Natural Frequency Shift of a Graphite Column by Gravity

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ABSTRACT

A gravity effect on the free vibration of a graphite column is studied. The core structure of a gas cooled reactor using prismatic type graphite fuel blocks consists of stacked hexagonal graphite blocks, such as the fuel blocks and neutron reflector blocks, forming a group of columns. A study of the free vibration characteristics of a graphite block column is the first step for a core internal structure dynamic analysis. Also it is important to check to see whether the effect of gravity is severe or not in a tall column structure.

INTRODUCTION

Graphite blocks, such as fuel assembly blocks, plenum blocks, and neutron reflector blocks, are the key components of a HTGR (High Temperature Gas Cooled Reactor). The idea of a HTGR using prismatic graphite blocks started several decades ago and a few reactor systems of this type have been designed. The GT-MHR (Gas Turbine-Modular Helium Reactor)[1] and the HTTR (High Temperature Test Reactor)[2, 3] are typical of them. The reactor cores of the GT-MHR and HTTR are composed of columns of stacked hexagonal graphite fuel blocks. The vertically neighboring graphite fuel blocks are constrained by dowel pins and dowel sockets by each other and they form the graphite column. And each column has gaps between neighboring columns and stands on the core bottom structure by itself. The cross section of the prismatic block is a hexagonal one with many holes for fuel elements, coolant pathways, and handling tools. The prismatic type core structure is one of the design candidates of the NHDD (Nuclear Hydrogen Development and Demonstration) project[4] which is on going at KAERI (Korea Atomic Energy Institute).

A seismic analysis of the reactor core structure of a HTGR is an important design issue and has a long history. However less attention has been paid to this area after the studies of the HTGR in the 70’s and early 80’s. The structural integrity of the blocks under seismic loadings is one of the key design issues for a reactor system. The vibrations of the graphite block columns induced by earthquakes may cause solid impacts between the graphite blocks, and lead to structural integrity problems. Although the academic understandings still have a long way to go, some important progress was achieved in the 70’s by engineers. A researcher at General Atomic Company, T. H. Lee, presented a methodology for analyzing the nonlinear response of a column of stacked prismatic fuel blocks of the HTGR in 1975[5]. He derived a set of equations of motion, and obtained the numerical results for a free vibration and a forced vibration. In his work for a column of stacked graphite blocks between two side walls, he showed the feasibility of a numerical study for multi-columns of stacked graphite blocks. In parallel with Lee’s study of on column models, seismic tests of one-fifth scale planar array models of a HTGR core structure were conducted by B. E. Olsen, A. J. Neylan and W. Gorholt, and their test results with comparative results of numerical analyses were presented in 1976[5]. JAEA started their own program on the seismic analyses and tests for a HTGR in the 70s. The seismic tests of a single column half scale model were conducted in 1976, a seven columns model in 1977, a vertical two dimensional model from 1978 to 1979, as well as a horizontal two dimensional model, and some other models and relevant cases[7]. In 1979 T. Ikushima and T. Nakazawa presented their results[8] on a seismic analysis of a column of stacked fuel blocks of a HTTR and compared them to the seismic test results of a half scale column model. The numerical model is similar to that of Lee’s study[5], but the parameters in their analysis were from their scaled model tests and their results were verified.

As summarized above, seismic analyses and tests for HTGR systems had been conducted with great effort. However, unfortunately, documents open to the public describing such studies are not so abundant. Also there are some questions arising from reviewing the literatures surveyed. One is whether the gravity effect has been investigated or not. Although the core structure of a HTGR is composed of blocks and each block behaves like a rigid body, the vertically stacked blocks may behave like a single column when the amplitude of the lateral motion is small. And it is known that the downward axial force exerted by the gravitational force in a heavy long vertical column structure leads to a negative stiffness effect on a flexural bending motion and it shifts the natural frequencies[9, 10]. Generally the gravity effect is not considered in not so tall a structure, because the gravity effect term makes the analysis and design complicated. But proving that it is small and negligible for the structure is a prerequisite requirement. The gravity effect should be considered in the analysis and the design of a core structure, if an investigation says it is not negligible. This paper presents a simple investigation of the gravity effect on a free vibration of graphite columns with different heights. The material properties of the nuclear grade graphite and the shape of the cross section of the fuel block of the GT-MHR are considered.

EQUATION OF MOTION

Governing equation of an undamped free vibration of a general slender vertical column is as follows[10]:

$$\frac{\partial^2}{\partial y^2}\left(EI\,\frac{\partial^2 x}{\partial y^2}\right)+m\frac{\partial^2 x}{\partial t^2}+\frac{\partial}{\partial y}\left(N\,\frac{\partial x}{\partial y}\right)=0 \tag{1}$$

where *EI*, *m*, *N*, *x* and *y* are the flexural stiffness, the mass per unit length, the axial force, the transverse displacement of a column and the vertical coordinate, respectively. The axial force term is from the gravitational force and as follows:

$$N(y)=-mg(L-y) \tag{2}$$

in which *g* is the gravitational acceleration, and *L* is the column height. With a uniform cross section, equation (1) may lead to the equation for a mode shape function *X(y)*:

$$EI\,\frac{d^4 X}{dy^4}-mg\,\frac{d}{dy}\left((L-y)\,\frac{dX}{dy}\right)-m\omega^2 X\;=0 \tag{3}$$

where *X(y)* is the mode shape function, ω is the circular natural frequency. The gravity effect term exerts a negative stiffness which varies with the vertical coordinate, and so except for some special cases it is hard to obtain an analytical solution for mode shapes and natural frequencies. However, a numerical method can be used to obtain the mode shapes and the natural frequencies of bending vibration of slender columns with the gravitational force.

NUMERICAL ANAYSIS

A free bending vibration analysis of graphite columns is performed with the commercial structural analysis code, ABAQUS. Nine cases with different heights, from 4m to 20m, are considered. The natural frequencies of the columns without and with the gravity effect are compared, and the effect of the column height is also studied. To consider the gravity effect in the cases with the gravity effect, a two-step analysis procedure is adopted. The first step is a static analysis that calculates the shortening of the column by its weight or gravity force and produces a modified bending stiffness of the column by a negative stiffness coming from the gravity effect. The second is a free bending vibration analysis that provides the mode shapes and the natural frequencies. The young's modulus, density, area and second moment of the cross section of the models are 10GPa, 1750kg/m^3, 6.42E-2m^2, 4.91E-4m^4 respectively. The first step of the analysis provides the shortening of column height by the gravity force for the cases of different initial column heights, and they are listed on Table 1. As one can see from the table the shortenings of the column heights are negligibly small for all the cases considered.

TABLE 1. Shortening of the column heights by the gravitational force

	Initial column height (m)	Shortening by gravity (mm)	Shortening ratio (%)
Case 1	4	0.02	0.0005
Case 2	6	0.05	0.0008
Case 3	8	0.10	0.0013
Case 4	10	0.15	0.0015
Case 5	12	0.22	0.0018
Case 6	14	0.29	0.0021
Case 7	16	0.38	0.0024
Case 8	18	0.49	0.0027
Case 9	20	0.60	0.0030

The second step is for the free vibration analysis, and it gives the vibration mode shapes and the natural frequencies for the cases. Typical mode shapes are shown in Fig. 1. The simplest way of checking the gravity effect on a vibration mode shape is to study the correlation coefficients between the mode shape vectors with and without the gravity force term. However, in this paper, the effect is so weak and all of the correlation coefficients are nearly unity, and so the vector norms of the difference vectors between the normalized mode shape vectors are calculated, see Table 2. Comparing to the norms of the mode shape vectors which are unity, the norms of the difference vectors are quite small. Although the norm values reveal some trends, they have no meaningful digits. From this simple investigation one can obtain a result that the gravity term does not seriously alter the mode shapes.

Different from the above results, the natural frequency is a function of the column height and the gravity force. The frequency analysis results are summarized on Table 3 and Fig. 2. The natural frequencies of the graphite columns without the gravitational force term and of different heights are shown in Fig. 2(a). The frequencies increase with the mode number, and a taller column has a lower natural frequency in each mode. The natural frequencies in the cases with the gravity force are shifted downward from those of the cases without the gravity force. The percent reduction ratios are shown in Fig. 2(b). Although the shortening of the column heights by the gravity force are quite small and negligible, the natural frequency reductions are not. The natural frequency reduction ratios increase with an increase of the column height, and they decrease with an increase of the mode number. Especially the reduction is remarkable for the first natural frequency, and for the tall column cases it is more drastic. In the case of the column height of 10m, the frequency reduction by the gravity force is about 2.5%, but it is over 22% when the height is 20m.

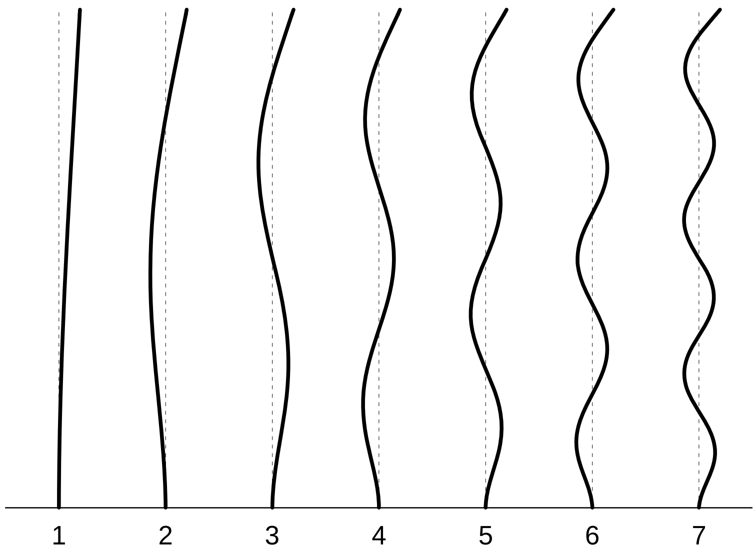

Fig. 1. Typical mode shapes of a graphite column

TABLE 2. Norms of the difference vector of the normalized mode shape vectors

	Difference Vector Norm								
Mode	1	2	3	4	5	6	7	8	9
Case 1	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003
Case 2	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008
Case 3	0.00019	0.00019	0.00019	0.00019	0.00019	0.00019	0.00019	0.00019	0.00019
Case 4	0.00037	0.00037	0.00037	0.00037	0.00037	0.00037	0.00037	0.00037	0.00037
Case 5	0.00065	0.00065	0.00065	0.00065	0.00065	0.00065	0.00065	0.00065	0.00065
Case 6	0.00103	0.00103	0.00103	0.00103	0.00103	0.00103	0.00103	0.00103	0.00103
Case 7	0.00155	0.00155	0.00155	0.00155	0.00155	0.00155	0.00155	0.00155	0.00155
Case 8	0.00223	0.00223	0.00223	0.00223	0.00223	0.00223	0.00223	0.00223	0.00223
Case 9	0.00309	0.00309	0.00309	0.00309	0.00309	0.00309	0.00309	0.00309	0.00309

CONCLUSIONS

The effects of the gravity on a free transverse vibration of graphite columns were studied. In all the cases considered the shortenings of the graphite column height by the gravity were below an order of a millimeter and thus are negligible. The variation in the column height does not alter the vibration mode shapes notably, but it does the natural frequencies. The natural frequency is a decay function of the column height and the gravity. The frequencies increase with the mode number, and a higher column has lower natural frequencies. The natural frequencies in the cases of with the gravity are shifted downward. The reduction ratios decrease with a mode number increase, and they increase with a column height increase. The frequency shifting is remarkable for the first natural frequency, and for the taller column cases it is more drastic. The downward shift is caused by the negative bending stiffness coming from the gravity effect, and it is over 22% when the graphite column is taller than 20m. A careful consideration as to whether the gravity effects shall be included or not is required before a seismic analysis and design when the columns of stacked graphite blocks composing a reactor core are taller than 10m.

TABLE 3. Natural frequency with and without the gravity effect

Mode		Natural Frequency (Hz)						
		1	2	3	4	5	6	7
Case 1	Without gravity	5.51	33.9	92.1	174	274	340	517
	With gravity	5.50	33.9	92.1	174	274	340	517
	Reduction ratio (%)	0.16	0.02	0.01	0.01	0.00	0.00	0.00
Case 2	Without gravity	2.45	15.2	42.1	80.9	131	190	258
	With gravity	2.44	15.2	42.1	80.9	131	190	258
	Reduction ratio (%)	0.54	0.08	0.03	0.01	0.02	0.01	0.00
Case 3	Without gravity	1.38	8.61	23.9	46.4	75.6	111	152
	With gravity	1.36	8.60	23.9	46.4	75.6	111	152
	Reduction ratio (%)	1.28	0.18	0.07	0.03	0.02	0.02	0.01
Case 4	Without gravity	0.88	5.52	15.4	29.9	49.0	72.5	99.9
	With gravity	0.86	5.50	15.4	29.9	49.0	72.4	99.9
	Reduction ratio (%)	2.52	0.35	0.13	0.07	0.04	0.03	0.02
Case 5	Without gravity	0.61	3.84	10.7	20.9	34.3	50.9	70.4
	With gravity	0.59	3.82	10.7	20.9	34.3	50.8	70.4
	Reduction ratio (%)	4.40	0.60	0.21	0.12	0.07	0.05	0.04
Case 6	Without gravity	0.45	2.82	7.88	15.4	25.3	37.6	52.2
	With gravity	0.42	2.80	7.86	15.4	25.3	37.6	52.2
	Reduction ratio (%)	7.10	0.96	0.35	0.19	0.12	0.08	0.06
Case 7	Without gravity	0.35	2.16	6.04	11.8	19.5	28.9	40.2
	With gravity	0.31	2.13	6.01	11.8	19.4	28.9	40.2
	Reduction ratio (%)	10.80	1.44	0.53	0.29	0.17	0.12	0.09
Case 8	Without gravity	0.27	1.71	4.78	9.34	15.4	22.9	31.9
	With gravity	0.23	1.67	4.74	9.30	15.4	22.9	31.9
	Reduction ratio (%)	15.81	2.06	0.75	0.40	0.25	0.17	0.12
Case 9	Without gravity	0.22	1.38	3.87	7.57	12.5	18.6	25.9
	With gravity	0.17	1.35	3.83	7.53	12.5	18.6	25.9
	Reduction ratio (%)	22.51	2.83	1.03	0.55	0.34	0.24	0.17

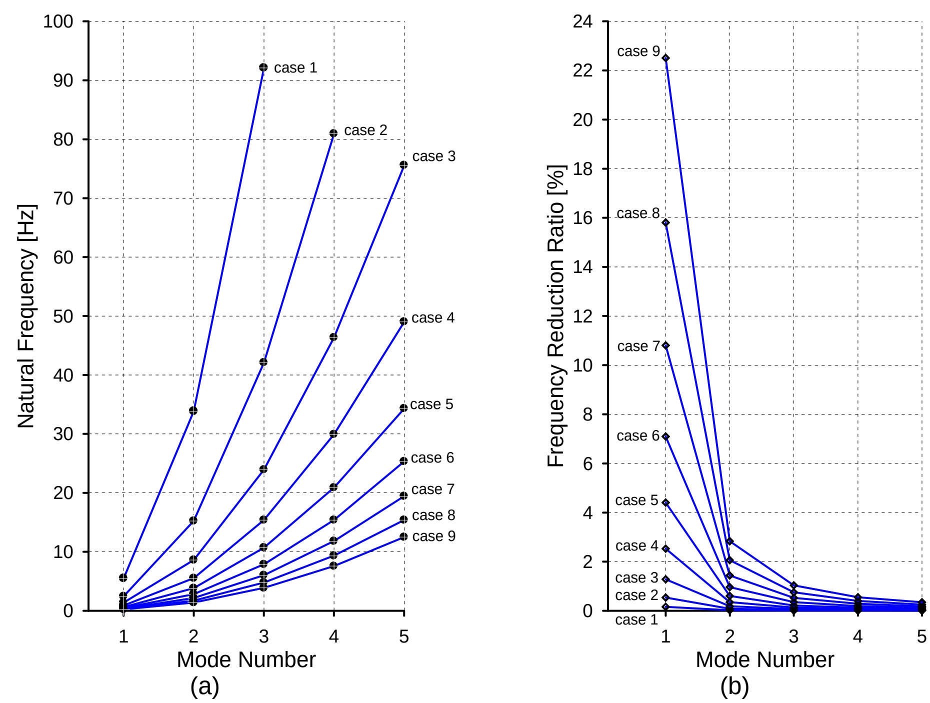

Fig. 2. The frequency analysis results; (a) natural frequencies of the graphite columns of different heights, (b) percent ratios of the natural frequency reduction by the gravitational force

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