Creep of Intersecting Cylindrical Shells Under Uniform Pressure

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Abstract

The authors make use of an axisymmetric shell model with pressure and edge loads chosen to simulate the loadings at the crotch point of two normally intersecting shells of equal diameter and thickness. An assessment of this model in the case of elastic analysis has already been presented by the authors.

The present paper has a two-fold purpose. First of all, the steady state creep rates, as functions of the exponent of the power law of creep rate versus stress and of the radius-to-thickness ratio, are calculated. Here, the constitutive relationship accounts for elastic plus secondary creep strains only. This parameter study of the steady state creep response gives a designer some approximate preliminary data from which he might estimate possible creep damage. Secondly, a more detailed analysis, which takes into account thickness change due to creep, is presented to estimate the elapsed time to rupture.

1. Introduction

Structures exposed to steady tensile stresses and high temperatures over long periods of time are subject to failure by either the accumulation of excessive creep strain or by creep rupture. The creep rupture phenomenon may be caused by the deterioration of the material creep strength with increasing time under adverse environment, or it may be caused by a necking instability. Tensile stresses much higher than the nominal membrane stresses occur in pressure vessels with unreinforced cutouts, nozzles, and shell intersections; therefore, the possibility of creep rupture of these components certainly does exist.

Owing to the redistribution of stress with increasing creep strain, elastic stress analysis is too conservative for predicting the stresses which occur over the lifetime of the structure. For a structural configuration which is reasonably insensitive to geometry change, the stress distribution approaches a steady state after creep strains, still of elastic order, have occurred. Therefore, a knowledge of the steady-state stress distribution under secondary creep, then, is valuable for estimating the life of a pressure vessel with cutouts, nozzles, and shell intersections under a high temperature environment. On the other hand, a few calculations which account for the secondary effects such as strain hardening and geometry change are useful for determining the limitations of the simplified models.

2. Method of Analysis

In reference [1] the calculation of the peak elastic stresses of the intersecting shell problem of Figure 1 is reduced to the analysis of a cylindrical shell of the same diameter and thickness under the axisymmetric loading shown in Figure 2. This axisymmetric model is based on satisfying the following assumptions:

1. The intersecting shells have identical diameter, thickness, and material properties.
2. The shells intersect normally.
3. The only external loading is uniform internal or external pressure.
4. The stress and strain distributions satisfy the thin shell assumptions that plane sections remain plane, transverse shear strains have negligible influence on the deformation, and transverse stresses have negligible influence on the shell mechanical behavior.
5. Circumferential bending strains are negligible.
6. Near the crotch stress gradients in the circumferential direction may be neglected in comparison with stress gradients in the meridianal and thickness directions.

The justification of the axisymmetric replacement model with the boundary conditions given in Figure 2 is discussed in reference [1]. The left edge of the shell of Figure 2 represents the crotch point of the intersecting shells with the given edge loads being derived from an overall equilibrium equation in reference [1]. The above assumptions do not restrict the choice of model for the mechanical behavior of the material; therefore, the axisymmetric shell is also a valid replacement model for the crotch region of intersecting shells exhibiting material nonlinearity, provided the above listed assumptions are satisfied. The use of the axisymmetric replacement model to treat intersecting shells of elastic-perfectly plastic material is discussed in reference [2].

The approximate creep response at points near the crotch of intersecting shells may then be calculated by means of an axisymmetric shell model with appropriate stress-strain-strain rate relations. In the first class of problems examined in this paper, it will be assumed that the overall deformations are sufficiently small so that geometry change may be neglected. With an elastic-power law creep strain model of mechanical behavior solutions for the stress redistribution from the elastic distribution to the stationary state are obtained. The stationary creep solution represents the stress state over a significant portion of the life of the vessel.

Constitutive relations which include elastic strains and secondary creep strains in a thin axisymmetric shell of isotropic material are

$$\dot{\varepsilon}_x = (\dot{\sigma}_x - \nu\dot{\sigma}_\theta)/E + U_o\sigma_e^{n-1} (\sigma_x - \sigma_\theta/2)s_o^n \quad (1)$$

$$\dot{\varepsilon}_\theta = (\dot{\sigma}_\theta - \nu\dot{\sigma}_x)/E + U_o\sigma_e^{n-1} (\sigma_\theta - \sigma_x/2)/s_o^n \quad (2)$$

where σ_e is the equivalent stress given by

$$\sigma_e = (\sigma_x{}^2 - \sigma_x\sigma_\theta + \sigma_\theta{}^2)^{1/2} \quad (3)$$

The boundary conditions appropriate for the axisymmetric model for the intersecting shell problem are

$$Q = pR/2,\ u_x = 0,\ \beta = 0 \quad (4)$$

at $x = 0$ and

$$Q = 0,\ M_x = 0,\ N_x = pR/2 \quad (5)$$

at large values of x, i.e. $x > 5\sqrt{(Rt)}$. Numerical solutions of this axisymmetric problem are obtained by means of the KSHEL computer program described in reference [3]. The loading is taken to be a step load applied to an initially stress free shell.

3. The Stationary State

With the assumption that the constitutive relations (1)-(3) apply to nominal stresses and strains based on the original dimensions of the shell, the results of the computer analysis describe the transition from the elastic stress distribution to the stationary state. Figure 3 shows a plot of the maximum dimensionless equivalent stress versus a dimensionless time parameter. From dimensional reasoning, it appears that such dimensionless stress versus dimensionless time curves depend on only three parameters. These are the Poisson's ratio ν, the radius-to-thickness ratio R/t, and the creep power law exponent n. The stationary dimensionless stresses depend only on the later two.

Table 1 gives values of the nondimensional stresses at the crotch point at the stationary state. The stationary creep rates at the crotch point may then be calculated from the stresses using constitutive relations (1)-(3) with the elastic strain rate terms omitted, i.e. $E\to\infty$. Calculations of these creep rates reveal that the stationary circumferential creep strain rate at the crotch point may be determined approximately by a formula of the type

$$\dot{\varepsilon}_\theta = U_0[a_1 + a_2(R/t)^{1/2}]^n(pR/s_0t)^n \quad (6)$$

where a_1 and a_2 are given as functions of n in Table 2.

The average creep rate in the thickness direction may readily be determined from the incompressibility condition

$$\dot{\varepsilon}_z = -(\dot{\varepsilon}_x + \dot{\varepsilon}_\theta) \tag{7}$$

applied to the average strain rates. Since in shell theory the strain rates vary linearly with the thickness coordinate, the thickness averages of the creep rates are simply the arithmetic means of the values at the inner and outer surfaces. Using stresses from Table 1 and the constitutive relations (1)-(3), it can be shown that the average creep rate in the thickness direction satisfies the approximate relationship:

$$\dot{\varepsilon}_z = -U_0[b_1 + b_2(R/t)^{1/2}]^n(pR/s_0t)^n \tag{8}$$

where values of b_1 and b_2 are given in Table 2. Since the thickness strain rate is negative, the wall undergoes thinning as the creep process takes place.

4. Creep Rupture

Necking instability and creep rupture of a ductile body under constant load can occur in a tensile structure which becomes thinner as the creeping motion takes place. As the wall thickness reduces, the true stresses increase, which, in turn, increase the creep rates and the rate of thinning. This phenomenon can be described by a model which predicts that the thickness will reduce to zero in a finite critical time. Of course, rupture takes place before this time is reached.

In the intersecting shells problem, necking and rupture will take place locally near the crotch point; therefore, the axisymmetric model is appropriate for estimating the time to failure. In a tensile creep instability analysis, constitutive relations (1)-(3) must be regarded as a relationship between true stresses and true strain rates based on the instantaneous configuration. However, the elastic terms may be omitted.

As a first and very crude model of the creep instability of intersecting shells, it is assumed that the thickness of the shells remains uniform in space, but decreases with increasing time. As a very conservative measure, the strain state at the crotch point will be used to determine the value of the wall thickness. Since elastic strains are neglected here and the wall is spacewise uniform, the stress distribution near the crotch becomes that of stationary state of a shell of reduced thickness. Since this thickness is assumed to be uniform, the results of the previous section may be applied here with the understanding that true stresses and strain rates are implied. With these assumptions, the time rate of thickness change may be calculated as

$$\dot{t} = t\dot{\varepsilon}_z \tag{9}$$

where the expression for $\dot{\varepsilon}_z$ may be taken from equation (8). Letting

$$\omega = t/t_0 \tag{10}$$

where t_0 is the original thickness, equations (8), (9), and (10) reduce to the differential equation and initial condition

$$d\omega/d\eta = -\omega^{1-n}[b_1 + b_2(R/t_0\omega)^{1/2}]^n \tag{11}$$

$$\omega = 1 \text{ at } \eta = 0 \tag{12}$$

Here, η is a nondimensional time parameter given by

$$\eta = \tau U_0(pR/s_0t_0)^n \tag{13}$$

Letting ω_f be the value of ω at which rupture takes place, the critical value of η then becomes

$$\eta_{cr} = \int_{\omega_f}^{1} \omega^{n-1}[b_1 + b_2(R/t_o\omega)^{1/2}]^{-n}\, d\omega \tag{14}$$

Two further simplifications may be made in order to obtain simple approximate expression for the critical time. By taking b_1 and ω_f equal to zero, the following is obtained

$$\eta_{cr} \approx 2\, b_2^{-n}(R/t_o)^{-n/2}/3n \tag{15}$$

These two approximations are justified since equation (14) is already a very crude model of the creep instability problem, b_1 is small in comparison with a term added to it, and the difference between the results for $\omega_f = 0$ and those for small values of ω_f is small.

The second method for treating the large strain creep analysis problem is a numerical one. The axisymmetric model of the shell and loading in the crotch region is employed and the resulting axisymmetric shell problem is solved numerically by the KSHEL computer program. However, to account for the reduction of wall thickness with increasing strains, the thickness profile is adjusted at each time step in accordance with equations (7) and (9). Hence, the shell analyzed over each time step is a variable thickness shell.

Figure 4 shows the results of a typical calculation of the thickness change at the crotch versus time. For the same shell and geometry equation (15) predicts that the critical time corresponds to $\eta = 3.5 \times 10^{-5}$. According to Figure 4 the thickness at the crotch point has already reduced to about 60 percent of its original value at this time and the shell will reach zero thickness at $\eta \approx 7 \times 10^{-5}$. Therefore, equation (15) gives a reasonable, but conservative, estimate of the critical time.

5. Conclusions

Significant redistribution of stress takes place at the crotch point of intersecting cylinders as creep strains reach elastic order. Therefore, a prediction of creep life on the basis of stresses calculated by elastic analysis would be too conservative for materials following a highly nonlinear creep law.

The creep strains in the crotch region of intersecting shells under internal pressure cause thinning of the wall so that the failure mode in the creep regime would be one of necking instability and eventually creep rupture at the crotch point. Some approximate formulas for estimating this time to rupture have been derived.

REFERENCES

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NOMENCLATURE

E	Young's modulus of elasticity
ν	elastic Poisson's ratio
U_o	material creep rate parameter
s_o	material creep strength
n	creep exponent, $n \geq 1$
σ_e	equivalent stress
p	internal pressure
Q	transverse shear force per unit circumference
β	rotation of shell normal
σ_x, σ_θ	stress components
$\varepsilon_x, \varepsilon_\theta$	strain components
N_x, N_θ	components of membrane stress resultant
M_x, M_θ	components of bending moment per unit length
R	shell radius
t	shell thickness
τ	time

Dots over variables indicate rates with respect to τ.
Subscript x refers to direction along meridian.
Subscript θ refers to circumferential direction.

TABLE 1. Nondimensional Stresses at the Crotch Point

R/t	n	$\sigma_x t/pR$		$\sigma_\theta t/pR$	
		Inside	Outside	Inside	Outside
25	1	-5.62	6.62	1.00	7.12
25	3	-4.85	5.29	-0.52	4.36
25	5	-4.50	4.77	-0.87	3.58
25	9	-4.15	4.33	-0.97	3.06
100	1	-11.75	12.75	1.00	13.25
100	3	-9.47	9.87	-1.64	7.88
100	5	-8.53	8.80	-2.03	6.46
100	9	-7.77	7.93	-2.30	5.39
225	1	-17.87	18.87	1.00	19.37
225	3	-14.07	14.45	-2.74	11.39
225	5	-12.60	12.85	-3.26	9.30
225	9	-11.50	11.60	-3.30	8.00

TABLE 2. Values of Constants in Equations (6) and (8)

n	a_1	a_2	b_1	b_2
1	0.75	0.613	0.75	0.307
3	0.68	0.553	0.78	0.501
5	0.63	0.539	0.67	0.519
9	0.57	0.535	0.57	0.526

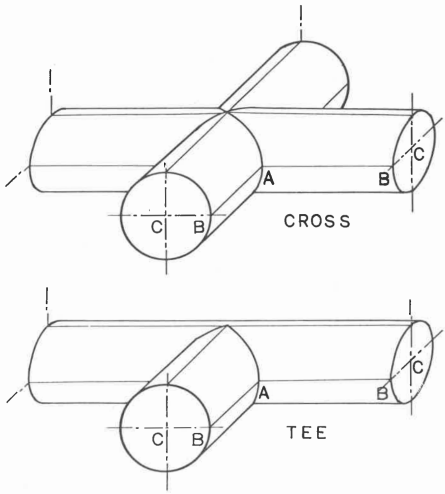

Figure 1. Normally Intersecting Cylindrical Shells

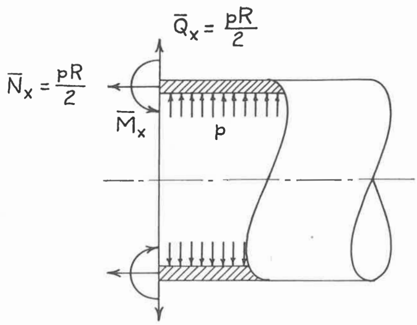

Figure 2. Axisymmetric Shell Model for Crotch Region of Intersecting Shells

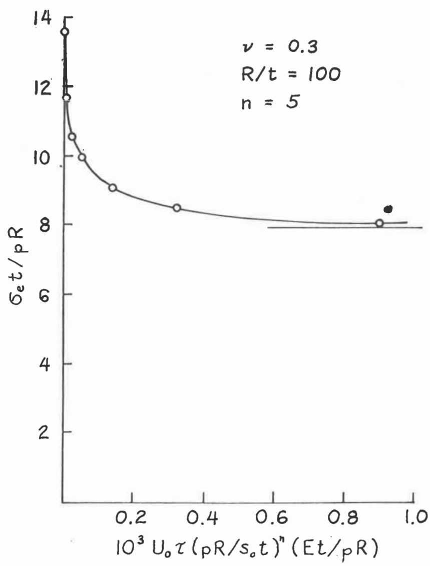

Figure 3. Dimensionless Equivalent Stress at Inside of Crotch Point Versus a Dimensionless Time Parameter

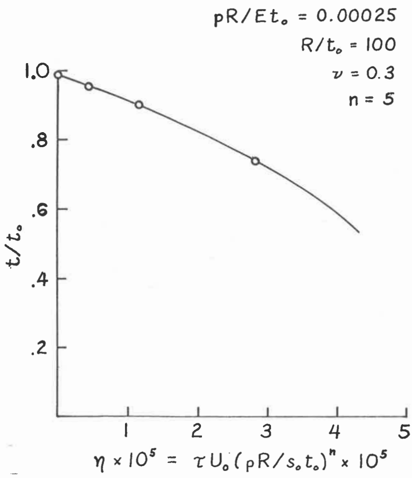

Figure 4. Ratio of Thickness at Crotch Point to Initial Thickness as a Function of Dimensionless Time η.