ASSESSMENT OF ACTUAL LOADING CONDITIONS OF STEAM GENERATORS AND HEAT EXCHANGERS ACCORDING TO PROBABILISTIC CRITERIA

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SUMMARY

Classical design codes are based on nominal conditions, e.g. pressure, temperature, material strength and tube diameter for tubes under pressure; local loads, hot spots, deviations in strength and geometry, etc., are compensated by the use of safety factors. More recent methods—in particular those for the assessment of steam generators and heat exchangers for nuclear applications—call for designs to meet actual loading conditions. Thus detailed investigations are required to determine realistic operational characteristics of the components to be assessed. However a "worst case" treatment on the basis of the above criteria would yield an over-conservative design which in special cases might even be worse than the classical one, e.g. in the case of heated tubes due to increased thermal stresses. As a consequence, with all the data or the spectra of data available, the assessment has to be executed according to probabilistic criteria resulting in a design with an acceptably low and, above all, known failure rate adjusted to fit into the overall plant concept.

In components to which the above criteria have been applied before—mainly pressure vessels—the design variables "pressure" and "temperature" generally are considered to be uncorrelated, the temperature is in most cases assumed to be constant. For heat exchangers and steam generators these simplifications are not acceptable by principle; on the contrary, even deviations in geometry like tube tolerances result in flow rate deviations with corresponding deviations in temperature and pressure. Thus a new procedure for the assessment according to actual loading conditions, e.g. by the application of probabilistic criteria, is proposed which can be used for heat exchangers and steam generators. It is characterised by the following features:

(a) In a series of thermal performance computations the effects of deviations of the relevant input data like operating conditions, geometry, etc. on the load determining factors pressure and temperature are calculated.

(b) For each case the relative deviations of the load determining factors are transformed into deviations of the variables to be assessed (e.g. in the case of tube into deviations of the required wall thickness); particular care has to be taken for the distinction between correlated and uncorrelated effects.

(c) The single effects according to (b) are superposed, beginning with the correlated ones; for this superposition the relevant rules have been developed. In this step the probabilistic character of the individual factors can be taken into account by the use of mean values and standard deviations for the input data, yielding the resulting design characteristics in similar form.

With this new probabilistic method the design of steam generators and heat exchangers can be accomplished to the high reliability standards required in nuclear power stations.

1. Introduction

The high demands on heat transfer equipment for increasing output and compactness, on the one hand, and for reliability and safety of operation on the other, have led to a critical reconsideration of design methods [1]. Classical design codes are based on nominal conditions, e.g. pressure, temperature, material strength and tube diameter for tubes under pressure; local loads, hot spots, deviations in strength and geometry, etc. are compensated by the use of safety factors. More recent methods - in particular those for the assessment of steam generators and heat exchangers for nuclear applications - call for designs to meet actual loading conditions. Thus, detailed investigations are required to determine realistic operational characteristics of the components to be assessed. However a "worst case" treatment on the basis of the above criteria would yield an over-conservative design which in special cases might even be worse than the classical one, e.g. in the case of heated tubes due to increased thermal stresses. As a consequence, with all the data or the spectra of data available, the assessment has to be made according to probabilistic criteria resulting in a design with an acceptably low and, above all, known failure rate adjusted to fit into the overall plant concept.

In components to which the above criteria have been applied before - mainly pressure vessels - the design variables "pressure" and "temperature" are generally considered to be uncorrelated and the temperature is in most cases assumed to be constant. For heat exchangers and steam generators these simplifications are not acceptable as a matter of principle; on the contrary, even deviations in geometry like tube tolerances result in flow rate deviations with corresponding deviations in temperature and pressure. Thus, a new procedure for the assessment according to actual loading conditions, e.g. by the application of probabilistic criteria, is proposed which can be used for heat exchangers and steam generators. It is limited strictly to steady state cases; the transient behaviour of heat transfer equipment is dealt with elsewhere [2]. Tubes under internal overpressure have been selected as typical components, but the method can be applied to other parts analogously.

By this new method the design of steam generators and heat exchangers can be made to meet the high reliability standards required in nuclear power stations.

2. Principles of probabilistic assessment

Actual loading conditions deviate in most cases from nominal conditions. Two different types of such deviations are to be considered:

- Systematic deviations, resulting from inaccuracies and simplifications of calculation methods, of material properties and of geometry. These deviations represent the difference between nominal conditions and the mean value of actual loading conditions (expectation).

- Random deviations on a statistical basis, e.g. the scattering of material strength, tolerances in dimensions of individual parts or due to manufacturing processes. These can be characterized quantitatively by the use of standard deviations. In the superposition of such deviations a different procedure has to be adopted for independent (uncorrelated) and correlated ones.

The general subject as defined above calls for a treatment according to probabilistic principles where all the input data are given in the form of expectations and standard deviations, yielding the resulting design characteristics - mainly the required tube wall thickness S_{req} - in similar form, i.e. as an expectation $\epsilon(S_{req})$ and a standard deviation $\sigma(S_{req})$.

A comparison of the required and the installed wall thickness (S_{req}, S_{inst}) gives the reliability of the tube [3]. On the basis of the oversizing U, the failure criterion can be expressed by

$$U = S_{inst} - S_{req} < 0 \qquad (1)$$

Due to the scattering of S_{inst}, S_{req} and U, however, failure may occur even when $\epsilon(U) > 0$.

This criterion has the advantage - in comparison with the conventional one based on the difference between strength and load - that changes and deviations will influence only one of the terms (S_{req}). This is of particular importance for the calculation procedure with respect to correlated and uncorrelated deviations. To quote a typical example, a temperature deviation would yield changes in strength as well as stress, the effect becoming less transparent.

The failure probability can only be determined if the distribution of U is known. According to basic rules of statistics ("zentraler Grenzwertsatz der Statistik" e.g. [4]), the generally adopted assumption of a normal distribution of U is justified [5, 6, 7]. Thus failure probability f is given by

$$f(U<0) = \frac{1}{\sqrt{2\pi}} \int_{u}^{\infty} e^{-t^2/2} dt \qquad (2)$$

where u is the relative oversizing

$$u = \frac{\epsilon(U)}{\sigma(U)} = \frac{\epsilon(S_{inst}) - \epsilon(S_{req})}{\sigma(S_{inst} - S_{req})} \qquad (3)$$

For the actual assessment the first step is the determination of the relative oversizing from expectation values and standard deviations (see Para. 3 and 4). The failure probability can then be calculated from Eq. (2), which, for convenience, is represented graphically in Fig. 1 [8].

3. Expectation of required wall thickness

3.1 Basic equations

The generally accepted formula for the assessment of the wall thickness of cylinders under pressure is used:

$$S_{req} = D\ P/(2K + P) \tag{4}$$

wherein D is the outside diameter, P the internal overpressure and K the material strength. In classical calculations, safety factors are to be applied to the material strength K. If one uses probabilistic methods, no such factor is to be used; the expectation of the wall thickness $\epsilon(S_{req})$ is thus given by

$$\epsilon(S_{req}) = \epsilon(D)\,\epsilon(P)/(2\,\epsilon(K) + \epsilon(P)) \tag{5}$$

It should be noted that actual expectation values of material strength are used, but not the values guaranteed by manufacturers which are often as low as 0.8 ϵ(K). Material scattering and other deviations are compensated by oversizing, see Eq. (3) and Para. 4.

3.2 Expectation of pressure

The expectation of pressure to be used in Eq. (5) is, as a matter of principle, the expectation of the operating pressure. However, systematic deviations of operating pressure may occur, in particular, in once-through steam generators with single tube runs required to facilitate tube plugging /‾9/. Furthermore, an adjustment of the flow rate in individual tubes to yield equal outlet temperatures will result in an uneven pressure distribution /‾10/.

These systematic deviations can be determined by steady state performance calculations, varying the input data. The calculations will yield pressure and temperature deviations simultaneously. These new pressures and temperatures are used for the determination of $\epsilon(S_{req})$ according to Eq. (5). It should be noted that deviations at certain discrete locations may also influence the design parameters at other locations (e.g. by their influence on mass flow rate).

3.3 Expectation of material strength and wall thickness

Material strength is dependent on operating temperature; thus, the determination of the expectation of material strength can be reduced to the determination of tube wall temperature.

Systematic deviations due to individual tube runs have been mentioned before as well as the possibility to arrive at equal outlet temperatures by adjusting the flow rate in individual tubes. However, certain temperature deviations will remain, mainly in the evaporator section. Another major effect to be considered is the uneven heat transfer along the tube perimeter which can differ by a factor 2 or more. Thus hot spots will be created, although heat conduction in circumferential direction will be of beneficial influence. Further effects to be studied individually are

- influence of by-pass flow;
- uneven temperature or flow distribution of heating medium;
- uneven distribution of heat transfer surface;
- degree of gas mixing [11].

4. Standard deviation of required wall thickness

Random deviations may be the result of one or more of the following or other causes:

- tolerances in tube wall thickness
- tolerances in tube diameter (inner and outer)
- tolerances in tube length
- tolerances due to manufacturing processes
- scattering of material strength
- scattering of thermal conductivity.

Similar steady state performance calculations with variation of input data as in Para. 3.2 have to be performed to determine the influence on the design parameters pressure and temperature.

A method has been elaborated determining the effect of a multitude of causes. The influence of cause (a) in a specific location j on the required wall thickness in the location i is represented by the coefficient a_{ij} (see Fig. 2). If cause (a) refers to a deviation in tube length in location j, the required increase of wall thickness at the location i $(\Delta S_{req})_i$ can be written

$$(\Delta S_{req})_{ija} = a_{ij}\ (\Delta L)_j \tag{6}$$

If cause (b) refers to tolerances in the installed tube wall thickness the relevant equation is

$$(\Delta S_{req})_{ijb} = b_{ij}\ (\Delta S_{inst})_j \tag{7}$$

Cause (c) may be the scattering of material strength; thus

$$(\Delta S_{req})_{ijc} = c_{ij}\ (\Delta K)_j \tag{8}$$

Since deviations of material strength can result only in deviations of the required wall thickness at the particular loaction of the deviation, only a direct influence exists, and

$c_{ij} = 0$ for $i \neq j$ and

$c_{ij} \neq 0$ for $i = j$

The coefficients are arranged advantageously in matrix form as in Fig. 2. The principal diagonal then gives the coefficients of direct influence.

One can define a stochastic variable like S_{req} in the following form

$$S_{req} = \Delta S_{req} + \text{const}$$

Thus follows

$$\sigma(S_{req}) = \sigma(\Delta S_{req})$$

and Eq. (6) can be transformed into

$$\sigma^2(S_{req})_{ija} = a_{ij}^2 \ \sigma^2(L)_j \qquad (9)$$

further Eq. (7) into

$$\sigma^2(S_{req})_{ijb} = b_{ij}^2 \ \sigma^2(S_{inst})_j \qquad (10)$$

and Eq. (8) into

$$\sigma^2(S_{req})_{iic} = c_{ii}^2 \ \sigma^2(K)_i \qquad (11)$$

c_{ii} is by definition dS/dK and from Eq. (4) $c_{ii} = - 2 D P/(2K + P)^2$.

To determine the value $\sigma(S_{inst} - S_{req})$ for each loaction as required for Eq. (3), the single standard deviations have to be superimposed for each tube run, taking into account existing correlations. This procedure is shown graphically in Fig. 3: Standard deviations of the required wall thickness are first calculated from the predetermined coefficients (e.g. from a_{ij}, using Eq. (9)). Then the correlated effects are summed up, as in Fig. 3 in the lines referring to $(S_{inst})_1$. The superposition of all uncorrelated effects is done by geometric addition (last column of Fig. 3).

5. Comparison of results with classical methods

5.1 Worked example

The above indicated method was applied for a characteristic HTR once-through steam generator designed according to TRD standards [12]. Operating and design data for the inlet and outlet of individual parts are summarized in Tab. I. A hot spot factor of 1.8 for uneven heat transfer along the tube perimeter has been selected as a typical systematic deviation. From the resulting hot spot temperature t_{HS} the expectation of the required wall thickness has been calculated using Eq. (5). For the sake of completeness the installed wall thickness is included.

The following were selected as typical examples for random deviations

(a) tolerances in tube length;
(b) tolerances in installed tube wall thickness;
(c) scattering of material strength.

To explain the results of the thermal calculations Tab. II shows in the first columns for each location the deviations (Δt and Δp) due to a nominal deviation of 1 m of tube length in the connection tubes between live steam header and superheater (location 5). The third column gives the resulting coefficient a_{i5}. Similar calculations yielded the remaining coefficients a_{ij} indicated in Tab. II. It can be seen that deviations in tube length are generally of less importance; a moderate influence can be found only in heated sections or parts with high pressure drop.

In Tab. II the coefficients for the two other causes (b) and (c) have also been included. Cause (c) is a "direct influence" deviation; thus all but the coefficients in the principal diagonal are equal to zero.

To arrive at the required standard deviations the following assumptions for input deviations have been made:

(a) $\sigma(L) = 10$ mm
(b) $\sigma(S_{inst}) = \pm 6$ %
(c) $\sigma(K) = \pm 10$ %

The standard deviations have been calculated with these assumptions and the coefficients of Tab. II. The results are given in Tab. III. Since installed and required tube wall thicknesses are correlated according to Eq. (7) and (10) in the principle diagonal of cause (b) values $(1-b_{ii})$ are listed instead of b_{ii}.

The last column of Tab. III is the summation of the effects of the three causes by geometric addition. Final results are given in Tab. IV: In column 5 the failure probability f according to Eq. (2) and Fig. 1 is listed.

5.2 Discussion

The original assessment has been carried out according to TRD standards. To provide for additional safety, the actual installed wall thickness S_{inst} has been selected as the next but one instead of the next bigger standard thickness (e.g. for the preheater tubes 2.3 mm have been selected instead of 2.0 mm which would have been satisfactory for the required 1.9 mm). This fact can be seen from column 7 of Tab. IV, where the failure criterion according to TRD rules is listed.

However, according to probabilistic criteria for certain locations, e.g. 3o, a relatively high failure probability results which cannot be accepted for a nuclear steam generator. On the other hand, a great number of the remaining sections have been designed on a too conservative basis, resulting in an uneconomic steam generator.

A similar picture is reflected by the "worst case" treatment, the results of which are given in the last part of Tab. IV. Maximum deviations ΔS_{max} have been calculated from the standard deviations of Tab. III by converting the standard distributions into equivalent uniform (rectangular) ones and direct summing up of the terms (contrary to the geometric addition of Tab. III). The required wall thickness is calculated using $\epsilon(S_{req})$ and ΔS_{max}. Also this approach indicates the wall thickness at the evaporator outlet (30) as being unsafe.

6. Conclusion

A new procedure for the assessment according to actual loading conditions by the application of probabilistic criteria is proposed which can be used for heat exchangers and steam generators. It is characterized by the following features:

(a) In a series of thermal performance computations the effects of deviations of the relevant input data, like operating conditions, geometry etc. on the load determining factors pressure and temperature are calculated.

(b) Systematic deviations determined in such calculations will be used to define expectation values of the variables to be assessed (e.g. in the case of tubes the required wall thickness).

(c) The computed random deviations of the load determining factors are transformed into deviations of the variables to be assessed; particular care has to be taken for the distinction between correlated and uncorrelated effects.

(d) The single effects according to (c) are superposed, beginning with the correlated ones; for this superposition the relevant rules have been developed. In this step the probabilistic character of the individual factors can be taken into account by the use of mean values and standard deviations for the input data, yielding the resulting design characteristics in similar form.

(e) The failure probability is calculated with the expectation values and the results of the superposition of individual effects.

(f) The failure criterion used is based on the difference between installed and required dimensions (e.g. in the case of tubes the wall thickness); thus a direct dependence of the failure probability from oversizing is made evident.

The results according to (b) and (c) can also be used for an assessment of classical criteria, e.g. a "worst case" treatment.

With this new method components of nuclear power plants like steam generators and heat exchangers can be designed to meet the required low failure probability to satisfy the overall reliability concept of the plant.

7. References

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Table I Design characteristics

		Design data (TRD)			Tube characteristics				Expectation values	
	Location 1	Operating pressure (ata)	Tube wall temperature (°C)	Design pressure (atü)	o.d. (mm)	Material No. (DIN 17007) (-)	S_{req} (TRD) (mm)	S_{inst} (mm)	$t_{HS} = \epsilon(t)$ (°C)	$\epsilon(S_{req})$ (mm)
Feed header	1i	232	180	250	21.3	5415	1.4	2.0	180	0.76[1]
Tail pipes outlet	1o	220	180	238	21.3	5415	1.3	2.0	180	0.73[1]
Preheater inlet	2i	220	199	238	21.3	5415	1.3	2.3	205	0.74[1]
Preheater outlet	2o	213	393	231	21.3	5415	1.9	2.3	403	1.00[1]
Evaporator inlet	3i	213	395	231	25.0	7380	1.9	3.2	407	1.03[1]
Evaporator outlet	3o	211	498	229	25.0	7380	2.7	3.2	517	2.16[2]
Superheater inlet	4i	211	502	229	25.0	4961	3.1	4.0	523	1.73[1]
Superheater outlet	4o	206	604	225	25.0	4961	3.5	4.0	618	2.5 [2]
Tail pipes inlet	5i	206	550	225	38.0	4961	4.9	5.6	550	2.74[1]
Live steam header	5o	190	550	208	38.0	4961	4.4	5.6	550	2.54[1]

[1] using for $\epsilon(K_s) = 1{,}2\ K_{s\ MIN}$

[2] using $\epsilon(K_{B/100\ 000}) = K_{B/100\ 000}$

Table II Deviation coefficients

	Deviation due to 1m deviation in tube length in Loc. 5			Deviation coefficient due to cause (a), for ΔL = 1m					Deviation coefficient due to cause (b), for ΔS_{inst} = 10%					Deviation coefficient due to cause (c), for $\Delta(K)$ = 10%				
Location[1]	Δt (deg)	Δp (at)	ΔS_{req} ($=a_{i5}$) (mm/m)	a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i5}	b_{i1}	b_{i2}	b_{i3}	b_{i4}	b_{i5}	c_{i1}	c_{i2}	c_{i3}	c_{i4}	c_{i5}
				(mm/m)					(mm/mm)					(mm/kp/mm^2)				
1i	0	0	0	0	0	0	0	0	0	0	0	0	0	0.03	0	0	0	0
1o	0	0	0	0	0	0	0	0	0	0	0	0	0	0.03	0	0	0	0
2i	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.03	0	0	0
2o	0.5	0	0	0	0	0	0	0	0	0.02	0	0.01	0.03	0	0.07	0	0	0
3i	0.5	0	0	0	0	0	0	0	0	0.01	0	0.01	0.02	0	0	0.06	0	0
3o	3.8	0	0.08	0	0.27	0.28	0.13	0.08	0.26	1.66	0.31	1.18	2.50	0	0	0.11	0	0
4i	3.8	0	0[2]	0	0	0	0	0	0	0	0	0	0	0	0	0	0.16	0
4o	2.3	0	0.05	0	0.14	0.14	0.20	0.05	0.13	0.81	0.15	0.58	1.20	0	0	0	0.20	0
5i	3.9	0	0[2]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.16
5o	3.9	0	0[2]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.15

[1] for definition see Tab. I

[2] since material strength is almost independent of temperature at design temperature

Table III Standard deviations and determination of $\sigma(S_{inst} - S_{req})$

Location [1]	Standard deviations due to cause (a)					Standard deviations due to cause (b)					Standard deviations due to cause (c)					$\sigma(S_{inst}-S_{req})$
	$\sigma(S_{req})_{i1a}$	$\sigma(S_{req})_{i2a}$	$\sigma(S_{req})_{i3a}$	$\sigma(S_{req})_{i4a}$	$\sigma(S_{req})_{i5a}$	$\sigma(S_{req})_{i1b}$	$\sigma(S_{req})_{i2b}$	$\sigma(S_{req})_{i3b}$	$\sigma(S_{req})_{i4b}$	$\sigma(S_{req})_{i5b}$	$\sigma(S_{req})_{i1c}$	$\sigma(S_{req})_{i2c}$	$\sigma(S_{req})_{i3c}$	$\sigma(S_{req})_{i4c}$	$\sigma(S_{req})_{i5c}$	
	(mm)					(mm)					(mm)					(mm)
1i	0	0	0	0	0	0.120[2]	0	0	0	0	0.090	0	0	0	0	0.150
1o	0	0	0	0	0	0.120[2]	0	0	0	0	0.086	0	0	0	0	0.148
2i	0	0	0	0	0	0	0.138[2]	0	0	0	0	0.089	0	0	0	0.164
2o	0	0	0	0	0	0	0.135[2]	0	0.001	0.004	0	0.120	0	0	0	0.181
3i	0	0	0	0	0	0	0.002	0.192[2]	0.002	0.004	0	0	0.124	0	0	0.229
3o	0	0.003	0.003	0.001	0.001	0.050	0.318	0.132[2]	0.227	0.480	0	0	0.164	0	0	0.654
4i	0	0	0	0	0	0	0	0	0.240[2]	0	0	0	0	0.198	0	0.311
4o	0	0.001	0.001	0.002	0.001	0.031	0.195	0.036	0.101[2]	0.288	0	0	0	0.217	0	0.425
5i	0	0	0	0	0	0	0	0	0	0.336[2]	0	0	0	0	0.196	0.389
5o	0	0	0	0	0	0	0	0	0	0.336[2]	0	0	0	0	0.186	0.384

[1] for definition see Tab. I

[2] calculated with $(1 - b_{ii})$ instead of b_{ii} because of correlation (Fig. 3)

Table IV Evaluation of results

Location [1]	S_{inst}	Probabilistic criterion			TRD standard		"worst case" treatment		
		$\epsilon(S_{req})$	u	f	S_{req}	$S_{inst} - S_{req}$	Δs_{max}	S_{req}	$S_{inst} - S_{req}$
	(mm)	(mm)	(-)	(-)	(mm)	(mm)	(mm)	(mm)	(mm)
1i	2.0	0.76	8.3	$<10^{-12}$	1.4	0.6	0.364	1.12	0.88
1o	2.0	0.73	8.6	$<10^{-12}$	1.3	0.7	0.357	1.09	0.91
2i	2.3	0.74	9.5	$<10^{-12}$	1.3	1.0	0.394	1.13	1.17
2o	2.3	1.00	7.2	$<10^{-12}$	1.9	0.4	0.450	1.45	0.85
3i	3.2	1.03	4.4	5×10^{-6}	1.9	1.3	0.561	1.59	1.61
3o	3.2	2.16	1.6	6×10^{-2}	2.7	0.5	2.387	4.55	-1.35
4i	4.0	1.73	4.7	1×10^{-6}	3.1	0.9	0.758	2.49	1.51
4o	4.0	2.5	3.5	2×10^{-4}	3.5	0.5	1.510	4.01	-0.01
5i	5.6	2.74	7.3	$<10^{-12}$	4.9	0.7	0.921	3.66	1.94
5o	5.6	2.54	8.0	$<10^{-12}$	4.4	1.2	0.905	3.45	2.15

[1] for definition see Tab. I

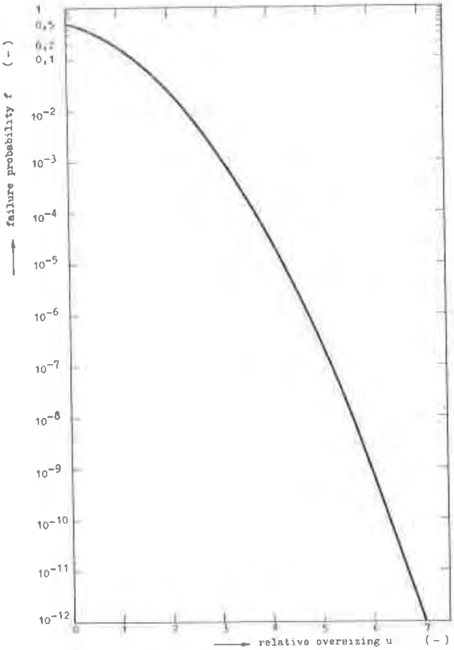
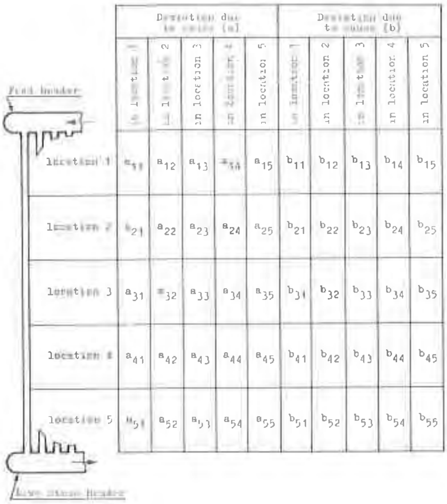
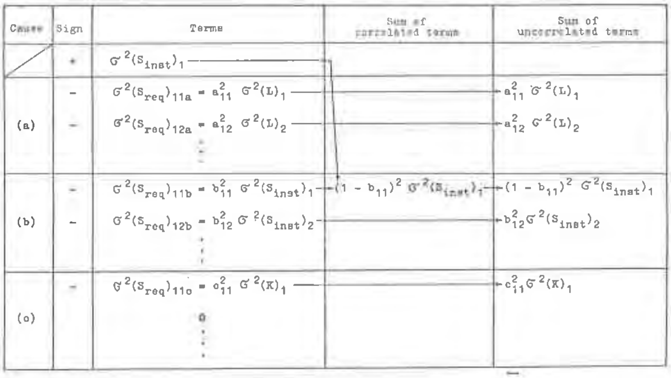


Fig. 1 Failure probability as function of relative oversizing according to Eq. (2)



	Deviation due to cause (a)					Deviation due to cause (b)				
	in location 1	in location 2	in location 3	in location 4	in location 5	in location 1	in location 2	in location 3	in location 4	in location 5
location 1	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	b_{11}	b_{12}	b_{13}	b_{14}	b_{15}
location 2	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	b_{21}	b_{22}	b_{23}	b_{24}	b_{25}
location 3	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	b_{31}	b_{32}	b_{33}	b_{34}	b_{35}
location 4	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	b_{41}	b_{42}	b_{43}	b_{44}	b_{45}
location 5	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	b_{51}	b_{52}	b_{53}	b_{54}	b_{55}

Fig. 2 Deviation coefficients a_{ij} and b_{ij}



Cause	Sign	Terms	Sum of correlated terms	Sum of uncorrelated terms
	+	$\sigma^2(S_{inst})_1$		
(a)	− −	$\sigma^2(S_{req})_{11a} = a_{11}^2\ \sigma^2(L)_1$ $\sigma^2(S_{req})_{12a} = a_{12}^2\ \sigma^2(L)_2$ ⋮		$a_{11}^2\ \sigma^2(L)_1$ $a_{12}^2\ \sigma^2(L)_2$
(b)	− −	$\sigma^2(S_{req})_{11b} = b_{11}^2\ \sigma^2(S_{inst})_1$ $\sigma^2(S_{req})_{12b} = b_{12}^2\ \sigma^2(S_{inst})_2$ ⋮	$(1 - b_{11})^2\ \sigma^2(S_{inst})_1$	$(1 - b_{11})^2\ \sigma^2(S_{inst})_1$ $b_{12}^2\sigma^2(S_{inst})_2$
(c)	−	$\sigma^2(S_{req})_{11c} = c_{11}^2\ \sigma^2(K)_1$ ⋮		$c_{11}^2\sigma^2(K)_1$

Fig. 3 Determination of $\sigma(S_{inst} - S_{req})$ for location 1