ABSTRACT

Adams, J.W. Green weight, volume and taper equations for Virginia pine (*Pinus virginiana)* in the Piedmont region of North Carolina.

Abstract: Virginia pine (*Pinus virginiana*) is a prolific pioneer tree species in the Piedmont region of North Carolina that has the potential to be a commercially important tree species. Reliable estimates of weight, stem volume and taper are needed for proper management of approximately 405,000 acres of Virginia pine presently located in the North Carolina Piedmont region. A study was conducted to derive merchantable green weight and merchantable volume equations to any upper stem diameter or height for Virginia pine (*Pinus virginiana*) across the Piedmont region of North Carolina. Models were derived from data collected at the North Carolina State University's Hill Demonstration Forest. For total and merchantable green weight models, 100 Virginia pine trees were destructively sampled and weighed. Fixed and mixed effects models were fit and prediction equations were developed for total green weight, green weight to any merchantable outside bark or inside bark diameters, and green weight to any upper merchantable height. Combined variable equations, nonlinear ratio equations and nonlinear exponential ratio equations were fit to these data. Using AIC and minus two log likelihood as the criterion for model fit, the mixed effects ratio model proved superior for predicting green weight to any upper merchantable height, while the mixed effects exponential ratio model was superior for predicting green weight to any upper diameter (outside or inside bark).

For merchantable volume, 105 Virginia pine trees were sampled to obtain outside and inside bark diameters to estimate stem volume. A combined variable equation was used to determine both inside and outside bark total volume. Fixed and mixed effects models were fit and prediction equations were developed for merchantable volume outside bark to any upper merchantable diameter outside bark and inside bark volume to any upper merchantable diameter inside bark. Equations to predict merchantable outside and inside bark volume to any upper merchantable stem height were also derived. Nonlinear ratio equations and nonlinear exponential ratio equations were fit to these data. Using AIC and minus two log likelihood as the criterion for model fit, the mixed effects ratio model proved superior for predicting merchantable outside bark volume and merchantable inside bark volume to any upper stem height, while the mixed effects exponential ratio model was superior for predicting merchantable outside bark volume to any upper stem diameter outside bark. A mixed effects exponential ratio model was also superior for predicting merchantable inside bark volume to any upper stem diameter inside bark. Taper equations were derived for the fixed effects models to predict diameter at any given height and to predict height at any given diameter for Virginia pine trees. The results of this research should be of interest to forest managers and private landowners in the Piedmont physiographic province of North Carolina and will enable foresters to develop more accurate estimates of weight or volume to any specified merchantable diameter or height limit for Virginia pine trees.

Green Weight, Volume and Taper Equations for Virginia Pine (*Pinus virginiana)* in the Piedmont Region of North Carolina

by

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BIOGRAPHY

John William Adams was born in 1971 in the town of Altavista, Va. He received his undergraduate degree from Liberty University in Lynchburg, Va. Upon graduation, he taught science at Ragsdale High School in Jamestown, N.C. It was during this time he met his beautiful wife Paige and was married on April 15th, 2000. John and Paige moved to Holly Springs, N.C. in August of 2003 when John started his Master of Science work at North Carolina State University. In October of 2004, Paige gave birth to their first son, Cooper.

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Table of Contents

List of Figures

List of Tables

Green Weight, Volume and Taper Equations for Virginia pine (*Pinus virginiana)* in the Piedmont Region of North Carolina

1. Introduction

Virginia pine (*Pinus virginiana*) is a prolific pioneer tree species in the Piedmont region of North Carolina that has potential to be a commercially important tree species. Reliable equations to determine estimates of weight, stem volume, and taper are needed for proper management of the approximate 405,000 acres of Virginia pine presently located in the piedmont region. Mills utilize Virginia pine for pulp in production of Oriented Strand Board (OSB) due to the trees long fibers and good pulping qualities. With the continuing drop of pulp prices in the current market, it is important for landowners to have an accurate estimate of weight and volume to assure top dollar for their product. These equations are needed for efficient utilization standards in determining merchantable weight and volume to any upper diameter or height limit.

Tree stems are used for a variety of products and considerable effort has been devoted to developing accurate weight and volume estimation procedures for any standard of utilization or merchantable height. Of the models available for green weight estimates of Virginia pine in the Piedmont region of North Carolina, many lack the versatility to predict weight to any merchantable height or diameter limit (Bullock and Burkhart 2003). According to Avery and Burkhart (2002), green weight equations are advantageous because delivery of fresh wood to the mill is encouraged. Weight scaling of log trucks saves time for both buyer and seller, and is more objective than manual scaling. While volume tables do exist for Virginia pine, (Clark 1994, Slocum 1953) none allow the determination of merchantable volume to any desired upper stem diameter outside bark (dob) or diameter inside bark (dib). The volume equation developed by Clark (1994) was derived from segmented stem profile equations integrated to determine volume between any two heights. A segmented stem profile separates the stem into sections and uses a different equation form to determine the volume of each segment. Volume tables to saw-log merchantable height and 7 in dob top were developed by Clark (1994). Separate ratio equations were also developed to determine volume between 7 inch dob top and 4 inch dob top. Independent variables needed to determine outside and inside bark volume include dob and dib at breast height, dob and dib at 17.3 ft, and total height (Clark 1994). The equations developed by Clark do not allow determination of volume to any merchantable upper stem diameter. An ad hoc method of determining volume currently used by some forest managers incorporates a model for loblolly pine (*Pinus taeda* L.) volume and deducts a percentage from the total estimated volume to arrive at an estimate of total volume for Virginia pine. This ad hoc deduction method is used for saw timber but is generally not utilized for pulpwood. The research presented will allow a greater amount of specificity in volume determination to managers working in the Piedmont Region of North Carolina.

The purpose of this research is to derive new yield models for Virginia pine. This will enable more accurate estimates of weight or volume to any specified merchantable diameter or height limit. Linear fixed effects combined variable equations, nonlinear fixed and mixed effects ratio and exponential ratio models will be evaluated for predicting total and merchantable weight and volume of Virginia pine. Implicit taper functions are derived from fixed effects weight and volume equations to provide equations to predict diameter or height up the stem. The results of this research should be of interest to forest managers and private landowners in the Piedmont physiographic province of North Carolina.

2. Data

Virginia pine trees were sampled at the North Carolina State University's (NCSU) Hill Demonstration Forest located in Durham County, N.C. (Figure 1). A total of 105 trees were sampled during May – June of 2004 from plantation and natural stands of Virginia pine as well as mixed pine – hardwood stands. Only Virginia pine trees with a minimum diameter at breast height of 4.5 inches with no forked stem and a canopy position of dominant or co-dominant were sampled. A histogram of the distribution of stem diameter at breast height is provided in Figure 2. Data collected on each tree sampled were: age at stump height, age at breast height (4.5 ft), diameter at breast height (D), diameter outside bark (dob) and diameter inside bark (dib) up the stem at pre-determined points to an approximate 3 in dob top, height to live crown and total height. Prior to felling, trees were marked at 0.5, 2.5, 4.5 and 7.0 ft. These heights were chosen and marked to identify stump height for felling, minimize the effect of butt swell, and identify D. The 7.0 ft height was marked for ease of measurements for the remaining sections. After being felled, the tree was delimbed and the bole was marked from the 7.0 ft height in 3 ft increments to a 3 in dob top. A merchantable height of 17.3 ft was also marked and measured for diameter outside bark and diameter inside bark but no weight was taken at this height. A height of 17.3 ft was chosen to allow for a 1 ft stump, 16 ft log and 0.3 ft kerf. Diameter outside and inside bark were measured by placing a 20th scale engineer's ruler (in) across the stump and the top of each bolt with two measurements taken perpendicular to each other. The diameter measurements were averaged to give an estimate of diameter at that height. Age in years was determined by counting rings at the stump and at breast height.

Figure 1: Natural range of *Pinus virginiana* and location of NCSU's Hill Demonstration Forest, Durham County, N.C.

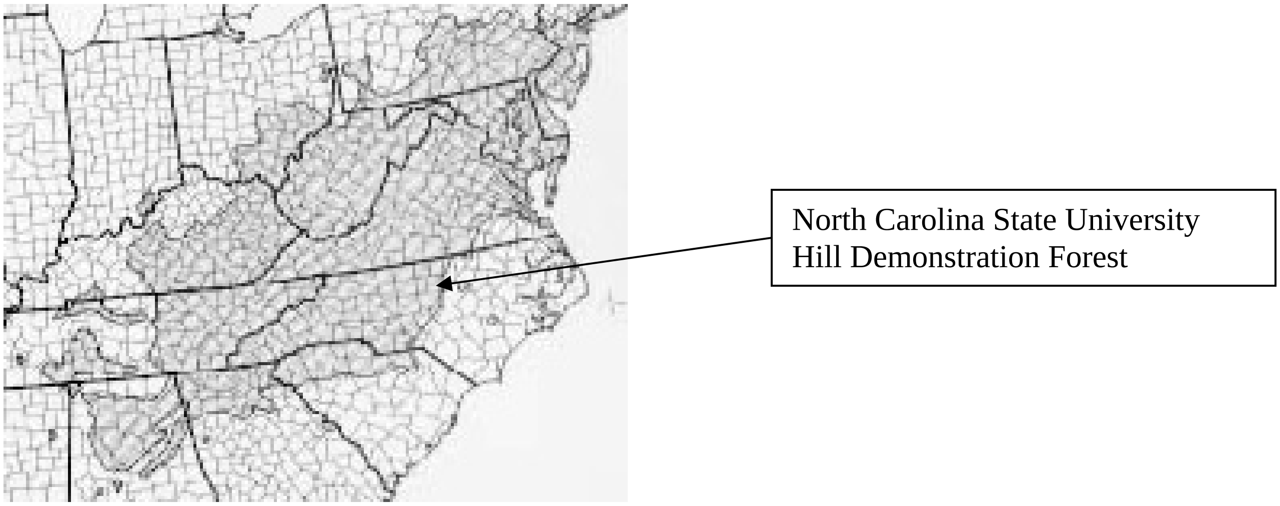
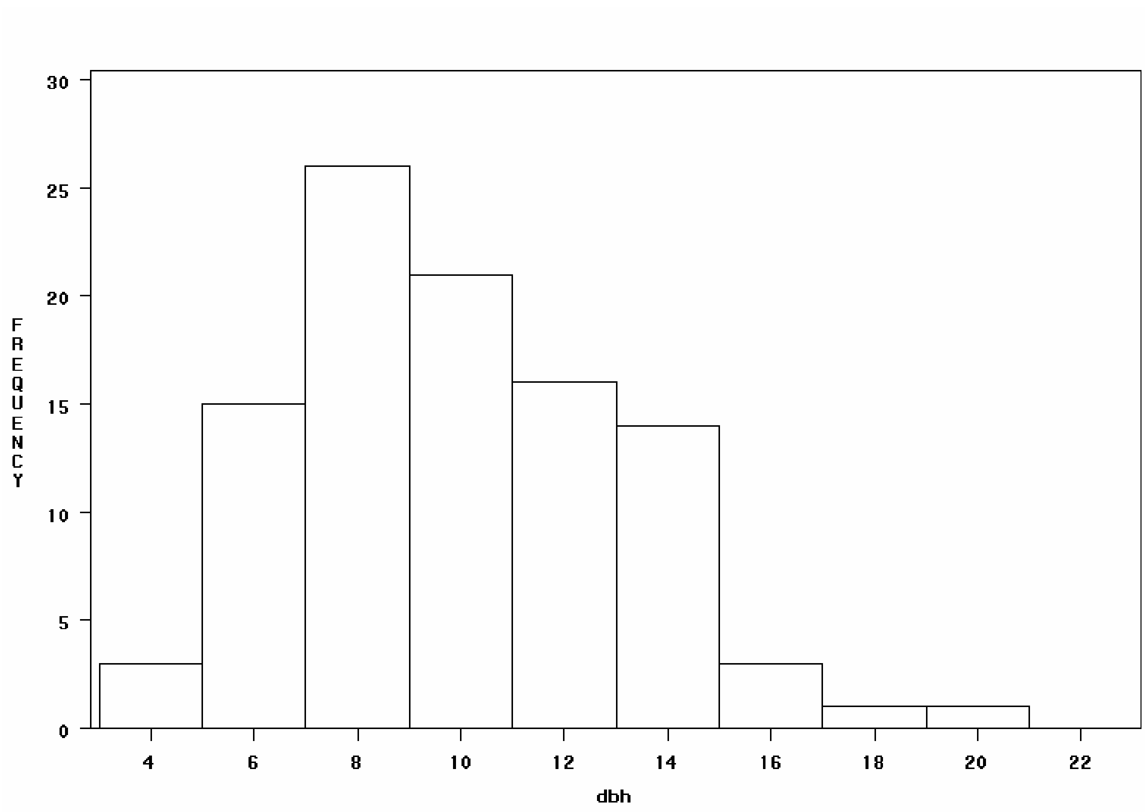


Figure 2: Histogram of stem diameter at breast height of Virginia pine sampled in this research.



After bucking the bole into 3 ft sections, green weights to a 3 in outside bark top diameter were recorded in the field on a portable bench scale (placed on plywood for stability and levelness) to the nearest 0.1 pound. The two sections created from obtaining diameters at 17.3 ft were weighed together. Cookies used to determine moisture content were taken at D, the bottom of the middle bolt (as determined from the distance to the 3 inch outside bark top height), and from the bottom of the bolt at the 3 inch outside bark top limit. The sizes of the cookies varied, but most were from 1 to 3 inches in height. Green weights of each cookie were taken to the nearest 0.1 lb using the same portable bench scale. The samples were then placed in bags and transported to the lab for dry weight analysis. The cookies were oven dried at 103 degrees Celsius until a stable weight was obtained for a 24 hour period, measured to 0.01 lb.

Smalian's formula (Avery and Burkhart 2002) was used to determine the cubic ft volume of each bolt. All statistical analysis was performed in SAS (SAS 2000). Summary statistics for the data are presented in Table 1.

Table 1: Summary Statistics for the 105 Virginia pine trees used in this study.

	n	Mean	SD	Min	Max	SE	CV
A_s	103	37.74	9.86	29.0	63.0	0.971	26.57
H_3	105	55.15	11.15	27.0	85.5	1.088	19.02
H	105	67.04	9.34	47.6	95.0	0.912	13.67
D	105	9.89	3.19	4.45	20.93	0.311	31.01
V_{tot}	105	20.84	17.99	2.22	127.89	1.637	72.11
W_{tot}	100	1214.5	923.67	129	5002.4	89.893	70.62
MC_{dbh}	93	43.99	8.88	17	99.41	0.921	20.18
MC_{middle}	95	52.93	10.69	27.07	99.62	1.097	20.19
$MC_{3\ inch\ top}$	86	53.45	8.15	14.12	74.20	0.880	15.25

where:

SD = standard deviation
SE = standard error
CV = coefficient of variation
A_s = age in years at stump
H_3 = height to 3 inch dob top, ft
H = total height, ft
D = diameter at breast height, in
V_{tot} = total volume, ft^3
W_{tot} = total green weight, lb
MC_{dbh} = moisture content as a percent of dry weight at D
MC_{middle} = moisture content as a percent of dry weight at approximate middle of stem total height
$MC_{3\ inch\ top}$ = moisture content as a percent of dry weight at 3 in top, dob

3. Green Weight Modeling

3.1 Total Green Weight Fixed Effects Modeling

Total stem green weight to an approximate 3 in. outside bark top diameter was estimated using a combined variable equation (Avery and Burkhart 2002) estimated as a function of diameter at breast height (D) and total stem height (H). Of the 105 trees in the data set, only 99 were used for green weight modeling because broken scale prevented weight measurement for 6 trees. The linear combined variable form used to model the total green weight to a 3 in. outside bark top diameter was:

$$W_{tot} = \beta_0 + \beta_1 (D^2 H) + \varepsilon \qquad (1)$$

where

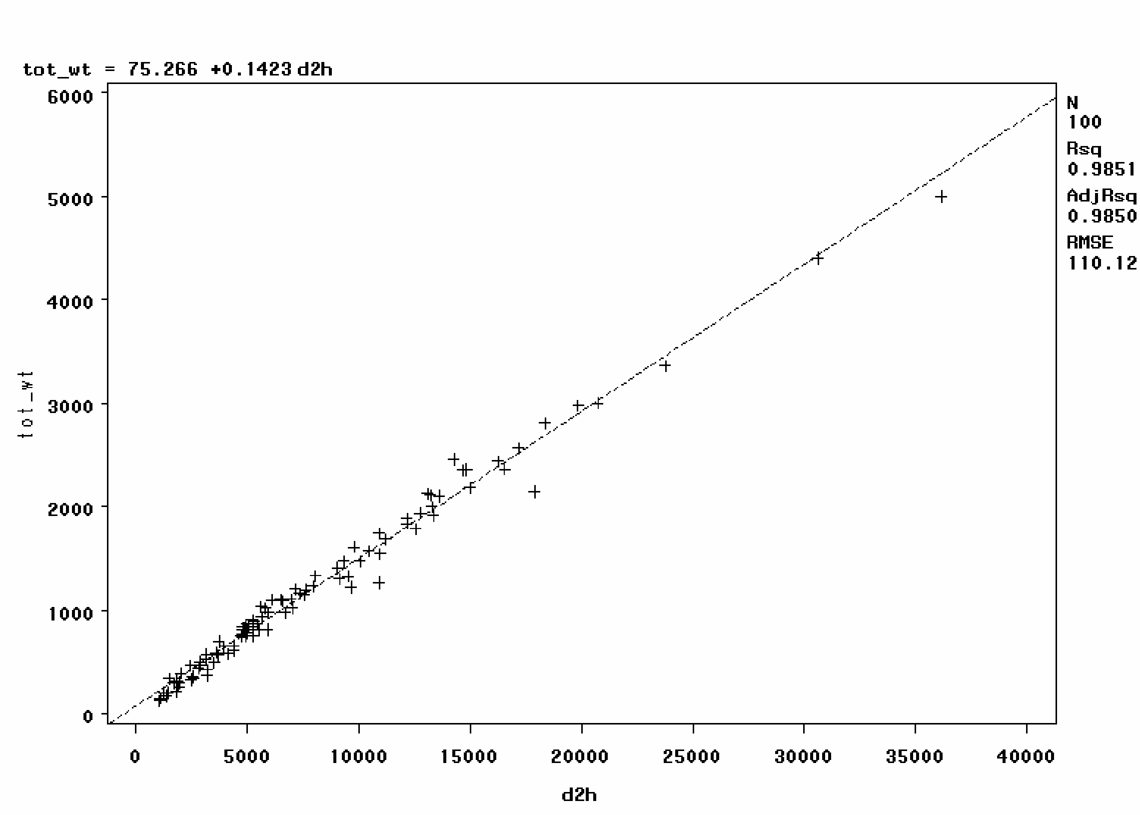
W_{tot} = total green weight of stem to a 3 in. outside bark top diameter, lb
β_0, β_1 = parameters to be estimated
ε = error term

The resulting prediction equation was:

$$\hat{W}_{tot} = 75.266 + 0.1423(D^2 H) \qquad (2)$$

The model fit the data well with a coefficient of determination (R^2) value of 0.9851 and a root mean square error ($RMSE$) of 110.12 lb. A graph of the data and resulting prediction line shows a strong relationship between D^2H and total green weight (Figure 3).

Figure 3: Linear Regression of Total Outside Bark Green Weight



where

tot_wt = total outside bark green weight to an approximate 3 in. outside bark top diameter, lb
d2h = (diameter at breast height squared) *(total height)

3.2 Ratio Form - Fixed Effects

Ratios of merchantable green weight to total green weight were derived at specified heights up the stem for each tree using the following equation (Burkhart 1977):

$$R_w = \frac{W_{mer}}{W_{tot}} \tag{3}$$

where

R_w = weight ratio

W_{mer} = green weight to any upper merchantability limit, lb

This research used the following ratio form equations to model the weight ratio:

$$R_{w,ob} = 1 + \beta_1 \left(\frac{dob^{\beta_2}}{D^{\beta_3}} \right) + \varepsilon \tag{4}$$

$$R_{w,ib} = 1 + \beta_4 \left(\frac{dib^{\beta_5}}{D^{\beta_6}} \right) + \varepsilon \tag{5}$$

where

$R_{w,ob}$ = weight ratio of outside bark (ob) measurements

$R_{w,ib}$ = weight ratio of inside bark (ib) measurements

dob = upper diameter limit, outside bark, in

dib = upper diameter limit, inside bark, in

β_i = coefficients to be estimated, i = 1,…, 6

ε = error term

To determine the merchantable green weight, Equation 3 was rearranged such that the merchantable green weight is the product of predicted total stem green weight multiplied by the predicted ratio of merchantable green weight to total green weight. (Equations 6 and 7)

$$\hat{W}_{dob} = \hat{W}_{tot}(\hat{R}_{w,ob}) \tag{6}$$

$$\hat{W}_{dib} = \hat{W}_{tot}(\hat{R}_{w,ib}) \tag{7}$$

where

$W_{dob} =$ merchantable green weight to any specified upper dob, lb
$W_{dib} =$ merchantable green weight to any specified upper dib, lb

Burkhart's (1977) nonlinear model form, Equations 4 and 5, was used to determine the weight ratio outside bark (o.b.) and weight ratio inside bark (i.b.) and was conditioned such that when diameter at the top is equal to zero, the ratio will equal one and the merchantable green weight will equal the total stem green weight. Ratio equations were inserted into Equations 6 and 7 and used nonlinear regression techniques to model these trends (Table 2). Maximum correlation as indicated by the correlation matrix between coefficients $\hat{\beta}_2$ and $\hat{\beta}_3$ was 0.86 and 0.89 between coefficients $\hat{\beta}_5$ and $\hat{\beta}_6$. Though correlation between these coefficients seems high, convergence was not inhibited.

In many instances, the desired height is known but not the diameter at this height. Cao and Burkhart's (1980) ratio model form (Equation 8) was used to determine the weight ratio for green weight to any specified height (R_h).

$$R_h = \left(1 + \alpha_1\left(\frac{(H-h)^{\alpha_2}}{H^{\alpha_3}}\right)\right) + \varepsilon \tag{8}$$

where:

h = upper height limit, ft
α_i = coefficients to be estimated, i = 1,2,3

R_h was inserted into Equation 3 and used to determine green weight o.b. to any specified height:

$$\hat{W}_h = \hat{W}_{tot}(\hat{R}_h) \tag{9}$$

where:

W_h = green weight to any specified upper merchantable height, lb

Nonlinear regression techniques were used to fit the data to this model (Table 2). Maximum correlation, as indicated by the correlation matrix between estimated coefficients $\hat{\alpha}_2$ and $\hat{\alpha}_3$, was 0.51.

3.3 Exponential Ratio Form - Fixed Effects

Van Deusen et al. (1981) notes that Burkhart's (1977) equation is unbounded as diameter increases and could yield illogical volume estimates under certain conditions. Tasissa et al. (1997) agreed with Van Deusen et al. (1981) and added that Burkhart's (1977) formula works well in determining merchantable weight and volume to upper stem diameters and heights of practical use, but it does not work well with lower merchantable heights. Bullock and Burkhart (2003) added that exponential models are doubly constrained such that as the upper diameter limit goes to zero, the ratio goes to one and the predicted green weight goes to the total green weight. As the lower diameter limit goes to infinity, the ratio goes to zero and the predicted green weight goes to zero. In this study, we used nonlinear regression techniques to test the Tasissa et al. (1997) exponential ratio model form, a modification of the Van Deusen et al. (1981) exponential ratio form equation for determining merchantable green weight outside bark to any upper dob and merchantable green weight outside bark to any upper dib (Equations 10 and 11)

Table 2: Parameter estimates and fit statistics for the fixed effects nonlinear ratio and exponential ratio equations used to predict green weight outside bark to any upper stem diameter or height.

EQ	ESTIMATED COEFFICIENTS			RSS	RMSE	AIC	-2 LOG
6	$\hat{\beta}_1 = -0.6067$	$\hat{\beta}_2 = 3.1282$	$\hat{\beta}_3 = 2.9983$	37,211,679	138.39	24,715	24,707
7	$\hat{\beta}_4 = -0.8326$	$\hat{\beta}_5 = 3.3959$	$\hat{\beta}_6 = 3.3196$	38,882,493	141.46	24,801	24,793
9	$\hat{\alpha}_1 = -0.7823$	$\hat{\alpha}_2 = 2.0352$	$\hat{\alpha}_3 = 1.9761$	2,340,639	34.70	19,332	19,324
10	$\hat{\beta}_7 = -0.9404$	$\hat{\beta}_8 = 5.6707$	$\hat{\beta}_9 = 5.3741$	23,516,974	110.01	23,822	23,814
11	$\hat{\beta}_{10} = -1.4980$	$\hat{\beta}_{11} = 5.8488$	$\hat{\beta}_{12} = 5.6393$	30,227,272	124.72	24,311	24,303
12	$\hat{\alpha}_4 = -1.0973$	$\hat{\alpha}_5 = 3.0504$	$\hat{\alpha}_6 = 2.8885$	4,329,770	47.20	20,529	20,521

EQ = Equation number
RSS = Residual Sum of Squares
RMSE = Root Mean Square Error, lb
AIC = Akaike's Information Criterion (lower number is better)
-2 LOG = Minus 2 log likelihood

with the estimated parameters listed in Table 2.

$$W_{dob} = \hat{W}_{tot}\left(\exp\left(\beta_7\left(\frac{dob^{\beta_8}}{D^{\beta_9}}\right)\right)\right) + \varepsilon \qquad (10)$$

$$W_{dib} = \hat{W}_{tot}\left(\exp\left(\beta_{10}\left(\frac{dib^{\beta_{11}}}{D^{\beta_{12}}}\right)\right)\right) + \varepsilon \qquad (11)$$

where :

exp = the base of the natural logarithm

β_i = coefficients to be estimated, *i* = 7,….,12

Maximum correlation as indicated by the correlation matrix between variables $\hat{\beta}_8$ and $\hat{\beta}_9$ for Equation 10 was 0.90; a correlation of 0.91 between variables $\hat{\beta}_{11}$ and $\hat{\beta}_{12}$ was found for Equation 11. Though correlation between these coefficients seems high, convergence was not inhibited.

As with the ratio model, when desired height is known the following equation was fitted for determining merchantable green weight outside bark to any upper height.

$$W_h = \hat{W}_{tot}\left(\exp\left(\alpha_4\left(\frac{(H-h)^{\alpha_5}}{H^{\alpha_6}}\right)\right)\right) + \varepsilon \qquad (12)$$

where

α_i = coefficients to be estimated, *i*=4,5,6

Nonlinear regression techniques were used to fit the data (Table 2).

3.4 Comparison of Fixed Effects Ratio and Exponential Ratio Form Equations

Residual sum of squares, root mean square error and AIC statistics were used to determine the best fit in comparing the ratio and exponential ratio equations for predicting outside bark green weight of Virginia pine to any upper diameter or height limit (Table 2). Since the exponential ratio RSS is 37% lower than the ratio model, it suggests the exponential ratio model (Equation 10) will yield a more accurate estimate of green weight to any upper dob. The RMSE AIC and -2 log likelihood fit statistics validated this result producing lower values than the ratio model.

For predicting the green weight to any upper dib, the exponential ratio model (Equation 11) will deliver the most accurate estimate of green weight to any upper dib because it has a RSS 23% lower than the ratio model. The RMSE AIC and -2 log likelihood fit statistics validated this result producing lower values than the ratio model.

When using any upper merchantable height limit as a predictor of outside bark green weight for Virginia pine, the RSS for the ratio form model was 46% lower than the exponential ratio model, indicating the ratio model (Equation 9) is a better predictor of Virginia pine green weight to any upper merchantable height limit. The RMSE AIC and -2 log likelihood fit statistics validated this result producing lower values than the exponential ratio model.

3.5 Green Weight - Mixed Effects Modeling

Data collected for volume, taper and weight equations are hierarchical with multiple measurements taken on one tree and thus violate the assumptions of independence and zero autocorrelation. (Garber and Maguire 2002, Neter et al. 1998, Rawlings et al. 1998). Independence is key to obtaining an unbiased estimate of the covariance matrix in regression (Gregorie and Schabenberger 1996, Kozak 1997, Valentine and Gregorie 2001). These correlations among measurements collected on a single tree were not accounted for until nonlinear mixed models employed by Gregoire and Schabenberger (1996). A mixed effects modeling framework, comprised of fixed and random effects, has the advantage of correctly estimating the covariance matrix in regression (Leites and Robinson 2003). Success in reducing the effect of autocorrelation in longitudinal forestry data using mixed effects models has been demonstrated in several recent studies (Garber and Maguire 2002, Biging 1985, Bullock and Burkhart 2003, Gregoire et al. 1995, Tassisa and Burkhart 1998, Calegario et al. 2004).

A random effect term to account for the natural variance within individual tree stems was added to the fixed effects for the model considered in this research. Initial parameter estimates for the mixed models were estimated by determining the parameter estimates of each individual stem using proc nlin in SAS and then identifying the mean and variance of each fixed effect. Initial values were entered into a mixed model and new fixed and random effects parameters were determined using maximum likelihood. Minus twice the log likelihood along with AIC were used to determine model fit and to compare the fit with the fixed effects models previously discussed.

3.6 Ratio Form - Mixed Effects

The merchantable o.b. and i.b. green weight fixed effects ratio forms (Equations 6 and 7) were modified with the addition of a random effect variable, u_i, with the following mixed effects models resulting:

$$W_{dob} = \hat{W}_{tot}\left(1+\left(\left(\beta_{13}+u_1\right)\left(\frac{dob^{\beta_{14}}}{D^{\beta_{15}}}\right)\right)\right)+\varepsilon \tag{13}$$

$$W_{dib} = \hat{W}_{tot}\left(1+\left(\left(\beta_{16}+u_2\right)\left(\frac{dib^{\beta_{17}}}{D^{\beta_{18}}}\right)\right)\right)+\varepsilon \tag{14}$$

where:

u_i = random effect variable; *i*=1,2

The fit statistics for predicting merchantable green weight to any upper diameter o.b. results for AIC were 24,682 and -2 log likelihood were 24,672, while the fit statistics for predicting merchantable green weight to any upper diameter i.b. produced AIC results of 24,620 and -2 log likelihood results of 24,610.

Cao and Burkhart's (1980) model form (Equation 9), for determining green weight to any specified height was also modified with the addition of a random effect variable with the following equation resulting:

$$W_h = \hat{W}_{tot}\left(1+\left(\left(\alpha_7+u_3\right)\left(\frac{\left(H-h\right)^{\alpha_8}}{H^{\alpha_9}}\right)\right)\right)+\varepsilon \tag{15}$$

Parameter estimates and fit statistics were determined for the ratio models (Table 3).

3.7 Exponential Ratio Form - Mixed Effects

Exponential ratio form Equations 10 and 11 were modified with a random effect variable:

$$W_{dob} = \hat{W}_{tot}\left(\exp\left(\left(\beta_{19}+u_4\right)\left(\frac{dob^{\beta_{20}}}{D^{\beta_{21}}}\right)\right)\right)+\varepsilon \tag{16}$$

$$W_{dib} = \hat{W}_{tot}\left(\exp\left(\left(\beta_{22}+u_5\right)\left(\frac{dib^{\beta_{23}}}{D^{\beta_{24}}}\right)\right)\right)+\varepsilon \tag{17}$$

To determine merchantable green weight to any specified height, Equation 12 was modified with the addition of a random effect variable (Equation 18).

$$W_h = \hat{W}_{tot}\left(\exp\left(\left(\alpha_{10}+u_6\right)\left(\frac{\left(H-h\right)^{\alpha_{11}}}{H^{\alpha_{12}}}\right)\right)\right)+\varepsilon \tag{18}$$

Parameter estimates for the exponential ratio model along with fit statistics are listed in Table 3.

3.8 Comparison of Mixed Effects Ratio and Exponential Ratio Form Equations

Mixed model analysis yielded the same ranking between the ratio and exponential ratio model forms as the fixed effects comparisons. The exponential ratio models produced lower AIC and -2 log likelihood values for predicting o.b. green weight to any upper merchantable diameter outside or inside bark.

For green weight to any merchantable dob using the mixed models, the AIC and – 2 log likelihood values for the exponential ratio model were 5% lower than the green weight dob ratio equations. This would indicate that the mixed effects exponential ratio models were better predictors for merchantable green weight to any upper dob.

The AIC and -2 log likelihood values for the mixed model exponential ratio model for merchantable green weight to any upper dib was 3% lower than the mixed model ratio equation. This would indicate that the mixed effects exponential ratio models were better predictors for merchantable green weight to any upper dib.

For green weight to any upper stem merchantable height, AIC and -2 log likelihood ratio models yielded a 7% lower value than the exponential ratio model equation indicating that the ratio model is the better predictor. See Table 3 for results comparison.

Table 3: Parameter estimates and fit statistics for the mixed effects model nonlinear ratio and exponential ratio model forms used to predict o.b. green weight to any upper stem diameter or height.

EQ	ESTIMATED COEFFICIENTS				AIC	-2 LOG
13	$\hat{\beta}_{13} = -0.6007$	$\hat{\beta}_{14} = 3.1397$	$\hat{\beta}_{15} = 3.0039$	$\hat{u}_1 = -0.02245$	24,682	24,672
14	$\hat{\beta}_{16} = -0.7758$	$\hat{\beta}_{17} = 3.4691$	$\hat{\beta}_{18} = 3.3552$	$\hat{u}_2 = 0.05167$	24,620	24,610
15	$\hat{\alpha}_7 = -0.8322$	$\hat{\alpha}_8 = 2.0338$	$\hat{\alpha}_9 = 1.9889$	$\hat{u}_3 = 0.01590$	19,018	19,008
16	$\hat{\beta}_{19} = -0.8704$	$\hat{\beta}_{20} = 5.9224$	$\hat{\beta}_{21} = 5.5634$	$\hat{u}_4 = 0.1284$	23,471	23,461
17	$\hat{\beta}_{22} = -1.3370$	$\hat{\beta}_{23} = 6.1350$	$\hat{\beta}_{24} = 5.8427$	$\hat{u}_5 = 0.2071$	23,946	23,936
18	$\hat{\alpha}_{10} = -1.2325$	$\hat{\alpha}_{11} = 3.0514$	$\hat{\alpha}_{12} = 2.9154$	$\hat{u}_6 = 0.05018$	20,331	20,321

u_i = random effect representing variation within individual stems
EQ = Equation number
AIC = Akaike's Information Criterion (lower number is better)
-2 LOG = - 2 log likelihood

3.9 Comparison of Fixed and Mixed Effects Models

Mixed models consistently yielded a lower AIC and -2 log likelihood value for all equation types for predicting outside bark green weight to any upper merchantable dob, dib or height. This indicates that the mixed models are better predictors for o.b. than the fixed effects models for determining merchantable o.b. green weight to any upper stem o.b. diameter. The difference in predicted values for merchantable o.b. green weight to any upper stem o.b. diameter between the fixed and mixed effects ratio and exponential ratio models using trees from the 25th percentile (D = 7.5 in, H = 66 ft, total green weight = 698 lb), 50th percentile (D = 9.8 in, H = 72 ft, total green weight = 1102 lb), and 75th percentile (D = 11.7 in, H = 71.5 ft, and total green weight = 1610 lb) of the data are illustrated in Figures 4, 5 and 6. While the ratio models for both fixed and mixed effects provide adequate predictions for a portion of the tree, they falsely predict a negative value for merchantable weight at very low heights near the stump and underestimate green weight as diameter decreases. The mixed and fixed effects exponential models both provide better estimates as diameter decreases up the stem with the mixed model prediction line more consistent with the observed data.

For predicting merchantable o.b. green weight to any upper stem i.b. diameter, the AIC and -2 log likelihood statistics were lower for the mixed effects models than the fixed effects models (Table 4). This indicates that the mixed models with the random effects are better predictors than fixed effects models for determining merchantable o.b. green weight to any upper stem i.b. diameter. When predicting merchantable o.b. green weight to any upper i.b. diameter, trees from the 25th percentile (D = 7.5 in, H = 66 ft, total green weight = 698 lb), 50th percentile (D = 9.8 in, H = 72 ft, total green weight = 1102 lb) and 75th percentile (D = 11.7 in, H = 71.5 ft, and total green weight = 1610 lb) of the data were used to illustrate Figures 7, 8 and 9. The trends observed in Figures 8 and 9 are similar to those discussed for Figures 4, 5, and 6 while Figure 7 shows no negative values for the mixed or fixed effects ratio models. Figures 7, 8 and 9 show that the mixed effects exponential ratio models provide better predictions in relation to the observed data.

The AIC and -2 log likelihood statistics for the mixed model used to predict o.b. green weight to any upper height were lower than the fixed effects model (Table 4). This indicates that the mixed effects model is the better predictor when determining merchantable o.b. green weight to any upper stem height. See Table 4 for results comparison. When predicting merchantable o.b. green weight to any upper stem height, Figures 10, 11 and 12 reveal the differences in predicted values for the fixed and mixed effects ratio and exponential ratio models using trees from the 25^{th} percentile (D = 7.5 in, H = 66 ft, total green weight = 698 lb), 50^{th} percentile (D = 9.8 in, H = 72 ft, total green weight = 1102 lb) and 75^{th} percentile (D = 11.7 in, H = 71.5 ft, and total green weight = 1610 lb) of the data. While all figures illustrate a close fit between the fixed and mixed effects ratio and exponential models, it is evident that the mixed effects ratio model maintains a closer relationship to the observed values and reveals that it is the best predictor. Additional figures including the 5^{th} and 95^{th} percentiles are in Appendix 1.

Table 4: Comparison of fit statistics for merchantable green weight equations in fixed effects and mixed effects models.

	FIXED EFFECTS			MIXED EFFECTS		
MODEL FORM	EQ	AIC	-2 LOG	EQ	AIC	-2 LOG
Ratio dob	6	24,715	24,707	13	24,682	24,672
Exp Ratio dob	10	23,822	23,814	16*	23,471	23,461
Ratio dib	7	24,801	24,793	14	24,620	24,610
Exp Ratio dib	11	24,311	24,303	17*	23,946	23,936
Ratio ht	9	19,322	19,324	15*	19,018	19,008
Exp Ratio ht	12	20,529	20,521	18	20,331	20,321

EQ = Equation number

-2 LOG = - 2 log likelihood

***** Indicates best formula between ratio and exponential ratio and between fixed and mixed effects

Figure 4: Predicted merchantable green weight up the stem comparing ratio dob and exponential ratio dob of both fixed and mixed effects models for a tree from the 25th percentile for total green weight with D = 7.5 in, H = 66 ft, total green weight = 698 lb.

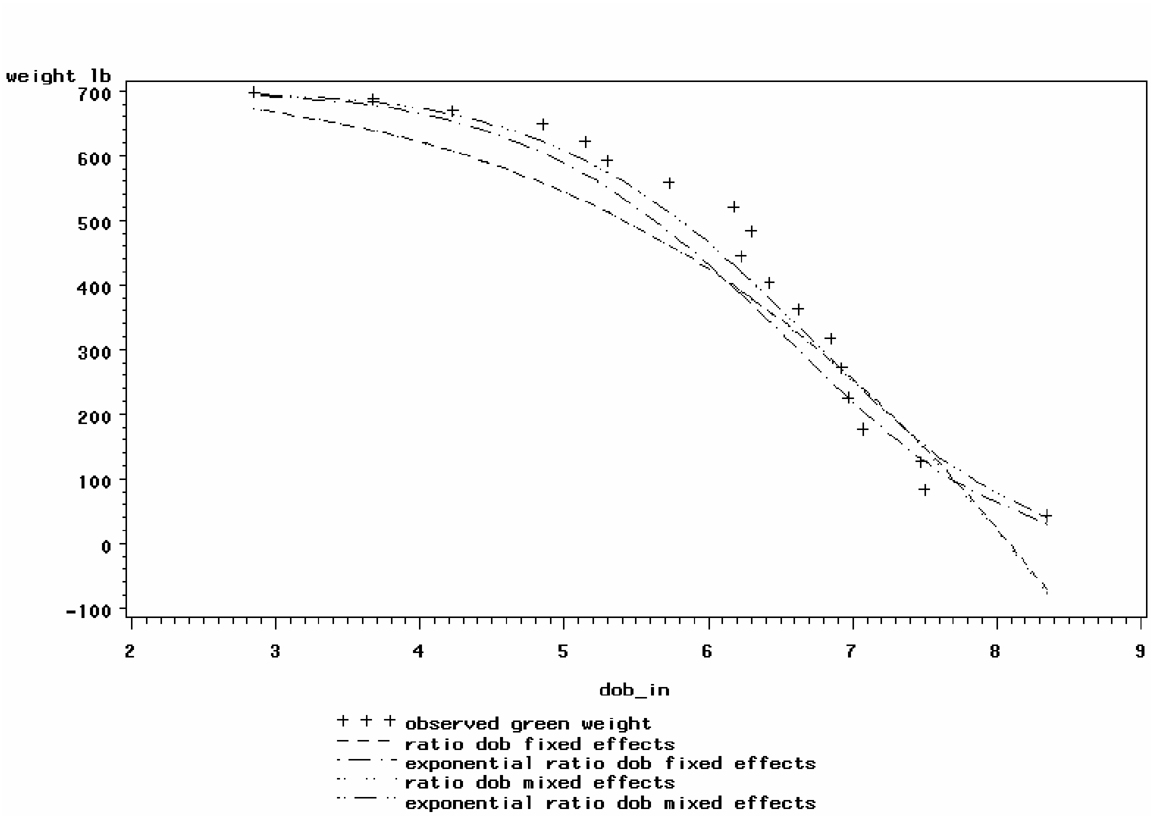


Figure 5: Predicted merchantable green weight up the stem comparing ratio dob and exponential ratio dob of both fixed and mixed effects models for a tree from the 50th percentile for total green weight with D = 9.8 in, H = 72 ft, total green weight = 1102 lb.

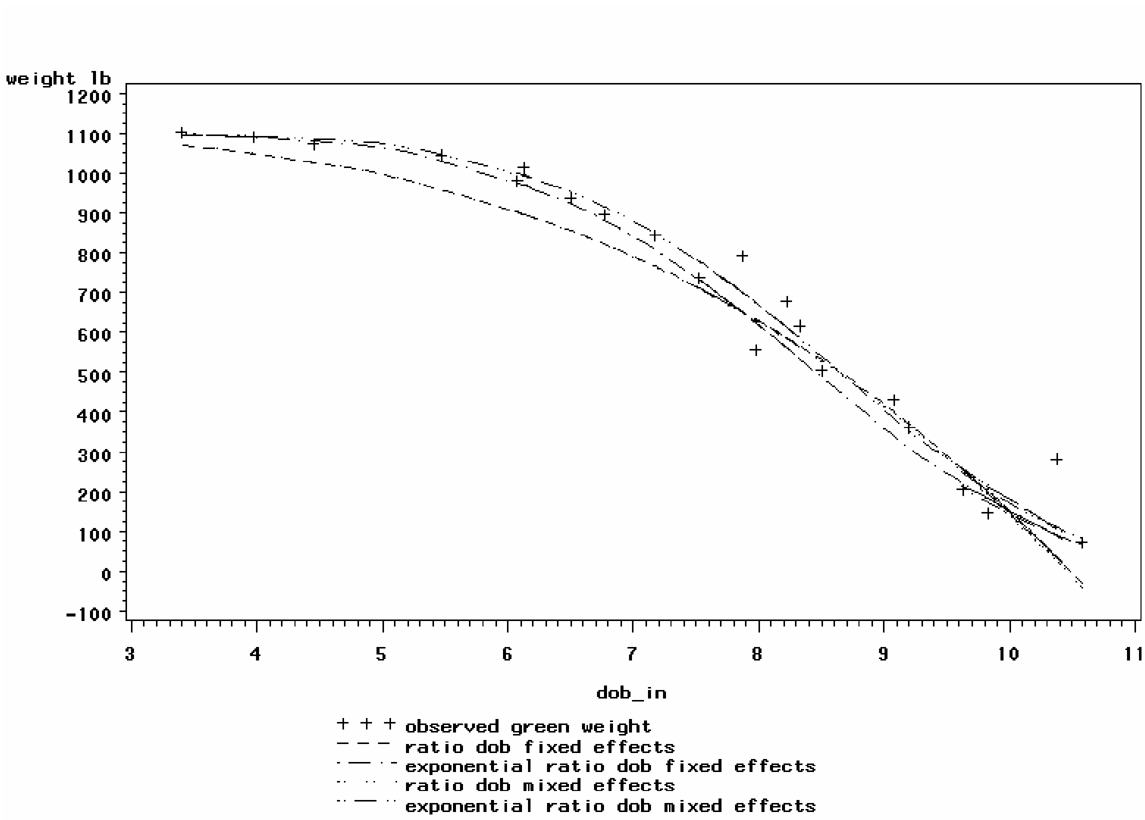


Figure 6: Predicted merchantable green weight up the stem comparing ratio dob and exponential ratio dob of both fixed and mixed effects models for a tree from the 75th percentile for total green weight with D = 11.7 in, H = 71.5 ft, total green weight = 1610.6 lb.

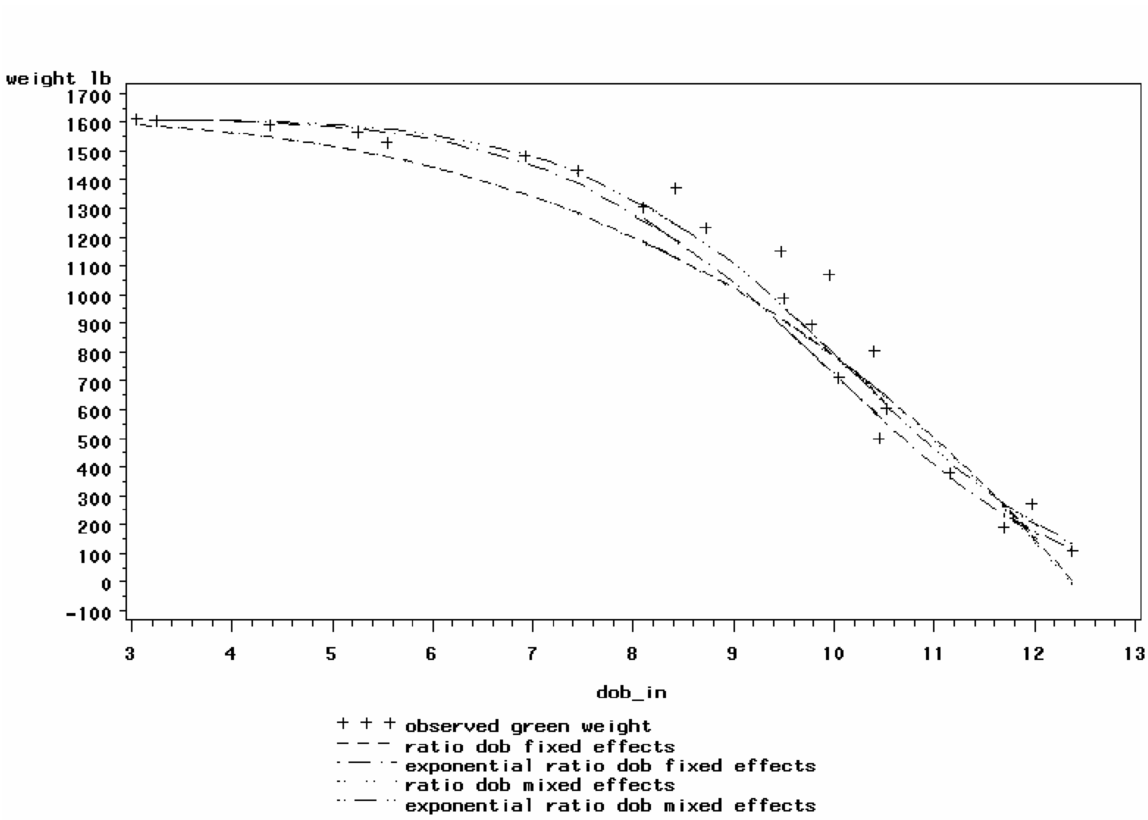


Figure 7: Predicted merchantable green weight up the stem comparing ratio dib and exponential ratio dib of both fixed and mixed effects models for a tree from the 25th percentile for total green weight with D =7.5 in, H = 66 ft, total green weight = 698 lb.

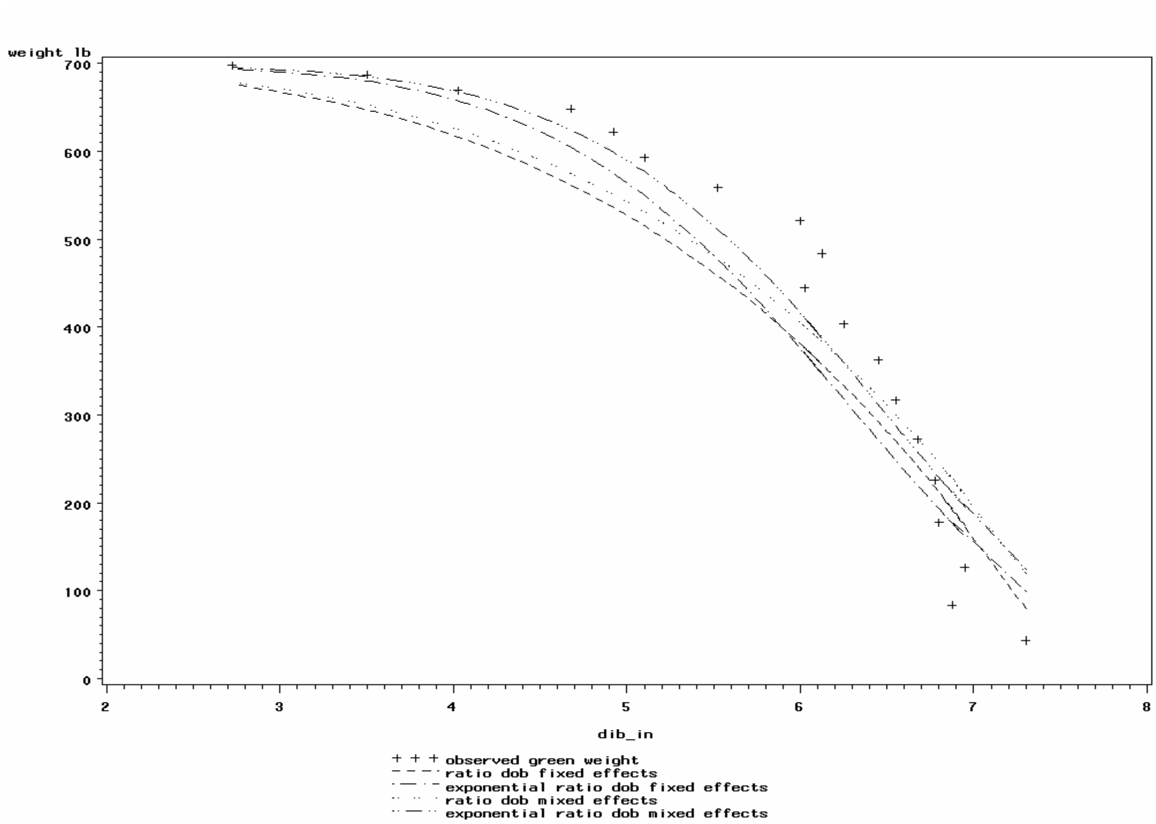


Figure 8: Predicted merchantable green weight up the stem comparing ratio dib and exponential ratio dib of both fixed and mixed effects models for a tree from the 50th percentile for total green weight with D = 9.8 in, H =72 ft, total green weight = 1102 lb.

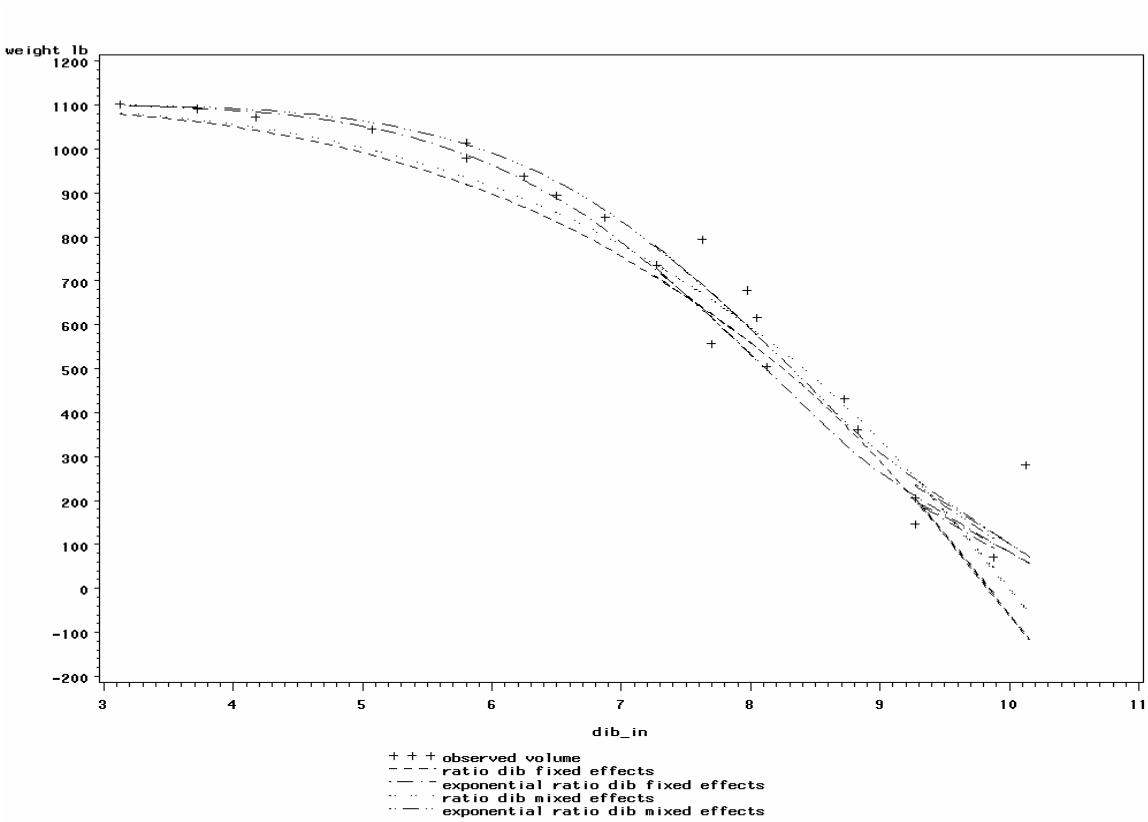


Figure 9: Predicted merchantable green weight up the stem comparing ratio dib and exponential ratio dib of both fixed and mixed effects models for a tree from the 75th percentile for total green weight with D = 11.7 in, H = 71.5 ft, total green weight = 1610 lb.

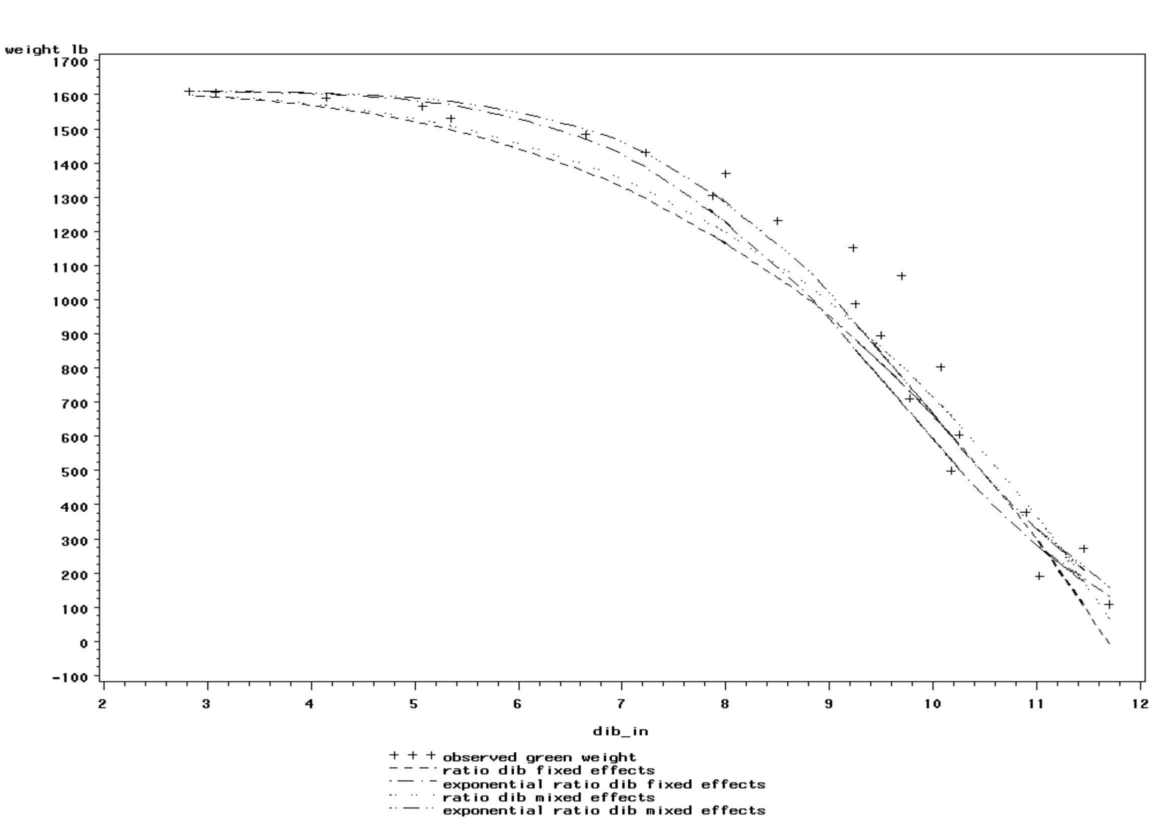


Figure 10: Predicted merchantable green weight up the stem comparing ratio ht and exponential ratio ht of both fixed and mixed effects models for a tree from the 25th percentile for total green weight with D =7.5 in, H= 66 ft, total green weight = 698 lb.

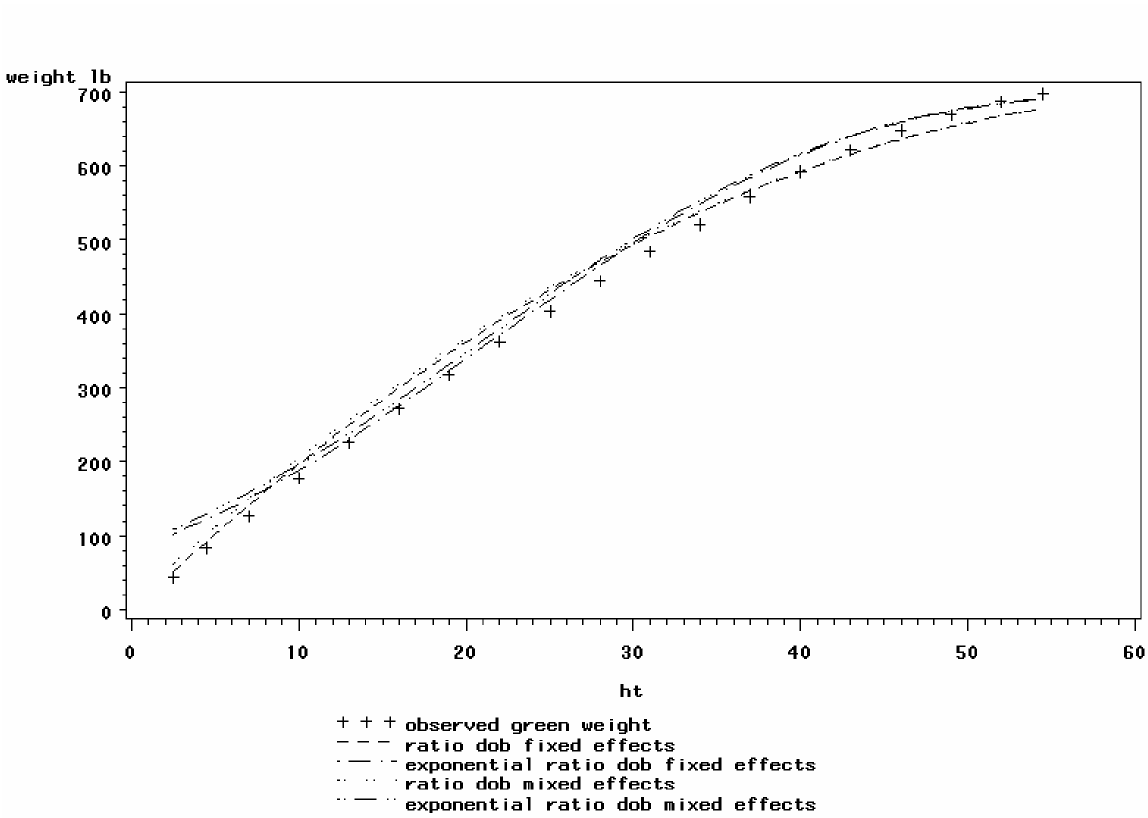


Figure 11: Predicted merchantable green weight up the stem comparing ratio ht and exponential ratio ht of both fixed and mixed effects models for a tree from the 50th percentile for total green weight with D = 9.8 in, H= 72 ft, total green weight = 1102 lb.

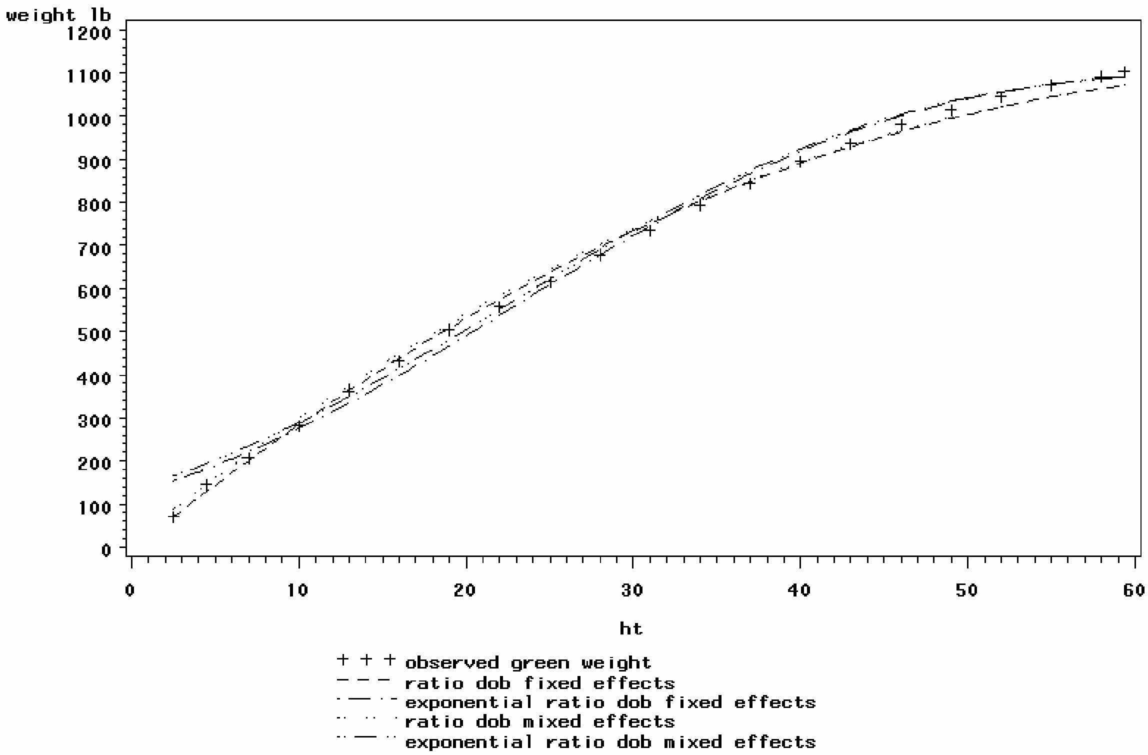
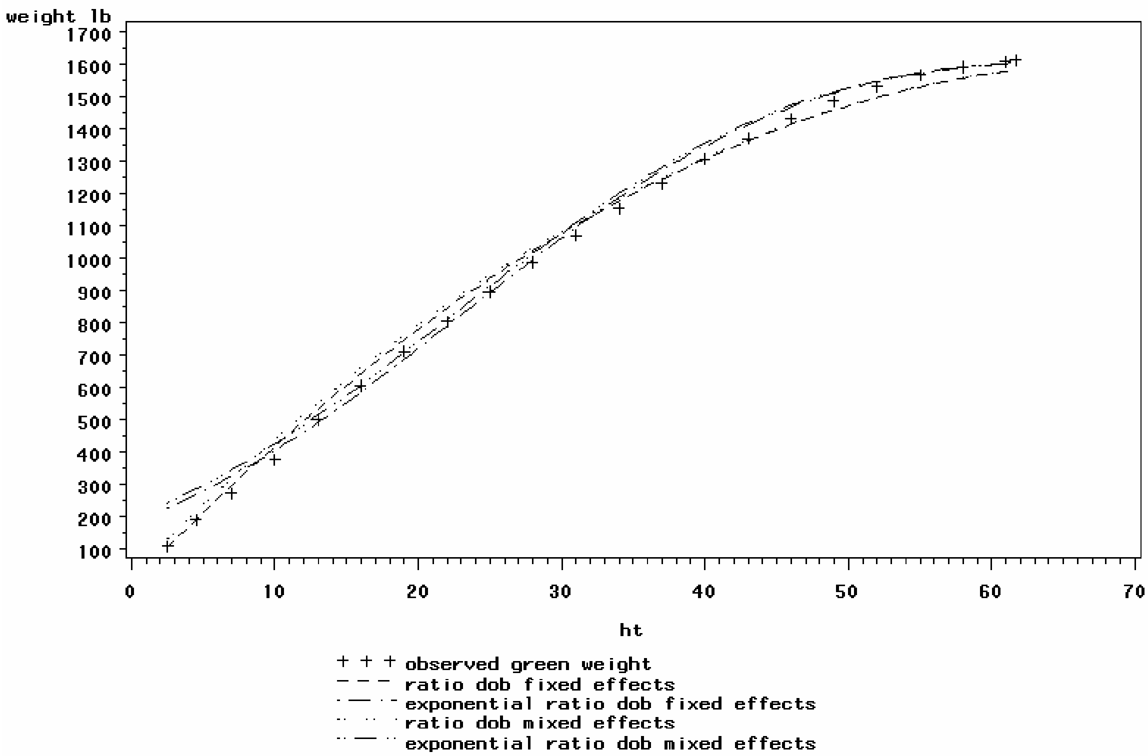


Figure 12: Predicted merchantable green weight up the stem comparing ratio ht and exponential ratio ht of both fixed and mixed effects models for a tree from the 75th percentile for total green weight with D = 11.7 in, H= 71.5 ft, total green weight = 1610 lb.



3.10 Implicit Taper Functions

Amateis and Burkhart (1987) recognized the implied taper relationship between the ratio equations used to predict height to any given diameter or diameter to any given height. Implicit taper functions were obtained by equating fits of Amateis and Burkhart's (1987) height-based and diameter-based merchantable volume equations and solving for the respective value. Tasissa et al. (1997) also observed this relationship and rearranged the exponential equation to yield similar implicit taper functions. Bullock and Burkhart (2003) utilized implicit taper functions when modeling the green weight of loblolly pine. Implicit taper functions provide an additional method for evaluating the fit and performance of the fixed effects ratio and exponential ratio models, while also predicting diameter at any height or height at any diameter. Merchantable green weight equations (Equations 4 and 8, along with 5 and 8), were equated and rearranged producing the following implicit taper functions derived for the ratio form to predict diameter outside bark (dob_r), diameter inside bark (dib_r) and height up the stem from diameter outside bark equations(ht_{ro}) and height up the stem from diameter inside bark equations (ht_{ri}).

$$dob_r = \left\{ \left(\frac{\alpha_1}{\beta_1} \right)^{\frac{1}{\beta_2}} D^{\frac{\beta_3}{\beta_2}} \left[\frac{(H-h)^{\frac{\alpha_2}{\beta_2}}}{H^{\frac{\alpha_3}{\beta_2}}} \right] \right\} + \varepsilon \qquad (19)$$

$$dib_r = \left\{ \left(\frac{\alpha_1}{\beta_4} \right)^{\frac{1}{\beta_5}} D^{\frac{\beta_6}{\beta_5}} \left[\frac{(H-h)^{\frac{\alpha_2}{\beta_5}}}{H^{\frac{\alpha_3}{\beta_5}}} \right] \right\} + \varepsilon \qquad (20)$$

$$ht_{ro} = H - \left\{ \left(\frac{\beta_1}{\alpha_1} \right)^{\frac{1}{\alpha_2}} \left(H^{\frac{\alpha_3}{\alpha_2}} \right) \left(\frac{dob^{\frac{\beta_2}{\alpha_2}}}{D^{\frac{\beta_3}{\alpha_2}}} \right) \right\} + \varepsilon \tag{21}$$

$$ht_{ri} = H - \left\{ \left(\frac{\beta_4}{\alpha_1} \right)^{\frac{1}{\alpha_2}} \left(H^{\frac{\alpha_3}{\alpha_2}} \right) \left(\frac{dib^{\frac{\beta_5}{\alpha_2}}}{D^{\frac{\beta_6}{\alpha_2}}} \right) \right\} + \varepsilon \tag{22}$$

Merchantable green weight equations (Equations 8 and 10, along with 9 and 10), were also equated and rearranged producing the following implicit taper functions derived for the exponential ratio form to predict diameter outside bark (dob_e), diameter inside bark (dib_e), height up the stem from outside bark equations (ht_{eo}), and height up the stem from inside bark equations (ht_{ei}).

$$dob_e = \left\{ \left(\frac{\alpha_4}{\beta_7} \right)^{\frac{1}{\beta_8}} D^{\frac{\beta_9}{\beta_8}} \left[\frac{(H-h)^{\frac{\alpha_5}{\beta_8}}}{H^{\frac{\alpha_6}{\beta_8}}} \right] \right\} + \varepsilon \tag{23}$$

$$dib_e = \left\{ \left(\frac{\alpha_4}{\beta_{10}} \right)^{\frac{1}{\beta_{11}}} D^{\frac{\beta_{12}}{\beta_{11}}} \left[\frac{(H-h)^{\frac{\alpha_5}{\beta_{11}}}}{H^{\frac{\alpha_6}{\beta_{11}}}} \right] \right\} + \varepsilon \tag{24}$$

$$ht_{eo} = H - \left\{ \left(\frac{\beta_7}{\alpha_4} \right)^{\frac{1}{\alpha_5}} \left(H^{\frac{\alpha_6}{\alpha_5}} \right) \left(\frac{dob^{\frac{\beta_8}{\alpha_5}}}{D^{\frac{\beta_9}{\alpha_5}}} \right) \right\} + \varepsilon \tag{25}$$

$$ht_{ei} = H - \left\{ \left(\frac{\beta_{10}}{\alpha_4} \right)^{\frac{1}{\alpha_5}} \left(H^{\frac{\alpha_6}{\alpha_5}} \right) \left(\frac{dib^{\frac{\beta_{11}}{\alpha_5}}}{D^{\frac{\beta_{12}}{\alpha_5}}} \right) \right\} + \varepsilon \tag{26}$$

The β_i and α_i values are obtained from the estimated coefficients for the ratio and exponential ratio models, and are given in Table 2. For example, a Virginia pine with a D of 9.8 in, H of 72 ft and a total green weight of 1102 lb, representing the 50th percentile of the data, was used to develop graphs of predicted diameter outside and inside bark up the stem and predicted height up the stem using diameter outside and inside bark equations. A graph of the predicted diameter outside bark up the stem, from the observed values, ratio and exponential ratio model forms, is presented for comparison in Figure 13. As observed in earlier model comparisons, the recommended exponential ratio model provides a better fit across the range of observed values. A graph of the predicted diameter inside bark up the stem, from the observed values, ratio and exponential ratio model forms, is presented for comparison in Figure 14. As observed in earlier model comparisons, the recommended exponential ratio model provides a better fit across the range of observed values. A graph of predicted height up the stem, derived from diameter outside bark equations, from the ratio and exponential ratio model forms, is presented for comparison in Figure 15. As observed in earlier model comparisons, the recommended ratio model provides a better fit across the range of observed values. A graph of predicted height up the stem, derived from diameter inside bark equations, from the ratio and exponential ratio model forms, is presented for comparison in Figure 16. As observed in earlier model comparisons, the recommended ratio model provides a better fit across the range of observed values.

Figure 13: Predicted diameter outside bark up the stem comparing ratio and exponential ratio fixed effects for a Virginia pine tree with D = 9.8 in, H = 72 ft, total green weight = 1102 lb, representing the 50th percentile of the data for total green weight.

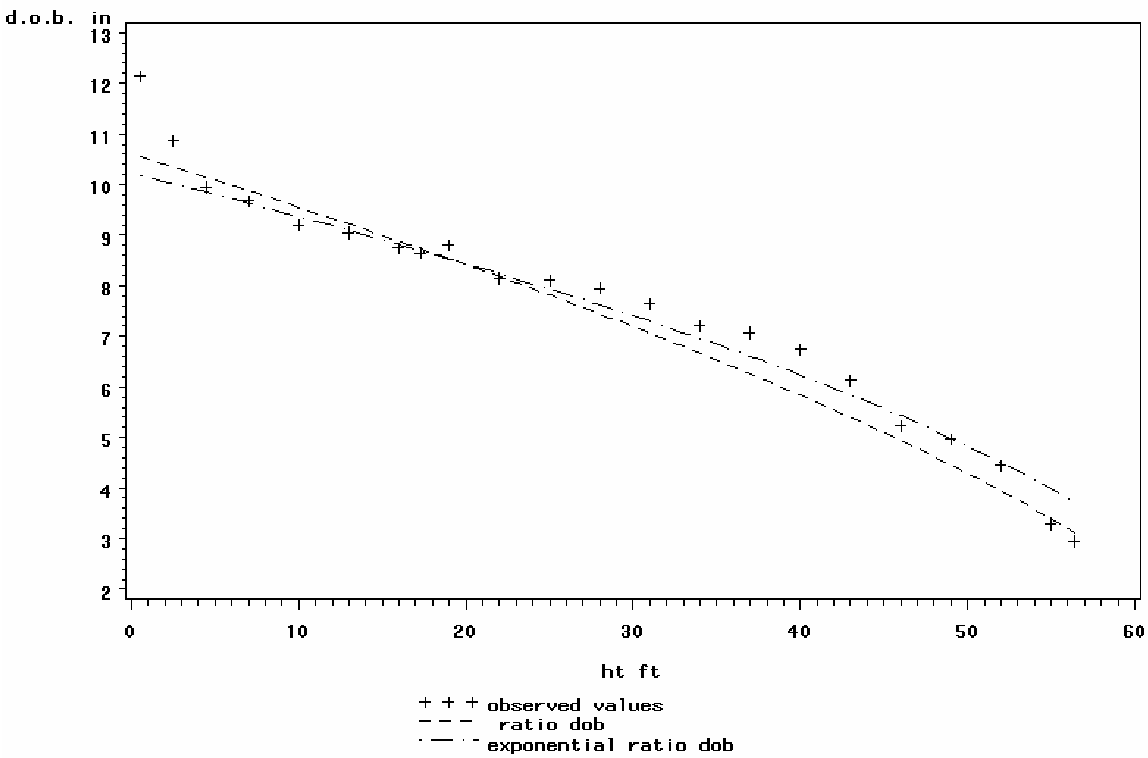


Figure 14: Predicted diameter inside bark up the stem comparing ratio and exponential ratio fixed effects for a Virginia pine tree with a D = 9.8 in, H = 72 ft, total green weight = 1102 lb, representing the 50^{th} percentile of the data for total green weight.

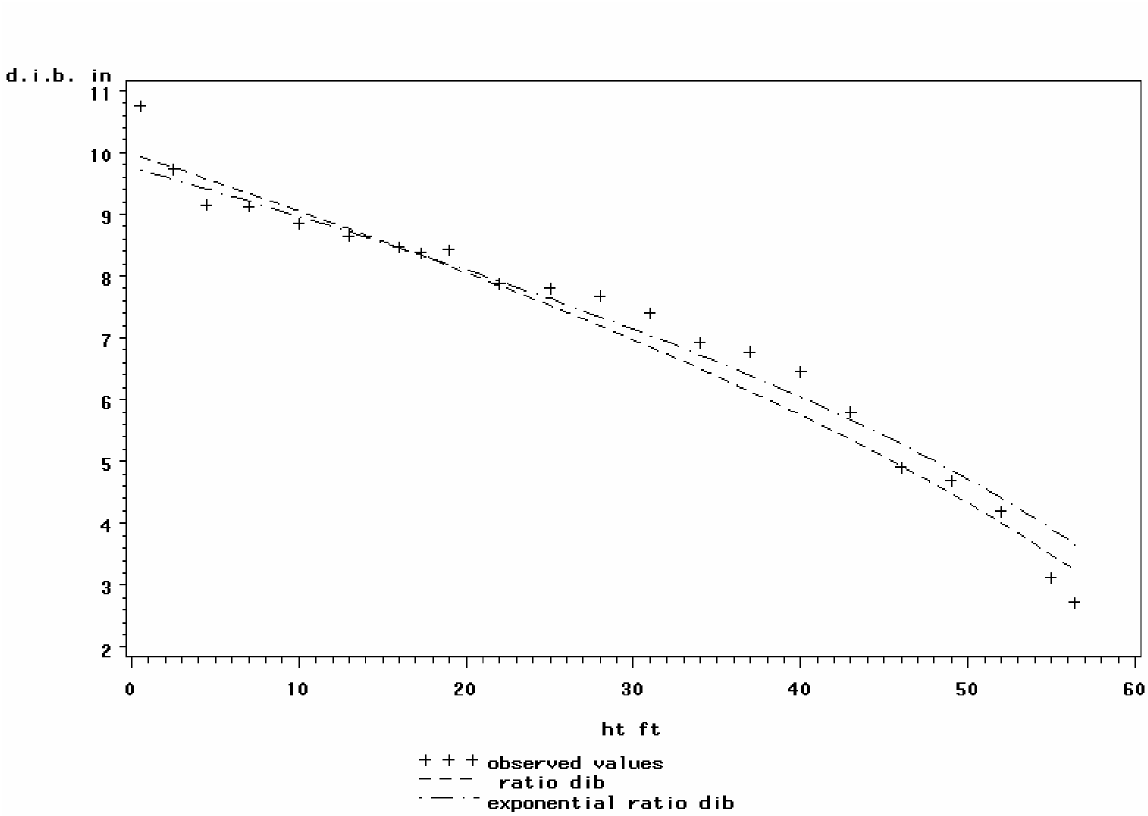


Figure 15: Predicted height up the stem to an approximate 3 in top, diameter outside bark, comparing ratio and exponential ratio fixed effects for a Virginia pine tree with a D = 9.8 in, H = 72 ft, total green weight = 1102 lb, representing the 50th percentile of the data for total green weight.

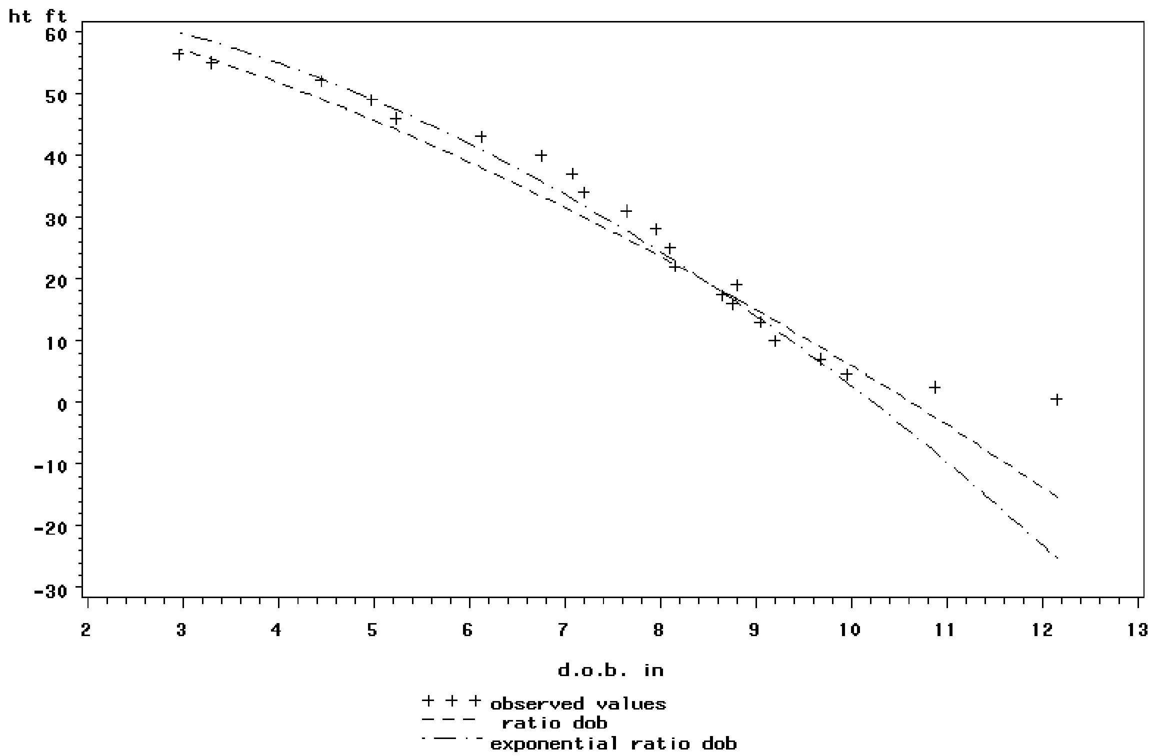
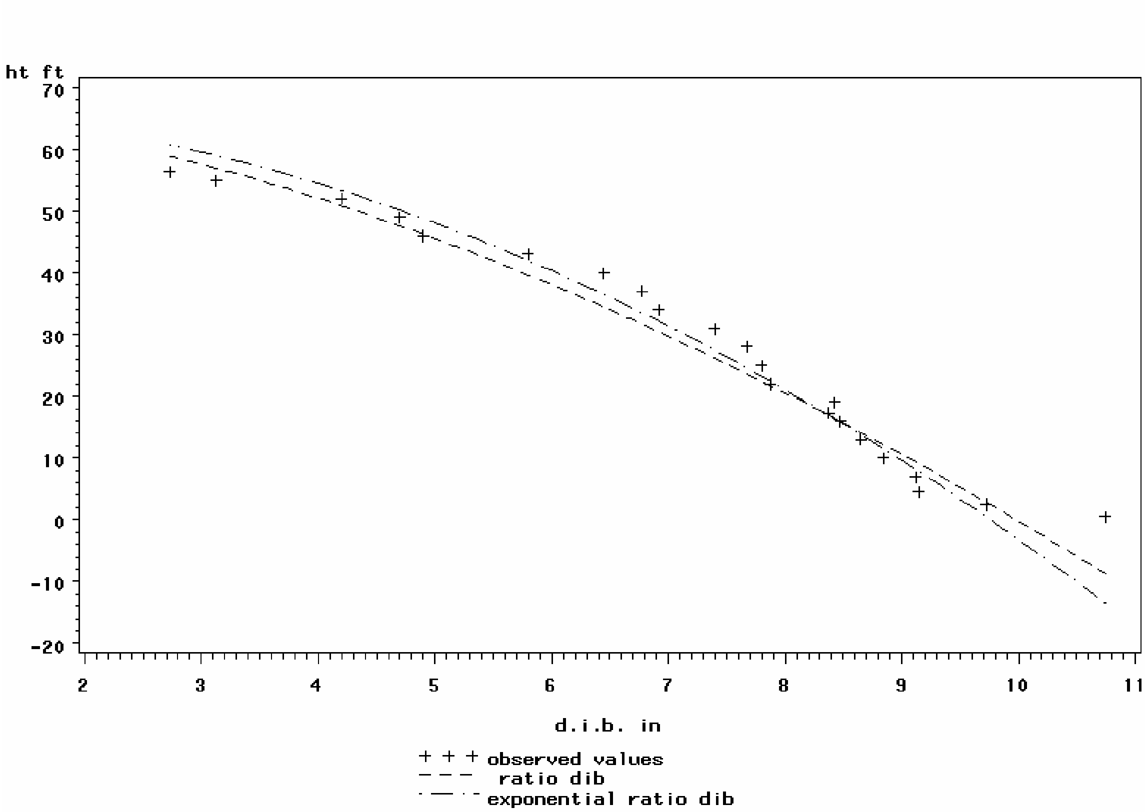


Figure 16: Predicted height up the stem to an approximate 3 in top, diameter inside bark, comparing ratio and exponential ratio fixed effects for a Virginia pine tree with a D = 9.8 in, H = 72 ft, total green weight = 1102 lb, representing the 50th percentile of the data for total green weight.



3.11 Conclusion and Recommendations for Green Weight Model Forms

The purpose of this research was to determine which model, fixed or mixed effects ratio or exponential ratio, would best predict merchantable o.b. green weight to any upper stem diameter (outside or inside bark), or height. Using RSS, RMSE, AIC and -2 log likelihood as standards of model fit for fixed effects models, the exponential ratio models proved to best predict merchantable green weight to any upper stem diameter. Using AIC and -2 log likelihood as standards of model fit for the mixed effects models, the exponential ratio models proved to best predict merchantable o.b. green weight to any upper stem diameter, (outside or inside bark), while the ratio model best predicted merchantable o.b. green weight to any upper stem height.

Overall, the mixed effects exponential ratio model is recommended when predicting merchantable green weight to any upper diameter limit, (o.b. or i.b.) Mixed effects ratio model is recommended for predicting green weight to any upper height.

4. Volume Modeling

4.1 Total Volume Fixed Effects Modeling

With utilization standards changing rapidly for tree stems (Cao and Burkhart 1980) precise estimates of volume to any specified merchantable diameter or height are essential to forest managers. Total tree volume estimates are often required to determine the wood content of standing trees, usually as an intermediate step in the computation of merchantable volume (Tasissa et al. 1997).

Smalian's formula (Avery and Burkhart 2002) was used to determine the cubic ft volume of each bolt (Equations 27). Cumulative volumes were added to obtain a value for the total stem volume to an approximate 3 inch top outside bark diameter and an approximate 3 inch top inside bark diameter.

$$V = \left(\frac{B + b}{2} \right) L \tag{27}$$

where:

V = Volume of bolt, ft^3
B = cross sectional area at bottom of bolt, ft^2
b = cross sectional area at top of bolt, ft^2
L = Length of bolt, ft

Volume for the stem section above a 3 inch top was derived using the conic formula (Equation 28) (Avery and Burkhart 2002). The top section volume was added to the cumulative volume to a 3 inch top to obtain a total volume for the stem.

$$V = \left(\frac{B}{3} \right) L \tag{28}$$

Total volume, inside and outside bark, was estimated using the combined variable equation (Avery and Burkhart 2002), with the total volume estimated as a function of diameter at breast height and total stem height. The full data set of 105 trees was used for volume calculations. Using the linear combined variable equation form presented in Equation 1, the resulting prediction equations for total volume o.b. and i.b. are:

$$\hat{V}_{tot,o.b.} = 1.06307 + 0.002706\left(D^2H\right) \tag{29}$$

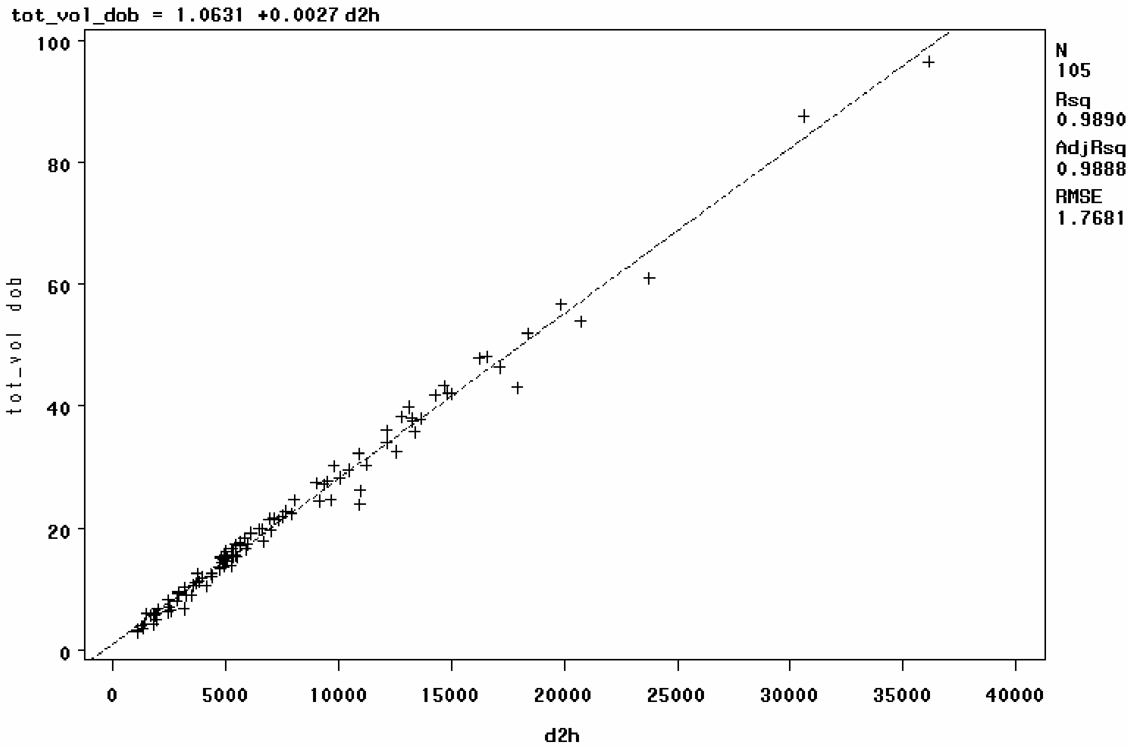
$$\hat{V}_{tot,i.b.} = 0.6812 + 0.002536\left(D^2H\right) \tag{30}$$

where:

$V_{tot,o.b.}$ = total volume outside bark, cu ft.
$V_{tot,i.b.}$ = total volume inside bark, cu ft.

For predicting total volume o.b., the model fit the data well with a coefficient of determination of 0.9890 and a RMSE of 1.768 cu ft. For predicting total volume i.b., the model fit well with a coefficient of determination of 0.9886 and a RMSE of 1.682 cu ft. Graphs of the data and the resulting prediction line show a strong relationship between D^2H and total volume o.b. and total volume i.b. (Figure 17 and Figure 18).

Figure 17: Linear regression of total outside bark volume

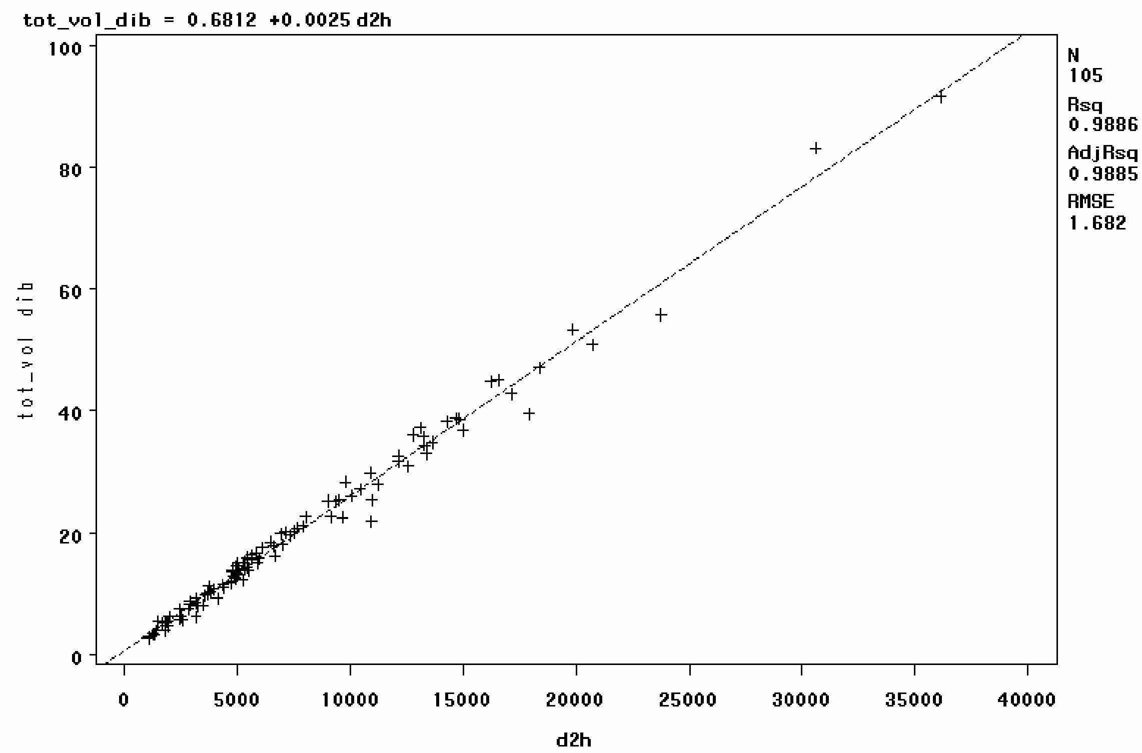


where

tot_vol dob = total o.b. volume, cu ft

d2h = (diameter at breast height)*(total height)

Figure 18: Linear regression of total inside bark volume



where

tot_vol dib = total i.b. volume, cu ft
d2h = (diameter at breast height)*(total height)

4.2 Ratio Form - Fixed Effects

Merchantable o.b and i.b. volume estimates were computed and ratios of merchantable o.b. and i.b. volume to total volume were rewritten from Equation 3 and developed up the stem at height increments previously specified (Equation 31).

$$R_v = \frac{V_{mer}}{V_{tot}} \tag{31}$$

Equations 4 and 5 have been rewritten for volume calculations and are presented below.

$$R_{v,o.b.} = 1 + \chi_1 \left(\frac{dob^{\chi_2}}{D^{\chi_3}} \right) + \varepsilon \tag{32}$$

$$R_{v,i.b.} = 1 + \chi_4 \left(\frac{dib^{\chi_5}}{D^{\chi_6}} \right) + \varepsilon \tag{33}$$

where

χ_i = Coefficients to be estimated $i = 1, \ldots, 6$
$R_{v,o.b.}$ = Ratio of cumulative o.b. volume divided by total o.b. volume
$R_{v,i.b.}$ = Ratio of cumulative i.b. volume divided by total i.b.volume

To determine merchantable o.b and i.b. volume, the ratio form was utilized where merchantable o.b. and i.b. volume is the product of the predicted total o.b. or i.b. volume

(Equation 29 and 30, respectively) multiplied by the predicted ratio. These formulas are presented in Equations 34 and 35.

$$V_{mer,o.b.} = \hat{V}_{tot,o.b.}\left(1 + \chi_7\left(\frac{dob^{\chi_8}}{D^{\chi_9}}\right)\right) + \varepsilon \tag{34}$$

$$V_{mer,i.b.} = \hat{V}_{tot,i.b.}\left(1 + \chi_{10}\left(\frac{dib^{\chi_{11}}}{D^{\chi_{12}}}\right)\right) + \varepsilon \tag{35}$$

where:

$V_{mer,o.b.}$ = Merchantable o.b. volume to any upper diameter o.b., cu ft
$V_{mer,i.b.}$ = Merchantable i.b. volume to any upper diameter i.b., cu ft

Nonlinear regression techniques were used to fit these models (Table 5). Maximum correlation for Equation 34 as indicated by the correlation matrix between variables $\hat{\chi}_8$ and $\hat{\chi}_9$ was 0.88. For Equation 35, the maximum correlation between variables $\hat{\chi}_{11}$ and $\hat{\chi}_{12}$ was 0.91. Though correlation between these coefficients seems high, convergence was not inhibited.

As stated earlier, often the desired merchantable height is known but not the diameter at this height. For this reason, a volume ratio equation, (similar to Equation 9) was fitted using total volume o.b. to determine merchantable outside bark volume to any specified upper stem height and total volume i.b. to determine merchantable inside bark volume to any specified height.

$$V_{h,o.b.} = \hat{V}_{tot,o.b.}\left(1 + \gamma_1\left(\frac{(H-h)^{\gamma_2}}{H^{\gamma_3}}\right)\right) + \varepsilon \tag{36}$$

$$V_{h,i.b.} = \hat{V}_{tot,i.b.}\left(1+\gamma_4\left(\frac{(H-h)^{\gamma_5}}{H^{\gamma_6}}\right)\right)+\varepsilon \qquad (37)$$

where

γ_i = coefficients to be estimated, $i=1,\ldots,6$

$V_{h,o.b.}$ = merchantable outside bark volume to any specified height, cu ft

$V_{h,i.b.}$ = merchantable inside bark volume to any specified height, cu ft

Nonlinear regression techniques were used to fit data to this model (Table 5). Maximum correlation, as indicated by the correlation matrix, between variables $\hat{\gamma}_2$ and $\hat{\gamma}_3$ was 0.56. For merchantable i.b. volume to an upper i.b. stem diameter, maximum correlation as indicated by the correlation matrix, between variables $\hat{\gamma}_5$ and $\hat{\gamma}_6$ was 0.56.

Table 5: Parameter estimates and fit statistics for the fixed effects ratio and exponential ratio equations for predicting merchantable volume to any upper stem diameter or height.

EQ	ESTIMATED COEFFICIENTS			RSS	RMSE	AIC	-2 LOG
34	$\hat{\chi}_7 = -0.6451$	$\hat{\chi}_8 = 3.3419$	$\hat{\chi}_9 = 3.2400$	15,837.7	2.65	10,799	10,791
35	$\hat{\chi}_{10} = -0.9047$	$\hat{\chi}_{11} = 3.6294$	$\hat{\chi}_{12} = 3.5825$	14,972.7	2.57	10,676	10,668
36	$\hat{\gamma}_1 = -1.0530$	$\hat{\gamma}_2 = 2.2552$	$\hat{\gamma}_3 = 2.2619$	692.6	0.554	3,744.8	3,736.8
37	$\hat{\gamma}_4 = -1.282$	$\hat{\gamma}_5 = 2.2489$	$\hat{\gamma}_6 = 2.2689$	614.3	0.522	3,474.9	3,466.9
38	$\hat{\chi}_{13} = -1.0592$	$\hat{\chi}_{14} = 6.1280$	$\hat{\chi}_{15} = 5.8798$	10,348.5	2.14	9,840	9,832
39	$\hat{\chi}_{16} = -1.7391$	$\hat{\chi}_{17} = 6.3587$	$\hat{\chi}_{18} = 6.1939$	11,827.2	2.29	10,145	10,137
40	$\hat{\gamma}_7 = -2.0008$	$\hat{\gamma}_8 = 3.3780$	$\hat{\gamma}_9 = 3.3528$	1,878.2	0.913	5,993.4	5,985.4
41	$\hat{\gamma}_{10} = -2.2831$	$\hat{\gamma}_{11} = 3.4024$	$\hat{\gamma}_{12} = 3.4007$	1,618.0	0.847	5,685.9	5,650.9

EQ = Equation number
RSS = Residual Sum of Squares
RMSE = Root Mean Square Error, ft^3
AIC = Akaike's Information Criterion (lower number is better)
-2 LOG = Minus 2 log likelihood

4.3 Exponential Ratio Form - Fixed Effects

While Burkhart's (1977) formula may be acceptable in determining merchantable volume to upper stem diameters and heights of practical use, illogical volume estimates may be given using the ratio form equation for lower merchantable heights. Exponential models are constrained by zero and one such that the ratio equals one as the upper diameter reaches zero (predicted volume equals total volume) and the ratio goes to zero as the lower diameter limit goes to infinity (predicted volume goes to zero).

The exponential ratio model presented by Tasissa et al. (1997) was estimated for the data using nonlinear regression techniques. The merchantable o.b. volume to any upper o.b. stem diameter is predicted in Equation 38 while equation 39 predicts the merchantable i.b. volume to any upper i.b. stem diameter. Estimated coefficients are listed in Table 5.

$$V_{mer,o.b.} = \hat{V}_{tot,o.b.}\left(\exp\left(\chi_{13}\left(\frac{dob^{\chi_{14}}}{D^{\chi_{15}}}\right)\right)\right) + \varepsilon \qquad (38)$$

$$V_{mer,i.b.} = \hat{V}_{tot,i.b.}\left(\exp\left(\chi_{16}\left(\frac{dib^{\chi_{17}}}{D^{\chi_{18}}}\right)\right)\right) + \varepsilon \qquad (39)$$

Maximum correlation for merchantable o.b. volume as indicated by the correlation matrix between variables $\hat{\chi}_{14}$ and $\hat{\chi}_{15}$ was 0.92. For merchantable i.b. volume, maximum correlation between variables $\hat{\chi}_{17}$ and $\hat{\chi}_{18}$ was 0.93. Though correlation between these coefficients seems high, convergence was not inhibited.

As with the ratio model, when desired height is known, a merchantable volume equation, (similar in form to Equation 12), was fitted to determine merchantable o.b. volume and merchantable i.b. volume to any specified height.

$$V_{h,o.b.} = \hat{V}_{tot,o.b.}\left(\exp\left(\gamma_7\left(\frac{(H-h)^{\gamma_8}}{H^{\gamma_9}}\right)\right)\right) + \varepsilon \tag{40}$$

$$V_{h,i.b.} = \hat{V}_{tot,i.b.}\left(\exp\left(\gamma_{10}\left(\frac{(H-h)^{\gamma_{11}}}{H^{\gamma_{12}}}\right)\right)\right) + \varepsilon \tag{41}$$

Nonlinear regression techniques were used to fit data (Table 5). Maximum correlation as indicated by the correlation matrix between variables $\hat{\gamma}_8$ and $\hat{\gamma}_9$ was 0.52. For merchantable i.b. volume to any upper stem height, the maximum correlation as indicated by the correlation matrix between variables $\hat{\gamma}_{11}$ and $\hat{\gamma}_{12}$ was 0.52.

4.4 Comparison of Fixed Effects Ratio and Exponential Ratio Form Equations

Residual sum of squares, root mean square error, AIC and -2 log likelihood statistics were used to determine the best fit in comparing ratio and exponential ratio equations for predicting the merchantable (o.b. or i.b.) volume of Virginia pine to any upper stem diameter or height. Since the exponential ratio RSS is 35% lower than the ratio model, it suggest this model (Equation 38) will deliver the most accurate estimate of merchantable o.b. volume. The RMSE, AIC and -2 log likelihood values were all lower for the exponential ratio model giving further validity to the results. For merchantable i.b. volume to any upper i.b. stem diameter, the RSS for the exponential ratio model was 22% lower than the ratio model for merchantable i.b. volume, suggesting this model (Equation 39), will deliver the most accurate estimate of merchantable i.b. volume to any upper stem diameter. The RMSE, AIC and -2 log likelihood were all lower for the exponential ratio model further validating the results.

When using any upper height limit as a predictor of merchantable o.b. volume, the ratio model produced a RSS 73% lower than the exponential ratio model. This indicates that the ratio model (Equation 36), is a better predictor of Virginia pine merchantable o.b. volume to any upper height limit. The RMSE, AIC and -2 log likelihood were all lower for the ratio model further validating the results.

For predicting merchantable i.b. volume to any upper stem height, the ratio equation produced an RSS value 72% lower than the exponential ratio model. This indicates the ratio model (Equation 37) is a better model for predicting merchantable i.b. volume to an upper height limit. The RMSE, AIC and -2 log likelihood values were all lower for the ratio model giving further validity to the results.

4.5 Volume - Mixed Effects Modeling

Thought was given to the problem of serial correlation which is known to exist in tree volume ratio models between successive observations (Reams 1994). Mixed effects modeling includes the addition of a random effect variable to account for the natural variation and correlation within individual tree stems. Previous equations were modified with the addition of a random effect variable. Model fit was assessed using the AIC and -2 log likelihood statistics.

4.6 Ratio Form - Mixed Effects

The ratio volume equations (o.b. and i.b.) previously fit (Equations 34 and 35), were modified with the addition of a random effect variable to account for natural variation within individual stems and are presented below:

$$V_{mer,o.b.} = \hat{V}_{tot,o.b.}\left(1+\left(\left(\chi_{19}+u_7\right)\left(\frac{dob^{\chi_{20}}}{D^{\chi_{21}}}\right)\right)\right)+\varepsilon \quad (42)$$

$$V_{mer,i.b.} = \hat{V}_{tot,i.b.}\left(1+\left(\left(\chi_{22}+u_8\right)\left(\frac{dib^{\chi_{23}}}{D^{\chi_{24}}}\right)\right)\right)+\varepsilon \quad (43)$$

where:

u_i = random effect variable; *i*=7,8

Parameters and fit were estimated for this model (Table 6).

Cao and Burkhart's (1980) formulas (Equations 36 and 37, respectively) for determining merchantable volume, (o.b. or i.b., respectively) to any specified upper stem height were also modified with the addition of a random effect variable and are presented in Equations 44 and 45.

$$V_{h,o.b.} = \hat{V}_{tot,o.b.}\left(1+\left(\left(\gamma_{13}+u_9\right)\left(\frac{\left(H-h\right)^{\gamma_{14}}}{H^{\gamma_{15}}}\right)\right)\right)+\varepsilon \quad (44)$$

$$V_{h,i.b.} = \hat{V}_{tot,i.b.}\left(1+\left(\left(\gamma_{16}+u_{10}\right)\left(\frac{\left(H-h\right)^{\gamma_{17}}}{H^{\gamma_{18}}}\right)\right)\right)+\varepsilon \quad (45)$$

Parameters and fit were estimated for this model (Table 6).

Table 6: Parameter estimates and fit statistics for the mixed effects ratio and exponential ratio equations for predicting merchantable volume to any upper stem diameter or height.

EQ	ESTIMATED COEFFICIENTS				AIC	-2 LOG
41	$\hat{\chi}_{19} = -0.6330$	$\hat{\chi}_{20} = 3.3600$	$\hat{\chi}_{21} = 3.2477$	$\hat{u}_{7} = 0.02822$	10,751	10,741
42	$\hat{\chi}_{22} = -0.8330$	$\hat{\chi}_{23} = 3.7215$	$\hat{\chi}_{24} = 3.6289$	$\hat{u}_{8} = -0.06313$	10,438	10,428
43	$\hat{\gamma}_{13} = -1.1959$	$\hat{\gamma}_{14} = 2.2536$	$\hat{\gamma}_{15} = 2.2891$	$\hat{u}_{9} = 0.02623$	3,250	3,240
44	$\hat{\gamma}_{16} = -1.2780$	$\hat{\gamma}_{17} = 2.2472$	$\hat{\gamma}_{18} = 2.2956$	$\hat{u}_{10} = 0.0262$	3,072	3,062
45	$\hat{\chi}_{25} = -0.9978$	$\hat{\chi}_{26} = 6.4699$	$\hat{\chi}_{27} = 6.1584$	$\hat{u}_{11} = 0.1626$	9,424	9,414
46	$\hat{\chi}_{28} = -1.5772$	$\hat{\chi}_{29} = 6.8035$	$\hat{\chi}_{30} = 6.5446$	$\hat{u}_{12} = 0.2865$	9,634	9,624
47	$\hat{\gamma}_{19} = -2.6824$	$\hat{\gamma}_{20} = 3.3787$	$\hat{\gamma}_{21} = 3.4196$	$\hat{u}_{13} = -0.1123$	5,823	5,813
48	$\hat{\gamma}_{22} = -3.0679$	$\hat{\gamma}_{23} = 3.4032$	$\hat{\gamma}_{24} = 3.4684$	$\hat{u}_{14} = 0.1268$	5506	5,496

EQ = Equation number
$u_i =$ random effect representing variation between individual stems
AIC = Akaike's Information Criterion (lower number is better)
-2 LOG = Minus 2 log likelihood

4.7 Exponential Ratio Form – Mixed Effects

Exponential ratio form models (Equations 38 and 39) were modified with the inclusion of a random effect variable and are listed below:

$$V_{mer,o.b.} = \hat{V}_{tot,o.b.}\left(\exp\left(\left(\chi_{25} + u_{11}\right)\left(\frac{dob^{\chi_{26}}}{D^{\chi_{27}}}\right)\right)\right) + \varepsilon \tag{46}$$

$$V_{mer,i.b.} = \hat{V}_{tot,i.b.}\left(\exp\left(\left(\chi_{28} + u_{12}\right)\left(\frac{dib^{\chi_{29}}}{D^{\chi_{30}}}\right)\right)\right) + \varepsilon \tag{47}$$

Parameters and fit were estimated for this model (Table 6).

As with the ratio model, when desired height is known, Equations 48 and 49 were fitted to determine merchantable o.b. volume and merchantable i.b. volume to any specified height.

$$V_{h,o.b.} = \hat{V}_{tot,o.b.}\left(\exp\left(\left(\gamma_{19} + u_{13}\right)\left(\frac{\left(H-h\right)^{\gamma_{20}}}{H^{\gamma_{21}}}\right)\right)\right) + \varepsilon \tag{48}$$

$$V_{h,i.b.} = \hat{V}_{tot,i.b.}\left(\exp\left(\left(\gamma_{22} + u_{14}\right)\left(\frac{\left(H-h\right)^{\gamma_{23}}}{H^{\gamma_{24}}}\right)\right)\right) + \varepsilon \tag{49}$$

Parameters and fit were estimated for this model (Table 6).

4.8 Comparison of Mixed Effects Ratio and Exponential Ratio Form Equations

Ranking of the ratio and exponential ratio form equations for determining merchantable volume (o.b. or i.b.) to any upper stem diameter or height was the same as the mixed effects model forms for predicting green weight to any merchantable upper stem diameter or height. For merchantable o.b. volume, the AIC and -2 log likelihood for the exponential ratio model was 12% lower than the ratio equations. These fit statistics indicate that the exponential ratio model is a better predictor when determining merchantable outside bark volume to any upper stem diameter. The AIC and -2 log likelihood for the exponential ratio model for merchantable i.b. volume was 8% lower than the ratio equation. This indicates that the exponential model is a better predictor for merchantable i.b. volume to any upper diameter. For merchantable o.b. volume to any upper o.b. stem height, ratio equations yielded 47% lower AIC and -2 log likelihood valued than the exponential ratio equations. This indicates that the ratio model is a better predictor for determining merchantable o.b. volume to any upper o.b. stem height. AIC and -2 log likelihood ratio model values for merchantable inside bark volume to any upper i.b. height were 45% lower than the exponential ratio model. These lower values indicate that the ratio model equation is the better predictor for determining merchantable inside bark volume to any upper i.b. stem height. Comparison results listed in Table 6.

4.9 Comparison of Fixed and Mixed Effects Models

Mixed models consistently yielded a lower AIC and -2 log likelihood value for all equation types for predicting volume to any upper stem diameter or height. This indicates that the mixed models with the random effects terms are better predictors than the fixed effect models. The difference in predicted values for merchantable o.b. volume to any upper diameter outside bark between the fixed and mixed effects ratio and exponential ratio models are illustrated in Figures 19 20 and 21 using trees from the 25^{th} percentile (D = 7.5, H = 66.2), 50^{th} percentile (D = 9, H = 68), and the 75^{th} percentile (D = 12.8, H = 73.8). While the ratio models for both fixed and mixed effects provide adequate predictions for a portion of the tree, they falsely predict a negative value for merchantable volume in Figure 20 at very low heights near the stump and underestimate volume as diameter decreases near the top of the stem. The exponential ratio models provide better estimates as diameter decreases up the stem with the mixed effects exponential ratio model prediction line more consistent with the observed data.

For predicting merchantable i.b. volume to any upper stem diameter i.b., the AIC and -2 log likelihood fit statistics were lower than the fixed effects models (Table 7). This indicates that the mixed models with the random effects term are better predictors than the fixed effect models. Using trees from the 25th percentile (D = 7.5, H = 66.2), 50th percentile (D = 9, H = 68), and the 75th percentile (D = 12.8, H = 73.8), Figures 22, 23, and 24 show similar results as Figures 19, 20, and 21, with Figures 23 and 24 ratio models falsely predicting a negative value for merchantable i.b. volume and the mixed effects exponential ratio model providing a better estimate as diameter decreases up the stem in relation to the observed data.

The AIC and -2 log likelihood fit statistics for the mixed model in regards to predicting merchantable o.b. volume to any upper height were lower than the fixed effects model (Table 7). When determining merchantable o.b. volume to any upper stem height, Figures 25, 26, and 27, using trees from the 25th percentile (D = 7.5, H = 66.2), 50th percentile (D = 9, H = 68), and the 75th percentile (D = 12.8, H = 73.8), show a close relationship to the original data between all models. The mixed effects ratio model, however, shows the closest prediction line in relation to the observed data.

The ranking of the models is the same when predicting merchantable i.b. volume to any upper stem height. The mixed effects AIC and -2 log likelihood fit statistics were lower for merchantable i.b. volume to any upper height (Table 7), thus indicating the mixed effects models are the better predictors. Similar to the merchantable o.b. volume to any upper height (Figures 25, 26, and 27), Figures 28, 29, and 30 show a close relationship between all models with the mixed effects ratio model providing the closest prediction to the original data using trees from the 25th percentile (D = 7.5, H = 66.2), 50th percentile (D = 9, H = 68), and the 75th percentile (D = 12.8, H = 73.8). Additional figures including the 5th and 95th percentile are in Appendix 1.

Table 7: Comparison of fixed effects and mixed effects models for determining merchantable volume to any upper stem diameter outside bark, inside bark, or height up the stem.

	FIXED EFFECTS			MIXED EFFECTS		
MODEL FORM	EQ	AIC	-2 LOG	EQ	AIC	-2 LOG
Ratio dob	33	10,799	10,791	41	10,751	10,741
Exp Ratio dob	37	9,840	9,832	45*	9,424.2	9,414
Ratio dib	34	10,676	10,668	42	10,438	10,428
Exp Ratio dib	38	10,145	10,137	46*	9,633.8	9,624
Ratio ht dob	35	3,744.8	3,736.8	43*	3,250.1	3,240
ExpRatiohtdob	39	5,993.4	5,985.4	47	5,822.5	5,813
Ratio ht dib	36	3,474.9	3,466.9	44*	3,072	3,062
ExpRatioht dib	40	5,658.9	5,650.9	48	5,506.1	5,496

EQ = equation number

- 2 LOG = - 2 log likelihood

* indicates best formula between ratio and exponential ratio and between fixed and mixed effects models

Figure 19: Predicted merchantable o.b. volume up the stem to an upper diameter o.b. comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D=7.5 in and H = 66.2 ft, representing the 25th percentile of the data for total volume.

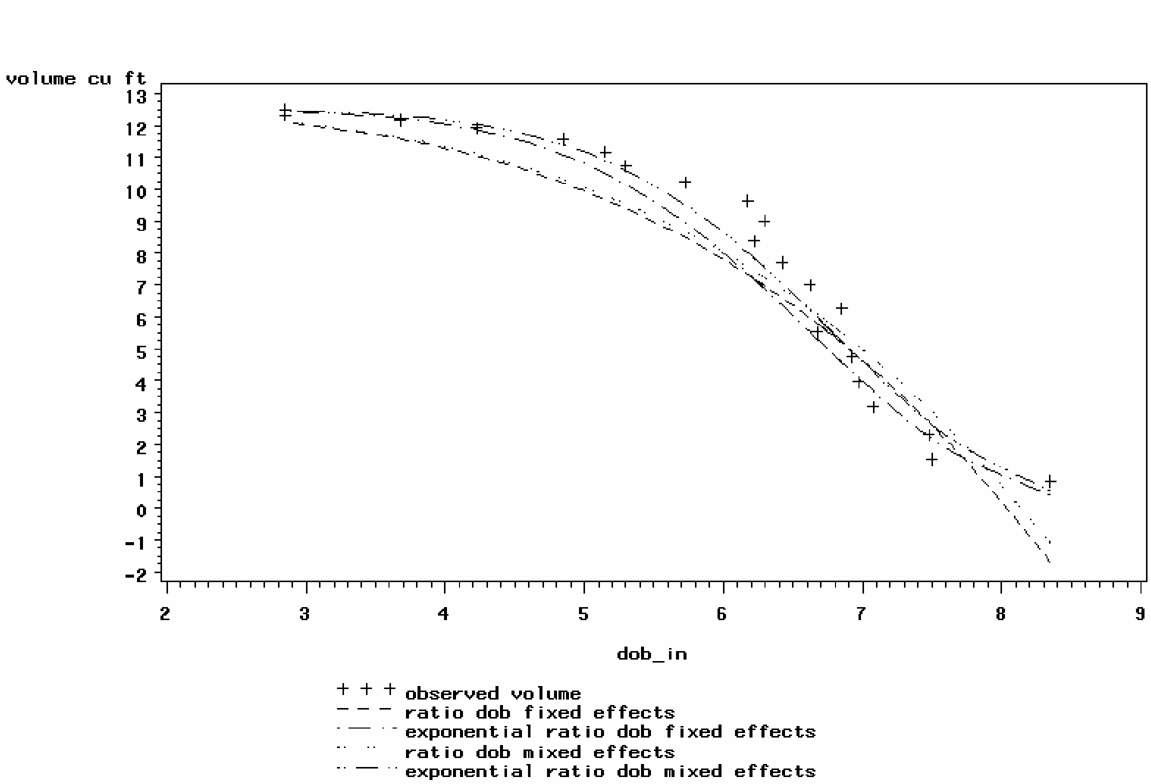


Figure 20: Predicted merchantable o.b. volume up the stem to an upper diameter o.b. comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D=9 in and H = 68 ft, representing the 50th percentile of the data for total volume.

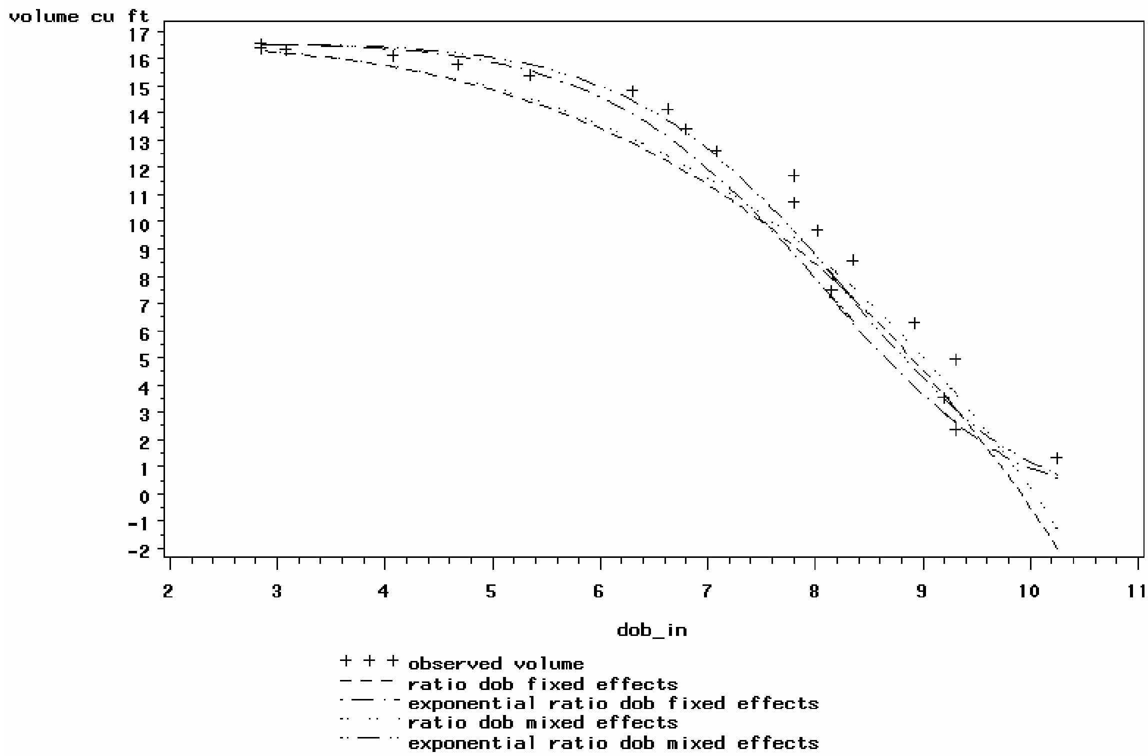


Figure 21: Predicted merchantable o.b. volume up the stem to an upper diameter o.b. comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D = 12.1 in and H = 70.4 ft, representing the 75th percentile of the data for total volume.

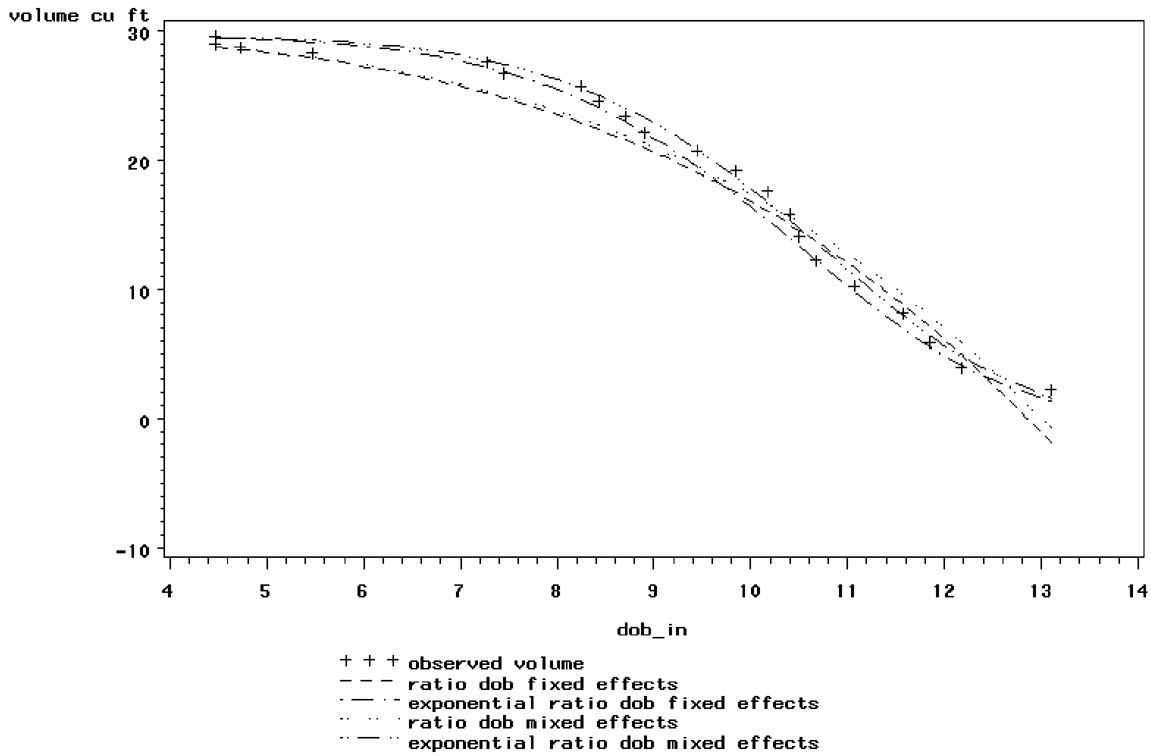


Figure 22: Predicted merchantable i.b. volume up the stem to an upper diameter i.b. comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D = 7.5 in and H = 66.2 ft, representing the 25th percentile of the data for total volume.

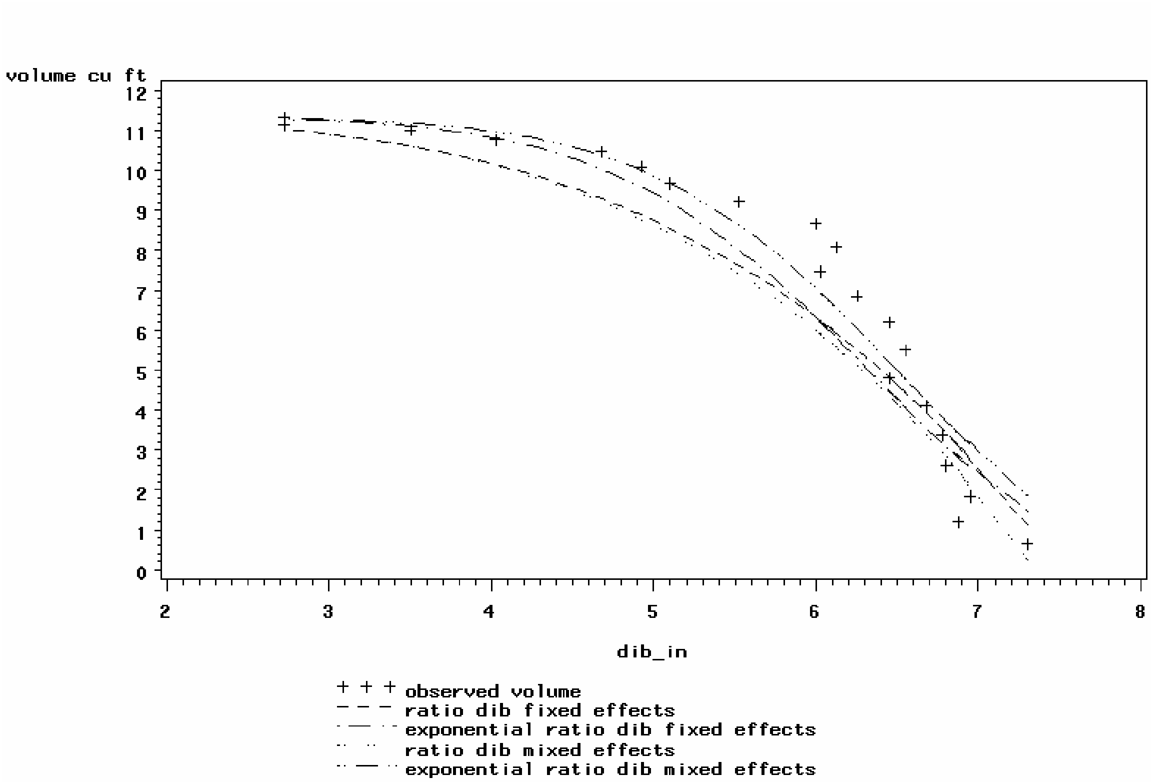


Figure 23: Predicted merchantable i.b. volume up the stem to an upper diameter i.b. comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D=9 in and H = 68 ft, representing the 50th percentile of the data for total volume.

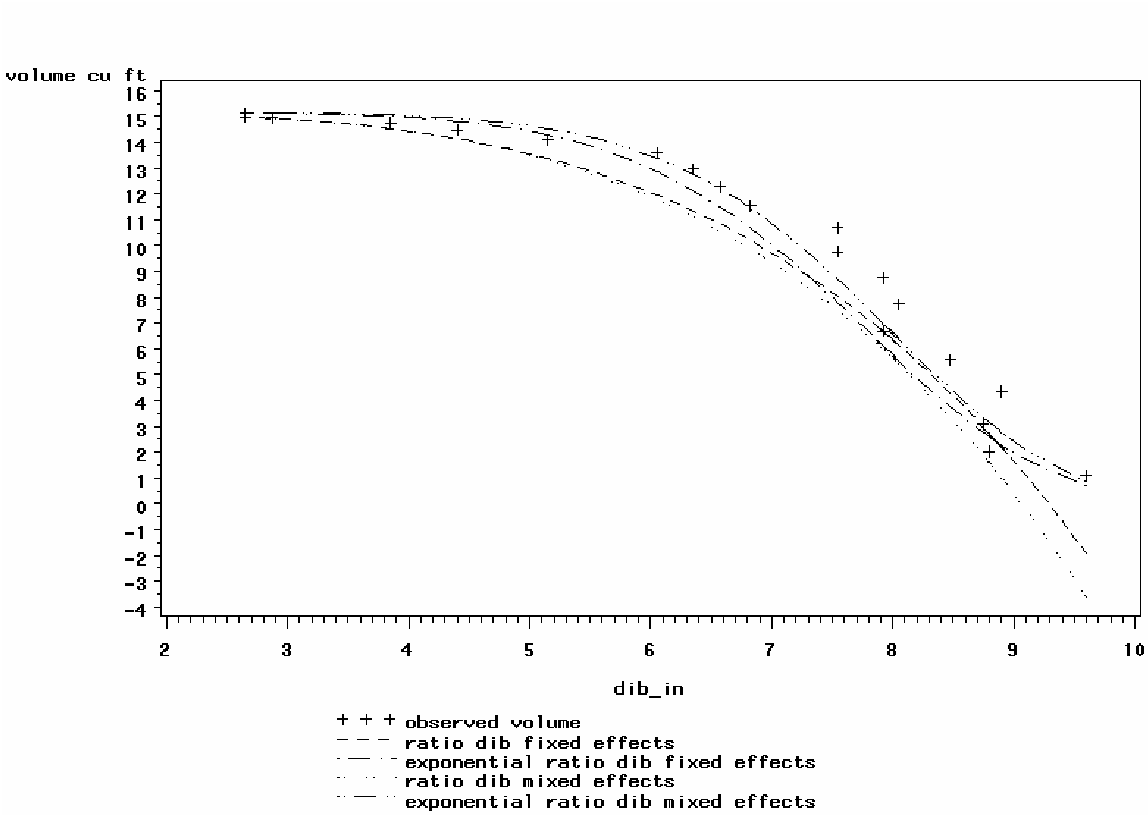


Figure 24: Predicted merchantable i.b. volume up the stem to an upper diameter i.b. comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D=12.2 in and H = 70.4 ft, representing the 75th percentile of the data for total volume

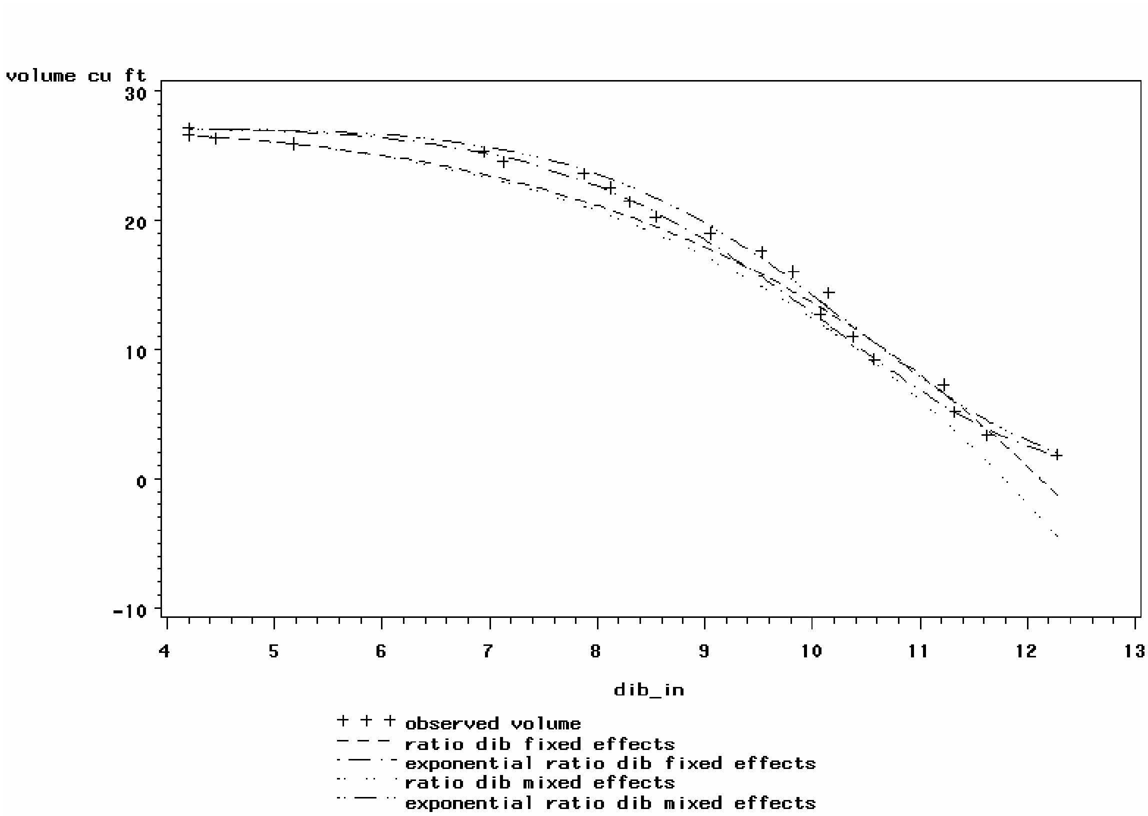


Figure 25: Predicted merchantable o.b. volume up the stem to an upper height comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D = 7.5 in and H = 66.2 ft, representing the 25th percentile of the data for total volume.

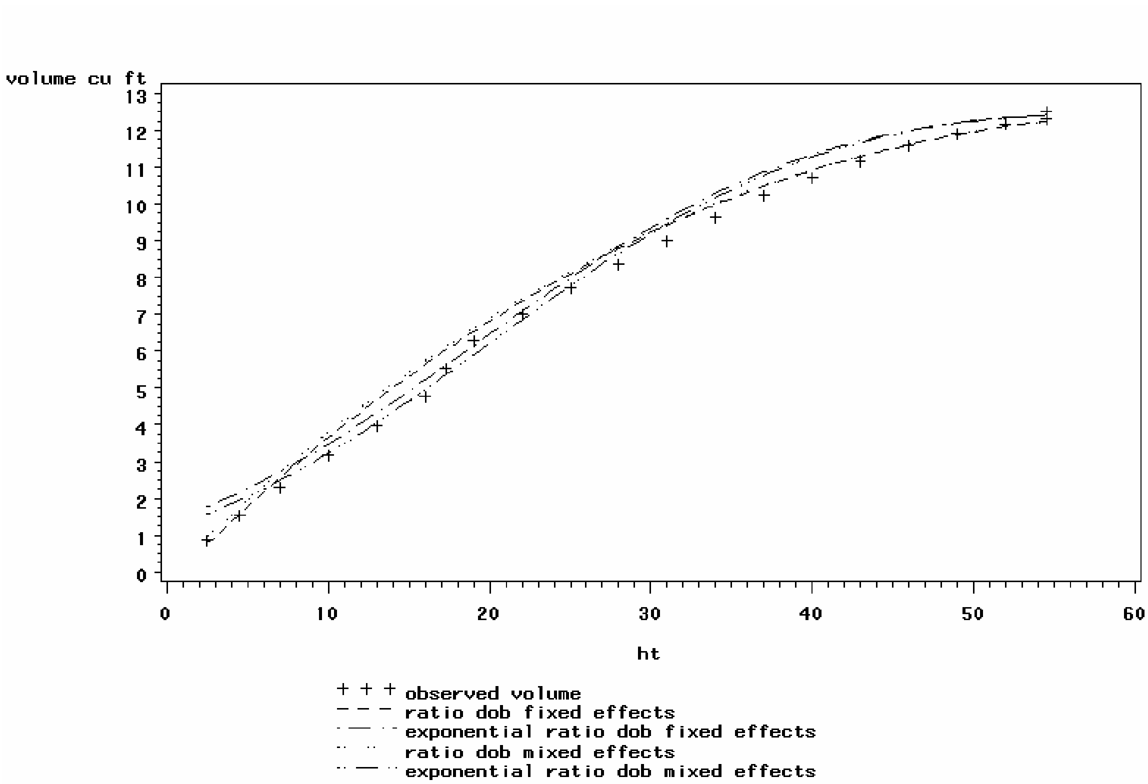


Figure 26: Predicted merchantable o.b. volume up the stem to an upper height comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D=9 in and H = 68 ft, representing the 50th percentile of the data for total volume.

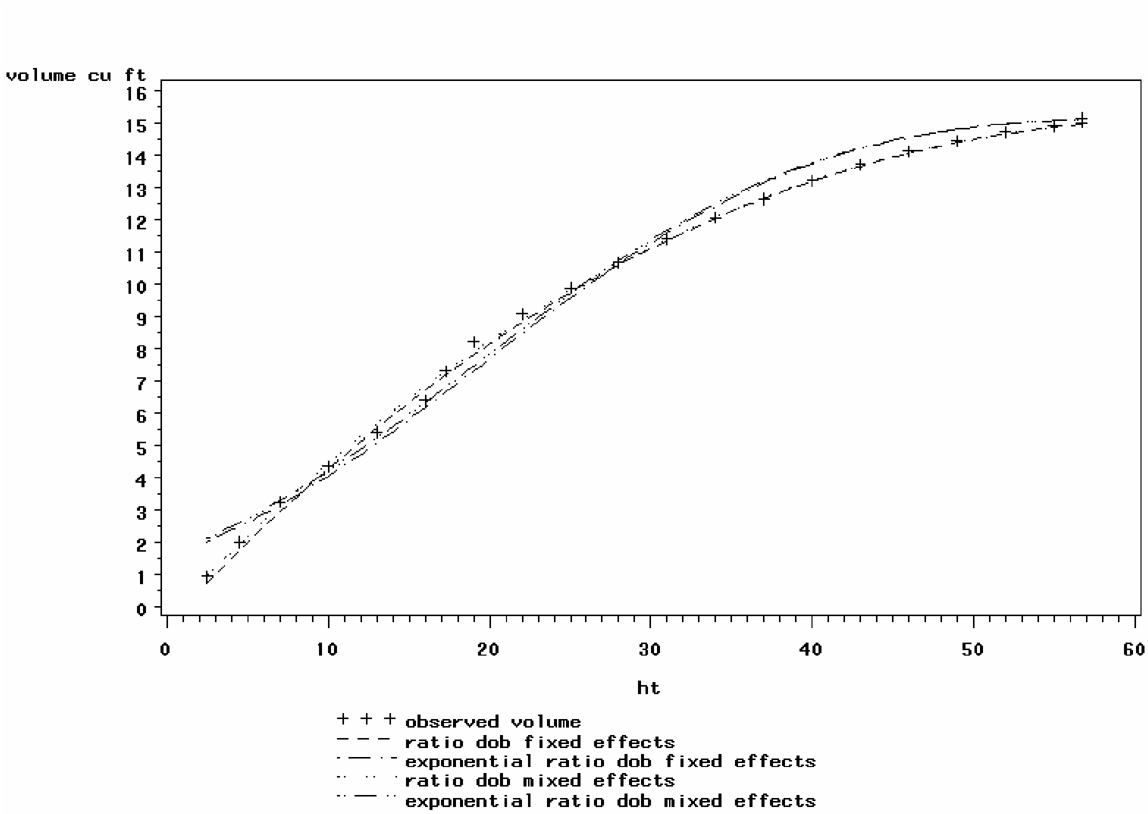


Figure 27: Predicted merchantable o.b. volume up the stem to an upper height comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D=12.2 in and H = 70.4 ft, representing the 75th percentile of the data for total volume

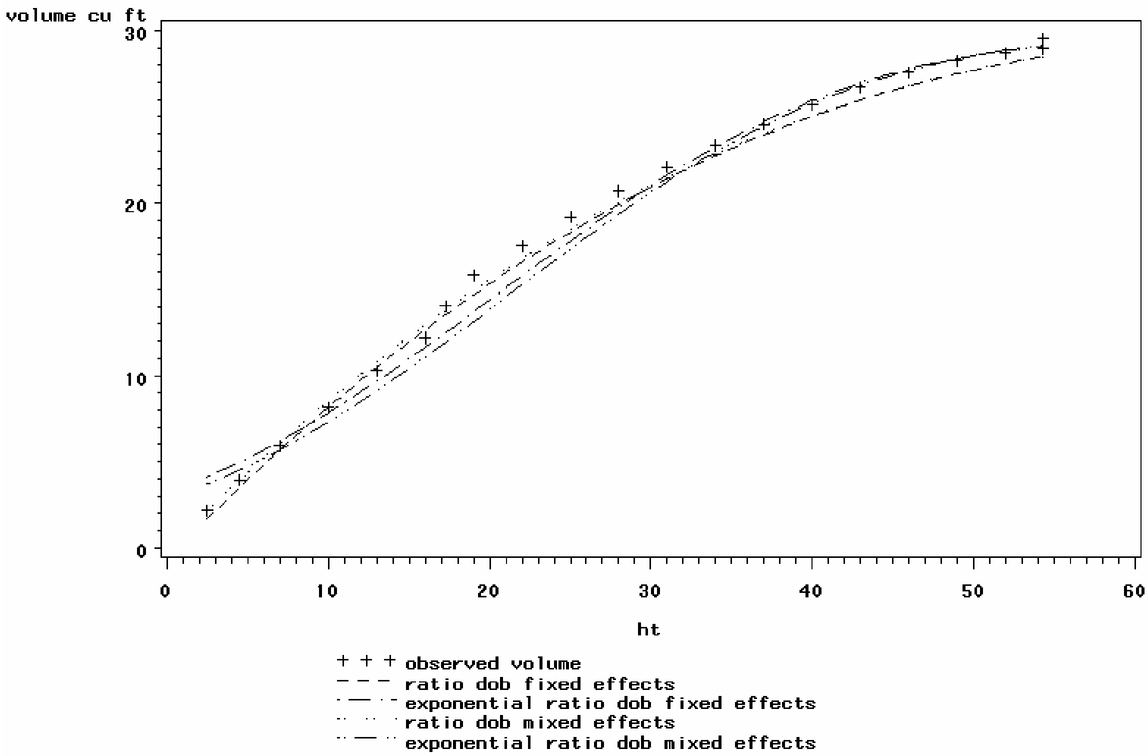


Figure 28: Predicted merchantable i.b. volume up the stem to an upper height comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D = 7.5 in and H = 66.2 ft, representing the 25th percentile of the data for total volume.

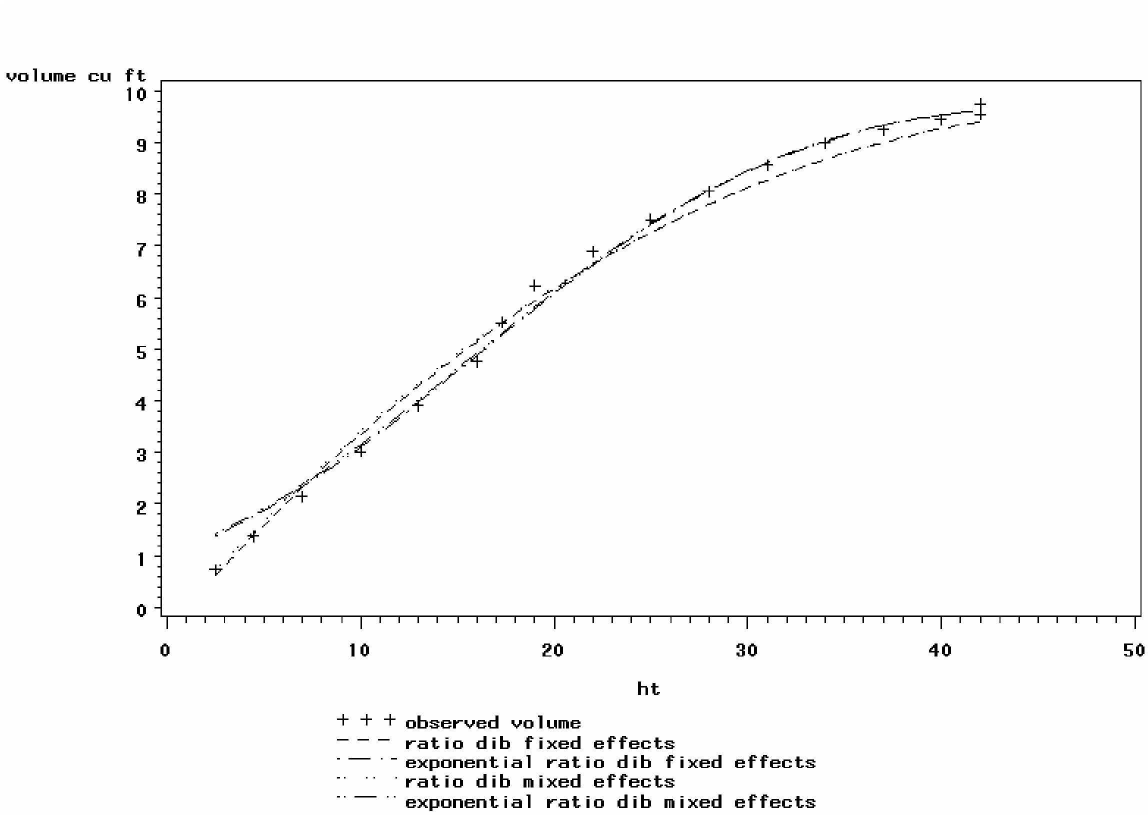


Figure 29: Predicted merchantable i.b. volume up the stem to an upper height comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D=9 in and H = 68 ft, representing the 50th percentile of the data for total volume.

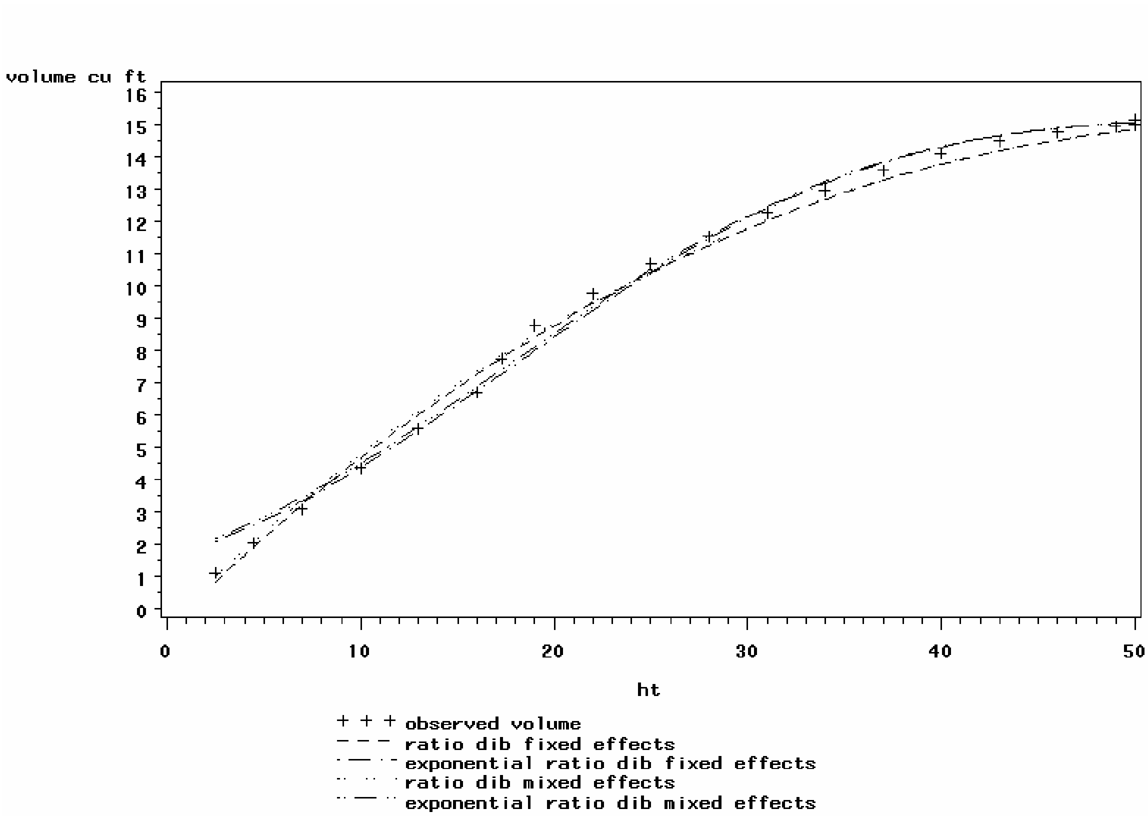
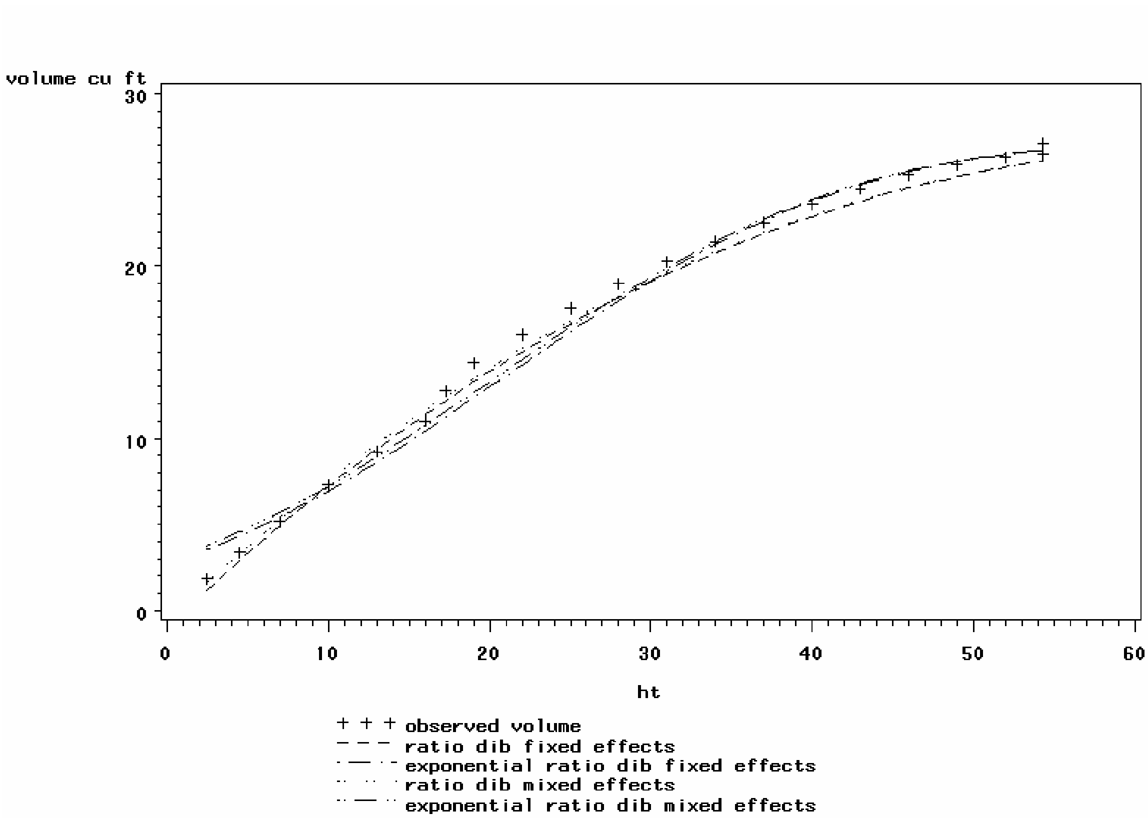


Figure 30: Predicted merchantable i.b. volume up the stem to an upper height comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D=12.2 in and H = 70.4 ft, representing the 75th percentile of the data for total volume



4.10 Implicit Taper Equations

Merchantable volume equations (Equations 36, 37, 38 and 39), were equated and rearranged producing the following implicit taper functions derived for the ratio form to predict diameter outside bark (dob_r), diameter inside bark (dib_r) and height up the stem from diameter outside bark equations (h_{or}) and height up the stem from diameter inside bark equations (h_{ir}). All χ_i and γ_i coefficients were obtained from Table 5.

$$dob_r = \left\{ \left(\frac{\gamma_1}{\chi_7} \right)^{\frac{1}{\chi_8}} D^{\frac{\chi_9}{\chi_8}} \left[\frac{(H-h)^{\frac{\gamma_2}{\chi_8}}}{H^{\frac{\gamma_3}{\chi_8}}} \right] \right\} + \varepsilon \tag{50}$$

$$dib_r = \left\{ \left(\frac{\gamma_4}{\chi_{10}} \right)^{\frac{1}{\chi_{11}}} D^{\frac{\chi_{12}}{\chi_{11}}} \left[\frac{(H-h)^{\frac{\gamma_5}{\chi_{11}}}}{H^{\frac{\gamma_6}{\chi_{11}}}} \right] \right\} + \varepsilon \tag{51}$$

$$h_{or} = H - \left\{ \left(\frac{\chi_7}{\gamma_1} \right)^{\frac{1}{\gamma_2}} \left(H^{\frac{\gamma_3}{\gamma_2}} \right) \left(\frac{dob^{\frac{\chi_8}{\gamma_2}}}{D^{\frac{\chi_9}{\gamma_2}}} \right) \right\} + \varepsilon \tag{52}$$

$$h_{ir} = H - \left\{ \left(\frac{\chi_{10}}{\gamma_4} \right)^{\frac{1}{\gamma_5}} \left(H^{\frac{\gamma_6}{\gamma_5}} \right) \left(\frac{dib^{\frac{\chi_{11}}{\gamma_5}}}{D^{\frac{\chi_{13}}{\gamma_5}}} \right) \right\} + \varepsilon \tag{53}$$

Merchantable volume equations (Equations 40, 41, 42 and 43), were also equated and rearranged producing the following implicit taper functions derived from the exponential ratio form to predict diameter outside bark (dob_e), diameter inside bark (dib_e)

and height up the stem from outside bark equations (h_{oe}) and height up the stem from inside bark equations (h_{ie}). All χ_i and γ_i coefficients were obtained from Table 5.

$$dob_e = \left\{ \left(\frac{\gamma_7}{\chi_{13}} \right)^{\frac{1}{\chi_{14}}} D^{\frac{\chi_{15}}{\chi_{14}}} \left[\frac{(H-h)^{\frac{\gamma_8}{\chi_{14}}}}{H^{\frac{\gamma_9}{\chi_{14}}}} \right] \right\} + \varepsilon \qquad (54)$$

$$dib_e = \left\{ \left(\frac{\gamma_{10}}{\chi_{16}} \right)^{\frac{1}{\chi_{17}}} D^{\frac{\chi_{18}}{\chi_{17}}} \left[\frac{(H-h)^{\frac{\gamma_{11}}{\chi_7}}}{H^{\frac{\gamma_{12}}{\chi_{17}}}} \right] \right\} + \varepsilon \qquad (55)$$

$$h_{oe} = H - \left\{ \left(\frac{\chi_{13}}{\gamma_7} \right)^{\frac{1}{\gamma_8}} \left(H^{\frac{\gamma_9}{\gamma_8}} \right) \left(\frac{dob^{\frac{\chi_{14}}{\gamma_8}}}{D^{\frac{\chi_{15}}{\gamma_8}}} \right) \right\} + \varepsilon \qquad (56)$$

$$h_{ie} = H - \left\{ \left(\frac{\chi_{16}}{\gamma_{10}} \right)^{\frac{1}{\gamma_{11}}} \left(H^{\frac{\gamma_{12}}{\gamma_{11}}} \right) \left(\frac{dib^{\frac{\chi_{17}}{\gamma_{11}}}}{D^{\frac{\chi_{18}}{\gamma_{11}}}} \right) \right\} + \varepsilon \qquad (57)$$

A Virginia pine tree with a D of 9.2 in, H of 68.1 ft, 3 in o.b. top height of 57.2 representing the 50^{th} percentile of the data was used to develop graphs of predicted diameter outside and inside bark up the stem and predicted height up the stem using diameter outside and inside bark equations. A graph of the predicted diameter outside bark up the stem, comparing the observed values, ratio and exponential ratio model forms, is presented in Figure 31. The graphs shows the exponential ratio dob provides a better fit to the observed data at the lower and middle sections of the tree while ratio provides a better fit as diameter decreases near the 3 in top. A graph of the predicted diameter inside bark up the stem, comparing the observed values, ratio and exponential ratio model forms, is presented in Figure 32. The graph shows the exponential ratio dib model provides a better fit as compared to the observed data for the lower and upper sections of the tree while the ratio dib provides a better fit for a small section in the middle of the tree. A graph of predicted height up the stem, using diameter outside bark equations, comparing the observed values, ratio and exponential ratio model forms, is presented in Figure 33. The graph shows that with the exception of the lower section of the bole, the exponential ratio dob model provides a better fit to the observed data as diameter decreases as compared to the ratio dob model. A graph of predicted height up the stem, using diameter inside bark equations, comparing the observed values, ratio and exponential ratio model forms, is presented in Figure 34. The ratio model provides a better fit, when compared to the observed values, for the lower section of the bole while the exponential ratio model provides a better fit as diameter decreases up the stem.

Figure 31: Predicted diameter outside bark up the stem comparing ratio and exponential ratio fixed effects model for a Virginia pine with a D=9.2 in, H = 68.1 ft, 3 in top = 57.2 ft representing the 50th percentile of the data for total o.b. volume.

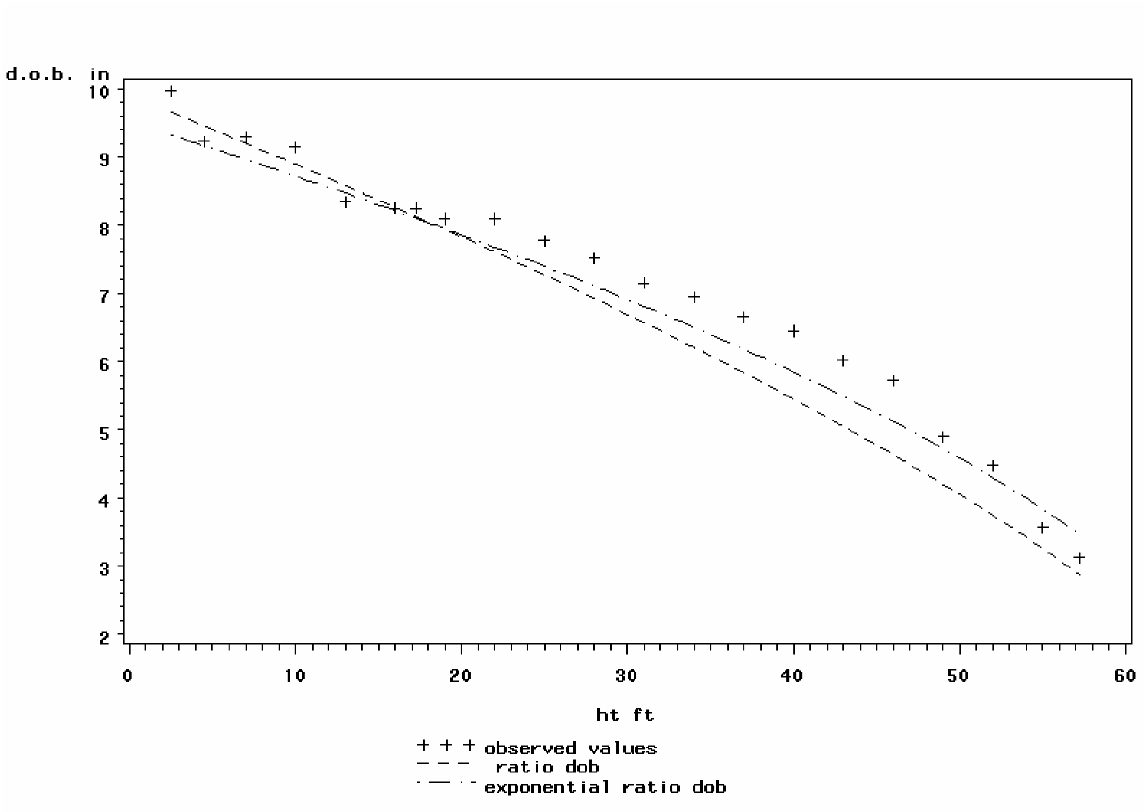


Figure 32: Predicted diameter inside bark up the stem comparing ratio and exponential ratio fixed effects model for a Virginia pine with a D = 9.2 in, H = 68.1 ft, 3 in top = 57.2 ft representing the 50th percentile of the data for total i.b. volume.

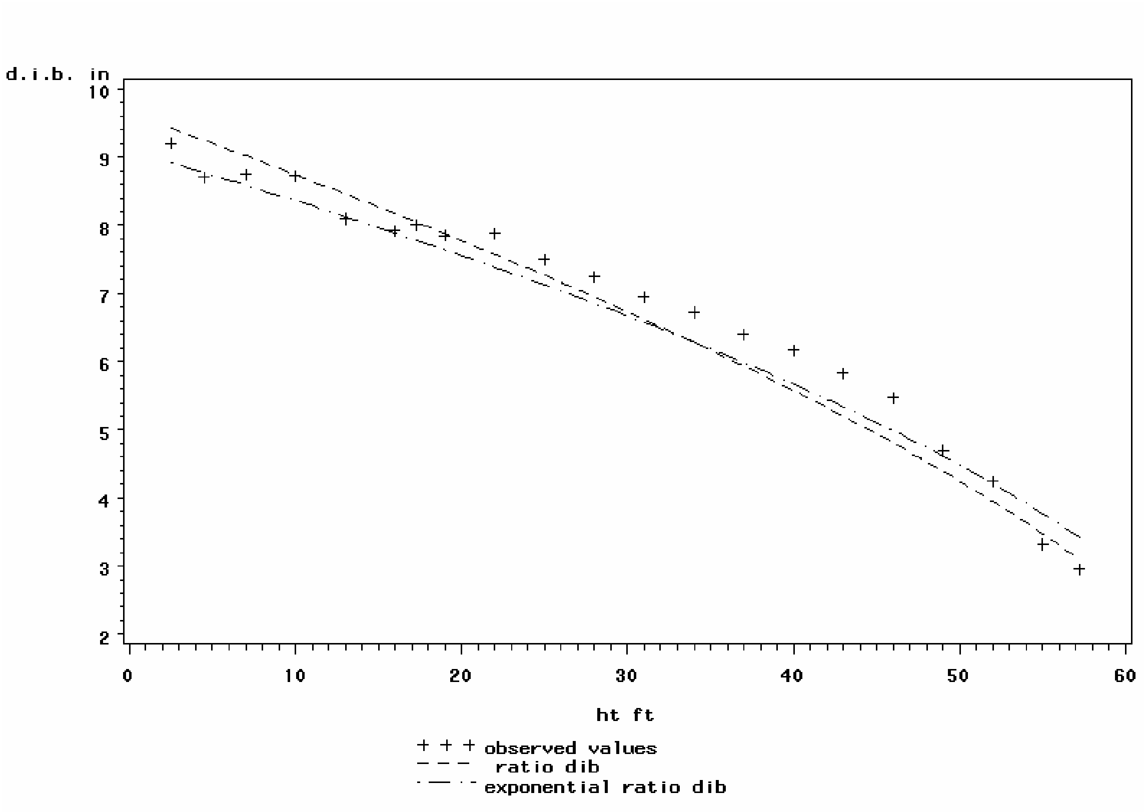


Figure 33: Predicted height up the stem, diameter outside bark, comparing ratio and exponential ratio fixed effects model for a Virginia pine with a D = 9.2 in, H = 68.1 ft, 3 in top = 57.2 ft representing the 50th percentile of the data for total o.b. volume.

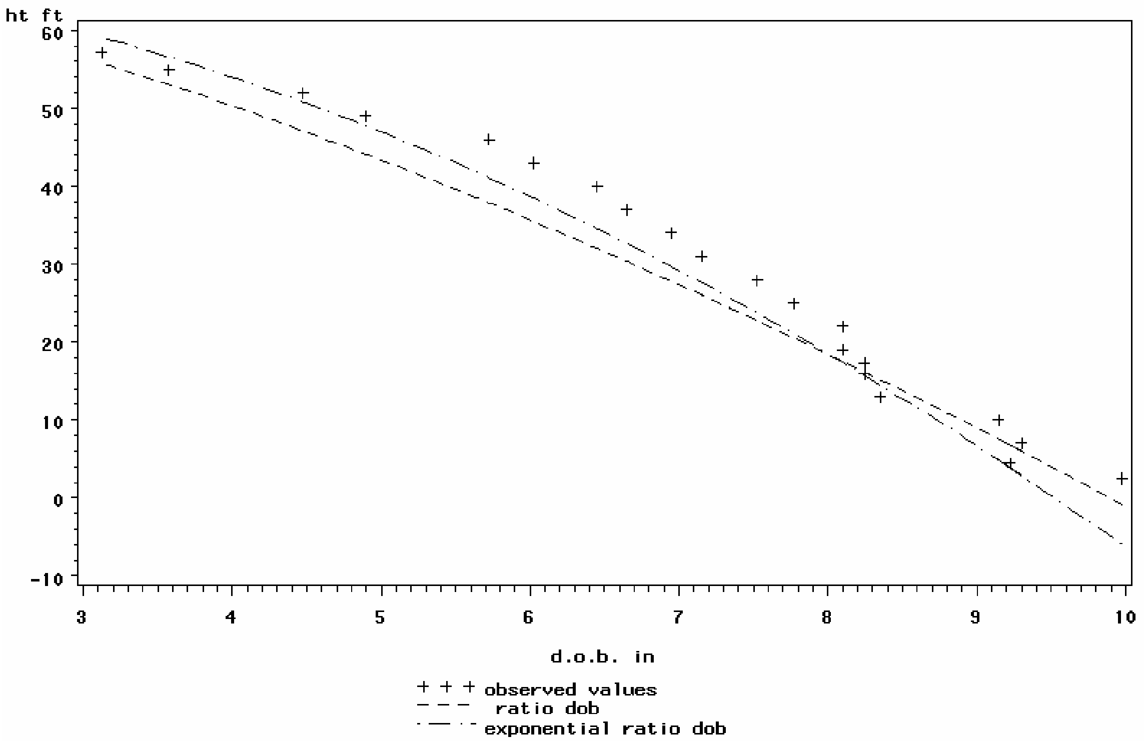


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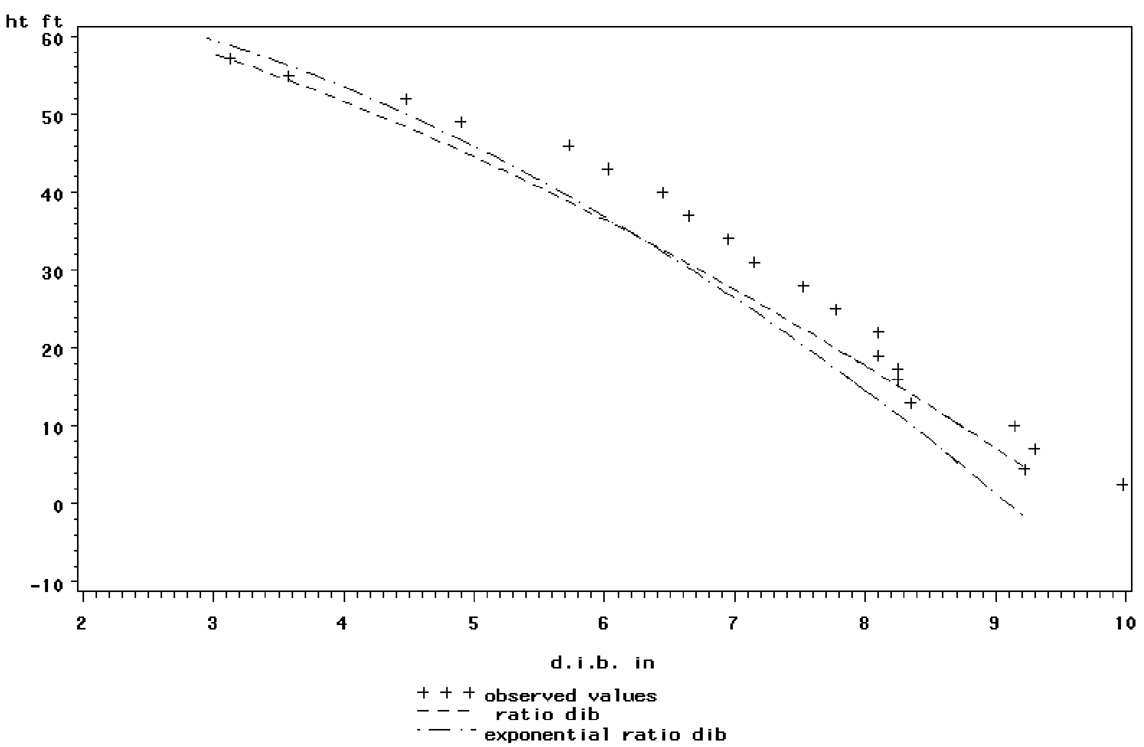
Figure 34: Predicted height up the stem, diameter inside bark, comparing ratio and exponential ratio fixed effects model for a Virginia pine with a D = 9.2 in, H = 68.1 ft, 3 in top = 57.2 ft representing the 50th percentile of the data for total i.b. volume.



4.11 Weight to Volume Ratio

A relationship of pounds per cubic foot volume was developed. For each stem, the means of the ratio for total weight to a 3 inch top divided by total volume dob to a 3 inch top was recorded. A relationship of 54.558 lb per cubic ft was determined with a standard deviation of 3.42 lb per cubic ft, standard error of 0.0740. A maximum ratio of 78.642 lb per cubic ft and minimum ratio of 48.107 lb per cubic ft were recorded.

4.12 Conclusion and Recommendations for Merchantable Volume Model Forms

Developing an accurate model for determining merchantable volume (o.b. or i.b.), to any upper stem diameter or height is important to the forest industry. This research focused on determining which model, fixed or random effects, ratio or exponential ratio, would best predict merchantable volume (o.b. or i.b.) to any upper stem diameter or height. The exponential ratio models proved to best predict merchantable volume (o.b. or i.b.), to any upper stem diameter while the ratio model best predicted merchantable volume (o.b. or i.b.), to any upper stem height using RSS, RMSE, AIC and -2 log likelihood as standards of model fit for fixed effects models. Using AIC and -2 log likelihood as standards of model fit for mixed effects models, the exponential ratio models proved to best predict merchantable volume (o.b. or i.b.) to any upper stem diameter while the ratio model best predicted merchantable volume (o.b. or i.b.) to any upper stem height. Overall, this research indicates that the mixed effects models are more reliable for determining merchantable volume, (o.b or i.b.) to any upper stem diameter or height.

5. Discussion

Results of this research recommend the use of the mixed effects models when predicting merchantable o.b. green weight or volume to any upper stem diameter (outside or inside bark) or height. For predicting merchantable o.b. green weight to any upper stem diameter (outside bark or inside bark), merchantable outside bark volume to any upper stem diameter o.b. or merchantable inside bark volume to any upper stem diameter i.b., the mixed effects exponential ratio equations are recommended. For predicting merchantable o.b. green weight to any upper stem height, merchantable outside bark volume to any upper stem height or merchantable inside bark volume to any upper stem height, the mixed effects ratio equations are recommended. When compared to the results from Clark (1994), where volume tables were derived for predicting stemwood volume to a 4 inch dob top in pulpwood trees based on dbh and total tree height for Virginia pine South-wide, the equations presented in this research predicted an average of 9.4 cu ft more volume over the range of data at dbh than those from Clark (1994). When compared to Slocum (1953) Piedmont region volume o.b. table to 4 inch top, the equations presented in this research predicted an average of 2.9 cu ft more volume over the range of data at dbh. The predicted values obtained from the equations developed with this research produced a mean difference of .0368 cu ft from the observed values with a standard deviation of .0535 cu ft. Clark (1994) South-wide average volume tables are less accurate and underestimate volume o.b. in the piedmont region and should only be used when a species volume table is not available for a desired geographic region. See Table 8 for recommended equations from this research for determining green weight or volume to any upper merchantable diameter or height with estimated parameter coefficients.

These equations provide another tool for forest land managers, procurement foresters, and timber purchasers at local mills and allow them to produce better estimates of total and merchantable green weight and volume for Virginia pine. These equations, however, are limited to Virginia pine in the Piedmont region of North Carolina and caution is urged when using them outside of the range of data used for parameter estimation.

Table 8: Recommended equations with estimated parameter coefficients

Green weight to upper stem dob	$\hat{W}_{dob} = \left(75.266 + 0.1423 D^2 H\right)\left(\exp\left(\left(-0.8704 + 0.0\right)\left(\frac{dob^{5.9224}}{D^{5.5634}}\right)\right)\right)$
Green weight to upper stem dib	$\hat{W}_{dib} = \left(75.266 + 0.1423 D^2 H\right)\left(\exp\left(\left(-1.3370 + 0.0\right)\left(\frac{dib^{6.1350}}{D^{5.8427}}\right)\right)\right)$
Green weight to upper stem height	$\hat{W}_{h} = \left(75.266 + 0.1423 D^2 H\right)\left(1 + \left(\left(-0.8322 + 0.0\right)\left(\frac{\left(H-h\right)^{2.0338}}{H^{1.9889}}\right)\right)\right)$
Volume o.b. to upper stem dob	$\hat{V}_{mer,o.b.} = \left(1.0631 + 0.0027 D^2 H\right)\left(\exp\left(\left(-0.9978 + 0.0\right)\left(\frac{dob^{6.4699}}{D^{6.1584}}\right)\right)\right)$
Volume i.b. to upper stem dib	$\hat{V}_{mer,i.b.} = \left(0.6812 + 0.0025 D^2 H\right)\left(\exp\left(\left(-1.5772 + 0.0\right)\left(\frac{dib^{6.8035}}{D^{6.5446}}\right)\right)\right)$
Volume o.b. to upper stem height	$\hat{V}_{h,o.b.} = \left(1.0631 + .0027 D^2 H\right)\left(1 + \left(\left(-1.1959 + 0.0\right)\left(\frac{\left(H-h\right)^{2.2536}}{H^{2.2891}}\right)\right)\right)$
Volume i.b. to upper stem height	$\hat{V}_{h,i.b.} = \left(0.6812 + 0.0025 D^2 H\right)\left(1 + \left(\left(-1.2780 + 0.0\right)\left(\frac{\left(H-h\right)^{2.2472}}{H^{2.2956}}\right)\right)\right)$

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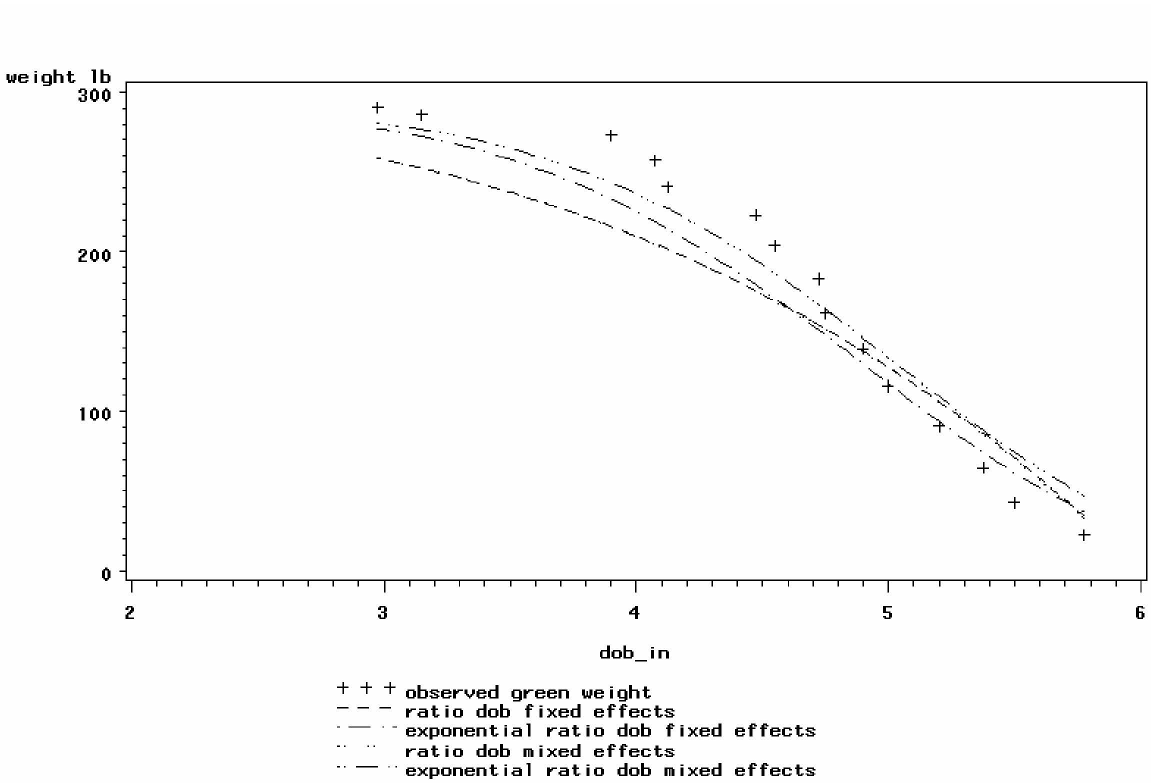
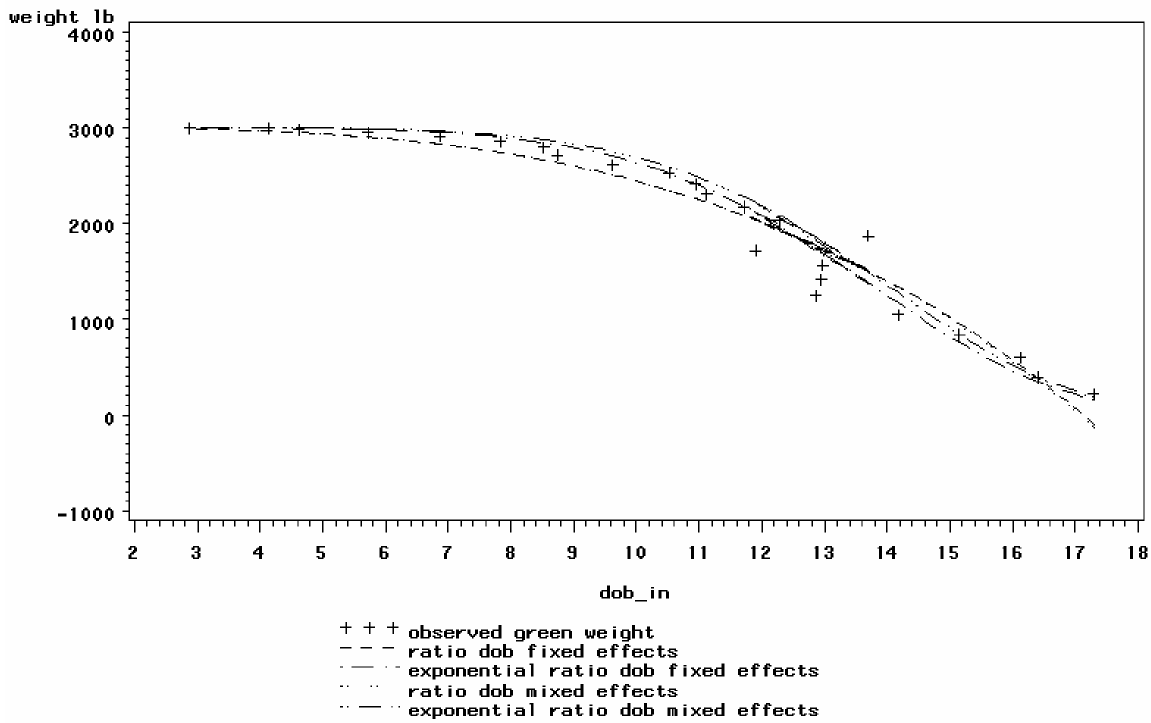
Appendix I: Additional Figures

Green Weight Figures – 5th and 95th percentile

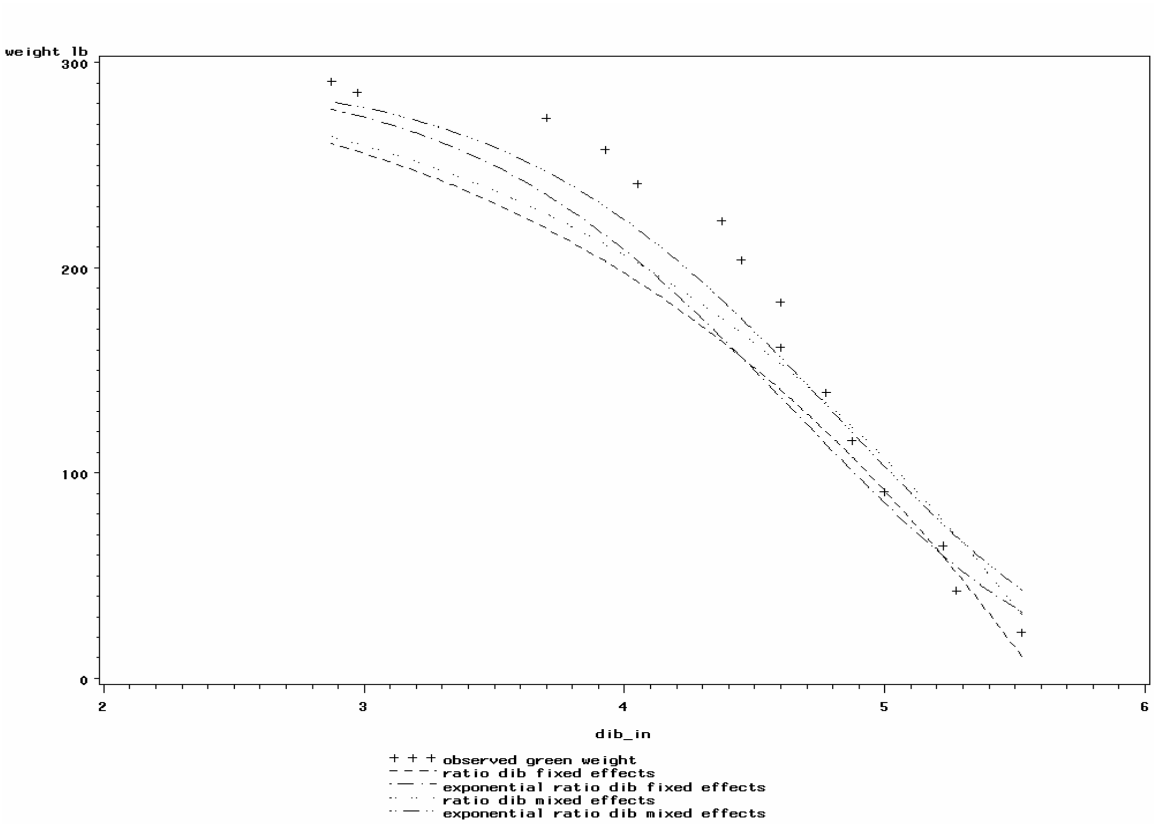
Predicted merchantable green weight up the stem comparing ratio dob and exponential ratio dob of both fixed and mixed effects models for a tree from the 5th percentile for total green weight with D = 5.5 in, H = 55.7 ft, total green weight = 290.8 lb.



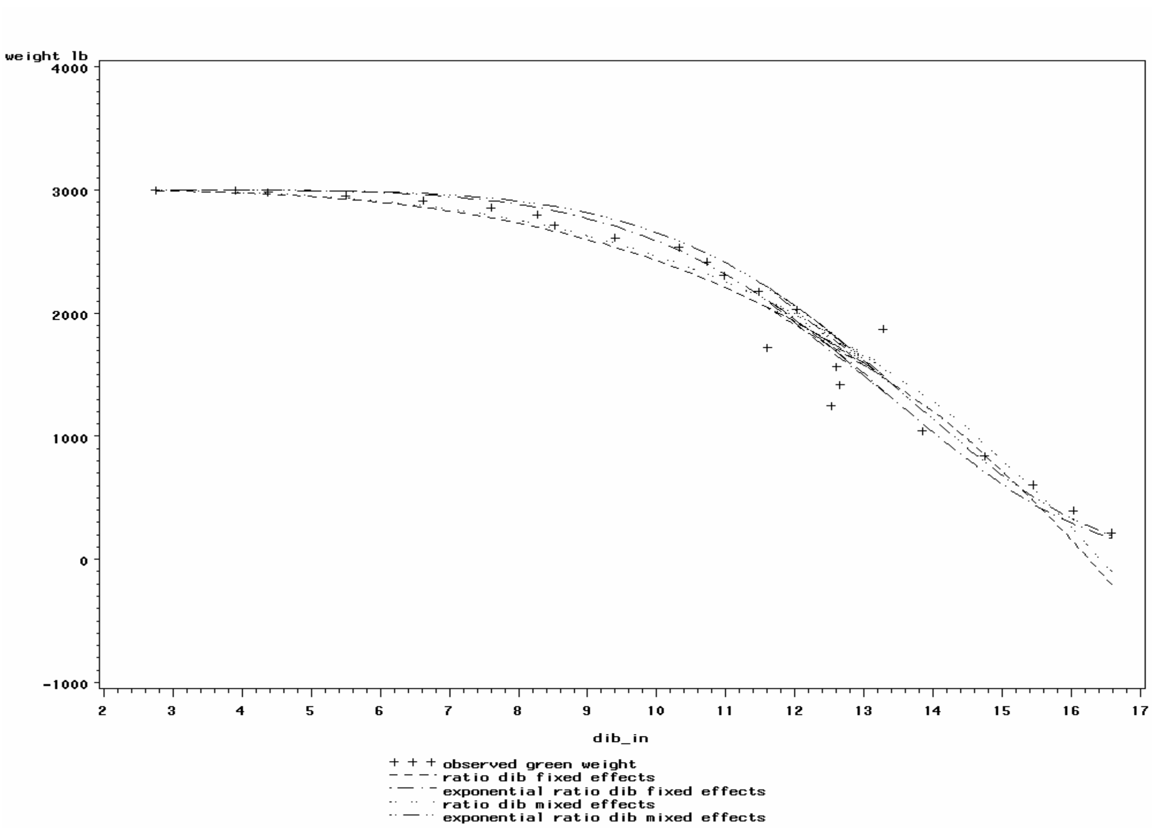
Predicted merchantable green weight up the stem comparing ratio dob and exponential ratio dob of both fixed and mixed effects models for a tree from the 95th percentile for total green weight with D = 16.4 in, H = 77 ft, total green weight = 3001.9 lb.



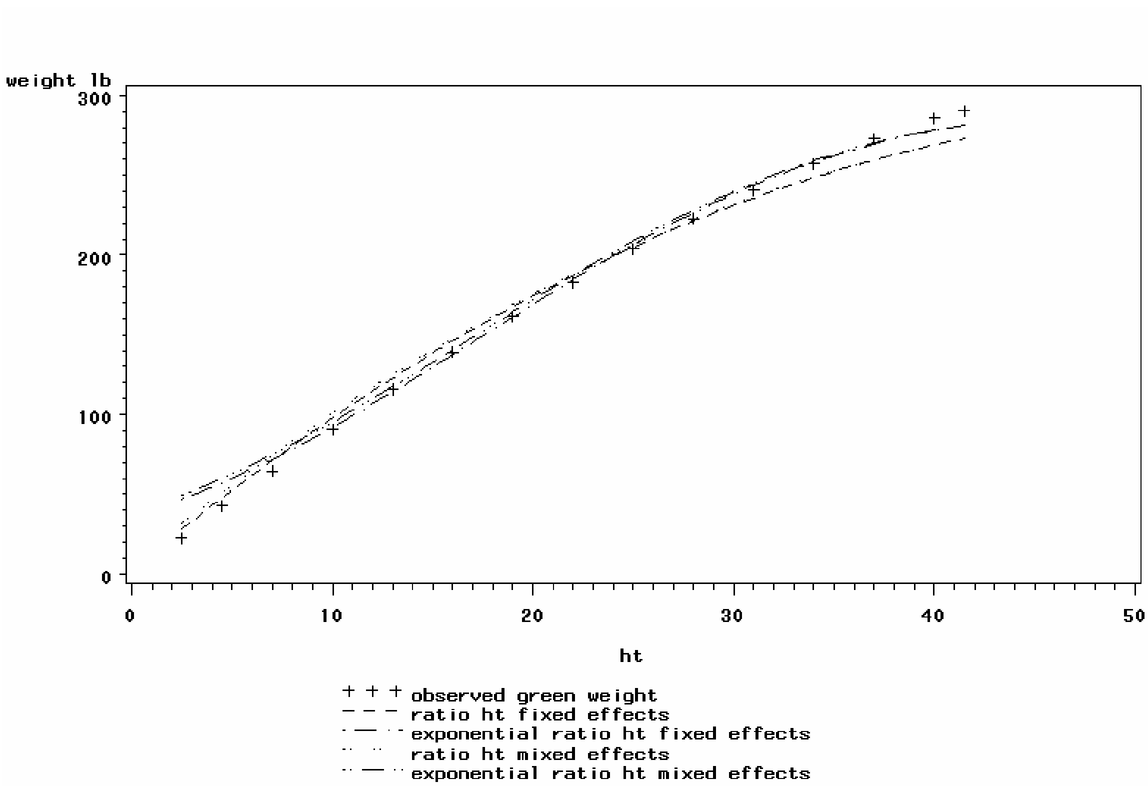
Predicted merchantable green weight up the stem comparing ratio dib and exponential ratio dib of both fixed and mixed effects models for a tree from the 5th percentile for total green weight with D = 5.5 in, H = 55.7 ft, total green weight = 290.8 lb.



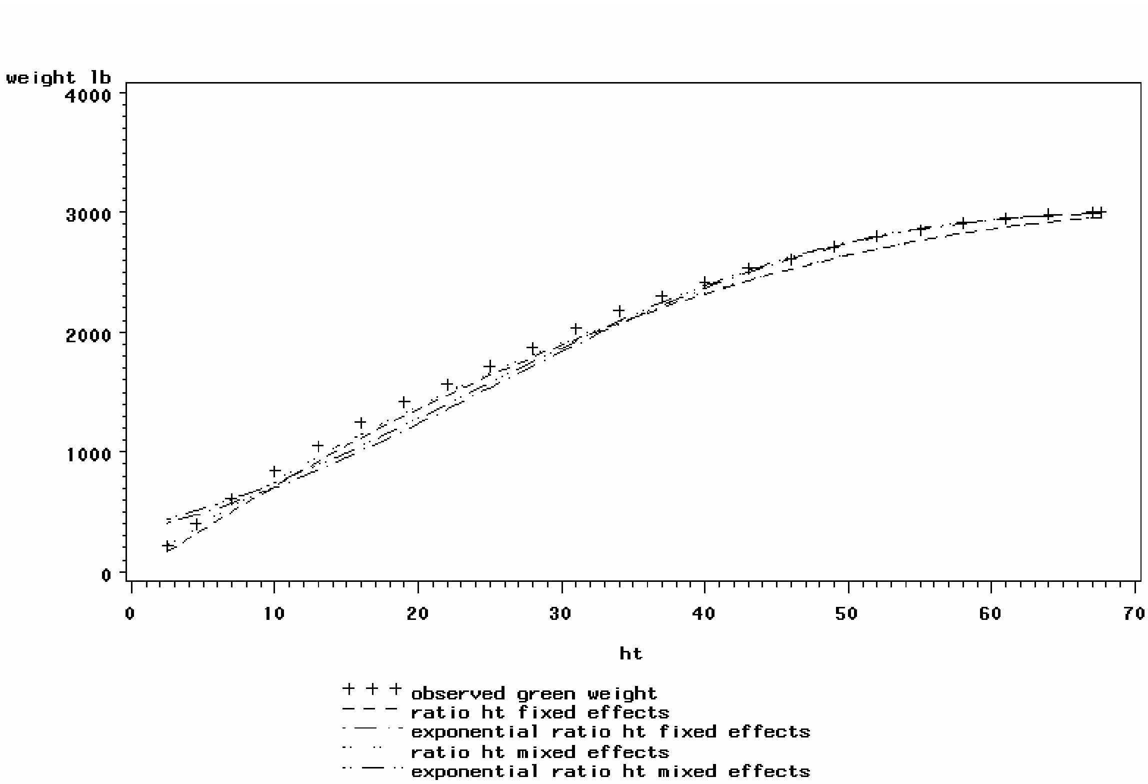
Predicted merchantable green weight up the stem comparing ratio dib and exponential ratio dib of both fixed and mixed effects models for a tree from the 95th percentile for total green weight with D = 16.4 in, H = 77 ft, total green weight = 3001.9 lb.



Predicted merchantable green weight up the stem comparing ratio ht and exponential ratio ht of both fixed and mixed effects models for a tree from the 5th percentile for total green weight with D = 5.5 in, H = 55.7 ft, total green weight = 290.8 lb.



Predicted merchantable green weight up the stem comparing ratio ht and exponential ratio ht of both fixed and mixed effects models for a tree from the 95th percentile for total green weight with D = 16.4 in, H = 77 ft, total green weight = 3001.9 lb.



Volume Figures – 5th and 95th percentile

Figure 35: Predicted merchantable o.b. volume up the stem to an upper diameter comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D = 5.5 in and H = 55.7 ft, representing the 5th percentile of the data for total volume.

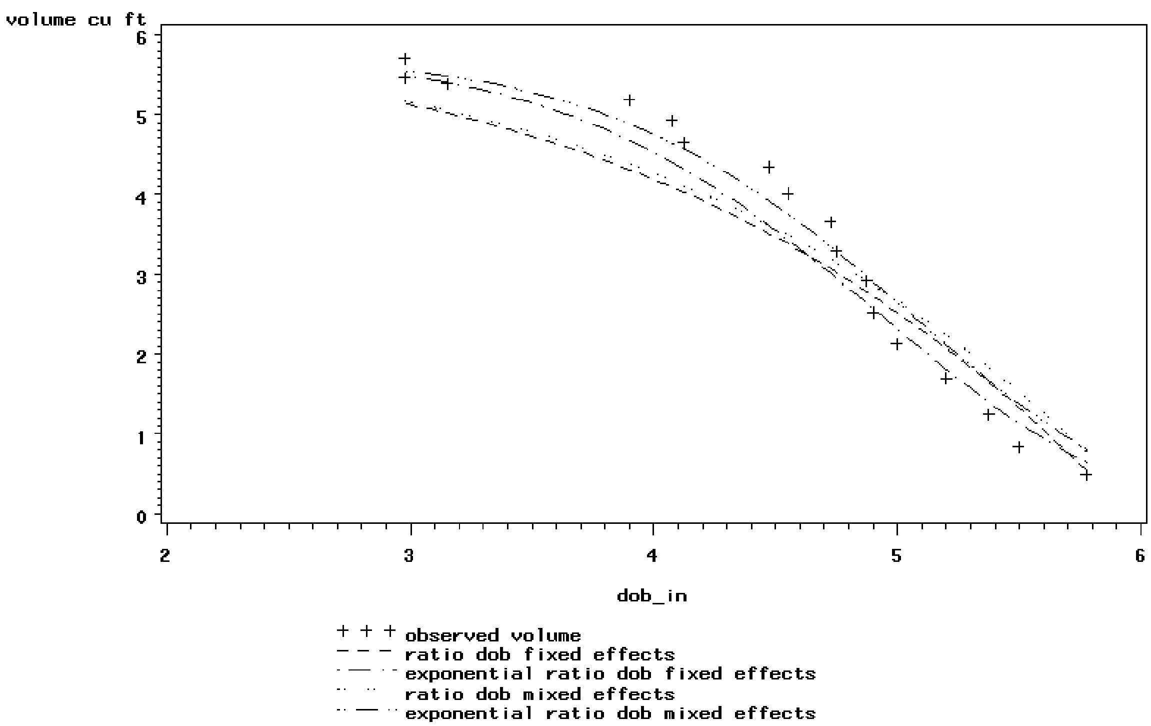


Figure 36: Predicted merchantable o.b. volume up the stem to an upper diameter comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D = 16.4 in and H = 77 ft, representing the 95th percentile of the data for total volume.

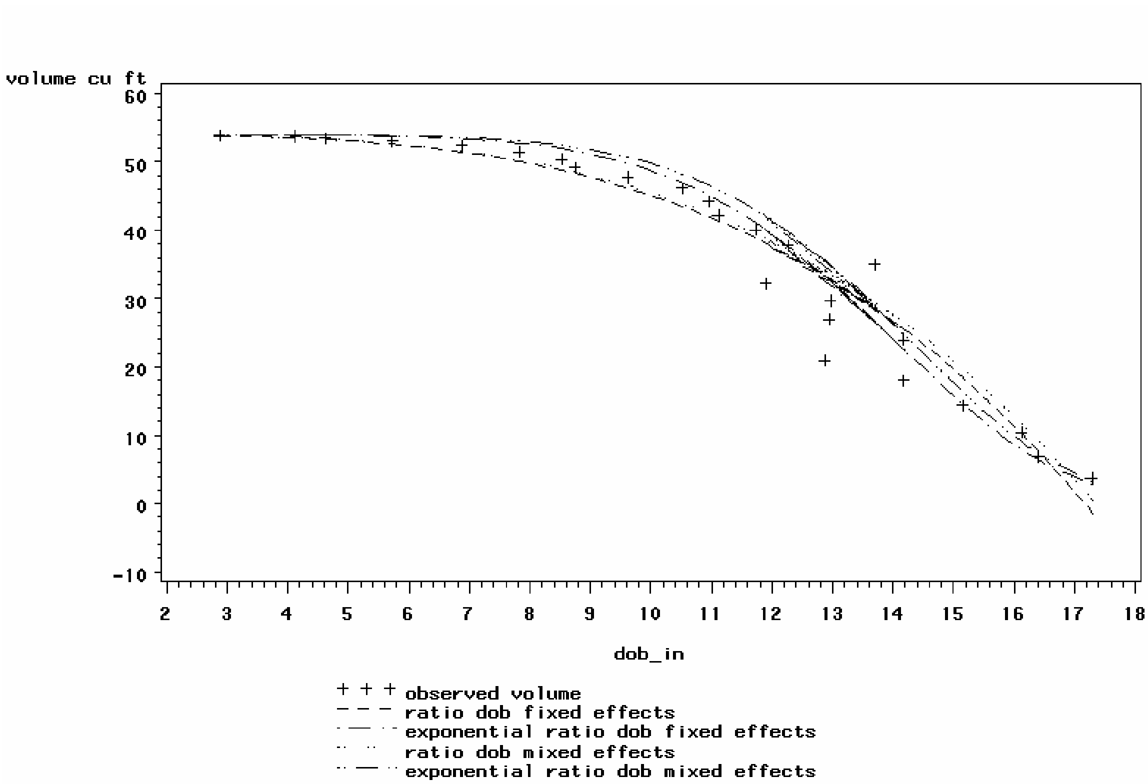


Figure 37: Predicted merchantable i.b. volume up the stem to an upper diameter comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D = 5.5 in and H = 55.7 ft, representing the 5th percentile of the data for total volume.

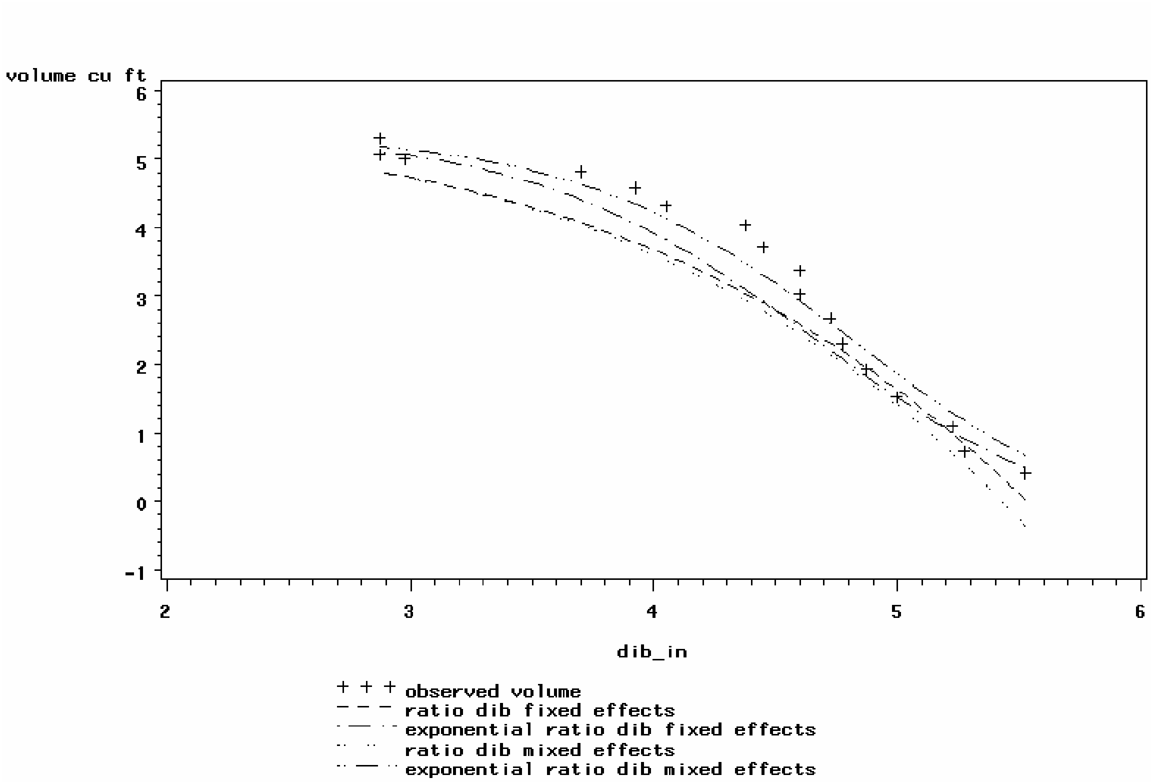


Figure 38: Predicted merchantable i.b. volume up the stem to an upper diameter comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D = 16.4 in and H = 77 ft, representing the 95th percentile of the data for total volume.

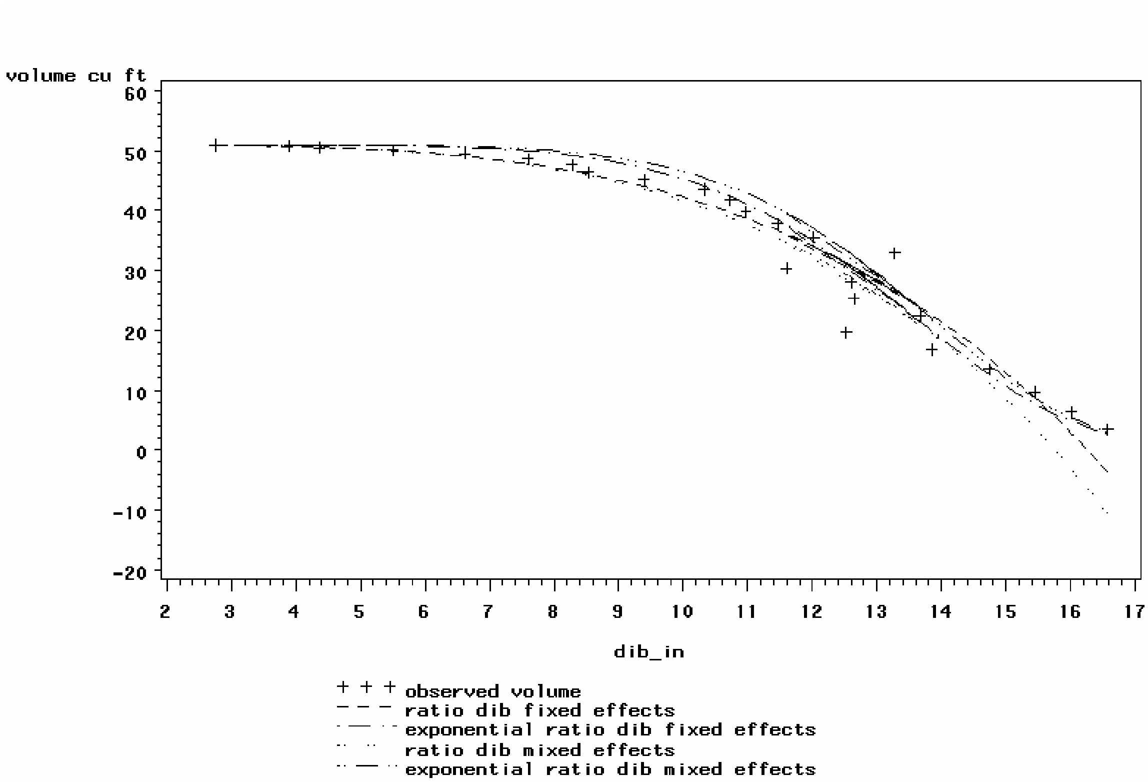


Figure 39: Predicted merchantable o.b. volume up the stem to an upper height comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D = 5.5 in and H = 55.7 ft, representing the 5th percentile of the data for total volume.

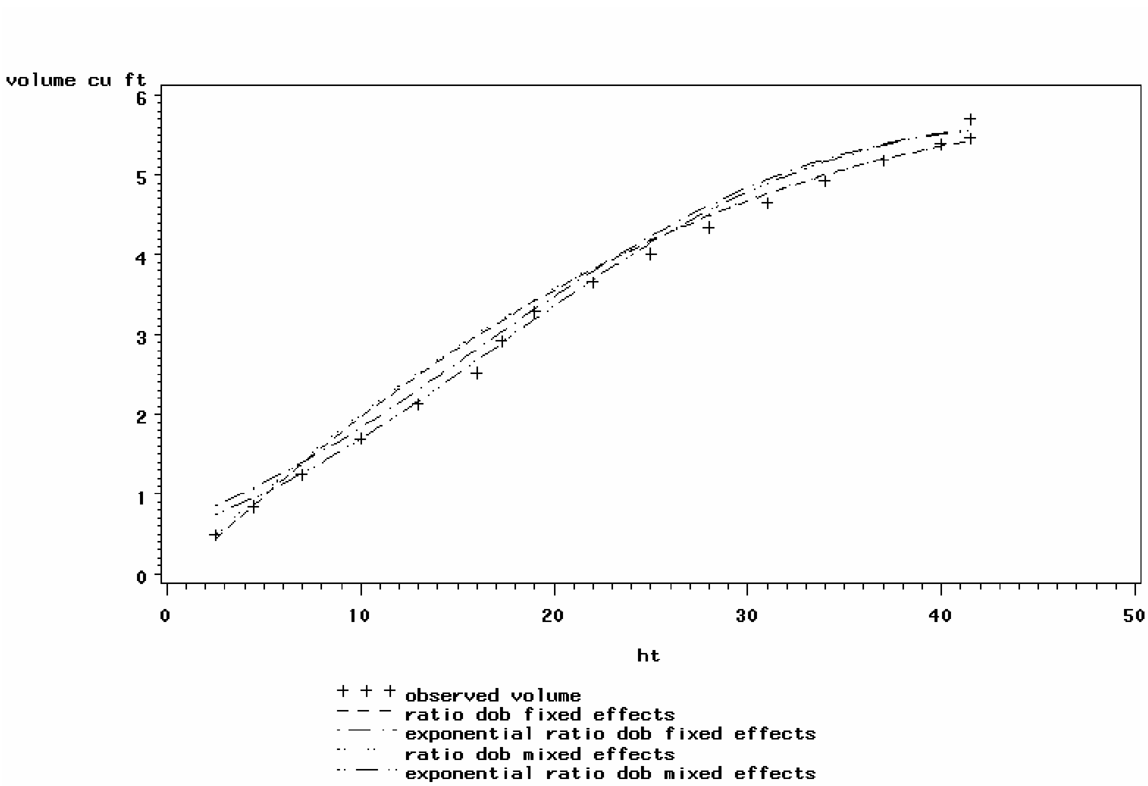


Figure 40: Predicted merchantable o.b. volume up the stem to an upper height comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D = 16.4 in and H = 77 ft, representing the 95th percentile of the data for total volume.

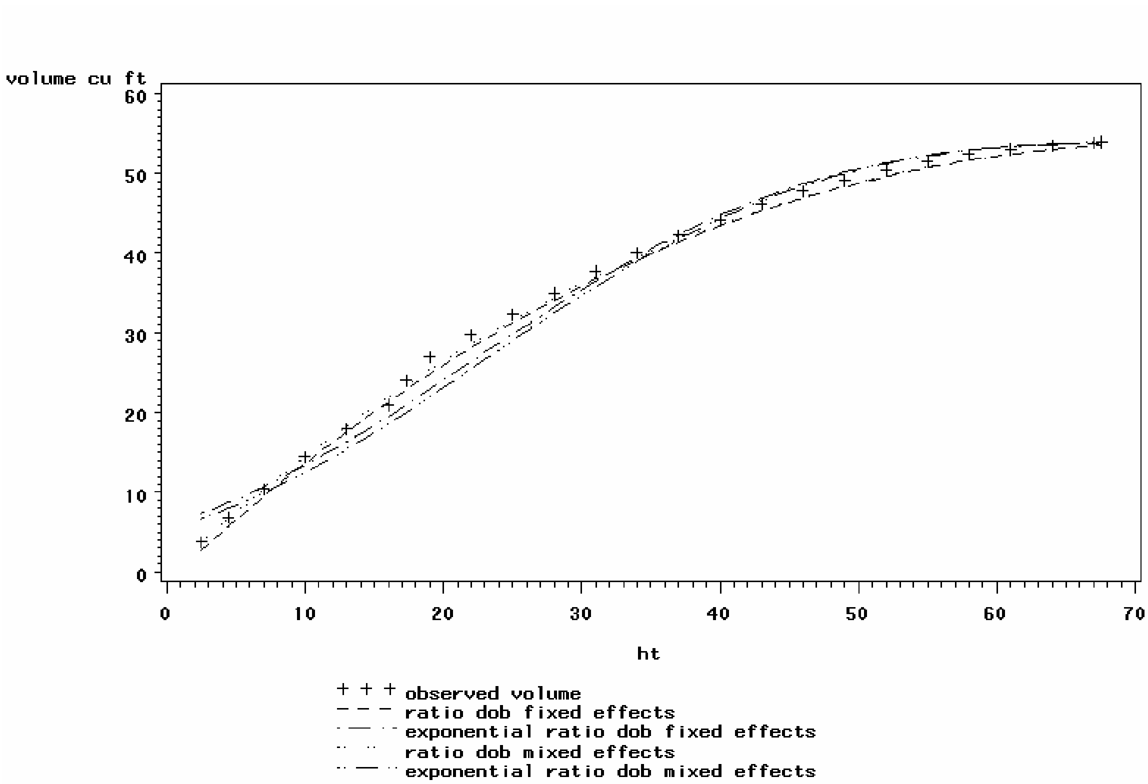


Figure 41: Predicted merchantable i.b. volume up the stem to an upper height comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D = 5.5 in and H = 55.7 ft, representing the 5th percentile of the data for total volume.

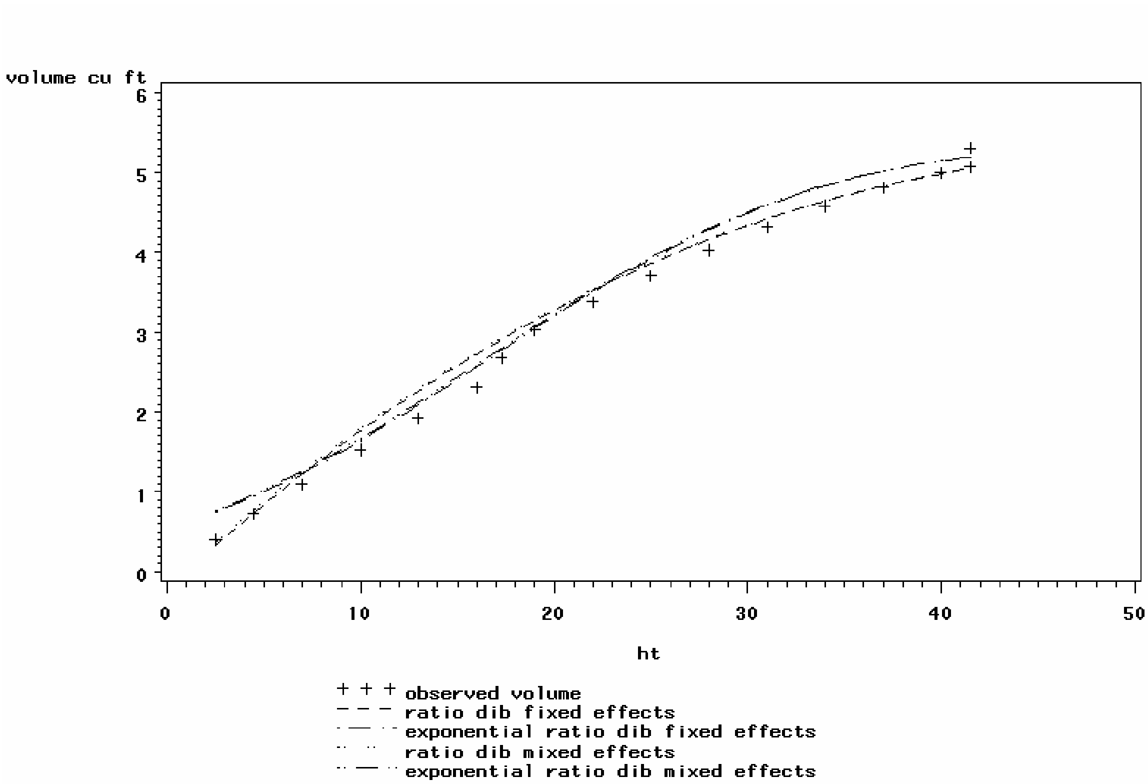
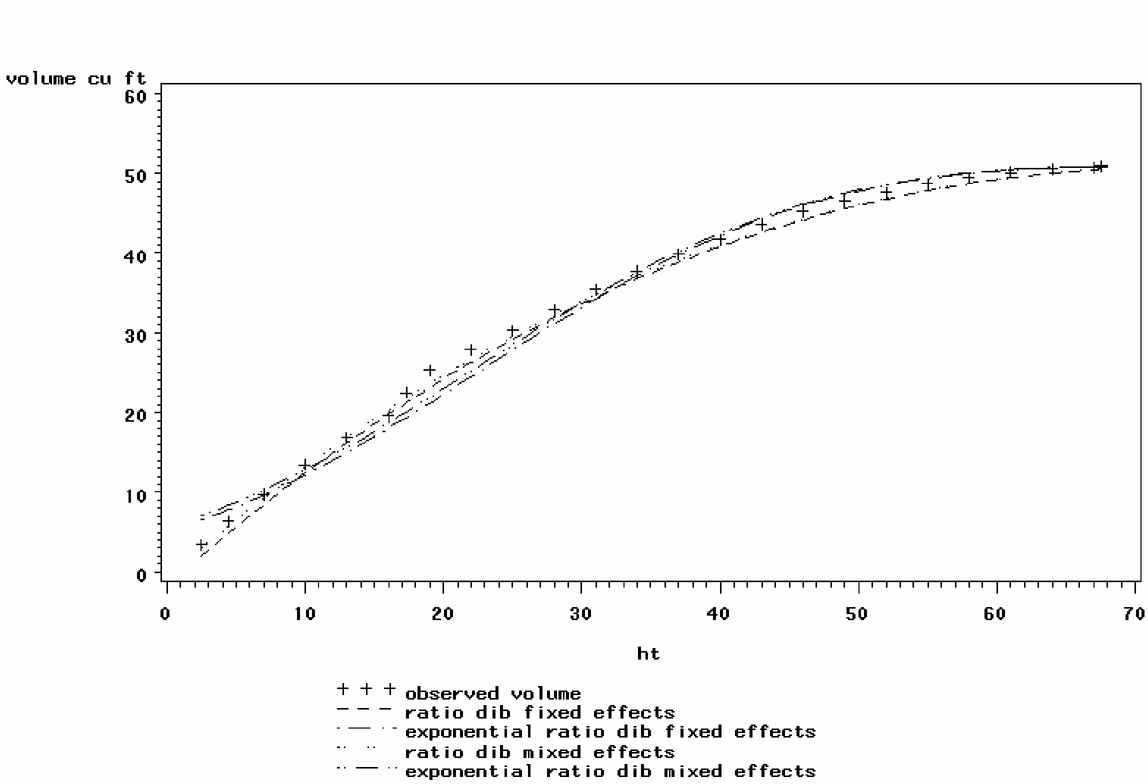


Figure 42: Predicted merchantable i.b. volume up the stem to an upper height comparing ratio and exponential ratio forms of both fixed and mixed effects models for a Virginia pine tree with D = 16.4 in and H = 77 ft, representing the 95th percentile of the data for total volume.



Appendix II: SAS Code

Green Weight Fixed Effects

```
data vp.vpw;
 set vp.vp;
 R = cum_wt/tot_wt;
 Rpred_ratio_dob= 1 -
(0.606693737*((dob_in**3.1282190424)/(dbh**2.9982625124)));
 Rpred_ratio_dib=1- 0.832619489*(dib_in**3.395907618
/dbh**3.3196481029);
 Rpred_ratio_ht = 1 - 0.782280777*(((tot_ht -
ht)**2.0352055325)/(tot_ht**1.9761290029));
 Rpred_eratio_dob = exp((-
.940363122*(dob_in)**5.6706672123)/(dbh**5.3740524716));
 Rpred_eratio_dib = exp((-
1.497996631*(dib_in)**5.8488368459)/(dbh**5.6392833058));
 Rpred_eratio_ht = exp(-1.097332683*(((tot_ht-
ht)**3.0504455141)/(tot_ht**2.8884850628)));
/*All of these pred_values are R values to be used in the green weight
merchantable estimate (gwme)
 add beta values from output and insert them here to get predicted
values so can compare
 to observed values in one table.*/
/* diff = dob_in - dbh; if I want to find the diff between 2 variables,
reinsert this line*/

 d2=dbh*dbh;
 d2h=d2*tot_ht;
 twe =75.266+0.1423*d2h;/*total weight estimate*/
 gwme = tot_wt*R;       /*green weight merchantable estimate. this line
was used in the
 original code so I could determine the coefficients used above in the
pred_ratio lines.*/
 if ht = . then delete;
 if ht = 0.5 then delete;
 if ht = 17.3 then delete;
 if tree_no_=65 then delete;
 if tree_no_=66 then delete;
 if tree_no_=67 then delete;
 if tree_no_=68 then delete;
 if tree_no_ =69 then delete; /*deleted these because no weight for
these trees*/
 rel_ht=ht/tot_ht;      /* relative height on a scale of 0 to 1*/
 rel_diameter=dob_in/dbh; /*relative diameter on a scale of 0 to 1*/

 wtrestm=twe*pred_ratio_dob; /* merc weight estimate with ratio*/
 wterestm=twe*pred_eratio_dob; /*merc weight estimate with e ratio*/
 wto = tot_wt;                /*total weight observed - placed here
for ease of reference*/
 run;
```

```
 /*new model with gwme*/
   proc nlin maxiter=200 data=vp.vpw outest=vp.ratio_wt_parms_dob;
 /*outest =vp.ratio_wt_parms_dob   gives me larger number of sig digits
in output*/
 parameters b1=1 b2=1 b3=1;
 model gwme = tot_wt*(1+b1*((dob_in**b2)/(dbh**b3)));
 output out=vp.gwme_ratio_resid_dob p=gwme_pred1 r=gwme_res1;
 /*output out= the name of your new output lib, p = predicted values r
= residuals*/
 /*diff=dib_in-dbh; add this line to create a column that gives the
difference
   between the dib and dbh or any other variables*/
 title'ratio dob';
 run;

/*code for ratio dib*/
proc nlin maxiter=200 data=vp.vpw outest=vp.ratio_wt_parms_dib;
/*outest = gives me larger number of sig digits in output*/
parameters b4=1 b5=1 b6=1;
model gwme = tot_wt*(1+b4*((dib_in**b5)/(dbh**b6)));
output out=vp.gwme_ratio_resid_dib p=gwme_pred2 r=gwme_res2;
/*output out= the name of your new output lib, p = predicted values r =
residuals*/
/*diff=dib_in-dbh; add this line to create a column that gives the
difference
  between the dib and dbh or any other variables*/
title' ratio dib';
run;

/*code for ratio ht*/
proc nlin maxiter=300 data=vp.vpw outest=vp.ratio_wt_parms_ht;
parameters a1=1 a2=1 a3=1;
model gwme= tot_wt*(1+a1*(((tot_ht - ht)**a2)/(tot_ht**a3)));
output out=vp.gwme_ratio_residht p=gwme_pred3 r=gwme_res3;
title'ratio ht';
run;

/*code for exponential ratio dob*/

proc nlin maxiter=200 data=vp.vpw outest=vp.eratio_wt_parms_dob;
parameters b7=1 b8=1 b9=1;
model gwme = tot_wt*(exp(b7*((dob_in**b8)/(dbh**b9))));
output out=vp.gwme_res_eratio_dob p=gwme_pred4 r=gwme_res4;
title'exponential ratio dob';
run;

/*code for exponential ratio dib*/
proc nlin maxiter=200 data=vp.vpw outest=vp.eratio_wt_parms_dib;
parameters b10=1 b11=1 b12=1;
model gwme = tot_wt*(exp((b10*(dib_in)**b11)/(dbh**b12)));
output out=vp.gwme_res_eratio_dib p=gwme_pred5 r=gwme_res5;
title'exponential ratio dib';
run;
```

```
/*code for exponential ratio ht*/
proc nlin maxiter=300 data=vp.vpw outest=vp.eratio_wt_parms_ht;
parameters a4=1 a5=1 a6=1;
model gwme=tot_wt*(exp(a4*(((tot_ht-ht)**a5)/(tot_ht**a6))));
output out=vp.gwme_res_eratio_ht p=gwme_pred6 r=gwme_res6;
title'exponential ratio ht';
run;
```

Green Weight Mixed-Effects

```
data vp.vpwmixed;
set  vp.vpw;
gwmd=tot_wt*R;
gwmdpr=(1-45.4307*(dob_in**3.1545/dbh**4.8595));
gwmh=tot_wt*R;
H=tot_ht;
run;

/* ratio equation dob*/
proc nlmixed data=vp.vpwmixed;
parms b1=-.6067 b2=3.1282 b3=2.99 su=.7715641 se=138.3;
x=(b1+u1);
z=(x*(dob_in**b2/dbh**b3));
R=(1+z); /*R = ratio including the z and the fixed effects betas*/
model gwmd ~ normal(tot_wt*R,se*se); /*gwmd = green weight merchantable
diameter*/
random u1 ~ normal(0,su*su) subject = tree_no_;
title'ratio dob';
run;

/*ratio equation dib*/
proc nlmixed data=vp.vpwmixed;
parms b4=-.8326 b5=3.3959 b6=3.3196 su=.9045340237 se=141.46;
x=(b4+u2);
z=(x*(dib_in**b5/dbh**b6));
R=(1+z); /*R = ratio including the z and the fixed effects betas*/
model gwmd ~ normal(tot_wt*R,se*se); /*gwmd = green weight merchantable
diameter*/
random u2 ~ normal(0,su*su) subject = tree_no_;
title'ratio dib';
run;

/*ratio height*/

proc nlmixed data=vp.vpwmixed;
parms a1=-.7823 a2=2.0352 a3=1.9761 su=.815756 se=34.70;
x=(a1+u3);
z=(x*((H-ht)**a2/H**a3));
R=(1+z); /*R = ratio including the z and the fixed effects betas*/
model gwmh ~ normal(tot_wt*R,se*se); /*gwmd = green weight merchantable
height*/
```

```
random u3 ~ normal(0,su*su) subject = tree_no_;
title'ratio height';
run;

/*exponential ratio dob*/
proc nlmixed data=vp.vpwmixed;
parms b7=-.9404 b8=5.6707 b9=5.3741 su=3.192164 se=110.01;
x=(b7+u4);
z=(x*(dob_in**b8/dbh**b9));
R=exp(z);
model gwmd ~ normal(tot_wt*R,se*se); /*gwmd = green weight merchantable
diameter*/
random u4 ~ normal(0,su*su) subject = tree_no_;
title'exponential ratio dob';
run;

/*exponential ratio dib*/

proc nlmixed data=vp.vpwmixed;
parms b10=-1.4980 b11=5.8488 b12=5.6393 su=3.7565716 se=124.72;
x=(b10+u5);
z=(x*(dib_in**b11/dbh**b12));
R=exp(z);
model gwmd ~ normal(tot_wt*R,se*se); /*gwmd = green weight merchantable
diameter*/
random u5 ~ normal(0,su*su) subject = tree_no_;
title'exponential ratio dib';
run;

/*exponential ratio height*/

proc nlmixed data=vp.vpwmixed;
parms a4=-1.0973 a5=3.0504 a6=2.8885 su=1.501733632 se=47.20;
x=(a4+u6);
z=(x*((H-ht)**a5/H**a6));
R=exp(z);
model gwmh ~ normal(tot_wt*R,se*se); /*gwmh = green weight merchantable
height*/
random u6 ~ normal(0,su*su) subject = tree_no_;
title'exponential ratio height';
run;
```

Total Volume Fixed Effects

```
proc nlin maxiter=200 data=vp.vpvm4 outest=vp.ratio_v_parms_dob;
parameters b13=1 b14=1 b15=1;
model omvoldob = tot_vol_dob*(1+b13*(dob_in**b14/dbh**b15));
output out=vp.rres_dobv p=pred_ratio_dobvolume r=res_ratio_dobvolume;
/*output out= the name of your new output lib, p = predicted values r =
residuals*/
/*diff=dib_in-dbh; add this line to create a column that gives the
difference
  between the dib and dbh or any other variables*/
title'volume ratio dob';
run;
```

```
proc gplot data=vp.rres_dobv;
plot res_ratio_dobvolume*pred_ratio_dobvolume;
title 'res vs pred ratio dob volume';
run;

/* proc gplot plots the residuals vs predicted*/


/*ratio volume dib*/
proc nlin maxiter=200 data=vp.vpvm4 outest=vp.ratio_v_parms_dib;
parameters b16=1 b17=1 b18=1;
model omvoldib =tot_vol_dib* (1+b16*(dib_in**b17/dbh**b18));
output out=vp.rres_dibv p=pred_ratio_dibvolume r=res_ratio_dibvolume;
/*output out= the name of your new output lib, p = predicted values r =
residuals*/
/*diff=dib_in-dbh; add this line to create a column that gives the
difference
  between the dib and dbh or any other variables*/
title'volume ratio dib';
run;

proc gplot data=vp.rr_dibv;
plot res_ratio_dibvolume*pred_ratio_dibvolume;
title 'res vs pred ratio dib volume';
run;


/*ratio volume height dob*/
proc nlin maxiter=300 data=vp.vpvm4 outest=vp.ratio_v_parms_dob_ht;
parameters a7=1 a8=1 a9=1;
model omvolhtdob=tot_vol_dob * (1+a7*((tot_ht - ht)**a8/tot_ht**a9));
output out=vp.rres_dobhtv p=pred_ratio_dobht r=res_ratio_dobht;
title'volume ratio height dob';
run;
/*Rdob=cum_vol_dob/tot_vol_dob*/
proc gplot data=vp.rres_dobhtv;
plot res_ratio_dobht*pred_ratio_dobht;
title 'res vs pred ratio ht dob volume';
run;


/*ratio volume height dib*/
proc nlin maxiter=300 data=vp.vpvm4 outest=vp.ratio_v_parms_dib_ht;
parameters a10=1 a11=1 a12=1;
model omvolhtdib = tot_vol_dib* (1+a10*((tot_ht -
ht)**a11/tot_ht**a12));
output out=vp.rres_dibhtv p=pred_ratio_dibht r=res_ratio_dibht;
title'volume ratio height dib';
run;
/*Rdib=cum_vol_dib/tot_vol_dib*/
proc gplot data=vp.rres_dibhtv;
plot res_ratio_dibht*pred_ratio_dibht;
title 'res vs pred ratio ht dib volume';
run;
```

```
/* CODE FOR EXPONENTIAL RATIOS BEGINS HERE*/

/*Exponential ratio volume dob*/
proc nlin maxiter=200 data=vp.vpvm4 outest=vp.ratio_ev_parms_dob;
parameters b19=1 b20=1 b21=1;
model omvoldob = tot_vol_dob* (exp(b19*(dob_in)**b20/dbh**b21));
output out=vp.erres_dobv p=pred_eratio_dobvolume
r=res_eratio_dobvolume;
title' exponential ratio volume dob';
run;
/*Rdob=cum_vol_dob/tot_vol_dob)*/

proc gplot data=vp.erres_dobv;
plot res_eratio_dobvolume*pred_eratio_dobvolume;
title 'res vs pred exponential ratio dob volume';
run;

/*Exponential ratio volume dib */
proc nlin maxiter=200 data=vp.vpvm4 outest=vp.ratio_ev_parms_dib;
parameters b22=1 b23=1 b24=1;
model omvoldib = tot_vol_dib *(exp(b22*(dib_in)**b23/dbh**b24));
output out=vp.erres_dibv p=pred_eratio_dibvolume
r=res_eratio_dibvolume;
title' exponential ratio volume dib';
run;
/*Rdob=cum_vol_dob/tot_vol_dob)*/

proc gplot data=vp.erres_dibv;
plot res_eratio_dibvolume*pred_eratio_dibvolume;
title 'res vs pred exponential ratio dib volume';
run;

/*Exponential ratio volume ht dob*/
proc nlin maxiter=300 data=vp.vpvm4 outest=vp.ratio_ev_parms_dob_ht;
parameters a13=1 a14=1 a15=1;
model omvolhtdob =tot_vol_dob * (exp(a13*((tot_ht-
ht)**a14/(tot_ht**a15))));
output out=vp.erres_dobhtv p=pred_eratio_dobht r=res_eratio_dobht;
title'exponential ratio volume ht dob';
run;
proc gplot data=vp.erres_dobhtv;
plot res_eratio_dobht*pred_eratio_dobht;
title 'res vs pred exponential ratio dob height';
run;

/*Exponential ratio volume ht dib*/
proc nlin maxiter=300 data=vp.vpvm4 outest=vp.ratio_ev_parms_dib_ht;
parameters a16=1 a17=1 a18=1;
model omvolhtdib =tot_vol_dib * (exp(a16*((tot_ht-
ht)**a17/(tot_ht**a18))));
output out=vp.erres_dibhtv p=pred_eratio_dibht r=res_eratio_dibht;
title'exponential ratio volume ht dib';
run;
proc gplot data=vp.erres_dibhtv;
plot res_eratio_dibht*pred_eratio_dibht;
title 'res vs pred exponential ratio dib height';
```

```
run;
quit;
```

Total Volume Mixed-Effects

```
data vp.vpvm4mixed;
set  vp.vpvm4;
volmdob=tot_vol_dob*Rdob;
volmdib=tot_vol_dib*Rdib;
volmhdob=tot_vol_dob*Rdob;
volmhdib=tot_vol_dib*Rdib;
H=tot_ht;
run;

/* ratio equation dob*//*this uses the dbldog instead of default
quanew|*/
proc nlmixed data=vp.vpvm4mixed;
parms b13=-.6451 b14=3.3419 b15=3.2400 su=.8336821 se=2.65252;
x=(b13+u7);
z=(x*(dob_in**b14/dbh**b15));
Rdob=(1+z); /*R = ratio including the z and the fixed effects betas*/
model volmdob ~ normal(tot_vol_dob*Rdob,se*se); /*gwmd = green weight
merchantable dob*/
random u7 ~ normal(0,su*su) subject = tree_no_;
title'volume ratio dob';
run;


/*ratio equation dib*/
proc nlmixed data=vp.vpvm4mixed;
parms b16=-.9407 b17=3.6294 b18=3.5825 su=1.010291 se=2.57848;
x=(b16+u8);
z=(x*(dib_in**b17/dbh**b18));
Rdib=(1+z); /*R = ratio including the z and the fixed effects betas*/
model volmdib ~ normal(tot_vol_dib*Rdib,se*se); /*gwmd = green weight
merchantable diameter*/
random u8 ~ normal(0,su*su) subject = tree_no_;
title'volume ratio dib';
run;

/*ratio height dob*/

proc nlmixed data=vp.vpvm4mixed lis=3;
parms a7=-1.0530 a8=2.2552 a9=2.2619 su=1.132907 se=.554707;
x=(a7+u9);
z=(x*((H-ht)**a8/H**a9));
Rdob=(1+z); /*R = ratio including the z and the fixed effects betas*/
model volmhdob ~ normal(tot_vol_dob*Rdob,se*se); /*gwmd = green weight
merchantable height*/
random u9 ~ normal(0,su*su) subject = tree_no_;
title'ratio height dob';
run;

/*ratio height dib*/
```

```
proc nlmixed data=vp.vpvm4mixed;
parms a10=-1.1282 a11=2.2489 a12=2.2689 su=1.136348 se=.522302;
x=(a10+u10);
z=(x*((H-ht)**a11/H**a12));
Rdib=(1+z); /*R = ratio including the z and the fixed effects betas*/
model volmhdib ~ normal(tot_vol_dib*Rdib,se*se); /*gwmd = green weight
merchantable height*/
random u10 ~ normal(0,su*su) subject = tree_no_;
title'ratio height dib';
run;

/*exponential ratio dob*/
proc nlmixed data=vp.vpvm4mixed;
parms b19=-1.0592 b20=6.1280 b21=5.8798 su=4.153727 se=2.144131;
x=(b19+u11);
z=(x*(dob_in**b20/dbh**b21));
Rdob=exp(z);
model volmdob ~ normal(tot_vol_dob*Rdob,se*se); /*gwmdob = green weight
merchantable diameter*/
random u11 ~ normal(0,su*su) subject = tree_no_;
title'volume exponential ratio dob';
run;

/*exponential ratio dib*/

proc nlmixed data=vp.vpvm4mixed;
parms b22=-1.7391 b23=6.3587 b24=6.1939 su=4.003467797 se=2.2917024;
x=(b22+u12);
z=(x*(dib_in**b23/dbh**b24));
Rdib=exp(z);
model volmdib ~ normal(tot_vol_dib*Rdib,se*se); /*gwmd = green weight
merchantable diameter*/
random u12 ~ normal(0,su*su) subject = tree_no_;
title'volume exponential ratio dib';
run;

/*exponential ratio height dob*/

proc nlmixed data=vp.vpvm4mixed;
parms a13=-2.0008 a14=3.3780 a15=3.3528 su=1 se=.9134549;
x=(a13+u13);
z=(x*((H-ht)**a14/H**a15));
Rdob=exp(z);
model volmhdob ~ normal(tot_vol_dob*Rdob,se*se); /*gwmh = green weight
merchantable height*/
random u13 ~ normal(0,su*su) subject = tree_no_;
title'exponential ratio height dob';
run;

/*exponential ratio height dib*/

proc nlmixed data=vp.vpvm4mixed;
parms a16=-2.2831 a17=3.4024 a18=3.4007 su=.8476437 se=.8476437;
x=(a16+u14);
z=(x*((H-ht)**a17/H**a18));
Rdib=exp(z);
```

```
model volmhdob ~ normal(tot_vol_dib*Rdib,se*se); /*gwmh = green weight
merchantable height*/
random u14 ~ normal(0,su*su) subject = tree_no_;
title'exponential ratio height dib';
```