Problems Associated with the Design of Notched Components

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Summary

Since all fatigue cracks originate at some form of stress raiser, the problem of the design of notched components is very real. Because of the extremes of notch severities, sizes and stress levels encountered by practical components, a wide range of different design techniques are required. Combinations of these parameters which lead to initiation dominated lives can be handled by relating conditions at a notch root to an equivalent smooth (un-notched) specimen by some form of stress/strain analysis, perhaps based on Neuber's approximation. In this case the saturated cyclic stress/strain response is the most important characterisation of the material. For some other situations approaches based on crack propagation have been successful. In order to do this, the stress intensities of the cracked notch geometries are required, together with crack propagation data for the material in question. However, an important class of problem exists where due attention must be given to the initiation of a crack at the notch root, its development in the plastic zone generated by the notch and its subsequent growth to failure as a "long crack" controlled by crack tip plasticity.

The first stage of a general solution to this problem has been the prediction and verification of the stress levels required to cause failure in notched components. In particular the prediction of the stress levels which permit initiation, but not propagation to failure have been identified by a simple design equation. This is the regime of the so called "non-propagating" crack. This rule also allows for the size effect observed in geometrically similar notches of varying size. Thus the artificially high fatigue limits observed from small laboratory sized specimens can be reduced to account for the larger sizes of actual components.

The next aid to design has been the construction of charts relating life, notch acuity and stress range. Situations have been identified and marked as regions on these charts where either the initiation or propagation approach by itself proves to give an adequate prediction of fatigue life. Thus in these situations the design process can be greatly simplified.

1. Introduction

1.1 Infinite Life Prediction

Fatigue is a consequence of irreversible plastic deformation. The general sequence of events during fatigue at notches is first the initiation and then the early growth in a plastic zone due to the notch, where the crack is totally enclosed in a plastically deformed region. This is followed by 'long crack' growth where the plasticity is localised to the crack tip region and growth rates may be quantified by linear elastic fracture mechanics parameters, (LEFM). Depending on the values of notch acuity and applied stress level the extremes of the above stages may separately dominate the process. If these combinations of parameters can be identified, the design process can be greatly simplified.

The predictions of the stress levels which just cause initiation are much easier than the more general task of predicting finite lifetimes. Since a measure of the stress level needed to just cause cyclic yielding in a smooth specimen is the plain fatigue limit, $\Delta\sigma_e$, a simple conservative estimate for this initiation stress at a notch of elastic stress concentration factor, k_T, is $\Delta\sigma_e/k_T$, see Fig. 1. But experimental values lie increasingly above this value for increasing notch acuity. By incorporating the stress gradient effect ahead of the notch root Smith and Miller [1] proposed a stress level for initiation of $\Delta\sigma_e/K_{Fatigue}$, where $K_{Fatigue} = (1 + 7.69\sqrt{\frac{D}{\rho}})^{\frac{1}{2}}$, D being the notch depth and ρ its root radius.

However, a crack once initiated does not necessarily grow to failure. Because the stress gradient ahead of a sharp notch is steep, a situation can arise in which the crack has initiated and grown out of the notch plastic zone, but has insufficient crack tip plasticity to propagate through the bulk elastic field. A convenient measure of this crack tip plasticity required for propagation is the threshold stress intensity factor, ΔK_{TH}. This critical condition corresponds to an applied stress level [2] of approximately $0.5\,\frac{\Delta K_{TH}}{\sqrt{D}}$, which defines a region on Fig. 1 in which non-propagating cracks are expected. Estimates have been made for the lengths of such cracks [3].

The important conclusions arising from the above are that for sharp notches, the propagation stress rather than the initiation value is important, and that the size of the testpiece used to determine the propagation fatigue limit is important, since the propagation stress varies inversely with $\sqrt{D}$.

As far as the prediction of the life of a component loaded above the safe limits defined in Fig. 1, the general approach is to consider the three distinct stages i.e. initiation, early growth in the notch plastic zone and 'elastic' long crack growth, Fig. 2. However, great difficulty arises in the demarkation between initiation and early plastic growth. Microscopically, a distinction might be possible between Stage 1, Mode II type growth under shear stresses and Stage 2, Mode I growth normal to the principal tensile stresses, Forsyth [4]. There is much evidence that Stage I does not occur at sharp notches and its identification is only possible subsequent to a test. Mechanically, the growth controlled by notch plasticity can be separated from long crack growth, but again experimentally this separation is hard to observe.

1.2 Finite Life Prediction

In order to predict the life of a notched member, four conditions have to be fulfilled:

1) The predicted growth rates for short and long crack growth must correlate with crack length in a similar manner to that of the component.

2) The crack length at which crack tip plasticity begins to control the growth rate must be accurately known.

3) The fraction of the predicted life spent in each stage must correlate with the component.

4) The total life of all three stages, when summed, must give an adequate approximation to the component's life.

Conditions 1) and 2) can be compared with experiment simply by plotting growth rates against crack length while condition 4) is illustrated by using stress versus cycles plots. In order to show the fraction of the predicted life spent in each stage, condition 3), it is necessary to plot stress versus normalised life, N/N_T, where N_T is the total predicted or experimental life.

If all four conditions are satisfied then a reasonably accurate life prediction can be achieved, as shown in the next section.

2. Experiments, Analysis and Results

Two types of edge notches, Fig. 3, in mild steel plates have been tested. The elastic stress concentrations were estimated as 6.5 and 15. The specimens were tested under constant amplitude loading with an $R(=\Delta\sigma_{min}/\Delta\sigma_{max})$ value slightly greater than zero. Short cracks were measured using cellulose acetate replicating tape [5] while long cracks were measured with an optical microscope. The Paris Law, cyclic stress-strain and strain-life curves were obtained. Mean stress had no noticeable effect on crack growth rates. Failure was arbitrarily defined as a crack of length 12 mm from the notch root.

The initiation life was estimated by correlating the notch root strains with the strain-life curve, a method summarised by Dowling [6]. The notch root strains were calculated by combining the cyclic stress-strain curve with Neuber's Rule [7]. Previously, the initiation crack length has been defined in a number of ways; the most popular being an arbitrarily short crack length that can be measured experimentally. Alternatively, if the length at which the propagation threshold changes from being controlled by the plain fatigue limit, $\Delta\sigma_e$, to the threshold stress intensity, ΔK_{th}, [8] is used as the initiation length, the problems of applying LEFM to short cracks can be overcome. This length, ℓ_0, is given by:

$$\ell_0 = \frac{1}{\pi}\left\{\frac{\Delta K_{th}}{F\Delta\sigma_e}\right\}^2 \qquad \text{eq. (1)}$$

This length term has been used by El Haddad et al [9] to artificially modify the crack length for application of a strain based intensity factor to short cracks.

Since ℓ_0 is usually small, it may not be possible to obtain experimental values for the initiation life. This can be overcome by including a portion of short crack growth with the initiation prediction and comparing this value with experimental results for the same crack length. This will be illustrated later.

Short crack growth is considered to be controlled by the actual length of the crack and the strains generated by the notch. A strain intensity is therefore employed, eq. (2), combined with a growth law, to estimate the growth rates in this region.

$$\Delta K_\varepsilon = f(\Delta\varepsilon, \sqrt{\ell}) \qquad \text{eq. (2)}$$

where $\Delta\varepsilon$ is the strain at distance ℓ ahead of an uncracked notch.

The theoretical elastic stress distribution ahead of a notch can be estimated by the simple approximation proposed by Neuber [7]:

$$\Delta\sigma_{\ell} = \Delta\sigma_{max} \left\{\frac{\rho}{\rho + 4\ell}\right\}^{\frac{1}{2}} \qquad \text{eq. (3)}$$

where $\Delta\sigma_{\ell}$ is the local stress at a distance ℓ ahead of an uncracked notch. By subsituting the cyclic yield stress, $\Delta\sigma_{cy}$, for $\Delta\sigma_{\ell}$ an estimate for the length of the cyclic plastic zone can be obtained:

$$\ell_{cy} = \frac{\rho}{4}\{\mu^2 - 1\} \qquad \text{eq. (4)}$$

where μ is given by $\frac{k_T\ \Delta\sigma_{nom}}{\Delta\sigma_{cy}}$. Eq. (4) has been shown to be useful in this context by Hammouda et al [10] and by Cameron and Smith [3], to predict the lengths of non-propagating cracks.

Eq. (2) combined with Neubers Rule, the cyclic stress-strain curve and fitted into a growth law, provides an estimate for the growth rates which are controlled by the strain generated by the notch. If long crack growth rates, obtained by assuming a crack of length $\ell + D$, are superimposed then plots of $d\ell/dN$ versus ℓ can be generated. Fig. (3) shows such plots together with the experimental results for the two notches at two stress levels. The agreement between theory and experiment is satisfactory, both in growth rates and the change over between notch plasticity control to crack tip plasticity control. It is concluded that conditions 1) and 2) are satisfied by this technique.

In order to satisfy condition 3) it is necessary to produce plots of the form of Figs. (4) and (5). Here the life spent in each stage is normalised by the total life. The full lines in Figs. (4) and (5) represent the following:

Initiation: The fraction of life, estimated from strain-cycles data, required to initiate a crack of length ℓ_o. ℓ_o is estimated from eq. (1).

Initiation + Plastic Growth: The fraction of the total life required to initiate a crack and grow it to the length of the notch plastic zone. The extent of the notch plastic zone is estimated from eq. (4).

The remaining area of Figs. (4) and (5) is the fraction of life spent in long crack growth controlled by crack tip plasticity.

Since a portion of plastic growth can be incorporated with the initiation life the problem of defining an experimental initiation crack length is illiminated. The predicted life for this stage can be matched to the first experimentally observed crack which may vary in length over the stress ranges considered. Figs. (4) and (5) illustrate this technique for comparison with experimental data. The theoretical predictions are thus joined by a dotted line since, strictly speaking, they are discontinuous.

Figs. (4) and (5) show that condition (3) is satisfied. For the sharper notch non-propagating cracks are predicted and have been observed. However, this will not be discussed further.

Finally, condition 4) is shown to be satisfied by S-N plots, Figs. (6) and (7). The agreement between the experimental results and the theoretical prediction is satisfactory. The majority of the error is incurred in the estimation of the initiation life. It is doubtful if this could be improved due to the number of variables that play an important role in this stage.

3. Discussion

The technique of breaking down the life of a notched member into three distinct stages has been shown to assist the prediction of fatigue life. However, Figs. (4) and (5) imply that an estimate for all three stages may be unnecessary for an adequate prediction. For the blunter notch at low stresses, only initiation need be considered while at high stresses and for non-propagating cracks at the sharper notch, only the cycles spent in the notch plastic zone are required. There is also an intermediate area for the sharper notch, where an estimate based on long crack growth may be sufficient. In this region a design calculation need only consider growth controlled by crack tip plasticity and hence the simplification implied by LEFM.

4. Conclusions

1) The stress levels for initiation and propagation of a fatigue crack at a notch can be predicted.
2) The fatigue life of a notched member is best estimated by splitting the life into three stages; initiation, early crack growth controlled by notch plasticity and long crack growth controlled by crack tip plasticity.
3) Although a three stage process is fundamentally more correct, it has been shown that a reasonable estimate for the fatigue life may be obtained, under certain conditions, by consideration of only one stage.
4) The very real experimental problems in defining crack initiation are overcome by including a portion of early crack growth with the initiation life. This is for comparative purposes only.

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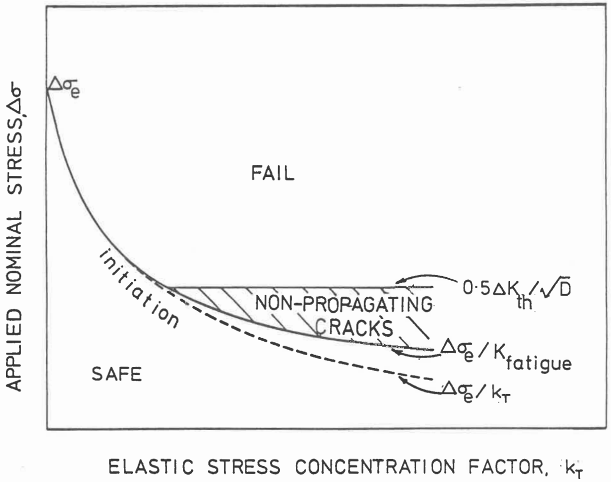

Fig. 1: Fatigue regimes of notched components.

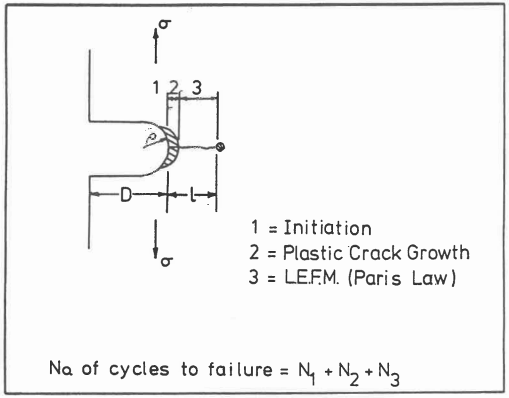

Fig. 2: Three stage process of notch fatigue.

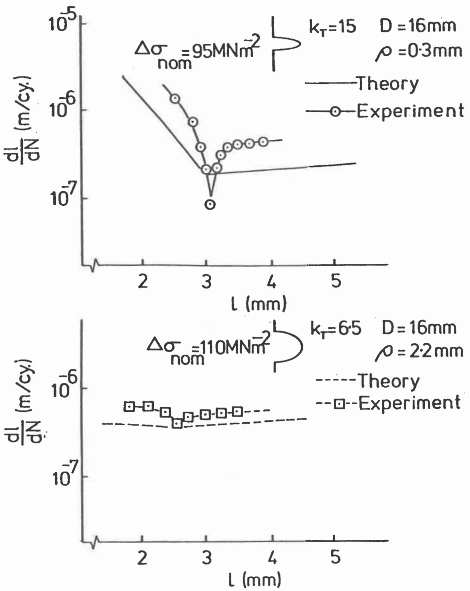

Fig. 3: Crack growth behaviour near notch roots.

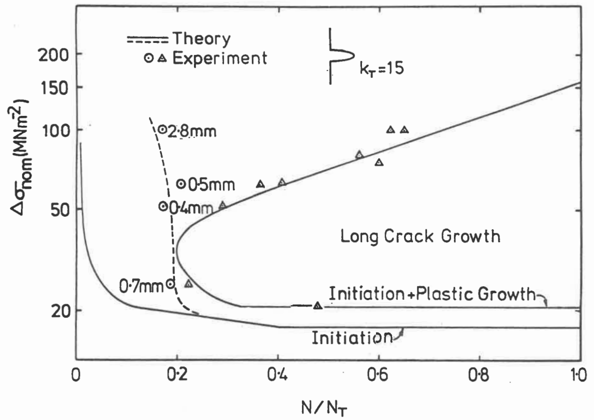

Fig. 4: Nominal stress versus normalised life for an edge notch, $k_T = 15$.

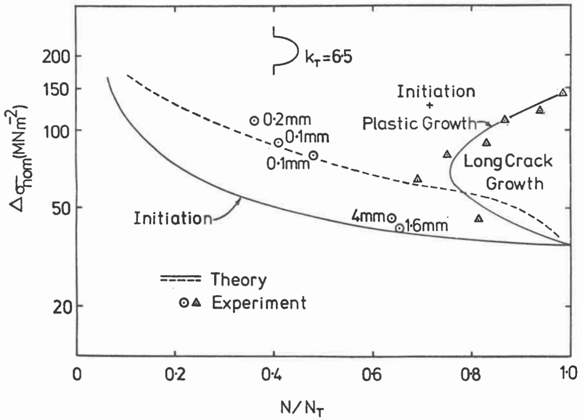

Fig. 5: Nominal stress versus normalised life for an edge notch, $k_T = 6.5$.

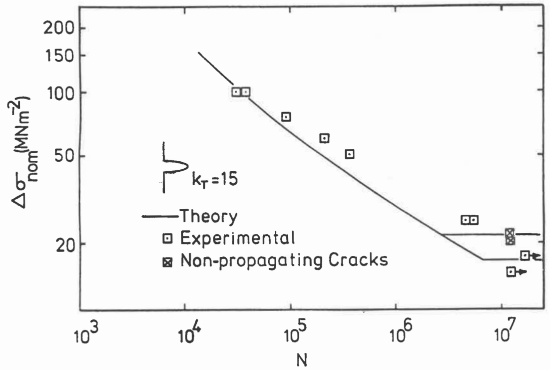

Fig. 6: Stress/Life for edge notch, $k_T = 15$.

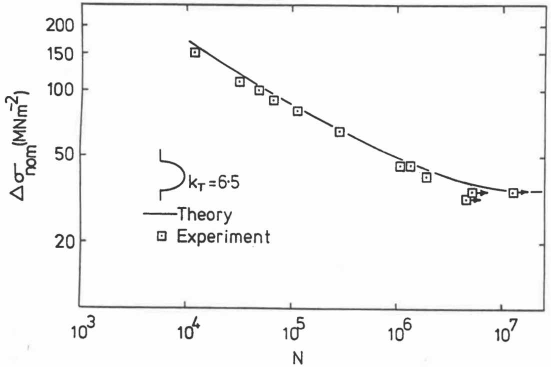

Fig. 7: Stress/Life for edge notch, $k_T = 6.5$.