LARGE DEFLECTIONS OF VISCOELASTIC ORTHOTROPIC CYLINDRICAL SHELLS

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ABSTRACT

Using the convolutional variational principle of minimum potential energy for viscoelastic bodies the basic equations of large deflection theory for viscoelastic anisotropic shallow shells are derived. These equations correspond to Kármán-Donnel equations for elastic shallow shells. Then the solution of boundary value problems is transformed into the solution of a system of non-linear integro-differential equations. Further the symmetrical buckling of a cylindrical shallow shell, which is uniformly compressed in the axial direction, is analysed. It is shown that, in general, there can occur not only infinite critical time but also finite critical times.

1. INTRODUCTION

The modern non-linear theory of viscoelasticity, noted by its utmost generality, is not very convenient for the solution of boundary value problems. Green and Rivlin [1] have shown that general operators of constitutive equations may be expressed to any desired approximation by the sum of multiple integral operators. The first order approximation by a single integral operator generalizes the infinitisimal theory of viscoelasticity to finite deformation theory of viscoelasticity. Brilla [2] has shown that also this theory is very complicated for the solution of boundary value problems.

In this paper we shall deal with the further simplification of the general theory for two dimmensional bodies corresponding to large deflection theory of elastic plates and shells. Thus we shall assume that deflections of shells are not small in comparison with the thickness of the shell but are still small as compared with other dimensions.

2. CONSTITUTIVE EQUATIONS

Consider quasi-static problems in which inertia forces due to deformations are negligible. The constitutive equation of an arbitrary linear viscoelastic material can be written in the form

$$\sigma^{ij} = H^{ijkl}\,\varepsilon_{kl}\,,$$

where H^{ijkl} represents a tensor operator. On the basis of the assumptions of Onsager's theory for linear rheological models, this tensor operator is symmetric. In addition, on the basis of the second law of thermodynamics it may be proved that this operator is positive definite. H^{ijkl} can have an integral or differential form. Then according to Brilla [3,4,5] constitutive equation /1/ assume the following forms

$$H^{(r)}\sigma^{ij} = H^{ijkl}_{(s)}\ \varepsilon_{kl} \qquad /2/$$

or

$$Q^{(r)}\varepsilon_{ij} = Q^{(s)}_{ijkl}\ \sigma^{kl} \qquad /3/$$

where

$$H^{(r)} = \prod_{n=1}^{r}\left(\frac{\partial}{\partial t} + \varkappa_n\right), \qquad Q^{(r)} = \prod_{n=1}^{r}\left(\frac{\partial}{\partial t} + \lambda_n\right) \qquad /4/$$

are scalar operators and

$$H^{ijkl}_{(s)} = \sum_{n=1}^{s} H^{ijkl}_{(n)}\ \frac{\partial^n}{\partial t^n}\,, \qquad Q^{(s)}_{ijkl} = \sum_{n=1}^{s} Q^{(n)}_{ijkl}\ \frac{\partial^n}{\partial t^n} \qquad /5/$$

are tensor operators, $\varkappa \geq 0$ are inverse relaxation times, $\lambda_n \geq 0$ are inverse retardation times and $K^{(o)} = 1$, $Q^{(o)} = 1$. As it was proved by Brilla [3] tensor operators can not be on both sides of Eqs./2-3/.

In the case of a homogeneous relaxation spectrum Eq./5/ assume the form

$$H^{ijkl}_{(s)} = H^{ijkl}\sum_{n=1}^{s} h_n\,\frac{\partial^n}{\partial t^n}\,, \qquad Q^{(s)}_{ijkl} = Q_{ijkl}\sum_{n=1}^{s} g_n\,\frac{\partial^n}{\partial t^n}\ . \qquad /6/$$

3. BASIC EQUATIONS OF A VISCOELASTIC SHALLOW SHELL

The basic equations of the large deflection theory for viscoelastic anisotropic shallow shells follow from the convolutional variational principle of minimum potential energy derived for viscoelastic bodies by Brilla [2].

$$\Pi = \int_0^t\!\!\iiint_V \frac{1}{2}\sigma^{\alpha\beta}(\tau)\,\varepsilon_{\alpha\beta}(t-\tau)\,dV\,d\tau - \int_0^t\!\!\iint_S q(t)\,w(t-\tau)\,dS\,d\tau\,, \qquad /7/$$

where q is the transverse load, w the normal /transverse/ displacement of the shell and the indices α , β assume the values 1, 2 .

According to the assumptions of the shallow shell theory we have

$$\varepsilon_{\alpha\beta} = \frac{1}{2}\left(u_{\alpha/\beta} + u_{\beta/\alpha} + w_{/\alpha}\,w_{/\beta}\right) + b_{\alpha\beta}\,w - x_3\,w_{/\alpha\beta} \qquad /8/$$

where $b_{\alpha\beta}$ is the curvature tensor, u_α the tangential displacements and

$$\varepsilon^{(0)}_{\alpha\beta} = \frac{1}{2}\left(u_{\alpha/\beta} + u_{\beta/\alpha} + w_{/\alpha}\, w_{/\beta}\right) \qquad /9/$$

are middle surface strains. Commas dentoe covariant differentiation with respect to surface coordinates ξ_1, ξ_2.

Integrating Eq./7/ with respect to x_3 through the thickness of the shell $(-h/2,\ h/2)$ we obtain

$$\int_0^t\!\!\iint_S \left\{\frac{1}{2}\left[-M^{\alpha\beta}(\tau)\, w_{/\alpha\beta}(t-\tau) + N^{\alpha\beta}(\tau)\, \varepsilon^{(0)}_{\alpha\beta}(t-\tau)\right] - q(\tau)\, w(t-\tau)\right\} dS\, d\tau. \qquad /10/$$

We shall consider the combination of the following boundary conditions

$$w = 0, \qquad w_{/n} = 0 \qquad \text{on } \partial S, \qquad /11/$$

or

$$w = 0, \qquad M_{nn} = 0 \qquad \text{on } \partial S, \qquad /12/$$

and

$$N_{nn} = 0, \qquad N_{ns} = 0 \qquad \text{on } \partial S, \qquad /13/$$

or

$$u_1 = 0, \qquad u_2 = 0 \qquad \text{on } \partial S. \qquad /14/$$

Then the basic equations for viscoelastic shallow shells can be obtained from the convolutional principle of minimum potential energy constrained by the condition Eq./9/

$$\int_0^t\!\!\iint_S \Big\{\frac{1}{2}\left[-M^{\alpha\beta}(\tau)\, w_{/\alpha\beta}(t-\tau) + N^{\alpha\beta}(\tau)\, \varepsilon_{\alpha\beta}(t-\tau)\right] - N^{\alpha\beta}(\tau)\Big[\varepsilon^{(0)}_{\alpha\beta}(t-\tau) - \frac{1}{2}\big(u_{\alpha/\beta}(t-\tau) + u_{\beta/\alpha}(t-\tau) + w_{/\alpha}(t-\tau)\, w_{/\beta}(t-\tau)\big) - b_{\alpha\beta}\, w(t-\tau)\Big] - q(t)\, w(t-\tau)\Big\}\, dS\, d\tau \qquad /15/$$

where $N^{\alpha\beta}$ is the Lagrange multiplies.

Using the last term of Eq./8/ and constitutive equations we have

$$M^{\alpha\beta} = -D^{\alpha\beta\gamma\delta}\, w_{/\gamma\delta} \qquad /16/$$

where

$$D^{\alpha\beta\gamma\delta} = \frac{h^3}{12} H^{\alpha\beta\gamma\delta} \qquad /17/$$

Similarly it holds

$$h\varepsilon^{(0)}_{\alpha\beta} = G_{\alpha\beta\gamma\delta}\, N^{\gamma\delta} \qquad /18/$$

where

$$G_{\alpha\beta\gamma\delta} = (H^{\alpha\beta\gamma\delta})^{-1}. \qquad /19/$$

Introducing the stress function

$$h\sigma^{\alpha\beta} - N^{\alpha\beta} = \epsilon^{\alpha\gamma}\epsilon^{\beta\delta} F_{,\gamma\delta}\,, \qquad /20/$$

where $\epsilon^{\alpha\beta}$ is the alternating tensor we arrive at

$$\mathcal{E}^{(o)}_{\alpha\beta} = G_{\alpha\beta\gamma\delta}\ \epsilon^{\gamma\mu}\epsilon^{\delta\nu} F_{,\mu\nu} \qquad /21/$$

After inserting of Eqs./16,20/ into Eq./15/ the calculus of variation leads to the basic equations of viscoelastic anisotropic shallow shells.

$$K^{\alpha\beta\gamma\delta} F_{,\alpha\beta\gamma\delta} = h\epsilon^{\alpha\gamma}\epsilon^{\beta\delta}(b_{\alpha\beta}\, w_{,\gamma\delta} + \tfrac{1}{2}\, w_{,\alpha\delta}\, w_{,\beta\gamma}), \qquad /22/$$

$$D^{\alpha\beta\gamma\delta} w_{,\alpha\beta\gamma\delta} = q + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta} F_{,\gamma\delta}\ (w_{,\alpha\beta} - b_{\alpha\beta}), \qquad /23/$$

where we have denoted

$$K^{\alpha\beta\gamma\delta} = \epsilon^{\alpha\mu}\epsilon^{\beta\nu}\epsilon^{\gamma\varkappa}\epsilon^{\delta\rho}\ G_{\mu\nu\varkappa\rho}\,. \qquad /24/$$

As a special case we shall consider a viscoelastic shallow shell of a Zener material with the homogeneous relaxation spectrum of Maxwell element /Fig.1/.

According to Brilla [4] the Laplace transform of the constitutive equation has the following form

$$(p + k)\,\tilde{\sigma}^{\alpha\beta} = (pE^{\alpha\beta\gamma\delta} + kE_1^{\alpha\beta\gamma\delta})\,\tilde{\mathcal{E}}_{\gamma\delta} \qquad /25/$$

where

$$E^{\alpha\beta\gamma\delta} = E_1^{\alpha\beta\gamma\delta} + E_2^{\alpha\beta\gamma\delta} \qquad /26/$$

$E^{\alpha\beta\gamma\delta}$ is tensor of moduli of elasticity and symbols with tildas denote Laplace transforms.

Solving this equation we obtain

$$\Delta(p)\,\tilde{\mathcal{E}}_{\alpha\beta} = (p + k)\, F_{\alpha\beta\gamma\delta}\ \tilde{\sigma}^{\gamma\delta} \qquad /27/$$

where, in general, the determinant $\Delta(p) = |pE^{\alpha\beta\gamma\delta} + kE_1^{\alpha\beta\gamma\delta}|$ is a polynomial of p of degree 3 and the adjoint matrix $F_{\alpha\beta\gamma\delta}(p)$ is a p - matrix of degree 2 .

Expanding Eq./27/ in partial fractions, denoting the roots of the determinantal equation $\Delta(p) = 0$ by $-\lambda_n$, we obtain for different roots

$$\tilde{\varepsilon}_{\alpha\beta} = \sum_{n=1}^{3} \frac{p+\kappa}{p+\lambda_n} A_{\alpha\beta\gamma\delta}(\lambda_n)\, \tilde{\sigma}^{\gamma\delta}, \qquad /28/$$

where

$$A_{\alpha\beta\gamma\delta}(\lambda_n) = \frac{F_{\alpha\beta\gamma\delta}(-\lambda_n)}{\Delta^{(1)}(-\lambda_n)} \qquad /29/$$

and $\Delta^{(1)}(-\lambda_n)$ is the first derivative of $\Delta(p)$ with respect to p for $p = -\lambda_n$.
Equ./28/ can be written in the form

$$A\tilde{\varepsilon}_{\alpha\beta} = (p+\kappa)\, A_{\alpha\beta\gamma\delta}\, \tilde{\sigma}^{\gamma\delta}, \qquad /30/$$

where

$$A = p^3 + A_2 p^2 + A_1 p + A_0, \qquad /31/$$

$$A_{\alpha\beta\gamma\delta} = A^{(2)}_{\alpha\beta\gamma\delta} p^2 + A^{(1)}_{\alpha\beta\gamma\delta} p + A^{(0)}_{\alpha\beta\gamma\delta}. \qquad /32/$$

and

$$A_3 = 1, \qquad A_2 = \lambda_1 + \lambda_2 + \lambda_3,$$

$$A_2 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3, \quad A_0 = \lambda_1\lambda_2\lambda_3,$$

$$A^{(2)}_{\alpha\beta\gamma\delta} = \sum_{n=1}^{3} A_{\alpha\beta\gamma\delta}(\lambda_n),$$

$$A^{(1)}_{\alpha\beta\gamma\delta} = (\lambda_2+\lambda_3) A_{\alpha\beta\gamma\delta}(\lambda_1) + (\lambda_1+\lambda_3) A_{\alpha\beta\gamma\delta}(\lambda_2) + (\lambda_1+\lambda_2) A_{\alpha\beta\gamma\delta}(\lambda_3),$$

$$A^{(0)}_{\alpha\beta\gamma\delta} = \lambda_2\lambda_3 A_{\alpha\beta\gamma\delta}(\lambda_1) + \lambda_1\lambda_3 A_{\alpha\beta\gamma\delta}(\lambda_2) + \lambda_1\lambda_2 A_{\alpha\beta\gamma\delta}(\lambda_3).$$

After inversion it holds

$$\left(\frac{\partial^3}{\partial t^3} + A_2\frac{\partial^2}{\partial t^2} + A_1\frac{\partial}{\partial t} + A_0\right)\varepsilon_{\alpha\beta} = \qquad /33/$$

$$\left(\frac{\partial}{\partial t} + \kappa\right)\left(A^{(2)}_{\alpha\beta\gamma\delta}\frac{\partial^2}{\partial t^2} + A^{(1)}_{\alpha\beta\gamma\delta}\frac{\partial}{\partial t} + A^{(0)}_{\alpha\beta\gamma\delta}\right)\sigma^{\gamma\delta}$$

The basic equations of large deflections of a shallow shell of a Zener material then assume the form

$$\left(\frac{\partial}{\partial t} + k\right) K^{\alpha\beta\gamma\delta} F_{,\alpha\beta\gamma\delta} = \qquad /34/$$

$$= \sum_{n=0}^{3} A_n \frac{\partial^n}{\partial t^n} \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\left(b_{\alpha\beta}\, w_{,\gamma\delta} + \frac{1}{2} w_{,\alpha\delta}\, w_{,\beta\gamma}\right)$$

$$\left(k D_1^{\alpha\beta\gamma\delta} + \frac{\partial}{\partial t} D^{\alpha\beta\gamma\delta}\right) w_{,\alpha\beta\gamma\delta} = \qquad /35/$$

$$= \left(k + \frac{\partial}{\partial t}\right)\left[q + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\left(w_{,\alpha\beta} - b_{\alpha\beta}\right)F_{,\gamma\delta}\right],$$

where

$$K^{\alpha\beta\gamma\delta} = \epsilon^{\alpha\mu}\epsilon^{\beta\nu}\epsilon^{\gamma\kappa}\epsilon^{\delta\rho} A_{\mu\nu\kappa\rho}. \qquad /36/$$

These equations correspond to Donnel-Kármán equation for elastic shallow shells.

4. INTEGRO-DIFFERENTIAL EQUATIONS OF SHALLOW ANISOTROPIC VISCOELASTIC SHELLS

When dealing with boundary value problems for large deflections of shallow viscoelastic shells it is advantageous to replace the original problem by a solution of non-linear integro-differential equations.

We consider the basic equations of shallow viscoelastic shells Equ./22,23/. Applying formally Laplace transformation we arrive at

$$\tilde{D}^{\alpha\beta\gamma\delta}\, \tilde{w}_{,\alpha\beta\gamma\delta} = \tilde{q} + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\left(\widetilde{w_{,\alpha\beta}\, F_{,\gamma\delta}} - b_{\alpha\beta}\, \widetilde{F_{,\gamma\delta}}\right), \qquad /37/$$

$$\tilde{K}^{\alpha\beta\gamma\delta}\, \tilde{F}_{,\alpha\beta\gamma\delta} = h\epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\left(b_{\alpha\beta}\, \tilde{w}_{,\gamma\delta} + \frac{1}{2}\widetilde{w_{,\alpha\delta}\, w_{,\beta\gamma}}\right), \qquad /38/$$

where the Laplace transform, as it is denoted by tildas, is applied to the whole non-linear terms and not to single terms, separately.

Denoting Green functions of the left hand sides of Equ./37,38/ with appropriate boundary conditions by $\tilde{G}_1$, $\tilde{G}_2$ we obtain

$$\tilde{w} = \iint_S \tilde{G}_1(x_1 - \xi_1, x_2 - \xi_2, p)\left[\tilde{q} + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\left(\widetilde{w_{,\alpha\beta} F_{,\gamma\delta}} - b_{\alpha\beta}\tilde{F}_{,\gamma\delta}\right)\right] d\xi_1\, d\xi_2, \qquad /39/$$

$$\tilde{F} = \iint_S \tilde{G}_2(x_1 - \xi_1, x_2 - \xi_2, p)\, h\epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\left(b_{\alpha\beta}\, \tilde{w}_{,\gamma\delta} + \frac{1}{2}\widetilde{w_{,\alpha\delta}\, w_{,\beta\gamma}}\right) d\xi_1\, d\xi_2 . \qquad /40/$$

Using convolution theorem we find that the inverse transform gives

$$w = \int_0^t \iint_S G_1(x_1-\xi_1, x_2-\xi_2, t-\tau)\left[q + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}(w_{,\alpha\beta} - b_{\alpha\beta})F_{,\gamma\delta}\right] d\xi_1\, d\xi_2\, d\tau \qquad /41/$$

$$F = \int_0^t \iint_S G_2(x_1-\xi_1, x_2-\xi_2, t-\tau)\, h\, \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\left(b_{\alpha\beta}\, w_{,\gamma\delta} + \frac{1}{2} w_{,\alpha\delta}\, w_{,\beta\gamma}\right) d\xi_1\, d\xi_2\, d\tau\,. \qquad /42/$$

Thus we have arrived at a system of non-linear integro-differential equations for shallow viscoelastic shells.

This system can be solved by the method of successive approximation. As the first approximation we take the linear solution for

$$w_1 = \int_0^t \iint_S G_1(x_1-\xi_1, x_2-\xi_2, t-\tau)\, q(\xi_1, \xi_2, \tau)\, d\xi_1\, d\xi_2\, d\tau \qquad /43/$$

and

$$F_1 = \int_0^t \iint_S G_2(x_1-\xi_1, x_2-\xi_2, t-\tau)\, h\, \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\left(b_{\alpha\beta}\, w_{1,\gamma\delta} + \frac{1}{2} w_{1,\alpha\delta}\, w_{1,\beta\gamma}\right) d\xi_1\, d\xi_2\, d\tau\,. \qquad /44/$$

Continuing this process we find as the nth approximation

$$w_n = \int_0^t \iint_S G_1(x_1-\xi_1, x_2-\xi_2, t-\tau)\left[q + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}(w_{n-1,\alpha\beta} - b_{\alpha\beta})F_{n-1,\gamma\delta}\right] d\xi_1\, d\xi_2\, d\tau\,, \qquad /45/$$

$$F_n = \int_0^t \iint_S G_2(x_1-\xi_1, x_2-\xi_2, t-\tau)\, h\, \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\left(b_{\alpha\beta}\, w_{n,\gamma\delta} + \frac{1}{2} w_{n,\alpha\delta}\, w_{n,\beta\gamma}\right) d\xi_1\, d\xi_2\, d\tau\,. \qquad /46/$$

In the case of a homogenenous relaxation spectrum the integro-differential equations /41,42/ assume the form

$$\sum_{n=1}^{s} h_n \frac{\partial^n}{\partial t^n} w = \iint_S G_1(x_1-\xi_1, x_2-\xi_2) H^{(n)}\left[q + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}(w_{,\alpha\beta} - b_{\alpha\beta})F_{,\gamma\delta}\right] d\xi_1\, d\xi_2 \qquad /47/$$

$$H^{(r)} F = \iint_S G(x_1-\xi_1, x_2-\xi_2) \sum_{n=1}^{s} h_n \frac{\partial}{\partial t^n} \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\left(b_{\alpha\beta}\, w_{,\gamma\delta} + \frac{1}{2} w_{,\alpha\delta}\, w_{,\beta\gamma}\right) d\xi_1\, d\xi_2 \qquad /48/$$

where G_1, G_2 are Green functions of the elastic shallow shell. This system of integro-differential equations can be solved by the method of successive approximations, too.

5. STABILITY OF A CYLINDRICAL VISCOELASTIC SHELL

Due to viscoelastic behaviours the deformations of a viscoelastic shell increase with time. This process can lead to instability of the shell. In what follows we shall deal with the stability of a cylindrical viscoelastic shell uniformly compressed in the axial direction.

In the known way we obtain from the basic equations of large deflection theory of viscoelastic shallow shells the differential equations for linear buckling

$$D^{\alpha\beta\gamma\delta} w_{,\alpha\beta\gamma\delta} + \epsilon^{\alpha\beta}\epsilon^{\beta\delta} b_{\alpha\beta} F_{,\gamma\delta} - w_{,\alpha\beta} N_{\alpha\beta} = 0, \qquad /49/$$

$$K^{\alpha\beta\gamma\delta} F_{,\alpha\beta\gamma\delta} - h\epsilon^{\alpha\gamma}\epsilon^{\beta\delta} b_{\alpha\beta} w_{,\gamma\delta} = 0. \qquad /50/$$

We shall consider an orthotropic viscoelastic cylinder uniformly compressed in the axial direction. Then buckling symmetrical with respect to the axis of the cylinder may occur at a certain value of the compressive stress.

In discussing the stability of a cylinder we assume that the generator of the shell is vertical and parallel to x_1 - axis, x_2 is in the direction of the tangent to the normal cross section and x_3 in the direction of the normal to the shell.

Then

$$b_{11} = 0, \qquad b_{22} = \frac{1}{R} \qquad /51/$$

and

$$N_{11} = -N, \qquad N_{12} = N_{22} = 0. \qquad /52/$$

Thus the differential equations for linear buckling become

$$K_{2222} F_{,1111} - \frac{h}{R} w_{,11} = 0, \qquad /53/$$

$$D_{1111} w_{,1111} + \frac{1}{R} F_{,11} + N w_{,11} = 0. \qquad /54/$$

Eliminating w we arrive at

$$\frac{R}{h} D_{1111} K_{2222} F_{,111111} + \frac{1}{R} F_{,11} + \sigma R K_{2222} F_{,1111} = 0; \qquad /55/$$

where

$$\sigma = \frac{N}{h}. \qquad /56/$$

In order to simplify the following analysis we restrict our attention to a shell of Zener material with homogeneous relaxation spectrum /Fig.1/. Putting Eq./26/

$$E_2^{\alpha\beta\gamma\delta} = K_2 E_1^{\alpha\beta\gamma\delta} \qquad /57/$$

and

$$\mathcal{K} = 1 + \mathcal{K}_2 \tag{58}$$

we find that

$$\left(\frac{\partial}{\partial t} + \mathcal{K}_1\right)\sigma^{\alpha\beta} = \left(\mathcal{K}\frac{\partial}{\partial t} + \mathcal{K}_1\right)E^{\alpha\beta\gamma\delta}\,\varepsilon_{\gamma\delta} \tag{59}$$

and

$$\left(\mathcal{K}\frac{\partial}{\partial t} + \mathcal{K}_1\right)\varepsilon_{\alpha\beta} = C_{\alpha\beta\gamma\delta}\left(\frac{\partial}{\partial t} + \mathcal{K}_1\right)\sigma^{\gamma\delta}. \tag{60}$$

Thus using the contracted notation we have

$$D_{11} = \frac{\mathcal{K}\frac{\partial}{\partial t} + \mathcal{K}_1}{\frac{\partial}{\partial t} + \mathcal{K}_1}\,\frac{h^2}{12}E_{11}\,, \qquad K_{22} = \frac{\frac{\partial}{\partial t} + \mathcal{K}_1}{\mathcal{K}\frac{\partial}{\partial t} + \mathcal{K}_1}\,C_{22}\,, \tag{61}$$

where

$$C_{22} = \frac{E_{11}}{E_{11}E_{22} - E_{12}^2}\,. \tag{62}$$

Then the differential equation for buckling becomes

$$\frac{h^2}{12}E_{11}C_{22}\left(\mathcal{K}\frac{\partial}{\partial t} + \mathcal{K}_1\right)F_{,111111} + \frac{1}{R^2}\left(\mathcal{K}\frac{\partial}{\partial t} + \mathcal{K}_1\right)F_{,11} + $$
$$+ C_{22}\left(\frac{\partial}{\partial t} + \mathcal{K}_1\right)\sigma F_{,1111} = 0. \tag{63}$$

We shall assume that F has the form

$$F = f(t)\sin\lambda x\,. \tag{64}$$

Hence

$$\left[\left(\frac{h^2}{12}E_{11}C_{22}\lambda^4 + \frac{1}{R^2}\right)\mathcal{K} - C_{22}\lambda^2\sigma\right]\dot{f} + $$
$$+\left[\left(\frac{h^2}{12}E_{11}C_{22}\lambda^4 + \frac{1}{R^2} - C_{22}\lambda^2\sigma\right)\mathcal{K}_1 - C_{22}\lambda^2\dot{\sigma}\right]f = 0 \tag{65}$$

The solution of this differential equation is

$$f = C\exp\left[-\int_0^t \frac{\left(\frac{h^2}{12}E_{11}C_{22}\lambda^4 + \frac{1}{R^2} - \lambda^2C_{22}\sigma\right)\mathcal{K}_1 - \lambda^2C_{22}\dot{\sigma}}{\left(\frac{h^2}{12}E_{11}C_{22}\lambda^2 + \frac{1}{R^2}\right)\mathcal{K} - \lambda^2C_{22}\sigma}\,dt\right]. \tag{66}$$

The stability of the cylinder depends on the sign of the exponent. For negative values of the exponent the deflection f decreases as time goes on. and the shell is stable. For positive values of exponent f increases with time and the shell is unstable. The exponent is negative for small values σ and then the numerator changes its sign. We shall analyse the exponent for $\sigma = \mathrm{const}$. Then $\dot{\sigma} = 0$.

The numerator has its minimum at

$$\lambda^2 = \frac{6\sigma}{h^2 E_{11}} \tag{67}$$

and changes the sign at

$$\sigma_{cr}^{(1)} = \frac{h}{R}\sqrt{\frac{E_{11}}{3C_{22}}}\,, \tag{68}$$

which in the case of isotropy gives

$$\sigma_{cr}^{(1)} = \frac{Eh}{R}\,\frac{1}{\sqrt{3(1-\nu^2)}}\,. \tag{69}$$

The denominator has its minimum at

$$\lambda^2 = \frac{6\sigma}{h^2 E_{11} k} \tag{70}$$

and is equal to zero at

$$\sigma_{cr}^{(2)} = \frac{hk}{R}\sqrt{\frac{E_{11}}{3C_{22}}} = k\sigma_{cr}^{(1)}\,, \tag{71}$$

which for an isotropic shell gives

$$\sigma_{cr}^{(2)} = \frac{Ehk}{R}\,\frac{1}{\sqrt{3(1-\nu^2)}} = k\sigma_{cr}^{(1)} \tag{72}$$

It is obvious that for $\sigma = \sigma_{cr}^{(2)}$ the velocity of the deflection becomes infinite. Now we can make the following statement.

The viscoelastic shell is stable for $0 \le \sigma \le \sigma_{cr}^{(1)}$, unstable with infinite critical time for $\sigma_{cr}^{(1)} < \sigma < \sigma_{cr}^{(2)}$ and unstable with instant loos of stability, it is with finite critical time, for $\sigma = \sigma_{cr}^{(2)}$.

When comparing $\sigma_{cr}^{(1)}$ and $\sigma_{cr}^{(2)}$ with the corresponding critical stress of the elastic shell, it is obvious, that $\sigma_{cr}^{(1)}$ corresponds to the critical stress of the elastic element E_1 of Zener model /Fig.1/ and $\sigma_{cr}^{(2)}$ corresponds to the critical stress of both springs $E_1 + E_2$.

That fact shows that the statement by B.Venkatraman in the discusion to Bychawski [6] on nonexistence of finite critical times for linear viscoelastic structures is true only for some viscoelastic models.

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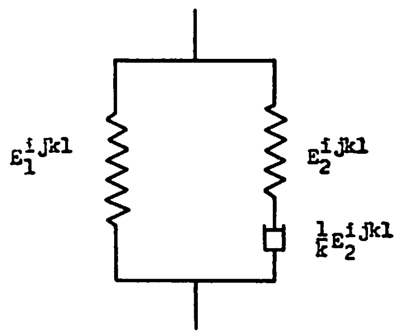

FIG.1