_0(\omega)\, F(\omega) \tag{5}$$

the corresponding stationary process $\zeta_0^*(t)$ can be generated. Furthermore, $\zeta_0^*(t)$ is fractionally folded at the $\pm$1G level to produce $\zeta_2^*(t)$ and the artificial earthquake $z_2^*(t)$ is obtained as $z_2^*(t) = g(t)\zeta_2^*(t)$.

Figure 3 shows the $z_2^*(t)$ thus generated, while Fig. 4 the smoothed power spectral density function $\bar{S}_2^*(\omega)$ of the underlying stationary process $\zeta_2^*(t)$. In Ref. 3, it was observed that there is little difference between $z_2(t)$ and $z_2^*(t)$. At the same time, it was also observed that not only the fluctuation of the spectral density function $\bar{S}_2^*(\omega)$ is much less extensive than that of the raw spectral density $S_2^*(\omega)$ associated with $\zeta_2^*(t)$, but also the values of $\bar{S}_2^*(t)$ are generally above those of the Kanai-Tajimi spectral density function $S_0(\omega)$ at the higher frequency ranges. (See Fig. 4 and note that the smooth solid curve indicates the Kanai-Tajimi spectral density function.)

For smoothing, the following moving average technique is used in approximation: the averaging is made on four values of $S_2^*(\omega)$ at ω_i, ω_{i+1}, ω_{i+2} and ω_{i+3} with the average values plotted at ω_{i+1} as $\bar{S}_2^*(\omega)$. It is noted that the frequency increment $\Delta\omega$ (rad/sec) between the two successive frequencies ω_i and ω_{i+1} is very small; $\Delta\omega = 0.314$ rad/sec.

There are two reasons that justify such an averaging. First, a better estimate for the spectral density at a frequency value can be obtained as an average involving the raw spectral density values at its neighboring frequencies (e.g., Bendat and Piersol [5]). Second, the product of the power spectral density and the square of the absolute frequency response function represents the response power spectral density under the assumed stationarity of the input processes. The integral of the response power spectral density will then provide the mean square value of the response. Therefore, if the absolute frequency response function is smooth over the frequency band in which each moving average is taken, the proposed smoothing is justifiable in the sense that the integral is expected to be more or less the same with or without smoothing. Each frequency band over which such a smoothness is required must be larger than $3\Delta\omega$ for the current four point average. In view of the fact that $\Delta\omega$ is fairly small, a smoothing involving even more than four frequency points will not be unreasonable. The justification just given is based on the stationarity assumption. However, it also applies, at least in approximation, to the case involving nonstationary input processes when the nonstationarity is characterized by an envelope function as shown in Fig. 2 which has a relatively dominant stationary segment. The major point here is that such a smoothing involving only four frequency points has considerably reduced the extent of fluctuation of $S_2^*(\omega)$ particularly in the higher frequency ranges and, at the same time, has enhanced the power spectral density so that the smoothed density $\bar{S}_2^*(\omega)$ is much closer to and even above $S_0(\omega)$ in some frequency ranges. In fact, the most adverse difference between $S_0(\omega)$ and and $\bar{S}_2^*(\omega)$ in Fig. 4 is observed in the neighborhood of f = 0.2 Hz with $\bar{S}_2^*(\omega)$ being approximately 15% below the target.

The above observation leads to the following recommendation, albeit preliminary: an additional requirement to NRC RG 1.60 be introduced in such a way that for an artificial ground acceleration time history to be acceptable for design, it must not only produce response spectra for designated damping ratios that envelope from above the specified response spectra but also possess a power spectral density which is no less than the prescribed percentage of the target spectral density everywhere in the frequency range considered. The percentage may be 85% and the target spectral density may be of the form of a one-sided Kanai-Tajimi with $S_0 = 1100$ in^2/sec^3, $\omega_g = 10.66$ rad/sec and $\zeta_g = 0.98$.

The time history (Fig. 3) generated on the basis of the enhanced spectral density $S_0^*(\omega)$ as the spectral density, generally satisfies NRC RG 1.60 for damping ratios 0.5, 2, 3, 5, 7 and 10% (see [3]). In fact, Fig. 5 plots the response spectra associated with this time history for a damping ratio of 2%.

Some comments are offered in [3] with respect to the validity of using the power spectral density functions associated with the stationary process $\ddot{\zeta}_2(t)$ in order to examine if the power content is smoothly distributed over the frequency range.

4. Summary and Conclusions

A method of generating artificial ground acceleration histories is presented. The method consists of (1) Generating a sample function $\ddot{\zeta}_0^*(t)$ of a stationary Gaussian process with zero mean and prescribed power spectral density function function $S_0^*(\omega)$ where $S_0^*(\omega)$ is obtained from the "initial" power spectral density $S_0(\omega)$ by enhancement; (2) Constructing a sample function $\ddot{\zeta}_2(t)$ by fractionally folding $\ddot{\zeta}_0(t)$ beyond the ±1G level; and (3) Multiplying $\ddot{\zeta}_2(t)$ by a non-dimensional deterministic envelope function g(t) to produce the artificial acceleration history $\ddot{z}_2(t) = g(t)\ddot{\zeta}_2(t)$.

The duration of the earthquake is assumed to be 20 seconds and the envelope function g(t) is in the shape of an isosceles trapezoid with a 10 second duration of constancy at the level of unity, preceded by a 5 second period of linear rise from zero and followed by a 5 second linear decay to zero.

The Kanai-Tajimi form is used for the initial power spectral density function. Unlike most artificial earthquakes designed to satisfy the prescribed response spectra only, the proper choice of values of the parameters involved in the Kanai-Tajimi spectrum (S_0 = 1100 in^2/sec^3, ω_g = 10.66 rad/sec and ζ_g = 0.98) and use of the above procedures (1) - (3) produced acceptable acceleration histories with the smoothed power spectral density function which maintains the spectral density sufficiently close to the initially prescribed spectral density throughout the frequency range.

A preliminary suggestion was also made as to the requirements for the initial power spectral density function to be added to those of NRC RG 1.60 in order to ensure both response and power spectrum requirements.

5. References

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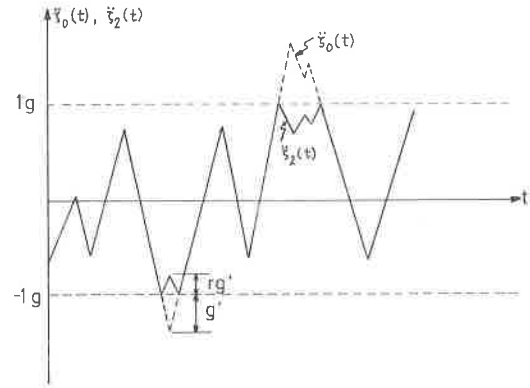
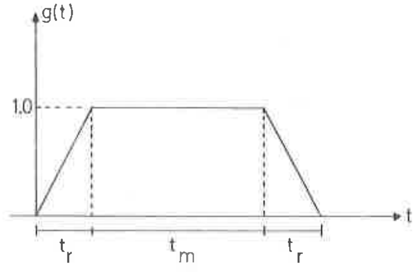
Notice

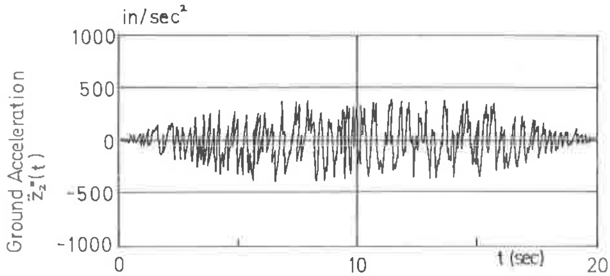
The views expressed in this paper are those of the authors. The technical contents of this paper have neither been approved nor disapproved by the US Nuclear Regulatory Commission.



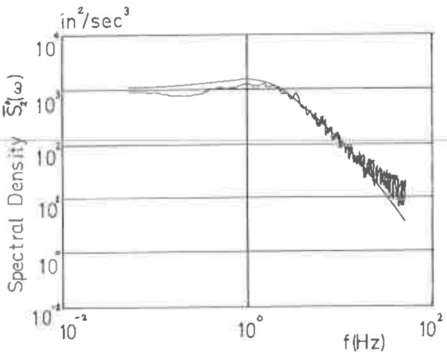
1. Fractionally-Folded Time Histories



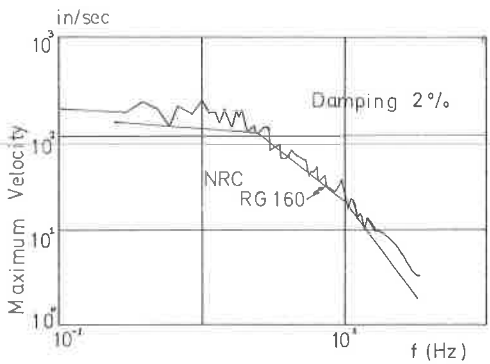
2. Deterministic Envelope Function



3. Acceleration Time History



4. Spectral Density Functions



5. Velocity Spectrum