Nonlinear Structural Seismic Analysis Through an Equivalent Linearization

P. Guihot, D. Brochard, R. J. Gibert
CEA-CEN Saclay, Gif sur Yvette, France

1. INTRODUCTION

The seismic methods available for non linear behaviour of structures can be divided in two groups: the outright methods and the non linear methods.

In the first case a linear calculation is performed and the results are modified in order to take into account the non-linearity. This method is easy to apply but allows only an approximate knowledge of the structure response.

The non linear methods needs the response calculation to specific seismic time histories. The high costs of such methods limit their application to approximate method verification.

In this context, equivalent linearization methods give an interesting alternative and are more and more used (To, 1987 ; Ziegler, 1987). The linearization method that we proposed is applied to a one degree of freedom oscillator with shock and a random stationary excitation.

This method has been applied to weak and strong non linearities to demonstrate its capabilities but also its limits to represent real structures. The method accuracy is verified against Monte-Carlo simulations and the solution to the Fokker-Planck equation. We can note that the procedure can be extended to the analysis of multidegree of freedom (Ziegler, 1987) and asymmetric (Spanos, 1980) systems.

2. FORMULATION

Let us consider an harmonic one degree of freedom (D.O.F.) oscillator with the characteristics m,c,k (see figure (1)) ; the mass displacement is restrained by two stops.

The oscillator motion equation is

(1) $$m\ddot{x} + c\dot{x} + kx + f(x) = W$$

where f(x) represent the non linear force given by

$$f(x) = k'\{(x-e)H(x-e) + (x+e)H(-x-e)\}$$

with k' = stop stiffness
e = gap

and H = Heaviside function
W = White noise excitation.

The excitation is supposed to be a stationary gaussian process with zero mean value, such as

$$< W(t).W(t+\tau) > = \delta(\tau)$$

where δ is the Dirac function and the angular bracket < > denotes the operator of mathematical expectation.

2.1 - Linear system determination

The simplest equivalent linear model is a one D.O.F. oscillator ; its motion is given by:

(2) $$m\ddot{\chi} + c\dot{\chi} + k\chi + k_{eq}\chi = W$$

Because the non linear restoring force f depends only on non linear stiffness, the equivalent linear system gives only an additional stiffness. In the same way, if the non linear restoring force corresponds to a non linear damping, only a equivalent damping appears.

By this linearization, a systematic error is made. By example, the equivalent displacement χ does not verify exactly equation (1):

(3) $$m\ddot{\chi} + c\dot{\chi} + k\chi + f(\chi) = W + \varepsilon$$

ε is a representation of the error which can be written:

(4) $$\varepsilon = f(\chi) - k_{eq}\chi$$

Obviously the optimal k_{eq} value is associated to the minimum error ε.

This error, being a random process, is calculated by a mean square minimization, but other criteria could be used (Iwan, 1972). The equivalent stiffness is then given by:

$$\frac{d}{d\,k_{eq}} <\varepsilon^2(t)> = 0 \qquad (5) \quad k_{eq} = <\chi.f(\chi)>.<\chi^2>^{-1}$$

2.2 - Equivalent stiffness calculation

The excitation was a gaussian probability law, so the response is also gaussian and is expressed by:

(6) $$p(\chi) = \frac{1}{\sqrt{2\pi}\,\sigma_\chi} \exp\left[- \frac{(\chi - <\chi>)^2}{2\,\sigma_\chi^2} \right] \text{ where } \sigma_\chi 2 = <\chi 2>$$

Due to the symmetry: $<\chi> = 0$ and $<\dot{\chi}> = 0$.

The χ and $\dot{\chi}$ variances are obtained from equation (2):

(7) $$\left| \begin{array}{l} m<\ddot{\chi}\chi> + c<\dot{\chi}\chi> + (k + k_{eq}) <\chi\chi> = <W\chi> \\ m<\ddot{\chi}\dot{\chi}> + c<\dot{\chi}\dot{\chi}> + (k + k_{eq}) <\chi\dot{\chi}> = <W\dot{\chi}> \end{array} \right.$$

and owing to the stationary properties of W and χ, the system (7) is transformed in:

$$(8)\quad \begin{cases} \langle\dot{\chi}^2\rangle = c^{-1}\ \langle W\dot{\chi}\rangle \\ \langle\chi^2\rangle = (k + k_{eq})^{-1}\ \{m/c\ \langle W\dot{\chi}\rangle + \langle W\chi\rangle\} \end{cases}$$

Making use of the fact that W is delta correlated, one can show that:

$$(9)\qquad \langle W\chi\rangle = 0 \quad \text{and} \quad \langle W\dot{\chi}\rangle = \{2m\}^{-1}$$

So the system (8) is reduced to:

$$(10)\qquad \langle\dot{\chi}^2\rangle = (2mc)^{-1} \quad \text{and} \quad \langle\chi^2\rangle = \{2c(k + k_{eq})\}^{-1}.$$

and the equivalent oscillator characteristics are calculated by resolving the coupled equations (11) and (12). These two equations are obtained from equations (5) and (10).

$$(11)\qquad k_{eq} = k'[1 - \text{erf}\ (\frac{e}{\sqrt{2}\ \sigma_\chi})] \quad \text{and} \quad \langle\dot{\chi}^2\rangle = \frac{k + k_{eq}}{m}\ \langle\chi^2\rangle$$

$$(12)\qquad \langle\chi^2\rangle - \frac{1}{2c}\ \{\ k + k'[1 - \text{erf}\ (\frac{e}{\sqrt{2}\ \sigma_\chi})]\ \}^{-1} = 0\ .$$

where erf is the error function such as: $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)dt.$

The implicit equation (12) is solved by the Newton-Raphson method (calculation of function zeros) which allow to obtain with an excellent precision in a few number of iterations.

3. METHOD APPLICATION

This linearization method was applied to calculate the equivalent oscillators associated to various values of c, k, k' and the gap e. The response characteristics were compared to the non linear ones in order to analyse the method validity.

3.1 - Comparison tools

The non linear response characteristics can be obtained in two ways:

1) Non linear simulations of the shock oscillator and after averaging calculation of the response statistical values.
2) For the particular case of a shock oscillator and a random stationary excitation, the probability density which is given by the Fokker-Planck equation can be solved (Lin, 1967) and then the probabilistics of the displacement x and the velocity $\dot{x}$ can be calculated.

The probability density is equal to:

$$(13)\qquad p(x,\dot{x}) = B\ \exp\ \{\ \frac{-1}{2\sigma_o^{\ 2}}\ \frac{2U}{k} - \frac{\dot{x}^2}{2\sigma_{\dot{x}}^{\ 2}}\ \}$$

where $U = \frac{1}{2}\ kx^2 + \frac{1}{2}\ k'\ \{(x-e)H(x-e) + (x+e)H(-x-e)\}$,

and $\sigma_o^{\ 2} = (2ck)^{-1}$, $\sigma_{\dot{x}} = \sigma_o\ (k/m)^{\frac{1}{2}}$

B is a normalization such as: $\int_o^\infty \int_o^\infty p(x,\dot{x})dx\ d\dot{x} = 1$.

3.2 - Time history analysis

The figure (2) shows the probability density for a reduced gap η equal to 0.5. (This gap η is a non dimensional parameter chosen because it is a representation of the non linearity level). One can notice:

- a very good agreement between the simulation results and the theoretical results, obtained by the Fokker-Planck equation solution,
- a non gaussian probability density ; this induces discrepancies between the equivalent model and the non linear model (fig. (2) is an extreme case.).

The response maximum values are important for the structure design, so a comparison of standard deviation and maximum value has been performed (figs. 3 and 4). When the behaviour is weakly non linear the equivalent linear results are very good. The maximum discrepancy on figure (3) is 10%, but the difference increases with stiffness ratio k'/k.

3.3 - Spectral analysis

The figure (5) represents the power spectral density (P.S.D.) of the response as a function of η. When the non linearity is important, the resonance peak is replaced by a plate shape which is associated to a frequency scattering.
The physical explanation to the phenomenon is simple. One can divide the oscillator cycle in two periods: one where we have shocks, and the other one without shock ; we call the last one "flight", the shock stiffness k' being important in front of the oscillator stiffness k, the shock duration is small in front of the flight duration and can be neglected if $\eta \geqslant 0.2$. In addition, we note that the total energy of the oscillator can have important variation in time and when this energy is important (small), the velocity during flight is great (small), the flight duration is small (great) and the associated frequency is high (small) (see figure 6).
A comparison of the P.S.D. calculated from the equivalent model and from non linear model is shown on figure (7) for a typical non linear case. As the differences are very important, it seems necessary to see if an other one D.O.F. linear system could give a better result. So a mean square minimization has been performed on the simulation P.S.D. to determine the oscillator characteristics c_1 and k_1. This other "equivalent" oscillator P.S.D. is presented on figure (7) too. One can notice that the c_1 damping does not allow a better representation of the shape of the P.S.D. than the previous oscillator.

4. CONCLUSION

A method for equivalent linearization for oscillator with gaps under random excitation is proposed. The proposed method gives for various parameters of the response an error estimation and consequently shows its validity domain. It was shown that some characteristics cannot be representated by a single D.O.F. model when the non linearity is important.

REFERENCES

Iwan, W.D. and Patula, E.J. "The Merit of Different Error Minimization Criteria in Approximate Analysis". Journal of Applied Mechanics, (1972), pp. 257-262.

Lin, Y.K. "Probabilistic Theory of Structural Dynamics". Mac Grow-Hill, New York, (1967).

Spanos, P.T.D. "Formulation of Stochastic Linearization for Symmetric and Assymmetric M.D.O.F. Non Linear Systems". Journal of Applied Mechanics, (1980), Vol. 47, p. 209.

To, C.W.S., "Random Vibration of Non Linear Systems". Shock and Vibration Digest, (1987), Vol. 19, pp. 3-9.

Ziegler, F. and Shueller, G.I., "Non Linear Stochastic Dynamic Engineering Systems". Springer Verlag, Berlin, (1987).

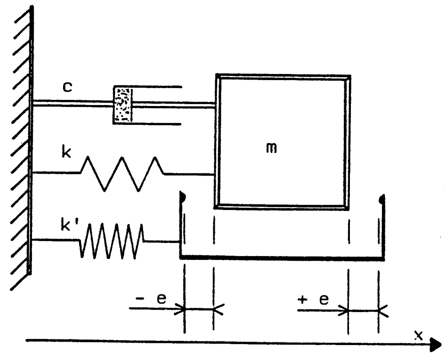

Fig. 1 - Harmonic oscillator with stop stiffness

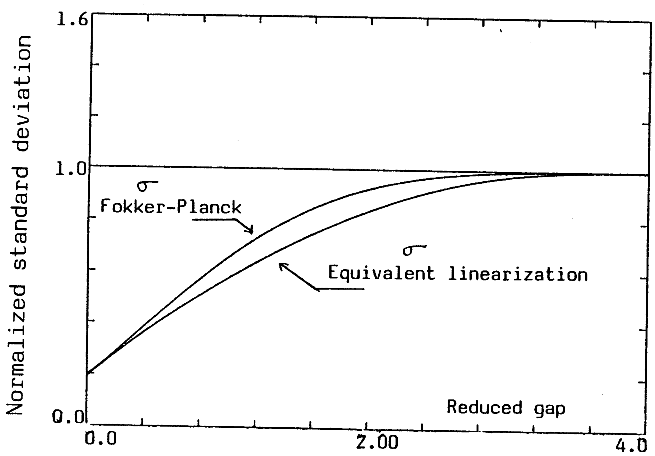

Fig. 2 - Evolution of the standard deviation with the reduced gap

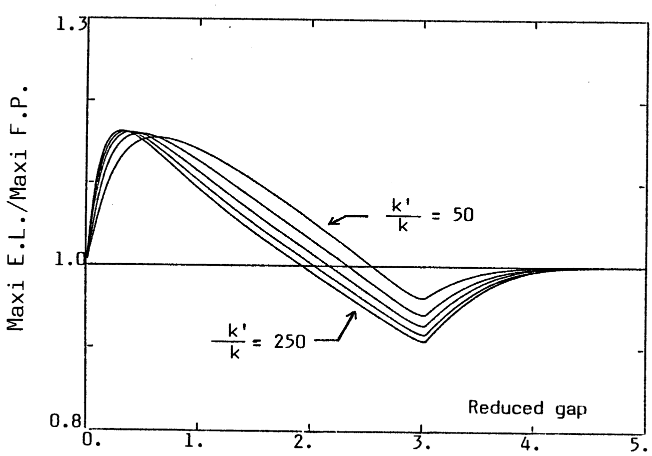

Fig. 3 - Evolution of maximal displacements

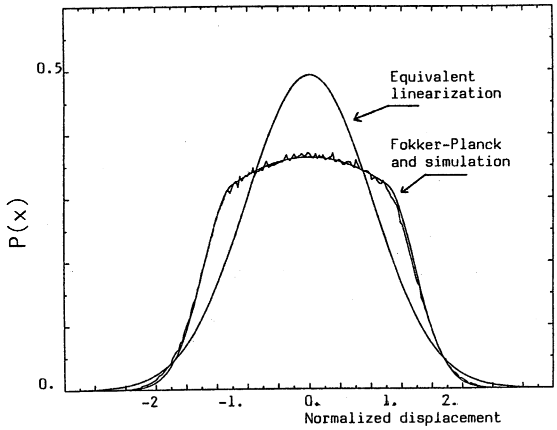

Fig. 4 - Displacement probability density (e = 1 ; η = 0.5)
- Equivalent linearization
- Fokker-Planck
- Simulation

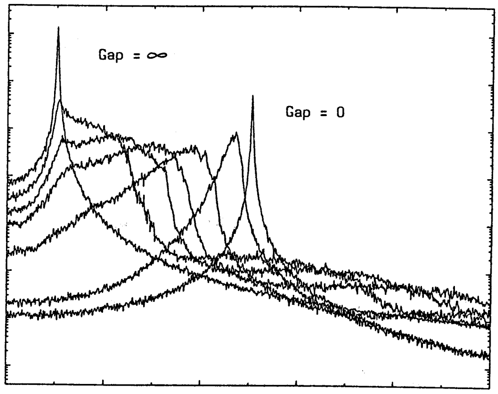

Fig. 5 - P.S.P. evolution as a function of the normalized gap
$\eta = \infty$; 1 ; 0.6 ; 0.4 ; 0.2 ; 0.05 ; 0

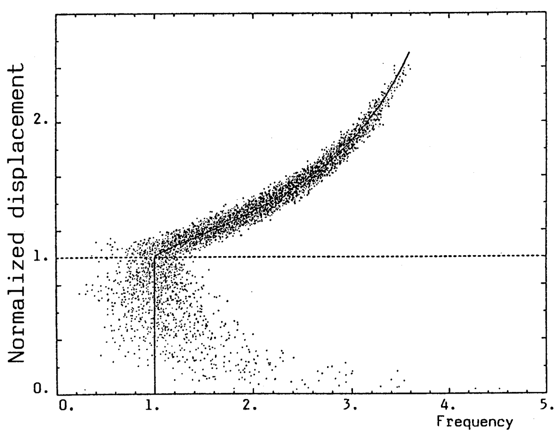

Fig. 6 - Relation between the maximal displacement of one cycle to the equivalent frequency of this cycle

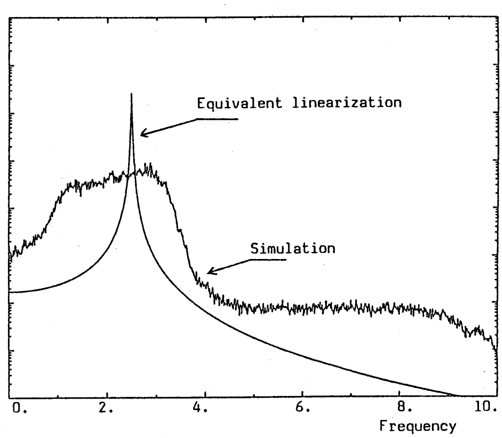

Fig. 7 - Power spectral density
- Simulation with η = 0.5
- Equivalent linearization