The Extension and Verification of the Bree Diagram

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Summary

The mechanical and cyclic thermal loading of a reactor fuel can has been analysed by Bree. The objective in this analysis was to define stress regimes for combinations of cyclic thermal stresses (due to temperature gradients through the fuel can wall) and continuous stresses (due to internal pressure). These regimes indicate states of stress in which the fuel can exhibits elastic, shakedown, alternating plasticity or ratchetting response to loading.

The analysis of Bree, which treated the fuel can as a thin walled cylinder, showed that an equivalent state of stress existed in a doubly symmetric beam section subjected to cyclic bending to a fixed radius of curvature with a continuous axial load. This analogy has been used in this work to experimentally verify the Bree diagram for elastic/perfectly-plastic materials. The boundary on the diagram defining the onset of ratchetting has been found to agree well with experimental data although the rate of accumulation of strain was found to be in poor agreement with that predicted by Bree.

The concepts of Bree have been used to generate three alternative diagrams numerically. These diagrams cover the alternative loading situations of alternating pressure and alternating thermal load (a) in phase and (b) out of phase and (c) continuous thermal loading with alternating pressure load.

A numerical study has also been undertaken to examine the response of a strain-hardening material subjected to Bree loading. A non-linear analysis has been used employing the Mróz hardening model and this has yielded lines on the Bree diagram of constant total strain accumulation in the ratchetting regime.

1. Introduction

The strain behaviour of the fuel can of a fast nuclear reactor has been studied by Bree [1] and others (see e.g. [2] and [3]). In this study, Bree considered the fuel can as a thin walled cylinder subjected to a steady internal pressure together with an alternating thermal gradient across the can wall. Although in this analysis Bree approximated the state of stress as purely axial the analysis was most useful since it led to a simple analytical solution being obtained. Thus, a qualitative understanding of the strain behaviour of the can was obtained. Bree considered an elastic/perfectly-plastic material in the initial analysis but extended the work to include strain-hardening effects using a linear hardening model. Later, Mulcahy [2] obtained a closed form solution for a linear strain-hardening material having an idealised Bauschinger effect.

The paper of Bree, which presented a plot of thermal stress against the pressure stress, depicting zones of strain behaviour has given rise to what is now termed the "Bree Diagram". The usefulness to designers of such a diagram is evident since only the fictitious elastic thermal stress and the pressure stress need be known for the diagram to yield the strain behaviour of the fuel can e.g. shakedown, ratchetting or alternating plasticity. As a consequence, the Bree Diagram has become a popular design tool.

There is little evidence that any attempt has been made to verify this diagram experimentally. Two possible reasons for this omission are the difficulty in creating thermal stresses of sufficient magnitude and the problem of strain measurement at elevated temperatures. In this present work these problems have been circumvented by substituting bending stresses for the thermal stresses. It can be shown that although the total strain distribution differs to that of Bree, the ratchet strains generated by this loading are analogous to those envisaged by Bree. This has enabled the strain behaviour of thin strips of an elastic/perfectly-plastic steel to be studied for varying "thermal" and "pressure" stresses.

Although Bree considered the case of continuous pressure loading with alternating thermal loads there are design conditions in which this loading sequence is not applicable. A numerical study has been undertaken and reported here to produce equivalent Bree diagrams for the three alternative loading sequences. This work is restricted to an elastic/perfectly-plastic materials idealisation.

Few materials, if any, are truly elastic/perfectly-plastic. Idealisations such as linear strain-hardening are useful to gain a qualitative understanding of how hardening affects the performance of the structure. In this work, linear hardening, piecewise linear and non-linear hardening have been studied numerically with various models of hardening. The strain behaviour of a fuel can made of a material such as stainless steel can thus be predicted and some such results are included for non-linear hardening using the Mroz hardening model.

For all the numerical work reported here the original idealisations of Bree (i.e. the uniaxial model etc.) have been retained.

2. A Technique for Experimental Verification of the Bree Diagram

The reasons which cause an experimental verification of the diagram to be difficult have been given. However, it is evident that the state of stress is similar to that of a beam which carries a constant axial load together with an alternating change in curvature. This change in curvature can be related to the thermal stress and was given by Bree as

$$\sigma_t = \tfrac{1}{2}\rho E t \qquad (1)$$

when the beam is deformed from zero curvature to a specific value ρ. The thermal strain however is no longer present but this does not influence the magnitude of the ratchet strains.

It is thus simpler to undertake experiments in which a material is subjected to a constant axial stress (representing the pressure stress) together with an alternating change in curvature (representing the thermal stress). Fig. 1 shows the experimental equipment used in which a thin strip of shim steel was formed over a wheel of specific radius. To each end of the test strip a chain which passed over pulleys was attached and to these chains deadweights were fixed.

The test strip was chosen initially to have dimensions 0.20" (5 mm) wide x .020" (.5 mm) thick in order that strain gauges could be attached to the strip and so that sufficient stresses could be generated. With such a width to thickness ratio there was a danger that the strip would behave as a plate and not as a beam. However, strain measurements made in the transverse direction showed that beam theory was applicable.

The experimental technique was to attach the test strip at one end to the chain carrier and with the full deadweight acting on the strip to form it over the wheel and attach the second chain carrier. The wheel, which was driven electrically, turned alternately clockwise and anti-clockwise to bring about 10" (250 mm) of the test strip to the wheel's curvature and return it to zero curvature. The experimental equipment shown in Fig. 1 was capable of producing axial stresses up to the yield stress of the material, and by changing wheels, bending stresses in the range $8\sigma_y > \sigma_B > 1.9\sigma_y$ could be produced. By attaching a strain gauge to the free surface of the test strip the surface strain could be measured in the axial direction.

Some preliminary experimentation was undertaken to establish the effect of friction, between wheel and test strip, during the process of bringing the strip into contact with the wheel. It was found that a peak strain was produced as the strain-gauged area came into contact with the wheel. Some experiments were performed in which the strain-gauged area was repeatedly brought to this peak strain position and returned to zero curvature. Using a virgin test strip for a number of such experiments showed that the results obtained were not repeatable, i.e. the ratchet strains obtained under the same loading conditions could vary significantly from one test to the next. The cause of this phenomenon was attributed to small but significant variations locally in the yield stress of the material. To measure the overall change in length of the strip was thus preferable since this would give an average strain for the test section rather than a local strain. Thus, strain gauged displacement transducers were incorporated in the rig to measure the overall extension of the test strip with a resolution equivalent to a surface mounted strain gauge, i.e. 1μ. Using such transducers the results obtained were repeatable to better than 1%.

3. Results from the Experimentation

The experimental results reported here were obtained using mild steel test strips for which a wet chemical analysis showed C = 0.077, Mg = 0.40 and no other alloying elements greater than 0.03% present. The heat treatment for the steel was a one hour soak at 650°C followed by a slow furnace cool. After this heat treatment the material exhibited a yield stress of 202 MN/m^2 (13.0 $tonf/in^2$) and at this stress the strain was greater than 40,000μ, thus indicating perfect-plasticity. The elastic modulus was recorded as 172 GN/m^2 (11,137 $tonf/in^2$).

In all experimental work concerned with cyclic strain performance, alternating plasticity and shakedown can not be differentiated. Only ratchetting can be clearly identified and distinguished from elastic, shakedown or alternating plasticity behaviour. This limitation is not too problematic since it is the boundary of the ratchetting regime of the Bree diagram that is of most interest to designers. Consequently, the results presented here only attempt to define this boundary of the ratchetting regime.

The locations on the Bree diagram which were examined experimentally are given in Table I. For each location a virgin test strip was used. Having assembled the strip into the wheel rig, the wheel was rotated to bring about 200 mm of the strip onto the wheel and counter-rotated to bring the strip back to its starting position. Displacement transducers were used to give the overall change in length of the strip and from this the strain in the test section was calculated.

Fig. 2 is an example of the data collected, showing the accumulation of strain for a wheel size giving σ_t/σ_y = 3.71 and for a range of deadweights giving σ_p/σ_y in the range $0.354 \leq \sigma_p/\sigma_y \leq 0.567$. From such data the rate of accumulation of strain over the first few cycles was calculated. These rates of accumulation are plotted for all data in Fig. 3 together with the rate which Bree predicted.

From Fig. 3 the onset of ratchetting can be seen clearly. Interpolation of this experimental data has yielded a boundary of the ratchetting regime and this is shown in Fig. 4 together with the Bree line. Also shown in Fig. 4 are lines of constant ratchetting strain rate.

4. Numerical Extension of the Bree Diagram

Several computer programs have been written to model the behaviour of the fuel can as idealised by Bree. These programs are categorised by the material model employed and two types of material behaviour are reported here. These are (a) an elastic/perfectly-plastic material and (b) a non-linear strain-hardening material based on a typical stress/strain curve for stainless steel type 316. A detailed description of these programs will not be given here except to describe the hardening models employed. For the elastic/perfectly-plastic analysis the material was considered to yield at $\pm \sigma_y$ regardless of the magnitude of strain. For the non-linear strain-hardening analysis the concept of the model of Mroz [4] was employed. The stress/strain curve used for this analysis is shown in Fig.5, together with the Osgood/Ramberg [5] descriptors of the curve. It is emphasised that the yield stress for this material has been taken as the first departure from linearity.

(a) Elastic/Perfectly-Plastic Material. Bree gave a solution for the case of continuous pressure (primary) load together with alternating thermal (secondary) loading. Three other combinations of loading are of interest to designers. These are

(i) Alternating pressure and thermal loading in phase.

(ii) Alternating pressure and thermal loading out of phase.

(iii) Continuous thermal loading with an alternating pressure load.

Numerical solutions have been obtained for these three cases and are presented here in Bree diagram form in Fig. 6 together with the original Bree diagram. The notation used in these diagrams is that used by Bree [1].

(b) Non-Linear Strain-Hardening Material

Stainless steel is becoming an increasingly popular choice of material for reactor components. However, the low yield stress but high ultimate stress presents designers with a difficult choice of design stress. Over the past few years much work has been directed to quantifying the mechanical properties of this range of steels (in both cyclic and monotonic modes). However, relatively little effort has been devoted to establishing a criterion for shakedown for this class of materials.

The objective of this analysis was to study numerically the cyclic performance of a typical stainless steel for loading conditions corresponding to those of Bree. The computer programs written used a non-linear description (Osgood and Ramberg [5]) of the uniaxial stress/strain curve of stainless steel type 316. The curve was obtained from a sample of the steel in a solution treated condition. The hardening model used was in concept that of Mroz [4]. This model requires that the stress/strain curve be constructed in a piecewise-linear fashion but for this work has been modified to retain the essential features of the model but to use a non-linear description of the curve.

Identification of regimes of strain behaviour is now not as straight forward as for the perfectly-plastic material. For example, Fig. 7 shows the computed strain accumulation for four examples of primary and secondary loads. In each case the strain behaviour is initially ratchetting but this degenerates to a condition of zero strain accumulation within a few cycles. In addition the diagram is no longer bounded at $\sigma_p/\sigma_y = 1$. Since for the strain-hardening material this is not a collapse condition.

Fig. 8 shows lines of constant total strain accumulation. For example, at a diagram location $\sigma_t/\sigma_y = 4.0$ and $\sigma_p/\sigma_y = 1.2$ the total strain accumulated is 100μ. Once this point of strain accumulation is reached the structure appears to "shakedown", i.e. accumulates no further strain, although suffers repeated plastic strains.

5. Discussion

The experimental technique used to verify the Bree diagram has proved to be a reliable and repeatable method of experimentation. It has the primary disadvantage that test strips have to be manufactured by rolling and shearing which is outside the domain of normal workshop facilities. Thus, commercially available strip must be used with consequently restricted choice. The mild steel strip used in this investigation was as near elastic/perfectly-plastic as could be hoped but the cross-section of the strip was not suitable for any form of cyclic testing and so no information concerning the Bauschinger effect etc., could be obtained.

The rates of accumulation of strain which are presented in Fig. 3 are those occurring over the first few cycles of loading. These rates decay with increasing cycles. The results presented in Fig. 3 are in poor agreement with the ratchetting rates predicted by Bree even when comparing the initial slopes. If the "steady state" rates were compared the agreement would be much worse. This disagreement may be attributed to the material cyclically hardening (which was not included in the original analysis of Bree) or to friction effects in the experimentation. The point of onset of ratchetting however, is in good agreement with that proposed by Bree and it is useful to note that the Bree analysis is thus a safe one.

The "alternative" Bree diagrams presented here permit the strain behaviour to be identified for three other loading sequences. It is again of interest to note that for a continuous thermal stress with alternating pressure stress, ratchetting and alternating plasticity are excluded from the diagram. The diagram depicting thermal and pressure stresses applied in phase is probably the most useful of these alternatives and the original Bree diagram has often been incorrectly used to design for this loading. However, it appears that the boundary of ratchetting on the original Bree diagram is conservative when compared with the alternative diagrams.

The work reported here on the non-linear analysis has shown the strain behaviour of one particular strain-hardening material. It is hoped that several such materials will be studied in the near future and the effects of varying the hardening law will also be studied. The Mroz model used for this work has been shown to agree well with experimental data for first cycle loading. Any constitutive law should ideally also include cyclic hardening or softening effects and it is hoped in future work to include such phenomenon.

6. Conclusions

The line of the Bree diagram defining the onset of ratchetting has been shown to be in good agreement with experimental data. The rate of ratchetting, however, is not in good agreement. Three diagrams for alternative loading have been produced to supplement the Bree diagram and an example of strain accumulation has been given for a non-linear strain-hardening material.

References

[1] BREE, J., "Elastic-Plastic Behaviour of Thin Tubes Subject to Internal Pressure and Intermittent High-Heat Fluxus with Application to Fast-Nuclear Fuel Elements", J. of Strain Analysis, Vol. 2, No. 3, 1967.

[2] MULCAHY, T. M., "Thermal Ratchetting of a Beam Element Having an Idealised Bauschinger Effect", J. of Engineering Materials and Technology, Vol. 98, 1976.

[3] BURGREEN, D., "Structural Growth Induced by Thermal Cycling", Trans. ASME, J. of Basic Engineering, Vol. 90, 1968.

[4] MROZ, Z., "On the Description of Anisotropic Hardening", J. Mech. Phys. Solids, 15, (1967).

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Table I. Experimental Locations on the Bree Diagram.

σ_t/σ_y	σ_p/σ_y					
5.050	.071	.142	.213	.283	.354	.425	-
3.710	.213	.283	.354	.425	.496	.567	-
3.328	.213	.283	.354	.425	.496	.567	.638
2.996	.283	.354	.425	.496	.567	.638	-
1.943	.425	.496	.567	.638	.708	.779	-

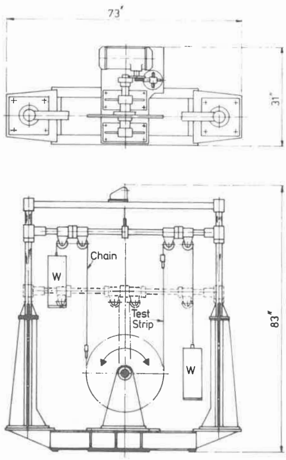

Fig. 1 Experimental Wheel Rig.

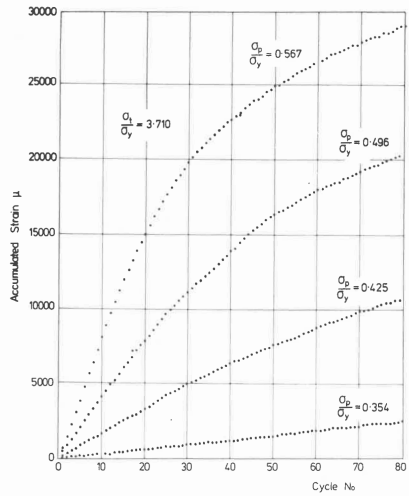

Fig. 2 Experimental Strain Accumulation with Cycling.

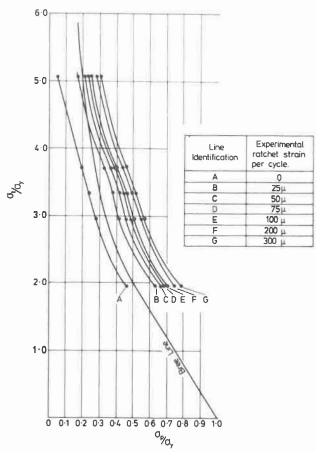

Fig. 3 Experimental Ratchet Strain/Cycle Compared with Bree (1).

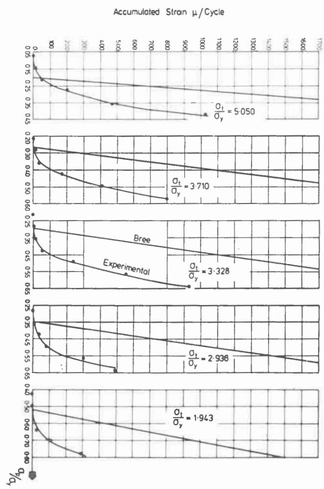

Fig. 4 Lines of Constant Ratchet Strain/Cycle Compared with Bree (1).

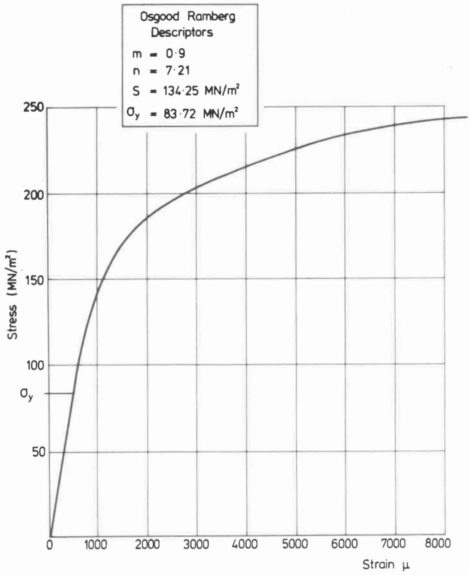

Fig. 5 Uniaxial Stress-Strain Curve for Stainless Steel Type 316.

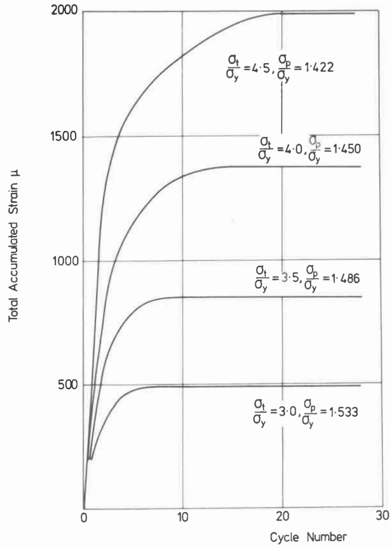

Fig. 6 Alternative Bree Diagrams.

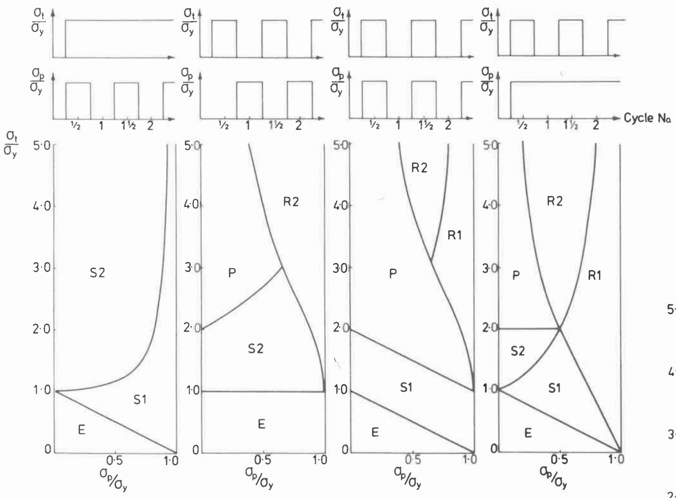

Fig. 7 Computed Accumulation of Strain with Cycling for Stainless Steel 316.

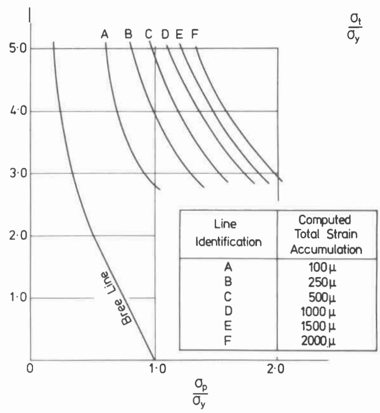

Fig. 8 Computed Lines of Constant Total Ratchet Strain for Stainless Steel 316.