ABSTRACT

KANE, ROBERT F. Cost Reduction, Quality Improvement, and Distribution. (Under the direction of John J. Seater.)

In chapter 1 I propose a trade and growth model in which the transition from autarky to trade is endogenous. It uses a tractable two-country trade and growth model that yields closed-form solutions for the entire system including the timing of the transition. The transition is guaranteed as long as the two countries have different steady-state growth rates. If countries have identical fundamentals, but different initial conditions, the transition from autarky to to trade might or might not occur. In addition this chapter provides new insight into the effects of trading frictions on growth. Non-prohibitive trading frictions have transitory effects on growth but do not affect the long-run growth rate. The tractability of the framework allows for a full analysis of the welfare effects of trading frictions. A reduction in non-prohibitive trading frictions is always welfare improving, the removal of prohibitive frictions has ambiguous effects.

In chapter 2 I analyze the effects of alternative distribution technologies on endogenous innovation, in particular cost reduction and quality improvement. Quality improvement and cost reduction have long been treated as identical ways of changing the quality adjusted price of a good; with quality improvement working by increasing the denominator, and cost reduction reducing the numerator. However, the data tells a different story. Firms engage in both quality improvement and cost reduction. In addition, there are significant cross country differences in R&D expenditure shares. The aim of this paper is to show that under realistic conditions, cost reduction and quality improvement are inherently different. When distribution is modeled using the standard 'iceberg' formulation the usual isomorphism is present. When the distribution technology is modeled

using a per-unit formulation cost reduction can no longer be a source of long-run growth while quality improvement can.

In chapter 3 I develop a model of endogenous innovation where firms undertake in two in-house R&D activities; cost reduction and quality improvement. I use a modern Schumpeterian growth model so that the scale effect is absent. The purpose of this is two-fold. First, it yields a more complete picture of growth. The steady-state growth rates of cost reduction and quality improvement are jointly determined. There are numerous steady-states, firms may engage in both R&D activities, they may engage in only one, or they might do neither. Second, the introduction of an irreducible unit cost (distribution costs) generates interesting results. Similar to the result of chapter 2; absent technical progress in distribution, cost reduction cannot be a source of long-run growth, while quality improvement can. Introducing technical progress in distribution allows for cost reduction in the steady-state, in fact in a steady-state with cost reduction, the rate of cost reduction must equal the rate of technical progress in distribution.

The common theme throughout all chapters is the interaction between growth and trading frictions. In chapter 1, growth renders prohibitive trading frictions non-prohibitive which generates an endogenous transition from autarky to trade. In chapter 2 I show that the assumptions regarding trading frictions (per-unit vs iceberg) dramatically affects the results for growth. In chapter 3 I show that if there are any irreducible per-unit trading frictions, economies will devote an increasing share of R&D expenditures towards quality improvement.

Cost Reduction, Quality Improvement, and Distribution

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DEDICATION

To my parents.

BIOGRAPHY

The author was born in Syracuse, New York and was raised in Oswego, New York. Prior to attending NCSU he received degrees in economics and math at his undergraduate SUNY Oswego. He enjoys research and cookies.

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TABLE OF CONTENTS

LIST OF FIGURES

Chapter 1

Shipping the Good Intermediates Out, Eventually

1.1 Introduction

It is common practice in trade models for authors to assume that countries "open up" to trade; this means that there is a reduction in shipping costs, tariffs, or a removal of an artificial trade barrier. This leaves a surprising gap in the literature; the transition from autarky to trade is based on an exogenous event. In early trade models the authors had to proceed in this manner because the models were static. It is impossible for static models to account for the origin of trade other than an exogenous event. To put it simply, existing trade models provide answers to why countries trade, but not why they begin to trade.

In this paper I develop a theory of the emergence of trade. The main building blocks of the model are trading frictions (ad-valorem) and dynamic comparative advantage. The model built consists of two countries and two tradable goods; homogenous final output

and quality differentiated intermediate goods. The intermediate good firms engage in costly R&D to improve product quality and entrepreneurs invest to create new goods. At all times firms are legally entitled to purchase goods from either country, however, they must pay the trading frictions. With sufficiently large frictions, the countries remain in autarky.

Clearly the removal of artificial trade barriers plays an important role in explaining the growth in world trade.[1] Another potential explanation is improvements in the shipping technology and reductions in tariffs, Yi (2003) develops a theory in which tariff reductions explains a large part of the increase in world trade. The purpose of this paper is to introduce an alternative explanation; dynamic comparative advantage. As long as the countries have different rates of technical progress, the transition from autarky to trade is guaranteed, even holding tariffs and the shipping technology constant.

There is a growing literature on the dynamic effects of trade on growth. An important result from this literature is that trade has the potential to be welfare reducing. Young (1991) and Redding (1999) provide models in which trade might reduce the growth rate. Intuitively, the trading pattern is determined by current states. Agents do not internalize the growth effects of their specialization decision. There are the usual static gains from trade but there is the potential for dynamic loses. Interestingly when trade occurs in this endogenous fashion, there are no static gains from trade, however, there is no possibility dynamic losses. Hence, trade is less likely to be welfare reducing. Strictly speaking this framework predicts that trade might reduce the welfare of the country importing the intermediate goods. Intuitively, this country losses their dividend income. However,

[1]In the Edo period Japan was essentially in autarky because of trade policies, it was not until the Meiji Restoration that Japan began significant trade. In addition, the Embargo Act of 1807 essentially made the United States an autarkic country until it was removed in 1809.

they are guaranteed to have dynamic gains from trade.[2]

The dynamic trade model utilized is based on Kane, Ji, and Seater (2013). The underlying growth framework of that model is Peretto (2007). This class of growth models is extremely tractable and yields closed-form solutions for the transitional dynamics. In addition, they have received strong empirical support.[3] In this framework, endogenous entry kills the scale effect. Therefore any effects that trade and trading frictions have on growth will not work through the scale effect channel. This is important because the importance of nullifying the scale effect goes far beyond the realm of population. Removing the scale effects changes the effect of taxes on growth. This is important because Stokey and Rebelo (1995) present evidence that is inconsistent with early endogenous growth models, in particular, these models predicted counter factually large effects of taxes. By removing the scale effect, the models remove the inappropriate scale effect channel for taxes to effect growth.

The framework is trying to capture the essential facts presented by Eaton and Kortum (2001) in an endogenous growth model in the most tractable way possible. Eaton and Kortum (2001) show that the production of intermediates as well as R&D expenditure is highly concentrated among a few countries. Eaton and Kortum (2001) develop a theoretical model to explain these facts. In their model the country with the lowest quality adjusted price of intermediates will incompletely-specialize in their production, the other country shuts down production. However, quality evolves according to an exogenous process. Like Eaton and Kortum (2001), Kane, Ji, and Seater (2013) is based on comparative advantage and the country with the lowest quality adjusted price incompletely specializes in their production, however, there is endogenous innovation (horizontal and

[2]This deserves much more discussion. However, for clarity I postpone doing so until section 4.

[3]See: Ang and Madsen (2011) , Laincz and Peretto (2006), Madsen (2008) , Madsen, Ang, and Banerjee (2010), and Ha and Howitt (2007).

vertical).

My model also yields new insight into the effects of tariffs and other trading frictions on growth. Specifically, while tariffs and shipping costs have transitory effects on growth, the steady-state growth rate is unaffected by them. A reduction in tariffs or shipping costs will increase the growth rate in the short-run, however, once endogenous entry occurs, the growth rate returns to the original steady state value. An interesting feature of this model is the differences between a reduction in prohibitive frictions to non-prohibitive levels and reducing non-prohibitive frictions. Prohibitive frictions have ambiguous welfare effects, however, non-prohibitive frictions are always welfare reducing.

In summary, this paper contributes to the literature in two regards. First, it provides a theoretical framework in which the transition from autarky to trade is endogenous. In particular rather than relying on falling shipping costs or the removal of a trade barrier, trading frictions are held constant. Instead, endogenous quality improvement induces comparative advantage sufficiently large to overwhelm the trading frictions. The second contribution is the analysis of the effects of trading frictions on growth and welfare. A fall in trading frictions only has temporary effects on the growth rate. Interestingly, the welfare effects of trading frictions on growth is related to entry costs. When entry costs are large, the welfare effects of a reduction in trading frictions will be larger. Intuitively, the increase in the growth rate disappears as firms enter, the larger the entry cost larger the gain from the transition.

The model is organized as follows. In section 2 I setup the model with trade and provide the trading pattern conditions. In section 3 I solve the model in autarky and describe the transition to trade. In section 4 I analyze the model in trade and provide welfare results. Finally I conclude in section 5.

1.2 Model Setup

The economy is open in that Home and Foreign are legally entitled to trade with each other. Time subscripts are suppressed when confusion does not arise.

1.2.1 Households

The economy is populated by a representative agent who operates with the following preferences:

$$U\left(C_k\right) = \int_0^{\infty} e^{-\rho t} \log\left(\frac{C_{kt}}{L_{kt}}\right) dt. \tag{1.1}$$

Households maximize lifetime utility (3.1) subject to the flow version of their budget constraint

$$\dot{N}_k V_k + C_k = \int_0^{N_k} \pi_{ki} di + w_k L_k, \tag{1.2}$$

where π_{ki} is dividend income, w_k is the wage rate, L_k is population size, C_k is aggregate consumption, and $\dot{N}_k V_k$ is investment expenditure in country k.

The household optimization implies the usual Euler equation

$$\widehat{C}_k = r_k - \rho. \tag{1.3}$$

1.2.2 Final output

Home firms producing final output operate with the following technology:

$$Y_h = \int_0^{N_h} X_{hi}^{\theta} \left(Z_{hi}^{\delta} Z_h^{1-\delta} l_{hhi}\right)^{1-\theta} di + \int_0^{N_f} X_{fj}^{\theta} \left(Z_{fj}^{\delta} Z_f^{1-\delta} l_{hfj}\right)^{1-\theta} dj.$$

Home final good producers may use Home intermediates X_{hi} or imported Foreign intermediates X_{fj}. Attached to the intermediate goods is it's product quality, i.e X_{hi} has attached quality Z_{hi} as well as the spillover from the average quality of the inputs Z_h. Likewise if the firm decides to buy the foreign intermediate X_{fj} they receive the attached quality Z_{fj} as well as the spillover Z_f. Firms also choose how much labor to use along with each intermediate good. For good X_{hi}, firms choose how much labor to use , if they are using the foreign good X_{fj} they choose how much labor to use along with it l_{hfj}. The subscripts denote the fact that the countries are using Home's labor to use with a Home or Foreign input, and the last subscript indicates the specific good.

Foreign firms operate with identical technology. However, it is worthwhile to discuss the notation:

$$Y_f = \int_0^{N_h} X_{hi}^{\theta} \left(Z_{hi}^{\delta} Z_h^{1-\delta} l_{fhi}\right)^{1-\theta} di + \int_0^{N_f} X_{fj}^{\theta} \left(Z_{fj}^{\delta} Z_f^{1-\delta} l_{ffj}\right)^{1-\theta} dj,$$

the only difference is that now the subscripts for the labor. The first subscript now describes the fact that it is Foreign labor.

1.2.3 Pattern of Specialization

In this section I discuss conditions under which each trading regime occurs.

Home final good producers profits are:

$$\pi_{yh} = \int_0^{N_h} X_{hi}^{\theta} \left(Z_{hi}^{\delta} Z_h^{1-\delta} l_{hhi}\right)^{1-\theta} di + \int_0^{N_f} X_{fj}^{\theta} \left(Z_{fj}^{\delta} Z_f^{1-\delta} l_{hfj}\right)^{1-\theta} dj$$
$$- \int_0^{N_h} \left(P_{xh} X_{hi} + w_h l_{hhi}\right) di - \int_0^{N_f} \left(\left(P_{xf}(1+\tau_h)\, X_{fj} + w_h l_{hfj}\right) dj,\right.$$

to obtain the good, the final good producers pay $P_{xf}(1+\tau_h)$.

The first order conditions imply a corner solution with respect to the use of Home versus Foreign inputs. Home producers will choose to purchase domestically (from Home) iff:

$$\frac{P_{xh}}{Z_h^{\frac{1-\theta}{\theta}}} < \frac{P_{xf}(1+\tau_h)}{Z_f^{\frac{1-\theta}{\theta}}}. \tag{1.4}$$

Foreign final good producers maximize profits[4]:

$$\pi_{yf} = \int_0^{N_h} X_{hi}^{\theta}\left(Z_{hi}^{\delta} Z_h^{1-\delta} l_{fhi}\right)^{1-\theta} di + \int_0^{N_f} X_{fj}^{\theta}\left(Z_{fj}^{\delta} Z_f^{1-\delta} l_{ffj}\right)^{1-\theta} dj$$
$$- \int_0^{N_f} \left(P_{xf} X_{fi} + w_f l_{fhi}\right) di - \int_0^{N_h} \left(\left(P_{xh}(1+\tau_f) X_{hi} + w_f l_{ffi}\right) di\right.$$

The solution implies that Foreign purchase the intermediates domestically (from Foreign) iff:

$$\frac{P_{xf}}{Z_f^{\frac{1-\theta}{\theta}}} < \frac{P_{xh}(1+\tau_f)}{Z_h^{\frac{1-\theta}{\theta}}}. \tag{1.5}$$

Both (1.4) and (1.5) show the role of trading frictions in influencing the trading pattern. If $\tau_f = \tau_h = 0$, trade must occur.[5] When the trading frictions are positive, the quality adjusted price of the goods must be sufficiently different to compensate for the trading frictions.

The price of the intermediate goods are simply the marked up unit costs. Using this, I rewrite the inequalities (1.4) and (1.5) in terms of deep parameters

[4]In other circumstances I would have to multiply production by the price P_{yf} , since I allow the trade of final output $P_{yf} = P_{yh} = 1$

[5]This is the standard Ricardian explanation of trade.

$$\frac{A_h}{Z_h^{\frac{1-\theta}{\theta}}} < \frac{A_f\left(1+\tau_h\right)}{Z_f^{\frac{1-\theta}{\theta}}};$$

$$\frac{A_f}{Z_f^{\frac{1-\theta}{\theta}}} < \frac{A_h\left(1+\tau_f\right)}{Z_h^{\frac{1-\theta}{\theta}}},$$

if the current parameter and state configuration are such that the inequalities are in this direction, the economy remains in autarky. The intuition is simple, the quality adjusted prices are not different enough to compensate for the shipping costs. As long as the inequalities are such that both countries prefer to buy the intermediates domestically, they are in autarky. Now I proceed to describe the model in autarky.

1.3 Autarky

Final output is produced in a competitive market with the following technology:

$$Y_t = \int_0^{N_t} X_{it}^{\theta}\left(Z_{it}^{\delta} Z_t^{1-\delta} l_{it}\right)^{1-\theta} di. \tag{1.6}$$

Firms produce final output by using intermediate goods (X_{it}) and labor (l_{it}). Attached to each intermediate good X_{it}, is the quality of the input Z_{it}, and the product quality spillover, Z .

The profits are:

$$\pi_{yt} = P_{yt}\int_0^{N_t} X_{it}^{\theta}\left(Z_{it}^{\delta} Z_t^{1-\delta} l_{it}\right)^{1-\theta} di - \int_0^{N_t}\left(w_t l_{it} + P_{xit} X_{it}\right) di.$$

I choose the numeraire to be the price of final output, henceforth I set $P_{yt} = 1$. When

confusion does not arise I suspend time subscripts. Profit maximization implies:

$$P_{xi} = \theta X_i^{\theta-1} \left(Z_i^{\delta} Z^{1-\delta} l_i\right)^{1-\theta}, \tag{1.7}$$

$$w = (1-\theta) X_i^{\theta} \left(Z_i^{\delta} Z^{1-\delta}\right)^{1-\theta} l_i^{-\theta}. \tag{1.8}$$

These imply:

$$\int_0^N P_{xi} X_i di = \theta Y, \tag{1.9}$$

$$\int_0^N w l_i di = (1-\theta) Y. \tag{1.10}$$

1.3.1 Intermediate goods

Intermediate goods are produced by monopolistically competitive firms who operate with a technology that requires A units of final output to produce one unit of the intermediate good. These firms engage in costly R&D to improve product quality, the R&D technology is described by:

$$\dot{Z}_i = \alpha_z R_i, \tag{1.11}$$

where α_z measures the effectiveness of R&D in improving product quality.

Firms profits are thus:

$$\pi_{xi} = X_i \left(P_{xi} - A\right) - \phi Z - R_i, \tag{1.12}$$

the firms have to pay a fixed operating cost ϕZ, and pay the endogenous fixed sunk cost

R_i in R&D expenditure. Firms maximize their lifetime value:

$$V_{it} = \int_0^\infty e^{-r(s)s}\pi(s)ds, \tag{1.13}$$

where $r(s) = \frac{1}{s}\int_0^s r(v)dv$ is the average interest rate.

The current-value Hamiltonian for the firms dynamic optimization is:

$$H = X_i\left(P_{xi} - A\right) - \phi Z - R_i + \eta\alpha_z R_i,$$

the first order conditions imply:

$$P_x = \frac{A}{\theta},$$

an interior solution for quality improvement implies,

$$\eta = \frac{1}{\alpha_z}.$$

The return to R&D is:

$$r_z = \alpha_z\delta\left(\frac{\theta^2}{A}\right)^{\frac{1}{1-\theta}}\left(\frac{1-\theta}{\theta}\right)AZ_i^{\delta-1}Z^{1-\delta}l_i,$$

in symmetric equilibrium:

$$r_z = \alpha_z\delta\left(\frac{\theta^2}{A}\right)^{\frac{1}{1-\theta}}\left(\frac{1-\theta}{\theta}\right)A\frac{L}{N}. \tag{1.14}$$

1.3.2 Entry

Firms enter the market subject to an entry costs. Following Peretto, I assume that the entry costs are proportional to the volume of production at the time of entry i.e to enter at time t, firms must pay βX_{it} in start up costs. In addition, firms enter at the average quality level[6]:

$$V_{it} = \beta X_{it}, \tag{1.15}$$

simple differentiation of (1.13) implies:

$$r_n = \frac{\pi_{xi}}{V} + \widehat{V},$$

substituting in (1.12) and (3.13) gives the rate of return to entry:

$$r_n = \frac{X_i A\left(\frac{1-\theta}{\theta}\right) - \phi Z - R_i}{\beta X_i} + \widehat{X}. \tag{1.16}$$

1.3.3 General equilibrium

The household budget constraint can be written as:

$$\widehat{N} = \frac{\pi_i}{V} + \frac{w_h L}{NV} - \frac{C}{NV}, \tag{1.17}$$

(1.9) implies

$$\widehat{N} + \widehat{X} = \widehat{Y}, \tag{1.18}$$

substituting (1.17) into (1.18) yields

[6]This simplifying assumption preserves symmetry at all times.

$$\widehat{Y} = \frac{\pi_i}{V} + \frac{w_h L}{NV} - \frac{C}{NV} + \widehat{X}, \tag{1.19}$$

using (3.3) and (1.19) gives the growth rate of the consumption output ratio:

$$\widehat{\left(\frac{C}{Y}\right)} = \frac{C}{\beta\frac{\theta^2}{A}Y} - \frac{1-\theta}{\beta\frac{\theta^2}{A}} - \rho,$$

this is an unstable differential equation. This implies that $c = \frac{C}{Y}$ jumps to its steady state value:

$$c = 1 - \theta + \beta\frac{\theta^2}{A}\rho.$$

Simple differentiation of (1.6) implies

$$\widehat{Y} = \widehat{N} + \theta\widehat{X} + (1-\theta)\left[\widehat{Z} - \widehat{N}\right] = \widehat{N} + \theta\left[\widehat{Z} - \widehat{N}\right] + (1-\theta)\left[\widehat{Z} - \widehat{N}\right] = \widehat{Z},$$

this implies $\widehat{C} = \widehat{Y} = \widehat{Z}$. Then the Euler equation yields the rate of quality improvement:

$$\widehat{Z} = \alpha_z\delta\left(\frac{\theta^2}{A}\right)^{\frac{1}{1-\theta}}\left(\frac{1-\theta}{\theta}A\right)\frac{L}{N} - \rho, \tag{1.20}$$

note that there exists a critical value of N in which $\widehat{Z} = 0$. If $N > \bar{N}$ then the non-negativity implies $R_i = 0$, where:

$$\bar{N} = \alpha_z\delta\left(\frac{\theta^2}{A}\right)^{\frac{1}{1-\theta}}\left(\frac{1-\theta}{\theta}A\right)\frac{L}{\rho},$$

as long as the number of firms is below this threshold there will be vertical innovation.

1.3.4 Growth rates

Substituting (1.18) into (1.16) yields:

$$\widehat{N} = \frac{X_i \left(\frac{1-\theta}{\theta}\right) A - \phi Z - R_i}{\beta X_i} - \rho, \tag{1.21}$$

substituting (1.7) and (1.20) into (1.21) yields:

$$\widehat{N} = \begin{cases} \frac{\left(\frac{\theta^2}{A}\right)^{\frac{1}{1-\theta}} L\left(\frac{1-\theta}{\theta}A\right)(1-\delta) - N\left(\phi - \frac{1}{\alpha_z}\rho\right)}{\beta\left(\frac{\theta^2}{A}\right)^{\frac{1}{1-\theta}} L} - \rho & : R_i > 0 \\ \frac{\left(\frac{\theta^2}{A}\right)^{\frac{1}{1-\theta}} L\left(\frac{1-\theta}{\theta}\right)A - N\phi}{\beta\left(\frac{\theta^2}{A}\right)^{\frac{1}{1-\theta}} L} - \rho & : R_i = 0 \end{cases}, \tag{1.22}$$

I am now able to fully characterize the equilibrium. The rate of entry and growth rate of quality are:

$$\widehat{N} = \frac{\left(\frac{\theta^2}{A}\right)^{\frac{1}{1-\theta}} L\left(\frac{1-\theta}{\theta}A\right)(1-\delta) - N\phi + N\frac{1}{\alpha_z}\rho}{\beta\left(\frac{\theta^2}{A}\right)^{\frac{1}{1-\theta}} L} - \rho,$$

$$\widehat{Z} = \alpha_z \delta \left(\frac{\theta^2}{A}\right)^{\frac{1}{1-\theta}} \left(\frac{1-\theta}{\theta}\right) A\frac{L}{N} - \rho.$$

The growth rate of product quality is declining in the number of firms N. Entry reduces incumbents market share, and hence reduce the incentive to innovate. The rate of entry is also decreasing in N.[7] The intuition here comes from the fact that the fixed operating cost ϕ puts a cap on the number of firms.[8]

Steady state growth is only possible if entry shuts down before quality improvement

[7]For the rate of entry to be declining in N, the model requires $\phi > \frac{1}{\alpha_z}\rho$. Intuitively, this is required because entry is reducing incentive to innovate, and is hence reducing one component of the fixed costs and this leads to a tendency to increase entry. However, more entry puts pressure on the use of resources. This assumption guarantees that the latter effect dominates.

[8]For a discussion of fixed operating costs and their effect on entry see Peretto and Connolly (2007).

shuts down.[9] There is an N such that, the rate of entry is equal to 0 (I denote it as N_{ss}). Setting (1.21) to zero I obtain:

$$N_{ss} = \frac{\left(\frac{\theta^2}{A}\right)^{\frac{1}{1-\theta}} L\left[\frac{1-\theta}{\theta} A(1-\delta) - \beta\rho\right]}{\left(\phi - \frac{1}{\alpha_z}\rho\right)}. \tag{1.23}$$

To obtain the steady state growth rate of quality, substitute (1.23) into (1.20):

$$\widehat{Z}_{ss} = \frac{\delta(1-\theta)A\left(\alpha_z\phi - \rho\right)}{(1-\theta)A(1-\delta) - \theta\beta\rho} - \rho. \tag{1.24}$$

It is intuitively obvious why the scale effect is absent, the number of firms is linear in population size. However, the entry costs effect the rate of entry. With costless entry, entry occurs instantaneously.

Recall the conditions for autarky:

$$\frac{A_h}{Z_h^{\frac{1-\theta}{\theta}}} < \frac{A_f\left(1+\tau_h\right)}{Z_f^{\frac{1-\theta}{\theta}}} \tag{1.25}$$

$$\frac{A_f}{Z_f^{\frac{1-\theta}{\theta}}} < \frac{A_h\left(1+\tau_f\right)}{Z_h^{\frac{1-\theta}{\theta}}} \tag{1.26}$$

These inequalities are not time invariant. The quality of the intermediate goods are increasing, there is no reason to believe that the qualities are growing at the same rate. In section 2) I derived the autarky growth rate (2.29). There is no reason to believe that R&D efficiencies are identical across countries, in other words $\alpha_{zh} \neq \alpha_{zf}$. This implies

[9] This is guaranteed to occur if $\rho\left[(1-\theta)A(1-\delta) - \theta\beta\rho\right] < \delta\left(1-\theta\right)A\left(\alpha_z\phi - \rho\right)$

that:

$$\widehat{Z}_h = \frac{\delta(1-\theta)A_h\left(\alpha_{zh}\phi-\rho\right)}{(1-\theta)A_h(1-\delta)-\theta\beta\rho}-\rho \neq \widehat{Z}_f = \frac{\delta(1-\theta)A_f\left(\alpha_{zf}\phi-\rho\right)}{(1-\theta)A_f(1-\delta)-\theta\beta\rho}-\rho.$$

It turns out there are two possibilities that must be examined, Home growing faster, or Foreign growing faster. Since the case is symmetric, henceforth I proceed under the assumption that Home is growing faster.

1.3.5 Transitioning out

Rearranging (1.25) and (1.26) I obtain:

$$\left(\frac{A_h}{A_f\left(1+\tau_h\right)}\right)^{\frac{\theta}{1-\theta}} < \frac{Z_h}{Z_f}, \tag{1.27}$$

$$\left(\frac{A_f}{A_h\left(1+\tau_f\right)}\right)^{\frac{\theta}{1-\theta}} < \frac{Z_f}{Z_h}. \tag{1.28}$$

Since Home is growing faster, the direction of inequality (1.27) is reinforced. Intuitively, Home began with goods that are of lower quality adjusted price, since their product quality is growing faster than Foreign they will never want to buy intermediate goods from Foreign. However, the inequality (1.28) must reverse.

The intuition can be seen in figure 1.1. I plot the quality adjusted price that Foreign firms pay for domestic and imported capital.[10] At $T = 0$, both countries are legally able to trade, however, both Home and Foreign find it cheaper to buy domestically. Since Home is growing faster, the quality adjusted price of Home's capital is falling faster than Foreign's. At $T*$, the quality adjusted price of domestic and imported capital facing

[10]Since Home is growing faster than Foreign, Home will never import capital from Foreign, hence I ignore the quality adjusted prices facing Home.

Foreign firms is equal. Once this happens, the countries begin trading.

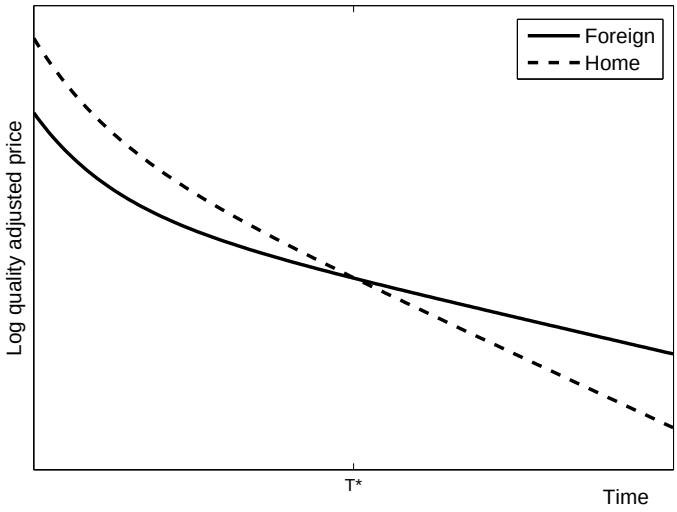


Figure 1.1: log of quality adjusted price

Proposition 1: *If two countries begin in a no trade regime and both countries are in their steady state, the country that is more efficient at R&D will specialize in the production of the R&D good.*

Proof: The left hand side of (1.25) and (1.26) is constant. The right hand side of (1.25) is increasing, therefore the first inequality is being reinforced. However, the right hand side of(1.26) is decreasing. In fact, the right hand side asymptotically approaches zero, in finite time (1.28) will reverse, and Foreign will begin importing intermediates from Home.

1.3.6 Timing

The model yields an implicit solution for $T*$. The closed form solution for Home and Foreign's intermediate good quality as a function of time is given by (1.29)[11]:

$$Z_{kT} = Z_{k0} \exp\left[\widehat{Z}_{ss}T + \frac{\beta(\widehat{Z}_{ss}+\rho)\theta}{(1-\theta)A_k(1-\delta)-\theta\beta\rho}\left(\frac{N_{ssk}-N_{k0}}{N_{k0}}\right)\left(1-\exp^{-\left(\frac{(1-\theta)A_k(1-\delta)-\theta\beta\rho}{\theta\beta}\right)T}\right)\right] \tag{1.29}$$

where k denotes the country, N_{k0} and Z_{k0} denote the initial values of the number of firms and quality.

The $T*$ in figure 1.1 corresponds to the amount of time it takes for the quality adjusted prices to become equal. The exact solution is given by setting the quality adjusted price of imported capital to domestic capital this yields (1.30):

$$\begin{aligned}
&\ln\left(\left(\frac{A_f}{A_h\left(1+\tau_f\right)}\right)^{\frac{\theta}{1-\theta}}\frac{Z_{h0}}{Z_{f0}}\right) = \\
&\widehat{Z}_{fss}T^* + \frac{\beta_f\left(\widehat{Z}_{fss}+\rho\right)\theta\left(\frac{N_{ssf}}{N_{f0}}-1\right)\left(1-\exp^{-\left(\frac{(1-\theta)A_f(1-\delta)-\theta\beta_f\rho}{\theta\beta_f}\right)T^*}\right)}{(1-\theta)A_f\left(1-\delta\right)-\theta\beta_f\rho} \\
&-\widehat{Z}_{hss}T^* - \frac{\beta_h\left(\widehat{Z}_{hss}+\rho\right)\theta\left(\frac{N_{ssh}}{N_{h0}}-1\right)\left(1-\exp^{-\left(\frac{(1-\theta)A_h(1-\delta)-\theta\beta_h\rho}{\theta\beta_h}\right)T^*}\right)}{(1-\theta)A_h\left(1-\delta\right)-\theta\beta_h\rho}.
\end{aligned} \tag{1.30}$$

Unfortunately there is no closed form solution to the general case. However, there are two interesting experiments that provide insight. The first is to assume that both countries are at their respective steady states. The second is to assume that the countries are identical in every way except for the initial number of firms. These are very different experiments.

[11] See the appendix 1 for the derivation

The first deals with what Grossman and Helpman (1990) define as comparative advantage in R&D. In the second there is no long-run comparative advantage in R&D, however, the countries grow at different rates because of transitional growth.

If both countries are at their respective steady states, (1.30) is simplified to:

$$T^* = \frac{ln\left(\left(\frac{A_h\left(1+\tau_f\right)}{A_f}\right)^{\frac{\theta}{1-\theta}} \frac{Z_{f0}}{Z_{h0}}\right)}{\widehat{Z}_{hss} - \widehat{Z}_{fss}} \tag{1.31}$$

Proposition 2: *Assuming the countries are at their respective steady states the amount of time it takes to make the transition is declining in the difference in steady state growth rates and increasing in the ratio of the initial quality adjusted price of imported to domestically produced intermediates*

Proof: Simple differentiation of (1.31).

Upon reflection, an interesting feature of (1.31) is that a reduction in trading frictions will speed up the transition, it does not necessarily make the transition instantaneous. This is a new mechanism for trading frictions to affect welfare. A reduction in trading frictions speeds up the transition and leads the economy to enjoy the effects of trade sooner.

In the second experiment I assume that both countries have identical fundamentals but different initial conditions. In particular I assume that Home is receiving extra transitional growth (i.e $N_{h0} < N_{f0}$), other than this all parameters are identical. Transitional growth is not sufficient to guarantee that the countries will make the transition from autarky to trade. As long as

$$\frac{\ln\left[1+\tau_f\right]\left[\left(1-\theta\right)A\left(1-\delta\right)-\theta\beta\rho\right]}{\beta\left(\widehat{Z}_{ss}+\rho\right)\left(1-\theta\right)N_{ss}\left(\frac{1}{N_{h0}}-\frac{1}{N_{f0}}\right)}>1-\exp^{-\left(\frac{(1-\theta)A(1-\delta)-\theta\beta\rho}{\theta\beta}\right)T}, \tag{1.32}$$

the countries will remain in autarky. The right hand side of (1.32) is increasing over time, and asymptotically approaches 1. If the left-hand side is less than one, the economy will make the transition, otherwise they will remain in autarky forever.

If a solution exists, (1.30) is simplified to:

$$\ln\left[1+\tau_f\right]=\frac{\beta\left(\widehat{Z}_{ss}+\rho\right)\left(1-\theta\right)\left(1-\exp^{-\left(\frac{(1-\theta)A(1-\delta)-\theta\beta\rho}{\theta\beta}\right)T^*}\right)}{\left(1-\theta\right)A\left(1-\delta\right)-\theta\beta\rho}N_{ss}\left(\frac{N_{f0}-N_{h0}}{N_{f0}N_{h0}}\right). \tag{1.33}$$

The preceding experiments rely on different mechanisms to generate different cross-country growth rates. The first relied on different fundamentals (i.e R&D efficiencies) but imposed identical initial conditions. The second assumed that countries had identical fundamentals but different initial conditions. When countries have different steady state growth rates the transition is guaranteed. Transitional growth does not guarantee the transition. However, the transitional and steady state growth might have opposing signs.[12] This leads to the question, can the "wrong" country specialize in the production of intermediates? The answer is yes, however, with sufficiently large trading frictions the transitional growth effect is dominated by steady state growth differences.

Next, I solve for a sufficiently large trading friction that prevents the less R&D efficient country from specializing in the production of intermediates. Throughout the paper I have assumed that Home is growing faster than Foreign, I continue to do so for steady state growth rates, however, I allow for Foreign to be growing faster on the transition

[12]For instance, the United Kingdom might have a higher steady-state growth rate than China but be growing slower because of transitional forces.

path. Clearly if Home is receiving no transitional growth, this will give a "lower bound" for a friction to prevent Foreign from specializing. Therefore to find a friction that is sufficiently large I assume that Home is receiving no transitional growth.

Imposing zero transitional growth for Home, as long as:

$$\frac{\beta_f\left(\widehat{Z}_{fss}+\rho\right)\theta\left(\frac{N_{ssf}-N_{f0}}{N_{f0}}\right)\left(1-\exp^{-\left(\frac{(1-\theta)A_f(1-\delta)-\theta\beta_f\rho}{\theta\beta_f}\right)T}\right)}{(1-\theta)A_f(1-\delta)-\theta\beta_f\rho}-\left(\widehat{Z}_{hss}-\widehat{Z}_{fss}\right)T \;<\; L+\frac{\theta\ln(1+\tau_f)}{1-\theta}, \quad (1.34)$$

where I define:

$$L=-\ln\left(\left(\frac{A_f}{A_h}\right)^{\frac{\theta}{1-\theta}}\frac{Z_{h0}}{Z_{f0}}\right),$$

holds, Home will purchase intermediates domestically. The left hand sound of (1.34) is bounded with respect to T, therefore for τ_f large enough, the country less efficient at R&D will not specialize in the production of intermediates. The intuition is simple, in early stages of development transitional growth is able to dominate different steady state growth differences. As the transitional growth disappears, the steady state growth differences dominate.

To provide more intuition behind the model I provide some simple calibration results. For simplicity I assume:

$$\left(\frac{N_{ssf}}{N_{f0}}-1\right)=\left(\frac{N_{ssh}}{N_{h0}}-1\right)=0,$$

$$\widehat{Z}_{hss}-\widehat{Z}_{fss}=0.05\%,$$

$$\frac{\theta}{1-\theta} = 1,$$

$$\frac{\frac{A_h\left(1+\tau_f\right)}{Z_{h0}}}{\frac{A_f}{Z_{f0}}} = 2,$$

the first assumption implies that the transitional growth effects are identical for both countries (and equal to zero), the second implies that Home is growing 0.05 percentage points faster than Foreign, the third implies that the intermediates good share is 0.5, lastly I assume that in the initial conditions, the quality adjusted price of imported capital is double than domestically produced capital. These assumptions imply that:

$$T* = \frac{ln\left(2\right)}{.05\%} = 1386$$

these results indicate that the timing of endogenously overcoming the barriers can be fairly long.

In order to further investigate the effects of trading frictions on the timing of the regime switch I consider a new set of calibrated values. Here I assume that:

$$\frac{A_f}{Z_{f0}} = \frac{A_h}{Z_{h0}},$$

$$\tau_f = 0.3,$$

the current calibration assumes that the initial quality adjusted prices (without shipping costs) are equal,and that trading frictions increase the price of the good 30% (all other parameters are the same as the previous example). This new set of parameters imply:

$$T* = \frac{ln\left(1+\tau_f\right)}{\widehat{Z}_{hss} - \widehat{Z}_{fss}},$$

$$T* = \frac{ln\,(1.3)}{.05\%} = 524.$$

This experiment assumes that countries begin with the same initial conditions. Initially Home and Foreign have very similar intermediate goods, there is no comparative advantage. Given Home is growing modestly faster than Foreign (by 0.05 percentage points), it will take 524 years to overcome a 30% barrier to trade.

1.4 Trade

The countries are now in a trading regime. First I analyze the evolution of the economy in response to trade. The tractability of the framework allows a full analysis of the transitional dynamics and welfare effects. When countries begin trading in this endogenous way, there are no static gains from trade, instead all gains come from dynamics. Trade has different effects for Home and Foreign.

1.4.1 Intermediate goods

The demand for intermediate goods are:

$$X_{hi} = \left(\frac{\theta}{P_{xhi}}\right)^{\frac{1}{1-\theta}} Z_{hi}^{\delta} Z_h^{1-\delta} l_{hh},$$

and

$$X_{hi_{export}} = \left(\frac{\theta}{P_{xhi}\,(1+\tau_f)}\right)^{\frac{1}{1-\theta}} Z_{hi}^{\delta} Z_h^{1-\delta} l_{fh}.$$

The total demand is thus:

$$X_{hitotal} = Z_{hi}^{\delta} Z_h^{1-\delta} \left[\left(\frac{\theta}{P_{xhi}} \right)^{\frac{1}{1-\theta}} l_{hh} + \left(\frac{\theta}{P_{xhi}\left(1+\tau_f\right)} \right)^{\frac{1}{1-\theta}} l_{fh} \right]. \tag{1.35}$$

Firms maximize their lifetime value subject to their R&D technology (2.27) and the demand (3.6). Their current value Hamiltonian is:

$$H = X_{hitotal}\left(P_{xhi} - A_h\right) - \phi Z - R_{hi} + \eta \alpha_{zh} R_i,$$

following the same steps as in section 2) I obtain:

$$r_z = \alpha_{zh}\delta \frac{1-\theta}{\theta} A_h \left(\frac{\theta^2}{A_h} \right)^{\frac{1}{1-\theta}} \left(\frac{L_h + \left(\frac{1}{1+\tau_f} \right)^{\frac{1}{1-\theta}} L_f}{N} \right),$$

$$P_{xhi} = \frac{A_h}{\theta}.$$

Firms enter the market subject to the entry cost:

$$V_{it} = \beta X_{ittotaldemand}.$$

The rate of return to entry is given by:

$$r_n = \frac{\pi_{xi}}{V} + \widehat{V}.$$

Following the steps in section 2) I obtain:

$$\widehat{Y}_h = \frac{\pi_{xi}}{V} + \frac{w_h}{NV} - \frac{C_h}{NV} + \widehat{X}_h.$$

The Euler equation implies:

$$\widehat{C}_h = \frac{\pi_{xi}}{V} + \widehat{X}_h - \rho.$$

These two together imply that the Home consumption output ratio $c_h = \frac{C_h}{Y_h}$ is constant:

$$c_h = 1 - \theta + \frac{\rho\beta\left(\frac{\theta}{P_{xhi}}\right)\left(L_h + \left(\frac{1}{1+\tau_f}\right)^{\frac{1}{1-\theta}} L_f\right)}{L_h},$$

this implies that

$$\widehat{C}_h = \widehat{Y}_h = \widehat{Z}_h,$$

the rate of quality growth at the instant of trade is thus:

$$\widehat{Z}_{ht} = \alpha_{zh}\delta\frac{1-\theta}{\theta}A_h\left(\frac{\theta^2}{A_h}\right)^{\frac{1}{1-\theta}}\frac{\left[L_h + \left(\frac{1}{1+\tau_f}\right)^{\frac{1}{1-\theta}} L_f\right]}{N_{ht}} - \rho. \tag{1.36}$$

Now I discuss the entry process when countries begin trading. Following the steps used to derive (1.21) I obtain:

$$\widehat{N} = \frac{\pi_{hi}}{V} - \rho,$$

or:

$$\widehat{N}_{trade} = \frac{X_{hitotal}\left(\frac{1-\theta}{\theta}A\right) - \phi Z - R_i}{\beta X_{hitotal}} - \rho. \tag{1.37}$$

The steady state number of firms is thus:

$$N_{sstrade} = \frac{\left[\frac{1-\theta}{\theta}A_h(1-\delta) - \beta\rho\right]\left(\frac{\theta^2}{A_h}\right)^{\frac{1}{1-\theta}}\left[L_h + \left(\frac{1}{1+\tau_f}\right)^{\frac{1}{1-\theta}} L_f\right]}{(\phi - \frac{1}{\alpha_z}\rho)}. \tag{1.38}$$

Substituting (1.38) into (1.36) I obtain the steady state growth rate of the economy:

$$\widehat{Y}_{hss} = \widehat{Y}_{fss} = \widehat{Z}_{hss} = \frac{\delta(1-\theta)A_h(\alpha_{zh}\phi - \rho)}{(1-\theta)A_h(1-\delta) - \theta\beta_h\rho} - \rho, \tag{1.39}$$

note that (1.39) is identical to the steady state growth rate of Home in autarky. Also note that the steady state growth rate is invariant to trading frictions and the size of either economy. This is because of the market structure in this framework. The export demand from Foreign increases the market size of Home, however, this export demand also increases profits which induces entry; this is what nullifies the scale effect.

Proposition 4: *Absent transportation costs, the steady state number of firms in the world is linear in aggregate population; with transportation costs it is not.*

Proof: Observe that (1.38) is modified to:

$$N_{ss} = \frac{\left[\frac{1-\theta}{\theta}A_h(1-\delta) - \beta\rho\right]\left(\frac{\theta^2}{A_h}\right)^{\frac{1}{1-\theta}}\left[L_h + L_f\right]}{(\phi - \frac{1}{\alpha_z}\rho)}$$

when $\tau_f = 0$. Clearly, with the shipping costs the steady state number of firms is not linear in aggregate population. Thus the presence of trade might create difficulties for empirical investigations into the relationship between the number of firms and population size. This point is important, because a common misunderstanding of this class of models is that there is an assumed linear relationship between the number of firms and population size.[13] If I were to simply assume that the number of firms is linear in aggregate population size (i.e $N_{ss} = \xi\left(L_h + L_f\right)$), the steady state growth rate is dramatically

[13]Jones (1999) makes this claim.

altered:

$$\widehat{Z}_{hssfalse} = \frac{\alpha_z \delta \frac{1-\theta}{\theta} A_h \left(\frac{\theta^2}{A_h}\right)^{\frac{1}{1-\theta}} \left[L_h + \left(\frac{1}{1+\tau_f}\right)^{\frac{1}{1-\theta}} L_f\right]}{\xi\left(L_h + L_f\right)} - \rho, \tag{1.40}$$

the true steady state growth rate (1.39) and (1.40) have completely different interpretations. For instance, (1.39) tells us that migration from Foreign to Home would not influence the long-run growth rate, while it would increase (1.40). In addition, note that the shipping cost is not present in (1.39), but it is in (1.40).

1.4.2 Welfare

In this section I derive the welfare effects of trade for Home and Foreign. The welfare effects can be broken up into three parts; transitional growth, steady state growth, and static effects.

Home

Before the countries begin trading Home's consumption can be found from (1.2), rearranging:

$$C_h = N_h \pi_{hi} + w_h L_h - N_h V_h \widehat{N}_h$$

Substituting in (1.22) yields:

$$C_{hautarky} = \left((1-\theta) + \beta\rho\left(\frac{\theta^2}{A_h}\right)\right)\left(\frac{\theta^2}{A_h}\right)^{\frac{\theta}{1-\theta}} L_h Z_h.$$

In trade this is modified to:

$$C_{htrade} = \left((1-\theta) + \beta\rho \left(\frac{\theta^2}{A_h} \right) \left(1 + \left(\frac{1}{1+\tau_f} \right)^{\frac{1}{1-\theta}} \frac{L_f}{L_h} \right) \right) \left(\frac{\theta^2}{A_h} \right)^{\frac{\theta}{1-\theta}} L_h Z_h$$

The difference:

$$\frac{C_{htrade} - C_{hautarky}}{L_h} = \beta_h \rho \left(\frac{\theta^2}{A_h \left(1+\tau_f\right)} \right)^{\frac{1}{1-\theta}} \frac{L_f}{L_h} Z_h,$$

is unambiguously positive. This corresponds to extra dividend income the country gets from servicing two countries. Home thus receives static gains from trade.

In addition, I analyze the dynamic gains Home reaps. To do so, I compare the lifetime utility Home receives under the trading regime to had they remained in autarky(see Appendix 2 for detailed derivations).

Had Home remained in autarky:

$$U_{hautarky} = \frac{\widehat{Z}_{hss}}{\rho^2} + \left(\frac{\beta \left(\widehat{Z}_{hss} + \rho \right) \theta}{\rho \left(1-\theta\right) A_h \left(1-\delta\right)} \right) \left(\frac{N_{hss} - N_{h0}}{N_{h0}} \right) + M_{h0}, \tag{1.41}$$

where:

$$M_{h0} = \frac{\log\left(Z_{k0}\right) + \log\left(\left((1-\theta) + \beta\rho \left(\frac{\theta^2}{A_h} \right) \right) \left(\frac{\theta^2}{A_h} \right)^{\frac{\theta}{1-\theta}} L_{ht} \right)}{\rho}.$$

After the countries begin trading the lifetime utility of Home is:

$$U_{htrade} = \frac{\widehat{Z}_{ss}}{\rho^2} + \frac{\beta \left(\widehat{Z}_{ss} + \rho \right) \theta}{\rho \left(1-\theta\right) A \left(1-\delta\right)} \left(\frac{N_{sstrade} - N_{h0}}{N_{h0}} \right) + M_{h0trade}, \tag{1.42}$$

where:

$$M_{h0trade} = \frac{\log\left(Z_{h0}\right) + \log\left[\left(1-\theta+\beta\rho\left(\frac{\theta}{P_{xhi}}\right)^{\frac{1}{1-\theta}}\left(1+\left(\frac{1}{1+\tau_f}\right)^{\frac{1}{1-\theta}}\frac{L_f}{L_h}\right)\right)\left(\frac{\theta}{P_{xh}}\right)^{\frac{\theta}{1-\theta}}L_h\right]}{\rho}.$$

Subtracting (1.41) from (1.42) gives the change in lifetime utility:

$$U_{h\triangle_{trade}} = \frac{\log\left(\frac{(1-\theta)+\beta\rho\left(\frac{\theta^2}{A_h}\right)\left(1+\left(\frac{1}{1+\tau_f}\right)\frac{L_f}{L_h}\right)}{(1-\theta)+\beta\rho\left(\frac{\theta^2}{A_h}\right)}\right)}{\rho} + \frac{\beta\left(\widehat{Z}_{hss}+\rho\right)\theta\left(\frac{N_{sstrade}-N_{hss}}{N_{h0}}\right)}{\rho\left(1-\theta\right)A\left(1-\delta\right)}$$

The first term is unambiguously positive. Home gains extra dividend income. The second term is also unambiguously positive. Home is now receiving faster transitional growth. Note that these welfare gains are for arbitrary values of N_{h0}. Not only is there additional transitional growth to the new steady state, the transitional growth the country was receiving in autarky is amplified.

Assuming Home was at their steady state before trade:

$$U_{h\triangle_{trade}} = \frac{\beta\theta\left(\widehat{Z}_{hss}+\rho\right)\left(\frac{1}{1+\tau_f}\right)^{\frac{1}{1-\theta}}\left(\frac{L_f}{L_h}\right)}{\left(1-\theta\right)\left(1-\delta\right)A_h} + \frac{\log\left(\frac{(1-\theta)+\beta\rho\left(\frac{\theta^2}{A_h}\right)\left(1+\left(\frac{1}{1+\tau_f}\right)\frac{L_f}{L_h}\right)}{(1-\theta)+\beta\rho\left(\frac{\theta^2}{A_h}\right)}\right)}{\rho},$$

the gains from the transition are increasing in the size of intermediate importing country (Foreign). Intuitively trade increases market size, how much trade increases market size is related to the relative size of the importing country. If the importing country is small compared to the exporting country, their wont be very much additional transitional dynamics. (again, in this scenario I assumed Home was at their steady state before trade,

ie $N_{h0} = N_{ss}$)

Foreign

Had Foreign remained in autarky:

$$U_{fautarky} = \frac{\widehat{Z}_{fss}}{\rho^2} + \beta\left(\widehat{Z}_{fss} + \rho\right)\left(\frac{\theta}{\rho\left(1-\theta\right)A_f\left(1-\delta\right)}\right)\left(\frac{N_{fss} - N_{f0}}{N_{f0}}\right) + M_{f0}, \quad (1.43)$$

where:

$$M_{f0} = \frac{\log\left(Z_{f0}\right) + \log\left(\left(\left(1-\theta\right) + \beta\rho\left(\frac{\theta^2}{A_f}\right)\right)\left(\frac{\theta^2}{A_f}\right)^{\frac{\theta}{1-\theta}} L_f\right)}{\rho}.$$

After the countries begin trading the lifetime utility of Foreign is:

$$U_{ftrade} = \frac{\widehat{Z}_{hss}}{\rho^2} + \left(\frac{\beta\left(\widehat{Z}_{hss} + \rho\right)\theta}{\rho\left(1-\theta\right)A_h\left(1-\delta\right)}\right)\left(\frac{N_{sstrade} - N_{h0}}{N_{h0}}\right) + M_{ftrade}, \quad (1.44)$$

where:

$$M_{f0trade} = \frac{\log\left(Z_{h0}\right) + \log\left(\left(1-\theta\right)\left(\frac{\theta^2}{\left(1+\tau_f\right)A_h}\right)^{\frac{\theta}{1-\theta}} L_{ft}\right)}{\rho}.$$

Subtracting (1.43) from (1.44) yields:

$$U_{f\triangle_{trade}} = \frac{\widehat{Z}_{hss} - \widehat{Z}_{fss}}{\rho^2} + \frac{\beta\theta\left[\left(\widehat{Z}_{hss}+\rho\right)\left(\frac{N_{sstrade}-N_{h0}}{N_{h0}A_h}\right) - \left(\widehat{Z}_{fss}+\rho\right)\left(\frac{N_{fss}-N_{f0}}{N_{f0}A_f}\right)\right]}{\rho\left(1-\theta\right)\left(1-\delta\right)}$$
$$\frac{\log\left(\frac{\frac{Z_{h0}}{\left(A_h(1+\tau)\right)^{\frac{\theta}{1-\theta}}}}{\frac{Z_{f0}}{A_f^{\frac{\theta}{1-\theta}}}}\right) + \log\left(\frac{1-\theta}{1-\theta+\beta\rho\left(\frac{\theta^2}{A_f}\right)}\right)}{\rho},$$

the quality adjusted prices are equal at the moment of trade, this implies:

$$U_{f\triangle_{trade}} = \frac{\widehat{Z}_{hss} - \widehat{Z}_{fss}}{\rho^2} + \frac{\beta\theta\left[\left(\widehat{Z}_{hss}+\rho\right)\left(\frac{N_{sstrade}-N_{h0}}{N_{h0}A_h}\right) - \left(\widehat{Z}_{fss}+\rho\right)\left(\frac{N_{fss}-N_{f0}}{N_{f0}A_f}\right)\right]}{\rho\left(1-\theta\right)\left(1-\delta\right)}$$
$$\rho\log\left(\frac{1-\theta}{1-\theta+\beta\rho\left(\frac{\theta^2}{A_f}\right)}\right).$$

In general the welfare effects on Foreign can be positive or negative. The steady-state growth of quality improvement is higher in Home than Foreign. Foreign also gets the transitional growth induced in trade, but they lose the transitional growth had they remained in autarky. They also lose their dividend income.

Assuming Home and Foreign were at their respective steady states, the following guarantees gains from trade:

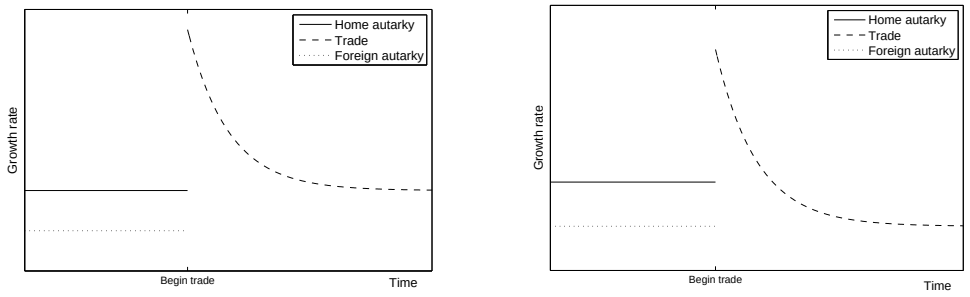
$$\frac{\widehat{Z}_{hss} - \widehat{Z}_{fss}}{\rho} + \left(\frac{\beta_h\theta\left(\widehat{Z}_{hss}+\rho\right)}{\left(1-\theta\right)\left(1-\delta\right)A_h}\right)\left(\frac{1}{1+\tau_f}\right)^{\frac{1}{1-\theta}}\frac{L_f}{L_h} > \log\left(\frac{\left(1-\theta\right)+\beta_f\rho\left(\frac{\theta^2}{A_f}\right)}{1-\theta}\right)$$

This says that the dynamic gains from trade (higher steady state growth and transition)

has to be greater than the loss of income. One thing to notice is that the change in static income (and transitional dynamics) should be relatively small compared to the effect of the change in steady state growth. The change in steady state growth rates is being divided by ρ, given plausible values of ρ , this term can be very large.

1.4.3 Endogenous transition vs Exogenous

As mentioned in the introduction, there are fundamental differences between an endogenous transition and exogenous. In particular, when trade happens in this endogenous way, dynamic gains from trade are guaranteed. With an exogenous reduction in tariffs or shipping costs, the less efficient R&D country can specialize in the production of R&D goods. There are two possible outcomes depicted in figure 1.2.



(a) Endogenous transition or Exogenous (b) Exogenous

Figure 1.2: The dynamic effects of trade on growth

Suppose that at T_0 there is an exogenous reduction in τ. Then whoever has the lowest quality adjusted price of the intermediate goods specializes in their production.

- Case 1:

$$\frac{A_h}{Z_{h0}^{\frac{1-\theta}{\theta}}} > \frac{A_f}{Z_{f0}^{\frac{1-\theta}{\theta}}}$$

Foreign specializes in the production of capital goods. Since Foreign has a lower R&D efficiency than Home the steady state growth rate of Home will be lower after trade, as depicted in figure 1.2b

- Case 2:

$$\frac{A_h}{Z_{h0}^{\frac{1-\theta}{\theta}}} < \frac{A_f}{Z_{f0}^{\frac{1-\theta}{\theta}}}$$

In this scenario, Home is better at R&D than Foreign and also has a lower quality adjusted price at T_0. Thus the response to trade is given by figure 1.2a.

In case 1, a reduction in trading frictions τ can be welfare reducing. Intuitively, the intermediate good importing country might receive static gains from trade but necessarily gets dynamic losses. In case 2, a reduction in τ has the same static effects as in case 1, however, the importing county will have dynamic gains.

1.4.4 Dynamic Effects of frictions

In this section I assume that the countries are in the trading regime. Recall that the steady state growth rate (1.39) is unaffected by the frictions. However, that does not mean that the frictions do not affect growth. In fact, the frictions have temporary effects. Assume at T_0 there is an exogenous reduction of τ from τ_1 to τ_0. Instantaneously the growth rate jumps from:

$$\widehat{Z}_{h0} = \alpha_{zh}\delta\frac{1-\theta}{\theta}A_h\left(\frac{\theta^2}{A_h}\right)^{\frac{1}{1-\theta}}\frac{\left[L_h + \left(\frac{1}{1+\tau_0}\right)^{\frac{1}{1-\theta}}L_f\right]}{N_{ht}} - \rho$$

to:

$$\widehat{Z}_{h0} = \alpha_{zh}\delta\frac{1-\theta}{\theta}A_h\left(\frac{\theta^2}{A_h}\right)^{\frac{1}{1-\theta}}\frac{\left[L_h + \left(\frac{1}{1+\tau_1}\right)^{\frac{1}{1-\theta}}L_f\right]}{N_{ht}} - \rho$$

However, this is a partial equilibrium result. Entrepreneurs see profit opportunities and enter until we reach the new steady state. A reduction in frictions will cause the growth rate to jump up. The size of the jump depends on several things. However, one of the most interesting is the relative size of Foreign's market. If exports make up a large portion of Home's market, a reduction in the friction will have a larger effect. The growth effect is temporary, the dynamics tells us that the growth rate will decline back to the steady state value (1.39).

The welfare effects of the trading frictions are easily calculable:

$$U_{h\triangle friction} = \frac{\beta\left(\widehat{Z}_{ss}+\rho\right)\theta L_f}{\rho\left(\left(1-\theta\right)A\left(1-\delta\right)\right)}\left(\frac{\left(\frac{1}{1+\tau_{f_1}}\right)^{\frac{1}{1-\theta}} - \left(\frac{1}{1+\tau_{f_0}}\right)^{\frac{1}{1-\theta}}}{L_h + \left(\frac{1}{1+\tau_{f_0}}\right)^{\frac{1}{1-\theta}}L_f}\right) + M_{h\tau_1} - M_{h\tau_0},$$

where:

$$M_{h\tau} = \frac{\log\left(Z_{h0}\right) + \log\left(\left(\left(1-\theta\right) + \beta\rho\left(\frac{\theta^2}{A_h}\right)^{\frac{1}{1-\theta}}\left(1 + \left(\frac{1}{1+\tau_f}\right)^{\frac{1}{1-\theta}}\frac{L_f}{L_h}\right)\right)\left(\frac{\theta^2}{A_h}\right)^{\frac{\theta}{1-\theta}}L_h\right)}{\rho}.$$

A reduction in the friction will have a larger impact on welfare if the entry cost β is high. The static income effect gives higher dividend income to Home, dividend income is increasing in the entry cost. In addition, the higher entry cost slows down the rate of entry which yields more transitional growth. Unsurprisingly, the larger the importing country is the larger the impact of a reduction in the friction. This again works through

the static income effect and the transitional growth.

1.5 Discussion

The model developed in this paper delivers several new results on the relation between trade, growth and trading frictions. This paper proposed a theory of the emergence of trade. Rather than relying on falling shipping costs or reductions in tariffs, endogenous quality improvement is what leads countries to begin trading. As long as the countries have different R&D efficiencies the countries will make the transition from autarky to trade. In fact, even if countries have identical fundamentals the transition might still occur. What matters is that the autarkic growth rate of the countries is different. If one of the countries is receiving faster transitional growth than the other country the countries might still make the transition. In addition, this paper provides new insight into the effects of trading frictions on growth. Trading frictions do not affect the steady-state growth rate, instead their effects are transitional. A reduction in trading frictions will increase the market size of existing firms, this increases their incentive to do R&D. However, the fall in the trading frictions induces entry, which nullifies the scale effect.

A point worth emphasizing is that while transitional dynamics complicate matters, they have the potential to have important effects. One of the results of this paper is that if countries are at their respective steady-states, the country that is more efficient at R&D will incompletely specialize in the production of the intermediates. The country importing the intermediates will necessarily receive faster steady-state growth under the trade regime than had they remained in autarky. However, allowing the countries to out of their steady-states, this result disappears. If Foreign is receiving extra transitional growth they might end up producing the intermediates. The tractability of the framework

allows me to find sufficiently large trading frictions to rule out this second scenario which is likely welfare sub-optimal.

Chapter 2

Higher Quality Apples, or Lower Cost?

2.1 Introduction

The purpose of this paper is to study the interaction between distribution and endogenous innovation (quality improvement and cost reduction). In particular this paper focuses on the effects of different distribution technologies. When distribution is modeled using the familiar 'iceberg' technology, cost reduction and quality improvement are isomorphic. However, when distribution is modeled using a per-unit technology the isomorphism is destroyed. Furthermore, if there is any component of the distribution technology that is per-unit, cost reduction cannot be a source of long-run growth while quality improvement can.

It is not controversial that product quality has increased immensely for many goods. For example, automobile quality has increased in terms of tangible features; increased efficiency, fuel economy, and safety. Other more difficult to quantify features have been added as well; blue-tooth connectivity, power-steering, multi-media devices, and so forth.

It is also not be controversial to state that the manufacturing productivity of workers

has also increased. One of the most blatant examples of this is the computer industry. This can seen by the outdated "Machrone's" law. In the 1980's the editor of the popular "PC Magazine" Bill Machrone observed that the price of new computer models was staying roughly constant at $5,000. His law was: "The computer you want, always costs five thousand dollars". This "law" is clearly not valid anymore even without adjusting for inflation.

Machrone's "law" deals with two issues that are at the heart of this paper. First, new computer models are being created that are of ever increasing quality. Second, at the time, the nominal price of new editions of computers was roughly constant. Of course, adjusting for inflation this meant that the real price of computers were falling. There were two distinct events occurring falling prices and ever increasing product quality.

The growth literature has successfully incorporated different types of research and development. Quality improvement, variety-expansion, and cost reduction all have been explored. Early on, these models were seen as substitutes and simple isomorphisms of each other[1]. Later, the problems of scale effects were successfully eliminated by Peretto (1998) and Howitt (1999) by allowing vertical and horizontal innovation to occur simultaneously. With Peretto focusing on cost reduction, and Howitt focusing on quality improvement.

Despite recent interest in product quality[2], the differences between cost reduction and quality improvement have been ignored. This is surprising because firms engage in both cost reduction and quality improvement. Furthermore, there are significant cross-country differences regarding R&D expenditures. Mansfield (1988) finds that American

[1]See Barro and i Martin (2003) for a discussion.

[2]Product quality has been investigated empirically by Hallak and Schott (2008) and Hummels and Klenow (2005). The theoretical literature on product quality is far too large to list; notable papers include: Grossman and Helpman (1991), Segerstrom, Anant, and Dinopoulos (1990), and Aghion and Howitt (1992).

firms devote two-thirds of their R&D expenditures to product innovation while Japanese firms only devote one-third, with the remaining R&D expenditures going to cost reduction. Unfortunately, Mansfield does not distinguish between quality improvement and product creation[3]. Scherer (1980) analyzes R&D expenditure in more detail. He finds that American firms devote 59% of their research expenditure to improving the quality of existing products, 28% to developing new products and 13% to developing new processes. Modern endogenous growth models make no attempt to explain this data. In fact, there is little hope they can because in most models quality improvement and cost reduction are isomorphic.

As argued above, the average price of computers declined substantially since their creation. However, casual observation tells us that the rate of price reduction has slowed in recent years. This trend has been observed by industry insiders, in a recent Wall Street Journal article Worthen (2010) quoted chief executive of Intel Paul Otellini "They aren't going to fall to $99" regarding laptops at an investors meeting.

The data agrees with Paul Otellini's statement. In data made available by the BLS it is evident that the rate of cost reduction in computer and electronic manufacturing is declining. From 1990 to 2000 the average annual rate of unit cost reduction was 10%, from 2001 to 2010 the average rate was only 2.9%.

It is not surprising that the rate of price reduction fell, this phenomenon is well known. In fact, Burns (1934) documented this trend in data going back to 1870. Gort and Klepper (1982) show this is present in modern data as well. However, these features have never been incorporated or explained in the growth literature[4].

We generalize the cost function facing firms to incorporate two distinct unit costs;

[3]Product innovation includes both quality improvement and product proliferation.

[4]Interestingly, when Burns (1934) discusses growth, product quality is not mentioned. This is surprising considering how much emphasis has been placed on product quality in the modern growth literature.

manufacturing costs which are reducible through costly R&D, and distribution costs that firms take as given. By instituting this distribution cost, the isomorphism between cost reduction and quality improvement is destroyed. Most notably, cost reduction cannot be a source of long run growth, while quality improvement is left unharmed.

In addition to the distribution costs, there are other justifications for the irreducible unit costs. Instituting an additional irreducible unit cost generalizes the cost function facing firms; rather than endogenous cost reduction leading to unit costs that approach zero, instead they are allowed to approach a positive constant[5]. However, more insights can be gained from the distribution cost interpretation. Accordingly, we proceed under the distribution cost interpretation. In fact, the original motivation of this paper came from analyzing the impact of per unit shipping costs on cost reduction and quality improvement.

This is not the first paper that imposes an irreducible unit cost; Sutton (2007) institutes an exogenous per-unit 'materials' cost in a static trade model with costly R&D. In his model, the material cost sets a lower bound for the price of a good. The price consumers pay for a good can never be lower than the exogenous materials cost. This breaks the isomorphism between cost and quality in a trade setting. In essence, firms that are incredibly efficient and have a very low production cost are not the same as firms that have an extremely high quality because the price has a lower bound; the materials cost, while quality has no such bound.

The mechanism in Sutton (2007) is also present in this model. Firms can do R&D to reduce their unit cost, this reduces the price that consumers pay and increases their demand for the good. However, the presence of this per-unit distribution cost creates a

[5]This generalization becomes important if there is any component of the unit cost that cannot be reduced.

lower bound for the price. No matter how much R&D firms do, the price consumers pay for the good will never be lower than the distribution cost, this puts a limit on demand. In a quality improvement model, quality has no limit, and hence there is no limit to demand.

The reason we break with Sutton and focus on distribution costs rather than 'materials' cost is that, firms might be able to reduce the amount of materials they need in terms of production. For example, in Peretto (2010) technology augments not only the labor but also the materials required for production. In this context, even though the price paid by the consumers is influenced by materials cost, R&D can reduce the amount of materials required[6].

However, both this model and Sutton's have an alternative justification; the irreducible component institutes a lower bound for the unit cost. For Sutton's case this lower bound on materials makes absolute sense; producing a sheet of paper will always require some minimum amount of pulp, a laptop will always require some minimum amount of glass. In this model, it is also entirely reasonable that there is a lower bound to the distribution costs; a worker has to load and secure each item on a truck, stock-boys have to place the item on the shelves, and finally consumers have to spend time driving to the store to purchase the item[7].

The growth literature has entirely ignored distribution costs. Currently, growth economists can say that included in the unit cost is the cost of producing the good and distributing to the customers. This of course is a heroic assumption. Firms do cost reduction in manufacturing, however, these improvements in manufacturing productivity

[6]According to Warr, Schandl, and Ayres (2008) the natural resource energy intensity of GDP for the U.K declines at an average rate of 2.4% per year.

[7]As mentioned previously even ignoring distribution costs, the irreducible component can be justified as a minimum manufacturing unit cost.

are unlikely to have any impact on the productivity of distribution. By arguing that the unit costs include the distribution costs, one is implicitly assuming that improvements in manufacturing productivity increases the distribution efficiency.

It appears that distribution costs should not be ignored, empirically they are surprisingly large. Burstein, Neves, and Rebelo (2003) find that over 40% of the retail price of a typical consumer good is comprised of distribution costs. Argentina's distribution appeared to be quite worse than the United States, with distribution costs that make up 60% of the retail price. Feenstra (1998) gives the following example to show how large distribution costs can be: "...the doll is produced in Asia at a cost of one dollar per unit (35 cents for labor and 65 cents for materials). It costs an additional dollar of distribution services to get the doll to the US via Hong Kong. Mattel makes one dollar of profit from each doll. The sale price is $10 dollars of which $7 pay for transportation, marketing, and retail services in the US. Production costs are a mere 10% of the retail price of the product."

The purpose of this paper is to analyze the effect of an irreducible unit cost in a cost reduction and quality improvement setting. To analyze this issue, it is imperative that we use an endogenous growth model, since the issue raised deals with the effects of an irreducible unit cost on research and development and their effects on long term growth. Peretto and Connolly (2007) is particularly well suited for this task, it is the most tractable endogenous growth model without scale effects available[8]. In addition, this class of models has received strong empirical support in recent years see: Ang and Madsen (2011) , Laincz and Peretto (2006), Madsen (2008) , Madsen, Ang, and Banerjee (2010), and Ha and Howitt (2007).

[8]The authors are able to obtain closed form transitional dynamics, since are adding a complication it is natural to use the most tractable model available.

To some it might sound strange to discuss distribution costs in an autarkic model, however, firms have to ship their items to their destinations. In fact, shipping costs per mile is much more expensive on land than by boat. To discuss this model in the context of international trade would distract from the main points; although the results are very relevant to the field of international economics. In particular, this model predicts that countries that face high shipping costs will do less cost reduction; accordingly, the share of R&D expenditures going to quality improvement will be higher in these countries.

All of these results lead to the importance of thinking about physical units. In the cost reduction model our world becomes overrun by physical units of intermediate goods. In the quality improvement model, the number of physical units remains the same, instead the products are of ever increasing quality. The cost reduction model essentially says that in 100 years consumers will be buying much more computers than we are today. In the quality improvement model, the number of computers individuals buy remains constant, just with higher processing power and storage space. If one takes into account that one has to ship these goods, it is not surprising at all that quality improvement is different than cost reduction.

Hallak and Schott (2008) have created a novel approach to estimate product quality. One of their findings is that some countries engage in quality improvement while others engage in price reduction, this compliments the results Mansfield (1988) obtains. These results indicate that the theoretical differences between cost reduction and quality improvement must be analyzed. Our model yields interesting transitional dynamics that are present in cost reduction, that are not present in quality improvement. This model suggests that a country with low manufacturing productivity like Japan (in the 1980's), is more likely to do cost reduction than a more developed country like the United States.

This paper contributes to the literature on several levels. First, this work sheds light

on important theoretical differences between cost reduction and quality improvement. In particular, cost reduction cannot be a driver of long run growth, while quality improvement can. The mechanism at work gives a new explanation for the falling rates of price reduction documented in the data. Secondly, we show the fundamental difference between the traditional iceberg transportation cost, and the less often utilized per-unit transportation costs. The difference is not innocuous, this results suggests that models of trade and growth will be drastically altered depending on the transportation cost assumptions.

The paper is organized as follows. In section 2 we briefly describe the general framework that will be used in the following sections, and show that absent the distribution cost, the quality improvement and cost reduction models are equivalent. In section 3 we analyze the cost reduction model and explore the difference between iceberg shipping costs and per-unit. Furthermore, I show that the results hold in a learning by doing model as well. Finally, in section 4 we analyze the quality improvement model. We conclude in section 5.

2.2 The model

We utilize Peretto and Connolly (2007) as the base model. We break with them in two regards. First, we institute a per unit distribution cost. Second, we allow quality improvement; we assume that there is a quality attached to the intermediate goods consumers buy. In this section, we show that absent the distribution cost, the cost reduction and quality improvement models are qualitatively identical. Although the growth rates are identical under both formulations, the mechanism is different. The purpose of this section is to provide the intuition behind both models

In both forms preferences are described by:

$$U = \int_0^{\infty} e^{-\rho t} log(C_t) dt \tag{2.1}$$

In the original form the aggregator is:

$$C = \left[\int_0^N (x_i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

we modify this to:

$$C = \left[\int_0^N (Q_i^{\theta} x_i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \tag{2.2}$$

In this setting, individuals buy intermediate goods x_i, attached to each intermediate is its quality Q_i. It is assumed that $\epsilon > 1$ and $1 \geq \theta \geq 0$. If $\theta = 0$ then the goods quality does not effect consumers utility, and firms have no incentive to invest in quality improvement.

Intuitively, the way cost reduction and quality improvement models work is different[9]. In the cost reduction model, growth occurs because people buy more of the intermediate goods as the goods become cheaper. In the quality improvement model people do not buy more goods, the price remains constant; instead people buy the same amount of goods that are growing in quality over time.

All of the derivations occur later in the paper, but it is worthwhile to briefly discuss the growth rates under both formulations. For the cost reduction model, we set $\theta = 0$, imposing symmetry (3.2) yields:

$$C = N^{\frac{\epsilon}{\epsilon-1}} x$$

[9]The isomorphism between cost reduction and quality improvement is commonly attributed to Spence (1984). He points out that if one views quality improvement as delivering the same amount of services for a lower cost, cost reduction and quality improvement are formally equivalent.

In the steady state as long as there is no population growth, there is no variety-expansion. Simple differentiation implies:

$$\widehat{C} = \widehat{x} = \sigma \widehat{Z}$$

the growth rate of the consumption good is equal to the growth rate of manufacturing productivity.

In the quality improvement model, there is no increase in manufacturing productivity (we do so by imposing $\sigma = 0$). Instead, quality improvement drives everything. In this case, symmetry implies:

$$C = N^{\frac{\epsilon}{\epsilon-1}} Q^{\theta} x$$

Just as before, there is no variety-expansion. Simple differentiation yields:

$$\widehat{C} = \theta \widehat{Q}$$

The growth rate of consumption under cost reduction is:

$$\widehat{C} = \alpha_z \frac{\sigma^2 LE(\epsilon - 1)}{\epsilon N_{ss}} - \sigma\rho$$

While the growth rate of consumption under quality improvement is:

$$\widehat{C} = \alpha_q \frac{\theta^2 LE(\epsilon - 1)}{\epsilon N_{ss}} - \theta\rho$$

As long as the efficiency of labor is the same at cost reduction and quality improvement i.e $\alpha_q = \alpha_z$ and $\sigma = \theta$ the rate of growth in the consumption aggregate will be identical regardless which method we use.

It is worth to discuss the differences between cost reduction and quality improvement

at an intuitive level. In the cost reduction model, the price goes down and people buy more physical units of the goods. In the quality improvement model, the number of physical units remain constant while the quality is increasing. If we take the case of computers, both models seem relevant. The computers we currently use are far better than the computers used 10 years ago, and the cost of computers have decreased dramatically. In addition, according to the BLS, the number of computers per household doubled from 2000 to 2008.

2.3 Cost Reduction

In this section we set $\theta = 0$, so quality does not influence consumers decisions. There are two potential sources of growth, variety-expansion and cost reduction. As will be seen, the fixed operating costs shut down variety-expansion, and the distribution cost shuts down cost reduction as a source of long run growth.

2.3.1 Households

Households maximize (3.1) subject to the aggregator and budget constraint, repeated here for convenience:

$$C = \left[\int_0^N (x_i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$$

$$\dot{A} = rA + W - E \tag{2.3}$$

where $E = \int_0^N P_i x_i di$ is consumption expenditure. Henceforth, we use the wage as the numeraire and set $W_t = 1$.

The dynamic optimization requires

$$\frac{\dot{E}}{E} = r - \rho \tag{2.4}$$

and also implies the usual per capita demands:

$$x_i = E\frac{P_i^{-\epsilon}}{\int_0^N P_i^{1-\epsilon} di} \tag{2.5}$$

2.3.2 Technology

The productivity of workers in manufacturing has improved dramatically throughout human history. There are several explanations for this, one common explanation is the accumulation of public knowledge, another plausible explanation is disembodied productivity knowledge. Following Peretto, we allow the accumulation of disembodied knowledge that is unique to each firm. In this setting, firms engage in costly R&D to improve the manufacturing efficiency of their own workers.

While there is little doubt that workers are much more efficient at manufacturing goods, the same cannot be said about the shipping industry. This is not surprising, the shipping industry is governed by fundamentally different technology than manufacturing. In fact, we should expect to see relatively little technical progress in the shipping industry given its market structure. According to Davies (1986), the shipping industry meets the criteria of a contestable market[10].

The differentiated consumption goods are manufactured by monopolisticly competitive firms who operate with the following technology:

[10]Davies is in the context of international trade, but one would expect his results should carry over to autarkic distribution.

$$h(Z_i)X_i + \phi = l_{xi} \tag{2.6}$$

firms hire l_{xi} units of labor to produce and distribute their goods. The only difference between this model and Peretto and Connolly (2007) is this specification for the marginal cost $h(Z_i)$. In the original setting $h(Z_i) = Z_i^{-\sigma}$, we modify this to $h(Z_i) = Z_i^{-\sigma} + S$, where $Z_i^{-\sigma}$ is the unit cost of production, and the unit cost of distributing the good is S. The fixed operating cost of the firm is ϕ.

$$\dot{Z}_i = \alpha_z Z l_{zi} \tag{2.7}$$

$$\dot{N} = \beta N L_n \tag{2.8}$$

Equation (2.7) describes how manufacturing productivity increases, α_z measures how effective scientists are at increasing the productivity. Entry is described by equation (2.8), β measures the difficulty of new product creation.

The new feature we add is the distribution cost. We separate unit cost of production and distribution because they are governed by fundamentally different technologies. Of course, a large part of distribution is shipping the goods to the customers. Virtually every good produced has to be on a truck at some point in the delivery. The unit labor cost of shipping a good by truck has remained relatively stable over the past 23 years, see Fig. 2.1.

We use per-unit transportation costs as opposed to the traditional (and convenient) "iceberg" costs to model distribution. Hummels and Skiba (2004) have shown in the context of international trade that iceberg shipping costs are strongly rejected by the

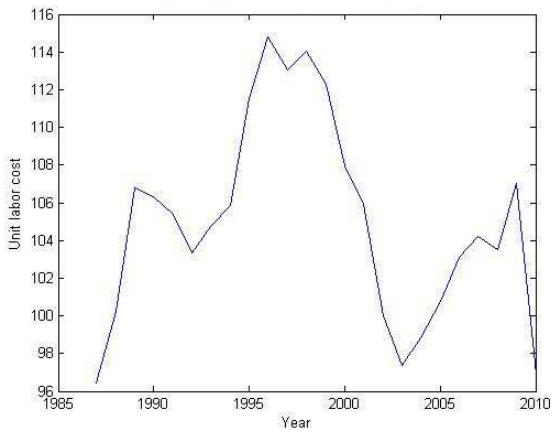


Figure 2.1: Unit labor cost of long distance trucking. Source: Bureau of Labor Statistics

data[11]. Moreover, they find a strong confirmation of the "Alchian-Allen" theorem. On a theoretical basis this is not surprising; by imposing iceberg shipping costs, modelers are imposing that the shipping technology is identical to the production technology.

If someone was to model shipping cost using iceberg formulation, they would be led to the conclusion that the assembly line dramatically improved Ford's shipping productivity. This conclusion of course is nonsense [12]. The difference between iceberg and per-unit transportation costs is explored below.

For expositional purposes we have assumed that shipping is done in house, the model can be modified to incorporate an additional shipping industry without changing the results. Suppose the price of shipping one unit is P_s and that each unit shipped requires

[11] In addition Irarrazabal, Moxnes, and Opromolla (2010) strongly reject iceberg shipping cost.

[12] To see this, look at the profit function under the iceberg formulation.
$\pi = P(1-S)X - l_x$
$\pi = PX - l_x - PSX$
The third term is the aggregate expenditure on shipping. The optimal price is $P = \frac{\epsilon}{(\epsilon-1)(1-s)} Z^{-\sigma}$. Thus we can rewrite profits:
$\pi = \frac{\epsilon}{(\epsilon-1)(1-s)} Z^{-\sigma} X - l_x - \frac{\epsilon}{(\epsilon-1)(1-s)} Z^{-\sigma} SX$
The per-unit expenditure on shipping is $\frac{\epsilon S}{(\epsilon-1)(1-S)} Z^{-\sigma}$, this is decreasing in manufacturing productivity Z_i.

S units of labor. Zero profits in the shipping industry requires $P_s X = SX$. Nothing fundamental changes in the model by adding a shipping industry, so it is omitted.

2.3.3 Firms

The firms instantaneous profits are given by:

$$\pi_i = P_i X_i - l_{xi} - l_{zi}$$

Substituting in the demands and technology, we obtain the profit maximizing price:

$$P_i = \frac{\epsilon}{\epsilon - 1}(Z^{-\sigma} + S) \tag{2.9}$$

Firms not only markup their production unit cost, but also their delivery cost. Interestingly, we can foresee some results by looking at the demands. Substituting the optimal price into the demand function (2.5) we obtain:

$$X_i = \frac{LE(\epsilon - 1)}{\epsilon} \frac{(Z_i^{-\sigma} + S)^{-\epsilon}}{\int_0^N (Z_i^{-\sigma} + S)^{1-\epsilon} di} \tag{2.10}$$

Differentiating (2.10) with respect to Z_i gives the elasticity of demand:

$$\frac{\partial X_i}{\partial Z_i} \frac{Z_i}{X_i} = \sigma\epsilon \frac{Z_i^{-\sigma}}{Z_i^{-\sigma} + S} \tag{2.11}$$

If $S = 0$, as in the original framework, the demand is isoelastic with respect to the productivity term Z_i. In this case, this means that a 1% increase in Z_i leads to a $\sigma\epsilon$% increase in demand. The σ controls how effective improving Z_i is at reducing the unit cost, and the ϵ controls how effective price reduction is at increasing demand. Without

the shipping cost, a 1% increase in Z_i leads to a σ% decrease in the price. The σ% decrease in price leads to a ϵ% increase in demand.

When $S > 0$ the demand is no longer isoelastic, as Z_i increases the elasticity (2.11) falls. A 1% increase in Z_i no longer leads to a σ% decrease in the price. Instead the elasticity of P_i with respect to Z_i is given by:

$$\frac{\partial P_i}{\partial Z_i}\frac{Z_i}{P_i} = -\sigma\left(\frac{Z_i^{-\sigma}}{Z_i^{-\sigma}+S}\right) \tag{2.12}$$

(2.12) tells us that the elasticity of price reduction with respect to Z_i is always lower than σ, and is declining in Z_i. Since it becomes increasingly difficult to decrease the price, this makes it increasingly difficult to increase demand. This is what shuts down cost reduction as a source of long run growth. In fact, the ratio

$$\frac{Z_i^{-\sigma}}{Z_i^{-\sigma}+S} \tag{2.13}$$

is important for intuition. $Z_i^{-\sigma}$ is the component of the unit costs that can be reduced, S is the component that cannot. This ratio, (2.13) is the percentage of the unit costs that may be reduced. As the percentage of the unit cost that can be reduced declines, it becomes increasingly difficult to decrease the aggregate unit cost. The dynamics are analyzed formally in the next section.

2.3.4 Dynamics

Firms maximize the present discounted value of profit:

$$V_i = \int_0^\infty e^{-\int_0^t r(s)ds}\left[P_iX_i - l_{xi} - l_{zi}\right]dt \tag{2.14}$$

The firms current value Hamiltonian is given by (2.15).

$$H = P_i X_i - l_{xi} - l_{zi} + \eta_{zi}\alpha_z Z l_{zi} \tag{2.15}$$

R&D investment l_{zi} and the price P_i are the firms control variables, and the firms knowledge stock Z_i is the state variable. The co-state variable η_{zi} represents the value of the firms innovation.

The linearity of the Hamiltonian leads to:

$$l_{zi} = \begin{cases} 0 & : \eta_{zi} < \frac{1}{\alpha_z Z} \\ \frac{L_z}{N} & : \eta_{zi} = \frac{1}{\alpha_z Z} \\ \infty & : \eta_{zi} > \frac{1}{\alpha_z Z} \end{cases}$$

if the value of the innovation is greater than the cost, firms will want to do infinite R&D, if the value of innovation is less than the cost firms will not engage in R&D. The former is inconsistent with the general equilibrium conditions, and is thus ruled out.

Assuming an interior solution, the return to innovation is given by:

$$r_z = \frac{H_{zi}}{\eta_{zi}} + \widehat{\eta}_{zi}$$

where

$$H_{zi} = \frac{\sigma L E (\frac{\epsilon(Z_i^{-\sigma}+S)}{\epsilon-1})^{-\epsilon}}{\int_0^N P_i^{1-\epsilon} di} Z_i^{-\sigma-1}$$

Imposing symmetry we obtain the rate of return of cost reduction:

$$r_z = \alpha_Z \frac{\sigma L E(\epsilon-1)}{\epsilon N}\left(\frac{Z^{-\sigma}}{Z^{-\sigma}+S}\right) - \widehat{Z} \tag{2.16}$$

From (2.16), we see that the rate of return is increasing in α_z, L, E, ϵ, σ. It is decreasing in Z_i, N and S. Immediately it is clear that constant cost reduction is inconsistent with a constant interest rate.

I now derive the rate of return to entry. In order to do so differentiate (3.7) and obtain:

$$r_n = \frac{\pi_i}{V_i} + \frac{\dot{V}_i}{V_i} \tag{2.17}$$

It must be the case that the (3.7) is equal to the entry cost. If the value of the firm is greater than the entry cost there would be infinite entry and this would violate the resource constraint. If the entry cost were greater than the value of the firm no one would enter. Since we focus on the regime with entry, it must be the case that:

$$V_i = \frac{1}{\beta N} \tag{2.18}$$

substituting (2.18) into (3.12) we obtain

$$r_n = \beta N \left(\frac{LE}{\epsilon N} - \phi - l_{zi} \right) - \widehat{N}$$

the resource constraint implies:

$$r_n = \beta L(E-1) \tag{2.19}$$

Substituting this into (2.4)

$$\widehat{E} = \beta L(E-1) - \rho$$

This is an unstable differential equation, the transversality conditions require that E

jumps to its steady state value:

$$E^* = \frac{\rho}{\beta L} + 1$$

This implies that at all times:

$$r = \rho$$

Substituting this into (2.16)

$$\widehat{Z} = \alpha_Z \frac{\sigma L E^* (\epsilon - 1)}{\epsilon N} \left(\frac{Z^{-\sigma}}{Z^{-\sigma} + S} \right) - \rho \tag{2.20}$$

This equation tells us that the growth rate in productivity is declining in two states: N and Z. If $s = 0$ then (2.20) is only decreasing in N. The negative relation between cost reduction and the number of firms is because entry reduces the market share of firms and hence decreases the incentive for R&D. The negative relation between cost reduction and unit cost comes from the fact that as Z grows, the shipping cost bears a larger percentage of of the price paid by consumers. Hence for a given labor effort the effectiveness of R&D in the cost reduction sector declines.

Eventually the productivity growth must stop. As mentioned above there are two forces that cause this, endogenous entry and the the fact that cost reduction becomes less effective as the unit cost decreases. In fact, even if we counter-factually hold N fixed, productivity growth has to go to zero.

The growth rate of N is given by:

$$\widehat{N} = \begin{cases} \beta \left[\frac{LE^*}{\epsilon} + N(\frac{\rho}{\alpha_z} - \phi) - \frac{\sigma L E^* (\epsilon - 1)}{\epsilon} \left(\frac{Z^{-\sigma}}{Z^{-\sigma} + S} \right) \right] - \rho & : l_z > 0 \\ \beta \left[\frac{LE^*}{\epsilon} - N\phi \right] - \rho & : l_z = 0 \end{cases} \tag{2.21}$$

It is worthwhile to understand what the irreducible unit costs are doing. If $S = 0$ then

the first branch of (2.21) is modified to:

$$\beta \left[LE^* \frac{1-\sigma\left(\epsilon-1\right)}{\epsilon} + N(\frac{\rho}{\alpha_z} - \phi) \right]$$

The productivity term drops out. The reason for this is simple, absent the shipping cost the productivity of vertical R&D is constant in the level of Z. When shipping cost are imposed, the effectiveness of vertical R&D declines leading the economy to reallocate its labor to other sectors.

Balanced growth implies that variety-expansion and cost reduction must stop. There are two reasons for this. Variety-expansion has to stop because of the fixed operating cost. Cost reduction has to stop because the presence of the distribution cost. The relation between entry and fixed operating cost is quite simple, as N grows, more labor is devoted to the manufacturing sector paying the fixed cost. The relation between cost reduction and the distribution cost is similar. As firms reduce the cost they lower the price, increasing the demand. The amount of physical units sold increases, and the amount of labor devoted to shipping increases.

It is worthwhile to examine what happens to labor allocations. Substituting the demands and technology, we obtain:

$$L_x = \frac{LE^*\left(\epsilon-1\right)}{\epsilon}\left(\frac{S}{Z^{-\sigma}+S}\right) + \frac{LE^*\left(\epsilon-1\right)}{\epsilon}\left(\frac{Z^{-\sigma}}{Z^{-\sigma}+S}\right) + N\phi$$

L_x is the total amount of labor used in manufacturing and distributing the goods to consumers. The first term is total amount of labor used in manufacturing the good, the second term is the amount of labor used in distributing the good. As productivity rises, firms devote more labor to ship the goods to the consumers, and less to actual

production.

2.3.5 Iceberg vs Per-Unit

Here we explore the difference in detail between iceberg and per-unit transportation costs. It is the presence of per-unit transportation costs that shuts down cost reduction as a source of long run growth. If we institute iceberg costs instead of per-unit this result is not present. The reason for this is simple, by modeling transportation with iceberg technology, modelers are implicitly assuming that manufacturing productivity growth translates into productivity growth in distribution.

An increase in manufacturing productivity makes it cheaper to produce the goods. When shipping a unit of a good, iceberg shipping costs imply that $S\%$ of the good melts. The equivalent per-unit cost of shipping is an additional $\frac{S}{1-S} = m$ units of the good that have to be produced. Since the firms engage in cost reduction, the extra m goods required to ship 1 unit to the consumer becomes cheaper. Below is the amount of labor required to produce and deliver one unit of output to the consumer:

$$1 + m = Z_i^{-\sigma} + Z_i^{-\sigma} m$$

As Z_i increases, the labor required to ship the unit of the good decreases. This is exactly why endogenous growth can occur with iceberg shipping costs and not per-unit. Essentially, one can think of iceberg shipping costs as per-unit where the per-unit cost is declining in manufacturing productivity, in fact, necessarily linearly.

In order for endogenous growth to be possible from cost reduction, the per-unit distribution cost S has to decline at the same rate as the per-unit cost (2.20) repeated for

convenience:

$$\widehat{Z} = \alpha_Z \frac{\sigma L E^* (\epsilon - 1)}{\epsilon N} \left(\frac{Z^{-\sigma}}{Z^{-\sigma} + S} \right) - \rho$$

Hiding in the assumption of iceberg shipping costs is the assumption that the per-unit shipping cost is declining in the level of manufacturing productivity. If we impose that the per-unit shipping cost is linear in manufacturing productivity growth, as is assumed by iceberg shipping costs we obtain:

$$\widehat{Z} = \alpha_Z \frac{\sigma L E^* (\epsilon - 1)}{\epsilon N} \left(\frac{Z^{-\sigma}}{Z^{-\sigma} + Z^{-\sigma} A} \right) - \rho = \alpha_Z \frac{\sigma L E^* (\epsilon - 1)}{\epsilon N} \left(\frac{1}{1 + A} \right) - \rho$$

In this case, endogenous growth can occur.

On an applied note, as long as the price of the good is not changing, modeling shipping cost as per-unit or iceberg does not matter, they are essentially the same. However, whenever the price of the good changes, iceberg shipping costs and per-unit will give dramatically different results. If the true nature of shipping cost is per-unit, estimating shipping cost with the iceberg technology will be misleading. Suppose an industry is doing cost reduction and they pay a time invariant per-unit shipping cost. If an applied economist uses iceberg shipping cost, they will be led to the conclusion that shipping costs rose[13].

2.3.6 Learning by Doing

In this section we show that the same results regarding cost reduction and distribution costs hold under another well accepted growth model. In particular the learning by doing framework utilized by Lucas (1988) and Krugman (1987) . We keep everything identical

[13]Indeed, the iceberg "equivalent" would in fact rise, but not because of fundamental factors.

as before, with the exception of the technology. We modify the technology to:

$$l_{mi} = \frac{X_i}{H_m} \tag{2.22}$$

$$l_{si} = \frac{X_i}{H_s} \tag{2.23}$$

where H_s and H_m are the stocks of knowledge in shipping and manufacturing. There is no R&D, and entry is costless. We assume that there is learning by doing in manufacturing;

$$\dot{H}_m = \delta_m H_m l_m, \tag{2.24}$$

but not in shipping (H_m is fixed)[14]. As before the wage is the numeraire.

The demands are:

$$X_i = \frac{L}{NP} \tag{2.25}$$

The profits are:

$$\pi_i = \frac{LP_i^{-\epsilon}}{\int_0^N P_i^{1-\epsilon} di} \left(P_i - \frac{1}{H_m} - \frac{1}{H_s} \right) - \phi$$

Profit maximization yields:

$$P_i = \frac{\epsilon}{\epsilon - 1} \left(\frac{1}{H_m} + \frac{1}{H_s} \right).$$

Zero profits imply:

$$N = \frac{L}{\phi\epsilon}.$$

[14]This can be relaxed. For brevity I omit this form the current discussion. In chapter 3, I allow for technical progress in distribution.

Substituting (2.22) into (2.24)

$$\widehat{H}_m = \phi\delta_m\left(\epsilon - 1\right)\left(\frac{H_s}{H_s + H_m}\right)$$

Recall that the human capital in distribution (H_s) is fixed. As the stock of manufacturing knowledge increases the growth rate in the stock of knowledge declines. There cannot be perpetual growth through learning by doing if there is any component of the unit cost that is irreducible. If there are no distribution costs, (i.e $H_m = \infty$) manufacturing productivity can be sustained and is equal to:

$$\widehat{H}_m = \phi\delta_m\left(\epsilon - 1\right)$$

2.4 Quality improvement

Now we allow $\theta > 0$, so that quality becomes relevant. we also set $\sigma = 0$, this makes unit cost of production equal to one, and kills cost reduction as a source of growth. In this section there are two possible sources of growth; variety-expansion, and quality improvement. Just as in section 3) variety-expansion is killed by the fixed operating cost. Quality improvement, unlike cost reduction, is not killed by the distribution cost.

2.4.1 Households

Households maximize their lifetime utility (3.1) subject to the aggregator and budget constraint, which are repeated here for convenience:

$$C = \left[\int_0^N (Q_i^{\theta} x_i)^{\frac{\epsilon-1}{\epsilon}} d\dot{\imath}\right]^{\frac{\epsilon}{\epsilon-1}}$$

$$\dot{A} = rA + W - E$$

The Euler equation is unmodified:

$$\frac{\dot{E}}{E} = r - \rho$$

The rest remains the same as in section 3, except the demands are now:

$$E\frac{P_i^{-\epsilon}Q_i^{\theta(\epsilon-1)}}{\int_0^N P_i^{1-\epsilon}Q_i^{\theta(\epsilon-1)}di} = x_i$$

2.4.2 Firms

Here, the technologies are essentially unmodified from section 3) with the exception of $\sigma = 0$, and replacing the R&D equation for cost reduction (2.7), with (2.26):

$$l_{xi} = X_i(1+S) + \phi$$

$$\dot{Q} = \alpha_q Q l_{qi} \tag{2.26}$$

$$\dot{N} = \beta N L_n$$

by setting $\sigma = 0$ the unit cost $Z_i^{-\sigma} = 1$, regardless of the level of Z_i.

Firms solve their maximization problem:

$$H = P_i X_i - l_{xi} - l_{qi} + \eta_q \alpha_q Q l_{qi}$$

Following the same methods as in section 3 we obtain the following prices:

$$P_i = \frac{\epsilon(1+S)}{\epsilon - 1}$$

$$r_q = \alpha_q \frac{LE\theta(\epsilon-1)}{\epsilon N} - \widehat{Q}$$

$$r_n = \beta L(E-1)$$

As before, $r = \rho$, so

$$\widehat{Q} = \frac{LE\alpha_q\theta(\epsilon-1)}{\epsilon N} - \rho \tag{2.27}$$

There is a critical number of firms that implies $\widehat{Q} = 0$

$$\bar{N} = \alpha_q \frac{LE\theta(\epsilon-1)}{\rho\epsilon}$$

If $N < \bar{N}$ vertical innovation is positive and equal to (2.27). If N reaches the critical value $l_q = 0$ and vertical innovation stops.

$$\widehat{N} = \begin{cases} \beta\left[LE^{\frac{1-\theta(\epsilon-1)}{\epsilon}} - N(\phi - \frac{\rho}{\alpha_q})\right] - \rho & : l_q > 0 \\ \beta\left[\frac{LE}{\epsilon} - N\phi\right] - \rho & : l_q = 0 \end{cases}$$

There are two possible cases, $\widehat{N} = 0$ while the economy is still doing vertical innovation. The alternative is vertical innovation shuts down and $\widehat{N} = 0$ after vertical innovation stops. We focus on the case in which horizontal innovation stops first[15]. In this case

$$N^* = \frac{\beta\left[LE^{\frac{1-\theta(\epsilon-1)}{\epsilon}}\right] - \rho}{\beta(\phi - \frac{\rho}{\alpha_q})} \tag{2.28}$$

Substituting (2.28) into (2.27) we obtain the steady state growth rate of quality:

[15]See the appendix for details.

$$\widehat{Q} = \frac{(\alpha_q \phi - \rho)\,\theta(\epsilon - 1)}{1 - \theta(\epsilon - 1) - \frac{\epsilon\rho}{\beta L}} - \rho \tag{2.29}$$

(2.29), is identical to the steady state manufacturing productivity growth in section 3) if $S = 0$.

2.4.3 Comparison of Cost and Quality

An interesting comparison can be made analyzing the demand functions. Using the setup of section 3) the demand functions are:

$$x_i = E\frac{P_i^{-\epsilon}}{\int_0^N P_i^{1-\epsilon} di}$$

Symmetry implies:

$$\widehat{x} = \widehat{P} - \widehat{N}$$

The amount of physical units purchased by consumers is increasing in the price. This is how growth occurs in cost reduction models. As firms become more efficient, the unit cost declines, which leads to a lower price and hence increased consumption.

In section 4) the demand functions are

$$x_i = E\frac{P_i^{-\epsilon} Q_i^{\theta(\epsilon-1)}}{\int_0^N P_i^{1-\epsilon} Q_i^{\theta(\epsilon-1)} di}$$

symmetry implies:

$$\widehat{x} = -\widehat{N}$$

With symmetry, firms are increasing the quality as the same rate, this leaves the demand for an individual good unchanged. The way growth occurs in quality improvement

is through increasing the quality of the goods that people buy, not by increasing the physical units sold. This is exactly the mechanism that makes cost reduction and quality improvement different.

We can also compare the elasticities of demand. The elasticity of demand with respect to quality improvement is isoelastic:

$$\frac{\partial X_i}{\partial Q_i}\frac{Q_i}{X_i} = \theta(\epsilon - 1)$$

The elasticity of demand with respect to productivity is not isoelastic. Increasing manufacturing productivity has less effect on demand as manufacturing productivity rises:

$$\frac{\partial X_i}{\partial Z_i}\frac{Z_i}{X_i} = \sigma\epsilon\left(\frac{Z_i^{-\sigma}}{Z_i^{-\sigma} + S}\right).$$

2.5 Conclusion

We learn several things from adding the distribution costs. First, the presence of these irreducible unit cost destroys the isomorphism between cost reduction and quality improvement. Absent technical progress in distribution, cost reduction cannot be a driver of long run growth. Second, important differences between iceberg and per-unit transportation costs are analyzed.

This model suggests that countries with low manufacturing productivity are more likely to engage in cost reduction. Once manufacturing productivity gets high enough, cost reduction shuts down. If the economy is to continue to grow it must somehow transition to quality improvement. This seems to be broadly consistent with the facts regarding the history of the United States and other developed countries. The industrial

revolution brought huge advances in manufacturing productivity. However, it appears that we have transitioned to an economy that is improving product quality rather than manufacturing productivity. Scherer (1980) provides evidence that is consistent with this view, in the United States the share of R&D expenditures going to quality improvement is 59% with only 13% going to cost reduction.

In summary, cost reduction and quality improvement are not simple isomorphisms of each other. For cost reduction and quality improvement to be isomorphic, the entire unit cost must be reducible. If there is any irreducible component of the unit cost, cost reduction cannot be a source of endogenous growth. Given the data on the per-unit cost of shipping goods within the United States (see Fig. 2.1), the introduction of an irreducible unit cost is not merely a theoretical exercise, but a necessary generalization.

Chapter 3

Quality Biased Technical Progress

3.1 Introduction

In this paper, we develop a model in which firms may engage in quality improvement, cost reduction, and variety expansion. The purpose is two-fold. First, firms engage in quality improvement and cost reduction simultaneously. As of yet, there have been no endogenous growth models studying all R&D activities simultaneously. By including both we are able to study any potential interactions between the two. Second, and more importantly, we study the long-run relationship between distribution costs and productivity growth. we are using the term distribution to include every unit cost that firms pay besides manufacturing. This by itself is a worthwhile endeavor. In this paper we can also answer the question, how is manufacturing productivity related to the distribution efficiency? Absent technical progress in distribution, firms stop doing cost reduction and instead put all of their R&D expenditure towards quality improvement. Over time, the distribution cost bears a larger portion of the price and therefore cost reduction becomes less effective. The distribution costs effects the steady-state production efficiency. When

the distribution costs are large, firms do less cost reduction and have a higher steady-state manufacturing unit cost. We also engage in the experiment of having technical progress in distribution. In a steady-state with cost reduction the rate of cost reduction must equal the growth rate in shipping productivity. It is worthwhile to point out that in both cases, manufacturing productivity and distribution productivity are fundamentally linked.

The underlying framework is Peretto and Connolly (2007), this is a tractable growth model and has received strong empirical support[1]. In the original framework there is no quality improvement, we break with them by allowing firms to do quality improvement. Even without the distribution costs we find interesting results. In the steady-state firms may engage in; cost reduction and quality improvement, a combination of the two, or nothing. The model is analytically tractable and yields the full transitional dynamics. Countries might go through a transition in which all R&D activities are positive, and eventually stop one or both.

In interesting empirical work Scherer (1980) finds that American firms devote 59% of their research expenditure to improving the quality of existing products, 28% to developing new products and 13% to developing new processes. Furthermore, there are significant cross-country differences regarding R&D expenditures. Mansfield (1988) finds that American firms devote two-thirds of their R&D expenditures to product innovation while Japanese firms only devote one-third, with the remaining R&D expenditures going to cost reduction. Unfortunately, Mansfield does not distinguish between quality improvement and product creation[2]. As of yet, growth models cannot explain these facts because no models have included all types of R&D.

[1]See: Madsen (2008) , Laincz and Peretto (2006) , and Ha and Howitt (2007).

[2]Product innovation includes both quality improvement and product proliferation.

There have been three sources of growth in the endogenous growth literature; cost reduction, quality improvement, and variety expansion[3]. However, there are no growth models that have incorporated quality improvement, cost reduction, and variety expansion simultaneously. This can be traced back to Spence (1984), who writes:

> Suppose that products deliver services to consumers. Let s be the services and P(s) be the inverse demand. Services are delivered through goods. Let x be the quantity of goods, and c(x) be the cost function. Let f(q) be the quantity of services per unit of the good. Then s = f(q)x, and the cost of delivering services s is c(s/f(q)). If f'(q) $>$ 0, and q is raised through R&D of the product development kind, then the effect is to reduce the costs of the service. Thus formally this kind of product development is equivalent to cost reduction.

In the trade and growth literature, cost reduction and quality improvement are viewed as simple substitutes for each other[4]. In the I.O literature the differences between cost-reduction and quality improvement have been studied; Sutton (2007) develops a static trade model with endogenous R&D in quality improvement and cost reduction[5]. The central idea of that paper is that with a per unit "materials cost", cost reduction and quality improvement are different. There is a minimum quality to make firms viable but not a minimum productivity. Intuitively, if productivity is very low, significantly low wages can make firms viable. However, if quality is too low, low wages might not be enough to compensate, the materials cost puts a lower bound on the price. The purpose of this paper is different. In my paper firms pay labor to manufacture the goods and to distribute them, the mechanism used by Sutton is not present in my paper. Firms pay the same wages to workers distributing the goods as the workers manufacturing the goods. Instead we analyze the impact of having two fundamentally different technologies,

[3]See: Romer (1986) , Grossman and Helpman (1991) , Howitt (1999) , Peretto (1998).

[4]Melitz (2003) makes this claim.

[5]Levin and Reiss (1988) explore the interaction between spillovers and cost reduction and quality improvement.

manufacturing and distribution. In addition my model is dynamic, therefore we can analyze how R&D expenditure shares evolve over time.

In my second chapter, we show that cost reduction cannot be a source of long run growth while quality improvement can. The purpose of this paper is to analyze a complete model with all activities active. Interestingly, the model predicts that R&D becomes increasingly biased towards quality improvement. The share of R&D expenditures going towards quality improvement is increasing throughout, and in the steady state all of R&D is directed towards quality improvement.

In summary this paper contributes to the literature in several ways. First, the model is more complete than existing models in that it has all types of R&D; cost reduction, quality improvement, and variety expansion. The growth rate of product quality and the growth rate of manufacturing productivity are jointly determined, they both affect the steady-state number of firms and hence each other. Second, we incorporate distribution costs. Interestingly, absent technical progress in distribution, the only source of long run growth is quality improvement. Third, the model introduces interesting testable implications. In particular, over time R&D expenditures should become increasingly biased towards quality improvement.

The paper is organized as follows. In section 2 we develop a model with cost reduction, quality improvement, and variety expansion. In section 3 we analyze the full model with positive distribution costs. First we hold distribution productivity fixed, and then we allow for technical progress in distribution. we conclude in section 4.

3.2 The model

This model builds upon Peretto and Connolly (2007) in two ways. In this section, we generalize Peretto and Connolly (2007) to include quality improvement, rather than just cost reduction and variety expansion. Second, in the following section we introduce distribution costs and analyze their effects.

3.2.1 Households

The economy is populated by a representative agent who has lifetime utility

$$U = \int_0^\infty e^{-\rho t} log(C_t) dt. \tag{3.1}$$

Consumption is determined by the following aggregator

$$C = \left[\int_0^N (Q_i^\theta x_i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}. \tag{3.2}$$

In this setting, individuals buy intermediate goods x_i, attached to each intermediate is its quality Q_i. It is assumed that $\epsilon > 1$ and $1 \geq \theta \geq 0$. If $\theta = 0$ then the goods quality does not effect consumers utility, and firms have no incentive to invest in quality improvement.

Consumers maximize (3.1) subject to (3.2) and their budget constraint:

$$\dot{A} = rA + W - E,$$

where all variables are in per-capita terms and,

$$E = \int_0^N P_i x_i di.$$

The maximization implies the usual Euler equation,

$$\widehat{E} = r - \rho, \tag{3.3}$$

and demands,

$$x_i = \frac{E P_i^{-\epsilon} Q_i^{\theta(\epsilon-1)}}{\int_0^N P_i^{1-\epsilon} Q_i^{\theta(\epsilon-1)} di}. \tag{3.4}$$

Notice that there are two ways to increase demand. An increase in product quality will increase demand with elasticity $\theta(\epsilon - 1)$, a reduction in price will increase demand with elasticity ϵ.

3.2.2 Technology

Each firm produces with the technology:

$$l_{xi} = Z_i^{-\sigma} X_i + \phi, \quad 0 < \sigma < 1 \ , \ \phi > 0, \tag{3.5}$$

where X_i is output, $Z_i^{-\sigma}$ is the unit cost of production, ϕ is a fixed operating cost, and l_{xi} is the total amount of labor employed by firm i. Firms improve their manufacturing productivity Z_i through R&D. The total demand facing the firm is given by,

$$X_i = \frac{L E P_i^{-\epsilon} Q_i^{\theta(\epsilon-1)}}{\int_0^N P_i^{1-\epsilon} Q_i^{\theta(\epsilon-1)} di}. \tag{3.6}$$

notice that an increase in own product quality or a reduction in the price of good i will increase the demand for the good.

Firms maximize their lifetime value,

$$V_i = \int_0^\infty e^{-\int_0^t r(s)ds} \pi_{it} dt \tag{3.7}$$

where,

$$\pi_{it} = X_{it}\left(P_i - Z_i^{-\sigma}\right) - \phi - l_{zi} - l_{qi},$$

and,

$$\dot{Z}_i = \alpha_z Z l_{zi}; \tag{3.8}$$

$$\dot{Q}_i = \alpha_q Q l_{qi}. \tag{3.9}$$

Where l_{zi} is R&D expenditure in cost reduction, and l_{qi} is R&D expenditure in quality improvement.

The associated Hamiltonian with the firms maximization is given by

$$H = X_{it}\left(P_i - Z_i^{-\sigma}\right) - \phi - l_{zi} - l_{qi} + \eta_z \alpha_z Z l_{zi} + \eta_q \alpha_q Q l_{qi},$$

the linearity of the Hamiltonian yields:

$$\eta_z = \frac{1}{\alpha_z Z}.$$

and

$$\eta_q = \frac{1}{\alpha_q Q}.$$

The firms profit maximizing price is given by:

$$P_i = \frac{\epsilon}{\epsilon - 1} Z_i^{-\sigma}$$

The rates of return are:

$$r_q = \frac{H_q}{\eta_q} + \widehat{\eta}_q = \alpha_q Q \left(\frac{LE\theta(\epsilon-1)Q_i^{\theta(\epsilon-1)-1} Z_i^{\sigma(\epsilon-1)}}{\epsilon \int_0^N Q_i^{\theta(\epsilon-1)} Z_i^{\sigma(\epsilon-1)} di} \right) - \widehat{Q},$$

and

$$r_z = \frac{H_z}{\eta_z} + \widehat{\eta}_z = \alpha_z Z \left(\frac{LE\sigma\left(\epsilon-1\right) Q_i^{\theta(\epsilon-1)} Z_i^{\sigma(\epsilon-1)-1}}{\epsilon \int_0^N Q_i^{\theta(\epsilon-1)} Z_i^{\sigma(\epsilon-1)} di} \right) - \widehat{Z},$$

Imposing symmetry, the rates of return to quality improvement and cost reduction are,

$$r_q = \frac{LE\alpha_q \theta(\epsilon-1)}{\epsilon N} - \widehat{Q}, \tag{3.10}$$

and

$$r_z = \frac{LE\alpha_z \sigma(\epsilon-1)}{\epsilon N} - \widehat{Z}. \tag{3.11}$$

Note that (3.10) and (3.11) are qualitatively identical. This is not surprising, as the model stands cost reduction and quality improvement are isomorphic. The intuition is simple. Looking at firms profit:

$$\pi = \frac{LE}{\epsilon \int_0^N P_i^{1-\epsilon} Q_i^{\theta(\epsilon-1)} di} \left(\frac{\epsilon}{\epsilon - 1} \right)^{1-\epsilon} Q_i^{\theta(\epsilon-1)} Z_i^{\sigma(\epsilon-1)} - \phi - l_{zi} - l_{qi}$$

a one percent increase in product quality will increase profits by $\theta(\epsilon - 1)$ percent, a one percent increase in productivity will increase profits by $\sigma(\epsilon - 1)$ percent.

Entrepreneurs create new goods that compete with existing products. The rate of return to entry is given by the usual asset-pricing equation,

$$r_n = \frac{\pi}{V} + \widehat{V} \tag{3.12}$$

which says that the rate of return to product creation is equal to the discounted flow of profits plus capital appreciation. It is obtained by differentiating (3.7).

The entry process is described by[6]:

$$\dot{N} = \beta N L_n. \tag{3.13}$$

It must be the case that

$$V \leq \frac{1}{\beta N}$$

with strict equality when $\dot{N} > 0$. Otherwise there would be infinite entry. Assuming an interior solution,

$$r_n = \beta N \pi - \widehat{N}.$$

3.2.3 General Equilibrium

The resource constraint:

$$L = N(l_{xi} + l_{zi} + l_{qi}) + L_n$$

[6]See Peretto and Connolly (2007) for a discussion of alternative entry cost specifications.

Substituting this into (3.12), yields:

$$r_n = \beta L\,(E-1) \tag{3.14}$$

Substituting (3.14) into (3.3) yields,

$$\widehat{E} = \beta L\,(E-1) - \rho,$$

which is an unstable differential equation. Therefore E jumps to the steady state value,

$$E^* = \frac{\rho}{\beta L} + 1. \tag{3.15}$$

Which of course means that

$$r = \rho.$$

Now we are able to fully characterize the equilibrium. The growth rate of product quality is given by:

$$\widehat{Q} = \begin{cases} \frac{LE^*\alpha_q\theta(\epsilon-1)}{\epsilon N} - \rho & : N < \bar{N}_q \\ 0 & : N > \bar{N}_q \end{cases} \tag{3.16}$$

where

$$\bar{N}_q = \frac{LE^*\alpha_q\theta(\epsilon-1)}{\epsilon\rho}.$$

Intuitively, there exists a threshold of firms in which the market becomes too diluted to justify R&D quality improvement.

The growth rate of productivity is given by:

$$\widehat{Z} = \begin{cases} \frac{LE^*\alpha_z\sigma(\epsilon-1)}{\epsilon N} - \rho & : N < \bar{N}_z \\ 0 & : N < \bar{N}_q \end{cases} \tag{3.17}$$

where:

$$\bar{N}_z = \frac{LE^*\alpha_z\sigma(\epsilon-1)}{\epsilon\rho}.$$

Similar to the case for quality improvement there exists a threshold in which if there are too many firms, incumbents stop doing R&D.

Proposition 1: If the rate of quality improvement (cost reduction) is higher than the rate of cost reduction (quality improvement), it will remain that way forever.

Proof: Assume that the rate of quality improvement is higher than the rate of cost reduction:

$$\widehat{Q} - \widehat{Z} = \frac{LE^*\alpha_q\theta(\epsilon-1) - LE^*\alpha_z\sigma(\epsilon-1)}{\epsilon N} > 0,$$

Since this is positive, it will be positive for all N . In fact, if $\alpha_q\theta > \alpha_z\sigma$, the rate of quality improvement will be higher than cost reduction.

This proposition is trivial but deserves discussion. In this basic model, if firms devote more R&D resources quality improvement than cost reduction it will remain that way forever. We will see in the next section that the presence of distribution cost removes this result.

The growth rate of the number of firms is given by:

$$\widehat{N} = \beta N \left(X_{it}\left(P_i - Z_i^{-\sigma}\right) - \phi - l_{zi} - l_{qi}\right) - \rho. \tag{3.18}$$

Taking account of all of the possibilities, variety expansion is given by:

$$\widehat{N} = \begin{cases} \frac{\beta L E^*}{\epsilon}\left[1-\theta(\epsilon-1)-\sigma(\epsilon-1)\right] - \beta N\left[\phi - \left(\frac{1}{\alpha_z}+\frac{1}{\alpha_q}\right)\rho\right] - \rho & : l_q > 0, l_z > 0 \\ \frac{\beta L E^*}{\epsilon}\left[1-\theta(\epsilon-1)\right] - \beta N\left(\phi - \frac{1}{\alpha_q}\rho\right) - \rho & : l_q > 0, l_z = 0 \\ \frac{\beta L E^*}{\epsilon}\left[1-\sigma(\epsilon-1)\right] - \beta N\left[\phi - \frac{1}{\alpha_z}\rho\right] - \rho & : l_q = 0, l_z > 0 \\ \frac{\beta L E^*}{\epsilon} - \beta N\phi - \rho & : l_q = 0, l_z = 0 \end{cases} \tag{3.19}$$

If cost reduction or quality improvement shuts down they stay shut down forever (unless the system is shocked). The dynamics are cumbersome but are worthwhile to discuss. There exists four different steady states. In this first, both R&D activities are positive. In the second, firms engage in quality improvement without cost reduction. In the third, firms engage in cost reduction but not quality improvement. Lastly, firms may do no R&D in the steady state.

3.2.4 Case 1: Both R&D activities active in the steady-state

On the transition path, the growth rate of the number of firms is determined by the first branch of (3.19). A necessary condition for both R&D activities to be active in the steady state is that

$$\phi > \left(\frac{1}{\alpha_z}+\frac{1}{\alpha_q}\right)\rho,$$

this condition guarantees that $\widehat{N}$ is decreasing over time. The next condition is that the steady-state number of firms is lower than the thresholds for no R&D, i.e:

$$N_{ss} < \bar{N}_q, \bar{N}_z,$$

the exact parameters restrictions is:

$$\frac{(\rho+\beta L)\left(1-(\theta+\sigma)(\epsilon-1)\right)-\epsilon\rho}{\phi-\left(\frac{1}{\alpha_z}+\frac{1}{\alpha_q}\right)\rho}<\text{Min}\left[\frac{(\rho+\beta L)\,\alpha_q\theta(\epsilon-1)}{\rho},\frac{(\rho+\beta L)\,\alpha_z\sigma(\epsilon-1)}{\rho}\right].$$

This is depicted in figure 3.1:

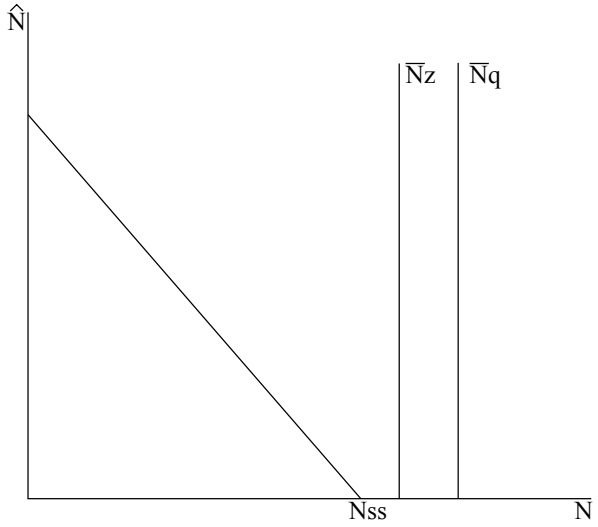


Figure 3.1: Dynamics case 1

The steady-state number of firms is:

$$N_{ss}=\frac{\beta LE^*\left(1-(\theta+\sigma)(\epsilon-1)\right)-\epsilon\rho}{\epsilon\beta\left(\phi-\left(\frac{1}{\alpha_z}+\frac{1}{\alpha_q}\right)\rho\right)}.$$

Substituting this into (3.16) and (3.17) yields:

$$\widehat{Q}_{ss}=\frac{\alpha_q\theta(\epsilon-1)\left(\phi-\left(\frac{1}{\alpha_z}+\frac{1}{\alpha_q}\right)\rho\right)}{1-(\theta+\sigma)(\epsilon-1)-\frac{\epsilon\rho}{\beta LE^*}}-\rho, \tag{3.20}$$

$$\widehat{Z}_{ss} = \frac{\alpha_z \sigma (\epsilon - 1) \left(\phi - \left(\frac{1}{\alpha_z} + \frac{1}{\alpha_q} \right) \rho \right)}{1 - (\theta + \sigma)(\epsilon - 1) - \frac{\epsilon \rho}{\beta L E^*}} - \rho. \tag{3.21}$$

In the steady-state the growth rate of product quality and the growth rate of productivity need not be equal. If the economy is better at quality improvement than cost reduction, firms will tend to devote more R&D expenditures to quality improvement rather than cost reduction.

Proposition 2: An increase in the R&D efficiency of cost reduction (quality improvement) increases the rate of both cost reduction and quality improvement. Furthermore, if:

$$\phi > \text{Max} \left\{ \frac{\alpha_q \theta \rho \alpha_z^{-2}}{\sigma} + \frac{\rho}{\alpha_q}, \frac{\alpha_z \sigma \rho \alpha_q^{-2}}{\theta} + \frac{\rho}{\alpha_z} \right\}$$

then the steady-state growth rate of cost reduction (quality improvement) increases more than quality improvement (cost reduction).

Proof: See the appendix.

In this class of models, R&D expenditure is one of the fixed cost that determines the number of firms. [7] If firms become better at cost reduction R&D expenditure will increase, this will decrease the steady state number of firms and hence increase the rate of quality improvement.

Consumption growth is easily calculable:

$$\widehat{C} = \frac{\epsilon}{\epsilon - 1} \widehat{N} + \theta \widehat{Q} + \widehat{X}$$

Using (3.4) we obtain:

[7] Peretto (1996) discusses this point in greater detail in a model without entry cost or quality improvement.

$$\widehat{C} = \frac{1}{\epsilon - 1}\widehat{N} + \theta\widehat{Q} + \sigma\widehat{Z}.$$

The growth rates of Z and Q are very similar, and enter the growth rate in identical fashions. In the steady state,

$$\widehat{C}_{ss} = \left(\theta^2\alpha_q + \sigma^2\alpha_z\right)\left(\frac{(\epsilon-1)\left(\phi - \left(\frac{1}{\alpha_z} + \frac{1}{\alpha_q}\right)\rho\right)}{1 - (\theta+\sigma)(\epsilon-1) - \frac{\epsilon\rho}{\beta LE^*}}\right) - (\sigma+\theta)\rho.$$

In fact, the tractability of the model allows me to solve the complete system. When both R&D activities are positive,

$$N_t = \frac{A}{B + D\exp^{-AT}}, \tag{3.22}$$

where;

$$A = \beta LE^*\left[1 - \theta(\epsilon-1) - \sigma(\epsilon-1)\right] - \epsilon\rho,$$

$$B = \epsilon\beta\left[\phi - \left(\frac{1}{\alpha_z} + \frac{1}{\alpha_q}\right)\rho\right],$$

$$D = \frac{\beta LE^*\left[1 - \theta(\epsilon-1) - \sigma(\epsilon-1)\right] - \epsilon\rho}{N_0} - \epsilon\beta\left[\phi - \left(\frac{1}{\alpha_z} + \frac{1}{\alpha_q}\right)\rho\right].$$

In that case,

$$Q_t = Q_0\exp^{\left[\widehat{Q}_{ss}T + \frac{\epsilon(\widehat{Q}_{ss}+\rho)}{1-\theta(\epsilon-1)-\sigma(\epsilon-1)-\frac{\epsilon\rho}{\beta LE^*}}\left(\frac{N_{ss}-N_{k0}}{N_{k0}}\right)\left(1-\exp^{-\left(\frac{\beta LE^*}{\epsilon}[1-\theta(\epsilon-1)-\sigma(\epsilon-1)]-\rho\right)T}\right)\right]}$$

and:

$$Z_t = Z_0 \exp\left[\widehat{Z}_{ss}T + \frac{\epsilon\left(\widehat{Z}_{ss}+\rho\right)}{1-\theta(\epsilon-1)-\sigma(\epsilon-1)-\frac{\epsilon\rho}{\beta LE^*}}\left(\frac{N_{ss}-N_{k0}}{N_{k0}}\right)\left(1-\exp^{-\left(\frac{\beta LE^*}{\epsilon}[1-\theta(\epsilon-1)-\sigma(\epsilon-1)]-\rho\right)T}\right)\right]$$

That is the entire system. The consumption is:

$$C = \left[\int_0^N (Q_i^\theta x_i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}},$$

imposing symmetry:

$$C = N^{\frac{\epsilon}{\epsilon-1}} Q_i^\theta x_i.$$

The lifetime utility is:

$$U = \int_0^\infty e^{-\rho t} \ln(N^{\frac{\epsilon}{\epsilon-1}} Q_i^\theta x_i) dt.$$

Substituting in (3.4) yields

$$U = \int_0^\infty e^{-\rho t} \ln(N^{\frac{1}{\epsilon-1}} Q_i^\theta Z_i^\sigma) dt.$$

So:

$$U = \int_0^\infty e^{-\rho t} \left(\frac{1}{\epsilon-1}\ln(N) + \sigma \ln(Z_i) + \theta \ln(Q_i)\right).$$

This can be separated into:

$$U = J + K + L,$$

where

$$J = \int_0^\infty e^{-\rho t} \left(\frac{\epsilon}{\epsilon-1}\ln(N)\right)$$

$$K = \sigma \int_0^{\infty} e^{-\rho t} \left(\ln\left(Z_i\right)\right)$$

$$L = \theta \int_0^{\infty} e^{-\rho t} \ln\left(Q_i\right)$$

$$K = \frac{\sigma \ln Z_0}{\rho} + \frac{\sigma \widehat{Z}_{ss}}{\rho^2} + \frac{\sigma\left(\frac{N_{ss}-N_{k0}}{N_{k0}}\right)\epsilon\left(\widehat{Z}_{ss}+\rho\right)}{\rho\left[1-\theta(\epsilon-1)-\sigma(\epsilon-1)\right]}$$

$$L = \frac{\theta \ln Z_0}{\rho} + \frac{\theta \widehat{Z}_{ss}}{\rho^2} + \frac{\theta\left(\frac{N_{ss}-N_{k0}}{N_{k0}}\right)\epsilon\left(\widehat{Q}_{ss}+\rho\right)}{\rho\left[1-\theta(\epsilon-1)-\sigma(\epsilon-1)\right]}$$

The lifetime utility is thus:

$$U = \frac{\sigma \ln Z_0 + \theta \ln Q_0}{\rho} + \frac{\sigma \widehat{Z}_{ss} + \theta \widehat{Q}_{ss}}{\rho^2} + \left(\frac{N_{ss}-N_{k0}}{N_{k0}}\right)\epsilon \frac{\sigma\left(\widehat{Z}_{ss}+\rho\right)+\theta\left(\widehat{Q}_{ss}+\rho\right)}{\rho\left[1-\theta(\epsilon-1)-\sigma(\epsilon-1)\right]} + J$$

3.2.5 Case 2: Only quality improvement

Looking at the branches it is possible that cost reduction shuts down and quality improvement remains active. Necessary and sufficient conditions for this to occur is that

$$\bar{N}_z < N_{ss} < \bar{N}_q.$$

The exact parameter restrictions are:

$$\frac{\beta L E^* \sigma(\epsilon-1)\left(\alpha_z \phi - \frac{\alpha_z}{\alpha_q}\rho\right)}{\rho} < \beta L E^*\left[1-\theta(\epsilon-1)\right] - \epsilon\rho < \frac{\beta L E^* \theta(\epsilon-1)\left(\alpha_q \phi - \rho\right)}{\rho}.$$

The transition can be seen in figure 3.2:

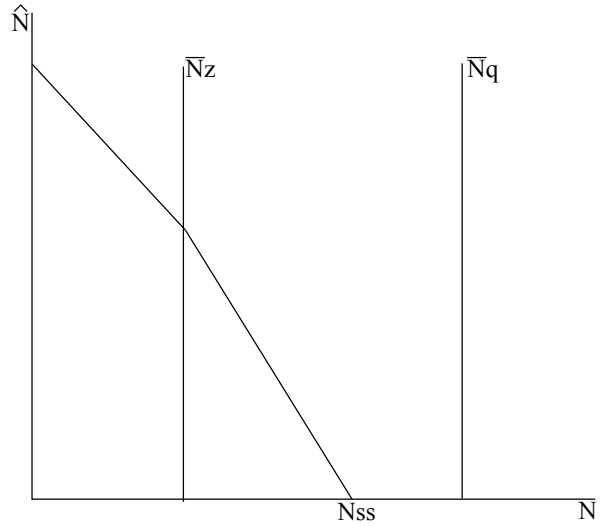


Figure 3.2: Dynamics Case 2

The number of firms is given by the first branch of (3.19), once N reaches $\bar{N}_z$ the dynamics are described by the second branch. In that case the steady state number of firms is given by:

$$N_{ss} = \frac{\beta L E^* \left[1 - \theta(\epsilon - 1)\right] - \epsilon\rho}{\epsilon\beta\left(\phi - \frac{1}{\alpha_q}\rho\right)}$$

and the steady state growth rate of product quality is given by:

$$\widehat{Q}_{ss} = \frac{\theta\left(\epsilon - 1\right)\left(\phi\alpha_q - \rho\right)}{1 - \theta\left(\epsilon - 1\right) - \frac{\epsilon\rho}{\beta L E^*}} - \rho.$$

Note that the parameters concerning cost reduction are absent. That does not mean they do not contribute to welfare, it is possible to have a regime in which firms begin engaging in all activities, the return to cost reduction eventually falls so low that they

stop doing it. The households enjoy the transitional growth, and the steady-state level of productivity affects welfare.

3.2.6 Case 3: Only Cost Reduction

Looking at the branches it is possible that quality improvement shuts down and cost reduction remains active. Necessary and sufficient conditions for this to occur is that

$$\bar{N}_q < N_{ss} < \bar{N}_z.$$

The exact parameter restrictions are:

$$\frac{\beta L E^* \theta(\epsilon-1)\left(\alpha_q \phi - \frac{\alpha_q}{\alpha_Z}\rho\right)}{\rho} < \beta L E^* \left[1-\sigma(\epsilon-1)\right] - \epsilon\rho < \frac{\beta L E^* \sigma(\epsilon-1)\left(\alpha_z \phi - \rho\right)}{\rho}.$$

This dynamics can be seen in figure 3.3:

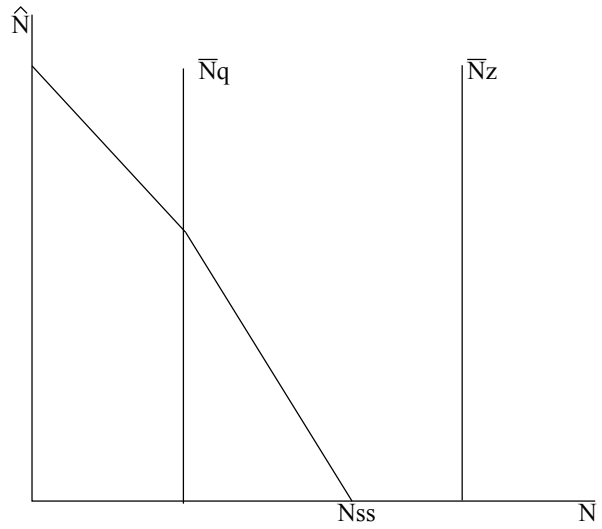


Figure 3.3: Dynamics Case 3

When cost reduction dies, variety expansion is driven by the third branch of (3.19). In that case the steady state number of firms is given by:

$$N_{ss} = \frac{\beta L E^* \left[1 - \sigma(\epsilon - 1)\right] - \epsilon\rho}{\epsilon\beta\left(\phi - \frac{1}{\alpha_z}\rho\right)}$$

the steady-state growth rate of product quality is given by:

$$\widehat{Z}_{ss} = \frac{\sigma\left(\epsilon - 1\right)\left(\phi\alpha_z - \rho\right)}{1 - \sigma\left(\epsilon - 1\right) - \frac{\epsilon\rho}{\beta L E^*}} - \rho.$$

In this steady-state all growth is driven by cost reduction. Does this mean that quality improvement is irrelevant? No, the households enjoy transitional growth for quality improvement. Moreover, the level of product quality affects welfare.

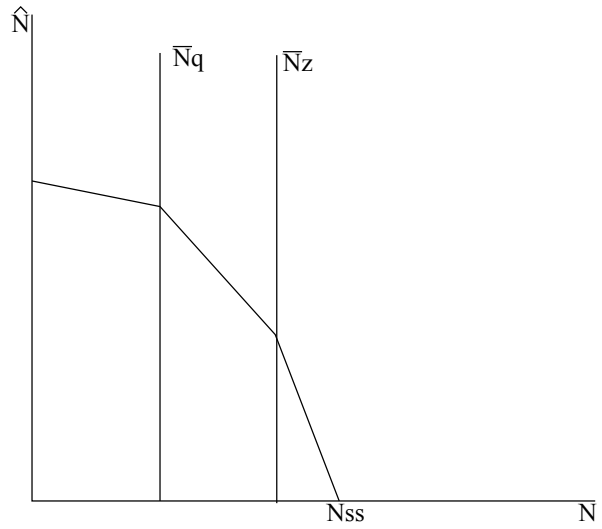


Figure 3.4: Dynamics Case 4

3.2.7 Case 4: Nothing Active

It is possible that none of the R&D activities are positive. In this case,

$$\bar{N}_q, \bar{N}_z < N_{ss}$$

where:

$$N_{ss} = \frac{\beta L E^* - \epsilon\rho}{\beta\phi}.$$

In this scenario, neither R&D is active in the steady state. Intuitively, the market becomes too saturated. The dynamics can be seen in figure 3.4:

For simplicity we focus on a case in which quality improvement shuts down first and then cost reduction. Under this scenario, the growth rate of the number of firms is given by the first branch of (3.19) then the 3rd branch, and then the fourth. In this steady-state there is no growth.

3.2.8 Discussion

There are many possible regimes. Countries may engage in both cost reduction and quality improvement, quality improvement without cost reduction, cost reduction without quality improvement, or no R&D activities. This already takes a step into answering questions raised by Mansfield (1988). One potential explanation into why the U.S engaged in more quality improvement than Japan is that the R&D efficiency of quality improvement is higher in the U.S than it was in Japan.

A simple point worthwhile making is that if cost reduction and or quality improvement shuts down, the prices and qualities become fixed. For example, if we're in a regime where we eventually transition to a steady-state with only quality growth the prices eventually become fixed.

3.3 Distribution costs

In this section we add distribution costs, and analyze their effects. It turns out that they add additional transition dynamics that might be relevant for the Japanese U.S R&D expenditure differences.

3.3.1 Positive Distribution Costs

In this section we generalize the technology (3.5) to:

$$l_{xi} = \left(Z_i^{-\sigma} + S\right) X_i + \phi, \qquad 0 < \sigma < 1 \ , \ \phi > 0, \tag{3.23}$$

where as before $Z_i^{-\sigma}$ is the unit cost of manufacturing, ϕ is a fixed operating cost, X_i is the total amount of goods produced and l_{xi} is the total amount of labor used by

firms. The addition, is the per unit distribution cost S . In this section we treat S as fixed, this restriction is relaxed in the next section. It is worthwhile to note that this per unit distribution cost is non-negligible. Shipping costs are primarily per unit[8], and they a make up a large portion of the price of typical consumer goods[9]. As set up, the distribution cost has a fundamentally different technology than manufacturing. Firms can increase their manufacturing technology through costly R&D, but they take the distribution productivity as given.

Just as in section 2, firms maximize their lifetime value subject to the R&D technologies (3.8), (3.9), demand (3.6) , and production technology (3.23). In symmetric equilibrium, the prices and rate of returns are:

$$P_i = \frac{\epsilon}{\epsilon - 1}\left(Z_i^{-\sigma} + S\right),$$

$$r_z = \alpha_z \frac{LE\sigma\left(\epsilon - 1\right)}{\epsilon N}\left(\frac{Z^{-\sigma}}{Z^{-\sigma} + S}\right) - \widehat{Z},$$

$$r_q = \alpha_q \frac{LE\theta\left(\epsilon - 1\right)}{\epsilon N} - \widehat{Q}.$$

There are several worthwhile things to point out. First, the price is no longer isoelastic with respect to the manufacturing productivity. This can be seen by:

$$\frac{dp}{p}\frac{Z}{dz} = -\sigma\left(\frac{Z_i^{-\sigma}}{Z_i^{-\sigma} + S}\right)$$

when $S = 0$, a one percent increase in manufacturing productivity reduces the price by σ

[8]See Hummels and Skiba (2004) and Irarrazabal, Moxnes, and Opromolla (2010).
[9]See Burstein, Neves, and Rebelo (2003) and Irarrazabal, Moxnes, and Opromolla (2010).

. When $S > 0$, a one percent increase in manufacturing productivity will reduce the price by less than σ percent. As the manufacturing productivity rises, the elasticity falls. This is related to the Alchian-Allen theorem Armen A. Alchian (1964). For a given per unit friction, lower priced goods are affected more. As the manufacturing productivity rises, the price falls and the distribution cost makes up a larger portion of the price. Second, the rate of return to cost reduction is a function of the ratio of reducible unit costs to total unit cost. If there are high unit costs the return to cost reduction is lower. This has the potential to explain some empirical facts brought forth by Levin and Reiss (1988). They point out that the chemical processing and semi-conductor industries are efficient at cost reduction:, while assembly industries are less efficient. The distribution cost of chemical processing and semi-conductor industries might have a lower distribution cost than the assembly industry. Third, the return to quality improvement is independent of the distribution cost.

Proceeding the same way as section 2, $r = \rho$ and $E^* = \frac{\rho}{\beta L} + 1$. This yields the growth rates:

$$\widehat{Z} = \frac{LE^*\alpha_z\sigma\left(\epsilon - 1\right)}{\epsilon N}\left(\frac{Z^{-\sigma}}{Z^{-\sigma} + S}\right) - \rho,$$

$$\widehat{Q} = \frac{LE^*\alpha_q\theta\left(\epsilon - 1\right)}{\epsilon N} - \rho.$$

Proposition 3: With fixed distribution technology, the difference in growth rates ($\widehat{Q} - \widehat{Z}$) , can be positive or negative. It can switch from being negative to positive. Once positive, it will remain non-negative forever.

Proof: See the appendix.

This exemplifies why the distribution sector is worthwhile to add. On the transition

path, firms may do more cost reduction than quality improvement. Proposition 1 showed that without distribution costs the amount of cost reduction will remain higher forever. With the distribution cost, firms do less cost reduction over time as the manufacturing productivity rises the per unit shipping cost bears a larger portion of the price and cost reduction becomes less effective at increasing demand.

The analysis becomes considerably more complicated. The simplest place to start is quality improvement. As long as $N < \bar{N}_q$, quality improvement is positive and equal to (3.16), where

$$\bar{N}_q = \frac{\alpha_q L E^* \theta(\epsilon - 1)}{\epsilon \rho},$$

just as in section 2 if there are too many firms, the market lacks enough concentration and firms do no quality improvement.

Cost reduction is positive when:

$$\frac{\alpha_z \sigma L E^* (\epsilon - 1)}{\epsilon N} \left(\frac{Z^{-\sigma}}{Z^{-\sigma} + S} \right) > \rho,$$

endogenous entry reduces the rate or return to cost reduction in addition so does the level of manufacturing productivity. Therefore if the number of firms is high and firms are efficient at production, firms will not engage in cost reduction.

Taking account of all of the possibilities, variety expansion is given by:

$$\widehat{N} = \begin{cases} \frac{\beta L E^*}{\epsilon} \left[1 - \theta(\epsilon - 1) - \sigma(\epsilon - 1) \left(\frac{Z^{-\sigma}}{Z^{-\sigma} + S} \right) \right] - \beta N \left[\phi - \left(\frac{1}{\alpha_z} + \frac{1}{\alpha_q} \right) \rho \right] - \rho & : l_q > 0, l_z > 0 \\ \frac{\beta L E^*}{\epsilon} \left[1 - \theta(\epsilon - 1) \right] - \beta N \left(\phi - \frac{1}{\alpha_q} \rho \right) - \rho & : l_q > 0, l_z = 0 \\ \frac{\beta L E^*}{\epsilon} \left[1 - \sigma(\epsilon - 1) \left(\frac{Z^{-\sigma}}{Z^{-\sigma} + S} \right) \right] - \beta N \left[\phi - \frac{1}{\alpha_z} \rho \right] - \rho & : l_q = 0, l_z > 0 \\ \frac{\beta L E^*}{\epsilon} - \beta N \phi - \rho & : l_q = 0, l_z = 0 \end{cases}$$

The rate of cost reduction is declining in N and Z. The growth of N is increasing in Z and declining in N . There are therefore two loci:

$$\dot{N} = 0 \implies \frac{\beta L E^* \left[1 - \theta\left(\epsilon - 1\right) - \sigma\left(\epsilon - 1\right)\left(\frac{Z^{-\sigma}}{Z^{-\sigma}+S}\right)\right] - \epsilon\rho}{\beta\epsilon\left[\phi - \left(\frac{1}{\alpha_z} + \frac{1}{\alpha_q}\right)\rho\right]} = N$$

$$\dot{Z} = 0 \implies \alpha_z \frac{\sigma L E^* \left(\epsilon - 1\right)}{\epsilon\rho}\left(\frac{Z^{-\sigma}}{Z^{-\sigma} + S}\right) = N$$

The dynamics can be seen in figure 3.5.

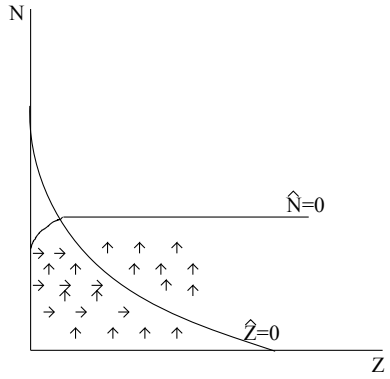


Figure 3.5: Dynamics

Cost reduction must shut down. Even for a constant number of firms, when Z increases the economy would transition to the $\dot{Z}$ locus.

To prevent the analysis from becoming taxonomic we focus on a regime with quality improvement active, i.e $\bar{N_q} > N_{ss}$. In this case:

$$N_{ss} = \frac{\beta L E^* \left(1 - \theta\left(\epsilon - 1\right)\right) - \epsilon\rho}{\epsilon\beta\left(\phi - \frac{1}{\alpha_q}\rho\right)},$$

and

$$\widehat{Q}_{ss} = \frac{\theta(\epsilon-1)\left(\alpha_q\phi-\rho\right)}{1-\theta\left(\epsilon-1\right)-\frac{\epsilon\rho}{\beta LE^*}} - \rho.$$

Neither cost-reduction or variety expansion can be sustained in the long-run. Both require replication of a fixed factor. Variety expansion increases pressure on the resource constraint because of the fixed operating cost, cost reduction puts pressure on the resource constraint because of the irreducible unit cost. Why does quality improvement not suffer from this problem? An increase in product quality does not require addition resources to be enjoyed.

While there are no closed form solutions to the system, it is worthwhile to discuss the transition in more detail. In particular the R&D expenditure share of quality improvement is:

$$R\&D_q = \frac{Nl_q}{Nl_q + Nl_z + L_n},$$

on the transition path this is a number between 0 and 1, but in the steady-state it goes to 1.

3.3.2 Technical Progress in Distribution

In this section we allow exogenous technical progress in distribution. In particular we posit:

$$\widehat{S} = -M,$$

where M is a positive constant.

Repeating the growth rates:

$$\widehat{Q} = \alpha_q \frac{LE^*\theta\left(\epsilon-1\right)}{\epsilon N} - \rho,$$

$$\widehat{Z} = \alpha_z \frac{LE^* \sigma (\epsilon - 1)}{\epsilon N} \left(\frac{Z^{-\sigma}}{Z^{-\sigma} + S} \right) - \rho,$$

and

$$\widehat{N} = \beta N \left(X_{it} \left(P_i - Z_i^{-\sigma} - S \right) - \phi - l_{zi} - l_{qi} \right) - \rho.$$

Just as before there exists thresholds $\bar{N}_q$ and $\bar{N}_z$ such that $\widehat{Q}$ and $\widehat{Z}$ equal zero. To prevent this analysis from becoming to taxonomic we focus on a regime with both activities active.

Constant growth of quality requires N to be constant, therefore in the steady-state:

$$N_{ss} = \frac{\beta LE^* \left[1 - \theta (\epsilon - 1) - \sigma (\epsilon - 1) \left(\frac{Z^{-\sigma}}{Z^{-\sigma} + S} \right) \right] - \epsilon \rho}{\epsilon \beta \left[\phi - \left(\frac{1}{\alpha_z} + \frac{1}{\alpha_q} \right) \rho \right]}. \tag{3.24}$$

Since N is constant in the steady-state,

$$\frac{d \left(\frac{Z^{-\sigma}}{Z^{-\sigma} + S} \right)}{dt} = 0,$$

or

$$\frac{-\sigma Z^{-\sigma} \left(Z^{-\sigma} + S \right) \widehat{Z} + Z^{-\sigma} \left(\sigma Z^{-\sigma} \widehat{Z} - S \widehat{S} \right)}{\left(Z^{-\sigma} + S \right)^2} = 0$$

This implies:

$$\sigma \widehat{Z}_{ss} = M,$$

note that if $M = 0$, the steady-state growth rate of cost reduction must be zero. This by itself is worthy of discussion. If cost reduction is active in the steady-state, it must be the case that the rate of cost reduction is equal to the rate of technical progress in distribution. The intuition is that if the growth rate of manufacturing productivity is higher than the rate of technical progress in distribution, the return to cost reduction

falls. Likewise, if the rate of technical progress in distribution is higher than the rate of cost reduction, the rate of return to cost reduction will rise.

Using this fact, yields the steady-state growth rate of productivity:

$$\sigma\widehat{Z} = M = \frac{LE^{*}\alpha_z\sigma^2\left(\epsilon-1\right)}{\epsilon N_{ss}}\left(\frac{Z^{-\sigma}}{Z^{-\sigma}+S}\right) - \sigma\rho \tag{3.25}$$

substituting (3.24) into (3.25) yields:

$$\left(\frac{Z^{-\sigma}}{Z^{-\sigma}+S}\right)_{ss} = \frac{\left(M+\sigma\rho\right)\left[1-\theta\left(\epsilon-1\right)-\frac{\epsilon\rho}{\beta LE^{*}}\right]}{\alpha_z\sigma^2\left(\epsilon-1\right)\left[\phi-\left(\frac{1}{\alpha_z}+\frac{1}{\alpha_q}\right)\rho\right]+\left(M+\sigma\rho\right)\sigma\left(\epsilon-1\right)} \tag{3.26}$$

(3.26) deserves discussion. Recall that the numerator is the manufacturing unit cost, the denominator is the total unit cost. The left hand side of (3.26) is the ratio of manufacturing costs to total unit cost. If the rate of technical progress in shipping is high, the steady-state ratio will be high. If the R&D efficiency of cost reduction is high, the ratio will be lower.

The steady-state number of firms:

$$N_{ss} = \frac{LE^{*}\left(1-\theta\left(\epsilon-1\right)-\frac{\epsilon\rho}{\beta LE^{*}}\right)}{\epsilon\left[\phi+\frac{M}{\alpha_z\sigma}-\frac{\rho}{\alpha_q}\right]},$$

it is worthwhile to point out that the steady-state number of firms depends negatively on the rate of productivity growth in shipping. Again, this boils down to the endogenous market structure of the model. An increase in the shipping productivity increases firms incentive to do cost reducing R&D which is an endogenous fixed sunk cost which affects the number of firms. Another thing interesting to point out is that an increase in the

efficiency of R&D in quality improvement reduces the steady-state number of firms. An increase in the R&D efficiency of cost reduction reduces increases the number of firms. This is because the rate of cost reduction has to equal the rate of technical progress in distribution. If firms are very efficient at R&D they will not have to do as much R&D in cost reduction.

The steady-state growth rate of product quality:

$$\widehat{Q}_{ss} = \frac{\alpha_q \theta (\epsilon - 1) \left(\phi + \frac{M}{\alpha_z \sigma} - \frac{\rho}{\alpha_q} \right)}{1 - \theta (\epsilon - 1) - \frac{\epsilon \rho}{\beta L E^*}} - \rho,$$

which is increasing in the rate of technical progress in shipping. This boils down to the fact that if there is a lot technical progress in shipping, firms must do more cost reduction which reduces the steady-state number of firms. The steady-state consumption growth rate is:

$$\widehat{C} = \sigma M + \frac{\alpha_q \theta^2 (\epsilon - 1) \left(\phi + \frac{M}{\alpha_z \sigma} - \frac{\rho}{\alpha_q} \right)}{1 - \theta (\epsilon - 1) - \frac{\epsilon \rho}{\beta L E^*}} - \theta \rho$$

3.4 Conclusion

This paper proposed a model with R&D in variety expansion, cost reduction and quality improvement. We analyze two cases. In the first, distribution costs are absent. In the second, we analyze the effects of distribution costs.

Assuming technical progress in distribution might buy realism in some regard, and is a worthwhile theoretical exercise. In particular technical progress in distribution makes cost reduction in the steady-state possible, however this result should be taken with a grain of salt. Constant productivity growth in distribution implies that the cost of shipping goods must fall to zero asymptotically. One who is familiar with basic physics

principles should be deeply bothered by this. If there is any irreducible unit cost, cost reduction cannot be a source of long-run growth.

One of the contributions of this paper is it generates a relationship between manufacturing productivity and shipping productivity previously not seen in the literature. With high distribution costs firms do less cost reduction and in the steady-state they will have a higher unit cost. In addition, the growth rate of manufacturing productivity and shipping technology are fundamentally linked. If there is going to be steady-state cost reduction, it must equal the rate of technical progress in shipping.

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APPENDICES

Appendix A

A.1

In this section I derive the closed form solution for Z_t in both autarky and trade.

In autarky:

$$\dot{N}_k \beta \left(\frac{\theta^2}{A_k}\right)^{\frac{1}{1-\theta}} L_k = N_k \left(\left(\frac{\theta^2}{A_k}\right)^{\frac{1}{1-\theta}} L_k \left[\left(\frac{1-\theta}{\theta} A_k\right)(1-\delta) - \beta\rho \right] - N_k \left(\phi_k - \frac{1}{\alpha_{kz}}\rho\right) \right)$$

Define:

$$D_k = \beta \left(\frac{\theta^2}{A_k}\right)^{\frac{1}{1-\theta}} L_k$$

$$E_k = \left(\frac{\theta^2}{A_k}\right)^{\frac{1}{1-\theta}} L_k \left[\left(\frac{1-\theta}{\theta} A_k\right)(1-\delta) - \beta\rho \right]$$

$$F_k = \left(\phi_k - \frac{1}{\alpha_{kz}}\rho\right)$$

Entry is governed by:

$$\dot{N}_k D_k = N_k \left(E_k - N_k F_k\right)$$

$$\frac{\dot{N}}{N\left(E-NF\right)}=\frac{1}{D}$$

Using partial fractal decomposition:

$$\left(\frac{1}{EN}+\frac{F}{E\left(E-NF\right)}\right)\dot{N}=\frac{1}{D}$$

$$\int_{N_0}^{N_t}\frac{dN}{N}+E\int_{N_0}^{N_t}\frac{dN}{\left(E-NF\right)}=\int_0^T\frac{E}{D}dt$$

$$ln\left(\frac{N_t}{N_0}\right)-ln\left(\frac{E-N_tF}{E-N_0F}\right)=\frac{E}{D}T$$

$$\left(\frac{N_t}{N_0}\right)\left(\frac{E-N_0F}{E-N_tF}\right)=\exp^{(\frac{E}{D})T}$$

$$N_t=\frac{EN_0}{FN_0+\left(E-N_0F\right)\exp^{-(\frac{E}{D})T}}$$

Interestingly this can be written as:

$$N_t=\frac{N_{ss}N_0}{N_0+\left(N_{ss}-N_0\right)\exp^{-(\frac{E}{D})T}}$$

Now

$$\frac{dZ}{Z}=\left(\alpha_z\delta\left[\left(\frac{\theta^2}{A}\right)^{\frac{1}{1-\theta}}\left(\frac{1-\theta}{\theta}A\right)\right]L\frac{1}{N}-\rho\right)dt$$

$$G_k=\alpha_{kz}\delta\left[\left(\frac{\theta^2}{A_k}\right)^{\frac{1}{1-\theta}}\left(\frac{1-\theta}{\theta}A_k\right)\right]L_k$$

$$ln\left(\frac{Z_t}{Z_0}\right)=\int_0^T\left[G\left(\frac{FN_0+\left(E-N_0F\right)\exp^{-(\frac{E}{D})t}}{EN_0}\right)-\rho\right]dt$$

$$ln\left(\frac{Z_t}{Z_0}\right)=\frac{G}{E}FT-\rho T+\frac{G\left(E-N_0F\right)}{EN_0}\int_0^T\exp^{-(\frac{E}{D})t}\,dt$$

$$Z_{kT} = Z_{k0} \exp\left[\left(\frac{GF}{E} - \rho\right)T + \frac{D_k G_k \left(E - N_{k0} F_k\right)}{E_k^2 N_{k0}} \left(1 - \exp^{-\left(\frac{E_k}{D_k}\right)T}\right)\right]$$

$$Z_{kT} = Z_{k0} \exp\left[\widehat{Z}_{ss} T + \frac{\beta\left(\widehat{Z}_{ss} + \rho\right)\theta}{(1-\theta)A_k(1-\delta) - \theta\beta\rho}\left(\frac{N_{ssk} - N_{k0}}{N_{k0}}\right)\left(1 - \exp^{-\left(\frac{(1-\theta)A_k(1-\delta) - \theta\beta\rho}{\theta\beta}\right)T}\right)\right]$$

With trade:

$$\widehat{N} = \frac{\left[\left(\frac{\theta}{P_{xhi}}\right)^{\frac{1}{1-\theta}} L_h + \left(\frac{\theta}{P_{xhi}(1+\tau_f)}\right)^{\frac{1}{1-\theta}} L_f\right]\left(\frac{1-\theta}{\theta}A\right)(1-\delta) - N\left(\phi - \frac{1}{\alpha_z}\rho\right)}{\beta\left[\left(\frac{\theta}{P_{xhi}}\right)^{\frac{1}{1-\theta}} L_h + \left(\frac{\theta}{P_{xhi}(1+\tau_f)}\right)^{\frac{1}{1-\theta}} L_f\right]} - \rho$$

or:

$$\dot{N} D_{trade} = N\left(E_{trade} - N F_{trade}\right)$$

where:

$$D_{trade} = \beta\left[\left(\frac{\theta}{P_{xhi}}\right)^{\frac{1}{1-\theta}} L_h + \left(\frac{\theta}{P_{xhi}\left(1+\tau_f\right)}\right)^{\frac{1}{1-\theta}} L_f\right]$$

$$E_{trade} = \left[\left(\frac{\theta}{P_{xhi}}\right)^{\frac{1}{1-\theta}} L_h + \left(\frac{\theta}{P_{xhi}\left(1+\tau_f\right)}\right)^{\frac{1}{1-\theta}} L_f\right]\left[\left(\frac{1-\theta}{\theta}A\right)(1-\delta) - \beta\rho\right]$$

$$F_{trade} = \left(\phi - \frac{1}{\alpha_{zh}}\rho\right)$$

$$G_{trade} = \alpha_{zh}\delta\frac{1-\theta}{\theta}A\left[\left(\frac{\theta}{P_{xh}}\right)^{\frac{1}{1-\theta}} L_h + \left(\frac{\theta}{P_{xh}\left(1+\tau_f\right)}\right)^{\frac{1}{1-\theta}} L_f\right]$$

$$Z_{ttrade} = Z_{0trade} \exp\left[\left(\frac{G_{trade}F_{trade}}{E_{trade}} - \rho\right)T + \frac{D_{trade}G_{trade}\left(E_{trade} - N_0 F_{trade}\right)}{E_{trade}^2 N_0}\left(1 - \exp^{-\left(\frac{E_{trade}}{D_{trade}}\right)T}\right)\right]$$

A.2

In this section I derive the welfare effects.

$$U\left(C_k\right)=\int_0^{\infty} e^{-\rho t}\left(\log\left(Z_{ht}\right)dt+\log\left(\left(1-\theta\right)+\beta\rho\left(\frac{\theta}{P_{xhi}}\right)\right)\left(\frac{\theta}{P_{xh}}\right)^{\frac{\theta}{1-\theta}}L_{ht}\right)$$

I define:

$$M_{h0}=\frac{\log\left(Z_{k0}\right)+\log\left(\left(1-\theta\right)+\beta\rho\left(\frac{\theta}{P_{xhi}}\right)\right)\left(\frac{\theta}{P_{xh}}\right)^{\frac{\theta}{1-\theta}}L_{ht}}{\rho}$$

$$M_{k1}=\frac{D_kG_k\left(E_k-N_{k0}F_k\right)}{E_k^2N_{k0}}$$

$$U\left(C_k\right)=\widehat{Z}_k\int_0^{\infty}e^{-\rho t}tdt+M_{k1}\int_0^{\infty}e^{-\rho t}\left(1-e^{-\left(\frac{E_k}{D_k}\right)t}\right)dt+M_{k0}$$

After making the relevant substitutions and integrating:

$$U_{hautarky}=\frac{\widehat{Z}_{hss}}{\rho^2}+\left(\frac{\beta\left(\widehat{Z}_{hss}+\rho\right)\theta}{\rho\left(1-\theta\right)A_h\left(1-\delta\right)}\right)\left(\frac{N_{hss}-N_{h0}}{N_{h0}}\right)+M_{h0}$$

After trade:

$$U\left(C_k\right)=M_{1trade}\int_0^{T}e^{-\rho t}tdt+M_{2trade}\int_0^{T}e^{-\rho t}\left(1-e^{-\left(\frac{E_{trade}}{Dtrade}\right)t}\right)dt+M_{0trade}$$

Where:

$$M_{h0trade} = \frac{\log\left(Z_{h0}\right) + \log\left(\left(1-\theta\right)\left(\frac{\theta}{P_{xh}}\right)^{\frac{\theta}{1-\theta}} + \beta\rho\left(\left(\frac{\theta}{P_{xhi}}\right)^{\frac{1}{1-\theta}} + \left(\frac{\theta}{P_{xhi}(1+\tau_f)}\right)^{\frac{1}{1-\theta}}\frac{L_f}{L_h}\right)L_h\right)}{\rho}$$

$$M_{1trade} = \left(\frac{G_{trade}F_{trade}}{E_{trade}} - \rho\right)$$

$$M_{2trade} = \frac{D_{trade}G_{trade}\left(E_{trade} - N_{k0}F_{trade}\right)}{E_{trade}^2 N_{k0}}$$

So:

$$U_{htrade} = \frac{\widehat{Z}_{trade}}{\rho^2} + \frac{\beta\left(\widehat{Z}_{ss} + \rho\right)\theta}{\rho\left(\left(1-\theta\right)A\left(1-\delta\right)\right)}\left(\frac{N_{sstrade} - N_{h0}}{N_{h0}}\right) + M_{h0trade}$$

The welfare effects are thus:

$$U_{h\triangle} = \frac{\log\left(\frac{\left(1-\theta\right)+\beta\rho\left(\frac{\theta^2}{A_h}\right)\left(1+\left(\frac{1}{1+\tau_f}\right)\frac{L_f}{L_h}\right)}{\left(1-\theta\right)+\beta\rho\left(\frac{\theta^2}{A_h}\right)}\right)}{\rho} + \frac{\beta\left(\widehat{Z}_{hss} + \rho\right)\theta}{\rho\left(\left(1-\theta\right)A\left(1-\delta\right)\right)}\left(\frac{N_{sstrade} - N_{hss}}{N_{h0}}\right)$$

The first term is unambiguously positive. Home gains extra dividend income. The second term is also unambiguously positive. Home is now receiving faster transitional growth. Note that these welfare gains are for arbitrary values of N_{h0}. Not only is there additional transitional growth to the new steady state, the transitional growth the country was receiving in autarky is amplified.

Assuming Home was at their steady state before trade:

$$U_{h\triangle} = \left(\frac{\beta\theta\left(\widehat{Z}_{hss} + \rho\right)}{(1-\theta)(1-\delta)A_h} \right) \left(\frac{1}{1+\tau_f} \right)^{\frac{1}{1-\theta}} \frac{L_f}{L_h} + \frac{\log\left(\frac{(1-\theta)+\beta\rho\left(\frac{\theta^2}{A_h}\right)\left(1+\left(\frac{1}{1+\tau_f}\right)\frac{L_f}{L_h}\right)}{(1-\theta)+\beta\rho\left(\frac{\theta^2}{A_h}\right)} \right)}{\rho}$$

the gains from the transition are increasing in the size of intermediate importing country (Foreign). Intuitively trade increases market size, how much trade increases market size is related to the relative size of the importing country. If the importing country is small compared to the exporting country, their wont be very much additional transitional dynamics. (again, in this scenario I assumed Home was at their steady state before trade, ie $N_{h0} = N_{ss}$)

The derivation for Foreign is very similar and is therefore omitted.

Appendix B

Following the methods described in Peretto and Connolly (2007), we describe how entry stops.

Recall the variety-expansion growth rate:

$$\widehat{N} = \begin{cases} \beta \left[LE \left(\frac{1-\theta(\epsilon-1)}{\epsilon} \right) - N(\phi - \frac{\rho}{\alpha_q}) \right] - \rho & : l_q > 0 \\ \beta \left[\frac{LE}{\epsilon} - N\phi \right] - \rho & : l_q = 0 \end{cases} \tag{B.1}$$

Assuming $\alpha_q \phi > \rho$ the growth rate is declining in N. From here there are two possible cases:

1. $\widehat{N} = 0$ before N reaches $\bar{N}$

   (a) In this case, variety-expansion shuts down and vertical expansion occurs indefinitely

2. $\widehat{N}$ is positive when N reaches $\bar{N}$

   (a) In this case, vertical expansion shuts down, and $\widehat{N}$ is governed by the second branch of (B.1)

(b) $\widehat{N} = 0$ implies that $N_{ss} = \frac{\beta \frac{LE}{\epsilon} - \rho}{\beta \phi}$

If $\alpha_q \phi < \rho$ the growth rate is increasing in N. In this case, N reaches $\bar{N}$ and is then governed by the second branch of (B.1).

Appendix C

C.1

Proof of Proposition 2.

The steady-state growth rates are:

$$\widehat{Q}_{ss} = \frac{\alpha_q \theta (\epsilon - 1) \left(\phi - \left(\frac{1}{\alpha_z} + \frac{1}{\alpha_q} \right) \rho \right)}{1 - (\theta + \sigma)(\epsilon - 1) - \frac{\epsilon \rho}{\beta L E^*}} - \rho, \tag{C.1}$$

$$\widehat{Z}_{ss} = \frac{\alpha_z \sigma (\epsilon - 1) \left(\phi - \left(\frac{1}{\alpha_z} + \frac{1}{\alpha_q} \right) \rho \right)}{1 - (\theta + \sigma)(\epsilon - 1) - \frac{\epsilon \rho}{\beta L E^*}} - \rho. \tag{C.2}$$

Differentiating (C.1) and (C.2)with respect to α_q yields:

$$\frac{d\widehat{Q}_{ss}}{d\alpha_q} = \frac{\theta (\epsilon - 1) \left(\phi - \frac{\rho}{\alpha_z} \right)}{1 - (\theta + \sigma)(\epsilon - 1) - \frac{\epsilon \rho}{\beta L E^*}} > 0$$

$$\frac{d\widehat{Z}_{ss}}{d\alpha_q} = \frac{\alpha_z \sigma (\epsilon - 1) \rho \alpha_q^{-2}}{1 - (\theta + \sigma)(\epsilon - 1) - \frac{\epsilon \rho}{\beta L E^*}} > 0$$

If

$$\phi > \frac{\alpha_z \sigma \rho \alpha_q^{-2}}{\theta} + \frac{\rho}{\alpha_z}$$

then

$$\frac{d\widehat{Q}_{ss}}{d\alpha_q} > \frac{d\widehat{Z}_{ss}}{d\alpha_q}$$

Differentiating (C.1) and (C.2)with respect to α_z yields:

$$\frac{d\widehat{Q}_{ss}}{d\alpha_z} = \frac{\alpha_q\theta\left(\epsilon-1\right)\rho\alpha_z^{-2}}{1-\left(\theta+\sigma\right)\left(\epsilon-1\right)-\frac{\epsilon\rho}{\beta LE^*}} > 0$$

$$\frac{d\widehat{Z}_{ss}}{d\alpha_z} = \frac{\sigma\left(\epsilon-1\right)\left(\phi-\frac{\rho}{\alpha_q}\right)}{1-\left(\theta+\sigma\right)\left(\epsilon-1\right)-\frac{\epsilon\rho}{\beta LE^*}} > 0$$

If

$$\phi > \frac{\alpha_q\theta\rho\alpha_z^{-2}}{\sigma} + \frac{\rho}{\alpha_q}$$

then

$$\frac{d\widehat{Z}_{ss}}{d\alpha_z} > \frac{d\widehat{Q}_{ss}}{d\alpha_z}$$

Then for

$$\phi > \text{Max}\left\{\frac{\alpha_q\theta\rho\alpha_z^{-2}}{\sigma} + \frac{\rho}{\alpha_q}, \frac{\alpha_z\sigma\rho\alpha_q^{-2}}{\theta} + \frac{\rho}{\alpha_z}\right\}$$

$$\frac{d\widehat{Z}_{ss}}{d\alpha_z} > \frac{d\widehat{Q}_{ss}}{d\alpha_z}, \quad \frac{d\widehat{Q}_{ss}}{d\alpha_q} > \frac{d\widehat{Z}_{ss}}{d\alpha_q}$$

C.2

Proof of Proposition 3.

If

$$\alpha_q\theta < \alpha_z\sigma\left(\frac{Z^{-\sigma}}{Z^{-\sigma}+S}\right)$$

the rate of cost reduction will be higher than the rate of quality improvement. The left

hand side is a constant, the right hand side is declining over time. When the productivity becomes high enough, the inequality reverses.