ABSTRACT

STURGILL, BRADLEY SCOTT. Essays on Factor Shares, Development Accounting, and Factor-Eliminating Technical Change. (Under the direction of John J. Seater.)

The stability of factor shares has long been considered one of the "stylized facts" of macroeconomics. However, the relationship between cross-country factor shares and economic development is dependent on how factor shares are measured. Most factor share studies acknowledge only two factors of production: total capital and total labor. The failure to acknowledge more than two factors yields misleading results. In the first essay I disentangle physical capital's share from natural capital's share and human capital's share from unskilled labor's share. Results reveal that non-reproducible factor shares decrease with the stage of economic development, and reproducible factor shares increase with the stage of economic development. This suggests that studies relying on the macroeconomic paradigm of constant factor shares should be revisited.

Development accounting nearly always assumes the constancy of factor shares. In the second essay I perform the development accounting exercise but allow factor shares to vary and distinguish between reproducible and non-reproducible factors. My approach yields results that stand in stark contrast to those previously attained. The general consensus is that at least half of the cross-country variation in output per worker accrues to the Total Factor Productivity (TFP) residual. With my approach, the majority of variation in output per worker accrues to factor shares, specifically physical capital's share and natural capital's share. Depending on the approach used to compute factor shares, TFP's explanatory power decreases by as much as 61 percentage points. This evidence does not, however, diminish the role of technical change. Rather, the evidence indicates the importance of acknowledging a new type of technical change, one that impacts factor shares.

Peretto and Seater (2009) develop a theory of factor eliminating technical progress that predicts a systematic relationship between factor shares and output per worker. The first essay verifies this systematic variation, and the second essay revisits one of many macroeconomic exercises that assume such variation does not exist: the estimation of the TFP residual. In the third essay, I extend the Peretto and Seater model by incorporating endogenous saving. Endogenous saving alters the model so that the possibility of a Solow

Steady state is eliminated. All equilibrium paths lead to a production function that asymptotically becomes AK.

Essays on Factor Shares, Development Accounting, and Factor-Eliminating Technical Change

by
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DEDICATION

To my parents

BIOGRAPHY

Bradley Scott Sturgill was born in Winston-Salem, North Carolina on May 31, 1981 to Tony and Karon Sturgill. Brad grew up in Walnut Cove, North Carolina and attended South Stokes High School. He earned a bachelors degree in Economics at Appalachian State University in 2003. After finishing his undergraduate studies, he immediately began pursuing his graduate degree at North Carolina State University. Brad served as a graduate instructor and taught introductory economics at NCSU for three years. He completed the final two years of his graduate work while teaching full time as a Visiting Instructor in the Economics Department at Appalachian State University. Brad will continue his academic career as an Assistant Professor in the Economics Department at Grand Valley State University in Allendale, Michigan.

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Chapter 1

The Relationship between Cross-Country Factor Shares and the Stage of Economic Development

1.1 Introduction

Capital shares and labor shares are typically treated as constant parameters. For example, Hall and Jones (1999), in an investigation of the role of productivity in explaining cross-country differences in output per worker, assume that capital shares and labor shares are constant across countries and equal to 1/3 and 2/3 respectively. Some studies, such as Gollin (2002), present empirical evidence in support of constant factor shares across countries. Others, such as Zuleta (2008a), conclude that factor shares vary across countries. Despite conflicting empirical evidence and despite the doubts about the constancy of factor shares expressed by Keynes (1939) and Solow (1958), most researchers accept Kaldor's (1961) submission that factor shares are constant as a "stylized fact" of macroeconomics.

Factor shares are not constant when factors of production are properly defined and measured. The key step is making a distinction between reproducible factors and non-reproducible factors. In most factor share studies, only two factors of production, capital and labor, are acknowledged. Failure to acknowledge more than two factors yields results and conclusions that are misleading at best. When discussing capital, economists generally refer to physical or human capital—physical capital being tools, machinery, and structures, and human capital encompassing education, health, and training. However, standard capital share measures include the fractions of income paid to physical capital as well as natural capital, which encompasses all natural resources including land, minerals, and oil. Physical capital and natural capital are two distinct factors. Physical capital is reproducible, meaning it can be accumulated, whereas natural capital is non-reproducible and can not be accumulated.[1] Therefore, any claim about the standard capital share and how it relates to the stage of economic development is really a claim about two separate factor shares and their collective relationship with the stage of economic development. Likewise, standard measures of labor's share entangle the fraction of income paid to human capital, a reproducible factor, and unskilled labor, a non-reproducible factor.

In this first chapter, I disentangle physical capital's share from natural capital's share and human capital's share from unskilled labor's share. Though recent empirical work has made progress in this area, my measurement techniques represent a clear departure from the literature, and I provide a more complete and comprehensive analysis of the relationship between factor shares and the stage of economic development. There is strong evidence that non-reproducible factor shares decrease with the stage of economic development, and reproducible factor shares increase with the stage of economic development. This finding has theoretical and empirical implications. First, it provides support for theoretical growth models, such as those presented by Peretto and Seater (2008) and Zuleta (2008b), that incorporate factor eliminating technical progress. Secondly, it suggests that any theoretical or empirical study relying on Kaldor's claim that factor shares are constant should be revisited.

The remainder of the chapter is organized as follows. Section 1.2 provides the theoretical and empirical backdrop for my analysis. In Section 1.3, I disentangle physical capital's share from natural capital's share and formally analyze the cross-country relationship between output per worker and each factor share. I disentangle human capital's share from unskilled labor's share in Section 1.4 and perform a similar cross-country

[1]Non-reproducible factors are those factors with which an economy is endowed. Reproducible factors have to be produced.

analysis. I make some comments and discuss the departure of my paper from the related literature in Section 1.5. Section 1.6 concludes.

1.2 Motivation and Related Literature

1.2.1 Theoretical Background

The work of Cobb and Douglas (1928) and Kaldor (1961) suggesting that factor shares were constant created a paradigm in macroeconomics. However, new theories and a general refinement in the way we think about factors and factor shares call into question the precedent set forth by Cobb and Douglas and Kaldor. Recent work in endogenous growth theory distinguishes between reproducible and non-reproducible factors and explores the idea that technical change can alter factor shares. These theoretical advances yield specific predictions about the systematic relationship between the stage of economic development and both reproducible and non-reproducible factor shares across countries.

Perpetual growth requires that the marginal products of reproducible factors of production be bounded away from zero (Jones and Manuelli, 1997). This means that the non-reproducible factors must either be augmented or eliminated. Virtually all analyses focus on augmentation. However, Peretto and Seater (2008) develop a theory of endogenous growth that focuses on factor elimination. Factor intensities are allowed to change endogenously via spending on R&D, and this serves as the catalyst for growth. As economies advance, non-reproducible factors of production become less important, and reproducible factors of production become more important. In other words, their theory predicts that non-reproducible factor intensities should decrease with output per worker, and reproducible factor intensities should increase with output per worker.[2]

The Peretto and Seater theory allows for monopolistic competition in the intermediate goods sector. As a result, firms earn excess profits, and payments to the factors of production do not exhaust firm revenues. Consequently, factor intensities and factor shares,

[2] The term "factor intensity" refers to the elasticity of output with respect to a factor of production.

though related, are not equivalent. However, to the extent that factor shares measured using national income account data are reasonable estimates of factor intensities, the theory suggests that non-reproducible factor shares should decrease with output per worker, and reproducible factor shares should increase with output per worker.

A factor share is the portion of total income that is paid to a factor of production, and factors of production are often separated into two broad categories: total capital and total labor. Total capital generally encompasses physical capital and natural capital, and total labor generally encompasses human capital and unskilled labor. That being said, total capital and total labor are *not* the factors considered by Peretto and Seater. They consider reproducible and non-reproducible factors. Reproducible factors include physical capital and human capital, the former of which typically gets lumped in with total capital and the latter with total labor. Non-reproducible factors include natural capital and unskilled labor, the former of which typically gets lumped in with total capital and the latter with total labor. On the surface, this may seem like a trivial difference in the allocation of inputs between composite categories. However, the theory's implications pertain specifically to the relationship between an economy's stage of economic development and non-reproducible and reproducible factor shares. Therefore, any empirical evidence in support of or against a correlation between output per worker and either total capital's share or total labor's share neither validates nor nullifies the theory. The theory is silent on the relationship between an economy's stage of economic development and either total capital's share or total labor's share.

In a related vein of the literature, Zuleta (2008b) develops an endogenous growth model in which growth occurs via capital using and labor saving technological progress. Like Peretto and Seater, Zuleta incorporates endogenous factor intensities. The key differences between Zuleta's model and that of Peretto and Seater are: Zuleta solves the social planner problem whereas Peretto and Seater consider the market solution; the saving rate is endogenous in Zuleta and exogenous in Peretto and Seater; and the cost of new technologies is modeled differently in the two studies. However, from an empirical standpoint, Zuleta's model yields the same testable implications pertaining to factor shares, namely that reproducible factor shares are positively related to the stage of economic development, and non-reproducible factor shares are negatively related to the stage of economic development.[3]

Hansen and Prescott (2002), who build on Galor and Weil (2000), propose a model of transition from a primitive to an advanced economy. In their model, advancements in the stage of development, which occur via exogenous technical progress, are accompanied by decreases in land's share. Land, like other natural capital, is non-reproducible, so the prediction of Hansen and Prescott's model is consistent with the aforementioned theories that suggest non-reproducible factor shares should fall with output per worker.

1.2.2 Empirical Background

The simplest labor share calculation is computed as the fraction of real GDP attributed to employee compensation. Capital's share is then computed as the residual, $1-\left(\frac{Employee\ Compensation}{GDP}\right)$. It has been argued, most notably by Gollin (2002), that the aforementioned method, which Gollin refers to as *naïve*, is misleading because published numbers on employee compensation omit the income flowing to the self-employed. Assuming that a portion of self-employed income represents labor income, the consequence of this omission is estimation of labor's share that is too low and estimation of capital's share that is too high, especially in developing countries where self-employment is prevalent.

Gollin's Contribution Gollin adjusts for this omission by including the operating surplus of private unincorporated enterprises (*OSPUE*) in the computation of labor's share. The idea is

[3] Boldrin and Levine (2002) and Zeira (1998, 2006) develop models similar to that of Zuleta. Technical advancement occurs via substitution of capital for labor. Boldrin and Levine's model predicts that labor's share should decrease with economic development. Zeira's model, though it makes no explicit predictions about the relationship between factor shares and economic development, predicts a positive correlation between the capital to output ratio and economic development.

that most self-employed people do not operate incorporated enterprises, and, consequently, capital income and labor income of the self-employed are encompassed by *OSPUE*. Gollin allocates *OSPUE* to labor and capital using three different adjustments and concludes that accounting for the income of the self-employed via *OSPUE* yields results indicative of stable factor shares across countries.

Gollin arrives at his conclusion without performing any formal analysis to test for correlation between either capital's share or labor's share and real GDP per capita. Instead, the stability claim is based on the observation that the adjustments using *OSPUE* yield capital shares that are clustered in a range from .15 to .40. Such a range, which represents almost a three-fold difference, is nontrivial, especially in the context of empirical estimation of production functions where factor shares often appear as exponents.

Of the 31 countries for which Gollin computes labor's share, real GDP per capita is available from version 6.2 of the Penn World Tables for 26 of them. Omitting Botswana, which is an outlier in the data,[4] and then regressing Gollin's adjusted labor shares on real GDP per capita for the remaining 25 observations yields slope coefficients that, statistically, are no different from zero. Thus, given Gollin's approach, a more formal analysis reveals there are no inconsistencies between his claim and his data, and indeed factor shares appear to be constant across countries. Regression results are reported in columns 1 and 3 of Table 1.1.

Irrespective of these results and the importance of adjusting for self-employed income, aspects of Gollin's methodology are questionable. First, real GDP per worker, as opposed to real GDP per capita, is a better measure of economic development and should be

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[4] Botswana's labor share is the lowest among all countries regardless of the adjustment used by Gollin. Specifically, the value is 0.368 and 0.341 for Adjustment 1 and Adjustment 2 respectively. Including Botswana, the mean labor share for Adjustment 1 is 0.745, and the mean labor share for Adjustment 2 is 0.675. The standard deviation is 0.110 and 0.107 for Adjustment 1and Adjustment 2 respectively. Omitting Botswana, the mean increases to 0.758, and the standard deviation falls to 0.087 for Adjustment 1. For Adjustment 2, the mean increases to 0.686 and the standard deviation falls to 0.089.

used when testing for systematic variation in factor shares across countries.[5] Secondly, the data from Gollin's paper used in the aforementioned regression analysis are not a true cross-section because the year for which the 25 data points are observed is not constant across countries. The year of observation ranges from 1977 to 1992.

Gollin also plots pooled cross-country and time series data. His conclusion is that the scatter plots indicate no systematic variation in labor shares across countries. Again, he provides no statistical proof for his claim, and though a lack of correlation seems obvious to the naked eye, the presence of heteroskedasticity is also obvious. Gollin acknowledges that the variance in factor shares is higher for the poorer countries than for the richer countries, and he admits that data quality may be a problem. However, he quickly dismisses this potential problem without mentioning its potential consequences. Heteroskedasticity yields biased standard errors and creates a situation where t statistics are no longer t distributed. Failure to control for heteroskedasticity could lead to incorrect conclusions about the statistical significance of slope coefficients.[6] A thorough analysis should consider the sensitivity of the results to heteroskedasticity before making conclusions about the nature of the cross-country variation in factor shares.

Bernanke and Gurkaynak Extension Using the Gollin framework, and specifically Gollin's adjustment 2, Bernanke and Gurkaynak (2001) estimate average labor shares over the period 1980-1995. They increase the number of countries for which labor shares can be calculated by constructing an *imputed OSPUE* measure. This measure is substituted in place of actual *OSPUE* for countries that report only total operating surplus and do not distinguish between the surplus of corporate enterprises and private unincorporated enterprises. In

[5] I regressed Gollin's adjusted labor shares on real GDP per worker instead of real GDP per capita, and the qualitative results are unchanged. See columns 2 and 4 in Table 1.1. Thus, in this specific case, the results are not dependent on how the stage of economic development is measured. The relationship between real GDP per capita and real GDP per worker is considered in detail in Section 1.5 of the paper, and more detail as to why real GDP per worker is a more appropriate measure is given.

[6] Dawson et al. (2001) analyze data quality induced heteroskedasticity, and their findings suggest that the issue is not just a technicality that should be addressed in order to simply comply with econometric theory; it is a problem that, if not controlled for, can lead to incorrect interpretation of empirical results.

addition, when *OSPUE* can not be imputed, Bernanke and Gurkaynak compute average labor's share using what they refer to as the *labor force correction.* I discuss the *imputed OSPUE* and *labor force correction* measures in detail later herein.

Bernanke and Gurkaynak "find no systematic tendency for country labor shares to vary with real GDP per capita." However, they reference no statistical tests in support of their claim. They simply observe that most labor shares in their 54 country sample lie between 0.60 and 0.80. In fact, Bernanke and Gurkaynak's table 10, the table which reports total labor shares, does not even include data on real GDP per capita.

Statistical evidence calls into question the validity of Bernanke and Gurkaynak's claim. I gathered data on real GDP per capita from version 6.2 of the Penn World Tables and computed the average level of real GDP per capita over the period 1980-1995 for each country in the Bernanke and Gurkaynak sample. A simple linear regression reveals a positive and statistically significant relationship at the 1% level between the average labor share reported in Bernanke and Gurkaynak's table 10 and average real GDP per capita.[7] In addition, the relationship remains statistically significant and positive after controlling for heteroskedasticity using White corrected standard errors. Therefore, statistical evidence indicates that Bernanke and Gurkaynak's claim is unwarranted. Unless Bernanke and Gurkaynak performed an unreported statistical analysis using a measure other than average real GDP per capita, which would itself seem unwarranted given that the reported labor share data is averaged, the claim directly conflicts with the data. A scatter plot is provided in Figure 1.1[8], and regression results are reported in Table 1.2.[9]

[7] I also ran the same regression substituting average real GDP per worker for average real GDP per capita. Average labor's share is positively and significantly correlated with average real GDP per worker at the 1% level, and the point estimates change very little as can be seen in Table 1.2.

[8] The International Organization for Standardization's (ISO) three-letter country codes are used as data markers in all plots.

[9] For the plot and the regression analysis, I used the labor shares computed with "Actual *OSPUE*" wherever possible. These numbers are reported in column 4 of Bernanke and Gurkaynak's Table 10. When "Actual *OSPUE*" was not available for a country, I used the labor shares computed with *imputed OSPUE*, which are reported in column 5 of Bernanke and Gurkaynak's Table 10. When *imputed* OSPUE was not available for a country, I used the labor shares computed via the *labor force correction*, which are reported in column 6 of Bernanke and Gurkaynak's Table 10 under the heading "*LF*."

An Argument for Attributing all Self-Employed Income to Capital Though the argument for allocating self-employed income to labor and capital is sound, an argument in support of the *naïve* measure also has merit. The *naïve* measure attributes all self-employed income to capital. This is reasonable only if one acknowledges a self-employed person as a unit of capital. Such acknowledgement may seem unwarranted at first pass, and it is likely that the reader's main objection to categorizing a self-employed person as a unit of capital is the physical distinction between physical capital and labor. After all, a self-employed individual, just like an employee, is indeed a person, and the contribution to production comes from the human body. Physical capital on the other hand encompasses machines, buildings, tools, etc., and these things are inanimate, durable inputs that must be produced. Such sentiments arise from the typical textbook definitions of labor and capital. However, this paper focuses on measuring the fractions of income that get paid to the inputs used in production. From an income allocation perspective, a self-employed person is very similar to a unit of physical capital.

The crucial question is whether self-employed income comes from a residual or from a commitment. That is, does a self-employed person's income come from the funds left over after all expenses have been paid, or, does the self-employed person make a commitment to pay himself a wage? Employers make a commitment to pay employees a wage, and to the extent that employers want to retain employees, they take on risk because the commitment is legally binding irrespective of the firm's revenue. If a self-employed person makes a commitment to pay himself a wage, there is no net risk nor is there a potential net gain or net loss, because the individual is betting against himself. Therefore, the self-employed person has no incentive to make a commitment to pay himself a wage. Such a commitment is not going to result in a larger amount of income because the commitment can only be kept if revenue less expenses exceeds the wage, and revenue less expenses belongs to the self-employed person anyway. Regardless of any commitment to oneself, the amount of income a self-employed person brings in is a residual. Therefore, it can be argued that self-employed income should be treated as residual income and categorized as operating surplus just as residual income in the corporate sector. Operating surplus, which is defined as "the excess of value added over the sum of compensation of employees, consumption of fixed capital, and net indirect taxes" by the United Nations Yearbook of National Account Statistics, is considered part of capital compensation.

Gollin reports *naïve* labor share estimates in his table 2 along side his adjusted labor share estimates. A simple linear regression reveals that the *naïve* measure is positively and significantly related to real GDP per capita. A statistically significant positive relationship is also present when the analysis is performed using real GDP per worker instead of real GDP per capita. The regression results are reported in columns 5 and 6 of Table 1.1.

The absence of a systematic relationship between factor shares and the stage of economic development is a result that rests on an adjustment to the commonly used calculation that reflects a more detailed treatment of the data. However, the underlying premise for the self-employment adjustment is questionable, and thus the aforementioned analyses are incomplete at best.

1.3 Decomposition of Total Capital's Share

The key omission in the aforementioned empirical studies is acknowledgement of more than two factors of production. When Gollin performs his analysis and concludes that factor shares do not systematically vary with real GDP per capita, the driving force of his result is the adjustment made for the income of the self-employed. Bernanke and Gurkaynak's results emanate from the same adjustment. Regardless of the validity of this adjustment, using the standard measures of capital and labor to study the empirical relationship between factor shares and economic development is misleading if one fails to acknowledge the composite nature of the factors. Standard accounting lumps non-reproducible and reproducible factors together in composite categories. Specifically, capital's share encompasses the share of income paid to both physical capital and natural capital, and labor's share encompasses the share of income paid to both unskilled labor and human capital. The reproducible shares need to be separated from the non-reproducible shares, and the relationship between a single factor share, not a composite share, and economic development should be analyzed.

I focus first on disentangling physical capital's share from natural capital's share. Let α denote physical capital's share, and let γ denote natural capital's share. The starting point is the computation of total capital's share, $\alpha + \gamma$, and there are numerous ways to proceed. I consider three methods. The first two make adjustments for the income of the self-employed and the third does not. It turns out that the qualitative results are robust with respect to the treatment of self-employed income.

1.3.1 Total Capital's Share, *OSPUE Adjustment*

I begin by computing total capital's share according to Bernanke and Gurkaynak's variation of Gollin's adjustment 2, which I refer to as the *OSPUE adjustment*. This computation, which is given by

$$(\alpha + \gamma)_{OSPUE} = 1 - \left(\frac{Employee\ Compensation}{GDP - Indirect\ Taxes - imputed\ OSPUE} \right), \tag{1.1}$$

is an indirect measure of total capital's share, and, specifically, it is the perfect competition counterpart to total labor's share, because it is the residual remaining after total labor's share is computed and subtracted from one.

Implicit Assumptions and Data Subtracting *OSPUE* from GDP in equation (1.1) implies that self-employed income is dispersed between labor and capital in the same manner that corporate sector income is dispersed between the two factors. In other words, the share of labor income in *OSPUE* is assumed to be the same as the share of labor income generated in the corporate sector.[10]

Ideally, *Indirect Taxes*, which include but are not limited to taxes on fixed assets and taxes on the total wage bill, should be allocated to capital or labor compensation depending on the tax type.[11] However, most countries only report an aggregate tax value without any detailed breakdown of the various taxes. Therefore, it is impossible to know exactly how *Indirect Taxes* should be dispersed. By subtracting *Indirect Taxes*, the implicit assumption is that the fraction of *Indirect Taxes* attributable to capital compensation is equivalent to capital's share, and the fraction of *Indirect Taxes* attributable to labor compensation is equivalent to labor's share.

Note that it is *imputed OSPUE* rather than *OSPUE* that enters equation (1.1). Though operating surplus can be broken down into corporate, unincorporated, public and private components, 1997 is the last year for which the U.N. Yearbook of National Accounts reports *OSPUE*. As is discussed later, data availability prevents me from disentangling physical capital's share from natural capital's share for any year except 2000. Therefore, I need *OSPUE* for the year 2000, so I impute it following the method of Bernanke and Gurkaynak (2001).

The *imputed OSPUE* measure is computed as the share of non-corporate employees in the labor force multiplied by private sector income. Implicit in this calculation is the assumption that the fraction of private sector income attributable to corporations is equivalent to the fraction of the labor force employed by corporations. Private sector income is the sum

[10] Gollin also computes total labor's share assuming that all self-employed income is labor income; this is his Adjustment 1. However, assuming that all income of the self-employed is paid to labor is equivalent to assuming that the self employed do not use capital. This seems an unrealistic assumption and would undoubtedly overstate labor's share of national income. Adjustment 1 may be more reasonable for a poor, developing country, but Gollin acknowledges that even in poor countries, the self employed tend to have substantial amounts of capital in their businesses.

[11] Income received by firms and not paid to owners in the form of excess profits should be paid to the factors that generate the output. Thus, for the purpose of estimating factor shares, it is misleading to treat the income received by firms and paid to the government in the form of indirect taxes as anything other than income attributed to factors of production. Doing so would skew the analysis and yield factor share estimates that account for something less than one hundred percent of factor generated income.

of corporate and non-corporate income, and it can also be interpreted as the sum of operating surplus and corporate employee compensation. Several different pieces of data, all of which come from either the International Labor Organization's (ILO) LABORSTA database or the ILO's 2005 Yearbook of Labor Statistics, are used to perform the calculations needed to arrive at the *imputed OSPUE* measure.[12]

Data for *Employee Compensation* and *Indirect Taxes* comes from table 2.3 of the 2006 version of the United Nations Yearbook of National Account Statistics. *Employee Compensation* is defined as "the income accruing to employees as remuneration for their work for domestic production." Moreover, it is the "sum of wage and salary accruals and of supplements to wages and salaries." *Indirect Taxes* are "taxes chargeable to the cost of production or sale of goods and services." Such taxes include, among other things, import and export duties, excise, sales, entertainment, real estate, and land taxes. *GDP* is reported in table 1.1 of the United Nations yearbook.

Results and Analysis Total capital share estimates computed via the *OSPUE* adjustment are presented in Table 1.3 for the 33 countries for which the necessary data are available for the year 2000. The same shares are depicted graphically in Figure 1.2 where they are plotted against real GDP per worker. Real GDP per worker data comes from version 6.2 of the Penn World Tables.[13] Figure 1.2 suggests a quadratic relationship between total capital's share

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[12] First, I calculate the corporate share of the labor force by dividing *Paid Employment* by the labor force, which I compute by summing *Employment* and *Unemployment*. The share of non-corporate employees is computed as one minus the corporate share of the labor force. To obtain *imputed OSPUE*, the share of non-corporate employees is then multiplied by total corporate sector income, which is the sum of *Gross Operating Surplus* and *Employee Compensation*. *Gross Operating Surplus* and *Employee Compensation* come from the 2006 version of the United Nation's Yearbook of National Account Statistics. *Paid Employment*, *Employment*, and *Unemployment* come from the ILO publications.

[13] 'Worker' in real GDP per worker refers to an individual, not a labor hour. Specifically, in the Penn World Tables, 'worker' has a census definition based on the *Economically Active Population.* The underlying worker data employed by Summers and Heston in the Penn World Tables comes from the ILO, and according to the ILO, the *Economically Active Population* "comprises all persons of either sex above a specified age who furnish the supply of labor for the production of economic goods and services as defined by the System of National Accounts (SNA), during a specified time reference period." Note that this definition of 'worker' includes the employed, the unemployed, and those seeking work for the first time.

and real GDP per worker. Formal regression analysis supports this. Consider the following regression equation:

$$(\alpha + \gamma)_{OSPUE_i} = \psi_0 + \psi_1 u_i + \psi_2 u_i^2 + \varepsilon_i \tag{1.2}$$

where u_i is a coded independent variable that takes the form

$$u_i = \frac{y_i - \bar{y}}{s_y}, \tag{1.3}$$

ε_i is the error term, and *i* indexes the country. y_i is real GDP per worker in country *i*, and $\bar{y}$ is the average value of y in the sample. s_y is the standard deviation of the y values. The coded variable, *u*, is used in place of y in order to reduce the multicollinearity inherent in polynomial regression models.[14]

Though OLS estimation of equation (1.2) reveals a negative and statistically insignificant estimate of ψ_1, an F test indicates that the quadratic model is statistically useful. The estimate of ψ_2 is positive and significant at the 5% level indicating upward concavity. The estimated slope coefficient, $\hat{\psi}_1 + 2\hat{\psi}_2 u$, is negative for lower *u* values and positive for higher *u* values. This implies that, among lower income countries, total capital's share tends to decrease as output per worker increases, and among higher income countries, total capital's share tends to increase as output per worker increases. Estimation results are reported in Table 1.4.

Drawing final conclusions about the relationship between total capital's share and real GDP per worker at this point would be premature. In any cross-country study, data quality is a concern. The general consensus is that the quality of economic data increases

[14] Minimizing the effects of multicollinearity is important because multicollinearity increases the likelihood of rounding errors in the standard errors and can sometimes have an effect on the sign of regression coefficients.

with the level of economic development. Failure to control for any systematic variation in data quality across countries could significantly impact the observed relationship between total capital's share and real GDP per worker. Specifically, if data quality is systematically related to total capital's share, then the squared residuals produced by estimation of equation (1.2) will fluctuate with data quality. If real GDP per worker and data quality are correlated, the squared residuals will fluctuate with real GDP per worker and introduce heteroskedasticity into the estimation of equation (1.2). Further precautions should be taken to ensure the observed relationship between total capital's share and real GDP per worker is representative of the actual relationship and not a mere artifact of systematic cross-country variation in data quality.

To formally test for heteroskedasticity, I estimate

$$e_i^2 = \delta_0 + \delta_1 \left(\alpha \hat{+} \gamma\right)_{OSPUE_i} + \delta_2 \left(\alpha \hat{+} \gamma\right)_{OSPUE_i}^2 + \mu_i \tag{1.4}$$

where the e_i are the regression residuals from OLS estimation of equation (1.2) and the $\left(\alpha \hat{+} \gamma\right)_{OSPUE_i}$ are the OLS fitted values. The null hypothesis of no heteroskedasticity is a joint hypothesis that δ_1 and δ_2 are equivalent and equal to zero. The accompanying alternative to the null is that at least one of the coefficients is not zero. I obtain an F-statistic of 3.215, which is insignificant, and conclude that the data are not plagued by heteroskedasticity.[15]

The quadratic relationship between total capital's share and real GDP per worker is neither supported nor contradicted by economic theory. Total capital's share is an empirical measure that is often used by researchers who have intentions of estimating physical capital's share. However, as noted earlier, total capital's share is the sum of physical capital's share and natural capital's share. The aforementioned relationship is meaningful only because it suggests that physical capital's share, natural capital's share or both are systematically related to output per worker; it is not very meaningful in and of itself. Separating physical

[15] I use the same technique to test for heteroskedasticity in the estimation of all regression equations that follow.

capital's share from natural capital's share is a logical and necessary progression if the true nature of the relationship between each of these shares and the stage of economic development is to be revealed.[16]

1.3.2 Physical Capital's Share, *OSPUE Adjustment*

To isolate physical capital's share, I follow the approach of Caselli and Feyrer (2007). Define total wealth as the sum of physical capital and natural capital so that $W = K + N$. *W* is total wealth; *K* denotes the value of the aggregate stock of physical capital; and *N* denotes the value of the aggregate stock of natural capital. Like Caselli and Feyrer, I assume that differences in capital gains for natural and physical capital are negligible so that all units of wealth pay the same return, r_w. Given this notation, total capital's share can be expressed as $\frac{r_w W}{Y}$, which, after substituting for *W*, is equivalent to $\frac{r_w(K+N)}{Y}$ where *Y* is aggregate output or GDP. This last term can be rewritten as the sum of two terms, $\frac{r_w K}{Y} + \frac{r_w N}{Y}$, the first of which is physical capital's share and the second of which is natural capital's share.[17] Each share can be expressed as a function of total capital's share by multiplying and dividing by total wealth. Focusing for now on physical capital's share, such manipulation yields the following:

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[16] Even if the composite relationship were insignificant, a systematic relationship between each factor share and the stage of economic development could not be ruled out. The two shares summed together may not exhibit a statistically significant correlation with the stage of economic development if a positive correlation is compensated by a negative correlation.

[17] Caselli and Feyrer refer to $\frac{r_w K}{Y}$ as 'reproducible' capital's share. Though physical capital is a reproducible factor of production, so is human capital, and human capital is not encompassed by $\frac{r_w K}{Y}$. Thus, Caselli and Feyrer's usage of 'reproducible' capital's share is misleading. 'Reproducible' capital's share refers to a share broader in scope than the one they actually compute.

$$\frac{r_w K}{Y} = \frac{K}{W} \cdot \frac{r_w W}{Y}.$$

$$\Rightarrow \alpha_{OSPUE} = \frac{K}{W} \cdot (\alpha + \gamma)_{OSPUE} \tag{1.5}$$

Thus, physical capital's share is proportional to the fraction of wealth attributable to physical capital. In accordance with equation (1.5), estimates of α_{OSPUE} can be obtained by combining my estimates of $(\alpha + \gamma)_{OSPUE}$ from Section 1.3.1 with estimates of $\frac{K}{W}$, which can be computed using the wealth data reported in Appendix 2 of The World Bank (2006).

The World Bank splits national total wealth for the year 2000, and only the year 2000, into three components: natural capital, produced capital and intangible capital. Total wealth is estimated as the present value of future consumption. The value of the produced capital stock is computed from historical investment data using the perpetual inventory method. Natural capital is valued according to data on physical stocks of natural resources and estimates of resource rents. Intangible capital, which encompasses human capital, social capital, property rights, efficiency of the judicial system, and effectiveness of government, is measured as the residual remaining after subtracting natural and produced capital from total wealth.

Total capital's share does not include income paid to human capital nor the value of any other element soaked up by The World Bank's intangible capital residual. Therefore, The World Bank's total wealth measure, which includes intangible capital, is too broad and can not be used to estimate W. In addition, produced capital's value, as reported by The World Bank, encompasses the value of urban land. Land, regardless of how it is used in production, should not be interpreted as physical capital. Unlike physical capital, land can not be produced. Thus, The World Bank's estimates of produced capital's value are inappropriate estimates of K. In the context of this analysis, urban land should be categorized as natural capital.

To convert the raw data provided by the World Bank into data appropriate for estimation of $\frac{K}{W}$, I proceed as Caselli and Feyrer do. First, I obtain measures of the value of the aggregate stock of physical capital, K. The World Bank follows Kunte (1998) and assumes for each country a value of urban land equal to 24 percent of the value of the aggregate stock of physical capital. So, produced capital's value equals $K + .24K$, and estimates of K are derived by dividing The World Bank's estimates of produced capital's value by 1.24. Since the value of N as reported by The World Bank does not include urban land but the value of N as defined herein does, it follows that urban land's value should be reallocated. To do this, I take The World Bank's estimates of produced capital's value and subtract the newly obtained estimates of K to obtain urban land values. I then add these urban land values to The World Bank's estimates of N to obtain corrected estimates of N. W is then estimated as the sum of the estimate of K and the corrected estimate of N. It follows that the estimate of a country's physical capital share of wealth, $\frac{K}{W}$, is computed by dividing the estimate of K by the estimate of W.[18]

Estimates of α_{OSPUE} for the year 2000 are presented in Table 1.5 and plotted against real GDP per worker in Figure 1.3.[19] I regress α_{OSPUE} on an intercept and real GDP per worker, and OLS estimation reveals a positive and statistically significant slope coefficient at the 5% level. This indicates that physical capital's share, as predicted, is positively correlated with the stage of economic development across countries. Regression results are presented in column 1 of Table 1.7.

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[18] The World Bank reports all of its data in dollars per capita.

[19] α_{OSPUE} is estimated for 31 countries. This is two fewer than the 33 for which total capital's share, $(\alpha + \gamma)_{OSPUE}$, was estimated. The sample is smaller because wealth data is not available for the Czech Republic and Poland.

1.3.3 Natural Capital's Share, *OSPUE Adjustment*

Natural capital's share can be expressed in general terms as

$$\frac{r_W N}{Y} = \frac{N}{W} \cdot \frac{r_W W}{Y}$$

$$\Rightarrow \gamma_{OSPUE} = \frac{N}{W} \cdot (\alpha + \gamma)_{OSPUE} \;, \tag{1.6}$$

but given estimates of total capital's share and physical capital's share, it is easier and equivalent to treat natural capital's share as a residual and back out estimates according to

$$\gamma_{OSPUE} = (\alpha + \gamma)_{OSPUE} - \alpha_{OSPUE} \,. \tag{1.7}$$

Table 1.6 presents estimates of γ_{OSPUE}. These estimates are plotted against real GDP per worker in Figure 1.4. The scatter plot seems to indicate a negative correlation between γ_{OSPUE} and real GDP per worker, which is to be expected given the non-reproducible nature of natural capital. This is supported by OLS estimation, which indicates a negative and statistically significant relationship between the two variables at the 1% level. The regression results are reported in column 2 of Table 1.7.

1.3.4 Total Capital's Share, *Labor Force Correction*

Incorporating *OSPUE* in the estimation of total capital's share makes the assumption that the shares of labor and capital income in *OSPUE* are equivalent to the shares of labor and capital income in the corporate sector. An alternative to the *OSPUE adjustment*, which involves no guesswork as to how *OSPUE* should be divided between labor and capital, is to impute the labor compensation of the self-employed.

Employee Compensation encompasses the labor compensation of only individuals who work in the corporate sector. To account for the income of the self-employed, *Employee Compensation* can be scaled up by the ratio of the total labor force to the number of workers in the corporate sector. This yields an estimate of *all* labor income because labor force numbers include the self-employed. This method, which is Gollin's adjustment 3 and Bernanke and Gurkaynak's *labor force correction*, is referred to as the *labor force correction* herein. Total capital's share is computed as

$$(\alpha+\gamma)_{labor\ \ force\ \ correction} = 1 - \left(\frac{Employee\,Compensation}{(Corporate\ \ Share\ \ of\ \ Labor\ Force)\cdot(GDP - Indirect\,Taxes)} \right). \quad (1.8)$$

Implicit in the *labor force correction* estimate of total capital's share is the assumption that corporate and non-corporate workers receive the same average compensation. Like the *OSPUE adjustment*, the *labor force correction* does not measure total capital's share directly, but rather as the residual remaining after subtracting total labor's share from one.

The data sources for *Employee Compensation*, *GDP*, and *Indirect Taxes* are the same as those used for the *OSPUE adjustment*. The *Corporate Share of the Labor Force* is computed by dividing *Paid Employment*, which comes from the ILO's LABORSTA database, by the *labor force*, which I compute by summing *employment* and *unemployment*, both of which also come from the LABORSTA database.

Estimates of total capital's share computed in accordance with the *labor force correction* are presented in Table 1.8[20] for the 33 countries for which the necessary data are available for the year 2000. The shares are plotted against real GDP per worker in Figure 1.5. Column 1 of Table 1.10 presents estimation results that indicate a statistically significant quadratic relationship between total capital's share and real GDP per worker. The

[20]The sample of countries is identical to the sample for which the *OSPUE* adjustment was employed because the data constraints for computing total capital's share are the same. Specifically, the data needed to compute the *Corporate Share of Labor Force* is a subset of the data needed to compute *imputed OSPUE*.

coefficient on the squared term is positive, indicating upward concavity, and it is statistically significant at the 5% level. However, heteroskedasticity is present and should be controlled for. I begin by exploring the relationship between data quality and real GDP per worker.

In an appendix to their paper accompanying version 6.1 of the Penn World Tables, Summers and Heston (2004) provide proxies for data quality. Each country is assigned a numerical quality grade based on three criteria. The first is the Variance Measure, which Summers and Heston define as the variance of price level estimates. For each country, many estimates of the price level are considered, and a country is assigned a 1 for high variance between estimates and up to a 5 for low variance between estimates. The lower the variance among the alternative price level estimates, the more reliable the data is assumed to be. The second criterion is the Benchmark Measure, and it considers the number of times a country has participated in a benchmark study. A country receives a 0 if it has never served as a benchmark country, a 1 for one benchmark or a quasi-benchmark, and a 2 for more than one benchmark. More benchmarks are assumed to be associated with better data quality. The third criterion is the Data Rank Measure. Based on the assumption that the resources used to gather data statistics increase with income, Summers and Heston put countries into six income groups and assign a score of 1-6 where 1 corresponds to the poorest countries and 6 corresponds to the richest countries. Given these three criteria, the Numerical Quality Score is computed by summing twice the Variance Measure, the Benchmark Measure and the Data Rank Measure. A higher Numerical Quality Score is assumed to indicate better data quality.

As is evidenced by column 1 of Table 1.9, the Summers and Heston Numerical Quality Score is positively and significantly related to real GDP per worker. That said, a systematic relationship between data quality and total capital's share would help explain the presence of heteroskedasticity in the data. This specific form of heteroskedasticity could be controlled for by incorporating data quality into a Weighted Least Squares (WLS) analysis. However, regressing total capital's share on an intercept and the quality score does not yield a statistically significant slope coefficient. See column 3 of Table 1.10.

In light of this, I compute White corrected standard errors. Such an approach is a cure-all because it does not require knowledge of the specific form of heteroskedasticity. The *t* statistics reported in column 2 of Table 1.10 incorporate White corrected standard errors.[21] The conclusions about the relationship between total capital's share and real GDP per worker are unchanged; the squared term remains statistically significant at the 5% level after controlling for heteroskedasticity.

1.3.5 Physical and Natural Capital's Share, *Labor Force Correction*

In the context of the *labor force correction*, physical capital's share is given by

$$\alpha_{labor\ force\ correction} = \frac{K}{W} \cdot (\alpha + \gamma)_{labor\ force\ correction} . \tag{1.9}$$

Estimates of $\alpha_{labor\ force\ correction}$ are presented in Table 1.11 and plotted against real GDP per worker in Figure 1.6. Standard OLS estimation indicates a positive and statistically significant relationship between physical capital's share and real GDP per worker at the 1% level. The regression results are reported in column 1 of Table 1.13.

Subtracting physical capital share estimates from total capital share estimates yields the natural capital share estimates reported in Table 1.12. Figure 1.7 plots natural capital's share against real GDP per worker. OLS estimation indicates that natural capital's share and real GDP per worker are negatively related at the 1% level. The regression results are reported in column 2 of Table 1.13.

1.3.6 Total Capital's Share, *No Adjustment for Self-employed Income*

In the third method, no adjustment for the omission of self-employed income in the NIPA *Employee Compensation* data is made. I treat self-employed income as capital

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[21] A Wald test rather than an F-test is reported in column 2 of Table 1.10. The Wald statistic, unlike the F-statistic, controls for heteroskedasticity.

income. Bernanke and Gurkaynak (2001) and Gollin (2002) argue that acknowledging some portion of self-employed income as labor income is necessary to compute labor and capital shares correctly. I have argued previously that treating all self-employed income as capital income has merit, and so this third method, which I refer to as *No Adjustment*, is presented as a valid approach rather than a naïve baseline from which proper measures emanate.

The EU KLEMS Project (2007)[22] defines capital compensation as

$$Cap = OS + IT - LC^S \tag{1.10}$$

where *Cap* denotes capital compensation, *IT* denotes indirect taxes, *OS* denotes gross operating surplus, and LC^S denotes labor compensation of the self-employed. It follows that total capital's share according to the EU KLEMS project is $\frac{Cap}{GDP}$.[23] Since I am assuming that self-employed income is capital income, I make a slight modification to the EU KLEMS capital compensation measure and do not subtract LC^S from *OS*. Total capital's share is computed as

$$(\alpha + \gamma)_{No\ Adjustment} = \frac{OS + IT}{GDP}. \tag{1.11}$$

Note that implicit in equation (1.11) is the assumption that all indirect taxes are related to capital. As discussed previously, indirect taxes should be dispersed between

[22] The following is a portion of the EU KLEMS Project description which can be found at www.euklems.net. "This project aims to create a database on measures of economic growth, productivity, employment creation, capital formation and technological change at the industry level for all European Union member states from 1970 onwards." The project is funded by the European Commission.

[23] Total capital's share is typically estimated as a residual. However, Blanchard (1997) computes total capital's share directly in a time series analysis. The data necessary to compute total capital's share in a cross-country setting as Blanchard did in a time series setting is not available for the year 2000. However, the EU KLEMS approach is very similar to Blanchard's method, and I thank Olivier Blanchard for making me aware of the EU KLEMS technique.

capital and labor according to the type of tax, but detailed tax data is rarely available, and most countries only report an aggregate tax value. That being said, I follow the default procedure of the EU KLEMS Project and allocate all production taxes to capital compensation.

Also, equation (1.11) is a direct measure of total capital's share. Total capital's share computed via the *OSPUE adjustment* and the *labor force correction* are indirect measures. Specifically, total capital's share is what remains once total labor's share is subtracted from 1. Therefore, the previous two methods assume perfect competition, but the third does not.

Table 2.3 of the 2006 version of the United Nations Yearbook of National Account Statistics is the data source for *OS*. As discussed previously, data for *IT* and *GDP* are also reported in this United Nation's publication. For the year 2000, the necessary data for computing estimates of $(\alpha + \gamma)_{No\ Adjustment}$ are available for 80 countries. The shares are reported in Table 1.14 and plotted against real GDP per worker in Figure 1.8.

Formal regression analysis reveals a negative and statistically significant relationship between total capital's share and real GDP per worker once heteroskedasticity is controlled for. The regression results in column 5 of Table 1.15 indicate that the Summers and Heston Numerical Quality Score is negatively and significantly related to total capital's share at the 5% level.[24] Given that the Numerical Quality Score is significantly related to real GDP per worker, as is revealed by regression results reported in column 2 of Table 1.9, data quality's systematic relationship with total capital's share is at least partially responsible for the heteroskedasticity.

I correct for the heteroskedasticity associated with data quality by implementing WLS. First, I estimate

$$e^2 = \lambda_0 + \lambda_1 Numerical\ Quality\ Score + v \tag{1.12}$$

[24] Column 6 of Table 1.15 reports the results of regressing total labor's share on each of the three components used to derive the Numerical Quality Score. It is evident that the Variance Measure is the driving force behind the significant relationship between the Numerical Quality Score and total capital's share.

by OLS, where e represents the residuals yielded by regressing total capital's share on real GDP per worker, y. Given the coefficient estimates, $\hat{\lambda}_0$ and $\hat{\lambda}_1$, I define the weighting term for country i as

$$w_i = \frac{1}{\sqrt{\hat{\lambda}_0 + \hat{\lambda}_1 Numerical\ Quality\ Score_i}}. \tag{1.13}$$

Weighted least squares estimates are obtained by applying OLS to

$$\frac{(\alpha + \gamma)_{No\ Adjustment\ i}}{w_i} = \theta_0 \left[\frac{1}{w_i}\right] + \theta_1 \left[\frac{y_i}{w_i}\right] + error_i \tag{1.14}$$

for $i = 1...n$ where n is the number of countries in the sample.

The results from estimation of (1.14) are reported in column 3 of Table 1.15. The coefficient on real GDP per worker is negative and significant at the 10% level. When WLS is performed using the three components from which the Numerical Quality Score is derived, the coefficient on real GDP per worker is significant at the 5% level.[25] See column 4 of Table 1.15.

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[25] In this case, I first estimate $e^2 = \lambda_0 + \lambda_1 Variance\ Measure + \lambda_2 Benchmark\ Measure + \lambda_3 Data\ Rank\ Measure + v$. The weighting term for country i is defined as $w\_2_i = \frac{1}{\sqrt{\hat{\lambda}_0 + \hat{\lambda}_1 Variance\ Measure + \hat{\lambda}_2 Benchmark\ Measure + \hat{\lambda}_3 Data\ Rank\ Measure}}$. Weighted least squares estimates are then obtained by applying OLS to $\frac{(\alpha + \gamma)_{No\ Adjustment_i}}{w\_2_i} = \theta_0 \left[\frac{1}{w\_2_i}\right] + \theta_1 \left[\frac{y_{worker_i}}{w\_2_i}\right] + error_i$.

1.3.7 Physical and Natural Capital's Share, *No Adjustment for Self-employed Income*

Physical capital's share, $\alpha_{No\ Adjustment}$, is estimated as $\frac{K}{W} \cdot (\alpha + \gamma)_{No\ Adjustment}$. The necessary data is available for 59 countries. Table 1.16 reports the estimates, and Figure 1.9 depicts the relationship between the estimates and real GDP per worker. OLS estimation, which is reported in column 1 of Table 1.18, reveals a positive and statistically significant relationship between physical capital's share and real GDP per worker at the 1% level. Thus, the empirical evidence supports the theoretical prediction of a positive correlation between physical capital's share and the stage of economic development across countries.

Table 1.17 presents estimates of natural capital's share associated with the *No Adjustment* method, and these estimates, just like their *OSPUE adjustment* and *labor force correction* counterparts, are computed as the residuals remaining after physical capital's share is subtracted from total capital's share. The relationship between the estimates and real GDP per worker is depicted in Figure 1.10. Regressing natural capital's share on an intercept and real GDP per worker using standard OLS reveals a negative and statistically significant slope coefficient at the 1% level. Regression results are reported in column 2 of Table 1.18.

1.4 Decomposition of Total Labor's Share

I turn now to disentangling unskilled labor's share from human capital's share. Cross country estimates of total labor's share incorporate *Employee Compensation*. *Employee Compensation* conflates the income paid to unskilled labor and the income paid to human capital. My approach involves estimating the income paid to unskilled labor and then computing unskilled labor's share. Human capital's share is the residual left over after subtracting unskilled labor's share from total labor's share. Three methods, each corresponding to a different treatment of self-employed income in the computation of total labor's share, are considered. The treatment of self-employed income for each method discussed below is identical to the treatment of self-employed income for the same method in Section 1.3.

1.4.1 Total Labor's Share, *OSPUE Adjustment*

Let η denote unskilled labor's share and let β denote human capital's share. Assuming that self-employed income is allocated to labor and capital in the same proportions as corporate sector income, total labor's share can be computed as

$$(\eta + \beta)_{OSPUE} = \frac{Employee\ Compensation}{GDP - Indirect\ Taxes - imputed\ OSPUE}. \quad (1.15)$$

The components of equation (1.15) and their data sources have already been discussed. Estimates of $(\eta + \beta)_{OSPUE}$ for 2000 are presented in Table 1.19 and plotted against real GDP per worker in Figure 1.11. The sample consists of the same 33 countries for which estimates of $(\alpha + \gamma)_{OSPUE}$ were presented. The *OSPUE* adjusted total labor share estimate is the perfect competition counterpart to the *OSPUE* adjusted total capital share estimate. Therefore, the estimates sum to one, and statistical inference reveals a quadratic relationship between total labor's share and real GDP per worker. The only difference is that the inference for total labor's share indicates downward concavity instead of upward concavity. Nonetheless, for completeness, regression results are presented in Table 1.20.

1.4.2 Unskilled Labor's Share, Accounting for the Self-employed

Ashenfelter and Jurajda (2001) collect average hourly gross wage rates for McDonalds's restaurants across 27 countries for the year 2000.[26] The McDonald's rates represent different compensations for identical jobs, and the authors use the rates to perform

[26] The wages, in general, are collected for McDonalds restaurants in large, urban areas.

cross-country wage comparisons.[27] I use the average McDonald's wage rate to proxy for the compensation paid to an unskilled unit of labor. Such a proxy is reasonable because the wage rates that are collected are for basic entry level jobs, and these jobs do not require experience or any type of formal education or training. Employees generally begin working as crew members and are assigned to specific food preparation stations. They are then rotated through various stations and then to the sales counter where they work as cashiers. The wages are comparable across countries because the duties performed by entry level employees are identical across countries. McDonald's restaurants operate with a standardized protocol for employee work. The preparation of food is extremely mechanized, and the equipment used varies little across restaurants within and between countries.

Given knowledge of hours worked and the number of workers in a country, the average hourly unskilled wage rate can be converted to a total wage bill under the hypothetical scenario that all workers in a country are compensated at the unskilled wage rate. This hypothetical wage bill as a fraction of total output is my estimate of unskilled labor's share.

I obtain average hours worked per worker in the year 2000 from table 4A in the Yearly Statistics section of the ILO's LABORSTA website.[28] This series is usually presented in terms of the average number of hours worked per week, though in a few cases, hours worked per month are reported. The type of worker encompassed by the reported averages varies from country to country. Some averages are computed based on total employment, which includes employees and self-employed workers, and some are computed based on paid employment, which includes only employees.

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[27] McDonalds wages are different within countries and within cities. Ashenfelter and Jurajda note that these differences are usually related to full-time/part-time status and seniority. They control for both issues when compiling their data.

[28] For a few countries, average hours worked data is not reported in table 4A of the LABORSTA website. In these cases I obtain data from the ILO's October Inquiry and compute a weighted average using the number of workers employed. The October Inquiry reports average hours of work per week or per month, for up to 159 occupations. Table 2B in the Yearly Statistics section of the LABORSTA database reports employment numbers categorized by industry. I weight the average hours worked for each occupation by the fraction of employees who work in the industry of which the particular occupation belongs.

To compute the total unskilled wage bill for each country in the year 2000, I first multiply the average hourly McDonald's wage rate for an individual by the average number of hours worked. I then multiply by either 52 or 12, depending on whether average hours worked is reported in per week or per month form respectively. This yields the average yearly compensation of an unskilled worker in 2000. Finally, *Employment*, which is reported in table 2A in the Yearly Statistics section of the LABORSTA database, is multiplied by average yearly compensation of an unskilled worker to obtain the total unskilled wage bill.

Two implicit assumptions associated with my approach should be noted. First, recall that average hours worked pertains to total employment for some countries and only paid employment for others. The LABORSTA database makes it clear as to which workers are included in the reported data, but when I create the average yearly compensation of an unskilled worker, I treat all average hours worked data the same. I do not distinguish between average hours worked for total employment and average hours worked for paid employment. Thus, I am assuming that average hours worked by employees is equivalent to average hours worked by the self-employed. Secondly, since *Employment* encompasses employed and self-employed workers, multiplying average yearly compensation by *Employment* means I am assuming that employed and self-employed workers command equivalent wages.

By construction, the unskilled wage bill already incorporates the labor income of unskilled self-employed workers. There is no need to make any sort of adjustment by subtracting *OSPUE*, and the unskilled wage bill is just divided by *GDP* less *Indirect Taxes* so that unskilled labor's share is given by

$$\eta_{se} = \frac{\textit{Unskilled Wage Bill}}{\textit{GDP} - \textit{Indirect Taxes}} \tag{1.16}$$

where the *se* subscript denotes *self-employed* and indicates that the estimate accounts for the income of the self-employed. The data needed to estimate η_{se} is available for 15 countries, and the estimates are presented in column 1 of Table 1.21.[29] Figure 1.12 plots these estimates against real GDP per worker. OLS estimation reveals a negative relationship between unskilled labor's share and the stage of economic development. These results are presented in column 1 of Table 1.22, and the slope coefficient is statistically significant at the 5% level.

1.4.3 Human Capital's Share, *OSPUE Adjustment*

Human capital's share is computed as a residual and given by

$$\beta_{OSPUE} = (\eta + \beta)_{OSPUE} - \eta_{se}. \tag{1.17}$$

Of the 15 countries for which η_{se} could be computed, only 10 of them overlap with countries for which $(\eta + \beta)_{OSPUE}$ could be computed. Column 2 of Table 1.21 presents the estimates of β_{OSPUE}. Figure 1.13 plots these estimates against real GDP per worker. The regression results reported in column 2 of Table 1.22 reveal a positive slope coefficient, which is in line with theoretical predictions, but the coefficient is statistically insignificant. Thus, inference based on the 10 country full sample indicates no systematic relationship between human capital's share and the stage of economic development. However, Germany's human capital share, which takes on a value of 0.243, the lowest in the sample, is an outlier.[30] With real

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[29] For clarity, an example of the computation of unskilled labor's share for Canada is given below. As can be seen in Table 1.21, unskilled labor's share in Canada is equal to 0.192. I arrive at this number in the following manner. The average hourly gross wage rate for McDonald's cashier and crew workers was equal to 6.95 Canadian dollars in 2000. Average hours worked per week by a worker in 2000, which I compute as a weighted average using the ILO's October Inquiry, is 36.9. *Employment* equals 14,764,200 in 2000. Therefore, the unskilled wage bill is equal to 6.95*36.9*52*14,764,200=1.969×10^{11}. GDP in Canada for the year 2000 is 1.07658×10^{12}, and *Indirect Taxes* equal 5.1691×10^{10}. Thus, unskilled labor's share in Canada in the year 2000 is $\frac{1.969 \times 10^{11}}{1.07658 \times 10^{12} - 5.1691 \times 10^{10}} = 0.192$.

[30] Germany is unique in the sense that most of its economic prosperity is generated by activity in the western part of the country. Even after reunification, the standard of living remains significantly higher in the former West German States. The West's prosperity is undoubtedly responsible for the country's high level of output

GDP per worker just over \$51,000, the corresponding human capital share of 0.243 stands out in Figure 1.13.[31] Because there are only 10 observations, data points that take on extreme values relative to the others in the sample have a substantial impact on the OLS estimation. When Germany is omitted, the slope coefficient remains positive and becomes statistically significant at the 5% level. See column 3 of Table 1.22 for the regression results omitting Germany.

Though this result would be more appealing had it been obtained with a larger sample, the implications of the result should not be dismissed. In spite of the small sample size, the positive correlation is confirmed statistically for real GDP per worker that ranges from about \$16,600 in Russia all the way up to \$67,000 in the U.S. So, the systematic relationship between human capital's share and real GDP per worker that exists when Germany is omitted is not specific to a cluster of countries at similar stages of economic development.

1.4.4 Total Labor's Share, *Labor Force Correction*

Total labor's share computed via the *labor force correction* is given by

$$(\eta+\beta)_{labor\ force\ correction} = \frac{Employee\ Compensation}{(Corporate\ Share\ of\ Labor\ Force)\cdot(GDP - Indirect\ Taxes)}. \quad (1.18)$$

The necessary data sources have already been discussed. Table 1.23 reports the estimate of $(\eta+\beta)_{labor\ force\ correction}$ for the year 2000, and Figure 1.14 depicts the relationship between these estimates and real GDP per worker. OLS estimation reveals a statistically significant quadratic relationship. Regression results are reported in Table 1.24.

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per worker. The economic conditions in the East along with my specific methodology may be responsible for the share result. If the average McDonald's wage rate for Germany overstates the wage earned by individuals in the East, then the estimate of unskilled labor's share for Germany is too high and consequently human capital's share is too low.

[31] The regression line shown in Figure 1.13 is derived after omitting Germany.

1.4.5 Unskilled Labor's Share and Human Capital's Share, *Labor Force Correction*

The only difference between the labor share analysis that employs the *OSPUE adjustment* and that which employs the *labor force correction* is the way in which total labor's share is adjusted to account for self-employed income. The unskilled wage bill used to estimate unskilled labor's share in Section 1.4.2 encompasses the labor income of the unskilled self employed, and the wage bill is not dependent on assumptions pertaining to the division of self-employed income between capital and labor as is total labor's share. Therefore, the estimate of unskilled labor's share is the same for the *labor force correction* as it is for the *OSPUE adjustment*.

Human capital's share, on the other hand, is a residual and is dependent on whether the *OSPUE adjustment* or the *labor force correction* is used to compute total labor's share. The *labor force correction* estimates of human capital's share are presented in Table 1.25 and plotted against real GDP per worker in Figure 1.15. When the full sample of 10 observations is considered, regressing human capital's share on an intercept and real GDP per worker yields a positive but insignificant slope coefficient. Results are reported in column 1 of Table 1.26.

Germany's human capital share is the lowest in the sample at 0.257, and the outlying nature of this estimate can be seen in Figure 1.15.[32] I report the OLS estimation results when Germany is omitted from the sample in column 2 of Table 1.26. The slope coefficient is positive and is now significant at the 10% level.

The remaining nine human capital share values in the reduced sample correspond to real GDP per worker values that range from a minimum of $16,642 in Poland to a maximum of $67,078 in the USA. This is a difference in real GDP per worker of over $50,000. This

[32] Germany is omitted in the derivation of the regression line shown in Figure 1.15.

means the result is not specific to a cluster of developed or undeveloped countries, and so the positive correlation should not be dismissed simply because the sample is small.

1.4.6 Total Labor's Share, *No Adjustment for Self-employed Income*

The estimate of total labor's share computed without any adjustment for self-employed income is given by

$$(\eta + \beta)_{No\ Adjustment} = \frac{Employee\ Compensation}{GDP}. \tag{1.19}$$

This measure is the so called *naïve* measure reported by Bernanke and Gurkaynak (2001) and Gollin (2002). These authors consider the estimate $(\eta + \beta)_{No\ Adjustment}$ to be naïve because it attributes the income of the self-employed to capital income. An argument in favor of such an attribution has been discussed previously herein. The total labor share estimate given by equation (1.19) is derived in similar spirit to the total capital share estimate given by equation (1.11), and just like the total capital share measure, the total labor share measure assumes all taxes associated with production are allocated to capital.

Table 27 presents estimates of $(\eta + \beta)_{No\ Adjustment}$ for the 93 countries for which the necessary data is available. The relationship between the estimates and real GDP per worker is depicted graphically in Figure 1.16. Regression results indicate a statistically significant quadratic relationship between total labor's share and real GDP per worker. The coefficient on the squared term is negative, indicating downward concavity, and it is statistically significant at the 1% level. Estimation results are reported in Table 1.28.

1.4.7 Unskilled Labor's Share and Human Capital's Share, *No Adjustment for Self-employed Income*

Since total labor's share computed via equation (1.19) treats self-employed income as capital income, unskilled labor's share computed in accordance with this approach should reflect the unskilled labor compensation of only employees. Therefore, the average yearly compensation of an unskilled worker is computed just as it was for the *OSPUE adjustment* and the *labor force correction*, but it is multiplied by *Paid Employment* rather than *Employment* to arrive at the unskilled wage bill.

The implicit assumption that employees and self-employed workers work an equivalent amount of hours on average is still present in this approach because there is no change in the computation of average yearly compensation. However, the implicit assumption that employees and self-employed workers command equivalent wages is no longer present because the average compensation of unskilled workers in this case is scaled up by *Paid Employment*, which only encompasses employees. *Employment* encompasses employees and self-employed workers.

Also, to obtain unskilled labor's share, the unskilled wage bill is only divided by *GDP,* not *GDP - Indirect Taxes*. This is consistent with the assumption that all taxes on production are allocated to capital compensation. Estimates of unskilled labor's share for which an adjustment for self-employed income is not made are reported in column 1 of Table 1.29 and plotted against real GDP per worker in Figure 1.17.

Regressing unskilled labor's share on an intercept and real GDP per worker using OLS estimation yields a slope coefficient that is statistically insignificant. The estimation results, which are reported in column 1 of Table 1.30, do not support the theoretical prediction that unskilled labor's share decreases as the stage of economic development increases.[33]

[33] Germany's unskilled labor share of 0.352 is the highest in the sample and an outlier in Figure 1.17. However, omitting Germany does not change the qualitative results.

Human capital shares are computed as residuals, and the estimates are reported in column 2 of Table 1.29. Figure 1.18 suggests a strong, positive correlation between human capital's share and real GDP per worker, and statistical analysis confirms the correlation. Regressing human capital's share on an intercept and real GDP per worker yields a positive slope coefficient that is significant at the 1% level. The results are reported in column 2 of Table 1.30 and indicate that human capital's share, as predicted by theory, is positively and systematically related to the stage of economic development.

1.5 Remarks

1.5.1 Key Departures and Contributions

The cross-country analysis of factor shares presented herein is more complete than the analyses of Zuleta (2008a) and Caselli and Feyrer (2007), and techniques that I employ represent clear departures from these studies. First, I decompose both total capital's share and total labor's share into reproducible and non-reproducible components. Caselli and Feyrer only separate physical capital's share from natural capital's share. They do not address total labor's share and its components. Zuleta decomposes total capital's share and total labor's share, but when analyzing total capital's share he only separates land's share from physical capital's share. Other natural resources including oil, natural gas and minerals are encompassed by the typical total capital share measure and should be distinguished from physical capital. My analysis, just as that of Caselli and Feyrer, makes this distinction and separates physical capital's share from natural capital's share, not just land's share. That being said, each of the two aforementioned studies contains a crucial element that the other study omits. I incorporate elements from both studies into a single, comprehensive analysis.

Second, to disentangle physical capital's share from natural capital's share I use the same wealth data published by The World Bank (2006) for the year 2000 that Caselli and Feyrer use. But, instead of combining total capital share estimates for the period 1980-1995 with the wealth data for 2000, I compute and then combine total capital share estimates for

2000 with the wealth data for 2000. Caselli and Feyrer implicitly assume that average total capital's share over the period 1980-1995 is equivalent to total capital's share for the year 2000. Factor shares are not constant over time, so such an assumption can yield misleading results.[34] If one compares the physical capital share estimates reported in Caselli and Feyrer's Table II to physical capital share estimates reported in my Table 1.5, it is obvious that the time period used for total capital share estimation is nontrivial for some countries.

Consider Ireland, Greece and Egypt for example. For these three countries, I obtain physical capital share estimates equal to 0.327, 0.309 and 0.237 respectively. Caselli and Feyrer report physical capital share estimates for these countries equal to 0.18, 0.15, and 0.10. For Greece and Egypt, my estimates are more than double those reported by Caselli and Feyrer. For Ireland, my estimate is nearly double that reported by Caselli and Feyrer. Though the relationship between physical capital's share and real GDP per worker reported by Caselli and Feyrer is qualitatively consistent with the relationship I report, the quantitative results are very different. My technique is methodologically more appealing because I back out physical capital share estimates using wealth data and total capital share estimates from the same time period, the year 2000. Caselli and Feyrer's technique is less appealing because they use different time periods.

I control for heteroskedasticity, and, when warranted, incorporate data quality into my estimation. It turns out that the presence of heteroskedasticity only has a qualitative impact on the estimation results involving total capital's share computed via the method that makes no adjustment for self-employed income. In this case, prior to controlling for heteroskedasticity, the relationship between total capital's share and real GDP per worker is statistically insignificant. After incorporating the Summers and Heston data quality measures into a weighted least squares estimation, the relationship becomes statistically significant. Although all other results are unaffected, identifying and controlling for the presence of heteroskedasticity adds credibility to my approach and my inference. In any cross-country

[34] See Sato (1970), Bound and Johnson (1995), Blanchard (1997), and Krueger (1999) for evidence of variation in factor shares over time.

analysis, systematic variation in data quality is a concern. Knowing that the sign and significance of coefficient estimates are true reflections of the relationship between factor shares and real GDP per worker is imperative.

The most striking departure of my analysis from the current literature is the approach used to disentangle human capital's share from unskilled labor's share. Zuleta (2007) posits functional forms for factor shares and uses a Cobb-Douglas production function to estimate growth regressions that require cross-country data on human capital. He then takes the parameter estimates from these regressions and substitutes them into his factor share formulas in order to obtain estimates of human capital's share, unskilled labor's share, land's share and physical capital's share. Although Zuleta's results are, for the most part[35], consistent with the theoretical predictions discussed earlier, there are two aspects of his approach that are unappealing.

First, his factor share estimates are obtained via empirical estimation. Whenever possible, it is better to obtain estimates of economic variables by direct observation. Secondly, Zuleta incorporates human capital data into his analysis. Human capital is a difficult economic variable to proxy. Estimates of human capital vary from source to source depending on, among other things, schooling definitions, assumptions about returns to schooling, the decision about which age groups to use when computing average schooling measures, and assumptions pertaining to work experience.

Barro and Lee have contributed much to the empirical estimation of human capital, but their measures depend heavily on data for average years of education, which, as pointed out by Domenech and de la Fuente (2001), is "very noisy". Zuleta uses human capital data from Domenech and de la Fuente, and though Caselli (2005) argues that the Domenech and de la Fuente data set employs more comprehensive information than that of Barro and Lee (1993), the data set still involves a large degree of interpolation and projection. Domenech

[35] He only makes a distinction between physical capital's share and land's share. Total capital's share includes physical capital's share, land's share, and other natural resource shares. Therefore, Zuleta's physical capital share estimates are too large because they encompass returns to all natural resources, except land.

and de la Fuente even admit that construction of their data "involves a fair amount of guesswork."

I do not use statistical techniques or human capital proxies to obtain my share estimates.[36] Instead, using the definition of a factor share as a guide, I combine direct observations of unskilled wage rates with employment data to obtain estimates of unskilled labor's share.[37] Human capital's share is then the residual remaining after unskilled labor's share is subtracted from total labor's share. Though my technique involves the assumption that average McDonalds' cashier and crew wages represent average unskilled labor compensation, my estimates, unlike Zuleta's estimates, are not functions of statistically estimated parameters that are subject to measurement error and dependent on the functional form of a production function.

1.5.2 Real GDP per Worker vs. Real GDP per Capita

On a much different note, I relate factor shares to real GDP per worker, not real GDP per capita. Though not equivalent, real GDP per worker and real GDP per capita are closely related. The BLS (2007) relates real GDP per capita and real GDP per worker in the following way:

$$\frac{Y}{Pop} = \left(\frac{L}{Pop}\right)\left(\frac{Y}{L}\right) \tag{1.20}$$

where *Pop* denotes population; *L* denotes the labor force; and *Y* denotes real GDP. Given this decomposition, real GDP per capita is a function of the labor force participation rate and

[36] For many empirical macroeconomic exercises, human capital proxies have to be used. For example, estimation of aggregate production functions and estimation of growth regressions typically require human capital data. Thus, the argument here is not that human capital is difficult to measure, and, as a result, human capital proxies should not be used in empirical work. The argument is that human capital is difficult to measure, and human capital proxies should not be used if another technique will suffice.

[37] Young and Zuleta (2008) directly compute unskilled labor's share in a time series framework. My approach, though conducted in a cross-country setting and very different in terms of data and computations, is conducted in a similar spirit.

real GDP per worker. It is because some people in the population do not work that real GDP per worker and real GDP per capita are not equivalent.

Generally, when macroeconomists analyze aggregate production, they focus on real GDP per worker. When the analysis is more welfare oriented, real GDP per capita is the more relevant measure. Either measure could effectively be used to proxy for the stage of economic development, but real GDP per capita, through its inclusion of the labor force participation rate, reflects social and political variables to a greater extent than does real GDP per worker. The labor force participation rate, $\frac{L}{Pop}$, could differ across countries as a result of differences in, among other things, labor laws, the degree of government intervention in the economy, and social acceptance of women in the workplace. Real GDP per worker is narrower in its focus and is a better measure of economic development. In addition, a factor share, by definition, is directly related to production. That said, if a per individual measure of output is going to be used to relate economic development to factor shares, real GDP per worker seems a better measure since real GDP per capita encompasses individuals who are not even part of the production process. Nonetheless, real GDP per worker and real GDP per capita move together, and though not reported, the qualitative results are unchanged when real GDP per capita is used in place of real GDP per worker.

1.5.3 The Treatment of Self-employed Income

Acceptance or rejection of the evidence presented in this Chapter, at least from a qualitative perspective, is not dependent on the reader's acceptance or rejection of my explanation for the treatment of self-employed income as capital income in the *No Adjustment* approach. If the reader agrees that a portion of self-employed income should be counted as employee compensation, then total capital's share computed via *No Adjustment* will be too high, and either the *OSPUE adjustment* or the *labor force correction* is more appealing. If the reader agrees that self-employed income is capital income, then he is more comfortable with total capital's share computed via *No Adjustment* and feels that the *OSPUE* *adjustment* and the *labor force correction* yield total capital share estimates that are too small. Either way, the qualitative relationships between factor shares and real GDP per worker are robust to the treatment of self-employed income, except in the case of unskilled labor's share.

Unskilled labor share estimates computed via the *OSPUE adjustment* or the *labor force correction* are negatively related to real GDP per worker. However, no systematic relationship exists between unskilled labor's share and real GDP per worker when the *No Adjustment* approach is used and self-employed income is treated as capital compensation. This does not imply that computing unskilled labor's share without making an adjustment for self-employed income is incorrect. Human capital's share, which is a residual and thus directly linked to the unskilled labor share estimate, is positively and significantly related to real GDP per worker across all three approaches. In other words, the human capital share estimate is a function of the unskilled labor share estimate, and results indicate that, regardless of how unskilled labor's share is computed, human capital's share is positively related to output per worker, which corresponds to economic theory. The absence of a systematic relationship between unskilled labor's share and output per worker when the *No Adjustment* approach is used may just be a result of too few observations.

1.5.4 Statistical Tests

Finally, I determine the significance levels of slope coefficients. Others, for whatever reason, do not perform any statistical tests to support their conclusions. Gollin (2002) and Bernanke and Gurkaynak (2001) make claims about the relationship between share estimates and output per worker by eyeballing data tables and scatter plots. I use two-tailed tests to determine the significance levels of slope coefficients for all analyses pertaining to either total labor's share or total capital's share. The purpose here is to ascertain whether there is any relationship, be it positive or negative, between share estimates and output per worker. Theory yields no predictions about the relationship, so the alternative hypothesis is that the slope coefficient differs from zero.

On the other hand, theory yields specific predictions about the nature of the relationship between non-reproducible and reproducible factor shares and output per worker. Therefore, the significance levels of slope coefficients are determined using one-tailed tests for all analyses pertaining to either physical capital's share, natural capital's share, unskilled labor's share or human capital's share. The purpose here is to ascertain whether there is a positive or negative relationship. The alternative hypothesis is that the slope coefficient is greater than zero if the factor share is reproducible and less than zero if the factor share is non-reproducible.

1.6 Conclusion

Skepticism about the constancy of factor shares dates back to the time of Keynes and Solow, but only recently have theoretical analyses like that of Peretto and Seater (2008) and Zuleta (2008b) yielded specific predictions about the systematic relationship between cross-country factor shares and the stage of economic development. I provide empirical evidence consistent with these theoretical claims, and, specifically, my results reveal that non-reproducible factor shares decrease with the stage of economic development, and reproducible factor shares increase with the stage of economic development. This result suggests that factor eliminating technical progress is a potentially important phenomenon, and incorporation of such progress into models of economic growth should be considered.

Theoretical or empirical studies that incorporate the assumption of constant factor shares should be revisited. Researchers rarely make a distinction between reproducible and non-reproducible factors. As a result, the shares that are typically considered are composite shares that conflate the fractions of income paid to fundamentally different factors of production. A very common approach is to combine all factors of production into one of two categories: capital or labor. The standard capital share measure conflates physical capital's share and natural capital's share. The standard labor share measure conflates human capital's share and unskilled labor's share. Failure to acknowledge the composite nature of the standard share measures can yield misleading conclusions. The results presented in Chapter

1 reveal that the systematic relationship between composite shares and the stage of economic development is different from the systematic relationship between a single, non-reproducible or reproducible share and the stage of economic development. Kaldor (1961), whose “stylized facts” are often cited, concluded that factor shares were constant over time and across countries without making a distinction between reproducible and non-reproducible factors. This distinction turns out to be very important.

One macroeconomic exercise that virtually always assumes constancy of factor shares is the estimation of Total Factor Productivity (TFP). Examples in the literature include Young (1995), Klenow and Rodriguez-Clare (1997), Hall and Jones (1999), Caselli (2005), and Baier, Dwyer and Tamura (2006). In Chapter 2, I revisit the estimation of TFP, acknowledging variation in factor shares and making a distinction between non-reproducible and reproducible factors. Specifically, I compare the fraction of cross-country variation in economic performance attributable to variation in TFP to the fraction of cross-country variation in economic performance attributable to variation in factors and factor shares. Rather than assume factor shares are constant across countries, I allow factor shares to vary in accordance with the estimates presented here in Chapter 1.

Table 1.1: Total Labor's Share-Gollin's Data

	Regression Equation					
	Adjustment 1		Adjustment 2		Naive Measure	
Variable	**1**	**2**	**3**	**4**	**5**	**6**
Intercept	0.803***	0.798***	0.675***	0.664***	0.362***	0.345***
	(37.104)	(33.040)	(27.256)	(25.851)	(9.279)	(8.113)
Real GDP per capita	-3.358E-06	---	2.841E-06	---	1.319E-05***	
	(-1.536)		(1.135)		(3.348)	
Real GDP per worker	---	-8.342E-07	---	1.232E-06	---	4.554E-06***
		(-1.088)		(1.509)		(3.371)
Adjusted R^2	0.054	0.008	0.012	0.012	0.298	0.320
Sample	25 obs.	23 obs.	25 obs.	23 obs.	25 obs.	23 obs.

--The dependent variable is total labor's share. Adjustment 1 treats all self-employed income as labor income, and Adjustment 2 treats self-employed income as comprising the same mix of labor and capital income as the corporate and government sectors of the economy.

--The naiive measure treats all self-employed income as capital income.

--t-statistics computed using the typical OLS standard errors are in parantheses.

--Though not reported, I also computed White corrected standard errors, and the corresponding t-statistics do not change the conclusions about the overall significance of a variable nor the level of significance.

--* indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level.

Table 1.2: Average Total Labor's Share vs. Average Real GDP per Capita, 1980-1995
Bernanke and Gurkaynak Data

	Regression Equation	
Variable	**1**	**2**
Intercept	0.605***	0.599***
	(30.036)	(26.634)
	[25.673]	[22.072]
Average Real GDP per capita	5.587E-06***	---
	(2.873)	
	[2.925]	
Average Real GDP per worker		2.159E-06***
		(2.783)
		[2.736]
Adjusted R^2	0.120	0.113
Sample	54 obs.	54 obs.

--The dependent variable is average total labor's share over the period 1980-1995 and comes directly from Bernanke and Gurkaynak (2001).
--Real GDP per capita data comes from version 6.2 of the Penn World Tables.
--t-statistics computed using the typical OLS standard errors are in parantheses. t-statistics computed using White corrected standard errors are in brackets.
--* indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level
--The t-statistics are such that the significance levels of the coefficient parameters are unchanged no matter the standard error used.

Table 1.3: Total Capital's Share, 2000 (*OSPUE Adjustment*)

Country	Total Capital's Share	Country	Total Capital's Share
Australia	0.384	Japan	0.256
Austria	0.398	Korea, Republic Of	0.332
Belgium	0.340	Mauritius	0.354
Botswana	0.534	Mexico	0.518
Canada	0.334	Netherlands	0.418
Costa Rica	0.345	New Zealand	0.418
Czech Republic	0.472	Norway	0.526
Denmark	0.408	Panama	0.361
Egypt	0.538	Poland	0.379
Finland	0.418	Portugal	0.326
France	0.376	Russia	0.485
Germany	0.360	Singapore	0.443
Greece	0.443	Spain	0.306
Hungary	0.400	Sweden	0.351
Ireland	0.497	Trinidad and Tobago	0.409
Israel	0.313	U.S.A	0.320
Italy	0.408		

Source: Author's Calculations.

Table 1.4: Total Capital's Share - *OSPUE Adjustment*

Variable	
Intercept	0.361***
	(20.886)
u	-0.011
	(-0.910)
u^2	0.04***
	(2.890)
F-test for overall significance of regression	5.735
	[3.316]
Adjusted R^2	0.228
F-test for no heteroskedasticity	3.215
	[3.316]
Sample	33 obs.

--Dependent variable is Total Capital's share.
--*u* is a coded independent variable used in place of real GDP per worker.
--t-statistics are in parantheses.
--brackets are 5% critical values of the F distribution.
--*indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level.

Table 1.5: Physical Capital's Share, 2000 (*OSPUE Adjustment*)

Country	Physical Capital's Share	Country	Physical Capital's Share
Australia	0.219	Japan	0.204
Austria	0.293	Korea, Republic Of	0.251
Belgium	0.261	Mauritius	0.271
Botswana	0.318	Mexico	0.289
Canada	0.164	Netherlands	0.305
Costa Rica	0.137	New Zealand	0.154
Denmark	0.287	Norway	0.291
Egypt	0.237	Panama	0.200
Finland	0.284	Portugal	0.236
France	0.273	Russia	0.186
Germany	0.273	Singapore	0.357
Greece	0.309	Spain	0.222
Hungary	0.245	Sweden	0.249
Ireland	0.327	Trinidad and Tobago	0.105
Israel	0.231	U.S.A	0.218
Italy	0.302		

Source: Author's Calculations.

Table 1.6: Natural Capital's Share, 2000 (*OSPUE Adjustment*)

Country	Natural Capital's Share	Country	Natural Capital's Share
Australia	0.165	Japan	0.052
Austria	0.106	Korea, Republic Of	0.080
Belgium	0.079	Mauritius	0.083
Botswana	0.217	Mexico	0.230
Canada	0.170	Netherlands	0.114
Costa Rica	0.207	New Zealand	0.264
Denmark	0.121	Norway	0.235
Egypt	0.301	Panama	0.161
Finland	0.134	Portugal	0.091
France	0.103	Russia	0.299
Germany	0.087	Singapore	0.086
Greece	0.134	Spain	0.084
Hungary	0.156	Sweden	0.102
Ireland	0.170	Trinidad and Tobago	0.304
Israel	0.081	U.S.A	0.102
Italy	0.106		

Source: Author's Calculations.

Table 1.7: Physical Capital's Share and Natural Capital's Share - *OSPUE Adjustment*

Variable	Dependent Variable: Physical Capital's Share	Dependent Variable: Natural Capital's Share
Intercept	0.200***	0.251***
	(6.880)	(7.656)
real GDP per worker, y	1.162E-06**	-2.421E-06***
	(1.791)	(-3.307)
Adjusted R^2	0.069	0.249
F-test for no heteroskedasticity	0.511	0.205
	[3.340]	[3.340]
Sample	31 obs.	31 obs.

--t-statistics are in parantheses.
--brackets are 5% critical values of the F distribution.
--*indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level.

Table 1.8: Total Capital's Share, 2000 (*Labor Force Correction*)

Country	Total Capital's Share	Country	Total Capital's Share
Australia	0.370	Japan	0.264
Austria	0.387	Korea, Republic Of	0.282
Belgium	0.315	Mauritius	0.311
Botswana	0.505	Mexico	0.494
Canada	0.320	Netherlands	0.408
Costa Rica	0.315	New Zealand	0.410
Czech Republic	0.457	Norway	0.520
Denmark	0.396	Panama	0.296
Egypt	0.451	Poland	0.334
Finland	0.396	Portugal	0.289
France	0.363	Russia	0.470
Germany	0.346	Singapore	0.443
Greece	0.390	Spain	0.276
Hungary	0.349	Sweden	0.337
Ireland	0.482	Trinidad and Tobago	0.409
Israel	0.291	U.S.A	0.320
Italy	0.372		

Source : Author's Calculations

Table 1.9: Data Quality

Variable	Sample: *Labor Force Correction*	Sample: *No Adjustment*
Intercept	9.302***	9.088***
	(9.973)	(11.760)
real GDP per worker	1.411E-04***	9.620E-05***
	(6.603)	(4.194)
Adjusted R^2	0.571	0.175
Sample	33 obs.	79 obs.
correlation coefficient	0.765	0.431

--Dependent variable is the Summers and Heston Numerical Quality Score.
--The sample considered in column 2 corresponds only to the analysis that involves total capital's share. The "No adjustment" sample in the analysis of total labor's share includes more countries.
--The correlation coefficient is computed for The Numerical Quality score and real GDP per worker.
--t-statistics are in parantheses.
--*indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level.

Table 1.10: Total Capital's Share - *Labor Force Correction*

	Regression Equation			
Variable	**1**	**2**	**3**	**4**
Intercept	0.339***	0.339***	0.458***	0.507***
	(18.484)	(21.081)^W	(7.110)	(5.883)
u	0.002	0.002		
	(0.151)	(0.124)^W		
u^2	0.037**	0.037**		
	(2.550)	(2.198)^W		
Numerical Quality Score			-5.561E-03	
			(-1.321)	
Variance Measure				-0.022
				(-1.431)
Benchmark Measure				-0.031
				(-0.854)
Data Rank Measure				0.006
				(0.484)
Adjusted R^2	0.131	0.131	0.023	-0.004
F-test for no heteroskedasticity	4.952			
	[3.316]			
F-test for overall significance of regression	3.412			0.962
	[3.316]			[2.934]
Wald test for overall significance of regression		6.424		
		{5.991}		
Sample	33 obs.	33 obs.	33 obs.	33 obs.

--Dependent variable is total capital's share computed using the *labor force correction* .
--*u* is a coded independent variable used in place of real GDP per worker.
--Variance Measure, Benchmark Measure, and Data Rank Measure are the three individual criterion employed by Summers and Heston to compute the Numerical Quality Score.
--t-statistics are in parantheses.
--W indicates t-statistics computed using White corrected standard errors.
--[ ] are 5% critical values of the F distribution
--{ } are 5% critical values of the X2 distribution
--*indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level

Table 1.11: Physical Capital's Share, 2000 (*Labor Force Correction*)

Country	Physical Capital's Share	Country	Physical Capital's Share
Australia	0.211	Japan	0.211
Austria	0.284	Korea, Republic Of	0.214
Belgium	0.242	Mauritius	0.238
Botswana	0.300	Mexico	0.275
Canada	0.157	Netherlands	0.297
Costa Rica	0.126	New Zealand	0.151
Denmark	0.279	Norway	0.287
Egypt	0.198	Panama	0.164
Finland	0.269	Portugal	0.209
France	0.264	Russia	0.180
Germany	0.262	Singapore	0.357
Greece	0.272	Spain	0.201
Hungary	0.213	Sweden	0.239
Ireland	0.317	Trinidad and Tobago	0.105
Israel	0.216	U.S.A	0.218
Italy	0.275		

Source: Author's Calculations

Table 1.12: Natural Capital's Share, 2000 (*Labor Force Correction*)

Country	Natural Capital's Share	Country	Natural Capital's Share
Australia	0.159	Japan	0.053
Austria	0.103	Korea, Republic Of	0.068
Belgium	0.073	Mauritius	0.073
Botswana	0.205	Mexico	0.219
Canada	0.163	Netherlands	0.111
Costa Rica	0.190	New Zealand	0.259
Denmark	0.117	Norway	0.232
Egypt	0.253	Panama	0.132
Finland	0.127	Portugal	0.080
France	0.099	Russia	0.290
Germany	0.084	Singapore	0.086
Greece	0.118	Spain	0.076
Hungary	0.136	Sweden	0.098
Ireland	0.165	Trinidad and Tobago	0.304
Israel	0.076	U.S.A	0.102
Italy	0.097		

Source: Author's Calculations

Table 1.13: Physical Capital's Share and Natural Capital's Share - *Labor Force Correction*

	Dependent Variable	
	Physical Capital's Share	**Natural Capital's Share**
Variable		
Intercept	0.167***	0.223***
	(6.151)	(6.796)
real GDP per worker, y	1.574E-06***	-1.983E-06***
	(2.587)	(-2.698)
Adjusted R^2	0.159	0.173
F-test for no heteroskedasticity	0.372	0.380
	[3.340]	[3.340]
Sample	31 obs.	31 obs.

--t-statistics are in parantheses

--brackets are 5% critical values of the F distribution

--*indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level.

Table 1.14: Total Capital's Share, 2000 (*No Adjustment*)

Country	Total Capital's Share	Country	Total Capital's Share
Algeria	0.744	Kuwait	0.746
Armenia	0.481	Kyrgyzstan	0.171
Australia	0.419	Latvia	0.364
Austria	0.387	Lesotho	0.808
Azerbaijan	0.722	Lithuania	0.349
Bahrain	0.640	Malta	0.317
Belgium	0.381	Mauritius	0.534
Belarus	0.461	Mexico	0.605
Botswana	0.619	Moldova	0.298
Brazil	0.348	Mozambique	0.349
Bulgaria	0.540	Namibia	0.507
Burkina Faso	0.714	Netherlands	0.387
Cameroon	0.725	Netherlands Antilles	0.334
Canada	0.422	New Zealand	0.545
Chile	0.411	Nicaragua	0.306
Colombia	0.330	Niger	0.107
Costa Rica	0.462	Norway	0.448
Croatia	0.282	Oman	0.575
Cuba	0.484	Panama	0.494
Czech Republic	0.487	Paraguay	0.232
Denmark	0.332	Peru	0.669
Egypt	0.495	Philippines	0.741
Estonia	0.436	Poland	0.485
Finland	0.399	Portugal	0.372
France	0.377	Qatar	0.693
Germany	0.366	Romania	0.474
Greece	0.585	Russia	0.484
Honduras	0.543	Saudi Arabia	0.704
Hungary	0.407	Senegal	0.695
Iceland	0.292	Singapore	0.567
Iran	0.447	Slovakia	0.476
Ireland	0.493	Slovenia	0.258
Israel	0.364	South Africa	0.429
Italy	0.501	Spain	0.410
Ivory Coast	0.276	Sri Lanka	0.488
Japan	0.501	Sweden	0.325
Jordan	0.466	Trinidad and Tobago	0.627
Kazakhstan	0.352	Ukraine	0.425
Kenya	0.539	U.S.A	0.407
Korea, Republic Of	0.459	Venezuela	0.472

Source: Author's Calculations

Table 1.15: Total Capital's Share - *No Adjustment*

	Regression Equation					
	1	**2**	**3**	**4**	**5**	**6**
Variable			**WLS 1**	**WLS 2**		
Intercept	0.476***	0.476***	0.484***	0.487	0.571***	0.566***
	(16.962)	(14.326)^W	(17.797)	(16.003)	(12.625)	(12.450)
real GDP per worker, y	-3.396E-07	-3.396E-07	-1.027E-06*	-1.150E-06**	---	---
	(-0.407)	(-0.376)^W	(-1.730)	(-1.996)		
Numerical Quality Score	---	---	---	---	-8.714E-03**	---
					(-2.410)	
Variance Measure	---	---	---	---	---	-0.031***
						(-2.770)
Benchmark Measure	---	---	---	---	---	0.032
						(1.189)
Data Rank Measure	---	---	---	---	---	-0.009
						(-0.793)
F-test for overall significance of regression	---	---	---	---	---	2.892
						[2.727]
Adjusted R^2	-0.011	-0.011	0.745	0.951	0.058	0.068
F-test for no heteroskedasticity	7.792	---	---	---	---	---
	[3.115]					
Sample	80 obs.	80 obs.	79 obs.	79 obs.	79 obs.	79 obs.

--Dependent variable is Total Capital's Share.

--W indicates t-statistics computed using White corrected standard errors.

--WLS 1 is weighted least squares estimation using just the Numerical Quality Score.

--WLS 2 is weighted least squares estimation using the three individual criterion employed by Summers and Heston in computing the the Numerical Quality Score.

--Regression equations 3 through 6 only include 79 observations because proxies for data quality are unavailable for Netherlands Antilles.

--Brackets are 5% critical values of the F distribution.

--t-statistics are in parantheses. *indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level.

Table 1.16: Physical Capital's Share, 2000 (*No Adjustment*)

Country	Physical Capital's Share	Country	Physical Capital's Share
Algeria	0.239	Korea, Republic Of	0.348
Australia	0.239	Latvia	0.207
Austria	0.284	Lesotho	0.563
Belgium	0.293	Mauritius	0.408
Botswana	0.368	Mexico	0.337
Brazil	0.165	Moldova	0.137
Bulgaria	0.264	Mozambique	0.087
Burkina Faso	0.232	Namibia	0.288
Cameroon	0.158	Netherlands	0.281
Canada	0.208	New Zealand	0.200
Chile	0.164	Nicaragua	0.111
Colombia	0.114	Niger	0.011
Costa Rica	0.184	Norway	0.248
Denmark	0.233	Panama	0.273
Egypt	0.218	Paraguay	0.085
Estonia	0.263	Peru	0.328
Finland	0.271	Philippines	0.378
France	0.274	Portugal	0.269
Germany	0.277	Romania	0.250
Greece	0.408	Russia	0.185
Honduras	0.221	Senegal	0.243
Hungary	0.249	Singapore	0.458
Iran	0.069	South Africa	0.236
Ireland	0.324	Spain	0.298
Israel	0.269	Sri Lanka	0.303
Italy	0.371	Sweden	0.231
Ivory Coast	0.054	Trinidad and Tobago	0.161
Japan	0.400	U.S.A	0.277
Jordan	0.324	Venezuela	0.127
Kenya	0.169		

Source : Author's Calculations

Table 1.17: Natural Capital's Share, 2000 (*No Adjustment*)

Country	Natural Capital's Share	Country	Natural Capital's Share
Algeria	0.506	Korea, Republic Of	0.111
Australia	0.180	Latvia	0.158
Austria	0.103	Lesotho	0.245
Belgium	0.088	Mauritius	0.126
Botswana	0.251	Mexico	0.268
Brazil	0.183	Moldova	0.161
Bulgaria	0.276	Mozambique	0.261
Burkina Faso	0.482	Namibia	0.220
Cameroon	0.567	Netherlands	0.105
Canada	0.215	New Zealand	0.345
Chile	0.247	Nicaragua	0.195
Colombia	0.217	Niger	0.096
Costa Rica	0.277	Norway	0.200
Denmark	0.098	Panama	0.221
Egypt	0.277	Paraguay	0.147
Estonia	0.173	Peru	0.340
Finland	0.128	Philippines	0.363
France	0.103	Portugal	0.103
Germany	0.089	Romania	0.224
Greece	0.177	Russia	0.298
Honduras	0.322	Senegal	0.452
Hungary	0.158	Singapore	0.110
Iran	0.378	South Africa	0.193
Ireland	0.169	Spain	0.112
Israel	0.095	Sri Lanka	0.186
Italy	0.130	Sweden	0.094
Ivory Coast	0.222	Trinidad and Tobago	0.466
Japan	0.101	U.S.A	0.130
Jordan	0.142	Venezuela	0.345
Kenya	0.370		

Source : Author's Calculations

Table 1.18: Physical Capital's Share and Natural Capital's Share - *No Adjustment*

	Dependent Variable	
	Physical Capital's Share	**Natural Capital's Share**
Variable		
Intercept	0.200***	0.308***
	(9.277)	(14.028)
real GDP per worker, y	1.756E-06***	-3.238E-06***
	(2.732)	(-4.945)
Adjusted R^2	0.100	0.288
F-test for no heteroskedasticity	2.489	2.172
	[3.162]	[3.162]
Sample	59 obs.	59 obs.

--t-statistics are in parantheses.

--brackets are 5% critical values of the F distribution

--*indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level.

Table 1.19: Total Labor's Share, 2000 (*OSPUE Adjustment*)

Country	Total Labor's Share	Country	Total Labor's Share
Australia	0.616	Japan	0.744
Austria	0.602	Korea, Republic Of	0.668
Belgium	0.660	Mauritius	0.646
Botswana	0.466	Mexico	0.482
Canada	0.666	Netherlands	0.582
Costa Rica	0.655	New Zealand	0.582
Czech Republic	0.528	Norway	0.474
Denmark	0.592	Panama	0.639
Egypt	0.462	Poland	0.621
Finland	0.582	Portugal	0.674
France	0.624	Russia	0.515
Germany	0.640	Singapore	0.557
Greece	0.557	Spain	0.694
Hungary	0.600	Sweden	0.649
Ireland	0.503	Trinidad and Tobago	0.591
Israel	0.687	U.S.A	0.680
Italy	0.592		

Source: Author's Calculations.

Table 1.20: Total Labor's Share - *OSPUE Adjustment*

Variable	
Intercept	0.639***
	(36.995)
u	0.011
	(0.910)
u^2	-0.040***
	(-2.890)
F-test for overall significance of regression	5.735
	[3.316]
Adjusted R^2	0.228
F-test for no heteroskedasticity	3.215
	[3.316]
Sample	33 obs.

--Dependent variable is Total Labor's share.
--*u* is a coded independent variable used in place of real GDP per worker.
--t-statistics are in parantheses.
--brackets are 5% critical values of the F distribution.
--*indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level.

Table 1.21: Unskilled Labor's Share and Human Capital's Share, 2000 (*OSPUE Adjustment*)

Country	Unskilled Labor's Share	Human Capital's Share
Brazil	0.207	
Canada	0.192	0.474
Czech Republic	0.207	0.321
Germany	0.396	0.243
Hong Kong	0.086	
Japan	0.261	0.483
Korea, Republic Of	0.195	0.473
Philippines	0.500	
Poland	0.206	0.415
Russia	0.252	0.263
Singapore	0.141	0.416
Sweden	0.204	0.445
Thailand	0.410	
UK	0.241	
USA	0.172	0.508

Source: Author's Calculations.

Table 1.22: Unskilled Labor's Share and Human Capital's Share - *OSPUE Adjustment*

	Dependent Variable		
	Unskilled Labor's Share	Human Capital's Share	
			Omit Germany
Variable			
Intercept	0.347***	0.313***	0.302***
	(6.197)	(4.076)	(5.683)
real GDP per worker, y	-2.840E-06**	2.247E-06	3.049E-06**
	(-2.056)	(1.286)	(2.474)
Adjusted R^2	0.187	0.068	0.390
F-test for no heteroskedasticity	0.497	0.172	2.805
	[3.885]	[4.737]	[5.143]
Sample	15 obs.	10 obs.	9 obs.

--t-statistics are in parantheses.

--brackets are 5% critical values of the F distribution.

--*indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level.

Table 1.23: Total Labor's Share, 2000 (*Labor Force Correction*)

Country	Total Labor's Share	Country	Total Labor's Share
Australia	0.630	Japan	0.736
Austria	0.613	Korea, Republic Of	0.718
Belgium	0.685	Mauritius	0.689
Botswana	0.495	Mexico	0.506
Canada	0.680	Netherlands	0.592
Costa Rica	0.685	New Zealand	0.590
Czech Republic	0.543	Norway	0.480
Denmark	0.604	Panama	0.704
Egypt	0.549	Poland	0.666
Finland	0.604	Portugal	0.711
France	0.637	Russia	0.530
Germany	0.654	Singapore	0.557
Greece	0.610	Spain	0.724
Hungary	0.651	Sweden	0.663
Ireland	0.518	Trinidad and Tobago	0.591
Israel	0.709	U.S.A	0.680
Italy	0.628		

Source : Author's Calculations

Table 1.24: Total Labor's Share - *Labor Force Correction*

Variable	Regression Equation 1	2	3	4
Intercept	0.661***	0.661***	0.542***	0.493***
	(36.051)	(41.115)W	(8.400)	(5.725)
u	-1.874E-03	-1.874E-03		
	(-0.151)	(-0.124)W		
u ^{2}	-0.037**	-0.037**		
	(-2.550)	(-2.198)W		
Numerical Quality Score			5.561E-03	
			(1.321)	
Variance Measure				0.022
				(1.431)
Benchmark Measure				0.031
				(0.854)
Data Rank Measure				-0.006
				(-0.484)
Adjusted R^2	0.131	0.131	0.023	-0.004
F-test for no heteroskedasticity	4.952			
	[3.316]			
F-test for overall significance of regression	3.412			0.962
	[3.316]			[2.934]
Wald test for overall significance of regression		6.424		
		{5.991}		
Sample	33 obs.	33 obs.	33 obs.	33 obs.

--Dependent variable is total labor's share computed using the *labor force correction* .

--*u* is a coded independent variable used in place of real GDP per worker.

--Variance Measure, Benchmark Measure, and Data Rank Measure are the three individual criterion employed by Summers and Heston to compute the Numerical Quality Score.

--t-statistics are in parantheses.

--W indicates t-statistics computed using White corrected standard errors.

--[ ] are 5% critical values of the F distribution

--{ } are 5% critical values of the X2 distribution

--*indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level

Table 1.25: Human Capital's Share, 2000 (*Labor Force Correction*)

Country	Human Capital's Share
Canada	0.488
Czech Republic	0.336
Germany	0.257
Japan	0.474
Korea	0.522
Poland	0.460
Russia	0.278
Singapore	0.416
Sweden	0.459
USA	0.508

Source: Author's Calculations

Table 1.26: Human Capital's Share - *Labor Force Correction*

Variable		Omit Germany
Intercept	0.356***	0.345***
	(4.373)	(5.537)
real GDP per worker, y	1.572E-06	2.351E-06*
	(0.848)	(1.624)
Adjusted R^2	-0.032	0.170
F-test for no heteroskedasticity	0.148	3.094
	[4.737]	[5.143]
Sample	10 obs.	9 obs.

--t-statistics are in parantheses

--brackets are 5% critical values of the F distribution

--*indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level.

Table 1.27: Total Labor's Share, 2000 (*No Adjustment*)

Country	Total Labor's Share	Country	Total Labor's Share
Algeria	0.215	Kyrgyzstan	0.307
Armenia	0.427	Latvia	0.414
Australia	0.492	Lesotho	0.179
Austria	0.510	Lithuania	0.395
Azerbaijan	0.216	Malta	0.433
Bahamas	0.464	Mauritius	0.382
Bahrain	0.341	Mexico	0.313
Belarus	0.439	Moldova	0.319
Belgium	0.508	Mongolia	0.263
Bolivia	0.361	Mozambique	0.224
Botswana	0.291	Namibia	0.395
Brazil	0.405	Netherlands	0.507
Bulgaria	0.346	Netherlands Antilles	0.584
Burkina Faso	0.224	New Zealand	0.415
Cameroon	0.200	Nicaragua	0.335
Canada	0.506	Niger	0.163
Chile	0.404	Nigeria	0.174
Colombia	0.355	Norway	0.431
Costa Rica	0.449	Oman	0.264
Croatia	0.529	Panama	0.378
Cuba	0.366	Paraguay	0.388
Cyprus	0.466	Peru	0.244
Czech Republic	0.419	Philippines	0.259
Denmark	0.527	Poland	0.402
Egypt	0.287	Portugal	0.499
Estonia	0.456	Qatar	0.210
Fiji	0.353	Romania	0.411
Finland	0.472	Russia	0.402
France	0.519	Saudia Arabia	0.296
Germany	0.534	Senegal	0.185
Greece	0.314	Singapore	0.433
Honduras	0.457	Slovakia	0.413
Hong Kong	0.515	Slovenia	0.529
Hungary	0.428	South Africa	0.480
Iceland	0.551	Spain	0.495
Iran	0.234	Sri Lanka	0.512
Iraq	0.054	Sweden	0.552
Ireland	0.401	Tanzania	0.063
Israel	0.529	Thailand	0.304
Italy	0.392	Trinidad and Tobago	0.373
Ivory Coast	0.232	Tunisia	0.362
Japan	0.539	Turkey	0.292
Jordan	0.393	Ukraine	0.423
Kazakhstan	0.358	United Kingdom	0.558
Kenya	0.348	U.S.A	0.593
Korea, Republic Of	0.429	Venezuela	0.328
Kuwait	0.288		

Source: Author's Calculations

Table 1.28: Total Labor's Share - *No Adjustment*

Variable	
Intercept	0.433***
	(28.352)
u	0.094***
	(8.408)
u^2	-0.053***
	(-4.270)
F-test for overall significance of regression	35.451
	[3.098]
Adjusted R^2	0.428
F-test for no heteroskedasticity	0.132
	[3.098]
Sample	93 obs.

--Dependent variable is Total Labor's share.
--*u* is a coded independent variable used in place of real GDP per worker.
--t-statistics are in parantheses.
--brackets are 5% critical values of the F distribution
--*indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level

Table 1.29: Unskilled Labor's Share and Human Capital's Share, 2000 (*No Adjustment*)

Country	**Unskilled Labor's Share**	**Human Capital's Share**
Argentina	0.067	
Brazil	0.082	0.323
Canada	0.153	0.353
Columbia	0.058	0.297
Czech Republic	0.175	0.244
Germany	0.352	0.183
Hong Kong	0.066	0.449
India	0.039	
Japan	0.203	0.336
Korea, Republic Of	0.122	0.307
Philippines	0.228	0.031
Poland	0.148	0.254
Russia	0.214	0.188
Singapore	0.115	0.318
Sweden	0.198	0.353
Thailand	0.163	0.141
UK	0.223	0.335
USA	0.156	0.436

Source: Author's Calculations

Table 1.30: Unskilled Labor's Share and Human Capital's Share - *No Adjustment*

Variable	Unskilled Labor's Share	Human Capital's Share
Intercept	0.117***	0.153***
	(3.209)	(3.152)
real GDP per worker, y	1.110E-06	3.791E-06***
	(1.149)	(3.441)
Adjusted R^2	0.019	0.419
F-test for no heteroskedasticity	0.098	0.468
	[3.682]	[3.806]
Sample	18 obs.	16 obs.

--t-statistics are in parantheses.

--brackets are 5% critical values of the F distribution

-- * indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level.

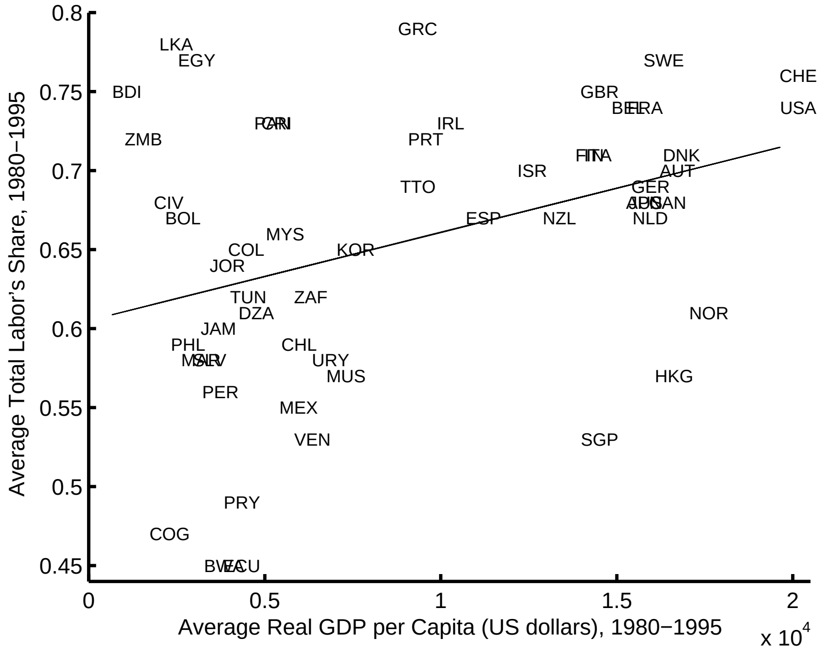

Figure 1.1—Average Total Labor's Share; Bernanke and Gurkaynak Data

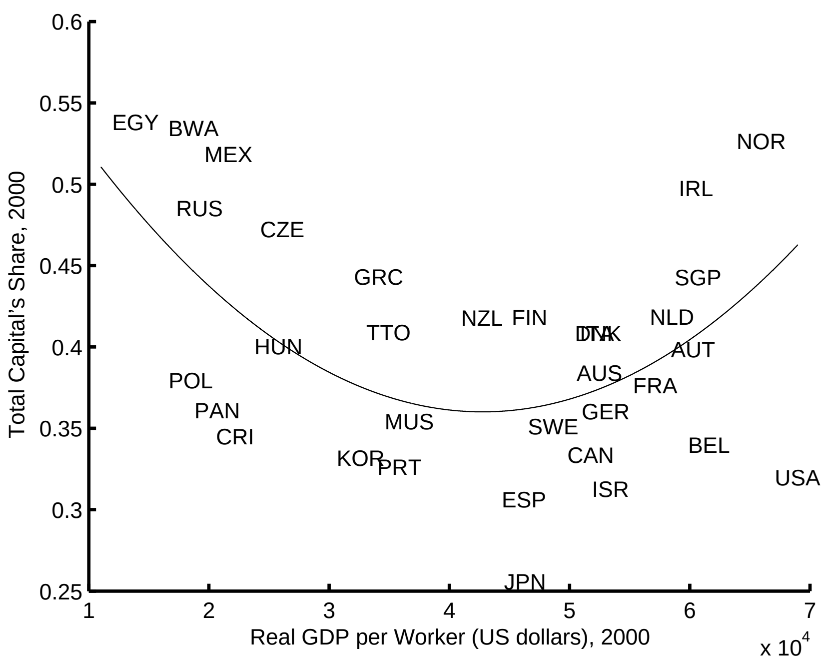

Figure 1.2—Total Capital's Share vs. Real GDP per Worker; *OSPUE Adjustment*

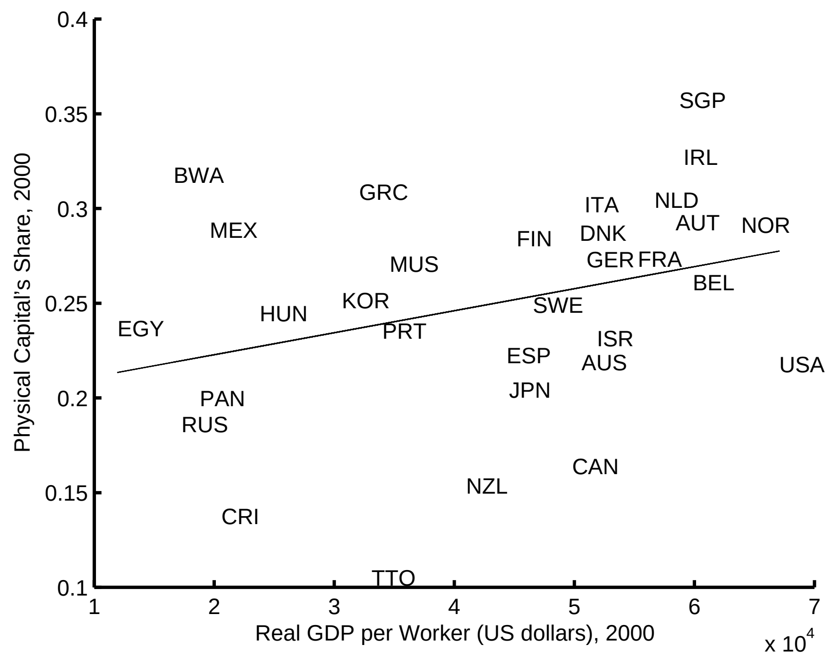

Figure 1.3—Physical Capital's Share vs. Real GDP per Worker; *OSPUE Adjustment*

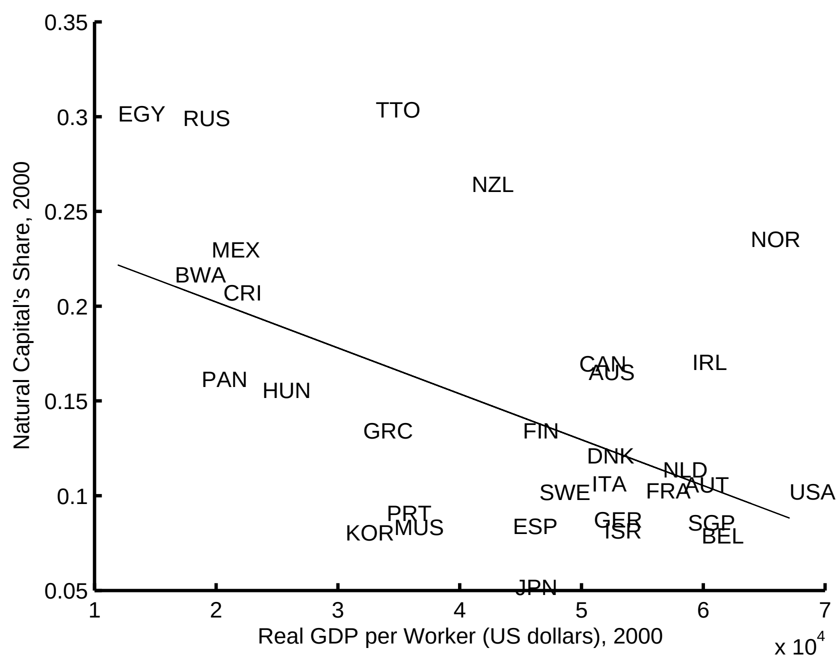

Figure 1.4—Natural Capital's Share vs. Real GDP per Worker; *OSPUE Adjustment*

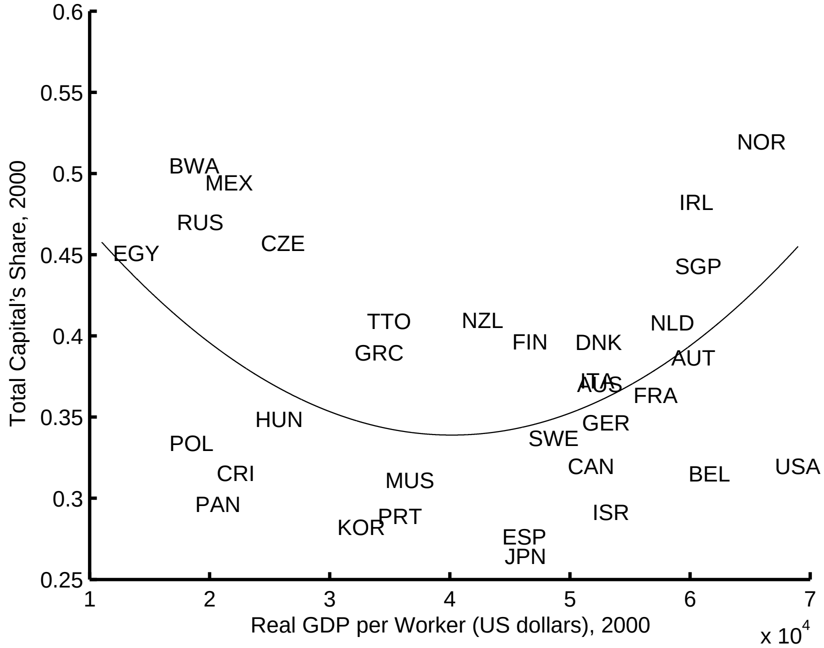

Figure 1.5—Total Capital's Share vs. Real GDP per Worker; *Labor Force Correction*

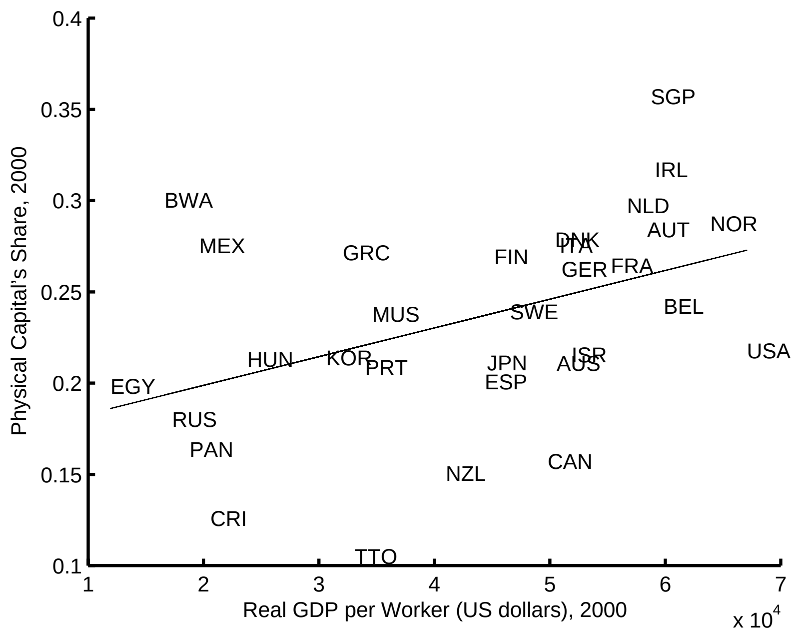

Figure 1.6—Physical Capital's Share vs. Real GDP per Worker; *Labor Force Correction*

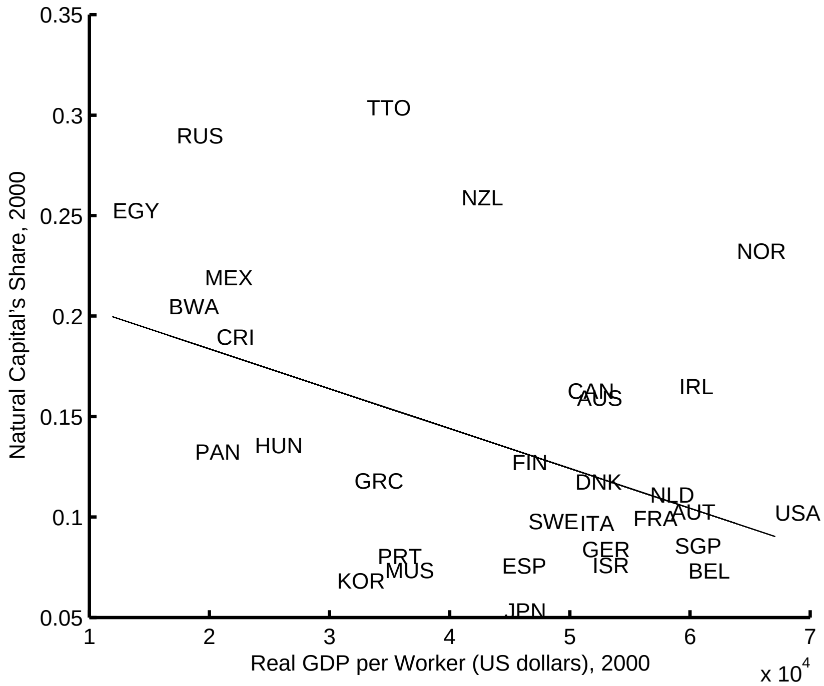

Figure 1.7—Natural Capital's Share vs. Real GDP per Worker; *Labor Force Correction*

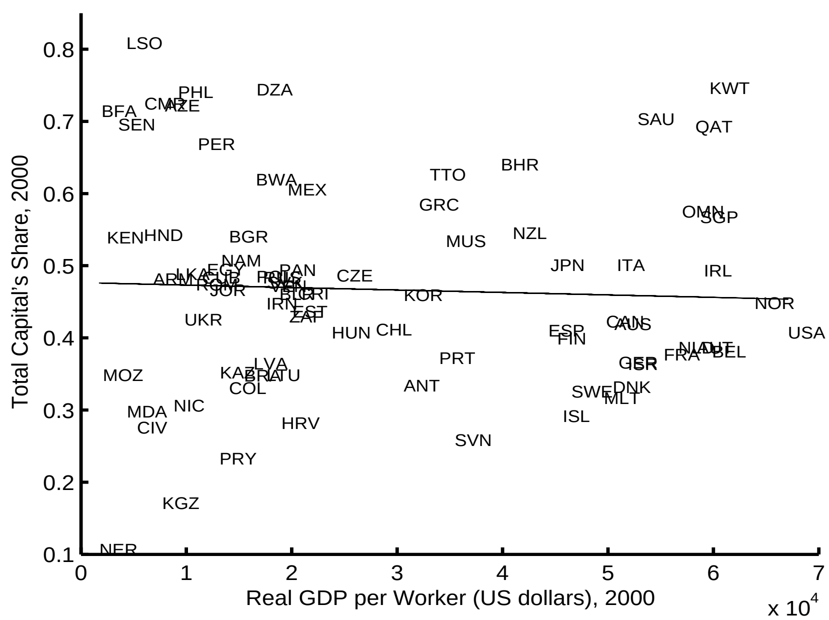

Figure 1.8—Total Capital's Share vs. Real GDP per Worker; *No Adjustment*

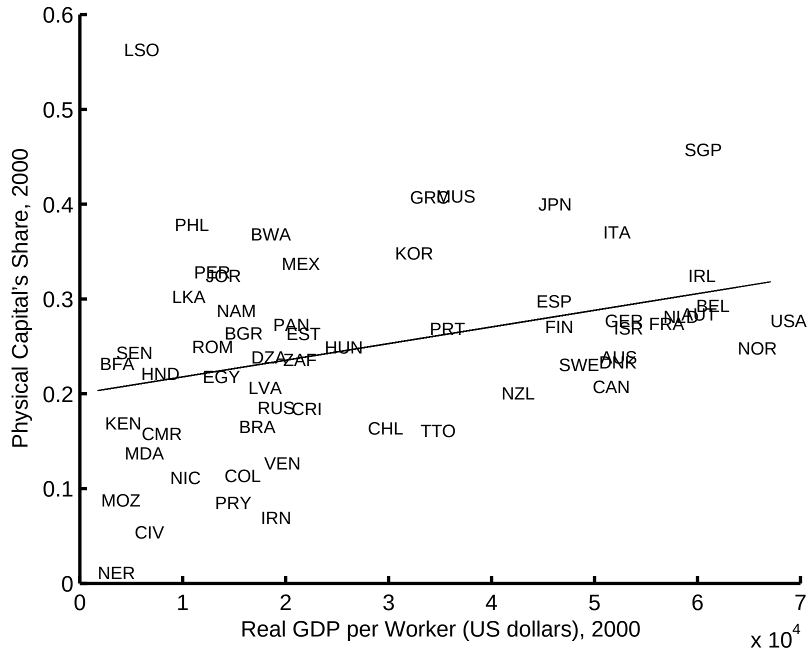

Figure 1.9—Physical Capital's Share vs. Real GDP per Worker; *No Adjustment*

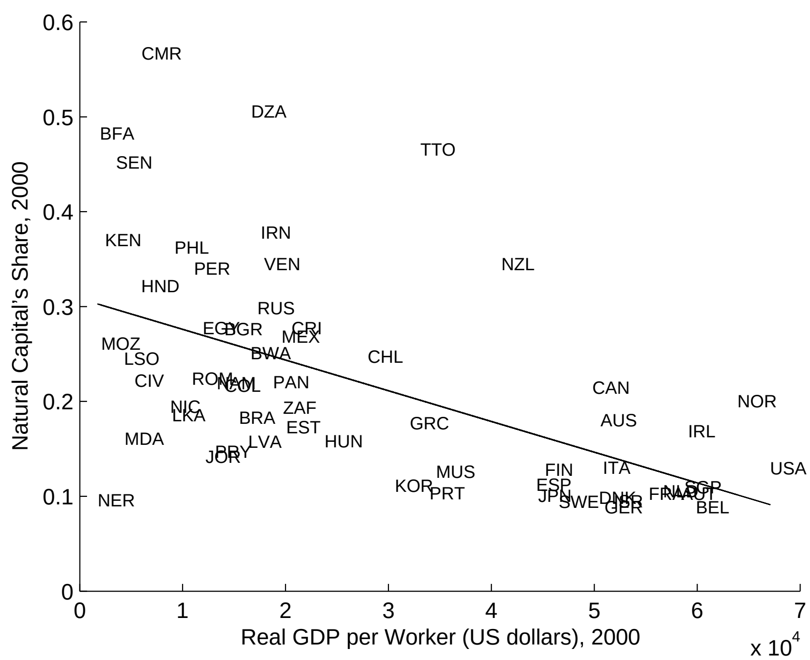

Figure 1.10—Natural Capital's Share vs. Real GDP per Worker; *No Adjustment*

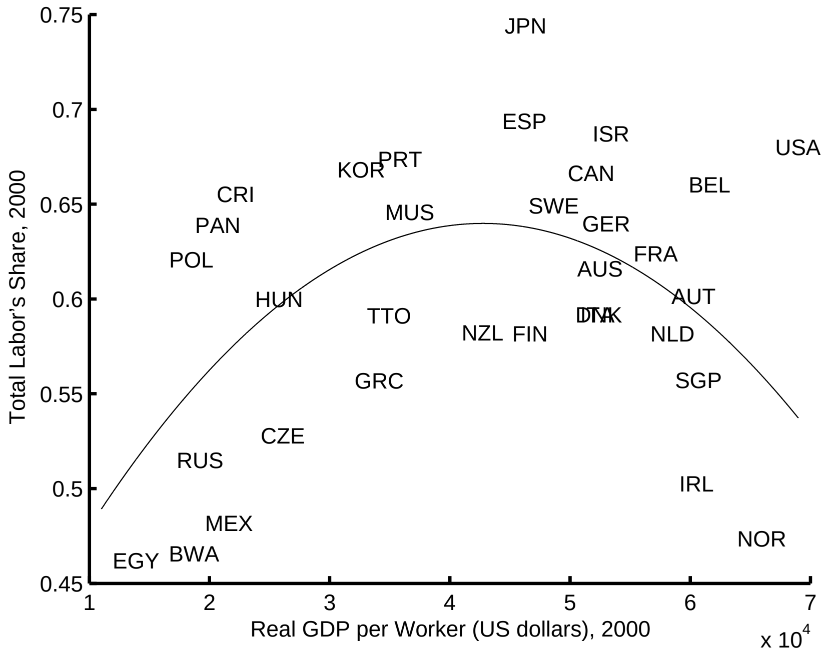

Figure 1.11—Total Labor's Share vs. Real GDP per Worker; *OSPUE Adjustment*

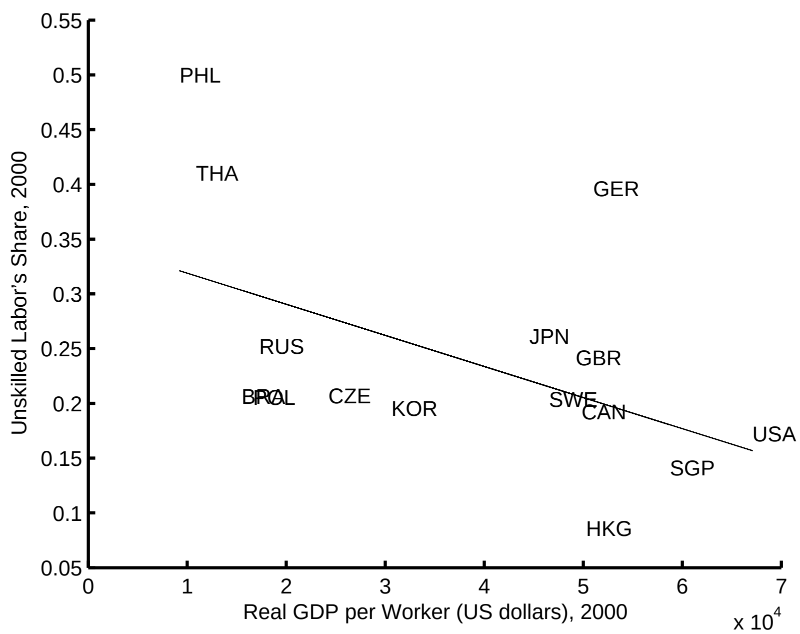

Figure 1.12—Unskilled Labor's Share vs. Real GDP per Worker; Adjusted for Self-employed Labor

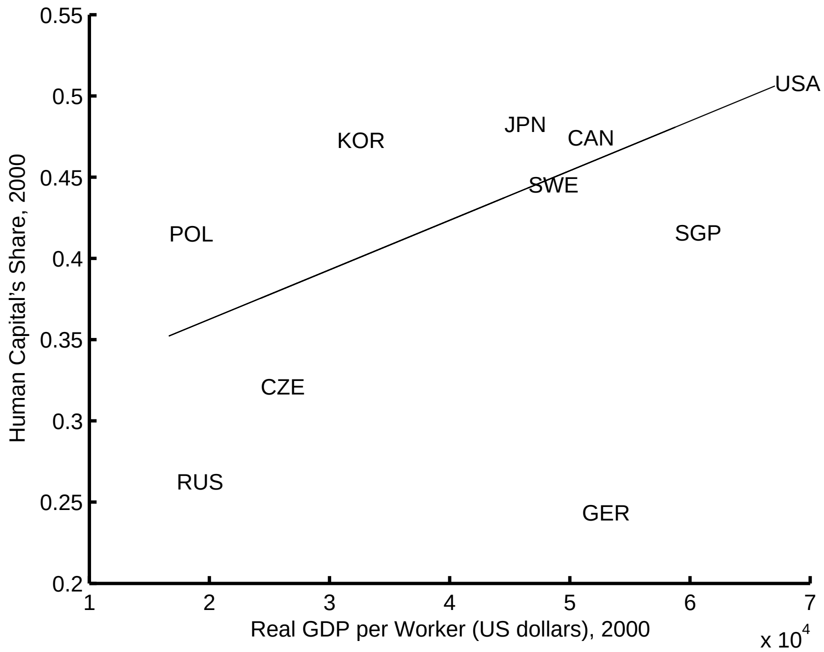

Figure 1.13—Human Capital's Share vs. Real GDP per Worker; *OSPUE Adjustment*

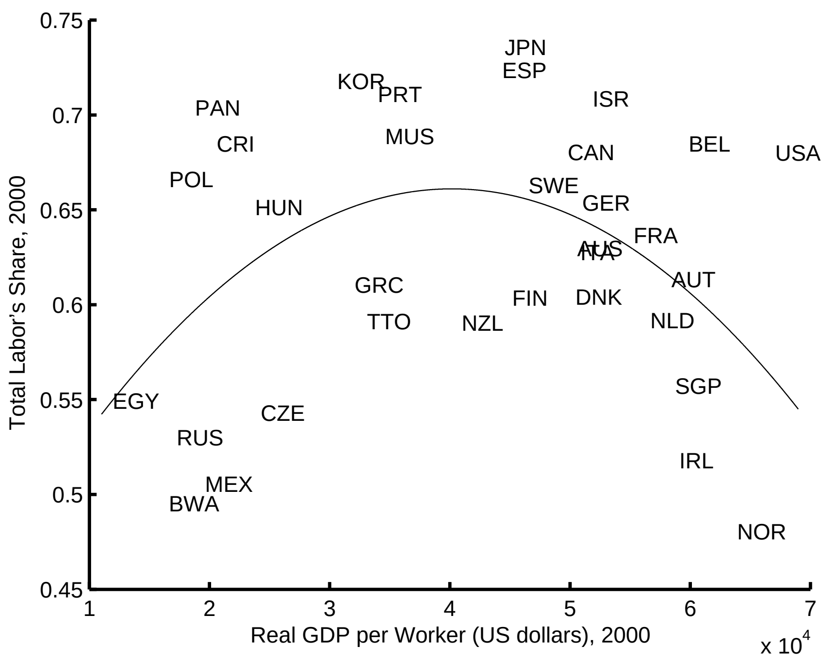

Figure 1.14—Total Labor's Share vs. Real GDP per Worker; *Labor Force Correction*

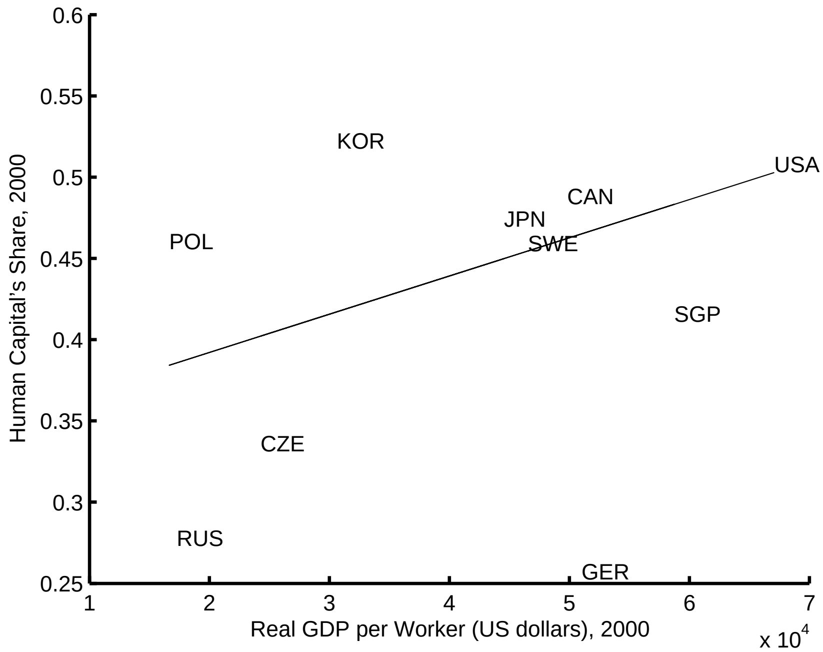

Figure 1.15—Human Capital's Share vs. Real GDP per Worker; ***Labor Force Correction***

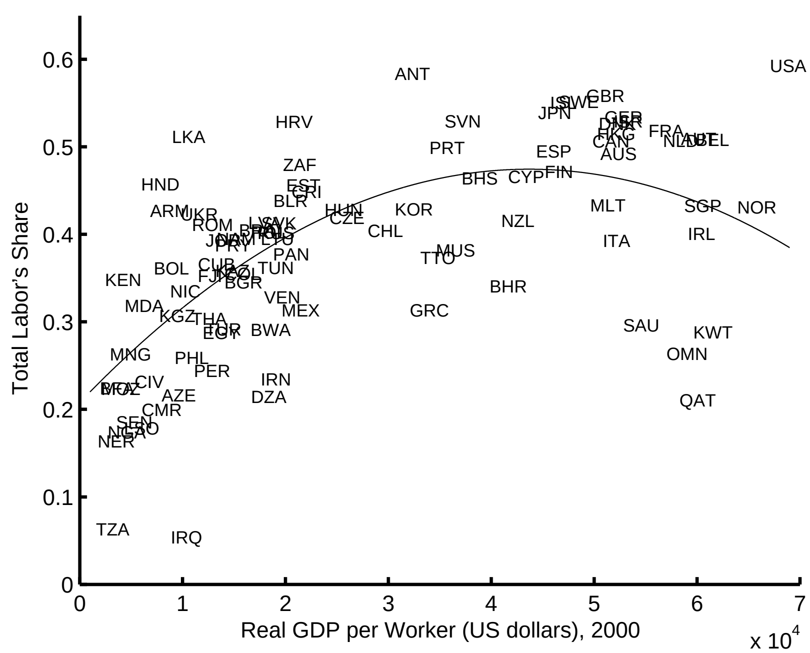

Figure 1.16—Total Labor's Share vs. Real GDP per Worker; ***No Adjustment***

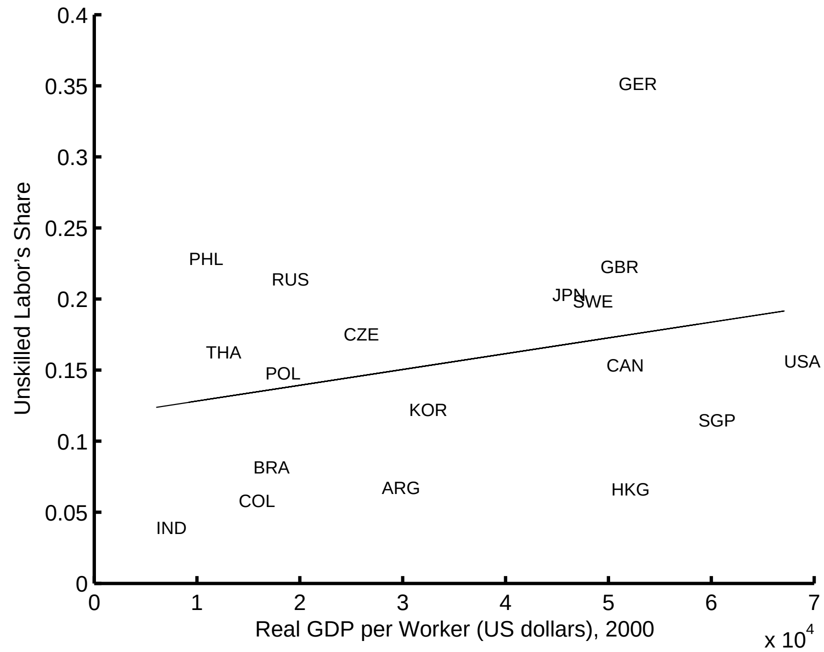

Figure 1.17—Unskilled Labor's Share vs. Real GDP per Worker; *No Adjustment*

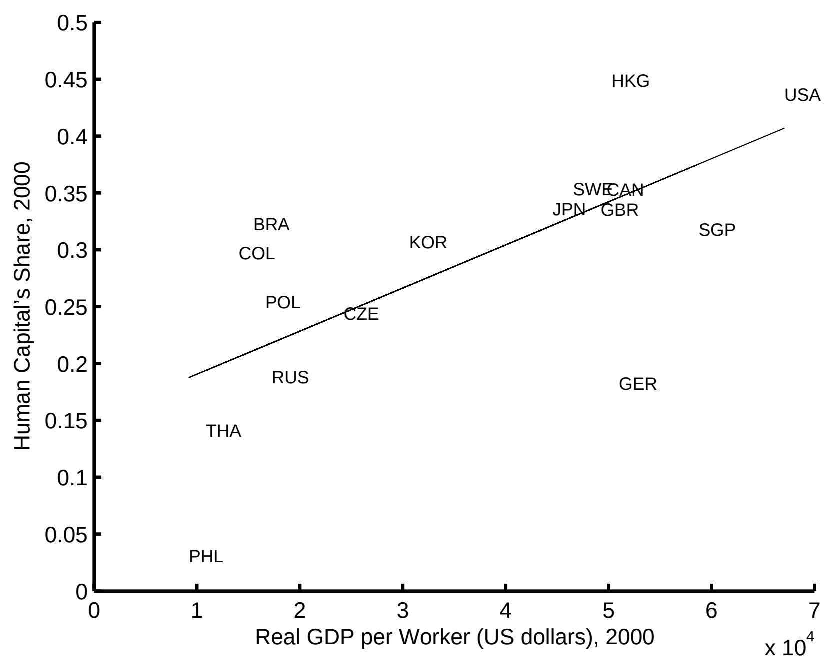

Figure 1.18—Human Capital's Share vs. Real GDP per Worker; *No Adjustment*

Chapter 2

Acknowledging Cross-Country Variability in Factor Shares: The Implications for Development Accounting

2.1 Introduction

The consensus view in the development accounting literature is that the Total Factor Productivity (TFP) residual, which is typically considered a proxy for technology, accounts for at least half of the variation in cross-country output per worker. This view has emerged despite numerous attempts to reduce residual variation via adjustments in the functional form of the production function and improvements in the accuracy of the data used to estimate these functions. Caselli (2005) takes inventory of these attempts, which include: improvements in the accuracy of human and physical capital measures; acknowledgement of embodied technical progress; acknowledgement of the prevalence of agriculture in developing countries; and consideration of non-neutral differences in technology.

Caselli's (2005) study is built upon the work of Mankiw, Romer, and Weil (1992), King and Levine (1994), Klenow and Rodriguez-Clare (1997), and Hall and Jones (1999). More recent contributions to the development accounting literature include Jones and Schneider (2006), Caselli and Coleman (2006), Weil (2007), and Vollrath (2009). Jones and Schneider incorporate cross-country differences in IQ, and Weil controls for cross-country differences in health. Both of these studies reduce residual variation via improvements in the measurement of human capital. Vollrath is able to reduce residual variation by controlling for the distribution of factors across agriculture and non-agriculture industries. Caselli and Coleman decompose the TFP residual and find that most of the residual variation is due to variation in the efficiency of skilled labor.

The aforementioned studies and virtually all development accounting studies assume that factor shares are constant. This assumption is a direct reflection of the macroeconomic paradigm created by Cobb and Douglas (1928) and Kaldor (1961). In Chapter 1 I distinguished between reproducible and non-reproducible factors, and I showed that factor shares are not constant and vary systematically with output per worker. In Chapter 2 I look at the implications of systematic variation in factor shares for the measurement of the TFP residual across countries. Specifically, I compare the fraction of cross-country variation in economic performance attributable to variation in the TFP residual to the fraction of cross-country variation in economic performance attributable to variation in factors and factor shares. Rather than assume factor shares are constant across countries, I allow factor shares to vary in accordance with the estimates presented in Chapter 1.

I find that the majority of variation in output per worker accrues to factor shares when factor shares are allowed to vary and a distinction between reproducible and non-reproducible factors is made. Variation in output per worker accruing to TFP falls by as much as 61 percentage points depending on the specific approach used to estimate factor shares. These results, in addition to suggesting that factor shares play an important role in development accounting, reveal the inappropriateness of forcing all technical progress to work through the TFP residual. Technical progress that manifests itself via changes in factor shares is certainly plausible, and my results provide strong evidence that such progress is at work and a prominent source of cross-country variation in output per worker.

The remainder of Chapter 2 is organized as follows. In Section 2.2, I estimate TFP residuals using the *OSPUE adjusted* factor share estimates from Chapter 1. I then analyze the impact of variable factor shares on the variation in output per worker accruing to observables and the TFP residual. The qualitative results are unchanged when factor shares computed via the *labor force correction* and *No Adjustment* approaches are used. I briefly discuss the results yielded by these other two factor share computations and make some additional remarks in Section 2.3. Section 2.4 concludes.

2.2 Implications for Development Accounting

2.2.1 The Production Function

Since I am introducing variable factor shares, something completely ignored until now[38], it is reasonable to carry out my analysis using a production function comparable to that which most development accounting studies employ. Therefore, I stick with the workhorse of the literature and employ a Cobb-Douglas functional form.[39]

Let production in country *i* be characterized by

$$Y_i = A_i K_i^{\alpha_i} N_i^{\gamma_i} \left(L_i h_i - L_i\right)^{\beta_i} L_i^{\eta_i} \tag{2.1}$$

where L is the number of workers and represents unskilled labor; h is a labor augmenting variable encompassing the level of education; and A is the TFP residual. The other variables in equation (2.1) were defined in Chapter 1. I take the average years of schooling for the population aged 15 and over from Barro and Lee (2001) and convert it into a proxy for human capital following Hall and Jones (1999). $h = e^{\phi(E)}$ where E is average years of schooling, and $\varphi(E)$ is piecewise linear with slope 0.117 for $E \leq 4$, 0.097 for $4<E\leq 8$, and 0.075 for $E>8$. The slope coefficients represent rates of return for education as reported by Psacharopoulos and Patrinos (2004). $Lh - L$ measures human capital and can be thought of as the difference between the effective workforce, which is the workforce augmented by education, and the basic workforce, which is not augmented. I use the *Economically Active*

[38]Caselli (2005) analyzes the impact of allowing shares to take on different constant values, but he does not let shares vary across countries.

[39] Mankiw, Romer, and Weil (1992), Klenow and Rodriguez Claire (1997), Hall and Jones (1999), Caselli (2005), Weil (2007), and Vollrath (2009) all employ the Cobb-Douglas form to carry out development accounting exercises.

Population, which is reported in the ILO's LABORSTA database, to proxy for *L*. Data sources for all other observable variables are the same as the data sources used in Chapter 1. All data is for the year 2000, and all results presented in Section 2.2 are derived using factor share estimates computed according to Chapter 1's *OSPUE Adjustment*.

2.2.2 The Impact on TFP Levels

Dividing both sides of equation (2.1) by *L* yields the per worker production function,

$$y_i = A_i k_i^{\alpha_i} n_i^{\gamma_i} \left(h_i - 1\right)^{\beta_i}, \tag{2.2}$$

where lower case letters represent per worker values. The typical development accounting approach involves the following: α_i and $\beta_i + \eta_i$ are assumed to equal 1/3 and 2/3 respectively for all *i*; human capital and unskilled labor are entangled in a single, composite measure; and natural capital is ignored so that γ_i equals zero for all *i*. Given equation (2.2), the TFP residual, *A*, can be computed in accordance with the typical approach as

$$A_i = \frac{y_i}{k_i^{1/3} h_i^{2/3}} \ . \tag{2.3}$$

Researchers often justify the constant exponent on physical capital of one third by noting that one third is consistent with the average "capital" share of national income for a broad sample of countries. But, the computations that lead to this value do not separate the income that gets paid to physical capital from the income that gets paid to natural capital. One third is the average value of total capital's share. So, not only is the systematic variation in cross-country factor shares ignored in the development accounting literature, the typical approach incorrectly assigns a factor exponent to a factor. Physical capital's share, not total capital's share, should be the exponent associated with physical capital.

Estimates of the typical TFP residual given by equation (2.3) are presented in Table 2.1 along with the two observable components of output per worker, $k^{1/3}$ and $h^{2/3}$. Notice that the TFP residual is very large relative to the *observables*. The average value of the TFP residual is 537, which is about 13 times larger than the average value of $k^{1/3}$. It is 291 times larger than the average value of $h^{2/3}$.

Including natural capital as a factor of production, treating human capital and unskilled labor as separate, imperfectly substitutable inputs, and allowing factor shares to vary yields the following TFP residual for country *i*:

$$A_i = \frac{y_i}{k_i^{\alpha_i} n_i^{\gamma_i} \left(h_i - 1\right)^{\beta_i}} . \tag{2.4}$$

Table 2.2 reports these residual values along with their observable counterparts for each country.[40] Notice that the average value of the TFP residual does not change a great deal when the typical development accounting assumptions are relaxed. In fact, statistically, the two values are equivalent; the t-statistic from a paired difference test is only equal to -0.58.

One might expect the average TFP residual to be lower in Table 2.2 because the residual encompasses fewer unobservable components. However, omitting natural capital and entangling human capital and unskilled labor in a single composite measure leads to an upward bias in the TFP residual that is offset by a downward bias created by the

[40] Recall that *OSPUE* adjusted physical and natural capital shares are computed for 31 countries in Chapter 1. Of these 31 countries only 8 of them have the data necessary for direct computation of human capital's share. I interpolate the missing human capital shares using the intercept and slope coefficients yielded by the regression of human capital's share on real GDP per worker. Because Germany's human capital share was an outlier and omitted from the aforementioned regression, I do not include Germany in the development accounting analysis presented in Section 2.2. Therefore, results in Tables 2.1, 2.2, and 2.3 are presented for a sample of 30 countries, not 31.

measurement error in physical capital's share.[41] When these biases are eliminated, there is very little net change in the average TFP residual.

2.2.3 Estimating the Variation in Output per Worker accruing to *observables* and TFP

Though the average level of the TFP residual is relatively unaffected when all factors of production are acknowledged and factor shares are allowed to vary, the fraction of variation in output per worker explained by variation in the TFP residual is impacted substantially. Henceforth, I omit the *i* subscript except where clarity requires it. Define $y_{observables} = k^{\alpha} n^{\gamma} (h-1)^{\beta}$ so that the per worker production function given by equation (2.2) can be rewritten as $y = Ay_{observables}$. The exact form of $y_{observables}$ will change as assumptions about factors and factor shares change, but in general, the variance of output per worker can be decomposed as follows:

$$\text{var}[\ln y] = \text{var}[\ln A] + \text{var}[\ln y_{observables}] + 2\,\text{cov}[\ln A, \ln y_{observables}]. \qquad (2.5)$$

How much of the variation in output per worker across countries is attributable to variation in *observables*, and how much is attributable to variation in the TFP residual? To answer this question, some assumption about the covariance term in equation (2.5) must be made. Any correlation between *observables* and the TFP residual implies that the variation in output per worker accruing to *observables* is correlated with the variation in output per worker accruing to the TFP residual. Therefore, the covariance term embodies pertinent

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[41] There are large differences between the values of $k_i^{1/3}$ and $k_i^{\alpha_i}$. For example, when the observed value of α rather than 1/3 is inserted as the exponent on Canada's k, the value of k raised to the exponent falls from 43.96 to 6.43. This is almost a seven fold difference. The average value of α in the sample is 0.256, a value less than 1/3. As a result, the average value of k^{α} equals 19.60, which is roughly half the size of the average value of $k^{1/3}$, which is 40.82.

interaction effects that need to be accounted for in some manner when determining the contribution of variation in each the observable and residual components to variation in output per worker. One option is to ignore the covariance and assume that the TFP residual is constant across countries. Caselli (2005) takes this approach. I find the approach unappealing because it yields relative variances that do not add up to one when the actual covariance between the TFP residual and *observables* is non-zero. Mankiw, Romer, and Weil (1992) allow the TFP residual to vary, but given their reliance on regression analysis to obtain input measures, their covariance term is zero by construction. Their approach is just as unappealing because the correlation between *observables* and the TFP residual is ignored.[42]

A more useful variance decomposition, which is suggested by Baier, Dwyer, and Tamura (2006)[43], is

$$\frac{\left(1-\rho^2_{obs.,A}\right)\mathrm{var}\left[\ln y_{observables}\right]}{\mathrm{var}[\ln y]}+\frac{\left\{sd\left[\ln A\right]+sd\left[\ln y_{observables}\right]\rho_{obs.,A}\right\}^2}{\mathrm{var}\left[\ln y\right]}=1. \qquad (2.6)$$

[42] In my sample, the statistical correlation between the TFP residual and *observables* ranges from -0.85 to 0.30 depending on the specific assumptions accompanying the production function. The bottom of Table 2.3 presents all relevant variance and covariance measures, and the last row in Table 2.3 provides the statistical correlation between *observables* and the TFP residual. To see why the relative variances are misleading if the correlation between the TFP residual and *observables* is ignored, consider the scenario yielded by the production function in column 1 of Table 8. The correlation between the TFP residual and *observables* equals 0.30 for this production function. $\frac{\mathrm{var}\left[\ln y_{observables}\right]}{\mathrm{var}\left[\ln y\right]}$ equals 0.49 in this case. To say that 49% of income variation is explained by *observables* is misleading because implicit in such a claim is that 51% of income variation is explained by *unobservables* -- i.e., the TFP residual. Given the data, this is not the case. $\frac{\mathrm{var}\left[\ln A\right]}{\mathrm{var}\left[\ln y\right]}$ equals 0.29, so, disregarding the covariance term*,* variation in *observables* and variation in *unobservables* together explain only 78% of the variation in income. That suggests that something other than *observables* or *unobservables* explains 22% of the variation in income. Such a scenario is illogical and stems from the fact that the covariance term does not equal zero.

[43] Baier, Dwyer, and Tamura (2006) use the decomposition in a growth accounting framework, but adjusting it for use in a development accounting framework is straightforward.

$\rho_{obs.,A}$ is the statistical correlation between *observables* and the TFP residual. *sd* denotes standard deviation. With this decomposition the covariance between the TFP residual and *observables* is not ignored. Rather, all of the correlation between *observables* and the TFP residual is attributed to the TFP residual. Also, the estimates of the relative variances sum to one, and interpreting each value is straightforward. The first term on the left side of equation (2.6) is the fraction of variation in output per worker attributable to variation in *observables*, and the second term is the fraction of variation in output per worker attributable to variation in the TFP residual.[44]

There is an economic justification for this decomposition. The level of economic development is dependent on certain economic fundamentals that are not explicitly accounted for in the production function. Differences in the accumulation of factors and differences in factor shares are undoubtedly going to impact differences in output per worker, but these differences are driven by differences in saving rates, R & D costs and production technologies, all of which are encompassed by the TFP residual. Thus, the TFP residual drives all of the variation in *observables*. Attributing all of the interaction effects embodied by the covariance to the TFP residual not only makes interpreting relative variance estimates easier, it is a reasonable approach from a theoretical standpoint.

[44] Since it is assumed that any relationship between *observables* and the TFP residual reflects effects of the TFP residual, the covariance term along with a fraction of the variation in *observables* is added to the variance of the TFP residual so that the fraction of variation in output per worker attributable to the TFP residual can be written: $\frac{\mathrm{var}[\ln A] + 2\,\mathrm{cov}[\ln y_{observables}, \ln A] + \mathrm{var}[\ln y_{observables}]\rho^2_{obs.,A}}{\mathrm{var}[\ln y]}$.

This expression is equivalent to the expression given by the second term in equation (2.6). The fraction of the variation in *observables* that gets allocated to the variation in the TFP residual is determined by the squared correlation, $\rho^2_{obs.,A}$. *Observables* and the TFP residual may be negatively correlated. Squaring the correlation ensures that variation in *observables* that reflects variation in the TFP residual is added to variation in the TFP residual. The fraction of variation in output per worker attributable to variation in *observables* can be written as: $\frac{\mathrm{var}[\ln y_{observables}] - \mathrm{var}[\ln y_{observables}]\rho^2_{obs.,A}}{\mathrm{var}[\ln y]}$. This expression is equivalent to the expression given by the first term in equation (2.6). The intuition is that any variation in *observables* that really reflects variation in the TFP residual should be attributed to variation in the TFP residual and, therefore, subtracted from the variation in *observables*.

2.2.4 Relative Variance Estimates: Typical Assumptions

Estimates of the relative variances given by the decomposition in equation (2.6) are presented in Table 2.3 for four different combinations of assumptions pertaining to the production function. Under typical development accounting assumptions, the production function given by equation (2.2) simplifies to $y = Ak^{1/3}h^{2/3}$. Given this functional form, 45% of the variation in output per worker is attributable to *observables*, and 55% is attributable to the TFP residual. This breakdown of explanatory power is consistent with the consensus view that *observables* account for at most 50% of the variation in cross-country output per worker (Caselli, 2005). This substantiates my method because no other study that I am aware of estimates relative variances according to equation (2.6). Mankiw, Romer, and Weil (1992) and Caselli (2005) ignore the covariance between the TFP residual and *observables*. Klenow and Rodriguez-Clare (1997) attribute half of the contribution of the covariance term to the TFP residual and half to *observables*.[45] Weil (2007) uses the most similar decomposition. He adds twice the covariance to the variance as I do, but he does not incorporate the correlation coefficient in any way.

2.2.5 Relative Variance Estimates: Allowing Factor Shares to Vary

As you move to the right in Table 2.3, the assumptions about the production function become increasingly consistent with reality. In the second column, factor shares are allowed to vary, but the other traditional development accounting assumptions still hold, so the production function is given by $y = Ak^{\alpha}h^{(\beta+\eta)}$. Allowing factor shares to vary has a substantial impact on the relative variance estimates. Of the variation in output per worker, 99% is now due to variation in *observables*, and only 1% is due to variation in the TFP residual. The TFP residual's explanatory power essentially disappears under traditional development accounting if factor shares are allowed to vary.

[45] Attributing half of the covariance term to the TFP residual and half to *observables* has no theoretical support. Klenow and Rodriguez-Clare just feel it is an "informative way of characterizing the data."

This result does not indicate that variation in factor shares absorbs the shift in explanatory power. It could be that allowing factor shares to vary simply serves as an avenue for the redistribution of explanatory power to the factors. Therefore, separating the variation in output per worker explained by *observables* into that accruing to factors and that accruing to factor shares is useful.

Decomposing the Variation in *Observables* This additional breakdown of explanatory power is a two step process. First, the variation in *observables* must be broken down into the variation attributable to each of the two observable components, $\alpha \ln k$ and $(\beta+\eta)\ln h$. The second step is breaking down the variation in each observable component into that accruing to the factor and that accruing to the factor share.

The variance of *observables* can be decomposed as follows:

$$\text{var}[\ln y_{observables}] = \text{var}[\alpha \ln k] + \text{var}[(\beta+\eta)\ln h] + 2\,\text{cov}[\alpha \ln k, (\beta+\eta)\ln h]. \qquad (2.7)$$

Uniquely estimating the fractions of variation in *observables* attributable to variation in $\alpha \ln k$ and variation in $(\beta+\eta)\ln h$ requires that some assumption about the covariance in equation (2.7) be made. No theory exists to guide this assumption. However, by considering two estimates, each of which attributes all of the correlation to either $\alpha \ln k$ or $(\beta+\eta)\ln h$, an upper and lower bound for the relative variances can be obtained.

Denote $\rho_{\alpha \ln k,\,(\beta+\eta)\ln h}$ as the statistical correlation between $\alpha \ln k$ and $(\beta+\eta)\ln h$. If all of the correlation between $\alpha \ln k$ and $(\beta+\eta)\ln h$ is attributed to $\alpha \ln k$, the relative variances can be computed according to the following decomposition:

$$\frac{\left(1-\rho^2_{\alpha \ln k,(\beta+\eta)\ln h}\right)\text{var}[(\beta+\eta)\ln h]}{\text{var}[\ln y_{observables}]} + \frac{\left\{sd[\alpha \ln k] + sd[(\beta+\eta)\ln h]\rho_{\alpha \ln k,(\beta+\eta)\ln h}\right\}^2}{\text{var}[\ln y_{observables}]} = 1. \qquad (2.8)$$

The variation in *observables* attributable to variation in $(\beta+\eta)\ln h$ is represented by the first term on the left hand side of equation (2.8). The second term represents the variation in *observables* attributable to variation in $\alpha \ln k$. Alternatively, all correlation between $\alpha \ln k$ and $(\beta+\eta)\ln h$ can be attributed to $(\beta+\eta)\ln h$, in which case the relative variance decomposition takes the form:

$$\frac{\left\{sd\left[(\beta+\eta)\ln h\right]+sd\left[\alpha \ln k\right]\rho_{\alpha \ln k,(\beta+\eta)\ln h}\right\}^2}{\operatorname{var}\left[\ln y_{observables}\right]}+\frac{\left(1-\rho^2_{\alpha \ln k,(\beta+\eta)\ln h}\right)\operatorname{var}\left[\alpha \ln k\right]}{\operatorname{var}\left[\ln y_{observables}\right]}=1. \qquad (2.9)$$

As in equation (2.8), the first and second terms in equation (2.9) can be interpreted as the fractions of variation in *observables* attributable to $(\beta+\eta)\ln h$ and $\alpha \ln k$ respectively.

In order to break down the variation in *observables*, and ultimately the variation in output per worker, into that accruing to factors and that accruing to factor shares, the variation attributable to factors and factor shares must be extracted from the overall variation in each of the two observable components, $(\beta+\eta)\ln h$ and $\alpha \ln k$. Focusing first on $\alpha \ln k$, let E denote the expectations operator and let $\Delta\alpha=\alpha-E(\alpha)$ and $\Delta \ln k=\ln k-E(\ln k)$. Following the decomposition for the variance of a product presented by Goodman (1960) and Bohrnstedt and Goldberger (1969), the variance of $\alpha \ln k$ can be written

$$\begin{aligned}\operatorname{var}[\alpha \ln k] = {} & E^2(\alpha)\operatorname{var}[\ln k]+E^2(\ln k)\operatorname{var}[\alpha]+E\left[(\Delta\alpha)^2(\Delta \ln k)^2\right] \\ & +2E(\alpha)E\left[(\Delta\alpha)(\Delta \ln k)^2\right]+2E(\ln k)E\left[(\Delta \ln k)(\Delta\alpha)^2\right] \\ & +2E(\alpha)E(\ln k)\operatorname{cov}[\alpha, \ln k]-\operatorname{cov}^2[\alpha, \ln k].\end{aligned} \qquad (2.10)$$

The first and second terms on the right hand side of equation (2.10) can be thought of as the direct effects of variability in $\ln k$ and α respectively. The remaining terms encompass the interaction between $\ln k$ and α. To uniquely estimate the fractions of variation in $\alpha \ln k$ accruing to α and $\ln k$, some assumption about the interaction terms must be made. Again, no theory exists to guide such an assumption, but by considering two extreme decompositions, one in which all interaction is assumed to reflect variability in α and the other in which all interaction is assumed to reflect variability in $\ln k$, the possible range of relative variance estimates can be obtained.

In the first decomposition I assume that all interaction between $\ln k$ and α reflects variability in α. The relative variance decomposition is given by

$$\frac{E^2(\ln k)\text{var}[\alpha] + Interaction_{\alpha,\ln k} + E^2(\alpha)\text{var}[\ln k]\rho^2_{\alpha,\ln k}}{\text{var}[\alpha \ln k]} + \frac{(1-\rho^2_{\alpha,\ln k})E^2(\alpha)\text{var}[\ln k]}{\text{var}[\alpha \ln k]} = 1 \quad (2.11)$$

where

$$\begin{aligned} Interactio\ n_{\alpha,\ln k} = {} & E\left[(\Delta\alpha)^2(\Delta \ln k)^2\right] + 2E(\alpha)E\left[(\Delta\alpha)(\Delta \ln k)^2\right] + 2E(\ln k)E\left[(\Delta \ln k)(\Delta \ln \alpha)^2\right] \\ & + 2E(\alpha)E(\ln k)\text{cov}[\alpha \ln k] - \text{cov}^2[\alpha \ln k] \end{aligned}$$

and $\rho_{\alpha,\ln k}$ denotes the statistical correlation between α and $\ln k$. The first term on the left hand side of equation (2.11) represents the fraction of variation in $\alpha \ln k$ attributable to variation in α. The second term represents the fraction of variation attributable to $\ln k$.

Alternatively, if all of the interaction is assumed to reflect variability in $\ln k$, the relative variances can be estimated according to

$$\frac{(1-\rho^2_{\alpha,\ln k})E^2(\ln k)\text{var}[\alpha]}{\text{var}[\alpha \ln k]} + \frac{E^2(\alpha)\text{var}[\ln k] + Interaction_{\alpha,\ln k} + E^2(\ln k)\text{var}[\alpha]\rho^2_{\alpha,\ln k}}{\text{var}[\alpha \ln k]} = 1. \quad (2.12)$$

As in equation (2.11), the first term on the left hand side of equation (2.12) is the fraction of variation in $\alpha \ln k$ attributable to variation in α, and the second term is the fraction of variation in $\alpha \ln k$ attributable to variation in $\ln k$.

The variance decomposition of $(\beta+\eta)\ln h$ is identical to the decomposition given by equation (2.10), only $\beta+\eta$ appears in place of α, and $\ln h$ appears in place of $\ln k$. The same issue as to how the interaction terms should be treated arises, and since there is no theory for which to appeal, I follow the same methodology used with α and $\ln k$ to obtain estimates of the range of relative variances. The relative variance decompositions take the same form as those in equations (2.11) and (2.12), but $\beta+\eta$ and $\ln h$ take the place of α and $\ln k$, respectively.

Given the range of estimates for the variation in *observables* accruing to the two observable components and the range of estimates for the variation in each observable component accruing to the factor and factor share, estimates of the range of variation in output per worker accruing to each factor and factor share can be determined. For example, 99% of the variation in output per worker accrues to *observables*. Decomposing this variation in accordance with equations (2.8) and (2.9) indicates that 97-100% of the variation in *observables* accrues to $\alpha \ln k$. Of the variation in $\alpha \ln k$, the decompositions given by equations (2.11) and (2.12) reveal that 73-94% of that variation accrues to α. Therefore, the lower bound for the range of variation in output per worker accruing to α is given by the product (99%)(97%)(73%) = 70%. The upper bound for the range of variation in output per worker accruing to α is given by the product (99%)(100%)(94%) = 93%. Thus, variation in α accounts for 70-93% of the variation in output per worker. The ranges of variation in output per worker accruing to *k*, $\beta+\eta$ and *h* are determined in a similar manner. As reported in column 2 of Table 3, 6-27% of the variation in output per worker accrues to *k*, 0-1% accrues to $\beta+\eta$, and 0-2% accrues to *h*.

In light of these results, it can be concluded that the variation in output per worker accrues primarily to physical capital's share. Variation in physical capital per worker absorbs the second largest fraction of variation in output per worker. Variation in total labor's share and variation in the average level of human capital augmented labor together account for a relatively small portion of the variation in output per worker. The important revelation is that the explanatory power lost by the TFP residual is not redistributed to factors when factor shares are allowed to vary. It is the actual variation in factor shares and primarily the variation in physical capital's share that absorbs the TFP residual's lost explanatory power.

2.2.6 Relative Variance Estimates: Allowing Factor Shares to Vary and Distinguishing between Human Capital and Unskilled Labor

Though I allow factor shares to vary in the second column of Table 2.3, human capital and unskilled labor are still entangled in a single, composite measure, and natural capital is not acknowledged. In other words, no distinction between reproducible and non-reproducible factors has been made. Column 3 of Table 2.3 presents results based on $y = Ak^{\alpha}(h-1)^{\beta}$. Relative to the production function considered in Section 2.2.5, I have moved even further from the typical development accounting approach by treating human capital and unskilled labor as separate, imperfectly substitutable factors. Natural capital, however, is still omitted.

Following the decomposition given by equation (2.6), I find that variation in *observables* accounts for 99% of the variation in output per worker, and the remaining 1% is accounted for by variation in the TFP residual. I decompose the explanatory power of *observables* into that accruing to factors and factor shares following the steps described in Section 2.2.5. The only difference is that β and h-1 take the place of β+η and h respectively.

The conclusions change very little. Results indicate that most of the variation in output per worker still accrues to variation in physical capital's share. Variation in physical capital per worker absorbs the majority of the remaining variation in output per worker. The labor variables, even after distinguishing between unskilled labor and human capital, explain very little of the variation in output per worker.

2.2.7 Relative Variance Estimates: Including Natural Capital

I use my baseline production function, $y = Ak^{\alpha} n^{\gamma} (h-1)^{\beta}$, to obtain the results in column 4 of Table 2.3. None of the traditional development accounting assumptions is present. All factors of production, including natural capital, are acknowledged, reproducible factors are distinguished from non-reproducible factors, and factor shares are allowed to vary. In accordance with the relative variance decomposition given by equation (2.6), I find that 77% of the variation in output per worker accrues to *observables*, and 23% accrues to the TFP residual. The fraction of variation accruing to *observables* decreases relative to the same fraction in columns 2 and 3 because of the relatively large magnitude of the correlation between the TFP residual and *observables*. The intuition follows directly from equation (2.6). The fraction of variation in output per worker assumed to reflect variation in *observables* gets smaller as the magnitude of the correlation between *observables* and the TFP residual gets larger.

I follow a two step process analogous to that described in Section 2.2.5 to decompose the explanatory power of *observables* into that accruing to factors and that accruing to factor shares. The variance of *observables* can be expressed as

$$\begin{aligned} \mathrm{var}[\ln y_{observables}] = {} & \mathrm{var}[\alpha \ln k] + \mathrm{var}[\gamma \ln n] + \mathrm{var}[\beta \ln(h-1)] + 2\,\mathrm{cov}[\alpha \ln k, \gamma \ln n] \\ & + 2\,\mathrm{cov}[\alpha \ln k, \beta \ln(h-1)] + 2\,\mathrm{cov}[\gamma \ln n, \beta \ln(h-1)] \end{aligned} . \quad (2.13)$$

Uniquely estimating the variation in *observables* accruing to $\alpha \ln k$, $\gamma \ln n$, and $\beta \ln(h-1)$ requires assumptions about the interaction effects contained within the covariance terms in equation (2.13). Previously, there was only one covariance term to deal with at this stage in the process. Now there are three. However, it turns out that the last two terms on the right hand side of equation (2.13) are empirically negligible. Omitting these covariances yields

$$\widetilde{\mathrm{var}}[\ln y_{observables}] = \mathrm{var}[\alpha \ln k] + \mathrm{var}[\gamma \ln n] + \mathrm{var}[\beta \ln(h-1)] + 2\,\mathrm{cov}[\alpha \ln k, \gamma \ln n], \quad (2.14)$$

which is an extremely good approximation of the actual variance of *observables*. The actual variance equals 0.643, and the approximation equals 0.636.

In light of this result, I determine an upper and lower bound for the variation in *observables* accruing to each of the three components by considering two alternative relative variance decompositions. The first decomposition, which is given by

$$\frac{\left(1-\rho^2_{\alpha \ln k,\, \gamma \ln n}\right)\text{var}[\gamma \ln n]}{\widetilde{\text{var}}[\ln y_{observables}]} + \frac{\left\{sd[\alpha \ln k] + sd[\gamma \ln n]\rho_{\alpha \ln k, \gamma \ln n}\right\}^2}{\widetilde{\text{var}}[\ln y_{observables}]} + \frac{\text{var}[\beta \ln(h-1)]}{\widetilde{\text{var}}[\ln y_{observables}]} = 1, \quad (2.15)$$

attributes all of the correlation between $\alpha \ln k$ and $\gamma \ln n$ to $\alpha \ln k$. $\rho_{\alpha \ln k,\, \gamma \ln n}$ represents the statistical correlation between $\alpha \ln k$ and $\gamma \ln n$. The first, second, and third terms on the left hand side of equation (2.15) are the estimates of the variation in *observables* accruing to $\gamma \ln n$, $\alpha \ln k$ and $\beta \ln(h-1)$, respectively. If all correlation between $\alpha \ln k$ and $\gamma \ln n$ is attributed to $\gamma \ln n$, then the relative variance decomposition takes the form:

$$\frac{\left\{sd[\gamma \ln n] + sd[\alpha \ln k]\rho_{\alpha \ln k,\, \gamma \ln n}\right\}^2}{\widetilde{\text{var}}[\ln y_{observables}]} + \frac{\left(1-\rho^2_{\alpha \ln k,\, \gamma \ln n}\right)\text{var}[\alpha \ln k]}{\widetilde{\text{var}}[\ln y_{observables}]} + \frac{\text{var}[\beta \ln(h-1)]}{\widetilde{\text{var}}[\ln y_{observables}]} = 1. \quad (2.16)$$

The first, second and third terms on the left hand side of equation (2.16) have the same interpretations as the corresponding terms in equation (2.15).

Notice that the estimate of the variation in *observables* accruing to $\beta \ln(h-1)$ is the same in both decompositions. This is because the covariance between $\beta \ln(h-1)$ and each of the other two observable components is negligible and therefore ignored. The covariance between $\alpha \ln k$ and $\gamma \ln n$ is not negligible, and so the relative variance estimates for each of these components is dependent on the degree to which variation in one of the components reflects variation in the other. There is no theory suggesting that a specific fraction of the interaction between $\alpha \ln k$ and $\gamma \ln n$ be allocated to either $\alpha \ln k$ or $\gamma \ln n$. There are, however, two possible extremes: either all variation in $\alpha \ln k$ reflects variation in $\gamma \ln n$ or all variation in $\gamma \ln n$ reflects variation in $\alpha \ln k$. Thus, the relative variance estimates for $\gamma \ln n$ and $\alpha \ln k$ contained in the decompositions given by equations (2.15) and (2.16) serve as upper and lower bounds.

I break down the variation in each of the three observable components into that accruing to the factor and that accruing to the factor share as in Section 2.2.5. Equations (2.10), (2.11), and (2.12) pertain specifically to $\alpha \ln k$, but applying the methodology to $\beta \ln(h-1)$ and $\gamma \ln n$ is straightforward.

The break down of the explanatory power of *observables* indicates that most of the variation in output per worker accrues to physical capital's share and natural capital's share. As reported in column 4 of Table 2.3, 22-60% of the variation in output per worker accrues to natural capital's share, and 10-47% accrues to physical capital's share. Variation in physical capital accounts for 1-14% of the variation in output per worker, and each of the remaining variables accounts for no more than 2% of the variation.

2.2.8 Acknowledging a New Type of Technical Progress

The TFP residual is generally thought to encompass productivity and efficiency and is often interpreted as "the" indicator of technology. Thus, the typical result that the lion's share of variation in output per worker accrues to the TFP residual is usually interpreted as evidence of technology's importance in explaining cross-country differences in output per worker. But, the TFP residual also encompasses all sorts of biases and measurement errors that arise from misguided assumptions about the production process. Factor shares are not constant across countries, so assuming they are constant forces the actual variation in factor shares to be encompassed by variation in the TFP residual. In addition, the omission of natural capital and the amalgamation of human capital and unskilled labor are misspecifications of the production function, and variation in the TFP residual will reflect these misspecifications.

In my sample, variation in the TFP residual explains a little over half of the variation in output per worker when the typical development accounting approach is followed. When factor shares are treated as variables and a distinction between all reproducible and non-reproducible factors of production is made, the overwhelming majority of variation in output per worker accrues to factor shares, not the TFP residual. This result, however, does not diminish the role of technical change.

Changes in technology are not synonymous with changes in the TFP residual. The TFP residual picks up everything not explicitly accounted for by the production function. Moreover, the TFP residual enters the production function linearly; therefore, it appropriately accounts for technical progress of only a factor augmenting nature. There is no reason to believe that technical progress cannot manifest itself as a change in factor shares. In fact, there is a theoretical precedent for such progress. Peretto and Seater (2008) and Zuleta (2008b) develop endogenous growth models whereby technical progress occurs via changes in factor shares. This type of progress, which Peretto and Seater refer to as factor-eliminating technical progress, impacts the intensity with which factors of production are used. It does not impact the effectiveness or productivity of factors of production, and so it is fundamentally different from factor-augmenting technical progress. The development accounting results presented herein do not dismiss the importance of technology. Rather, the large degree of explanatory power accruing to factor shares indicates the importance of acknowledging factor eliminating technical progress, a new form of technical progress that impacts factor shares.

2.3. Remarks

2.3.1 Alternative Production Functions

An alternative to the Cobb-Douglas production function is the CES production function. The CES production function, which includes Cobb-Douglas as a special case, allows for non-neutral differences in technology. Though theoretically appealing, the CES specification presents empirical challenges. Caselli (2005) experiments with the CES function and finds that the development accounting results are very sensitive to the choice of the elasticity of substitution. This poses an empirical issue because, as noted by Caselli (2005), published estimates of the elasticity of substitution between capital and labor are neither "stable nor reliable." [46] That said, it is not obvious that any bias resulting from misspecification associated with forcing the elasticity of substitution to equal one across all countries in the Cobb-Douglas form is greater than the bias associated with the misspecification resulting from plugging in inaccurate measures of the elasticity of substitution for each country in a CES framework.[47]

Though technical progress explicitly accounted for by the CES function need not affect each factor of production proportionally, it is still factor augmenting. The residual parameters still enter the production function multiplicatively, and technical progress that impacts factor shares is still inappropriately accounted for if forced to work through the TFP

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[46] Studies that estimate the elasticity of substitution between capital and labor include: Arrow et al. (1961), Ferguson (1965), Sato (1970), Hamermesh (1993), Genc and Bairam (1998), Duffy and Papageorgiou (2000), and Boskin and Lau (2000). Some of these studies report estimates below one, and others report estimates above one.

[47] Caselli and Coleman (2006) use a CES aggregate of unskilled and skilled labor to model their labor input into production. They use Katz and Murphy's (1992) estimate of 1.4 for the elasticity of substitution between skilled and unskilled labor in the United States. They argue that this is reasonable, in part, because Autor et al (1998) conclude that the elasticity of substitution between skilled and unskilled labor is unlikely to fall outside the interval between one and two. That said, there is much more uncertainty surrounding the elasticity of substitution between capital and labor. Moreover, distinguishing between reproducible and non-reproducible factor shares as I do and analyzing the consequences of allowing these shares to vary across countries in a CES framework would require data on the elasticity of substitution between: natural capital and physical capital, physical capital and unskilled labor, physical capital and human capital, natural capital and human capital, natural capital and unskilled labor, and human capital and unskilled labor.

parameters. Thus, using a general CES function instead of a Cobb-Douglas function does not circumvent the entanglement of factor augmenting and factor eliminating technical progress that arises when factor shares are assumed constant.

Note also that the production structure I use in equation (2.1) consists of a single sector. Caselli (2005) and Vollrath (2009) estimate a two sector model in the spirit of Galor and Weil (2000) and Hansen and Prescott (2002). Both use two Cobb-Douglas production functions, one for agriculture and one for industry, to control for the distribution of factors across the two sectors. The agriculture sector in these models employs land, and the industrial sector does not. Both approaches have their merits, but neither accounts for natural resources such as minerals and oil that are used in industry. Minerals and oil are non-reproducible factors just as land is. Since both approaches completely overlook a portion of natural capital, neither can support an analysis aimed at evaluating the implications of variability in all reproducible and non-reproducible factor shares. My approach does not overlook any portion of reproducible or non-reproducible factors; it merely lumps all factors into a single aggregate production function.

2.3.2 Additional Results

Analyses identical to the one presented in Section 2.2 are also performed using factor share estimates obtained via the *labor force correction* and *No Adjustment* approaches from Chapter 1. These results are presented in Tables 2.4 through 2.8. The *labor force correction* sample consists of the same 30 countries that comprise the *OSPUE adjustment* sample. With the *No Adjustment* approach, the sample is larger and consists of 59 countries. The results are qualitatively equivalent. The finding that the majority of the variation in output per worker accrues to physical and natural capital's share when factor shares are allowed to vary is robust to the manner in which factor shares are computed.

2.4. Conclusion

I revisit the development accounting exercise, acknowledging variation in factor shares and making a distinction between reproducible and non-reproducible factors. The general consensus is that at least half of the variation in output per worker accrues to the TFP residual. Researchers have attempted a number of things in an effort to chip away at the TFP residual's explanatory power, but nothing has led to a substantial reduction in the importance of the TFP residual until now. The fraction of cross-country variation in output per worker accruing to the TFP residual drops from 55% when factors shares are assumed constant to a substantially lower 23% when factor shares are allowed to vary in accordance with my *OSPUE Adjusted* estimates from Chapter 1. For factor share estimates computed via the *labor force correction* and *No Adjustment* approaches, the TFP residual's explanatory power falls by an even larger margin. Cross-country variation in factor shares, completely ignored in the standard approach, explains the majority of variation in output per worker.

The shift in explanatory power does not diminish the role of technical progress. It does, however, indicate that most of the variation in output per worker accruing to technical progress is variation in factor-eliminating rather than factor-augmenting progress. That said, identifying and understanding the determinants of cross-country differences in factor shares is imperative to understanding cross-country differences in output per worker.

Table 2.1: Decomposition of Output per Worker - *OSPUE Adjustment*
Factor Shares Constant; Labor Components Entangled; and Natural Capital Omitted

Country	y	$k^{1/3}$	$h^{2/3}$	A
U.S.A	67078.860	50.530	2.167	612.705
Norway	63909.140	56.919	2.145	523.442
Belgium	59873.550	49.137	1.892	644.007
Ireland	59103.420	43.348	1.893	720.257
Singapore	58750.040	48.878	1.664	722.335
Austria	58441.050	49.415	1.801	656.782
Netherlands	56690.570	46.328	1.893	646.416
France	55285.960	47.132	1.753	668.955
Israel	51882.640	45.138	1.917	599.646
Italy	50853.040	46.704	1.678	648.860
Australia	50606.350	45.300	2.048	545.588
Denmark	50448.300	49.464	1.923	530.483
Canada	49815.630	43.958	2.121	534.426
Sweden	46544.490	45.732	2.098	485.022
Finland	45192.140	46.045	1.955	502.147
Japan	44563.230	61.015	1.904	383.513
Spain	44360.540	41.548	1.689	632.168
New Zealand	40976.960	39.054	2.133	491.845
Mauritius	34617.690	27.912	1.555	797.702
Portugal	34000.270	36.699	1.542	600.917
Trinidad and Tobago	33101.830	29.732	1.742	639.045
Greece	32069.690	38.038	1.830	460.776
Korea, Republic Of	30620.650	37.850	2.039	396.677
Hungary	23788.820	31.222	1.871	407.151
Costa Rica	20596.220	26.548	1.560	497.370
Mexico	19621.490	35.444	1.683	328.835
Panama	18798.390	27.108	1.819	381.275
Russia	17269.690	29.281	1.958	301.150
Botswana	16616.550	27.637	1.583	379.771
Egypt	11939.540	21.579	1.506	367.322
Average	41580.558	40.823	1.845	536.886
Standard Deviation	16081.332	9.777	0.196	129.625
correlation with y (logs)	1	0.894	0.497	0.745
correlation with A (logs)	0.745	0.409	-0.073	1

Table 2.2: Decomposition of Output per Worker - *OSPUE Adjustment*
Factor Shares Vary; Labor Components Separated; and Natural Capital Included

Country	y	k^{α}	n^{γ}	$(h\text{-}1)^{\beta}$	A
U.S.A	67078.860	12.961	3.079	1.489	1129.164
Norway	63909.140	34.127	16.494	1.448	78.431
Belgium	59873.550	21.099	2.286	1.252	991.572
Ireland	59103.420	40.423	6.141	1.252	190.203
Singapore	58750.040	64.537	2.406	1.059	357.494
Austria	58441.050	30.685	3.098	1.179	521.419
Netherlands	56690.570	33.269	3.314	1.249	411.772
France	55285.960	23.567	2.966	1.139	694.406
Israel	51882.640	14.058	2.331	1.260	1256.890
Italy	50853.040	32.565	3.046	1.076	476.441
Australia	50606.35	12.210	6.315	1.350	486.217
Denmark	50448.300	28.832	3.717	1.262	373.099
Canada	49815.630	6.425	6.899	1.418	792.503
Sweden	46544.490	17.396	2.936	1.374	663.429
Finland	45192.140	26.134	4.225	1.276	320.660
Japan	44563.230	12.424	1.760	1.265	1610.919
Spain	44360.540	12.021	2.354	1.082	1448.755
New Zealand	40976.960	5.417	21.083	1.385	259.040
Mauritius	34617.690	14.923	2.086	0.974	1141.764
Portugal	34000.270	12.749	2.444	0.963	1132.773
Trinidad and Tobago	33101.830	2.913	30.400	1.116	335.087
Greece	32069.690	29.080	3.872	1.175	242.374
Korea, Republic Of	30620.650	15.489	2.191	1.359	663.929
Hungary	23788.820	12.491	4.647	1.193	343.559
Costa Rica	20596.220	3.867	8.362	0.979	650.261
Mexico	19621.490	21.984	11.086	1.068	75.402
Panama	18798.390	7.219	4.777	1.155	472.087
Russia	17269.690	6.575	23.876	1.157	95.079
Botswana	16616.550	23.640	7.963	0.997	88.535
Egypt	11939.540	8.853	17.302	0.941	82.828
Average	41580.558	19.598	7.115	1.196	579.536
Standard Deviation	16081.332	13.285	7.336	0.153	430.812
correlation with y (logs)	1	0.465	-0.461	0.594	0.501
correlation with A (logs)	0.501	-0.128	-0.740	0.160	1

Table 2.3: Decomposition of Variation in Output per Worker - *OSPUE Adjustment*

	Production Function			
	Constant Shares	Variable Shares		
	N omitted; Labor Components Entangled	*N* omitted; Labor Components Entangled	*N* omitted; Labor Components Separated	Include *N*; Labor Components Separated
Variance Decomposition	$y=Ak^{1/3}h^{2/3}$	$y=Ak^{\alpha}h^{\beta+\eta}$	$y=Ak^{\alpha}(h-1)^{\beta}$	$y=Ak^{\alpha}n^{\gamma}(h-1)^{\beta}$
Variation in Output per Worker attributable to *Observables*	0.45	0.99	0.99	0.77
Variation accruing to α		0.70 - 0.93	0.67 - 0.90	0.10 - 0.47
Variation accruing to k		0.06 - 0.27	0.06 - 0.27	0.01 - 0.14
Variation accruing to $\beta+\eta$		0.00 - 0.01		
Variation accruing to β			0.00 - 0.02	0.00 - 0.01
Variation accruing h		0.00 - 0.02		
Variation accruing to h-1			0.02 - 0.06	0.01 - 0.02
Variation accruing to γ				0.22 - 0.60
Variation accruing to n				0 - 0.08
Variation in Output per Worker attributable to the TFP residual	0.55	0.01	0.01	0.23
Variances and Covariances				
var(ln(y))	0.224	0.224	0.224	0.224
var(ln(A))	0.065	0.355	0.391	0.828
var(α ln(k))	0.066	0.529	0.529	0.529
var(($\beta+\eta$)ln(h))	0.012	0.016		
var(β ln(h-1))			0.017	0.017
var(γ ln(n))				0.656
cov[ln(A), α ln(k)]	0.027	-0.355	-0.376	-0.093
cov[ln(A), ($\beta+\eta$)ln(h)]	-0.002	0.031		
cov[ln(A), β ln(h-1)]			0.012	0.016
cov[ln(A), γ ln(n)]				-0.546
cov[α ln(k), γ ln(n)]				-0.283
cov[αln(k), ($\beta+\eta$)ln(h)]	0.016	-0.014		
cov[αln(k), β ln(h-1)]			0.007	0.007
cov[γ ln(n), β ln(h-1)]				-0.004
Raw Correlation				
correlation coefficient, $\rho_{obs,\text{TFP}}$	0.30	-0.76	-0.78	-0.85

Table 2.4: Decomposition of Output per Worker - *Labor Force Correction*
Factor Shares Vary; Labor Components Separated; and Natural Capital Included

Country	y	k^{α}	n^{γ}	$(h-1)^{\beta}$	A
U.S.A	67078.860	12.961	3.079	1.489	1129.160
Norway	63909.140	32.609	15.908	1.458	84.482
Belgium	59873.550	16.905	2.152	1.258	1308.589
Ireland	59103.420	36.064	5.806	1.257	224.510
Singapore	58750.040	64.538	2.406	1.059	357.491
Austria	58441.050	27.729	2.996	1.183	594.760
Netherlands	56690.570	30.535	3.218	1.254	460.071
France	55285.960	21.194	2.860	1.142	798.924
Israel	51882.640	11.742	2.200	1.265	1587.672
Italy	50853.040	23.968	2.762	1.077	713.096
Australia	50606.350	11.169	5.914	1.357	564.597
Denmark	50448.300	26.052	3.572	1.267	427.859
Canada	49815.630	5.946	6.366	1.432	918.722
Sweden	46544.490	15.538	2.814	1.387	767.413
Finland	45192.140	22.030	3.918	1.282	408.558
Japan	44563.230	13.497	1.793	1.260	1461.617
Spain	44360.540	9.423	2.165	1.083	2007.286
New Zealand	40976.960	5.236	19.833	1.392	283.419
Mauritius	34617.690	10.753	1.908	0.973	1733.082
Portugal	34000.270	9.542	2.208	0.963	1676.614
Trinidad and Tobago	33101.830	2.913	30.400	1.117	334.585
Greece	32069.690	19.374	3.289	1.178	427.328
Korea, Republic Of	30620.650	10.291	1.949	1.403	1087.947
Hungary	23788.820	9.030	3.814	1.195	577.844
Costa Rica	20596.220	3.444	6.973	0.979	875.725
Mexico	19621.490	19.064	9.922	1.068	97.090
Panama	18798.390	5.063	3.608	1.157	889.744
Russia	17269.690	6.199	21.619	1.167	110.466
Botswana	16616.550	19.861	7.103	0.997	118.146
Egypt	11939.540	6.216	10.898	0.941	187.370
Average	41580.558	16.963	6.448	1.201	740.472
Standard Deviation	16081.332	12.768	6.875	0.157	543.303
correlation with y (logs)	1.000	0.539	-0.381	0.574	0.336
correlation with A (logs)	0.336	-0.203	-0.757	0.031	1.000

Table 2.5: Decomposition of Variation in Output per Worker - *Labor Force Correction*

	Production Function			
	Constant Shares	Variable Shares		
	N omitted; Labor Components Entangled	*N* omitted; Labor Components Entangled	*N* omitted; Labor Components Separated	Include *N*; Labor Components Separated
Variance Decomposition	$y = Ak^{1/3}h^{2/3}$	$y = Ak^{\alpha}h^{\beta+\eta}$	$y = Ak^{\alpha}(h\text{-}1)^{\beta}$	$y = Ak^{\alpha}n^{\gamma}(h\text{-}1)^{\beta}$
Variation in Output per Worker attributable to *Observables*	0.45	1.00	1.00	0.89
Variation accruing to α		0.60 - 0.95	0.56 - 0.93	0.14 - 0.53
Variation accruing to k		0.05 - 0.38	0.04 - 0.37	0.01 - 0.21
Variation accruing to $\beta+\eta$		0.00 - 0.01		
Variation accruing to β			0.00 - 0.03	0.00 - 0.01
Variation accruing h		0.00 - 0.02		
Variation accruing to h-1			0.02 - 0.09	0.01 - 0.02
Variation accruing to γ				0.25 - 0.64
Variation accruing to n				0.00 - 0.13
Variation in Output per Worker attributable to the TFP residual	0.55	0.00	0.00	0.11
Variances and Covariances				
var(ln(y))	0.224	0.224	0.224	0.224
var(ln(A))	0.065	0.322	0.359	0.814
var(α ln(k))	0.066	0.538	0.538	0.538
var(($\beta+\eta$)ln(h))	0.012	0.015		
var(β ln(h-1))			0.017	0.017
var(γ ln(n))				0.624
cov[ln(A), α ln(k)]	0.027	-0.338	-0.363	-0.135
cov[ln(A), ($\beta+\eta$)ln(h)]	-0.002	0.025		
cov[ln(A), β ln(h-1)]			0.006	0.004
cov[ln(A), γ ln(n)]				-0.539
cov[α ln(k), γ ln(n)]				-0.229
cov[αln(k), ($\beta+\eta$)ln(h)]	0.016	-0.013		
cov[αln(k), β ln(h-1)]			0.013	0.013
cov[γ ln(n), β ln(h-1)]				0.002
Raw Correlation				
correlation coefficient, $\rho_{obs,TFP}$	0.30	-0.76	-0.78	-0.86

Table 2.6: Decomposition of Output per Worker - *No Adjustment*
Factor Shares Constant; Labor Components Entangled; and Natural Capital Omitted

Country	y	$k^{1/3}$	$h^{2/3}$	A
U.S.A	67078.860	50.530	2.167	612.705
Norway	63909.140	56.919	2.145	523.442
Belgium	59873.550	49.137	1.892	644.007
Ireland	59103.420	43.348	1.893	720.257
Singapore	58750.040	48.878	1.664	722.335
Austria	58441.050	49.415	1.801	656.782
Netherlands	56690.570	46.328	1.893	646.416
France	55285.960	47.132	1.753	668.955
Israel	51882.640	45.138	1.917	599.646
Germany	51010.260	48.573	1.975	531.680
Italy	50853.040	46.704	1.678	648.860
Australia	50606.350	45.300	2.048	545.588
Denmark	50448.300	49.464	1.923	530.483
Canada	49815.630	43.958	2.121	534.426
Sweden	46544.490	45.732	2.098	485.022
Finland	45192.140	46.045	1.955	502.147
Japan	44563.230	61.015	1.904	383.513
Spain	44360.540	41.548	1.689	632.168
New Zealand	40976.960	39.054	2.133	491.845
Mauritius	34617.690	27.912	1.555	797.702
Portugal	34000.270	36.699	1.542	600.917
Trinidad and Tobago	33101.830	29.732	1.742	639.045
Greece	32069.690	38.038	1.830	460.776
Korea, Republic Of	30620.650	37.850	2.039	396.677
Chile	27994.580	28.161	1.719	578.405
Hungary	23788.820	31.222	1.871	407.151
Costa Rica	20596.220	26.548	1.560	497.370
Estonia	20074.110	31.955	1.925	326.427
South Africa	19759.940	23.728	1.569	530.792
Mexico	19621.490	35.444	1.683	328.835
Panama	18798.390	27.108	1.819	381.275
Venezuela	17913.330	29.329	1.620	376.904
Iran	17594.660	19.844	1.487	596.300
Russia	17269.690	29.281	1.958	301.150
Algeria	16661.230	29.691	1.493	375.935
Botswana	16616.550	27.637	1.583	379.771
Latvia	16372.110	28.972	1.925	293.633
Brazil	15470.100	25.933	1.446	412.495
Bulgaria	14070.380	22.021	1.904	335.522

Table 2.6 (continued)

Country	y	$k^{1/3}$	$h^{2/3}$	A
Colombia	14053.810	28.169	1.483	336.405
Namibia	13314.510	24.116	1.316	419.540
Paraguay	13149.930	19.972	1.573	418.578
Jordan	12239.140	24.528	1.649	302.592
Egypt	11939.540	21.579	1.506	367.322
Peru	11108.320	24.728	1.722	260.867
Romania	10908.750	23.673	1.908	241.486
Philippines	9229.470	17.484	1.788	295.223
Sri Lanka	8966.610	18.212	1.645	299.337
Nicaragua	8801.460	15.939	1.418	389.314
Cameroon	6022.730	15.365	1.318	297.400
Honduras	5975.600	19.320	1.439	214.982
Ivory Coast	5325.300	12.745	1.316	317.518
Moldova	4370.510	20.836	1.925	108.996
Lesotho	4317.090	17.901	1.387	173.919
Senegal	3542.310	11.999	1.220	241.960
Kenya	2458.240	11.390	1.384	155.955
Mozambique	2086.220	9.265	1.090	206.491
Burkina Faso	1962.170	11.264	1.316	132.374
Niger	1748.870	8.751	1.083	184.563
Average	27185.059	31.332	1.702	431.562
Standard Deviation	19874.078	13.292	0.270	167.381
correlation with y (logs)	1.000	0.959	0.761	0.889
correlation with A (logs)	0.889	0.739	0.442	1.000

Table 2.7: Decomposition of Output per Worker - *No Adjustment*
Factor Shares Vary; Labor Components Separated; and Natural Capital Included

Country	y	k^{α}	n^{γ}	$(h\text{-}1)^{\beta}$	A
U.S.A	67078.860	26.125	4.188	1.408	435.620
Norway	63909.140	20.207	10.879	1.351	215.197
Belgium	59873.550	30.575	2.527	1.196	647.803
Ireland	59103.420	39.182	6.047	1.195	208.741
Singapore	58750.040	208.310	3.079	1.044	87.700
Austria	58441.050	27.715	2.995	1.139	618.037
Netherlands	56690.570	25.500	3.026	1.190	617.455
France	55285.960	23.695	2.971	1.106	709.715
Israel	51882.640	21.658	2.677	1.192	750.741
Germany	51010.260	25.228	2.541	1.111	716.378
Italy	50853.040	71.781	3.922	1.057	170.888
Australia	50606.350	15.353	7.476	1.254	351.526
Denmark	50448.300	15.357	2.906	1.192	948.469
Canada	49815.630	10.544	11.539	1.297	315.762
Sweden	46544.490	14.113	2.714	1.287	944.717
Finland	45192.140	22.517	3.956	1.195	424.610
Japan	44563.230	138.590	3.023	1.178	90.302
Spain	44360.540	27.888	3.145	1.059	477.635
New Zealand	40976.960	9.064	53.372	1.260	67.248
Mauritius	34617.690	59.084	3.034	0.982	196.607
Portugal	34000.270	18.247	2.772	0.975	689.304
Trinidad and Tobago	33101.830	5.149	187.435	1.076	31.890
Greece	32069.690	85.482	5.972	1.112	56.473
Korea, Republic Of	30620.650	44.487	2.964	1.220	190.322
Chile	27994.580	5.161	13.189	1.060	387.972
Hungary	23788.820	13.066	4.776	1.114	342.203
Costa Rica	20596.220	6.115	17.173	0.988	198.557
Estonia	20074.110	15.427	5.612	1.124	206.210
South Africa	19759.940	9.404	6.045	0.992	350.425
Mexico	19621.490	36.942	16.606	1.039	30.780
Panama	18798.390	14.925	8.485	1.087	136.529
Venezuela	17913.330	3.618	46.486	1.014	105.091
Iran	17594.660	1.856	56.447	0.956	175.768
Russia	17269.690	6.545	23.692	1.110	100.343
Algeria	16661.230	11.325	250.544	0.959	6.123
Botswana	16616.550	39.097	11.077	0.998	38.435
Latvia	16372.110	8.054	4.718	1.116	385.897
Brazil	15470.100	5.011	6.079	0.907	559.965
Bulgaria	14070.380	11.577	13.128	1.106	83.742

Table 2.7 (continued)

Country	y	k^{α}	n^{γ}	$(h-1)^{\beta}$	A
Colombia	14053.810	3.121	10.067	0.938	476.911
Namibia	13314.510	15.607	7.674	0.872	127.485
Paraguay	13149.930	2.151	4.067	0.994	1511.560
Jordan	12239.140	22.506	3.462	1.022	153.656
Egypt	11939.540	7.436	13.775	0.968	120.405
Peru	11108.320	23.563	26.817	1.046	16.807
Romania	10908.750	10.695	8.199	1.100	113.072
Philippines	9229.470	25.706	22.129	1.010	16.060
Sri Lanka	8966.610	13.935	4.602	1.020	137.146
Nicaragua	8801.460	2.522	5.621	0.933	665.460
Cameroon	6022.730	3.643	215.853	0.890	8.609
Honduras	5975.600	7.118	19.664	0.945	45.157
Ivory Coast	5325.300	1.509	7.457	0.890	531.862
Moldova	4370.510	3.491	4.439	1.091	258.573
Lesotho	4317.090	130.645	6.815	0.926	5.238
Senegal	3542.310	6.136	38.479	0.839	17.882
Kenya	2458.240	3.424	19.922	0.927	38.852
Mozambique	2086.220	1.793	7.621	0.728	209.591
Burkina Faso	1962.170	5.387	47.376	0.898	8.564
Niger	1748.870	1.074	2.303	0.720	982.974
Average	27185.059	24.838	21.959	1.058	313.848
Standard Deviation	19874.078	36.425	47.889	0.140	312.927
correlation with y (logs)	1.000	0.581	-0.335	0.803	0.423
correlation with A (logs)	0.423	-0.155	-0.724	0.273	1.000

Table 2.8: Decomposition of Variation in Output per Worker - *No Adjustment*

	Production Function			
	Constant Shares	Variable Shares		
	N omitted; Labor Components Entangled	*N* omitted; Labor Components Entangled	*N* omitted; Labor Components Separated	Include *N*; Labor Components Separated
Variance Decomposition	$y=Ak^{1/3}h^{2/3}$	$y=Ak^{\alpha}h^{\beta+\eta}$	$y=Ak^{\alpha}(h\text{-}1)^{\beta}$	$y=Ak^{\alpha}n^{\gamma}(h\text{-}1)^{\beta}$
Variation in Output per Worker attributable to *Observables*	0.21	0.96	0.96	0.82
Variation accruing to α		0.44 - 0.88	0.41 - 0.88	0.16 - 0.52
Variation accruing to k		0.05 - 0.35	0.05 - 0.35	0.02 - 0.21
Variation accruing to $\beta+\eta$		0.00 - 0.22		
Variation accruing to β			0.00 - 0.04	0.00
Variation accruing h		0.00 - 0.23		
Variation accruing to h-1			0.01 - 0.31	0.01
Variation accruing to γ				0.24 - 0.55
Variation accruing to n				0.00 - 0.02
Variation in Output per Worker attributable to the TFP residual	0.79	0.04	0.04	0.18
Variances and Covariances				
var(ln(y))	0.970	0.970	0.970	0.970
var(ln(A))	0.201	0.918	0.943	1.937
var(α ln(k))	0.230	1.340	1.340	1.340
var(($\beta+\eta$)ln(h))	0.028	0.025		
var(β ln(h-1))			0.018	0.018
var(γ ln(n))				1.322
cov[ln(A), α ln(k)]	0.159	-0.757	-0.754	-0.250
cov[ln(A), ($\beta+\eta$)ln(h)]	0.033	0.021		
cov[ln(A), β ln(h-1)]			0.012	0.051
cov[ln(A), γ ln(n)]				-1.158
cov[α ln(k), γ ln(n)]				-0.504
cov[αln(k), ($\beta+\eta$)(ln(h))]	0.064	0.079		
cov[αln(k), β ln(h-1)]			0.077	0.077
cov[γ ln(n), β ln(h-1)]				-0.039
Raw Correlation				
correlation coefficient, $\rho_{obs,\text{TFP}}$	0.69	-0.62	-0.62	-0.74

Chapter 3

Endogenous Saving in a Model of Factor-Eliminating Technical Change

3.1 Introduction

Perpetual growth requires that the marginal products of reproducible factors of production be bounded away from zero (Jones and Manuelli, 1997). Virtually all theoretical studies satisfy this requirement for growth via the augmentation of non-reproducible factors of production. Peretto and Seater (2009), however, satisfy the requirement using a different mechanism. They develop an endogenous theory of factor elimination, whereby the non-reproducible factors of production are eliminated from the production process. Factor-augmenting technical progress, the typical engine of growth, is completely absent in their model. Instead, they allow factor intensities to change endogenously via spending on R&D, and this serves as the catalyst for growth.[48] In this paper, I extend the theory developed by Peretto and Seater by incorporating endogenous saving.

[48] Reproducible factors (physical capital, human capital) have to be produced and can therefore be accumulated. Non-reproducible factors (natural capital, unskilled labor) are those factors with which an economy is endowed. Since non-reproducible factors can not be accumulated they create diminishing returns to the reproducible factors of production. Augmenting the non-reproducible factors effectively increases the amount of non-reproducible factors. This offsets the drag that non-reproducible factors have on the marginal products of the reproducible factors, thereby sustaining the incentive for reproducible factors to be accumulated. With factor elimination, the economy develops a technology that uses only reproducible factors of production. Since non-reproducible factors become inessential to the production process, the fixed nature of non-reproducible factors no longer poses a threat to growth.

Peretto and Seater assume that households save a fixed fraction of their total income. There is no intertemporal utility maximization, so there is no consumption-saving choice. The general equilibrium dynamics have two possible outcomes. If the exogenous saving rate is high enough, the economy's production function becomes *AK* in the limit thereby supporting perpetual growth. If the saving rate is not sufficiently high, the economy goes to a Solow steady state with no growth and a standard production function with fixed factor intensities.

My model yields the same testable implications pertaining to cross-country factor shares, offers the same insight into the transition from a primitive to an advanced economy and provides the same resolution to the linearity critique as that of Peretto and Seater.[49] However, the equilibrium dynamics in my model have only one possible outcome—the economy achieves perpetual growth. Consumer optimization alters the model so that the possibility of a Solow Steady state is eliminated; all equilibrium paths lead to a production function that asymptotically becomes *AK*.

This extension, which is analogous to moving from the Solow model to the Cass model, enriches the theory. The primary new finding is that the saving rate, when chosen by optimizing households, is always high enough to support perpetual growth. The inclusion of consumer optimization also lays the foundation for the model to be extended in the future to analyze government policies. Government policies, in general, impact the incentive to save.

3.2 Factor Elimination with Endogenous Saving

The economy is comprised of households, competitive final good producers and monopolistically competitive intermediate good producers. My extension alters only the behavior of households, so final and intermediate good producers are characterized exactly as in Peretto and Seater (2009). I summarize the behavior of final and intermediate goods

[49] Their model predicts that non-reproducible factor shares decrease with output per worker, and reproducible factor shares increase with output per worker. The model lays out a purely endogenous explanation for the transition from a primitive to an advanced economy. The linearity requirement arises endogenously.

producers below in Sections 3.2.1 and 3.2.2. The equations characterizing producer behavior are important and will be needed for the equilibrium analysis, but there is nothing new, so I present the majority of equations in Sections 3.2.1 and 3.2.2 without detail or derivation.[50] My contribution begins with Section 3.2.3 where I introduce optimizing households. The general equilibrium dynamics of the model are analyzed in Section 3.3. Section 3.4 concludes.

3.2.1 Final Good Producers

Competitive firms produce the final good, *Y*, according to the technology

$$Y = \left[\int_0^1 X_i^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \ \varepsilon > 1, \tag{3.1}$$

where X_i is the quantity of intermediate good i and ε is the elasticity of substitution between intermediate goods. There is a fixed continuum of intermediate goods ranging from 0 to 1.[51] The final good is the numeraire so that $P_Y \equiv 1$. Denoting P_i as the price of X_i, maximization of profit

$$\pi_Y = Y - \int_0^1 P_i X_i di \tag{3.2}$$

subject to (3.1) yields the demand function

[50] See Section 3.2 and 3.3 of Peretto and Seater (2009) for a more thorough treatment of the optimizing behavior of firms.
[51] Holding fixed the number of intermediate goods rules out variety expansion as a potential source of growth.

$$X_i = YP_i^{-\varepsilon}. \tag{3.3}$$

3.2.2 Intermediate Good Producers

Intermediate firms, in addition to producing goods, invest in capital accumulation and conduct R&D to learn new production technologies. These new technologies, which arrive in the form of higher capital intensities, are the origins of growth in the model.

Production Each intermediate firm owns capital *K* and hires labor *L*. The term "capital" is used in a broad sense and encompasses all types of reproducible factors of production, including both human and physical capital. Likewise, *L*, though referred to as "labor" herein, encompasses all types of non-reproducible factors, including unskilled labor, land and natural resources. The firm produces output according to

$$X_i = AK_i^{q_i} L_i^{1-q_i}. \tag{3.4}$$

Total Factor Productivity (TFP) *A* is constant. Fixing *A* at a constant value rules out factor-augmenting technical progress, which is the typical engine of growth.

Research and Development The factor intensity parameter q_i is chosen from a set of known technologies $[0, \alpha_i]$, $0 \le \alpha_i \le 1$ where α_i is the technology frontier. Let R_i be R&D investment in units of the final good. R&D accumulates knowledge *Z* according to $\dot{Z}_i = R_i$. Knowledge is converted into technology via the relation $\alpha_i = h(Z_i)$, so $\dot{\alpha}_i = h'(Z_i) \cdot \dot{Z}_i = h'(Z_i) \cdot R_i = h'\left[h^{-1}(\alpha_i)\right] \cdot R_i$. Defining $f(\alpha_i) \equiv h'\left[h^{-1}(\alpha_i)\right]$, the technology frontier increases in response to the firm's R&D according to

$$\dot{\alpha}_i = \begin{cases} f(\alpha_i)\cdot R_i & \alpha_i < 1 \\ 0 & \alpha_i = 1 \end{cases} \quad . \tag{3.5}$$

As long as α_i is less than 1, it grows whenever the firm devotes resources to R&D. Once α_i reaches 1, it stays there regardless of the amount spent on R&D. The function $f(\alpha_i)$ is decreasing in α_i and represents the productivity of R&D.[52]

Investment in Capital Capital accumulates according to

$$\dot{K}_i = I_i - \delta K_i \tag{3.6}$$

where I_i is gross capital investment in units of the final good and δ is the depreciation rate.

The Firm's Optimization Problem The firm chooses the path of capital intensity, the price of the intermediate good, employment, investment and R&D to maximize the present value of its dividend payments:

$$\max_{\{a_{it},\, P_{it},\, L_{it},\, I_{it},\, R_{it}\}_{t=0}^{\infty}} \int_o^{\infty} \left(P_{it} X_{it} - w_t L_{it} - I_{it} - R_{it}\right) e^{-\bar{r}_t t} dt \ ,$$

where w_t is the wage rate at time *t*, $\bar{r}_t \equiv \frac{1}{t}\int_0^t r_u \, du$ is the average interest rate between time 0 and time *t*, and r_u is the instantaneous interest rate at time *u*. The optimization is subject to (3.3), (3.4), (3.5), (3.6), $I_{it} \geq 0$ and $R_{it} \geq 0$.

[52] Symmetric equilibrium requires $f'(\alpha_i) < 0$. Justification for this requirement and other details of the R&D function are discussed in Sections 3.3.2 and 3.5.5 of Peretto and Seater (2009).

It is useful to think of the intermediate firm as operating two divisions, production and investment, where investment consists of two departments, one for capital investment and one for R&D. The objective function can then be expressed as follows:

$$\max_{\{a_{it},\, P_{it},\, L_{it},\, I_{it},\, R_{it}\}_{t=0}^{\infty}} \int_{o}^{\infty} \left[\left(P_{it}X_{it} - w_t L_t - p_{K\,it}K_{it}\right) + \left(p_{K\,it}K_{it} - I_{it} - R_{it}\right)\right] e^{-\bar{r}_t t}\, dt \,.$$

The term inside the first set of parentheses is the instantaneous profit of the production division, and the second set of parentheses contains the instantaneous profit of the investment division. The production division rents capital from the investment division and pays the internal transfer price $p_{K\,i}$. The rental income received by the investment division is spent on investment in capital accumulation and R&D.

The firm's maximization problem is solved in two steps. The production division does not accumulate any assets, so nothing carries over from one period to the next. Thus, the production division faces a sequence of independent instantaneous profit maximization problems. It chooses P_i, L_i, and K_i taking w, $p_{K\,i}$, and α_i as given. The investment division then chooses I_i and R_i subject to the solution presented by the production division.

Production Division-Choice of *q*, *P*, *L* and *K* Making the usual symmetry assumption that all intermediate firms are identical allows the *i* subscript to be omitted. The production division solves

$$\max_{\{a_t,\, P_t,\, L_t,\, K_t\}_{t=0}^{\infty}} \int_0^{\infty} \left(P_t X_t - w_t L_t - p_{K\,t}K_t\right) e^{-\bar{r}_t t}\, dt \quad s.t.\, (3.3),\, (3.4)\,.$$

Since there is no time component associated with the production division's optimization problem, the *t* subscript can be omitted henceforth in equations characterizing the production division. That said, after eliminating P using the demand curve (3.3), the production division faces a sequence of independent instantaneous profit maximization problems of the form

$$\max_{\alpha, L, K} \pi = Y^{\frac{1}{\varepsilon}} X^{1-\frac{1}{\varepsilon}} - wL - p_K K \quad s.t. \,(3.4)\,.$$

The endogeneity of capital intensity adds an interesting twist to the production division's optimization problem. The production division has the option of using any capital intensity in the interval $q \in [0, \alpha]$. So, which capital intensity does it use? Zuleta (2009) determines that a firm will use the lowest, the highest, or the lowest and highest capital intensity. Peretto and Seater explain that as long as the firm can operate more than one plant, it will operate both extreme technologies. Let the lowest capital intensity be $\alpha_{\min} = 0$. The highest capital intensity is the technology frontier, and so $\alpha_{\max} = \alpha > 0$. The firm will divide its labor force between a plant that uses the "primitive" technology $q = 0$ and one that uses the "advanced" technology $q = \alpha$ to equalize the marginal product of labor across the two plants. As the technology frontier expands, the firm updates its labor allocation.

The allocation of labor across the two plants yields total firm output equal to

$$X = A\left[(L - l) + \left(K^{\alpha} l^{1-\alpha}\right)\right] \tag{3.7}$$

where L is total employment, $l \in [0, L]$ is labor allocated to the advanced plant and $L - l$ is labor used in the primitive plant. As is evident in equation (3.7), the capital intensity choice, through its impact on labor allocation, removes the essentiality of capital for all $\alpha < 1$. Therefore, an economy that knows only the primitive technology builds no capital and produces with labor only. A more advanced economy with $\alpha > 0$ and capital can still use the primitive technology and produce output even if its capital stock is depleted. The production division's problem can now be written

$$\max_{l,L,K} \pi = Y^{\frac{1}{\varepsilon}} X^{1-\frac{1}{\varepsilon}} - wL - p_K K \quad s.t. (3.7) .$$

There are three first-order conditions. One gives the demand for capital, another gives the demand for labor, and the third characterizes the optimal allocation of labor across the two plants. The first-order condition characterizing the optimal allocation of labor across the two plants yields $l = K(1-\alpha)^{\frac{1}{\alpha}}$. It is possible that $K(1-\alpha)^{\frac{1}{\alpha}}$ could exceed total employment L in which case the firm is constrained and allocates all its labor to the advanced plant. Taking this labor allocation constraint into account, the optimal allocation of labor across the two plants can be expressed as

$$l = \begin{cases} K(1-\alpha)^{\frac{1}{\alpha}} & K(1-\alpha)^{\frac{1}{\alpha}} < L \\ L & K(1-\alpha)^{\frac{1}{\alpha}} \geq L \end{cases} . \tag{3.8}$$

In like manner, the other two first-order conditions can be written in two parts as:

$$p_K = \begin{cases} Y^{\frac{1}{\varepsilon}} X^{-\frac{1}{\varepsilon}} \left(1-\frac{1}{\varepsilon}\right) A\alpha \left(\frac{K}{l}\right)^{\alpha-1} & K(1-\alpha)^{\frac{1}{\alpha}} < L \\ Y^{\frac{1}{\varepsilon}} X^{-\frac{1}{\varepsilon}} \left(1-\frac{1}{\varepsilon}\right) A\alpha \left(\frac{K}{L}\right)^{\alpha-1} & K(1-\alpha)^{\frac{1}{\alpha}} \geq L \end{cases} ; \tag{3.9}$$

$$w = \begin{cases} Y^{\frac{1}{\varepsilon}} X^{-\frac{1}{\varepsilon}} \left(1-\frac{1}{\varepsilon}\right) A & K(1-\alpha)^{\frac{1}{\alpha}} < L \\ Y^{\frac{1}{\varepsilon}} X^{-\frac{1}{\varepsilon}} \left(1-\frac{1}{\varepsilon}\right) A(1-\alpha)\left(\frac{K}{L}\right)^{\alpha} & K(1-\alpha)^{\frac{1}{\alpha}} \geq L \end{cases} . \tag{3.10}$$

The emergence of the aforementioned labor allocation constraint alters the expression for firm output. Defining $m(\alpha) \equiv \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}$, equation (3.8) can be used to rewrite equation (3.7) as

$$X = \begin{cases} A[L + m(\alpha)K] & K(1-\alpha)^{\frac{1}{\alpha}} < L \\ AK^{\alpha}L^{1-\alpha} & K(1-\alpha)^{\frac{1}{\alpha}} \geq L \end{cases} \quad . \tag{3.11}$$

Just like the equations above, this expression has two parts. If the labor allocation constraint is non-binding, then output is given by the top line in (3.11). If the labor allocation constraint is binding, then output is given by the bottom line in (3.11).

The function $m(\alpha)$, in addition to simplifying notation, plays an important role in the determination of the equilibrium dynamics in Section 3.3. The following properties will prove useful: $m' > 0$ and $m'' > 0$ for all $\alpha \in [0,1]$; $m(0) = 0$; $m(1) = 1$; $m'(0) = e^{-1}$ and $m'(1) = +\infty$. See the appendix for additional details pertaining to $m(\alpha)$.

Investment Division-Choice of *I* and *R* Taking the production division's decision as given, the investment division chooses *I* and *R* to maximize its present value:

$$\max_{\{I_t, R_t\}_{t=0}^{\infty}} \int_0^{\infty} (p_K K_t - I_t - R_t) e^{-\bar{r}_t t} dt \quad s.t.\ (3.5), (3.6), (3.9), K_0, \alpha_0 \quad .$$

The Hamiltonian associated with this optimal control problem is linear in *I* and *R*. This creates bang-bang control, which actually simplifies the solution by reducing the state space from four to two; the costate variables are constants, so only the paths of *K* and α need solving. The necessary conditions yield the following expressions for the returns to capital investment (net of depreciation) and R&D:

$$r_K \equiv \begin{cases} Y^{\frac{1}{\varepsilon}} X^{-\frac{1}{\varepsilon}} \left(1 - \frac{1}{\varepsilon}\right) Am(\alpha) - \delta & K(1-\alpha)^{\frac{1}{\alpha}} < L \\ Y^{\frac{1}{\varepsilon}} X^{-\frac{1}{\varepsilon}} \left(1 - \frac{1}{\varepsilon}\right) \alpha A \left(\frac{K}{L}\right)^{\alpha - 1} - \delta & K(1-\alpha)^{\frac{1}{\alpha}} \geq L \end{cases} ; \tag{3.12}$$

$$r_\alpha \equiv \begin{cases} Y^{\frac{1}{\varepsilon}} X^{-\frac{1}{\varepsilon}} \left(1 - \frac{1}{\varepsilon}\right) AKm'(\alpha) f(\alpha) & K(1-\alpha)^{\frac{1}{\alpha}} < L \\ Y^{\frac{1}{\varepsilon}} X^{-\frac{1}{\varepsilon}} \left(1 - \frac{1}{\varepsilon}\right) AL \left(1 + \ln \frac{K}{L}\right) \left(\frac{K}{L}\right)^{\alpha} f(\alpha) & K(1-\alpha)^{\frac{1}{\alpha}} \geq L \end{cases} . \tag{3.13}$$

3.2.3 Households

Households are identical and infinitely lived. They are endowed with a fixed amount of labor L, which they supply inelastically in a competitive market.[53] Households own firms and hold assets in the form of corporate equity and loans. Loans and corporate equity are assumed to be perfect substitutes as stores of value; thus, the two assets pay the same real rate of return, r_t. The economy is closed, so no assets are traded internationally.

Let E denote total equity held by households (i.e. ownership shares of intermediate goods firms). The value of the firm (i.e. price of ownership share), V, at time zero is given by

$$V(0) = \int_0^\infty D_t e^{-\bar{r}_t t} dt \tag{3.14}$$

where $D_t \equiv P_t X_t - w_t L_t - I_t - R_t$ is the dividend payment.

[53] Recall that "labor" encompasses all non-reproducible factors, including unskilled labor, land and natural resources.

The total value of equity in the economy is given by EV. Recall that the mass of intermediate goods firms is set at 1. Since the number of firms is fixed, total equity, E, is fixed. I normalize E to1 so that the value of equity holdings per worker is V/L. Let b represent loans per worker, and let a represent assets per worker. It follows that $a \equiv \left(b + \frac{V}{L}\right)$.

Assuming log utility, the household's optimization problem is:

$$\max_{\{c_t\}} \int_0^\infty \log c_t e^{-\rho t} \quad s.t. \quad \dot{a}_t = r_t a_t + w_t - c_t \tag{3.15}$$

where $c = \frac{C}{L}$ is consumption per worker, and ρ is the rate of time preference.[54] This setup yields the well known result:

$$\frac{\dot{C}}{C} = r - \rho. \text{ [55]} \tag{3.16}$$

The Hamiltonian associated with the optimal control problem given by (3.15) along with all necessary conditions are presented in the Appendix.

[54] Recall that "labor" L encompasses all non-reproducible factors, including unskilled labor, land and natural resources. Thus, $\frac{C}{L}$ is technically consumption per non-reproducible factor?

[55] $\frac{\dot{C}}{C} = \frac{\dot{c}}{c}$ because L is constant.

3.2.4 General Equilibrium

The economy has four markets: final good, intermediate good, labor, and asset. The requirements for equilibrium in the intermediate goods market and labor market are unchanged relative to Peretto and Seater (2009).

Since intermediate goods firms are monopolistically competitive, equilibrium in the intermediate goods market arises from each firm choosing its price, which is equivalent to choosing a point on the demand curve it faces. Recall that the final good is the numeraire so that $P_Y \equiv 1$, and final good producers are competitive and spend all of their revenue on intermediate goods. In symmetric equilibrium, all intermediate producers are the same. In light of the aforementioned, each intermediate producer sets $P = 1$, which given equations (3.1) through (3.3), yields the equilibrium condition in the intermediate goods market, $Y = X$.

Given that labor is supplied inelastically in a competitive market, equilibrium employment is the labor endowment, *L*. Combining this result with $Y = X$ and the demand for labor in (3.10), equilibrium in the labor market can be expressed:

$$w = \begin{cases} \left(1 - \frac{1}{\varepsilon}\right) A & K(1-\alpha)^{\frac{1}{\alpha}} < L \\ \left(1 - \frac{1}{\varepsilon}\right) A (1-\alpha) \left(\frac{K}{L}\right)^{\alpha} & K(1-\alpha)^{\frac{1}{\alpha}} \geq L \end{cases} . \tag{3.17}$$

Output *Y* is divided among consumption *C*, investment *I* in capital *K*, and investment *R* in R&D. The saving rate is not fixed, so unlike the model of Peretto and Seater, *C* is not a fixed fraction of *Y*. The market clearing condition in the final good market is $C + I + R = Y$, which can be rearranged as

$$Y - C = I + R . \tag{3.18}$$

Equation (3.18) states that saving in the economy must equal investment in capital accumulation and R&D. Given that $Y = X$, equations (3.5), (3.6), (3.11) and the definition of Z can be used to write the equilibrium condition in the final goods market (3.18) as

$$\dot{K} + \delta K + \dot{Z} = \dot{K} + \delta K + \frac{\dot{\alpha}}{f(\alpha)} = \begin{cases} A[L + m(\alpha)K] - C & K(1-\alpha)^{\frac{1}{\alpha}} < L \\ AK^{\alpha}L^{1-\alpha} - C & K(1-\alpha)^{\frac{1}{\alpha}} \geq L \end{cases} \tag{3.19}$$

Firms allocate the economy's saving between investment in capital accumulation and investment in R&D based on how r_K relates to r_α. Rewriting equations (3.12) and (3.13) to reflect general equilibrium yields:

$$r_K \equiv \begin{cases} \left(1 - \frac{1}{\varepsilon}\right) Am(\alpha) - \delta & K(1-\alpha)^{\frac{1}{\alpha}} < L \\ \left(1 - \frac{1}{\varepsilon}\right) \alpha A \left(\frac{K}{L}\right)^{\alpha - 1} - \delta & K(1-\alpha)^{\frac{1}{\alpha}} \geq L \end{cases} ; \tag{3.20}$$

$$r_\alpha \equiv \begin{cases} \left(1 - \frac{1}{\varepsilon}\right) AKm'(\alpha) f(\alpha) & K(1-\alpha)^{\frac{1}{\alpha}} < L \\ \left(1 - \frac{1}{\varepsilon}\right) AL \left(1 + \ln \frac{K}{L}\right) \left(\frac{K}{L}\right)^{\alpha} f(\alpha) & K(1-\alpha)^{\frac{1}{\alpha}} \geq L \end{cases} . \tag{3.21}$$

The bang-bang nature of the investment division's optimization problem yields three combinations for the (I, R) pair:

$$r_K > r_\alpha \Leftrightarrow I > 0,\ R = 0$$
$$r_K = r_\alpha \Leftrightarrow I > 0,\ R > 0$$
$$r_K < r_\alpha \Leftrightarrow I = 0,\ R > 0$$

In the first case, all saving is allocated to investment in capital accumulation, and in the third case, all saving is allocated to investment in R&D. When the two rates of return are equal, as in the second case, saving equals the sum of investment in capital accumulation and R&D.

Equilibrium in the asset market requires that the return to saving equal the return to investment. The Euler equation (3.16) characterizes the return to saving *r*. If $r_K > r_\alpha$, then the active return to investment is r_K, and equilibrium in the asset market is given by

$$\frac{\dot{C}}{C} = \begin{cases} \left(1 - \frac{1}{\varepsilon}\right) Am(\alpha) - \delta - \rho & K(1-\alpha)^{\frac{1}{\alpha}} < L \\ \left(1 - \frac{1}{\varepsilon}\right) \alpha A \left(\frac{K}{L}\right)^{\alpha - 1} - \delta - \rho & K(1-\alpha)^{\frac{1}{\alpha}} \geq L \end{cases} . \tag{3.22}$$

If $r_K < r_\alpha$, then the active return to investment is r_α, and equilibrium in the asset market is given by

$$\frac{\dot{C}}{C} = \begin{cases} \left(1 - \frac{1}{\varepsilon}\right) AKm'(\alpha) f(\alpha) - \rho & K(1-\alpha)^{\frac{1}{\alpha}} < L \\ \left(1 - \frac{1}{\varepsilon}\right) AL\left(1 + \ln\frac{K}{L}\right)\left(\frac{K}{L}\right)^{\alpha} f(\alpha) - \delta - \rho & K(1-\alpha)^{\frac{1}{\alpha}} \geq L \end{cases} . \tag{3.23}$$

If $r_K = r_\alpha$, then (3.22) and (3.23) are identical, and either equation can be used to depict equilibrium in the asset market.

3.3 Dynamic Analysis

Consumer optimization adds a third dimension to the model, and so the dynamic analysis requires three loci. A reasonable approach to analyzing the dynamic behavior of the economy is to construct a three-dimensional phase diagram in (α, K, C) space.[56] However, the equilibrium loci are difficult to draw in three dimensions, and even after successful construction of each locus in three-space, tractability remains a constraining factor. I analyze the dynamic behavior of the economy by constructing a sequence of two dimensional phase diagrams in (α, K) space, each corresponding to a different fixed value of *C*. It turns out that only one of the three equilibrium loci depends on *C*, so examining snap shots of the economy at different levels of *C* provides a convenient method for determining the laws of motion for α, *K*, and *C*.

Consumer optimization has no impact on the equilibrium values of r_K or r_α so the *arbitrage locus*, which I describe below, is identical to the arbitrage locus constructed by Peretto and Seater (2009). Along this locus, $r_K = r_\alpha$, and investment in both capital accumulation and R&D occurs. The second locus is the *stationarity locus*, along which total net investment $\dot{K} + \dot{Z} = \dot{K} + \dfrac{\dot{\alpha}}{f(\alpha)}$ is zero. Peretto and Seater also construct a *stationarity locus*, but the one I construct is different because saving, and thus investment, is dependent on *C* in the general equilibrium of my model. The third locus is the $\dot{C} = 0$ *locus*, and it is new.

Before constructing the loci, two additional characteristics of my approach should be noted. First, I posit the specific functional form $f(\alpha) = 1 - \alpha$ for the productivity of R&D. Such a specification is consistent with firms needing infinite knowledge to reach $\alpha = 1$. As a result of this specification, the *arbitrage* and $\dot{C} = 0$ *loci* that I describe below approach $\alpha = 1$

[56] For the household, choosing consumption is identical to choosing saving, so the third variable in the analysis need not be *C*; saving, the saving rate, and the consumption rate are also feasible options, at least theoretically. I considered these other variables, but the shape and position of the corresponding equilibrium loci could not be determined.

asymptotically. Moreover, the economy approaches $\alpha = 1$ asymptotically. There are other specifications for $f(\alpha)$ that satisfy the requirements for symmetric equilibrium but do not require infinite knowledge to achieve $\alpha = 1$. These specifications support an economy that reaches $\alpha = 1$ in finite time. From a qualitative perspective, the economy's dynamics are equivalent regardless of whether the economy reaches $\alpha = 1$ in the limit or in finite time.

Second, I focus the dynamic analysis on the interior equilibrium where the labor allocation constraint does not bind. That is, I analyze the case where $K(1-\alpha)^{\frac{1}{\alpha}} < L$. It may be that the economy always eventually leaves the constrained region, in which case, the analysis of the unconstrained region is the only analysis that matters for long run dynamics.[57] However, I have not analyzed the dynamics when the labor allocation constraint binds, so I can not say with certainty that the constrained case is irrelevant. I choose to focus on the unconstrained case because it is the simpler of the two to work with, and I wanted to keep things as simple as possible when introducing a third dimension to the model. That said, further analysis is needed and will be pursued in future work.

3.3.1 Equilibrium Loci

Substituting the unconstrained portions of r_K and r_α in (3.20) and (3.21) into the no arbitrage condition $r_K = r_\alpha$ yields the following result.

Proposition 1 ***Arbitrage Locus.*** *The arbitrage locus in (α, K) space is*

$$K = \begin{cases} 0 & 0 \le \alpha \le \overline{\alpha} \\ \dfrac{1}{m'(\alpha)(1-\alpha)}\left[m(\alpha) - \dfrac{\delta\varepsilon}{A(\varepsilon-1)}\right] & \overline{\alpha} < \alpha \le 1 \end{cases} \tag{3.24}$$

[57] In the two dimensional model of Peretto and Seater (2009), the relative slopes of the equilibrium loci are such that the economy always leaves the constrained region. An analogous result may or may not hold for the three dimensional model herein.

where, assuming $\frac{\delta\varepsilon}{A(\varepsilon-1)}<1$, $\overline{\alpha}\in(0,1)$ *solves* $m(\alpha)=\frac{\delta\varepsilon}{A(\varepsilon-1)}$. *The locus starts at zero and lies on the* $\alpha-$*axis for* $0\leq\alpha\leq\overline{\alpha}$. *For* $\overline{\alpha}<\alpha\leq 1$*, the locus is positive and increases to* $+\infty$ *as* $\alpha\rightarrow 1$.

The proof is given in Section 3.5.3 of the Appendix.

Imposing $\dot{K}+\dot{Z}=0$ in the unconstrained portion of (3.19) and solving for K yields the *stationarity locus* described below.

Proposition 2 ***Stationarity Locus.*** *The stationarity locus in* (α, K) *space is*

$$K=\frac{AL-C}{\delta-Am(\alpha)} \quad . \tag{3.25}$$

Assuming $\frac{\delta}{A}<1$, *the locus has an asymptote at* $\widetilde{\alpha}\in(0,1)$, *where* $\widetilde{\alpha}$ *solves* $\delta=Am(\alpha)$.

The shape and position of the locus in (α, K) *space depends on the value of C. There are three possibilities.*

i. $0\leq C<AL$: *For* $\alpha<\widetilde{\alpha}$, $K>0$, *and the locus starts at* $\frac{AL-C}{\delta}>0$ *and goes asymptotically to* $+\infty$ *as* $\alpha\rightarrow\widetilde{\alpha}$. *For* $\alpha>\widetilde{\alpha}$, $K<0$, *and the locus starts at* $-\infty$ *and rises to* $\frac{AL-C}{\delta-A}<0$ *at* $\alpha=1$.

ii. $C=AL$: *The* α *-axis and the vertical line* $\alpha=\widetilde{\alpha}$ *form the stationarity locus.*

iii. $C>AL$: *For* $\alpha<\widetilde{\alpha}$, $K<0$, *and the locus starts at* $\frac{AL-C}{\delta}<0$ *and goes asymptotically to* $-\infty$ *as* $\alpha\rightarrow\widetilde{\alpha}$. *For* $\alpha>\widetilde{\alpha}$, $K>0$, *and the locus starts at* $+\infty$ *and falls to* $\frac{AL-C}{\delta-A}>0$ *at* $\alpha=1$.

The proof is given in Section 3.5.4 of the Appendix.

The requirements for $\dot{C}=0$ differ depending on whether $r_K > r_\alpha$ or $r_K < r_\alpha$. Combining the condition $\dot{C}=0$ with the unconstrained portions of (3.22) and (3.23) yields the following result.

Proposition 3 $\dot{C}=0$ ***Locus.*** *For* $r_K > r_\alpha$, *the* $\dot{C}=0$ *locus in* (α, K) *space is the vertical line* $\alpha = \overset{*}{\alpha}$ *where, assuming* $\rho \le \left(\frac{\varepsilon-1}{\varepsilon}\right)A-\delta$, $\overset{*}{\alpha} \in (0,1]$ *solves* $m(\alpha)=\frac{(\rho+\delta)\varepsilon}{A(\varepsilon-1)}$. *For* $r_K < r_\alpha$, *the* $\dot{C}=0$ *locus in* (α, K) *space is*

$$K=\frac{\rho}{\left(1-\frac{1}{\varepsilon}\right)Am'(\alpha)(1-\alpha)}. \tag{3.26}$$

The locus starts at $\frac{\rho}{\left(1-\frac{1}{\varepsilon}\right)Ae^{-1}}>0$ *for* $\alpha=0$ *and increases to* $+\infty$ *as* $\alpha \to 1$.

The proof is given in Section 3.5.5 of the Appendix.

3.3.2 Phase Diagrams

Figures 3.1.1 through 3.1.5 present a sequence of phase diagrams, each depicting the shape and position of the equilibrium loci for a given value of *C*. In each diagram, I assume that $\frac{\delta\,\varepsilon}{A(\varepsilon-1)}<1$ and $0<\rho<\frac{A(\varepsilon-1)}{\varepsilon}-\delta$.[58] Though the diagrams are two dimensional and

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[58] The assumption $\frac{\delta\varepsilon}{A(\varepsilon-1)}<1$ prevents the arbitrage locus from lying entirely on the horizontal axis. This prevents a situation where all dynamic adjustment paths lead to $K=0$, an outcome inconsistent with human

constructed in (α, K) space, keep in mind that there are three variables that are evolving in this economy: α, *K and C*. Each diagram is drawn for a given *C*, but it is imperative that the reader not interpret *C* as remaining constant in each diagram. Each diagram reveals the laws of motion for α, *K* and *C* at all possible (α, K) combinations assuming some specific value for the current level of *C*.

The reader may find it useful to imagine a three dimensional figure where the loci are planes and surfaces, and then think about slicing that figure into cross-sections at different levels of *C*. The projection of those cross-sections on to (α, K) space is what is given in Figures 3.1.1 through 3.1.5.

In all figures, points below the arbitrage locus yield $r_\alpha < r_K$, so that $I = \dot{K} - \delta K > 0$ and $R = \dot{Z} = \dfrac{\dot{\alpha}}{1-\alpha} = 0$. All points above the locus yield $r_\alpha > r_K$, so that $I = 0$ and $R > 0$. Points on the locus yield $r_\alpha = r_K$, and the economy experiences $I > 0$ and $R > 0$. Only the branch of the stationarity locus for which *K* is positive is relevant to the equilibrium dynamics. Points that lie to the left, right, or on the relevant branch of the stationarity locus yield $\dot{K} + \dot{Z} < 0$, $\dot{K} + \dot{Z} > 0$ and $\dot{K} + \dot{Z} = 0$, respectively. Points above or below the $r_\alpha > r_K$ branch of the $\dot{C} = 0$ locus yield $\dot{C} > 0$ and $\dot{C} < 0$, respectively. Points to the left or right of the $r_\alpha < r_K$ branch of the $\dot{C} = 0$ locus yield $\dot{C} < 0$ and $\dot{C} > 0$, respectively.

The relation $\overset{*}{\alpha} > \overline{\alpha} > \widetilde{\alpha}$ always holds.[59] The two branches of the $\dot{C} = 0$ locus always intersect each other and the arbitrage locus at $\overset{*}{\alpha}$.[60] Recall that, of the three loci, only the stationarity locus is a function of *C*, so only the stationarity locus changes from one diagram to the next. For $C \leq AL$, the stationarity locus intersects only the $r_\alpha > r_K$ branch of

history. See Section 3.5.3 of the Appendix for more details. The assumption $0 < \rho < \dfrac{A(\varepsilon - 1)}{\varepsilon} - \delta$ is relaxed later in the paper.

59 See Section 3.5.6 of the Appendix for the proof.

60 See Section 3.5.7 of the Appendix for the proof.

the $\dot{C} = 0$ locus. For $C > AL$, the stationarity locus intersects both branches of the $\dot{C} = 0$ locus and the arbitrage locus.

In Figures 3.1.1, 3.1.2 and 3.1.3, the equilibrium dynamics are qualitatively identical. The loci divide all three phase planes into eight regions, labeled 1 through 8. In regions 1, 2, 3, 4 and 6, $r_\alpha > r_K$, so R&D is positive and gross capital investment is zero. The only difference across these regions is that total accumulation $\dot{K} + \dot{Z}$ is positive in regions 3, 4 and 6 but negative in regions 1 and 2. Since gross capital investment is zero, the difference in total accumulation has no impact on the laws of motion. Technology α grows, and the capital stock *K* falls because of depreciation. The dynamic adjustment paths point southeast. In regions 5, 7 and 8, $r_\alpha < r_K$, so gross capital investment is positive and R&D is zero. Technology α remains constant, and since $\dot{K} + \dot{Z} > 0$ in regions 5, 7 and 8, gross capital investment exceeds depreciation and the capital stock *K* grows. The dynamic adjustment paths point north.

Figures 3.1.1, 3.1.2 and 3.1.3 indicate that, for $C \leq AL$, any (α, K) combination puts the economy on a path where α approaches 1 and *K* approaches infinity. Along this path, *C* may increase or decrease; however, once α moves past $\overset{*}{\alpha}$, *C* is always increasing. Do α and *K* continue towards 1 and infinity, respectively, once *C* surpasses *AL*? What value does *C* approach? These questions are addressed by analyzing Figures 3.1.4 and 3.1.5, which depict the equilibrium loci for $C > AL$.

If $C > AL$, the relevant branch of the stationarity locus is a negatively sloped curve lying to the right of $\tilde{\alpha}$. The stationarity locus intersects the arbitrage locus and the $r_\alpha > r_K$ branch of the $\dot{C} = 0$ locus to the left of $\overset{*}{\alpha}$ in Figure 3.1.4.[61] The phase plane is divided into ten regions. As before, most of the dynamic adjustment paths point southeast or north.

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[61] Depending on the degree to which *C* exceeds *AL*, the intersection between the arbitrage locus and the stationarity locus may occur to the right or left of $\overset{*}{\alpha}$. For *C* that exceeds *AL* only slightly, the intersection occurs to the left. For *C* that exceeds *AL* substantially, as in Figure 3.1.5, the intersection occurs to the right.

However, the new position of the stationarity locus creates two interesting regions where the adjustment paths actually point south. These regions are labeled 3 and 4 in Figure 3.1.4. In both regions, $r_\alpha < r_K$, so gross capital investment is positive and R&D is zero. Technology α remains constant, and since $\dot{K} + \dot{Z} < 0$, depreciation exceeds gross capital investment and the capital stock K falls. Does K fall to zero and yield a no growth steady state where output is produced by the primitive sector only? It may seem as though the diagram reflects such a scenario, but further analysis reveals otherwise.

Suppose the economy is in region 3 of Figure 3.1.4. K is falling, α is constant and C is falling. Given the economy's resource constraint (3.19), when K falls to zero, C must equal AL. Thus, at the moment the economy hits the horizontal axis in region 3, the relevant phase diagram changes. The economy finds itself on the horizontal axis between $\overline{\alpha}$ and $\overset{*}{\alpha}$ in Figure 3.1.3. K starts increasing and C continues to fall while α remains constant. As a result of the further decline in C, the economy moves into region 5 of Figure 3.1.2. The economy will eventually run into the arbitrage locus, and when it does, α and K will both increase as the economy begins moving up and to the right along the arbitrage locus. Once α moves past $\overset{*}{\alpha}$, C will stop decreasing and begin increasing. At some point, C will rise to a level above AL, and this moves the analysis back to Figure 3.1.4 where the economy's (α, K) combination now lies to the right of $\overset{*}{\alpha}$ on the arbitrage locus. Note also that this position lies to the right of the stationarity locus.

All three variables are now increasing. The increase in C causes the stationarity locus to shift up and to the right.[62] The question remaining is whether the economy's position on the arbitrage locus will remain to the right of the stationarity locus. Consider the scenario that would play out if the (α, K) combination on the arbitrage locus did not evolve quickly enough to keep the economy to the right of the stationarity locus. The economy would find itself on the arbitrage locus in region 5 of Figure 3.1.5. Technology α stops growing and K

[62] See Section 3.5.4 of the Appendix.

declines. The decline in K eventually moves the economy into region 4 of Figure 3.1.5. All the while, however, C continues increasing. Once the economy's path hits the horizontal axis, $K = 0$, $Y = AL$ and the economy's resource constraint dictates that C must jump downward to AL. Such a jump violates the first order condition underlying equation (3.22); when C is growing, a fall in C is not possible. Therefore, any path that positions the economy to the right of $\overset{*}{\alpha}$ and to the left of the stationarity locus is not an equilibrium path. α and K evolve quickly enough to keep the economy's position on the arbitrage locus to the right of the stationarity locus. This result leaves paths leading to perpetual growth in α, K and C as the only possibilities.

3.3.3 The Impact of ρ

The rate of time preference ρ is positively correlated with $\overset{*}{\alpha}$ and the slope and $\alpha = 0$ intercept of the $r_\alpha > r_K$ branch of the $\dot{C} = 0$ locus.[63] Consider the equilibrium dynamics for the lowest possible rate of time preference, $\rho = 0$.

Equilibrium Dynamics: $\rho = 0$

If $\rho = 0$, then $\overset{*}{\alpha} = \overline{\alpha}$, and the $r_\alpha > r_K$ branch of the $\dot{C} = 0$ locus consists of the horizontal axis and the vertical line $\alpha = 1$. Though the phase diagrams, which are presented in Figures 3.2.1 through 3.2.4, change slightly, the laws of motion for α and K are qualitatively identical to those described in Section 3.3.2. Note that a path that takes the economy to region 4 of Figure 3.2.4 is not an equilibrium path because it violates the first

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[63] Since the phase diagrams are drawn for fixed values of C, ρ can not influence the stationarity locus in the phase diagrams via its impact on C. The value of ρ alters the point in time at which a two dimensional snap shot of the stationarity locus holds. However, the economy's qualitative dynamics are not dependent on the position of the stationarity locus at an exact point in time.

order condition associated with household optimization.[64] All equilibrium paths eventually lead to perpetual growth in α and *K*. All paths also lead to perpetual growth in *C*. In fact, there are no points along any equilibrium path where *C* declines. Growth in *C* is always positive, and this is the main distinction from the analysis in Section 3.3.2 that assumes $0 < \rho < \frac{A(\varepsilon - 1)}{\varepsilon} - \delta$.

Equilibrium Dynamics: $\rho = \frac{A(\varepsilon - 1)}{\varepsilon} - \delta$

Now consider the opposite extreme where ρ is high. Figures 3.3.1 through 3.3.4 present the sequence of phase diagrams corresponding to $\rho = \frac{A(\varepsilon - 1)}{\varepsilon} - \delta$. Note that $\overset{*}{\alpha} = 1$, and the $\dot{C} = 0$ locus ($r_\alpha > r_K$ branch) does not intersect the arbitrage locus. Both loci are asymptotic to $\alpha = 1$.[65]

All equilibrium paths eventually lead to perpetual growth in α and *K*, but *C* falls to zero as $t \to \infty$. Region 6 of Figure 3.3.4 may seemingly contradict perpetual growth in α and *K*, but region 6 is similar to region 3 of Figure 3.1.4. The laws of motion in region 6 do not yield a steady state at $K = 0$. Once *K* reaches zero, output and consumption equal *AL*, but consumption continues to fall creating the saving that is necessary to generate positive growth in *K*. *K* grows forever, and technology α starts growing and then grows forever once the economy reaches the arbitrage locus.

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[64] Such a path would take the economy to a primitive state where $K = 0$ and $Y = C = AL$. *C* would have to jump down to *AL* at the moment the economy hits the horizontal axis. Such a jump is not possible because *C* is always growing in region 4.

[65] The equilibrium dynamics are qualitatively identical if $\rho > \frac{A(\varepsilon - 1)}{\varepsilon} - \delta$. The only difference relative to the scenario where $\rho = \frac{A(\varepsilon - 1)}{\varepsilon} - \delta$ is that $\overset{*}{\alpha}$ does not exist, and so there is no $r_\alpha < r_K$ branch of the $\dot{C} = 0$ locus. However, the $r_\alpha > r_K$ branch of the $\dot{C} = 0$ locus and the arbitrage locus still have the same shapes, and both approach $\alpha = 1$ asymptotically.

Graphically, the decline in consumption eventually moves the analysis from region 6 of Figure 3.3.4 to region 5 of Figure 3.3.3 and then to region 5 of Figure 3.3.2. From an analytical perspective, the only difference between region 6 in Figure 3.3.4 and region 3 in Figure 3.1.4 is the ensuing path of *C*. For the $\rho = \frac{A(\varepsilon - 1)}{\varepsilon} - \delta$ scenario, the decline in *C* to a level below *AL*, which prevents the economy from stagnating at $K = 0$, is a permanent rather than temporary decline.

The result that the economy (α and *K*) grows forever despite the high value of ρ is unique. In the standard *AK* model, saving is not high enough to offset depreciation when ρ exceeds the critical value. Therefore, regardless of where *K* starts, it eventually falls to zero. Since $Y = AK$, $K = 0$ implies that $Y = 0$ and $C = 0$. Thus, a ρ that is too high kills growth in the *AK* model, and the economy eventually disappears. Since history shows that long run growth is positive, growth economists just assume that ρ lies below the critical value when working with the *AK* model.[66] In my model, no such assumption is necessary; the economy achieves perpetual growth no matter the value of ρ.[67]

Because capital is not essential to production in my model, output and consumption are still positive if $K = 0$. Specifically, $Y = AL$ and $C \leq AL$ if $K = 0$.[68] The continual decline in *C* induced by the high value of ρ prevents the capital stock from stagnating at zero. As *C* falls below the level consistent with $K = 0$, saving (*Y-C*) increases, and this generates the investment necessary to keep *K* growing. The essentiality of capital present in the *AK* model removes the possibility of generating investment once *K* falls to zero.

K and α go to infinity and 1, respectively, as $t \to \infty$. How can it be optimal for consumption to fall to zero while output rises? The simple answer is that the preferences of households make it optimal. When the economy reaches the arbitrage locus, *K* and α both

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[66] See Chapter 4 of Barro and Sala-i-Martin (2004).

[67] It is important to note that perpetual growth refers to perpetual "economic growth," and "economic growth" refers to growth in output, not growth in consumption.

[68] At $K = 0$, $C < AL$ if $\dot{\alpha} > 0$, which holds for $\alpha < \overline{\alpha}$. $C = AL$ if $\dot{\alpha} = 0$, which holds for $\alpha > \overline{\alpha}$.

increase, and this causes the decline in C to slow down.[69] That is, the growth rate of C becomes less negative. So, consumption, though falling, does not fall as fast, yielding higher levels of consumption at earlier points on the time path. This is exactly what the household wants. The high rate of time preference implies that households have a strong preference for consumption earlier rather than later.

Though the "high ρ" scenario in my model is perfectly feasible from a theoretical perspective, it makes little sense from a historical and practical standpoint. My result says that economic agents will produce output forever but are not smart enough to consume at a rate that can sustain their existence. Though this outcome is theoretically optimal, when has an economy ever lacked such consumptive foresight?

The situation is analogous to the high ρ scenario in the *AK* model. In the *AK* model, a high ρ, though it violates no optimality conditions, is ruled out because the outcome does not correspond to economic history. A high ρ can be ruled out for the same reason in the model herein. A ρ that is too high causes the economy in my model to disappear just like it does in the *AK* model. However, the *AK* economy disappears because capital and output fall to zero. The economy in my model disappears not because output disappears, but because consumption goes to zero and households cease to exist.

3.4 Conclusion

I extend the endogenous growth model developed by Peretto and Seater by incorporating consumer optimization. All equilibrium paths lead to an economy that is asymptotically *AK*. In Peretto and Seater's model, the economy becomes *AK* and enjoys perpetual growth only if the exogenous saving rate is sufficiently high. If it is too low, the economy goes to a Solow steady state. In my model, the saving rate, which is chosen by optimizing households, is always high enough to support perpetual growth.

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[69] On the arbitrage locus, $r_K = r_\alpha$ so equations (3.22) and (3.23) are equivalent. It is apparent from equation (3.22) that if α increases the decline in C will slow down.

Though the inclusion of consumer optimization enriches the theory, the primary implications and insights do not change. The model still predicts that non-reproducible factor shares should decrease with the level of economic development, and reproducible factor shares should increase with the level of economic development. The model offers the same, purely endogenous explanation for the transition from a primitive to an advanced economy. The model provides the same resolution to the linearity critique.

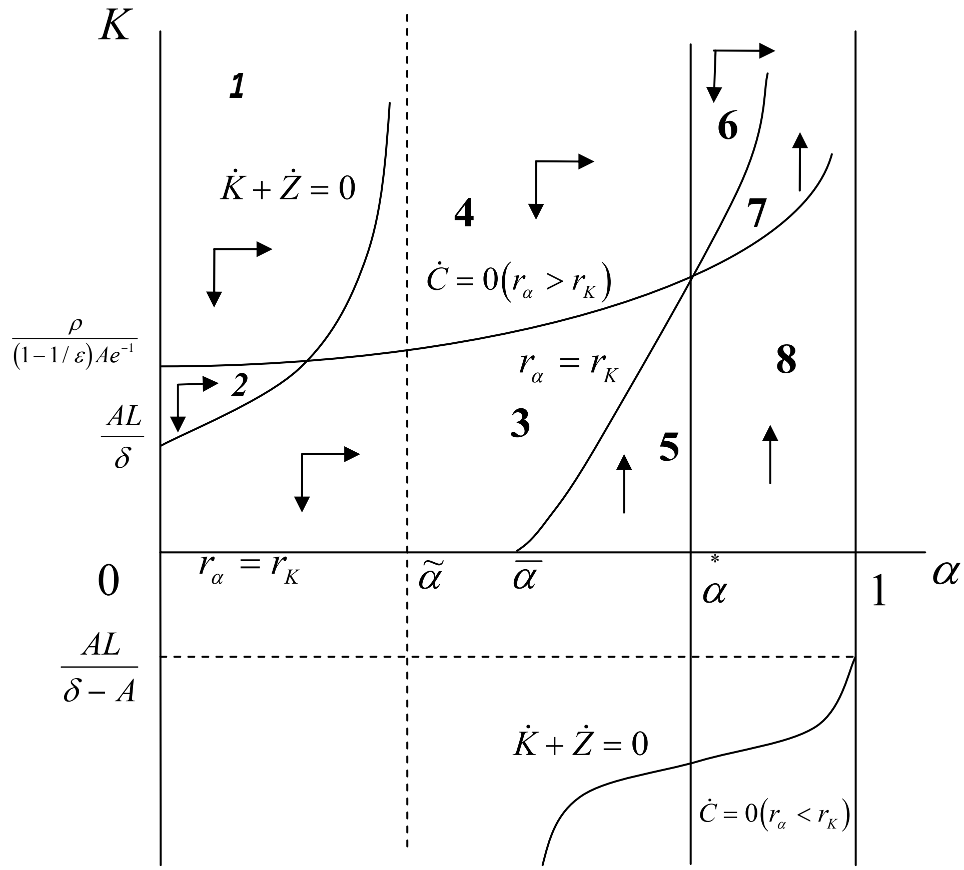

Figure 3.1.1: Phase Diagram; $\boldsymbol{C=0,\ 0<\rho<\frac{A(\varepsilon-1)}{\varepsilon}-\delta}$

Notes:

Region 1: $\dot{K}+\dot{Z}<0$	Region 2: $\dot{K}+\dot{Z}<0$	Region 3: $\dot{K}+\dot{Z}>0$	Region 4: $\dot{K}+\dot{Z}>0$
$r_\alpha > r_K$	$r_\alpha > r_K$	$r_\alpha > r_K$	$r_\alpha > r_K$
$\dot{C}>0$	$\dot{C}<0$	$\dot{C}<0$	$\dot{C}>0$
Region 5: $\dot{K}+\dot{Z}>0$	Region 6: $\dot{K}+\dot{Z}>0$	Region 7: $\dot{K}+\dot{Z}>0$	Region 8: $\dot{K}+\dot{Z}>0$
$r_\alpha < r_K$	$r_\alpha > r_K$	$r_\alpha < r_K$	$r_\alpha < r_K$
$\dot{C}<0$	$\dot{C}>0$	$\dot{C}>0$	$\dot{C}>0$

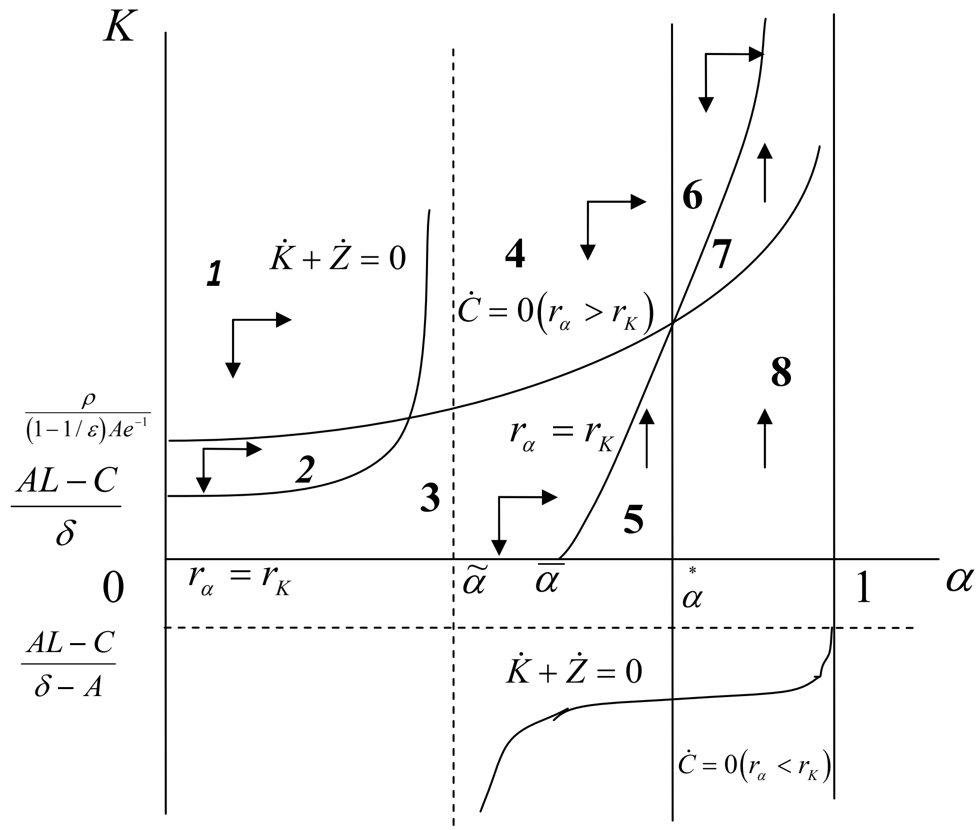

Figure 3.1.2: Phase Diagram; $0 < C < AL$, $0 < \rho < \frac{A(\varepsilon-1)}{\varepsilon} - \delta$

Notes:

Region 1: $\dot{K}+\dot{Z}<0$	Region 2: $\dot{K}+\dot{Z}<0$	Region 3: $\dot{K}+\dot{Z}>0$	Region 4: $\dot{K}+\dot{Z}>0$
$r_\alpha > r_K$	$r_\alpha > r_K$	$r_\alpha > r_K$	$r_\alpha > r_K$
$\dot{C}>0$	$\dot{C}<0$	$\dot{C}<0$	$\dot{C}>0$

Region 5: $\dot{K}+\dot{Z}>0$	Region 6: $\dot{K}+\dot{Z}>0$	Region 7: $\dot{K}+\dot{Z}>0$	Region 8: $\dot{K}+\dot{Z}>0$
$r_\alpha < r_K$	$r_\alpha > r_K$	$r_\alpha < r_K$	$r_\alpha < r_K$
$\dot{C}<0$	$\dot{C}>0$	$\dot{C}>0$	$\dot{C}>0$

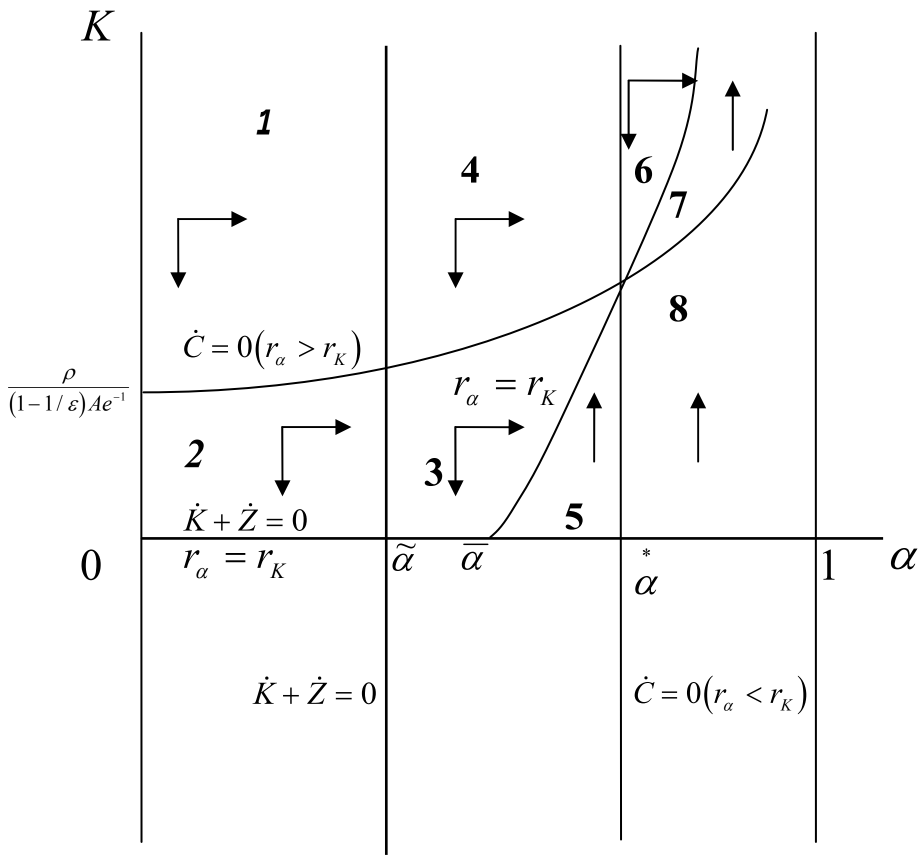

Figure 3.1.3: Phase Diagram; $C = AL$, $0 < \rho < \frac{A(\varepsilon - 1)}{\varepsilon} - \delta$

Notes:

Region 1 : $\dot{K} + \dot{Z} < 0$	Region 2 : $\dot{K} + \dot{Z} < 0$	Region 3 : $\dot{K} + \dot{Z} > 0$	Region 4 : $\dot{K} + \dot{Z} > 0$
$r_\alpha > r_K$	$r_\alpha > r_K$	$r_\alpha > r_K$	$r_\alpha > r_K$
$\dot{C} > 0$	$\dot{C} < 0$	$\dot{C} < 0$	$\dot{C} > 0$

Region 5 : $\dot{K} + \dot{Z} > 0$	Region 6 : $\dot{K} + \dot{Z} > 0$	Region 7 : $\dot{K} + \dot{Z} > 0$	Region 8 : $\dot{K} + \dot{Z} > 0$
$r_\alpha < r_K$	$r_\alpha > r_K$	$r_\alpha < r_K$	$r_\alpha < r_K$
$\dot{C} < 0$	$\dot{C} > 0$	$\dot{C} > 0$	$\dot{C} > 0$

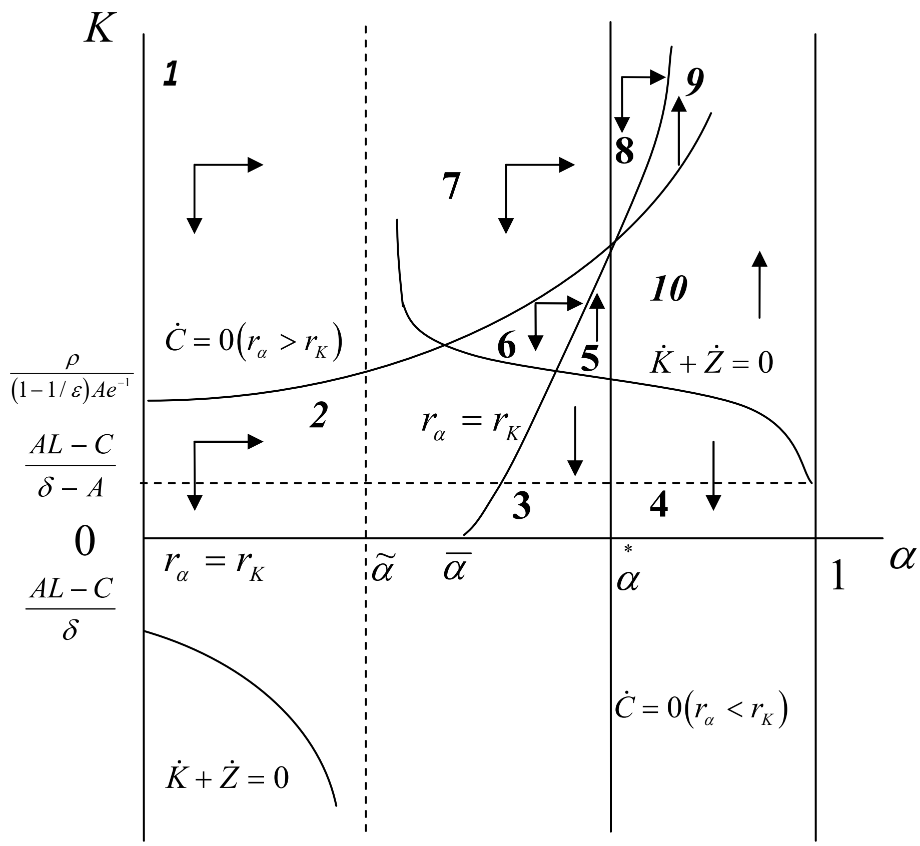

Figure 3.1.4: Phase Diagram; $C > AL$ **,** $0 < \rho < \frac{A(\varepsilon - 1)}{\varepsilon} - \delta$

Notes:

Region 1: $\dot{K}+\dot{Z}<0$	Region 2: $\dot{K}+\dot{Z}<0$	Region 3: $\dot{K}+\dot{Z}<0$	Region 4: $\dot{K}+\dot{Z}<0$	Region 5: $\dot{K}+\dot{Z}>0$
$r_\alpha > r_K$	$r_\alpha > r_K$	$r_\alpha < r_K$	$r_\alpha < r_K$	$r_\alpha < r_K$
$\dot{C}>0$	$\dot{C}<0$	$\dot{C}<0$	$\dot{C}>0$	$\dot{C}<0$
Region 6: $\dot{K}+\dot{Z}>0$	Region 7: $\dot{K}+\dot{Z}>0$	Region 8: $\dot{K}+\dot{Z}>0$	Region 9: $\dot{K}+\dot{Z}>0$	Region 10: $\dot{K}+\dot{Z}>0$
$r_\alpha > r_K$	$r_\alpha > r_K$	$r_\alpha > r_K$	$r_\alpha < r_K$	$r_\alpha < r_K$
$\dot{C}<0$	$\dot{C}>0$	$\dot{C}>0$	$\dot{C}>0$	$\dot{C}>0$

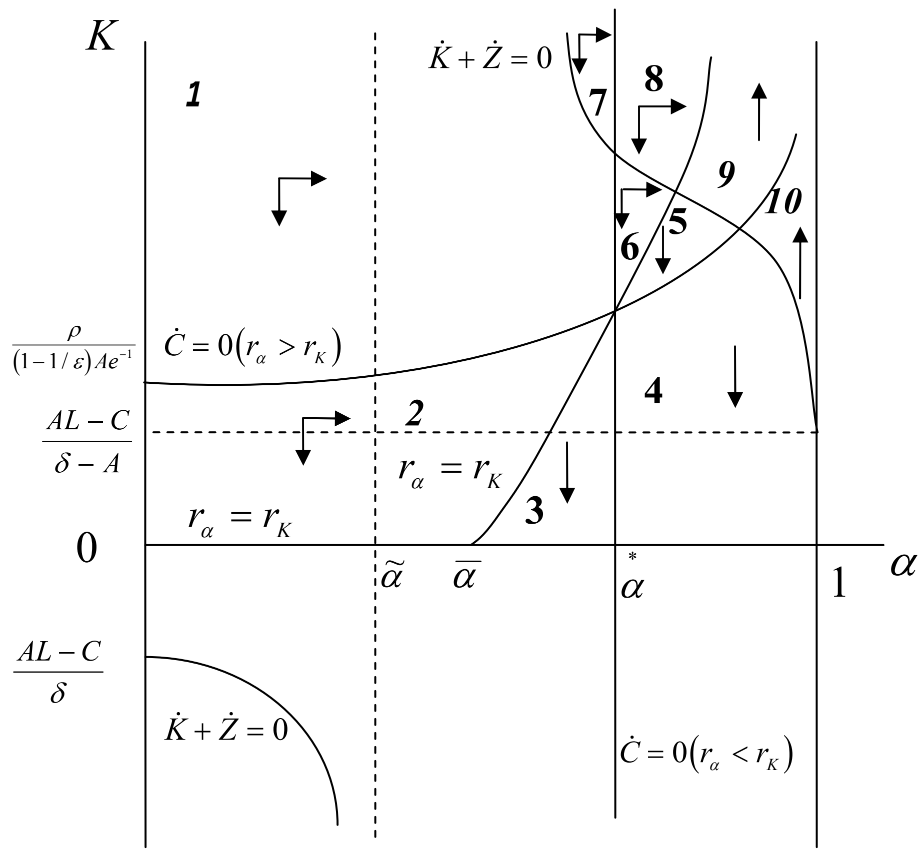

Figure 3.1.5: Phase Diagram; $C >> AL$, $0 < \rho < \frac{A(\varepsilon-1)}{\varepsilon} - \delta$

Notes:

Region 1: $\dot{K}+\dot{Z}<0$, $r_\alpha > r_K$, $\dot{C}>0$	Region 2: $\dot{K}+\dot{Z}<0$, $r_\alpha > r_K$, $\dot{C}<0$	Region 3: $\dot{K}+\dot{Z}<0$, $r_\alpha < r_K$, $\dot{C}<0$	Region 4: $\dot{K}+\dot{Z}<0$, $r_\alpha < r_K$, $\dot{C}>0$	Region 5: $\dot{K}+\dot{Z}<0$, $r_\alpha < r_K$, $\dot{C}>0$
Region 6: $\dot{K}+\dot{Z}<0$, $r_\alpha > r_K$, $\dot{C}>0$	Region 7: $\dot{K}+\dot{Z}>0$, $r_\alpha > r_K$, $\dot{C}>0$	Region 8: $\dot{K}+\dot{Z}>0$, $r_\alpha > r_K$, $\dot{C}>0$	Region 9: $\dot{K}+\dot{Z}>0$, $r_\alpha < r_K$, $\dot{C}>0$	Region 10: $\dot{K}+\dot{Z}>0$, $r_\alpha < r_K$, $\dot{C}>0$

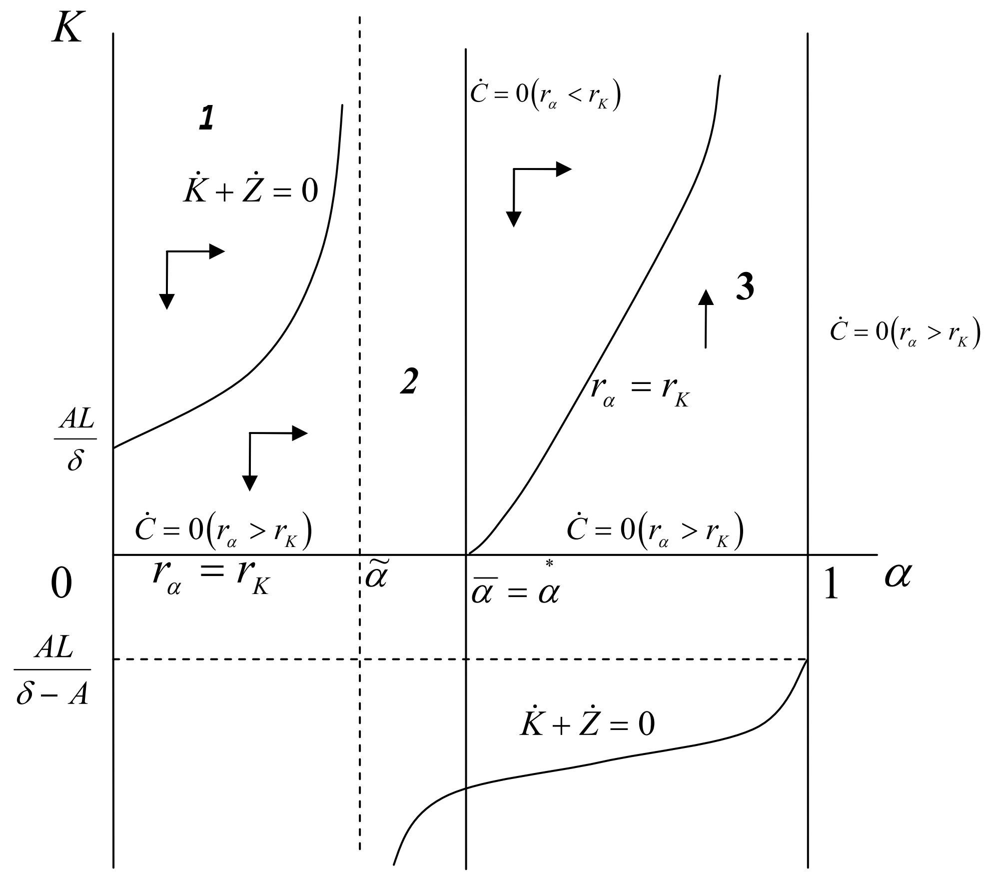

Figure 3.2.1: Phase Diagram; $\boldsymbol{C=0}$, $\boldsymbol{\rho}=0$

Notes:

Region 1: $\dot{K}+\dot{Z}<0$	Region 2: $\dot{K}+\dot{Z}>0$	Region 3: $\dot{K}+\dot{Z}>0$
$r_\alpha > r_K$	$r_\alpha > r_K$	$r_\alpha < r_K$
$\dot{C}>0$	$\dot{C}>0$	$\dot{C}>0$

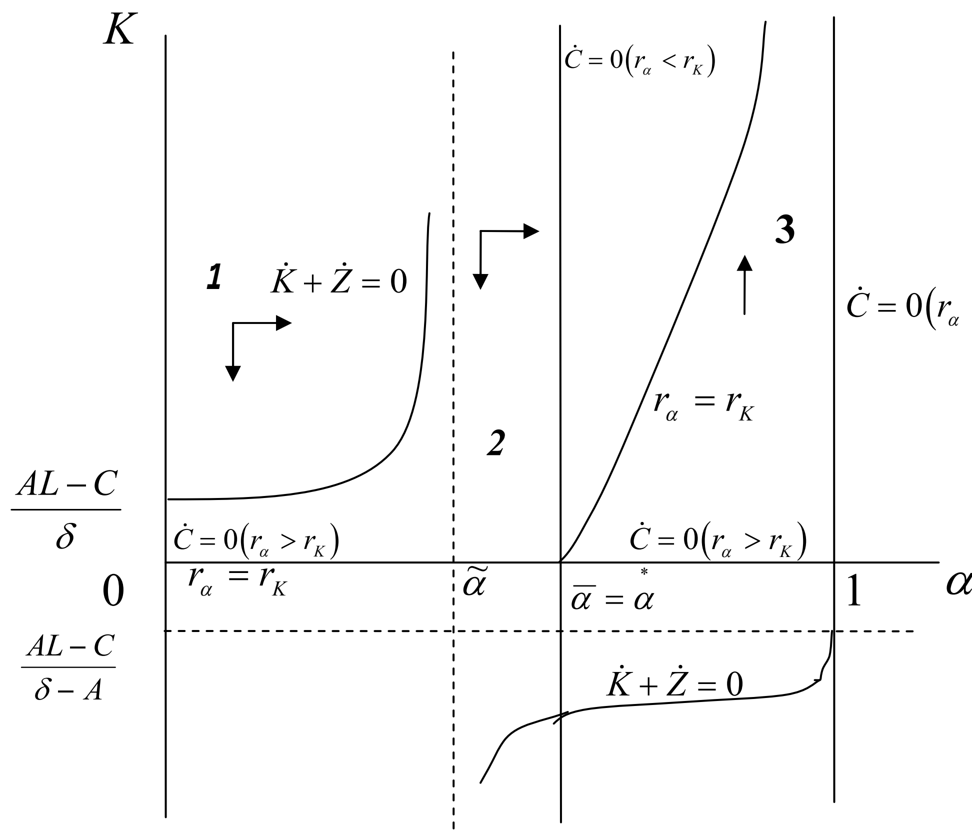

Figure 3.2.2: Phase Diagram; $0 < C < AL$, $\rho = 0$

Notes:

Region 1: $\dot{K}+\dot{Z}<0$	Region 2: $\dot{K}+\dot{Z}>0$	Region 3: $\dot{K}+\dot{Z}>0$
$r_\alpha > r_K$	$r_\alpha > r_K$	$r_\alpha < r_K$
$\dot{C}>0$	$\dot{C}>0$	$\dot{C}>0$

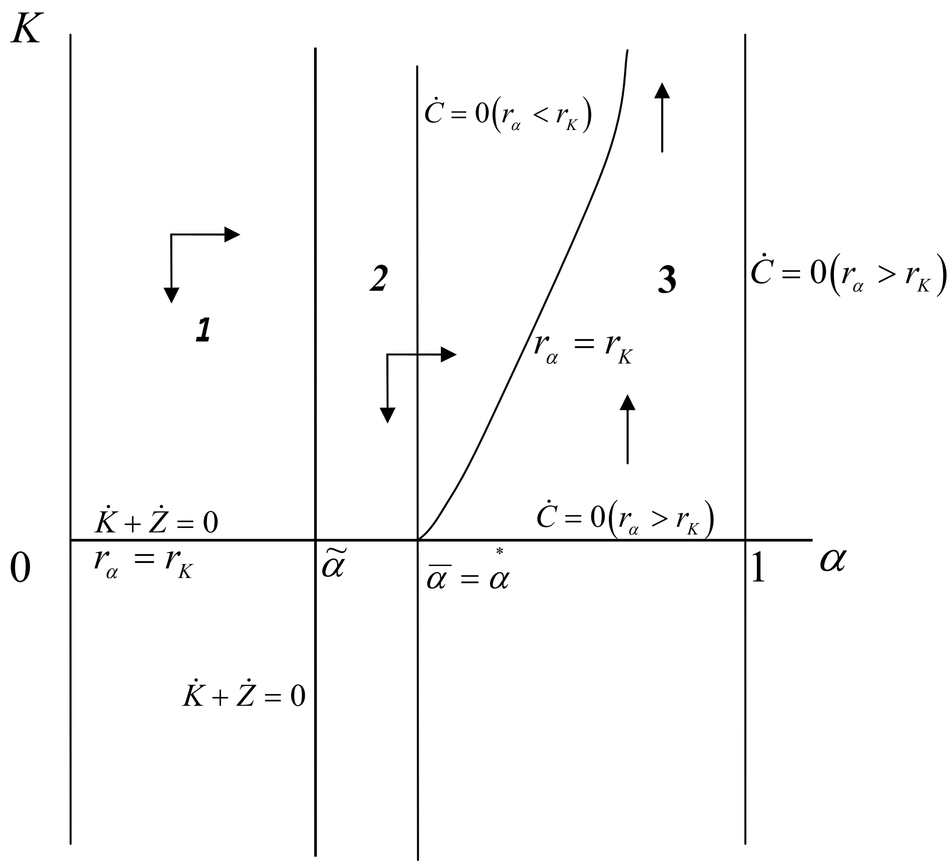

Figure 3.2.3: Phase Diagram; $C = AL$, $\rho = 0$

Notes:

Region 1 : $\dot{K} + \dot{Z} < 0$	Region 2 : $\dot{K} + \dot{Z} > 0$	Region 3 : $\dot{K} + \dot{Z} > 0$
$r_\alpha > r_K$	$r_\alpha > r_K$	$r_\alpha < r_K$
$\dot{C} > 0$	$\dot{C} > 0$	$\dot{C} > 0$

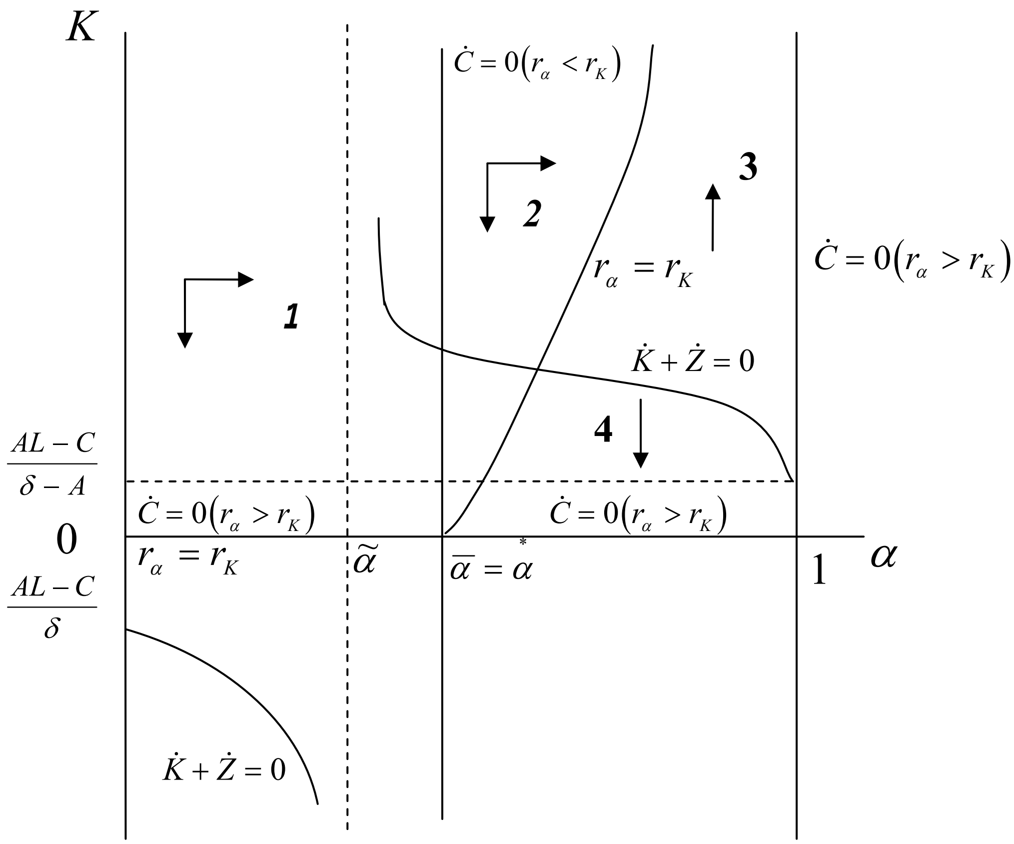

Figure 3.2.4: Phase Diagram; $C > AL$, $\rho = 0$

Notes:

Region 1: $\dot{K} + \dot{Z} < 0$	Region 2: $\dot{K} + \dot{Z} > 0$	Region 3: $\dot{K} + \dot{Z} > 0$	Region 4: $\dot{K} + \dot{Z} < 0$
$r_\alpha > r_K$	$r_\alpha > r_K$	$r_\alpha < r_K$	$r_\alpha < r_K$
$\dot{C} > 0$	$\dot{C} > 0$	$\dot{C} > 0$	$\dot{C} > 0$

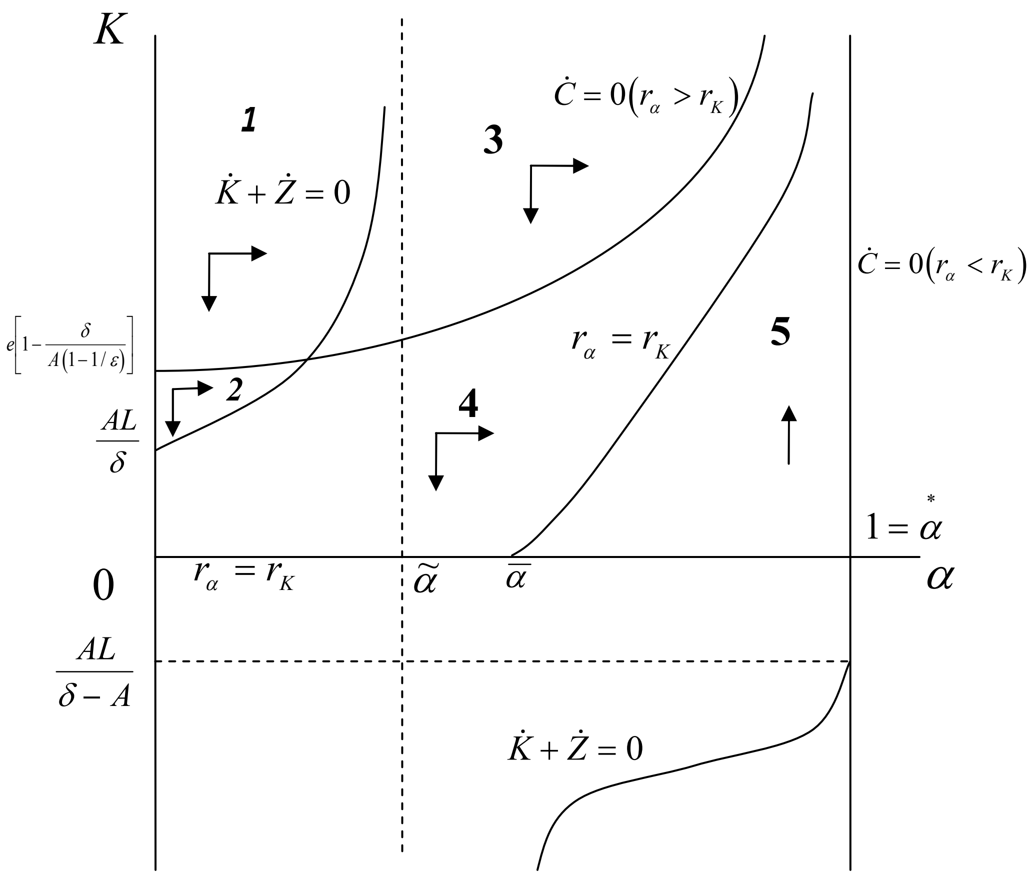

Figure 3.3.1: Phase Diagram; $C=0$, $\rho=\frac{A(\varepsilon-1)}{\varepsilon}-\delta$

Notes:

Region 1: $\dot{K}+\dot{Z}<0$, $r_\alpha > r_K$, $\dot{C}>0$

Region 2: $\dot{K}+\dot{Z}<0$, $r_\alpha > r_K$, $\dot{C}<0$

Region 3: $\dot{K}+\dot{Z}>0$, $r_\alpha > r_K$, $\dot{C}>0$

Region 4: $\dot{K}+\dot{Z}>0$, $r_\alpha > r_K$, $\dot{C}<0$

Region 5: $\dot{K}+\dot{Z}>0$, $r_\alpha < r_K$, $\dot{C}<0$

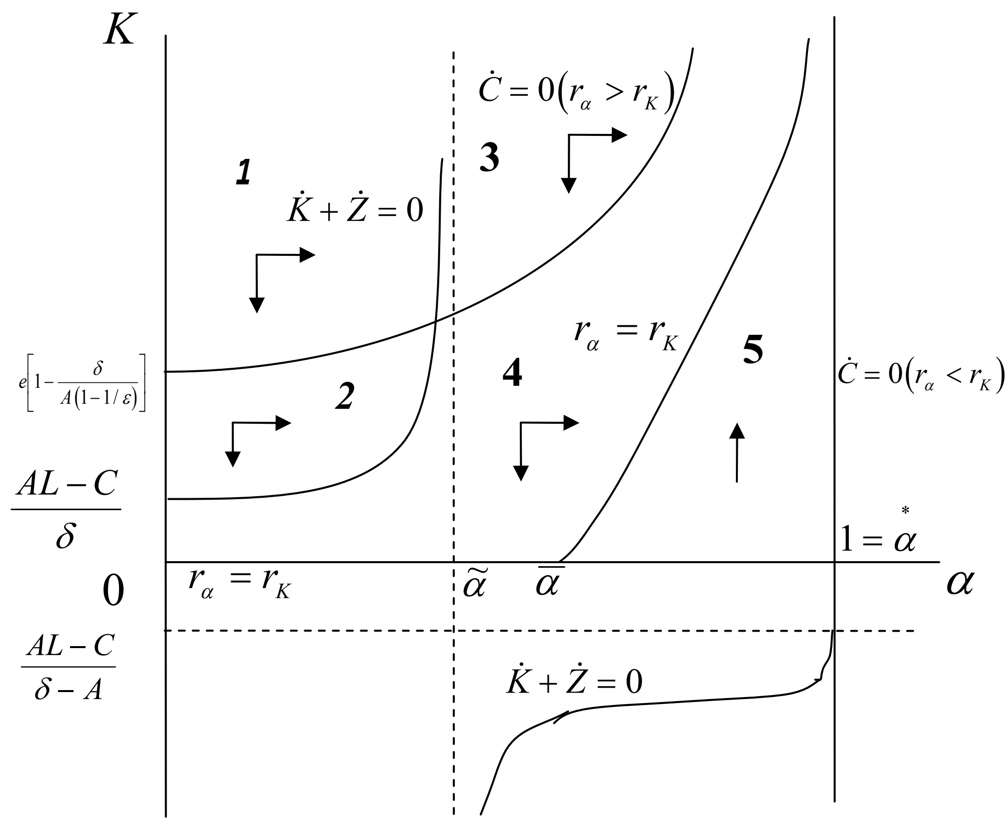

Figure 3.3.2: Phase Diagram; $0 < C < AL$, $\rho = \frac{A(\varepsilon - 1)}{\varepsilon} - \delta$

Notes:

Region 1: $\dot{K} + \dot{Z} < 0$	Region 2: $\dot{K} + \dot{Z} < 0$	Region 3: $\dot{K} + \dot{Z} > 0$	Region 4: $\dot{K} + \dot{Z} > 0$
$r_\alpha > r_K$	$r_\alpha > r_K$	$r_\alpha > r_K$	$r_\alpha > r_K$
$\dot{C} > 0$	$\dot{C} < 0$	$\dot{C} > 0$	$\dot{C} < 0$

Region 5: $\dot{K} + \dot{Z} > 0$

$r_\alpha < r_K$

$\dot{C} < 0$

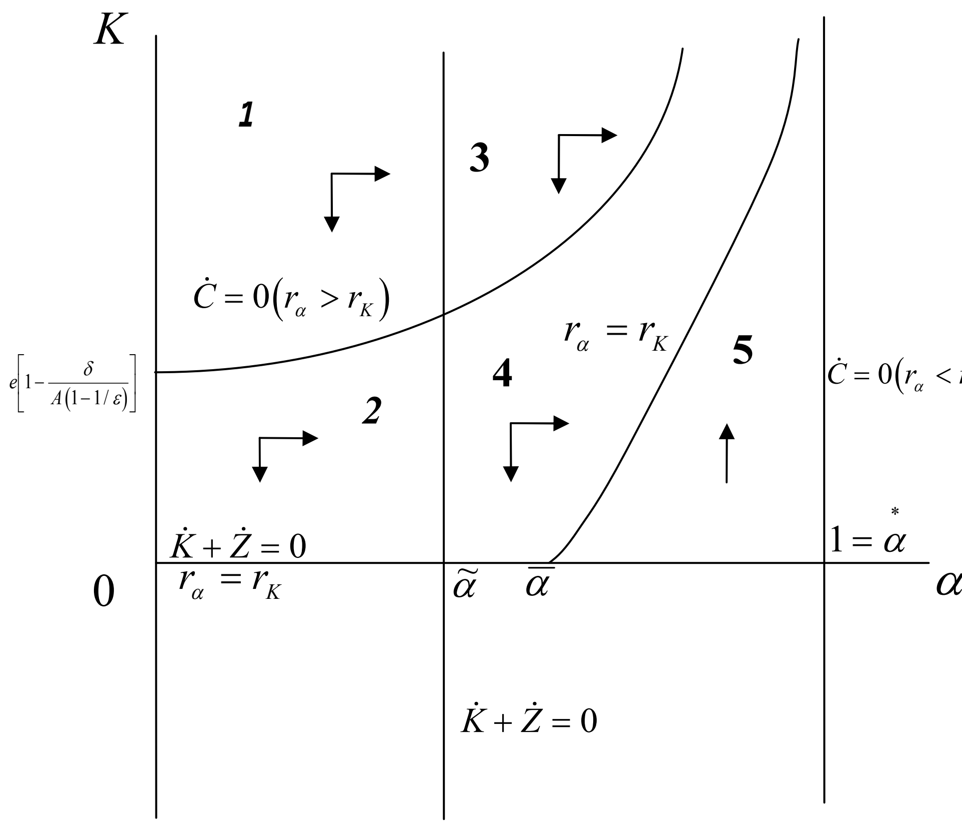

Figure 3.3.3: Phase Diagram; $\boldsymbol{C = AL}$ **,** $\boldsymbol{\rho = \frac{A(\varepsilon - 1)}{\varepsilon} - \delta}$

Notes:

Region 1: $\dot{K} + \dot{Z} < 0$, $r_\alpha > r_K$, $\dot{C} > 0$

Region 2: $\dot{K} + \dot{Z} < 0$, $r_\alpha > r_K$, $\dot{C} < 0$

Region 3: $\dot{K} + \dot{Z} > 0$, $r_\alpha > r_K$, $\dot{C} > 0$

Region 4: $\dot{K} + \dot{Z} > 0$, $r_\alpha > r_K$, $\dot{C} < 0$

Region 5: $\dot{K} + \dot{Z} > 0$, $r_\alpha < r_K$, $\dot{C} < 0$

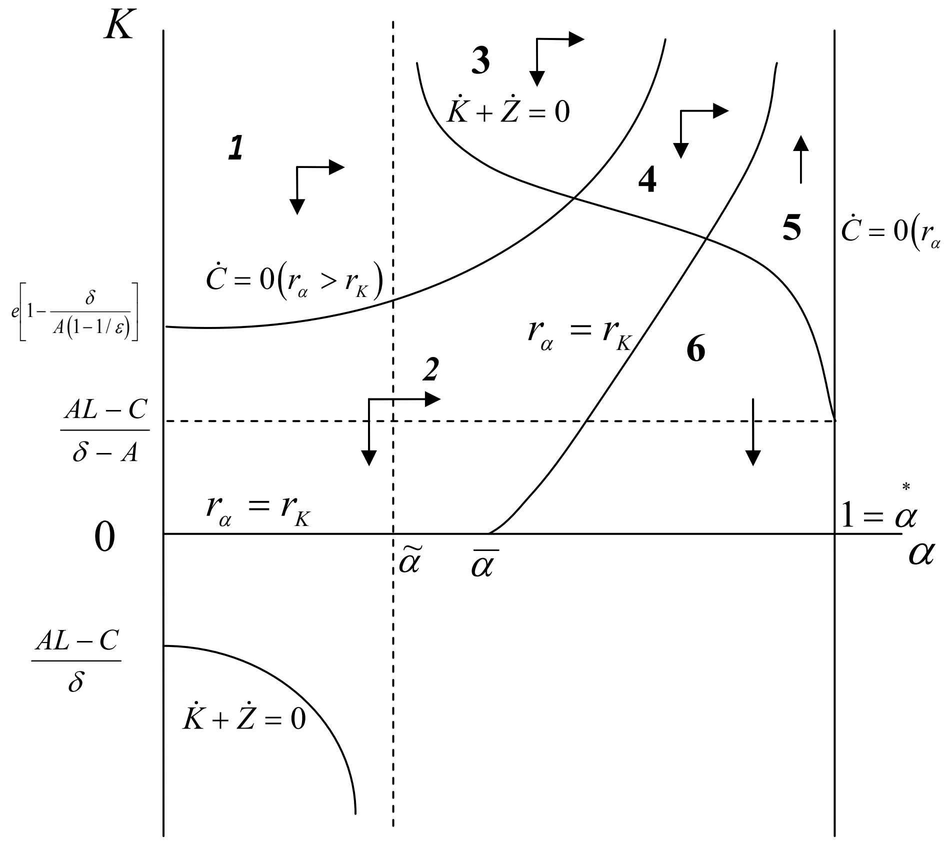

Figure 3.3.4: Phase Diagram; $C > AL$, $\rho = \frac{A(\varepsilon-1)}{\varepsilon} - \delta$

Notes:

Region 1	Region 2	Region 3	Region 4
$\dot{K}+\dot{Z}<0$	$\dot{K}+\dot{Z}<0$	$\dot{K}+\dot{Z}>0$	$\dot{K}+\dot{Z}>0$
$r_\alpha > r_K$	$r_\alpha > r_K$	$r_\alpha > r_K$	$r_\alpha > r_K$
$\dot{C}>0$	$\dot{C}<0$	$\dot{C}>0$	$\dot{C}<0$

Region 5	Region 6
$\dot{K}+\dot{Z}>0$	$\dot{K}+\dot{Z}<0$
$r_\alpha < r_K$	$r_\alpha < r_K$
$\dot{C}<0$	$\dot{C}<0$

3.5 Chapter 3 Appendix

3.5.1 The function $m(\alpha)$

The function

$$m(\alpha) = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}, \ \alpha \in [0,1]$$

is monotonically increasing in α. It has first and second derivatives:

$$m'(\alpha) = \frac{m(\alpha)}{\alpha^2} \ln\frac{1}{1-\alpha} > 0;$$

$$m''(\alpha) = \frac{m(\alpha)}{\alpha^2}\left\{\left[\frac{\ln\frac{1}{1-\alpha}}{\alpha^2} - \frac{2}{\alpha}\right]\ln\frac{1}{1-\alpha} - (1-\alpha)\right\} > 0.$$

The following forms are useful:

$$m(\alpha) = \alpha \exp\left[\frac{\ln(1-\alpha)}{\frac{\alpha}{1-\alpha}}\right];$$

$$m'(\alpha) = \frac{-\ln(1-\alpha)}{\alpha}\exp\left[\frac{\ln(1-\alpha)}{\frac{\alpha}{1-\alpha}}\right].$$

In addition,

$$m(0) = 0;$$
$$m(1) = 1;$$
$$m'(0) = e^{-1};$$
$$m'(1) = +\infty.$$

3.5.2 Household's Necessary Conditions for Optimization

The current-value Hamiltonian is:

$$H = \log c + \psi(ra + w - c)$$

The necessary conditions are:

$$\dot{a} = \frac{\partial H}{\partial \psi} = ra + w - c \tag{3.27}$$

$$\dot{\psi} = -\frac{\partial H}{\partial a} + \rho\psi = \psi(\rho - r) \tag{3.28}$$

$$\text{FOC: } \frac{\partial H}{\partial c} = \frac{1}{c} - \psi = 0 \tag{3.29}$$

$$\text{Transversality: } \lim_{t \to \infty} \psi_t a_t e^{-\rho t} = 0 \tag{3.30}$$

3.5.3 Arbitrage Locus

Setting the unconstrained portions of (3.20) and (3.21) equal to each other and solving for *K* yields

$$K = \frac{1}{m'(\alpha)(1-\alpha)}\left[m(\alpha) - \frac{\delta\varepsilon}{A(\varepsilon-1)}\right]. \tag{3.31}$$

If $\dfrac{\delta\,\varepsilon}{A(\varepsilon-1)} \geq 1$, then the bracketed expression in equation (3.31) is non-positive for all feasible values of $\alpha \in [0,1]$. Therefore, the $K \geq 0$ constraint binds, and the arbitrage locus collapses to the α-axis. All dynamic adjustment paths lead to $K = 0$, and the economy always moves to a primitive state where $Y = AL$. Such a scenario does not correspond to human history, so I rule it out by assuming $\frac{\delta \varepsilon}{A(\varepsilon - 1)} < 1$.

By the properties of $m(\alpha)$ and assuming $\frac{\delta \varepsilon}{A(\varepsilon - 1)} < 1$, there is a value $\overline{\alpha} \in (0,1)$ that solves $m(\alpha) = \frac{\delta\varepsilon}{A(\varepsilon - 1)}$. Recall that $\varepsilon > 1$; thus, the existence of $\overline{\alpha}$ requires $\frac{\delta}{A} < \frac{\varepsilon - 1}{\varepsilon} < 1$. If $0 \leq \alpha \leq \overline{\alpha}$, the bracketed expression is either negative or equal to zero, and since K cannot be negative, the arbitrage locus lies on the horizontal axis. For $\overline{\alpha} < \alpha \leq 1$, the arbitrage locus lies in the portion of (α, K) space where capital is positive. The locus is upward sloping because $m'(\alpha) > 0$ and $\frac{d[m'(\alpha)(1-\alpha)]}{d\alpha} < 0$. The locus goes asymptotically to infinity as $\alpha \to 1$ because $\lim_{\alpha \to 1} m'(\alpha)(1-\alpha) = 0$. Figure 3.4.1 illustrates the locus. The proofs for $\frac{d[m'(\alpha)(1-\alpha)]}{d\alpha} < 0$ and $\lim_{\alpha \to 1} m'(\alpha)(1-\alpha) = 0$ are given below.

Proof: $\frac{d[m'(\alpha)(1-\alpha)]}{d\alpha} < 0$

$$\frac{d[m'(\alpha)(1-\alpha)]}{d\alpha} = m''(\alpha)(1-\alpha) - m'(\alpha)$$

Substituting for $m'(\alpha)$ and $m''(\alpha)$ from Section 3.5.1 of the Appendix and then rearranging terms yields:

$$m''(\alpha)(1-\alpha) - m'(\alpha) = \frac{m(\alpha)}{\alpha^2} \ln\frac{1}{1-\alpha}\left[(1-\alpha)\left(\frac{\ln\frac{1}{1-\alpha}}{\alpha^2} - \frac{2}{\alpha}\right) - 1\right] - \frac{m(\alpha)(1-\alpha)^2}{\alpha^2}$$

The part in square brackets in the first term on the right hand side of the above equation is negative. This makes the first term negative, and since the second term is positive, the entire expression on the right hand side is negative.

Proof: $\lim_{\alpha\to 1} m'(\alpha)(1-\alpha)=0$

Substituting for $m'(\alpha)$ from Section 3.5.1 of the Appendix, $\lim_{\alpha\to 1} m'(\alpha)(1-\alpha)$ can be expressed

$$\lim_{\alpha\to 1} -\frac{\ln(1-\alpha)}{\alpha}\exp\left[\frac{\ln(1-\alpha)}{\frac{\alpha}{1-\alpha}}\right](1-\alpha)$$

$$=\underbrace{\lim_{\alpha\to 1} -\frac{\ln(1-\alpha)}{\frac{\alpha}{1-\alpha}}}_{A} \bullet \underbrace{\lim_{\alpha\to 1}\exp\left[\frac{\ln(1-\alpha)}{\frac{\alpha}{1-\alpha}}\right]}_{B}$$

Consider the limit given by expression *A*.

$$\lim_{\alpha\to 1} -\frac{\ln(1-\alpha)}{\frac{\alpha}{1-\alpha}} \overset{by\ L'Hopital's\ Rule}{=} \lim_{\alpha\to 1}\frac{\frac{1}{1-\alpha}}{\frac{1}{(1-\alpha)^2}}=\lim_{\alpha\to 1}(1-\alpha)=0$$

Consider the limit given by expression *B*.

$$\lim_{\alpha\to 1}\exp\left[\frac{\ln(1-\alpha)}{\frac{\alpha}{1-\alpha}}\right]=\exp\left[\lim_{\alpha\to 1}\frac{\ln(1-\alpha)}{\frac{\alpha}{1-\alpha}}\right] \overset{by\ L'Hopital's\ Rule}{=} \exp\left[\lim_{\alpha\to 1}(\alpha-1)\right]=\exp[0]=1$$

The product of expression *A* and expression *B* is zero; thus, $\lim_{\alpha\to 1} m'(\alpha)(1-\alpha)=0$.

3.5.4 Stationarity Locus

Imposing $\dot{K} + \dot{Z} = 0$ and solving for *K*, the unconstrained portion of (3.19) can be expressed

$$K = \frac{AL - C}{\delta - Am(\alpha)} \quad . \tag{3.32}$$

This is the *stationarity locus*. Assuming $\frac{\delta}{A} < 1$, there is an $\tilde{\alpha} \in (0, 1)$ such that $\delta = Am(\alpha)$. Since $m(\alpha)$ is monotonically increasing in α, $\delta > Am(\alpha)$ if $\alpha < \tilde{\alpha}$, and $\delta < Am(\alpha)$ if $\alpha > \tilde{\alpha}$. Holding *C* fixed, the first and second derivatives of the stationarity locus are

$$\frac{\partial K}{\partial \alpha} = \frac{Am'(\alpha)(AL - C)}{(\delta - Am(\alpha))^2}$$

$$\frac{\partial^2 K}{\partial \alpha^2} = \frac{Am''(\alpha)(AL - C)(\delta - Am(\alpha)) + A^2 m'(\alpha)^2 (AL - C)}{(\delta - Am(\alpha))^3}$$

The shape and position of the locus in (α, K) space depends on the value of *C*.

If $C = 0$, the stationarity locus takes the form $K = \frac{AL}{\delta - Am(\alpha)}$. The asymptote $\alpha = \tilde{\alpha}$ splits the locus into two branches. For $\alpha < \tilde{\alpha}$, the locus lies in the portion of the phase plane where capital is positive. For $\alpha > \tilde{\alpha}$, the locus lies in the portion of the phase plane where capital is negative. The first derivative is positive for all α, and the second derivative is positive for $\alpha < \tilde{\alpha}$. That said, the left branch of the stationarity locus holds for $0 \le \alpha < \tilde{\alpha}$, starts at $K = \frac{AL}{\delta}$ for $\alpha = 0$, increases at an increasing rate and goes asymptotically to infinity as $\alpha \to \tilde{\alpha}$. The right branch of the locus, which holds for $\tilde{\alpha} < \alpha \le 1$, starts at $-\infty$

immediately to the right of $\tilde{\alpha}$ and rises to $K = \frac{AL}{\delta - A} < 0$ at $\alpha = 1$. The sign of $\frac{\partial^2 K}{\partial \alpha^2}$ is ambiguous for $\alpha > \tilde{\alpha}$. However, I know that the slope of the locus equals $+\infty$ immediately to the right of $\tilde{\alpha}$ and at $\alpha = 1$, so the second derivative must change sign an odd number of times. None of the equilibrium dynamics depend on how many times the second derivative changes sign; I have chosen to depict the right branch of the locus with a single inflection point.

The two branches of the stationarity locus have the same general shape if $0 < C < AL$. The left branch of the stationarity locus lies in the region of the phase plane where capital is positive. It starts at $K = \frac{AL - C}{\delta}$ for $\alpha = 0$ and goes asymptotically to infinity as $\alpha \to \tilde{\alpha}$. The right branch of the locus lies in the region of the phase plane where capital is negative. It starts at $-\infty$ immediately to the right of $\tilde{\alpha}$ and rises to $K = \frac{AL - C}{\delta - A} < 0$ at $\alpha = 1$ with the second derivative changing sign an odd number of times. Relative to the $C = 0$ scenario, the $\alpha = 0$ intercept is smaller, the $\alpha = 1$ intercept is larger and the slope of both branches is less positive. That is, the left branch starts closer to the α axis and is flatter. The right branch ends closer to the α axis and is flatter.

If $C = AL$, then the unconstrained portion of (3.19) can be expressed $\dot{K} + \dot{Z} = [Am(\alpha) - \delta]K$. Therefore, $\dot{K} + \dot{Z} = 0$ is satisfied if $K = 0$ or if $\alpha = \tilde{\alpha}$, and so the stationarity locus is comprised of the horizontal axis and the vertical line $\alpha = \tilde{\alpha}$.

If $C > AL$, the stationarity locus is given by $K = \frac{AL - C}{\delta - Am(\alpha)}$. The asymptote $\alpha = \tilde{\alpha}$ splits the locus into two branches. For $\alpha < \tilde{\alpha}$, the locus lies in the portion of the phase plane where capital is negative. For $\alpha > \tilde{\alpha}$, the locus lies in the portion of the phase plane where capital is positive. The first derivative is negative for all α, and the second derivative is negative for $\alpha < \tilde{\alpha}$. Therefore, the left branch starts at $K = \frac{AL - C}{\delta} < 0$ for $\alpha = 0$, decreases at an increasing rate and falls to $-\infty$ as $\alpha \to \widetilde{\alpha}$. The right branch of the locus starts at $+\infty$ immediately to the right of $\widetilde{\alpha}$ and falls to $K = \frac{AL - C}{\delta - A} > 0$ at $\alpha = 1$. The sign of the second derivative is ambiguous, but I know the first derivative equals $-\infty$ at $\alpha = 1$. Therefore, the second derivative must change sign an odd number of times for $\alpha > \widetilde{\alpha}$. None of the equilibrium dynamics depend on how many times the derivative changes sign; I have chosen to depict the right branch of the locus with a single inflection point. For $C > AL$, as C increases, the $\alpha = 0$ intercept gets smaller, the $\alpha = 1$ intercept gets larger and the slope of both branches becomes more negative.

Figures 3.5.1 through 3.5.4 illustrate the various shapes and positions of the stationarity locus.

3.5.5 $\dot{C} = 0$ Locus

The requirements for $\dot{C} = 0$ differ depending on whether $r_K > r_\alpha$ or $r_K < r_\alpha$. Consider first the case of $r_K > r_\alpha$. The unconstrained portion of (3.22) and the condition $\dot{C} = 0$ imply

$$m(\alpha) = \frac{(\rho + \delta)\varepsilon}{A(\varepsilon - 1)}.$$

If $\frac{(\rho + \delta)\varepsilon}{A(\varepsilon - 1)} \leq 1$, which implies $\rho \leq \left(\frac{\varepsilon - 1}{\varepsilon}\right)A - \delta$, then by the properties of $m(\alpha)$ there is a value $\overset{*}{\alpha} \in (0,1]$ that solves $m(\alpha) = \frac{(\rho + \delta)\varepsilon}{A(\varepsilon - 1)}$. The vertical line $\alpha = \overset{*}{\alpha}$ is the $\dot{C} = 0$ locus if $r_K > r_\alpha$.

The unconstrained portion of (3.23) when combined with the condition $\dot{C} = 0$ yields the $\dot{C} = 0$ locus for $r_K < r_\alpha$:

$$K = \frac{\rho}{\left(1-\frac{1}{\varepsilon}\right)Am'(\alpha)(1-\alpha)} \quad . \tag{3.33}$$

The locus starts at $K = \frac{\rho}{\left(1-\frac{1}{\varepsilon}\right)Ae^{-1}} > 0$ for $\alpha = 0$. The first and second derivatives are

$$\frac{dK}{d\alpha} = \frac{-\rho\left(1-\frac{1}{\varepsilon}\right)A\left[m''(\alpha)(1-\alpha)-m'(\alpha)\right]}{\left[\left(1-\frac{1}{\varepsilon}\right)Am'(\alpha)(1-\alpha)\right]^2} \quad ;$$

$$\frac{d^2K}{d\alpha^2} = \frac{-\rho\left\{m'(\alpha)(1-\alpha)^2 m'''(\alpha) - 2\left[m''(\alpha)m'(\alpha)(1-\alpha) + m''(\alpha)^2(1-\alpha)^2 + m'(\alpha)^2\right]\right\}}{\left(1-\frac{1}{\varepsilon}\right)A\left[m'(\alpha)(1-\alpha)\right]^3} \quad .$$

In Section 3.5.3 of the Appendix, I showed that $m''(\alpha)(1-\alpha) - m'(\alpha) < 0$ and $\lim_{\alpha \to 1} m'(\alpha)(1-\alpha) = 0$. Given those results, the first derivative is positive, and the locus increases to $+\infty$ as $\alpha \to 1$. The second derivative is complicated and difficult to sign, but I know the slope must eventually increase at an increasing rate since the locus starts at a finite value at $\alpha = 0$ and approaches $\alpha = 1$ asymptotically. Whether the slope always or eventually increases at an increasing rate has no qualitative impact on the equilibrium dynamics.

The shape of the $\dot{C} = 0$ locus and its position in (α, K) space is independent of the value of *C*. Figures 3.6.1 and 3.6.2 depict the two branches. I depict the $r_K < r_\alpha$ branch in Figure 3.6.2 as always increasing at an increasing rate.

3.5.6 Relative position of $\bar{\alpha}$, $\tilde{\alpha}$ and $\overset{*}{\alpha}$

Recall the following: $\bar{\alpha}$ solves $m(\alpha)=\frac{\delta\varepsilon}{A(\varepsilon-1)}$; $\tilde{\alpha}$ solves $m(\alpha)=\frac{\delta}{A}$; and $\overset{*}{\alpha}$ solves $m(\alpha)=\frac{(\rho+\delta)\varepsilon}{A(\varepsilon-1)}$. The inequality $\frac{(\rho+\delta)\varepsilon}{A(\varepsilon-1)}>\frac{\delta\varepsilon}{A(\varepsilon-1)}$ holds because $(\rho+\delta)>\delta$. Recall that $\varepsilon>1$, so $\frac{\varepsilon}{\varepsilon-1}>1$. This implies $\frac{\delta\varepsilon}{A(\varepsilon-1)}>\frac{\delta}{A}$. Therefore, the following relationship holds: $\frac{(\rho+\delta)\varepsilon}{A(\varepsilon-1)}>\frac{\delta\varepsilon}{A(\varepsilon-1)}>\frac{\delta}{A}$. Since $m(\alpha)$ is increasing in α, it follows that

$$\overset{*}{\alpha}>\bar{\alpha}>\tilde{\alpha}\,. \tag{3.34}$$

3.5.7 Intersection of Arbitrage Locus and $\dot{C}=0$ Locus

Because the $r_K>r_\alpha$ branch of the $\dot{C}=0$ locus is a vertical line at $\overset{*}{\alpha}$, the intersection of this branch and the arbitrage locus is trivial; the two curves intersect at $\overset{*}{\alpha}$. To see where the $r_K<r_\alpha$ branch of the $\dot{C}=0$ locus intersects the arbitrage locus, set equations (3.24) and (3.26) equal to each other and solve for α:

$$\frac{\rho}{\left(1-\frac{1}{\varepsilon}\right)Am'(\alpha)(1-\alpha)}=\frac{1}{m'(\alpha)(1-\alpha)}\left[m(\alpha)-\frac{\delta\varepsilon}{A(\varepsilon-1)}\right].$$

$$\Rightarrow\frac{\rho}{\left(1-\frac{1}{\varepsilon}\right)A}=m(\alpha)-\frac{\delta\varepsilon}{A(\varepsilon-1)}$$

$$\Rightarrow m(\alpha)=\frac{(\rho+\delta)\varepsilon}{A(\varepsilon-1)}$$

As noted above in Section 3.5.5 of the Appendix, $\overset{*}{\alpha}$ solves the equation $m(\alpha) = \frac{(\rho + \delta)\varepsilon}{A(\varepsilon - 1)}$.

Thus, the $r_K < r_\alpha$ branch of the $\dot{C} = 0$ locus intersects the arbitrage locus at $\overset{*}{\alpha}$.

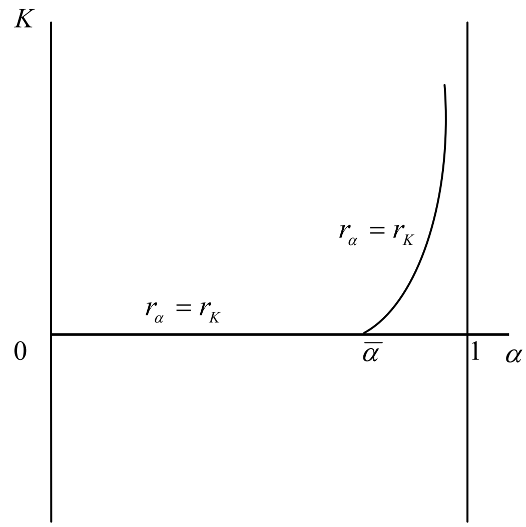

Figure 3.4.1: Arbitrage Locus

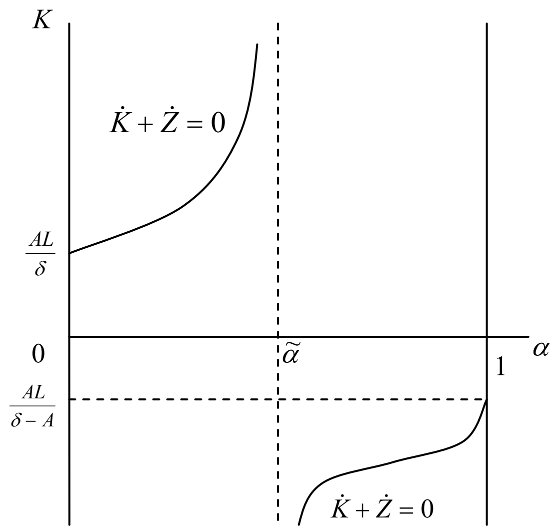

Figure 3.5.1: Stationarity Locus, *C* = 0

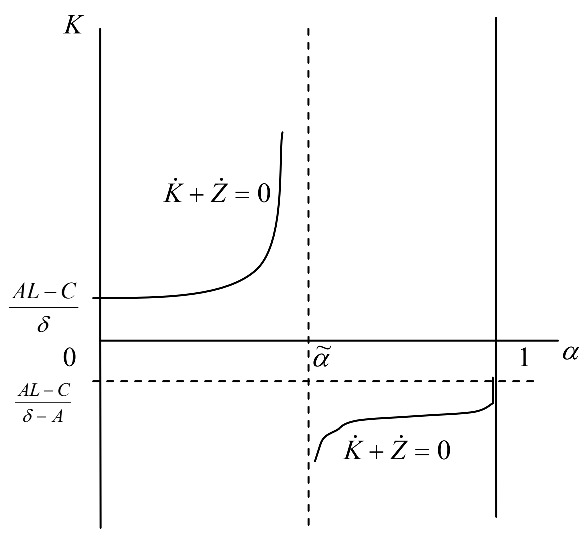

Figure 3.5.2: Stationarity Locus, $0 < C < AL$

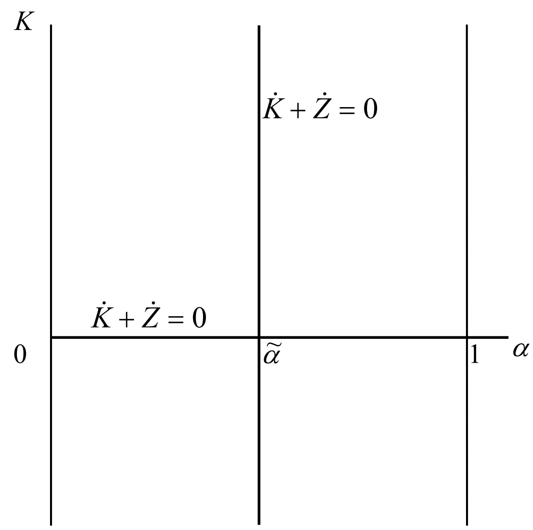

Figure 3.5.3: Stationarity Locus, $C = AL$

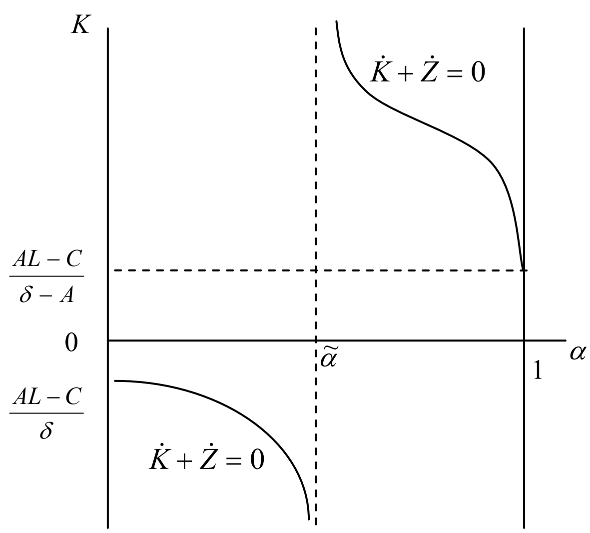

Figure 3.5.4: Stationarity Locus, $C > AL$

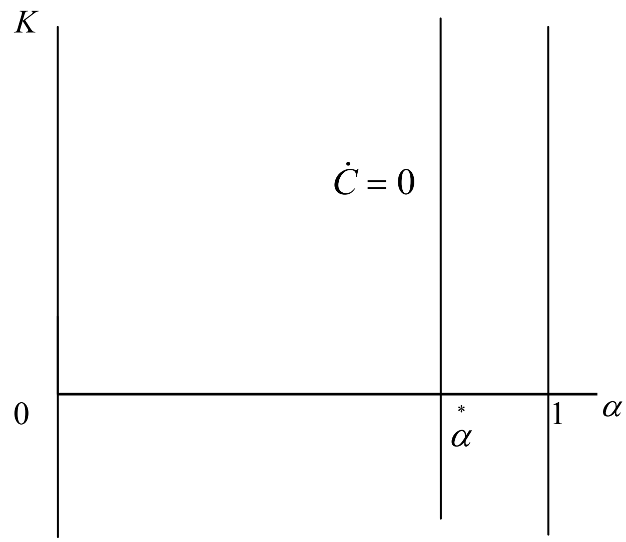

Figure 3.6.1: $r_\alpha < r_K$ branch of $\dot{C} = 0$ Locus

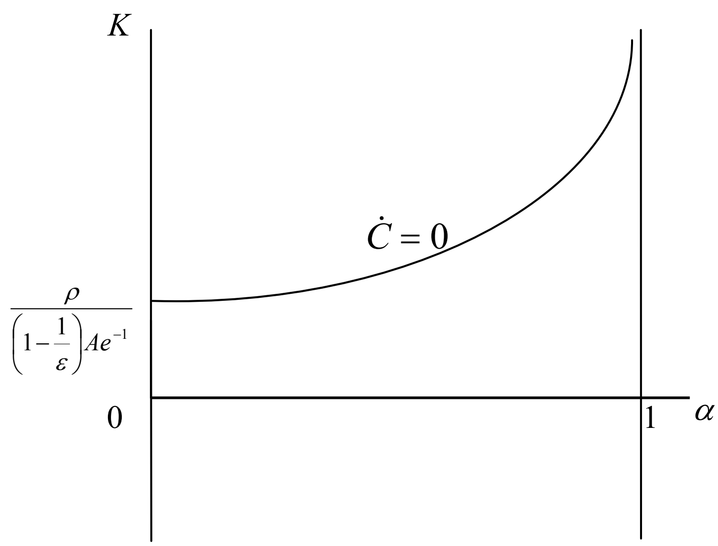

Figure 3.6.2: $r_\alpha > r_K$ branch of $\dot{C} = 0$ Locus

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