TRANSIENT THERMAL STRESSES IN A TRANSVERSELY ISOTROPIC THICK PLATE WITH A CYLINDRICAL HOLE DUE TO SURFACE HEAT-GENERATIONS

Y. SUGANO, Y. TAKEUTI

Department of Mechanical Engineering, Faculty of Engineering, University of Osaka Prefecture, Mozu Umemachi, Sakai, Osaka 591, Japan

SUMMARY

As a result of the increased usage of anisotropic materials in nuclear reactor components operating at elevated temperature, the interest in anisotropic thermal stress problem of thick plate and cylinder has grown considerably in recent years. However, most of the published works in this problem has been discussed about the steady-state thermal stresses for the case of prescribed surface temperature by the use of the displacement potential functions typified by the work of Sharma (See: Sharma,B., Trans.ASME,E, 80(1958),86). The thermal stress problem in anisotropic body with heat-generations is of considerable technological importance, particularly to the design of structural elements in nuclear reactors.

In this paper, an exact solution is given for the transient thermal stresses in a transversely isotropic thick plate with a cylindrical hole due to surface heat-generations expressed as arbitrary functions of radial position and time on the plane surfaces. The purpose of this paper is to present the results of theoretical analysis which considers the effects of the thermal and mechanical anisotropies of the material on the temperature and thermal stress fields in the thick plate.

The temperature field is exactly determined by a combined use of the generalized finite Fourier transform and the unconventional Hankel transform based on the Weber-Orr expansion.

The associated thermal stress field is analysed by the use of a set of stress functions closely related to the Love's function for the axially symmetric problem of isotropic bodies. The thermal stress components are expressed in terms of the stress functions, and certain constants which satisfy quadratic equations involving elastic constants. The traction-free boundary conditions on the plane surfaces and the cylindrical hole are rigorously satisfied.

Numerical calculations are carried out for the various values of the ratio of the thermal conductivity coefficients, the ratio of the Young's moduli and the ratio of the linear thermal expansion coefficients in the axial and radial directions.

From the results obtained, it is concluded that the magnitude of the maximum thermal stress is highly sensitively to the anisotropy of the thermal conductivity coeffecient but rather insensitive to the anisotropies of the Young's modulus and the linear thermal expansion coefficient.

1. Introduction

As a result of the increased usage of anisotropic materials in nuclear reactor components operating at elevated temperature, the interest in anisotropic thermal stress problem has grown considerably in recent years. However, all the published works which deal with this problem, except for that of the steady-state thermal stresses in an infinite, transversely isotropic, circular cylinder with a spherical cavity by Atsumi and Itou [5], have been discussed about the steady-state thermal stresses in plates and cylinders with no hole by the use of the displacement potential functions typified by the work of Sharma [1-4].

Then, in our previous papers [6,7], the authors have dealt with the transient thermal stress problem in a finite, transversely isotropic, circular cylinder due to a general axisymmetric heating.

In this paper, an exact solution is given for the transient thermal stresses in a transversely isotropic, thick plate with a cylindrical hole due to surface heat-generations expressed as arbitrary functions of radial position and time on the plane surfaces.

2. Temperature field

Consider a transversely isotropic, thick plate of thickness l with a cylindrical hole of radius a, on the assumption that the plate, initially at the same uniform temperature T_0 as the surrounding medium, is suddenly subjected to axisymmetric surface heat-generations $Q_1(r,t)$ and $Q_2(r,t)$ on a portion of the lower and upper plane surfaces, respectively and linear heat transfer over the remainder, and that the cylindrical hole is thermally insulated.

Using cylindrical polar coordinates (r,θ,z) and taking the z-axis along the axis of symmetry of transversely isotropic material as shown in Fig.1, we may express the heat conduction equation and the boundary and initial conditions in the dimensionless forms as follows:

$$\bar{T},_{\rho\rho} + \rho^{-1}\bar{T},_{\rho} + k^2\bar{T},_{\zeta\zeta} = \bar{T},_{\tau} \tag{1}$$

$$\bar{T},_{\zeta} - m_1\bar{T} = -\bar{Q}_{10}f_1(\rho)w_1(\tau) \quad on\ \zeta=0 \quad (2) \qquad \bar{T},_{\zeta} + m_2\bar{T} = \bar{Q}_{20}f_2(\rho)w_2(\tau) \quad on\ \zeta=\bar{l} \quad (3)$$

$$\bar{T},_{\rho} = 0 \quad on\ \rho=1 \quad (4) \qquad \bar{T} = 0 \quad at\ \tau=0 \quad (5)$$

The regularly condition is

$$\bar{T},\ \bar{T},_{\rho} \longrightarrow 0 \qquad as \quad \rho \to \infty \tag{6}$$

where $k^2=\lambda_z/\lambda_r$ is the ratio of thermal conductivity coefficients in the axial and radial directions, respectively, and $\bar{T},_{\tau}$ denotes $\partial\bar{T}/\partial\tau$. In addition, the following dimensionless quantities and heat-generation functions were introduced

$$\left.\begin{aligned} &\rho=r/a,\quad \zeta=z/a,\quad \bar{l}=l/a,\quad \tau=\kappa t/a^2,\quad \bar{T}(\rho,\zeta,\tau)=\{T(r,z,t)-T_0\}/(T_0\bar{Q}_0) \\ &\bar{Q}_0=Q_0a/(\lambda_zT_0),\quad m_j=h_ja/\lambda_z,\quad \bar{Q}_{j0}=Q_{j0}/Q_0\ (j=1,2) \end{aligned}\right\} \tag{7}$$

$$Q_1(r,t)=Q_{10}f_1(\rho)w_1(\tau), \qquad Q_2(r,t)=Q_{20}f_2(\rho)w_2(\tau) \tag{8}$$

where κ is the thermal diffusivity, m_j are the Biot number and Q_0 is a characteristic heat-generation per unit time and per unit area which is taken to be Q_{10} or Q_{20}.

By the boundary conditions (2)-(5) and the regularly condition (6), the generalized finite Fourier transform

$$\hat{T}(\rho,p_i,\tau)=\int_0^{\bar{l}} \Psi(\zeta,p_i)\bar{T}(\rho,\zeta,\tau)d\zeta \tag{9}$$

and the Hankel transform based on the Weber-Orr expansion

$$T^*(\alpha,p_i,\tau)=\int_1^{\infty} \rho U_0(\rho,\alpha)\hat{T}(\rho,p_i,\tau)/D(\alpha)d\rho \tag{10}$$

when applied to eqs.(1) and (5) yield

$$T^*,_{\tau} + G_k(\alpha,p_i)T^* = k^2F^*(\alpha,p_i,\tau) \quad (11) \qquad T^*=0 \quad at\ \tau=0 \quad (12)$$

where

$$\Psi(\zeta,p_i)= p_i\cos p_i\zeta + m_1\sin p_i\zeta, \quad U_n(\rho,\alpha)=J_n(\alpha\rho)Y_1(\alpha)-J_1(\alpha)Y_n(\alpha\rho)$$
$$D(\alpha)=J_1^2(\alpha)+Y_1^2(\alpha) \quad G_k(\alpha,p_i)=\alpha^2+ k^2p_i^2 \quad (13)$$

$$F^*(\alpha,p_i,\tau)=\int_1^\infty \rho U_0(\rho,\alpha)F(\alpha,p_i,\tau)/D(\alpha)d\rho$$
$$F(\rho,p_i,\tau)= \sqrt{(p_i^2+m_1^2)/(p_i^2+m_2^2)}.p_i\bar{Q}_{20}f_2(\rho)w_2(\tau)+p_i\bar{Q}_{10}f_1(\rho)w_1(\tau) \quad (14)$$

$J_n(\ )$ and $Y_n(\ )$ are the Bessel functions of first and second kinds of order n, respectively, and p_i are the positive, real roots of the following equations,

$$\tan p_i\bar{l} = p_i(m_1+ m_2)/(p_i^2- m_1m_2) \quad (15)$$

The solution of eq.(11) satisfying eq.(12) may be taken as

$$T^*= k^2\Theta(\tau) \quad (16)$$

where

$$\Theta(\tau)= e^{-G_k(\alpha,p_i)}\int_0^\tau F^*(\alpha,p_i,\tau')e^{G_k(\alpha,p_i)\tau'}d\tau' \quad (17)$$

Inversions of the Hankel and generalized finite Fourier transforms using

$$\hat{T}(\rho,p_i,\tau)= \int_0^\infty \alpha U_0(\rho,\alpha)T^*(\alpha,p_i,\tau)d\alpha \quad (18)$$

and

$$\bar{T}(\rho,\zeta,\tau)= \sum_{i=1}^{\infty} \xi_i^2\Psi(\zeta,p_i)\hat{T}(\rho,p_i,\tau) \quad (19)$$

and eq.(16) yield the final temperature function as

$$\bar{T}(\rho,\zeta,\tau)= k^2\sum_{i=1}^{\infty} \xi_i^2\Psi(\zeta,p_i)\int_0^\infty \alpha U_0(\rho,\alpha)\Theta(\tau)d\alpha \quad (20)$$

where

$$\xi_i^2=2C_{i2}/\{C_{i1}(\bar{l}C_{i2}+m_2)+ m_1C_{i2}\}, \quad C_{i1}=p_i^2+ m_1^2, \quad C_{i2}=p_i^2+ m_2^2. \quad (21)$$

3. Thermal stresses

The relations between thermal stresses and strains induced in the transversely isotropic thick plate with a hole by the temperature distribution (20) are given as

$$\varepsilon_{rr}=s_{11}\sigma_{rr}+s_{12}\sigma_{\theta\theta}+s_{13}\sigma_{zz}+\alpha_rT, \quad \varepsilon_{\theta\theta}=s_{12}\sigma_{rr}+s_{11}\sigma_{\theta\theta}+s_{13}\sigma_{zz}+\alpha_rT$$
$$\varepsilon_{zz}=s_{13}\sigma_{rr}+s_{13}\sigma_{\theta\theta}+s_{33}\sigma_{zz}+\alpha_zT, \quad \varepsilon_{rz}=s_{44}\sigma_{rz} \quad (22)$$

where s_{ij} are the elastic constants, T is the temperature rise from the reference temperature, and α_r and α_z are the linear thermal expansion coefficients in radial and axial directions, respectively.

We may relate the dimensionless thermal stresses to a set of dimensionless stress functions, $\bar{\Omega}$ and $\bar{\Phi}$ [8] satisfying

$$\bar{\Omega},_{\rho\rho}+\rho^{-1}\bar{\Omega},_{\rho}+ x_1\bar{\Omega},_{\zeta\zeta}= \eta\bar{T} \quad (23)$$

$$\bar{\Phi},_{\rho\rho}+\rho^{-1}\bar{\Phi},_{\rho}+ x_2\bar{\Phi},_{\zeta\zeta}= \varepsilon^{-1}(\bar{\Omega},_{\zeta\zeta}- \gamma\bar{T}) \quad (24)$$

where

$$\varepsilon=(s_{11}s_{33}-s_{13}^2)/\{s_{11}(s_{11}-s_{12})\}, \quad \gamma=(\alpha_zs_{11}-\alpha_rs_{13})/\{\alpha_r(s_{11}-s_{12})\}, \quad \eta=\gamma x_1-1 \quad (25)$$

and x_1 and x_2 are the roots of

$$(s_{13}^2-s_{11}s_{33})x^2+2\{s_{13}(s_{11}-s_{12})+s_{11}s_{44}\}x-(s_{11}^2-s_{12}^2)= 0. \quad (26)$$

The relations between the dimensionless thermal stress components and the dimensionless stress functions are given by the following equations

$$\bar{\sigma}_{rr}=\bar{\Phi},_{\zeta\zeta}+\rho^{-1}(y_1\bar{\Phi},_{\rho}+\bar{\Omega},_{\rho}), \quad \bar{\sigma}_{\theta\theta}=-a_1(\bar{\Phi},_{\rho\rho}+\rho^{-1}\bar{\Phi},_{\rho})-a_2\bar{\Phi},_{\zeta\zeta}-\rho^{-1}(y_1\bar{\Phi},_{\rho}+\bar{\Omega},_{\rho})-\bar{T}$$
$$\bar{\sigma}_{zz}=\bar{\Phi},_{\rho\rho}+\rho^{-1}\bar{\Phi},_{\rho} \quad \bar{\sigma}_{rz}=\bar{\Phi},_{\rho\zeta} \quad (27)$$

where

$$a_1=s_{13}/s_{11}, \qquad a_2=s_{12}/s_{11} \tag{28}$$

and y_1 and y_2 are the roots of

$$s_{11}(s_{12}-s_{11})y^2+s_{11}s_{44}y+\{s_{13}(2s_{13}+s_{44})-s_{33}(s_{11}+s_{12})\}=0 \tag{29}$$

Then, the dimensionless thermal stresses $\bar{\sigma}_{ij}$ $(i,j=r,\theta,z)$ are related to the actual components by

$$\sigma_{ij}=\alpha_r T_0 \bar{Q}_0 \bar{\sigma}_{ij}/s_{11} \tag{30}$$

Since the lower and upper plane and cylindrical surfaces of the thick plate are assumed to be traction-free, the boundary conditions on the stress field become

$$\bar{\sigma}_{rr}=0 \quad on\ \rho=1 \quad (31) \qquad \bar{\sigma}_{rz}=0 \quad on\ \rho=1 \quad (32) \qquad \bar{\sigma}_{zz}=0 \quad on\ \zeta=0 \quad (33)$$

$$\bar{\sigma}_{rz}=0 \quad on\ \zeta=0 \quad (34) \qquad \bar{\sigma}_{zz}=0 \quad on\ \zeta=\bar{l} \quad (35) \qquad \bar{\sigma}_{rz}=0 \quad on\ \zeta=\bar{l} \quad (36)$$

The general solutions of eqs.(23) and (24) are taken in the following forms, respectively

$$\bar{\Omega}=\sum_{i=1}^{\infty}\frac{A_{1i}\Psi(\zeta,p_i)K_0(p_i\sqrt{x_1}\rho)}{p_i^2K_0(p_i\sqrt{x_1})}+\int_0^{\infty}\frac{U_0(\alpha,\zeta)N_1(\alpha,\zeta)}{\alpha^2}d\alpha-\eta k^2\sum_{i=1}^{\infty}\xi_i^2\Psi(\zeta,p_i)\int_0^{\infty}\frac{\alpha U_0(\rho,\alpha)\theta(\tau)}{D(\alpha)G_1(\alpha,p_i)}d\alpha \tag{37}$$

$$\bar{\Phi}=\sum_{i=1}^{\infty}\frac{\Psi(\zeta,p_i)}{p_i^2}\left[\frac{A_{2i}K_0(p_i\sqrt{x_2}\rho)}{K_0(p_i\sqrt{x_2})}-\frac{A_{1i}K_0(p_i\sqrt{x_1}\rho)}{\varepsilon x_{12}K_0(p_i\sqrt{x_1})}\right]+\int_0^{\infty}\frac{U_0(\rho,\alpha)[N_2(\alpha,\zeta)-N_1(\alpha,\zeta)/\varepsilon x_{12}]}{\alpha^2}d\alpha$$

$$-\varepsilon^{-1}\sum_{i=1}^{\infty}\Psi(\zeta,p_i)\int_0^{\infty}\frac{\alpha U_0(\rho,\alpha)H(\alpha,p_i,\tau)}{D(\alpha)}d\alpha \tag{38}$$

where

$$\left.\begin{aligned}&G_1(\alpha,p_i)=\alpha^2+p_i^2x_1,\quad G_2(\alpha,p_i)=\alpha^2+p_i^2x_2,\quad \theta(\tau)=D(\alpha)\Theta(\tau),\quad x_{12}=x_1-x_2\\&H(\alpha,p_i,\tau)=k^2\xi_i^2\{\eta p_i^2-\gamma G_1(\alpha,p_i)\}\theta(\tau)/\{G_1(\alpha,p_i)G_2(\alpha,p_i)\}\end{aligned}\right\} \tag{39}$$

$$\left.\begin{aligned}N_1(\alpha,\zeta)&=\frac{B_1(\alpha)\cosh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_1}]}{\sinh(\alpha\bar{l}/2\sqrt{x_1})}+\frac{C_1(\alpha)\sinh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_1}]}{\cosh(\alpha\bar{l}/2\sqrt{x_1})}\\N_2(\alpha,\zeta)&=\frac{B_2(\alpha)\cosh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_2}]}{\sinh(\alpha\bar{l}/2\sqrt{x_2})}+\frac{C_2(\alpha)\sinh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_2}]}{\cosh(\alpha\bar{l}/2\sqrt{x_2})}\end{aligned}\right\} \tag{40}$$

where A_{ji}, $B_j(\alpha)$ and $C_j(\alpha)$ $(j=1,2)$ are unknowns to be determined by eqs.(31)-(36).

Substituting eqs.(37) and (38) into eq.(27), we obtain the following expressions for the stress components as

$$\bar{\sigma}_{rr}=-\sum_{i=1}^{\infty}\Psi(\zeta,p_i)\{A_{2i}[\frac{y_1\sqrt{x_2}K_1(p_i\sqrt{x_2}\rho)}{p_i\rho K_0(p_i\sqrt{x_2})}+\frac{K_0(p_i\sqrt{x_2}\rho)}{K_0(p_i\sqrt{x_2})}]+A_{1i}[(1-\frac{y_1}{\varepsilon x_{12}})\frac{\sqrt{x_1}K_1(p_i\sqrt{x_1}\rho)}{p_i\rho K_0(p_i\sqrt{x_1})}$$

$$-\frac{K_0(p_i\sqrt{x_1}\rho)}{\varepsilon x_{12}K_0(p_i\sqrt{x_1})}]\}+\int_0^{\infty}\{[\frac{U_0(\rho,\alpha)}{x_2}-\frac{y_1U_1(\rho,\alpha)}{\alpha\rho}]N_2(\alpha,\zeta)-[\frac{U_0(\rho,\alpha)}{\varepsilon x_{12}x_1}+(1-\frac{y_1}{\varepsilon x_{12}})\frac{U_1(\rho,\alpha)}{\alpha\rho}]N_1(\alpha,\zeta)\}d\alpha$$

$$-\varepsilon^{-1}\sum_{i=1}^{\infty}\Psi(\zeta,p_i)\int_0^{\infty}[\![p_i^2U_0(\rho,\alpha)+\alpha\rho^{-1}U_1(\rho,\alpha)\{y_1+\varepsilon\eta G_2(\alpha,p_i)/[\eta p_i^2-\gamma G_1(\alpha,p_i)]\}]\!]\,\alpha$$

$$\cdot H(\alpha,p_i,\tau)/D(\alpha)d\alpha \tag{41}$$

$$\bar{\sigma}_{\theta\theta}=-\sum_{i=1}^{\infty}\Psi(\zeta,p_i)\{A_{2i}[(a_1x_2-a_2)\frac{K_0(p_i\sqrt{x_2}\rho)}{K_0(p_i\sqrt{x_2})}-\frac{y_1\sqrt{x_2}K_1(p_i\sqrt{x_2}\rho)}{p_i\rho K_0(p_i\sqrt{x_2})}]-A_{1i}[\frac{(a_1x_1-a_2)K_0(p_i\sqrt{x_1}\rho)}{\varepsilon x_{12}K_0(p_i\sqrt{x_1})}$$

$$+(1-\frac{y_1}{\varepsilon x_{12}})\frac{\sqrt{x_1}K_1(p_i\sqrt{x_1}\rho)}{p_i\rho K_0(p_i\sqrt{x_1})}]\}+\int_0^{\infty}\{[(a_1-\frac{a_2}{x_2})U_0(\rho,\alpha)+\frac{y_1U_1(\rho,\alpha)}{\alpha\rho}]N_2(\alpha,\zeta)$$

$$-[(a_1-\frac{a_2}{x_1})\frac{U_0(\rho,\alpha)}{\varepsilon x_{12}} -(1-\frac{y_1}{\varepsilon x_{12}})\frac{U_1(\rho,\alpha)}{\alpha\rho}]N_1(\alpha,\zeta)\}d\alpha -\varepsilon^{-1}\sum_{i=1}^{\infty}\Psi(\zeta,p_i)\int_0^\infty [\![\{(a_1\alpha^2+a_2p_i^2)+\varepsilon G_1(\alpha,p_i)$$

$$\cdot G_2(\alpha,p_i)/[\eta p_i^2-\gamma G_1(\alpha,p_i)]\}U_0(\rho,\alpha)+\alpha\rho^{-1}U_1(\rho,\alpha)\{y_1+\varepsilon\eta G_2(\alpha,p_i)/[\eta p_i^2-\gamma G_1(\alpha,p_i)]\}]\!]$$

$$\cdot\alpha H(\alpha,p_i,\tau)/D(\alpha)d\alpha \tag{42}$$

$$\bar{\sigma}_{zz}=\sum_{i=1}^{\infty}\Psi(\zeta,p_i)[\frac{A_{2i}x_2K_0(p_i\sqrt{x_2}\rho)}{K_0(p_i\sqrt{x_2})}-\frac{A_{1i}x_1K_0(p_i\sqrt{x_1}\rho)}{\varepsilon x_{12}K_0(p_i\sqrt{x_1})}]$$

$$-\int_0^\infty U_0(\rho,\alpha)[N_2(\alpha,\zeta)-N_1(\alpha,\zeta)/\varepsilon x_{12}]d\alpha -\varepsilon^{-1}\sum_{i=1}^{\infty}\Psi(\zeta,p_i)\int_0^\infty \alpha^3U_0(\rho,\alpha)H(\alpha,p_i,\tau)/D(\alpha)d\alpha \tag{43}$$

$$\bar{\sigma}_{rz}=\sum_{i=1}^{\infty}\chi(\zeta,p_i)[\frac{A_{2i}\sqrt{x_2}K_1(p_i\sqrt{x_2}\rho)}{K_0(p_i\sqrt{x_2})}-\frac{A_{1i}\sqrt{x_1}K_1(p_i\sqrt{x_1}\rho)}{\varepsilon x_{12}K_0(p_i\sqrt{x_1})}]$$

$$+\int_0^\infty U_1(\rho,\alpha)[N_4(\alpha,\zeta)/\sqrt{x_2}-N_3(\alpha,\zeta)/\varepsilon x_{12}\sqrt{x_1}]d\alpha-\varepsilon^{-1}\sum_{i=1}^{\infty}p_i\chi(\zeta,p_i)\int_0^\infty \alpha^2U_1(\rho,\alpha)H(\alpha,p_i,\tau)/D(\alpha)d\alpha \tag{44}$$

where

$$\chi(\zeta,p_i)=m_1\cos p_i\zeta-p_i\sin p_i\zeta \tag{45}$$

By the substitution of eq.(41) into eq.(31) and the application of the generalized finite Fourier transform defined eq.(9) to the resulting equation, it is found that zero normal stress on the hole is assured if

$$A_{2i}[y_1\sqrt{x_2}K_{2i}/p_i+1]+A_{1i}[(1-y_1/\varepsilon x_{12})\sqrt{x_1}K_{1i}/p_i-(\varepsilon x_{12})^{-1}]+(2\xi_i^2/\pi)\int_0^\infty [B_2(\alpha)R_2(\alpha,p_i)$$

$$+C_2(\alpha)S_2(\alpha,p_i)-B_1(\alpha)R_1(\alpha,p_i)/\varepsilon x_{12}-C_1(\alpha)S_1(\alpha,p_i)/\varepsilon x_{12}]/\alpha d\alpha=(2/\pi\varepsilon)\int_0^\infty p_i^2H(\alpha,p_i,\tau)/D(\alpha)d\alpha$$

$$(i=1,2,\dots) \tag{46}$$

where

$$K_{1i}=K_1(p_i\sqrt{x_1})/K_0(p_i\sqrt{x_1}),\qquad K_{2i}=K_1(p_i\sqrt{x_2})/K_0(p_i\sqrt{x_2}) \tag{47}$$

$$\left.\begin{aligned}R_j(\alpha,p_i)&=\{\alpha[p_i+\Psi(\bar{l},p_i)]/\sqrt{x_j}-p_i\coth(\alpha\bar{l}/2\sqrt{x_j})[\chi(\bar{l},p_i)-m_1]\}/G_j(\alpha,p_i)\\ S_j(\alpha,p_i)&=\{\alpha[\Psi(\bar{l},p_i)-p_i]/\sqrt{x_j}-p_i\tanh(\alpha\bar{l}/2\sqrt{x_j})[\chi(\bar{l},p_i)+m_1]\}/G_j(\alpha,p_i)\end{aligned}\quad(j=1,2)\right\} \tag{48}$$

By the substitution of eq.(44) into eq.(32), it is immediately found that zero shear stress on the hole is satisfied if

$$A_{2i}\sqrt{x_2}K_{2i}-A_{1i}\sqrt{x_1}K_{1i}/\varepsilon x_{12}=0 \qquad (i=1,2,\dots) \tag{49}$$

By the substitution of eq.(43) into eq.(33) and the application of the Hankel transform defined gy eq.(10) to the resulting equation, it is found that zero normal stress on the lower plane surface is assured if

$$2\sum_{i=1}^{\infty}p_i^2[A_{2i}x_2\sqrt{x_2}K_{2i}/G_2(\alpha,p_i)-A_{1i}x_1\sqrt{x_1}K_{1i}/\{\varepsilon x_{12}G_1(\alpha,p_i)\}]+D(\alpha)[B_2(\alpha)\coth(\alpha\bar{l}/2\sqrt{x_2})$$

$$-B_1(\alpha)\coth(\alpha\bar{l}/2\sqrt{x_1})/(\varepsilon x_{12})-C_2(\alpha)\tanh(\alpha\bar{l}/2\sqrt{x_2})+C_1(\alpha)\tanh(\alpha\bar{l}/2\sqrt{x_1})/(\varepsilon x_{12})]$$

$$=(\pi/\varepsilon)\sum_{i=1}^{\infty}p_i\alpha^3H(\alpha,p_i,\tau) \tag{50}$$

By the substitution of eq.(44) into eq.(34) and the use of the following Hankel transform

$$F(\alpha)=\int_1^\infty \rho U_1(\rho,\alpha)f(\rho)/D(\alpha)d\rho, \tag{51}$$

it is found that zero shear stress on the lower plane surface is assured if

$$2m_1\sum_{i=1}^{\infty}[A_{2i}\sqrt{x_2}K_{2i}/G_2(\alpha,p_i)-A_{1i}\sqrt{x_1}K_{1i}/\{\varepsilon x_{12}G_1(\alpha,p_i)\}]+\pi D(\alpha)[B_2(\alpha)/\sqrt{x_2}-B_1(\alpha)/(\varepsilon x_{12}\sqrt{x_1})$$

$$-C_2(\alpha)/\sqrt{x_2}+C_1(\alpha)/(\varepsilon x_{12}\sqrt{x_1})]/\alpha=-m_1(\pi/\varepsilon)\sum_{i=1}^{\infty}p_i\alpha H(\alpha,p_i,\tau) \tag{52}$$

Similarly, zero normal stress and zero shear stress on the upper plane surface are assured if both the following equations are satisfied

$$2\sum_{i=1}^{\infty} p_i\Psi(\bar{l},p_i)[A_{2i}x_2\sqrt{x_2}K_{2i}/G_2(\alpha,p_i)-A_{1i}x_1\sqrt{x_1}K_{1i}/\{\varepsilon x_{12}G_1(\alpha,p_i)\}]+\pi D(\alpha)[B_2(\alpha)\coth(\alpha\bar{l}/2\sqrt{x_2})$$
$$-B_1(\alpha)\coth(\alpha\bar{l}/2\sqrt{x_1})/(\varepsilon x_{12})+C_2(\alpha)\tanh(\alpha\bar{l}/2\sqrt{x_2})-C_1(\alpha)\tanh(\alpha\bar{l}/2\sqrt{x_1})/(\varepsilon x_{12})]$$
$$=(\pi/\varepsilon)\sum_{i=1}^{\infty}\Psi(\bar{l},p_i)\alpha^3H(\alpha,p_i,\tau) \tag{53}$$

$$2\sum_{i=1}^{\infty}\chi(\bar{l},p_i)[A_{2i}\sqrt{x_2}K_{2i}/G_2(\alpha,p_i)-A_{1i}\sqrt{x_1}K_{1i}/(\varepsilon x_{12}G_1(\alpha,p_i))]-\pi D(\alpha)[B_2(\alpha)/\sqrt{x_2}-B_1(\alpha)/(\varepsilon x_{12}\sqrt{x_1})$$
$$+C_2(\alpha)/\sqrt{x_2}-C_1(\alpha)/(\varepsilon x_{12}\sqrt{x_1})]/\alpha=-(\pi/\varepsilon)\sum_{i=1}^{\infty}p_i\chi(\bar{l},p_i)\alpha H(\alpha,p_i,\tau) \tag{54}$$

Solving eqs.(50) and (52)-(54) for the unknowns $B_j(\alpha), C_j(\alpha)$ $(j=1,2)$ and eliminating A_{2i} in the results by using eq.(49), we can express these unknowns simply in terms of A_{1i} as

$$\left.\begin{aligned}
&B_1(\alpha)/\alpha=\{2\sum_{i=1}^{\infty}A_{1i}\sqrt{x_1}K_{1i}[T_2(\alpha,p_i)-T_1(\alpha,p_i)]-\pi x_{12}\sum_{i=1}^{\infty}V_2(\alpha,p_i)\alpha H(\alpha,p_i,\tau)\}/[2\pi D(\alpha)L(\alpha)]\\
&B_2(\alpha)/\alpha=\{2\sum_{i=1}^{\infty}A_{1i}\sqrt{x_1}K_{1i}[X_2(\alpha,p_i)-X_1(\alpha,p_i)]/(\varepsilon x_{12})-(\pi/\varepsilon)\sum_{i=1}^{\infty}V_1(\alpha,p_i)\alpha H(\alpha,p_i,\tau)\}\\
&\qquad/[2\pi D(\alpha)L(\alpha)]\\
&C_1(\alpha)/\alpha=\{2\sum_{i=1}^{\infty}A_{1i}\sqrt{x_1}K_{1i}[W_2(\alpha,p_i)-W_1(\alpha,p_i)]-\pi x_{12}\sum_{i=1}^{\infty}Z_2(\alpha,p_i)\alpha H(\alpha,p_i,\tau)\}/[2\pi D(\alpha)M(\alpha)]\\
&C_2(\alpha)/\alpha=\{2\sum_{i=1}^{\infty}A_{1i}\sqrt{x_1}K_{1i}[Y_2(\alpha,p_i)-Y_1(\alpha,p_i)]/(\varepsilon x_{12})-(\pi/\varepsilon)\sum_{i=1}^{\infty}Z_1(\alpha,p_i)\alpha H(\alpha,p_i,\tau)\}\\
&\qquad/[2\pi D(\alpha)M(\alpha)]
\end{aligned}\right\} \tag{55}$$

where

$$\left.\begin{aligned}
&T_j(\alpha,p_i)=\{p_ix_j[p_i+\Psi(\bar{l},p_i)]/\alpha\sqrt{x_2}-\coth(\alpha\bar{l}/2\sqrt{x_2})[m_1-\chi(\bar{l},p_i)]\}/G_j(\alpha,p_i)\\
&X_j(\alpha,p_i)=\{p_ix_j[p_i+\Psi(\bar{l},p_i)]/\alpha\sqrt{x_1}-\coth(\alpha\bar{l}/2\sqrt{x_1})[m_1-\chi(\bar{l},p_i)]\}/G_j(\alpha,p_i)\\
&W_j(\alpha,p_i)=\{p_ix_j[\Psi(\bar{l},p_i)-p_i]/\alpha\sqrt{x_2}+\tanh(\alpha\bar{l}/2\sqrt{x_2})[m_1+\chi(\bar{l},p_i)]\}/G_j(\alpha,p_i)\\
&Y_j(\alpha,p_i)=\{p_ix_j[\Psi(\bar{l},p_i)-p_i]/\alpha\sqrt{x_1}+\tanh(\alpha\bar{l}/2\sqrt{x_1})[m_1+\chi(\bar{l},p_i)]\}/G_j(\alpha,p_i)\\
&V_j(\alpha,p_i)=\alpha[p_i+\Psi(\bar{l},p_i)]/\sqrt{x_j}+\coth(\alpha\bar{l}/2\sqrt{x_j})p_i[m_1-\chi(\bar{l},p_i)]\\
&Z_j(\alpha,p_i)=\alpha[\Psi(\bar{l},p_i)-p_i]/\sqrt{x_j}-\tanh(\alpha\bar{l}/2\sqrt{x_j})p_i[m_1+\chi(\bar{l},p_i)]\\
&L(\alpha)=\coth(\alpha\bar{l}/2\sqrt{x_1})/\sqrt{x_2}-\coth(\alpha\bar{l}/2\sqrt{x_2})/\sqrt{x_1},\ M(\alpha)=\tanh(\alpha\bar{l}/2\sqrt{x_1})/\sqrt{x_2}-\tanh(\alpha\bar{l}/2\sqrt{x_2})/\sqrt{x_1}
\end{aligned}\right\} \tag{56}$$

The substitution of eqs.(49) and (55) into eq.(46) yields a system of simultaneous equations to be solved for the unknowns, A_{1i} as follows:

$$A_{1i}\sqrt{x_1}K_{1i}[\varepsilon x_{12}/p_i+(\sqrt{x_2}K_{2i})^{-1}-(\sqrt{x_1}K_{1i})^{-1}]/\varepsilon x_{12}+(2\xi_i^2/\pi^2)\sum_{n=1}^{\infty}A_{1n}\sqrt{x_1}K_{1n}/(\varepsilon x_{12})\int_0^{\infty}\{R_2(\alpha,p_i)$$
$$\cdot[X_2(\alpha,p_n)-X_1(\alpha,p_n)]/L(\alpha)+S_2(\alpha,p_i)[Y_2(\alpha,p_n)-Y_1(\alpha,p_n)]/M(\alpha)-R_1(\alpha,p_i)[T_2(\alpha,p_n)-T_1(\alpha,p_n)]$$
$$/L(\alpha)-S_1(\alpha,p_i)[W_2(\alpha,p_n)-W_1(\alpha,p_n)]/M(\alpha)\}/D(\alpha)d\alpha=(\xi_i^2/\pi\varepsilon)\int_0^{\infty}\{\sum_{n=1}^{\infty}\alpha H(\alpha,p_n,\tau)[R_2(\alpha,p_i)V_1(\alpha,p_n)$$
$$/L(\alpha)+S_2(\alpha,p_i)Z_1(\alpha,p_n)/M(\alpha)-R_1(\alpha,p_i)V_2(\alpha,p_n)/L(\alpha)-S_1(\alpha,p_i)Z_2(\alpha,p_n)/M(\alpha)]-2p_i^2H(\alpha,p_i,\tau)/\xi_i^2\}$$
$$/D(\alpha)d\alpha \qquad (i=1,2,\dots) \tag{57}$$

Once the unknowns, A_{1i} at each prescribed time are determined explicitly, we can easily obtain the thermal stress components from the following equations which are derived by the substitution of eqs.(49) and (55) into eqs.(41)-(44).

$$\bar{\sigma}_{rr}=-\sum_{i=1}^{\infty}\Psi(\zeta,p_i)\frac{A_{1i}\sqrt{x_1}K_{1i}}{\varepsilon x_{12}}\Big[\frac{y_1K_1(p_i\sqrt{x_2}\rho)}{p_i\rho K_1(p_i\sqrt{x_2})}+\frac{K_0(p_i\sqrt{x_2}\rho)}{\sqrt{x_2}K_1(p_i\sqrt{x_2})}+\frac{(\varepsilon x_{12}-y_1)K_1(p_i\sqrt{x_1}\rho)}{p_i\rho K_1(p_i\sqrt{x_1})}-\frac{K_0(p_i\sqrt{x_1}\rho)}{\sqrt{x_1}K_1(p_i\sqrt{x_1})}\Big]$$
$$+\frac{1}{\pi}\int_0^{\infty}\sum_{i=1}^{\infty}\frac{A_{1i}\sqrt{x_1}K_{1i}}{\varepsilon x_{12}D(\alpha)}\{[\frac{\alpha U_0(\rho,\alpha)}{x_2}-\frac{y_1U_1(\rho,\alpha)}{\rho}]I_1(\alpha,p_i)-[\frac{\alpha U_0(\rho,\alpha)}{x_1}+\frac{(\varepsilon x_{12}-y_1)U_1(\rho,\alpha)}{\rho}]$$
$$\cdot I_2(\alpha,p_i)\}d\alpha-(2\varepsilon)^{-1}\int_0^{\infty}\sum_{i=1}^{\infty}\alpha H(\alpha,p_i,\tau)[[\alpha U_0(\rho,\alpha)/x_2-y_1U_1(\rho,\alpha)/\rho]I_3(\alpha,p_i)-[\alpha U_0(\rho,\alpha)/x_1$$

$$+(\varepsilon x_{12}-y_1)U_1(\rho,\alpha)/\rho]I_4(\alpha,p_i)-2\Psi(\zeta,p_i)\{[p_i^2U_0(\rho,\alpha)+\alpha y_1U_1(\rho,\alpha)/\rho]+\varepsilon\eta\alpha G_2(\alpha,p_i)U_1(\rho,\alpha)/[\eta p_i^2-\gamma G_1(\alpha,p_i)]\}]]/D(\alpha)d\alpha \tag{58}$$

$$\bar{\sigma}_{\theta\theta}=-\sum_{i=1}^{\infty}\Psi(\zeta,p_i)\frac{A_{1i}\sqrt{x_1}K_{1i}}{\varepsilon x_{12}}\Big[\frac{(a_1x_2-a_2)K_0(p_i\sqrt{x_2}\rho)}{\sqrt{x_2}K_1(p_i\sqrt{x_2})}-\frac{y_1K_1(p_i\sqrt{x_2}\rho)}{p_i\rho K_1(p_i\sqrt{x_1})}-\frac{(a_1x_1-a_2)K_0(p_i\sqrt{x_1}\rho)}{\sqrt{x_1}K_1(p_i\sqrt{x_1})}$$
$$-\frac{(\varepsilon x_{12}-y_1)K_1(p_i\sqrt{x_1}\rho)}{p_i\rho K_1(p_i\sqrt{x_1})}\Big]+\frac{1}{\pi}\int_0^{\infty}\sum_{i=1}^{\infty}\frac{A_{1i}\sqrt{x_1}K_{1i}}{\varepsilon x_{12}D(\alpha)}\{[(a_1-a_2/x_2)\alpha U_0(\rho,\alpha)+\frac{y_1U_1(\rho,\alpha)}{\rho}]I_1(\alpha,p_i)$$
$$-[(a_1-a_2/x_1)\alpha U_0(\rho,\alpha)-\frac{(\varepsilon x_{12}-y_1)U_1(\rho,\alpha)}{\rho}]I_2(\alpha,p_i)\}d\alpha-(2\varepsilon)^{-1}\int_0^{\infty}\sum_{i=1}^{\infty}\alpha H(\alpha,p_i,\tau)[[(a_1-a_2/x_2)$$
$$\cdot\alpha U_0(\rho,\alpha)+y_1U_1(\rho,\alpha)/\rho]I_3(\alpha,p_i)-[(a_1-a_2/x_1)\alpha U_0(\rho,\alpha)-(\varepsilon x_{12}-y_1)U_1(\rho,\alpha)/\rho]I_4(\alpha,p_i)$$
$$+2\Psi(\zeta,p_i)\{[(a_1\alpha^2+a_2p_i^2)U_0(\rho,\alpha)+y_1\alpha U_1(\rho,\alpha)/\rho]+[G_1(\alpha,p_i)U_0(\rho,\alpha)+\eta\alpha U_1(\rho,\alpha)/\rho]\varepsilon G_2(\alpha,p_i)$$
$$/[\eta p_i^2-\gamma G_1(\alpha,p_i)]\}]]/D(\alpha)d\alpha \tag{59}$$

$$\bar{\sigma}_{zz}=\sum_{i=1}^{\infty}\Psi(\zeta,p_i)\frac{A_{1i}\sqrt{x_1}K_{1i}}{\varepsilon x_{12}}\Big[\frac{\sqrt{x_2}K_0(p_i\sqrt{x_2}\rho)}{K_1(p_i\sqrt{x_2})}-\frac{\sqrt{x_1}K_0(p_i\sqrt{x_1}\rho)}{K_1(p_i\sqrt{x_1})}\Big]-\frac{1}{\pi}\int_0^{\infty}\frac{\alpha U_0(\rho,\alpha)}{D(\alpha)}\sum_{i=1}^{\infty}\frac{A_{1i}\sqrt{x_1}K_{1i}}{\varepsilon x_{12}}$$
$$\cdot[I_1(\alpha,p_i)-I_2(\alpha,p_i)]d\alpha+(2\varepsilon)^{-1}\int_0^{\infty}\alpha^2U_0(\rho,\alpha)\sum_{i=1}^{\infty}H(\alpha,p_i,\tau)[I_3(\alpha,p_i)-I_4(\alpha,p_i)+2\alpha\Psi(\zeta,p_i)]$$
$$/D(\alpha)d\alpha \tag{60}$$

$$\bar{\sigma}_{rz}=\sum_{i=1}^{\infty}\chi(\zeta,p_i)\frac{A_{1i}\sqrt{x_1}K_{1i}}{\varepsilon x_{12}}\Big[\frac{K_1(p_i\sqrt{x_2}\rho)}{K_1(p_i\sqrt{x_2})}-\frac{K_1(p_i\sqrt{x_1}\rho)}{K_1(p_i\sqrt{x_1})}\Big]+\frac{1}{\pi}\int_0^{\infty}\frac{\alpha U_1(\rho,\alpha)}{D(\alpha)}\sum_{i=1}^{\infty}\frac{A_{1i}\sqrt{x_1}K_{1i}}{\varepsilon x_{12}}$$
$$\cdot[I_5(\alpha,p_i)-I_6(\alpha,p_i)]d\alpha-(2\varepsilon)^{-1}\int_0^{\infty}\alpha^2U_1(\rho,\alpha)\sum_{i=1}^{\infty}H(\alpha,p_i,\tau)[I_7(\alpha,p_i)-I_8(\alpha,p_i)+2p_i\chi(\zeta,p_i)]$$
$$/D(\alpha)d\alpha \tag{61}$$

where

$$\left.\begin{aligned}
&I_1(\alpha,p_i)=[X_2(\alpha,p_i)-X_1(\alpha,p_i)]\cosh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_2}]/[L(\alpha)\sinh(\alpha\bar{l}/2\sqrt{x_2})]+[Y_2(\alpha,p_i)-Y_1(\alpha,p_i)]\\
&\sinh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_2}]/[M(\alpha)\cosh(\alpha\bar{l}/2\sqrt{x_2})],\quad I_2(\alpha,p_i)=[T_2(\alpha,p_i)-T_1(\alpha,p_i)]\cosh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_1}]\\
&/[L(\alpha)\sinh(\alpha\bar{l}/2\sqrt{x_1})]+[W_2(\alpha,p_i)-W_1(\alpha,p_i)]\sinh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_1}]/[M(\alpha)\cosh(\alpha\bar{l}/2\sqrt{x_1})]\\
&I_3(\alpha,p_i)=V_1(\alpha,p_i)\cosh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_2}]/[L(\alpha)\sinh(\alpha\bar{l}/2\sqrt{x_2})]+Z_1(\alpha,p_i)\sinh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_2}]\\
&/[M(\alpha)\cosh(\alpha\bar{l}/2\sqrt{x_2})],\quad I_4(\alpha,p_i)=V_2(\alpha,p_i)\cosh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_1}]/[L(\alpha)\sinh(\alpha\bar{l}/2\sqrt{x_1})]\\
&+Z_2(\alpha,p_i)\sinh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_1}]/[M(\alpha)\cosh(\alpha\bar{l}/2\sqrt{x_1})],\quad I_5(\alpha,p_i)=[X_2(\alpha,p_i)-X_1(\alpha,p_i)]\\
&\cdot\sinh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_2}]/[L(\alpha)\sqrt{x_2}\sinh(\alpha\bar{l}/2\sqrt{x_2})]+[Y_2(\alpha,p_i)-Y_1(\alpha,p_i)]\cosh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_2}]\\
&/[M(\alpha)\sqrt{x_2}\cosh(\alpha\bar{l}/2\sqrt{x_2})],\quad I_6(\alpha,p_i)=[T_2(\alpha,p_i)-T_1(\alpha,p_i)]\sinh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_1}]/[L(\alpha)\sqrt{x_1}\\
&\cdot\sinh(\alpha\bar{l}/2\sqrt{x_1})]+[W_2(\alpha,p_i)-W_1(\alpha,p_i)]\cosh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_1}]/[M(\alpha)\sqrt{x_1}\cosh(\alpha\bar{l}/2\sqrt{x_1})]\\
&I_7(\alpha,p_i)=V_1(\alpha,p_i)\sinh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_2}]/[L(\alpha)\sqrt{x_2}\sinh(\alpha\bar{l}/2\sqrt{x_2})]+Z_1(\alpha,p_i)\cosh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_2}]\\
&/[M(\alpha)\sqrt{x_2}\cosh(\alpha\bar{l}/2\sqrt{x_2})],\quad I_8(\alpha,p_i)=V_2(\alpha,p_i)\sinh[\alpha(\zeta-\bar{l}/2)\sqrt{x_1}]/[L(\alpha)\sqrt{x_1}\sinh(\alpha\bar{l}/2\sqrt{x_1})]\\
&+Z_2(\alpha,p_i)\cosh[\alpha(\zeta-\bar{l}/2)/\sqrt{x_1}]/[M(\alpha)\sqrt{x_1}\cosh(\alpha\bar{l}/2\sqrt{x_1})]
\end{aligned}\right\} \tag{62}$$

4. Numerical results and discussion

To illustrate the foregoing analysis, numerical calculations were carried out for the following three cases of the transversely isotropic material constants and a special surface heat-generation as shown in Fig.2

Case 1 : k^2=2.0, 1.0(Isotropy), 0.5, E=1.0, F=0.4504, α=1.0,

Case 2 : k^2=1.0, E=2.0, 1.0(Isotropy), 0.5, F=0.4504, α=1.0,

Case 3 : k^2=1.0, E=1.0, F=0.4504, α=2.0, 1.0(Isotropy), 0.5,

where E, F and α are E_z/E_r, G_{rz}/E_r and α_z/α_r, respectively, E_r and E_z are the Young's moduli in the radial and axial directions, respectively, and G_{rz} is the shear modulus which characterizes change of angle between the radial and axial directions. Then the elastic constants, s_{11}, s_{12}, s_{13},... are given by the following equations

$$s_{11}=1/E_r,\quad s_{12}=-\nu_{r\theta}/E_r,\quad s_{13}=-\nu_{rz}/E_z,\quad s_{33}=1/E_z,\quad s_{44}=1/G_{rz} \tag{63}$$

Here the Poisson's ratios $\nu_{r\theta}$ and ν_{rz} for all cases mentioned above were taken as 0.11. In the case of Fig.2, the functions $f_1(\rho)$ and $w_1(\tau)$ may be expressed as

$$f_1(\rho)=H(\rho-\bar{c}_1)-H(\rho-\bar{c}_2),\quad w_1(\tau)=H(\tau). \tag{64}$$

where $H(\ )$ is the Heaviside step function and $\bar{c}_1$ and $\bar{c}_2$ denote c_1/a and c_2/a, respectively. The function $\theta(\tau)$ involved in eqs.(57)-(61) can be obtained as

$$\theta(\tau)=p_i\bar{Q}_{10}[\bar{c}_2U_1(\bar{c}_2,\alpha)-\bar{c}_1U_1(\bar{c}_1,\alpha)][1-e^{-G_k(\alpha,p_i)\tau}]/\alpha G_k(\alpha,p_i). \tag{65}$$

The temperature and thermal stress fields were calculated by truncating the infinite series at the first twenty terms and evaluating numerically the infinite integrals by the use of a modified Simpson's rule with a variable upper integration limit and variable step size in eqs.(20) and (58)-(61). Then the absolute values of the normal and shear stresses on the lower and upper plane surfaces and the cylindrical hole obtained were less than 3% of that of the maximum stress. In the numerical calculations, the following values were used

$$\bar{l}=2,\quad m_1=m_2=0.1,\quad \bar{c}_1=2.0,\quad \bar{c}_2=2.5,\quad \bar{Q}_{10}=1.$$

Fig.3 shows the effect of the anisotropy of thermal conductivity coefficient on the temperature distributions on the lower and upper plane surfaces of the plate. It is shown that the greater the ratio of thermal conductivity coefficients k^2, the greater are the temperature rise and the temperature gradients in the radial and axial directions.

Figs.4-6 show the radial variation of only the circumferential stress on the lower and upper plane surfaces of the plate. The circumferential stress at the center of the surface heat-generation on the lower surface is the maximum thermal stress in the plate subjected to the surface heat-generation as shown in Fig.2. These figures illustrate the effects of the anisotropies of the thermal conductivity coefficient, the young's modulus and the linear thermal expansion coefficient on the stress distribution, respectively. It is clear from Fig.4 that an increase in the ratio k^2 causes an increase in the magnitude of the circumferential stress in the heat-generating region. It is clear from Figs.5 and 6 that the magnitude of the circumferential stress is rather insensitive to the anisotropies of the Young's modulus and the linear thermal expansion coefficient.

References

[1] Sharma,B., "Thermal Stresses in Transversely Isotropic Semi-infinite Elastic Solids", J. Appl. Mech. 25, 86-88(1958).

[2] Akoz,A.Y., Tauchert,T.R., "Thermal Stresses in an Orthotropic Elastic Semispace", J. Appl. Mech. 39, 87-90(1972).

[3] Tauchert,T.R., Akoz,A.Y., "Thermal Stresses in an Orthotropic Elastic Slab due to prescribed Surface Temperature", J. Appl. Mech. 41, 222-228(1974).

[4] Tauchert,T.R., Akoz,A.Y., "Stationary Temperature and Stress Fields in an Anisotropic Elastic Slab", J. Appl. Mech. 42, 647-650(1975).

[5] Atsumi,A., Itou,S., "Axisymmetric Thermal Stresses in a Transversely Isotropic Circular Cylinder Having a Spherical Cavity", J. Appl. Mech. 43, 431-433(1976).

[6] Takeuti,Y., Noda,N., "A General Treatise on the Three-Dimensional Thermoelasticity of Curvilinear Aeolotropic Solids", J. Thermal Stresses 1, 25-39(1978).

[7] Sugano,Y., "Transient Thermal Stresses in a Transversely Isotropic Finite Circular Cylin-

der due to an Arbitrary Internal Heat-Generation", Int. J. Engng. Sci. (to be published).

[8] Singh,A., "Axisymmetrical Thermal Stresses in Transversely Isotropic Bodies", Arch. Mech. Stosw. 12, 287-304(1960).

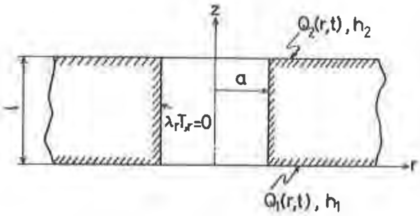

Fig.1. Transversely isotropic thick plate with arbitrary surface heat-generations.

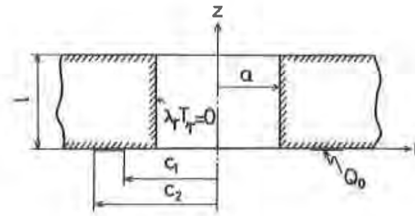

Fig.2. Transversely isotropic thick plate with a uniform heat-generation on finite region of lower plane surface.

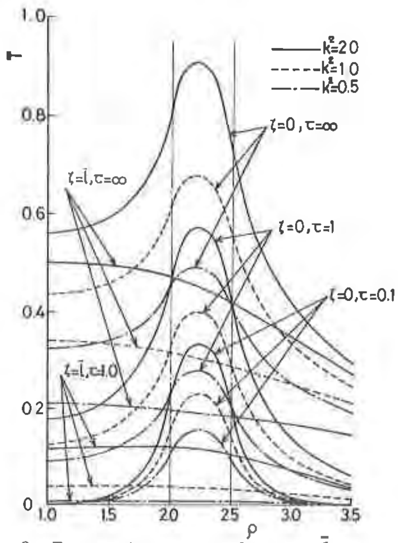

Fig.3. Temperature on $\zeta=0$ and $\zeta=\bar{l}$ for the cases of $k^2=2.0, 1.0$ and 0.5.

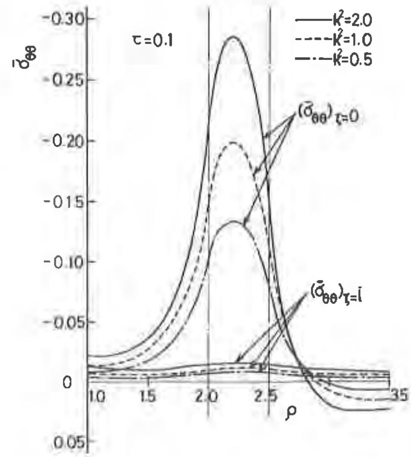

Fig.4. Circumferential stress on $\zeta=0$ and $\zeta=\bar{l}$ for Case 1.

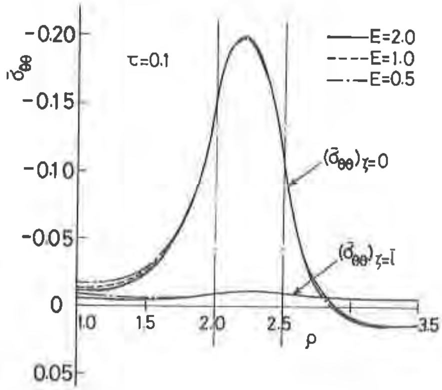

Fig.5. Circumferential stress on $\zeta=0$ and $\zeta=\bar{l}$ for Case 2.

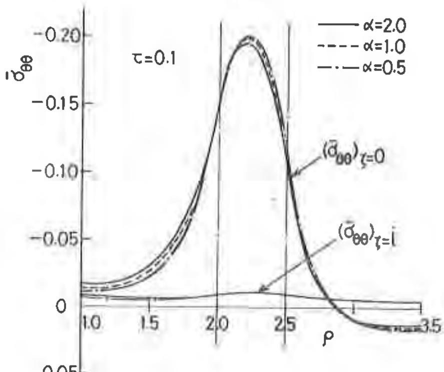

Fig.6. Circumferential stress on $\zeta=0$ and $\zeta=\bar{l}$ for Case 3.